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Statistical Properties and Applications of Dynamic Speckles
Produced by Double and Multiple Scatterings of Light

by

Takashi Okamoto

Contents

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Dynamic Speckles Produced by Double and
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Introductory Remarks

The subject of the book is devoted to the study of a new interesting phenomenon when the highly coherent laser light illuminates an optically rough surface of a volume medium with refractive index variations. The reflected incoherent light forms a random granular pattern in free and image spaces. This pattern, which is not called a speckle pattern by strictly collected speakers, arises from the random interference of wavelets scattered from many sub-wavelength regions in the illuminated object. Figure 1.1 shows an example of the speckle pattern produced in free space by illuminating an average pool with the monochromatic light of wavelength 633 nm from an Ar laser. The random interference pattern also appears at the image plane of the object, and

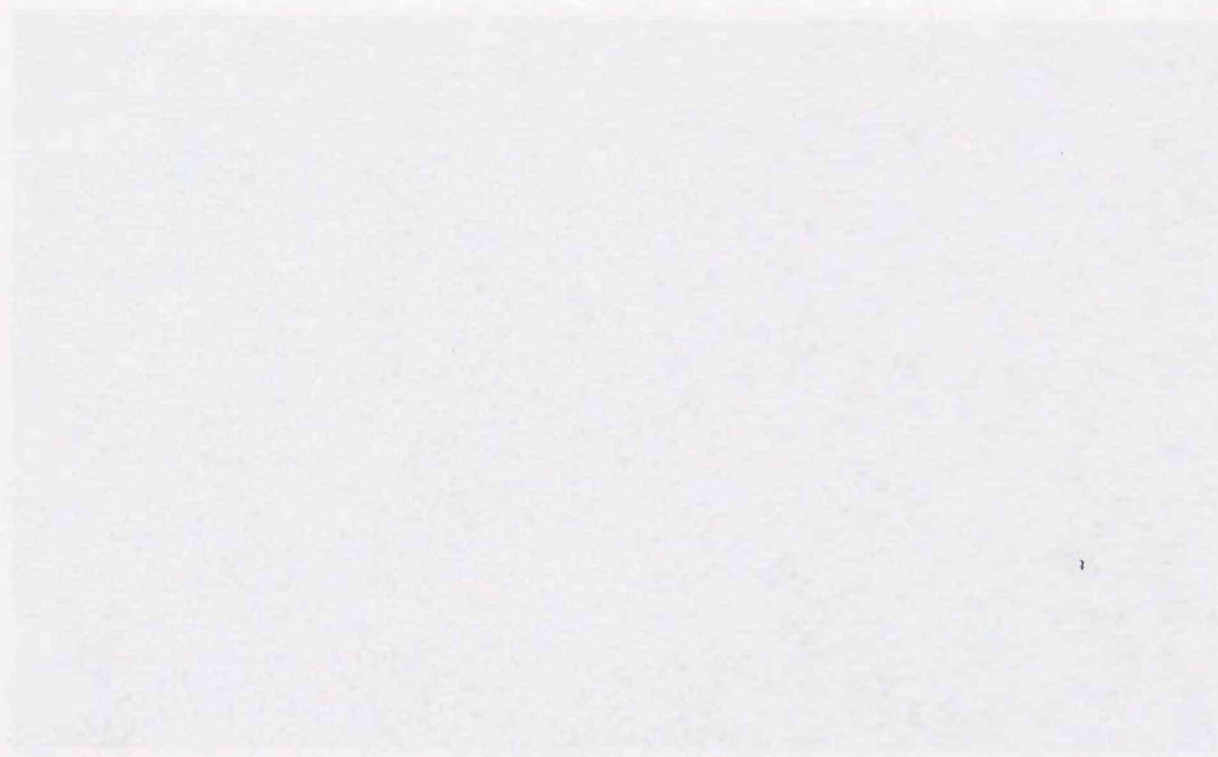


Figure 1.1 A random pattern seen in free space

Chapter 1

Introductory Remarks

The advent of the laser allowed us to observe a new interesting phenomenon: when the highly coherent laser light illuminates an optically rough surface or a volume medium with refractive index variations, the reflected or transmitted light forms a random granular pattern in free and image spaces. This pattern, which is now called a *speckle* pattern or simply called speckles, arises from the random interference of waves scattered from many microscopic regions in the illuminated object. Figure 1.1 shows an example of the speckle patterns produced in free space by illuminating an orange peel with the monochromatic light of wavelength 514.5 nm from an Ar laser. The random interference pattern also appears at the image plane of the object, and

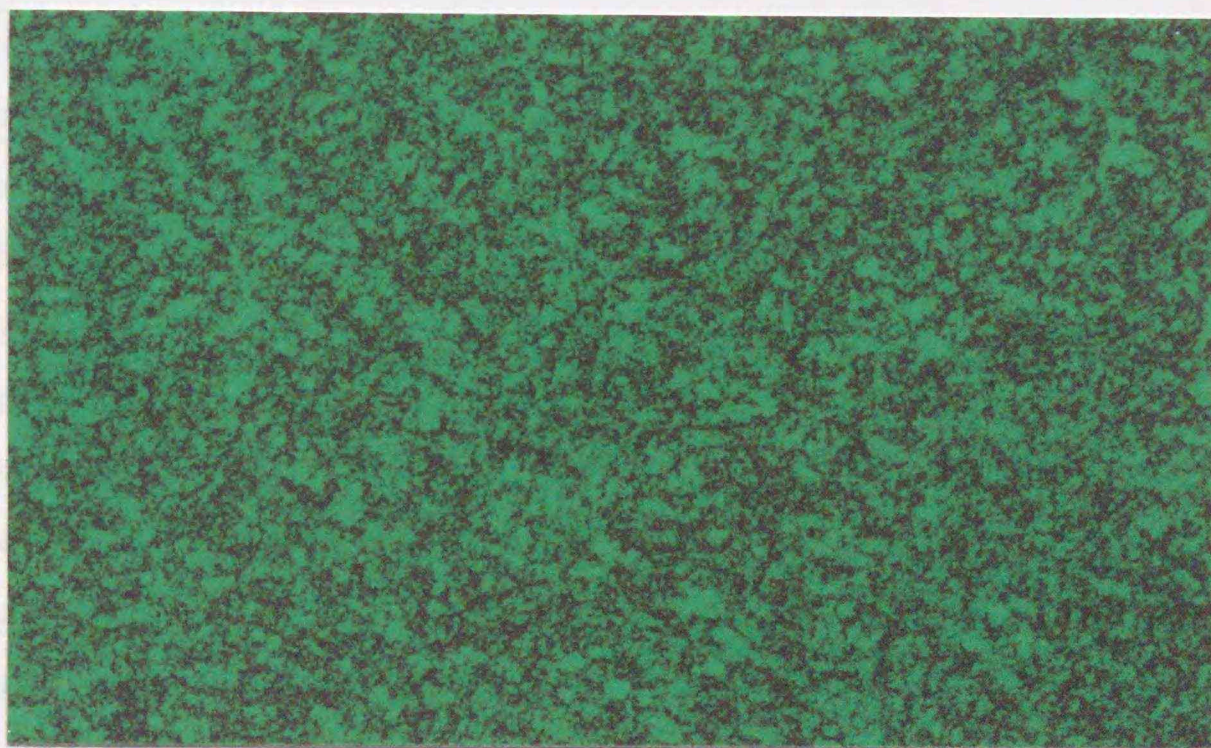


Figure 1.1. A speckle pattern from an orange peel

it bears no obvious relationship with the macroscopic characteristics of the object. This highly noisy appearance of speckle patterns creates serious problems when a laser source is applied to imaging systems. A numerous studies have been conducted on the reduction of such speckle noises in the coherent imaging system.

In the field of optical metrology, the speckle phenomenon provides a new measure for determination of the microscopic properties of the object. However, if the perfectly coherent light illuminates a relatively large area of the diffuse object and if the light scattered from the object experiences phase fluctuations greater than 2π , the statistics of the resultant speckle pattern are independent of the microscopic structure of the object. Hence, as long as the waves, each of which is scattered once by the object, form a speckle pattern, the information on the spatial structure of the object is obtained when at least one of the conditions above are not satisfied (for example, polychromatic illumination, or objects such as relatively smooth surfaces).

The speckle phenomenon caused by the single scattering of light has been extensively investigated since the invention of the CW laser in the early 1960s. A number of studies on the fundamental statistics and applications of laser speckle patterns are summarized and cited in the book edited by Dainty [1]. The diversity of applications of speckles to metrology can be found in the book edited by Erf [2], which offers various techniques for measurements of many physical quantities, such as surface roughness, displacement, strain, vibration, and motion paths. For the dynamical aspect of speckle patterns, Asakura and Takai [3] and Yoshimura [4] review the studies concerned with the statistical properties of dynamic speckles and their application to velocity measurements. The speckle pattern fluctuating in time is also observed in colloidal systems. Dynamic light scattering techniques make use of the temporal fluctuations of speckles arising from the motion of scattering particles. A good reference in this field is given by Berne and Pecora [5].

The speckles that obscure the detail in the image stimulated the investigations on the double scattering by two diffuse screens, which had already been applied to a grainless screen [6]. Leifer *et al.* [7] proposed the back-projection system in which two ground-glass screens are placed with their ground surfaces almost in contact and one is moved slowly in its own plane relative to the other. They showed that the resulting speckles are continuously changing and smoothed out by the persistence of vision. Lowenthal and Joyeux [8] evaluated theoretically and experimentally the

double-diffuser system by expressing the quality of images in terms of the ratio of the mean intensity of the image to the standard deviation of the speckle intensity (the S/N ratio). Experiments on the wavelength decorrelation of speckles from two ground-glass diffusers were reported by George and Jain [9]. A comparison between various techniques for the reduction of speckles is given by Rawson *et al.* [10], who concluded that illumination through two close diffuse-screens in relative motion is the most effective way of eliminating speckle patterns.

The efforts to formulate the statistical properties of doubly-scattered speckles have been made since 1980s. The probability of the intensity of speckles produced by illuminating a diffuser with a speckle pattern was studied by Fried [11], who introduced the concept of *speckled speckle* and considered the safety of the eyes exposed to the light forming ordinary speckle and speckled-speckle patterns. He showed that the probability densities of the speckled-speckle intensity are K distributions under certain conditions. The K -distribution of the intensity is also observed in the speckle pattern formed by the laser light after propagation through the turbulent atmosphere [12]. O'Donnell [13] and Barakat and Salawitch [14] reported theoretical analyses on the probabilities and correlations of doubly-scattered amplitude and intensity. Their studies revealed the non-Gaussian and non-stationary behavior of the speckles from the double scattering. Newman [15] verified the results obtained by O'Donnell and made a comment on the temporal correlation properties of the doubly-scattered light. General two-state correlation functions for speckles from a cascade of two thin diffusers were derived by Shirley and George [16], and they considered the effects of wavelength decorrelation and displacements of the diffusers. Reddy and Rao [17] investigated the correlation of singly- and doubly-scattered speckles under partially coherent illumination. The application of doubly-scattered speckles to the measurement of surface roughness was intensively studied by Iwai and Shigeta [18] and Yoshimura *et al.* [19–23].

Except for the studies on speckle reduction, all the reports mentioned above are concerned with the static properties of speckle patterns. The temporal properties of speckled speckles or speckles by the double scattering have been studied in various fields of optics. In astronomy, Roddier *et al.* [24] derived an expression for the lifetime of stellar speckles and examined the theoretical results by using two moving diffusers to simulate thin atmospheric layers. When a multimode optical fiber is used

as a guide for the laser light, the light emitted from the exit face of the fiber forms a speckle pattern because of the interference among different modes of the light propagating within the core. Takai and Asakura [25, 26] investigated the space-time correlation properties of speckled-speckle patterns produced by illuminating a moving diffuser with the light from the optical fibers. Fujii *et al.* [27, 28] examined the power spectra of speckle signals detected by an optical-fiber probe which consists of two fibers for illumination and detection. The double-diffuser model simplifies the complicated multiple scattering taken place inside the biological tissue. Illumination of a reflecting diffuser through a transparent diffuser gives rise to the triply-scattered dynamic speckles when one of the diffusers is moved in its own plane. Aizu, Iwai and Asakura [29–31] studied the temporal correlations of such speckles and found unique properties arising from the interference between singly- and triply-scattered speckle fields. The dynamic speckles doubly scattered by two moving diffusers were investigated by Okamoto and Asakura [32–36].

A multiple-diffuser model is considered to be a straightforward extension of the double-diffuser model. The multiple phase-screen model, which consists of a cascade of diffusers modulating only the phase of light waves, has been employed to the study of wavefront distortions and intensity variations in the light propagating through the atmosphere [37–41]. Owing to the weak scattering characteristics of the atmosphere, the weak random-phase screens were used for simulating the refractive index variations in the atmospheric turbulence. For strong scattering media, such as dense colloidal suspensions and biological tissue, it is appropriate to employ a set of strong diffusers, each of which completely randomizes the phase of the incident light. Okamoto and Asakura [42–44] analyzed the dynamic properties of multiply-scattered speckles from a series of such diffusers and found some typical relationships between the motion of diffusers and the resultant fluctuations in intensity of the transmitted light.

A more direct approach to the multiple scattering from concentrated particulate media is the diffusing-wave spectroscopy proposed by Pine *et al.* [45]. This technique utilizes the diffusive nature of photons propagating in strong disordered media. The Brownian motion of particles in dense colloidal systems was studied by making use of the temporal intensity fluctuations in the light multiply scattered by the system [46–57].

These two types of multiple scattering seem to be different phenomena on the surface, since the speckles from a cascade of diffusers arise from the wavefront distortion continuously given across the plane of each diffuser, and those from the colloidal system arise from the scattering by many discrete particles. However, as far as the dynamic properties of speckle patterns are concerned, the time-varying speckles resulting from these two different scattering mechanisms bear some similarities in their temporal behaviors.

The purpose of this dissertation is to investigate the statistical properties of dynamic speckles produced by double and multiple scatterings of light from the diffusers and colloidal systems. In the dissertation, the theoretical and experimental studies on three different types of laser speckles are described: (i) speckled speckles and doubly-scattered speckles from a single and double phase screens, (ii) multiply-scattered speckles from a series of phase screens, and (iii) multiply-scattered speckles from particle suspensions.

Chapter 2 serves the fundamental statistics of static and dynamic speckle patterns produced by a single phase screen. By starting from the origins of the speckle phenomenon, various basic formulae are derived for the later use. Throughout the dissertation, the phase screens considered are assumed to be the ones that randomize completely the phase of incident light to yield no unscattered component of the transmitted light.

Chapters 3-6 are concerned with the statistical properties of speckled speckles and doubly-scattered speckles. In Chapter 3, the speckle signals detected by an optical fiber probe are investigated. If a multimode optical fiber is used as an illuminating fiber, the light incident on a moving diffuser forms a speckle pattern and speckled speckles fluctuating in time arise at the input face of a detecting fiber. An expression is derived for the power spectrum of the intensity fluctuations, and the effects of the core diameters of illuminating and detecting fibers are examined. The typical phenomena predicted by the theory are verified by experiments. Chapter 4 introduces a cascade of two diffusers moving in their own planes. The first diffuser is regarded as a disturbing medium when the motion of the second diffuser is to be measured. The dependence of doubly-scattered speckles in the diffraction field on the motion of the diffusers is evaluated theoretically and experimentally from the temporal correlation function of their intensities. Chapter 5 deals with the same

diffuser system as in Chapter 4, but a single-lens imaging system is employed for detection. The intensity fluctuations of speckle patterns formed at the image plane of the second diffuser are experimentally studied, and qualitative explanations for the results are given. Space-time correlation properties of the speckle patterns from the double diffusers with an imaging system are formulated in Chapter 6. The theory developed allows the focus of the imaging lens to be placed at any position. Therefore, the first diffuser hidden behind the second diffuser is also considered as an object to be measured. Experiments are conducted to confirm the theoretical results on the relationship between the motion of diffusers and that of speckles detected in image space.

The statistical properties of multiply-scattered speckles from the diffusers are investigated theoretically in Chapters 7 and 8. Chapter 7 considers the motion of speckle patterns formed in the diffraction field by the light propagating through many transparent moving diffusers. Some statistical quantities describing the speckle motion are introduced and evaluated under different scattering orders and velocities of the diffusers. The theoretical results are derived for the particular velocity distributions chosen so as to simulate the velocity gradients commonly observed in the flow of multiple scattering media. Chapter 8 is concerned with the speckle patterns in image space produced by the multiple-diffuser system. The intensity fluctuations of image speckles are examined by varying the focal position and aperture size of the imaging system as well as the velocity distribution of the diffusers. The possibility of measuring different velocity components of a multiple scattering medium in motion is discussed.

Chapters 9 and 10 are devoted to the experimental study on the multiple light scattering from particle suspensions. Chapter 9 deals with the colloidal system in which particles in the Brownian motion are translating uniformly across the incident beam. The experimental results concerning the properties of the imaging system employed are compared with the theoretical ones obtained by the multiple-diffuser model in Chapter 8. The discussion is also given to the effects of polarization, the incident form of the beam, and the concentration of scattering particles. The velocity dependence of the multiply-scattered speckles is explained by considering the scattering volume formed inside the suspensions. In Chapter 10, the cross-correlation properties of dynamic speckles produced by different wavelengths of light are inves-

Chapter 2

Fundamental Statistics of Speckle Patterns

2.1 Introduction

Laser speckles arise as a consequence of the random interference of electromagnetic waves. The rigorous solutions for the properties of speckle patterns are obtained by calculating Maxwell's equations under appropriate boundary conditions. However, this calculation is rather complex and time consuming because of the randomness involved in scattering media that give rise to speckles. The scattering properties of such random media are suitably described by random variables which obey a stochastic process. Therefore, the statistical treatment is necessary for the study of laser speckles. A statistical model for the formation of speckles allows us to understand intuitively the phenomena of random interference of light.

In this section, the basic properties of speckle patterns are presented by introducing statistical descriptions for a diffuse object and resulting optical fields. First, the origins of speckle formation are described using the concept of a random walk in the complex plane. The probabilities of intensity and phase for fully-developed speckle patterns are obtained by applying the central limit theorem for the amplitude of the speckle field. Next, the autocorrelation function of intensities detected at two different points in space is considered. The spatial properties of speckle patterns, such as the average speckle size, are obtained by evaluating the autocorrelation function. Finally, the temporal properties of dynamic speckle patterns are investigated by analyzing the space-time cross-correlation function and power spectral distribution of intensity fluctuations. In particular, attention is focused on the relationship between the motion of a diffuse object and the temporal fluctuations of resultant speckle patterns.

Throughout this dissertation, theoretical analysis is made under the assumption

that the optical waves of concern are perfectly monochromatic and linearly polarized. This assumption enables the speckle field to be treated as scalar waves.

2.2 Formation of a Speckle Pattern

2.2.1 Origins of Speckle

Typical diffuse objects giving rise to speckles have surfaces which are rough on the scale of an optical wavelength ($\lambda \sim 0.5 \mu\text{m}$). When the monochromatic light illuminates such a surface, the waves reflected from many scattering elements on the surface of the object interfere at a given point in the observation plane (see Fig. 2.1(a)). The optical path lengths from these scattering elements to the observation point differ by more than several wavelengths if the large height variations of the surface exist. The intensity of light at the observation point is determined by the coherent addition of these waves with different phases. Since the phase of each wave varies in a random fashion with the position of the observation point, the intensity also varies in space, resulting in a granular pattern that is called speckle.

The speckles appear in image space by using an imaging system. The image of a point on the surface is described by the amplitude point spread function, which has a finite extent even for an aberration-free imaging system. As shown in Fig. 2.1(b), if the spread functions arising from many different scattering points overlap with one another, a speckle pattern arises by the superposition of many de-phased spread functions at each image point. Thus, in principle, no speckle pattern is formed by a high resolution imaging system which can perfectly resolve each microscopic elements of the surface.

The light transmitted through or reflected from colloidal suspensions also forms a speckle pattern. As illustrated in Fig. 2.1(c), the optical disturbance at a given point results from interference of waves leaving many scattering particles located randomly in a surrounding medium. Thus, the same argument as for rough surfaces can be applied for the formation of speckles from particle suspensions. An image of the particle suspension also shows a speckle appearance if the overlapping occurs between the images of many scattering particles.

2.2.2. Random Walk in the Complex Plane

For a monochromatic wave, the wave vector $\mathbf{k}$ is constant for the light polarisation component of the wave. The wave vector $\mathbf{k}$ is defined as $\mathbf{k} = \frac{2\pi}{\lambda} \mathbf{\hat{k}}$ and $\mathbf{\hat{k}}$ is a unit vector in the direction of propagation.

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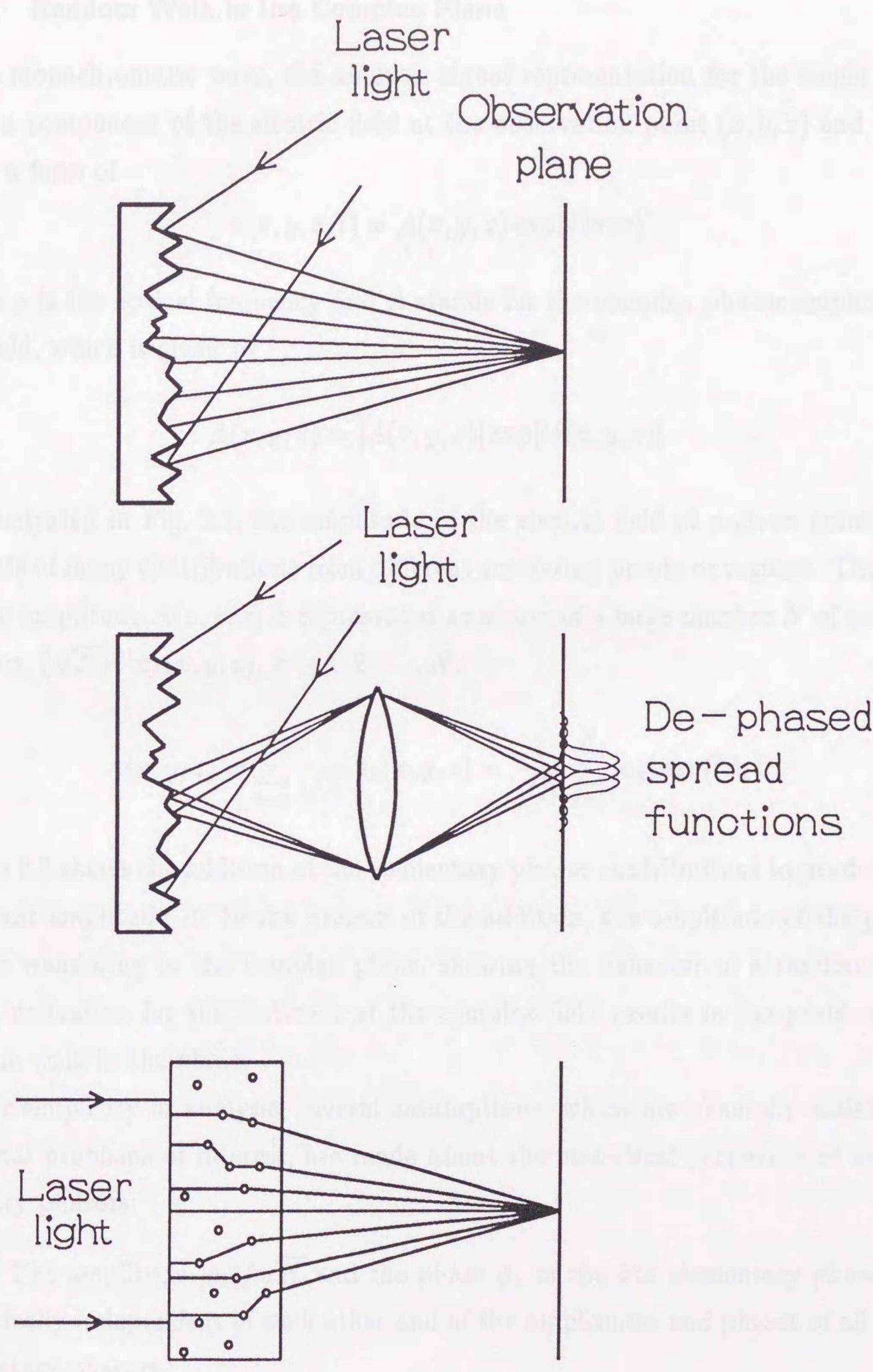


Figure 2.1. Speckle formation by a rough surface in (a) the diffraction field and (b) the image field, and (c) by a particle suspension in the diffraction field.

2.2.2 Random Walk in the Complex Plane

For a monochromatic wave, the analytic signal representation for the single polarization component of the electric field at the observation point (x, y, z) and time t takes a form of

$$u(x, y, z; t) = A(x, y, z) \exp(i2\pi\nu t), \quad (2.1)$$

where ν is the optical frequency and A stands for the complex phasor amplitude of the field, which is given by

$$A(x, y, z) = |A(x, y, z)| \exp[i\theta(x, y, z)]. \quad (2.2)$$

As illustrated in Fig. 2.1, the amplitude of the electric field at a given point (x, y) consists of many contributions from different scattering points or regions. Thus, the phasor amplitude $A(x, y, z)$ is represented as a sum of a large number N of complex phasors, $(\sqrt{N})^{-1}a_k(x, y, z)$, $k = 1, 2, \dots, N$:

$$A(x, y, z) = \sum_{k=1}^N \frac{1}{\sqrt{N}} a_k(x, y, z) = \frac{1}{\sqrt{N}} \sum_{k=1}^N |a_k| \exp(i\phi_k). \quad (2.3)$$

Figure 2.2 shows the addition of the elementary phasor contributions to produce the resultant amplitude A . In the process of the addition, the amplitude of the phasor sum is wandering in the complex plane, showing the behavior of a random walk. Thus, derivation for the statistics of the complex field results in the problem of a random walk in the plane.

For simplicity of analysis, several assumptions, which are generally satisfied in practical problems of interest, are made about the statistical properties of the elementary phasors:

(i) The amplitude $|a_k|/\sqrt{N}$ and the phase ϕ_k of the k th elementary phasor are statistically independent of each other and of the amplitudes and phases of all other elementary phasors.

(ii) The phases ϕ_k are all uniformly distributed on the interval $(-\pi, \pi)$ (that is, the surface is rough compared with a wavelength so that the phase excursions are many times 2π radians).

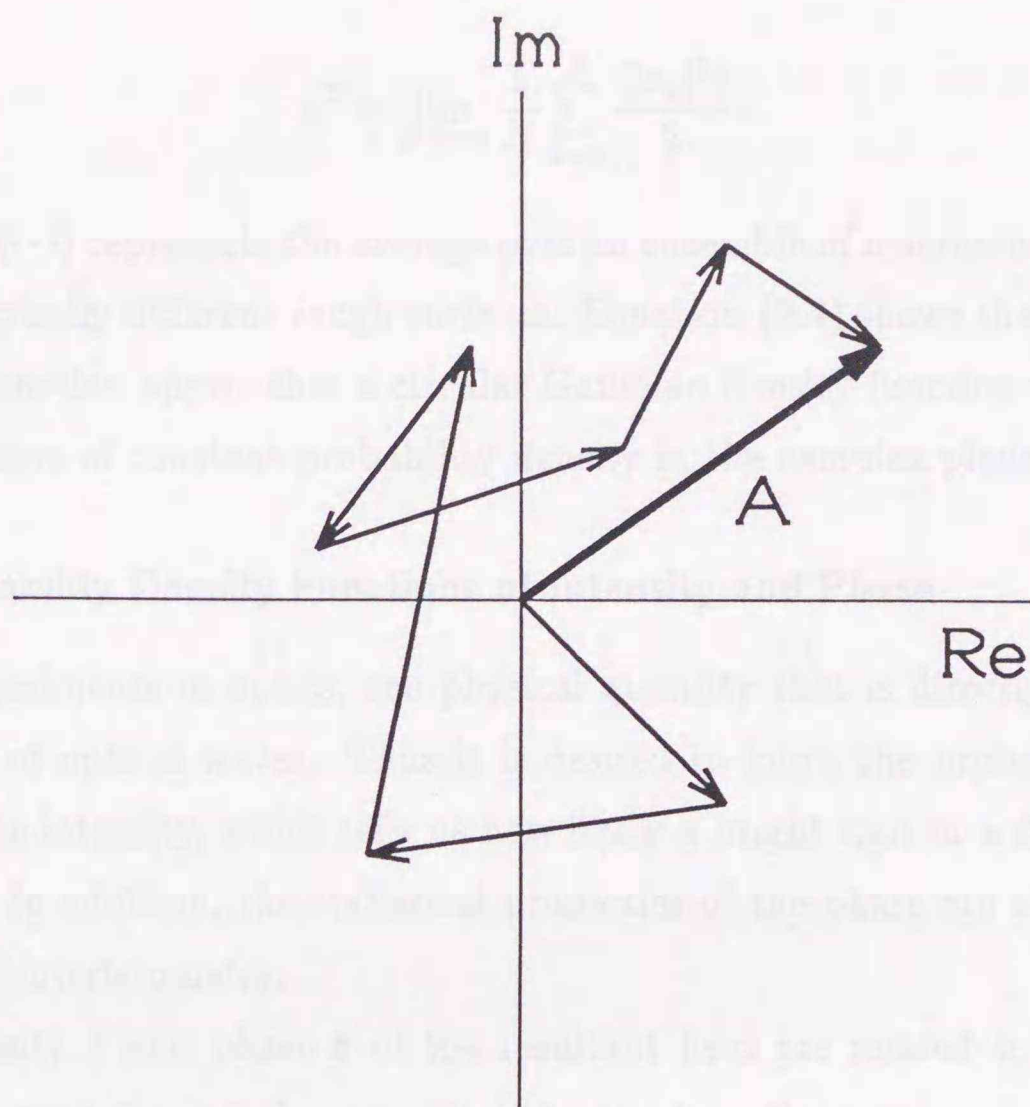


Figure 2.2. Random walk in the complex plane

We now derive the joint probability density function of real and imaginary parts A_{re} and A_{im} of the resultant phasor amplitude, which will be used to calculate the probability density functions of the intensity and phase.

First, from assumption (i), A_{re} and A_{im} are also sums of N independent random contributions. Since the number N is large, the conditions for satisfying the central limit theorem are satisfied. Hence, A_{re} and A_{im} are approximately normal (Gaussian) random variables. Next, by making use of assumption (ii) together with assumption (i), it is readily shown that A_{re} and A_{im} have zero means, identical variances, and are uncorrelated. Finally, these results are combined to yield the joint Gaussian probability density function of real and imaginary parts of the complex amplitude:

$$p_{re,im}(A_{re}, A_{im}) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{A_{re}^2 + A_{im}^2}{2\sigma^2}\right), \quad (2.4)$$

where

$$\sigma^2 = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \frac{\langle |a_k|^2 \rangle}{2}. \quad (2.5)$$

In Eq. (2.5), $\langle \dots \rangle$ represents the average over an ensemble of macroscopically similar but microscopically different rough surfaces. Equation (2.4) shows that, as $N \rightarrow \infty$, the density function approaches a circular Gaussian density function which has the circular contours of constant probability density in the complex plane.

2.2.3 Probability Density Functions of Intensity and Phase

For most experiments in optics, the physical quantity that is directly measured is the intensity of optical waves. Thus it is desired to know the probability density function of the intensity, which tells us how likely a bright spot or a dark area is to be observed. In addition, the statistical properties of the phase are also important in the field of interferometry.

The intensity I and phase θ of the resultant field are related to the real and imaginary parts of the complex amplitude by the transformation

$$I = A_{re}^2 + A_{im}^2, \quad (2.6)$$

$$\theta = \tan^{-1} \frac{A_{im}}{A_{re}}. \quad (2.7)$$

The usual techniques for transformations of random variables (for example, see [58]) are applied to calculate the joint probability density function of I and θ , which is expressed as

$$p_{I,\theta}(I, \theta) = p_{re,im}(\sqrt{I} \cos \theta, \sqrt{I} \sin \theta) |J|, \quad (2.8)$$

where $|J|$ is the modulus of the Jacobian

$$|J| = \left| \det \begin{bmatrix} \frac{\partial A_{re}}{\partial I} & \frac{\partial A_{re}}{\partial \theta} \\ \frac{\partial A_{im}}{\partial I} & \frac{\partial A_{im}}{\partial \theta} \end{bmatrix} \right| = \frac{1}{2} \quad (2.9)$$

and $\det[\dots]$ denotes the determinant. Substituting Eq. (2.4) in Eq. (2.8), we find

$$p_{I,\theta}(I, \theta) = \frac{1}{4\pi\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right) \quad I \geq 0, \quad -\pi \leq \theta < \pi, \quad (2.10)$$

and zero otherwise.

The integration of Eq. (2.10) with respect to θ over the interval $(-\pi, \pi)$ yields the marginal probability density function of the intensity:

$$p_I(I) = \frac{1}{2\sigma^2} \exp\left(-\frac{I}{2\sigma^2}\right) \quad I \geq 0, \quad (2.11)$$

and zero otherwise. Similarly, the integration with respect to I over the interval $(0, \infty)$ yields the marginal probability density function of the phase:

$$p_\theta(\theta) = \frac{1}{2\pi} \quad -\pi \leq \theta < \pi, \quad (2.12)$$

and zero otherwise. In summary, the intensity of a polarized speckle pattern follows a *negative exponential* law, while the phase follows a *uniform* law.

From Eq. (2.11), the mean value, the second moment, and the variance of the intensity are given, respectively, by

$$\langle I \rangle = 2\sigma^2, \quad (2.13)$$

$$\langle I^2 \rangle = 2\langle I \rangle^2, \quad (2.14)$$

$$\sigma_I^2 = \langle I^2 \rangle - \langle I \rangle^2 = \langle I \rangle^2. \quad (2.15)$$

Thus the contrast of a speckle pattern defined by the ratio $C = \sigma_I / \langle I \rangle$ is always unity. A speckle pattern having a contrast of one is called a *fully-developed* speckle pattern. The reciprocal of the contrast, $\langle I \rangle / \sigma_I$, is considered as the signal-to-noise ratio (S/N) of the image of an object. If the surface of the object is optically rough, the image with S/N=1 is obtained and provides no information about the macroscopic characteristics of the object. That accounts for many speckle reduction techniques being proposed in the study of coherent imagery.

2.3 Spatial Properties of Speckle Intensity Variations

In the previous section, the statistical properties of speckles detected at a single point were considered. The probability density function of the intensity describes how likely a given brightness of the scattered light is to be measured. However, in

order to describe the spatial structure of the speckle pattern, we need to know the correlation properties of intensity variations in space. In this section, expressions for the autocorrelation function of the speckle intensity are given in general forms.

2.3.1 Autocorrelation Function of Speckle Intensity

Consider the geometry for producing the speckle field shown in Fig. 2.3. A diffuse object is located in the plane u . The monochromatic and linearly polarized light is incident on the diffuse object, and the scattered light is transmitted through a linear optical system to the observation plane x . If the object and the optical system do not change the state of polarization of the optical field, the complex amplitude $A(x)$ of the speckle field at the observation point x is expressed as

$$A(x) = \int_{-\infty}^{\infty} E(u)O(u)T(u,x)du, \quad (2.16)$$

where $E(u)$ is the amplitude distribution of the incident light, $O(u)$ the phase modulation introduced by the diffuse object, and $T(u,x)$ the propagation function (or the impulse response function) of the optical system.

The purpose of this section is to calculate the autocorrelation function of the intensity distribution $I(x) = |A(x)|^2$, which is given by

$$\Gamma_I(x_1, x_2) = \langle I(x_1)I(x_2) \rangle$$

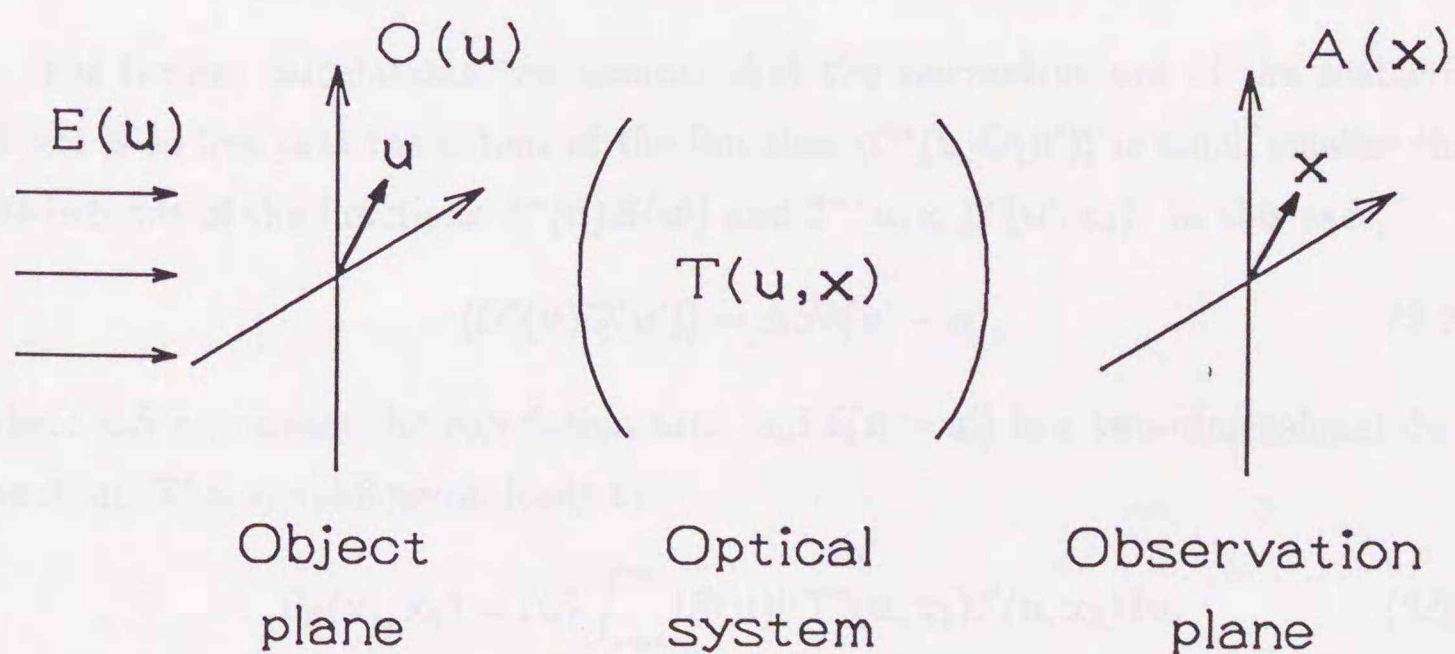


Figure 2.3. Geometry for producing the speckle field

$$= \langle A^*(\mathbf{x}_1)A(\mathbf{x}_1)A(\mathbf{x}_2)A^*(\mathbf{x}_2) \rangle, \quad (2.17)$$

where $*$ denotes the complex conjugate and the average is taken over an ensemble of diffuse objects. Substitution of Eq. (2.16) into Eq. (2.17) results in the general formula for the autocorrelation function of the intensity:

$$\begin{aligned} \Gamma_I(\mathbf{x}_1, \mathbf{x}_2) = & \iiint_{-\infty}^{\infty} E^*(\mathbf{u})E(\mathbf{u}')E(\mathbf{u}'')E^*(\mathbf{u}''') \\ & \times \langle O^*(\mathbf{u})O(\mathbf{u}')O(\mathbf{u}'')O^*(\mathbf{u}''') \rangle \\ & \times T^*(\mathbf{u}, \mathbf{x}_1)T(\mathbf{u}', \mathbf{x}_1)T(\mathbf{u}'', \mathbf{x}_2)T^*(\mathbf{u}''', \mathbf{x}_2) d\mathbf{u}d\mathbf{u}'d\mathbf{u}''d\mathbf{u}'''. \end{aligned} \quad (2.18)$$

The fourth-order moment $\langle O^*(\mathbf{u})O(\mathbf{u}')O(\mathbf{u}'')O^*(\mathbf{u}''') \rangle$ of a random variable $O(\mathbf{u})$ in the integrand in Eq. (2.18) takes a very complex form even in the cases of relatively simple scattering characteristics of objects. Fortunately enough, for circular complex Gaussian fields, $\Gamma_I(\mathbf{x}_1, \mathbf{x}_2)$ can be expressed by the autocorrelation function $\Gamma_A(\mathbf{x}_1, \mathbf{x}_2) = \langle A^*(\mathbf{x}_1)A(\mathbf{x}_2) \rangle$ of the complex amplitude $A(\mathbf{x})$ as [59]

$$\begin{aligned} \Gamma_I(\mathbf{x}_1, \mathbf{x}_2) = & \Gamma_A(\mathbf{x}_1, \mathbf{x}_1)\Gamma_A(\mathbf{x}_2, \mathbf{x}_2) + |\Gamma_A(\mathbf{x}_1, \mathbf{x}_2)|^2 \\ = & \langle I(\mathbf{x}_1) \rangle \langle I(\mathbf{x}_2) \rangle \left[1 + \left| \frac{\Gamma_A(\mathbf{x}_1, \mathbf{x}_2)}{\langle I(\mathbf{x}_1) \rangle^{1/2} \langle I(\mathbf{x}_2) \rangle^{1/2}} \right|^2 \right]. \end{aligned} \quad (2.19)$$

Thus the problem is reduced to the evaluation of the equation

$$\begin{aligned} \Gamma_A(\mathbf{x}_1, \mathbf{x}_2) = & \iint_{-\infty}^{\infty} E^*(\mathbf{u})E(\mathbf{u}') \langle O^*(\mathbf{u})O(\mathbf{u}') \rangle \\ & \times T^*(\mathbf{u}, \mathbf{x}_1)T(\mathbf{u}', \mathbf{x}_2) d\mathbf{u}d\mathbf{u}'. \end{aligned} \quad (2.20)$$

For further calculations, we assume that the microstructure of the scattering object is so fine that the extent of the function $\langle O^*(\mathbf{u})O(\mathbf{u}') \rangle$ is much smaller than the extends of the functions $E^*(\mathbf{u})E(\mathbf{u}')$ and $T^*(\mathbf{u}, \mathbf{x}_1)T(\mathbf{u}', \mathbf{x}_2)$. In this case,

$$\langle O^*(\mathbf{u})O(\mathbf{u}') \rangle = \Delta S \delta(\mathbf{u}' - \mathbf{u}), \quad (2.21)$$

where ΔS represents the correlation area and $\delta(\mathbf{u} - \mathbf{u}')$ is a two-dimensional delta function. This simplification leads to

$$\Gamma_A(\mathbf{x}_1, \mathbf{x}_2) = \Delta S \int_{-\infty}^{\infty} |E(\mathbf{u})|^2 T^*(\mathbf{u}, \mathbf{x}_1)T(\mathbf{u}, \mathbf{x}_2) d\mathbf{u}. \quad (2.22)$$

We evaluate Eq. (2.22) in two different optical geometries.

2.3.2 Free-Space Geometry

For the free-space propagation geometry, the propagation function is expressed in the Fresnel approximation as [60]

$$T(\mathbf{u}, \mathbf{x}) = \frac{1}{i\lambda R} \exp\left(\frac{i\pi}{\lambda R} |\mathbf{u} - \mathbf{x}|^2\right), \quad (2.23)$$

where λ is the optical wavelength and R is the distance from the object plane to the observation plane. By substituting Eq. (2.23) into Eq. (2.22), the autocorrelation function of the complex amplitude becomes

$$\begin{aligned} \Gamma_A(\mathbf{x}_1, \mathbf{x}_2) &= \frac{\Delta S}{\lambda^2 R^2} \exp\left[\frac{i\pi}{\lambda R} (|\mathbf{x}_2|^2 - |\mathbf{x}_1|^2)\right] \\ &\times \int_{-\infty}^{\infty} |E(\mathbf{u})|^2 \exp\left(-\frac{i2\pi}{\lambda R} \mathbf{u} \cdot \mathbf{X}\right) d\mathbf{u}, \end{aligned} \quad (2.24)$$

where $\mathbf{X} = \mathbf{x}_2 - \mathbf{x}_1$. Using Eq. (2.19), we obtain

$$\Gamma_I(\mathbf{X}) = \langle I \rangle^2 \left[1 + \frac{\left| \int_{-\infty}^{\infty} |E(\mathbf{u})|^2 \exp\left(-\frac{i2\pi}{\lambda R} \mathbf{u} \cdot \mathbf{X}\right) d\mathbf{u} \right|^2}{\int_{-\infty}^{\infty} |E(\mathbf{u})|^2 d\mathbf{u}} \right], \quad (2.25)$$

where $\langle I \rangle = \langle I(\mathbf{x}_1) \rangle = \langle I(\mathbf{x}_2) \rangle$. Equation (2.25) consists of a term corresponding to the average intensity and a term corresponding to the autocorrelation function of the fluctuating component $\Delta I = I - \langle I \rangle$ of the speckle intensity. The average size of speckles can reasonably be defined as the width of the autocorrelation function of intensity fluctuations ΔI . With reference to Eq. (2.25), the average speckle size is determined by the Fourier transform of the intensity distribution $|E(\mathbf{u})|^2$ of the light incident on the surface of the object. Thus, the speckle pattern becomes finer as the illuminating area becomes larger.

2.3.3 Imaging Geometry

Consider next the imaging system in which a single thin lens is located in the plane $\mathbf{x}'$ a distance d_o away from the object plane $\mathbf{u}$ and a distance d_i from the observation plane $\mathbf{x}$. The impulse response function of this imaging system is given by [61]

$$T(\mathbf{u}, \mathbf{x}) = \frac{1}{\lambda^2 d_o d_i} \exp\left[\frac{i\pi}{\lambda} \left(\frac{|\mathbf{u}|^2}{d_o} + \frac{|\mathbf{x}|^2}{d_i}\right)\right]$$

$$\times \int_{-\infty}^{\infty} P(\mathbf{x}') \exp \left[\frac{i\pi}{\lambda} \left(\frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{f} \right) |\mathbf{x}'|^2 \right] \\ \times \exp \left[-\frac{i2\pi}{\lambda} \left(\frac{\mathbf{u}}{d_o} + \frac{\mathbf{x}}{d_i} \right) \cdot \mathbf{x}' \right] d\mathbf{x}', \quad (2.26)$$

where $P(\mathbf{x}')$ represents the pupil function of the imaging system and f is the focal length of the system. When the intensity distribution of the incident light is uniform ($|E(\mathbf{u})|^2 = 1$), the autocorrelation function of the complex amplitude becomes

$$\Gamma_A(\mathbf{x}_1, \mathbf{x}_2) = \frac{\kappa \Delta S}{(\lambda^2 d_o d_i)^2} \exp \left[\frac{i\pi}{\lambda d_i} (|\mathbf{x}_2|^2 - |\mathbf{x}_1|^2) \right] \\ \times \int_{-\infty}^{\infty} |P(\mathbf{x}')|^2 \exp \left(-\frac{i2\pi}{\lambda d_i} \mathbf{x}' \cdot \mathbf{X} \right) d\mathbf{x}', \quad (2.27)$$

where κ is a proportionality constant. Substitution of Eq. (2.27) into Eq. (2.19) yields

$$\Gamma_I(\mathbf{X}) = \langle I \rangle^2 \left[1 + \frac{\left| \int_{-\infty}^{\infty} |P(\mathbf{x}')|^2 \exp \left(-\frac{i2\pi}{\lambda d_i} \mathbf{x}' \cdot \mathbf{X} \right) d\mathbf{x}' \right|^2}{\int_{-\infty}^{\infty} |P(\mathbf{x}')|^2 d\mathbf{x}'} \right]. \quad (2.28)$$

Comparison of Eq. (2.28) with Eq. (2.25) shows that, instead of the intensity distribution of illuminating light, the intensity transmittance $|P(\mathbf{x}')|^2$ of the lens pupil determines the coarseness of the speckle pattern in the imaging geometry. It should be noted that the autocorrelation function is independent of any aberrations affecting the phase of $P(\mathbf{x}')$. Moreover, Eq. (2.28) is also independent of the distance d_o between the object and the lens. That is, the statistical properties of a speckle pattern in image space are not affected by the condition whether the object is in focus or out of focus. This result indicates that the speckle size does not correspond to the point spread of the imaging system, since the point spread function measured in image space becomes broader as the surface of the object moves away from the focal plane of the lens.

The size of speckles can readily be evaluated if the function $|E(\mathbf{u})|^2$ or $|P(\mathbf{x}')|^2$ is specified. However, we do not perform a further calculation, and only make the general statement that the speckle size is related to the angle that the area of illumination or of the lens pupil subtends at a point in the observation plane.

2.4 Temporal Properties of Speckle Intensity Variations

In the previous section, only the spatial properties of a static speckle pattern are considered. If the diffuse object changes with time, the speckle pattern also evolves. In this section, the analysis is extended to include the temporal properties of such a dynamic speckle pattern. Attention is restricted to Gaussian speckles which satisfy the relation (2.19). Moreover, an ergodic process (for example, see [62]) is assumed for the speckle fields. Thorough reviews on the study of dynamic speckles appear in references [3, 4].

2.4.1 Space-Time Cross-Correlation Function of Speckle Intensity

For Gaussian speckles, the temporal behavior of dynamic speckles is fully described by the cross-correlation function of speckle fields measured at different positions and times,

$$\Gamma_A(\mathbf{x}_1, \mathbf{x}_2; t_1, t_2) = \langle A^*(\mathbf{x}_1, t_1) A(\mathbf{x}_2, t_2) \rangle. \quad (2.29)$$

Consider a rigid body object that has a rough plane surface translating in its own plane at a constant velocity $\mathbf{v}$. In this case, the phase modulation function of the object is given by $O(\mathbf{u} - \mathbf{v}t)$, and its correlation function of Eq. (2.21) is rewritten as

$$\langle O^*(\mathbf{u} - \mathbf{v}t_1) O(\mathbf{u}' - \mathbf{v}t_2) \rangle = \Delta S \delta(\mathbf{u}' - \mathbf{u} - \mathbf{v}\tau), \quad (2.30)$$

where $\tau = t_2 - t_1$. By substituting Eq. (2.30) into Eq. (2.20), Eq. (2.29) takes a form of

$$\Gamma_A(\mathbf{x}_1, \mathbf{x}_2; \tau) = \Delta S \int_{-\infty}^{\infty} E^*(\mathbf{u}) E(\mathbf{u} + \mathbf{v}\tau) T^*(\mathbf{u}, \mathbf{x}_1) T(\mathbf{u} + \mathbf{v}\tau, \mathbf{x}_2) d\mathbf{u}. \quad (2.31)$$

In order to evaluate Eq. (2.31), we assume that a coherent Gaussian beam with the wavelength λ illuminates the object. The incident field $E(\mathbf{u})$ is then given by

$$E(\mathbf{u}) = \frac{w_0}{w} \exp \left[i \left(\frac{2\pi z}{\lambda} + \frac{\pi |\mathbf{u}|^2}{\lambda R_0} \right) \right] \exp \left(-\frac{|\mathbf{u}|^2}{w^2} \right), \quad (2.32)$$

where w_0 is the radius of the beam waist, w and R_0 are, respectively, the radius of the beam spot and the wavefront-curvature radius in the object plane, and z is the

distance from the position of the beam waist to the object plane. By combining Eqs. (2.19), (2.23), (2.31), and (2.32), we obtain the space-time correlation function of the speckle intensity for the free-space geometry,

$$\Gamma_I(\mathbf{X}, \tau) = \langle I \rangle^2 \left\{ 1 + \exp \left[-\frac{(\tau - \tau_d)^2}{\tau_c^2} \right] \exp \left(\frac{\tau_d^2}{\tau_c^2} - \frac{|\mathbf{X}|^2}{\zeta^2} \right) \right\}, \quad (2.33)$$

where the correlation time τ_c , the time lapse τ_d , and the average radius ζ of speckles are given by

$$\frac{1}{\tau_c} = \left(\frac{\sigma^2}{\zeta^2} + \frac{1}{w^2} \right)^{1/2} |\mathbf{v}|, \quad (2.34)$$

$$\tau_d = \frac{\tau_c^2}{\zeta^2} \sigma \mathbf{v} \cdot \mathbf{X} \quad (2.35)$$

and

$$\zeta = \frac{\lambda R}{\pi w} \quad (2.36)$$

with

$$\sigma = 1 + \frac{R}{R_0}. \quad (2.37)$$

When $\mathbf{X} = 0$ ($\mathbf{x}_1 = \mathbf{x}_2$), Eq. (2.33) reduces to

$$\Gamma_I(0, \tau) = \langle I \rangle^2 \left[1 + \exp \left(-\frac{\tau^2}{\tau_c^2} \right) \right]. \quad (2.38)$$

Hence the correlation time τ_c represents the maximum time interval in which the intensities measured at the same point but different time instances can correlate with each other. In the chapters to follow, the most important physical quantity to be studied is the reciprocal $1/\tau_c$ of the correlation time, which measures the fluctuating rate of the speckle intensity. Another time scale, τ_d , corresponds to the time delay required for a speckle to translate from the position $\mathbf{x}_1$ to the position closest to the point $\mathbf{x}_2 = \mathbf{x}_1 + \mathbf{X}$.

For imaging geometry, we employ a Gaussian soft aperture with radius a as a pupil function $P(\mathbf{x}')$ of the lens:

$$P(\mathbf{x}') = \exp \left(-\frac{|\mathbf{x}'|^2}{a^2} \right). \quad (2.39)$$

Substituting Eq. (2.39) into Eq. (2.26), we have the impulse response function of the imaging system with the Gaussian aperture,

$$T(\mathbf{u}, \mathbf{x}) = \frac{-\pi a^2}{\lambda^2 d_o d_i} \exp \left[\frac{i\pi}{\lambda} \left(\frac{|\mathbf{u}|^2}{d_o} + \frac{|\mathbf{x}|^2}{d_i} \right) \right] \exp(i \tan^{-1} \varepsilon) \\ \times \exp \left(-i\varepsilon \left| \frac{\mathbf{u}}{\rho_o} + \frac{\mathbf{x}}{\rho_i} \right|^2 \right) \exp \left(- \left| \frac{\mathbf{u}}{\rho_o} + \frac{\mathbf{x}}{\rho_i} \right|^2 \right), \quad (2.40)$$

where ρ_o and ρ_i represent the widths of the point spread functions in the object and the observation planes, respectively, which are given by

$$\rho_o = \left(\frac{\lambda d_o}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (2.41)$$

and

$$\rho_i = \left(\frac{\lambda d_i}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (2.42)$$

with ε being a defocusing parameter,

$$\varepsilon = \frac{\pi a^2}{\lambda} \left(\frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{f} \right). \quad (2.43)$$

Substitution of Eq. (2.31) together with Eqs. (2.32) and (2.40) into Eq. (2.19) results in the space-time correlation function of the speckle intensity for the imaging geometry,

$$\Gamma_I(\mathbf{X}, \tau) = \langle I \rangle^2 \left\{ 1 + \exp \left[-\frac{(\tau - \tau_d)^2}{\tau_c^2} \right] \exp \left(\frac{\tau_d^2}{\tau_c^2} - \frac{|\mathbf{X}|^2}{\zeta^2} \right) \right\}, \quad (2.44)$$

where

$$\frac{1}{\tau_c} = \left(\frac{B^2}{\zeta^2} + \frac{\mu^2}{a^2} \right)^{1/2} |\mathbf{v}|, \quad (2.45)$$

$$\tau_d = \frac{\tau_c^2}{\zeta^2} B \mathbf{v} \cdot \mathbf{X} \quad (2.46)$$

and

$$\zeta = \frac{\lambda d_i}{\pi a} \quad (2.47)$$

with

$$\mu = 1 + \frac{d_o}{R_0} \quad (2.48)$$

and

$$B = \mu \left(\frac{d_i}{f} - 1 \right) - (\mu - 1) \frac{d_i}{d_o}. \quad (2.49)$$

In deriving Eqs. (2.44)–(2.49), it has been assumed that the dimensions of the illuminated area on the surface are much larger than the resolution limit of the optical system. The correlation function for the case of the plane wave illumination is readily obtained by making $R_0 \rightarrow \infty$.

2.4.2 Characteristics of Speckle Motion

First, let us consider the intensity fluctuations measured at a single point. With reference to Eqs. (2.34) and (2.45), the fluctuating rate of the speckle intensity is proportional to the moving velocity of the object in both free space and imaging geometries. Thus, the velocity of a rough object can be measured from the intensity fluctuations of scattered light that is detected at a single point, providing that the values of parameters which characterize the optical system are known. However, as will be seen in Chapters 4–8, this is not always the case that the speckle pattern arises from two or more diffuse objects.

Next, the properties of speckle motion is examined in terms of the change in its spatial structure. The peak value of the cross-correlation functions of Eqs. (2.33) and (2.44) is obtained at $\tau = \tau_d$, and it decreases with an increase of the distance $|\mathbf{X}|$. Now the correlation distance X_c can reasonably be defined by the distance at which the correlation peak drops to e^{-1} times the maximum value, i.e.,

$$\exp \left(\frac{\tau_d^2}{\tau_c^2} - \frac{|\mathbf{X}|^2}{\zeta^2} \right) = e^{-1}. \quad (2.50)$$

The correlation distance takes the maximum value $X_{c,max}$ when $\mathbf{X}$ is parallel to $\mathbf{v}$. Under this condition, evaluation of Eq. (2.50) yields

$$X_{c,max} = (\zeta^2 + \sigma^2 w^2)^{1/2} \quad (2.51)$$

for the free space geometry, and

$$X_{c,max} = \left[\zeta^2 + \left(\frac{B}{\mu} \right)^2 a^2 \right]^{1/2} \quad (2.52)$$

for the imaging geometry.

Dynamic speckle patterns exhibit two different temporal changes, which are characterized by the ratio of the maximum correlation distance $X_{c,max}$ to the average speckle width ζ . When $X_{c,max}/\zeta \gg 1$, the speckle pattern moves as a whole without any other significant change in the pattern. This motion is called a *translation* of speckles, and typically occurs in the diffraction field under divergent beam illumination or in the image plane of an object. On the contrary, when $X_{c,max}/\zeta \approx 1$, the individual speckles change their shapes independently of each other, without any displacement of their positions. This motion is called *boiling* of speckles, and typically occurs in the far-field of an object or the focal plane of an imaging system under plane-wave illumination. Most dynamic speckle patterns contain both translating and boiling elements.

Finally, the moving velocity V of the speckle pattern is found, by evaluating $V = X/\tau_d$ under the condition of translating speckles ($X_{c,max}/\zeta \gg 1$), to be

$$V = \sigma v \quad (2.53)$$

for the free space geometry, and

$$V = B v \quad (2.54)$$

for the imaging geometry. Although the velocity of the pattern depends on the illumination and observation conditions, it is proportional to the velocity of the object. However, it should be noted that this quantity provides no physical meaning in the case of boiling speckles, in contrast with the reciprocal of the correlation time that has a proportional relationship with the object velocity in any types of speckle motion.

2.4.3 Power Spectral Distribution of Speckle Intensity Fluctuations

When we deal with time-varying signals, it is often convenient to analyze them in the time frequency domain. For the speckles that have been considered, the power

spectrum of the temporal intensity fluctuations is given by a Fourier transform of its autocorrelation function [59],

$$S(f) = \int_{-\infty}^{\infty} \Gamma_I(0, \tau) \exp(-i2\pi f\tau) d\tau. \quad (2.55)$$

This relationship is known as the *Wiener-Khinchin theorem*. Note that the spectrum in this case does not refer to the optical frequency of light waves, but it represents the frequency distribution of time-varying intensity fluctuations. Substitution of Eq. (2.38) into Eq. (2.55) yields the power spectrum of the intensity of speckles produced by a moving diffuse object:

$$S(f) = \langle I \rangle^2 [\delta(f) + \sqrt{\pi} \tau_c \exp(-\pi^2 \tau_c^2 f^2)], \quad (2.56)$$

where τ_c is given by Eq. (2.34) for the free space geometry and Eq. (2.45) for the imaging geometry. In the cases that Eq. (2.55) cannot be evaluated analytically, a numerical evaluation can be performed by means of the fast Fourier transform (FFT) algorithm.

In this chapter, the fundamental properties of a speckle pattern have been examined in terms of the statistics of its spatio-temporal intensity variations. The important results obtained for speckles from optically rough surfaces are summarized as follows:

- (i) A polarized speckle pattern obeys negative exponential statistics.
- (ii) The average size of speckles is determined by the angle that the area of illumination or of the lens pupil subtends at a point in the plane forming the speckle field.
- (iii) The fluctuating rate of the speckle intensity and the translating velocity of a speckle pattern are proportional to the moving velocity of the object.
- (iv) Translating or boiling motion of speckles is observed depending on the parameters of the optical system producing the dynamic speckles.

Chapter 3

Dynamic Speckles Formed under Speckle-Pattern Illumination from an Optical Fiber

3.1 Introduction

When moving scatterers pass across a coherent light beam, they produce a moving random interference pattern, or so-called dynamic speckle, in the surrounding field. In Chapter 2, it was shown that the moving velocity of the pattern depends on various parameters of the optical system, such as the spot size and the radius of curvature of the incident beam, and the location of the detector. The most important and useful fact in the phenomenon of dynamic speckles is that the velocity of the pattern is always linearly proportional to that of the scatterers. Therefore the behavior of the dynamic speckle field around the scatterers has been extensively investigated by many authors (see the review papers, [3, 4]) aiming at measuring the velocity of the scattering objects. Several workers have attempted to employ optical fibers in the experimental system because of the flexibility of the fibers in transmitting the light between the object and the measuring system [63, 64]. In most of these studies, an optical fiber is employed for the detection of scattered light, not for the illumination of the object. To the author's knowledge, a few studies have been conducted on the statistical properties of speckles produced under illumination from an optical fiber [25, 26].

In this chapter, the power spectral distribution (PSD) of speckle signals obtained using an optical-fiber probe [27, 65] is analyzed. This probe consists of two fibers, one for illuminating the object and the other for detecting the speckle signals. When a single-mode fiber is employed for an illuminating fiber, the light field incident on the object is approximated as a spherical wave, and the resultant speckle signals are the same as those obtained with the direct illumination of a diverging laser

beam. However, the light emanating from a multimode fiber produces a speckle pattern as a result of the interference among a number of different modes of the light propagating within the core. The speckle pattern formed by illuminating the scatterers with such a speckle pattern is referred to as speckled speckle [11], and its temporal properties are expected to be different from those of normal speckles. First, a general formula describing the PSD of the speckle signal is given, and then it is evaluated for various optical parameters of the fibers. Several limitations of the present optical-fiber probe for velocity measurements are pointed out.

3.2 Theoretical Analysis

Figure 3.1 illustrates the optical system for the speckle field detection treated in the present theoretical analysis. The scatter plate moving with velocity $\mathbf{v}$ in the plane $\mathbf{p}$ is illuminated by the light emerging from the exit end of fiber F_1 located in the plane $\mathbf{u}$. According to the transverse motion of the plate, the dynamic speckle field is produced in the diffraction field and is detected by another fiber F_2 placed in the plane $\mathbf{r}$. In Fig. 3.1, the illuminating and detecting fibers are shown to be located on opposite sides of the plate. It makes no difference if they are placed on the same side.

By neglecting the absorption and the scattering losses of the light in its transmission through the fiber, the light intensity variation $I_S(t)$ obtained from the exit end of the detecting fiber is given by

$$I_S(t) = \int_{-\infty}^{\infty} I(\mathbf{r}, t) A(\mathbf{r}) d\mathbf{r}, \quad (3.1)$$

where $I(\mathbf{r}, t)$ is the speckle intensity distribution in the plane of detection and $A(\mathbf{r})$ is the aperture function of the input end of the detecting fiber. The PSD of the time-varying signal $I_S(t)$ is expressed as a Fourier transform of the correlation function of $I_S(t)$:

$$S(f) = \int_{-\infty}^{\infty} \langle I_S(t) I_S(t + \tau) \rangle \exp(-i2\pi f\tau) d\tau. \quad (3.2)$$

From Eq. (3.1), the correlation function in Eq. (3.2) is denoted by

$$\Gamma_{I_S}(\tau) = \langle I_S(t) I_S(t + \tau) \rangle$$

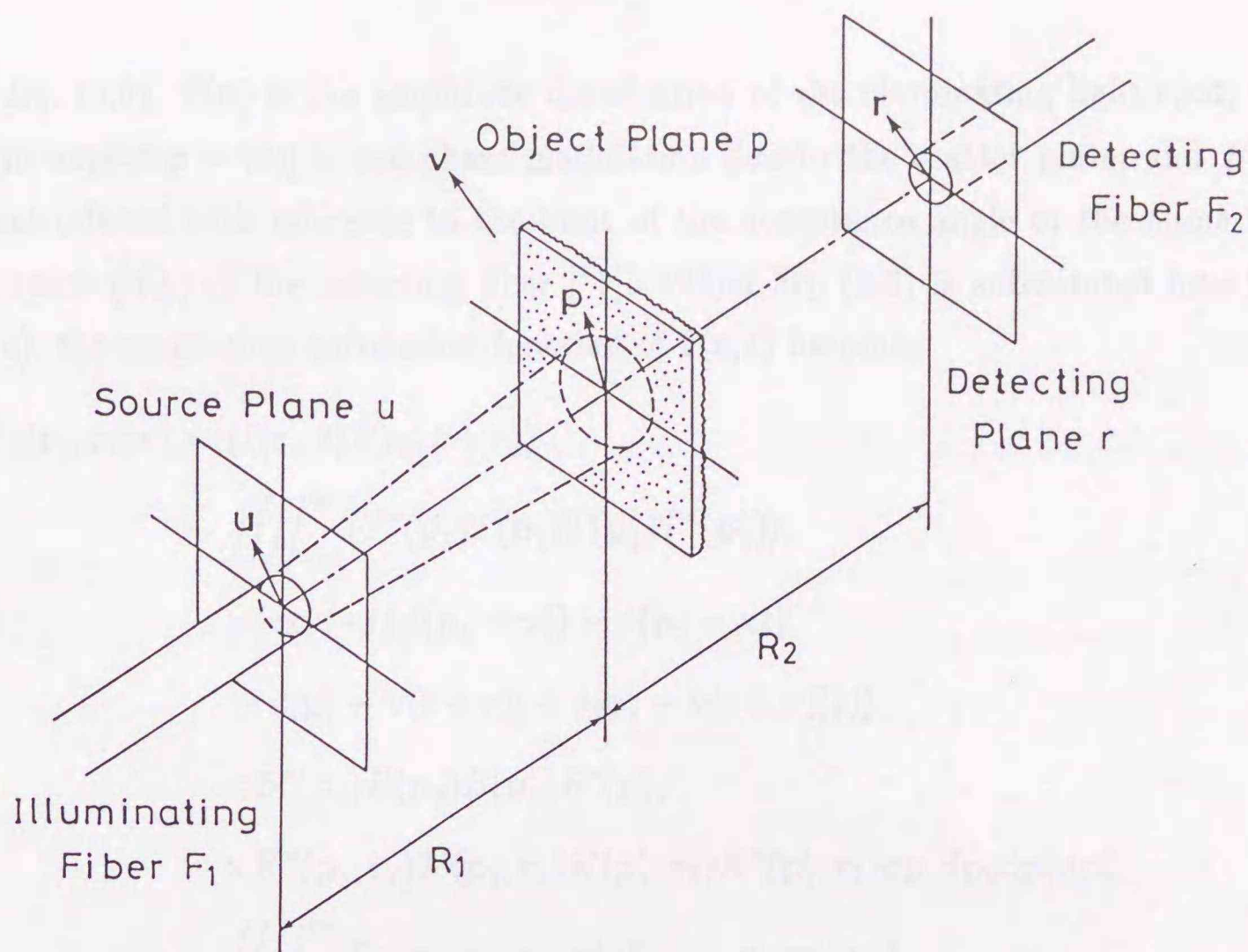


Figure 3.1. Schematic diagram of the optical system for the dynamic speckle field detection with an optical-fiber probe.

$$= \iint_{-\infty}^{\infty} \langle I(\mathbf{r}_1, t) I(\mathbf{r}_2, t + \tau) \rangle A(\mathbf{r}_1) A(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2. \quad (3.3)$$

In Eq. (3.3) the intensity distribution $I(\mathbf{r}, t)$ in the plane of detection is expressed as

$$I(\mathbf{r}, t) = \left| \int_{-\infty}^{\infty} U(\mathbf{p}, t) K(\mathbf{p}, \mathbf{r}) d\mathbf{p} \right|^2, \quad (3.4)$$

where $K(\mathbf{p}, \mathbf{r})$ is the propagation function of the light from the scatter plate to the plane of detection. In the present analysis, the fiber is placed in the Fresnel diffraction field. Then the function $K(\mathbf{p}, \mathbf{r})$ becomes

$$K(\mathbf{p}, \mathbf{r}) = (1/i\lambda R_2) \exp(i\pi |\mathbf{p} - \mathbf{r}|^2 / \lambda R_2). \quad (3.5)$$

The function $U(\mathbf{p}, t)$ in Eq. (3.4) is the complex amplitude distribution of the light after passing the scatter plate and is expressed as

$$U(\mathbf{p}, t) = T(\mathbf{p}) \exp[i\phi(\mathbf{p} - \mathbf{v}t)] B(\mathbf{p}). \quad (3.6)$$

In Eq. (3.6), $T(\mathbf{p})$ is the amplitude distribution of the illuminating light spot, the term $\exp[i\phi(\mathbf{p} - \mathbf{v}t)]$ is the phase modulation due to the scatter plate, and $B(\mathbf{p})$ is introduced with reference to the limit of the acceptance angle or the numerical aperture (NA) of the detecting fiber [65]. When Eq. (3.6) is substituted into Eq. (3.4), the space-time correlation function of $I(\mathbf{r}, t)$ becomes

$$\begin{aligned}
\Gamma_I(\mathbf{r}_1, \mathbf{r}_2; \tau) &= \langle I(\mathbf{r}_1, t) I(\mathbf{r}_2, t + \tau) \rangle \\
&= \iiint \iiint_{-\infty}^{\infty} \langle T^*(\mathbf{p}_1) T(\mathbf{p}_2) T(\mathbf{p}'_1) T^*(\mathbf{p}'_2) \rangle \\
&\quad \times \langle \exp(-i\{\phi(\mathbf{p}_1 - \mathbf{v}t) - \phi(\mathbf{p}_2 - \mathbf{v}t) \\
&\quad - \phi[\mathbf{p}'_1 - \mathbf{v}(t + \tau)] + \phi[\mathbf{p}'_2 - \mathbf{v}(t + \tau)]\}) \rangle \\
&\quad \times B^*(\mathbf{p}_1) B(\mathbf{p}_2) B(\mathbf{p}'_1) B^*(\mathbf{p}'_2) \\
&\quad \times K^*(\mathbf{p}_1, \mathbf{r}_1) K(\mathbf{p}_2, \mathbf{r}_1) K(\mathbf{p}'_1, \mathbf{r}_2) K^*(\mathbf{p}'_2, \mathbf{r}_2) d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\mathbf{p}'_2 \\
&= \iiint \iiint_{-\infty}^{\infty} \Gamma_T(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) \Gamma_\phi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) \\
&\quad \times \Gamma_B(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) \Gamma_K(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2; \mathbf{r}_1, \mathbf{r}_2) d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\mathbf{p}'_2. \quad (3.7)
\end{aligned}$$

By assuming that the amplitude distribution $T(\mathbf{p})$ of the illuminating spot is a complex Gaussian random variable [25], the correlation function $\Gamma_T(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2)$ can be divided into two terms, i.e.,

$$\begin{aligned}
\Gamma_T(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) &= \langle T^*(\mathbf{p}_1) T(\mathbf{p}_2) \rangle \langle T(\mathbf{p}'_1) T^*(\mathbf{p}'_2) \rangle \\
&\quad + \langle T^*(\mathbf{p}_1) T(\mathbf{p}'_1) \rangle \langle T(\mathbf{p}_2) T^*(\mathbf{p}'_2) \rangle. \quad (3.8)
\end{aligned}$$

The correlation function $\Gamma_T(\mathbf{p}_1, \mathbf{p}_2)$ of the complex amplitude distribution $T(\mathbf{p})$ in the illuminating spot emerging from an optical fiber was studied by Takai and Asakura [25], using a quasi-homogeneous source model, and is given by

$$\begin{aligned}
\Gamma_T(\mathbf{p}_1, \mathbf{p}_2) &= \frac{(\pi a_0 \zeta_2)^2}{(\lambda R_1)^2} I_0 \exp[-i\pi(|\mathbf{p}_1|^2 - |\mathbf{p}_2|^2)/\lambda R_1] \\
&\quad \times \exp(-|\mathbf{p}_1 + \mathbf{p}_2|^2/2q^2) \exp(-|\mathbf{p}_2 - \mathbf{p}_1|^2/2\zeta_1^2), \quad (3.9)
\end{aligned}$$

where a_0 is the core radius of the illuminating fiber and determines the average speckle size ζ_1 in the plane of the scatter plate as

$$\zeta_1 = \lambda R_1 / \pi a_0. \quad (3.10)$$

The spot radius q in the plane of the scatter plate is determined by the NA and the core diameter of the illuminating fiber and is expressed as

$$\begin{aligned} q &= \lambda R_1 / \pi \zeta_2, \\ 1/\zeta_2^2 &= 1/a_0^2 + 1/\zeta_0^2, \end{aligned} \quad (3.11)$$

where ζ_0 is the average speckle size in the exit end of the illuminating fiber and is given by the NA of the illuminating fiber [25].

