



HOKKAIDO UNIVERSITY

Title	Properties of Radiation from Partially Coherent Sources and Its Propagation through a Periodic Structure
Author(s)	Shirai, Tomohiro
Degree Grantor	北海道大学
Degree Name	博士(工学)
Dissertation Number	甲第3406号
Issue Date	1994-03-25
DOI	https://doi.org/10.11501/3076677
Doc URL	https://hdl.handle.net/2115/50018
Type	doctoral thesis
File Information	000000272710.pdf



Properties of Radiation from Partially Coherent Sources
and Its Propagation Through a Periodic Structure

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Properties of Radiation from Partially Coherent Sources and Its Propagation Through a Periodic Structure

Dissertation submitted to
Hokkaido University
for the doctor degree
(Engineering)

by

Tomohiro Shirai

1994

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CHAPTER 1

Introductory Remarks

1.1 Overview

Studies of radiation and propagation of light have long been the subject of great interest, primarily in conjunction with radiative energy transfer in radiometry. Traditional treatments of formulating energy transport in optical fields were made empirically on the basis of geometrical optics in which radiant energy is assumed to be propagated along a geometrical trajectory of light [1]. They have thus been developed, by neglecting the wave nature of light, around incoherent thermal sources and had no obvious connection with the Maxwell's theory of electromagnetic waves. However, there exists a distinct evidence of exhibiting that the wave nature of light plays a key role in radiometry. To observe it succinctly, let us consider a simple example [2].

Suppose that one compares the angular distributions of radiation from a conventional thermal source and from a typical laser. It is known that, under ordinary circumstances, the thermal source radiates in accordance with the Lambert's cosine law ([3], Sec. 4.8.1) in the sense that the radiant intensity of the source is proportional to the function of $\cos \theta$ in a direction making an angle θ with the normal axis to the radiating surface. In this case, the angular distribution of radiation is appreciably broad. On the other hand, the light from the laser is largely collected in a very narrow cone in the forward direction and, hence, it may be expedient to state that the laser has an apparent directionality in contrast to the thermal source. These phenomena regarding the energy flow from the source cannot be satisfactorily manifested within the framework of conventional radiometry with incoherent sources. As is widely recognized, the above sources of two types are different from each other in coherence which is the statistical properties of light waves: the thermal source is spatially almost incoherent whereas the laser source is spatially highly

coherent. Therefore, this example appears to reveal that there is an intimate relationship between the state of spatial coherence of a source and the behavior of energy flow from the source. In fact, considerable progress has recently been made to elucidate the foundations of radiometry from the standpoint of optical coherence theory.

The pioneering work to incorporate the concept of coherence into the conventional radiometry was conducted by Walther in 1968 [4]. Introducing a planar source of any state of coherence, he reexamined the radiometric quantities such as radiance and radiant intensity under the consideration of coherence properties of the source. A further refinement of his theory was conducted by Marchand and Wolf [5]. Since the publication of these first papers, a great deal of effort has been devoted to the investigation of radiometry with partially coherent sources and, as a result, the profound and significant connection between conventional radiometry and optical coherence theory has been up to now justified [6–18]. Excellent reviews in this area are given, though they are equally concerned only with an early stage of the development, by Baltes [19], Wolf [2], Baltes *et al.* [20] and Friberg [21]. It is worth emphasizing here that these researches have also led to some important developments in coherence theory itself, as well as in radiometry.

In a series of such investigations, there has been a particular interest in the interrelation between the coherence properties of a source and the angular distribution of the intensity from the source, i.e., the directionality of a partially coherent source. The first attempt for elucidating it was conducted by Wolf and Carter [22], who analyzed the directionality of a statistically homogeneous model source. Further extensions of this study were subsequently made on the basis of a homogeneous bounded model source [23] and a quasi-homogeneous model source [24]. These fundamental considerations have become a good motivation to yield an interesting investigation below.

It is believed, in general, that completely spatially coherent sources are required to produce a highly directional light beam, like a laser beam. This notion holds true in a sense, as was alluded previously. However, it was demonstrated by Collett and Wolf [25–27] that such is not always the case under certain conditions. They showed in their equivalence theorem that sources of different states of coherence may produce identical distributions of intensity throughout the far field and, eventually, arrived

at the conclusion that complete spatial coherence is not an indispensable condition for the attainment of high directionality. In this connection, it was also predicted theoretically that certain kinds of sources may give rise to exactly the same far-field intensity distribution as a fully coherent Gaussian laser. These noticeable predictions were verified experimentally soon afterwards [28–32]. Subsequent contributions in this field have been made in parallel with the introduction of model sources of new types for analysis [33–48] and a wide variety of partially coherent sources have been thoroughly explored with the close relation to their directionality.

Concurrently with the above studies regarding the directionality, the spatial coherence properties of the field produced by a source have also been examined for some selected types of the partially coherent sources [24, 32, 38, 39, 44]. Such information about the spatial coherence of the radiated field is required typically for imaging problems with partially coherent light [49] and inverse source problems [20, 50–52] and, thus, is worth investigating especially in considering practical applications of the partially coherent sources. A striking study with respect to this subject was made very recently by Shirai and Asakura [49, 53, 54] in association with a method of controlling the spatial coherence properties of the radiated field using a spatial filter.

To find further characteristics in radiation from partially coherent sources, Carter and Wolf [24] introduced a new concept of radiation efficiency. Radiation efficiency of the source is defined as a ratio of the total energy radiated by the source to the source-integrated intensity and acts as a measure for quantifying the feature of radiation from the source. Various types of planar sources [24, 55–58] and three-dimensional ones [59–61] have been examined on the basis of radiation efficiency. Consequently, it has become obvious that radiation efficiency is sensitively influenced by the behavior of light waves in the source region in comparison with the other measures, such as the directionality and the coherence properties of the radiated field. Hence, the results obtained in this analysis are expected to provide a somewhat new insight into the phenomena of radiation from partially coherent sources. Actually, the intrinsic difference in radiation between planar sources and three-dimensional ones, which have never been found before, has been clarified in both the study posed originally by Friberg [59] and its modification made by Shirai and Asakura [61].

In the last few years, renewed attention has been paid to the study of radiometry with partially coherent sources. This is due to the recent discovery of changes in the spectrum of radiation induced by the spatial coherence properties of the source (a brief review of this discovery was given by Wolf [62]). It has generally been taken for granted that the spectrum of light remains unchanged on propagation through the free space and, until recently, this belief has never been suspected. Nevertheless, Wolf predicted not long ago in his theoretical study [63] that the spectrum of radiation from a source may change on propagation, even in free space, depending not only on the source spectrum but also on the coherence properties of the source. This prediction was verified experimentally shortly after the publication of Wolf [64]. Nowadays this phenomenon of correlation-induced spectral changes is often referred to as the Wolf effect.

In the successive investigations on this effect, it has been shown theoretically [65–69] that the spectral changes caused in this way may be such as to yield the frequency shifts in the spectrum either towards lower frequencies (i.e., red shifts) or towards higher frequencies (i.e., blue shifts) and, this prediction has also been verified by some experiments [70–73]. An elegant experiment with acoustical sources, which also demonstrated the spectral shifts induced by the source correlation, have been performed by Bocko *et al.* [74]. In addition to the shifting properties of the spectrum, it is generally expected that changes in the spectrum may consist of, for example, narrowing and broadening a spectral line or generating several lines from a single line. Such changes have been examined theoretically [75, 76], and confirmed experimentally [77, 78]. As a continuation of these works, numerous publications both theoretically [79–98] and experimentally [99, 100] dealing with this phenomenon have appeared hitherto; they are concerned with the related phenomenon observed in a certain optical arrangement, such as an optical fiber [90], a periodic sequence of lenses [91], a linear time-invariant system [92] and others [93–96], and the application of this phenomenon to the inverse problems [97, 98]. Because of the well-known analogy that exists between the processes of radiation and scattering, it has been also demonstrated that similar phenomena may arise when a polychromatic light is scattered by a medium whose scattering potential fluctuates in space [101–103] (i.e., static light scattering) and also in time [104–107] (i.e., dynamic light scattering).

By taking into account the extensive investigations cited above, the discovery of the Wolf effect seems to give a conclusive impact on various areas of optics and other fields which make use of electromagnetic and other types of radiation. As a matter of fact, the significance of the Wolf effect is just becoming unsuspected in the fields of communications, metrology and information processing. In particular, these developments are likely to cause a new evolution for astronomy in relation to the cosmological redshifts associated with the expansion of a universe [65, 108].

On the other hand, independent of the above studies connected with the energy transfer in radiometry, there has also been an enormous interest in the studies of light propagation through a certain kind of medium or even in free space. Among them, the phenomena that occur on the propagation of light through a periodic structure have received particular attention not only in the fields of optics but also in various branches of physics.

From the historical viewpoint, the advent of lasers in 1960's has raised an interest in the study of light propagation through a periodic structure. Let us consider that laser resonators typically consist of two flat or curved mirrors set up facing each other and that the propagating light in the resonator may bounce back and forth between the two mirrors [109, 110]. Insofar as such actions of light are concerned, the laser resonator has many features in common with a periodic structure like an iterated sequence of apertures and that of lenses. Accordingly, laser resonator modes have been examined theoretically in accordance with integral equations which are derived by considering the laser resonator equivalent to the periodic structure of this type [109–113].

In a similar fashion, the spatial coherence properties of light propagating through the same structure have also been investigated [114–116]. It was shown in these studies that the light being initially spatially partially coherent, or even incoherent, may become completely spatially coherent under certain conditions by the process of light propagation through the structure. On the basis of such findings, Wolf succeeded in theoretically accounting for the physical origin of the spatial coherence of laser radiation immediately after the advent of lasers [114]. As a continuation of Wolf, Streifer [115] and Gori [116] dealt with some specific arrangements for analysis and examined the rapidity with which the spatial coherence grows up in the periodic structure. Very recently, a certain generalization of their treatment of

the problem has been carried out by Shirai and Asakura [117], who introduced a periodic sequence of uniform apertures separated by lenses and manifested the effect of successive diffractions by the hard-edged apertures on the behaviors in spatial coherence of the propagating light.

These works concerning both the modes and the spatial coherence of laser radiation were established under the quasi-monochromatic approximation ([3], Sec. 10.4) on the basis of the mode theory in the space-time domain. This method of analysis is entirely adequate because most of the lasers radiate the nearly monochromatic light. Unfortunately, however, information relevant to the spectral aspect of laser radiation cannot be extracted from such treatment due to the assumption of the quasi-monochromaticity. To circumvent this difficulty, the mode theory of laser resonators in the space-frequency domain have been established by Wolf and Agarwal [118] on the basis of the new theory of Wolf [119–121] dealing with the coherent-mode representation of the fields of any state of coherence in the space-frequency domain. The development of the mode theory in the space-frequency domain has led to an interesting investigation attempted by Shirai and Asakura [91] regarding the spectral aspect of the propagating light in a periodic sequence of lenses under the consideration of correlation-induced spectral changes (i.e., the Wolf effect).

Irrespective of laser resonators, the practical significance of the periodic structure have been reencountered, especially in recent years, in the fields of integrated optics [122] and microoptics [123]. In these fields, a wide variety of optical devices with a certain periodicity have been developed mostly in order to find a new principle that serves as a means of certain optical processing. The typical configuration may be a dielectric planar waveguide corrugated along its surface acting, for example, as a spectral filter, a mode coupler, a lens and others [124, 125]. Recent advances in optical information processing [126] have broadened the possibility of such devices and the area of their applications.

So far the spatial coherence properties of the propagating light through the devices with the periodicity, such as the preceding waveguide, have never been explored on the ground that the light confined is expected to be highly spatially coherent due to the tiny structure of such devices. However, the feature of the spatial coherence play a significant role when considering the multimode structure as a further generalization of the problem. Although the general structure of the

periodic devices employed for various processings is different in structure from the periodic structures associated with laser resonators, both structures are considered to be essentially equivalent at least in the sense that the propagating light suffers repeated perturbation from the structure. Therefore, it may be reasonable to state that the early works of Wolf [114], Streifer [115] and Gori [116] provide a certain insight into the behavior of light in the general structure of devices with the periodicity: the trends in spatial coherence of light in these different kinds of structures are expected to bear some similarities. Under such consideration, Shirai and Asakura have examined the spatial coherence of light propagating through a periodic system of apertures and lenses [117] and that of spatial filters and lenses [127], which are also regarded as the direct continuation of those early works.

