Phenomenology of supersymmetry $SU(5)$ GUT with neutrinophilic Higgs boson

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Among three typical energy scales, a neutrino mass scale ($m_\nu \sim 0.1$ eV), a grand unified theory (GUT) scale ($M_{\rm GUT} \sim 10^{16}$ GeV), and a TeV scale ($M_{NP} \sim 1$ TeV), there is the fascinating relation of $M_{NP} \simeq \sqrt{m_\nu \cdot M_{\rm GUT}}$. The TeV scale, M_{NP}, is a new physics scale beyond the standard model which is regarded as "supersymmetry" in this article. We investigate the phenomenology of the supersymmetry $SU(5)$ GUT with neutrinophilic Higgs, where the above relation is realized dynamically when neutrinos are Dirac particles. As a remarkable feature of this model, accurate gauge coupling unification can be achieved in keeping with a proton stability. We also evaluate flavor changing processes in quark/lepton sectors.

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I. INTRODUCTION

There are three typical energy scales, a neutrino mass scale ($m_\nu \sim 0.1$ eV), a GUT scale ($M_{\rm GUT} \sim 10^{16}$ GeV), and a TeV scale ($M_{NP} \sim 1$ TeV) which is a new physics scale beyond the standard model (SM) and regarded as supersymmetry (SUSY) in this article. Among these three scales, we notice a fascinating relation,

$$M_{NP}^2 \simeq m_\nu \cdot M_{\rm GUT}. \quad (1.1)$$

Is this relation an accident, or providing a clue to the underlying new physics ? We take a positive stance toward the latter possibility.

As for a neutrino mass m_ν, its smallness is still a mystery, and it is one of the most important clues to finding new physics. Among many possibilities, a neutrinophilic Higgs doublet model suggests an interesting explanation of the smallness by a tiny vacuum expectation value (VEV) [1–15]. This VEV from a neutrinophilic Higgs doublet is of $\mathcal{O}(0.1)$ eV that is suitable for Dirac neutrino mass [3,4,6,8], and we focus on this situation here.[1] We can see that the relation of Eq. (1.1) is realized by two fundamental parameters, $M_{\rm GUT}$ and M_{NP} [16]. The neutrino mass is much smaller than other fermions, since its origin is the tiny VEV from the different (neutrinophilic) Higgs doublet. The introduction of Z_2 symmetry distinguishes the neutrinophilic Higgs from the SM-like Higgs, where m_ν is surely generated only through the VEV of the neutrinophilic Higgs. The SUSY extension of the neutrinophilic Higgs doublet model is considered in Refs. [7,11,12,15]. Since the neutrino Yukawa couplings are not necessarily tiny anymore, some related research has been done, such as, collider phenomenology [8,10], low energy thermal leptogenesis [11,12], cosmological constraints [13],[2] and so on.

On the other hand, SUSY is the most promising candidate of new physics beyond the SM because of the excellent success of gauge coupling unification. Thus, the SUSY SM well fits the GUT scenario as well as the existence of a dark matter candidate.

There are some attempts that try to realize the relation in Eq. (1.1). One example is to derive m_ν from a higher dimensional operator in the SUSY framework [17,18]. Another example is to take a setup of matter localization [19] in a warped extra dimension [20]. These scenarios are interesting, but the model in this article is much simpler and contains no additional scales other than M_{NP}, m_ν, and $M_{\rm GUT}$. (For other related papers, see, for example, [21,22].)

In this article, we investigate the phenomenology of a SUSY $SU(5)$ GUT with the neutrinophilic Higgs ($SU(5)_{H_\nu}$) model proposed in Ref. [16].[3] Usually, SUSY neutrinophilic Higgs doublet models have the tiny mass scale of soft Z_2-symmetry breaking (ρ, $\rho' = \mathcal{O}(10)$ eV shown in Refs. [11,12,15]). This *additional* tiny mass scale plays a crucial role in generating the tiny neutrino mass, however, its origin is completely unknown (assumption). In other words, the smallness of m_ν is just replaced by that of Z_2-symmetry breaking mass parameters, and this is not an essential explanation of tiny m_ν. This is a common and serious problem that exists in neutrinophilic Higgs doublet models in general. It can be noted that this problem can be solved by Ref. [16], where two scales of $M_{\rm GUT}$ and M_{NP} naturally induce the suitable magnitude of m_ν through the relation of Eq. (1.1), and does not require any additional scales. The model contains a pair of new neutrinophilic Higgs doublets with GUT-scale masses, and the Z_2 symmetry is broken by the TeV-scale dimensionful couplings of these new doublets to the ordinary SUSY Higgs doublets. Once the ordinary Higgs doublets obtain VEVs ($v_{u,d}$) by the usual electroweak symmetry breaking, they trigger

[1]The Majorana neutrino case is also possible [1,2,5,9–12], but additional scales of Majorana masses make the realization of Eq. (1.1) complicated.