When the scatter plate is optically rough, the following approximation may be used for the correlation function $\Gamma_\phi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2)$ of its phase distribution [65]:

$$\begin{aligned} \Gamma_\phi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) &= \delta(\mathbf{p}_2 - \mathbf{p}_1) \delta(\mathbf{p}'_2 - \mathbf{p}'_1) \\ &\quad + \delta(\mathbf{p}_1 - \mathbf{p}'_1 + \mathbf{v}\tau) \delta(\mathbf{p}_2 - \mathbf{p}'_2 + \mathbf{v}\tau) \\ &\quad (\mathbf{p}_1 \neq \mathbf{p}_2). \end{aligned} \quad (3.12)$$

Substituting Eqs. (3.5), (3.8), (3.9), and (3.12) into Eq. (3.7), we have the space-time correlation function $\Gamma_I(\mathbf{r}_1, \mathbf{r}_2; \tau)$ of the speckle intensity $I(\mathbf{r}, t)$, given by

$$\Gamma_I(\mathbf{r}_1, \mathbf{r}_2; \tau) = c_1 + c_2 + F_1(\mathbf{r}_1, \mathbf{r}_2; \tau) + F_2(\mathbf{r}_1, \mathbf{r}_2; \tau), \quad (3.13)$$

where

$$c_1 = c_0 \left| \int_{-\infty}^{\infty} |B(\mathbf{p})|^2 \exp(-2|\mathbf{p}|^2/q^2) d\mathbf{p} \right|^2, \quad (3.14)$$

$$\begin{aligned} c_2 &= c_0 \iint_{-\infty}^{\infty} |B(\mathbf{p}_1)|^2 |B(\mathbf{p}_2)|^2 \exp(-|\mathbf{p}_1 + \mathbf{p}_2|^2/q^2) \\ &\quad \times \exp(-|\mathbf{p}_2 - \mathbf{p}_1|^2/\zeta_1^2) d\mathbf{p}_1 d\mathbf{p}_2, \end{aligned} \quad (3.15)$$

$$F_1(\mathbf{r}_1, \mathbf{r}_2; \tau) = c_0 h(\tau) \iint_{-\infty}^{\infty} f_1(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) d\mathbf{p}_1 d\mathbf{p}_2, \quad (3.16)$$

$$h(\tau) = \exp(-|\mathbf{v}\tau|^2/\zeta_1^2),$$

$$\begin{aligned} f_1(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) &= B^*(\mathbf{p}_1) B(\mathbf{p}_1 + \mathbf{v}\tau) B^*(\mathbf{p}_2 + \mathbf{v}\tau) B(\mathbf{p}_2) \\ &\quad \times \exp[-(|2\mathbf{p}_1 + \mathbf{v}\tau|^2 + |2\mathbf{p}_2 + \mathbf{v}\tau|^2)/2q^2] \\ &\quad \times \exp[-i2\pi(1/R_1 + 1/R_2)(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}\tau/\lambda] \\ &\quad \times \exp[i2\pi(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{p}_2 - \mathbf{p}_1)/\lambda R_2], \end{aligned}$$

$$F_2(\mathbf{r}_1, \mathbf{r}_2; \tau) = c_0 \iint_{-\infty}^{\infty} f_2(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) d\mathbf{p}_1 d\mathbf{p}_2, \quad (3.17)$$

$$\begin{aligned} f_2(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) = & B^*(\mathbf{p}_1)B(\mathbf{p}_1 + \mathbf{v}\tau)B^*(\mathbf{p}_2 + \mathbf{v}\tau)B(\mathbf{p}_2) \\ & \times \exp(-|\mathbf{p}_2 - \mathbf{p}_1|^2/\zeta_1^2) \\ & \times \exp[-(|\mathbf{p}_1 + \mathbf{p}_2|^2 + |\mathbf{p}_1 + \mathbf{p}_2 + 2\mathbf{v}\tau|^2)/2q^2] \\ & \times \exp[-i2\pi(1/R_1 + 1/R_2)(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}\tau/\lambda] \\ & \times \exp[i2\pi(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{p}_2 - \mathbf{p}_1)/\lambda R_2], \\ c_0 = & [(\pi a_0 \zeta_0)/(\lambda^2 R_1 R_2)]^4 I_0^2. \end{aligned} \quad (3.18)$$

By substituting Eqs. (3.13)–(3.18) into Eq. (3.3) and changing the order of integrations, the correlation function $\Gamma_S(\tau)$ of the detected signal $I_S(t)$ can be written in the form

$$\begin{aligned} \Gamma_S(\tau) = & \iint_{-\infty}^{\infty} [c_1 + c_2 + F_1(\mathbf{r}_1, \mathbf{r}_2; \tau) + F_2(\mathbf{r}_1, \mathbf{r}_2; \tau)] A(\mathbf{r}_1) A(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \\ = & c_1 + c_2 \\ & + c_0 h(\tau) \iint_{-\infty}^{\infty} d\mathbf{p}_1 d\mathbf{p}_2 \iint_{-\infty}^{\infty} f_1(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) A(\mathbf{r}_1) A(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \\ & + c_0 \iint_{-\infty}^{\infty} d\mathbf{p}_1 d\mathbf{p}_2 \iint_{-\infty}^{\infty} f_2(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2; \tau) A(\mathbf{r}_1) A(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \end{aligned} \quad (3.19a)$$

$$= c_1 + c_2 + g_1(\tau) + g_2(\tau), \quad (3.19b)$$

where $g_1(\tau)$ and $g_2(\tau)$ correspond to the third and fourth terms, respectively, of Eq. (3.19a). The amplitude transmittance of the light is again assumed to be uniform in the core region of the detecting fiber [65], i.e.,

$$A(\mathbf{r}) = (1/\pi a^2) \text{circ}(|\mathbf{r}|/a), \quad (3.20)$$

where $2a$ is the core diameter of the detecting fiber. By using Eq. (3.20), the integration for $\mathbf{r}_1$ and $\mathbf{r}_2$ in Eq. (3.19a) may be written as

$$\begin{aligned} & \iint_{-\infty}^{\infty} \exp[i2\pi(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{p}_2 - \mathbf{p}_1)/\lambda R_2] A(\mathbf{r}_1) A(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \\ & = \left| \int_{-\infty}^{\infty} A(\mathbf{r}) \exp[-i2\pi(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{r}/\lambda R_2] d\mathbf{r} \right|^2 \\ & = [2J_1(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)/(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)]^2, \end{aligned} \quad (3.21)$$

where $J_1(x)$ is a Bessel function of the first order. When Eq. (3.21) is substituted into Eq. (3.19a), $g_1(\tau)$ and $g_2(\tau)$ become

$$\begin{aligned}
g_1(\tau) = & c_0 h(\tau) \iint_{-\infty}^{\infty} B^*(\mathbf{p}_1) B(\mathbf{p}_1 + \mathbf{v}\tau) B^*(\mathbf{p}_2 + \mathbf{v}\tau) B(\mathbf{p}_2) \\
& \times \exp[-(|2\mathbf{p}_1 + \mathbf{v}\tau|^2 + |2\mathbf{p}_2 + \mathbf{v}\tau|^2)/2q^2] \\
& \times \exp[-i2\pi(1/R_1 + 1/R_2)(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}\tau/\lambda] \\
& \times [2J_1(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)/(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)]^2 d\mathbf{p}_1 d\mathbf{p}_2 \quad (3.22)
\end{aligned}$$

and

$$\begin{aligned}
g_2(\tau) = & c_0 \iint_{-\infty}^{\infty} B^*(\mathbf{p}_1) B(\mathbf{p}_1 + \mathbf{v}\tau) B^*(\mathbf{p}_2 + \mathbf{v}\tau) B(\mathbf{p}_2) \\
& \times \exp(-|\mathbf{p}_2 - \mathbf{p}_1|^2/\zeta_1^2) \\
& \times \exp[-(|\mathbf{p}_1 + \mathbf{p}_2|^2 + |\mathbf{p}_1 + \mathbf{p}_2 + 2\mathbf{v}\tau|^2)/2q^2] \\
& \times \exp[-i2\pi(1/R_1 + 1/R_2)(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{v}\tau/\lambda] \\
& \times [2J_1(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)/(2\pi a|\mathbf{p}_2 - \mathbf{p}_1|/\lambda R_2)]^2 d\mathbf{p}_1 d\mathbf{p}_2. \quad (3.23)
\end{aligned}$$

To evaluate Eqs. (3.14), (3.15), (3.22) and (3.23) analytically, we approximate the virtual spot distribution $B(\mathbf{p})$ to be Gaussian, i.e.,

$$B(\mathbf{p}) = \exp(-|\mathbf{p}|^2/s^2), \quad (3.24)$$

where s is the radius of the virtual spot [65]. The autocorrelation function $\Gamma_S(\tau)$ of the speckle signal is then calculated from Eq. (3.19b), the terms of which are given by (see the Appendix of Section 3.6)

$$c_1 = (c_0 \pi^2/4)(1/q^2 + 1/s^2)^{-2}, \quad (3.25)$$

$$c_2 = (c_0 \pi^2/4)(1/q^2 + 1/s^2)^{-1}(1/\zeta_1^2 + 1/s^2)^{-1}, \quad (3.26)$$

$$\begin{aligned}
g_1(\tau) = & c \exp[-(1/\zeta_1^2 + 1/s^2)v^2\tau^2] \\
& \times \int_0^\infty x^{-1} \exp[-(1/q^2 + 1/s^2)x^2] J_1^2(2\pi ax/\lambda R_2) \\
& \times J_0(2\pi(1 + R_2/R_1)v\tau x/\lambda R_2) dx \quad (3.27)
\end{aligned}$$

and

$$\begin{aligned}
 g_2(\tau) = & c \exp[-(1/q^2 + 1/s^2)v^2\tau^2] \\
 & \times \int_0^\infty x^{-1} \exp[-(1/\zeta_1^2 + 1/s^2)x^2] J_1^2(2\pi ax/\lambda R_2) \\
 & \times J_0(2\pi(1 + R_2/R_1)v\tau x/\lambda R_2) dx
 \end{aligned} \tag{3.28}$$

where

$$c = (\pi^2 a_0^2 \zeta_0^2 I_0 q s)^2 / [2a^2 \lambda^6 R_1^4 R_2^2 (q^2 + s^2)], \tag{3.29}$$

$$v = |\mathbf{v}|, \tag{3.30}$$

and $J_0(x)$ is a Bessel function of zero order. The PSD of $I_S(t)$ is finally obtained by using Eq. (3.2) as the Fourier transform of the above equations:

$$\begin{aligned}
 S(f) = & c \int_{-\infty}^\infty [g_1(\tau) + g_2(\tau)] \exp(-i2\pi f\tau) d\tau + (c_1 + c_2)\delta(f) \\
 = & S_a(f) + S_b(f) + (c_1 + c_2)\delta(f),
 \end{aligned} \tag{3.31}$$

where

$$S_a(f) = c \int_{-\infty}^\infty g_1(\tau) \exp(-i2\pi f\tau) d\tau, \tag{3.32}$$

$$S_b(f) = c \int_{-\infty}^\infty g_2(\tau) \exp(-i2\pi f\tau) d\tau. \tag{3.33}$$

Before showing the theoretical curves of the PSD, the properties of the above equations are first examined. Note that, in Eqs. (3.27) and (3.28), the variable τ is always multiplied by v , i.e.,

$$g_{1,2}(\tau) = g_{1,2}(v\tau).$$

The PSD given by the sum of the Fourier transforms of these functions has the variable f/v [66], and, therefore, $1/v$ becomes the scaling factor. Hence the cutoff frequency defined by the half-width of the PSD of the speckle signal is seen to be linearly proportional to the velocity of the scatter plate.

In the limit of $\zeta_0 \rightarrow \infty$, Eq. (3.31) gives the PSD function for the case of the scatter plate illuminated with a single-mode fiber. In this case Eq. (3.11) becomes

$$\lim_{\zeta_0 \rightarrow \infty} 1/\zeta_2^2 = \lim_{\zeta_0 \rightarrow \infty} (1/a_0^2 + 1/\zeta_0^2) = 1/a_0^2,$$

$$q = \lambda R_1 / \pi a_0 = \zeta_1.$$

Then Eqs. (3.27) and (3.28) may be simplified as

$$\begin{aligned} g_1(\tau) &= g_2(\tau) \\ &= c \exp[-(1/q^2 + 1/s^2)v^2\tau^2] \\ &\quad \times \int_0^\infty x^{-1} \exp[-(1/q^2 + 1/s^2)x^2] J_1^2(2\pi ax/\lambda R_2) \\ &\quad \times J_0(2\pi(1 + R_2/R_1)v\tau x/\lambda R_2) dx. \end{aligned} \quad (3.34)$$

3.3 Theoretical Results and Discussion

To obtain the theoretical curves of the PSD of the speckle signals, the equations obtained in the previous section are now evaluated as a function of the following variables:

- (1) Moving velocity v of the object scatter plate.
- For the illuminating fiber,
- (2) Core diameter $2a_0$,
 - (3) Illuminating spot diameter $2q$,
 - (4) Distance R_1 from its exit end to the plate.

For the detecting fiber,

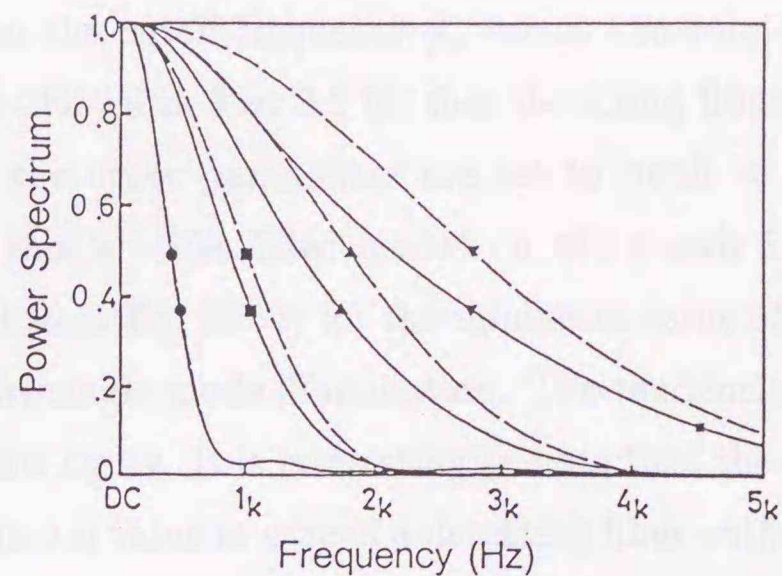
- (5) Core diameter $2a$,
- (6) Virtual spot size $2s$,
- (7) Distance R_2 from the plate to its input end.

As was explained in the previous section, the velocity v of the scatter plate has no effect on the profile of the PSD but is directly proportional to the spread of the PSD or its cutoff frequency f_c . For example, when the velocity of the plate becomes twice as fast as the original value, the cutoff frequency of the speckle signal detected by

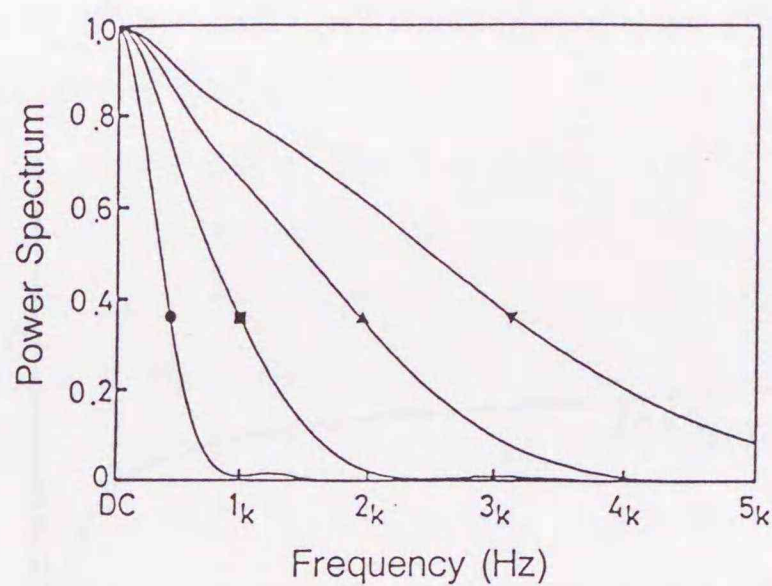
the probe is simply doubled. Therefore, we focus in this section on the effect of the other variables (2)–(7) on the profile of the PSD.

The theoretical curves of the PSD as a function of the core diameters, $2a_0$ and $2a$, of the illuminating and detecting fibers are evaluated by means of a numerical method including the fast Fourier transform (FFT). For simplicity, the distance R_1 in Fig. 3.1 is set equal to R_2 throughout the following evaluations. This condition of $R_1 = R_2 = R$ is reasonable for the simple fiber probe introduced in [65].

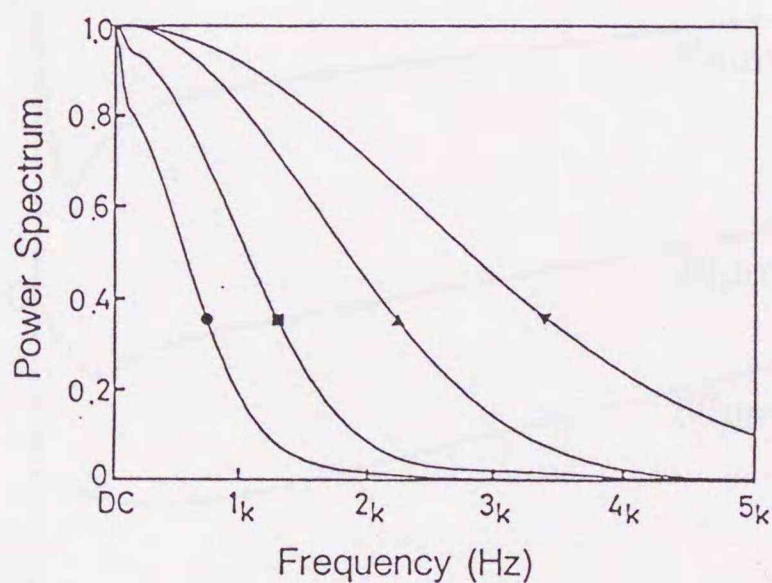
Figure 3.2 shows examples of the theoretical curves of the PSD for various values of the core diameters of the fiber pair. For simplicity, the size of the illumination spot $2q$ on the scatter plate is set equal to that of the virtual spot $2s$. The values of the parameters other than the core diameters are $R = 10$ mm, $2q = 2s = 2$ mm, and the velocity $v = 20$ mm/s of the scatter plate. The dashed lines in Fig. 3.2(a) show the theoretical results calculated from Eq. (3.34) for the limiting case of the illumination with a single-mode fiber. In this limit, the speckle pattern in the plane of detection is translated according to the motion of the scatter plate without changing its granular form (speckle translation or freezing [3, 67]). It is seen from Fig. 3.2 that the spread of the PSD or its cutoff frequency f_c is inversely proportional to the core diameter $2a$ of the detecting fiber. This is due to the spatial integration of the moving speckle field over the core region. With an increase in the core diameter of the illuminating fiber, the speckle size in the illuminating spot becomes smaller, and then the moving speckle field in the plane of detection begins to boil [3, 67] according to the motion of the scatter plate. This effect of speckle boiling produces other low-frequency signals, which are mixed with the original signal because of the translation of the speckle pattern. The solid lines in Figs. 3.2(a) and 3.2(b) show this process with the illuminating fibers of core diameters $2a_0 = 20$ μm [Fig. 3.2(a)] and $2a_0 = 50$ μm [Fig. 3.2(b)]. The PSD curves for this case are seen to be narrower than those for the single-mode fiber because of the relative increase of the low-frequency component. With a further increase in the core diameter of the illuminating fiber, the speckle size in the illuminating spot becomes very small. Then the speckle field in the plane of detection boils fast, even with only a small displacement of the scatter plate [8]. Figure 3.2(c) shows this effect with the illuminating fiber of core diameter $2a_0 = 400$ μm , where the PSD curves are seen to be broader than those in Figs. 3.2(a) and 3.2(b).



(a)



(b)



(c)

Figure 3.2. Theoretical curves of the PSD of the speckle signal as a function of the core diameters of the illuminating and detecting fibers. The dashed lines in (a) show PSD's with Gaussian-beam illumination; the solid lines show those with the illuminating fiber having a core diameter of $2a_0 = 20 \mu\text{m}$. Symbols show the different core diameters $2a$ of the detecting fibers: $50 \mu\text{m}$ (●), $20 \mu\text{m}$ (■), $10 \mu\text{m}$ (▲), $5 \mu\text{m}$ (▼). (b) and (c) show similar results for illuminating fibers with core diameters of $2a_0 = 50 \mu\text{m}$ and $2a_0 = 400 \mu\text{m}$, respectively. Other parameters are fixed: the illuminating spot size $2q = 2 \text{ mm}$, the virtual spot size $2s = 2 \text{ mm}$, and the velocity of the scatter plate $v = 20 \text{ mm/s}$.

The variation of the cutoff frequency f_c versus the core diameter $2a_0$ of the illuminating fiber is plotted in Fig. 3.3 for four detecting fibers having different core diameters. Again the other parameters are set to be $R = 10$ mm, $2q = 2s = 2$ mm, and $v = 20$ mm/s. The filled circles on the y axis indicate the cutoff frequencies calculated from Eq. (3.34) for the minimum value of the core diameter $2a_0$ corresponding to the single-mode illumination. The tendencies mentioned above are clearly seen from this figure. It is interesting to note that the cutoff frequency drops sharply to the minimum value in case of a detecting fiber with a small core diameter. The core diameter of the illuminating fiber giving the minimum cutoff frequency in-

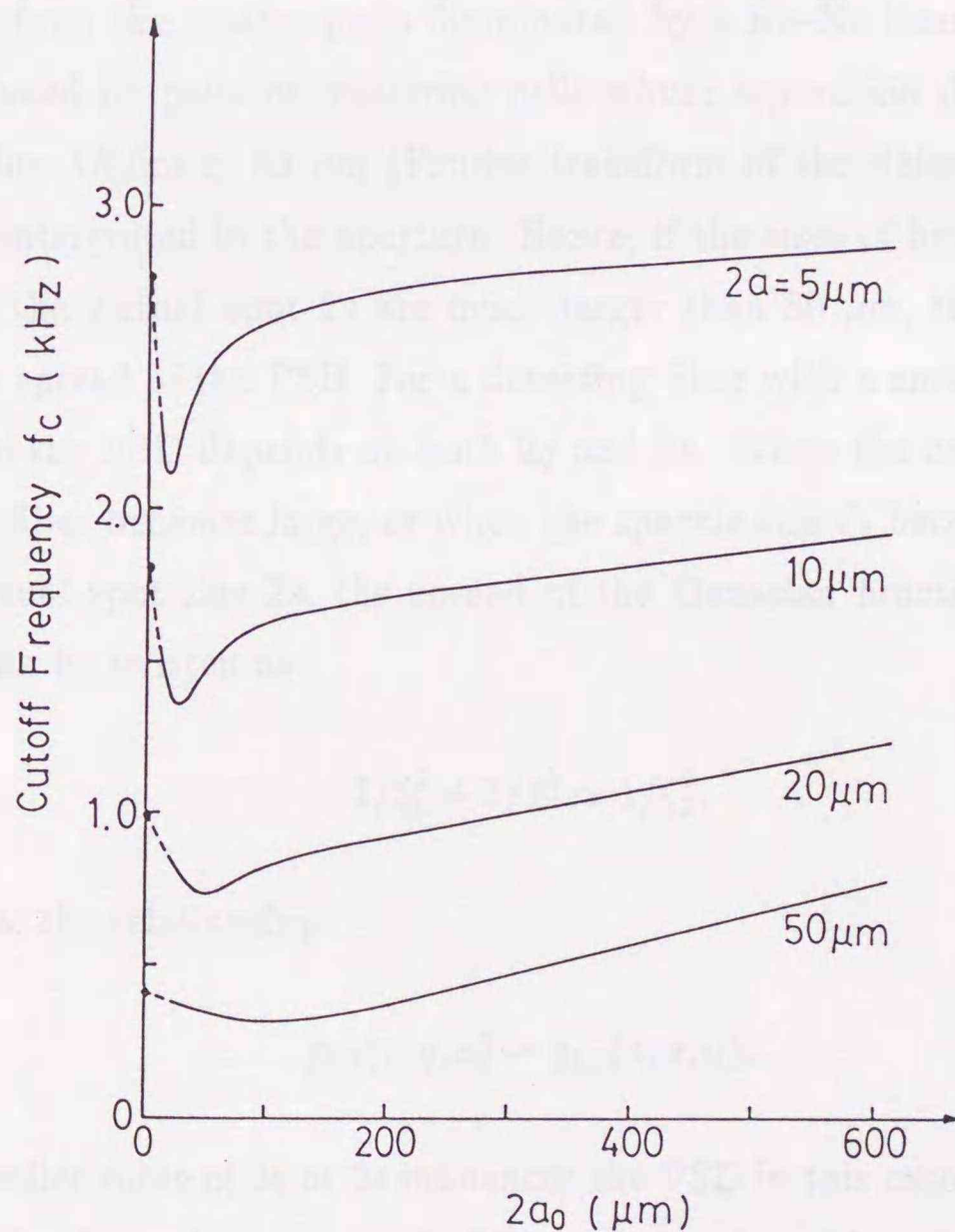


Figure 3.3. Theoretical values of the cutoff frequency f_c of the PSD as a function of the core diameter $2a_0$ of the illuminating fiber. The four curves correspond to the speckle signals detected through the four different fibers with core diameters of $2a = 5, 10, 20, 50$ μm . Other parameters are the same as those in Fig. 3.2.

creases with an increase in the core diameter $2a$ of the detecting fiber. Moreover, the relative depth at the minimum cutoff frequency is inversely proportional to the core diameter $2a$. Although this phenomenon may result from the spatial integration of speckles in the core region of the detecting fiber, a clear explanation for such complicated variations of the cutoff frequency has not yet been given.

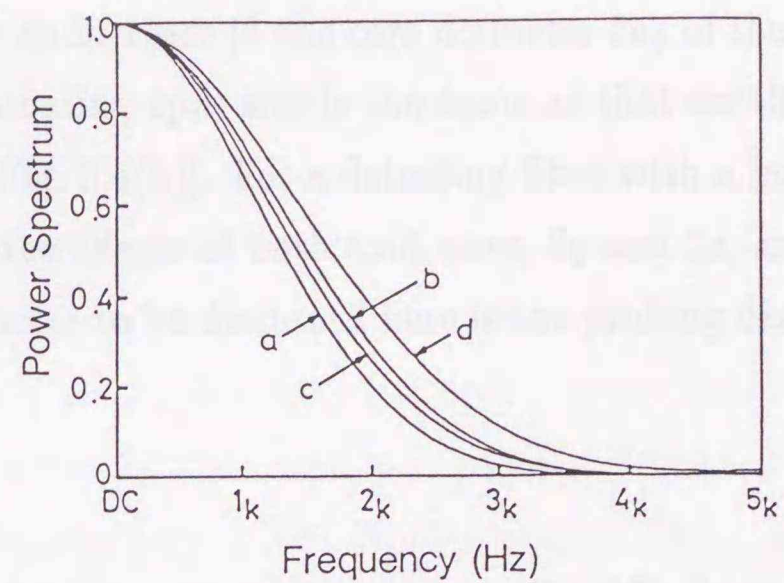
Next, we examine the variation of the PSD against the size of the illuminating spot and the virtual spot in the plane of the scatter plate. For a detecting fiber with a large core diameter, the effect of the virtual spot size is negligible [65], and the spread of the PSD is determined mainly by the Fourier transform of the amplitude transmittance at the input end of the detecting fiber. As an example, consider the case when a detecting fiber with a core diameter of $2a = 50 \mu\text{m}$ is placed a distance $R = 10 \text{ mm}$ from the scatter plate illuminated by a He-Ne laser. The interference fringes produced by pairs of scattering cells whose separation distances are larger than the value $\lambda R/\pi a \simeq 80 \mu\text{m}$ (Fourier transform of the detecting aperture) are all spatially integrated in the aperture. Hence, if the sizes of both the illumination spot $2q$ and the virtual spot $2s$ are much larger than $80 \mu\text{m}$, these spots have no effect on the spread of the PSD. For a detecting fiber with a smaller core diameter, the spread of the PSD depends on both $2q$ and $2s$. When the core diameter of the illuminating fiber becomes large, or when the speckle size ζ_1 becomes much smaller than the virtual spot size $2s$, the spread of the Gaussian functions in Eqs. (3.27) and (3.28) can be written as

$$1/\zeta_1^2 + 1/s^2 \simeq 1/\zeta_1^2.$$

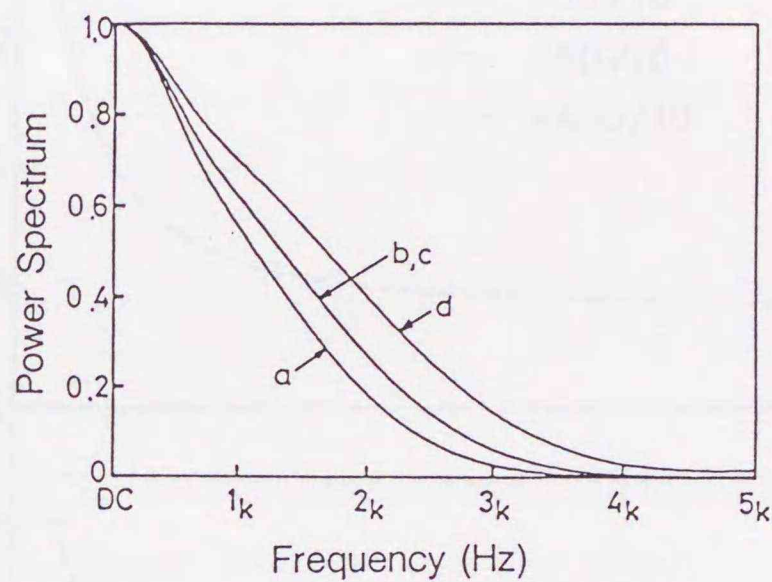
Then we have the relationship

$$g_{1,2}(\tau, q, s) = g_{1,2}(\tau, s, q).$$

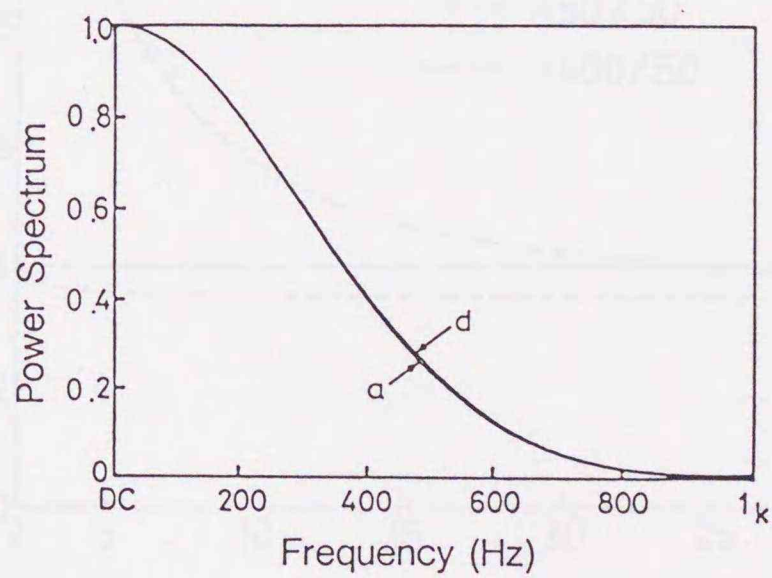
Thus the smaller value of $2q$ or $2s$ influences the PSD in this case. Figure 3.4 shows examples of the theoretical curves of PSD's calculated as a function of $2q$ and $2s$ for typical values of the core diameters of both illuminating and detecting fibers. The distance R and the velocity v are chosen to be 10 mm and 20 mm/s , respectively. As was mentioned above, it can be seen from Fig. 3.4(a), which shows the results for a pair of thinner fibers ($2a_0 = 10 \mu\text{m}$ and $2a = 10 \mu\text{m}$), that the spread of the PSD



(a)



(b)



(c)

Figure 3.4. Dependence of PSD's on the illuminating spot size $2q$ and the virtual spot size $2s$. The core diameters $2a_0/2a$ are (in micrometers) (a) 10/10, (b) 50/10, and (c) 50/50. Other parameters are the same as those in Fig. 3.2. The four curves correspond to different spot sizes: $2q/2s = 1/1$ (curve a), $1/4$ (curve b), $4/1$ (curve c), and $4/4$ (curve d).

decreases with a decrease in the virtual spot size $2s$ as well as in the illuminating spot size $2q$. With an increase in the core diameter $2a_0$ of the illuminating fiber, the effect on the illuminating spot size is the same as that on the virtual spot size [see curves b and c in Fig. 3.4(b)]. For a detecting fiber with a large core diameter, Fig. 3.4(c) shows that the effects of both spot sizes, $2q$ and $2s$, are almost negligible.

The last parameter to be discussed here is the probing distance R to the moving

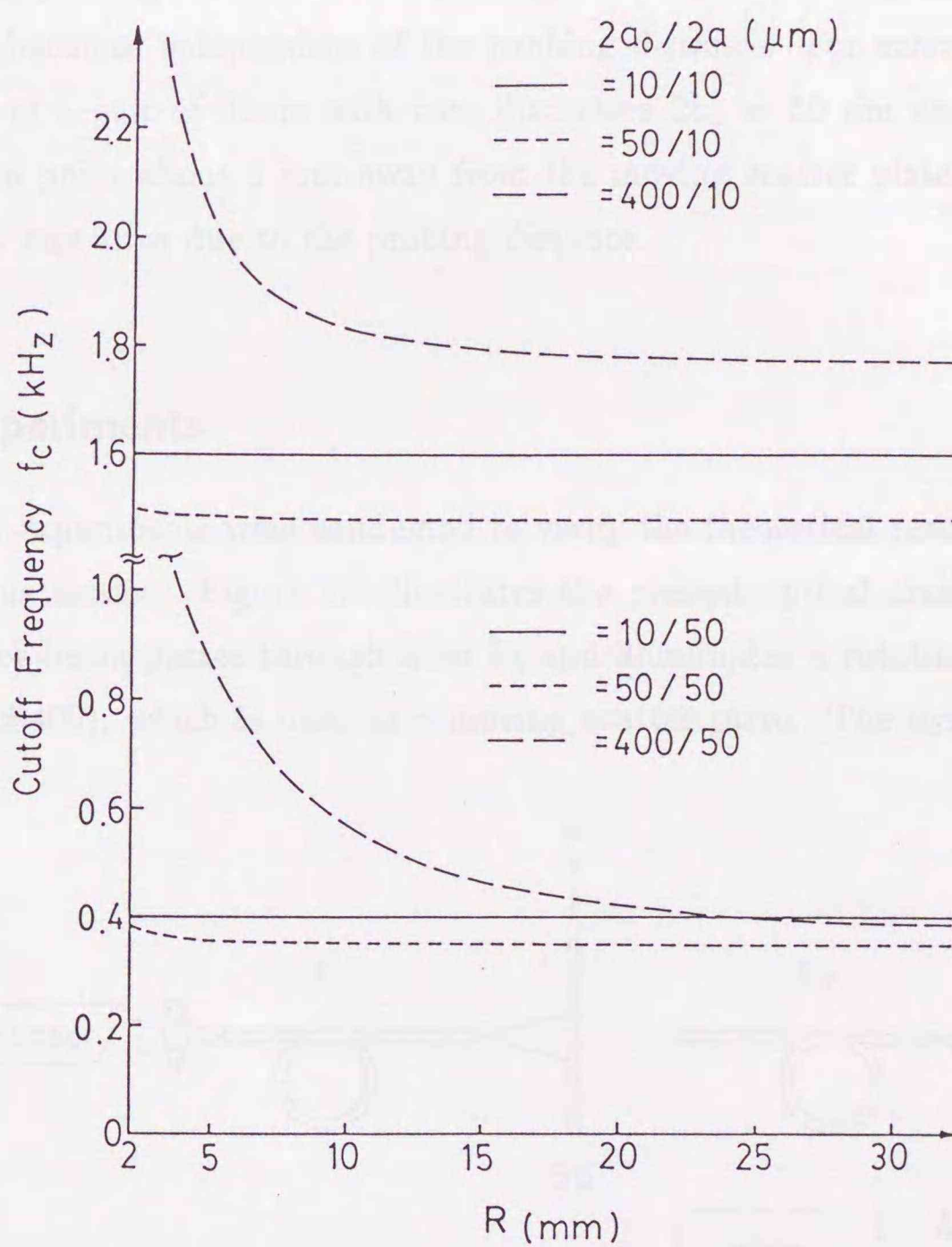


Figure 3.5. Cutoff frequency f_c of the PSD plotted as a function of the probing distance R for various core diameters of the fiber pairs characterized by $2a_0/2a$. Here the diverging angle of the illuminating beam and the acceptance angle of the detecting fiber are $2q/R = 0.2$ and $2s/R = 0.2$. The velocity of the scatter plate is 20 mm/s.

scatter plate. Figure 3.5 shows theoretical curves of the cutoff frequency variation against the probing distance R for several different combinations of illuminating and detecting fibers. The beam divergence angle $2q/R$ of the illuminating fiber, the acceptance angle $2s/R$ of the detecting fiber, and the velocity v are 0.2, 0.2, and 20 mm/s, respectively. With a decrease in the distance R , the speckle size in the plane of the scatter plate decreases and, as was mentioned above for the results in Fig. 3.2, the cutoff frequency gradually takes a higher value. This tendency is remarkable in case of the illuminating fiber with a large core. Figure 3.5 also shows the lower limit of the probing distance to the moving scatter plate. At this limit, the cutoff frequency becomes independent of the probing distance. For example, the probe consisting of a pair of fibers with core diameters $2a_0 = 50 \mu\text{m}$ and $2a = 50 \mu\text{m}$ can reach a point about 5 mm away from the moving scatter plate without being affected by any error due to the probing distance.

3.4 Experiments

Several experiments were conducted to verify the theoretical results obtained in the previous section. Figure 3.6 illustrates the present optical arrangement. The He-Ne laser beam passes through fiber F_1 and illuminates a rotating ground glass plate Sp (#400), which is used as a moving scatter plate. The dynamic speckles

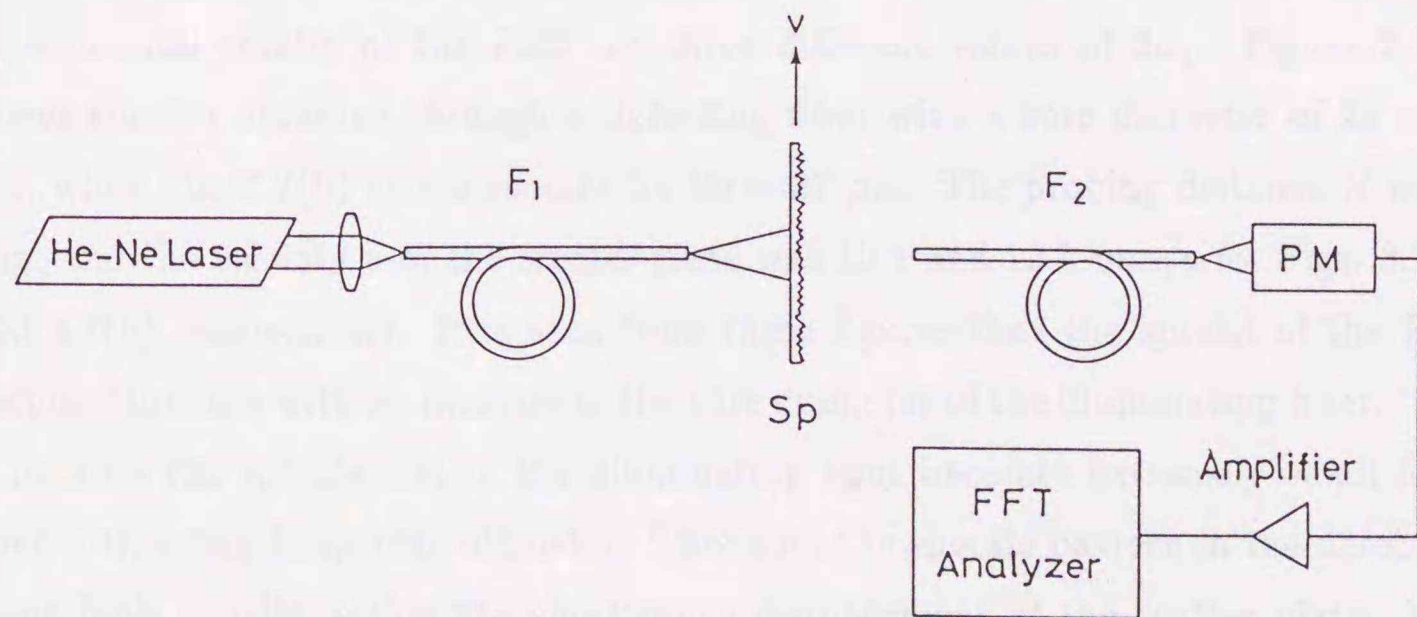


Figure 3.6. Experimental arrangement of the speckle field detection with an optical-fiber probe.

Table 3.1. Properties of the Fibers^a

	Core Diameters $2a$ (μm)					
	10	30	37	50	80	600
Diameters of Illuminating Spots $2q$ (mm)	—	0.74	—	0.74	1.49	1.19
Virtual Spots $2s$ (mm)	1.61	—	3.92	—	—	—

^a $R = 6$ mm.

produced by the scatter plate are received by another fiber, F_2 . The speckle signal is detected at the exit end of fiber F_2 by a photomultiplier, PM, and its PSD is measured by using a FFT analyzer. In the present experiment, several fiber cords with different values of core diameter and numerical aperture were used. Although not many combinations of the illuminating and detecting fibers have been used, most features of the PSD variation revealed from the theoretical results are verified in the following experiments. The core diameters of the illuminating fiber F_1 were chosen to be 30, 50, 80, and 600 μm , and those of the detecting fiber F_2 were 10 and 37 μm . The diameters of the illuminating spot $2q$ on the scatter plate and the virtual spot $2s$ were measured experimentally [65] and are shown in Table 3.1.

In the first experiment, the effect of the core diameter $2a_0$ of the illuminating fiber on the PSD of speckle signals was studied. Figure 3.7 shows examples of experimental results of the PSD for three different values of $2a_0$. Figure 3.7(a) shows spectra obtained through a detecting fiber with a core diameter of $2a = 10$ μm , while Fig. 3.7(b) shows spectra for $2a = 37$ μm . The probing distance R was 6 mm, and the velocity v of the scatter plate was 10.1 and 19.5 mm/s for Figs. 3.7(a) and 3.7(b), respectively. It is seen from these figures that the spread of the PSD becomes broader with an increase in the core diameter of the illuminating fiber. This is because the speckle size in the illuminating spot becomes extremely small for a fiber with a very large core diameter. Therefore the speckle pattern in the detecting plane boils rapidly within the short-range displacement of the scatter plate. It is interesting to note from Fig. 3.7(b) that the spread of the PSD for the fiber with an 80- μm diameter is narrower than that for the fiber with the 30- μm diameter. This is

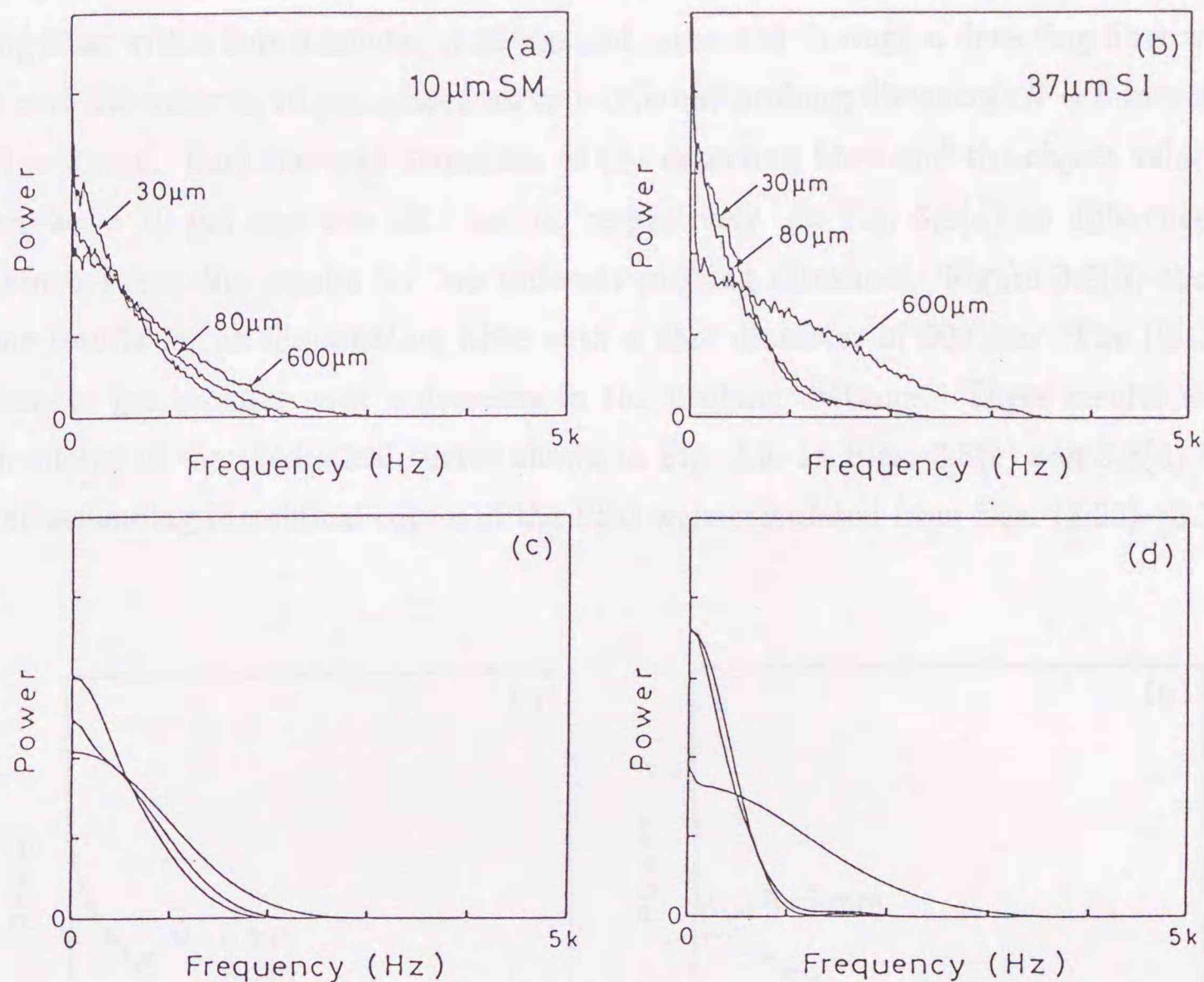


Figure 3.7. Variations of the PSD as a function of the core diameter of the illuminating fibers. (a) and (b) are the experimental results detected by the fibers with core diameters of $2a = 10 \mu\text{m}$ and $2a = 37 \mu\text{m}$, respectively. The velocity of the scatter plate is (a) $v = 10.1 \text{ mm/s}$ and (b) $v = 19.5 \text{ mm/s}$. (c) and (d) are the corresponding theoretical curves.

shown in Fig. 3.2. Comparison of Figs. 3.7(a) and 3.7(b) also shows that the spread of the PSD becomes narrower with an increase in the core diameter of the detecting fiber. Note that the velocity of the scatter plate in Fig. 3.7(a) is nearly twice that in Fig. 3.7(b). The corresponding theoretical curves of the PSD were evaluated by substituting the given parameters into Eqs. (3.25)–(3.33) and are plotted in Figs. 3.7(c) and 3.7(d). The theoretical results in Figs. 3.7(c) and 3.7(d) are in excellent agreement with the experimental curves in Figs. 3.7(a) and 3.7(b).

In the second experiment, the variation of the PSD as a function of the probing distance R and the core diameter $2a_0$ of the illuminating fiber was studied. Figure

3.8(a) shows the results of the PSD for the speckle signals produced by an illuminating fiber with a core diameter of $50\text{ }\mu\text{m}$ and measured through a detecting fiber with a core diameter of $10\text{ }\mu\text{m}$ placed at two different probing distances, $R = 6\text{ mm}$ and $R = 3\text{ mm}$. Here the core diameters of the detecting fiber and the object velocity are $2a = 10\text{ }\mu\text{m}$ and $v = 10.1\text{ mm/s}$, respectively. In Fig. 3.8(a) no difference is seen between the results for two different probing distances. Figure 3.8(b) shows the results for an illuminating fiber with a core diameter of $600\text{ }\mu\text{m}$. The PSD is seen to get broader with a decrease in the probing distance. These results were predicted in the theoretical curves shown in Fig. 3.5. In Figs. 3.8(c) and 3.8(d) the corresponding theoretical curves of the PSD were calculated from Eqs. (3.25)–(3.33)

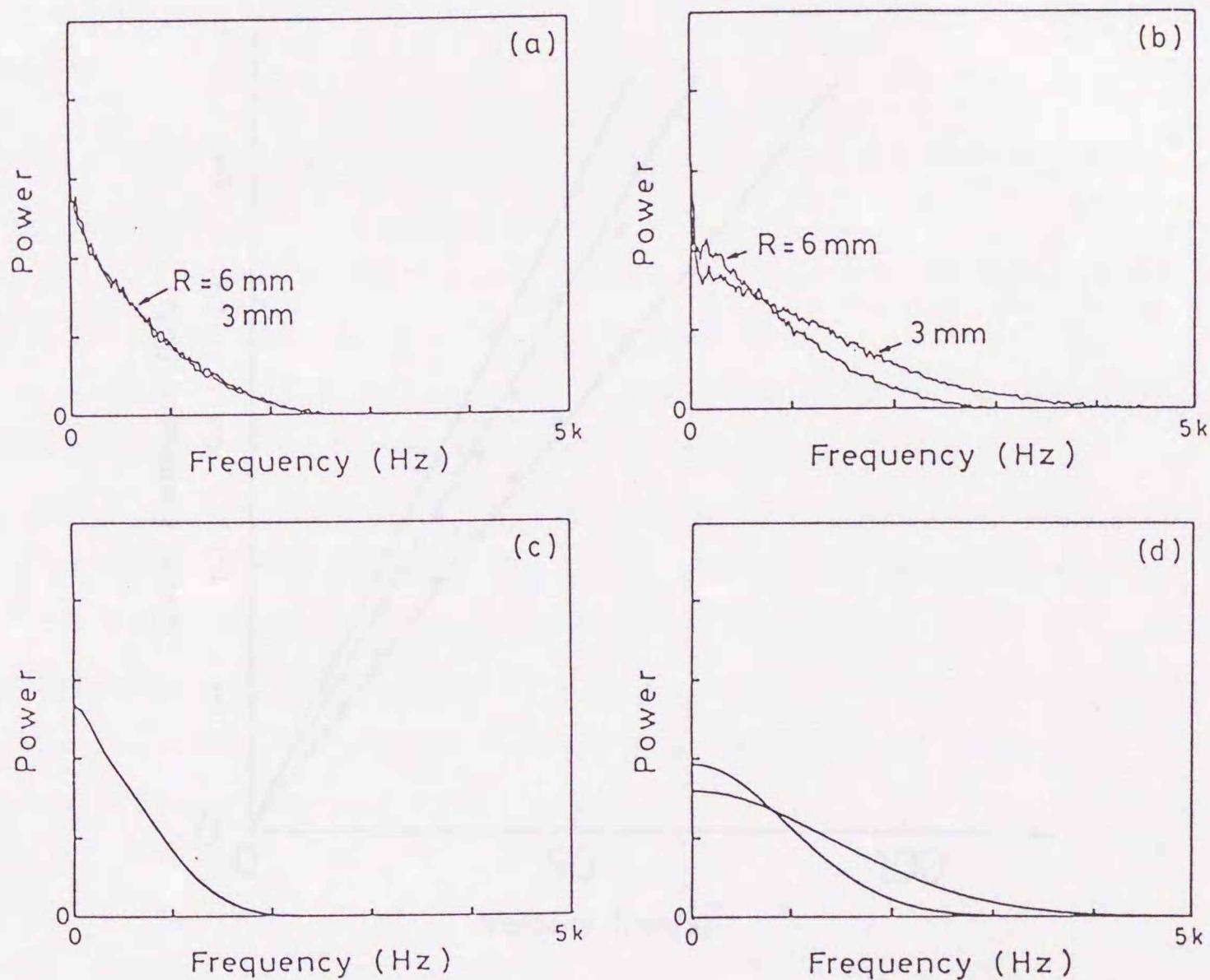


Figure 3.8. Variations of the PSD as a function of the probing distance. (a) and (b) are the experimental results for the illuminating fiber with core diameters of $2a_0 = 50\text{ }\mu\text{m}$ and $2a_0 = 600\text{ }\mu\text{m}$. The velocity of the scatter plate is $v = 10.1\text{ mm/s}$. (c) and (d) are the corresponding theoretical curves.

and plotted for comparison. Excellent agreement between the experimental and the theoretical results is observed in Fig. 3.8.

In the last experiment, the variation of the cutoff frequency against the moving velocity of the object was studied. The results are shown in Fig. 3.9 for three different speckle sizes in the illumination spot. The Gaussian beam illumination corresponds to the case of no speckle variation in the illuminating spot, while the fibers with the core diameters of $30\text{ }\mu\text{m}$ and $80\text{ }\mu\text{m}$ produce smaller speckle sizes in the spot. It is found that the cutoff frequency is linearly proportional to the moving velocity of the object, and that the proportionality constant depends on the speckle size in the illuminating spot.

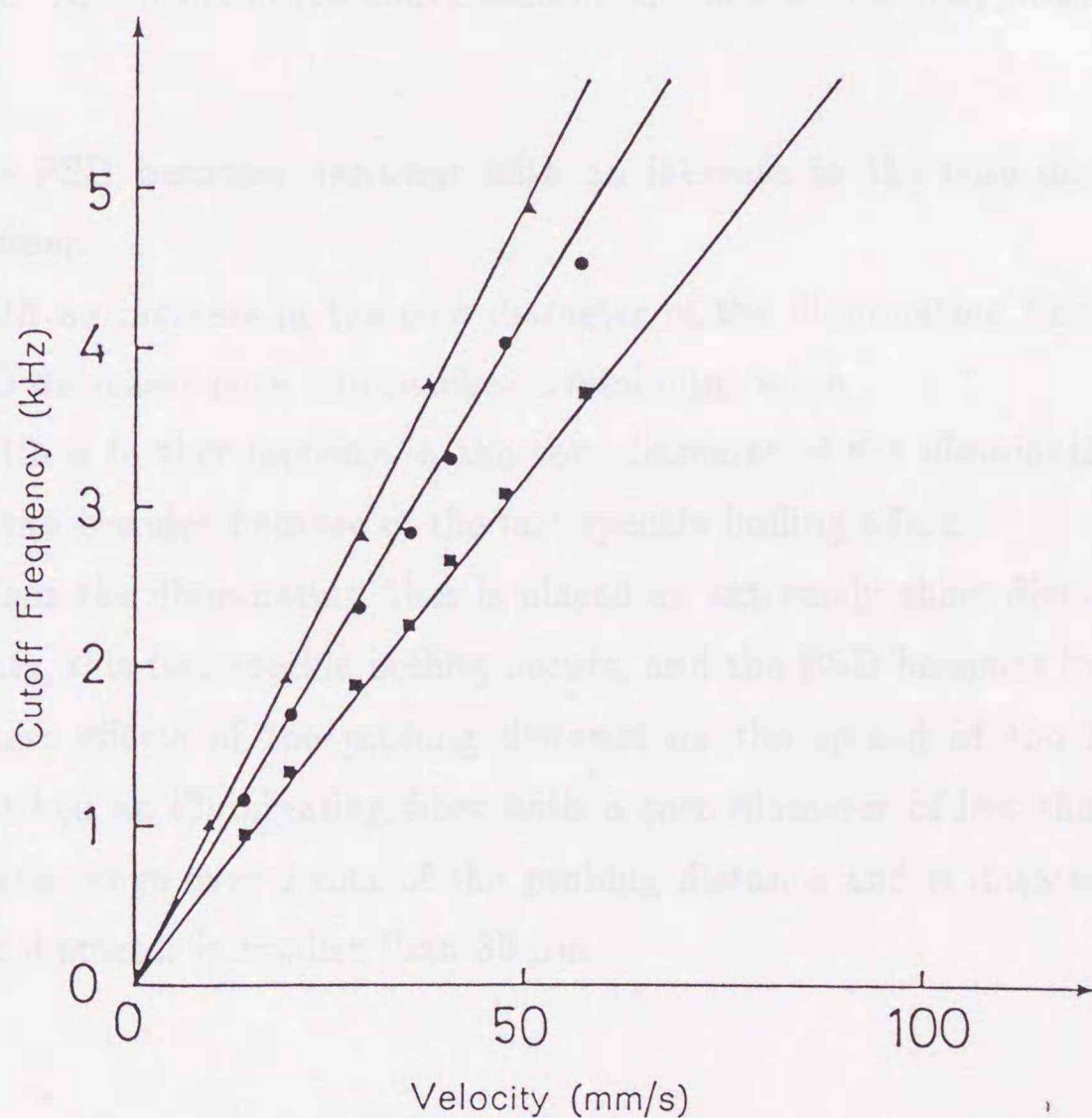


Figure 3.9. Variations of the cutoff frequency of the PSD against the moving velocity of the scatter plate for Gaussian beam illumination (▲) and illumination from optical fibers with core diameters of $2a_0 = 30\text{ }\mu\text{m}$ (■) and $2a_0 = 80\text{ }\mu\text{m}$ (●). The core diameter of a detecting fiber is $2a = 10\text{ }\mu\text{m}$.

3.5 Conclusion

A general equation for the PSD of the speckle signal obtained with an optical-fiber probe was derived as a function of the core diameter of the illuminating fiber, the illuminating spot size at the moving scatter plate, the core diameter and the NA of the detecting fiber, the probing distance of the fiber from the scatter plate, and the velocity of the plate. The equation was evaluated for various values of five important parameters: the core diameters of both the illuminating and detecting fibers, the sizes of the illuminating and the virtual spots in the plane of the scatter plate, and the probing distance. Several experiments were conducted to measure the PSD's of speckle signals by means of various pairs of optical fibers. The experimental results were compared with the corresponding theoretical ones, and excellent agreement was shown. The effects of the above parameters on the PSD may be summarized as follows:

- (i) The PSD becomes narrower with an increase in the core diameter of the detecting fiber.
- (ii) With an increase in the core diameter of the illuminating fiber, the spread of the PSD decreases once and reaches a minimum value.
- (iii) With a further increase in the core diameter of the illuminating fiber, the PSD becomes broader because of the fast speckle boiling effect.
- (iv) When the illuminating fiber is placed an extremely short distance from the scatter plate, this fast speckle boiling occurs, and the PSD becomes broader.
- (v) These effects of the probing distance on the spread of the PSD become negligible when an illuminating fiber with a core diameter of less than $100\text{ }\mu\text{m}$ is placed in the range over 2 mm of the probing distance and is detected by a fiber whose core diameter is smaller than $30\text{ }\mu\text{m}$.

3.6 Appendix: Derivations of Eqs. (3.27) and (3.28)

By substituting Eq. (3.24) into Eqs. (3.15), (3.22), (3.23) and changing the variables to $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$ and $\mathbf{p}' = \mathbf{p}_1 - \mathbf{p}_2$, the double integrals over $\mathbf{p}_1$ and $\mathbf{p}_2$ in these equations become a product of two simple integrals over $\mathbf{p}$ or $\mathbf{p}'$. Then, by expressing the

vectors p and v as

$$\begin{aligned} p &= (x \cos \theta, x \sin \theta), \\ v &= (v \cos \phi, v \sin \phi), \end{aligned} \quad (3.35)$$

Eqs. (3.14) and (3.15) become

$$\begin{aligned} c_1 &= c_0 \left\{ \int_0^{2\pi} d\theta \int_0^\infty x dx \exp[-2(1/q^2 + 1/s^2)x^2] \right\}^2 \\ &= (c_0 \pi^2/4)(1/q^2 + 1/s^2)^{-2}, \end{aligned} \quad (3.36)$$

$$\begin{aligned} c_2 &= \frac{c_0}{4} \left\{ \int_0^{2\pi} d\theta \int_0^\infty x dx \exp[-2(1/q^2 + 1/s^2)x^2] \right\}^2 \\ &\quad \times \left\{ \int_0^{2\pi} d\theta \int_0^\infty x dx \exp[-2(1/\zeta_1^2 + 1/s^2)x^2] \right\}^2 \\ &= (c_0 \pi^2/4)(1/q^2 + 1/s^2)^{-1}(1/\zeta_1^2 + 1/s^2)^{-1}. \end{aligned} \quad (3.37)$$

Equations (3.22) and (3.23) may be written as

$$g_1(\tau) = g_{11}(\tau)g_{12}(\tau)g_{13}(\tau), \quad (3.38)$$

$$g_2(\tau) = g_{21}(\tau)g_{22}(\tau)g_{23}(\tau), \quad (3.39)$$

where

$$g_{11}(\tau) = (c_0/4) \exp[-(1/q^2 + 1/\zeta_1^2 + 2/s^2)v^2\tau^2], \quad (3.40)$$

$$g_{21}(\tau) = (c_0/4) \exp[-(2/q^2 + 2/s^2)v^2\tau^2], \quad (3.41)$$

$$\begin{aligned} g_{12}(\tau) &= g_{22}(\tau) \\ &= \int_0^\infty x dx \exp[-(1/q^2 + 1/s^2)x^2] \\ &\quad \times \int_0^{2\pi} d\theta \exp[-2(1/q^2 + 1/s^2)xv\tau \cos(\theta - \phi)]. \end{aligned} \quad (3.42)$$

Using the formulas

$$\int_0^{2\pi} \exp(\pm z \cos \theta) d\theta = 2\pi I_0(z)$$

[where $I_0(z)$ is a modified Bessel function of the first order] and

$$\int_0^\infty x \exp(-a^2 x^2) I_0(bx) dx = (1/2a^2) \exp(b^2/4a^2),$$

we have

$$g_{12}(\tau) = g_{22}(\tau) = \pi(1/q^2 + 1/s^2)^{-1} \exp[(1/q^2 + 1/s^2)v^2\tau^2]. \quad (3.43)$$

$g_{13}(\tau)$ becomes

$$\begin{aligned} g_{13}(\tau) &= \int_0^\infty x dx \exp[-(1/q^2 + 1/s^2)x^2] \left[\frac{2J_1(2\pi ax/\lambda R_2)}{2\pi ax/\lambda R_2} \right]^2 \\ &\quad \times \int_0^{2\pi} d\theta \exp[-i2\pi(1 + R_2/R_1)xv\tau \cos(\theta - \phi)/\lambda R_2] \\ &= \frac{2\lambda^2 R_2^2}{\pi a^2} \int_0^\infty x^{-1} \exp[-(1/q^2 + 1/s^2)x^2] \\ &\quad \times J_1^2(2\pi ax/\lambda R_2) J_0[2\pi(1 + R_2/R_1)v\tau x/\lambda R_2] dx, \end{aligned} \quad (3.44)$$

where the formula

$$\int_0^{2\pi} \exp(ix \cos a) \exp(ina) da = \frac{2\pi}{i^{-n}} J_n(x)$$

has been used. Similarly, $g_{23}(\tau)$ becomes

$$\begin{aligned} g_{23}(\tau) &= \frac{2\lambda^2 R_2^2}{\pi a^2} \int_0^\infty x^{-1} \exp[-(1/\zeta_1^2 + 1/s^2)x^2] \\ &\quad \times J_1^2(2\pi ax/\lambda R_2) J_0[2\pi(1 + R_2/R_1)v\tau x/\lambda R_2] dx. \end{aligned} \quad (3.45)$$

Substituting Eqs. (3.40), (3.43), and (3.44) into Eq. (3.38), and Eqs. (3.41), (3.43), and (3.45) into Eq. (3.39), we have Eqs. (3.27) and (3.28).

Chapter 4

Doubly-Scattered Speckles Produced by a Pair of Moving Diffusers

4.1 Introduction

Theoretical studies have recently been carried out on the statistical properties of doubly-scattered coherent light. In the field of laser speckles, several researchers [11–16] have examined the probability density function of doubly-scattered speckles or the spatial correlation function of their intensity variations. However, these analyses are mainly focused on their static properties, and little attention has been paid to the dynamic behaviour of doubly-scattered speckles although several studies [25–28] have been reported in connection with dynamic speckles due to a moving diffuse object illuminated by light from a multimode optical fiber.

Dynamic properties of doubly- or multiply-scattered speckles have already been studied experimentally, aiming for speckle reduction [7–10] or measurements of the activity of biological objects, such as skin blood flow [68, 69], movement of microorganisms in a large volume [70], or cyclosis in citrus fruits [71]. These multiply-scattered speckles due to biological movements (so-called *bio-speckles* [72]) are very difficult to investigate analytically because living things have very complicated structures. Thus, double or triple scattering models have been presented to simplify the multiple scattering phenomena [29–31, 72]. These models consist of two diffusers, one motionless and the other moving with a constant velocity, which represent human skin or the surface of a plant. The experimental results have shown that the linear dependence of the velocity on doubly- or triply-scattered speckle fluctuations is still preserved even in the presence of a motionless diffuser. Since the above models have only a single moving diffuser, they are not suitable for characterizing the multiple scattering phenomena from objects such as a large number of micro-

organisms. Therefore it is strongly desired to investigate the properties of dynamic speckles produced by a set of moving diffusers.

The purpose of this chapter is to study the statistical properties of doubly-scattered speckles produced by a cascade of two diffusers moving with constant velocities [32]. The relationship is examined between the velocity of each diffuser and the time-varying speckle intensity detected at a single point. The problem is first formulated theoretically, and then it is evaluated for various parameters. Several theoretical predictions and the relevant experiments are discussed in connection with velocity measurements.

4.2 Theory

4.2.1 Correlation Function of Doubly-Scattered Speckles

Figure 4.1 illustrates the configuration for analyzing the speckles from a cascade of two moving diffusers. Two diffusers D_1 and D_2 are separated by spacing R_1 and move with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ at planes $\mathbf{u}_1$ and $\mathbf{u}_2$ respectively. A Gaussian laser beam propagating in the field F_1 illuminates the diffuser D_1 with radius R of the wavefront curvature and radius w of the beam spot at the plane $\mathbf{u}_1$. According to the transverse motion of the diffusers D_1 and D_2 , the dynamic speckles produced by the moving diffuser D_1 in the diffraction field F_2 are further modulated by the moving diffuser D_2 to form the doubly-scattered dynamic speckles in the diffraction field F_3 . In the following analysis, the space-time correlation functions of the speckle fields produced in the fields F_2 and F_3 is formulated.

The Gaussian beam propagating in the field F_1 with wavelength λ has an amplitude given by

$$E_1(\mathbf{u}_1) = \frac{w_0}{w} \exp\left(i\frac{2\pi}{\lambda}z\right) \exp\left(-\frac{|\mathbf{u}_1|^2}{w^2}\right) \exp\left(i\frac{\pi|\mathbf{u}_1|^2}{\lambda R}\right), \quad (4.1)$$

where w_0 and w are the radii of the beam at its waist and illuminating planes, and z is the distance between the illuminating plane $\mathbf{u}_1$ and the position of the beam waist. By assuming a random phase screen which modulates only the phase of the illuminating light, the amplitude distribution immediately behind the diffuser D_1

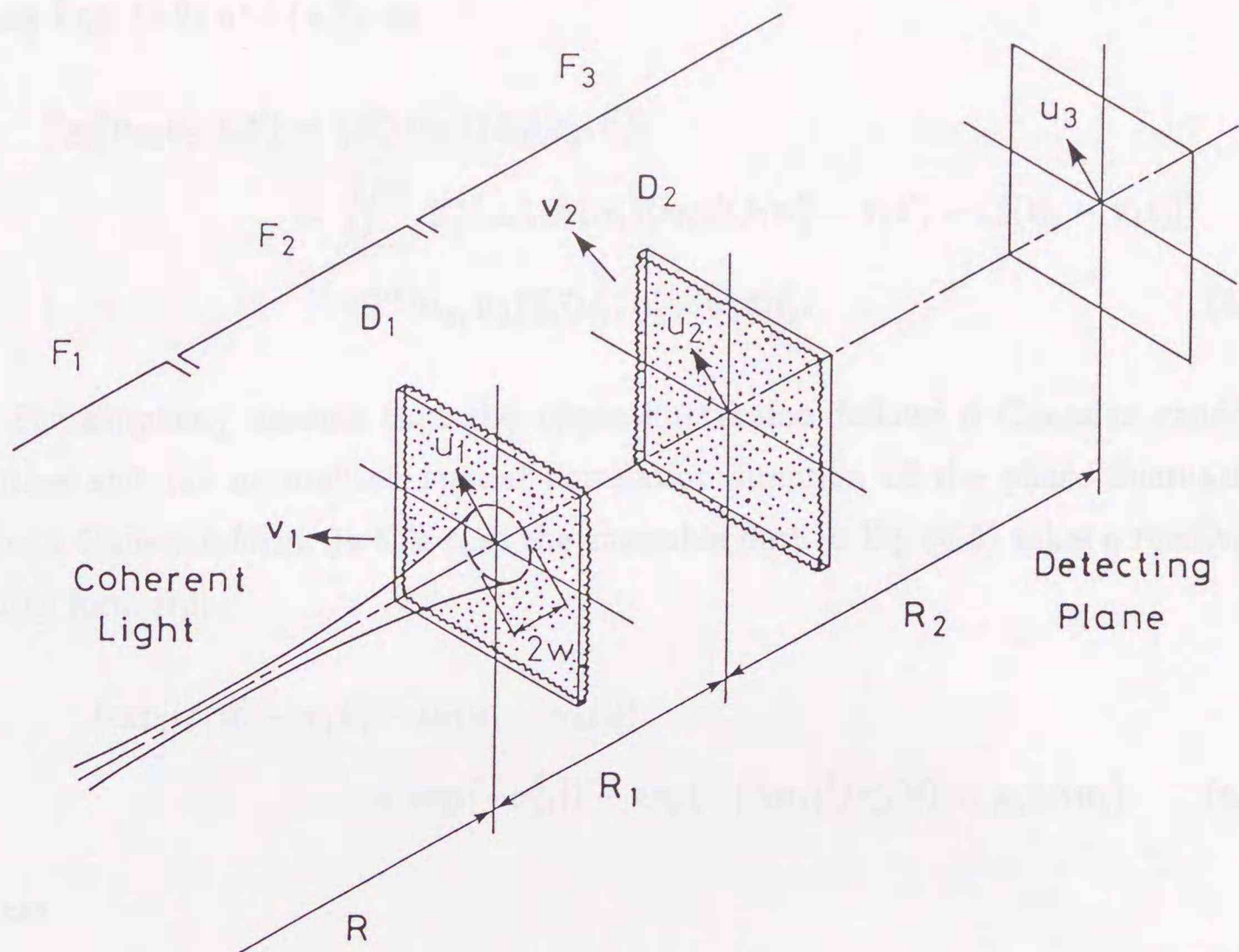


Figure 4.1. Schematic diagram of the optical configuration for analyzing a doubly-scattered speckle field.

can be written as

$$U_1(\mathbf{u}_1, t) = E_1(\mathbf{u}_1) \exp[i\phi(\mathbf{u}_1 - \mathbf{v}_1 t)], \quad (4.2)$$

where $\phi(\mathbf{u}_1 - \mathbf{v}_1 t)$ denotes the phase modulation due to the diffuser D_1 . Moreover, the amplitude distribution at the plane $\mathbf{u}_2$ can be expressed as

$$E_2(\mathbf{u}_2, t) = \int_{-\infty}^{\infty} U_1(\mathbf{u}_1, t) T_1(\mathbf{u}_1, \mathbf{u}_2) d\mathbf{u}_1, \quad (4.3)$$

where $T_1(\mathbf{u}_1, \mathbf{u}_2)$ represents the propagation function of the light amplitude from the plane $\mathbf{u}_1$ to the plane $\mathbf{u}_2$. In the present analysis, the plane $\mathbf{u}_2$ is taken in the Fresnel diffraction field and hence $T_1(\mathbf{u}_1, \mathbf{u}_2)$ is given by a well-known form

$$T_1(\mathbf{u}_1, \mathbf{u}_2) = \frac{1}{i\lambda R_1} \exp\left(\frac{i\pi|\mathbf{u}_1 - \mathbf{u}_2|^2}{\lambda R_1}\right). \quad (4.4)$$

The space-time correlation function of the complex amplitude E_2 is then written by

using Eqs. (4.2) and (4.3), as

$$\begin{aligned}\Gamma_{E_2}(\mathbf{u}_2, \mathbf{u}'_2; t, t') &= \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}'_2, t') \rangle \\ &= \iint_{-\infty}^{\infty} E_1^*(\mathbf{u}_1) E_1(\mathbf{u}'_1) \langle \exp[i\phi(\mathbf{u}'_1 - \mathbf{v}_1 t') - i\phi(\mathbf{u}_1 - \mathbf{v}_1 t)] \rangle \\ &\quad \times T_1^*(\mathbf{u}_1, \mathbf{u}_2) T_1(\mathbf{u}'_1, \mathbf{u}'_2) d\mathbf{u}_1 d\mathbf{u}'_1.\end{aligned}\quad (4.5)$$

For simplicity assume that the phase fluctuation follows a Gaussian random process and the normalized spatial correlation function of the phase fluctuation takes a Gaussian form. In this case the ensemble part in Eq. (4.5) takes a relatively simple form [73]

$$\begin{aligned}\langle \exp[i\phi(\mathbf{u}'_1 - \mathbf{v}_1 t') - i\phi(\mathbf{u}_1 - \mathbf{v}_1 t)] \rangle \\ = \exp\{-\sigma_{p1}^2[1 - \exp(-|\Delta\mathbf{u}_1|^2/r_{c1}^2)]\} \equiv \mu_1(\Delta\mathbf{u}_1),\end{aligned}\quad (4.6)$$

where

$$\Delta\mathbf{u}_1 = \mathbf{u}'_1 - \mathbf{u}_1 - \mathbf{v}_1(t' - t).$$

In Eq. (4.6), σ_{p1} and r_{c1} indicate the standard deviation and the correlation length of phase fluctuations respectively. When an object of rough surface is chosen as a diffuser, r_{c1} becomes equivalent to the correlation length of the surface height variations. Further, if the random phase screen is assumed to take a value of σ_{p1} larger than π , μ_1 in Eq. (4.6) approximately reduces to [74]

$$\mu_1(\Delta\mathbf{u}_1) = \exp(-|\Delta\mathbf{u}_1|^2/\beta_1^2),\quad (4.7)$$

where β_1 stands for the effective correlation length defined by

$$\beta_1 = r_{c1}/\sigma_{p1}.\quad (4.8)$$

By substituting Eqs. (4.1), (4.4) and (4.7) into Eq.(4.5) and by replacing $t' - t$ with τ , the correlation function of Eq.(4.5) becomes

$$\Gamma_{E_2}(\mathbf{u}_2, \mathbf{u}'_2; \tau) = \left(\frac{w_0}{\lambda R_1 w}\right)^2 \exp\left[\frac{i\pi(|\mathbf{u}'_2|^2 - |\mathbf{u}_2|^2)}{\lambda R_1}\right]$$

$$\begin{aligned}
& \times \iint_{-\infty}^{\infty} \exp\left(-\frac{|\mathbf{u}'_1|^2 + |\mathbf{u}_1|^2}{w^2}\right) \exp\left[\frac{i\pi(|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2)}{\lambda R}\right] \\
& \times \exp\left(-\frac{|\mathbf{u}'_1 - \mathbf{u}_1 - \mathbf{v}_1\tau|^2}{\beta_1^2}\right) \exp\left[-\frac{i2\pi(\mathbf{u}'_1 \cdot \mathbf{u}'_2 - \mathbf{u}_1 \cdot \mathbf{u}_2)}{\lambda R_1}\right] \\
& \times \exp\left[\frac{i\pi(|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2)}{\lambda R_1}\right] d\mathbf{u}_1 d\mathbf{u}'_1. \quad (4.9)
\end{aligned}$$

By changing the variables $\mathbf{u}_1$ and $\mathbf{u}'_1$ in Eq. (4.9) to

$$\mathbf{u}_+ = \mathbf{u}'_1 + \mathbf{u}_1, \quad \mathbf{u}_- = \mathbf{u}'_1 - \mathbf{u}_1, \quad (4.10)$$

Eq. (4.9) is modified to

$$\begin{aligned}
\Gamma_{E_2}(\mathbf{u}_2, \mathbf{u}'_2; \tau) &= \left(\frac{w_0}{2\lambda R_1 w}\right)^2 \exp\left[\frac{i\pi(|\mathbf{u}'_2|^2 - |\mathbf{u}_2|^2)}{\lambda R_1}\right] \\
&\times \int_{-\infty}^{\infty} d\mathbf{u}_- \exp\left(-\frac{|\mathbf{u}_-|^2}{2w^2}\right) \exp\left(-\frac{|\mathbf{u}_- - \mathbf{v}_1\tau|^2}{\beta_1^2}\right) \exp\left[-\frac{i\pi(\mathbf{u}'_2 + \mathbf{u}_2) \cdot \mathbf{u}_-}{\lambda R_1}\right] \\
&\times \int_{-\infty}^{\infty} d\mathbf{u}_+ \exp\left(-\frac{|\mathbf{u}_+|^2}{2w^2}\right) \exp\left\{-\frac{i\pi[(1 + K_1)\mathbf{u}_- - (\mathbf{u}'_2 - \mathbf{u}_2)] \cdot \mathbf{u}_+}{\lambda R_1}\right\}, \quad (4.11)
\end{aligned}$$

where

$$K_1 = R_1/R. \quad (4.12)$$

To evaluate Eq. (4.11), the following formula is used

$$\begin{aligned}
& \int_{-\infty}^{\infty} \exp(-A_1|\mathbf{u}|^2) \exp(-A_2|\mathbf{u}' - \mathbf{u}|^2) \exp(i\mathbf{B} \cdot \mathbf{u}) d\mathbf{u} \\
&= \frac{\pi}{A_1 + A_2} \exp\left(\frac{iA_2\mathbf{B} \cdot \mathbf{u}'}{A_1 + A_2}\right) \exp\left[-\frac{|\mathbf{B}|^2 + 4A_1A_2|\mathbf{u}'|^2}{4(A_1 + A_2)}\right]. \quad (4.13)
\end{aligned}$$