Throughout this section, extensive studies relating to the radiation and propagation of partially coherent light have been surveyed. Some of them have directly motivated the investigations collected in this dissertation and others have been the good tutorials for proceeding the course of the current research activity. Finally, it is important to keep in mind that the core of the research topics cited in this section is to elucidate the role of intrinsic fluctuations of light waves (i.e., the coherence properties of light) on the phenomena of interest.

1.2 Organization

The main purpose of this dissertation is to investigate theoretically the features of radiation and propagation of light, by taking into account the stochastic behaviors of light wave, on the basis of classical theory of partial coherence. Particular interest in the whole analyses is on the phenomena resulting from the spatial coherence property of light, rather than the temporal aspect of coherence. The body of materials in this dissertation may be divided into two parts: the former part (Chapters 4–7) is concerned with radiation from various types of partially coherent sources and the latter one (Chapters 8–10) deals with propagation of the partially coherent light through a periodic structure. Ten chapters follow the Introductory Remarks.

As a preliminary consideration, Chapters 2 and 3 provide a brief review of some underlying concepts indispensable for the studies in the subsequent chapters.

Specifically, some basic elements of optical coherence theory are outlined in Chapter 2, while the fundamentals of conventional radiometry with incoherent sources and those of generalized radiometry with partially coherent sources are summarized in Chapter 3.

Chapter 4 deals with the radiation properties of spatially partially coherent planar sources from the standpoint of the radiated field properties. As a model source for analysis, a Gaussian-correlated Schell-model source occupying a finite planar region with the uniform intensity distribution is introduced under the quasi-monochromatic approximation. The intensity distribution and the spatial coherence property of the radiated field produced in both the Fresnel and the Fraunhofer regions of the source are examined for the sources with various states of spatial coherence. In addition to these analyses, a spatial filtering method over the source using a source filter is discussed with the view of controlling the spatial coherence of the radiated field. Here, the source filter is a kind of the spatial filter placed just behind the source and changes the intensity distribution across the source. Two kinds of the source filter are introduced as simple examples and their effects on the spatial coherence are demonstrated theoretically.

Chapters 5 and 6 are concerned with radiation efficiency of partially coherent sources which is defined as a ratio of the radiated energy flux to the source-integrated intensity. In Chapter 5, a general expression is derived for radiation efficiency of partially coherent planar sources with arbitrary intensity distributions. To illustrate some numerical examples, a model source whose spatial coherence is specified by properly arranged Bessel functions is newly proposed. By making use of such model sources, radiation efficiencies of the sources with various intensity distributions are investigated quantitatively as a function of both the coherence condition and the intensity distribution across the source. This chapter also includes the related analysis for a quasi-homogeneous source as a limiting case of such model sources. In this context, the quasi-homogeneous source is characterized by the properties that the spatial variation of the source intensity is sufficiently large in comparison with the effective width of the source coherence. It is to be noted here that an exact form of the source intensity is of no importance as far as the radiation properties from the quasi-homogeneous source are concerned. Owing to the adequate selection of the source model, a physical origin of variation in radiation efficiency arising from the

source characteristics is satisfactorily manifested in terms of the spatial frequency spectrum of the source associated with evanescent waves. On the other hand, in Chapter 6, the model source for analysis is assumed to be a homogeneous, partially coherent source localized in a finite three-dimensional region and its spatial coherence is specified by the same functional forms as those adopted in Chapter 5. After establishing a general expression for determining the radiation efficiency of three-dimensional sources, a number of representative curves are illustrated as a function of the coherence condition of the source. The diversity in radiation between the planar (i.e., two-dimensional) source and the three-dimensional one is also discussed by comparing the results obtained here with those obtained in the corresponding analysis of Chapter 5.

Chapter 7 is devoted to the study on the radiation from three-dimensional, anisotropic Gaussian Schell-model sources. The model source of this kind can represent, by an appropriate selection of the source parameters, a wide variety of partially coherent sources ranging from planar sources to spherically symmetric three-dimensional sources. Accordingly, the analysis in this chapter allows us to clarify the difference in radiation between planar sources and three-dimensional ones as a supplement of the comparison study conducted in Chapter 6. In analysis, expressions are derived for both the radiant intensity and the far-field spectrum produced by the source. On the basis of the spectrum expression, the shift of the spectral line on light propagation is explored with reference to correlation-induced spectral changes of the Wolf effect. As some numerical examples, the spectral shifts are illustrated in a polar diagram as a function of the observation angle for various source parameters.

In Chapter 8, the spatial coherence of light passing through a periodic sequence of lenses separated by apertures with uniform transmittances is studied on the basis of the mode theory in the space-time domain. The initial light launched into the periodic structure is assumed to be incoherent under the quasi-monochromatic approximation. The propagation law of the mutual intensity passing through the structure is formulated in terms of prolate spheroidal wave functions that are the normal modes of the periodic structure under consideration. On the basis of such formulation, trends in the behaviors of spatial coherence and intensity distribution of light resulting from the propagation through the structure are analyzed. In particular, the variation in spatial coherence is examined physically with relation to

the eigenvalues of the integral equation for determining the normal modes of the periodic structure.

Chapter 9 deals with the spatial coherence of light passing through a periodic sequence of confocal lenses separated by Gaussian amplitude filters on the basis of the mode theory in the space-frequency domain. The initial light incident on the periodic structure is assumed to be synthesized by a finite superposition of the Hermite-Gaussian modes that are the normal modes of the periodic structure in question and, consequently, it is partially coherent. The source of this kind may provide certain information to deduce some general properties of multimode lasers. Various characteristics of the source and the field emerging from the structure are examined, especially from the viewpoint of the homogeneity of the spatial coherence which serves to characterize the dependence of spatial coherence on positions across the field.

In Chapter 10, changes in the spectrum of light passing through the same periodic structure as that introduced in Chapter 9 are investigated theoretically by making use of the modal expansion of the field within the framework of coherence theory in the space-frequency domain. The initial light launched into the structure is assumed to be a polychromatic, spatially incoherent light with a single spectral line. By considering generally that the initially incoherent light becomes partially coherent after its propagation through the structure and by referring to the recent studies on correlation-induced spectral changes of the Wolf effect, the trends in spectral modifications due to the light propagation are satisfactorily manifested. The contribution of each mode in the total spectral changes is also discussed mainly from the Wolf effect. Representative results of the analysis are illustrated by some numerical examples.

Finally, the main results extracted from the whole analyses in this dissertation are summarized in Chapter 11 as Concluding Remarks.

CHAPTER 2

Basic Elements of Optical Coherence Theory

2.1 Introduction

Every optical field observed in nature has certain fluctuations that behave generally in a stochastic manner. The fluctuations are, however, not directly measurable by means of the existing detectors due to their enormous rapidity. Instead, the correlations between the fluctuations of light waves can be detected in some experiments involving interference effects and play an important role in many respects rather than the fluctuations themselves. Actually, it is known that, in general, a description in terms of the correlations, most commonly in terms of the second-order correlations, is entirely adequate for elucidating the practical phenomena encountered in various experiments.

Optical coherence theory gives basically a foundation for describing the behaviors of light waves with the aid of statistical methods, namely, on the basis of the correlations between the fluctuations of light waves. An attractive feature of this theory is the fact that it operates with measurable quantities only (i.e., correlation function and time-averaged intensity). This is in contrast with the elementary theory of optical waves, where the basic quantity is the wave amplitude that is not measurable because of its great rapidity.

The origins of the modern concept of optical coherence can be found in the late nineteenth and early twentieth centuries. Among them, the earliest investigations of coherence appear to be conducted by Verdet [128, 129] who demonstrated that an extended primary source considered to be incoherent could produce interference fringes under certain conditions. Subsequently, von Laue [130, 131] first introduced the quantitative measure of the correlations in the optical field, and Berek [132–135] studied the image formation in the microscope with relation to the concept of coherence. Further contributions were carried out by van Cittert [136, 137] and Zernike

[138], who defined independently the degree of coherence of light from an extended source in a manner that is directly connected with experiment. Shortly after their works, Hopkins [139, 140] established a sophisticated theory of image formation on the basis of coherence properties of light. Later researches by Blanc-Lapierre and Dumontet [141] and Wolf [142–144] further generalized the concept of coherence in order to deal with more complex situations and in order to formulate the theory on a rigorous basis. It is worth noting that the particular importance of these investigations was on the introduction of a precise measure of correlations between the fluctuations of light waves at two points, i.e., the second-order correlations. This second-order theory provides an unified framework for the treatment of most commonly occurring coherence phenomena, as was also mentioned previously. A fuller historical account for the above survey is given by Born and Wolf ([3], Chap. 10), Mandel and Wolf [145], and Peřina [146].

In this chapter, some basic elements of optical coherence theory are briefly summarized as a preliminary for the subsequent chapters where the main subjects are established in accordance with this theory. Owing to the extensive works by several authors, there exist a number of sources on the subject of optical coherence theory. In what follows, we refer frequently to the articles of Born and Wolf ([3], Chap. 10), Mandel and Wolf [145], Beran and Parrent [147], Peřina [146], Marathay [148], and Goodman [149]. Attention is focussed here and in the subsequent chapters on the second-order coherence phenomena without taking into account the coherence of order higher than 2.

2.2 Definitions of Several Coherence Functions

In order to characterize the field correlations in the theory of second-order coherence, three quantities are generally used: mutual coherence function, mutual intensity and cross-spectral density function. The mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ is defined by ([3], Chap. 10)

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \langle V(\mathbf{r}_1, t + \tau) V^*(\mathbf{r}_2, t) \rangle. \quad (2.1)$$

Here, $V(\mathbf{r}_i, t)$ describes the complex analytic signal representing the scalar field (assumed here to be stationary and ergodic) at a point specified by a position vector $\mathbf{r}_i$ at a time t , the angular brackets denote either time average or an ensemble average, and the asterisk denotes the complex conjugate. By the normalization of $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$, we have the complex degree of coherence $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ as

$$\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \frac{\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)}{\sqrt{I(\mathbf{r}_1)}\sqrt{I(\mathbf{r}_2)}} \quad (2.2)$$

where

$$I(\mathbf{r}_i) = \Gamma(\mathbf{r}_i, \mathbf{r}_i; 0), \quad (i = 1, 2) \quad (2.3)$$

is the averaged intensity at a position $\mathbf{r}_i$. It is shown by making use of the Schwartz inequality that the absolute value of $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ given by Eq.(2.2) fulfills the inequality

$$0 \leq |\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)| \leq 1, \quad (2.4)$$

where the limiting value of zero and unity represents, respectively, the complete incoherence and the complete coherence. The physical implications of this quantity are mentioned in Section 2.4. The classical theory of second-order coherence was established originally by means of the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ or the complex degree $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ of coherence and, consequently, $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ has been traditionally used in the analysis of all elementary phenomena arising from interference and diffraction. In particular, under the quasi-monochromatic approximation which implies that the spectral width $\Delta\nu$ of light is small compared to the mean frequency $\bar{\nu}$ (detailed discussion on this approximation is presented in Section 2.4.2), the mathematical formulation of these phenomena takes a simpler form by introducing the mutual intensity $J(\mathbf{r}_1, \mathbf{r}_2)$ defined by

$$J(\mathbf{r}_1, \mathbf{r}_2) = \Gamma(\mathbf{r}_1, \mathbf{r}_2; 0), \quad (2.5)$$

in place of $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$. In this case, the complex degree of coherence defined by Eq.(2.2) is also independent of the parameter τ and, hence, we use the abbreviated notation $\gamma(\mathbf{r}_1, \mathbf{r}_2)$ in place of $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ and call it the complex degree of spatial coherence. The mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ and the mutual intensity

$J(\mathbf{r}_1, \mathbf{r}_2)$ are the correlation functions in the space-time domain representing the field correlations.