[2]The setup in Ref. [13] is different from the usual neutrinophilic Higgs doublet models, since it includes a light Higgs particle.

[3]A similar model was suggested in Ref. [23], where lepton flavor violation was also roughly estimated.

VEVs for the neutrinophilic Higgs doublets of $v_{u,d}M_{NP}/M_{\rm GUT}(\sim m_\nu)$. Then, $\mathcal{O}(1)$ Yukawa couplings of the neutrinophilic doublets to LN (L: lepton doublet, N: right-handed neutrino) give neutrino masses of the proper size. We can also obtain a GUT embedding of the SUSY neutrinophilic Higgs doublet model, which realizes the relation, $m_\nu \sim v_{u,d}M_{NP}/M_{\rm GUT}$, dynamically. As a remarkable feature of this model, accurate gauge couplings can be unified in keeping with a proton stability. Flavor changing processes are also a sensible aspect of this model. In general, flavor violation in the charged lepton sector is related to that in the quark sector because the lepton doublet and the right-handed down-type quark are contained in the same multiplet in $SU(5)$ GUT. Particularly, neutrino oscillation directly contributes flavor violations in both sectors. One of our purposes is to evaluate such flavor violating processes.

This paper is organized as follows. In Sec. II, we review the SUSY $SU(5)_{H_\nu}$ model. In Secs. III and IV, we discuss gauge coupling unification, and investigate flavor violations in the SUSY $SU(5)_{H_\nu}$ model. These sections are the main parts of this article. In Sec. V, we present a summary.

II. SUSY $SU(5)$ GUT WITH NEUTRINOPHILIC HIGGS

Before showing the SUSY $SU(5)_{H_\nu}$ model [16], we show the SUSY neutrinophilic Higgs doublet model. This has a specific parameter region which is different from Refs. [11,12,15]. We introduce Z_2 parity, where only vector-like neutrinophilic Higgs doublets and right-handed neutrino have odd charge. The superpotential of the Higgs sector is given by

$$\mathcal{W}_h = \mu H_u H_d + M H_\nu H_{\nu'} - \rho H_u H_{\nu'} - \rho' H_\nu H_d. \qquad (2.1)$$

H_ν ($H_{\nu'}$) is a neutrinophilic Higgs doublet, and H_ν has a Yukawa interaction of $LH_\nu N$, which induces a tiny Dirac neutrino mass through the tiny VEV of $\langle H_\nu\rangle$. This is the origin of the smallness of the neutrino mass, and we concentrate on a Dirac neutrino scenario, i.e., $m_\nu \simeq \langle H_\nu\rangle = \mathcal{O}(0.1)$ eV. On the other hand, $H_{\nu'}$ does not couple with any matter. H_u and H_d are Higgs doublets in the minimal supersymmetric standard model, and quarks and charged lepton obtain their masses through $\langle H_u\rangle$ and $\langle H_d\rangle$. Note that this structure is guaranteed by the Z_2 symmetry. Differently from conventional neutrinophilic Higgs doublet models, here we take M as the GUT scale and μ, ρ, ρ' $\mathcal{O}(1)$ TeV. The soft Z_2-parity breaking parameters, ρ and ρ', might be induced from the SUSY breaking effects (see below), and we regard ρ and ρ' as the mass parameters of new physics scale, $M_{NP} = \mathcal{O}(1)$ TeV. We remind that the usual SUSY neutrinophilic doublet models take ρ, $\rho' = \mathcal{O}(10)$ eV (for $\mathcal{O}(1)$ TeV B terms) [11,12,15]. This additional tiny mass scale plays a crucial role in generating the tiny neutrino mass; however, its origin is just an assumption. Thus, the smallness of m_ν is just replaced by that of ρ and ρ'. This is a common and serious problem that exists in neutrinophilic Higgs doublet models in general. The present model solves this problem, in which two scales of $M_{\rm GUT}$ and M_{NP} induce the suitable magnitude of m_ν dynamically, and does not require any additional scales, such as $\mathcal{O}(10)$ eV. This is one of the most important points in this model.