Then Eq. (4.11) becomes

$$\begin{aligned}
\Gamma_{E_2}(\mathbf{u}_2, \mathbf{u}'_2; \tau) &= \frac{1}{P} \left(\frac{\pi w_0}{\lambda R_1}\right)^2 \exp\left[\frac{i\pi}{\lambda R_1} \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1}\right) (|\mathbf{u}'_2|^2 - |\mathbf{u}_2|^2)\right] \\
&\times \exp\left[-\frac{P_1(P_2 + P_3)}{2P} \tau^2 |\mathbf{v}_1|^2\right] \exp\left[-\frac{i\pi P_1}{\lambda R_1 P} \tau \mathbf{v}_1 \cdot (\mathbf{u}'_2 + \mathbf{u}_2)\right] \\
&\times \exp\left[\frac{P_1 P_3}{P} \frac{1}{1 + K_1} \tau \mathbf{v}_1 \cdot (\mathbf{u}'_2 - \mathbf{u}_2)\right]
\end{aligned}$$

$$\times \exp\left(-\frac{|\mathbf{u}'_2 - \mathbf{u}_2|^2}{2\zeta_2^2}\right) \exp\left(-\frac{|\mathbf{u}'_2 + \mathbf{u}_2|^2}{2q_2^2}\right), \quad (4.14)$$

where ζ_2 and q_2 represent the average speckle size and the extent of the speckle field at the plane $\mathbf{u}_2$, which are defined by

$$\zeta_2 = \frac{\lambda R_1}{\pi w} \left(\frac{P}{P_1 + P_2}\right)^{1/2}, \quad q_2 = \frac{\lambda R_1}{\pi} P^{1/2}. \quad (4.15)$$

In Eqs. (4.14) and (4.15), P_1 , P_2 , P_3 and P are given by

$$P_1 = \frac{2}{\beta_1^2}, \quad P_2 = \frac{1}{w^2}, \quad P_3 = \left(\frac{\pi w}{\lambda R_1}\right)^2 (1 + K_1)^2$$

and

$$P = P_1 + P_2 + P_3. \quad (4.16)$$

When the second moving diffuser D_2 is placed at the plane $\mathbf{u}_2$ in the Fresnel diffraction field F_2 , the light amplitude immediately behind the diffuser D_2 is written as

$$U_2(\mathbf{u}_2, t) = E_2(\mathbf{u}_2, t) \exp[i\phi(\mathbf{u}_2 - \mathbf{v}_2 t)]. \quad (4.17)$$

In the present analysis, our attention is focused on the speckles produced under a free-space propagation geometry. In this case, the complex amplitude at the detecting plane $\mathbf{u}_3$ placed R_2 away from the diffuser D_2 in the Fresnel diffraction field F_3 can be expressed as

$$E_3(\mathbf{u}_3, t) = \int_{-\infty}^{\infty} U_2(\mathbf{u}_2, t) T_2(\mathbf{u}_2, \mathbf{u}_3) d\mathbf{u}_2, \quad (4.18)$$

where

$$T_2(\mathbf{u}_2, \mathbf{u}_3) = \frac{1}{i\lambda R_2} \exp\left(\frac{i\pi|\mathbf{u}_2 - \mathbf{u}_3|^2}{\lambda R_2}\right). \quad (4.19)$$

By using Eqs. (4.17) and (4.18), the correlation function of the complex amplitude is written as

$$\begin{aligned} \Gamma_{E_3}(\mathbf{u}_3, \mathbf{u}'_3; \tau) &= \langle E_3^*(\mathbf{u}_3, t) E_3(\mathbf{u}'_3, t + \tau) \rangle \\ &= \iint_{-\infty}^{\infty} \Gamma_{E_2}(\mathbf{u}_2, \mathbf{u}'_2; \tau) \mu_2(\mathbf{u}'_2 - \mathbf{u}_2 - \mathbf{v}_2 \tau) \\ &\quad \times T_2^*(\mathbf{u}_2, \mathbf{u}_3) T_2(\mathbf{u}'_2, \mathbf{u}'_3) d\mathbf{u}_2 d\mathbf{u}'_2. \end{aligned} \quad (4.20)$$

In Eq. (4.20) the same approximation as used in Eq. (4.7) has been taken to characterize the diffuser D_2 . Then, the normalized spatial correlation function of the phase fluctuation for the diffuser D_2 takes the form

$$\mu_2(\mathbf{u}'_2 - \mathbf{u}_2 - \mathbf{v}_2\tau) = \exp(-|\mathbf{u}'_2 - \mathbf{u}_2 - \mathbf{v}_2\tau|^2/\beta_2^2), \quad (4.21)$$

where β_2 denotes the effective correlation length of the diffuser D_2 defined by

$$\beta_2 = r_{c2}/\sigma_{p2}, \quad (4.22)$$

in which σ_{p2} and r_{c2} denote the standard deviation and the correlation length of phase fluctuations respectively. By substituting Eqs. (4.14), (4.19) and (4.21) into Eq. (4.20) and by performing an evaluation in the same way used to derive Eq. (4.14), the correlation function of doubly-scattered dynamic speckles is finally obtained as

$$\begin{aligned} \Gamma_{E_3}(\mathbf{u}_3, \mathbf{u}'_3; \tau) = & \frac{1}{PQ} \left(\frac{\pi w_0}{\lambda R_1} \right)^2 \left(\frac{\pi q_2}{\lambda R_2} \right)^2 \\ & \times \exp \left\{ \frac{i\pi}{\lambda R_2} \left[1 - \frac{Q_3}{Q} \frac{1}{1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right)} \right] (|\mathbf{u}'_3|^2 - |\mathbf{u}_3|^2) \right\} \\ & \times \exp \left\{ -\frac{P_1}{2} \left[1 - \frac{P_1}{Q} \left(1 + \frac{1}{K_2} \right)^2 \right] \tau^2 |\mathbf{v}_1|^2 + \frac{P_1 Q_1}{Q} \right. \\ & \quad \times \left(1 + \frac{1}{K_2} \right) \tau^2 \mathbf{v}_1 \cdot \mathbf{v}_2 - \frac{Q_1(Q_2 + Q_3)}{2Q} \tau^2 |\mathbf{v}_2|^2 \Big\} \\ & \times \exp \left\{ -\left[\frac{i\pi}{\lambda R_2} \frac{P_1}{Q} \left(1 + \frac{1}{K_2} \right) \tau \mathbf{v}_1 + \frac{i\pi}{\lambda R_2} \frac{Q_1}{Q} \tau \mathbf{v}_2 \right] \cdot (\mathbf{u}'_3 + \mathbf{u}_3) \right\} \\ & \times \exp \left\{ -\left[\frac{P_1}{K_2} \left(1 - \frac{Q_3}{Q} \frac{1 + K_2}{1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right)} \right) \right. \right. \\ & \quad \times \left. \left. \tau \mathbf{v}_1 - \frac{Q_1 Q_3}{Q} \frac{1}{1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right)} \tau \mathbf{v}_2 \right] \cdot (\mathbf{u}'_3 - \mathbf{u}_3) \right\} \\ & \times \exp \left(-\frac{|\mathbf{u}'_3 - \mathbf{u}_3|^2}{2\zeta_3^2} \right) \exp \left(-\frac{|\mathbf{u}'_3 + \mathbf{u}_3|^2}{2q_3^2} \right), \end{aligned} \quad (4.23)$$

where

$$K_2 = R_2/R_1, \quad (4.24)$$

and ζ_3 and q_3 represent the average speckle size and the extent of the speckle field at the plane $\mathbf{u}_3$ which are defined by

$$\zeta_3 = \frac{\lambda R_2}{\pi q_2} \left(\frac{Q}{Q_1 + Q_2} \right)^{1/2}, \quad q_3 = \frac{\lambda R_2}{\pi} Q^{1/2}. \quad (4.25)$$

In Eqs. (4.23) and (4.25), Q_1 , Q_2 , Q_3 , and Q are given by

$$Q_1 = \frac{2}{\beta_2^2}, \quad Q_2 = \frac{1}{\zeta_2^2}, \quad Q_3 = \left(\frac{\pi q_2}{\lambda R_2} \right)^2 \left[1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right) \right]^2,$$

$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{2}{\beta_2^2} + \left(\frac{2}{\beta_1^2} + \frac{1}{w^2} \right) \left(1 + \frac{1}{K_2} \right)^2 + \left(\frac{\pi w}{\lambda R_1} \right)^2 \left[(1 + K_1) \left(1 + \frac{1}{K_2} \right) - 1 \right]^2. \quad (4.26)$$

Since the dynamic properties of the speckle intensity are of practical interest, it is necessary to find the correlation function of the intensity fluctuations. For the purpose of this calculation, we make the two important assumptions that (1) the speckle field due to the first diffuser D_1 obeys a circular complex Gaussian process and that (2) the speckle field illuminating the second diffuser D_2 contains more than 15 speckle grains [13]. The first assumption has already been made in deriving Eq. (4.7); namely, the diffuser D_1 introduces phase fluctuations greater than π . The second assumption can be satisfied if a large number of independent scatterers exist within the illuminating area of the Gaussian beam. These assumptions enable us to consider that the doubly-scattered speckle field also obeys a circular complex Gaussian process. In this case, the normalized correlation function of speckle intensity fluctuations is given by the correlation function of the complex amplitude Γ_{E_3} as

$$\begin{aligned} \gamma_{\Delta I_3}(\mathbf{u}_3, \mathbf{u}'_3; \tau) &= \langle \Delta I_3(\mathbf{u}_3, 0) \Delta I_3(\mathbf{u}'_3, \tau) \rangle / \langle I_3(0, 0) \rangle^2 \\ &= |\Gamma_{E_3}(\mathbf{u}_3, \mathbf{u}'_3; \tau)|^2 / |\Gamma_{E_3}(0, 0; 0)|^2 \\ &= \exp \left\{ -P_1 \left[1 - \frac{P_1}{Q} \left(1 + \frac{1}{K_2} \right)^2 \right] \tau^2 |\mathbf{v}_1|^2 + \frac{2P_1 Q_1}{Q} \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left(1 + \frac{1}{K_2}\right) \tau^2 \mathbf{v}_1 \cdot \mathbf{v}_2 - \frac{Q_1(Q_2 + Q_3)}{Q} \tau^2 |\mathbf{v}_2|^2 \Big\} \\
& \times \exp \left\{ -2 \left[\frac{P_1}{K_2} \left(1 - \frac{Q_3}{Q} \frac{1 + K_2}{1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right)} \right) \tau \mathbf{v}_1 \right. \right. \\
& \quad \left. \left. - \frac{Q_1 Q_3}{Q} \frac{1}{1 + K_2 \left(1 - \frac{P_3}{P} \frac{1}{1 + K_1} \right)} \tau \mathbf{v}_2 \right] \cdot (\mathbf{u}'_3 - \mathbf{u}_3) \right\} \\
& \times \exp \left(-\frac{|\mathbf{u}'_3 - \mathbf{u}_3|^2}{\zeta_3^2} \right) \exp \left(-\frac{|\mathbf{u}'_3 + \mathbf{u}_3|^2}{q_3^2} \right), \quad (4.27)
\end{aligned}$$

where $\Delta I_3 = I_3 - \langle I_3 \rangle$. In the following analysis, we examine the dynamic properties of doubly-scattered intensity fluctuations based on the second-order statistics described by Eq. (4.27).

4.2.2 Correlation Time of Doubly-Scattered Speckles

Now we investigate the temporal behaviour of the doubly-scattered speckle intensity detected at a single point because it gives us useful information about the velocity measurements of the moving diffusers. If the detecting point is set along the optical axis, the autocorrelation function of the speckle intensity fluctuations becomes

$$\begin{aligned}
\gamma_{\Delta I_3}(0, 0; \tau) = & \exp \left\{ -P_1 \left[1 - \frac{P_1}{Q} \left(1 + \frac{1}{K_2} \right)^2 \right] \tau^2 |\mathbf{v}_1|^2 + \frac{2P_1 Q_1}{Q} \right. \\
& \times \left. \left(1 + \frac{1}{K_2} \right) \tau^2 \mathbf{v}_1 \cdot \mathbf{v}_2 - \frac{Q_1(Q_2 + Q_3)}{Q} \tau^2 |\mathbf{v}_2|^2 \right\}. \quad (4.28)
\end{aligned}$$

The most fundamental and useful parameter for velocity measurements may be the correlation time τ_c defined by the $1/e$ correlation time of speckle intensity fluctuations. From Eq. (4.28), τ_c is given by

$$\begin{aligned}
\tau_c = & \left\{ P_1 \left[1 - \frac{P_1}{Q} \left(1 + \frac{1}{K_2} \right)^2 \right] |\mathbf{v}_1|^2 - \frac{2P_1 Q_1}{Q} \left(1 + \frac{1}{K_2} \right) \mathbf{v}_1 \cdot \mathbf{v}_2 \right. \\
& \left. + \frac{Q_1(Q_2 + Q_3)}{Q} |\mathbf{v}_2|^2 \right\}^{-1/2}. \quad (4.29)
\end{aligned}$$

Substitution of Eqs. (4.16) and (4.26) into Eq. (4.29) yields the final result given by

$$1/\tau_c = (A_1|\mathbf{v}_1|^2 - 2A_2\mathbf{v}_1 \cdot \mathbf{v}_2 + A_3|\mathbf{v}_2|^2)^{1/2}, \quad (4.30)$$

where

$$\left. \begin{aligned} A_1 &= \frac{2}{\beta_1^2} \left(\frac{2}{\beta_2^2} + B \right) / \left[\frac{2}{\beta_1^2} \left(1 + \frac{1}{K_2} \right)^2 + \frac{2}{\beta_2^2} + B \right], \\ A_2 &= \frac{2}{\beta_1^2} \frac{2}{\beta_2^2} \left(1 + \frac{1}{K_2} \right) / \left[\frac{2}{\beta_1^2} \left(1 + \frac{1}{K_2} \right)^2 + \frac{2}{\beta_2^2} + B \right], \\ A_3 &= \frac{2}{\beta_2^2} \left[\frac{2}{\beta_1^2} \left(1 + \frac{1}{K_2} \right)^2 + B \right] / \left[\frac{2}{\beta_1^2} \left(1 + \frac{1}{K_2} \right)^2 + \frac{2}{\beta_2^2} + B \right], \\ B &= \frac{1}{w^2} \left(1 + \frac{1}{K_2} \right)^2 + \left(\frac{\pi w}{\lambda R_1} \right)^2 \left[(1 + K_1) \left(1 + \frac{1}{K_2} \right) - 1 \right]^2. \end{aligned} \right\} \quad (4.31)$$

Before analyzing Eqs. (4.30) and (4.31) in detail, the correlation time for the single scattering case is evaluated. By setting $\mathbf{v}_1 = \mathbf{v}_2$ and $R_1 = 0$, Eq. (4.30) reduces to

$$\frac{1}{\tau_c} = \left\{ \left(\frac{2}{\beta_1^2} + \frac{2}{\beta_2^2} \right)^{-1} + \left[\frac{1}{w^2} + \left(\frac{\pi w}{\lambda R_2} \right)^2 \left(1 + \frac{R_2}{R} \right)^2 \right]^{-1} \right\}^{-1/2} |\mathbf{v}_1|. \quad (4.32)$$

Equation (4.32) can also be obtained by the same process used to derive Eq. (4.30) from Eq. (4.23). That is, by calculating the normalized correlation function of speckle intensity fluctuations from Eq. (4.14) and by inserting $\mathbf{u}_2 = \mathbf{u}'_2 = 0$ to evaluate the $1/e$ correlation time, the reciprocal of the correlation time is given as

$$\frac{1}{\tau_c} = \left\{ \frac{\beta_1^2}{2} + \left[\frac{1}{w^2} + \left(\frac{\pi w}{\lambda R_1} \right)^2 \left(1 + \frac{R_1}{R} \right)^2 \right]^{-1} \right\}^{-1/2} |\mathbf{v}_1|. \quad (4.33)$$

Comparison of Eqs. (4.32) and (4.33) leads to a conclusion that Eq. (4.30) is valid not only for the doubly-scattered speckles but also for the singly-scattered ones, provided that the effective correlation length β is estimated as

$$\beta = \left(\frac{\beta_1^2 \beta_2^2}{\beta_1^2 + \beta_2^2} \right)^{1/2}. \quad (4.34)$$

Equation (4.33) confirms that the reciprocal of the correlation time is proportional to the velocity of a diffuser regardless of the optical configurations which determine only the sensitivity of τ_c to the velocity. In addition, it is well known that the term in Eq. (4.33) including the correlation time of the diffuser can be neglected if it is small enough to form a circular Gaussian speckle field.

For the double diffusers, it is obvious from Eq.(4.30) that the correlation time τ_c of the doubly-scattered speckle intensity depends on the velocities of two moving diffusers. Moreover, the linear relationship no longer holds between the reciprocal value $1/\tau_c$ of the correlation time and the velocity $\mathbf{v}_1$ or $\mathbf{v}_2$ of each diffuser. However, when only a single diffuser moves, we can see that the value of $1/\tau_c$ is linearly proportional to the velocity of the moving diffuser. Consequently, the velocity of a diffuser can be measured from the correlation time even in the presence of another motionless diffuser. This result has already been confirmed experimentally [30].

Since the speckles from the two diffusers moving simultaneously are practically interesting, Eqs. (4.30) and (4.31) are examined in detail by setting the values of $\mathbf{v}_1$ and $\mathbf{v}_2$ to non-zeros. In addition to the variables $\mathbf{v}_1$ and $\mathbf{v}_2$, Eq. (4.31) consists of variables characterizing optical configurations. Therefore the correlation time is analyzed as a function of the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ by changing the illuminating and detecting conditions.

To obtain some interesting features of doubly-scattered speckles, two limiting cases are considered. The first case employs the following relations:

$$B \ll \frac{2}{\beta_1^2}, \quad B \ll \frac{2}{\beta_2^2}, \quad K_2 \gg 1. \quad (4.35)$$

In this case, Eq. (4.31) is simplified to

$$A_1 = A_2 = A_3 = 2/(\beta_1^2 + \beta_2^2), \quad (4.36)$$

and the correlation time takes the simple form

$$\frac{1}{\tau_c} = \left[\left(\frac{\beta_1^2 + \beta_2^2}{2} \right)^{-1/2} \right] |\mathbf{v}_1 - \mathbf{v}_2|. \quad (4.37)$$

From Eq. (4.37) it is found that $1/\tau_c$ is directly proportional to the absolute value of the velocity difference $\mathbf{v}_1 - \mathbf{v}_2$ of the two moving diffusers. Moreover, the reciprocal

of the proportional constant in Eq. (4.37) is a root-mean-square value of the effective correlation lengths of the two diffusers. Therefore, if the optical systems are realized to satisfy the relations (4.35), the velocity difference of two moving diffusers can be measured from the speckle intensity fluctuations detected at one point. From now on, an abbreviation of $|\mathbf{v}_1 - \mathbf{v}_2| = v_r$ will be used and is called a relative velocity. For the special case where the effective correlation lengths of the two diffusers are equal with $\beta_1 = \beta_2 = \beta$, the correlation time in Eq. (4.37) is further reduced to

$$1/\tau_c = \frac{1}{\beta} v_r. \quad (4.38)$$

In this case, the proportional constant becomes exactly the reciprocal value of the effective correlation length, which is proportional to the averaged slopes of surface roughness if scatter plates like ground-glass plates are used as diffuse objects.

In order to examine whether the illuminating and detecting conditions satisfying the relations (4.35) exist, we find the corresponding relations in terms of the following five variables describing the optical configurations:

1. Width w of the illuminating beam spot,
2. Ratio $K_1 = R_1/R$,
where R is the radius of wavefront curvature of the illuminating beam and R_1 is the spacing between the two diffusers,
3. Ratio $K_2 = R_2/R_1$,
where R_2 is the distance between the second diffuser and the detecting point,
4. Effective correlation lengths β_1 and β_2 of the two diffusers.

To simplify the analysis, the wavelength λ and the spacing R_1 are fixed. It is seen from the expression for B in Eq. (4.31) that the value of B takes a minimum at a certain finite width w_s of the beam spot. This value is easily obtained as

$$w_s = \left\{ \frac{1 + \frac{1}{K_2}}{\frac{\pi}{\lambda R_1} \left[(1 + K_1) \left(1 + \frac{1}{K_2} \right) - 1 \right]} \right\}^{1/2}. \quad (4.39)$$

By substituting Eq. (4.39) into Eq. (4.31), B becomes

$$B = \frac{2\pi}{\lambda R_1} \left(1 + \frac{1}{K_2}\right) \left[(1 + K_1) \left(1 + \frac{1}{K_2}\right) - 1 \right]. \quad (4.40)$$

Figure 4.2 shows the theoretical curves of the reciprocal of the correlation time τ_c as a function of the diffuser velocity v_1 for three values of the illumination spot width (a) $w = 10 \mu\text{m}$; (b) $w = 41.28 \mu\text{m}$; (c) $w = 200 \mu\text{m}$. In the following analysis, the scalar notation of v_1 and v_2 instead of $\mathbf{v}_1$ and $\mathbf{v}_2$ is used by restricting the moving direction of the second diffuser to take the same or the opposite direction of the first diffuser. The values of the other parameters are $v_2 = 1 \text{ mm/s}$, $K_1 = 0.5$, $K_2 = 10$, $\beta_1 = \beta_2 = 1 \mu\text{m}$, $R_1 = 5 \text{ mm}$ and $\lambda = 632.8 \text{ nm}$. It is obvious from Fig. 4.2 that, when the two diffusers move simultaneously, $1/\tau_c$ is not proportional to the velocity v_1 , rather $1/\tau_c$ decreases once with increasing v_1 , reaches a minimum, and then increases with a further increase of v_1 . Under the above settings of the parameters, the value of w , taken to minimize the value of B is $41.28 \mu\text{m}$, corresponding to curve

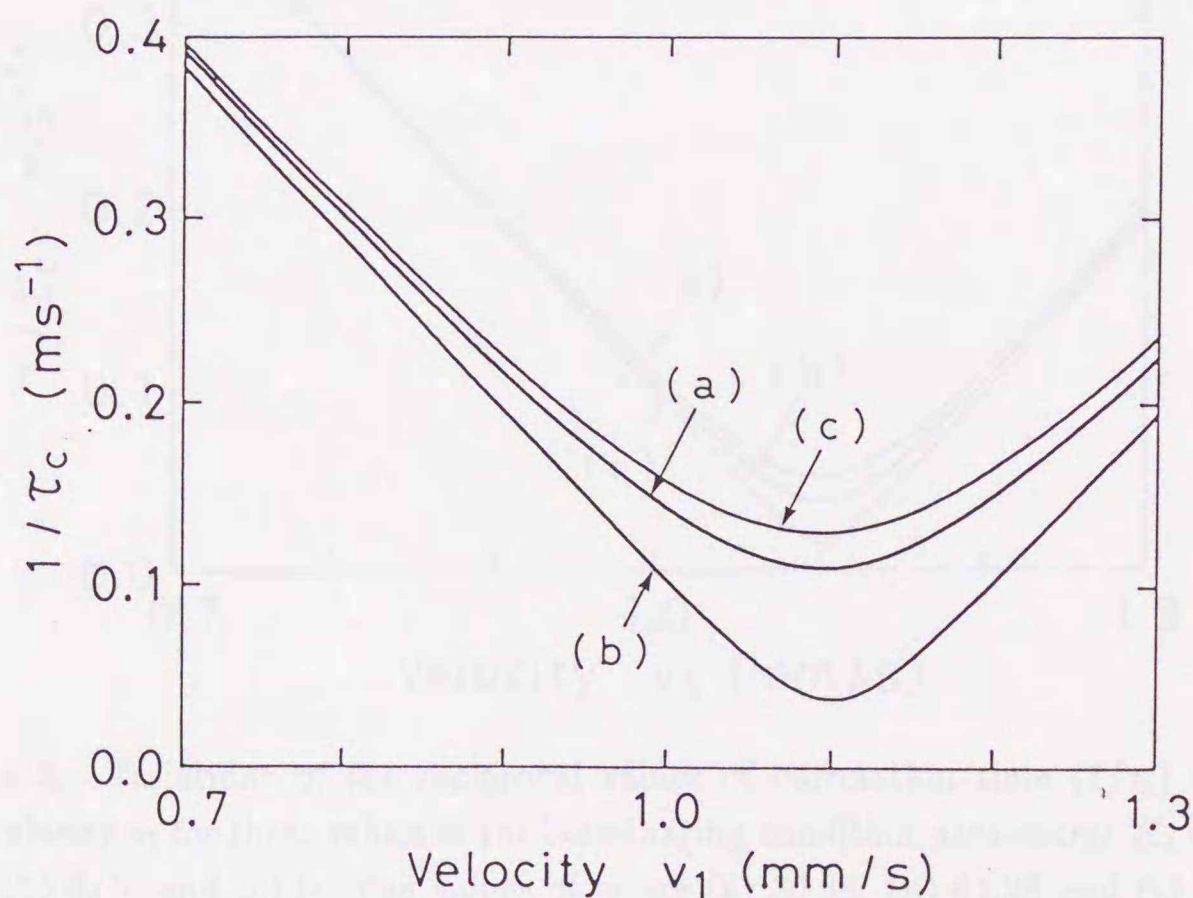


Figure 4.2. Variations of the reciprocal values of correlation time ($1/\tau_c$) as a function of the diffuser velocity v_1 for three values of the illumination spot width: $w =$ (a) 10, (b) 41.28 and (c) 200 μm . Other parameters are fixed: $K_1 = R_1/R = 5/10$, $K_2 = R_2/R_1 = 50/5$, $\beta_1 = \beta_2 = 1 \mu\text{m}$, $v_2 = 1 \text{ mm/s}$ and $\lambda = 632.8 \text{ nm}$.

(b) in Fig. 4.2. As is seen from the figure, the minimum values of $1/\tau_c$ become the smallest when the radius of the illumination spot is set equal to w_s .

The illuminating condition can also be varied by changing the parameter K_1 defined by the ratio of the spacing R_1 to the wavefront-curvature radius R . The variation of $1/\tau_c$ with the velocity v_1 of the first diffuser is plotted in Fig. 4.3 for the various values of K_1 : (a) $K_1 = 1$; (b) $K_1 = 0.5$; (c) $K_1 = 0$. The other parameters are equal to the values taken above, except that the illumination spot is set to be w_s calculated by Eq. (4.39) for each value of K_1 . Figure 4.3 shows that the minimum value of $1/\tau_c$ becomes smaller as the value of K_1 approaches zero. In other words, the sharpest bending of the $1/\tau_c$ curve is obtained if the diffusers are illuminated by a plane wave having the spot width w_s . This condition corresponds to the illumination at the beam waist with the waist radius of $w_0 = w_s$. In this case, the spot radius

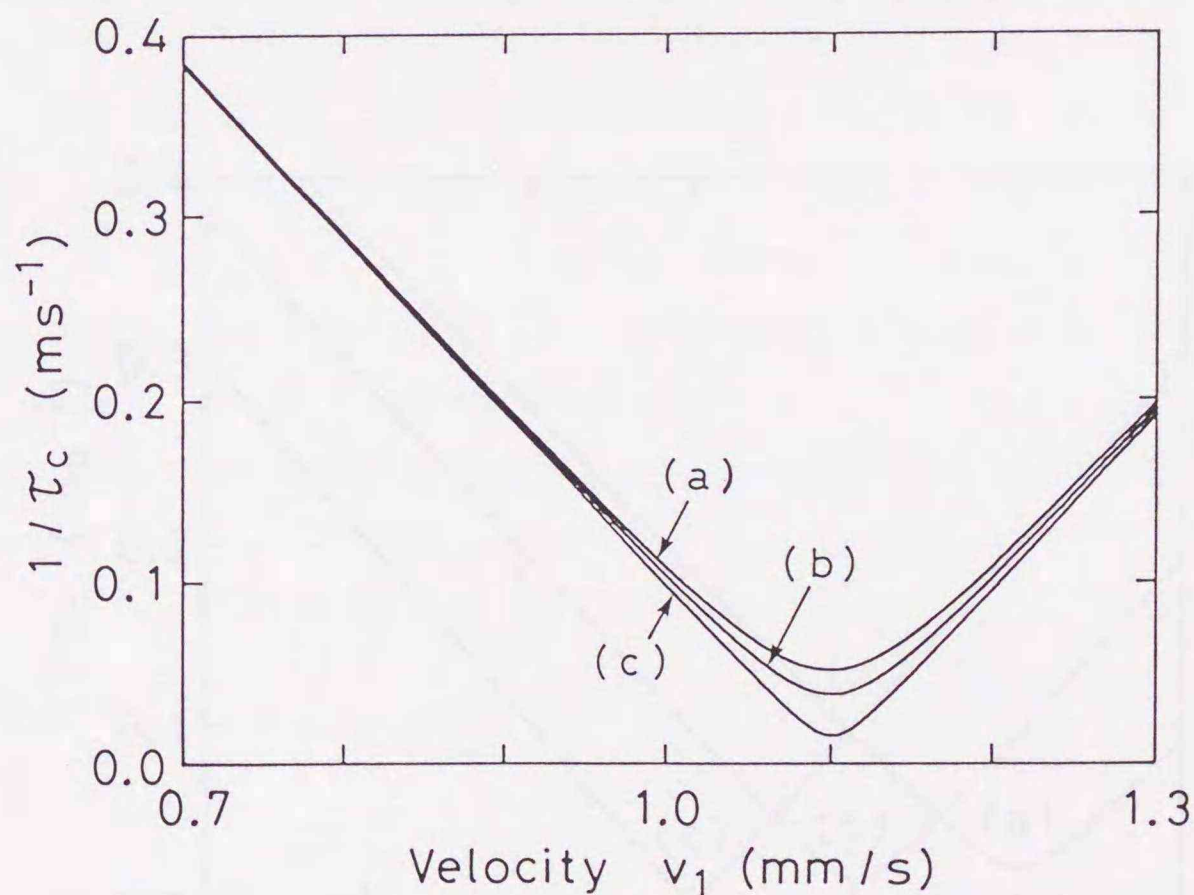


Figure 4.3. Variations of the reciprocal values of correlation time ($1/\tau_c$) versus the diffuser velocity v_1 for three values of the illuminating condition parameter: $K_1 = R_1/R =$ (a) 5/5, (b) 5/10 and (c) 0. The values of w are (a) 30.38, (b) 41.28 and (c) 105.3 μm . Other parameters are the same as those in Fig. 4.2.

w_s and B are found to be

$$w_s = \left(\frac{1 + \frac{1}{K_2}}{\frac{\pi}{\lambda R_1} \frac{1}{K_2}} \right)^{1/2}, \quad (4.41)$$

$$B = \frac{2\pi}{\lambda R_1} \frac{1}{K_2} \left(1 + \frac{1}{K_2} \right). \quad (4.42)$$

It is clearly seen from Eq. (4.42) that the value of B draws closer to zero when the value of K_2 approaches infinity. This fact indicates that all the conditions of the relations (4.35) can be satisfied at the same time. Therefore, if the detecting point is located in the far field of the second diffuser, the reciprocal value of the correlation time is directly proportional to the relative velocity v_r of the two moving diffusers. This result is shown in Fig. 4.4, where the values of $1/\tau_c$ are plotted against v_1 for three pairs of K_2 and w_s : (a) $(K_2, w_s) = (5, 77.74 \mu\text{m})$; (b) $(K_2, w_s) = (10, 105.3 \mu\text{m})$; (c) $(K_2, w_s) = (50, 710.3 \mu\text{m})$.

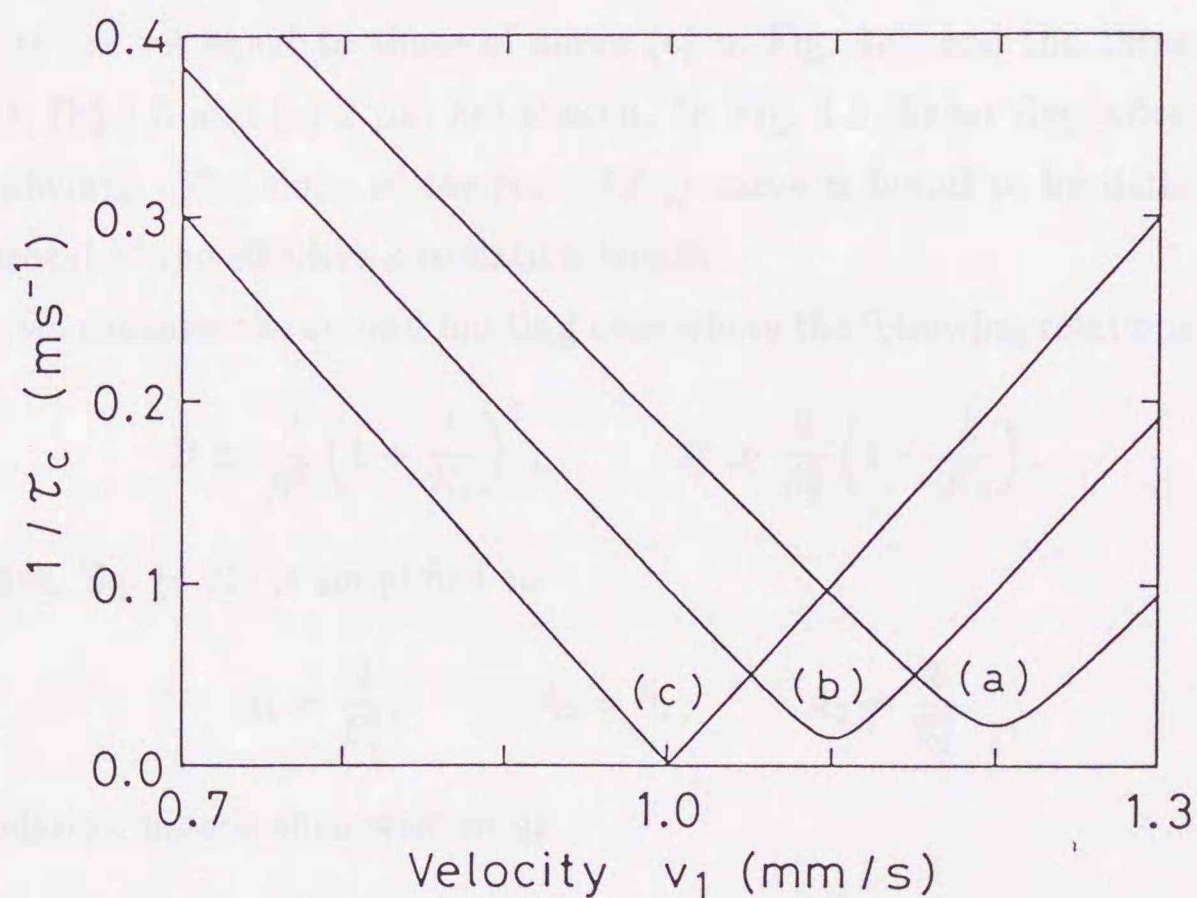


Figure 4.4. Variations of the reciprocal values of correlation time ($1/\tau_c$) versus the diffuser velocity v_1 for three values of the detecting condition parameter: $K_2 = R_2/R_1 =$ (a) 25/5, (b) 50/5 and (c) $5 \times 10^4/5$. The values of w are (a) 77.74, (b) 105.3 and (c) 710.3 μm , and the value of K_1 is fixed to be zero. Other parameters are the same as those in Fig. 4.2.

μm); (c) $(K_2, w_s) = (10^4, 710.3 \mu\text{m})$, and where the other parameters are set equal to those in Fig. 4.3 except that the value of K_1 is set to be zero. Curve (c) in Fig. 4.4 corresponds to the exact plots of Eq. (4.37). It is worth noting in Fig. 4.4 that the velocity v_1 at which $1/\tau_c$ takes a minimum approaches the velocity v_2 as the detecting point is set farther from the diffusers.

From the above analysis, it is found that, if speckle intensity fluctuations are detected at a single point in the far field under illumination at the beam waist with the width defined by Eq. (4.41), the relative velocity v_r of two moving diffusers can be measured by using the relation of Eq. (4.37) or Eq. (4.38). However, it should be noted that the roughness of the moving diffusers of interest has a strong effect on the correlation time of detected speckle signals. This is a significant feature of doubly-scattered speckles. On the contrary, for single scattered speckles, the roughness of a diffuser scarcely affects the statistical properties of speckle fluctuations as long as it introduces the phase fluctuations greater than 2π . This effect of roughness is illustrated in Fig. 4.5. In the figure, the curves of $1/\tau_c$ versus v_r are plotted for the various values of the effective correlation length β of the two diffusers. The parameters are set equal to those of curve (c) in Fig. 4.4, and the three values of $\beta =$ (a) 1, (b) 1.5 and (c) $2 \mu\text{m}$ are chosen. In Fig. 4.5, linear dependence of $1/\tau_c$ on v_r is obvious. The slope of the $(v_r - 1/\tau_c)$ curve is found to be determined by the reciprocal of the effective correlation length.

Next, we consider the second limiting case where the following relations are used:

$$B \gg \frac{2}{\beta_1^2} \left(1 + \frac{1}{K_2}\right)^2, \quad B \gg \frac{2}{\beta_2^2} \left(1 + \frac{1}{K_2}\right). \quad (4.43)$$

In this case, Eq. (4.31) is simplified to

$$A_1 = \frac{2}{\beta_1^2}, \quad A_2 = 0, \quad A_3 = \frac{2}{\beta_2^2}. \quad (4.44)$$

The correlation time is then written as

$$\frac{1}{\tau_c} = \left(\frac{2}{\beta_1^2} |\mathbf{v}_1|^2 + \frac{2}{\beta_2^2} |\mathbf{v}_2|^2 \right)^{1/2}. \quad (4.45)$$

If $\beta_1 = \beta_2 = \beta$, Eq. (4.45) can be rewritten in the form

$$1/\tau_c = (2/\beta)v_a, \quad (4.46)$$

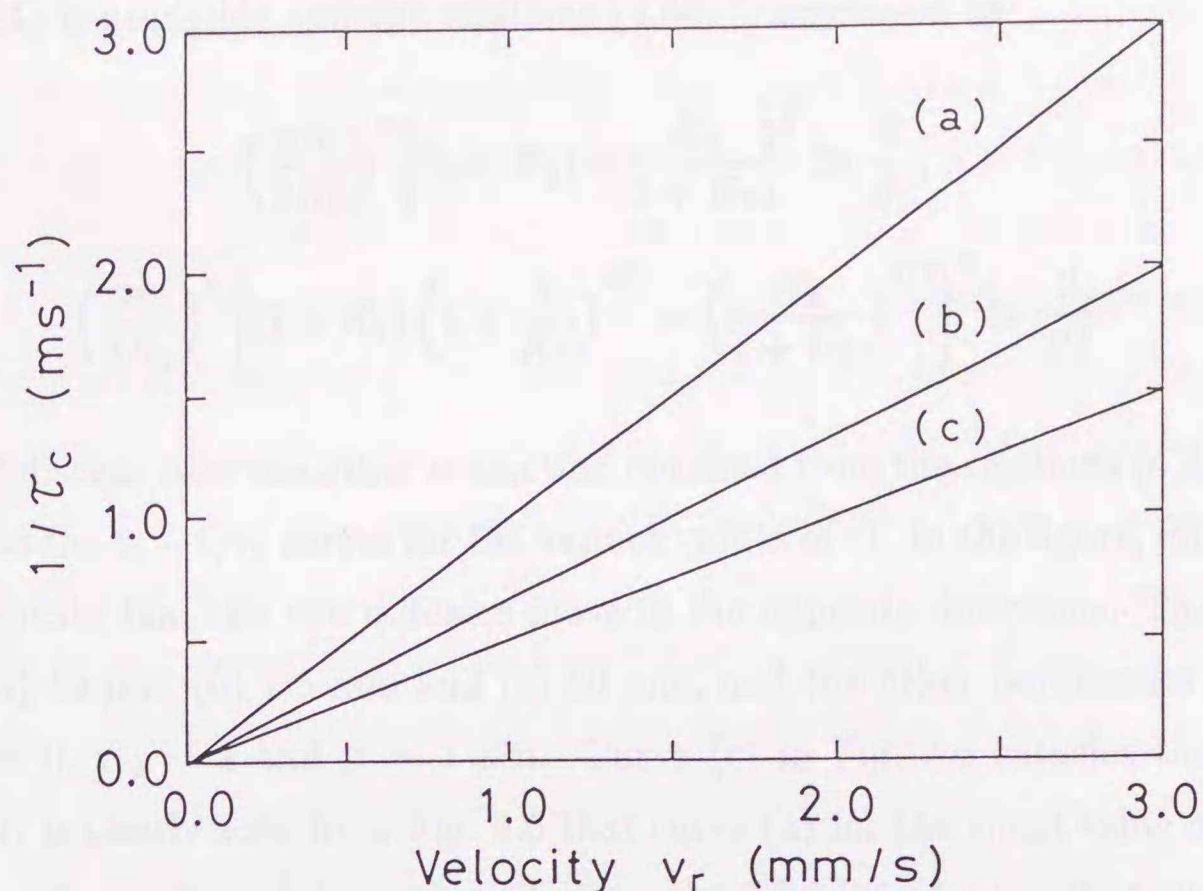


Figure 4.5. Reciprocal values of the correlation time plotted against the relative velocity $v_r = |\mathbf{v}_1 - \mathbf{v}_2|$ for three values of the effective correlation length: $\beta_1 = \beta_2 =$ (a) 1, (b) 1.5 and (c) 2 μm . Other parameters are the same as those in Fig. 4.4 except that the value of K_2 is fixed to be 10^4 .

where

$$v_a = \left(\frac{|\mathbf{v}_1|^2 + |\mathbf{v}_2|^2}{2} \right)^{1/2}. \quad (4.47)$$

Equation (4.46) indicates that the reciprocal of the correlation time is directly proportional to the root-mean-square value v_a of the velocities of two diffusers. In what follows, we examine the possibility that this relationship is realized in actual optical configurations. The same variables as discussed so far are chosen as the parameters: spot width w , illuminating condition K_1 , detecting condition K_2 and effective correlation lengths β_1 and β_2 .

As indicated by Eq. (4.31), the value of B becomes larger by setting the value of w much smaller or much larger than the values of β_1 and β_2 . The condition of $w \gg \beta_1$ and $w \gg \beta_2$ is probably more appropriate than $w \ll \beta_1$ and $w \ll \beta_2$ because a large number of independent scatterers are assumed to exist within the illuminating area. For $w \gg \beta_1$ and $w \gg \beta_2$, the first term on the right hand side of

Eq. (4.31) is negligible and the relations (4.43) is expressed as

$$\left(\frac{\pi w}{\lambda R_1}\right)^2 \left[(1 + K_1) - \frac{K_2}{1 + K_2} \right]^2 \gg \frac{2}{\beta_1^2},$$

$$\left(\frac{\pi w}{\lambda R_1}\right)^2 \left[(1 + K_1) \left(1 + \frac{1}{K_2}\right)^{1/2} - \left(\frac{K_2}{1 + K_2}\right)^{1/2} \right]^2 \gg \frac{2}{\beta_2^2}. \quad (4.48)$$

We will discuss later the other restriction obtained from the relations (4.48). Figure 4.6 shows the $v_1 - 1/\tau_c$ curves for the various values of w . In the figure, minus values of v_1 indicate that the two diffusers move in the opposite directions. The values of w are (a) 50 μm , (b) 1.5 mm and (c) 50 mm, and the other parameters are set to be $K_1 = 0$, $K_2 = 1$ and $\beta = 1 \mu\text{m}$. Curve (c) in Fig. 4.6 satisfies the relations (4.48). It is clearly seen from Fig. 4.6 that curve (a) for the small value of w shows the behaviour of the relative-velocity curve given by Eq. (4.37), while curve (c) for the large value of w is approximately given by Eq. (4.45) corresponding to the root-

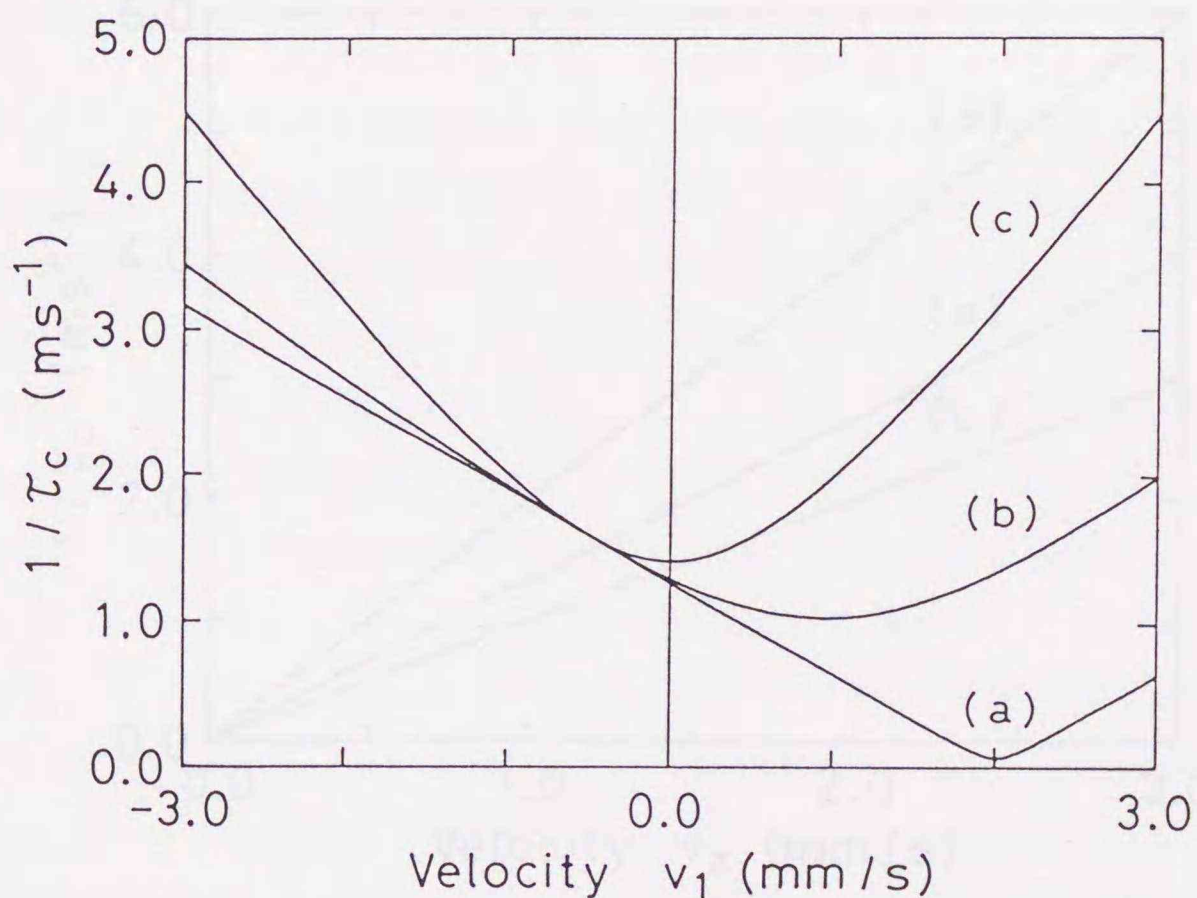


Figure 4.6. Variations of the reciprocal values of correlation time ($1/\tau_c$) as a function of the diffuser velocity v_1 for three values of the illumination spot width: $w =$ (a) 50 μm , (b) 1.5 mm and (c) 50 mm. Other parameters are fixed: $K_1 = R_1/R = 0$, $K_2 = R_2/R_1 = 5/5$, $\beta_1 = \beta_2 = 1 \mu\text{m}$, $v_2 = 1 \text{ mm/s}$ and $\lambda = 632.8 \text{ nm}$.

mean-square velocity (abbreviated as the r.m.s. velocity) curve. This tendency is observed when the two diffusers move simultaneously, and disappears if one of the diffusers is kept motionless. Figure 4.7 shows the theoretical curves of $1/\tau_c$ as a function of the r.m.s. velocity v_a for the various values of the effective correlation length β : (a) $\beta = 1 \mu\text{m}$; (b) $\beta = 1.5 \mu\text{m}$; (c) $\beta = 2 \mu\text{m}$. The values of the other parameters are set equal to those of curve (c) in Fig. 4.6. It is obvious from Fig. 4.7 that the reciprocal of the correlation time is proportional to the r.m.s. velocity and that the slopes of the curves are dependent on the effective correlation length of the diffusers. It is interesting to note that Figs. 4.5 and 4.7 appear to be very similar except for the variable of horizontal axis and the magnification of the vertical axis.

Before concluding the theoretical analysis, we discuss the possibility of measuring in practice the two quantities of the relative velocity and the r.m.s. velocity. The relative velocity can be measured because the necessary conditions for the beam waist illumination and the far-field detection can be easily realized. On the other

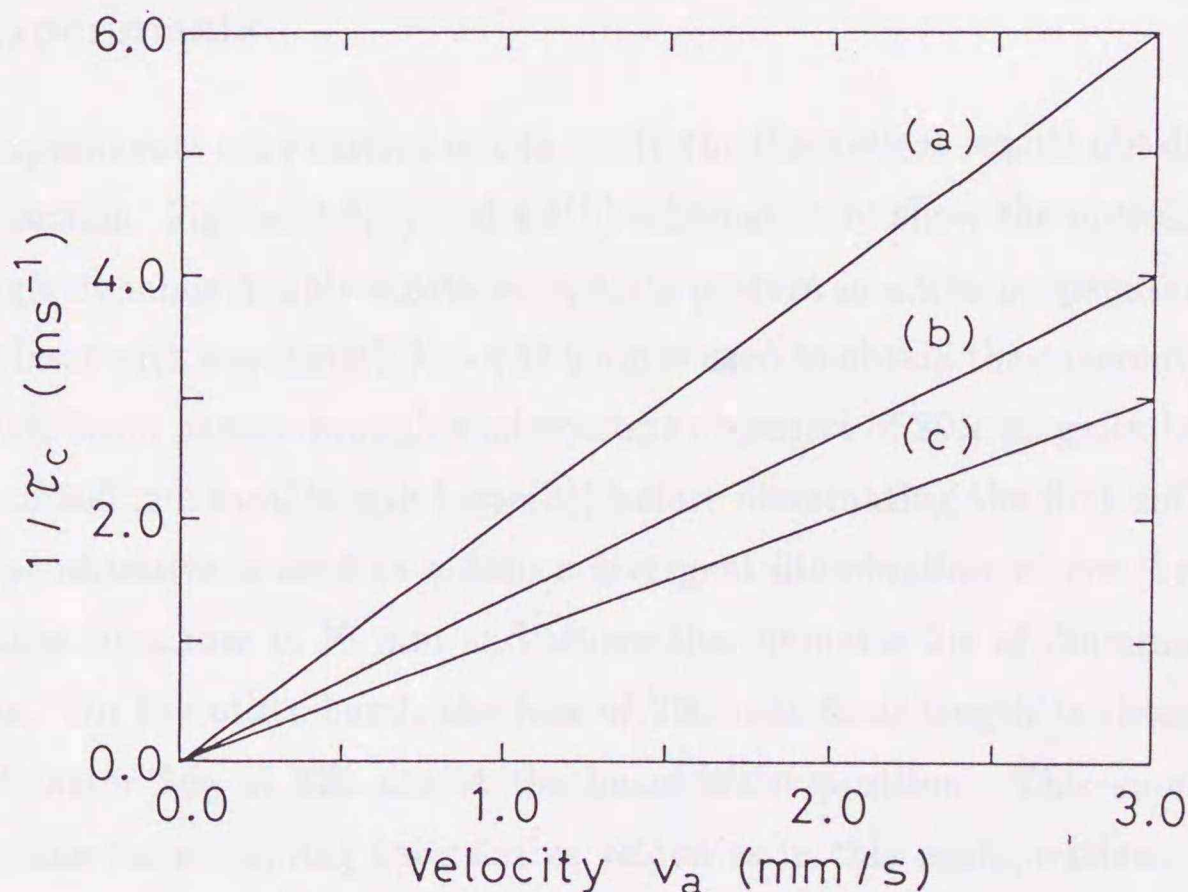


Figure 4.7. Reciprocal values of the correlation time plotted against the root-mean-square velocity v_a for three values of the effective correlation length: $\beta_1 = \beta_2 =$ (a) 1, (b) 1.5 and (c) $2 \mu\text{m}$. Other parameters are the same as those in Fig. 4.6 except that the value of w is fixed to be 50 mm.

hand, measurements of the r.m.s. velocity have some difficulties. To satisfy the relations (4.48), large values of K_1 and small values of K_2 are required. In other words, the diffusers to be measured should be illuminated by a divergent beam with a large spot width and the detector should be placed near the diffusers. However, these restrictions lead to several problems. For example, there may be some objects which do not have enough area to be illuminated, or there may be some situations where the detector cannot be set closely to the objects. The most serious problem is that the speckle signals detected near the object are integrated over a finite detector area because the speckle grains become very small as a result of the illumination over a wide area of the object. The statistical properties of these integrated speckles become different from those of ideal speckle signals considered above. If a very small detecting aperture is used to avoid this problem, then it may bring a decrease in the signal-noise ratio. All these problems make it impractical to measure the average velocities of moving diffusers from the correlation time of intensity fluctuations.

4.3 Experiments

Several experiments were carried out to verify the theoretical results obtained in the previous section. Figures 4.8(a) and 4.8(b) schematically show the optical setup for producing a dynamic doubly-scattered speckle pattern in a free-propagation system. A He-Ne laser with wavelength $\lambda = 632.8$ nm is used to obtain the coherent Gaussian beam. This beam passes through a microscope objective of $20\times$ magnitude (case(a)) or a lens of 200 mm focal length (case(b)) before illuminating the first diffuser. The microscope objective is used to obtain a divergent illumination where the radius R of wavefront-curvature is 10 mm and where the diameter $2w$ of illuminating spot is 1.0 mm. On the other hand, the lens of 200 mm focal length is chosen to give a spot diameter $2w_0$ of $220\ \mu\text{m}$ at the beam waist position. This spot size is a suitable value for measuring the relative velocities in this configuration. A second diffuser is placed R_1 from the first diffuser, and both diffusers are mounted on moving stages to be shifted continuously with constant velocities. Ground-glass plates having the various values of roughness (#120, #200, #500 and #600) are used as the diffusers, all of which meet the conditions mentioned in the previous

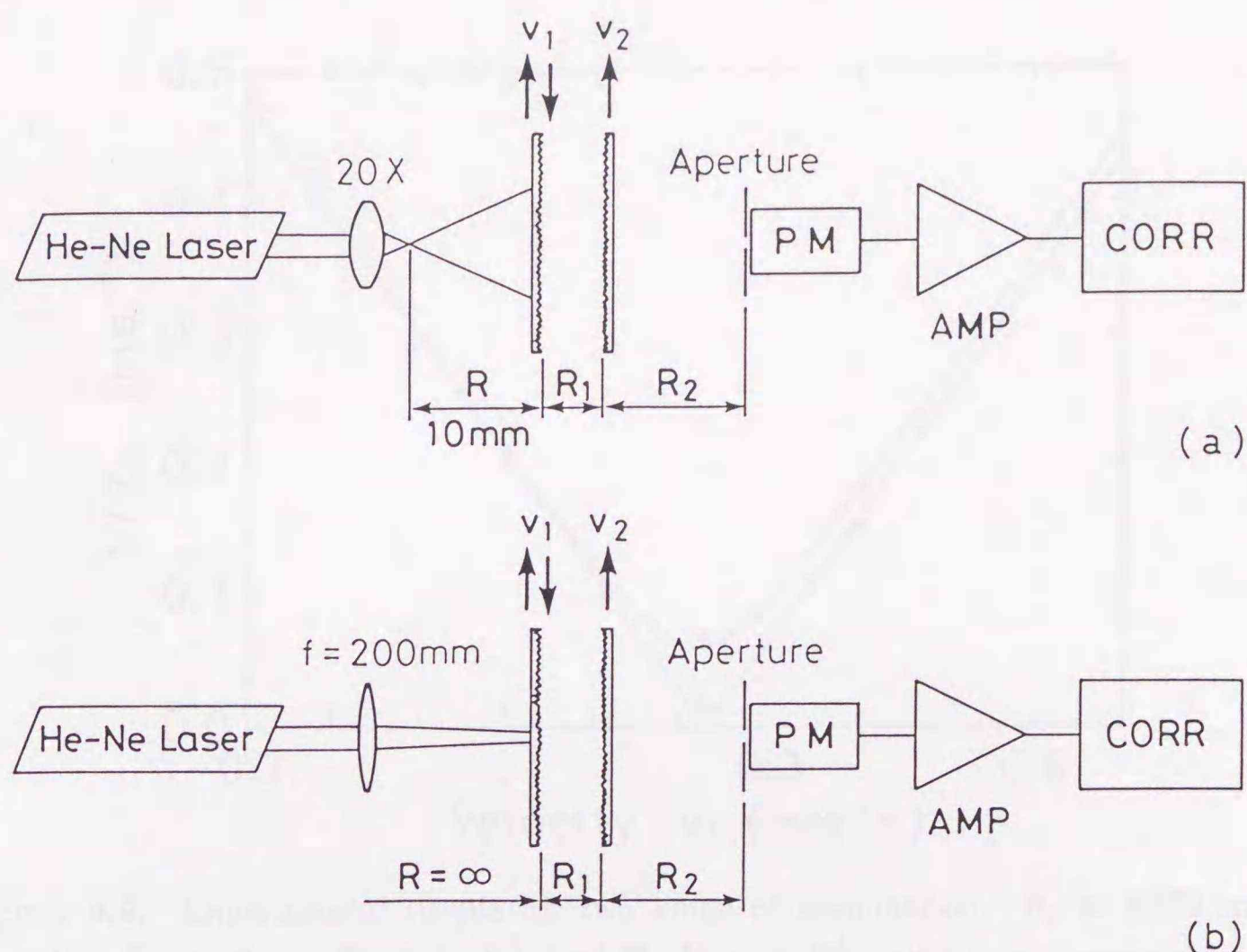


Figure 4.8. Optical setup: (a) divergent illumination and (b) beam-waist illumination.

section. The intensity fluctuations of speckles are detected by a photomultiplier PM via an aperture of diameter $10\text{ }\mu\text{m}$ located R_2 from the second diffuser. The time-varying speckle signals are amplified and fed into a FFT correlator CORR to measure the normalized autocorrelation function. The correlation time τ_c is finally obtained as the $1/e$ correlation time after averaging 64 autocorrelation functions, each of which is calculated by using 1024 sampling data.

The experiments were conducted aiming to measure only the relative velocity because the theoretical analysis has shown some difficulties of measuring the average velocity in practice. First, we examined the effect of the radius of wavefront-curvature and the size of illuminating spot on the correlation time. Figure 4.9 shows the experimental results obtained under the divergent illumination (indicated by ●) and the beam waist illumination (indicated by ■). The other parameters were set equal in both configurations: the spacing between the two ground-glass plates was 1 mm and the distance between the second ground-glass plate and the detecting aperture was 100 mm. Two ground-glass plates of #500 were used as the diffusers

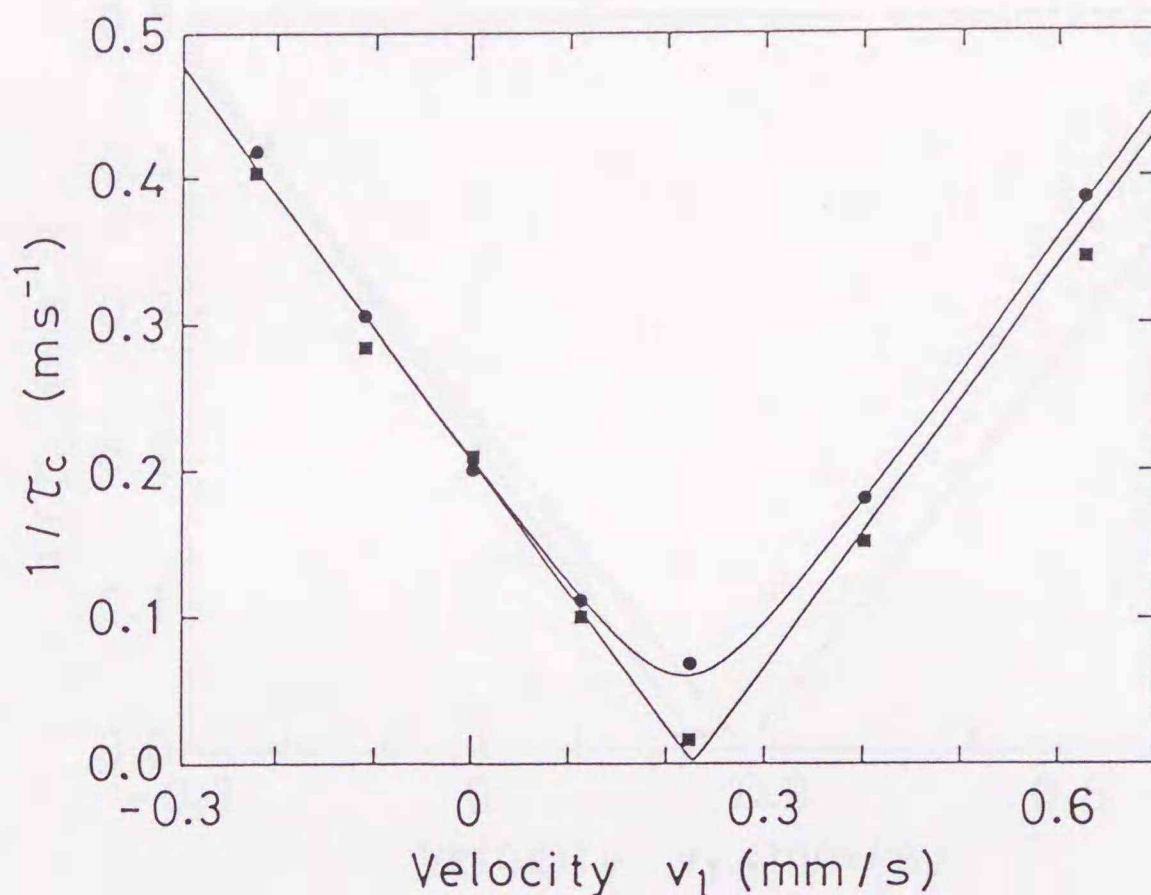


Figure 4.9. Experimental results for two kinds of illumination. $v_2 = 0.222$ mm/s, $K_2 = 100$, $\beta = 1.10$ μm ; ●, $K_1 = 0.1$, and ■, $K_1 = 0$. The solid lines correspond to the theoretical curves of Eq. (4.30).

and the moving velocity v_2 of the second diffuser was fixed at 0.222 mm/s. The solid lines in Fig. 4.9 indicate the theoretical curves evaluated by substituting the given parameters in both arrangements into Eqs. (4.30) and (4.31). It should be noted here that the values, β_1 and β_2 , of the effective correlation length of the ground-glass plates used in experiments were unknown. Thus they were chosen to give the best fit to the experimental results. In this case, β_1 and β_2 become 1.1 μm for #500 plates. This ambiguity, however, does not violate the significance of these experiments, because it is probably impossible to obtain the theoretical curves fitting to all the experimental values only by changing the values of β_1 or β_2 . The fit to the data is good enough to confirm the theoretical results.

Next, we examined the effect of the variation of distances R_1 and R_2 on the correlation time. The experimental results are shown in Fig. 4.10, where the values measured for the near-field detection ($R_1 = 10$ mm, $R_2 = 50$ mm) and the far-field detection ($R_1 = 1$ mm, $R_2 = 100$ mm) are indicated by symbols ▲ and ● respectively. The theoretical curves corresponding to the experimental data in Fig. 4.10 were obtained by the same way mentioned above. In the experiments under

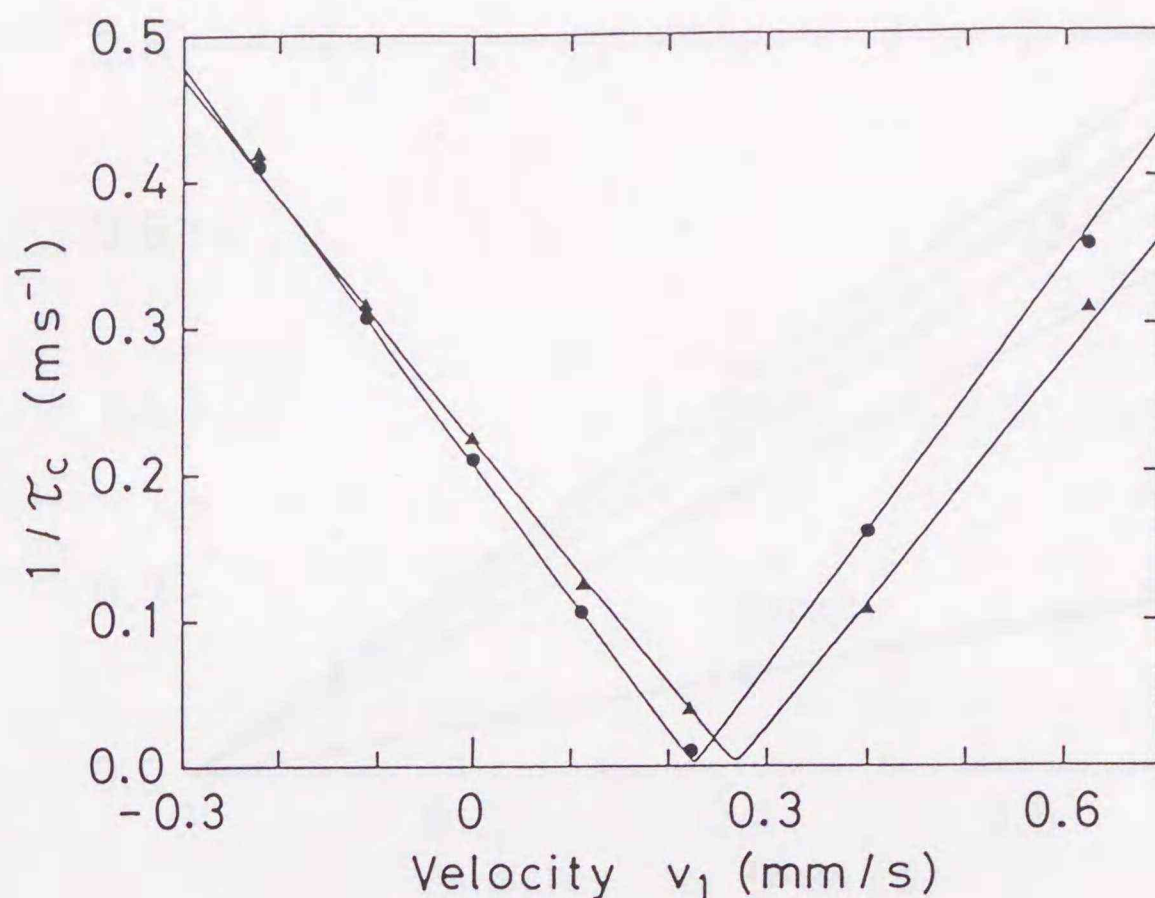


Figure 4.10. Experimental results for two kinds of diffuser spacing and detector position. The solid lines correspond to the theoretical curves of Eq. (4.30). $v_2 = 0.333$ mm/s, $K_1 = 0$, $\beta = 1.10$ μm ; ●, $K_2 = 100$, and ▲, $K_2 = 5$.

both the detecting conditions, the beam waist illumination shown in Fig. 4.8(b) was used and the other parameters were set equal to those given in Fig. 4.9. In addition, an aperture of diameter 2 μm instead of 10 μm was used for the near-field detection to avoid the integration of speckles. The experimental data in Fig. 4.10 show that an increase in the ratio $K_2 = R_2/R_1$ causes a decrease in the value of v_1 giving the minimum value of $1/\tau_c$.

Finally, we examined the roughness effect of the diffusers on the correlation time of singly- and doubly-scattered speckles. Since only the effect of roughness was our main concern, the velocity v_2 of the second diffuser was set to be zero and the divergent illumination shown in Fig. 4.8(a) was used in this experiment. Figure 4.11 shows the experimental results for four pairs of ground-glass plates having the different values of roughness, together with the fitted curves. The values of the effective correlation lengths of the ground-glass plates used were determined by the slopes of these corresponding curves and found to be 0.94, 1.01, 1.10 and 1.30 μm for #120 (indicated by ●), #200 (▲), #500 (■) and #600 (▼), respectively. As predicted in the theoretical analysis, it is clearly seen from Fig. 4.11 that the

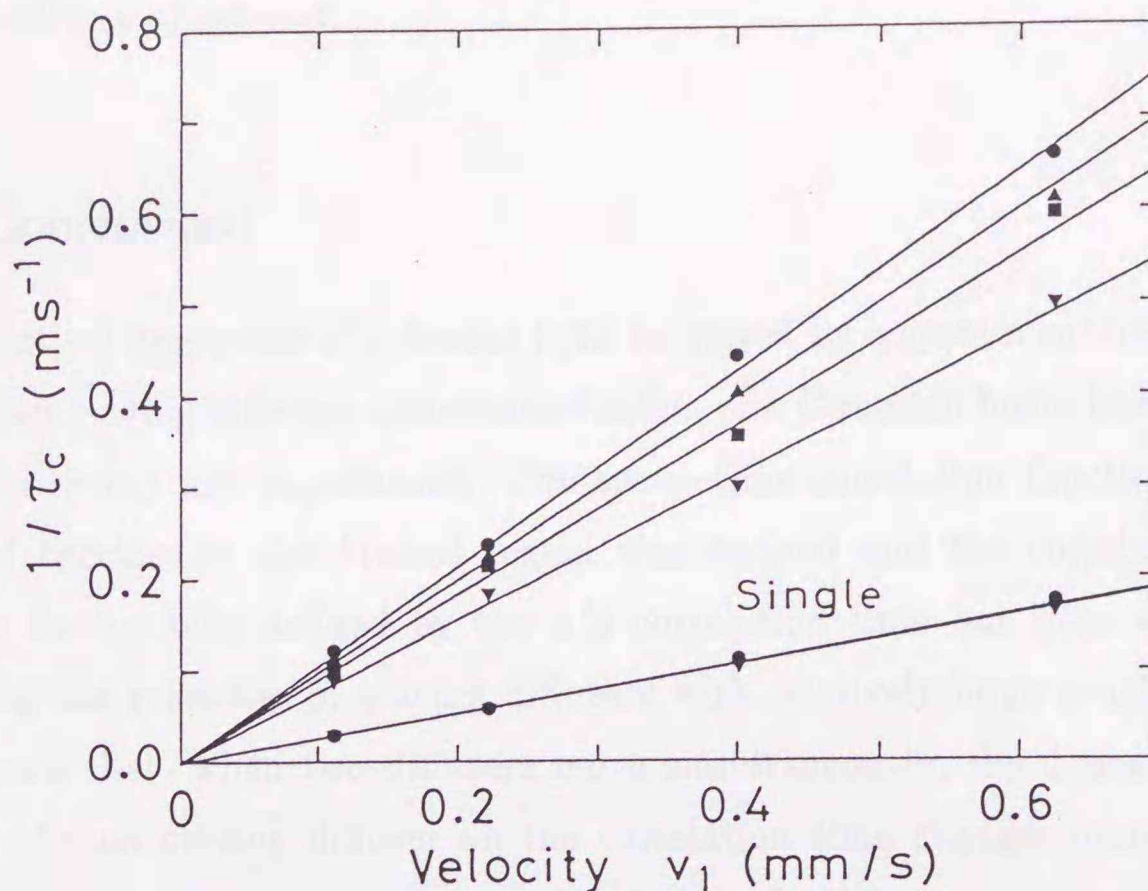


Figure 4.11. Experimental results for the various values of roughness of the ground-glass plates. The theoretical curves of Eq. (4.30) are evaluated for the double scattering case, and the solid line for the single scattering case is added for comparison. $v_2 = 0$, $K_1 = 0.1$, $K_2 = 100$; $\bullet$, $\beta = 0.94$; $\blacktriangle$, $\beta = 1.01$; $\blacksquare$, $\beta = 1.10$; and $\blacktriangledown$, $\beta = 1.30 \mu\text{m}$.

roughness of the diffusers strongly affects the correlation time of doubly-scattered speckles, while it has little effect on τ_c of singly-scattered speckles as long as the roughness is large enough.

Moreover, another feature of doubly-scattered speckles which has been pointed out by Iwai and Asakura [31] in connection with the velocity measurement of a single diffuser can be reconfirmed. Figure 4.11 shows that the time-varying intensity fluctuations of doubly-scattered speckles are more rapid than those of singly-scattered speckles. This result makes it advantageous to reduce the time of velocity measurements, if the zero-crossing counting method is employed. This zero-crossing counting method uses the fact that the number of the speckle intensities crossing the zero level per unit time is directly related to the velocity of the object. Hence, the more rapid the time-varying intensity fluctuations are, the shorter the unit time for counting the number of zeros can be set. This fact implies that, if the response of the electrical circuit counting the number of zeros is fast enough, the velocity can be measured more quickly by placing another motionless diffuser in front of the

moving diffuser of interest.

4.4 Conclusion

The statistical properties of coherent light scattered by a system consisting of a cascade of two moving diffusers under illumination of a Gaussian beam have been investigated in theory and experiment. The space-time correlation function of doubly-scattered speckles in the Fresnel region was derived and the correlation time of intensity fluctuations defined by the $1/e$ correlation time has been evaluated for measuring the velocities of moving diffusers with relatively large roughness. It has been shown that, when two diffusers move simultaneously, the dependence of the velocity of each moving diffuser on the correlation time changes according to optical arrangements, diffuser roughness, and the velocity of the other diffuser. Two variables related to the object velocity, namely the relative velocity and the root-mean-square velocity, have been introduced and the configurations of optical systems appropriate for measuring these quantities have been taken into consideration. The relative velocity of the two diffusers becomes proportional to the reciprocal of the correlation time when the diffuser is illuminated by a coherent Gaussian beam at the beam-waist position and when the intensity fluctuations are detected in the far field. In this case, illumination with a suitable spot size determined by the diffuser spacing and the position of the detector is also required. On the other hand, when a large area of the diffuser surface is illuminated by a divergent Gaussian beam and the intensity fluctuations are detected in the near field, the r.m.s. velocity of the two diffusers becomes proportional to the reciprocal of the correlation time. Several experiments were conducted to measure the relative velocities and excellent agreement between theoretical and experimental results was obtained. However, it is difficult to measure the r.m.s. velocity in the above configuration. Therefore it is required that other optical systems be studied.

Chapter 5

Speckles Formed in the Image Field under Dynamic Speckle-Pattern Illumination

5.1 Introduction

When a coherent light illuminates a moving diffuse object, a dynamic speckle pattern appears in the diffraction field or in the image field. Since this speckle pattern contains information about the motion of the diffuser, several methods have been proposed to measure the object velocity using the statistical properties of the speckle patterns [3]. For example, some methods are based on the fact that the decorrelation rate of speckle fluctuations is linearly proportional to the velocity of the moving object. However, this relation may no longer hold in the case where the illuminating light itself is modulated to become time-varying speckles by passing through a scattering medium in motion. Illuminating a diffuser with such a speckle pattern produces doubly- or multiply-scattered speckles whose statistical properties have been studied by many researchers [7–31, 75] particularly in the contexts of speckle reduction [7–10] and light scattering in atmospheric turbulence [24, 75]. To the author's knowledge, most analyses in these studies are mainly focused on the effect of speckle noise on the quality of images [7, 8, 10, 75] or the properties of instantaneous speckle patterns [11–23]. Although there are several reports [24, 29–31] concerning the velocity dependence of doubly- or multiply-scattered speckles, few studies have been conducted on the dynamic properties of doubly-scattered speckles appearing at the image plane of a moving diffuser with dynamic speckle illumination.