On the other hand, the correlation function characterizing the field correlations in the space-frequency domain is the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ [150] defined by

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle \hat{V}(\mathbf{r}_1, \omega) \hat{V}^*(\mathbf{r}_2, \omega) \rangle. \quad (2.6)$$

In this equation, $\hat{V}(\mathbf{r}_i, \omega)$ is the Fourier component of the field vibrations $V(\mathbf{r}_i, t)$ given by

$$\hat{V}(\mathbf{r}_i, \omega) = \int_{-\infty}^{\infty} V(\mathbf{r}_i, t) e^{j\omega t} dt, \quad (i = 1, 2) \quad (2.7)$$

and the angular brackets denote an ensemble average. Here, Eq.(2.7) must be understood in terms of generalized functions [150] because $V(\mathbf{r}_i, t)$ is, in general, not Fourier transformable. In terms of $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$, we can define the complex degree of spectral coherence, $\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)$, (also known as the complex degree of spatial coherence) given by

$$\mu(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{W(\mathbf{r}_1, \mathbf{r}_2; \omega)}{\sqrt{I_\omega(\mathbf{r}_1, \omega)} \sqrt{I_\omega(\mathbf{r}_2, \omega)}}, \quad (2.8)$$

where

$$I_\omega(\mathbf{r}_i, \omega) = W(\mathbf{r}_i, \mathbf{r}_i; \omega) \quad (i = 1, 2) \quad (2.9)$$

represents the averaged intensity (also known as the spectral density) at a frequency ω at a position $\mathbf{r}_i$. In a fashion similar to Eq.(2.4), the absolute value of Eq.(2.8) is found to satisfy the following relation

$$0 \leq |\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)| \leq 1. \quad (2.10)$$

In this case, the value of Eq.(2.10) determines the feature of spatial coherence of light at one particular frequency ω : $|\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)| = 0$ and 1 imply the complete spatial incoherence and the complete spatial coherence, respectively.

From Eqs.(2.6) and (2.7), together with Eq.(2.1), the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ can be necessarily described in terms of the mutual coherence

function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ as follows:

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) e^{j\omega\tau} d\tau. \quad (2.11)$$

This relationship is an optical analogue of the well-known Wiener-Khintchine theorem for stationary random processes. If the Fourier integral of Eq.(2.11) is performed inversely, an expression of $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ in terms of $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ can also be obtained, i.e.,

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \frac{1}{2\pi} \int_0^{\infty} W(\mathbf{r}_1, \mathbf{r}_2; \omega) e^{-j\omega\tau} d\omega. \quad (2.12)$$

Equations (2.11) and (2.12) reveal that the correlation functions in two domains provide essentially the same information associated with the field correlations. Nevertheless, the cross-spectral density function seems to have been used extensively in recent years in various studies concerning the second-order coherence phenomena. In general, the phenomena described by the function in the space-time domain, rather than those in the space-frequency domain, seem to be well understood, primarily because the correlation function in the space-time domain is directly measurable through a simple experiment and, hence, it provides a good intuitive understanding of the phenomena [151]. On the contrary, the spectral treatment based on the cross-spectral density function can become a powerful tool, in contrast to the description in the space-time domain, for studying the phenomena having a strong frequency-dependence, such as the interaction of light with matters and the laser beam propagation through the atmosphere. To this end, a suitable choice of the correlation functions is always required by taking into consideration the purpose of the problem to be analyzed.

2.3 Some Elementary Concepts

Although the coherence properties of the field are uniquely determined in terms of the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ defined by Eq.(2.1) (or, the complex degree of coherence $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ defined by Eq.(2.2)), it is often convenient to divide coherence effects into two classifications by arranging appropriately the above function: temporal coherence and spatial coherence. This section is devoted to a brief

remark on these two effects.

Following the previous section, we consider that $V(\mathbf{r}, t)$ denotes the complex analytic signal representing the scalar optical field at a point P specified by a position vector $\mathbf{r}$ in space and at an instance t in time. Let $A(\mathbf{r}, t)$ be a complex envelop function associated with $V(\mathbf{r}, t)$. Then, from the elemental theory of Fourier decomposition, it is readily expected that the amplitude and phase of $A(\mathbf{r}, t)$ are varying at a rate of the order $1/\Delta\nu$ with $\Delta\nu$ being the effective spectral width of $V(\mathbf{r}, t)$. Hence, the value of $A(\mathbf{r}, t)$ remains relatively constant during the time interval τ in the case of $\tau \ll 1/\Delta\nu$. With the above consideration, suppose now the optical fields $V(\mathbf{r}, t)$ and $V(\mathbf{r}, t + \tau)$ located at the same position P , but at different instants t and $t + \tau$. When the condition $\tau \ll 1/\Delta\nu$ holds, $V(\mathbf{r}, t)$ and $V(\mathbf{r}, t + \tau)$ are said to be highly correlated in the sense that they fluctuate basically in the same manner so that the behavior of $V(\mathbf{r}, t)$ can be deduced from the knowledge of $V(\mathbf{r}, t + \tau)$ and *vice versa*. Conversely, when $\tau \gg 1/\Delta\nu$, $V(\mathbf{r}, t)$ and $V(\mathbf{r}, t + \tau)$ are no longer correlated in the sense that $V(\mathbf{r}, t)$ fluctuates independently of $V(\mathbf{r}, t + \tau)$. The term of the temporal coherence is employed to represent the ability of the light to interfere with a delayed version (but not spatially shifted) of itself. Accordingly, the above phenomena can be rephrased as follows: $V(\mathbf{r}, t)$ and $V(\mathbf{r}, t + \tau)$ are highly temporally coherent when $\tau \ll 1/\Delta\nu$ whereas they are almost temporally incoherent when $\tau \gg 1/\Delta\nu$. It follows from the above discussion that the temporal coherence effects stem strongly from the finiteness of the spectral width. From the mathematical viewpoint, they are described in terms of the time-dependent part $\Gamma(\mathbf{r}, \mathbf{r}; \tau)$ (or, $\gamma(\mathbf{r}, \mathbf{r}; \tau)$) of the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$.

We next consider the two analytic signals $V(\mathbf{r}_1, t)$ and $V(\mathbf{r}_2, t)$ detected at two space points P_1 and P_2 specified by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, with the zero relative time delay. When the two points coincide, i.e., $P_1 = P_2$, the two waves are necessarily perfectly correlated. However, a certain loss in the degree of correlation is generally expected when P_1 and P_2 are separated apart. This kind of behaviors in correlation relevant to the ability of the light to interfere with the spatially shifted (but not delayed) version of itself is elucidated in terms of the notion of spatial coherence. This effect is directly related to the source size and, mathematically, described in terms of the space-dependent part $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau = 0)$ (or, $\gamma(\mathbf{r}_1, \mathbf{r}_2)$) of the mutual coherence function, i.e., the mutual intensity $J(\mathbf{r}_1, \mathbf{r}_2)$.

As is widely known, these effects of partial coherence are readily demonstrated in some simple experiments of interference, e.g., the Michelson's interferometry for measuring the temporal coherence and the Young's experiment for measuring the spatial coherence. However, it is never possible to separate completely the above two effects since temporal and spatial coherences are mutually connected. Yet, it is worth stressing from the practical point of view that a partial separation is possible under certain conditions. Consider a narrow band light, for example. The light field in this case may be nearly temporally coherent to a large extent and, within this area, the effect of partial coherence may be ascribed to the spatial coherence rather than the temporal coherence. More generally, under some assumptions, it may be shown that the complex degree of coherence, $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$, may be expressed, at least to a good approximation, as the product of two functions, one of which represents the spatial coherence (i.e., $\gamma(\mathbf{r}_1, \mathbf{r}_2)$), the other the temporal coherence (i.e., $\gamma(\mathbf{r}, \mathbf{r}; \tau)$). Such fields are said to be cross-spectrally pure and have been investigated by Mandel [150, 152].

The emphasis is placed throughout this dissertation on the spatial coherence effects rather than the temporal coherence, as mentioned in Chapter 1. In this connection, the detailed discussion on the spatial coherence is given in the next section with relation to the Young's experiment of interference.

2.4 The Laws of Interference with Partially Coherent Light

2.4.1 General formulation

The physical implication of the correlations between the optical fluctuations is elucidated from some experiments of interference. In this section, the simplest experiment of interference known as a Young's experiment is outlined to show the concept of the spatial coherence which plays a significant role in the present dissertation. In particular, it is demonstrated that the complex degree of coherence, $\gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$, is observable in the Young's experiment of interference.

Figure 2.1 shows a schematic diagram for an optical arrangement of the Young's interference experiment. In this figure, Σ represents a spatially extended polychromatic source of light. The light field from the source falls on an opaque screen A

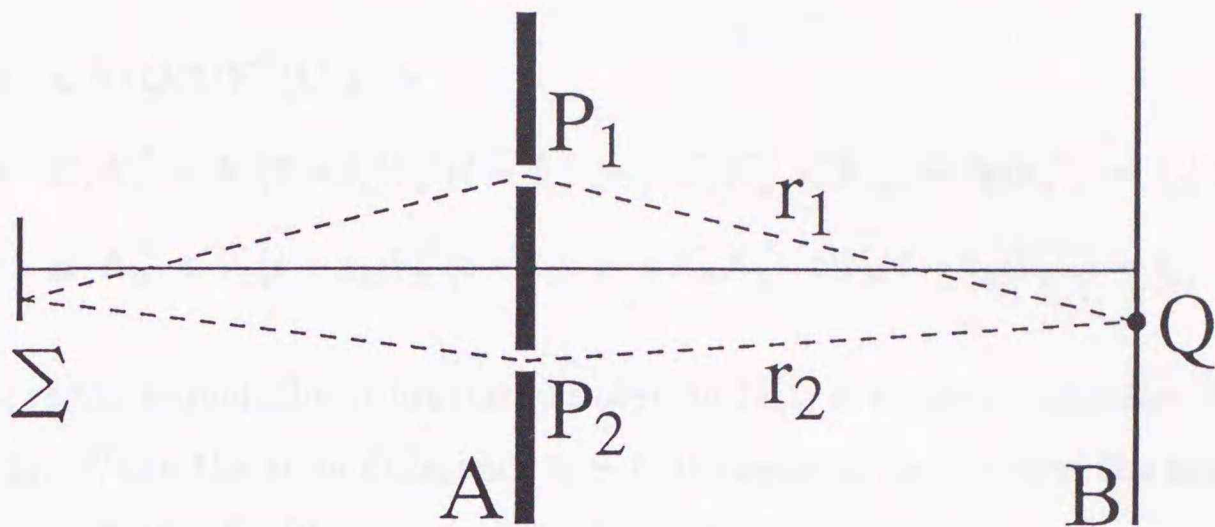


Figure 2.1. Young's interference experiment.

with two tiny pinholes at points P_1 and P_2 . An interference pattern, which is formed by the light passing through P_1 and P_2 , is observed at the second screen B located at some distance behind the screen A . In what follows, a mathematical description for an interference pattern is derived and some important aspects associated with it are discussed. To do so, the refractive index of the medium between the two screens is supposed to be unity. Moreover, we assume a stationary and ergodic optical field and neglect the polarization effect.

From the Maxwell's theory of electromagnetic waves, the light field $V(\mathbf{r}, t)$ is propagated in free space in accord with the scalar wave equation given by

$$\nabla^2 V(\mathbf{r}, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} V(\mathbf{r}, t). \quad (2.13)$$

Since this is a linear wave equation, the field at the point Q is represented as a superposition of the contribution from P_1 and P_2 , i.e.,

$$V(Q, t) = K_1 V(P_1, t - t_1) + K_2 V(P_2, t - t_2), \quad (2.14)$$

where $t_1 = r_1/c$ and $t_2 = r_2/c$ with c being the velocity of light in vacuum and K_1 and K_2 are purely imaginary constants depending on both the size of pinholes and the geometry of the arrangement. In this equation, the positions are specified by the symbols P_1 , P_2 and Q in place of the vector descriptions owing to the simplicity of the notation. By following the definition of the averaged intensity given by Eq.(2.3)

(and Eq.(2.1)), the resultant intensity at the point Q is written by

$$\begin{aligned} I(Q) &= \langle V(Q, t) V^*(Q, t) \rangle \\ &= K_1 K_1^* \langle V_1(t - t_1) V_1^*(t - t_1) \rangle + K_2 K_2^* \langle V_2(t - t_2) V_2^*(t - t_2) \rangle \\ &+ K_1 K_2^* \langle V_1(t - t_1) V_2^*(t - t_2) \rangle + K_2 K_1^* \langle V_2(t - t_2) V_1^*(t - t_1) \rangle. \end{aligned} \quad (2.15)$$

Here and in the sequel, the abbreviated notation $V_i(t)$ is employed instead of $V(P_i, t)$ for brevity. When the time difference $t_2 - t_1$ is replaced by τ under the assumption of stationary fields, Eq.(2.15) may be reduced to

$$I(Q) = |K_1|^2 I_1 + |K_2|^2 I_2 + 2 |K_1 K_2| \operatorname{Re}[\Gamma_{12}(\tau)], \quad (2.16)$$

where Re denotes the real part and

$$I_i = \langle V_i(t) V_i^*(t) \rangle, \quad (i = 1, 2) \quad (2.17)$$

$$\Gamma_{12}(\tau) = \langle V_1(t + \tau) V_2^*(t) \rangle \quad (2.18)$$

with $\Gamma_{12}(\tau)$ being the mutual coherence function defined by Eq.(2.1). Evidently, the term $|K_1|^2 I_1$ (or, $|K_2|^2 I_2$) in Eq.(2.16) is the observed intensity at the point Q when the light passes through only the pinhole at P_1 (or, P_2) under the condition that the other pinhole P_2 (or, P_1) is closed. Hence, the term $|K_1|^2 I_1$ (or, $|K_2|^2 I_2$) is replaced by the new symbol $I^{(1)}(Q)$ (or, $I^{(2)}(Q)$), i.e.,

$$I^{(i)}(Q) = |K_i|^2 I_i, \quad (i = 1, 2). \quad (2.19)$$

In addition, by introducing the complex degree of coherence, $\gamma_{12}(\tau)$, defined by Eq.(2.2), the final expression of the resultant intensity turns out to be written in the form

$$I(Q) = I^{(1)}(Q) + I^{(2)}(Q) + 2 \sqrt{I^{(1)}(Q) I^{(2)}(Q)} \operatorname{Re}[\gamma_{12}(\tau)]. \quad (2.20)$$

It is to be noted that, in the above equation, the third term represents an interference effect arising from the two pinholes. Moreover, the quantity $\operatorname{Re}[\gamma_{12}(\tau)]$ and hence $\operatorname{Re}[\Gamma_{12}(\tau)]$ can be basically determined from the experiment because the intensities $I(Q)$, $I^{(1)}(Q)$ and $I^{(2)}(Q)$ are readily observed using an ordinary detector.