Amazingly, stationary conditions make the VEVs of neutrinophilic Higgs fields to be

$$v_\nu = \frac{\rho v_u}{M}, \qquad v_{\nu'} = \frac{\rho' v_d}{M}. \qquad (2.2)$$

It is worth noting that they are induced dynamically through the stationary conditions, and their magnitudes are surely of $\mathcal{O}(0.1)$ eV. Since the masses of neutrinophilic Higgs H_ν and $H_{\nu'}$ are superheavy as the GUT scale, there are no other vacua (such as, $v_{u,d} \sim v_{\nu,\nu'}$) except for $v_{u,d} \gg v_{\nu,\nu'}$ [15]. Also, their heaviness guarantees the stability of the VEV hierarchy, $v_{u,d} \gg v_{\nu,\nu'}$, against radiative corrections [14,15]. This is because in the effective potential, H_ν and $H_{\nu'}$ inside loop diagrams are suppressed by their GUT-scale masses.

The model suggested in Ref. [16] has the GUT-scale mass of the neutrinophilic Higgs doublets in Eq. (2.1), so that it is naturally embedded into a GUT framework, and it is the SUSY $SU(5)_{H_\nu}$ model. A superpotential of a Higgs sector at the GUT scale is given by

$$\mathcal{W}_H^{\rm GUT} = M_0 {\rm tr}\Sigma^2 + \lambda {\rm tr}\Sigma^3 + H\Sigma\bar{H} + \Phi_\nu\Sigma\bar{\Phi}_\nu - M_1 H\bar{H} - M_2\Phi_\nu\bar{\Phi}_\nu, \qquad (2.3)$$

where Σ is an adjoint Higgs whose VEV reduces the GUT gauge symmetry into the SM. Φ_ν ($\bar{\Phi}_\nu$) is a neutrinophilic Higgs of (anti-)fundamental representation, which contains H_ν (H'_ν) in the doublet component [while the triplet component is denoted as T_ν ($\bar{T}_\nu$)]. Φ_ν and $\bar{\Phi}_\nu$ are odd under the Z_2 parity. H ($\bar{H}$) is a Higgs of (anti-)fundamental representation, which contains H_u (H_d) in the doublet component [while the triplet component is denoted as T ($\bar{T}$)]. The VEV of Σ and $M_{0,1,2}$ are all of $\mathcal{O}(10^{16})$ GeV; thus, we encounter the so-called triplet-doublet (TD) splitting problem. Some mechanisms have been suggested for a solution of TD splitting, but here we show a case where the TD splitting is realized just by a fine-tuning between $\langle\Sigma\rangle$ and M_1. That is, $\langle\Sigma\rangle - M_1$ induces the GUT scale masses of T, $\bar{T}$, in keeping with weak scale masses of H_u, H_d. This is a serious fine-tuning, so that we cannot expect a simultaneous fine-tuned cancellation between $\langle\Sigma\rangle$ and M_2. Thus, we consider the case where the TD splitting only works in H and $\bar{H}$, while it does not work in Φ_ν and $\bar{\Phi}_\nu$. This situation makes Eq. (2.3) become

$$\mathcal{W}_H^{\rm eff} = \mu H_u H_d + M H_\nu H_{\nu'} + M' T\bar{T} + M'' T_\nu \bar{T}_\nu. \qquad (2.4)$$

This is the effective superpotential of the Higgs sector below the GUT scale, and M, M', M'' are of $\mathcal{O}(10^{16})$ GeV, while $\mu = \mathcal{O}(1)$ TeV.