In Chapter 4, the velocity dependence of doubly-scattered speckles observed in the diffraction field has been analyzed by using two moving diffuse plates placed in parallel. It was found that the speckle intensity fluctuations measured by a point detector are influenced by the motion of both diffusers and that the velocity

dependence varies according to the size of an illuminating area or the position of the detector. Since the effect of the motion of the first diffuser is hardly negligible, it is very difficult to measure the velocity of the second diffuser placed behind the first, as long as the intensity fluctuations are detected in the diffraction field.

In this chapter, the dynamic properties of time-varying speckles produced at the image plane by the light passing through a cascade of two moving diffusers are investigated experimentally [33]. The motion of a speckle pattern illuminating the second diffuser is varied by changing the spot size of an illuminating light beam and the spacing between the two diffusers. The correlation time of speckle intensity fluctuations, which characterizes the rate of decorrelation, is measured as a function of the velocity of the first diffuser in order to estimate the effect of the motion of the illuminating speckle pattern on the resultant image speckles. The effect of the pupil size of an imaging lens is also discussed.

5.2 Experiments

Figure 5.1 shows the schematic diagram of the experimental setup. A He-Ne laser beam with wavelength $\lambda = 632.8$ nm directly illuminates a first diffuse plate D_1 with a spot diameter of 1.5 mm. A second diffuse plate D_2 is located at a certain distance from the first, and both diffusers are mounted on moving stages to translate at constant velocities in the same direction perpendicular to the optical axis. The spatial correlation lengths of these diffusers are about $11\ \mu\text{m}$, and their roughnesses were chosen to be larger than the wavelength of the laser light to eliminate the specular component. The doubly-scattered light is collected by a lens L to form time-varying speckles at the image plane of the second diffuser, where the detecting aperture A having a diameter of $2\ \mu\text{m}$ is placed on axis. A camera lens with a focal length of 55 mm is used as an imaging lens and is fixed at the distance 66 mm from the second diffuser and 330 mm from the detecting aperture to give an image magnification of 5. By fixing the lens position, the diameter of a lens pupil P determines the speckle size at the image plane and was chosen in such a way that the size of speckle grains is larger than that of the detecting aperture. The intensity fluctuations are detected via the aperture by a photomultiplier PM and

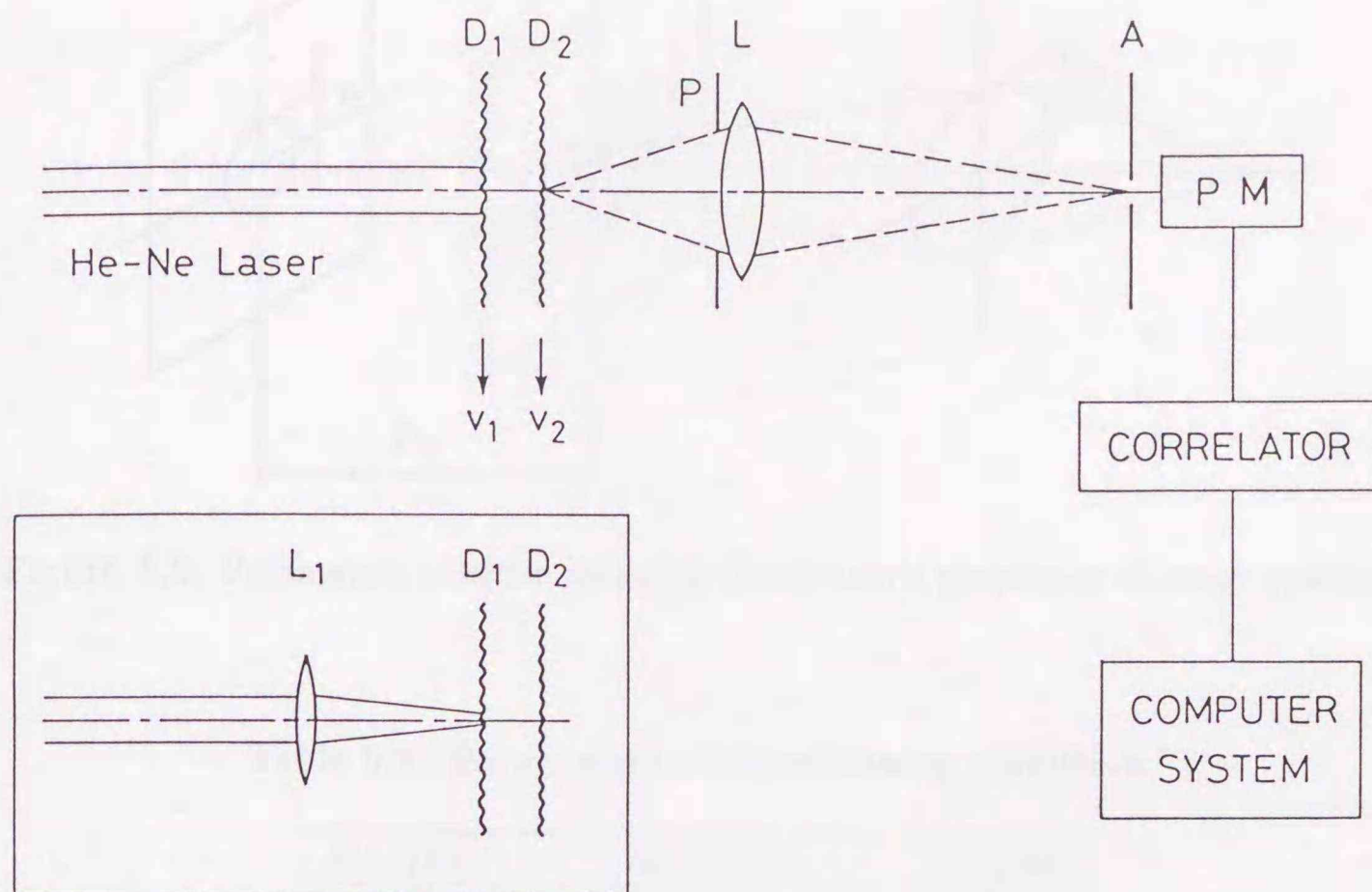


Figure 5.1. Schematic diagram of the experimental setup.

fed into a FFT correlator to measure their normalized autocorrelation function. The correlation time τ_c is obtained as the $1/e$ correlation time after averaging 64 autocorrelation functions, each of which is calculated by using 1024 sampling data.

To investigate the effect of the dynamic speckle pattern in front of the second diffuser, the size ζ and the motion of these speckles were varied by controlling the spacing R_1 between the two diffusers and the spot diameter $2w$ of the illuminating laser beam over the first diffuser D_1 . As shown in the inset of Fig. 5.1, a lens with a focal length of 200 mm was placed in front of the first diffuser to obtain the illuminating spot diameter smaller than that obtained under illumination of the direct beam. In addition, we also considered the effect of the diameter $2a$ of the pupil P which determines the diameter 2ρ of the point spread of the imaging lens L at the surface of the second diffuser. These parameters are illustrated schematically in Fig. 5.2 and the values used in experiments are shown in Tables 5.1 and 5.2.

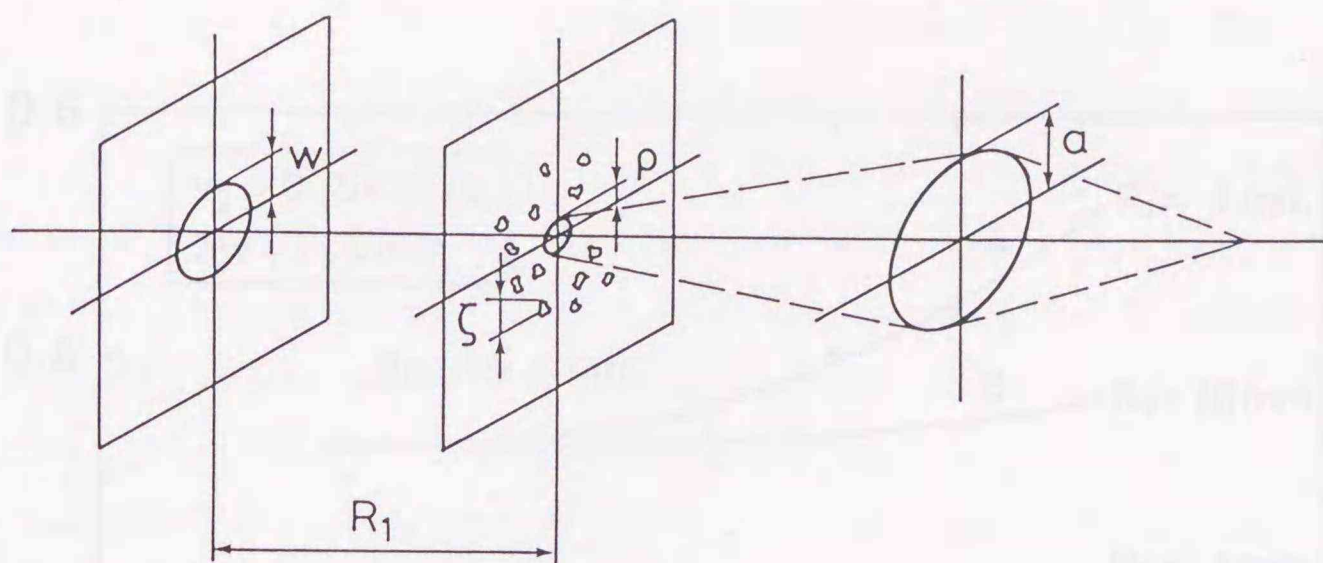


Figure 5.2. Parameters used for changing the dynamic properties of image speckles.

Table 5.1. Parameters of illuminating condition

$2w$ (μm)	R_1 (mm)	ζ (μm)
1500	1	1.37
	10	4.02
100	1	5.84
	10	57.0

Table 5.2. Parameters of detecting condition

$2a$ (mm)	ρ (μm)
19.4	1.77
1.72	20.0

5.3 Results and Discussion

Figure 5.3 shows the dependence of the reciprocal values of the correlation time τ_c of speckle intensity fluctuations on the velocity v_1 of the first diffuser obtained for the different values of (a) the spacing R_1 between the two diffusers and (b) the diameter $2w$ of the illuminating beam spot. The figure also shows the variation caused by changing the pupil diameter $2a$ of the imaging lens. The velocity v_2 of the second diffuser was fixed at 0.5 mm/s. The three kinds of the velocity dependence can be

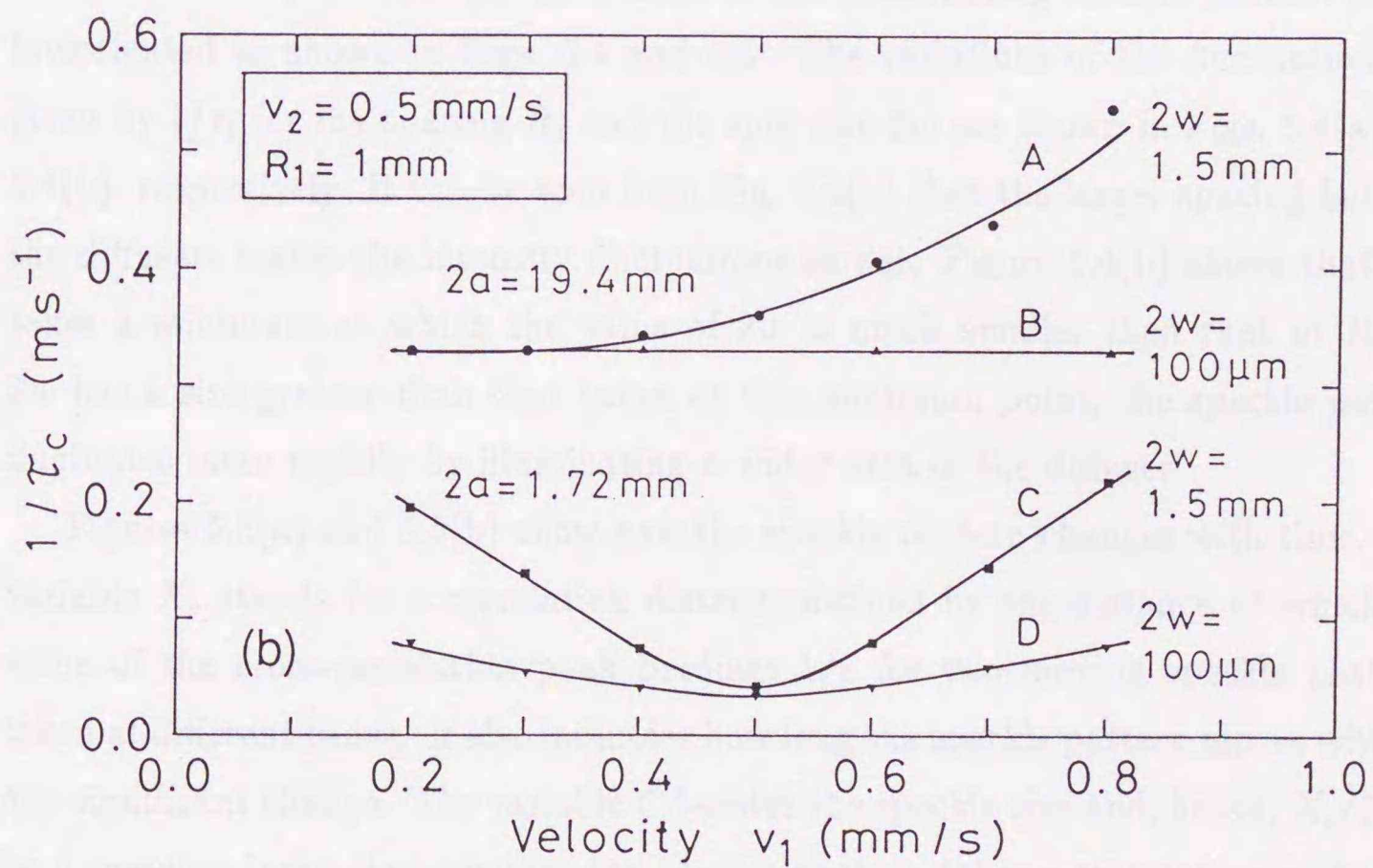
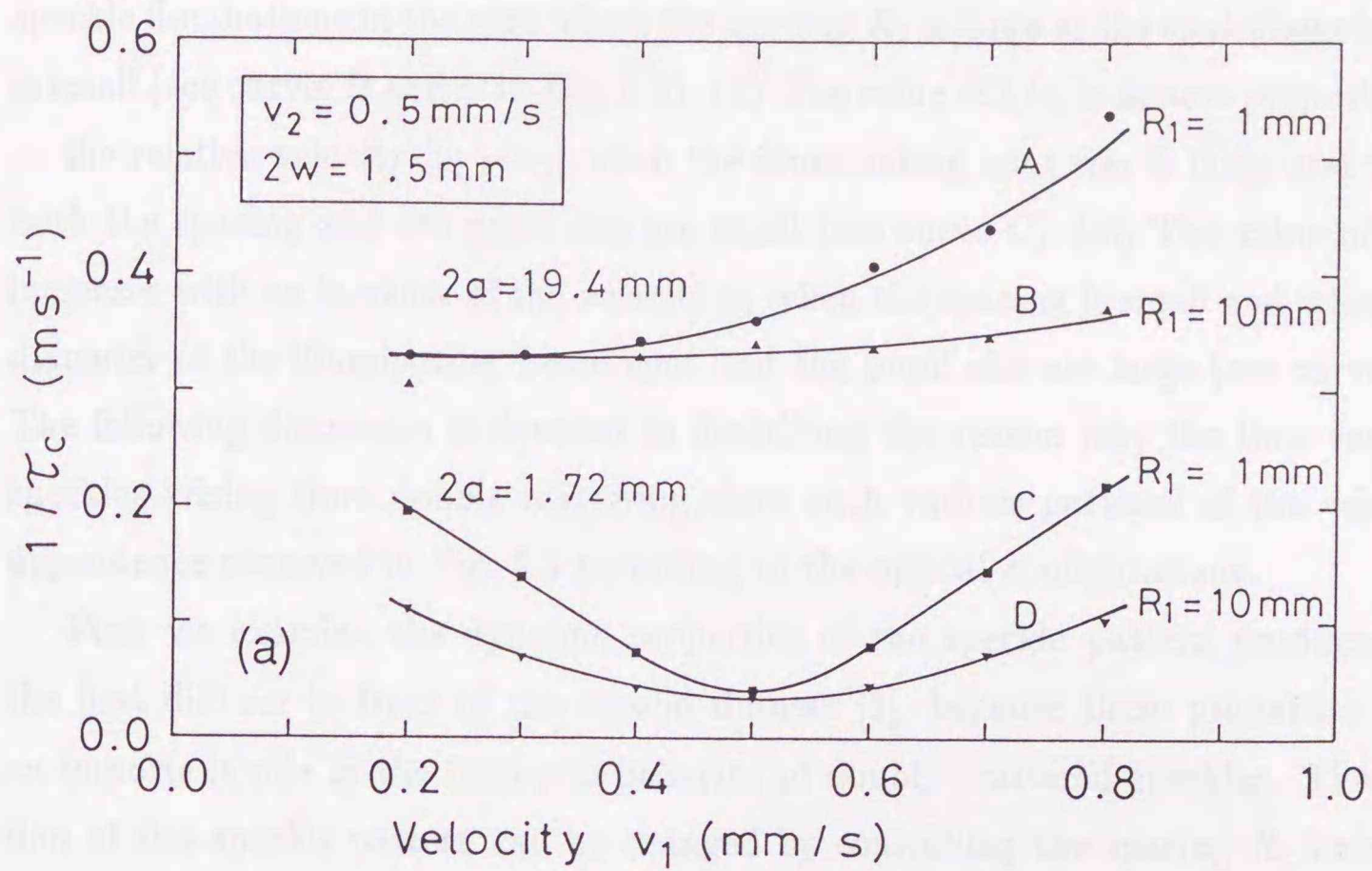


Figure 5.3. Variations of the reciprocal values $1/\tau_c$ of correlation time as a function of the velocity v_1 of the first diffuser (a) for the two different values of the spacing R_1 between the two diffusers and (b) for the two different values of the diameter $2w$ of the illuminating beam spot.

found from Fig. 5.3. (i) The motion of the first diffuser scarcely contributes to the speckle fluctuations in the case where the spacing R_1 is large or the spot diameter $2w$ is small (see curves B and D in Fig. 5.3). (ii) The value of $1/\tau_c$ is almost proportional to the relative velocity $|v_1 - v_2|$ when the illuminating spot size is large and when both the spacing and the pupil size are small (see curve C). (iii) The value of $1/\tau_c$ increases with an increase of the velocity v_1 when the spacing is small and when the diameter of the illuminating beam spot and the pupil size are large (see curve A). The following discussion is devoted to describing the reason why the time-varying speckles arising from double scattering show such various patterns of the velocity dependence observed in Fig. 5.3 according to the optical configurations.

First we examine the dynamic properties of the speckle pattern produced by the first diffuser in front of the second diffuser [3], because these properties play an important role in the temporal behavior of doubly-scattered speckles. The motion of this speckle pattern can be changed by controlling the spacing R_1 between the diffusers or the spot size $2w$ of the illuminating light. As a function of these two parameters, the dynamic properties of the illuminating speckle pattern can be investigated as shown in Figs. 5.4 and 5.5. The variations of the fluctuation rate given by $1/\tau_c$ for the spacing R_1 and the spot size $2w$ are shown in Figs. 5.4(a) and 5.4(b), respectively. It can be seen from Fig. 5.4(a) that the larger spacing between the diffusers makes the intensity fluctuations slower. Figure 5.4(b) shows that $1/\tau_c$ takes a minimum at which the value of $2w$ is much smaller than that of R_1 . If $2w$ has a size greater than that taken at this minimum point, the speckle pattern fluctuates more rapidly by illuminating a wider area of the diffuser.

Figures 5.5(a) and 5.5(b) show how the speckle pattern changes with time. The variable X_c stands for a correlation distance defined by the distance at which the value of the cross-correlation peak becomes $1/e$ for two moving speckle patterns taken at different times. It also indicates how long the speckle pattern moves without any significant change. The variable ζ denotes the speckle size and, hence, X_c/ζ can be a measure indicating whether the speckle pattern takes a translating or boiling motion. When the strong translating motion is observed, the value of X_c/ζ is large. When the boiling motion becomes dominant, it approaches one. From Figs. 5.5(a) and 5.5(b), it is found that the speckle pattern is translating in the case where the spacing R_1 is small or the spot diameter $2w$ is large. On the other hand, the

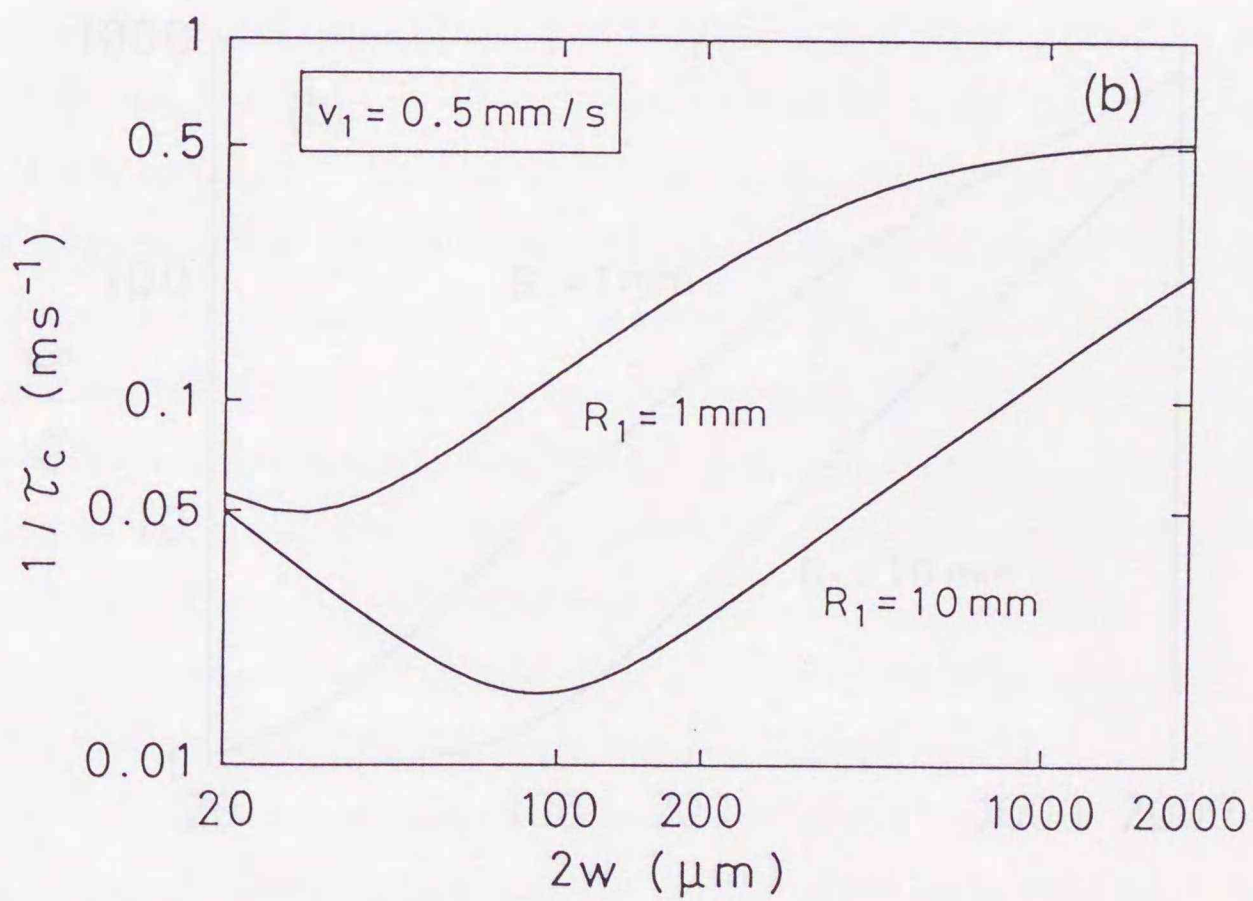
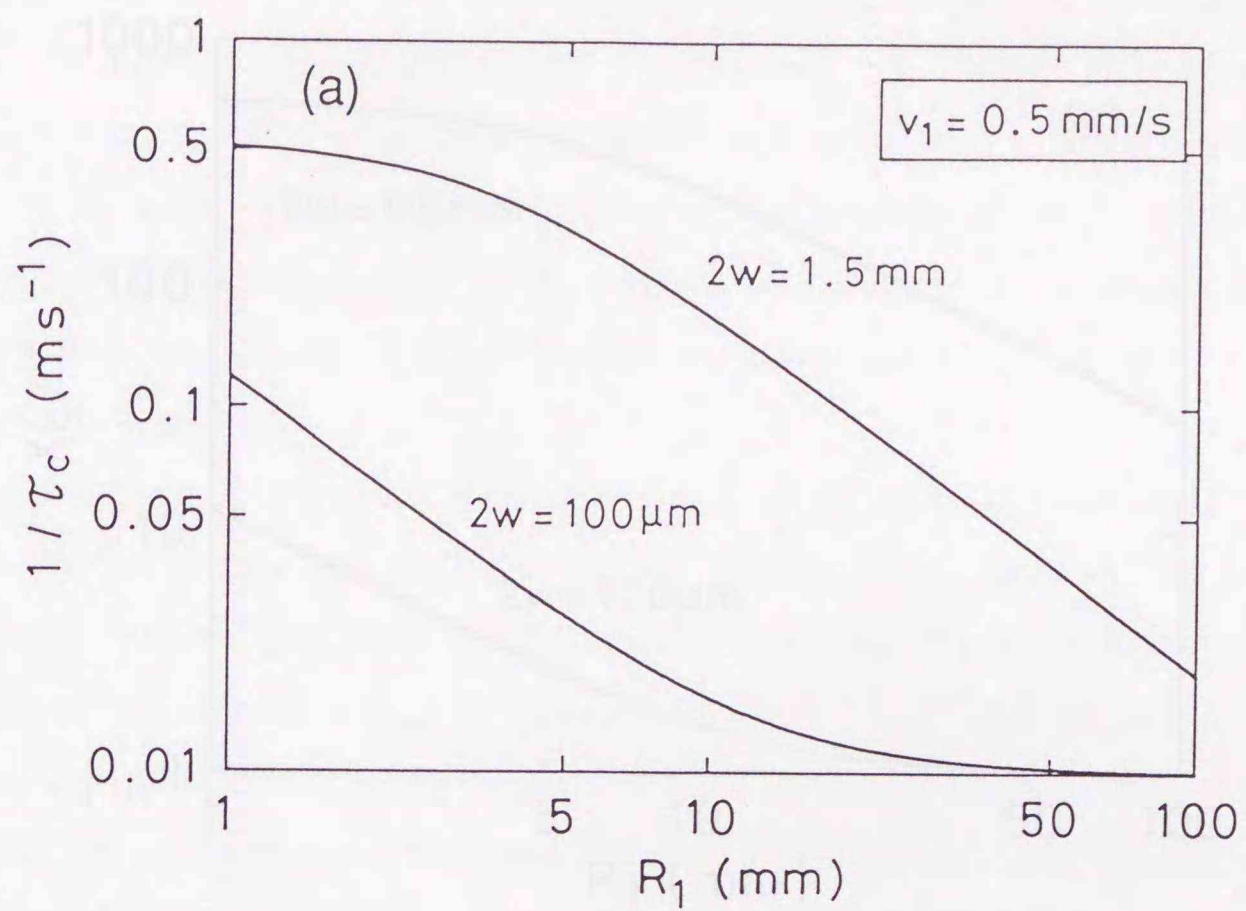


Figure 5.4. Variations of $1/\tau_c$ (corresponding to the fluctuating rate) of an illuminating speckle pattern (a) as a function of the spacing R_1 between the two diffusers and (b) as a function of the diameter $2w$ of the illuminating beam spot.

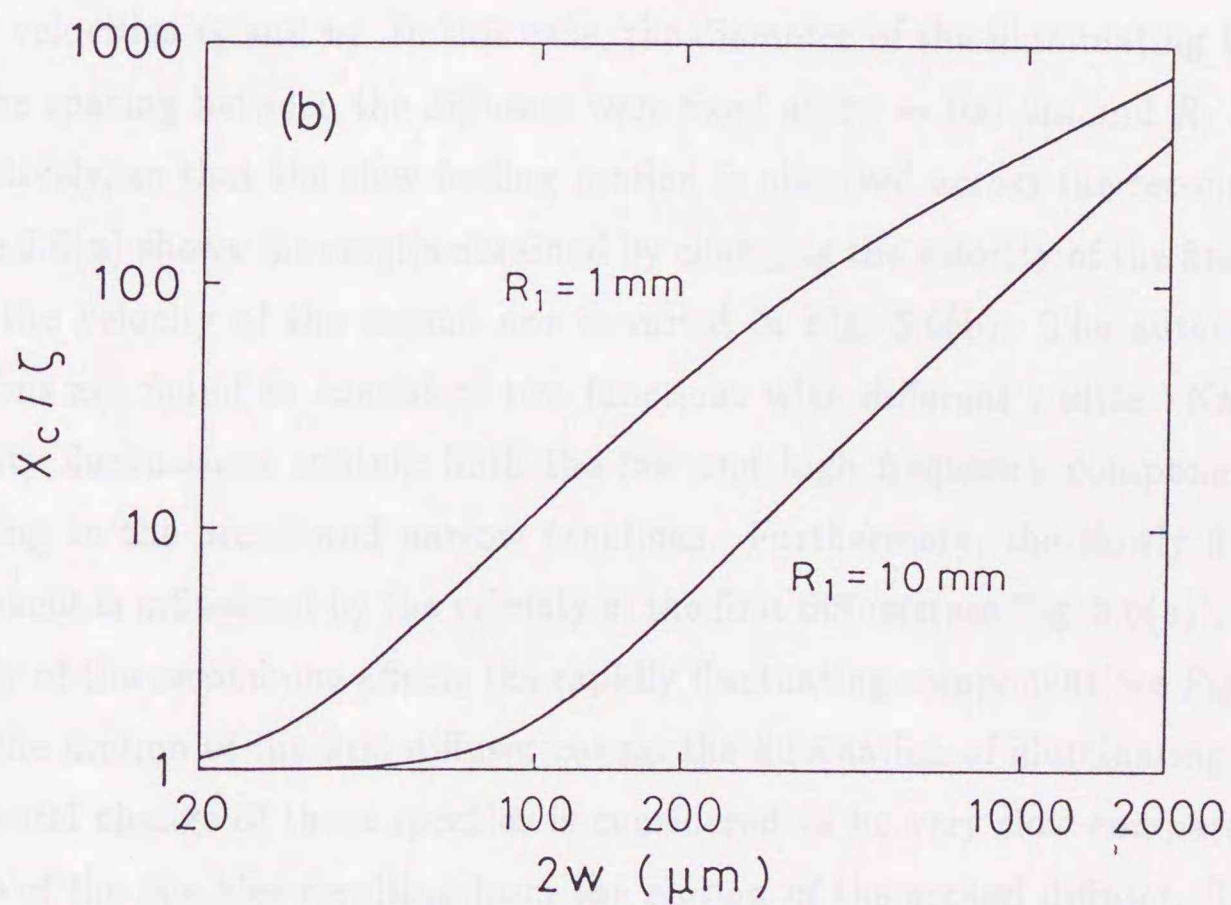
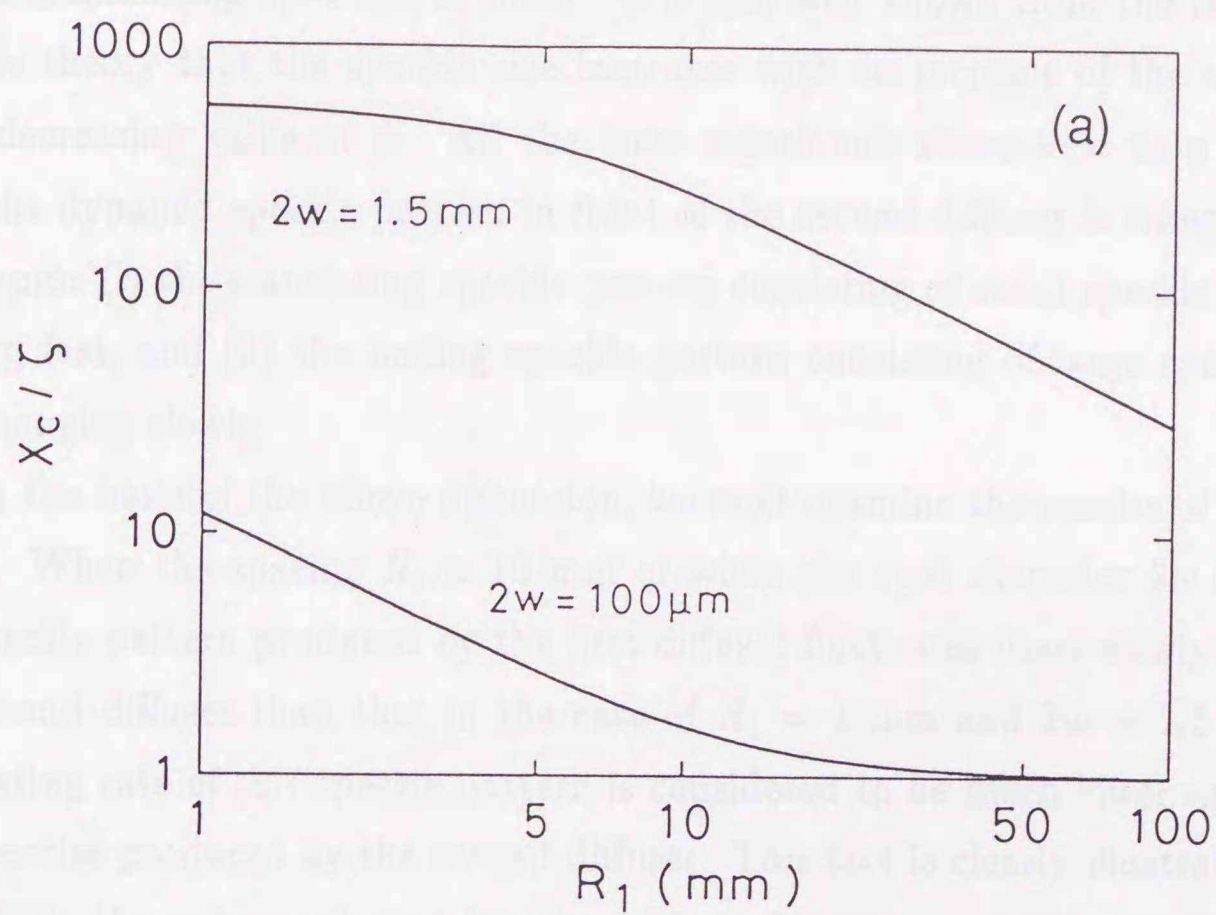


Figure 5.5. Variations of X_c/ζ (corresponding to the correlation distance divided by the speckle size) of an illuminating speckle pattern (a) as a function of the spacing R_1 between the two diffusers and (b) as a function of the diameter $2w$ of the illuminating beam spot.

boiling speckle pattern illuminates the second diffuser when the spacing is large or the illuminating spot size is small. It is also well known from the fundamental speckle theory that the speckle size increases with an increase of the value R_1 or with decreasing value of w . All the facts mentioned above lead to a conclusion that the dynamic speckle pattern in front of the second diffuser is categorized into two types: (i) the translating speckle pattern consisting of small speckle grains and moving fast, and (ii) the boiling speckle pattern consisting of large speckle grains and changing slowly.

On the basis of the above discussion, we next examine the results of Fig. 5.3 in detail. When the spacing R_1 is 10 mm or when the spot diameter $2w$ is 100 μm , the speckle pattern produced by the first diffuser fluctuates more slowly in front of the second diffuser than that in the case of $R_1 = 1$ mm and $2w = 1.5$ mm. The fluctuating rate of this speckle pattern is considered to be much lower than that of the speckles produced by the second diffuser. This fact is clearly illustrated in Fig. 5.6, where the autocorrelation functions of speckle intensity fluctuations obtained experimentally at the image plane of the second diffuser are shown for various values of the velocities v_1 and v_2 . In this case, the diameter of the illuminating beam spot and the spacing between the diffusers were fixed at $2w = 100$ μm and $R_1 = 10$ mm, respectively, so that the slow boiling motion is observed across the second diffuser. Figure 5.6(a) shows the results obtained by changing the velocity of the first diffuser, while the velocity of the second one is varied in Fig. 5.6(b). The autocorrelation functions are found to consist of two functions with different widths. Namely, the intensity fluctuations contain both the low and high frequency components corresponding to the broad and narrow functions. Furthermore, the slowly fluctuating component is influenced by the velocity of the first diffuser (see Fig. 5.6(a)), while the velocity of the second one affects the rapidly fluctuating component (see Fig. 5.6(b)). Since the motion of the first diffuser causes the fluctuation of illuminating speckles, a temporal change of these speckles is considered to be very slow compared with a change of the speckles resulting from the motion of the second diffuser. The correlation time τ_c has been defined by the time at which the correlation function takes the value of $1/e$ and, therefore, the value of τ_c is determined only by the velocity of the second diffuser.

The pupil size of an imaging lens is another important parameter. The light

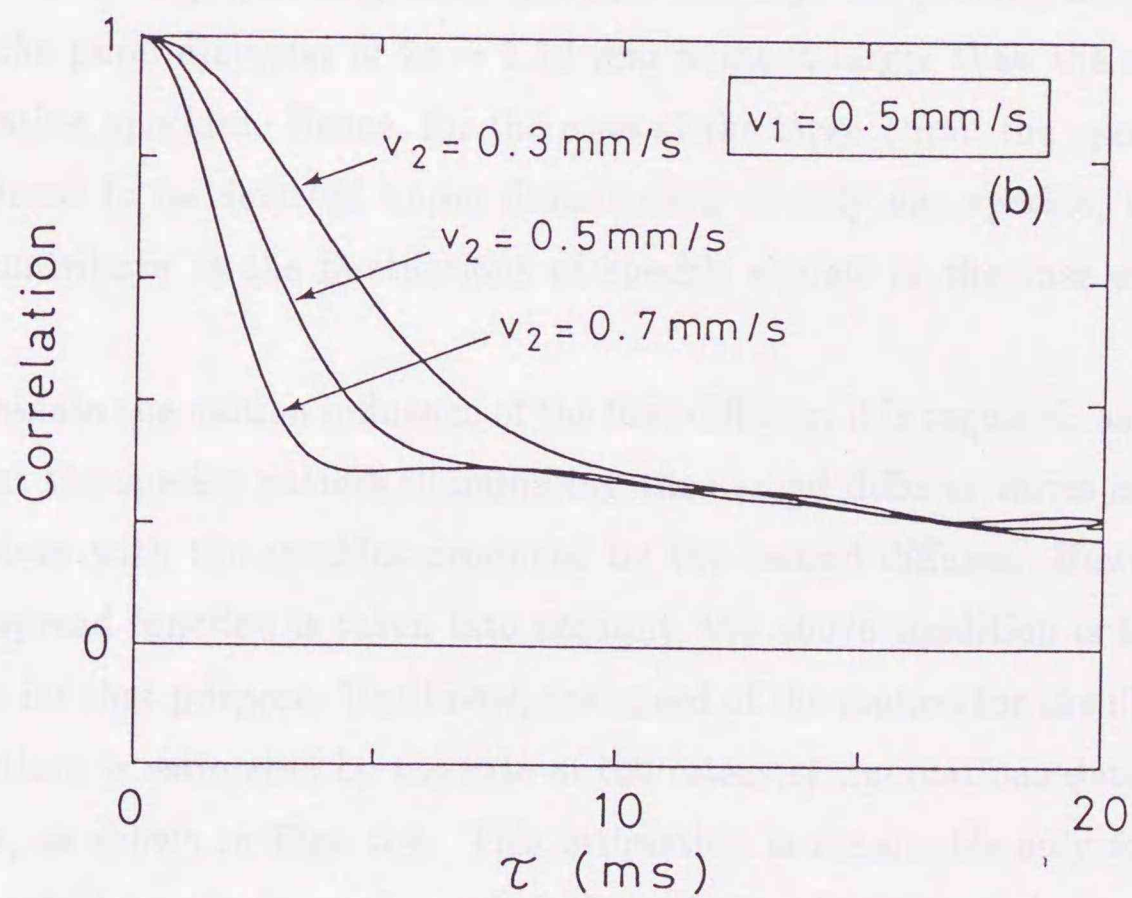
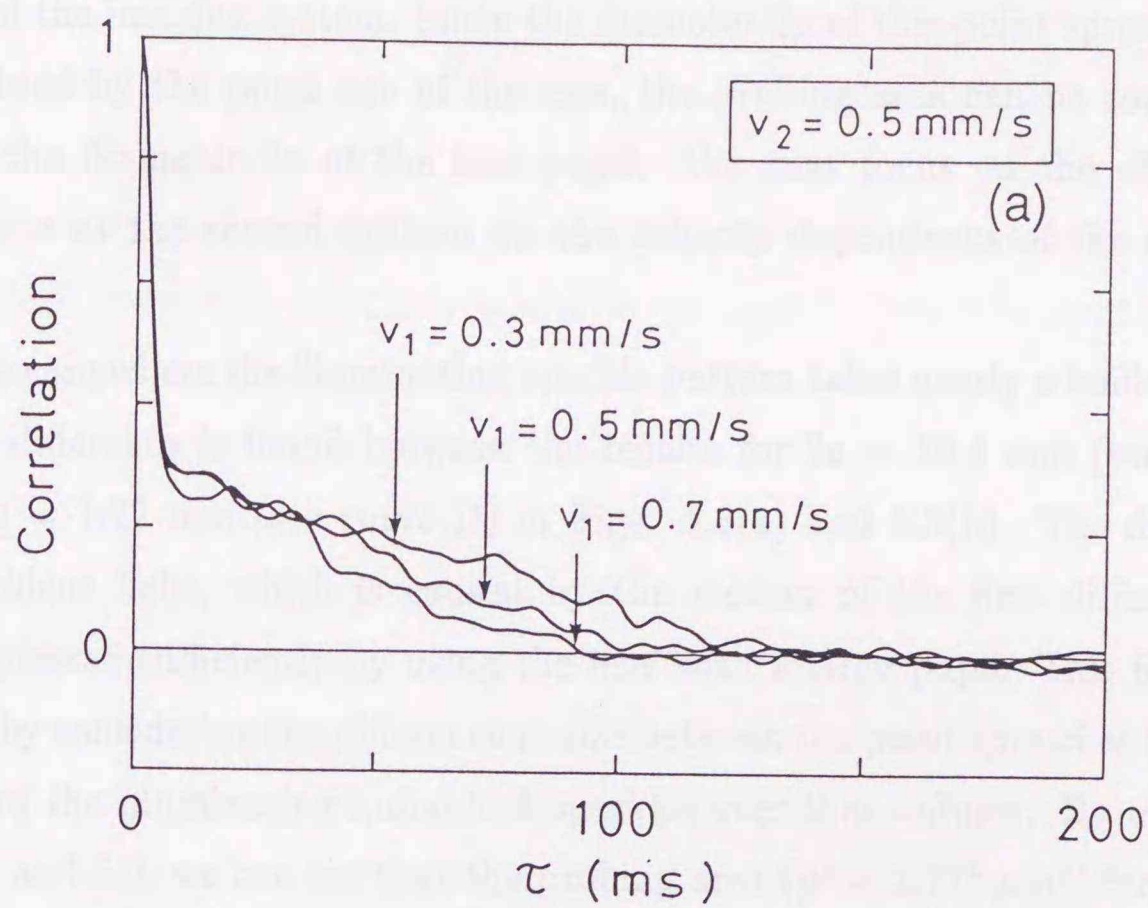


Figure 5.6. Autocorrelation functions of speckle intensity fluctuations obtained under illumination of a boiling speckle pattern for the two cases: (a) with the fixed velocity v_2 but with the three different velocities v_1 and (b) with the fixed velocity v_1 but with the three different velocities v_2 .

reaching the point detector located at the focal plane is limited by the point spread function of the imaging system. Since the diameter 2ρ of this point spread function is determined by the pupil size of the lens, the probing area can be controlled by changing the diameter $2a$ of the lens pupil. We next focus on the effect of the probing area at the second diffuser on the velocity dependence of the correlation time.

For the case where the illuminating speckle pattern takes nearly a boiling motion, the slight difference is found between the results for $2a = 19.4$ mm (see curve B) and for $2a = 1.72$ mm (see curve D) in Figs. 5.3(a) and 5.3(b). The disturbance of the incident light, which is caused by the motion of the first diffuser, seems to be suppressed sufficiently by using the lens with a large pupil. This fact can be explained by considering the difference in size between the point spread at the second diffuser and the illuminating individual speckles over that diffuser. By referring to Tables 5.1 and 5.2, we can see that the probing area ($\rho^2 = 1.77^2 \mu\text{m}^2$) for the pupil diameter of $2a = 19.4$ mm is much smaller than the area ($\zeta^2 = 4.02^2 \mu\text{m}^2$ or $5.84^2 \mu\text{m}^2$) of illuminating speckle grains. On the contrary, the probing area ($\rho^2 = 20^2 \mu\text{m}^2$) for the pupil diameter of $2a = 1.72$ mm is much larger than the average size of illuminating speckles. Hence, for the case of the large pupil, the speckle signals are considered to be detected under illumination of only one speckle, while many speckles contribute to the fluctuations of speckle signals in the case of the small pupil.

To eliminate the motion influence of the first diffuser, it is required, as mentioned before, that the speckle pattern illuminating the second diffuser varies much slowly in comparison with the speckles produced by the second diffuser. However, when the point spread function is taken into account, the above condition is found to be insufficient for that purpose. Until now, the speed of the motion for the illuminating speckle pattern is estimated by the rate of the intensity fluctuations detected at an ideal point, as shown in Figs. 5.4. This estimation is reasonable only for the case where the probing area is much smaller than the speckle size. The experimental result for the lens of the small pupil reveals that a motion of the gathering of individual speckles is not considered to be slow even if each speckle changes its shape slowly. In Chapter 4, it was found that the influence of the first diffuser cannot be easily removed in free-space geometry. Now this result can be explained

in terms of the probing area. Namely, if the intensity fluctuations are detected without the imaging lens, the detected signals contain the contributions from the whole speckles illuminating the second diffuser because no limitation is set for the probing area. Therefore, in order to measure the velocity of the second diffuser, it is necessary to employ an imaging system and to use the lens having a large pupil.

Figure 5.7 shows the experimental results obtained under the condition where the illuminating speckle pattern takes a boiling motion and where the large pupil is used for the imaging lens. The autocorrelation functions in Fig. 5.6(b) correspond to the case of $R_1 = 10$ mm in Fig. 5.7. In this case, the value of $1/\tau_c$ is linearly proportional to the velocity v_2 of the second diffuser even though the first diffuser is moving with a constant velocity of 0.5 mm/s. The data for $R_1 = 1$ mm, which were measured under the same condition as the data shown by curve B in Fig. 5.3(b), are affected slightly by the motion of the first diffuser. From Fig. 5.4(a), the fluctuating rate for $R_1 = 1$ mm is seen to be much higher than that in the case of $R_1 = 10$ mm. This difference of the rate seems to be the main reason why the two data set (the small and the large spacings) do not take the same values.

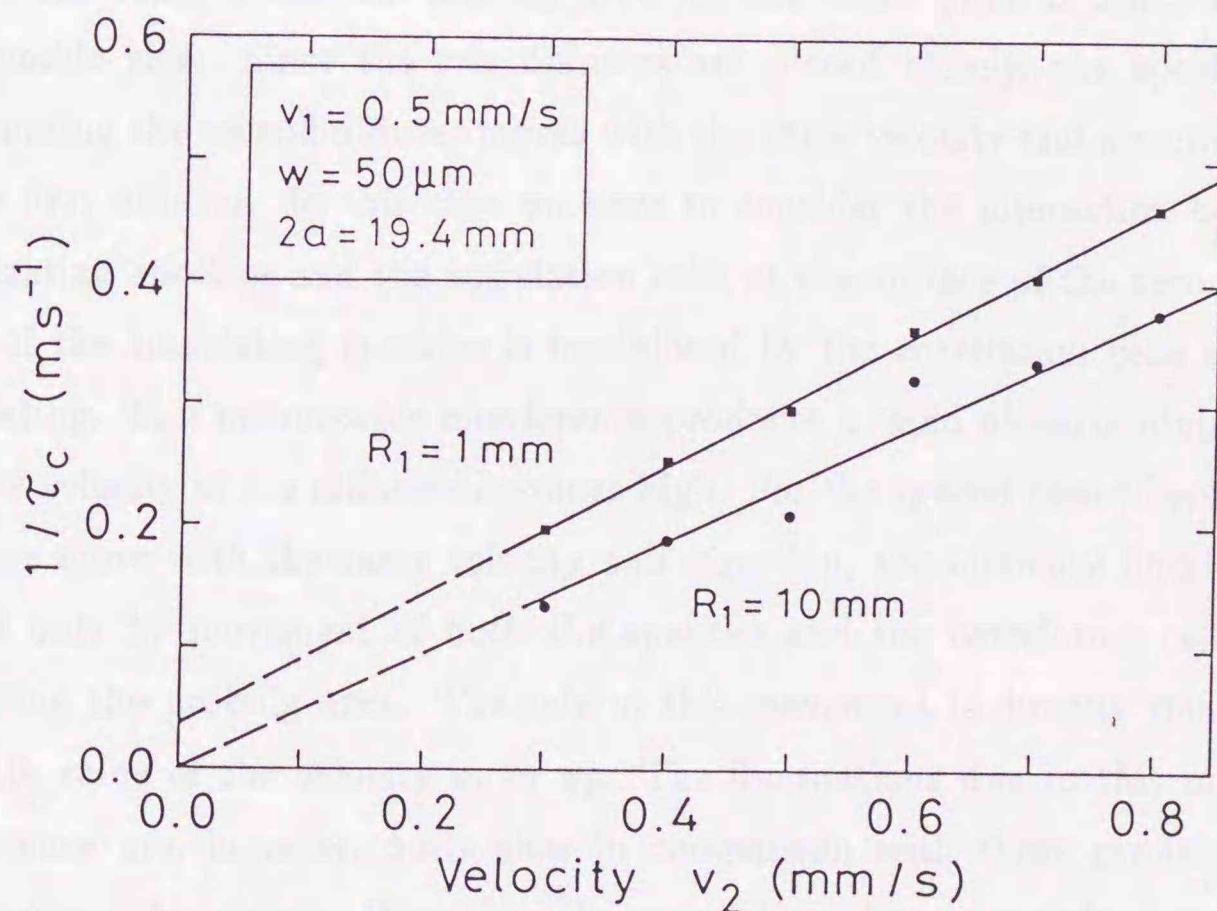


Figure 5.7. Linearity between the reciprocal $1/\tau_c$ of correlation time and the velocity v_2 of the second diffuser for the two different spacings R_1 .

Finally we examine the results obtained when the translating speckle pattern illuminates the second diffuser. The data for $2a = 19.4$ and 1.72 mm produced the curves in different forms as shown by curves A and C in Fig. 5.3. The value of $1/\tau_c$ is found to be almost proportional to the relative velocity $|v_1 - v_2|$ in the case of the small pupil, while it appears to increase with increasing v_1 for the large pupil. This difference can also be explained by using the dynamic properties of the illuminating speckle pattern and the probing area determined by the point spread function of the imaging lens. As mentioned earlier, the translating speckle pattern is fluctuating so rapidly that its motion cannot be neglected even if the point spread function is much narrower than the diameter of individual speckles in that pattern. In the experiment, the probing area ($\rho^2 = 1.77^2 \mu\text{m}^2$) for the large pupil is the same order in size as the speckle grains ($\zeta^2 = 1.37^2 \mu\text{m}^2$) as indicated in Tables 5.1 and 5.2. Although the moving direction of the speckle pattern is one of the important parameters characterizing the translating motion, this information is lost in this case because the light contributing to the detected signals is always emitted only from a single speckle grain. Therefore, the absolute value of the velocity v_1 determines the correlation time.

On the other hand, the probing area for the small pupil is much larger than the speckle size. Since the two diffusers are placed closely, the speckle pattern illuminating the second diffuser moves with the same velocity and direction as those of the first diffuser. In this case we have to consider the interaction between the illuminating speckles and the correlation cells at the surface of the second diffuser. Each of the translating speckles is modulated by the correlation cells that is also translating. This microscopic interference produces a rapid phase modulation if the relative velocity of the diffusers becomes high. For the special case where those two diffusers move with the same velocity and direction, the intensity fluctuations are caused only by movement of both the speckles and the correlation cells entering or leaving the probing area. The rate of this movement is directly related to the absolute value of the velocity v_1 or v_2 . The fluctuations due to this macroscopic interference are, however, quite slow in comparison with those produced by the microscopic interference. Therefore the correlation time is mainly determined by the relative velocity $|v_1 - v_2|$ rather than the absolute one of v_1 or v_2 .

We have revealed the three kinds of the typical tendencies appearing in the

Table 5.3. Three kinds of velocity dependence obtained under different conditions for illumination and detection^a

Illumination	Detection	$1/\tau_c$
$2w \ll R_1$	$\rho \ll \zeta$ ($2a \gg 1$)	proportional to $ v_2 $
$2w \approx R_1$	$\rho \gg \zeta$ ($2a \ll 1$)	proportional to $ v_1 - v_2 $
	$\rho \approx \zeta$	increasing with increasing $ v_1 $ or $ v_2 $

^a $2w$, Spot diameter of illuminating laser beam; R_1 , spacing between diffusers; ρ , radius of point spread function; ζ , radius of illuminating speckle; $2a$, diameter of lens pupil.

fluctuations of image speckles. Their features are summarized in Table 5.3. Since we used only one pair of sample diffusers, the size of the correlation cells for those diffusers has not been considered in the above discussion. For further analysis of double-scattering phenomena, it will be required that the parameters characterizing the diffuse objects must also be taken into consideration.

5.4 Conclusion

We have investigated experimentally the velocity dependence of coherent light scattered by a moving diffuser under illumination of dynamic speckles. The light scattered twice by a cascade of two diffusers was detected at the image plane of the second diffuser, and the correlation time was measured to estimate the rate of intensity fluctuations. It has been shown that, if the speckle pattern produced by the first diffuser takes a boiling motion and if a lens with a large pupil size is used, the motion of illuminating speckles has little effect on the fluctuations of doubly-scattered speckles. In this case, the reciprocal $1/\tau_c$ of the correlation time is proportional to the object velocity. When the translating speckles illuminate the second diffuser, the value of $1/\tau_c$ is proportional to the relative velocity of the two diffusers if the pupil size of the lens is so small that the point spread function is much broader than the individual speckles at the second diffuser. For the case of the large pupil, an increase of the velocity of each diffuser increases the value of $1/\tau_c$. Therefore, it is found that the three kinds of the velocity dependence can be obtained by controlling

the spot diameter of the illuminating light and the pupil diameter of the imaging lens. These results may provide an insight into more complicated phenomena such as multiply-scattered speckles.

Chapter 6

Effects of Imaging Properties on Doubly-Scattered Speckles

6.1 Introduction

When a light beam is scattered by a rough surface, the scattered light is composed of many small wavelets. These wavelets are scattered in all directions and their phases are random. This is the origin of the speckle pattern. The speckle pattern is a result of the interference of the scattered light. The speckle pattern is a random pattern and its intensity is proportional to the square of the amplitude of the scattered light. The speckle pattern is a result of the interference of the scattered light. The speckle pattern is a random pattern and its intensity is proportional to the square of the amplitude of the scattered light.

Along with the development of the laser, the study of speckles has become an important topic in optics. In the early days, the study of speckles was limited to the study of the intensity of the speckles. However, with the development of the laser, the study of speckles has become more and more complicated. The study of speckles is now a multidisciplinary field, involving optics, physics, and mathematics. The study of speckles is now a multidisciplinary field, involving optics, physics, and mathematics. The study of speckles is now a multidisciplinary field, involving optics, physics, and mathematics. The study of speckles is now a multidisciplinary field, involving optics, physics, and mathematics.

In Chapter 6, the effects of imaging properties on doubly-scattered speckles are studied. The study of doubly-scattered speckles is a more complicated problem than the study of singly-scattered speckles. The study of doubly-scattered speckles is a more complicated problem than the study of singly-scattered speckles. The study of doubly-scattered speckles is a more complicated problem than the study of singly-scattered speckles. The study of doubly-scattered speckles is a more complicated problem than the study of singly-scattered speckles.

Chapter 6

Effects of Imaging Properties on Doubly-Scattered Speckles

6.1 Introduction

When a coherent light propagates through a dense scattering medium in motion, a dynamic speckle pattern is observed in the diffraction and image fields. This speckle pattern results from multiple scattering due to a large number of moving particles or the temporal fluctuations of the refractive index in the medium. Therefore, the statistical properties of time-varying speckles provide us with useful information about the dynamic behavior of the scattering media.

Many studies [7–31, 75] have been conducted on doubly- or multiply-scattered speckles particularly in the contexts of speckle reduction [7–10] and light scattering in atmospheric turbulence [24, 75]. Most of these analyses deal with static speckle patterns [11–23], or dynamic speckles only at the image plane or Fourier plane of the imaging system since the moving diffusers employed in these studies are considered as a noise source producing the speckle noise added to the diffraction-limited image of the object concerned. However, it is also important to reveal the relationship between the motion of the diffusers and the fluctuations of the speckle intensity caused by them under more general conditions, including the effect of defocus, for the formation of dynamic speckles.

In Chapter 4, the velocity dependence of doubly-scattered speckles arising from a cascade of two moving diffusers observed in the Fresnel diffraction field was investigated theoretically and experimentally. The reciprocal of the correlation time, which is a measure of the fluctuating rate of the speckle intensity detected at a single point, shows a nearly linear relationship with the difference between the velocities of the diffusers in most of the cases. For speckles at the image plane, however, an

experimental study presented in Chapter 5 demonstrates that the intensity fluctuations show various kinds of dependence on the diffuser velocities, according to both scattering and detecting conditions.

In this chapter, we generalize the imaging conditions and investigate theoretically and experimentally the dynamic properties of time-varying speckles formed in an imaging system by passing light through a sequential pair of two moving diffusers [36]. The space-time correlation function for image speckle patterns is derived to examine the spatial properties of static speckle patterns and to estimate the influence of the motion of each diffuser on the fluctuating rate of the speckle intensity. It is found that the velocity of the diffuser in focus or the average velocity of the diffusers can be measured if the distance between the diffusers is much greater than the spot size of the illuminating beam and if an imaging lens with a small f -number (F) is used. On the other hand, when the distance between the diffusers and the beam-spot size are of the same order and when an imaging lens having a large f -number is used, the difference between the velocities of the diffusers can be measured. Experimental results are presented to confirm some of the theoretical results.

6.2 Theory

Figure 6.1 shows the optical arrangement used for evaluating the space-time correlation function of doubly-scattered speckle patterns. Two transparent diffusers D_1 and D_2 separated by a distance d move parallel to each other with constant velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ at planes $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively. In the following analysis, the diffusers are assumed to be random phase screens which modulate only the phase of the wave field. By illuminating the diffuser D_1 normally with a coherent Gaussian beam having the wavelength λ and the radius R of the wavefront curvature, a dynamic speckle pattern is produced at the image plane $\mathbf{x}$ of a lens L . The front focal plane of the lens L could be set in an arbitrary position and the distance from the plane $\mathbf{u}_2$ to the focal plane is indicated by δ .

In order to formulate in a relatively simple manner the correlation function of the speckle intensity at the image plane, it is assumed that the illuminating beam is linearly polarized and that the diffusers do not change the state of polarization of

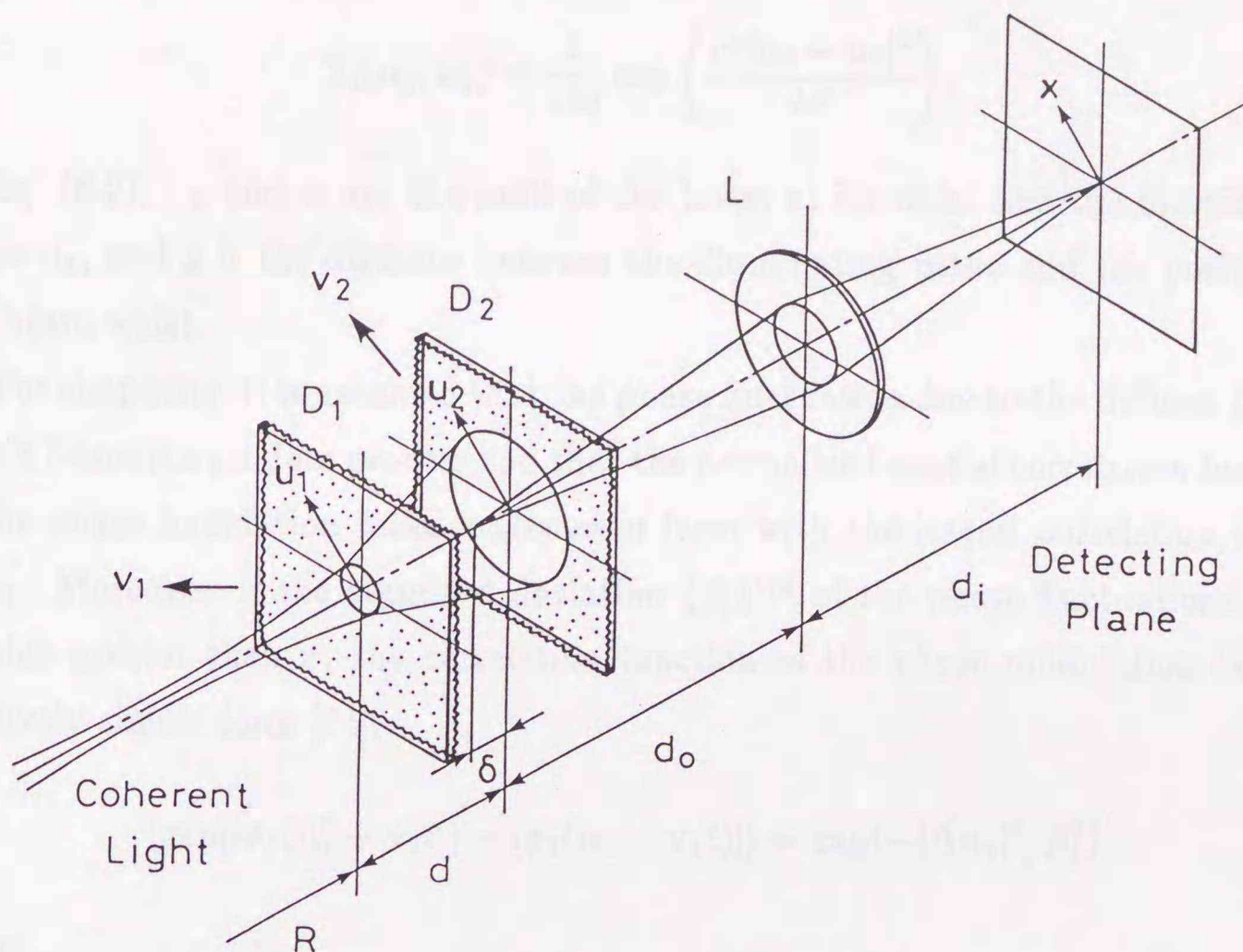


Figure 6.1. Schematic diagram of the optical configuration for analyzing a doubly-scattered speckle field.

the field. This assumption allows us to use the scalar wave approximation. It is also assumed that the illuminating speckle field in front of the plane u_2 obeys a complex Gaussian random process.

The intensity correlation function can be derived by using the space-time correlation function of the speckle field E_2 at the plane u_2 . Hence we first evaluate the correlation function of the speckle field E_2 , which is expressed as

$$\begin{aligned} \langle E_2^*(u_2, t) E_2(u'_2, t') \rangle &= \iint_{-\infty}^{\infty} E_1^*(u_1) E_1(u'_1) \langle \exp[i\phi(u'_1 - v_1 t') - i\phi(u_1 - v_1 t)] \rangle \\ &\quad \times T_1^*(u_1, u_2) T_1(u'_1, u'_2) du_1 du'_1, \end{aligned} \quad (6.1)$$

where $\langle \dots \rangle$ denotes the ensemble average. In Eq. (6.1), E_1 and T_1 stand for the incident field at the plane u_1 and the propagation function of the optical field from the plane u_1 to the plane u_2 , which are given, respectively, by

$$E_1(u_1) = \frac{w_0}{w} \exp\left(i\frac{2\pi}{\lambda}z\right) \exp\left(-\frac{|u_1|^2}{w^2}\right) \exp\left(i\frac{\pi|u_1|^2}{\lambda R}\right) \quad (6.2)$$

and

$$T_1(\mathbf{u}_1, \mathbf{u}_2) = \frac{1}{i\lambda d} \exp\left(\frac{i\pi|\mathbf{u}_1 - \mathbf{u}_2|^2}{\lambda d}\right). \quad (6.3)$$

In Eq. (6.2), w_0 and w are the radii of the beam at its waist and the illuminating plane $\mathbf{u}_1$, and z is the distance between the illuminating plane and the position of the beam waist.

For simplicity, it is assumed that the phase fluctuation due to the diffuser D_1 follows a Gaussian random process and that the normalized spatial correlation function of the phase fluctuation takes a Gaussian form with the lateral correlation length of α_1 . Moreover, if the standard deviation $\langle\phi_1^2\rangle^{1/2}$ of the phase fluctuations takes a value greater than π , the correlation function of the phase modulation takes a relatively simple form [74]:

$$\langle \exp[i\phi_1(\mathbf{u}'_1 - \mathbf{v}_1 t') - i\phi_1(\mathbf{u}_1 - \mathbf{v}_1 t)] \rangle = \exp(-|\Delta\mathbf{u}_1|^2/\beta_1^2), \quad (6.4)$$

where

$$\Delta\mathbf{u}_1 = \mathbf{u}'_1 - \mathbf{u}_1 - \mathbf{v}_1(t' - t) \quad (6.5)$$

and β_1 stands for the effective correlation length, given by

$$\beta_1 = \alpha_1/\langle\phi_1^2\rangle^{1/2}. \quad (6.6)$$

Substitution of Eqs. (6.2)–(6.5) together with $\tau = t' - t$ into Eq. (6.1) yields the space-time correlation function of the speckle field in front of the plane $\mathbf{u}_2$ [32]

$$\begin{aligned} \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}'_2, t + \tau) \rangle &= \frac{1}{P} \left(\frac{\pi w_0}{\lambda d} \right)^2 \exp \left\{ -\frac{i\pi}{\lambda d} \left[\left(1 - \frac{P_3}{\sigma P} \right) \right. \right. \\ &\quad \left. \left. \times (|\mathbf{u}_2|^2 - |\mathbf{u}'_2|^2) + \frac{P_1}{P} \mathbf{v}_1 \cdot (\mathbf{u}_2 + \mathbf{u}'_2) \tau \right] \right\} \\ &\times \exp \left[-\frac{P_1(P_2 + P_3)}{2P} |\mathbf{v}_1|^2 \tau^2 - \frac{P_1 P_3}{\sigma P} \mathbf{v}_1 \cdot (\mathbf{u}_2 - \mathbf{u}'_2) \tau \right] \\ &\times \exp \left(-\frac{|\mathbf{u}_2 + \mathbf{u}'_2|^2}{2q_2^2} \right) \exp \left(-\frac{|\mathbf{u}_2 - \mathbf{u}'_2|^2}{2\zeta_2^2} \right), \quad (6.7) \end{aligned}$$

where

$$\sigma = 1 + d/R, \quad (6.8)$$

and q_2 and ζ_2 , the extent of the speckle field and the mean speckle radius at the plane $\mathbf{u}_2$, respectively, are defined as

$$q_2 = \frac{\lambda d}{\pi} P^{1/2} \quad (6.9)$$

and

$$\zeta_2 = \frac{\lambda d}{\pi w} \left(\frac{P}{P_1 + P_2} \right)^{1/2}. \quad (6.10)$$

In Eqs. (6.7), (6.9) and (6.10),

$$P_1 = \frac{2}{\beta_1^2}, \quad P_2 = \frac{1}{w^2}, \quad P_3 = \left(\frac{\pi w \sigma}{\lambda d} \right)^2$$

and

$$P = P_1 + P_2 + P_3. \quad (6.11)$$

Using Eq. (6.7), we next derive the space-time correlation function of the intensity $I(\mathbf{x}, t)$ at the plane $\mathbf{x}$. This function is written as

$$\langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = \langle E^*(\mathbf{x}, t) E(\mathbf{x}, t) E(\mathbf{x}', t + \tau) E^*(\mathbf{x}', t + \tau) \rangle, \quad (6.12)$$

where $E(\mathbf{x}, t)$ is the field at the plane $\mathbf{x}$ and is given by

$$E(\mathbf{x}, t) = \int_{-\infty}^{\infty} E_2(\mathbf{u}_2, t) \exp[i\phi_2(\mathbf{u}_2 - \mathbf{v}_2 t)] T_2(\mathbf{u}_2, \mathbf{x}) d\mathbf{u}_2; \quad (6.13)$$

here, ϕ_2 is the phase modulation due to the diffuser D_2 and T_2 is the impulse response function of the lens L . By substituting Eq. (6.13) into Eq. (6.12), the intensity correlation function becomes

$$\begin{aligned} \langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = & \iiint \int_{-\infty}^{\infty} \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}'_2, t) E_2(\mathbf{u}''_2, t + \tau) E_2^*(\mathbf{u}'''_2, t + \tau) \rangle \\ & \times \langle \exp\{-i\phi_2(\mathbf{u}_2 - \mathbf{v}_2 t) + i\phi_2(\mathbf{u}'_2 - \mathbf{v}_2 t) \\ & + i\phi_2[\mathbf{u}''_2 - \mathbf{v}_2(t + \tau)] - i\phi_2[\mathbf{u}'''_2 - \mathbf{v}_2(t + \tau)]\} \rangle \\ & \times T_2^*(\mathbf{u}_2, \mathbf{x}) T_2(\mathbf{u}'_2, \mathbf{x}) T_2(\mathbf{u}''_2, \mathbf{x}') T_2^*(\mathbf{u}'''_2, \mathbf{x}') \\ & \times d\mathbf{u}_2 d\mathbf{u}'_2 d\mathbf{u}''_2 d\mathbf{u}'''_2. \end{aligned} \quad (6.14)$$

Since complex Gaussian statistics have been assumed for the speckle field at the plane $\mathbf{u}_2$, the fourth-order moment of the field is expressed by a combination of the second-order moments as

$$\begin{aligned} & \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}'_2, t) E_2(\mathbf{u}''_2, t + \tau) E_2^*(\mathbf{u}'''_2, t + \tau) \rangle \\ &= \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}'_2, t) \rangle \langle E_2(\mathbf{u}''_2, t + \tau) E_2^*(\mathbf{u}'''_2, t + \tau) \rangle \\ &+ \langle E_2^*(\mathbf{u}_2, t) E_2(\mathbf{u}''_2, t + \tau) \rangle \langle E_2(\mathbf{u}'_2, t) E_2^*(\mathbf{u}'''_2, t + \tau) \rangle. \end{aligned} \quad (6.15)$$

By using a Gaussian soft aperture with radius a for the imaging system, the impulse response function for a single lens geometry is given by

$$\begin{aligned} T_2(\mathbf{u}_2, \mathbf{x}) &= \frac{-\pi a^2}{\lambda^2 d_o d_i} \exp \left[\frac{i\pi}{\lambda} \left(\frac{|\mathbf{u}_2|^2}{d_o} + \frac{|\mathbf{x}|^2}{d_i} \right) \right] \exp(i \tan^{-1} \varepsilon) \\ &\times \exp \left(-i\varepsilon \left| \frac{\mathbf{u}_2}{\rho_2} + \frac{\mathbf{x}}{\rho} \right|^2 \right) \exp \left(- \left| \frac{\mathbf{u}_2}{\rho_2} + \frac{\mathbf{x}}{\rho} \right|^2 \right), \end{aligned} \quad (6.16)$$

where ρ_2 and ρ indicate the widths of the point spreads at planes $\mathbf{u}_2$ and $\mathbf{x}$, respectively, which are defined by

$$\rho_2 = \left(\frac{\lambda d_o}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (6.17)$$

and

$$\rho = \left(\frac{\lambda d_i}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (6.18)$$

with

$$\varepsilon = \frac{\pi a^2}{\lambda} \left(\frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{f} \right). \quad (6.19)$$

The standard deviation $\langle \phi_1^2 \rangle^{1/2}$ of phase fluctuations introduced by the diffuser D_1 has been assumed to have a value greater than π , but no restriction is imposed for the lateral correlation length of phase fluctuations. For the diffuser D_2 , however, more strict assumptions are taken into consideration. In addition to $\langle \phi_2^2 \rangle^{1/2} \gg 1$, the effective correlation length $\beta_2 = \alpha_2 / \langle \phi_2^2 \rangle^{1/2}$ is assumed to be much less than

the speckle size ζ_2 and the width ρ_2 of the point spread at the plane $\mathbf{u}_2$. These assumptions enable us to write the fourth-order moment of phase fluctuations as

$$\begin{aligned} & \langle \exp\{-i\phi_2(\mathbf{u}_2 - \mathbf{v}_2 t) + i\phi_2(\mathbf{u}'_2 - \mathbf{v}_2 t) + i\phi_2[\mathbf{u}''_2 - \mathbf{v}_2(t + \tau)] - i\phi_2[\mathbf{u}'''_2 - \mathbf{v}_2(t + \tau)]\} \rangle \\ &= \Delta S^2 [\delta(\mathbf{u}_2 - \mathbf{u}'_2) \delta(\mathbf{u}''_2 - \mathbf{u}'''_2) + \delta(\mathbf{u}_2 - \mathbf{u}''_2 + \mathbf{v}_2 \tau) \delta(\mathbf{u}'_2 - \mathbf{u}'''_2 + \mathbf{v}_2 \tau)], \end{aligned} \quad (6.20)$$

where ΔS is the area of correlation cells in the diffuser D_2 .