To make the further progress to obtain the significant nature of the fringe pattern, let us recall that $\gamma_{12}(\tau)$ can always be written in the form

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| \exp [j\{\alpha_{12}(\tau) - 2\pi\bar{\nu}\tau\}], \quad (2.21)$$

where

$$\alpha_{12}(\tau) = 2\pi\bar{\nu}\tau + \arg[\gamma_{12}(\tau)] \quad (2.22)$$

and $\bar{\nu}$ stands for a mean frequency of the light. Substitution of Eqs.(2.21) and (2.22) into Eq.(2.20) produces

$$I(Q) = I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)} |\gamma_{12}(\tau)| \cos[\alpha_{12}(\tau) - \delta(\tau)]. \quad (2.23)$$

where the time delay τ and the phase difference δ are given by

$$\tau = \frac{r_2 - r_1}{c} = t_2 - t_1, \quad (2.24)$$

$$\delta = 2\pi\bar{\nu}\tau = \frac{2\pi}{\bar{\lambda}}(r_2 - r_1) \quad (2.25)$$

with $\bar{\lambda}$ being the mean wavelength. If $|\gamma_{12}(\tau)|$ has the extreme value of unity, the intensity observed at Q is equivalent to that obtained when the fields across the two pinholes are strictly monochromatic with the wavelength $\bar{\lambda}$ and have the relative phase difference $\alpha_{12}(\tau)$. In this case, the fields at P_1 and P_2 are said to be coherent. On the other hand, if $\gamma_{12}(\tau)$ has the other extreme value of zero, the last term of Eq.(2.23) disappears. In this condition, any interference effects do not exist and thus the fields are said to be incoherent. If $|\gamma_{12}(\tau)|$ takes the value fulfilling the relation $0 < |\gamma_{12}(\tau)| < 1$, the fields are said to be partially coherent. Rearranging Eq.(2.23), we find

$$\begin{aligned} I(Q) &= |\gamma_{12}(\tau)| \left\{ I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)} \cos[\alpha_{12}(\tau) - \delta(\tau)] \right\} \\ &+ \{1 - |\gamma_{12}(\tau)|\} \{I^{(1)}(Q) + I^{(2)}(Q)\}. \end{aligned} \quad (2.26)$$

It is worth noting from the above equation that the resultant intensity observed at Q may be regarded as a mixture of coherent and incoherent superpositions of the

light from the two pinholes. More precisely, the terms in the first line may originate from the coherent superposition of two intensities $|\gamma_{12}(\tau)|I^{(1)}(Q)$ and $|\gamma_{12}(\tau)|I^{(2)}(Q)$ with the relative phase difference $\alpha_{12}(\tau) - \delta(\tau)$, while those in the second line may originate from the incoherent superposition of two intensities $[1 - |\gamma_{12}(\tau)|]I^{(1)}(Q)$ and $[1 - |\gamma_{12}(\tau)|]I^{(2)}(Q)$.

Let us now consider the visibility V of Michelson for quantifying the fringe quality, which is defined by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, \quad (2.27)$$

where I_{max} and I_{min} are the maximum and the adjacent minimum intensity of the fringe, respectively. In the vicinity of zero pathlength difference (i.e., $r_2 - r_1 = 0$ and hence $\alpha_{12}(\tau) = \delta(\tau) = 0$), it follows from Eq.(2.23) that the maximum and minimum intensities become

$$I_{max} = I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}|\gamma_{12}(\tau)|, \quad (2.28)$$

$$I_{min} = I^{(1)}(Q) + I^{(2)}(Q) - 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}|\gamma_{12}(\tau)|. \quad (2.29)$$

Therefore, substitution of Eqs.(2.28) and (2.29) into Eq.(2.27) yields

$$V = \frac{2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}}{I^{(1)}(Q) + I^{(2)}(Q)}|\gamma_{12}(\tau)|. \quad (2.30)$$

As a special case, we obtain for the fringe visibility the expression

$$V = |\gamma_{12}(\tau)|, \quad (2.31)$$

provided that $I^{(1)}(Q) = I^{(2)}(Q)$. This formula expresses that the degree of coherence $|\gamma_{12}(\tau)|$ is equal to the visibility V of the fringes and is directly measurable. This is the most important consequence of this section. When the two fields at P_1 and P_2 are mutually incoherent (i.e., $|\gamma_{12}(\tau)| = 0$), the fringes vanish (i.e., $V = 0$). On the other hand, the fringes become clear (i.e., $V = 1$) when the two fields are mutually coherent (i.e., $|\gamma_{12}(\tau)| = 1$). For an intermediate condition of the coherence (i.e., $0 < |\gamma_{12}(\tau)| < 1$), the fringes appear to be somewhat blurred (i.e., $0 < V < 1$).

2.4.2 Quasi-monochromatic approximation

In the formulation of the Young's interference experiment discussed in the previous section, the light field employed for analysis is polychromatic. In the subsequent discussion, the preceding theory of interference is reduced to a simple form by introducing the assumption of quasi-monochromaticity.

From the mathematical point of view, the light field is said to be quasi-monochromatic when its effective spectral width $\Delta\nu$ is much smaller than its mean frequency $\bar{\nu}$, i.e.,

$$\Delta\nu \ll \bar{\nu}. \quad (2.32)$$

This is the definition of the quasi-monochromatic light. In addition to this constraint, we further assume that the coherence time τ_c of light (which takes the value being the same order as $1/\Delta\nu$, see Section 2.3) is much greater than the time delay τ associated with the maximum pathlength difference encountered in the optical arrangement of Fig.2.1, i.e.,

$$\tau = \frac{|r_2 - r_1|}{c} \ll \tau_c. \quad (2.33)$$

Customarily, such light is said to satisfy the quasi-monochromatic conditions. In this case, the functions $\Gamma_{12}(\tau)$ and $\gamma_{12}(\tau)$ can be rewritten in the form

$$\Gamma_{12}(\tau) \simeq J_{12} \exp[-j2\pi\bar{\nu}\tau], \quad (2.34)$$

$$\gamma_{12}(\tau) \simeq \gamma_{12} \exp[-j2\pi\bar{\nu}\tau], \quad (2.35)$$

where $J_{12} = J(\mathbf{r}_1, \mathbf{r}_2)$ is the mutual intensity defined by Eq.(2.5) and $\gamma_{12} = \gamma_{12}(0) = \gamma(\mathbf{r}_1, \mathbf{r}_2; 0)$ is the complex degree of spatial coherence. As a result, by substituting Eq.(2.35) into Eqs.(2.30) and (2.31), the visibility V of the interference pattern may be written in the simple form

$$V = \begin{cases} \frac{2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}}{I^{(1)}(Q) + I^{(2)}(Q)} |\gamma_{12}| & \text{for } I^{(1)}(Q) \neq I^{(2)}(Q) \\ |\gamma_{12}| & \text{for } I^{(1)}(Q) = I^{(2)}(Q) \end{cases} \quad (2.36)$$

with V being independent of τ . Equation (2.36) reveals that the visibility of the interference pattern is uniquely determined in terms of the spatial coherence $|\gamma_{12}|$

instead of the space-time coherence $|\gamma_{12}(\tau)|$ and, thus, it takes a constant value across the observation region. For this reason, the temporal coherence effects may be neglected to a good approximation under these conditions, as was mentioned in Section 2.3.

2.5 Propagation Laws of Second-Order Coherence

2.5.1 Propagation of mutual coherence function

By considering that the detailed structures of optical waves undergo changes as the waves propagate through space, it is inferred that the correlations of the waves also undergo changes on propagation. As will be seen in the following, this is true. Moreover, it is important to keep in mind that this fact regarding the propagation of the correlations is the underlying physics for the main subject to be conducted in the subsequent chapters. In this section, we focus on a derivation of the propagation law for the mutual coherence function.

Figure 2.2 illustrates a schematic diagram for describing the propagation law of second-order coherence. In the figure, a fictitious surface A is illuminated by a polychromatic light from an extended source. Let us consider that the second fictitious surface B is illuminated by the light across the surface A either directly or through an optical system. We now show that the mutual coherence function for all pairs of points on the surface B can be determined in terms of that on the surface A .

To do so, the propagation equations for the mutual coherence function $\Gamma_{12}(\tau)$ are first derived. Taking the laplacian operator ∇_1^2 with respect to the position $\mathbf{r}_1$ directly to the definition of $\Gamma_{12}(\tau)$ given by Eq.(2.1), we obtain

$$\nabla_1^2 \Gamma_{12}(\tau) = \langle \nabla_1^2 V_1(t + \tau) V_2^*(t) \rangle, \quad (2.37)$$

where the operator ∇_1^2 is given by

$$\nabla_1^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2} \quad (2.38)$$

with $\mathbf{r}_1 = (x_1, y_1, z_1)$. From the wave equation of Eq.(2.13), the light field $V_1(t + \tau)$ turns out to obey the following equation:

$$\nabla_1^2 V_1(t + \tau) = \frac{1}{c^2} \frac{\partial^2 V_1(t + \tau)}{\partial (t + \tau)^2}. \quad (2.39)$$

By substituting Eq.(2.39) into Eq.(2.37) and by using the fundamental relation

$$\frac{\partial^2 V_1(t + \tau)}{\partial (t + \tau)^2} = \frac{\partial^2 V_1(t + \tau)}{\partial \tau^2}, \quad (2.40)$$

the governing equation for $\Gamma_{12}(\tau)$ becomes

$$\begin{aligned} \nabla_1^2 \Gamma_{12}(\tau) &= \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \langle V_1(t + \tau) V_2^*(t) \rangle \\ &= \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \Gamma_{12}(\tau). \end{aligned} \quad (2.41)$$

In a similar fashion, the second equation for $\Gamma_{12}(\tau)$ is readily obtained and becomes

$$\nabla_2^2 \Gamma_{12}(\tau) = \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \Gamma_{12}(\tau), \quad (2.42)$$

where

$$\nabla_2^2 = \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2} \quad (2.43)$$

with $\mathbf{r}_2 = (x_2, y_2, z_2)$.