Now let us consider an origin of the soft Z_2-parity breaking terms, $\rho H_u H_{\nu'}$ and $\rho' H_\nu H_d$, in Eq. (2.1). They play a crucial role in generating the marvelous relation in Eq. (1.1) as well as the tiny Dirac neutrino mass. Since the values of ρ, ρ' are of order $\mathcal{O}(1)$ TeV, they might be induced from the SUSY breaking effects. We consider some possibilities for this mechanism. One example is to take a noncanonical Kähler of $[S^\dagger(H_u H_{\nu'} + H_\nu H_d) + \text{H.c.}]_D$, where the F term of S could induce the ρ and ρ' terms effectively through the SUSY breaking scale as in Giudice-Masiero mechanism [24]. There might be other models that induce the ρ and ρ' terms in Eq. (2.1) without a singlet S.

III. GAUGE COUPLING UNIFICATION AND PROTON DECAY

In this section, we discuss a characteristic feature of the gauge coupling unification and the proton decay in the SUSY $SU(5)_{H_\nu}$ model by focusing on the role of T_ν and $\bar{T}_\nu$. As for the minimal SUSY $SU(5)$ GUT model, in order to unify the gauge couplings, the mass of T and $\bar{T}$ should be lighter than the GUT scale as 3.5×10^{14} GeV $\lesssim M' \lesssim 3.6 \times 10^{15}$ GeV due to threshold corrections [25]. However, to avoid rapid proton decay, M' must be heavier than the GUT scale ($M' > M_{\rm GUT}$). Hence, it is difficult to achieve both accurate gauge coupling unification and enough proton stability in the minimal SUSY $SU(5)$ GUT.

The situation changes in the SUSY $SU(5)_{H_\nu}$ model. In this model, a superpotential of the Yukawa sector is given by

$$\mathcal{W}_Y = \frac{1}{4} f_{u_{ij}} \psi_i \psi_j H + \sqrt{2} f_{d_{ij}} \psi_i \phi_j \bar{H} + f_{\nu_{ij}} \eta_i \phi_j \Phi_\nu \quad (3.1)$$

at the GUT scale, where i and j are family indices. ψ_i, ϕ_i, and η_i are 10-plet, $\bar{5}$-plet, and singlet in the $SU(5)$ gauge group, respectively, which are written in terms of minimal supersymmetric standard model fields as

$$\begin{aligned} \psi_i &= \{Q_i, e^{-i\phi_{u_i}} U_i, (V_{\rm KM})_{ij} \bar{E}_j\}, \\ \phi_i &= \{(V_D)_{ij} \bar{D}_j, \qquad (V_D)_{ij} L_j\}, \\ \eta_i &= \{e^{-i\phi_{\nu_i}} \bar{N}_i\}. \end{aligned} \quad (3.2)$$

Since Yukawa couplings are written as

$$\begin{aligned} f_{u_{ij}} &= f_{u_i} e^{i\phi_{u_i}} \delta_{ij}, \\ f_{d_{ij}} &= (V^*_{\rm KM})_{ik} f_{d_k} (V_D^\dagger)_{kj}, \\ f_{\nu_{ij}} &= f_{\nu_i} e^{i\phi_{\nu_i}} \delta_{ij}, \end{aligned} \quad (3.3)$$

the superpotential in this basis is given by

$$\begin{aligned} \mathcal{W}_Y = {} & f_{u_i} Q_i \bar{U}_i H_u + (V^*_{\rm KM})_{ij} f_{d_j} Q_i \bar{D}_j H_d + f_{d_i} \bar{E}_i L_i H_d \\ & + f_{u_j} (V_{\rm KM})_{ji} \bar{E}_i \bar{U}_j T - \frac{1}{2} f_{u_i} Q_i Q_i T \\ & + (V^*_{\rm KM})_{ij} f_{d_j} \bar{U}_i \bar{D}_j \bar{T} - (V^*_{\rm KM})_{ij} f_{d_j} Q_i L_j \bar{T} \\ & - f_{\nu_i} (V_D)_{ij} \bar{N}_i L_j H_\nu + f_{\nu_i} (V_D)_{ij} \bar{N}_i \bar{D}_j T_\nu, \end{aligned} \quad (3.4)$$