Substitution of Eqs. (6.15), (6.16) and (6.20) together with Eq. (6.7) into Eq. (6.14) finally results in the desired correlation function of the speckle intensity:

$$\begin{aligned} \langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle &= C \kappa_1 \exp\left(-\frac{1 - \kappa_1}{\rho^2} |\mathbf{x} + \mathbf{x}'|^2\right) \\ &\quad \times [\kappa_1 \Gamma_0(\mathbf{x}, \mathbf{x}') + \kappa_2 \Gamma_1(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1) \\ &\quad + \kappa_2 \Gamma_2(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_2) + \kappa_1 \Gamma_{12}(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \mathbf{v}_2)], \end{aligned} \quad (6.21)$$

where

$$\Gamma_0(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{1 - \kappa_1}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2\right), \quad (6.22)$$

$$\begin{aligned} \Gamma_1(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1) &= \exp\left\{-\frac{2}{\beta_1^2} [1 - \mu - \sigma(1 - \kappa_2)(1 - \nu)] |\mathbf{v}_1|^2 \tau^2\right\} \\ &\quad \times \exp\left[\frac{2\sigma(1 - \kappa_2)}{\rho \rho_2} \mathbf{v}_1 \cdot (\mathbf{x} - \mathbf{x}') \tau\right] \\ &\quad \times \exp\left(-\frac{1 - \kappa_2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2\right), \end{aligned} \quad (6.23)$$

$$\begin{aligned} \Gamma_2(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_2) &= \exp\left[-\left(\frac{1}{\kappa_1 \rho_2^2} + \kappa_2 A^2\right) |\mathbf{v}_2|^2 \tau^2\right] \\ &\quad \times \exp\left[\frac{2}{\rho} \left(\frac{1}{\rho_2} - \kappa_2 \varepsilon A\right) \mathbf{v}_2 \cdot (\mathbf{x} - \mathbf{x}') \tau\right] \\ &\quad \times \exp\left(-\frac{1 + \kappa_2 \varepsilon^2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2\right), \end{aligned} \quad (6.24)$$

$$\begin{aligned} \Gamma_{12}(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \mathbf{v}_2) &= \exp\left\{-\left\{\frac{2}{\beta_1^2} (1 - \kappa_1 \mu) |\mathbf{v}_1|^2 - 2\kappa_1 \mu [(1 - \nu)P + A'] \mathbf{v}_1 \cdot \mathbf{v}_2\right.\right. \\ &\quad \left.\left.+ \left(\frac{1}{\zeta_2^2} + \frac{1}{\rho_2^2} + \kappa_1 A^2\right) |\mathbf{v}_2|^2\right\} \tau^2\right\} \end{aligned}$$

$$\begin{aligned} & \times \exp \left\{ \frac{2}{\rho} \left[\frac{\pi \kappa_1 \varepsilon \mu \rho_2}{\lambda d} \mathbf{v}_1 + \left(\frac{1}{\rho_2} - \kappa_1 \varepsilon A \right) \mathbf{v}_2 \right] \cdot (\mathbf{x} - \mathbf{x}') \tau \right\} \\ & \times \exp \left(-\frac{1 + \kappa_1 \varepsilon^2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2 \right), \end{aligned} \quad (6.25)$$

$$C = \left(\frac{\pi \Delta S \rho_2^2}{2P} \right)^2 \left(\frac{\pi w_0}{\lambda d} \right)^4 \left(\frac{\pi a^2}{\lambda^2 d_o d_i} \right)^4 \quad (6.26)$$

with

$$A = \frac{\pi \rho_2}{\lambda} \left(\frac{\nu}{d} + \frac{1}{d_o} \right) - \frac{\varepsilon}{\rho_2} \quad (6.27)$$

and

$$A' = \frac{\pi \rho_2}{\lambda d} \left[\frac{\pi \rho_2}{\lambda} \left(\frac{1}{d} + \frac{1}{d_o} \right) - \frac{\varepsilon}{\rho_2} \right]. \quad (6.28)$$

The condition of $w \gg \alpha_1 > \beta_1$, which allows us to use the complex Gaussian statistics for the field E_2 , has been introduced to derive Eqs. (6.22)–(6.25). In Eqs. (6.21)–(6.25), κ_1 and κ_2 are the parameters determining the relationship between the width of the point spread and the extent of the illuminating speckle field or the size of illuminating speckles. They are expressed as

$$\kappa_1 = (1 + \rho_2^2/q_2^2)^{-1} \quad (6.29)$$

and

$$\kappa_2 = (1 + \rho_2^2/\zeta_2^2)^{-1}. \quad (6.30)$$

The parameter μ is defined by

$$\mu = \zeta_{2\infty}^2/\zeta_2^2 = \frac{2}{\beta_1^2} \left[\frac{2}{\beta_1^2} + \left(\frac{\pi w \sigma}{\lambda d} \right)^2 \right]^{-1}, \quad (6.31)$$

where

$$\zeta_{2\infty} = \frac{\lambda d}{\pi w}, \quad (6.32)$$

indicating the size of illuminating speckles when the second diffuser lies in the far region with respect to scattering from the first diffuser. Another parameter ν is

given by

$$\nu = 1 - \left(\frac{\sigma}{\zeta_2^2} \right) \left(\frac{2}{\beta_1^2} \right)^{-1} = \left[\frac{2}{\beta_1^2} + \sigma(\sigma - 1) \left(\frac{\pi w}{\lambda d} \right)^2 \right] \left[\frac{2}{\beta_1^2} + \left(\frac{\pi w \sigma}{\lambda d} \right)^2 \right]^{-1} \quad (6.33)$$

and represents the relationship between the correlation length β_1 of the field immediately behind the first diffuser and the correlation length (or the speckle size) ζ_2 of the intensity distribution immediately in front of the second diffuser. Therefore, the parameters μ and ν measure the change in the correlation length of the field propagating from the first diffuser to the second diffuser. The values of these parameters are determined mainly by the spot radius w of the illuminating beam and the distance d between the diffusers and, in the two limiting relations between the values of w and d , they become

$$\begin{aligned} \mu &= \begin{cases} 0 & w \gg d \\ 1 & w \ll d \end{cases} \quad (\sigma \neq 0), \\ \mu &= 1 \quad (\sigma = 0) \end{aligned} \quad (6.34)$$

and

$$\begin{aligned} \nu &= \begin{cases} 1 - 1/\sigma & w \gg d \\ 1 & w \ll d \end{cases} \quad (\sigma \neq 0), \\ \nu &= 1 \quad (\sigma = 0). \end{aligned} \quad (6.35)$$

In the case of $\sigma = 1$ (or $R \gg d$), μ and ν become exactly the same parameter.

Eq. (6.21) consists of four functions: the correlation function Γ_0 of the speckle intensity distribution; the space-time correlation functions Γ_1 and Γ_2 representing, respectively, the motion effects of the first and the second diffusers; and the space-time correlation function Γ_{12} representing the interference effect between the motions of the two diffusers. If a Gaussian approximation is employed not only for the field $E_2(\mathbf{x}, t)$ but also for the image field $E(\mathbf{x}, t)$, it is readily shown by evaluating the equation

$$\langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = \langle I(\mathbf{x}, t) \rangle \langle I(\mathbf{x}', t + \tau) \rangle + |\langle E^*(\mathbf{x}, t) E(\mathbf{x}', t + \tau) \rangle|^2, \quad (6.36)$$

that the functions Γ_1 and Γ_2 disappear while the functions Γ_0 and Γ_{12} still exist. Therefore, the image speckle field arising from double scattering does not usually

obey a Gaussian random process, and κ_2 is an important factor for determining the probability distributions of the doubly-scattered speckle field. Extensive studies have been made on the subject elsewhere (for example, [13–15]) so that we do not further discuss the probability aspect of the field but deal only with the correlation properties of the speckle intensity variations in the two limiting cases of $\kappa_2 = 0$ and 1.

6.2.1 Spatial-Correlation Function

Before analyzing the temporal behavior of speckle intensity, we briefly discuss the speckle size, which is one of the typical spatial properties of the intensity distribution. In what follows, the constant C in Eq. (6.21) is neglected and, as is mostly the case in practice, the illuminating area at the second diffuser is assumed to be much greater than the area of the point spread. The spatial-correlation function of the speckle intensity is then obtained by setting $\mathbf{v}_1 = \mathbf{v}_2 = 0$ and $\kappa_1 = 1$ in Eq. (6.21); thus

$$\begin{aligned} \langle I(\mathbf{x})I(\mathbf{x} + \mathbf{X}) \rangle = & 1 + \kappa_2 \exp \left(-\frac{1 - \kappa_2}{\rho^2} |\mathbf{X}|^2 \right) + \kappa_2 \exp \left(-\frac{1 + \kappa_2 \varepsilon^2}{\rho^2} |\mathbf{X}|^2 \right) \\ & + \exp \left(-\frac{1 + \varepsilon^2}{\rho^2} |\mathbf{X}|^2 \right), \end{aligned} \quad (6.37)$$

with $\mathbf{X} = \mathbf{x}' - \mathbf{x}$.

In order to have an insight into the intensity distribution, we consider two limiting cases: (a) the area of the point spread is much smaller than that of each speckle at the plane $\mathbf{u}_2$, namely $\kappa_2 = 1$; (b) the case opposite to (a), $\kappa_2 = 0$. In both cases, Eq. (6.37) takes the simplified form

$$\langle I(\mathbf{x})I(\mathbf{x} + \mathbf{X}) \rangle = 1 + \exp \left(-\frac{|\mathbf{X}|^2}{\zeta^2} \right), \quad (6.38)$$

where the constant factor has been neglected. In Eq. (6.38), ζ is the mean radius of speckles at the image plane $\mathbf{x}$ and is given by

$$\zeta = \frac{\lambda d_i}{\pi a}. \quad (6.39)$$

Therefore, the mean size of the speckles becomes the same as the width of the point spread of the lens at the image plane. In other words, for these limiting cases,

the doubly-scattered image speckles have exactly the same grain size as that of singly-scattered image speckles produced under the plane-wave illumination. This observation may be understood by considering the intensity distribution across the image plane as the image of the illuminating speckle pattern across the second diffuser modulated by the speckle pattern arising from the second diffuser.

In the case of $\kappa_2 = 1$, the image of each illuminating speckle is much broader than the resolution limit of the lens. Therefore, the spatial-correlation function for the intensity distribution at the image plane is determined only by its microscopic fluctuations due to the phase modulation of the second diffuser, and these fluctuations correspond to singly-scattered speckles obtained under the plane-wave illumination. On the other hand, if the condition $\kappa_2 = 0$ is satisfied, each speckle grain illuminating the second diffuser is unresolvable and, thus, the microscopic fluctuations of the intensity immediately in front of the second diffuser has little influence on the resultant speckle pattern at the image plane. Taking account of the condition $\kappa_1 = 1$, the correlation properties of the speckle pattern in this case also result in the same as those of the speckle pattern in the case of $\kappa_2 = 1$.

For the case when the values of ρ_2 and ζ_2 are the same order, the spatial-correlation function of the speckle intensity is affected by the speckle size of light incident on the second diffuser as well as the point-spread function for the imaging system.

6.2.2 Temporal-Correlation Function

We are now in a position to investigate the temporal behavior of intensity fluctuations. By assuming $\kappa_1 = 1$, and by substituting $\mathbf{x} = \mathbf{x}'$ into Eqs. (6.21)–(6.25), the terms depending on the spatial position disappear and the temporal-correlation function of intensity fluctuations at the image plane becomes

$$\begin{aligned} \langle I(t)I(t+\tau) \rangle = & 1 + \kappa_2 \exp \left\{ -\frac{2}{\beta_1^2} [1 - \mu - \sigma(1 - \kappa_2)(1 - \nu)] |\mathbf{v}_1|^2 \tau^2 \right\} \\ & + \kappa_2 \exp \left[-\left(\frac{1}{\rho_2^2} + \kappa_2 A^2 \right) |\mathbf{v}_2|^2 \tau^2 \right] \\ & + \exp \left\{ -\left\{ \frac{2}{\beta_1^2} (1 - \kappa_1 \mu) |\mathbf{v}_1|^2 - 2\mu[(1 - \nu)P + A'] \mathbf{v}_1 \cdot \mathbf{v}_2 \right. \right. \end{aligned}$$

$$+ \left(\frac{1}{\zeta_2^2} + \frac{1}{\rho_2^2} + A^2 \right) |\mathbf{v}_2|^2 \tau^2 \}. \quad (6.40)$$

The parameter κ_1 in the fourth term of Eq. (6.40) cannot be dropped, because when μ takes a value close to unity, a little change in the value of κ_1 affects the factor of $|\mathbf{v}_1|^2$ due to a very small value of β_1 . In order to extract some of the interesting results from the temporal-correlation function, it is appropriate to examine the two limiting cases of $\kappa_2 = 1$ and $\kappa_2 = 0$ used in the previous section. These cases will enable Eq. (6.40) to take a form consisting of various parameters used in practice.

When the speckle pattern illuminating the second diffuser is completely resolved by the imaging lens ($\kappa_2 = 1$), Eq. (6.40) can be rewritten as

$$\begin{aligned} \langle I(t)I(t+\tau) \rangle = & 1 + \exp \left[- \left(\frac{\sigma^2}{\zeta_2^2} + \frac{\mu}{w^2} \right) |\mathbf{v}_1|^2 \tau^2 \right] \\ & + \exp \left[- \frac{1}{\rho_0^2} \left(\frac{\nu\delta}{d} - 1 \right)^2 |\mathbf{v}_2|^2 \tau^2 \right] \\ & + \exp \left\{ - \frac{1}{\rho_0^2} \left[\mu\eta |\mathbf{v}_1|^2 - 2\mu \left[\frac{\sigma\rho_0^2}{\mu\zeta_2^2} + \frac{\delta}{d} \left(\frac{\delta}{d} - 1 \right) \right] \mathbf{v}_1 \cdot \mathbf{v}_2 \right. \right. \\ & \left. \left. + \left(\frac{\nu\delta}{d} - 1 \right)^2 |\mathbf{v}_2|^2 \right] \tau^2 \right\}, \end{aligned} \quad (6.41)$$

where

$$\eta = \frac{\frac{\sigma^2\rho_0^2}{\mu\zeta_2^2} \left[\frac{\sigma^2\rho_0^2}{\mu\zeta_2^2} \left(1 + \frac{\mu\zeta_2^2}{\sigma^2w^2} \right) + \frac{\delta^2}{d^2} \right]}{[\mu + \sigma(1-\nu)] \frac{\sigma^2\rho_0^2}{\mu\zeta_2^2} + \sigma(1-\nu) \frac{\delta^2}{d^2}} \quad (6.42)$$

and ρ_0 is the width of the point spread at the front focal plane of the lens, which is given by

$$\rho_0 = \frac{\lambda(d_o + \delta)}{\pi a}. \quad (6.43)$$

The relation $\rho_0 \leq \rho_2$ holds for most of the cases. Eq. (6.41) is derived by substituting Eqs. (6.17), (6.19), (6.27) and (6.28) into Eq. (6.40) and by eliminating the terms $1/a^2$, which usually has a value much smaller than the values of the terms $1/\zeta_2^2$ and $1/\rho_0^2$.

For further simplicity, we set another approximation—the radius w of the beam spot is much shorter than the distance d between the diffusers. In this case, $\mu = \nu = 1$ and Eq. (6.42) is drastically reduced to

$$\eta = \frac{\delta^2}{d^2}. \quad (6.44)$$

By substituting Eq. (6.44) into Eq. (6.41) and by using the relation $\rho_0^2 \ll \zeta_2^2$, Eq. (6.41) results in

$$\begin{aligned} \langle I(t)I(t+\tau) \rangle = & 1 + \exp \left[- \left(\frac{\sigma^2}{\zeta_2^2} + \frac{1}{w^2} \right) |\mathbf{v}_1|^2 \tau^2 \right] \\ & + \exp \left[- \frac{1}{\rho_0^2} \left(\frac{\delta}{d} - 1 \right)^2 |\mathbf{v}_2|^2 \tau^2 \right] \\ & + \exp \left\{ - \frac{1}{\rho_0^2} \left| \frac{\delta}{d} \mathbf{v}_1 - \left(\frac{\delta}{d} - 1 \right) \mathbf{v}_2 \right|^2 \tau^2 \right\}. \end{aligned} \quad (6.45)$$

We can now study from Eq. (6.45) the effect of defocus indicated by the parameter δ , which is the distance from the second diffuser to the front focal plane of the lens. If the imaging lens is focused on the first diffuser ($\delta = d$), Eq. (6.45) becomes

$$\langle I(t)I(t+\tau) \rangle = 2 + \exp \left[- \left(\frac{\sigma^2}{\zeta_2^2} + \frac{1}{w^2} \right) |\mathbf{v}_1|^2 \tau^2 \right] + \exp \left(- \frac{1}{\rho_0^2} |\mathbf{v}_1|^2 \tau^2 \right). \quad (6.46)$$

The first exponential term in Eq. (6.46) is a function of τ and is much broader than the second exponential term as long as the relation $\rho_0^2 \ll w^2$ holds. Hence, by eliminating the dc and slowly fluctuating components of the speckle intensity, the correlation time τ_c defined by the time at which the correlation value falls to e^{-1} is given by

$$1/\tau_c = v_1/\rho_0, \quad (6.47)$$

with $v_1 = |\mathbf{v}_1|$. The correlation time of Eq. (6.47) is equivalent to the time in which a group of the correlation cells within the area of the point spread changes to another group of cells by the motion of the first diffuser.

On the other hand, if the imaging lens is focused on the second diffuser ($\delta = 0$), Eq. (6.45) becomes

$$\langle I(t)I(t+\tau) \rangle = 1 + \exp \left[- \left(\frac{\sigma^2}{\zeta_2^2} + \frac{1}{w^2} \right) |\mathbf{v}_1|^2 \tau^2 \right] + 2 \exp \left(- \frac{1}{\rho_0^2} |\mathbf{v}_2|^2 \tau^2 \right). \quad (6.48)$$

The first exponential term again represents a slowly fluctuating component, providing that the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are of the same order of magnitude. Extracting only the rapidly fluctuating component leads to

$$1/\tau_c = v_2/\rho_0, \quad (6.49)$$

where $v_2 = |\mathbf{v}_2|$. From Eqs. (6.47) and (6.49), it can be concluded that the fluctuating rate of the speckle intensity at the image plane is proportional to the velocity of the diffuser in focus.

Next, we examine another limiting case where the imaging lens cannot resolve the illuminating speckle pattern ($\kappa_2 = 0$). In this case, Eq. (6.40) can be rewritten in the form

$$\begin{aligned} \langle I(t)I(t+\tau) \rangle = 1 + \exp \left\{ - \frac{1}{\rho_0^2} \left\{ \mu \eta |\mathbf{v}_1|^2 - 2\mu \left[\frac{\sigma \rho_0^2}{\mu \zeta_2^2} + \frac{\delta}{d} \left(\frac{\delta}{d} - 1 \right) \right] \mathbf{v}_1 \cdot \mathbf{v}_2 \right. \right. \\ \left. \left. + \left[\frac{\rho_0^2}{\zeta_2^2} + \left(\frac{\nu \delta}{d} - 1 \right)^2 \right] |\mathbf{v}_2|^2 \right\} \tau^2 \right\}. \end{aligned} \quad (6.50)$$

There are two cases where the above approximation can be applied. One is the case where the lens is out of focus even though it can resolve each illuminating speckle if it is focused on the second diffuser. Another is the case where the lens cannot resolve the illuminating speckles under the condition of its focus being set at any position. For the first case, η in Eq. (6.50) is again expressed by Eq. (6.44) under the condition that the radius w of the beam spot is much shorter than the distance d between the diffusers. Eq. (6.50) is then simplified to

$$\langle I(t)I(t+\tau) \rangle = 1 + \exp \left\{ - \frac{1}{\rho_0^2} \left| \frac{\delta}{d} \mathbf{v}_1 - \left(\frac{\delta}{d} - 1 \right) \mathbf{v}_2 \right|^2 \tau^2 \right\}. \quad (6.51)$$

If the focus is set on the first diffuser ($\delta = d$), Eq. (6.51) becomes

$$\langle I(t)I(t+\tau) \rangle = 1 + \exp \left(- \frac{1}{\rho_0^2} |\mathbf{v}_1|^2 \tau^2 \right). \quad (6.52)$$

The correlation time is given by Eq. (6.47). Therefore, the velocity v_1 of the first diffuser can be measured regardless of the value κ_2 if the relations $w \ll d$ and $\rho_0^2 \ll \zeta_2^2$ hold.

It is of interest to examine the case where the front focus of the lens is situated exactly in the middle of the diffusers. The temporal-correlation function, obtained by substituting $\delta = d/2$ into Eq. (6.51), is

$$\langle I(t)I(t+\tau) \rangle = 1 + \exp \left(-\frac{1}{4\rho_0^2} |\mathbf{v}_1 + \mathbf{v}_2|^2 \tau^2 \right). \quad (6.53)$$

The correlation time is then given by

$$1/\tau_c = v_a/\rho_0, \quad (6.54)$$

where v_a is the average velocity defined by

$$v_a = \left| \frac{\mathbf{v}_1 + \mathbf{v}_2}{2} \right|. \quad (6.55)$$

All the special cases analyzed so far have assumed the conditions $\rho_0^2 \ll \zeta_2^2$ and $w \ll d$. Other interesting properties may be obtained by evaluating the temporal-correlation function of Eq. (6.40) under the opposite conditions $\rho_0^2 \gg \zeta_2^2$ and $w \gg d$. For convenience, Eq. (6.50) is rewritten in the form

$$\begin{aligned} \langle I(t)I(t+\tau) \rangle = 1 + \exp \left\{ -\frac{1}{\zeta_2^2} \left\{ \sigma^2 \theta |\mathbf{v}_1|^2 - 2\sigma \left[1 + \frac{\mu \zeta_2^2 \delta}{\sigma \rho_0^2 d} \left(\frac{\delta}{d} - 1 \right) \right] \mathbf{v}_1 \cdot \mathbf{v}_2 \right. \right. \\ \left. \left. + \left[1 + \frac{\zeta_2^2}{\rho_0^2} \left(\frac{\nu \delta}{d} - 1 \right)^2 \right] |\mathbf{v}_2|^2 \right\} \tau^2 \right\}, \end{aligned} \quad (6.56)$$

where

$$\theta = \frac{1 + \frac{\mu \zeta_2^2}{\sigma^2 w^2} + \frac{\mu \zeta_2^2 \delta^2}{\sigma^2 \rho_0^2 d^2}}{\mu + \sigma(1-\nu) \left(1 + \frac{\mu \zeta_2^2 \delta^2}{\sigma^2 \rho_0^2 d^2} \right)}. \quad (6.57)$$

The relations $\rho_0^2 \gg \zeta_2^2$ and $w \gg d$ lead to $\mu = 0$, $\nu = 1 - (1/\sigma)$ and $\theta = 1$, finally giving

$$\langle I(t)I(t+\tau) \rangle = 1 + \exp \left(-\frac{1}{\zeta_2^2} |\sigma \mathbf{v}_1 - \mathbf{v}_2|^2 \tau^2 \right). \quad (6.58)$$

Equation (6.58) holds independently of the focal position of the imaging lens. The correlation time is then given by

$$1/\tau_c = |\sigma \mathbf{v}_1 - \mathbf{v}_2|/\zeta_2. \quad (6.59)$$

In Eq. (6.59), $\sigma \mathbf{v}_1$ is the translation velocity of the illuminating speckle pattern and, hence, the correlation time is equivalent to the time in which a group of correlation cells in the second diffuser within each of the illuminating speckles is replaced by another group due to the relative motion between the illuminating speckle pattern and the second diffuser. If a Gaussian beam illuminates the first diffuser at its waist position, Eq. (6.59) takes the form

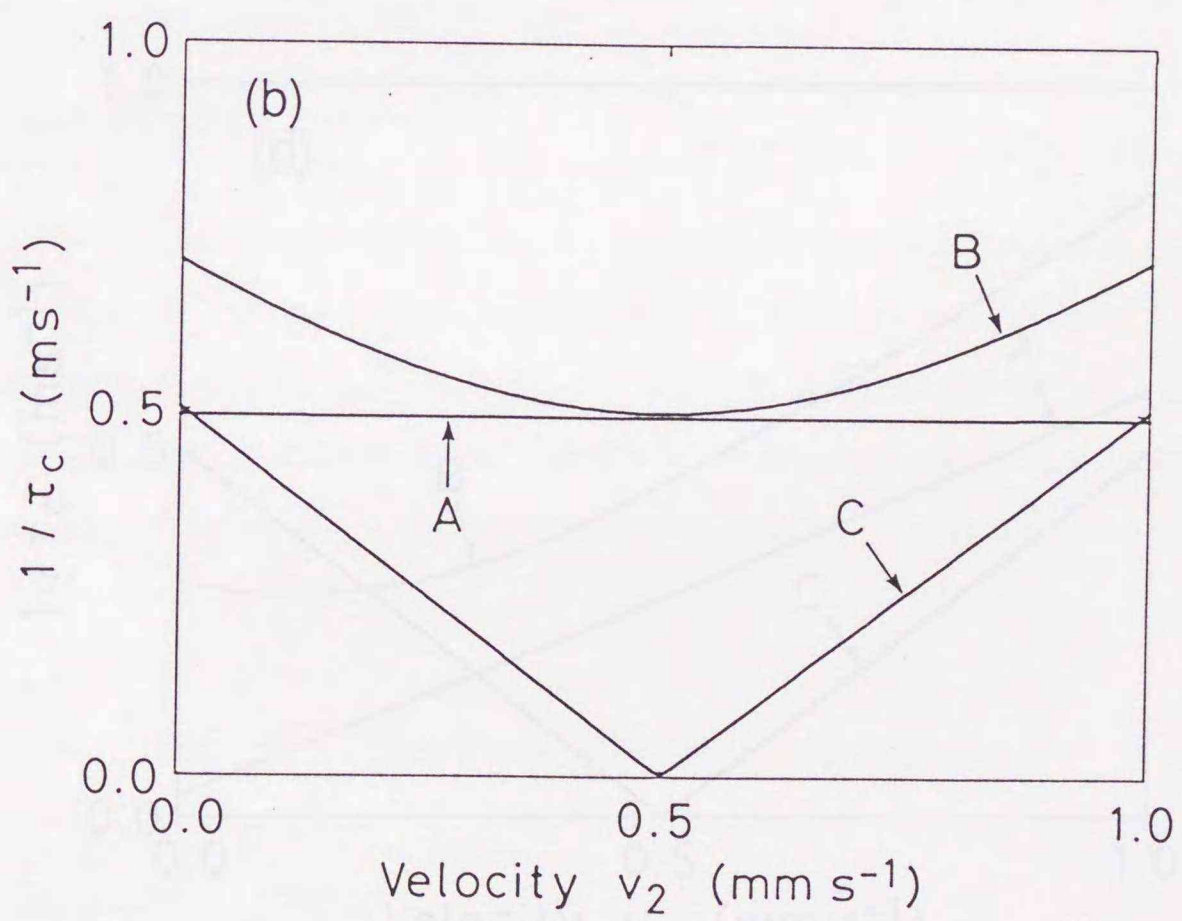
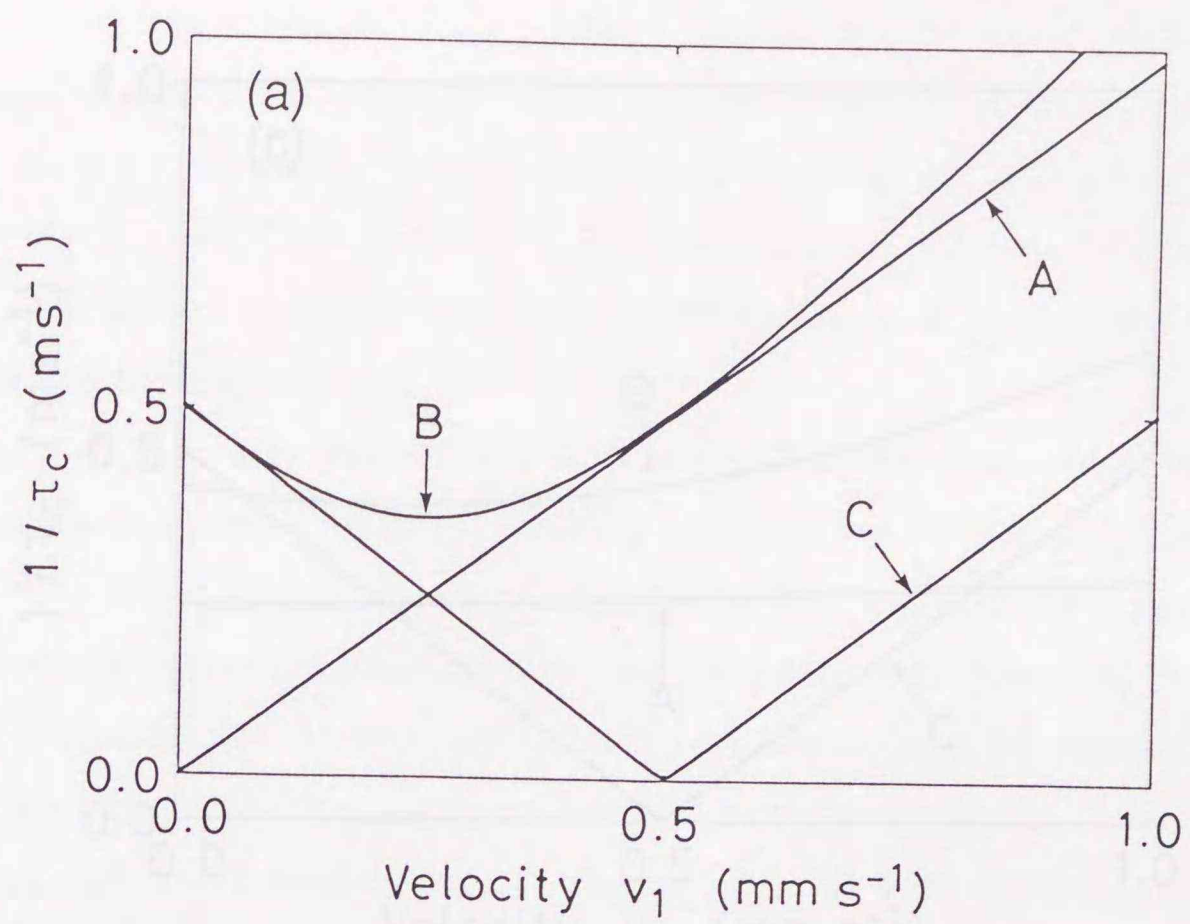
$$1/\tau_c = \Delta v/\zeta_2, \quad (6.60)$$

where Δv , the velocity difference between the diffusers, is given by

$$\Delta v = |\mathbf{v}_1 - \mathbf{v}_2|. \quad (6.61)$$

Figure 6.2 shows the theoretical curves of the reciprocal $1/\tau_c$ of the correlation time as a function of the velocity v_1 or v_2 for various values of w , d , ρ_0 and ζ_2 when the lens is focused on the first diffuser (Figs. 6.2(a) and 6.2(b)) and on the second diffuser (Figs. 6.2(c) and 6.2(d)). The diffusers are assumed to move in the same direction and the velocity v_2 is given by constant values of 0.5 mm/s in Figs. 6.2(a) and 6.2(c) and $v_1 = 0.5$ mm/s in Figs. 6.2(b) and 6.2(d). In each figure, curves A, B and C are obtained under the relations of $(w/d, \rho_0^2/\zeta_2^2) = (2 \times 10^{-3}, 10^{-4})$, $(0.2, 1)$ and $(0.2, 10^4)$, respectively.

Curves A in Fig. 6.2 clearly illustrate that the intensity fluctuations depend only on the motion of the diffuser in focus. In this case, the imaging lens has a narrow point-spread function and, therefore, it has a small f -number if the magnification of the lens is fixed. Moreover, the lens with a small f -number has a narrow focal depth so that a little defocusing may cause the image to be blurred. Therefore, the results confirm that not only the image but also the temporal fluctuations due to the diffuser set out of focus are smoothed to give the low-frequency component for the intensity fluctuations in which the high-frequency component caused by the diffuser in focus is predominant.



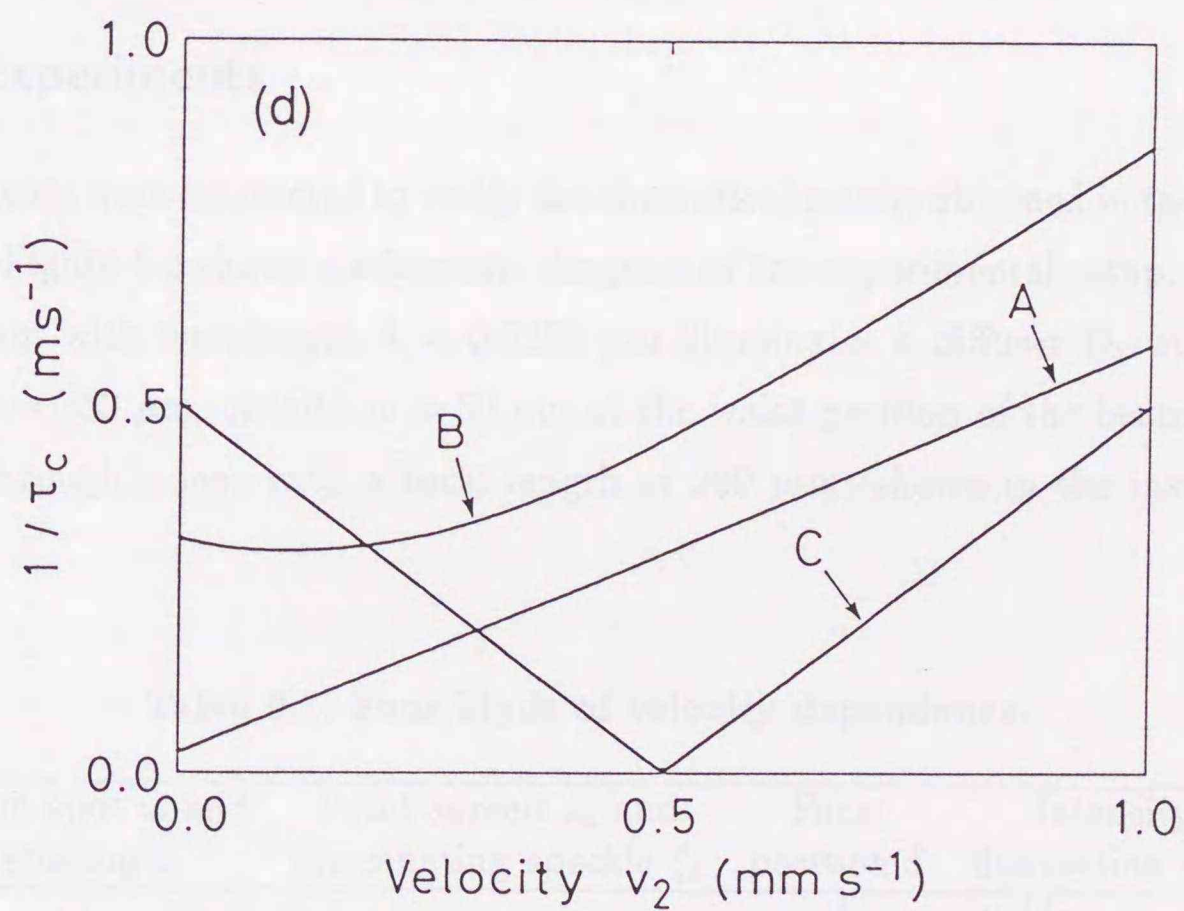
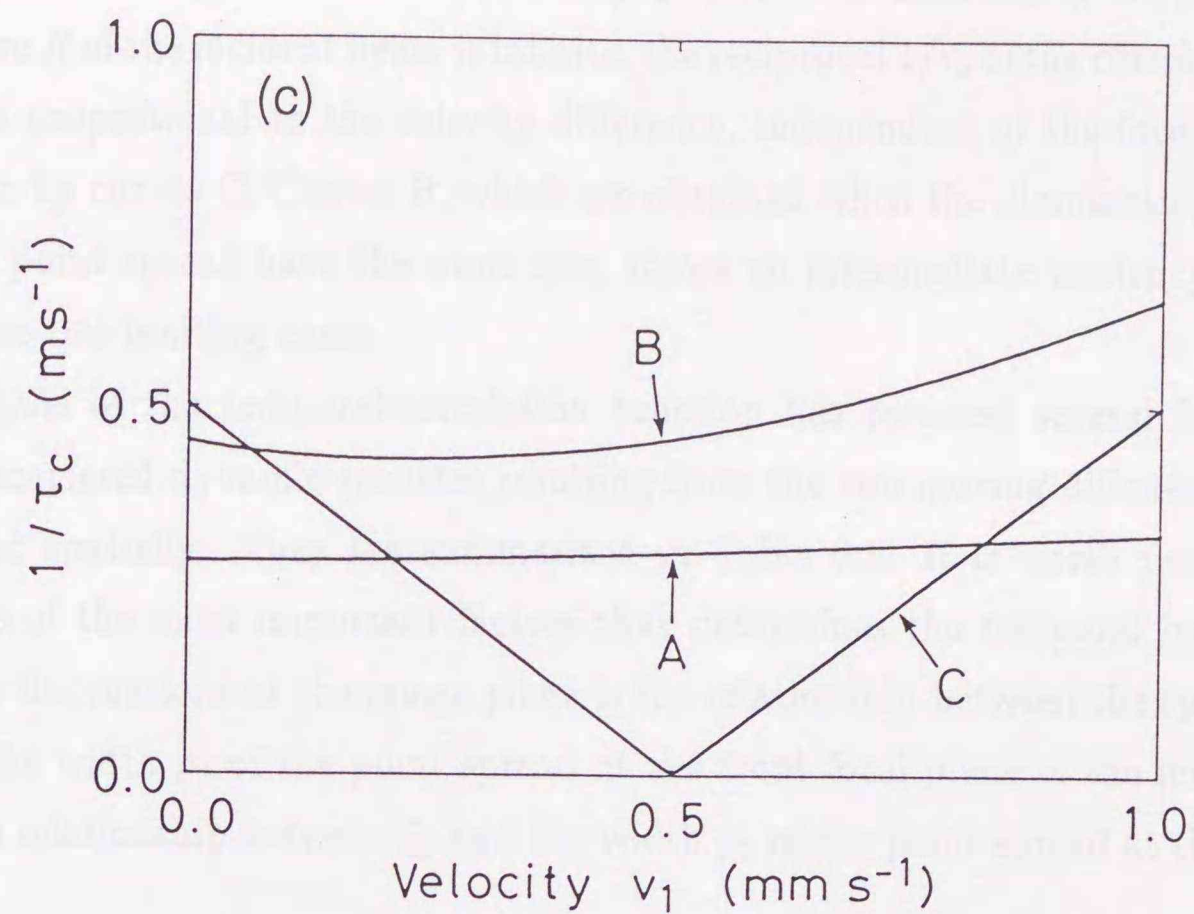


Figure 6.2. Variations of the reciprocal values ($1/\tau_c$) of the correlation time as a function of the diffuser velocities v_1 and v_2 for the cases where the focus is on the first diffuser ((a) and (b)) and on the second diffuser ((c) and (d)). Values of w , d , ζ_2 and ρ_0 are (A) 10 μm , 4.98 mm, 100 μm , 1 μm ; (B) 1 mm, 4.95 mm, 1 μm , 1 μm ; (C) 1 mm, 4.95 mm, 1 μm , 100 μm .

On the contrary, if the lens with a large f -number is used and if the phase front curvature R of the incident beam is infinite, the reciprocal $1/\tau_c$ of the correlation time becomes proportional to the velocity difference, independent of the focal position, as shown by curves C. Curves B, which are obtained when the illuminating speckles and the point spread have the same size, shows an intermediate tendency between the above two limiting cases.

Analysis of the temporal-correlation function has revealed several features of doubly-scattered dynamic speckles resulting from the two moving diffusers that are separated spatially. They are summarized in Table 6.1. It is worth pointing out that one of the most important factors that determines the temporal behavior of intensity fluctuations at the image plane is the relationship between the speckle size ζ_2 and the width ρ_0 of the point spread at the front focal plane of the lens, rather than the relationship between ζ_2 and the width ρ_2 of the point spread at the second diffuser.

6.3 Experiments

Experiments were conducted to verify the theoretical results obtained in the previous section. Figure 6.3 shows a schematic diagram of the experimental setup. A He-Ne laser beam with wavelength $\lambda = 0.6328 \mu\text{m}$ illuminates a diffuser D_1 with a spot radius $w = 750 \mu\text{m}$, or with $w = 50 \mu\text{m}$ at the waist position of the beam that has passed through a lens with a focal length of 200 mm (shown in the inset of Fig.

Table 6.1. Four kinds of velocity dependence.

Beam spot w and spacing d	Point spread ρ_0 and illuminating speckle ζ_2	Focal position δ	Intensity fluctuation $1/\tau_c$
$w \ll d$	$\rho_0^2 \ll \zeta_2^2$	d	$ \mathbf{v}_1 /\rho_0$
		$d/2$	$ (\mathbf{v}_1 + \mathbf{v}_2)/2 /\rho_0$
		0	$ \mathbf{v}_2 /\rho_0$
$w \gg d$	$\rho_0^2 \gg \zeta_2^2$	—	$ \sigma \mathbf{v}_1 - \mathbf{v}_2 /\zeta_2$

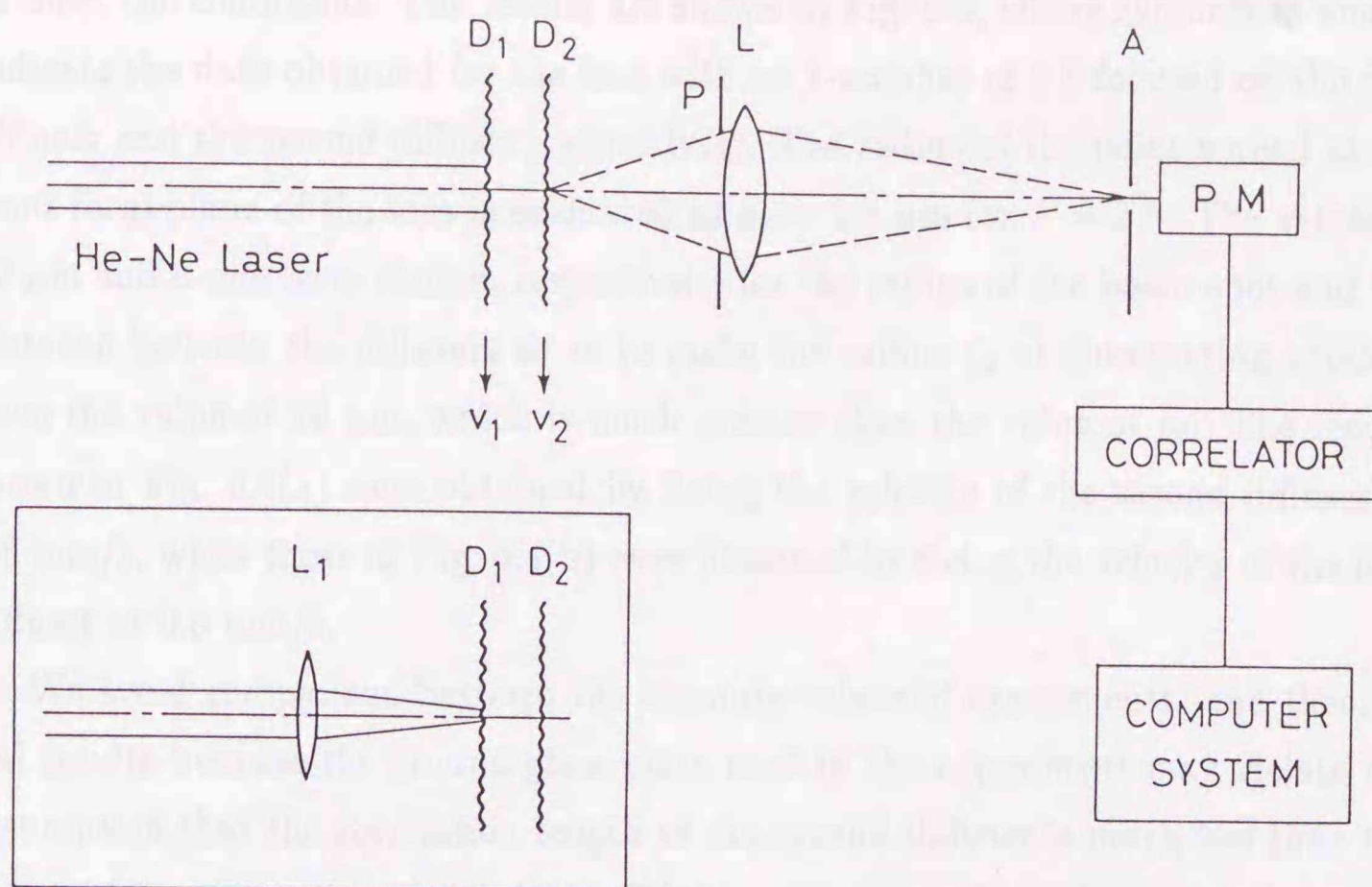


Figure 6.3. Experimental arrangement. The inset shows the beam-waist illumination.

6.3). Another diffuser D_2 is located at a certain distance from the first diffuser, and the diffusers are mounted on translation stages to move at constant velocities in the same direction perpendicular to the optical axis. Ground-glass plates used as the diffusers have almost the same roughness ($0.61 \mu\text{m}$) and correlation length ($4.2 \mu\text{m}$).

The doubly-scattered light is collected by a lens L to form time-varying speckles at the image plane, where a photomultiplier PM with a detecting aperture A is situated 330 mm away from the lens. A camera lens with a focal length of 55 mm is used as an imaging lens, so that the front focal plane exists 66 mm in front of the lens and the image magnification is 5. The diameters of the aperture A and the lens pupil P were chosen in such a way that the size of speckles at the image plane is much greater than that of the detecting aperture. The intensity fluctuations are detected via the aperture by the photomultiplier and fed into an FFT correlator to measure their normalized autocorrelation function. The correlation time τ_c is obtained as the e^{-1} -correlation time after averaging 64 autocorrelation functions, each of which is calculated by using 1024 sampling data.

We first tried to detect the velocity of each diffuser by setting the parameters

to meet the conditions. The results are shown in Fig. 6.4, where symbols $\bullet$ and $\blacktriangle$ indicate the data obtained for the lens with an f -number of 2.8 focused on the first diffuser and the second diffuser, respectively. The radius of the point spread at the front focal plane of the lens is evaluated as $\rho_0 = 1.8 \mu\text{m}$ for $F = 2.8$. The values of $50 \mu\text{m}$ and 5 mm were chosen, respectively, for the radius of the beam spot and the distance between the diffusers so as to make the radius ζ_2 of illuminating speckles have the value of $20 \mu\text{m}$, which is much greater than the value of ρ_0 . The results shown in Fig. 6.4(a) were obtained by fixing the velocity of the second diffuser at 0.5 mm/s , while those in Fig. 6.4(b) were obtained by fixing the velocity of the first diffuser at 0.5 mm/s .

We avoid comparison between the absolute values of experimental and theoretical results because the ground-glass plate used in the experiment may violate the assumption that the correlation length of the second diffuser is much less than the values of ζ_2 and ρ_0 . However, it is possible to verify some of the characteristic results predicted from the theory. Figure 6.4 clearly illustrates that the moving velocity of the diffuser in focus can be detected without being affected by the other diffuser out of focus under the conditions $w \ll d$ and $\rho_0^2 \ll \zeta_2^2$.

We next examined the velocity dependence of intensity fluctuations under the opposite conditions $w \gg d$ and $\rho_0^2 \gg \zeta_2^2$. Figure 6.5 shows the experimental results obtained for three different values of the defocusing parameter: $\blacksquare$, $\delta = -5 \text{ mm}$; $\circ$, $\delta = 0$; and, $\blacktriangle$, $\delta = 5 \text{ mm}$; (a) $v_1 < v_2$ and (b) $v_1 > v_2$. The lens with f -number of 32 was used, and the radius of the beam spot and the distance between the diffusers are $750 \mu\text{m}$ and 1 mm , respectively. From these values, the radii of the illuminating speckles and the point spread are calculated as $\zeta_2 = 1.4 \mu\text{m}$ and $\rho_0 = 20 \mu\text{m}$. The radius R of wavefront-curvature of the beam was chosen to be much greater than the distance d in order to detect the velocity difference $\Delta v = |v_1 - v_2|$. It is seen from Fig. 6.5 that, although the values of w and d are of the same order, the fluctuating rate of doubly-scattered speckles is proportional to the velocity difference, independent of the focal position.

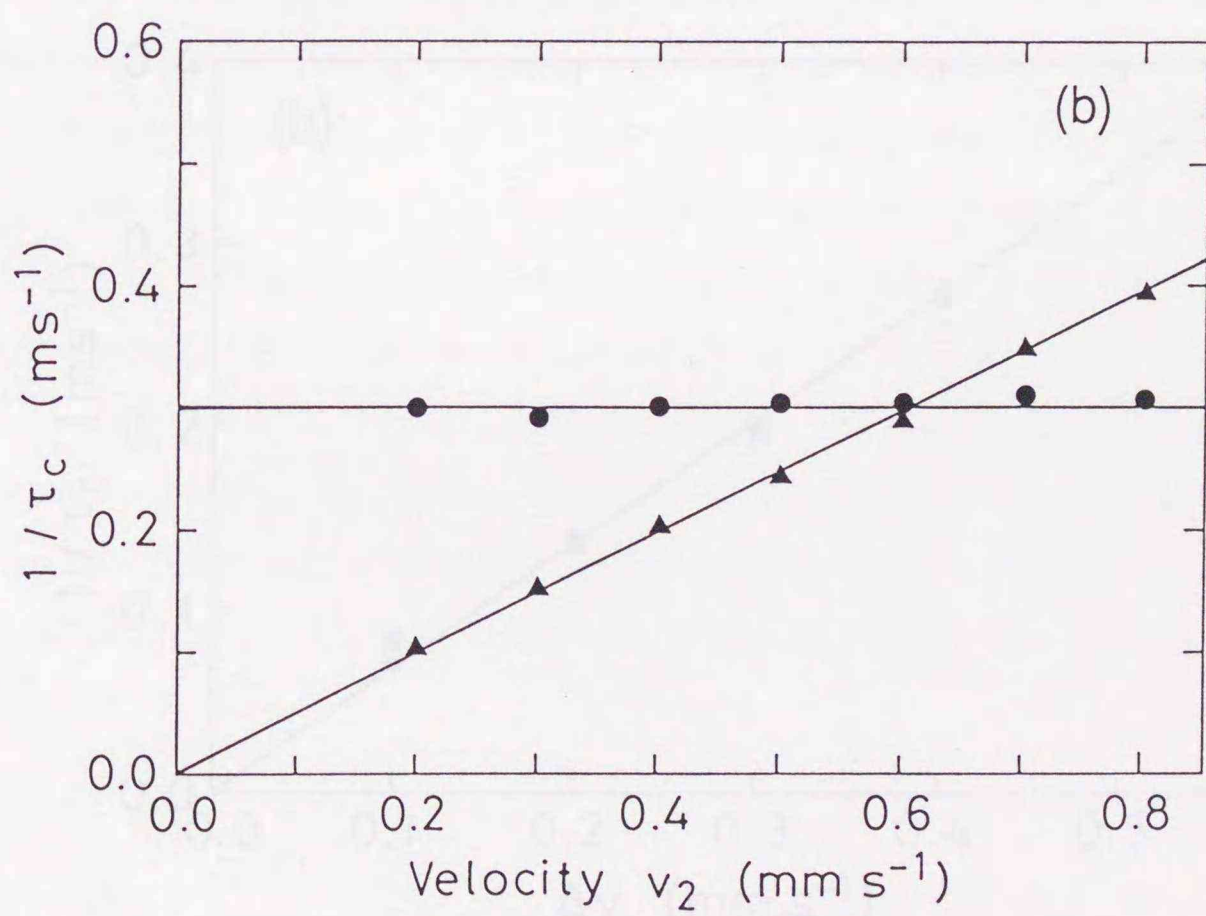
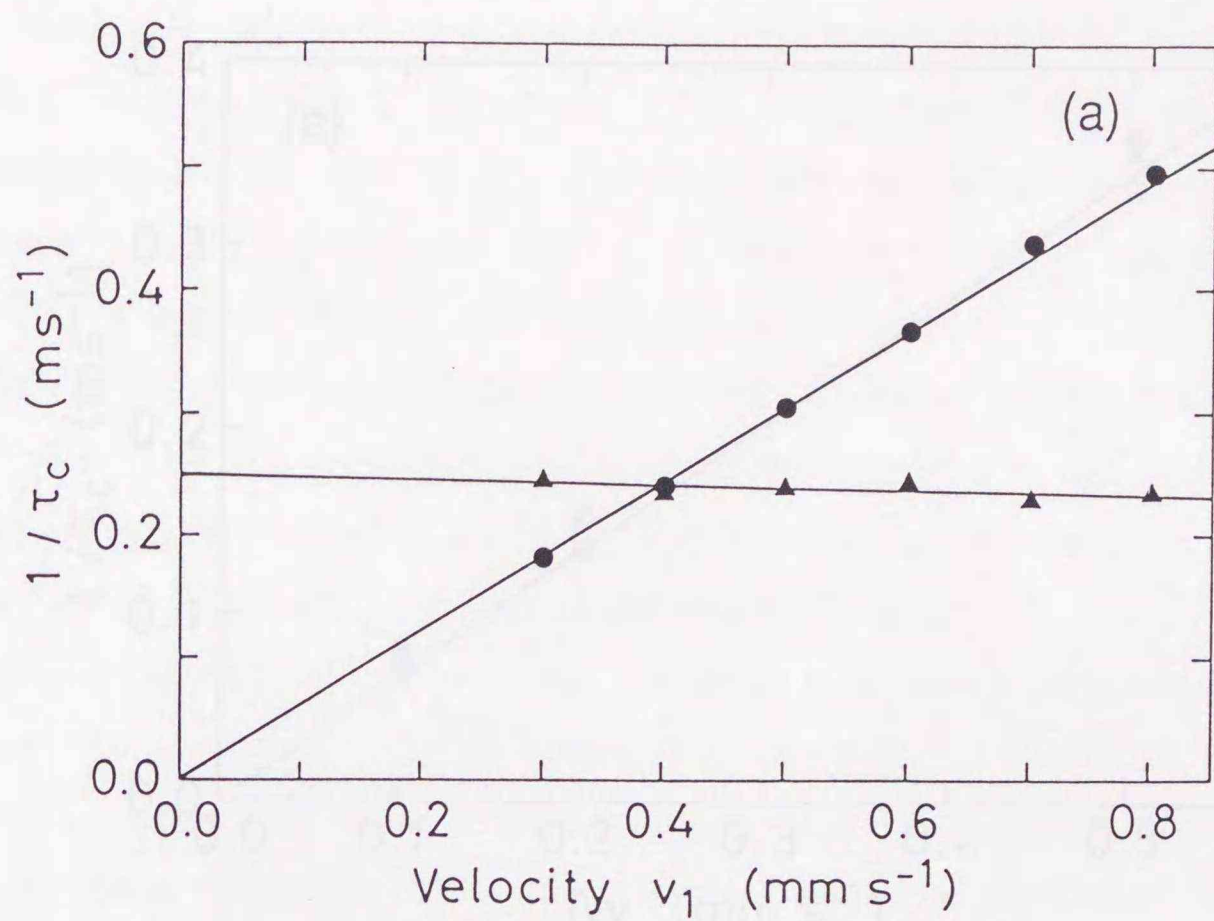


Figure 6.4. Experimental results obtained by using a lens of $F = 2.8$ and focusing on the first (●) and the second (▲) diffusers.

6.4 Conclusion

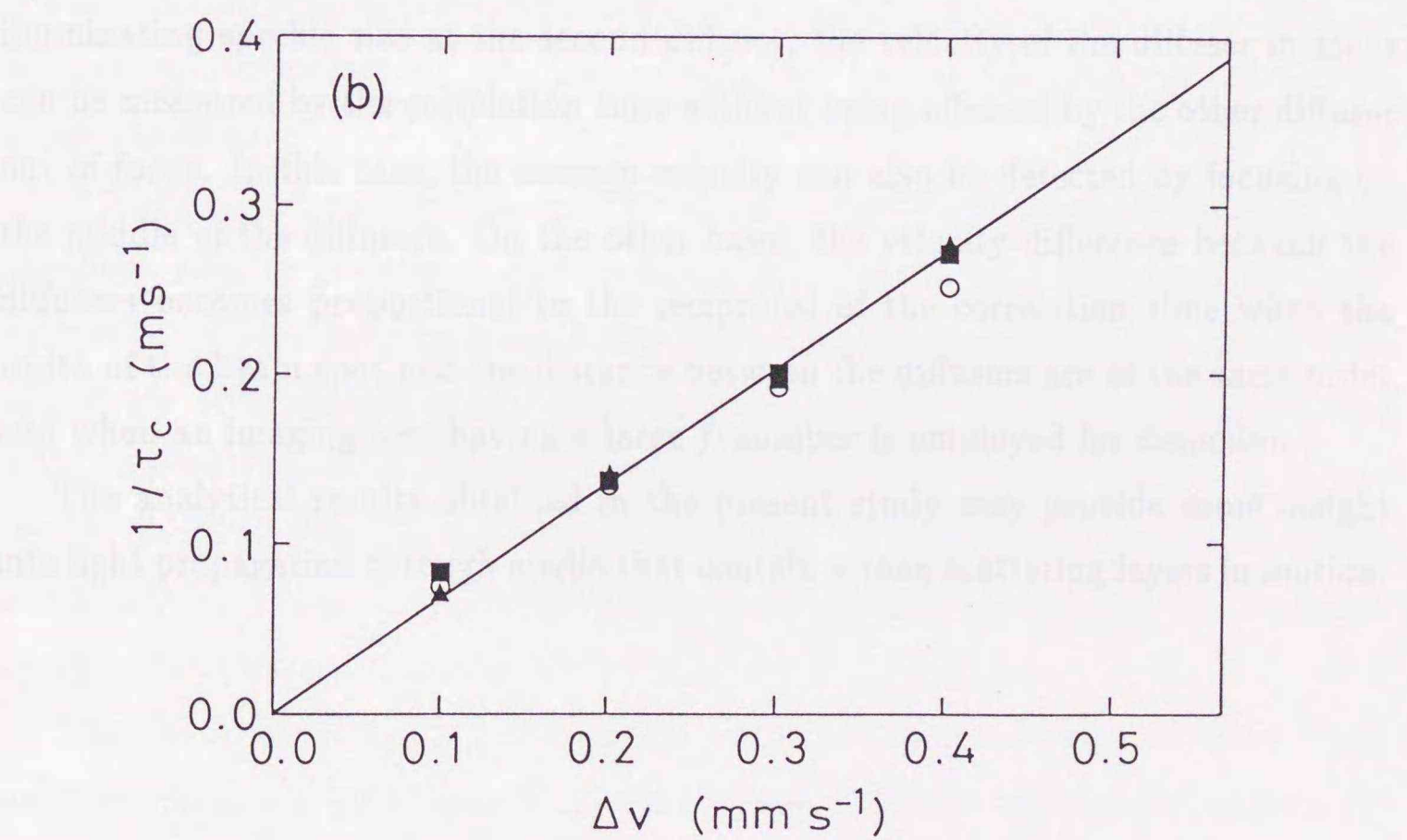
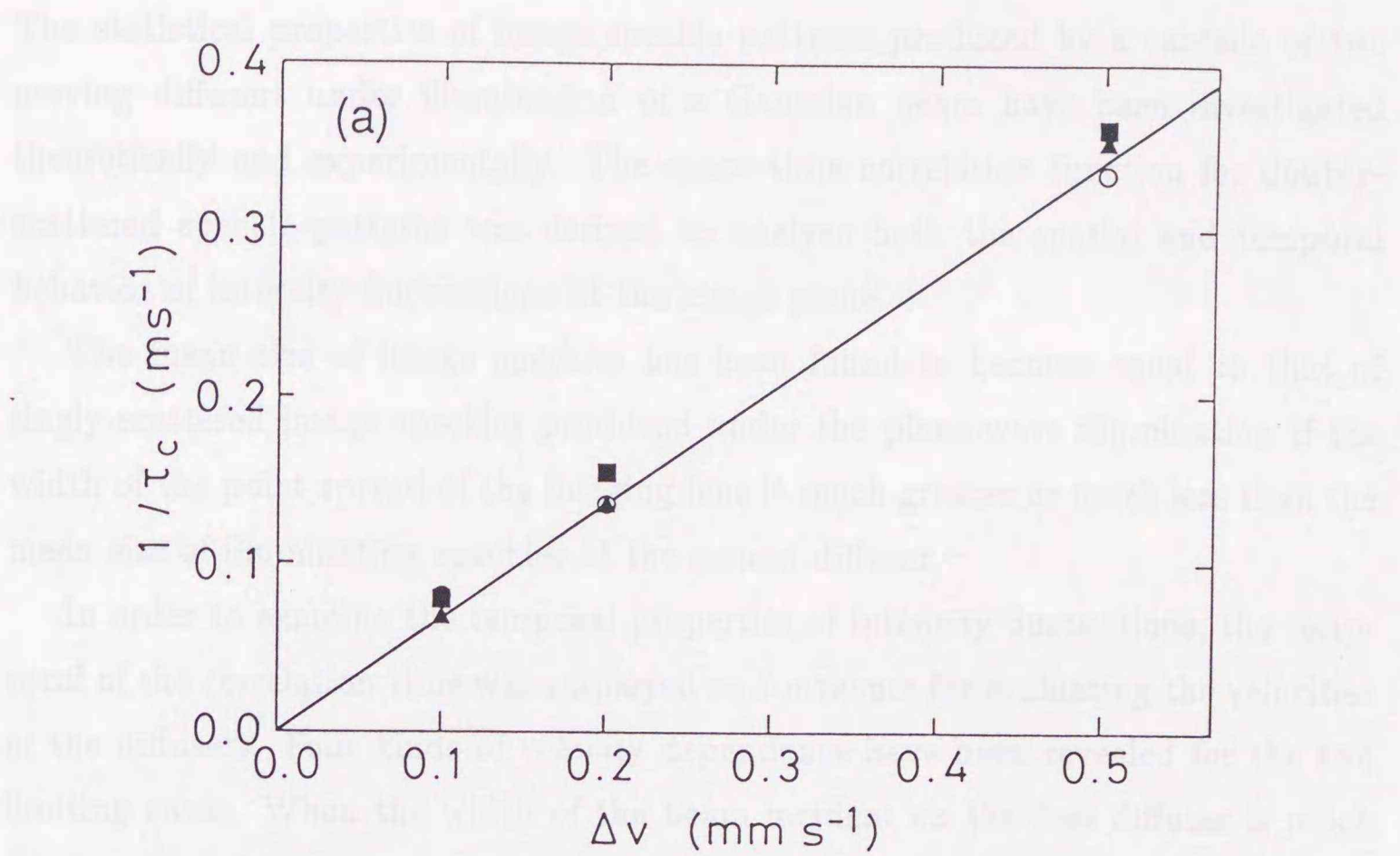


Figure 6.5. Experimental results obtained by using a lens of $F = 32$ for three positions of the front focal plane; ■, $\delta = -5$ mm; ○, $\delta = 0$; ▲, $\delta = 5$ mm.

6.4 Conclusion

The statistical properties of image speckle patterns produced by a cascade of two moving diffusers under illumination of a Gaussian beam have been investigated theoretically and experimentally. The space-time correlation function for doubly-scattered speckle patterns was derived to analyze both the spatial and temporal behavior of intensity fluctuations at the image plane.

The mean size of image speckles has been found to become equal to that of singly-scattered image speckles produced under the plane-wave illumination if the width of the point spread of the imaging lens is much greater or much less than the mean size of illuminating speckles at the second diffuser.

In order to examine the temporal properties of intensity fluctuations, the reciprocal of the correlation time was employed as a measure for evaluating the velocities of the diffusers. Four kinds of velocity dependence have been revealed for the two limiting cases. When the width of the beam incident on the first diffuser is much shorter than the distance between the diffusers and when an imaging lens with a small f -number is used so as to obtain a point-spread width much less than the illuminating speckle size at the second diffuser, the velocity of the diffuser in focus can be measured by the correlation time without being affected by the other diffuser out of focus. In this case, the average velocity can also be detected by focusing on the middle of the diffusers. On the other hand, the velocity difference between the diffusers becomes proportional to the reciprocal of the correlation time when the width of the beam spot and the distance between the diffusers are of the same order and when an imaging lens having a large f -number is employed for detection.

The analytical results obtained in the present study may provide some insight into light propagation through media that contain strong scattering layers in motion.

Chapter 7

Multiply-Scattered Speckles Produced by a Set of Moving Diffusers

7.1 Introduction

Speckle patterns are observed when coherent light is reflected by or transmitted through an optically rough surface, or when it propagates through a dense scattering medium. Different treatments must be adopted to analyze rigorously these two types of scattering phenomena. In some cases, however, the scattering from extended media can be approximated as successive scatterings from a large number of rough surfaces by introducing a set of random-phase screens. We use a model consisting of strongly scattering layers in motion and examine the dynamic properties of speckles arising from it.

A random-phase screen that modulates randomly the phase of the incident light has been used to investigate the scattering of coherent light from rough surfaces. In the context of laser speckle theory, the characteristics of rough surface of an object are often described by the statistical quantities of a random-phase screen which allows analytical study of the properties of the speckle pattern produced by the rough surface. Theoretical analyses [73, 76] have been carried out on the relationship between the statistical properties of speckles and the object roughness by using the phase screen model.

The phase screen model was extended to include the case where the light which has been scattered by a diffuser is again scattered by another diffuser and results in doubly-scattered speckles. Lowenthal and Joyeux [8] introduced two thin diffuse screens in the relative motion to reduce the speckle noise effectively and analyzed the quality of images. The statistics of doubly-scattered light have been extensively studied during the past 10 years [7–33] and most of the analyses employed random-

phase screens as diffuse objects.

The phase screen model has also been used in the study of atmospheric turbulence. Enhanced backscattering of coherent light through a random-phase screen was analyzed by Jakeman [77] and demonstrated experimentally by Tapster *et al.* [78]. Imaging of objects through a random medium was studied by Fante [79] and Mavroidis *et al.* [80–82], who used a thin diffuse screen for a model of refractive index fluctuations in the medium.

In the weak scattering regime, the extended medium can be represented as a series of weak-phase screens. The multiple phase screen technique was used by Lee and Harp [37], Roddier [38], Knepp [39] and Uscinski [40] to obtain intuitive understanding of wave propagation through the turbulent atmosphere and ionosphere. This approach was applied to the analysis of boiling speckle effects in stellar speckle interferometry [24] and to the study of fractal media [41] in which a series of fractal phase screens was used.

Although the multiple phase screen technique has become a common tool for investigating beam propagation in a weak-scattering medium and has been used even for diffraction analysis of bulk diffusers [83], speckle theories dealing with scattering from a number of deep random-phase screens in motion have not appeared in the literature. This model includes multiple-scattering effects and, therefore, it may be applied to volume scatterers such as particle suspensions flowing in laminar flow conditions.

In this chapter, the dynamic properties of speckle patterns are investigated which are produced by deep random-phase screens spaced some distance apart from each other and moving at constant velocities [43]. The space-time correlation functions of the time-varying speckle intensity are evaluated by extending the theory developed in Chapter 4. A set of recursion formulae is obtained for the correlation functions and for the three statistical quantities relating to the temporal behavior of speckle patterns: correlation time, correlation distance and speckle velocity.

Since we focus on the effects of motion of the phase screens, the other factors, such as the spatial coherence of illuminating light and the properties of phase modulation, will not be examined in detail. However, it is possible to change these factors since the correlation functions to be derived here include various parameters describing the illumination and scattering conditions.

7.2 Theoretical Analysis

Figure 7.1 shows the optical arrangement used for evaluating the space-time correlation function of multiply-scattered speckle patterns. Random-phase screens spaced in parallel with each other move at constant velocities in their own planes. By illuminating these diffusers with coherent light or spatially partially coherent light, a dynamic speckle pattern resulting from multiple scattering is produced at the observation plane located in the Fresnel diffraction region. In the following analysis, it is assumed that the illuminating light is linearly polarized and that the diffusers do not change the state of polarization of the field. These assumptions allow us to use the scalar wave approximation.

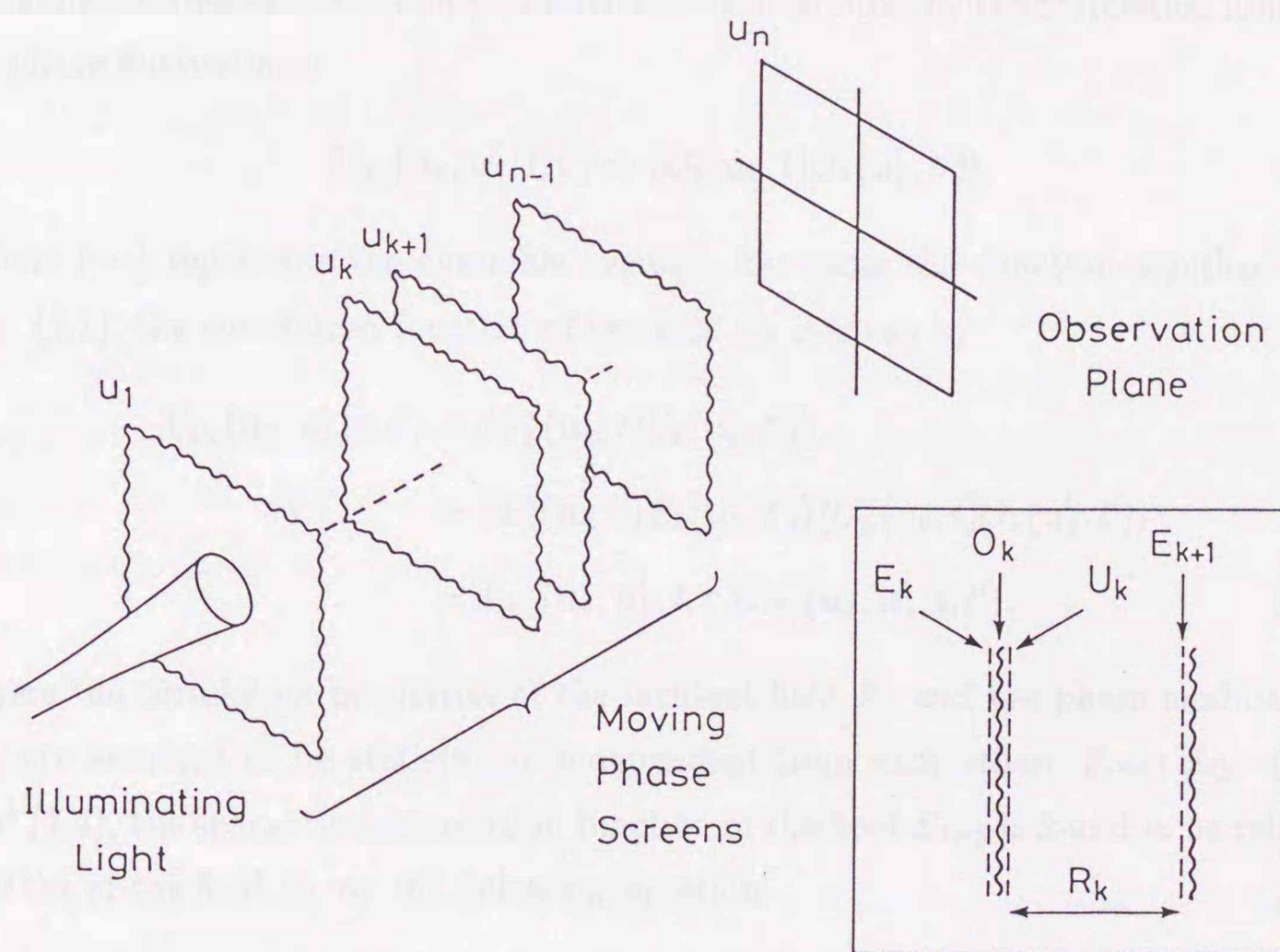


Figure 7.1. Schematic diagram for the analysis of speckles from multiple random-phase screens.

7.2.1 Propagation of the Space-Time Correlation Function

We consider a subset of the phase screens which is illustrated in the inset of Fig. 7.1. The optical field $U_k(\mathbf{u}_k, t)$ immediately behind the k th phase screen at the plane $\mathbf{u}_k$ is given by

$$U_k(\mathbf{u}_k, t) = E_k(\mathbf{u}_k, t)O_k(\mathbf{u}_k, t), \quad (7.1)$$

where $E_k(\mathbf{u}_k, t)$ is the field incident on the plane $\mathbf{u}_k$, $O_k(\mathbf{u}_k, t)$ is the phase modulation across the phase screen and $\mathbf{u}_k$ is a two-dimensional position vector at the plane $\mathbf{u}_k$. The field $E_{k+1}(\mathbf{u}_{k+1}, t)$ just in front of the $(k+1)$ th screen is expressed by using a propagation function $T_k(\mathbf{u}_k, t)$ as

$$E_{k+1}(\mathbf{u}_{k+1}, t) = \int_{-\infty}^{\infty} U_k(\mathbf{u}_k, t)T_k(\mathbf{u}_k, \mathbf{u}_{k+1})d\mathbf{u}_k. \quad (7.2)$$

The characteristics of random-phase screens are described by the correlation function of phase fluctuations:

$$\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t') = \langle O_k^*(\mathbf{u}_k, t)O_k(\mathbf{u}'_k, t') \rangle, \quad (7.3)$$

where $\langle \dots \rangle$ represents the ensemble average. By using this function together with Eq. (7.1), the correlation function of the field U_k is given by

$$\begin{aligned} \Gamma_{U_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t') &= \langle U_k^*(\mathbf{u}_k, t)U_k(\mathbf{u}'_k, t') \rangle \\ &= \langle E_k^*(\mathbf{u}_k, t)E_k(\mathbf{u}'_k, t') \rangle \langle O_k^*(\mathbf{u}_k, t)O_k(\mathbf{u}'_k, t') \rangle \\ &= \Gamma_{E_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t')\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t'), \end{aligned} \quad (7.4)$$

where the correlation properties of the incident field E_k and the phase modulation O_k are assumed to be statistically independent from each other. From Eqs. (7.2) and (7.4), the space-time correlation function of the field E_{k+1} is found to be related to that of the field E_k by the following equation:

$$\begin{aligned} \Gamma_{E_{k+1}}(\mathbf{u}_{k+1}, \mathbf{u}'_{k+1}, t, t') \\ = \iint_{-\infty}^{\infty} \Gamma_{E_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t')\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t')T_k^*(\mathbf{u}_k, \mathbf{u}_{k+1})T_k(\mathbf{u}'_k, \mathbf{u}'_{k+1})d\mathbf{u}_k d\mathbf{u}'_k. \end{aligned} \quad (7.5)$$

Thus, the correlation function of the speckle field resulting from the n th scattering can be evaluated by substituting the function Γ_{E_k} ($k = 1$ to n) into Eq. (7.5) successively.

7.2.2 Statistical Properties of Illumination and Phase Modulation

In order to evaluate Eq. (7.5), we specify the properties of illuminating light and random-phase screens, together with the propagation function.

