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じゃばら折りの一般化とその複雑さの研究

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2010/8/4

概要

日本での折紙の研究は，数学的な観点からの研究が先行してきた．しかし Lang, Domain，O’Rourke といった研究者の活躍により，最近 computational origami という言葉が市民権を得つつある．しかし理論計算機科学としての枠組みは，まだまだ未整備である．折紙を計算機科学の対象としたときの単純なモデルとは何だろうか．ここではごく単純な折紙モデルとして，次のものを考える．入力として与えられるのは，長さ n+1 の紙と，長さ n の M,V 上の文字列である．ここで M は山折り，V は谷折りを意味している．つまり長さ n+1 の紙の上に，等間隔に山/谷が指定されている．この文字列にしたがって紙を折る問題を考える．まず時間計算量に対しては，折りの回数が自然に対応すると考えることができる．この観点から，じゃばら折りとその一般化に対して，講演者らは，ほぼ最適な折りアルゴリズムを設計した．では領域計算量に対応する概念は何だろうか．これに対してはみんなが合意する概念は存在しない．講演者は，折り目に挟まる紙の枚数を基準として提案し，これに関する最適化問題を研究している．部分的な解は得られているものの，まだ残された研究課題の方が多いのが実状である．

Complexity of generalized pleat folding

Ryuhei Uehara
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or, ask Google with "uehara origami"

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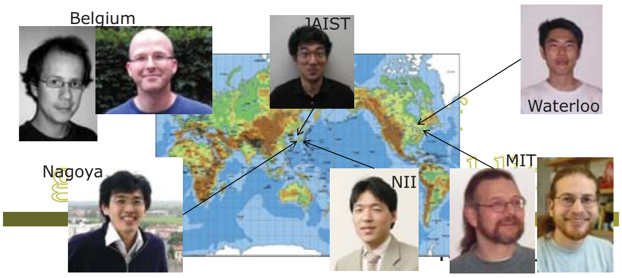

- Ryuhei Uehara: On Stretch Minimization Problem on Unit Strip Paper, 22nd Canadian Conference on Computational Geometry (CCCG 2010), Canada, 2010/8/9-11.
- Ryuhei Uehara: Stretch Minimization Problem of a Strip Paper, 5th International Conference on Origami in Science, Mathematics and Education (5OSME), Singapore, 2010/7/13-17.
- Jean Cardinal, Erik D. Demaine, Martin L. Demaine, Shinji Imahori, Tsuyoshi Ito, Masashi Kiyomi, Stefan Langerman, Ryuhei Uehara, and Takeaki Uno: Algorithmic Folding Complexity, Graphs and Combinatorics, accepted, 2010. (*ISAAC* 2009)

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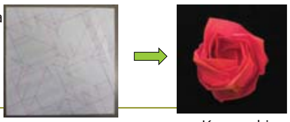
http://www.jaist.ac.jp/~ueha

Background story…



Kawasaki Rose

- At 4th Japan Meeting on Origami in Science, Mathematics, and Education, 2008/6/22,
  - Toshikazu Kawasaki, a mathematician and designer of Kawasaki rose, said that
    "For a mathematician, it is OK if solution *exists*."
  - A computer s
    - how to find
    - the cost to f
      - Goodness
      - Hardness
- Is there any computationa

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Pleat folding = 1D Origami

- Alternating foldings of *Mountain* and *Valley*
- Basic tool of Origami
- Many applications
- Extension to General Patterns and consider its complexity…

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Complexity of folding(?)

From the viewpoint of Computer Science

- Two resources of a computation model;
  1. time: the number of steps of operations
  2. space: the number of memory cells required to compute

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Complexity of folding(?)

From the viewpoint of Computer Science

- Two resources of Origami model?
  1. time…the number of *foldings* (operations)
     - J. Cardinal, E. D. Demaine, M. L. Demaine, S. Imahori, T. Ito, M. Kiyomi, S. Langerman, R. Uehara, and T. Uno: Algorithmic Folding Complexity, *Graphs and Combinatorics*, 2010.
  2. space…minimization of "*stretch*" that is
     - the number of papers between two hinged papers
       - R. Uehara: On Stretch Minimization Problem on Unit Strip Paper, *CCCG* 2010, Canada, 2010/8/9-11.
       - R. Uehara: Stretch Minimization Problem of a Strip Paper, *5OSME*, Singapore, 2010/7/13-17.

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New open problem

- Least **stretch** folding problem

  Input: Paper of length $n+1$ and $s \in \{M, V\}^n$
  Output: folded paper according to s
  Goal: Find a *good* folded state with few *stretch*
  - At each crease, the number of papers between the papers hinged at the crease is *stretch.*
  - Two minimization problems;
    - minimize maximum
    - minimize total (=average)

It seems simple,
… so easy??