where CP phases, ϕ_{u_i} and ϕ_{ν_i}, are omitted, for simplicity. The terms from the fourth to seventh in Eq. (3.4) cause proton decay, which also exists in the minimal SUSY $SU(5)$ GUT. Thus, we should take $M' > M_{\rm GUT}$ to avoid rapid proton decay. Meanwhile, the last term in Eq. (3.4) has nothing to do with the proton decay. Since T_ν and $\bar{T}_\nu$ contribute beta functions of $SU(3)_c \times U(1)_Y$, accurate gauge coupling unification is achieved with the T_ν and $\bar{T}_\nu$ threshold corrections with 3.5×10^{14} GeV $\lesssim M'' \lesssim 3.6 \times 10^{15}$ GeV. Therefore, the SUSY $SU(5)_{H_\nu}$ model can realize not only accurate gauge coupling unification but also proton stability. Remembering that M is the GUT scale, $\mathcal{O}(1)\%$ tuning between M and M'' is needed, but it can be happen. Or, no tuning is required when one of couplings is of $\mathcal{O}(0.01)$, for example, a coupling of $S^\dagger H_u H_{\nu'}$.

IV. FLAVOR CHANGING PROCESSES

Flavor changing in the lepton sector is related to that in the quark sector, since L and D are contained in a same multiplet in $SU(5)_{H_\nu}$, where, mixing angles in V_D are expected to be large, and masses of left-handed slepton and right-handed down-type squark get sizable radiative corrections in off-diagonal elements of flavor space. Leading log approximation makes the off-diagonal elements

$$(\delta m^2_{\tilde{L}})_{ij} \simeq -\frac{f^2_{\nu_k}}{8\pi^2} (V^*_D)_{ki} (V_D)_{kj} (3m_0^2 + A_0) \log\frac{M_P}{M}, \quad (4.1)$$

$$(\delta m^2_{\tilde{D}})_{ij} \simeq -\frac{f^2_{\nu_k}}{8\pi^2} (V^*_D)_{ki} (V_D)_{kj} (3m_0^2 + A_0) \log\frac{M_P}{M''}, \quad (4.2)$$

where M_P is the Planck scale, m_0 and A_0 are universal scalar mass and universal scalar trilinear coupling in the mSUGRA scenario. Equation (4.1) originates from the loop diagram of N and H_ν, where an energy scale in the renormalization group equations runs from M_P to M (H_ν, $\bar{H}_\nu$ mass). On the other hand, Eq. (4.2) is induced from the loop diagram of N and T_ν, which runs from M_P to M'' (T_ν, $\bar{T}_\nu$ mass). Notice that the loop effects in Eqs. (4.1) and (4.2) are different from those in $SU(5)$ with right-handed neutrinos ($SU(5)_{RN}$).[4] In the $SU(5)_{RN}$ model, neutrinos are Majorana, and the counterparts of Eqs. (4.1) and (4.2) are given by

$$\begin{aligned} (\delta m^2_{\tilde{L}})_{ij} \simeq {} & -\frac{f_{\nu_k} f_{\nu_m}}{8\pi^2} (V^*_D)_{ki} (V^*_M)_{lk} (V_M)_{lm} (V_D)_{mj} (3m_0^2 + A_0) \\ & \times \log\frac{M_P}{M_{N_l}}, \end{aligned} \quad (4.3)$$

$$(\delta m^2_{\tilde{D}})_{ij} \simeq -\frac{f^2_{\nu_k}}{8\pi^2} (V^*_D)_{ki} (V_D)_{kj} (3m_0^2 + A_0) \log\frac{M_P}{M'}, \quad (4.4)$$

where M_{N_l} is the diagonal Majorana mass of $N_l (l = 1, 2, 3)$. The mass matrix of N is diagonalized by the unitary matrix

[4] See, for example, [26].