Two kinds of illumination are considered; one is Gaussian beam illumination and the other is illumination from a Gaussian Shell-model source [84]. When the first phase screen is illuminated normally by a coherent Gaussian beam with wavelength λ , the incident field is given by

$$E_1(\mathbf{u}_1) = \frac{w_0}{w} \exp \left[i \left(\frac{2\pi z}{\lambda} + \frac{\pi |\mathbf{u}_1|^2}{\lambda R_0} \right) \right] \exp \left(-\frac{|\mathbf{u}_1|^2}{w^2} \right), \quad (7.6)$$

where R_0 is the wavefront curvature and z is the distance from the beam waist with radius w_0 to the plane $\mathbf{u}_1$ at which the radius of the beam is w . The correlation function of this incident field becomes

$$\begin{aligned} \Gamma_{E_1}(\mathbf{u}_1, \mathbf{u}'_1) &= \frac{w_0^2}{w^2} \exp \left[\frac{i\pi}{\lambda R_0} (|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2) \right] \exp \left(-\frac{|\mathbf{u}'_1 + \mathbf{u}_1|^2}{2w^2} \right) \\ &\times \exp \left(-\frac{|\mathbf{u}'_1 - \mathbf{u}_1|^2}{2w^2} \right). \end{aligned} \quad (7.7)$$

The other illuminating light to be considered is the one emanating from a Gaussian Shell-model source in which both the average intensity distribution and the degree of spatial coherence are assumed to be Gaussian. The mutual intensity at the source plane $\mathbf{u}_0$ is written in the form

$$\begin{aligned} \Gamma_{E_0}(\mathbf{u}_0, \mathbf{u}'_0) &= \langle E_0^*(\mathbf{u}_0) E_0(\mathbf{u}'_0) \rangle \\ &= \langle I_0 \rangle \exp \left(-\frac{|\mathbf{u}'_0 + \mathbf{u}_0|^2}{2q_0^2} \right) \exp \left(-\frac{|\mathbf{u}'_0 - \mathbf{u}_0|^2}{2\zeta_0^2} \right), \end{aligned} \quad (7.8)$$

where q_0 and ζ_0 represent the extent of the source and the spatial correlation length, respectively, and $\langle I_0 \rangle$ is the average intensity at the center of the source. Equation (7.8) is valid under the condition of $q_0 \gg \zeta_0$. When the first phase screen is located a distance R_0 from the source plane, the mutual intensity at the plane $\mathbf{u}_1$ is given by [25]

$$\Gamma_{E_1}(\mathbf{u}_1, \mathbf{u}'_1) = \langle I_1 \rangle \exp \left[\frac{i\pi}{\lambda R_0} (|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2) \right] \exp \left(-\frac{|\mathbf{u}'_1 + \mathbf{u}_1|^2}{2q_1^2} \right)$$

$$\times \exp\left(-\frac{|\mathbf{u}'_1 - \mathbf{u}_1|^2}{2\zeta_1^2}\right), \quad (7.9)$$

where

$$\langle I_1 \rangle = \left(\frac{\pi q_0 \zeta_0}{\lambda R_0}\right)^2 \langle I_0 \rangle, \quad (7.10)$$

and q_1 and ζ_1 are the extent of the illuminating field and the spatial coherence length (or the average speckle radius) defined by

$$q_1 = \frac{\lambda R_0}{\pi \zeta_0} \quad (7.11)$$

and

$$\zeta_1 = \frac{\lambda R_0}{\pi q_0}. \quad (7.12)$$

The phase modulation produced by the k th phase screen moving at a constant velocity $\mathbf{v}_k$ is given by

$$O_k(\mathbf{u}_k, t) = \exp[i\phi_k(\mathbf{u}_k - \mathbf{v}_k t)]. \quad (7.13)$$

For simplicity, assume that the phase fluctuations introduced by the phase screen follow a Gaussian random process and that the normalized spatial correlation function of phase fluctuations takes Gaussian form. In this case the space-time correlation function of Eq. (7.13) can be written in the form [73]

$$\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t') = \exp\{-\sigma_k[1 - \exp(-|\Delta\mathbf{u}_k|^2/\alpha_k^2)]\}, \quad (7.14)$$

where

$$\begin{aligned} \Delta\mathbf{u}_k &= (\mathbf{u}'_k - \mathbf{v}_k t') - (\mathbf{u}_k - \mathbf{v}_k t) \\ &= \mathbf{u}'_k - \mathbf{u}_k - \mathbf{v}_k \tau \end{aligned} \quad (7.15)$$

with $\tau = t' - t$. In Eq. (7.14), σ_k and α_k indicate the standard deviation and the correlation length of phase fluctuations, respectively. Since we confine our discussion to the case of strong scattering, it can be assumed that the value of σ_k is larger than π . Equation (7.14) is then approximately reduced to [74]

$$\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, \tau) = \exp(-|\Delta\mathbf{u}_k|^2/\beta_k^2), \quad (7.16)$$

where β_k stands for the effective correlation length defined by

$$\beta_k = \alpha_k / \sigma_k. \quad (7.17)$$

Finally, by using the Fresnel approximation, the propagation function from the plane $\mathbf{u}_k$ to the plane $\mathbf{u}_{k+1}$ becomes

$$T_k(\mathbf{u}_k, \mathbf{u}_{k+1}) = \frac{1}{i\lambda R_k} \exp\left(\frac{i\pi|\mathbf{u}_k - \mathbf{u}_{k+1}|^2}{\lambda R_k}\right). \quad (7.18)$$

7.2.3 Correlation Function for the Multiply-Scattered Speckle Field

Now we will evaluate the space-time correlation function of the multiply-scattered speckle field by using a method of mathematical induction. First, the following expression is assumed for the speckle field at the plane $\mathbf{u}_n$ resulting from $(n-1)$ th-order scattering:

$$\begin{aligned} \Gamma_{E_n}(\mathbf{u}_n, \mathbf{u}'_n, \tau) = & \langle I_n \rangle \exp\left(-\sum_{l=0}^{n-1} \sum_{m=0}^{n-1} A_n(l, m) \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 + \sum_{l=0}^{n-1} B_n(l) (\mathbf{u}'_n - \mathbf{u}_n) \cdot \mathbf{v}_l \tau\right) \\ & \times \exp\left[-\frac{i\pi}{\lambda R_{n-1}} \left(\sum_{l=0}^{n-1} C_n(l) (\mathbf{u}'_n + \mathbf{u}_n) \cdot \mathbf{v}_l \tau - D_n(|\mathbf{u}'_n|^2 - |\mathbf{u}_n|^2)\right)\right] \\ & \times \exp\left(-\frac{|\mathbf{u}'_n + \mathbf{u}_n|^2}{2q_n^2}\right) \exp\left(-\frac{|\mathbf{u}'_n - \mathbf{u}_n|^2}{2\zeta_n^2}\right) \quad (n \geq 1), \end{aligned} \quad (7.19)$$

where $\mathbf{v}_0 = 0$, and q_n and ζ_n are again the extent of the illuminating field and the average speckle radius at the plane $\mathbf{u}_n$, respectively. This expression is obtained as an extension of Eqs. (4.14) and (4.23) where the correlation functions of singly- and doubly-scattered speckle fields under Gaussian beam illumination have been evaluated.

At this stage, the values of the parameters $A_n(l, m)$, $B_n(l)$, $C_n(l)$, D_n , q_n and ζ_n in Eq. (7.19) are unknown. Therefore, it is necessary to determine the values of these parameters in such a way that Eq. (7.19) is suitably valid for any integer n greater than 0. Now we calculate the correlation function for the field at the plane $\mathbf{u}_{k+1}$ by assuming that Eq. (7.19) is true at $n = k$, and compare the parameters obtained for the plane $\mathbf{u}_{k+1}$ with those for the plane $\mathbf{u}_k$.

Substitution of Eqs. (7.15), (7.16), (7.18) and (7.19) into Eq. (7.5) results in the space-time correlation function for the plane $\mathbf{u}_{k+1}$:

$$\begin{aligned} \Gamma_{E_{k+1}}(\mathbf{u}_{k+1}, \mathbf{u}'_{k+1}, \tau) &= \langle I_{k+1} \rangle \exp \left(- \sum_{l=0}^k \sum_{m=0}^k A_{k+1}(l, m) \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 + \sum_{l=0}^k B_{k+1}(l) (\mathbf{u}'_{k+1} - \mathbf{u}_{k+1}) \cdot \mathbf{v}_l \tau \right) \\ &\times \exp \left[- \frac{i\pi}{\lambda R_k} \left(\sum_{l=0}^k C_{k+1}(l) (\mathbf{u}'_{k+1} + \mathbf{u}_{k+1}) \cdot \mathbf{v}_l \tau - D_{k+1} (|\mathbf{u}'_{k+1}|^2 - |\mathbf{u}_{k+1}|^2) \right) \right] \\ &\times \exp \left(- \frac{|\mathbf{u}'_{k+1} + \mathbf{u}_{k+1}|^2}{2q_{k+1}^2} \right) \exp \left(- \frac{|\mathbf{u}'_{k+1} - \mathbf{u}_{k+1}|^2}{2\zeta_{k+1}^2} \right). \end{aligned} \quad (7.20)$$

The parameters in Eq. (7.20) can be written in terms of those in Eq. (7.19), giving

$$\begin{aligned} A_{k+1}(l, m) &= A_k(l, m) - \frac{1}{2} P_k^{-1} B_k(l) B_k(m) - \gamma_k K_k (1 + K_k D_k) B_k(m) C_k(l) \\ &\quad + \frac{1}{2} \gamma_k K_k^2 (P_{k,1} + P_{k,2}) C_k(l) C_k(m), \end{aligned} \quad (7.21)$$

$$B_{k+1}(l) = \gamma_k (1 + K_k D_k) B_k(l) - \gamma_k K_k (P_{k,1} + P_{k,2}) C_k(l), \quad (7.22)$$

$$C_{k+1}(l) = P_k^{-1} B_k(l) + \gamma_k K_k (1 + K_k D_k) C_k(l), \quad (7.23)$$

$$D_{k+1} = 1 - \gamma_k (1 + K_k D_k), \quad (7.24)$$

$$q_{k+1} = \frac{\lambda R_k}{\pi} P_k^{1/2}, \quad (7.25)$$

$$\zeta_{k+1} = [\gamma_k (P_{k,1} + P_{k,2})]^{-1/2}, \quad (7.26)$$

where

$$A_k(l, k) = A_k(k, m) = 0, \quad (7.27)$$

$$A_k(k, k) = P_{k,1}/2, \quad (7.28)$$

$$B_k(k) = P_{k,1}, \quad (7.29)$$

$$C_k(k) = 0. \quad (7.30)$$

In Eqs. (7.21)–(7.30),

$$P_{k,1} = 2/\beta_k^2, \quad (7.31)$$

$$P_{k,2} = 1/\zeta_k^2, \quad (7.32)$$

$$P_{k,3} = \left(\frac{\pi q_k}{\lambda R_k} \right)^2 (1 + K_k D_k)^2 \quad (7.33)$$

and

$$P_k = P_{k,1} + P_{k,2} + P_{k,3} \quad (7.34)$$

are variables depending on the statistical properties of the incident light and the phase screens, and

$$K_k = R_k/R_{k-1} \quad (7.35)$$

and

$$\gamma_k = \langle I_{k+1} \rangle / \langle I_k \rangle = (1 + K_k D_k)^{-2} P_{k,3} / P_k \quad (7.36)$$

are the ratios between the two spacings and between the two average intensities, respectively. Equations (7.20)–(7.36) hold within the range of $k \geq 1$, $0 \leq l \leq k$ and $0 \leq m \leq k$. Equation (7.20) is found clearly to be expressed by Eq. (7.19) by substituting $n = k + 1$ into Eq. (7.19). This result indicates that, if Eq. (7.19) is valid for the speckle field at the plane $\mathbf{u}_k$, it is also true for the field $\mathbf{u}_{k+1}$ under the relations given by Eqs. (7.21)–(7.26).

The correlation function of the incident field is readily shown to be expressed by Eq. (7.19) for two kinds of illumination presented in Subsection 7.2.2. For a Gaussian beam illumination, a comparison between Eqs. (7.7) and (7.19) gives

$$A_1(0,0) = B_1(0) = C_1(0) = 0, \quad (7.37)$$

$$D_1 = 1 \quad (7.38)$$

and

$$q_1 = \zeta_1 = w \quad (7.39)$$

for the initial values of the parameters. For a Gaussian Shell-model beam, Equation (7.19) becomes the mutual intensity of Eq. (7.9) by substituting

$$A_1(0,0) = B_1(0) = C_1(0) = 0, \quad (7.40)$$

$$D_1 = 1, \quad (7.41)$$

$$q_1 = \frac{\lambda R_0}{\pi \zeta_0} \quad (7.42)$$

and

$$\zeta_1 = \frac{\lambda R_0}{\pi q_0} \quad (7.43)$$

into Eq. (7.19) with $n = 1$. Equation (7.19) thus gives the correlation function of the fields illuminated by both the Gaussian beam and the Gaussian Shell-model beam.

We have shown that (i) the correlation function expressed by Eq. (7.19) is suitably valid for $n = 1$ and that (ii) if Eq. (7.19) is true at $n = k$, it is also true at $n = k + 1$. These results lead to the conclusion that the space-time correlation function of the speckle field arising from multiple scattering of any order is given by Eq. (7.19) together with Eqs. (7.21)–(7.36).

As is seen from Eqs. (7.21)–(7.26), the parameters are expressed by a set of recursion formulae which appears to be very difficult to solve analytically, since the formula for each parameter includes not only that parameter itself but the other parameters as well. These recursion formulae, however, provide an expression that is useful to deal with when evaluating them numerically. Thus we do not perform a further calculation and use Eq. (7.19) as a final expression for the correlation function of the speckle fields.

7.2.4 Statistical Quantities Describing Speckle Motion

Since we are concerned with the intensity variations of dynamic speckle patterns, it is desirable to know the space-time correlation function of intensity fluctuations. The fluctuating component $\Delta I(\mathbf{u}_n, t)$ of the speckle intensity at point $\mathbf{u}_n$ and time t is given by

$$\Delta I_n(\mathbf{u}_n, t) = I_n(\mathbf{u}_n, t) - \langle I_n(\mathbf{u}_n, t) \rangle, \quad (7.44)$$

where

$$I_n(\mathbf{u}_n, t) = E_n(\mathbf{u}_n, t)E_n^*(\mathbf{u}_n, t) = |E_n(\mathbf{u}_n, t)|^2. \quad (7.45)$$

For simplicity, two assumptions are taken into consideration: (i) the speckle field due to the first phase screen obeys a circular complex Gaussian process and (ii)

every speckle field illuminating each phase screen contains a large number of speckle grains. These assumptions are satisfied simultaneously as long as a large number of independent phase-modulating cells exist within the illuminating area across each phase screen. This statement arises from two facts: (i) under the condition above, a deep random-phase screen produces a fully developed speckle pattern having lots of speckle grains ($q_2 \gg \zeta_2$) and its intensity fluctuation follows a circular complex Gaussian process; (ii) By referring to Eqs. (7.25)–(7.34), the relation of $q_k \gg \zeta_k$ holds for all k s greater than 2 once the relation $q_2 \gg \zeta_2$ has been satisfied. These assumptions enable us to consider that the multiply-scattered speckle field also obeys a circular complex Gaussian process (see Fig. 2 in [13]).

In such a case, the space-time correlation function of speckle intensity fluctuations can be written in terms of the correlation function of the speckle field as

$$\langle \Delta I_n(\mathbf{u}_n, t) \Delta I_n(\mathbf{u}'_n, t + \tau) \rangle = |\Gamma_{E_n}(\mathbf{u}_n, \mathbf{u}'_n, \tau)|^2. \quad (7.46)$$

Substitution of Eq. (7.19) into Eq. (7.46) leads to

$$\begin{aligned} & \langle \Delta I_n(\mathbf{u}_n, t) \Delta I_n(\mathbf{u}'_n, t + \tau) \rangle \\ &= \exp \left(-2 \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} A_n(l, m) \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 + 2 \sum_{l=0}^{n-1} B_n(l) (\mathbf{u}'_n - \mathbf{u}_n) \cdot \mathbf{v}_l \tau \right) \\ & \times \exp \left(-\frac{|\mathbf{u}'_n + \mathbf{u}_n|^2}{q_n^2} \right) \exp \left(-\frac{|\mathbf{u}'_n - \mathbf{u}_n|^2}{\zeta_n^2} \right), \end{aligned} \quad (7.47)$$

where the factor $\langle I_n \rangle^2$ has been omitted to normalize the function. In what follows the analysis is made within a local region whose area is much smaller than an area of the macroscopic extent q_n . The term $\exp(-|\mathbf{u}'_n + \mathbf{u}_n|^2/q_n^2)$ in Eq. (7.47) is then neglected since the value of the term is almost constant within that region.

Three important quantities describing the properties of dynamic speckles are derived from Eq. (7.47).

The correlation time τ_c measures how rapidly the intensity is changing at a specific point in a dynamic speckle pattern. By evaluating Eq. (7.47) at $\mathbf{u}_n = \mathbf{u}'_n$, the autocorrelation function is obtained as

$$\langle \Delta I_n(t) \Delta I_n(t + \tau) \rangle = \exp(-\tau^2/\tau_{c(n)}^2), \quad (7.48)$$

where the correlation time is given by

$$\tau_{c(n)} = \left(2 \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} A_n(l, m) v_l v_m \cos \psi_{lm} \right)^{-1/2} \quad (7.49)$$

in which $v = |\mathbf{v}|$, and ψ_{lm} is the angle between the directions of $\mathbf{v}_l$ and $\mathbf{v}_m$.

The correlation distance X_c is used to measure how long the speckle pattern translates without any appreciable change in the pattern. Equation (7.47) may be rewritten in the form

$$\langle \Delta I_n(\mathbf{u}_n, t) \Delta I_n(\mathbf{u}'_n, t + \tau) \rangle = \exp \left[-\frac{(\tau - \tau_{d(n)})^2}{\tau_{c(n)}^2} \right] \exp \left(\frac{\tau_{d(n)}^2}{\tau_{c(n)}^2} - \frac{|\mathbf{X}_n|^2}{\zeta_n^2} \right), \quad (7.50)$$

where $\mathbf{X}_n = \mathbf{u}'_n - \mathbf{u}_n$. In Eq. (7.50), $\tau_{d(n)}$ is the time lapse of the cross-correlation function given by

$$\begin{aligned} \tau_{d(n)} &= \frac{\sum_{l=0}^{n-1} B_n(l) \mathbf{v}_l \mathbf{X}_n}{2 \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} A_n(l, m) \mathbf{v}_l \mathbf{v}_m} \\ &= \tau_{c(n)}^2 X_n \sum_{l=0}^{n-1} B_n(l) v_l \cos \theta_l \end{aligned} \quad (7.51)$$

where $X_n = |\mathbf{X}_n|$ and θ_l denotes the angle between the vectors of $\mathbf{v}_l$ and $\mathbf{X}_n$. The peak value of the cross-correlation function is equivalent to the value of the second exponential term in Eq. (7.50). The correlation distance is defined by the distance at which the cross-correlation peak drops off to e^{-1} and hence

$$\exp \left(\frac{\tau_{d(n)}^2}{\tau_{c(n)}^2} - \frac{X_n^2}{\zeta_n^2} \right) = e^{-1}. \quad (7.52)$$

By substituting Eq. (7.51) in Eq. (7.52), the correlation distance is expressed by

$$X_{c(n)} = \left[1/\zeta_n^2 - \left(\tau_{c(n)} \sum_{l=0}^{n-1} B_n(l) v_l \cos \theta_l \right)^2 \right]^{-1/2}. \quad (7.53)$$

Since the time $\tau_{d(n)}$ is required for the speckles to translate over a distance X_n , the velocity V_n of the speckles moving across the plane $\mathbf{u}_n$ is obtained from Eq.

(7.51) as

$$V_n = X_n / \tau_{d(n)} = \left(\tau_{c(n)}^2 \sum_{l=0}^{n-1} B_n(l) v_l \cos \theta_l \right)^{-1}. \quad (7.54)$$

Equations (7.49), (7.53) and (7.54) are the main results of the analysis obtained here. The behavior of the speckle motion arising from a series of deep phase screens in motion are fully described by those equations. However, it is to be noted that the formulae given here can be used for the case in which the velocity of each phase screen has a definite value. Thus, when the velocity is treated as a random variable as well as the phase fluctuations due to the random-phase screens, the averaging operation will be needed after evaluation of the formulae for a number of the velocity distributions having the same statistical values.

7.3 Results and Discussion

We are now in a position to analyze in detail the motion of speckles due to multiple scattering. The space-time correlation function that was derived in Section 7.2 shows that the statistical properties of speckle patterns are affected by (i) the illumination geometry, (ii) the scattering properties of diffusers, (iii) the velocities of diffusers and (iv) the detection geometry. Since the temporal behavior of speckles is our main concern, we restrict ourselves to examining the effect of the velocity distribution of phase screens as well as the effect of multiple scattering, although the illuminating and detecting conditions have some effects on the speckle motion.

7.3.1 Effects of Multiple Scattering and Velocity Distributions

In most of the cases, a time-varying speckle pattern produced by many moving scatterers exhibits a quite different behavior from that produced by a single moving diffuser. This fact indicates that the number of scattering events affects the dynamic properties of resulting speckles. Moreover, in the case of multiple scatterers such as particle suspensions, the velocities of the scatterers are usually different from each other, and this velocity difference is also expected to have some influence on the behavior of dynamic speckles. Therefore, by changing both the number of the phase

screens and their velocity distribution, we examine the dependence of the three quantities describing the motion of multiply-scattered speckles on those parameters.

The geometry to be analyzed is shown schematically in Fig. 7.2. Random-phase screens having an effective correlation length of $3.183 \mu\text{m}$ are spaced equally in the range of 10 mm along the optical axis. A Gaussian beam illuminates the first phase screen at the position of its waist, whose diameter is 1 mm, and a speckle pattern is observed 100 mm away from the last phase screen. Since the number of phase screens is changed in such a way that the distance between the first and the last phase screens is always kept constant, the spacing between the neighboring screens decreases with an increase in the number of scattering events. This situation corresponds to increasing the density of scatterers in suspensions although the scattering from such volume scatterers is much more complex than our model.

Figure 7.3 shows the variations in the velocity distribution of the phase screens. The x - and y -axes are taken at the plane of the phase screens and each dot in Fig. 7.3

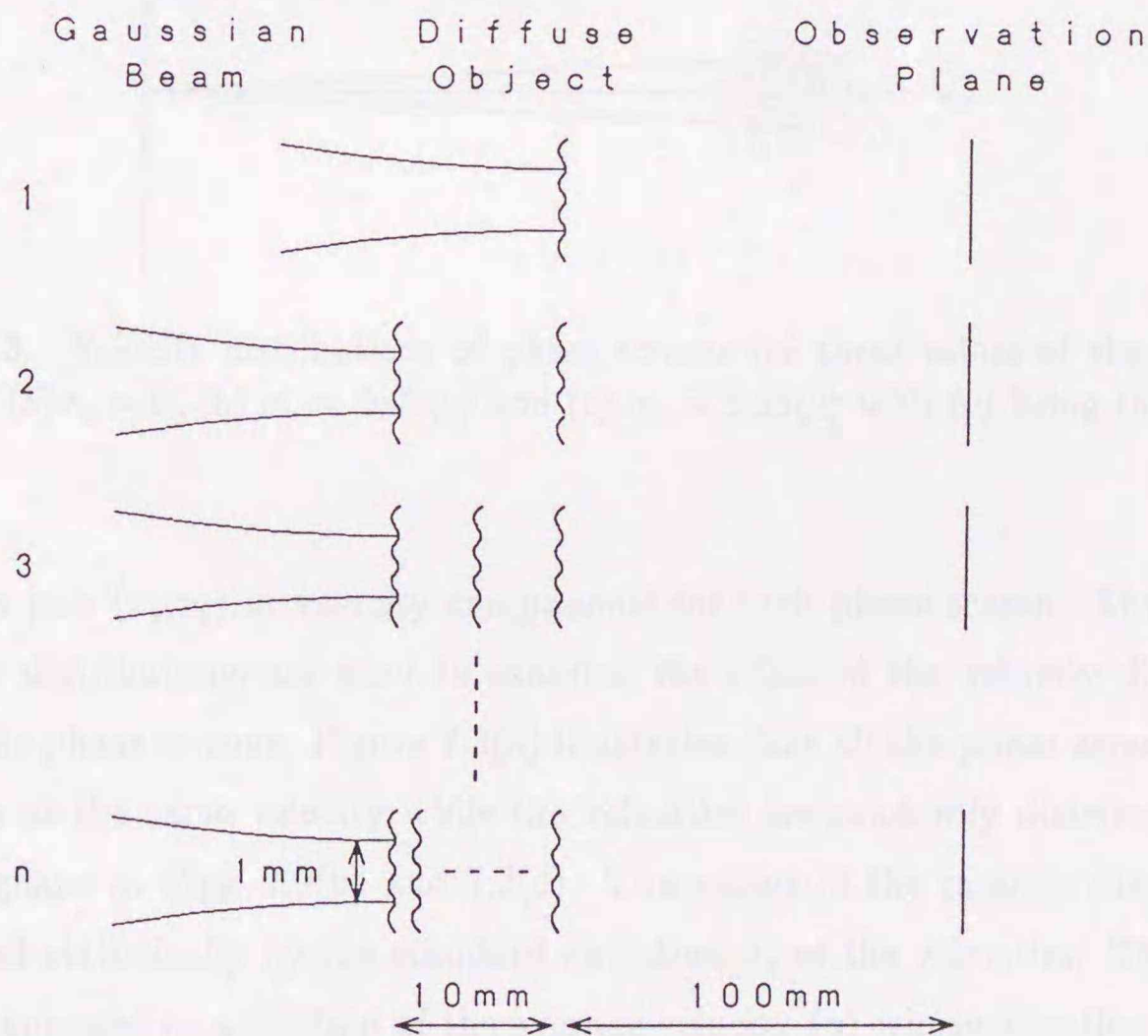


Figure 7.2. Geometries for various numbers of phase screens.

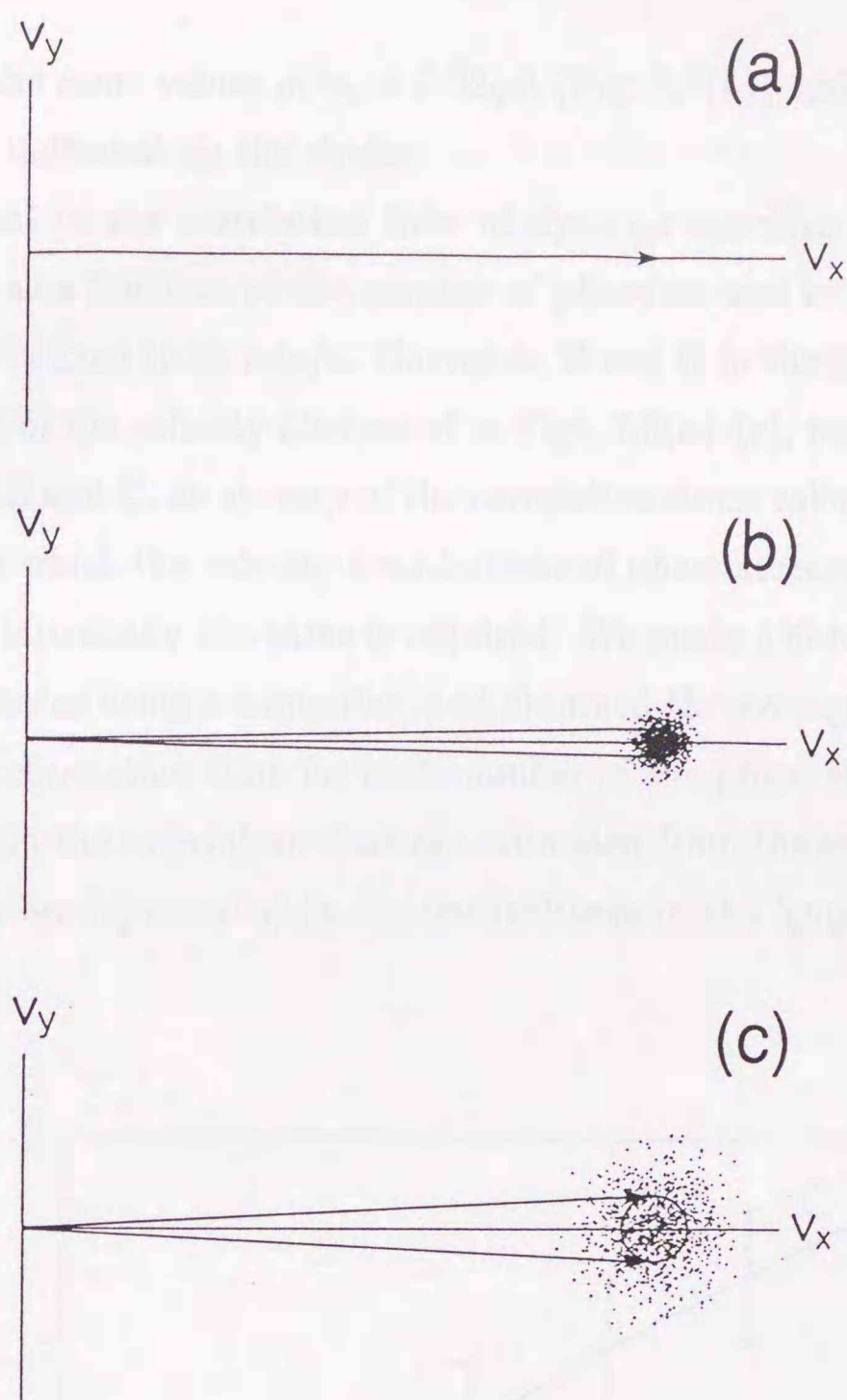


Figure 7.3. Velocity distributions of phase screens for three values of the standard deviation: (a) $\sigma_v = 0$, (b) $\sigma_v = 0.02\langle v \rangle$ and (c) $\sigma_v = 0.05\langle v \rangle$ with $\langle v \rangle$ being the average velocity.

indicates a pair (v_x, v_y) of velocity components for each phase screen. Three kinds of velocity distributions are used to examine the effect of the velocity differences between the phase screens. Figure 7.3(a) illustrates that all the phase screens move as a whole at the same velocity while the velocities are randomly distributed over the v_x - v_y plane in Figs. 7.3(b) and 7.3(c). The extent of the random distribution is estimated statistically by the standard deviation σ_v of the velocities. The values of σ_v are expressed as a fraction of the average velocity $\langle v \rangle$ whose direction is along the x -axis. In Figs. 7.3(b) and 7.3(c), the velocities are Gaussian distributed around the average velocity and the standard deviations of the x - and y -components of the

velocities have the same values of $\sigma_v = 0.02\langle v \rangle$ (Fig. 7.3(b)) and $\sigma_v = 0.05\langle v \rangle$ (Fig. 7.3(c)), and are indicated by the circles.

The reciprocal of the correlation time of dynamic speckles at the observation plane is plotted as a function of the number of phase screens in Fig. 7.4, where the average velocity is fixed to 10 mm/s. Curves A, B and C in the figure correspond to the distributions of the velocity illustrated in Figs. 7.3(a)–(c), respectively. In order to obtain curves B and C, an average of the correlation times calculated for a number of realizations in which the velocity distributions of phase screens are different from each other but statistically the same is required. We made 500 realizations for such a set of phase screens using a computer, and obtained the average and the standard deviation of the correlation time for each number of the phase screens. The ranges of the variations in the correlation time are estimated from the standard deviations, and these ranges are represented by the vertical bars in the figure.

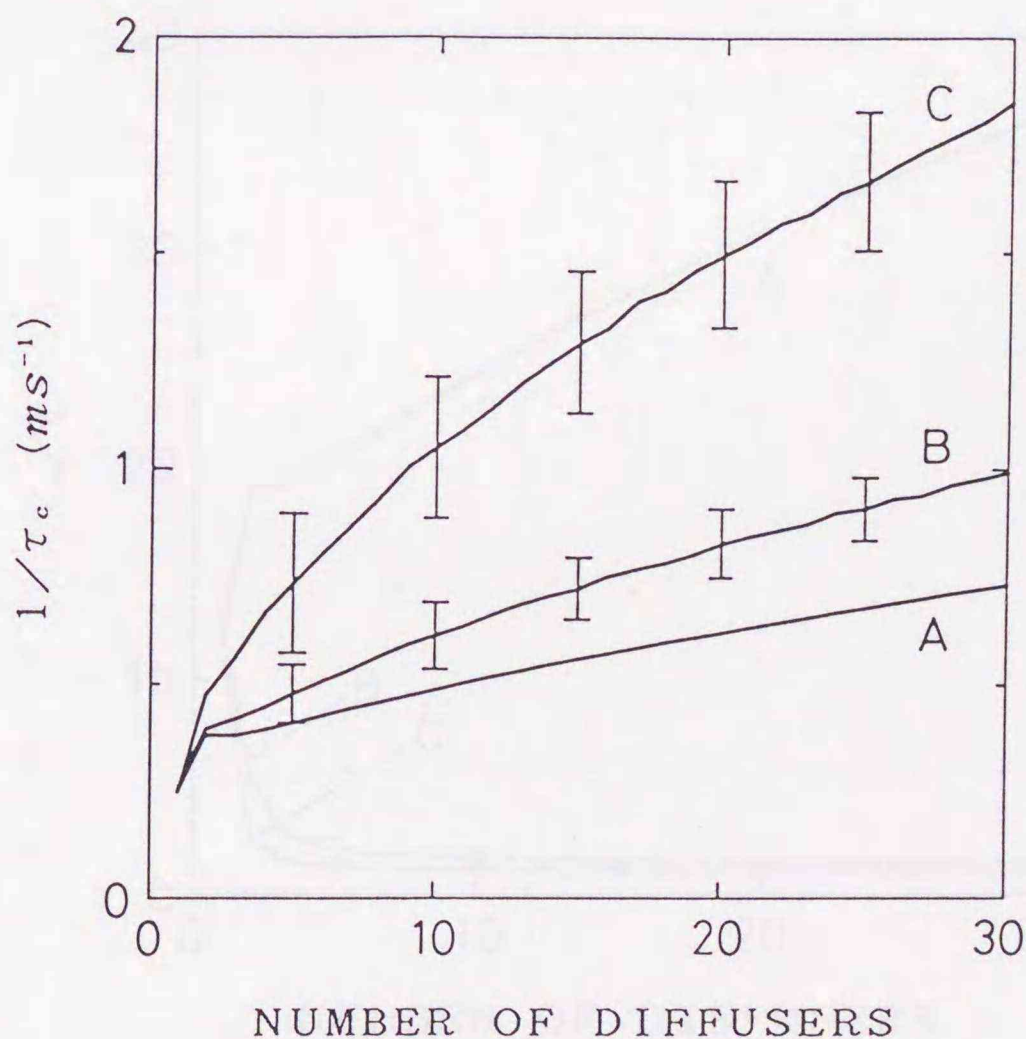


Figure 7.4. Reciprocal $1/\tau_c$ of the correlation time plotted against the number of phase screens for $\sigma_v = 0$ (A), $\sigma_v = 0.2$ mm/s (B) and $\sigma_v = 0.5$ mm/s (C) with $\langle v \rangle = 10$ mm/s and $\beta_k = 3.183\mu\text{m}$ for all k .

From Fig. 7.4 we can see that the intensity of the speckles changes more rapidly as the number of phase screens increases. Moreover, this fluctuating rate of the intensity shows a marked increase for the case where there are some variations in the velocity, compared with the case of no velocity variation. In other words, when the speckles arise from a large number of diffusers, a slight difference between the motions of the diffusers affects significantly the motion of the speckles they produce and yields rapid fluctuations in the intensity detected at a single point. These results, therefore, indicate that the rapid fluctuation of the intensity due to multiple scattering arises not just from the fact that the scattering occurs many times but from the inhomogeneity in the motion of scatterers.

The correlation distance gives us information about what kind of the speckle motion is taking place. Figure 7.5 shows the correlation distance normalized by the average speckle size as a function of the number of the phase screens. If a

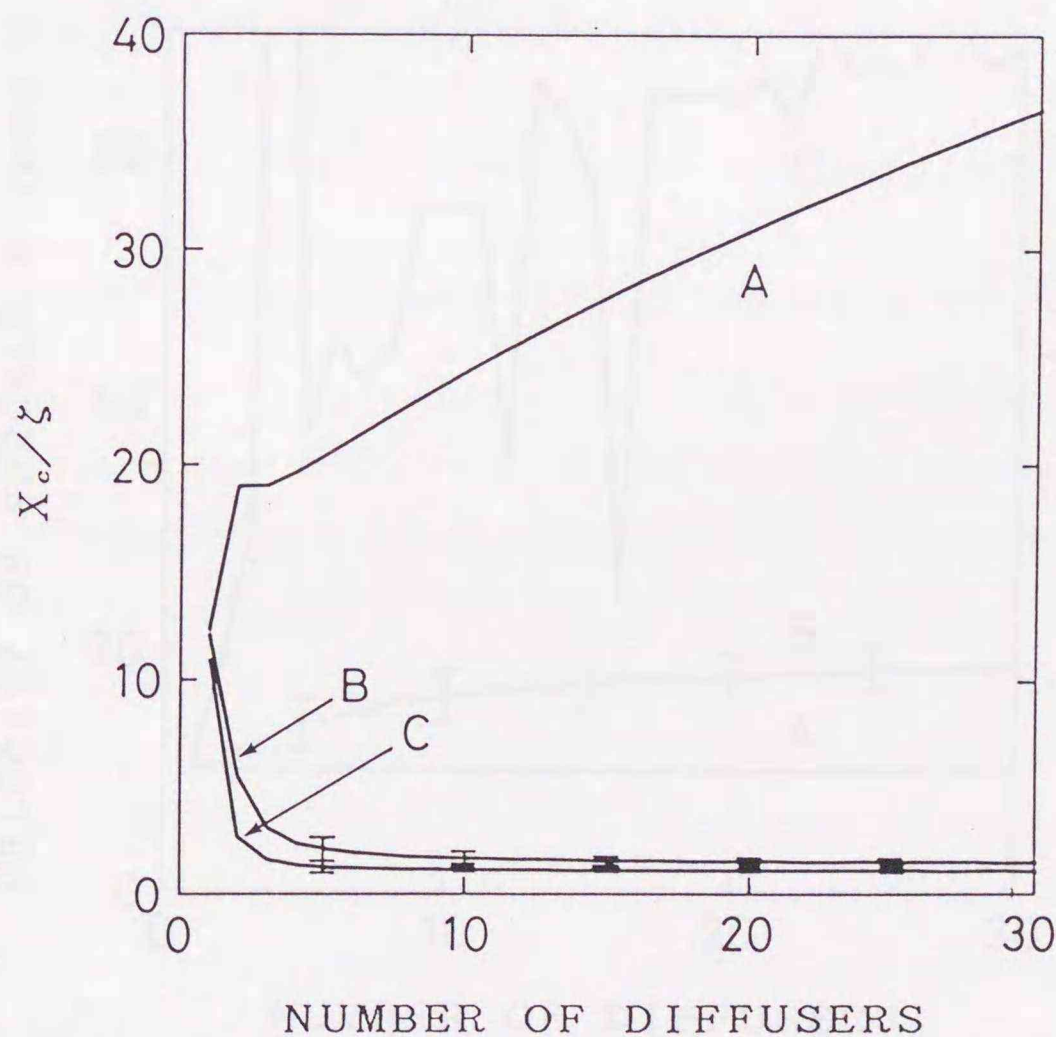


Figure 7.5. Correlation distance X_c divided by the average speckle size ζ versus the number of phase screens for $\sigma_v = 0$ (A), $\sigma_v = 0.2$ mm/s (B) and $\sigma_v = 0.5$ mm/s (C). The values of parameters are the same as those in Fig. 7.4.

speckle pattern is translating, the normalized correlation distance has a large value, while this distance becomes unity if a pure boiling motion of speckles occurs. The parameters are the same as those in Fig. 7.4, and the effect of variations in the velocity on the speckle motion is clearly demonstrated in Fig. 7.5. When all the phase screens are translating at the same velocity (see curve A), the speckle pattern arising from those screens is also translating. The increase of the normalized correlation distance shown by curve A is due to the decrease in the average speckle size ζ used as a normalized factor. On the other hand, when there are some variations in the velocity of the phase screens (see curves B and C), the boiling motion becomes dominant and, in particular, curve C drops to unity, indicating that the speckle pattern is completely boiling. Therefore, the variations in the velocity among the phase screens greatly affect the motion of the resulting speckle pattern.

The speckle velocity is plotted in Fig. 7.6 against the number of the phase screens.

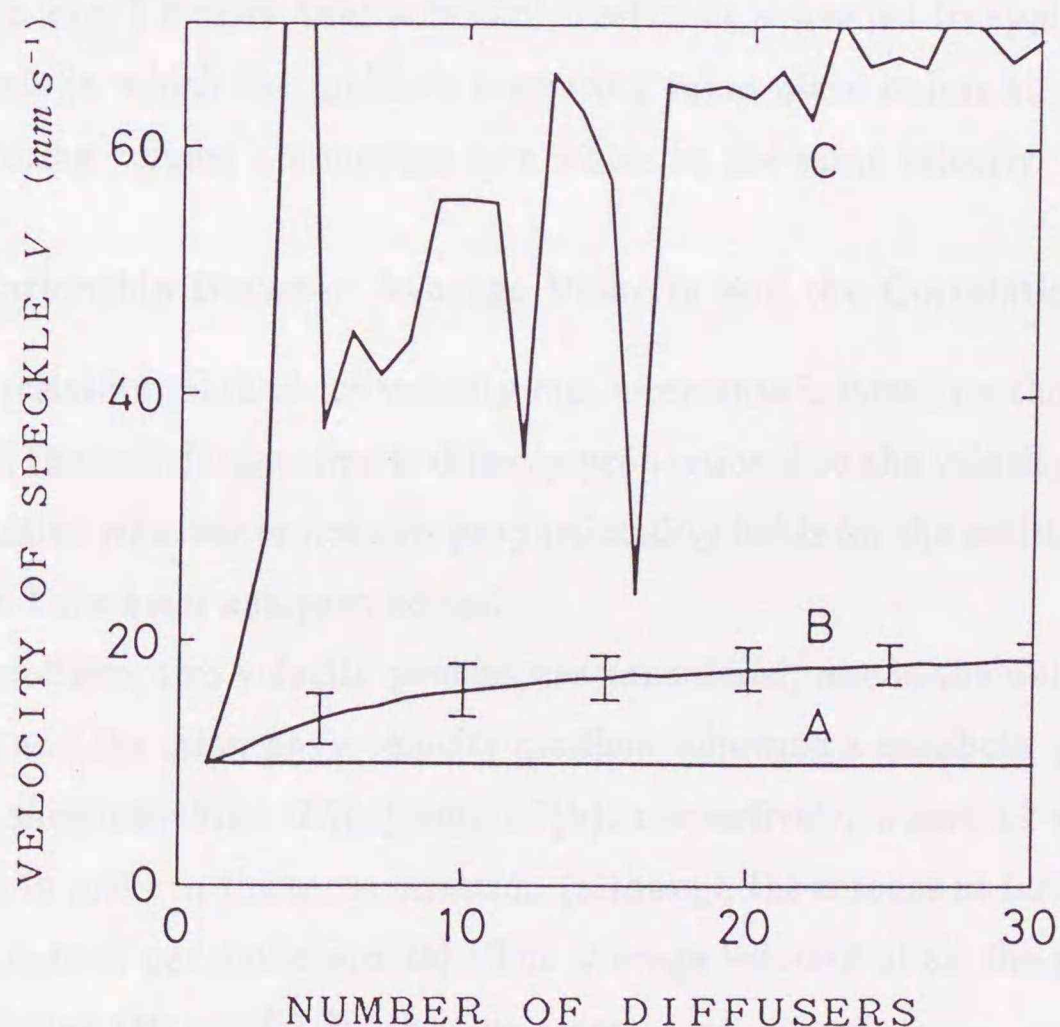


Figure 7.6. Speckle velocity V as a function of the number of phase screens for $\sigma_v = 0$ (A), $\sigma_v = 0.2$ mm/s (B) and $\sigma_v = 0.5$ mm/s (C). The values of parameters are the same as those in Fig. 7.4.

Again, the parameters are the same as those in Fig. 7.4. Curve A in Fig. 7.6 shows that the speckle pattern moves at the same velocity as that of the phase screens even in the case of multiple scattering. The speckle velocity given by curve B is different from that of curve A, because the dynamic speckle pattern in this case contains a boiling element which affects the translating velocity of the pattern. This effect is more noticeable for curve C. In this case, there is no translating element in the speckle motion and, hence, the translating velocity of a speckle pattern can no longer be considered as a meaningful quantity.

The deviation of the speckle velocity from the average velocity of diffusers creates a serious problem from the viewpoint of velocity measurements. There are three common methods for measuring the velocity of a moving diffuse object using dynamic speckles [3]. One is the autocorrelation method in which the correlation time is measured, and the other methods of cross-correlation and spatial filtering use the velocity of a speckle pattern. The latter two methods have an advantage that the absolute velocity can be obtained without calibration. However, the results illustrated in Fig. 7.6 show that a careful treatment is needed to apply these methods to objects in which the multiple scattering takes place unless all the scatterers in the scattering volume are moving as a whole at the same velocity.

7.3.2 Relationship Between Average Velocity and the Correlation Time

The autocorrelation method for velocity measurements is based on the fact that the reciprocal of the correlation time is directly proportional to the velocity of an object. Now we examine whether or not this proportionality holds for the multiple-scattering objects that have been analyzed so far.

In what follows, two velocity profiles are considered; one is the uniform velocity distribution and the other has a velocity gradient following a parabolic profile. These profiles are shown in Figs. 7.7(a) and 7.7(b), respectively, where 11 phase screens are assumed to move in the same direction (although the screens at both sides are at rest for the case of parabolic profile). The average velocity of all the phase screens is the same value (10 mm/s) for the two cases.

First, assume that there is no variation in the velocity of each diffuser. In other words, the diffusers have deterministic values in the velocity, not statistically distributed ones shown in Figs. 7.3(b) and 7.3(c). In this case, the reciprocal of the

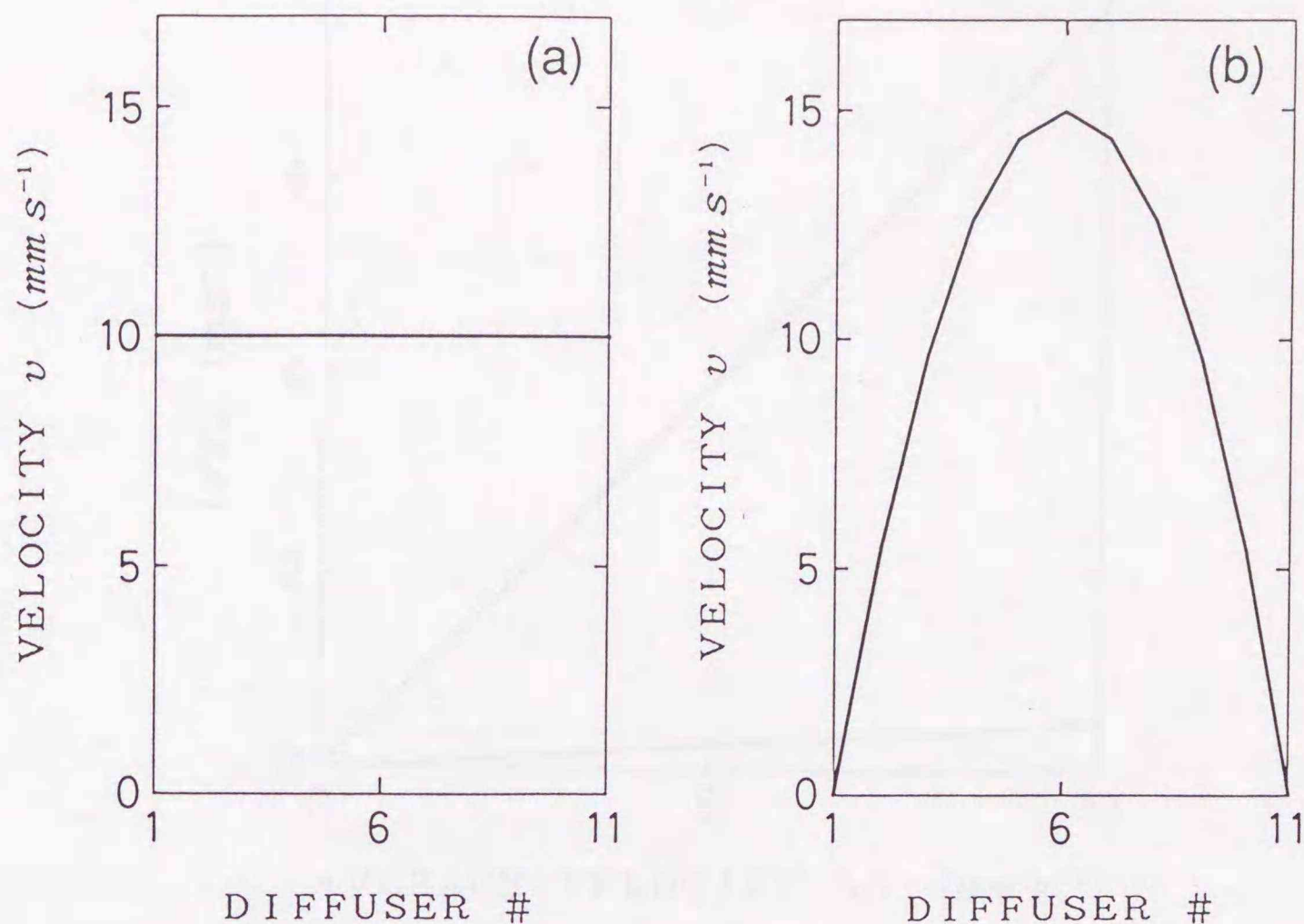


Figure 7.7. Velocity profiles of a series of phase screens: (a) uniform profile and (b) parabolic profile.

correlation time has a proportional relationship with the average velocity of all the phase screens as long as the velocity profile of these screens remains the same (see Fig. 7.8). This is readily shown in Eq. (7.49), where multiplying a constant value a by all of the velocities v_l and v_m results in $a(1/\tau_{c(n)})$ with ψ_{lm} being zero. Therefore, the autocorrelation method can also be applied to multiple-scattering objects whose velocity profiles do not change with time.

As is seen from Fig. 7.8, the proportional factor for the parabolic profile of the velocity (shown by curve B) is much larger than that for the profile with no gradient in the velocity (shown by curve A). This means that rapid fluctuation is observed in the speckle intensity when a velocity gradient exists in the scattering volume. Figure 7.8 clearly illustrates that the absolute value of the correlation time is determined not only by the average velocity of the diffusers but by the velocity profile of these diffusers.

Finally, we examine how the proportional relationship illustrated in Fig. 7.8 will

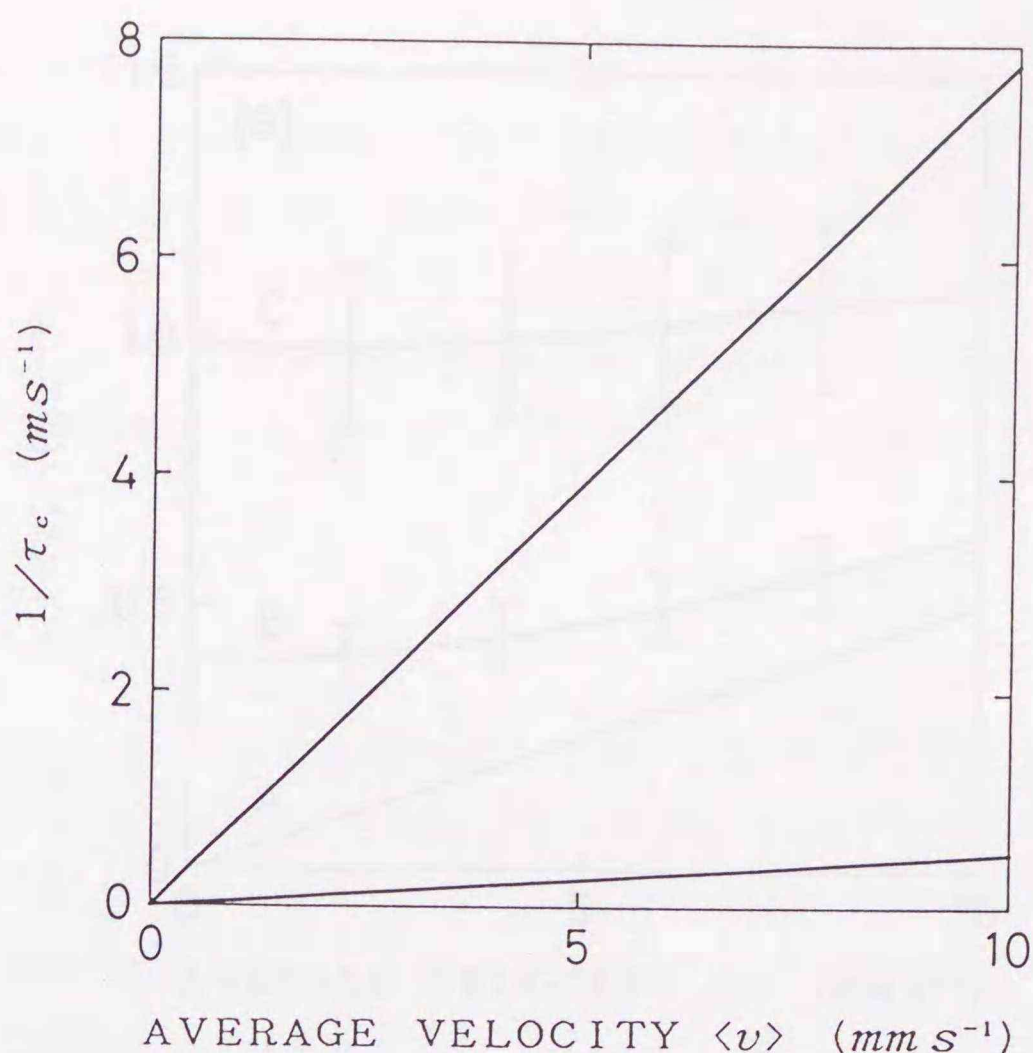


Figure 7.8. Linearity between the reciprocal $1/\tau_c$ of the correlation time and the average velocity $\langle v \rangle$ for a uniform velocity profile (A) and a parabolic velocity profile (B).

change if some randomness is added to the velocity of each phase screen. The changes in this relation for the two velocity profiles given by Figs. 7.7(a) and 7.7(b) are demonstrated in Figs. 7.9(a) and 7.9(b), respectively. Curves A in Figs. 7.9(a) and 7.9(b) are equivalent to the curves in Fig. 7.8, while curves B and C were obtained by allowing the velocity of each phase screen to be distributed randomly to the same extent shown in Figs. 7.3(b) and 7.3(c), respectively.

In the case where the average velocity of each phase screen has the same value (the case of no velocity gradient), the linearity observed in curve A in Fig. 7.9(a) has disappeared when the velocity variations become more than 2 % of the average velocity. In contrast, for the case of the parabolic velocity profile shown in Fig. 7.9(b), the variations in the velocity have little effect on the correlation time except in the region where the value of the average velocity is comparable to that of the extent of the velocity distribution. These results are not surprising, because the velocity differences between the phase screens are due to the velocity variations

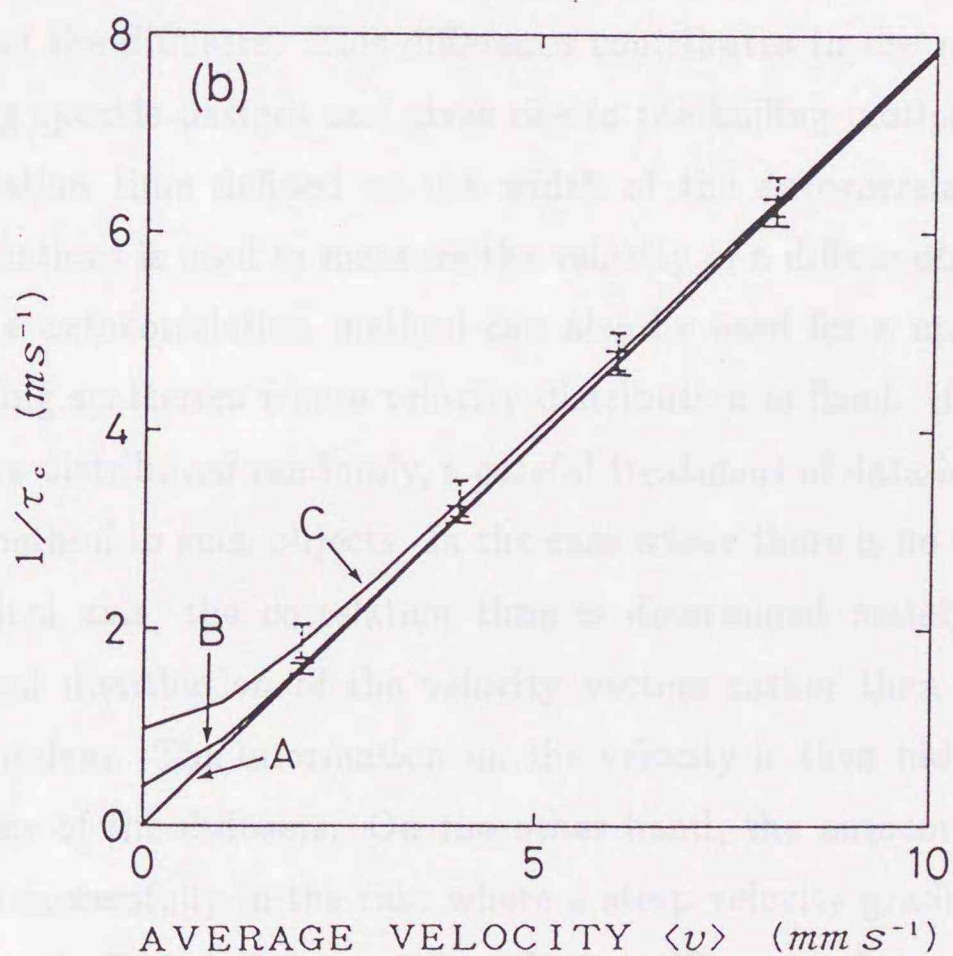
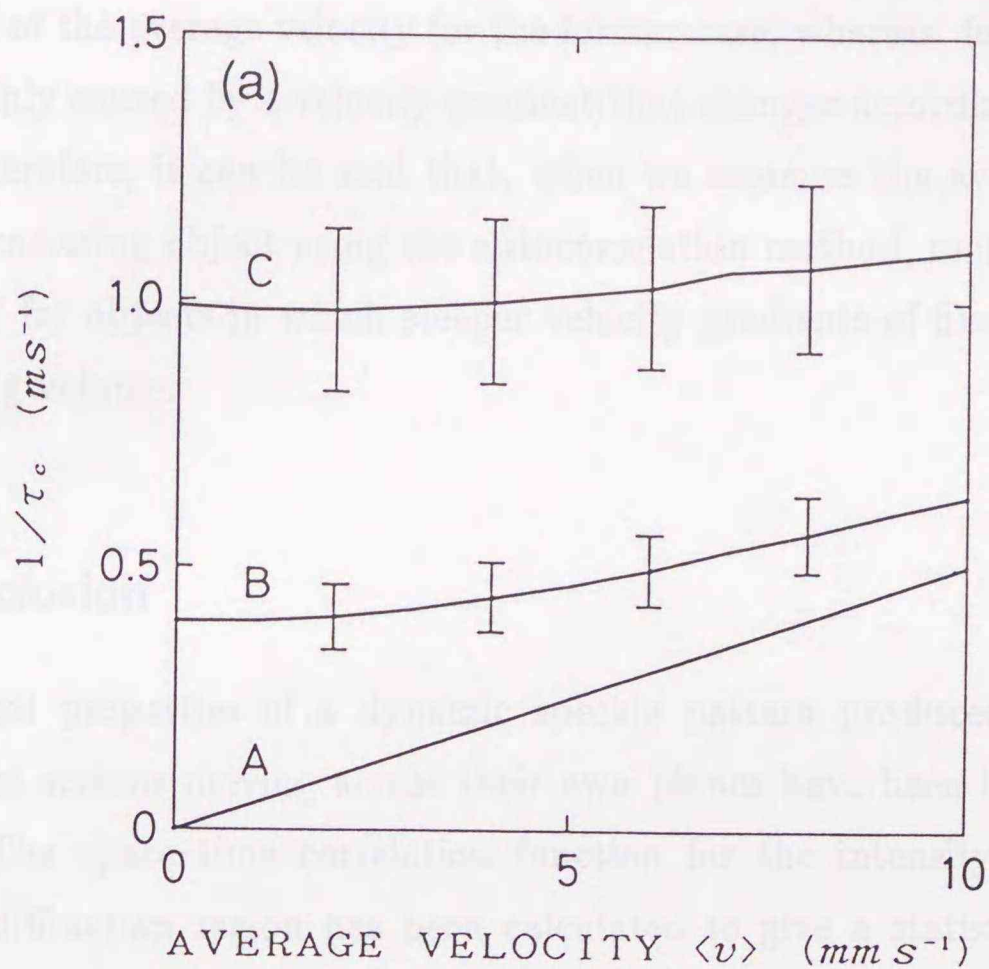


Figure 7.9. Dependence of the reciprocal $1/\tau_c$ of the correlation time on the average velocity $\langle v \rangle$ for $\sigma_v = 0$ (A), $\sigma_v = 0.2 \text{ mm/s}$ (B) and $\sigma_v = 0.5 \text{ mm/s}$ (C) obtained under (a) uniform and (b) parabolic profiles of the velocity.

independent of the average velocity for the former case, whereas, for the latter case, they are mainly caused by a velocity gradient that changes according to the average velocity. Therefore, it can be said that, when we measure the average velocity of a multiple scattering object using the autocorrelation method, more reliable values are expected for objects in which steeper velocity gradients of fixed shape exist in the scattering volume.

7.4 Conclusion

The statistical properties of a dynamic speckle pattern produced by a series of random-phase screens moving across their own planes have been investigated theoretically. The space-time correlation function for the intensity fluctuations in the Fresnel diffraction region has been calculated to give a statistical description of multiply-scattered speckle patterns. The most significant feature of multiply-scattered speckles is that the speckle motion is greatly affected by the difference in the velocities of the diffusers. This difference contributes to the rapid fluctuations of the resulting speckle pattern and gives rise to the boiling motion of the pattern.

The correlation time defined as the width of the autocorrelation function of intensity fluctuations is used to measure the velocity of a diffuse object. It has been shown that the autocorrelation method can also be used for a multiple scattering object containing scatterers whose velocity distribution is fixed. If the velocities of the diffusers are distributed randomly, a careful treatment of data is necessary when applying the method to such objects. In the case where there is no velocity gradient along the optical axis, the correlation time is determined mainly by the extent of the statistical distribution of the velocity vectors rather than by the absolute velocities themselves. The information on the velocity is then hidden by the non-uniform motions of the diffusers. On the other hand, the autocorrelation method can be applied successfully in the case where a steep velocity gradient exists in the scattering volume. This is because the velocity differences between the diffusers associated with the velocity gradient are much larger than those arising from a random distribution of the velocity.

The theoretical formulae presented in this paper take the forms which are best

suited to numerical computation. The statistical quantities we need to describe the temporal behavior of speckle patterns can readily be obtained by calculating the recursion formulae repeatedly.

Chapter 8

Effects of Imaging Properties on Multiply-Scattered Speckles

8.1 Introduction

The statistical properties of multiply-scattered light have been studied by many authors. In this chapter we shall consider the effects of imaging properties on the statistical properties of the scattered light. The statistical properties of the scattered light are determined by the properties of the object and the properties of the imaging system. The statistical properties of the scattered light are determined by the properties of the object and the properties of the imaging system. The statistical properties of the scattered light are determined by the properties of the object and the properties of the imaging system.

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Chapter 8

Effects of Imaging Properties on Multiply-Scattered Speckles

8.1 Introduction

The statistical properties of randomly scattered light produced by rough surfaces or random media are intimately related to the statistical variations of the scatterers. To evaluate these relationships theoretically, a phase-screen model has been widely used to represent the scattering characteristics of the objects. In the model, single or multiple random-phase screens are introduced to give random modulations to the phase of the incident light. The phase modulations also bring about amplitude and intensity variations in the light as it propagates some distance from the screens. Therefore, this model can be applied to examine statistically the properties of both wavefront distortions and speckle patterns.

The applications of the phase-screen model are mostly found in the study of atmospheric turbulence. Enhanced backscattering of coherent light [77, 78] and imaging of objects through a random medium [79–82] were studied by using a thin diffuse screen as a model of refractive-index fluctuations in the medium. In the weak scattering regime, an extended medium can be represented as a series of weak phase screens. The multiple phase-screen technique provides the intuitive understanding of how the amplitude variations develop as waves propagate through the medium. This approach was applied to the analysis of wave propagation through the turbulent atmosphere and ionosphere [37–40], boiling speckle effects in stellar speckle interferometry [24], and the evolution of a beam in fractal media [41].

The multiple phase-screen technique has become a common tool for investigating beam propagation in a weak scattering medium and was even used for the diffraction analysis of bulk diffusers [83]. In the context of laser speckle, however, most

analytical calculations are performed by using a single phase screen [73] or double phase screens [8, 11, 13, 14, 16, 21, 22, 32, 36]. For the scattering due to a number of deep random-phase screens, few studies appeared in the literature. Since each deep phase screen produces a fully developed speckle pattern, a series of such phase screens becomes a model for the strong scattering regime in which the light that has passed through a scatterer has no specular or coherent component.

In Chapter 7, the dynamic properties of speckle patterns in the Fresnel diffraction region produced by deep random-phase screens being spaced some distance apart from each other and moving at constant velocities were investigated. It has been shown that the velocity differences between the phase screens have a strong effect on both the decorrelation of the resulting speckle patterns and the fluctuating rate of the speckle intensity. In this chapter, the same scattering model is employed as in Chapter 7, but the analysis is made for the case of imaging geometry instead of free space geometry [44].

The space-time correlation functions are calculated for dynamic speckle patterns observed in image space to obtain the temporal autocorrelation functions which give the correlation time to be evaluated. Dependence of the correlation time on the velocities of the phase screens is then examined by using the focal position and resolution capability of the imaging system as parameters. Although attention is restricted to the properties of the imaging system, the effects of the other parameters, such as the spatial coherence of the incident light or the degree of phase distortion imposed by the phase screens, could also be analyzed by evaluating the correlation functions derived in the present study.

8.2 Theoretical Analysis

Figure 8.1 shows the schematic diagram of the optical arrangement used for evaluating the space-time correlation function of multiply-scattered speckle patterns in image space. Random-phase screens are spaced in parallel with each other and translate at constant velocities in their own planes. By illuminating these diffusers with coherent light or spatially-partially coherent light, a dynamic speckle pattern resulting from the multiple scattering is produced at the observation plane located behind

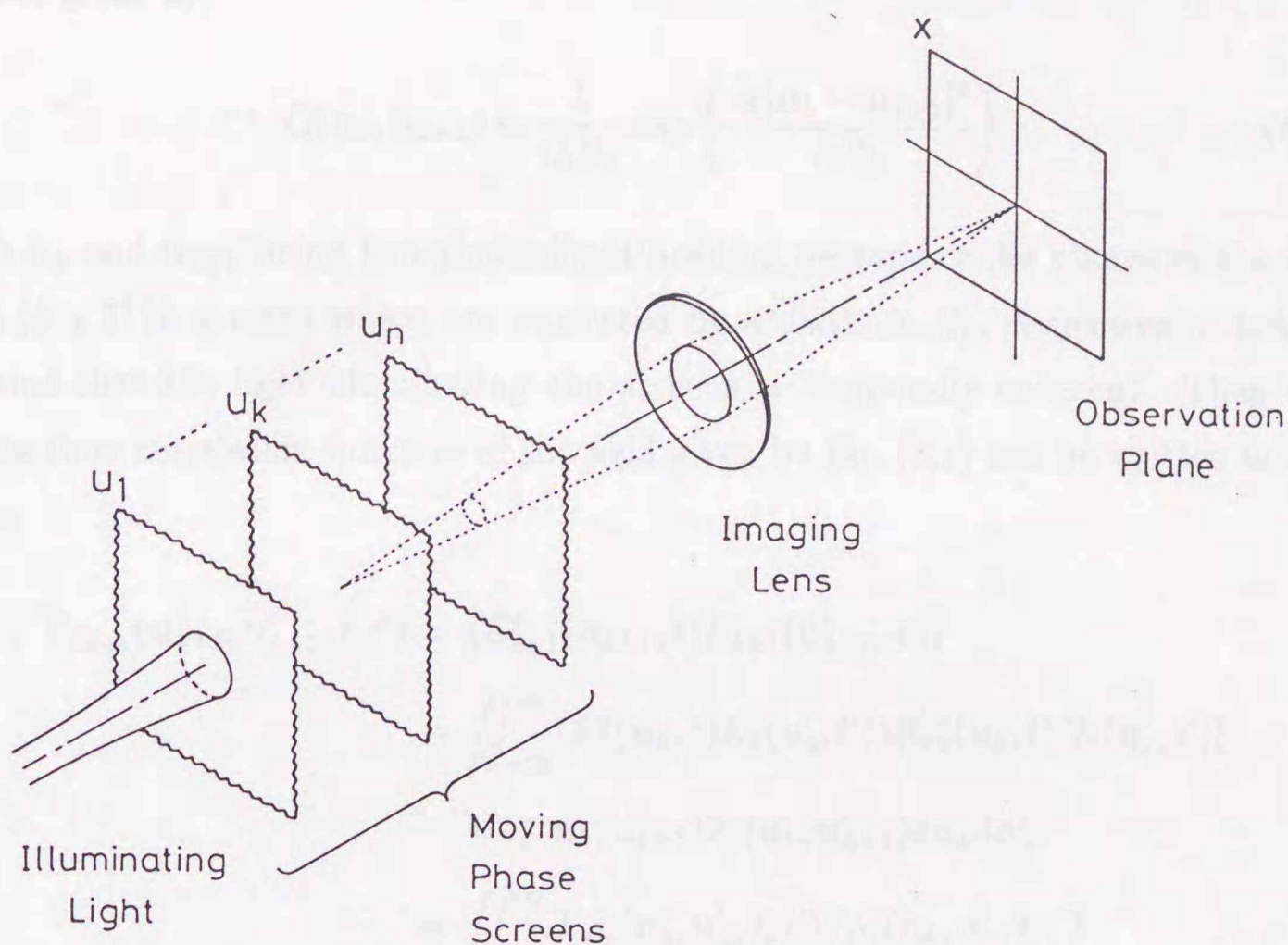


Figure 8.1. Schematic diagram for the analysis of image speckles from multiple random-phase screens.

an imaging lens. In the following analysis, it is assumed that the illuminating light is linearly polarized and that the diffusers do not change the state of polarization of the field. These assumptions allow us to use the scalar wave approximation.

8.2.1 Propagation of Light through Random Phase Screens

At the start, the space-time correlation function of the speckle field at each phase screen is evaluated using the Fresnel diffraction approximation. The field $E_{k+1}(\mathbf{u}_{k+1}, t)$ just in front of the $(k+1)$ th phase screen is expressed by

$$E_{k+1}(\mathbf{u}_{k+1}, t) = \int_{-\infty}^{\infty} E_k(\mathbf{u}_k, t) O_k(\mathbf{u}_k, t) T_k(\mathbf{u}_k, \mathbf{u}_{k+1}) d\mathbf{u}_k, \quad (8.1)$$

where $E_k(\mathbf{u}_k, t)$ is the field incident on the k th phase screen, $O_k(\mathbf{u}_k, t)$ is the phase modulation introduced by the k th phase screen, and $T_k(\mathbf{u}_k, \mathbf{u}_{k+1})$ is a Fresnel prop-

agator given by

$$T_k(\mathbf{u}_k, \mathbf{u}_{k+1}) = \frac{1}{i\lambda R_k} \exp\left(\frac{i\pi|\mathbf{u}_k - \mathbf{u}_{k+1}|^2}{\lambda R_k}\right) \quad (8.2)$$

with $\mathbf{u}_k$ and $\mathbf{u}_{k+1}$ being two-dimensional position vectors at the planes of the k th and $(k+1)$ th screens which are separated by a distance R_k , respectively. Let us assume that the light illuminating the screens is temporally coherent. Then the space-time correlation function of the field given by Eq. (8.1) can be written in the form

$$\begin{aligned} \Gamma_{E_{k+1}}(\mathbf{u}_{k+1}, \mathbf{u}'_{k+1}, t, t') &= \langle E_{k+1}^*(\mathbf{u}_{k+1}, t) E_{k+1}(\mathbf{u}'_{k+1}, t') \rangle \\ &= \iint_{-\infty}^{\infty} \langle E_k^*(\mathbf{u}_k, t) E_k(\mathbf{u}'_k, t') \rangle \langle O_k^*(\mathbf{u}_k, t) O_k(\mathbf{u}'_k, t') \rangle \\ &\quad \times T_k^*(\mathbf{u}_k, \mathbf{u}_{k+1}) T_k(\mathbf{u}'_k, \mathbf{u}'_{k+1}) d\mathbf{u}_k d\mathbf{u}'_k \\ &= \iint_{-\infty}^{\infty} \Gamma_{E_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t') \Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, t, t') \\ &\quad \times T_k^*(\mathbf{u}_k, \mathbf{u}_{k+1}) T_k(\mathbf{u}'_k, \mathbf{u}'_{k+1}) d\mathbf{u}_k d\mathbf{u}'_k, \end{aligned} \quad (8.3)$$

where $\langle \dots \rangle$ denotes an ensemble average. The correlation function of the speckle field resulting from the n th-order scattering can be evaluated by substituting successively the function Γ_{E_k} ($k = 1$ to n) into Eq. (8.3) if the correlation function of the incident light field at the first phase screen is specified.

For the light illuminating the first phase screen, two models are considered which have different spatial coherences; one is the completely coherent light with a Gaussian-beam type and another is the partially coherent light obtained from a Gaussian Shell-model source [84]. The correlation function of the incident light field with the Gaussian-beam type is given by

$$\begin{aligned} \Gamma_{E_1}(\mathbf{u}_1, \mathbf{u}'_1) &= \frac{w_0^2}{w^2} \exp\left[\frac{i\pi}{\lambda R_0}(|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2)\right] \exp\left(-\frac{|\mathbf{u}'_1 + \mathbf{u}_1|^2}{2w^2}\right) \\ &\quad \times \exp\left(-\frac{|\mathbf{u}'_1 - \mathbf{u}_1|^2}{2w^2}\right), \end{aligned} \quad (8.4)$$

where λ is the wavelength of the light, R_0 the wavefront curvature at the illuminating screen, and w_0 and w are the radii of the beam waist and the illuminating

spot, respectively. Note that Eq. (8.4) is not a statistically-defined function, but a deterministic one.

A Gaussian Shell-model source is often used to describe the mutual intensity of a quasi-homogeneous source in which the intensity profile of the source is much broader than its coherence interval. The mutual intensity of a Gaussian Shell-model source is written in the form

$$\begin{aligned}\Gamma_{E_0}(\mathbf{u}_0, \mathbf{u}'_0) &= \langle E_0^*(\mathbf{u}_0) E_0(\mathbf{u}'_0) \rangle \\ &= \langle I_0 \rangle \exp\left(-\frac{|\mathbf{u}'_0 + \mathbf{u}_0|^2}{2q_0^2}\right) \exp\left(-\frac{|\mathbf{u}'_0 - \mathbf{u}_0|^2}{2\zeta_0^2}\right),\end{aligned}\quad (8.5)$$

where q_0 and ζ_0 represent the extent of the source and the spatial coherence interval, respectively, and $\langle I_0 \rangle$ is the average intensity at the center of the source. The mutual intensity at the first phase screen a distance R_0 away from the source then becomes [25]

$$\begin{aligned}\Gamma_{E_1}(\mathbf{u}_1, \mathbf{u}'_1) &= \langle I_1 \rangle \exp\left[\frac{i\pi}{\lambda R_0}(|\mathbf{u}'_1|^2 - |\mathbf{u}_1|^2)\right] \exp\left(-\frac{|\mathbf{u}'_1 + \mathbf{u}_1|^2}{2q_1^2}\right) \\ &\quad \times \exp\left(-\frac{|\mathbf{u}'_1 - \mathbf{u}_1|^2}{2\zeta_1^2}\right),\end{aligned}\quad (8.6)$$

where q_1 and ζ_1 are the extent of the illuminating light field and the spatial correlation length at the first phase screen defined by

$$q_1 = \frac{\lambda R_0}{\pi \zeta_0} \quad (8.7)$$

and

$$\zeta_1 = \frac{\lambda R_0}{\pi q_0}, \quad (8.8)$$

and the average intensity is given by

$$\langle I_1 \rangle = \left(\frac{\pi q_0 \zeta_0}{\lambda R_0}\right)^2 \langle I_0 \rangle. \quad (8.9)$$

Since each phase screen is assumed to move at constant velocity $\mathbf{v}_k$ ($k = 1$ to n), the phase modulation of the light field introduced by the k th phase screen is

expressed as

$$O_k(\mathbf{u}_k, t) = \exp[i\phi_k(\mathbf{u}_k - \mathbf{v}_k t)], \quad (8.10)$$

where ϕ_k indicates the random phase fluctuations. Let us assume for simplicity that the phase fluctuations follow a Gaussian random process and that their normalized spatial correlation function takes a Gaussian form. Moreover, we confine the analysis to the case of deep random-phase screens for which the standard deviation of phase fluctuations is larger than π . These assumptions lead to a simple form for the correlation function of Eq. (8.10) [74]

$$\Gamma_{O_k}(\mathbf{u}_k, \mathbf{u}'_k, \tau) = \exp(-|\Delta \mathbf{u}_k|^2 / \beta_k^2), \quad (8.11)$$

where

$$\begin{aligned} \Delta \mathbf{u}_k &= (\mathbf{u}'_k - \mathbf{v}_k t') - (\mathbf{u}_k - \mathbf{v}_k t) \\ &= \mathbf{u}'_k - \mathbf{u}_k - \mathbf{v}_k \tau \end{aligned} \quad (8.12)$$

with $\tau = t' - t$, and β_k stands for the effective correlation length defined by

$$\beta_k = \alpha_k / \sigma_k \quad (8.13)$$

in which α_k and σ_k are the correlation length and the standard deviation of phase fluctuations, respectively.