These two wave equations (i.e., Eqs.(2.41) and (2.42)) governing the propagation of a mutual coherence function can be readily solved by making use of standard techniques, such as the Green's function method. Let us consider that P_1 and P_2 are two typical points on the surface A , and that Q_1 and Q_2 are two points on the surface B , as illustrated in Fig.2.2. In this figure, the distance between P_1 and Q_1 and that between P_2 and Q_2 are given by R_1 and R_2 , respectively. Furthermore, θ_1 and θ_2 are the angles that the lines P_1Q_1 and P_2Q_2 make with the normals to the surface A at P_1 and P_2 , respectively. With the above notation, the solution may be written, if the surface A is plane, in the form

$$\Gamma(Q_1, Q_2; \tau) = \frac{1}{(2\pi)^2} \iint_A \iint_A \frac{\cos \theta_1 \cos \theta_2}{R_1^2 R_2^2} \mathcal{D} \Gamma \left(P_1, P_2; \tau - \frac{R_1 + R_2}{c} \right) d^2 P_1 d^2 P_2 \quad (2.44)$$

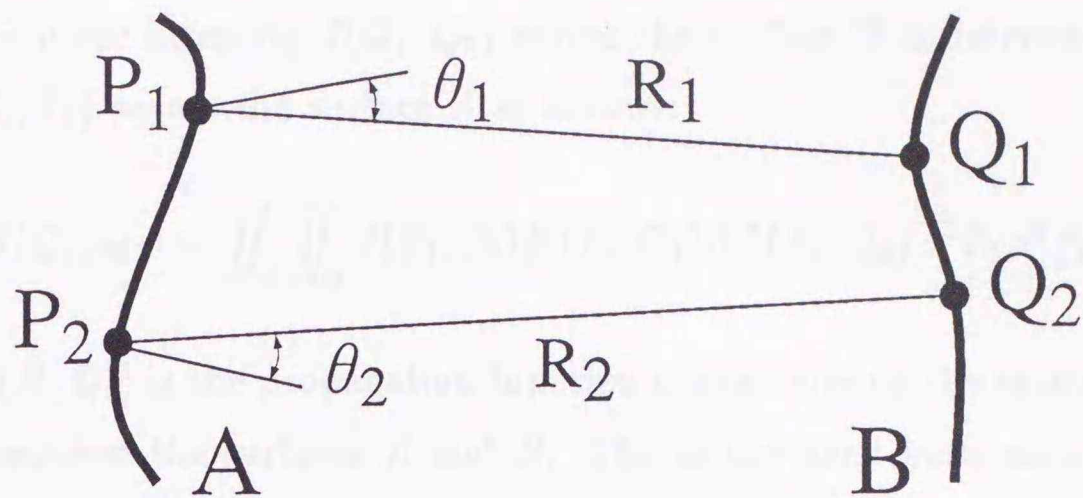


Figure 2.2. Geometry for propagation of second-order coherence.

where $\mathcal{D}$ denotes the differential operator given by

$$\mathcal{D} = 1 + \frac{R_1 - R_2}{c} \frac{\partial}{\partial \tau} - \frac{R_1 R_2}{c^2} \frac{\partial^2}{\partial \tau^2}. \quad (2.45)$$

In deriving Eq.(2.44) it was assumed that the field obeys the Sommerfeld radiation condition at infinity in the half-space containing the points Q_1 and Q_2 .

Equation (2.44) is the general solution for the mutual coherence function in a field produced by a plane polychromatic partially coherent source. It describes explicitly the propagation of the mutual coherence function in the form that the mutual coherence function at the surface B is determined in terms of the mutual coherence function at the surface A .

2.5.2 Propagation of mutual intensity

We now introduce the quasi-monochromatic conditions discussed in Section 2.4.2. In this case, on substituting Eq.(2.34) into Eqs.(2.41) and (2.42), we obtain the propagation equations for the mutual intensity J_{12} , i.e.,

$$\begin{cases} \nabla_1^2 J_{12} + \bar{k} J_{12} = 0 \\ \nabla_2^2 J_{12} + \bar{k} J_{12} = 0, \end{cases} \quad (2.46)$$

where $\bar{k} = 2\pi/\bar{\lambda}$ is the mean wavenumber and ∇_1^2 and ∇_2^2 are, respectively, given by Eqs.(2.38) and (2.43). As is widely known, these are Helmholtz equations. Accordingly, by making use of standard mathematical techniques for solving them (e.g.,

the Green's function method, as well as the previous section), it is readily found that the mutual intensity $J(Q_1, Q_2)$ across the surface B is determined in terms of that $J(P_1, P_2)$ across the surface A as follows:

$$J(Q_1, Q_2) = \iint_A \iint_A J(P_1, P_2) K(P_1, Q_1) K^*(P_2, Q_2) d^2 P_1 d^2 P_2, \quad (2.47)$$

where $K(P_i, Q_i)$ is the propagation function characterizing the optical system to be located between the surfaces A and B . This is the most basic expression required in this section for describing the propagation of the mutual intensity. The function $K(P, Q)$ can be, in general, readily obtained in deriving Eq.(2.47) once the optical system located between the two surfaces of A and B is given.

2.5.3 Propagation of cross-spectral density function

To obtain the propagation equation for the cross-spectral density function, we begin by recalling the relationship between the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ and the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$. According to Section 2.2, it is given by Eq.(2.12), i.e.,

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau) = \frac{1}{2\pi} \int_0^\infty W(\mathbf{r}_1, \mathbf{r}_2; \omega) e^{-j\omega\tau} d\omega. \quad (2.48)$$

Knowing the propagation equations for $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ from the discussion in Section 2.5.1, we can apply them to Eq.(2.48) and deduce the corresponding equations for the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$.

Substituting Eq.(2.48) into Eqs.(2.41) and (2.42), and interchanging the order of differentiation and integration, we obtain the expressions

$$\int_0^\infty \left[\nabla_1^2 - \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \right] W(\mathbf{r}_1, \mathbf{r}_2; \omega) e^{-j\omega\tau} d\omega = 0 \quad (2.49)$$

and

$$\int_0^\infty \left[\nabla_2^2 - \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \right] W(\mathbf{r}_1, \mathbf{r}_2; \omega) e^{-j\omega\tau} d\omega = 0 \quad (2.50)$$

where ∇_1^2 and ∇_2^2 are given by Eqs.(2.38) and (2.43), respectively. By applying the τ derivatives to the exponentials, Eqs.(2.49) and (2.50) become a set of Helmholtz

equations written in the forms

$$\begin{cases} \nabla_1^2 W(\mathbf{r}_1, \mathbf{r}_2; \omega) + k W(\mathbf{r}_1, \mathbf{r}_2; \omega) = 0 \\ \nabla_2^2 W(\mathbf{r}_1, \mathbf{r}_2; \omega) + k W(\mathbf{r}_1, \mathbf{r}_2; \omega) = 0, \end{cases} \quad (2.51)$$

where $k = 2\pi/\lambda = \omega/c$ is the wavenumber associated with a particular frequency ω , c being the speed of light in vacuum.

Equation (2.51) is essentially equivalent to Eq.(2.46) which represents the propagation law for the mutual intensity. The only difference is the expressions of the wavenumber in these equations: the mean wavenumber $\bar{k}$ is employed in Eq.(2.46) while one-particular value k of the wavenumber is incorporated in Eq.(2.51). Accordingly, the propagation law for the mutual intensity given by Eq.(2.47) can also be regarded as that for the cross-spectral density function, provided that the mean wavenumber $\bar{k}$ implicitly contained in Eq.(2.47) is replaced by one-particular value k of the wavenumber. It should be noted, however, that the physical meaning of the formulations based on these two functions (i.e., mutual intensity and cross-spectral density) are different from each other, even though each mathematical description bears a high analogy.

2.5.4 Van Cittert-Zernike theorem

One of the most important theorems in modern optics can be derived from the propagation law of the mutual intensity discussed in Section 2.5.2. This is the van Cittert-Zernike theorem which describes the one-to-one correspondence between the spatial coherence properties of light generated from an incoherent source and the intensity distribution across the source. The present section is devoted to a brief discussion on this theorem.

Let us consider that a planar quasi-monochromatic source Σ of a finite extent illuminates the plane A located parallel to the source in arbitrary free space, as illustrated in Fig.2.3. The source is assumed to consist of an extended collection of independent radiators. As usual, this type of the source is called an incoherent source. In this case, by recalling its definition, the mutual intensity of the field across the source may be approximately written in the form ([147], Sec. 4.4)

$$J(P_1, P_2) = I_0(P_1) \delta(P_1 - P_2), \quad (2.52)$$

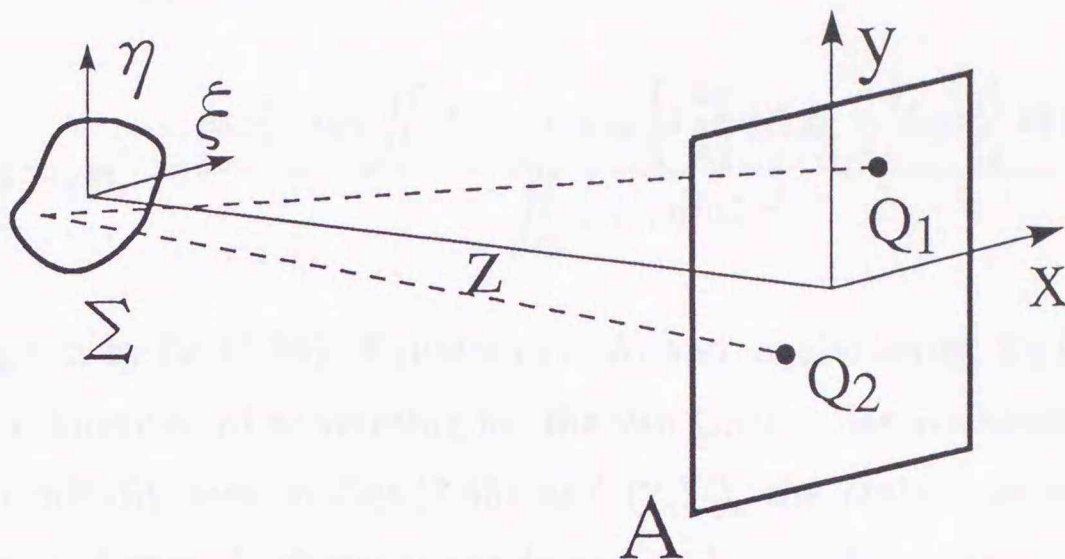


Figure 2.3. Illustrating the van Cittert-Zernike theorem.

where I_0 and $\delta(\cdot)$ imply the intensity distribution across the source and a two-dimensional Dirac delta function, respectively. When the plane A is located in the Fresnel diffraction field ([153], Sec. 4.1), the propagation function $K(P, Q)$ in Eq.(2.47) is known to become

$$K(P, Q) = \frac{1}{j\lambda z} \exp \left[j \frac{\pi}{\lambda z} |P - Q|^2 \right], \quad (2.53)$$

where z is the distance between the source and the plane A . Substitution of Eqs.(2.52) and (2.53) into Eq.(2.47) yields the required expression for the mutual intensity $J(Q_1, Q_2)$ across the observation plane.

To obtain an explicit expression for $J(Q_1, Q_2)$, we introduce the Cartesian coordinates given by

$$P = (\xi_i, \eta_i) \quad \text{and} \quad Q_i = (x_i, y_i), \quad (i = 1, 2) \quad (2.54)$$

and the new variables $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$. Then, we obtain for $J(Q_1, Q_2)$ the expression

$$J(x_1, y_1; x_2, y_2) = \frac{\exp[-j\psi]}{(\lambda z)^2} \iint_{\Sigma} I_0(\xi, \eta) \exp \left[j \frac{2\pi}{\lambda z} (\Delta x \xi + \Delta y \eta) \right] d\xi d\eta, \quad (2.55)$$

where ψ is given by

$$\psi = \frac{\pi}{\lambda z} \left[(x_2^2 + y_2^2) - (x_1^2 + y_1^2) \right]. \quad (2.56)$$

Normalization of Eq.(2.55) produces the complex degree of spatial coherence given by

$$\gamma(x_1, y_1; x_2, y_2) = \frac{\exp[-j\psi] \iint_{\Sigma} I_0(\xi, \eta) \exp \left[j \frac{2\pi}{\lambda z} (\Delta x \xi + \Delta y \eta) \right] d\xi d\eta}{\iint_{\Sigma} I_0(\xi, \eta) d\xi d\eta}, \quad (2.57)$$

where ψ is given by Eq.(2.56). Equation (2.55) and, equivalently, Eq.(2.57) are the mathematical formulas of accounting for the van Cittert-Zernike theorem.

As is evidently seen in Eqs.(2.55) and (2.57), the mutual intensity and the complex degree of spatial coherence are determined, aside from the factor $\exp(-j\psi)$ and scaling constants, by means of a two-dimensional Fourier transform of the intensity distribution I_0 across the source. Such a relationship is referred to as the van Cittert-Zernike theorem. This theorem leads to the important facts that, in general, an incoherent field can become a partially coherent field by the process of propagation and that the feature of the spatial coherence may be changed in accord with the intensity distribution across the source. The main consequence of this theorem stated above is quite similar to the relationship between the field across a coherently illuminated aperture and its diffracted field observed in the Fraunhofer region: the two-dimensional spatial Fourier transform of the source intensity determines the spatial coherence in the former case while that of the field across the aperture determines the field distribution across the observation plane in the latter case. It must be noted, however, that the Fourier transform relationship given by Eqs.(2.55) and (2.57) holds true when the observation plane is located at both the Fresnel and the Fraunhofer regions, as opposed to the diffraction phenomena. Moreover, the resultant field produced by the incoherent source is uniform while that produced by the coherently illuminated aperture has distinct non-uniformity due to the diffraction effect.