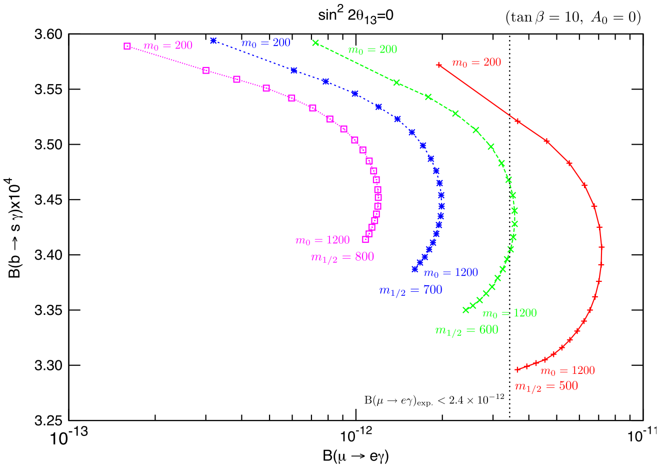

FIG. 1 (color online). Contour plot of $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\mu \to e\gamma)$ with $\sin^2 2\theta_{13} = 0$. Here, we take $200\ \mathrm{GeV} \leq m_0 \leq 1200\ \mathrm{GeV}$, $500\ \mathrm{GeV} \leq m_{1/2} \leq 800\ \mathrm{GeV}$, $A_0 = 0$, and $\tan\beta = 10$. The experimental upper bound for $\mathrm{B}(\mu \to e\gamma)$ is 2.4×10^{-12}.

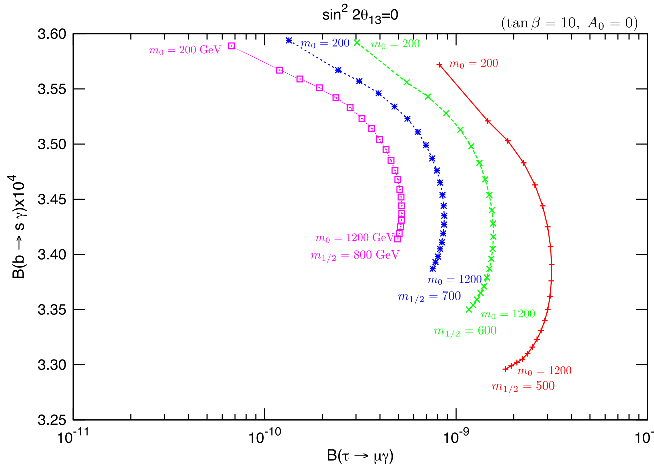

FIG. 3 (color online). Contour plot of $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\tau \to \mu\gamma)$ with $\sin^2 2\theta_{13} = 0$. Here, we take $200\ \mathrm{GeV} \leq m_0 \leq 1200\ \mathrm{GeV}$, $500\ \mathrm{GeV} \leq m_{1/2} \leq 800\ \mathrm{GeV}$, $A_0 = 0$, and $\tan\beta = 10$. The experimental upper bound for $\mathrm{B}(\tau \to \mu\gamma)$ is 4.4×10^{-8}.

V_M [27], which does not appear in Eq. (4.4) because M_{N_l} is usually assumed to be smaller than M' ($> M_{\mathrm{GUT}}$). By comparing the $SU(5)_{RN}$ model with the $SU(5)_{H_\nu}$ model, we can find an advantage in the latter model. This is a predictability, that is, flavor changing processes are strongly predicted since there are no degrees of freedom of V_M. This means that flavor violations in the charged lepton sector are directly related to those in the quark sector through the large flavor mixings in the neutrino sector. And, even if the mass matrix of the right-handed neutrinos is diagonal, magnitudes of $\delta m^2_{\tilde{L}}$ and $\delta m^2_{\tilde{D}}$ in the $SU(5)_{H_\nu}$ model are different from those in the $SU(5)_{RN}$ model. For example, Eq. (4.4) can be a few percent smaller than that in the $SU(5)_{H_\nu}$ model due to their log factors. The magnitude of the log factor is $\log\frac{M_P}{M'} \lesssim \log\frac{M_P}{M''}$, since M' must be larger than the GUT scale for the proton stability, and M'' must be of $\mathcal{O}(10^{14})$ GeV for accurate gauge coupling unification.

Let us show results of the numerical analyses in the $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $b \to s\gamma$ processes. Figures 1 and 2 show the correlations between the branching ratios of $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\mu \to e\gamma)$ with $\tan\beta = 10$ and $A_0 = 0$. In Fig. 1, $m_{1/2}$ is varied from 500 GeV to 800 GeV by 100 GeV. As for Fig. 2, $m_{1/2}$ is varied from 500 GeV to 1000 GeV by 100 GeV. m_0 is varied from 200 GeV to 1200 GeV by 100 GeV for each line. Here, the Higgs mass, calculated by FeynHiggs [28–31], is varied around 118 GeV which is not excluded by ATLAS [32] and CMS [33]. In Figs. 1 and 2, $\sin^2 2\theta_{13}$ is taken by 0 and 0.01,