No!!

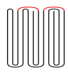
New open problem

- Simple non-trivial example

  Input: MMVMMVMVVVV

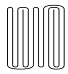
  M M V M M V M V V V V
  1 2 3 4 5 6 7 8 9 10 11 12

  The number of feasible folded states: **100**
  Goal: Find a *good* folded state with few *stretch*
  - The **unique** solution having min. max. value 3
    [5|4|3|6|7|1|2|8|10|12|11|9]



total=13

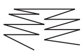
  - The **unique** solution having min. total value 11
    [5|4|3|1|2|6|7|8|10|12|11|9]



New open problem

- Least **stretch** folding problem

  Input: Paper of length $n+1$ and $s \in \{M, V\}^n$
  Output: folded paper according to s
  Goal: Find a *good* folded state with few *stretch*
  - Two minimization problems; max/total(average)
  - A few facts;
    - a pattern has a unique folded state iff it is pleats
    - solutions of {min max} and {min total} are different depending on a crease pattern.
    - there is a pattern having exponential combinations



M V M V M V M V M V M M V M V M V M V M V M

Least stretch folding problem

- Open problem:
  - Tractable/Intractable?
    - $\mathcal{NP}$-hard?
    - Poly-time solvable by Dynamic Programming?

    … up to now, I have no answer to this question ;-)

    - Then, exhaust search technique, that produces ***all*** possible folding ways, works?

average?

exponential pattern ⟷ pleats

No!!

Least stretch folding problem

- Partial answers;
  1. The number of folding ways for a <u>*random*</u> pattern
     - $\Theta(1.65^n)$ by experiments
     - $\Omega(1.53^n)$ and $O(2^n)$ by theoretical lower/upper bounds

     … so a naïve program runs *veeeerrrrrry* slow.

[Note]
These results are based on enumeration & rough counting, described hereafter…

Least stretch folding problem

1. Average number of folding ways for a random pattern = $f(n)/2^n$, where

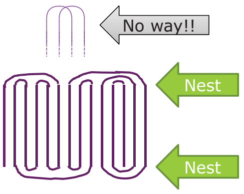
   $f(n)$ = # of folding ways (or folded states) of a paper of length $n+1$ (summation for all possible patterns)

- I give some bounds of $f(n)$:
  - "The On-Line Encyclopedia of Integer Sequences" tells us up to $n=28$ (by *enumeration*); $f(n) \sim \Theta(3.3^n)$.
  - I have upper/lower bounds;
    - upper bound: $f(n)=O(4^n)$
    - lower bound: $f(n)=\Omega(3.07^n)$

Least stretch folding problem

- Upper bound $f(n)$=O(4^n) comes from the Catalan number.

[Proof] If the paper of length n+1 is folded, the crease points should be nested.



(()())(()(()))

Combination of $n/2$ pairs of ()= Catalan Number $C_{n/2}$

×

·(()()())((·))

Combination of $n/2$ pairs of ()= Catalan Number $C_{n/2}$

Least stretch folding problem

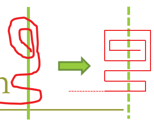
- Lower bound $f(n)=\Omega(3.07^n)$.

[Proof] We consider "folding of the last k+1 unit papers";

We let
- $f(n)$: the number of folding ways of length n+1
- $g(k)$: the number of folding ways of length k+1 s.t. the leftmost endpoint is not covered

Then, we have $f(n) \ge (g(k))^{\frac{n}{k-1}} = \left(g(k)^{1/(k-1)}\right)^n$

Least stretch folding problem



- Lower bound $f(n)=\Omega(3.07^n)$.

[Proof] We consider "folding of the last k+1 unit papers";

$g(k)$: the number of folding ways of length k+1 s.t. the leftmost endpoint is not covered

= "the number of ways a semi-infinite directed curve can cross a straight line k times",

A000682 in "The On-Line Encyclopedia of Integer Sequences".

From that site, we have $g(43)$=830776205506531894760.

Thus, by $f(n) \ge (g(k))^{\frac{n}{k-1}} = \left(g(k)^{1/(k-1)}\right)^n$

we have the lower bound.

also obtained by enumeration

Open problem and Future work

- Least stretch folding problem:
  - Tractable/Intractable?
    - $\mathcal{NP}$-hard?
    - Poly-time solvable by Dynamic Programming?
    - Fixed parameter tractable for "stretch≦k"?
  - What is the most complex pattern of M/V that has the most feasible folded states?

Hiro Ito gives hardness proof?

- Extension to...
  - non-unit length (operation-restricted linkage)
  - 2D (a kind of map folding)
- What "*space complexity*" of Origami is?
  - area for folding?