After a tedious calculation using Eqs. (8.2)–(8.4), (8.6) and (8.11), the following expression is obtained for the space-time correlation function of the speckle field just in front of the k th phase screen [43]

$$\begin{aligned} \Gamma_{E_k}(\mathbf{u}_k, \mathbf{u}'_k, \tau) &= \langle I_k \rangle \exp \left(- \sum_{l=0}^{k-1} \sum_{m=0}^{k-1} A_k(l, m) \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 + \sum_{l=0}^{k-1} B_k(l) (\mathbf{u}'_k - \mathbf{u}_k) \cdot \mathbf{v}_l \tau \right) \\ &\quad \times \exp \left[- \frac{i\pi}{\lambda R_{k-1}} \left(\sum_{l=0}^{k-1} C_k(l) (\mathbf{u}'_k + \mathbf{u}_k) \cdot \mathbf{v}_l \tau - D_k(|\mathbf{u}'_k|^2 - |\mathbf{u}_k|^2) \right) \right] \\ &\quad \times \exp \left(- \frac{|\mathbf{u}'_k + \mathbf{u}_k|^2}{2q_k^2} \right) \exp \left(- \frac{|\mathbf{u}'_k - \mathbf{u}_k|^2}{2\zeta_k^2} \right) \quad (k \geq 1). \end{aligned} \quad (8.14)$$

Variables $A_k(l, m)$, $B_k(l)$, $C_k(l)$, D_k , q_k and ζ_k in Eq. (8.14) take the form of recursion formulae:

$$A_{k+1}(l, m) = A_k(l, m) - \frac{1}{2}P_k^{-1}B_k(l)B_k(m) - \gamma_k K_k(1 + K_k D_k)B_k(m)C_k(l) + \frac{1}{2}\gamma_k K_k^2(P_{k,1} + P_{k,2})C_k(l)C_k(m), \quad (8.15)$$

$$B_{k+1}(l) = \gamma_k(1 + K_k D_k)B_k(l) - \gamma_k K_k(P_{k,1} + P_{k,2})C_k(l), \quad (8.16)$$

$$C_{k+1}(l) = P_k^{-1}B_k(l) + \gamma_k K_k(1 + K_k D_k)C_k(l), \quad (8.17)$$

$$D_{k+1} = 1 - \gamma_k(1 + K_k D_k), \quad (8.18)$$

$$q_{k+1} = \frac{\lambda R_k}{\pi} P_k^{1/2}, \quad (8.19)$$

$$\zeta_{k+1} = [\gamma_k(P_{k,1} + P_{k,2})]^{-1/2}, \quad (8.20)$$

where

$$A_k(l, k) = A_k(k, m) = 0, \quad (8.21)$$

$$A_k(k, k) = P_{k,1}/2, \quad (8.22)$$

$$B_k(k) = P_{k,1}, \quad (8.23)$$

$$C_k(k) = 0. \quad (8.24)$$

In Eqs. (8.15)–(8.24),

$$P_{k,1} = 2/\beta_k^2, \quad (8.25)$$

$$P_{k,2} = 1/\zeta_k^2, \quad (8.26)$$

$$P_{k,3} = \left(\frac{\pi q_k}{\lambda R_k}\right)^2 (1 + K_k D_k)^2 \quad (8.27)$$

and

$$P_k = P_{k,1} + P_{k,2} + P_{k,3} \quad (8.28)$$

are variables depending on the statistical parameters of the incident light and the phase screens, and

$$K_k = R_k/R_{k-1} \quad (8.29)$$

and

$$\gamma_k = \langle I_{k+1} \rangle / \langle I_k \rangle = (1 + K_k D_k)^{-2} P_{k,3} / P_k \quad (8.30)$$

are the ratios between the two neighboring spacings and between the two average intensities, respectively. Equations (8.14)–(8.30) are valid within the range of $k \geq 1$, $0 \leq l \leq k$ and $0 \leq m \leq k$. By setting the initial values

$$A_1(0,0) = B_1(0) = C_1(0) = 0, \quad (8.31)$$

$$D_1 = 1, \quad (8.32)$$

$$q_1 = \zeta_1 = w \quad (8.33)$$

for Gaussian beam illumination, or

$$A_1(0,0) = B_1(0) = C_1(0) = 0, \quad (8.34)$$

$$D_1 = 1, \quad (8.35)$$

$$q_1 = \frac{\lambda R_0}{\pi \zeta_0}, \quad (8.36)$$

$$\zeta_1 = \frac{\lambda R_0}{\pi q_0} \quad (8.37)$$

for illumination from a Gaussian Shell-model beam, the values of $A_k(l, m)$, $B_k(l)$, $C_k(l)$, D_k , q_k and ζ_k can be evaluated successively from $k = 1$ to $k = n$ to give the definite expression for the correlation function of the speckle field just in front of the last (n th) phase screen.

8.2.2 Intensity Correlation Functions for Image Speckles

To examine the statistical properties of speckle patterns, the statistics of a Gaussian process are often employed for theoretical analysis because the higher-order moments of a Gaussian variable are fully described by its second-order moment. However, for speckle patterns formed through an imaging system, the Gaussian approximation can only be applied for the case in which the size of the point spread of the imaging lens in object space is much larger than the correlation area of the phase variations across the object. Therefore, for the present analysis, the space-time correlation

function of the speckle intensity $I(\mathbf{x}, t)$ at the image plane should be calculated directly from the fourth-order moment of a complex amplitude $E(\mathbf{x}, t)$:

$$\langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = \langle E^*(\mathbf{x}, t) E(\mathbf{x}, t) E(\mathbf{x}', t + \tau) E^*(\mathbf{x}', t + \tau) \rangle. \quad (8.38)$$

The speckle field at the image plane is given by

$$E(\mathbf{x}, t) = \int_{-\infty}^{\infty} E_n(\mathbf{u}_n, t) O_n(\mathbf{u}_n, t) T(\mathbf{u}_n, \mathbf{x}) d\mathbf{u}_n, \quad (8.39)$$

where $E_n(\mathbf{u}_n, t)$ is the light field across the n th phase screen, $O_n(\mathbf{u}_n, t)$ the phase modulation introduced by the screen, and $T(\mathbf{u}_n, \mathbf{x})$ the point spread function of an imaging system. Substitution of Eq. (8.39) into Eq. (8.38) yields

$$\begin{aligned} \langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = & \iiint \int_{-\infty}^{\infty} \Gamma_{E_n}^{(4)}(\mathbf{u}_n, \mathbf{u}'_n, \mathbf{u}''_n, \mathbf{u}'''_n, \tau) \Gamma_{O_n}^{(4)}(\mathbf{u}_n, \mathbf{u}'_n, \mathbf{u}''_n, \mathbf{u}'''_n, \tau) \\ & \times \Gamma_T^{(4)}(\mathbf{u}_n, \mathbf{u}'_n, \mathbf{u}''_n, \mathbf{u}'''_n, \mathbf{x}, \mathbf{x}') d\mathbf{u}_n d\mathbf{u}'_n d\mathbf{u}''_n d\mathbf{u}'''_n, \end{aligned} \quad (8.40)$$

where

$$\Gamma_{E_n}^{(4)} = \langle E_n^*(\mathbf{u}_n, t) E_n(\mathbf{u}'_n, t) E_n(\mathbf{u}''_n, t + \tau) E_n^*(\mathbf{u}'''_n, t + \tau) \rangle, \quad (8.41)$$

$$\Gamma_{O_n}^{(4)} = \langle O_n^*(\mathbf{u}_n, t) O_n(\mathbf{u}'_n, t) O_n(\mathbf{u}''_n, t + \tau) O_n^*(\mathbf{u}'''_n, t + \tau) \rangle, \quad (8.42)$$

$$\Gamma_T^{(4)} = T^*(\mathbf{u}_n, \mathbf{x}) T(\mathbf{u}'_n, \mathbf{x}) T(\mathbf{u}''_n, \mathbf{x}') T^*(\mathbf{u}'''_n, \mathbf{x}'). \quad (8.43)$$

Strong phase fluctuations produced by a series of deep random-phase screens allow us to assume that the complex amplitude $E_n(\mathbf{u}_n, t)$ of the light which has been scattered by $n - 1$ times obeys a circular complex Gaussian process with zero mean (for a more precise argument, see Subsection 7.2.4). The fourth-order moment of the speckle field is then expressed by a combination of its second-order moments:

$$\begin{aligned} \Gamma_{E_n}^{(4)} = & \langle E_n^*(\mathbf{u}_n, t) E_n(\mathbf{u}'_n, t) \rangle \langle E_n(\mathbf{u}''_n, t + \tau) E_n^*(\mathbf{u}'''_n, t + \tau) \rangle \\ & + \langle E_n^*(\mathbf{u}_n, t) E_n(\mathbf{u}''_n, t + \tau) \rangle \langle E_n(\mathbf{u}'_n, t) E_n^*(\mathbf{u}'''_n, t + \tau) \rangle. \end{aligned} \quad (8.44)$$

By using a Gaussian soft aperture with radius a , the point spread function for single lens geometry is given by

$$T(\mathbf{u}_n, \mathbf{x}) = \frac{-\pi a^2}{\lambda^2 d_o d_i} \exp \left[\frac{i\pi}{\lambda} \left(\frac{|\mathbf{u}_n|^2}{d_o} + \frac{|\mathbf{x}|^2}{d_i} \right) \right] \exp(i \tan^{-1} \epsilon)$$

$$\times \exp \left(-i\varepsilon \left| \frac{\mathbf{u}_n}{\rho_n} + \frac{\mathbf{x}}{\rho} \right|^2 \right) \exp \left(- \left| \frac{\mathbf{u}_n}{\rho_n} + \frac{\mathbf{x}}{\rho} \right|^2 \right), \quad (8.45)$$

where ρ_n and ρ indicate the widths of the point spreads at the n th phase screen and at the image plane, respectively, which are defined by

$$\rho_n = \left(\frac{\lambda d_o}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (8.46)$$

and

$$\rho = \left(\frac{\lambda d_i}{\pi a} \right) (1 + \varepsilon^2)^{1/2} \quad (8.47)$$

with

$$\varepsilon = \frac{\pi a^2}{\lambda} \left(\frac{1}{d_o} + \frac{1}{d_i} - \frac{1}{f} \right). \quad (8.48)$$

Although we have made the assumption that the standard deviation of phase fluctuations introduced by each phase screen is larger than π , no restriction is imposed for the correlation length of phase fluctuations. For the last phase screen, however, the effective correlation length β_n is assumed to be much less than the speckle size ζ_n and the width ρ_n of the point spread at the last screen. These assumptions enable us to express the fourth-order moment of the phase modulation due to the last phase screen as

$$\Gamma_{O_n}^{(4)} = \Delta S^2 [\delta(\mathbf{u}_n - \mathbf{u}'_n) \delta(\mathbf{u}''_n - \mathbf{u}'''_n) + \delta(\mathbf{u}_n - \mathbf{u}''_n + \mathbf{v}_n \tau) \delta(\mathbf{u}'_n - \mathbf{u}'''_n + \mathbf{v}_n \tau)], \quad (8.49)$$

where ΔS denotes an area of correlation cells in the last phase screen.

By substituting Eqs. (8.44), (8.45) and (8.49), together with Eq. (8.14) evaluated at $k = n$, into Eq. (8.40), the space-time correlation function of the speckle intensity in image space results in the form

$$\begin{aligned} \langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle &= c \kappa_1 \exp \left(- \frac{1 - \kappa_1}{\rho^2} |\mathbf{x} + \mathbf{x}'|^2 \right) \\ &\times [\kappa_1 \Gamma_0(\mathbf{x}, \mathbf{x}') + \kappa_2 \Gamma_1(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \dots, \mathbf{v}_{n-1}) \\ &+ \kappa_2 \Gamma_2(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_n) + \kappa_1 \Gamma_{12}(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \dots, \mathbf{v}_{n-1}, \mathbf{v}_n)], \end{aligned} \quad (8.50)$$

where

$$\Gamma_0(\mathbf{x}, \mathbf{x}') = \exp \left(-\frac{1 - \kappa_1}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2 \right), \quad (8.51)$$

$$\begin{aligned} \Gamma_1(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \dots, \mathbf{v}_{n-1}) \\ = \exp \left\{ -\sum_{l=0}^{n-1} \sum_{m=0}^{n-1} [2A_n(l, m) - (1 - \kappa_2)\zeta_n^2 B_n(l)B_n(m)] \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 \right\} \\ \times \exp \left[\frac{2(1 - \kappa_2)\zeta_n^2}{\rho \rho_n} \sum_{l=0}^{n-1} B_n(l) \mathbf{v}_l \cdot (\mathbf{x} - \mathbf{x}') \tau \right] \exp \left(-\frac{1 - \kappa_2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2 \right), \end{aligned} \quad (8.52)$$

$$\begin{aligned} \Gamma_2(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_n) \\ = \exp \left[-\left(\frac{1}{\kappa_1 \rho_n^2} + \kappa_2 b^2 \right) |\mathbf{v}_n|^2 \tau^2 \right] \\ \times \exp \left[\frac{2}{\rho} \left(\frac{1}{\rho_n} - \kappa_2 \varepsilon b \right) \mathbf{v}_n \cdot (\mathbf{x} - \mathbf{x}') \tau \right] \exp \left(-\frac{1 + \kappa_2 \varepsilon^2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2 \right), \end{aligned} \quad (8.53)$$

$$\begin{aligned} \Gamma_{12}(\mathbf{x}, \mathbf{x}', \tau; \mathbf{v}_1, \dots, \mathbf{v}_{n-1}, \mathbf{v}_n) \\ = \exp \left\{ -\left\{ \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} [2A_n(l, m) + \kappa_1 a^2 C_n(l)C_n(m)] \mathbf{v}_l \cdot \mathbf{v}_m \right. \right. \\ \left. \left. - 2 \sum_{l=0}^{n-1} [B_n(l) + \kappa_1 a b C_n(l)] \mathbf{v}_l \cdot \mathbf{v}_n + \left(\frac{1}{\zeta_n^2} + \frac{1}{\rho_n^2} + \kappa_1 b^2 \right) |\mathbf{v}_n|^2 \right\} \tau^2 \right\} \\ \times \exp \left\{ \frac{2}{\rho} \left[\kappa_1 \varepsilon a \sum_{l=0}^{n-1} C_n(l) \mathbf{v}_l + \left(\frac{1}{\rho_n} - \kappa_1 \varepsilon b \right) \mathbf{v}_n \right] \cdot (\mathbf{x} - \mathbf{x}') \tau \right\} \\ \times \exp \left(-\frac{1 + \kappa_1 \varepsilon^2}{\rho^2} |\mathbf{x} - \mathbf{x}'|^2 \right) \end{aligned} \quad (8.54)$$

and

$$c = \langle I_n \rangle^2 \left(\frac{\pi \Delta S \rho_n^2}{2} \right)^2 \left(\frac{\pi a^2}{\lambda^2 d_o d_i} \right)^4 \quad (8.55)$$

with

$$a = \frac{\pi \rho_n}{\lambda R_{n-1}} \quad (8.56)$$

and

$$b = \frac{\pi \rho_n}{\lambda} \left(\frac{D_n}{R_{n-1}} + \frac{1}{d_o} \right) - \frac{\varepsilon}{\rho_n}. \quad (8.57)$$

In Eqs. (8.50)–(8.54), κ_1 and κ_2 are the parameters characterizing the relationships between the area of the point spread and the extent of the scattered light or the size of the speckles incident on the last phase screen:

$$\kappa_1 = (1 + \rho_n^2/q_n^2)^{-1} \quad (8.58)$$

and

$$\kappa_2 = (1 + \rho_n^2/\zeta_n^2)^{-1}. \quad (8.59)$$

Four functions (8.51)–(8.54) constituting Eq. (8.50) represent contributions from different combinations of the phase screens. The effect of motion of the last phase screen on the resultant speckle intensity is expressed by the functions Γ_2 and Γ_{12} while those of the other phase screens are expressed by the functions Γ_1 and Γ_{12} . This distinction of the last phase screen from the others stems from the restriction of the effective correlation length imposed only on the last phase screen. Although it would be possible to find an expression for the correlation function of the speckle intensity in more general cases, Eqs. (8.50)–(8.54) may satisfy the present analysis as long as the parameters in the equations are carefully chosen not to emphasize the effects of the last phase screen.

Since our attention is given to the analysis of the dynamic properties of the speckle intensity observed at a single point in the image plane, the temporal correlation function of

$$\begin{aligned} &\langle I(\mathbf{x}, t) I(\mathbf{x}, t + \tau) \rangle \\ &= c \kappa_1 \exp \left[-\frac{4(1 - \kappa_1)}{\rho^2} |\mathbf{x}|^2 \right] \\ &\times \left\{ \kappa_1 + \kappa_2 \exp \left\{ -\sum_{l=0}^{n-1} \sum_{m=0}^{n-1} [2A_n(l, m) - (1 - \kappa_2) \zeta_n^2 B_n(l) B_n(m)] \mathbf{v}_l \cdot \mathbf{v}_m \tau^2 \right\} \right. \\ &\quad \left. + \kappa_2 \exp \left[-\left(\frac{1}{\kappa_1 \rho_n^2} + \kappa_2 b^2 \right) |\mathbf{v}_n|^2 \tau^2 \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \kappa_1 \exp \left(- \left\{ \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} [2A_n(l, m) + \kappa_1 a^2 C_n(l) C_n(m)] \mathbf{v}_l \cdot \mathbf{v}_m \right. \right. \\
& \left. \left. - 2 \sum_{l=0}^{n-1} [B_n(l) + \kappa_1 ab C_n(l)] \mathbf{v}_l \cdot \mathbf{v}_n + \left(\frac{1}{\zeta_n^2} + \frac{1}{\rho_n^2} + \kappa_1 b^2 \right) |\mathbf{v}_n|^2 \right\} \tau^2 \right) \Big\} \quad (8.60)
\end{aligned}$$

will be used in the following sections to evaluate the fluctuating rate of the intensity.

At this point, we should briefly mention the statistics for the image speckles. When the speckle field at the image plane is governed by a Gaussian process, the formula

$$\langle I(\mathbf{x}, t) I(\mathbf{x}', t + \tau) \rangle = \langle I(\mathbf{x}, t) \rangle \langle I(\mathbf{x}', t + \tau) \rangle + |\langle E^*(\mathbf{x}, t) E(\mathbf{x}', t + \tau) \rangle|^2 \quad (8.61)$$

can be evaluated to give an expression consisting of only the functions of Γ_0 and Γ_{12} . This fact indicates that the Gaussian approximation cannot be applied if $\kappa_2 > 0$ or if the size of the point spread at the last phase screen is comparable with or less than the size of the speckles illuminating the screen. This non-Gaussian behavior has been extensively studied (for example, see [13, 14]) for complex amplitude variations of the light field produced by a diffuser with fluctuating illumination. Therefore, the properties of the probability density functions for such random fields will not be discussed any further in the present analysis.

8.3 Results and Discussion

In this section, the temporal correlation function in Eq. (8.60) will be evaluated to examine the relationship between the velocity distribution of moving phase screens and the resultant intensity fluctuations observed in image space. The intensity fluctuations measured at a single point are characterized in terms of the correlation time τ_c defined by the time at which the normalized temporal autocorrelation of intensity fluctuations falls to $1/e$. Since this statistical quantity is inversely proportional to the fluctuating rate of the speckle intensity, its reciprocal value measures the velocity of a diffuse object giving rise to the intensity fluctuations. However, this relationship has been confirmed only for the speckles scattered once or, in some special cases, scattered twice. It is thus interesting to know what kind of velocity

information is extracted from the multiply-scattered speckles produced by a number of moving diffusers.

When we analyze the speckles formed in imaging geometry, the point-spread function of an imaging lens plays an important role in determining the first- and the second-order statistics of speckle patterns. In the following analysis, the width of the point spread of the imaging lens as measured in object space will be used as a parameter, which is controlled by changing the aperture size of the lens with the focal length and the magnification of the lens kept fixed.

8.3.1 Effects of Focal Position

In Chapter 6, image speckles produced by double phase screens were studied. It has been shown that the velocity of each phase screen can be detected separately by focusing an imaging lens on the screen to be measured. In that case, the condition is required that the size of the point spread at the front focal plane of the lens be smaller than the size of speckles illuminating the second phase screen. Before discussing the case of multiple scattering, the results are presented for double scattering under the above condition.

Figure 8.2 shows the reciprocal of the correlation time of intensity fluctuations at the observation plane as a function of the front focal position of the lens. Two phase screens are moving in the same direction across the planes of $\delta = -20$ mm and $\delta = 0$ mm. In Fig. 8.2(a), the first screen has the velocities of 1 mm/s (curve A), 2 mm/s (curve B) and 3 mm/s (curve C) while the velocity of the second screen is fixed at 1 mm/s. On the other hand, the velocity of the second screen is varied in Fig. 8.2(b), keeping the velocity of the first screen fixed at 1 mm/s. The circles in the figure indicate the values of the correlation time obtained when the lens is focused on the screen the velocity of which is being changed. These values are found to increase linearly with an increase in the velocity. The results show that the velocity of the first screen is detected without being affected by the motion of the second screen when the lens is focused on the first screen (see the intersections of curves A, B and C with the vertical lines at $\delta = -20$ mm in Figs. 8.2(a) and 8.2(b)). The velocity of the second phase screen is also detected by focusing on it.

Results for the case of multiple scattering are shown in Fig. 8.3, where five phase screens are spaced 5 mm apart from each other and moving in the same direction.

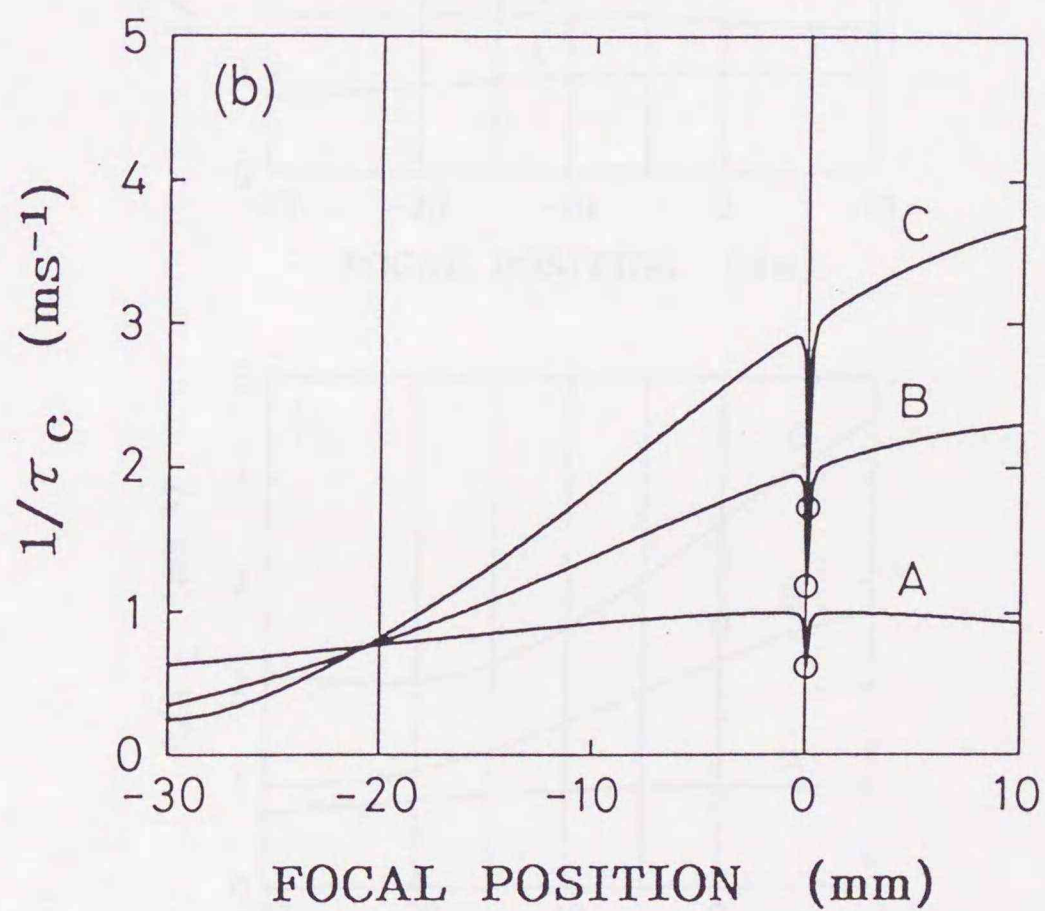
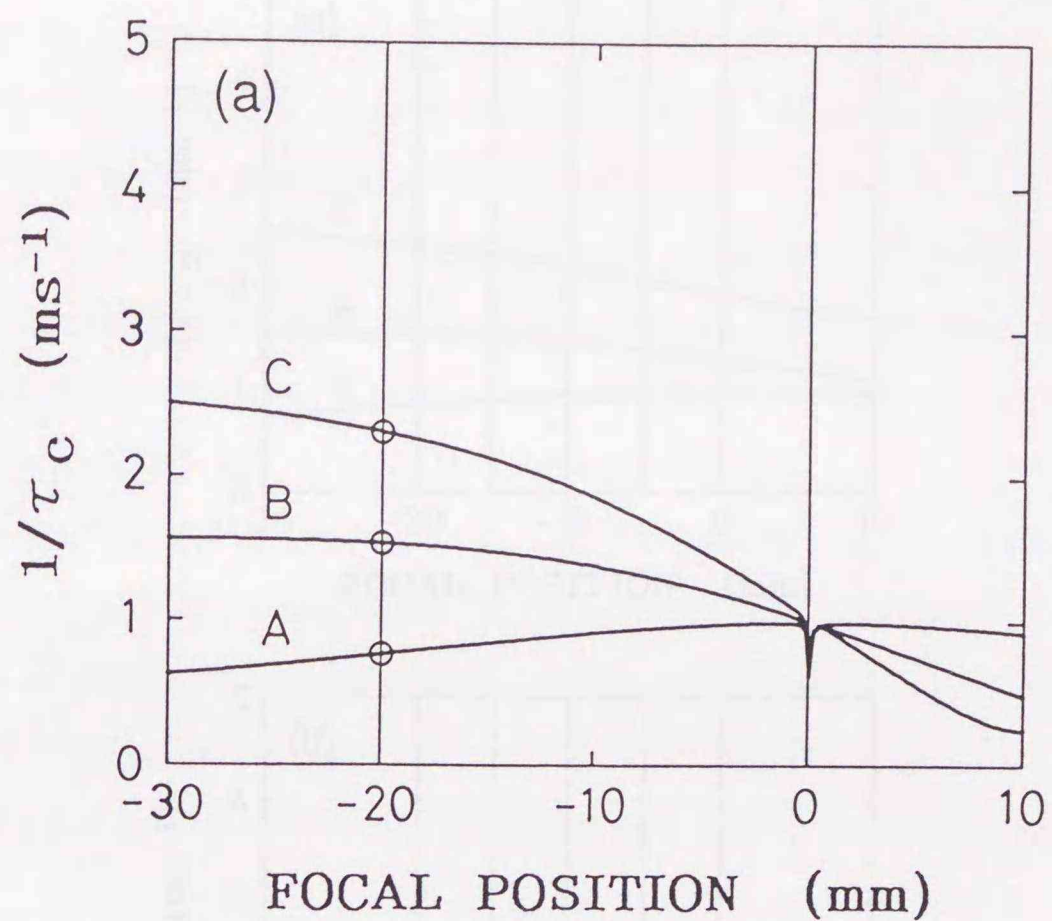


Figure 8.2. Reciprocal $1/\tau_c$ of the correlation time plotted against the front focal position δ of the lens for various values of (a) v_1 and (b) v_2 . Two vertical lines indicate the positions of phase screens. The values of parameters are as follows: $\lambda = 0.633 \mu\text{m}$, $w_0 = w = 0.5 \text{ mm}$, $R_0 = 1.54 \times 10^{25} \text{ mm}$, $\beta = 1.17 \mu\text{m}$, $a = 40.2 \text{ mm}$, $f = 100 \text{ mm}$, $d_i = 200 \text{ mm}$, $\zeta_n = 8.1 \mu\text{m}$ and $\rho_n = 1 \mu\text{m}$.

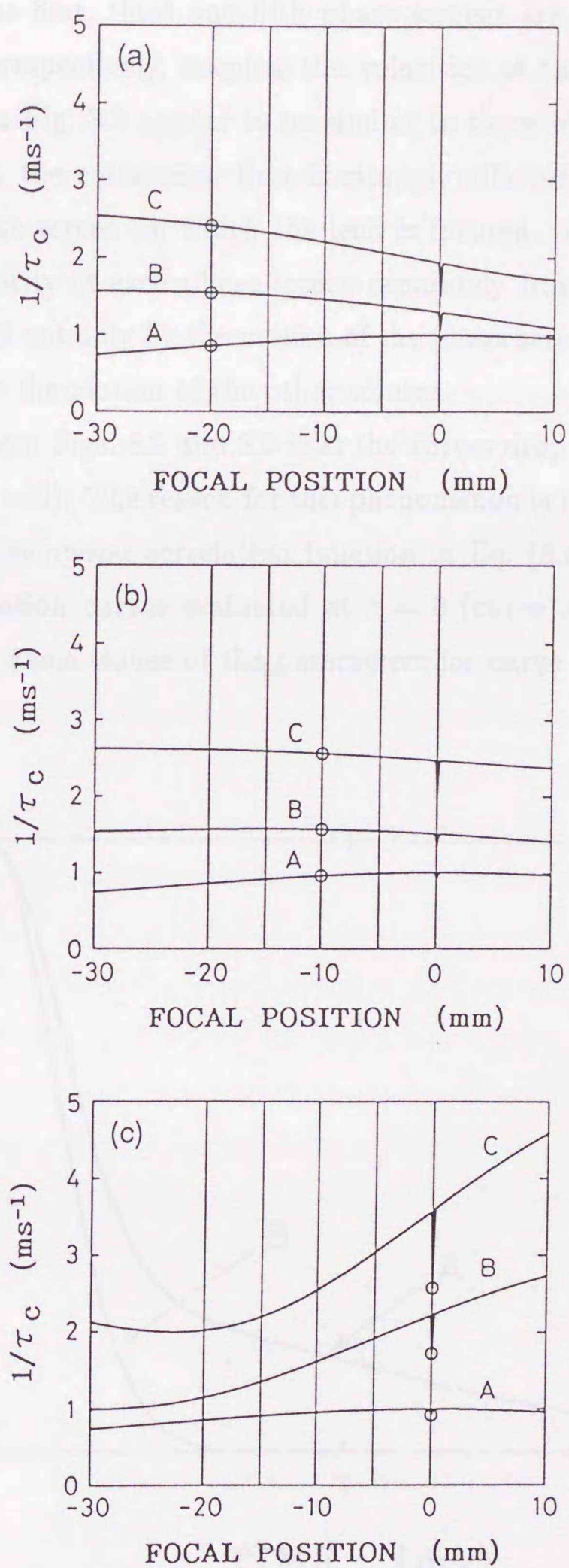


Figure 8.3. $1/\tau_c$ as a function of the focal position for various values of (a) v_1 , (b) v_3 and (c) v_5 obtained when five phase screens are located 5 mm apart from each other. The values of the parameters are the same as those in Fig. 8.2, except $\zeta_n = 1 \mu\text{m}$.

The velocities of the first, third and fifth phase screens are varied in Figs. 8.3(a), 8.3(b) and 8.3(c), respectively, keeping the velocities of the other phase screens fixed. The curves in Fig. 8.3 appear to be similar to those obtained for the double scattering. Namely, the correlation time is strongly affected by the change in the velocity of the phase screen on which the lens is focused. However, it is difficult to measure the velocity of each phase screen separately from the correlation time because it is affected not only by the motion of the phase screen located at the focal position but also by the motion of the other screens.

It can be seen from Figs. 8.2 and 8.3 that the curves drop suddenly at the plane of the last screen ($\delta = 0$). The reason for this phenomenon is related to the variation in the shape of the temporal correlation function in Eq. (8.60). Figure 8.4 shows the temporal correlation curves evaluated at $\delta = 0$ (curve A) and $\delta = -0.2$ mm (curve B) using the same values of the parameters for curve C in Fig. 8.3(c). The

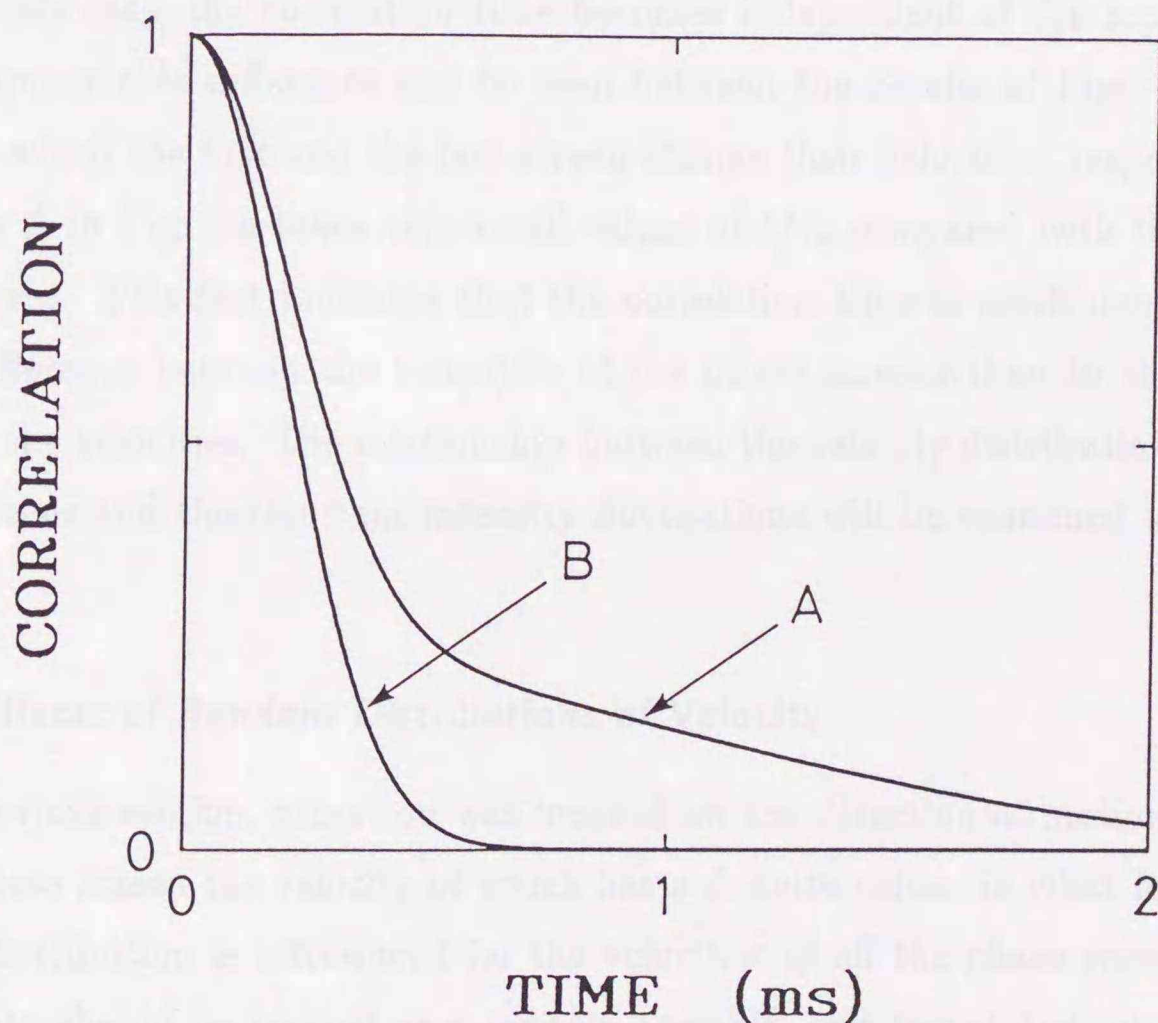


Figure 8.4. Plots of the temporal autocorrelation function for focused and defocused cases. Curves A and B are taken at $\delta = 0$ and -0.2 mm, respectively, using the same parameter for curve C in Fig. 8.3(c).

difference between the shapes of the two curves is caused mainly by the spreading of the diffraction spot of the lens at the plane of the last phase screen due to defocusing. The values of the variable κ_2 , which represents the ratio of the speckle size to the size of the point spread at the last phase screen (Eq. (8.59)), are 0.5 and 6.2×10^{-4} for curves A and B, respectively. By referring to Eq. (8.60), it is found that all the terms in Eq. (8.60) affect the correlation curves when the last phase screen is in focus, whereas the effects of the second and the third terms disappear when the screen is out of focus. The second term is usually a broader function of τ compared with the third and the fourth terms. Therefore, this broad function makes curve A broader than curve B and leads to a decrease in the value of $1/\tau_c$ at $\delta = 0$.

The correlation time has been shown to depend on the focal position of the lens when the size of its point spread at the front focus is less than or equal to the size of the speckles incident on the last phase screen. Results for the opposite case are shown in Fig. 8.5, where the values of optical parameters used are the same as those in Fig. 8.3 except that the size of the point spread is much larger than the speckle size. In this case, the correlation time becomes independent of the focal position and no appreciable difference can be seen between the results of Figs. 8.5(a) and 8.5(b) in which the first and the last screen change their velocities, respectively.

Curve A in Fig. 8.5 takes very small values of $1/\tau_c$ compared with those of the other curves. This fact indicates that the correlation time is much more sensitive to the difference between the velocities of the phase screens than to the absolute values of the velocities. The relationship between the velocity distribution of all the phase screens and the resultant intensity fluctuations will be examined in the next section.

8.3.2 Effects of Random Distributions of Velocity

In the previous section, attention was focused on the detection of motion of a particular phase screen the velocity of which has a definite value. In what follows, the random distribution is introduced for the velocities of all the phase screens. Then the velocity should be treated as a random variable, and two statistical quantities of the average velocity and the standard deviation of velocity are considered to characterize the motion of the scatterers.

First, let us consider the case where the average of the velocities of all the

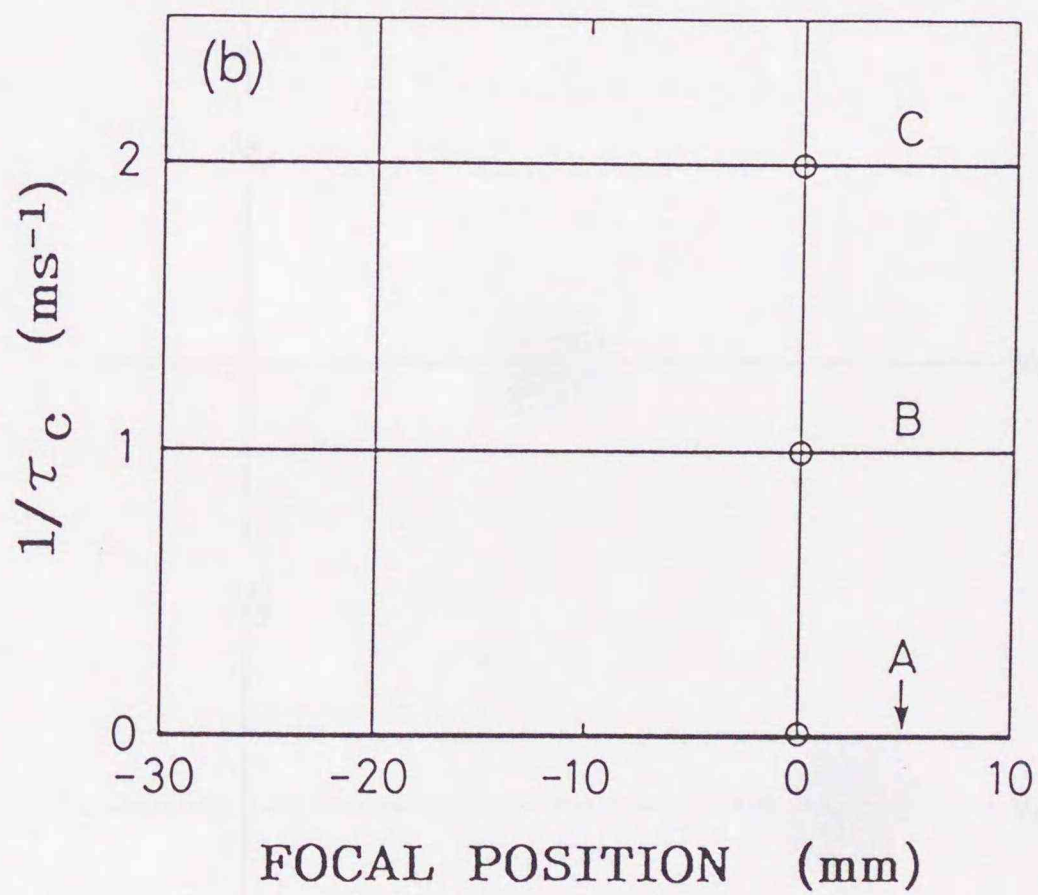
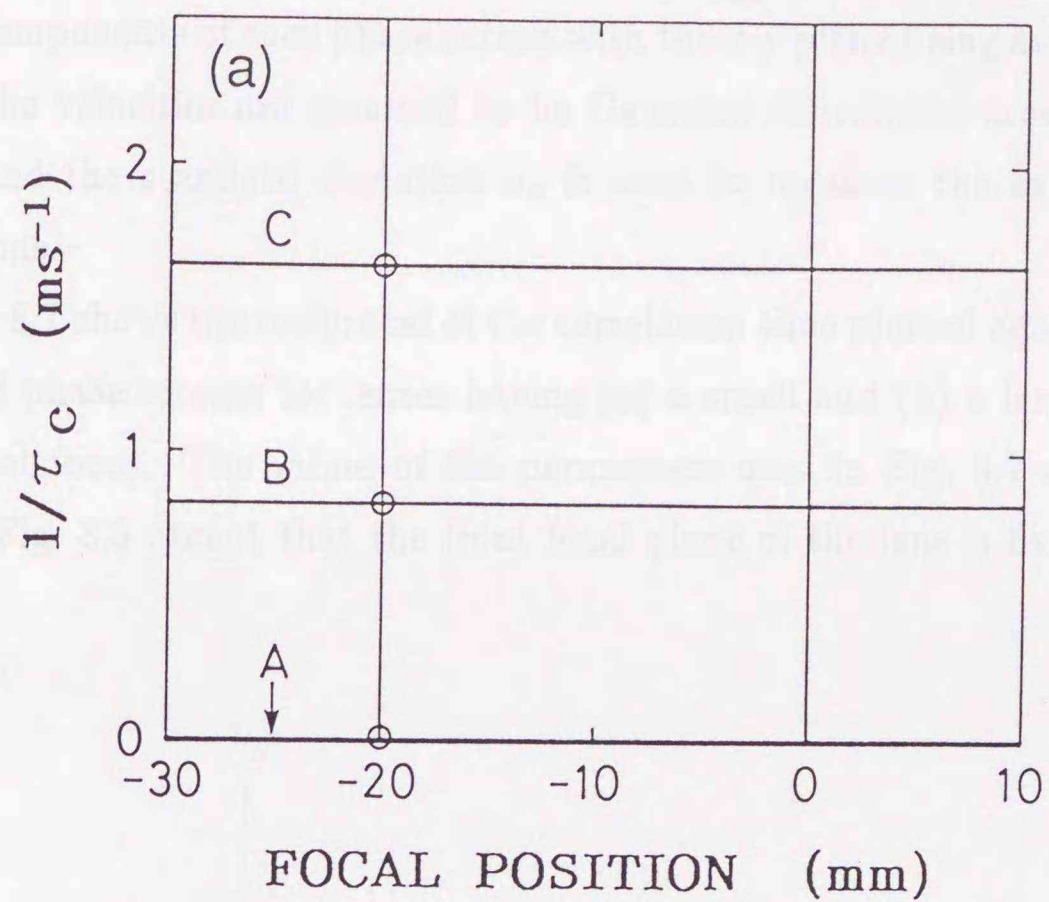


Figure 8.5. $1/\tau_c$ versus the focal position for various values of (a) v_1 and (b) v_5 in the case of five phase screens. The values of the parameters are the same as those in Fig. 8.3, except $\rho_n = 100 \mu\text{m}$.

phase screens varies while the extent of the velocity distribution is kept fixed. This situation is illustrated in Fig. 8.6, where each dot represents a pair (v_x, v_y) of the velocity components of each phase screen with the x-y plane being at the plane of the screen. The velocities are assumed to be Gaussian distributed around the average velocity and the standard deviation σ_v is used to measure the extent of velocity distributions.

Figure 8.7 shows the reciprocal of the correlation time plotted against the average velocity of phase screens for lenses having (a) a small and (b) a large point spread at the front focus. The values of the parameters used in Fig. 8.7 are the same as those for Fig. 8.3 except that the front focal plane of the lens is fixed on the third

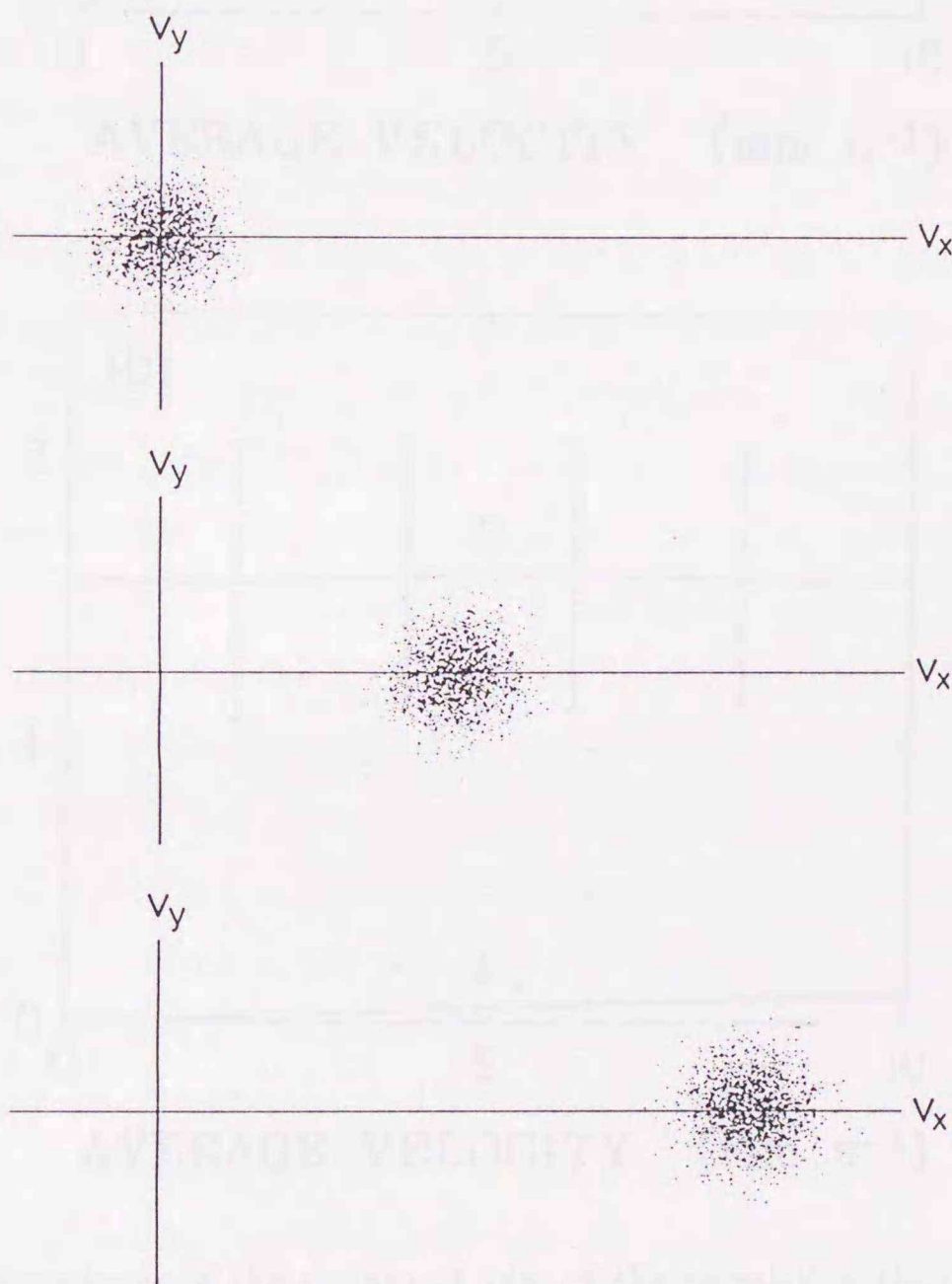


Figure 8.6. Velocity distributions of phase screens for different values of the average velocity $\langle v \rangle$. The standard deviation σ_v of velocity is fixed.

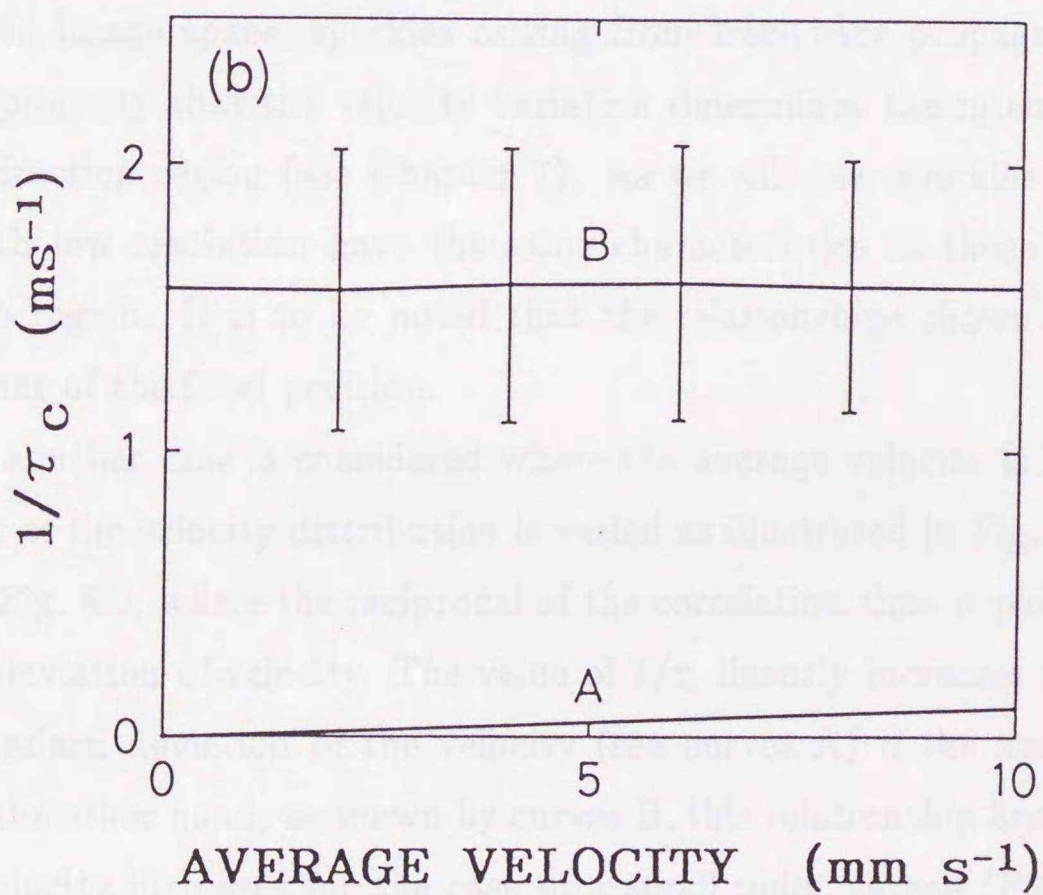
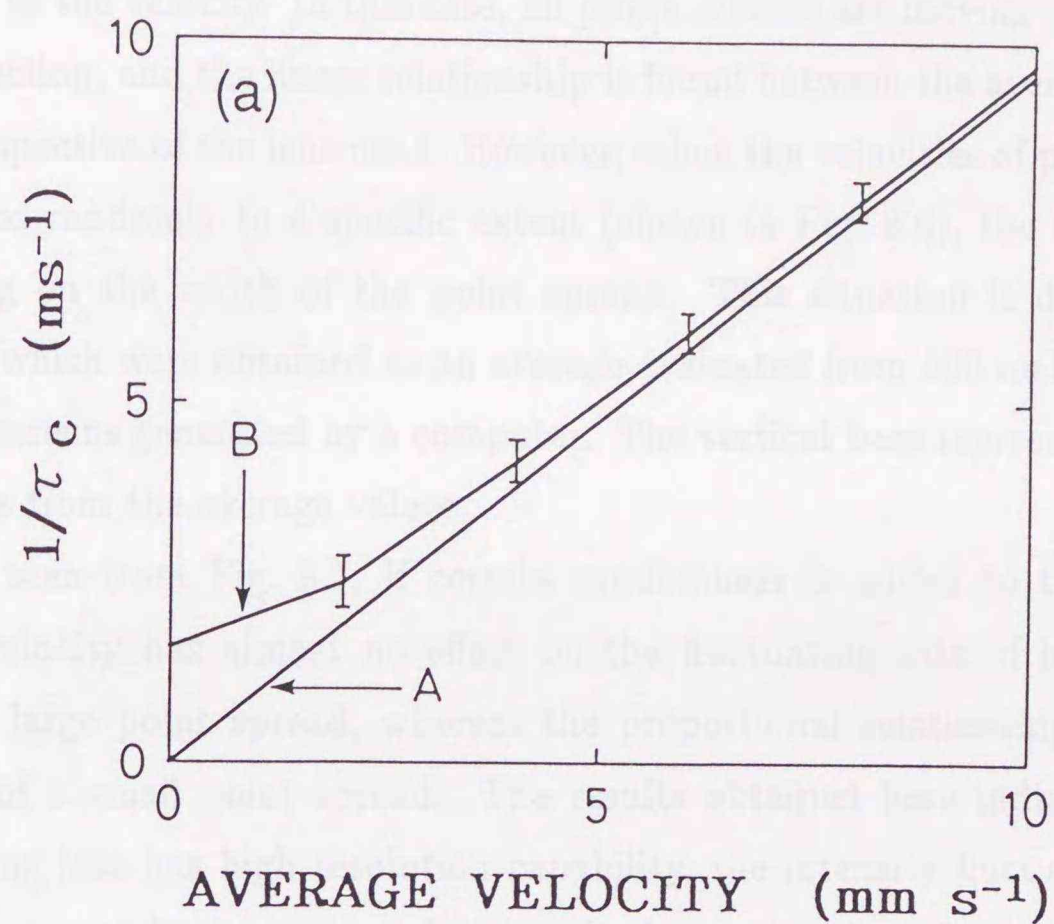


Figure 8.7. Dependence of the reciprocal $1/\tau_c$ of the correlation time on the average velocity for A, $\sigma_v = 0$, and B, $\sigma_v = 0.5 \text{ mm/s}$ obtained under (a) a small ($\rho_n = 1 \mu\text{m}$) and (b) a large ($\rho_n = 100 \mu\text{m}$) point spread.

phase screen. Curves A in Figs. 8.7(a) and 8.7(b) are obtained when there is no variation in the velocity. In this case, all phase screens are moving as a whole in the same direction, and the linear relationship is found between the average velocity and $1/\tau_c$, irrespective of the lens used. However, when the velocities of phase screens are distributed randomly to a specific extent (shown in Fig. 8.6), the relation changes depending on the width of the point spread. This situation is demonstrated by curves B which were obtained as an average evaluated from 500 realizations of a set of phase screens generated by a computer. The vertical bars represent the standard deviations from the average values.

As is seen from Fig. 8.7, if certain randomness is added to the velocity, the average velocity has almost no effect on the fluctuating rate of intensity for the case of a large point spread, whereas the proportional relationship still holds for the case of a small point spread. The results obtained here indicate that, when the imaging lens has high resolution capability, the intensity fluctuations of image speckles are mainly determined by the absolute velocities of phase screens rather than the velocity differences between them. This is characteristic of the speckles observed in image space; speckles arising from free-space propagation exhibit the opposite property that the velocity variation determines the intensity fluctuations in the diffraction region (see Chapter 7). As we will see, speckles imaged through a lens with low resolution have the same characteristics as those observed in the diffraction region. It is to be noted that the relationships shown in Fig. 8.7 hold independent of the focal position.

Next, another case is considered where the average velocity is kept fixed while the extent of the velocity distribution is varied as illustrated in Fig. 8.8. Results are shown in Fig. 8.9, where the reciprocal of the correlation time is plotted against the standard deviation of velocity. The value of $1/\tau_c$ linearly increases with an increase of the standard deviation of the velocity (see curves A) if the average velocity is zero. On the other hand, as shown by curves B, this relationship breaks down as the average velocity increases for the case of a small point spread (Fig. 8.9(a)), while the linear relationship still remains unchanged for the case of a large point spread (Fig. 8.9(b)).

Comparison between Figs. 8.7 and 8.9 leads to the conclusion that two sets of velocity information on the average velocity and the extent of velocity distribution

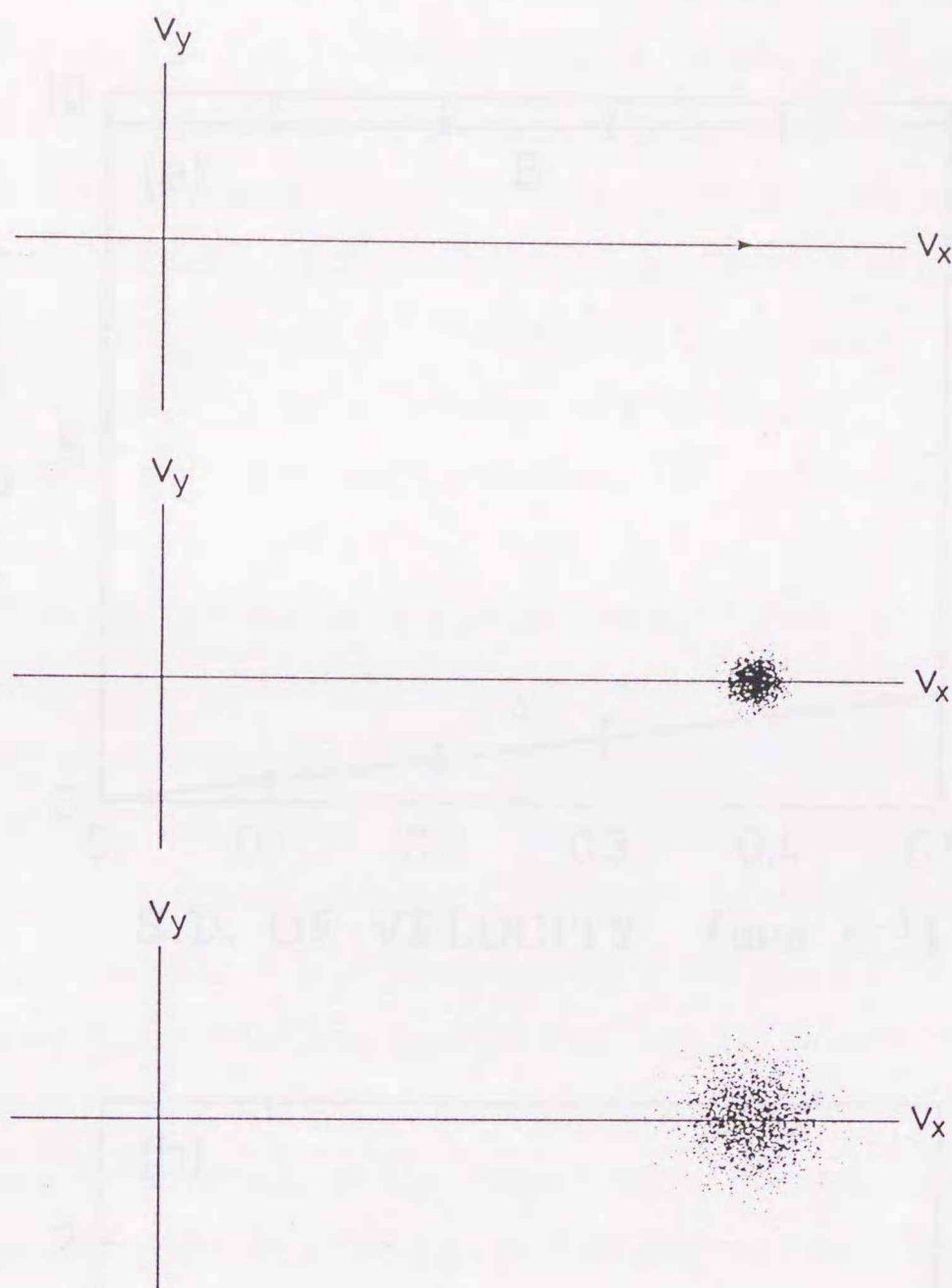


Figure 8.8. Variations in the extent σ_v of the velocity distribution of phase screens for a fixed value of the average velocity $\langle v \rangle$.

can be extracted from the intensity fluctuations of image speckles just by changing the size of the point spread at the front focal plane of the imaging lens. However, for the case where the values of the average velocity and the standard deviation of velocity are of the same order, both the quantities affect the measured values of the correlation time and their effects are hardly distinguishable from each other.

8.4 Conclusion

The statistical properties of dynamic speckles produced by a series of deep random-phase screens have been investigated for the case where the speckle patterns are

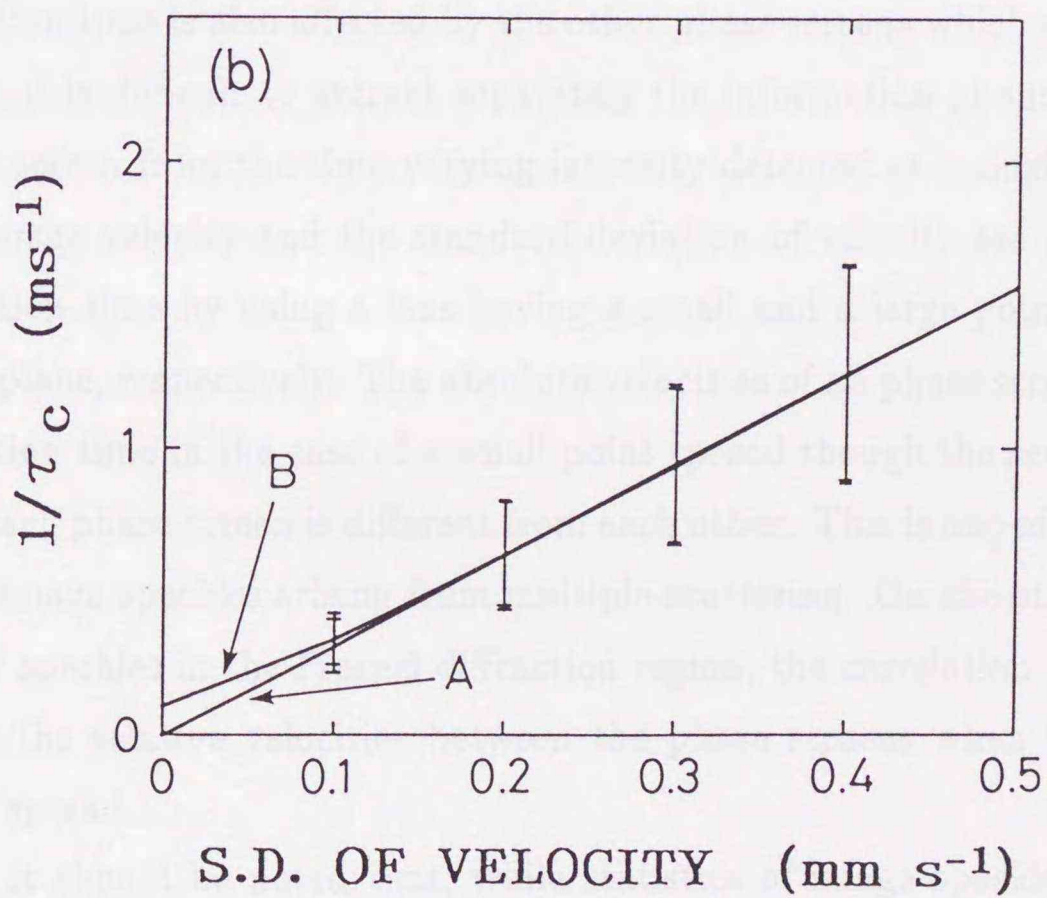
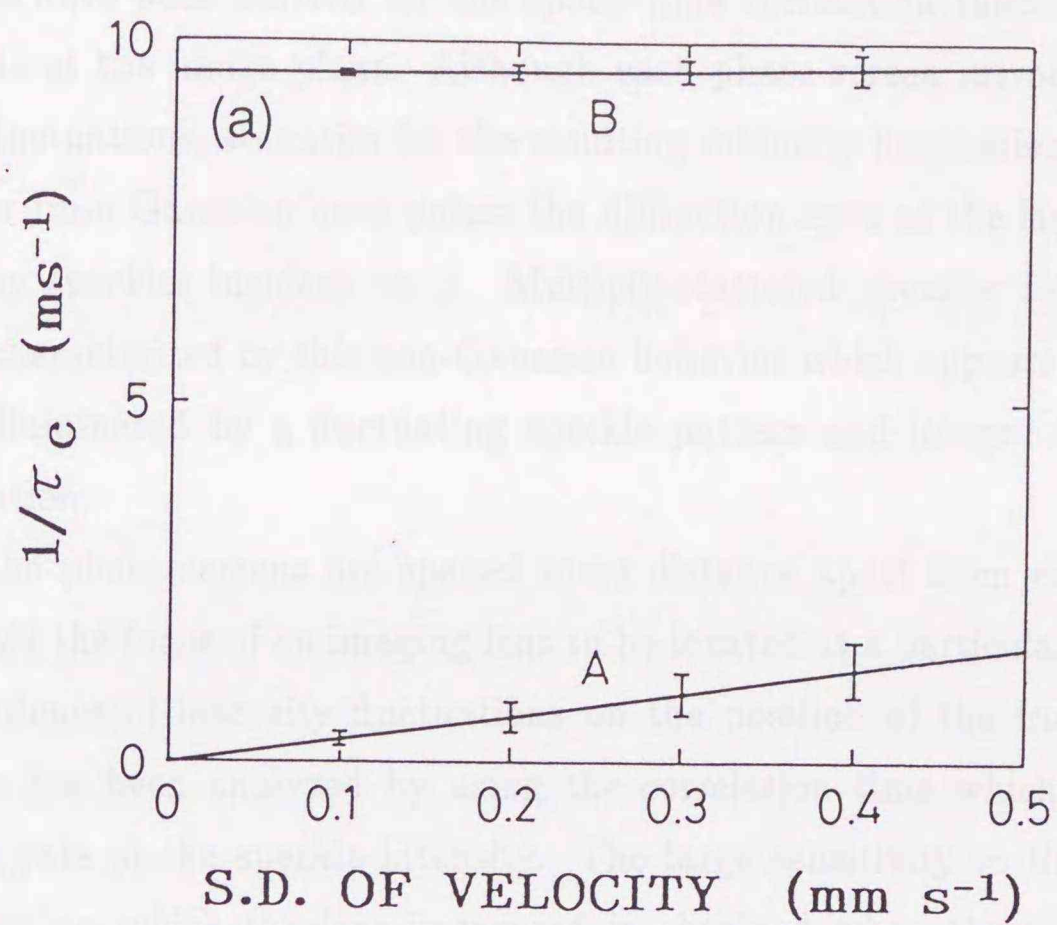


Figure 8.9. $1/\tau_c$ as a function of the standard deviation (s.d.) of velocity for A, $\langle v \rangle = 0$, and B, $\langle v \rangle = 10 \text{ mm/s}$. The sizes of the point spread are (a) $\rho_n = 1 \mu\text{m}$ and (b) $\rho_n = 100 \mu\text{m}$.

detected through an imaging system. By following the scalar wave approximation, expressions have been derived for the space-time correlation function of intensity fluctuations at the image plane. Although each phase screen introduces Gaussian intensity fluctuations, statistics for the resulting intensity fluctuations at the image plane differ from Gaussian ones unless the diffraction spot at the last phase screen spans many speckles incident on it. Multiply-scattered speckles formed in image space are characterized by this non-Gaussian behavior which appears when a diffuse object is illuminated by a fluctuating speckle pattern and imaged by a lens with high resolution.

Since the phase screens are spaced some distance apart from each other, this model allows the focus of an imaging lens to be located at a particular phase screen. The dependence of intensity fluctuations on the position of the front focal plane of the lens has been analyzed by using the correlation time which measures the fluctuating rate of the speckle intensity. The large sensitivity to the motion of a phase screen on which the lens is focused is obtained when the size of its point spread is small compared with the speckles illuminating the last screen. However, the correlation time is also affected by the other phase screens which are out of focus and, hence, it is difficult to extract separately the information about the motion of each phase screen from the time-varying intensity detected at a single point.

The average velocity and the standard deviation of velocity are measured from the correlation time by using a lens having a small and a large point spread at its front focal plane, respectively. The absolute velocities of all phase screens determine the correlation time in the case of a small point spread though the sensitivity to the motion of each phase screen is different from each other. This is one of the significant features of image speckles arising from multiple scattering. On the other hand, as is the case for speckles in the Fresnel diffraction region, the correlation time is greatly affected by the relative velocities between the phase screens when the lens has a large point spread.

Finally, it should be noted that, while statistics of image speckles are affected by the point spread at the diffuser closest to the lens, the relationship between the velocities of the diffusers and the fluctuations of the speckle intensity depends on the point spread at the front focal plane which is determined by the f -number and the magnification of the lens.

Chapter 9

Multiple Light Scattering from Particle Suspensions

9.1 Introduction

Dynamic light-scattering techniques have been widely used to study diffusion processes and fluid flows in colloidal and aerosol systems [5]. The intensity fluctuations of light scattered from small particles in a flow provide information about the Brownian motion of the particles as well as the motion of the flow. These techniques require a low concentration of the scattering particles to ensure that only the single scattering occurs. The dynamic properties of light scattered by the Brownian particles in a uniform flow [85–88] and in a shear flow [89, 90] have been studied in the single scattering limit from the viewpoints of particle sizing and measurements of velocities in laminar and turbulent flows. The results of these studies show that homodyne methods in which only the scattered light is detected are capable of measuring the diffusion coefficient and the velocity gradient of the flow. The uniform motion of the particles hardly affects the fluctuations of the light intensity in most of the optical configurations.

The contributions of diffusional and translational motions of the particles to the temporal fluctuations of the scattered light intensity have recently been investigated in the case of multiple scattering. Diffusing-wave spectroscopy [45] has extended the application of the above techniques to systems in which the light is strongly scattered. This technique was also applied to measurements of the laminar shear flow of dense colloidal particles [51]. The study showed that two time scales of the Brownian diffusion time and the shear relaxation time determine the decay of the autocorrelation function of intensity fluctuations. For the Brownian particles in a uniform flow, the influence of the translational motion of the particles is obscured by their diffusional motion as in the case of single scattering.

All the above investigations demonstrate that homodyne techniques are sensitive to the relative motion of the scatterers but insensitive to their collective motion. This statement is true when the scattering volume in a flow is large enough to neglect the intensity fluctuations arising from transits of the particles through the volume. However, if a small scattering volume is formed in a multiple scattering medium, the interplay is observed between the relaxation times for the relative and the collective motions of the particles.

In this chapter, the light multiply scattered from the particles conducting the Brownian motion in a layer moving transversely at a constant velocity is investigated experimentally. There is no velocity gradient or shear in the layer, and only two forms of the particle motion, diffusional and uniform motions, are considered in the system. The effect of the scattering volume on the correlation time of intensity fluctuations is analyzed by changing the illuminating form of a laser beam and the f -number of an optical imaging system used for the detection of scattered light. The properties of dynamic speckles in image space have already been analyzed for the light scattered from a single phase screen [4] or multiple phase screens [44]. A comparison is made between the time-varying intensities of speckles resulting from a translating phase screen and from particles undergoing the Brownian motion and the uniform translation. The concentration of particles changes the scattering volume in the multiple scattering regime, and its effect is also examined.

9.2 Experiments

Figure 9.1 shows a schematic diagram of the experimental setup used. Light from an argon-ion laser operating at the wavelength 514.5 nm is focused on the side of a sample cell via three lenses L_1 , L_2 and L_3 and a linear polarizer. The sample cell, which forms a layer of scattering media, has the dimensions of 65 mm length, 20 mm height and 1 mm thickness, and is mounted on a translation stage in order to move the scattering particles in the cell, uniformly in the direction perpendicular to the incident beam. The forward scattering light from the sample is collected by a camera lens L with a focal length of 135 mm and is detected by a photomultiplier PM through an analyzer the orientation of which is copolarized or cross-polarized

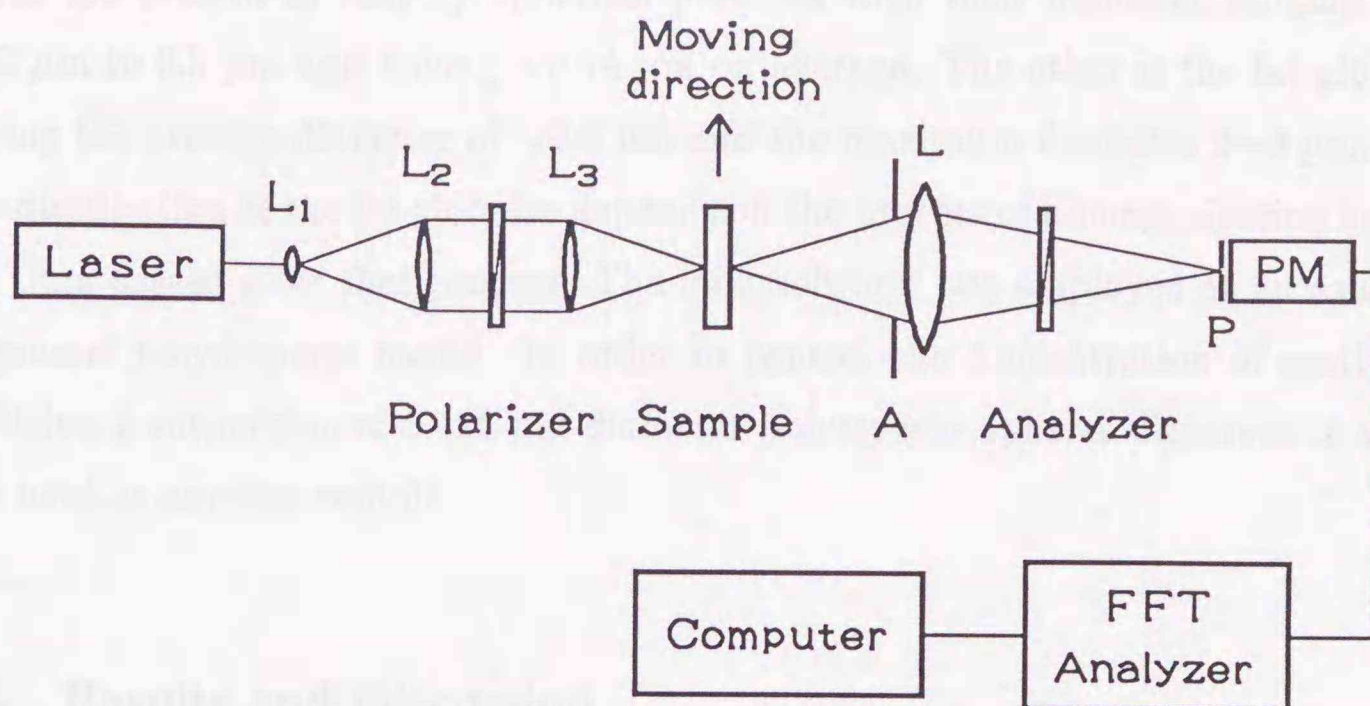


Figure 9.1. Schematic diagram of the experimental setup. L_1 , L_2 , L_3 and L , lenses; A , aperture; P , pinhole; PM, photomultiplier.

with the incident light.