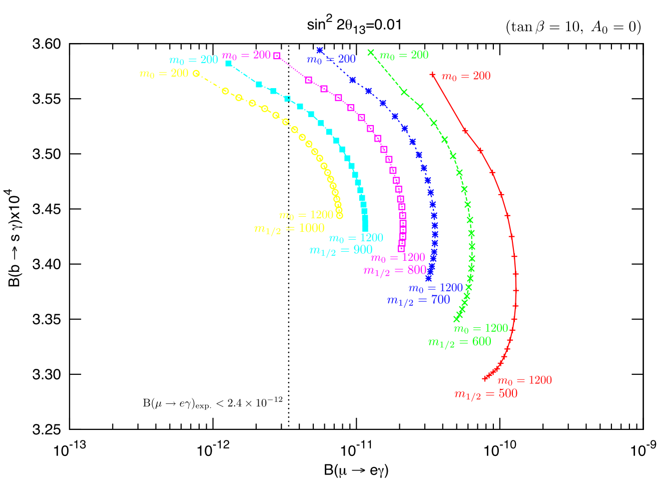

FIG. 2 (color online). Contour plot of $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\mu \to e\gamma)$ with $\sin^2 2\theta_{13} = 0.01$. We take $500\ \mathrm{GeV} \leq m_{1/2} \leq 1000\ \mathrm{GeV}$. The other parameters are the same as those in Fig. 1.

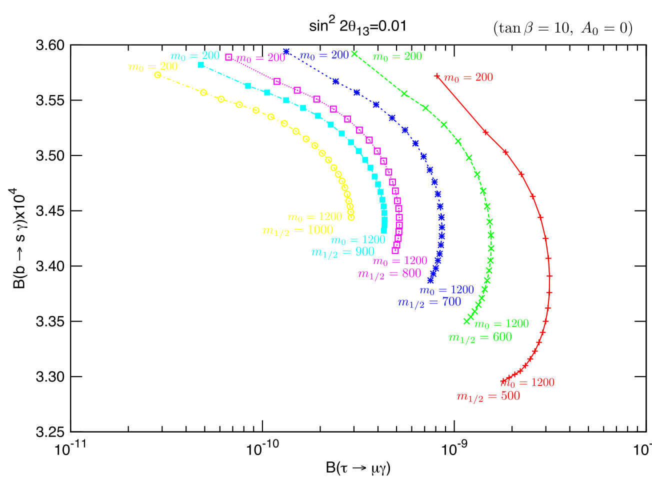

FIG. 4 (color online). Contour plot of $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\tau \to \mu\gamma)$ with $\sin^2 2\theta_{13} = 0.01$. We take $500\ \mathrm{GeV} \leq m_{1/2} \leq 1000\ \mathrm{GeV}$. The other parameters are the same as those in Fig. 3.

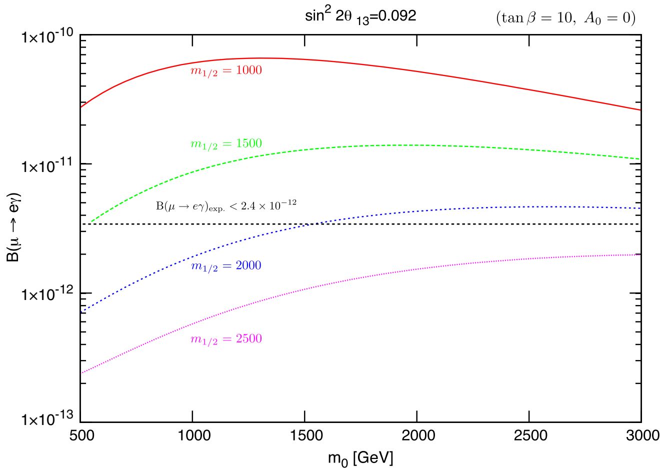

FIG. 5 (color online). $\mathrm{B}(\mu \to e\gamma)$ is plotted as a function of m_0 with $\sin^2 2\theta_{13} = 0.092$. We take $m_{1/2} = 1$ TeV, 1.5 TeV, 2 TeV, and 2.5 TeV.

respectively. We consider that the spectrum of neutrinos is hierarchical, and the ν_τ-Yukawa coupling is of $\mathcal{O}(1)$. The current upper bound on $\mathrm{B}(\mu \to e\gamma)$ is 2.4×10^{-12} by the MEG experiment [34]. Figure 2 shows that large θ_{13} is restricted in $\mu \to e\gamma$. In this parameter region, $\mathrm{B}(b \to s\gamma)$ does not change drastically because the $m_{1/2}$ dependence is larger than the m_0 dependence.