In closing this section, it should be borne in mind that one of the important applications of the van Cittert-Zernike theorem is a Michelson's stellar interferometer for measuring the angular diameter of stars. However, detailed discussion on this method is not presented here because this has no connection with the main subject of this dissertation.

CHAPTER 3

Fundamentals of Radiometry

3.1 Introduction

Radiometry is a science concerned with measurements of the light and pertains to one of the oldest branches of optics. It has undergone extensive developments and refinements over the last few centuries since the crucial contributions of Bouguer and Lambert in 1750's (a historical review of the matter is given by Geist [1]). The main purpose of the studies in this area is to formulate the transport of energy by the optical radiation. The conventional theory of a so-called radiative energy transfer was established on the basis of geometrical optics in which the light is regarded as the flow of luminous energy along the geometrical rays [154]. Attention in the conventional theory is, thus, only on the behavior of radiated fields produced by incoherent sources without taking into account the wave nature of light. Therefore, until recently, radiometry has been far removed from the mainstream of the modern theories of radiation, i.e., the Maxwell's electromagnetic theory and the quantum theory of radiation.

Since the publication of the first papers by Walther [4, 155] and Marchand and Wolf [5], considerable progress has been made to elucidate the relationship between the conventional radiometry and the electromagnetic theory of light. Consequently, it has become obvious that the notion of coherence relevant to the stochastic behavior of light waves plays an important role in radiometry. As a basic problem in a series of such investigations, the radiometric quantities, such as radiance, radiant emittance, and radiant intensity which are originally defined within the framework of conventional radiometry, have been generalized by incorporating with the coherence properties of light. Review articles on this subject have been provided by Wolf [2], Baltes *et al.* [20], Friberg [21], and Marathay ([148], Chap. 7).

In this chapter, some basic concepts of conventional radiometry and its gener-

alization associated with the coherence theory are briefly summarized as a primarily consideration for the subsequent investigations in this dissertation. For this purpose, we focus mainly on the definition of the basic radiometric quantities and the interrelationship among them, in spite of the broad topic of radiometry. To proceed further, we refer to the above-mentioned review articles.

In the following, after introducing the basic radiometric quantities in conventional radiometry, their generalized forms are presented in a new classification of radiometry, i.e., radiometry with partially coherent sources. Some important properties of the generalized quantities are discussed subsequently to enhance the difference and the analogy between conventional radiometry and generalized radiometry.

3.2 Conventional Radiometry

Let us consider a planar source occupying an area σ located in the plane $z = 0$ and radiating into the half-space $z > 0$, as illustrated in Fig.3.1. In conventional

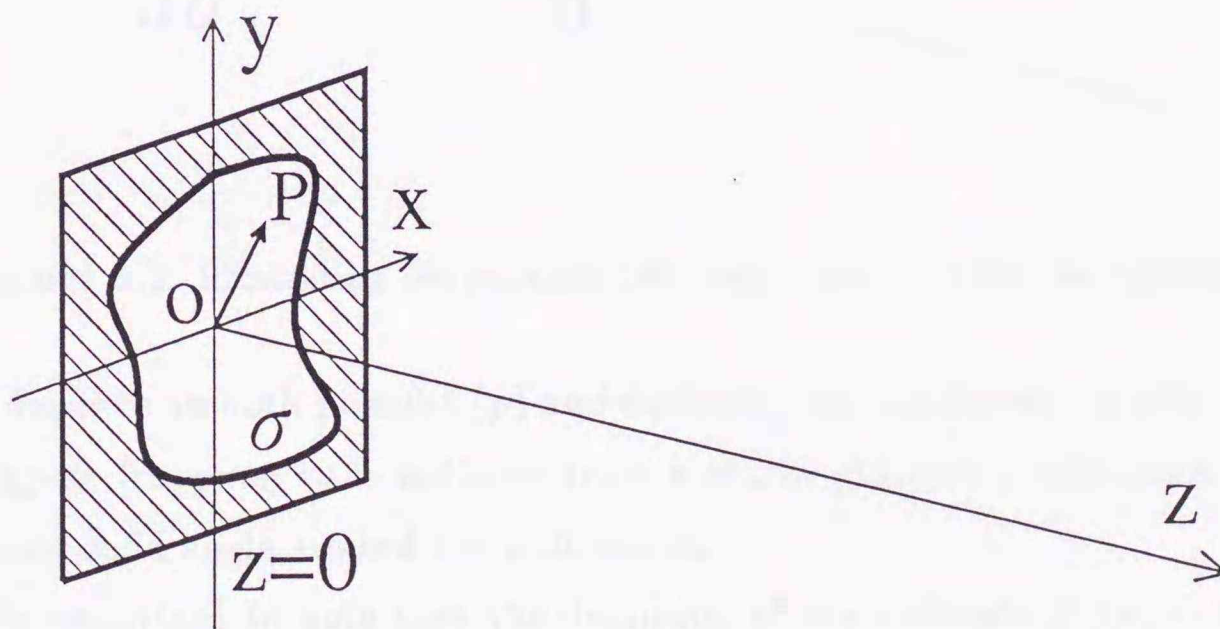


Figure 3.1. A planar source σ located at the plane $z = 0$ and radiating into the empty half-space $z > 0$.

radiometry, the central quantity of describing the light energy from the source is a radiance function $B_\omega(\rho, s)$ at frequency ω (also known as the specific intensity in

the theory of radiative energy transfer [154]). This function is defined through the relation

$$d\Phi_\omega = B_\omega(\rho, s) \cos \theta d\Omega d\sigma \quad (3.1)$$

where $d\Phi_\omega$ denotes the (time-averaged) amount of energy radiated by the source element $d\sigma$ at a point $P(\rho)$ specified by the two-dimensional position vector ρ , into an element $d\Omega$ of the solid angle around the direction specified by a three-dimensional unit vector s . Here θ is the angle which the unit vector s makes with the unit normal n to the source element $d\sigma$ (see Fig.3.2). Thus, the radiance $B_\omega(\rho, s)$, which, in

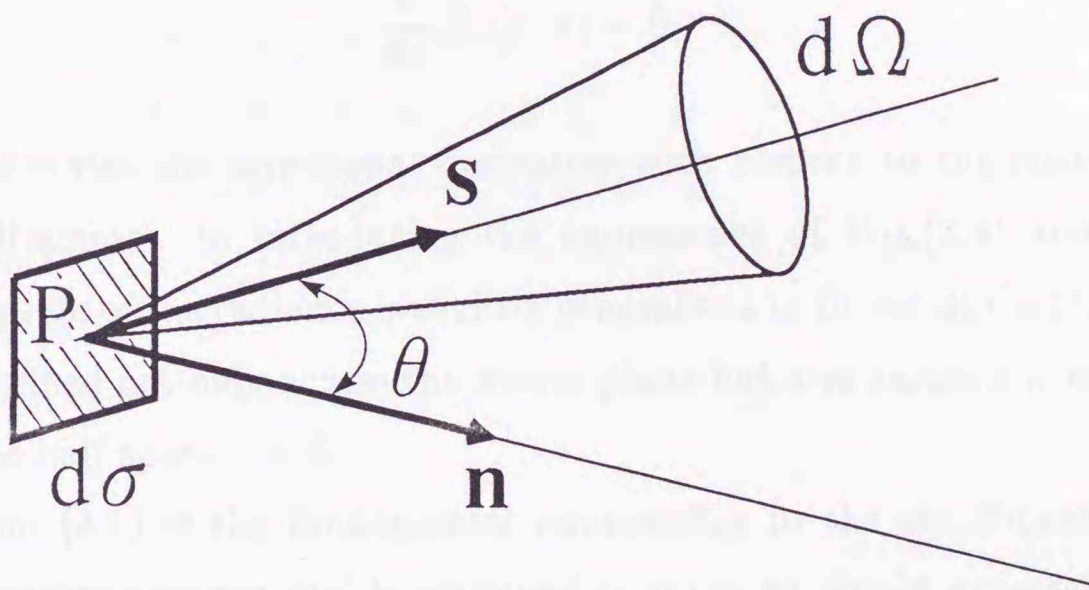


Figure 3.2. Illustrating the notation relating to the definition of radiance.

general, depends on both position (ρ) and direction (s), represents the rate at which the energy at frequency ω is radiated from a source element of unit area at $P(\rho)$ into an unit solid angle around the s -direction.

It is important to note that the definition of the radiance $B_\omega(\rho, s)$ given by Eq.(3.1) contains implicitly two assumptions: the radiance is necessarily a nonnegative quantity, i.e.,

$$B_\omega(\rho, s) \geq 0 \quad (3.2)$$

for all possible values of ρ and s and, vanishes in the source plane outside the source area σ , i.e.,

$$B_\omega(\rho, s) = 0, \quad \text{when } \rho \notin \sigma \quad (3.3)$$

for all possible s -directions. Moreover, recall here that, in conventional radiometry, the energy from the source in free space is assumed to be propagated along a straight line due to the initial postulate. It follows from the above fact that the radiance $B_\omega^{(z)}(\mathbf{r}, \mathbf{s})$ at a position $\mathbf{r} = (\boldsymbol{\rho}, z)$ in the half space $z > 0$ is related to the radiance $B_\omega(\boldsymbol{\rho}, \mathbf{s})$ in the source plane by the formula

$$B_\omega^{(z)}(\mathbf{r}, \mathbf{s}) = B_\omega[\boldsymbol{\rho} - (z/\cos\theta)\mathbf{s}_\perp, \mathbf{s}] \quad (3.4)$$

where $\mathbf{s} = (s_x, s_y, s_z)$ and $\mathbf{s}_\perp = (s_x, s_y)$. Equation (3.4) is essentially equivalent to the equation of radiative transfer in free space written in the form:

$$\frac{d}{ds} B_\omega(\mathbf{r}, \mathbf{s}) = 0, \quad (3.5)$$

where d/ds denotes the directional derivative with respect to the spatial variable $\mathbf{r}$ in the $\mathbf{s}$ -direction. In introducing the expressions of Eqs.(3.4) and (3.5), the original definition of the radiance is slightly generalized in the sense that the radiance function is defined not only across the source plane but also across a fictitious plane located in the half space $z > 0$.

Equation (3.1) is the fundamental relationship in the traditional treatment of radiative energy transfer and is employed in order to obtain expressions for the other radiometric quantities ([3], Sec. 4.8.1). To do this, $d\Phi_\omega$ in Eq.(3.1) should be decomposed in two different ways:

$$d\Phi_\omega = dE_\omega d\sigma = dJ_\omega d\Omega. \quad (3.6)$$

The above expression enhances the explicit dependence of $d\Phi_\omega$ on the element $d\Omega$ of the solid angle and the element $d\sigma$ of the source area. Comparing Eq.(3.6) with Eq.(3.1), we obtain the expressions

$$dE_\omega = \frac{d\Phi_\omega}{d\sigma} = B_\omega(\boldsymbol{\rho}, \mathbf{s}) \cos\theta d\Omega \quad (3.7)$$

and

$$dJ_\omega = \frac{d\Phi_\omega}{d\Omega} = B_\omega(\boldsymbol{\rho}, \mathbf{s}) \cos\theta d\sigma. \quad (3.8)$$

Then, the integral

$$E_{\omega}(\rho) = \int_{(2\pi)} B_{\omega}(\rho, s) \cos \theta d\Omega \quad (3.9)$$

taken over the 2π solid angle formed by all possible s -directions is referred to as the radiant emittance at the point $P(\rho)$, and the integral

$$J_{\omega}(s) = \cos \theta \int_{\sigma} B_{\omega}(\rho, s) d\sigma \quad (3.10)$$

taken over the source region σ is referred to as the radiant intensity in the s -direction. Due to these definitions, the radiant emittance denoted by $E_{\omega}(\rho)$ represents the energy at frequency ω radiated by the source per unit area around the point $P(\rho)$ whereas the radiant intensity denoted by $J_{\omega}(s)$ represents the energy at frequency ω radiated by the source per unit solid angle around the s -direction.