The lens used images the surface of the sample at a magnification of 5 onto the plane of a pinhole P which limits the light impinging upon the input face of the photomultiplier. The size of the detecting pinhole was chosen in such a way that the average number of speckles (or coherence area) included in the area of the pinhole becomes the same for different sizes of an aperture A . The pinholes with the diameters $20\text{ }\mu\text{m}$ and $200\text{ }\mu\text{m}$ were used for detection for the f -numbers 2.8 and 32 of the camera lens, respectively, with the number of speckles falling on the pinhole being ~ 3 in both cases.

The signal from the photomultiplier is fed to a FFT analyzer (CF-920, Ono Sokki), in which calculations of the autocorrelation function are repeated 32 times for different sets of 1024 sampled data to yield the ensemble-averaged autocorrelation function. The correlation time is then obtained using a computer as the decay time at the half maximum of average autocorrelation functions, and its mean and standard deviation are evaluated from five correlation times measured for the same sample at different times and positions.

Colloidal emulsions and suspensions were employed as scattering samples. A milk solution was made by diluting milk on the market with water at a concentration of 30 %. This solution contains two kinds of colloidal particles. One is the casein micelles

which are present in roughly spherical particles with their diameter ranging from $0.02\text{ }\mu\text{m}$ to $0.3\text{ }\mu\text{m}$ and having $\sim 0.14\text{ }\mu\text{m}$ on average. The other is the fat globules having the average diameter of $\sim 0.6\text{ }\mu\text{m}$ and the maximum diameter $2\sim 3\text{ }\mu\text{m}$. The size distribution of the fat globules depends on the process of homogenization and on the time passed after that process. The milk solution was employed as an example of general polydisperse media. In order to control the concentration of scattering particles, a suspension of $0.426\text{-}\mu\text{m}$ -diameter polystyrene spheres dispersed in water was used as another sample.

9.3 Results and Discussion

In this section, experimental results are presented for the effects of various parameters of the system on the dynamic properties of light scattered from milk solutions (Subsections 9.3.1, 9.3.2 and 9.3.3) and particle suspensions (Subsection 9.3.4). The results are compared with the ones obtained by a phase screen, and the difference between them is discussed. In addition, attention is paid to the relationship between the scattering volume and the intensity fluctuations occurred by the translational motion of the scatterers.

9.3.1 Illuminating Form and State of Polarization

Three illuminating forms of a laser beam incident on the sample cell are illustrated in Fig. 9.2. A spherical lens of the 200 mm focal length was used as lens L_3 for spot illumination, and a cylindrical lens of the same focal length for vertical- and horizontal-line illumination. The diameter of the spot illumination is set to be $17\text{ }\mu\text{m}$. The line illumination takes a form of the width $17\text{ }\mu\text{m}$ with the length 20 mm . The camera lens with the f -number 2.8 was used for light detection.

Figures 9.3(a) and 9.3(b) show the reciprocal of the correlation time of intensity fluctuations for the copolarized and cross-polarized scattered light, respectively, as a function of the translational velocity of the sample cell for the three kinds of illumination. A ground glass plate (#600) of roughness $0.61\text{ }\mu\text{m}$ was employed as a phase screen and was illuminated by a beam with the diameter $\sim 20\text{ mm}$, simulating the divergence of the incident light at the exit face of the sample cell filled with a

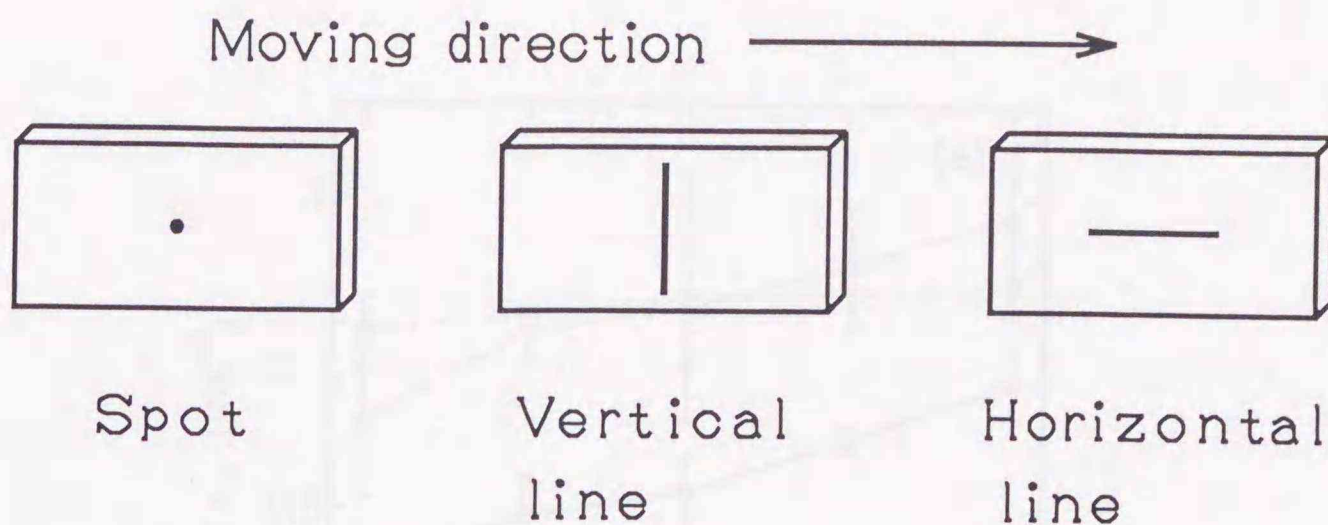


Figure 9.2. Three illuminating forms of a laser beam incident on the sample cell.

milk solution. The correlation time measured by using that plate is also plotted in Fig. 9.3(a).

For the light scattered from the phase screen, the correlation time of its intensity is inversely proportional to the moving velocity of the screen. This relationship is modified in the case of Brownian particles whose motion is diffusive. Owing to the Brownian motion, the intensity of light scattered by the milk solution fluctuates in time even if the sample cell is at rest. This fluctuation arises for any multiple-scattering media at any scattering angles, independent of the form of the incident beam.

However, the rate of intensity fluctuations due to the diffusional motion depends on the illuminating form of the beam. This observation stems from the fact that the correlation time becomes shorter as the number of multiple-scattering events increases [45]. The line forms of illumination cause the incident light to travel longer scattering paths than those for the spot form, leading to shorter correlation times.

The correlation times for the two kinds of line illumination are expected to coincide with each other when the sample cell is fixed, but disagreement is observed between them in Fig. 9.3. This is because the samples prepared had some variations in concentration and polydispersity of scattering particles. Even so, the difference between the spot and line forms of illumination is obvious.

As seen from Fig. 9.3(a), the values of $1/\tau_c$ for the spot and vertical-line illuminations increase at the same rate with an increase of the translational velocity of the

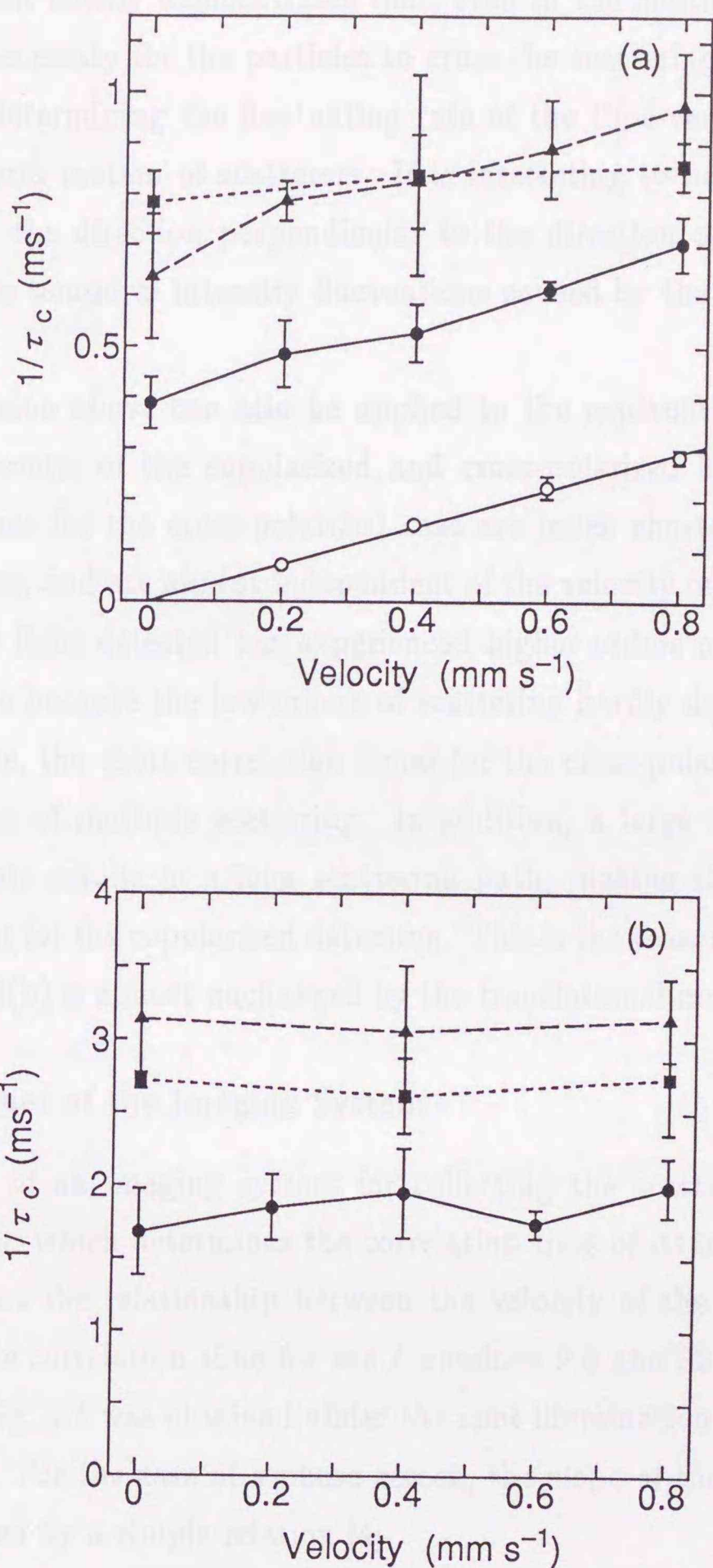


Figure 9.3. Reciprocal of the correlation time τ_c plotted against the translational velocity of a milk solution for the spot (●), vertical-line (▲) and horizontal-line (■) forms of illumination. The scattered light detected is copolarized (a) or cross-polarized (b) with the incident light. Open circles indicate the result for a ground glass plate.

cell, while the correlation time is almost constant for the horizontal-line illumination. This result clearly demonstrates that, even in the multiple-scattering regime, the distance necessary for the particles to cross the scattering volume is one of the main factors determining the fluctuating rate of the time-varying intensity arising from the uniform motion of scatterers. It is interesting to note that the extent of the volume in the direction perpendicular to the direction of the uniform motion only affects the temporal intensity fluctuations caused by the diffusional motion of the particles.

The discussion above can also be applied to the explanation for the difference between the results of the copolarized and cross-polarized light detections. The correlation times for the cross-polarized case are much shorter than those for the copolarized case, and are almost independent of the velocity of the sample cell. The cross-polarized light detected has experienced higher orders of scattering than the copolarized one because the low orders of scattering hardly depolarize the incident light. Therefore, the short correlation times for the cross-polarized case are caused by high degrees of multiple scattering. In addition, a large number of successive scattering events results in a long scattering path, making the scattering volume larger than that for the copolarized detection. This is the reason why the correlation time in Fig. 9.3(b) is almost unchanged by the translational motion of the particles.

9.3.2 F-number of the Imaging System

The f -number of an imaging system for collecting the scattered light is another important factor which determines the correlation time of its intensity fluctuations. Figure 9.4 shows the relationship between the velocity of the sample cell and the reciprocal of the correlation time for the f -numbers 2.8 and 32 of the imaging lens. The result in Fig. 9.4 was obtained under the spot illumination and the copolarized light detection. For the case of a phase screen, the slope of the fitting lines in Fig. 9.4 is determined by a simple relation [4]

$$1/\tau_c = v/\rho, \quad (9.1)$$

where v is the translational velocity of the screen and ρ represents the extent of the point spread function of the imaging system at its front focal plane in object space.

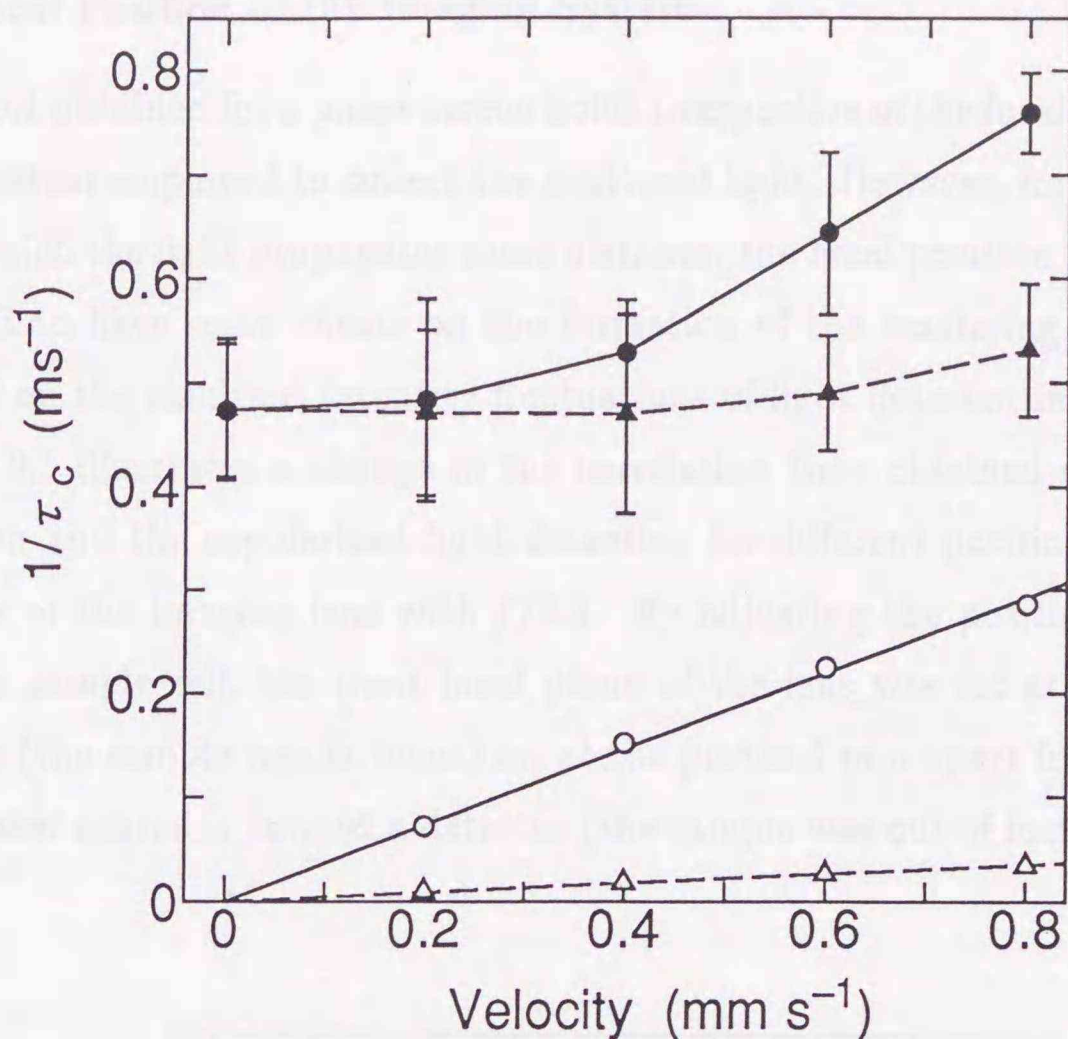


Figure 9.4. Dependence of the correlation time τ_c on the velocity of a ground glass plate ($\circ$, $\triangle$) and of a milk solution ($\bullet$, $\blacktriangle$) for f -numbers 2.8 (circles) and 32 (triangles) of the imaging lens.

Equation (9.1) holds as long as the value of ρ is much smaller than the extent of the illuminated area across the phase screen. If the imaging system used has a small f -number, a small point spread is obtained and rapid fluctuations of the scattered light intensity result.

In Fig. 9.4, the result of milk solutions exhibits the same tendency as that of a phase screen. The effect of the uniform motion of the particles on the fluctuations of the scattered light intensity is emphasized by use of an imaging system with a small f -number, and suppressed by a system of large f -number. This result has also been confirmed by the multiple phase screen model presented in Chapter 8, and it may be said that this is a general property of the scattered light detected through an imaging system. We avoid making a direct comparison between the rates of increase in the value of $1/\tau_c$ for a ground glass plate and for milk solutions because, as shown in Fig. 9.3(a), the rate varies in the latter case with the illuminating form of the incident beam.

9.3.3 Focal Position of the Imaging System

Equation 9.1 obtained for a phase screen holds irrespective of the focal position of the imaging system employed to collect the scattered light. However, for volume media through which the light propagates some distance, the focal position in object space is expected to have some effects on the formation of the scattering volume in the media and on the resultant intensity fluctuations of light detected in image space.

Figure 9.5 illustrates a change in the correlation time obtained under the spot illumination and the copolarized light detection for different positions of the front focal plane of the imaging lens with $f/2.8$. By adjusting the positions of the lens L_3 and the sample cell, the front focal plane of the lens was set at the surface of the sample (the sample was in focus) or, at the planes 5 mm apart from the surface toward a laser source or toward a detector (the sample was out of focus). The result

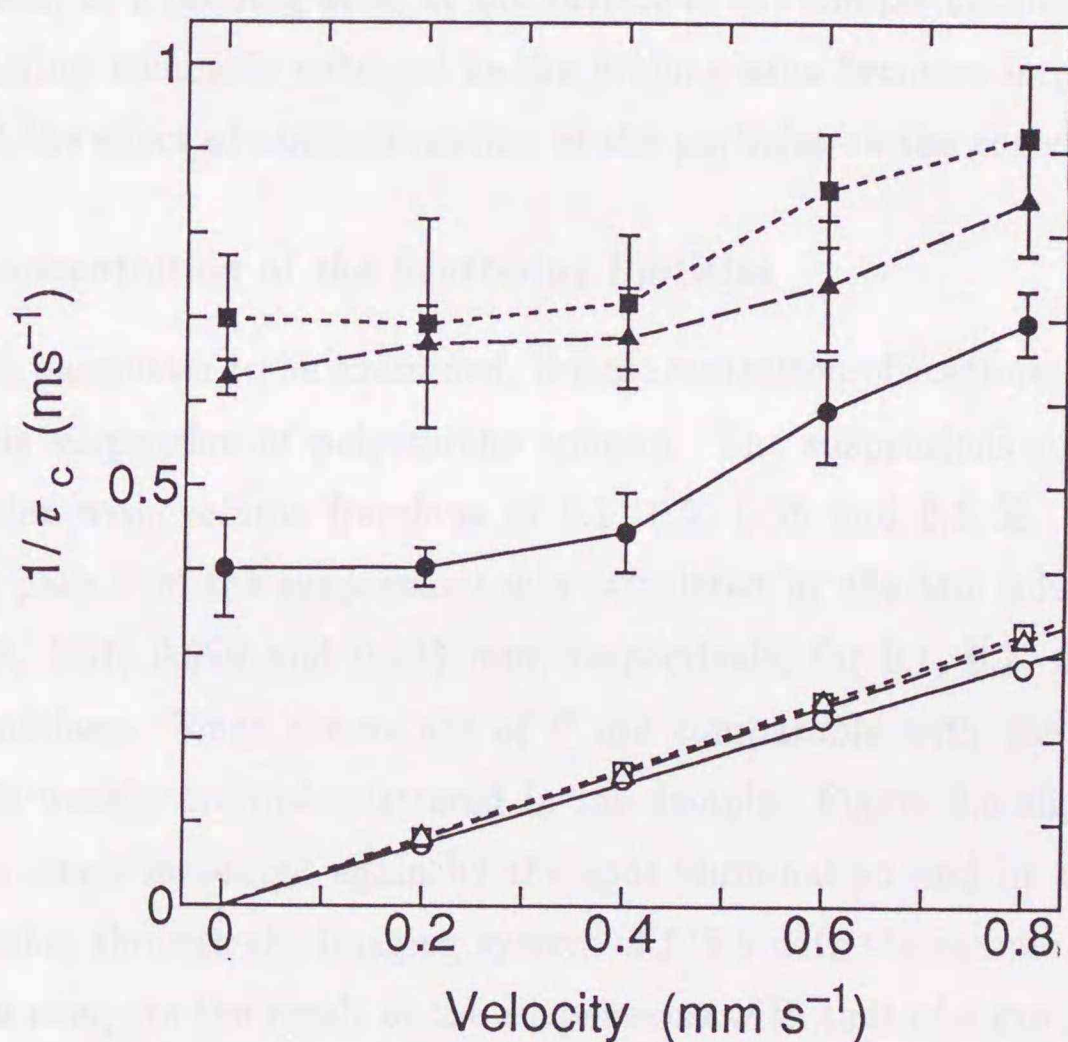


Figure 9.5. Reciprocal of the correlation time τ_c versus the velocity of a ground glass plate ($\circ$, $\triangle$, $\square$) and of a milk solution ($\bullet$, $\blacktriangle$, $\blacksquare$). The imaging lens is focused on the surface of the samples (circles), and on the planes 5 mm away from the surface toward a laser source (triangles) and toward a detector (squares).

of a ground glass plate demonstrates that the focal position of the detecting system has little influence on the correlation time, as expected.

For the case of milk solutions, however, the correlation time becomes shorter when the sample is out of focus. The explanation similar to the one given in Subsection 9.3.1 can be applied to this result. If the surface of the sample is located at a defocused plane, the point spread of the imaging system at that plane becomes larger than that at the focal plane. The imaging system then allows the scattered light emerging from a larger area of the sample surface to reach the detector. Since the incident light diverges in the sample starting from a small spot formed on the other side of the sample, the contributions of light having longer scattering paths increase and, consequently, the correlation time becomes shorter.

Although the values of $1/\tau_c$ increase with an increase of the translational velocity of the sample cell at all three positions of the focal plane, the rate of the increase is somewhat lower in the defocused case. This also results from the extension of a point spread, or a probing area, at the surface of the sample because of defocusing. The scattering volume is enlarged as the probing area becomes larger, making less significant the effect of uniform motion of the particles on the correlation time.

9.3.4 Concentration of the Scattering Particles

As the last parameter to be examined, the concentration of scatterers was varied for an aqueous suspension of polystyrene spheres. The suspensions prepared contain the particles with volume fractions of 0.1, 0.2, 0.35 and 0.5 %. The transport mean free path l^* of the suspensions was calculated by the Mie solution and found to be 2.02, 1.01, 0.576 and 0.403 mm, respectively, for 0.1, 0.2, 0.35 and 0.5 % volume fractions. Since the values of l^* are comparable with the sample depth, the light is weakly multiple-scattered in the sample. Figure 9.6 shows the plot of correlation times measured again by the spot illumination and by the copolarized light detection through the imaging system of $f/2.8$ with the sample being in focus. In order to compare the result of the suspensions with that of a ground glass plate, a longitudinally-expanded version of Fig. 9.6(a) is shown in Fig. 9.6(b), where a fitting line is plotted for experimental data of the plate.

From Fig. 9.6, we see that the correlation time becomes shorter as the particle concentration increases for all velocities of the scatterers. This result arises from

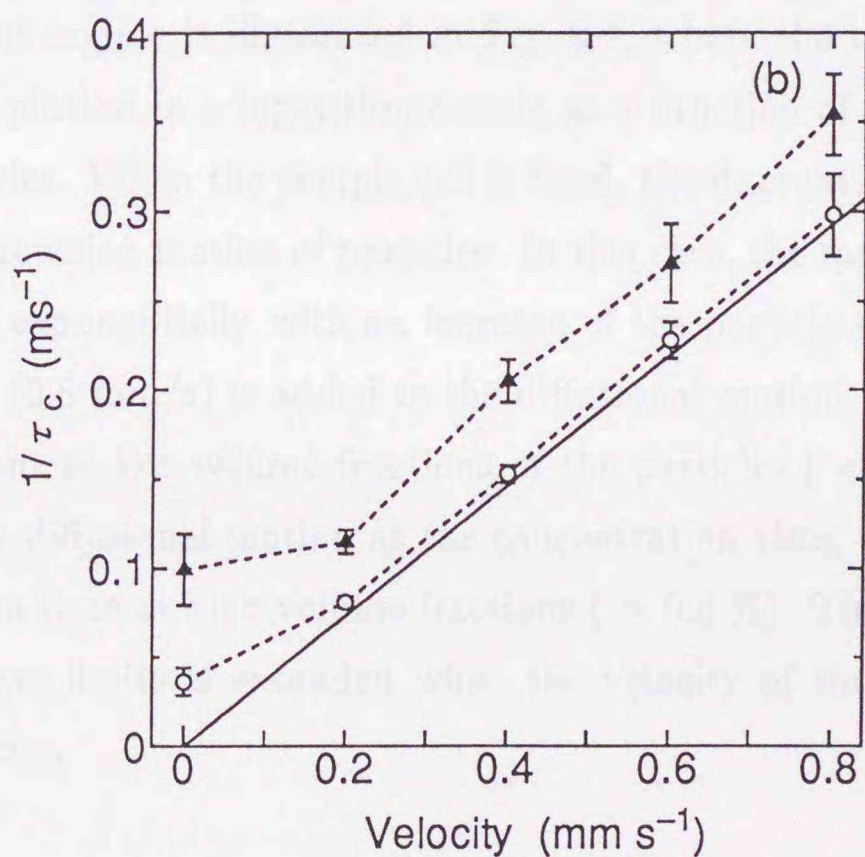
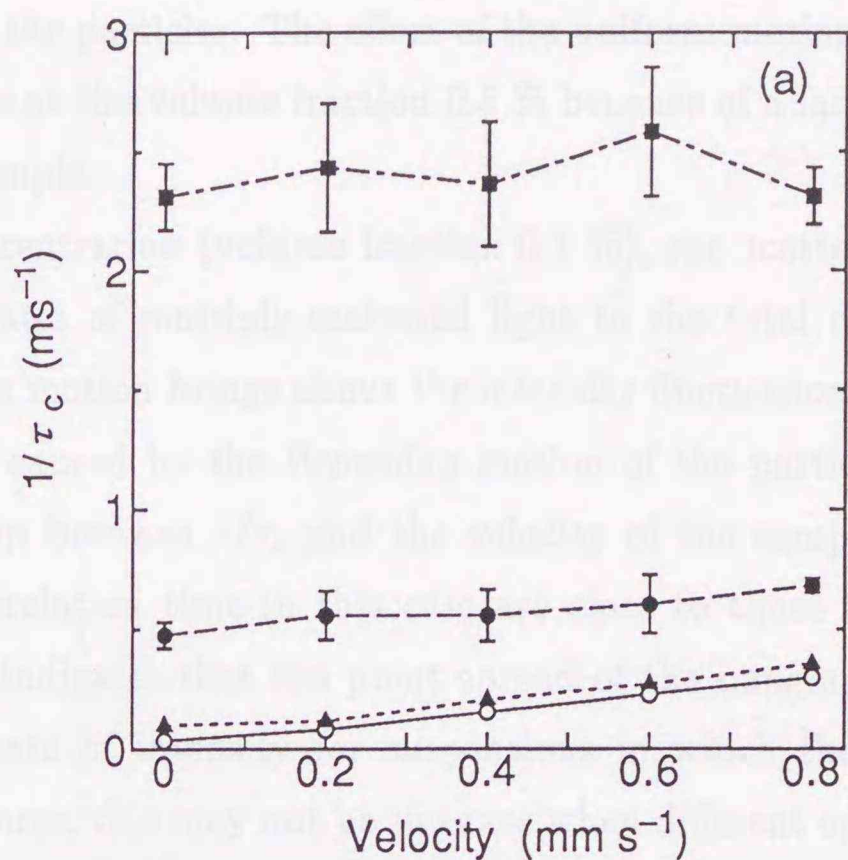


Figure 9.6. Reciprocal of the correlation time τ_c as a function of the translational velocity of suspensions for different volume fractions of the scattering particles: (a); 0.1 % (○), 0.2 % (▲), 0.35 % (●) and 0.5 % (■). The result for a ground glass plate is plotted in (b) for comparison, together with the data for volume fractions of 0.1 and 0.2 % of the suspensions.

the fact that the amount of multiple-scattering increases with an increase in the concentration of the particles. The effect of the uniform motion of the suspension is hardly noticeable at the volume fraction 0.5 % because of a large scattering volume formed in the sample.

At a low concentration (volume fraction 0.1 %), the scattering volume is small and the power ratio of multiply-scattered light to the total detected light is low. Then the uniform motion brings about the intensity fluctuations whose rate is much faster than that caused by the Brownian motion of the particles, resulting in the linear relationship between $1/\tau_c$ and the velocity of the sample. In addition, the values of the correlation time in this case are close to those for the ground glass plate. This fact indicates that the point spread of the imaging system determines the fluctuating rate of intensity for suspensions in which the weak scattering is dominant. Of course, this may not be the case when different optical geometries are employed. For example, the rate of intensity fluctuations caused by the diffusional motion of the scatterers varies with different angles of detection.

The transition from the uniform-motion dominant regime to the diffusional-motion dominant regime is illustrated in Fig. 9.7, where the reciprocal of the correlation time is plotted in a logarithmic scale as a function of the concentration of scattering particles. When the sample cell is fixed, the fluctuations of intensity arise only from the Brownian motion of particles. In this case, the rate of intensity fluctuations increases exponentially with an increase of the particle concentration. If the uniform motion (0.8 mm/s) is added to the diffusional motion, the correlation time is almost constant at low volume fractions of the particles (< 0.1 %), approaches the value for the diffusional motion as the concentration rises, and finally coincides with the diffusion time at high volume fractions (> 0.5 %). The transitional region between these two limits is extended when the velocity of the uniform motion of scatterers increases.

9.4 Conclusion

The dynamic properties of light scattered by a translating layer of the Brownian particles were investigated in the multiple scattering regime. The rate of intensity

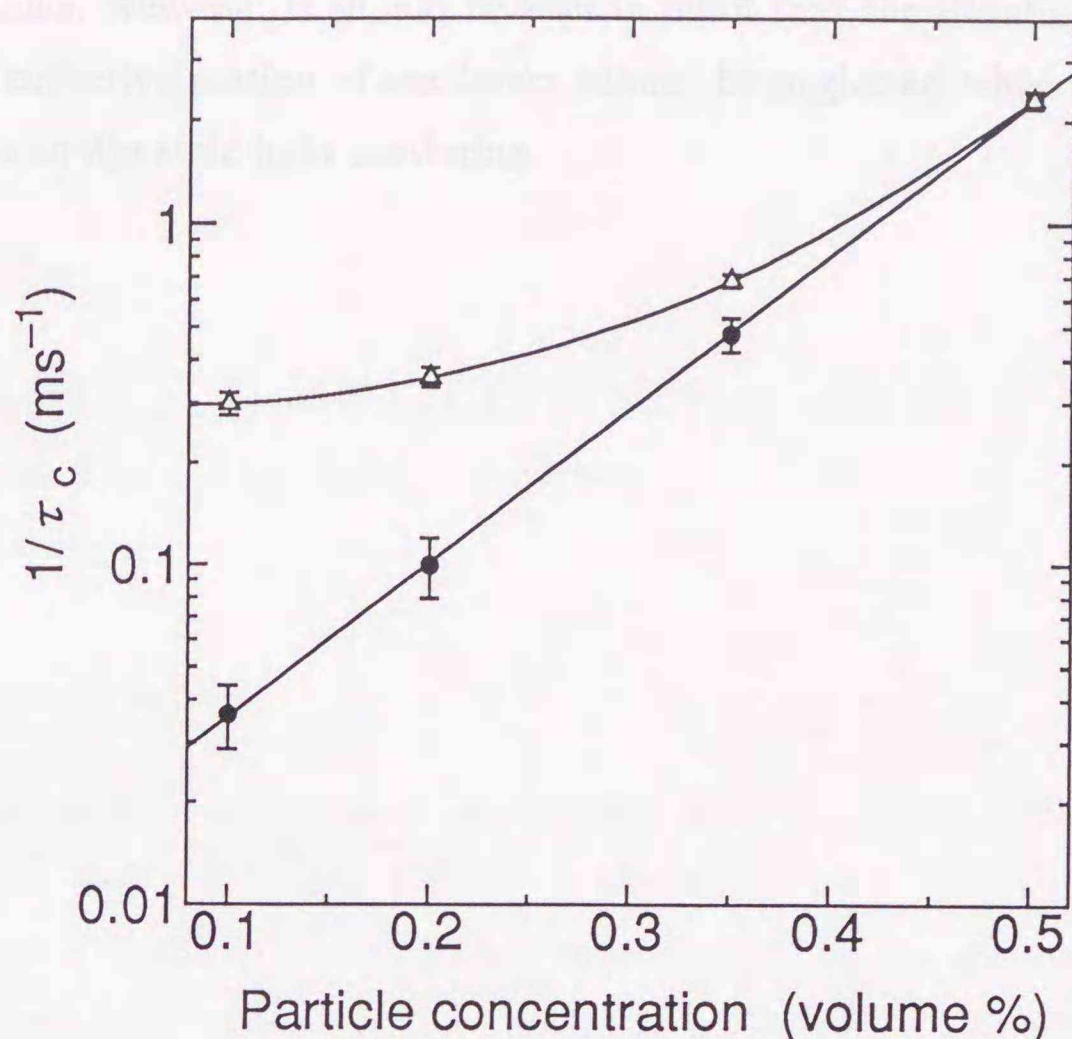


Figure 9.7. Dependence of the correlation time τ_c on the particle concentration. The sample cell is at rest (●) or translates at 0.8 mm/s (△).

fluctuations of the scattered light was quantified by the correlation time, the value of which reflects the motion of scatterers. The correlation time measured with a translating phase screen is simply determined by the velocity of the screen and the point spread function of an imaging system used to collect the scattered light. For the case of a scattering layer with finite thickness, however, the scattering volume formed in the layer also affects the temporal fluctuations of the light intensity. The effect of uniform motion of the particles becomes significant for optical systems having a small area of illumination and a small point spread of the imaging lens for collecting optics, both of which limit the scattering volume and suppress the fluctuations of light arising from the Brownian motion of the particles.

For a layer of suspensions in which the light is multiply scattered, an increase in the concentration of particles leads to larger scattering volumes and shorter correlation times as a result of the cumulative effect of scattering events occurred successively. Therefore, the uniform motion of the layer is less noticeable for higher particle

concentrations. However, it should be kept in mind that the situation may arise in which the collective motion of scatterers cannot be neglected when performing an experiment on dynamic light scattering.

Chapter 10

Application of Spectral Correlation Method to the Study of Particle Suspensions

10.1 Introduction

Two optical systems produced by Monod and his group with the laser light of different sub-lengths or different intensity variations [1, 2] show the degree of correlation between the spectra patterns depends on the scattering properties of the system, the spectral correlation method has been applied to study suspensions of the system [3-5]. In particular, this degree of correlation can be used to study the nature of scattering particles in a system. Thus, the other information that the particle suspension is obtained by applying this method to heterogeneous volume media [6, 7].

In the study of colloidal systems, the dynamic light scattering technique is widely used to characterize colloidal particles [8]. However, the application of this technique is limited to dilute suspensions in which the single scattering of light occurs. For a concentrated particle suspension, the scattered light from the sample appears as multiple beams which leads to a change in the shape of the autocorrelation function of intensity fluctuations and makes the interpretation of measured data. To overcome this problem, a new type of correlation technique was recently proposed [9, 10] to remove the multiple scattering effect. Although this technique employed two lengths of light for probing the dynamic particles, it should not be considered as an application of the spectral correlation method. The correlation properties of dual color light beams in an imaging geometry have not been fully investigated yet, and the spectral correlation method has a potential for offering a new measurement tool in the study of colloidal systems.

In this chapter, the principles and correlation properties of dynamic light scattering

Chapter 10

Application of Spectral Correlation Method to the Study of Particle Suspensions

10.1 Introduction

Two speckle patterns produced by illuminating a rough surface with the laser light of different wavelengths exhibit different intensity variations [91]. Since the degree of decorrelation between the speckle patterns depends on the scattering properties of the surface, the spectral speckle-correlation method has been applied to measurements of the surface roughness [92–102]. In principle, this decorrelation phenomenon can be observed for any kind of scattering materials giving rise to a speckle pattern. Thus, the other information rather than the surface roughness is obtained by applying this method to inhomogeneous volume media [103, 104].

In the study of colloidal systems, the dynamic light scattering technique is widely used to characterize colloidal particles [5]. However, the application of this technique is limited to dilute suspensions in which only the single scattering of light occurs. For a concentrated particle suspension, the scattered light from the sample experiences multiple scattering, which leads to a change in the shape of the autocorrelation function of intensity fluctuations and makes difficult the interpretation of measured data. To overcome this problem, a cross-spectral correlation technique was recently proposed [105, 106] to reduce the multiple scattering effect. Although this technique employs two wavelengths of light for probing the Brownian particles, it should not be considered as an application of the speckle method mentioned above. The correlation properties of dual color speckles formed in an imaging geometry have not been fully investigated yet, and the spectral speckle-correlation method has a potential for offering a new measurement tool in the study of colloidal systems.

In this chapter, the cross-spectral correlation properties of dynamic speckles pro-

duced by particle suspensions are investigated experimentally. A flowing Brownian motion system [87, 88] is realized by translating a sample suspension with a constant velocity. The effects of the Brownian and uniform motions of the particles on the temporal cross-correlation of intensity fluctuations are examined and compared with the case of the temporal autocorrelation of intensity fluctuations produced by the light of a single wavelength. It is shown that the effect of the Brownian motion is suppressed, independently of the amount of multiple scattering, by taking the two-wavelength cross-correlation of speckle intensity fluctuations.

10.2 Experiments

Figure 10.1 shows a schematic diagram of the experimental setup used to measure the spectral-correlation function of speckle intensity fluctuations produced by the particle suspension. A polychromatic argon-ion laser beam is collimated by the achromatic lens system L_1 - L_2 combined with the polarizer PL to illuminate a sample cell with the spot diameter 20 mm. The sample cell, which has the dimensions of 65 mm length, 20 mm height and 1 mm thickness, contains an aqueous suspension

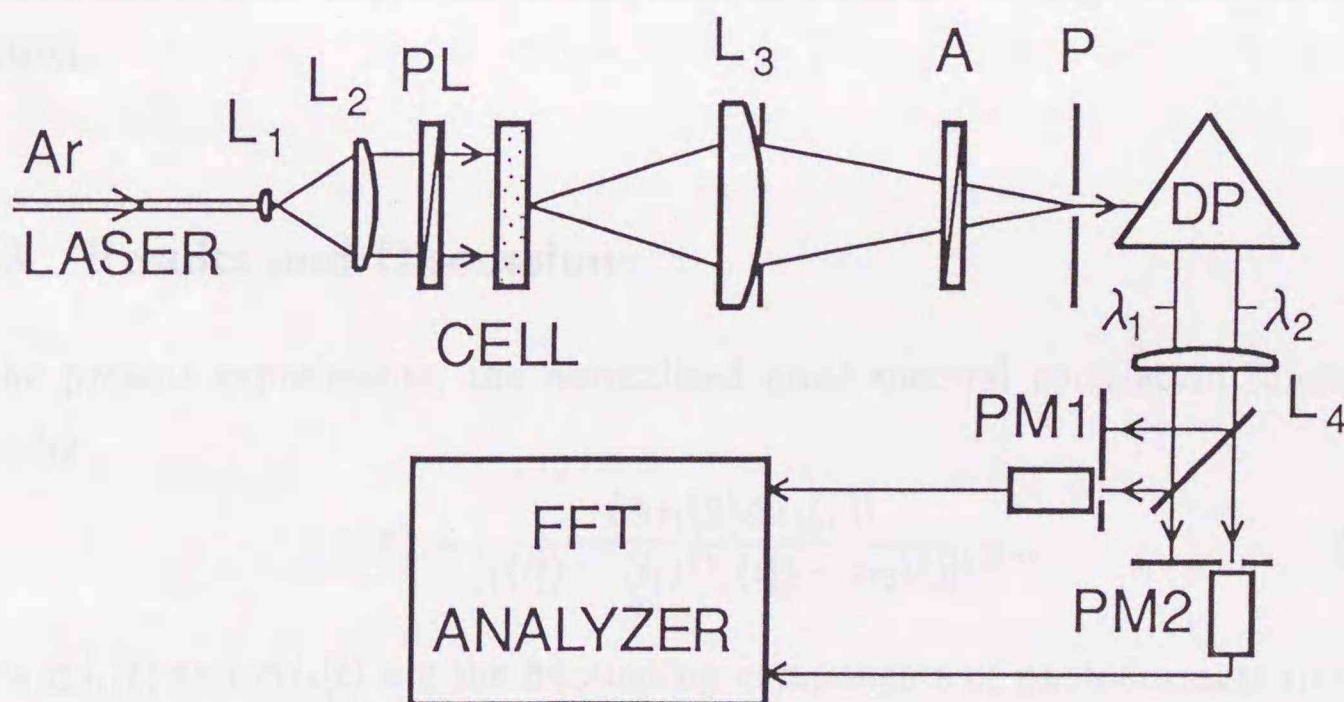


Figure 10.1. Schematic diagram of the experimental setup. L_1 , L_2 , L_3 and L_4 , achromatic lenses; PL, polarizer; A, analyzer; P, pinhole; DP, dispersion prism; PM1 and PM2, photomultipliers.

of 0.426- μm -diameter polystyrene microspheres, and is moved at a constant velocity by a translation stage in the direction perpendicular to the incident beam.

The interface between the suspension and the glass wall of the cell is imaged onto a pinhole P located on the optical axis with the magnification $\times 2.5$ by a camera lens L_3 via the analyzer A, which only allows the light copolarized with the incident light to enter the pinhole. The lens L_3 with a focal length of 135 mm and an f -number of 2.8 (the pupil diameter 48.2 mm) is located 189 mm away from the sample, collecting the light scattered at angles between -0.127 and 0.127 rad ($\pm 7.27^\circ$). This configuration makes many different scattering vectors be involved in the detected light, and reduces the phase fluctuations arising from the single scattering. As will be shown later, the suppression of the phase fluctuations can be considered as an advantage for velocity measurements.

A dispersion prism DP and an achromatic lens L_4 form the images of the pinhole with several different colors at different positions in the entrance planes of the photomultipliers PM1 and PM2. In the present experiment, two wavelengths, $\lambda_1 = 488$ nm and $\lambda_2 = 514.5$ nm, of the light were chosen by suitably positioned entrance apertures over the photomultipliers. The time-varying signals are fed into a FFT analyzer (CF-920, Ono Sokki), in which, after the zero-frequency component being eliminated, calculations of the cross-correlation function are repeated 64 times for different sets of 1024 sampled data to yield the ensemble-averaged cross-correlation function.

10.3 Results and Discussion

In the present experiments, the normalized cross-spectral correlation function is given by

$$\gamma(\tau) = \frac{\langle \Delta i_1(0) \Delta i_2(\tau) \rangle}{[(\langle i_1^2 \rangle - \langle i_1 \rangle^2)(\langle i_2^2 \rangle - \langle i_2 \rangle^2)]^{1/2}}, \quad (10.1)$$

where $\Delta i_1(t)$ and $\Delta i_2(t)$ are the fluctuating components of photocurrents $i_1(t)$ and $i_2(t)$ detected by the two photomultipliers, respectively, and $\langle \dots \rangle$ denotes the ensemble average. Note that the normalization is performed after elimination of the dc components of the signals. An example of the above functions obtained in the

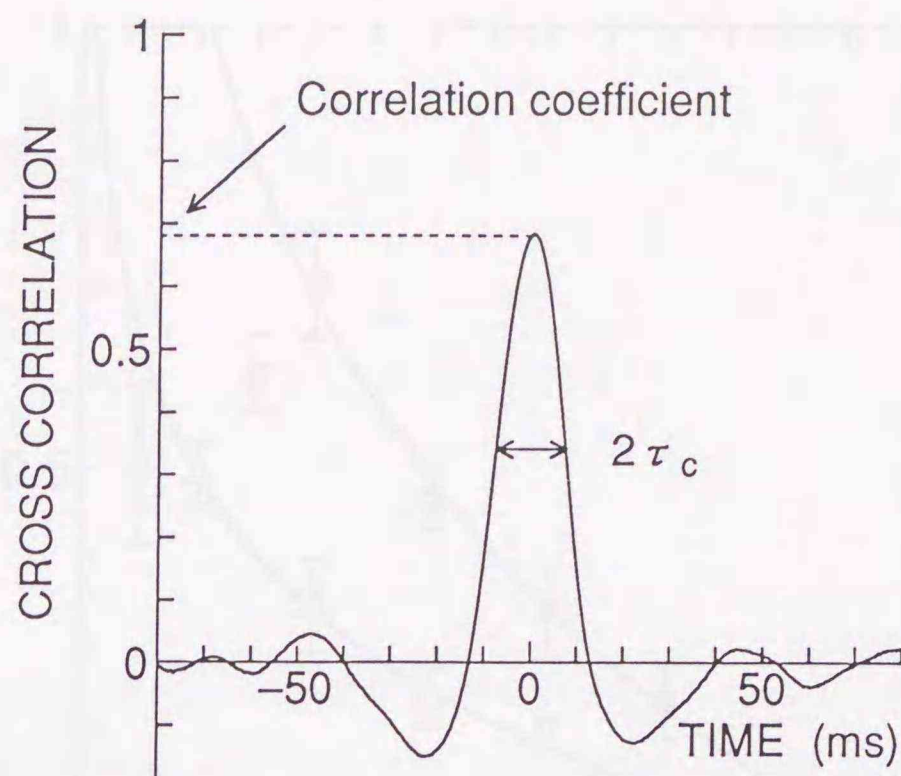


Figure 10.2. Cross-spectral correlation function of intensity fluctuations.

present setup is shown in Fig. 10.2. From the definition of Eq. (10.1), the peak value of the normalized cross-correlation function corresponds to the correlation coefficient of $\Delta i_1(t)$ and $\Delta i_2(t)$, and is given by $\gamma(0)$ for in-line optical configurations such as the one in Fig. 10.1. The correlation coefficient is determined by the number density of scatterers and the path length distribution of the scattered light propagating in the sample suspension. Hence, this quantity provides information about the scattering characteristics of the particles in the cell. The correlation time τ_c is measured from the half-width and half-maximum of the cross-correlation function given by Eq. (10.1) as well as those of the autocorrelation function of $\Delta i_1(t)$ or $\Delta i_2(t)$. The correlation time is intimately related to the motion of scattering particles. These two quantities were measured under the different conditions of particle suspensions.

10.3.1 Correlation Coefficient of Intensity Fluctuations

Figure 10.3 shows the correlation coefficient as a function of the volume concentration of particles obtained from the suspensions moving at 0.8 mm/s. The transport mean free path l^* of the samples was calculated with the wavelength 488 nm and found to be 19.1 mm for 0.01 % and 1.37 mm for 0.14 %. The optical depth, which is defined by L/l^* , L being the thickness of the sample, ranges from 0.0523 to 0.732, corresponding to the transition region from the single scattering regime to the mul-

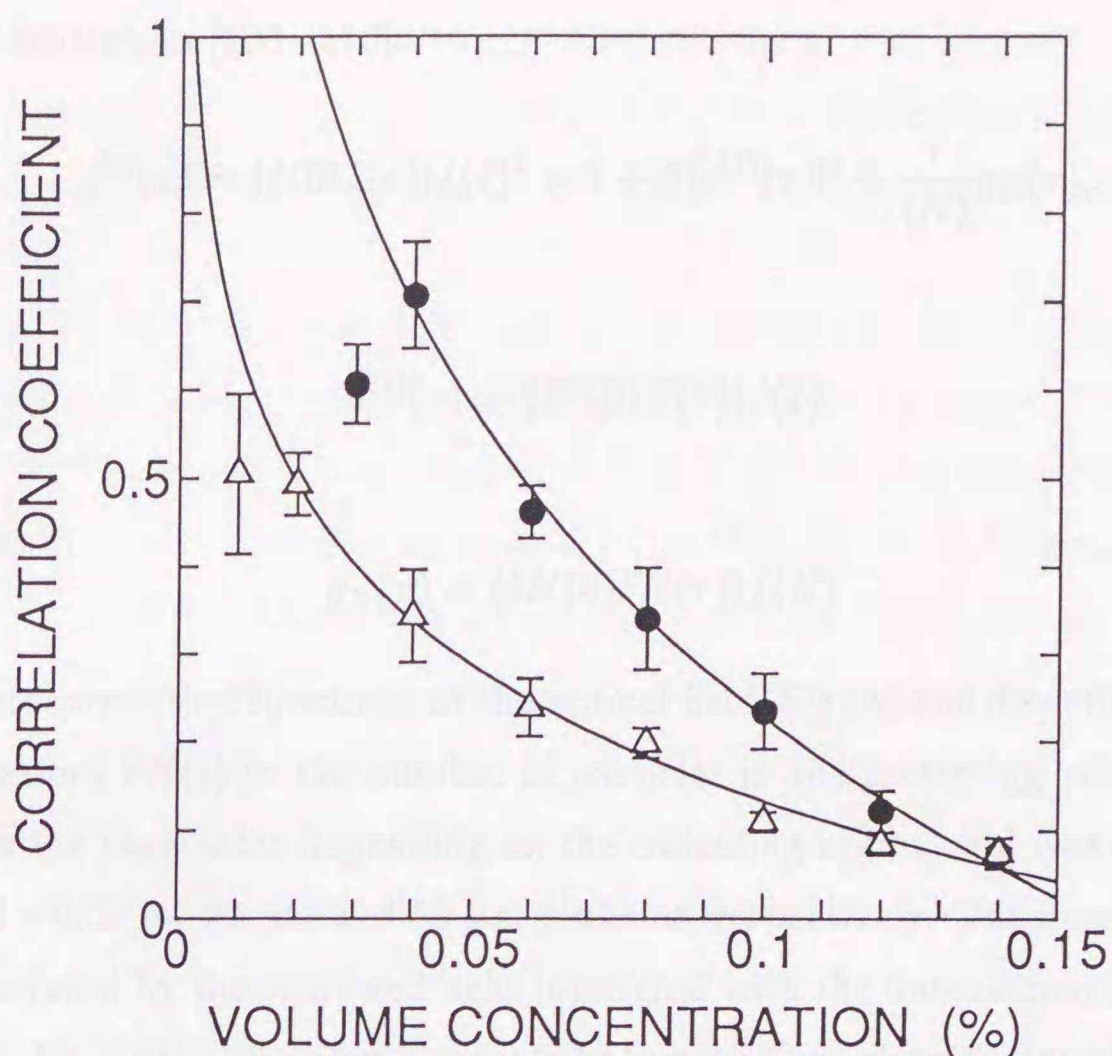


Figure 10.3. Correlation coefficient of intensity fluctuations as a function of the volume concentration of particles for the detecting pinholes with the diameters 20 μm ($\triangle$) and 50 μm ($\bullet$). The solid curves were computed using a least squares program.

multiple scattering one [107]. The number of speckles (or coherence area) falling on the detecting pinhole are ~ 12 and ~ 77 for diameters 20 μm and 50 μm of the pinhole, respectively. Thus, the speckle intensity detected is spatially integrated in both cases.

The best fits to the experimental data were obtained by using the fit function $C = a_0 + a_1 \ln V$, where C and V are the correlation coefficient and the volume fraction of particles, respectively. The fitted curves are drawn in Fig. 10.3 with the fitting parameters (a_0, a_1) of $(-1.41, -0.224)$ and $(-3.26, -0.505)$ for 20 μm and 50 μm pinholes, respectively. As shown in the figure, the correlation coefficient decreases with an increase of the particle concentration or, with an increase of the amount of multiple scattering. To understand this result, we consider two different types of the scattering process responsible for the fluctuations of intensity I detected by each of the photomultipliers. The normalized autocorrelation function of intensity

I may be written as [108–110]

$$g^{(2)}(\tau) = \langle I(0)I(\tau) \rangle / \langle I \rangle^2 = 1 + \beta |g^{(1)}(\tau)|^2 + \frac{1}{\langle N \rangle} g_N(\tau), \quad (10.2)$$

where

$$g^{(1)}(\tau) = \langle E^*(0)E(\tau) \rangle / \langle I \rangle \quad (10.3)$$

and

$$g_N(\tau) = \langle \delta N(0)\delta N(\tau) \rangle / \langle N \rangle \quad (10.4)$$

are the autocorrelation functions of the optical field $E(t)$ at the detecting plane and the fluctuations $\delta N(t)$ in the number of particles in the scattering volume, respectively. β is the parameter depending on the collecting optics, and was computed as ~ 0.07 and ~ 0.02 for $20 \mu\text{m}$ and $50 \mu\text{m}$ pinholes, respectively. The heterodyne component produced by the scattered light interfered with the unscattered one has been omitted in Eq. (10.2), because its contribution to the intensity correlation is less than that of the homodyne component (the second term of Eq. (10.2)) for volume concentrations of the samples we measured. Moreover, the cosine variations of the heterodyne term that would result from uniform motion of the particles detected at certain angles are averaged out owing to a wide angle of detection allowed by the present system.

The $g^{(1)}$ and g_N terms in Eq. (10.2) represent the intensity decorrelations due to the phase fluctuations of scattered light and the number fluctuations of particles, respectively. Since the number fluctuations are considered to be an incoherent phenomenon, no decorrelation is observed between the intensity fluctuations of light with different wavelengths. Thus, the g_N term contributes to the cross-correlation peak shown in Fig. 10.2. However, the phase fluctuations are caused by the coherent interference of light, and the cross-spectral intensity correlation only arises by the wavelets that have taken path lengths which are not much different from the path length of the unscattered beam. As the concentration of particles increases, the distribution of path lengths becomes broad because of the multiple scattering produced in the sample. As a result, the phase fluctuations due to the nearly-forward scattering light are completely obscured by those arising from the multiply scattered light which exhibits the spectral decorrelation. In addition, the component of

number fluctuations also decreases by $1/\langle N \rangle$, provided that the effective scattering volume measured by the cross-correlation remains the same (this is shown in the next section). These facts explain the change in the correlation coefficient shown in Fig. 10.3.

It is also found that larger detecting pinholes bring about an increase in the correlation coefficient. This is explained by the difference in the value of β in Eq. (10.2). A large detecting aperture suppresses the effect of phase fluctuations and makes the number fluctuations more dominant in the resulting intensity fluctuations. This is true under the condition that the average number $\langle N \rangle$ of particles in the measurement volume is not affected by the probing area on the sample. This condition is confirmed by the measurement of correlation time for the number fluctuations.

Discrepancies observed between the measured data and the fitting curves at concentrations of 0.01 % and 0.03 % are caused by the source and electronic noises superposed on the two signals independently of each other. The signal-to-noise ratio of the present system is more than 5 for the 0.1% volume fraction but reduces to ~ 2 for concentrations less than 0.03%. A decrease in the particle concentration brings about an increase in the level of the dc component of light detected, leading to a decrease in the ratio of the fluctuating component to the dc component. Thus, the reduction in the S/N ratio results since the strong dc background accompanies the noise of high levels.

10.3.2 Correlation Time of Intensity Fluctuations

We next examine the dependence of the cross-spectral correlation function of intensity fluctuations on the motion of particles in the suspension. Figure 10.4 shows the measured correlation functions for three different translating velocities of the sample. Comparison between curves B and C in the figure indicates that the correlation time becomes shorter as the velocity of the sample increases. Hence, rapid fluctuations of the intensity are obtained for fast translation of the particles. The interesting point to be mentioned is that no peak is observed in the cross-correlation function for the sample without the translating motion (curve A). This is explained by considering the scattering volume in the sample as well as the two different types of particle motion giving rise to the temporal intensity fluctuations.

In the weak multiple scattering regime, a large amount of light reaching the de-

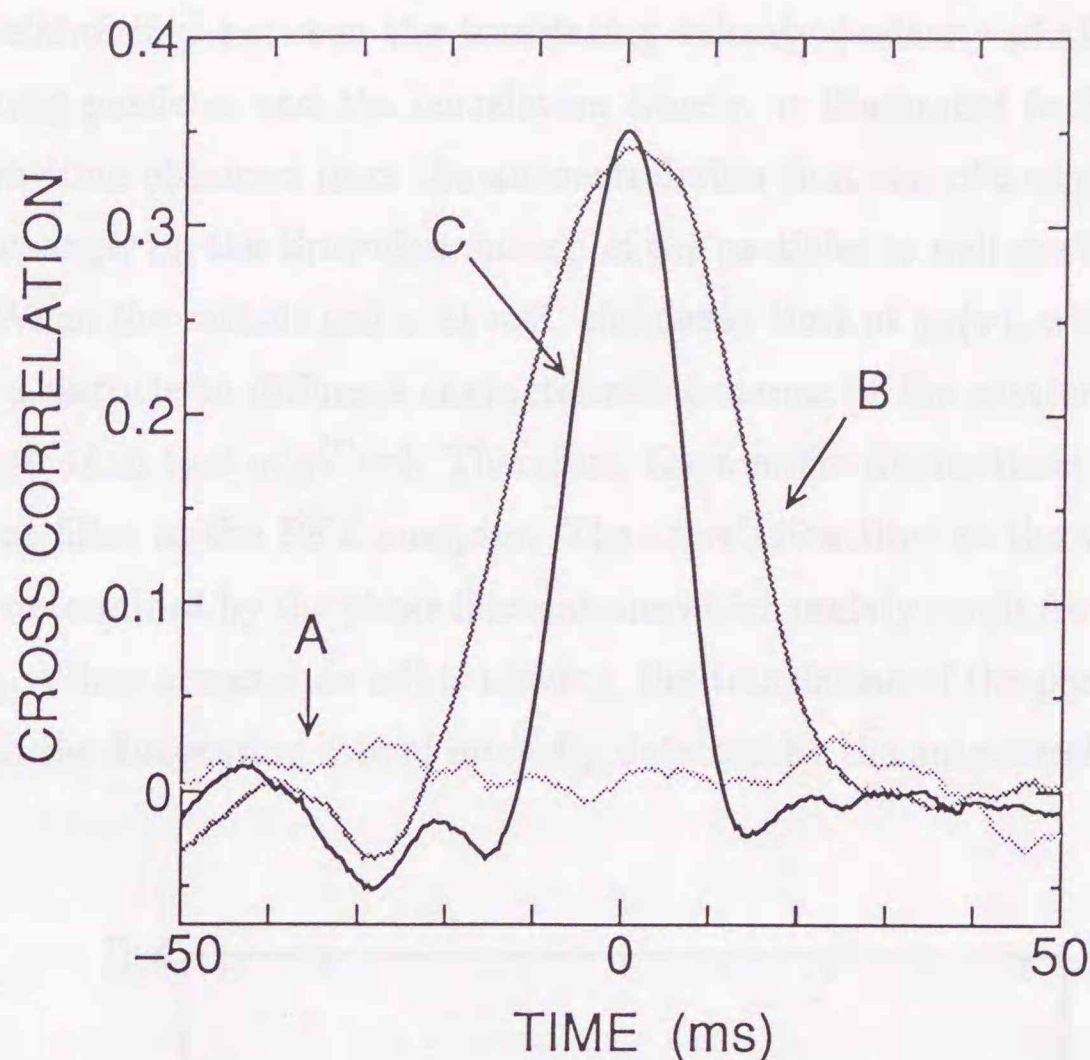


Figure 10.4. Cross-spectral correlation functions of intensity fluctuations for the translating velocities $v = 0$ (A), $v = 0.4$ mm/s (B) and $v = 0.8$ mm/s (C) of the sample cell. The concentration of the sample is 0.08 % and the diameter of the pinhole is 50 μm .

tector is unscattered or scattered by the particles in the forward direction (~ 50 % at the concentration 0.14 %). The path length of multiply scattered light in the sample becomes longer for higher orders of scattering and differs more from that of the unscattered and forward-scattered components. Then the phase fluctuations arising from the interference of light with those different path lengths completely decorrelate with each other by a small change of the wavelength. Consequently, the scattering volume measured by the cross-correlation is confined to a very thin narrow volume in the sample, although the scattered light diverges due to the multiple scattering. By referring to Eq. (10.2), the translating motion of the particles produces strong and rapid number fluctuations in the detected light because of a small cross section of the scattering volume and a small average number of the particles in it. On the contrary, the Brownian motion of the particles only gives rise to weak intensity fluctuations with very low frequencies, which are buried in the noise and cannot be detected.

The relationship between the translating velocity (velocity of the sample cell) of scattering particles and the correlation time τ_c is illustrated in Fig. 10.5. The correlation time obtained from the autocorrelation function of a single wavelength depends strongly on the Brownian motion of the particles as well as their translating motion. When the sample cell is at rest, the decay time of $g_N(\tau)$, which is the time taken by a particle to diffuse a characteristic distance of the scattering volume, is much longer than that of $g^{(1)}(\tau)$. Therefore, the number fluctuations are eliminated by a dc cut filter in the FFT analyzer. The correlation time at the velocity zero is thus only determined by the phase fluctuations which mainly result from the multiple scattering. When the sample cell is moving, the translation of the particles causes a decrease in the fluctuating rate of intensity detected by the autocorrelation method,

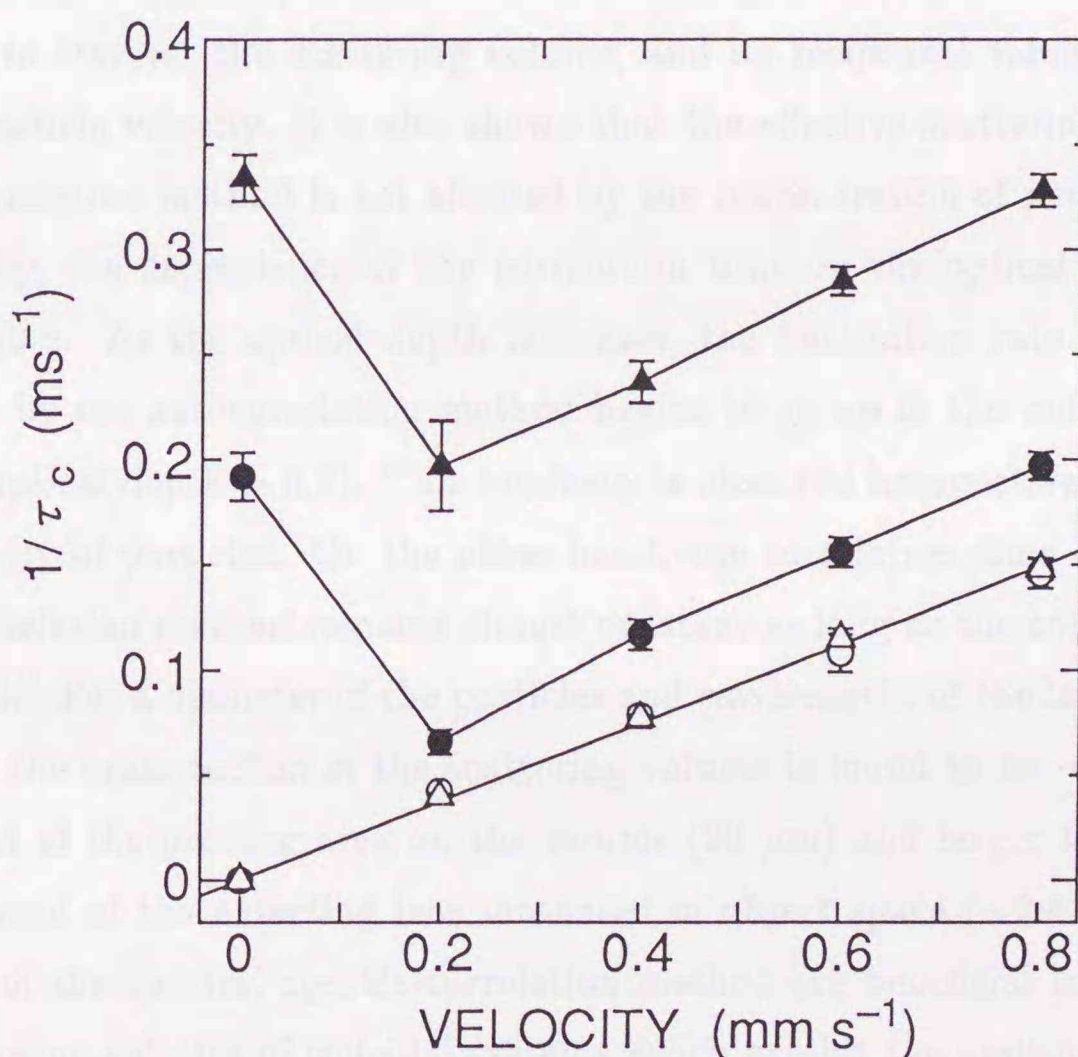


Figure 10.5. Reciprocal of the correlation time versus the translating velocity of particles. The correlation time is evaluated from the autocorrelation function of a single wavelength $\lambda = 488$ nm ($\bullet$, $\blacktriangle$) and the cross-correlation function of two wavelengths $\lambda_1 = 488$ nm and $\lambda_2 = 514.5$ nm ($\circ$, $\triangle$) for the volume concentrations of 0.02 % (circles) and 0.1 % (triangles). The diameter of the detecting pinhole is 50 μm .

even though their translating motion is superposed on the diffusive motion. This indicates that the number fluctuations are now in effect, the decay time of which is longer than that of the phase fluctuations for velocities of the translating cell used.

In contrast, the values of $1/\tau_c$ for the cross-correlation method is linearly proportional to the translating velocity of the particles, indicating that the effect of the Brownian motion is sufficiently suppressed. Moreover, its proportionality constant is the same for different concentrations of the particles. In the multiple scattering regime, an increase in the concentration of scatterers results in shorter correlation times. The result of the autocorrelation method demonstrates this tendency. However, this is only true when the diffusive motion of the scatterers is involved in the intensity fluctuations of detected light. As mentioned in the previous section, the number fluctuations mostly contribute to the spectral correlation of intensity fluctuations. Therefore, the correlation time is given by the time necessary for a particle to traverse the scattering volume, and its reciprocal value is proportional to the particle velocity. It is also shown that the effective scattering volume in the cross-correlation method is not affected by the concentration of particles.

Finally, the dependence of the correlation time on the optical depth is shown in Fig. 10.6. As the optical depth increases, the fluctuation rate of the intensity detected by the autocorrelation method begins to go up in the multiple scattering region (optical depth > 0.3). This tendency is observed irrespective of the translating velocity of particles. On the other hand, the correlation time obtained by the cross-correlation method remains almost constant as long as the correlation peak is detectable. For a diameter of the particles and wavelengths of the laser we used, the width of the cross section of the scattering volume is found to be $\sim 11 \mu\text{m}$, smaller than that of the probing area on the sample ($20 \mu\text{m}$) and larger than that of the point spread of the detecting lens measured in object space ($\sim 2.4 \mu\text{m}$). All these features of the spectral speckle-correlation method are beneficial to measurements of the flowing velocity of colloidal systems which exhibit the multiple scattering.

10.4 Conclusion

In this chapter, the spectral speckle-correlation method was applied to the suspensions of Brownian particles flowing uniformly at a constant velocity. An experimental investigation was performed, and qualitative explanations were given for the results obtained. The degree of the cross-spectral correlation of speckle intensity fluctuations decreases with an increase in the concentration of the particles. The spatial averaging by a detecting pinhole also changes the correlation coefficient of two-wavelength speckle patterns. The motion of colloidal particles was investigated by using the correlation times evaluated from the autocorrelation and cross-correlation functions. The Brownian motion of particles greatly affects the correlation time measured by the autocorrelation method, whereas the effect of that diffusive motion is sufficiently suppressed by the cross-spectral correlation method. As a result, the translating velocity of the particles can be measured in a wide range of the scattering regime including the multiple scattering.

Chapter 11

Concluding Remarks

The present dissertation has been concerned with the statistical properties of dynamic speckles produced by double and multiple scatterings of laser light from a series of random phase screens and colloidal suspensions. The studies were mainly focused on the relationship between the motion of objects and the autocorrelation and power spectrum of the resultant intensity fluctuations of speckles detected at a single point. The application to velocity measurements of the multiple scattering objects was also discussed.

In Chapter 2, the fundamental statistical properties of speckle patterns were reviewed and the basic theory for singly-scattered speckles was presented. The probability density functions of intensity and phase of the speckle fields derived using the concept of random walk in the complex plane were found, respectively, to follow a negative exponential law and a uniform law for strong diffusers. The spatio-temporal properties of time-varying speckle patterns are described by the space-time correlation function of intensity variations. The Fourier transform relationship was obtained between the autocorrelation function of spatial intensity variations at the observation plane and the macroscopic intensity distribution of light across the object surface for the free-space geometry, or across the lens pupil for an imaging geometry. The most important result concerning the temporal behavior of speckle patterns is that the fluctuating rate of speckle intensity and the translating speed of a speckle pattern are proportional to the moving velocity of the object giving rise to those speckles.

Chapters 3-6 were concerned with the statistical properties of speckled speckles and doubly-scattered speckles. In Chapter 3, a general expression was derived for the power spectral distribution (PSD) of the speckled-speckle signal obtained with

the optical-fiber probe which is used for the illumination of a moving diffuser as well as for the detection of scattered light. The spread of the PSD monotonically decreases with an increase in the core diameter of the detecting fiber, whereas the PSD once becomes narrower and then becomes broader with an increase of the core diameter of the illuminating fiber. The average size of speckles incident on the diffuser decreases as the illuminating core becomes larger, leading to a change in the motion of speckles at the detecting plane. The change of speckle motion together with the spatial-averaging effect of the detecting fiber cause the complex variation in shape and spread of the PSD. Although the PSD changes with the fiber probe used, the cutoff frequency of the speckle signal is proportional to the velocity of the object for a given probe.

In Chapter 4, the temporal properties of dynamic speckles produced in the diffraction field by a cascade of two moving diffusers were investigated theoretically and experimentally. The space-time correlation function of doubly-scattered speckle intensity was derived, and the correlation time was evaluated under various conditions of illumination and detection. It was shown that the reciprocal of the correlation time becomes proportional to the relative velocity of the two diffusers by illuminating the first diffuser with a suitable spot size of a Gaussian beam at its waist position and by detecting the scattered light from the second diffuser in the far field. However, the theoretical results revealed the difficulty in measuring the velocity of the second diffuser alone under illumination of a time-varying speckle pattern arising from the first diffuser.

In Chapter 5, an imaging geometry instead of the free-space geometry was employed in the double-diffuser system to examine experimentally the influence of the dynamic speckle illumination on the resultant intensity fluctuations arising at the image plane of the second diffuser. If the speckle pattern incident on the diffuse object is in boiling motion and if the light scattered from the object is detected through an imaging system with a large pupil size, the temporal variations of the illuminating light are suppressed, and the velocity of the object can be measured from the correlation time. In contrast, illumination with a translating speckle pattern and detection with a small-pupil imaging system allow the relative velocity of the diffusers to be measured.

The above results were confirmed in Chapter 6, where the space-time correlation

properties of speckle patterns formed in image space from the double diffusers were formulated, including the position of focus as a parameter. When the imaging system is focused on the first diffuser, the second diffuser is considered as a medium which disturbs the phase of light waves coming from the object. It was found that the influence of the second diffuser can be neglected if a small spot diameter of the illuminating beam and a large pupil diameter of the imaging system are employed. In this case, the velocity of the second diffuser is also measured just by focusing the imaging lens on the second diffuser. Under the opposite conditions, the velocity difference is proportional to the fluctuating rate of the speckle intensity, independently of the focal position.

Chapters 7 and 8 dealt with the statistical properties of multiply-scattered speckles from many diffusers. In Chapter 7, the space-time correlation function was calculated for the speckle patterns produced in the Fresnel diffraction region by a series of random-phase screens translating in their own planes. The theoretical results showed that even a small random variation in the velocities of the diffusers brings about a fast boiling motion of the speckle pattern. It follows that no velocity measurement technique by means of translating speckle patterns is applicable to such multiple-scattering objects. Even in the case of boiling speckles, the correlation time is a measure for the average velocity of the diffusers, providing that a regular form of velocity gradients exists among the diffusers. If there is no velocity gradient in the scattering volume, the correlation time is determined mainly by the extent of the statistical distribution of the velocity vectors, rather than the absolute value of the velocities.

The effects of imaging properties on multiply-scattered speckles were investigated in Chapter 8. It was found that the intensity fluctuations of image speckles are sensitive to the motion of a diffuser on which the imaging lens is focused, under the condition where the width of its point spread function in object space is small compared with that of the speckles illuminating the last diffuser. However, unlike the double-diffuser system, the other diffusers out of focus also affect the motion of resultant speckle patterns. One of the significant features of image speckles is that the average of the diffuser velocities which are distributed randomly is measured from the correlation time using an imaging system with a large pupil diameter. For a small diameter of the lens pupil, the correlation time is determined by the velocity

differences between the diffusers.

Chapters 9 and 10 were devoted to the experimental study on the multiple light scattering from particle suspensions. In the experiments, the suspensions that contain the particles undergoing the Brownian motion were moved uniformly at a constant velocity. In Chapter 9, the intensity of light multiply scattered from the colloidal suspensions was measured and the correlation time was evaluated for different parameters of the optical system and different properties of the sample suspensions. For multiple scattering media with finite thickness, the shape and size of the scattering volume formed in the media play an important role in determining the temporal fluctuations of scattered light. The effect of uniform motion of the particles becomes significant when the width of the scattering volume measured in the direction of the uniform motion decreases. The Brownian motion dominates the intensity fluctuations of scattered light at high concentrations of the particles because of the high orders of scattering which result in large scattering volumes and short correlation times.

In Chapter 10, the cross-correlation properties of dynamic speckles produced by different wavelengths of light were investigated. The degree of the cross-spectral correlation of speckle intensities decreases with an increase of the concentration of particles. This is due to the broadening of the light path distributions inside the sample suspensions. As shown in Chapter 9, the correlation time evaluated from the autocorrelation function of a single wavelength is greatly affected by the Brownian motion of particles. Experimental results showed that the effect of the diffusive motion is suppressed by measuring the correlation time evaluated from the correlated component of two intensity fluctuations with different wavelengths. This feature enables the translating velocity of the particles to be measured in the multiple scattering regime.

Dynamic light scattering phenomena from multiple scattering media have recently been the center of attention in the fields of optical metrology and medical applications as well as in the field of basic research. This dissertation provides with the basic relationship between the motion of diffuse objects and the statistical properties of resultant speckle fields. The phase-screen model combined with the transmission geometry allowed the doubly- and multiply-scattered speckles to be studied theoretically. Some of the theoretical results obtained by the model agreed

with the experimental results of colloidal suspensions. This fact indicates the validity of applying the diffuser model to discrete scatterers and the generality that the multiply-scattered light possesses.

Finally, it should be noted that the spatio-temporal properties of dynamic speckle patterns formed by the backscattered light from disordered media are of special importance in medical applications, such as blood flow monitoring. The theoretical analysis of multiple light scattering in the reflection geometry would be much more cumbersome than the analysis in the transmission one, but it will definitely bring some new insights into the formation of dynamic speckles from complex random media.

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