Figures 3 and 4 show the correlations between $\mathrm{B}(b \to s\gamma)$ and $\mathrm{B}(\tau \to \mu\gamma)$, whose parameters are the same as Figs. 1 and 2, respectively. $\mathrm{B}(\tau \to \mu\gamma)$ does not reach the experimental upper bound in this parameter region. [The experimental upper bound for $\mathrm{B}(\tau \to \mu\gamma)$ is 4.4×10^{-8} by *BABAR* experiment [35].] Note that a ratio of $\mathrm{B}(\tau \to \mu\gamma)/\mathrm{B}(\mu \to e\gamma)$ depends largely on θ_{13}, where other neutrino oscillation parameters are fixed. When θ_{13} becomes large, $\mathrm{B}(\tau \to \mu\gamma)/\mathrm{B}(\mu \to e\gamma)$ is closer to 10. This behavior is consistent with Ref. [23].

We do not consider the $\tau \to e\gamma$ process because the experimental upper bound for $\mathrm{B}(\tau \to e\gamma)$ is 3.3×10^{-8} [35] which is the same order as $\mathrm{B}(\tau \to \mu\gamma)$. The ratio of $\mathrm{B}(\tau \to e\gamma)/\mathrm{B}(\tau \to \mu\gamma)$ is roughly proportional to $(V_D)_{31}^2/(V_D)_{32}^2 < 1$. Hence, $\mathrm{B}(\tau \to \mu\gamma)$ is more stringent constraint than $\mathrm{B}(\tau \to e\gamma)$.

Finally, we comment on the Daya Bay experiment, which has measured a nonzero θ_{13} [36]. The best-fit value is given by $\sin^2 2\theta_{13} = 0.092$, and such a large mixing angle gives a more stringent constraint in $\mu \to e\gamma$. Figure 5 shows the m_0 and $m_{1/2}$ dependence of $\mathrm{B}(\mu \to e\gamma)$ with $\sin^2 2\theta_{13} = 0.092$. We can see that $m_{1/2}$ should be larger than 2 TeV in order not to exceed the experimental bound in Fig. 5. As for neutrinoless double beta decay, it is forbidden in our setup because the neutrinos are Dirac fermion with lepton number conservation.

V. SUMMARY

Among three typical energy scales, a neutrino mass scale, a GUT scale, and a TeV (SUSY) scale, there is the marvelous relation of Eq. (1.1). In this paper, we have investigated the phenomenology of the SUSY $SU(5)_{H_\nu}$ model proposed in Ref. [16]. This model realizes the relation of Eq. (1.1) dynamically as well as the suitable Dirac neutrino mass through the tiny VEV of neutrinophilic Higgs. First, we discussed the gauge coupling unification and the proton stability. Fascinatingly, the $SU(5)_{H_\nu}$ can realize not only accurate gauge coupling unification but also enough proton stability simultaneously, and this situation is hardly realized in the usual four-dimensional $SU(5)$ GUTs. Next, we investigated the correlations between $b \to s\gamma$ and $\mu \to e\gamma$, $\tau \to \mu\gamma$. We noted that $B(b \to s\gamma)$, $B(\mu \to e\gamma)$ and $B(\tau \to \mu\gamma)$ are correlated directly through the neutrino mixing in the $SU(5)_{H_\nu}$ model, which is an advantage of this model over the $SU(5)_{RN}$ model. As shown in Eq. (4.3), additional unknown degrees of freedom, *parameters in* V_M, are needed in the latter model. Therefore, flavor changing processes are strongly predicted in the $SU(5)_{H_\nu}$ model. As for the dependence of θ_{13}, $B(\mu \to e\gamma)$ depends largely on it, so that $B(\mu \to e\gamma)$ is strongly limited in large θ_{13}. On the other hand, we have shown that $B(b \to s\gamma)$ does not depend largely on θ_{13}.

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