Open problem and Future work

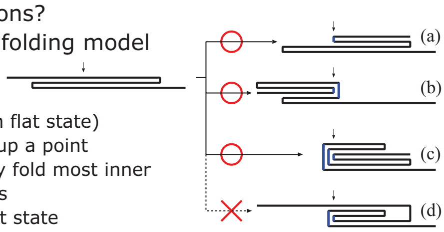
- Ext. of Least stretch folding problem:
  - non-unit length
    - First of all, there is a pattern that cannot be folded;

  - revisit unit length paper...
    - For any pattern, can it be folded?
      - "Yes"; by repeating "end-fold"
    - For any "folded state" of length 1, can it be folded?
      - If you allow *any folding*, it is "Yes" by
        - unlocked linkage in 2D ... reverse of bloom
        - universal theorem ... adding many creases
      - Is it "Yes" under simpler/reasonable model????

Universal theorem:

- Universal theorem of unit length folding in a simple folding model;
  - Input: a folded state of a paper strip of length n+1 to unit length
  - Question: Is it foldable on a suitable set of operations?
  - Simple folding model

1. (from flat state)
2. pick up a point
3. valley fold most inner layers
4. to flat state



Universal theorem of linkage…

- Universal theorem of unit length folding in a simple folding model;

  Any flat folded state of unit length can be made by a sequence of simple foldings of length at most $2n$.

- Related results;
  - Any M/V pattern can be flat folded by repeating "end-folding"
  - If the creases are not unit-length, some folded state cannot be folded by simple foldings:
  - If "any folding" is allowed, "Yes" even in non-unit length
    - comes from "no locked linkage in 2D"!!

Universal theorem of linkage…

[Theorem] For a strip of paper, any unit-length flat folded state can be made by a sequence of simple foldings

[Proof]

Key idea 1: P can be folded from a strip paper iff P can be unfolded to flat strip paper by reversing. So we show that any folded state P can be unfolded to flat by at most $2n$ simple unfoldings.

[Phase 1] unveil the endpoint
- unveil the covered endpoint and make it appear

[Phase 2] peal the paper from the endpoint
- unfold and extend the last flat part

Universal theorem of linkage…

[Proof]

[Phase 1] unveil the covered endpoint
- while we cannot see the endpoint, unfold the paper to expose the covered paper close to the endpoint.
- after at most n unfoldings, we can see the endpoint.

[Phase 2] peal the paper from the endpoint
- unfold and extend the last flat part that contains the endpoint.
- after at most n unfoldings, we obtain the flat paper.

[Note] At Phase 1, some new creases can be made by an "unfolding". Hence the total number of "unfolding" cannot be bounded by n.

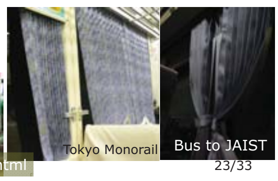
Universal theorem of linkage…

Open:

- The number of unfolding can be n?
- Characterization of "non-unfoldable" (non-unit length) origami by simple (un)folding.
- In simple folding model, no locked flat tree in 2D if edges are unit length?

Let's turn to the folding complexity of pleat folding

- Repeating of mountain and valley foldings
- Basic operation in some origami
- Many applications



Pleat folding

Repeating "folding in half" is the best way to make many creases.

- Pleat folding (in 1D)

- Naïve algorithm: n time folding is a trivial solution
- We have to fold at least $\log n$ times to make n creases
- More efficient ways…?
- General Mountain/Valley pattern?

- proposed at Open Problem Session on CCCG 2008 by R. Uehara.
- T. Ito, M. Kiyomi, S. Imahori, and R. Uehara: "*Complexity of pleats folding*", EuroCG 2009…

http://www.jaist.ac.jp/~uehara/etc/origami/

• Complexity of Pleat Fol

Model:
Paper has 0 thickness

[Main Motivation] Do we have to make n foldings to make a pleat folding with n creases??

1. The answer is "No"!
   - ***Any pattern*** **can be made by** $\lfloor n/2 \rfloor + \lceil \log n \rceil$ **foldings**
2. Can we make a pleat folding in $o(n)$ foldings?
   - Yes!! ...it can be folded in $O(\log^2 n)$ foldings.
3. Lower bound; $\log n$
   - $\Omega(\log^2 n/\log\log n)$ lower bound for pleat folding!!

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• Complexity of Pleat Folding

[Next Motivation] What about general pleat folding problem for a given M/V pattern of length n?

- ***Any pattern*** **can be made by** $\lfloor n/2 \rfloor + \lceil \log n \rceil$ **foldings**

1. Upper bound:
   Any M/V pattern can be folded by $(4+\varepsilon)\frac{n}{\log n} + o\left(\frac{n}{\log n}\right)$ foldings
2. Lower bound:
   Almost all mountain/valley patterns require $\frac{n}{3+\log n}$ foldings

[Note] Ordinary pleat folding is exceptionally easy pattern!