The three radiometric quantities defined above (i.e., the radiance $B_{\omega}(\rho, s)$, the radiant emittance $E_{\omega}(\rho)$, and the radiant intensity $J_{\omega}(s)$) describe, in common, rates of the energy transfer from the source and have certain characteristic properties. Specifically, not only the radiance (see Eq.(3.2)) but also all of these quantities must be nonnegative for all possible values of their arguments, and they are regarded as being measurable in principle. Furthermore, the radiance $B_{\omega}(\rho, s)$ and the radiant emittance $E_{\omega}(\rho)$ must have zero values at all points $P(\rho)$ in the source plane which lie outside the area occupied by the source (see also Eq.(3.3)).

The total energy radiated by the source is readily obtained from Eq.(3.1) as

$$\Phi_{\omega} = \int_{(2\pi)} d\Omega \int_{\sigma} d\sigma B_{\omega}(\rho, s) \cos \theta, \quad (3.11)$$

where the integrations extend over the 2π solid angle formed by all possible s -direction and over the source region σ . By making use of the expressions (3.9) and (3.10), Eq.(3.11) may be rewritten in two equivalent forms

$$\Phi_{\omega} = \int_{\sigma} E_{\omega}(\rho) d\sigma \quad (3.12)$$

and

$$\Phi_{\omega} = \int_{(2\pi)} J_{\omega}(s) d\Omega. \quad (3.13)$$

It is worth emphasizing here that Eq.(3.11), together with Eqs.(3.12) and (3.13), is derived on the basis of an implicit assumption that the total radiated energy is obtained by simply adding the contribution from all elements of the source and/or from all directions. This implies, in other words, that the various contributions are treated as if they were uncorrelated. Such a treatment of the radiated energy in optical fields is attributed to the initial postulate that conventional radiometry deals only with incoherent fields. However, by referring to the van Cittert-Zernike theorem discussed in Section 2.6, the radiation field produced by an incoherent source is no longer incoherent by the process of light propagation and becomes spatially partially coherent. Hence, the accuracy and the range of validity of conventional radiometry is quite ambiguous since the initial postulate itself is questionable. Actually, this fact motivated the study for connecting the conventional radiometry with the coherence theory, i.e., the study of so-called generalized radiometry or radiometry with partially coherent sources.

However, it is important to note finally that conventional radiometry has been successfully employed hitherto especially in astrophysics and in the field of atmospheric propagation, in spite of the fact that there is a certain requirement for modifying the conventional radiometry as was mentioned just above.

3.3 Radiometry with Partially Coherent Sources

3.3.1 Expressions for radiometric quantities

We consider a statistically stationary fluctuating planar source, either a primary or a secondary one, occupying a portion σ of the plane $z = 0$ and radiating into the half-space $z > 0$ (the geometry is the same as Fig.3.1). Since different temporal frequency components of a statistically stationary field are uncorrelated [150], it is sufficient to consider the transport of energy at one particular frequency ω of the field. Then, the statistical properties of the field are conveniently characterized by the correlation function in the space-frequency domain, i.e., the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$, rather than that in the space-time domain (i.e., the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$).

In order to determine the radiometric quantities associated with the partially

coherent source, we begin by dealing with the energy flow in the far zone of a source. Such a treatment of proceeding the present analysis is based on the fact that the energy flow from the source can be unambiguously described in terms of the intensity (also known as the spectral density) of the field only in the far zone of the source. In this region, the behavior of the energy flow originated from the partially coherent source can be readily elucidated with relation to the coherence properties of the source because the intensity (i.e., the spectral density) of the field produced by the source is determined, in terms of the source coherence, within the framework of coherence theory. In the following, the term of the intensity is replaced by that of the spectral density to distinguish it clearly from the radiometric concept of radiant intensity. It should be noted here that the spectral density itself is, in general, of little interest in radiometry since it represents neither an energy flow nor an energy density in any regions except for the far zone of the source.

Let us consider the energy flux vector $\mathbf{F}_\omega(\mathbf{r})$ in the scalar wave field at the point $P(\mathbf{r})$, which is essentially equivalent to the Poynting vector in the electromagnetic field ([148], Chap. 7). It is shown [33] that, in the far zone of the source, $\mathbf{F}_\omega(\mathbf{r})$ always points radially outwards from the source and that it is proportional to the spectral density. Accordingly, there exists a simple relationship between the energy flux vector $\mathbf{F}_\omega^{(\infty)}(r\mathbf{s})$ and the spectral density $I_\omega^{(\infty)}(r\mathbf{s})$ at any points $\mathbf{r} = r\mathbf{s}$ with a suitable choice of units:

$$\mathbf{F}_\omega^{(\infty)}(r\mathbf{s}) = I_\omega^{(\infty)}(r\mathbf{s}) \mathbf{s}, \quad (3.14)$$

where the superscript (∞) indicates far-zone values to be interpreted in the asymptotic sense as $kr \rightarrow \infty$ with $k = \omega/c$, c being the speed of light in vacuum, and $\mathbf{s}$ is a three-dimensional unit vector. The quantity $\mathbf{s} \cdot \mathbf{F}_\omega^{(\infty)}(r\mathbf{s})$ represents the rate at which the energy is radiated per unit area dS , located in the direction specified by the unit vector $\mathbf{s}$, of the surface of a large sphere of radius r centered at the origin O . By recalling the physical meaning of the radiant intensity and by using the relation

$$dS = r^2 d\Omega \quad (3.15)$$

with $d\Omega$ denoting the solid angle subtended by dS at the origin, the radiant intensity $J_\omega(\mathbf{s})$ is found to be related to the far-zone energy flux vector $\mathbf{F}_\omega^{(\infty)}(r\mathbf{s})$ by the

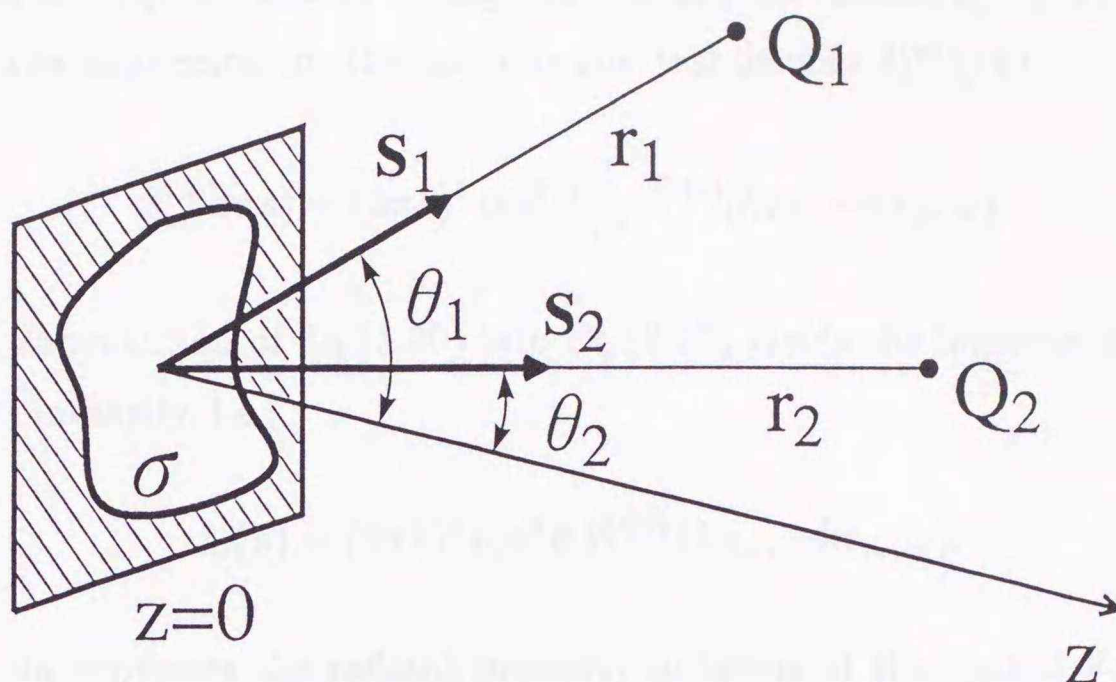


Figure 3.3. Illustration of the notation relating to Eq.(3.18).

formula

$$J_{\omega}(s) = r^2 s \cdot F_{\omega}^{(\infty)}(rs). \quad (3.16)$$

Substitution of Eq.(3.14) into Eq.(3.16) yields

$$J_{\omega}(s) = r^2 I_{\omega}^{(\infty)}(rs). \quad (3.17)$$

The far-zone spectral density $I_{\omega}^{(\infty)}(rs)$ is readily obtained in accord with the propagation law of the cross-spectral density function [156] given in the form

$$W^{(\infty)}(r_1 s_1, r_2 s_2; \omega) = (2\pi k)^2 \cos \theta_1 \cos \theta_2 \tilde{W}^{(0)}(k s_{1\perp}, -k s_{2\perp}; \omega) \frac{\exp[jk(r_1 - r_2)]}{r_1 r_2} \quad (3.18)$$

with $s_{1\perp}$ and $s_{2\perp}$ being the projections of the unit vectors s_1 and s_2 , respectively, onto the plane of the source and, θ_1 and θ_2 being the angle that s_1 and s_2 make with the normal to the source surface (see Fig.3.3). In this formula, $W^{(\infty)}$ is the cross-spectral density function in the far zone of the source and $\tilde{W}^{(0)}$ is the four-dimensional spatial Fourier transform of the cross-spectral density function $W^{(0)}$ in the source plane defined by

$$\tilde{W}^{(0)}(f_1, f_2; \omega) = \frac{1}{(2\pi)^4} \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} W^{(0)}(\rho_1, \rho_2; \omega) \exp[-j(f_1 \cdot \rho_1 + f_2 \cdot \rho_2)] d^2 \rho_1 d^2 \rho_2, \quad (3.19)$$

where f_1 and f_2 are two-dimensional (spatial-frequency) vectors. On setting $rs = r_1s_1 = r_2s_2$ in Eq.(3.18) and by taking into account the relation $I_\omega^{(\infty)}(rs) \equiv W^{(\infty)}(rs, rs; \omega)$, we obtain the expression for the far-zone spectral density $I_\omega^{(\infty)}(rs)$:

$$I_\omega^{(\infty)}(rs) = (2\pi k)^2 \cos^2 \theta \frac{1}{r^2} \tilde{W}^{(0)}(ks_\perp, -ks_\perp; \omega). \quad (3.20)$$

Therefore, substitution of Eq.(3.20) into Eq.(3.17) yields the important formula for the radiant intensity, i.e.,

$$J_\omega(s) = (2\pi k)^2 \cos^2 \theta \tilde{W}^{(0)}(ks_\perp, -ks_\perp; \omega). \quad (3.21)$$

This formula expresses the radiant intensity in terms of the spatial Fourier transform of the cross-spectral density function of the light across the source and, hence, establishes a relation between a radiometric quantity and a basic quantity of coherence theory. With the help of this formula (3.21), other radiometric quantities, i.e., the radiance $B_\omega(\rho, s)$ and the radiant emittance $E_\omega(\rho)$, can be readily derived in connection with the spatial coherence properties of the source.