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• De

Difficulty/Interest come from two kinds of *Parities*:
• "Face/back" determined by layers
• Stackable points having the same parity

Input: Paper of length $n+1$ and a string s in $\{M, V\}^n$
Output: Well-creased paper according to s at regular intervals.

Basic operations
1. Flat {mountain/valley} fold {all/some} papers at an integer point (= simple folding)
2. Unfold {all/rewind/any} crease points (= reverse of simple foldings)

Rules
1. Each crease point remembers the last folded direction
2. Paper is rigid except those crease points

Goal: **Minimize the number of folding operations**

Note: We ignore the cost of unfoldings

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• Upper bound of Unit FP (1)

- Any pattern can be made by $\lfloor n/2 \rfloor + \lceil \log n \rceil$ foldings

1. *M/V* fold at center point according to the assignment
2. Check the center point of the folded paper, and count the number of *M*s and *V*s (we have to take care that odd depth papers are reversed)
3. *M/V* fold at center point taking majority
4. Repeat steps 2 and 3
5. Unfold all (cf. on any model)
6. Fix all incorrect crease points one by one

Steps 1~4 require $\log n$ and step 6 requires $n/2$ foldings

□

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Upper bound of Pleat Folding(1)

[Observation]
If $f(n)$ foldings achieve n mountain foldings,
n pleat foldings can be achieved by $2f(n/2)$ foldings.

The following strategy works;
- Make $f(n/2)$ mountain foldings at odd points;
- Reverse the paper;
- Make $f(n/2)$ mountain foldings at even points.

We will consider the
"**mountain folding problem**"

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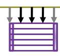
Mountain folding in $\log^2 n$ foldings



Step 1;
1. Fold in half until it becomes of length [vvv] ($\log n$-2 foldings)
2. Mountain fold 3 times and obtain [MMM]
3. Unfold; vMMMvvvvvMMMvvvvvMMMvvvvvMMMvvvvv...

Step 2; [MvvvvvM]
1. Fold in half until all "vvvvv"s are piled up ($\log n$-3 foldings)
2. Mountain fold 5 times [MMMMMMM], and unfold
3. vMvMMMMMMMvMvvvvvMvMMMMMMMvMvvvvvMvM

Step 3; Repeat step 2 until just one "vvvvv" remains
vMvMMMvMMMvMMMMMMMvMMMvMMMvMvvvvvMvM

Step 4; Mountain fold all irregular vs step by step.

• #iterations of Steps 2~3; $\log n$
• #valleys at step 4; $\log n$

#foldings in total~ $(\log n)^2$

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• Lower bound of Unit FP

[Thm] Almost all patterns but $o(2^n)$ exceptions require $\Omega(n/\log n)$ foldings.

{surface/reverse} × {front/back}

[Proof] A simple counting argument:

- # patterns with n creases $> 2^n/4 = 2^{n-2}$
- # patterns after k foldings <

  $(2\times n)\times(n+1)\times(2\times n)\times(n+1)\times\ldots\times(n+1)\times(2\times n)$

  M/V Position Possible unfoldings $<(2n(n+1))^k$
- We cannot fold most patterns after at most k foldings if

  $$\sum_{i=0}^{k}(2n(n+1))^i \le (2n(n+1)+1)^k < 2^{n-2}$$
- Letting

  $n \ge 2, k = O\left(\frac{n}{\log n}\right)$ we have $(2n(n+1)+1)^k = o(2^n)$ □

Any pattern can be folded in $cn/\log n$ foldings

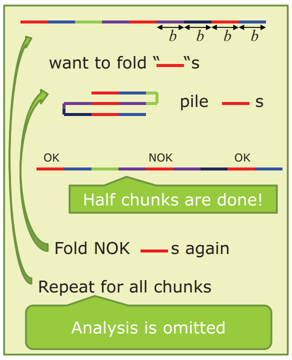
Select suitable b depending on n.

Prelim.

- Split into *chunks* of size b;
  1. Each chunk is small and easy to fold
  2. #kinds of different bs are not so big

Main alg.

- For each possible b
  1. pile the chunks of pattern b and mountain fold them
  2. fix the reverse chunks
  3. fix the boundaries



• Open Problems

- Pleat foldings
  - Make upper bound $O(\log^2 n)$ and lower bound $\Omega(\log^2 n/\underline{\log\log n})$ closer
- "**Almost all** patterns are difficult", but…
  - **No explicit M/V pattern** that requires $(cn/\log n)$ foldings
- When "unfolding cost" is counted in…
  - Minimize #foldings + #unfoldings