To obtain the expressions for other radiometric quantities associated with the partially coherent source, Eq.(3.21) is first rewritten by using Eq.(3.19) in the form

$$J_\omega(s) = \left(\frac{k}{2\pi}\right)^2 \cos^2 \theta \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} W^{(0)}(\rho_1, \rho_2; \omega) \exp[-jk s_\perp \cdot (\rho_1 - \rho_2)] d^2 \rho_1 d^2 \rho_2. \quad (3.22)$$

By changing the variables of integrations from ρ_1 and ρ_2 to ρ and ρ' with

$$\rho = \frac{\rho_1 + \rho_2}{2}, \quad \rho' = \rho_1 - \rho_2, \quad (3.23)$$

the expression (3.22) becomes

$$J_\omega(s) = \left(\frac{k}{2\pi}\right)^2 \cos^2 \theta \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} W^{(0)}\left(\rho + \frac{\rho'}{2}, \rho - \frac{\rho'}{2}; \omega\right) \exp[-jk s_\perp \cdot \rho'] d^2 \rho d^2 \rho'. \quad (3.24)$$

By recalling the implication of the radiance and by comparing Eq.(3.24) with Eq.(3.10), we may obtain the expression, which was first proposed by Walther in 1968 [4], for

the radiance associated with the partially coherent source (which is often referred to as the generalized radiance) as follows:

$$B_{\omega}(\rho, s) = \left(\frac{k}{2\pi}\right)^2 \cos \theta \iint_{-\infty}^{\infty} W^{(0)}\left(\rho + \frac{\rho'}{2}, \rho - \frac{\rho'}{2}; \omega\right) \exp[-j k s_{\perp} \cdot \rho'] d^2 \rho'. \quad (3.25)$$

In a similar way, the radiant emittance function associated with the partially coherent source (likewise, which is often referred to as the generalized radiant emittance) may be written, by substituting Eq.(3.25) into Eq.(3.9), in the form [5]

$$E_{\omega}(\rho) = \iint_{-\infty}^{\infty} W^{(0)}\left(\rho + \frac{\rho'}{2}, \rho - \frac{\rho'}{2}; \omega\right) K_{\omega}(\rho') d^2 \rho' \quad (3.26)$$

where

$$K_{\omega}(\rho') = \left(\frac{k}{2\pi}\right)^2 \int_{(2\pi)} \cos^2 \theta \exp(-k s_{\perp} \cdot \rho') d\Omega. \quad (3.27)$$

The integral on the right-hand side of Eq.(3.27) may be readily evaluated and becomes

$$K_{\omega}(\rho') = \frac{k^2}{2\sqrt{2\pi}} \frac{J_{3/2}(k|\rho'|)}{(k|\rho'|)^{3/2}} \quad (3.28)$$

with

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right). \quad (3.29)$$

Up to now, the generalized forms of the conventional radiometric quantities have been established: the generalized radiance $B_{\omega}(\rho, s)$ is given by Eq.(3.25), the generalized radiant emittance $E_{\omega}(\rho)$ by Eq.(3.26), and the generalized radiant intensity $J_{\omega}(s)$ by Eq.(3.24). All of them are associated with the coherence properties of the source through the cross-spectral density function $W^{(0)}(\rho_1, \rho_2; \omega)$ of light across the source. However, it is important to note that there exist a number of alternative expressions for these generalized radiometric quantities. Actually, for example, Walther further introduced another expression for the generalized radiance in 1973 [155]. Attempts of finding the other candidates for the generalized radiometric quantities are still of great interest due to the current requirement for exploring a new treatment of radiative energy transfer in partially coherent fields.

3.3.2 Special cases

As special cases of the generalized radiometric quantities Eqs.(3.24), (3.25) and (3.26) just derived in the previous section, this section is concerned with the two limiting cases when the planar source is either completely spatially incoherent or completely spatially coherent.

We first consider a completely spatially incoherent source. Such a source is characterized by the cross-spectral density function of the form ([147], Sec. 4.4)

$$W^{(0)}(\rho_1, \rho_2; \omega) = I_{\omega}^{(0)}(\rho_1) \delta(\rho_1 - \rho_2) \quad (3.30)$$

where $\delta(\cdot)$ is the two-dimensional Dirac delta function and

$$I_{\omega}^{(0)}(\rho) \geq 0 \quad (3.31)$$

is the spectral density across the source. In this case, the expression (3.25) for the generalized radiance reduces to

$$B_{\omega}(\rho, s) = \left(\frac{k}{2\pi}\right)^2 \cos \theta I_{\omega}^{(0)}(\rho). \quad (3.32)$$

Similarly, the expressions (3.26) and (3.27) become

$$E_{\omega}(\rho) = \left(\frac{k}{2\pi}\right)^2 I_{\omega}^{(0)}(\rho) \int_{(2\pi)} \cos^2 \theta d\Omega \quad (3.33)$$

and

$$J_{\omega}(s) = \left(\frac{k}{2\pi}\right)^2 \cos^2 \theta \iint_{-\infty}^{\infty} I_{\omega}^{(0)}(\rho) d^2 \rho. \quad (3.34)$$

The integral in Eq.(3.33) may be evaluated easily by expressing $d\Omega$ in spherical polar coordinates and takes the value $2\pi/3$. Therefore, Eq.(3.33) becomes

$$E_{\omega}(\rho) = \frac{k^2}{6\pi} I_{\omega}^{(0)}(\rho). \quad (3.35)$$

From Eqs.(3.32) and (3.35), it follows that the radiance $B_{\omega}(\rho, s)$ and the radiant emittance $E_{\omega}(\rho)$ take nonnegative values for a spatially completely incoherent source

and that they have zero values in the source plane outside the source. These results indicate that the generalized radiometry is not contradicted by the conventional radiometry in the limit of spatial incoherence as far as the properties of these radiometric quantities are concerned. However, the equation of radiative energy transfer (i.e., Eq.(3.5)) may not be rigorously fulfilled even in this limit.

We next consider a completely spatially coherent source. In this case, it is known [157, 158] that the cross-spectral density may be factorized in the form

$$W^{(0)}(\rho_1, \rho_2; \omega) = v^{(0)}(\rho_1, \omega) v^{(0)*}(\rho_2, \omega), \quad (3.36)$$

where $v^{(0)}(\rho, \omega)$ represents the amplitude distribution of light across the source and, thus, $v^{(0)}(\rho, \omega) = 0$ when the point ρ lies outside the source region. In this formula, the asterisk denotes the complex conjugate. With the cross-spectral density function of the source given by Eq.(3.36), the generalized radiance given by Eq.(3.25) becomes

$$B_\omega(\rho, s) = \left(\frac{k}{2\pi}\right)^2 \cos \theta \iint_{-\infty}^{\infty} v^{(0)}\left(\rho + \frac{\rho'}{2}, \omega\right) v^{(0)*}\left(\rho - \frac{\rho'}{2}, \omega\right) \exp[-jk s_\perp \cdot \rho'] d^2 \rho'. \quad (3.37)$$

The generalized radiant emittance may be determined by substituting Eq.(3.36) into Eq.(3.26) and becomes

$$E_\omega(\rho) = \frac{k^2}{2\sqrt{2\pi}} \iint_{-\infty}^{\infty} v^{(0)}\left(\rho + \frac{\rho'}{2}, \omega\right) v^{(0)*}\left(\rho - \frac{\rho'}{2}, \omega\right) \frac{J_{3/2}(k|\rho'|)}{(k|\rho'|)^{3/2}} d^2 \rho' \quad (3.38)$$

where $J_{3/2}(\cdot)$ is given by Eq.(3.29). Finally, the generalized radiant intensity of the completely spatially coherent source is written by using Eqs.(3.24) and (3.36) in the form

$$J_\omega(s) = (2\pi k)^2 \cos^2 \theta |\tilde{v}^{(0)}(k s_\perp, \omega)|^2, \quad (3.39)$$

where $\tilde{v}^{(0)}(f, \omega)$ is the two-dimensional spatial Fourier transform of $v^{(0)}(\rho, \omega)$ defined by

$$\tilde{v}^{(0)}(f, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} v^{(0)}(\rho, \omega) \exp[-j f \cdot \rho] d^2 \rho. \quad (3.40)$$

It is readily shown [5] from Eqs.(3.37) and (3.38) that both the generalized radiance $B_\omega(\rho, s)$ and the generalized emittance $E_\omega(\rho)$ may take negative values in this limit.

This feature of these quantities suggests that the generalized radiance $B_\omega(\rho, s)$ and the generalized radiant emittance $E_\omega(\rho)$ are, in general, not measurable. In this respect, the generalized radiometry has no connection with the conventional radiometry in this limit. It is worth stressing, however, that the generalized radiant intensity $J_\omega(s)$ defined by Eq.(3.24) can be shown to be nonnegative in both of these two limiting cases and is the basic measurable quantity that characterizes some of the radiation properties of a source of any state of coherence.

3.4 Some Properties of Radiometric Quantities

In conventional radiometry, the basic quantity of formulating the radiative energy transfer is the radiance function $B_\omega(\mathbf{r}, \mathbf{s})$. As mentioned in Section 3.3.1, at least two different definitions (both of which are due to Walther [4, 155]) have been given for the radiance in the context of the generalized radiometry. It has been pointed out in Section 3.3.2, however, that one of the above two functions cannot be regarded as a true measure for describing the energy flow because it may occasionally take negative values. Accordingly, we first consider briefly the existence of the generalized radiance that satisfies all the requirements postulated for the radiance in conventional radiometry.

From the discussion in the previous two sections, it is readily found that, in order to possess the conventional radiometric properties, the generalized radiance $B_\omega(\mathbf{r}, \mathbf{s})$ must necessarily fulfill the following four requirements for all possible values of its arguments $\mathbf{r}$ and $\mathbf{s}$:

- (I) $B_\omega(\mathbf{r}, \mathbf{s}) = \mathcal{L}\{W^{(0)}(\mathbf{r}_1, \mathbf{r}_2; \omega)\},$
- (II) $B_\omega(\mathbf{r}, \mathbf{s}) \geq 0,$
- (III) $B_\omega(\mathbf{r}, \mathbf{s}) = 0$ when $\mathbf{r} \notin \sigma,$
- (IV) $\cos \theta \int_\sigma B_\omega(\mathbf{r}, \mathbf{s}) d\sigma = [J_\omega(s)]_{phys. opt.},$

where $\mathcal{L}$ denotes a linear transformation. The first requirement expresses a physically quite natural fact that the cross-spectral density function $W^{(0)}(\mathbf{r}_1, \mathbf{r}_2; \omega)$ of light across the source is linearly mapped onto the generalized radiance. The second

and third requirements are directly attributed to the intrinsic properties of the radiance function in conventional radiometry (see Eqs.(3.2) and (3.3)): the generalized radiance should be nonnegative in any cases and have zero values at all points $\mathbf{r}$ across the source plane outside the source region. The fourth requirement expresses the condition that the radiometric expression (3.10) on the left-hand side should correctly represent the radiant intensity $[J_\omega(\mathbf{s})]_{phys. opt.}$ when the radiant intensity is calculated on the basis of physical optics, i.e., through Eq.(3.21). However, it has been proved by Friberg [159, 160] that there is no generalized radiance function that the four requirements (I)-(IV) are simultaneously satisfied for a planar source of any state of coherence (Friberg's theorem). Indeed, as discussed in Section 3.3.2, the generalized radiance function defined by Eq.(3.25) satisfies the requirements (I) and (IV) whereas it fails to satisfy the requirements (II) and (III).

This result has its root in the fact that the radiance $B_\omega(\mathbf{r}, \mathbf{s})$ is a function of both the position $\mathbf{r}$ and the direction $\mathbf{s}$, which are essentially Fourier conjugate variables from each other. More specifically, it is impossible to define rigorously the generalized radiance function due to the well-known uncertainty principle [161]. In analogy with quantum mechanics [161], this result also suggests that the generalized radiance can no longer be regarded as a measurable quantity. In this respect, a few remarks are succinctly made below in connection with quantum mechanics [2, 21].

Radiometry with partially coherent sources is somewhat similar in its mathematical structure to the phase-space representation of quantum mechanics. To put it more precisely, the generalized radiance defined by Eq.(3.25) is analogous to the Wigner function which is the distribution function in phase space [162] given by

$$f(x, p) = \int_{-\infty}^{\infty} \rho\left(x + \frac{1}{2}x', x - \frac{1}{2}x'\right) \exp\left(-j\frac{px'}{\hbar}\right) dx', \quad (3.41)$$

where x and p are respectively the position and the momentum variables of a particle moving in one dimension, $\rho(x, x')$ is the coordinate representation of the density operator, and $\hbar$ is the Planck's constant divided by 2π . It is known that the phase-space distribution functions cannot be considered as true probability densities and are often called quasi-probabilities, because they do not possess all the attributes of ordinary probabilities, e.g., they may take negative values. Nevertheless, the phase-space representation of quantum mechanics has proved a very powerful technique for

solving quantum-mechanical problems. In view of the formal analogy between the generalized radiometry and the phase-space representation of quantum mechanics, the generalized radiance function are also likely to find useful applications even though they cannot satisfy all the postulates of conventional radiometry.

In conclusion, it has been shown that the generalized radiance $B_{\omega}(\mathbf{r}, \mathbf{s})$ given by Eq.(3.25) (actually, also the generalized radiant emittance $E_{\omega}(\mathbf{r})$ in Eq.(3.26)) cannot be regarded as a true measure of the energy flow. Instead, the generalized radiant intensity $J_{\omega}(\mathbf{s})$ represents always correctly the energy per unit solid angle as in conventional radiometry and can be regarded as a measure for specifying the behavior of energy flow from the source, as was mentioned in Section 3.3.2. It should be noted here that, although the generalized radiance function may not have the simple intuitive physical implication, it can be employed successfully in calculating values of truly measurable quantities, such as the radiant intensity. However, the present dissertation is concerned only with the problems closely related to the generalized radiant intensity and does not take into consideration the generalized radiance, together with the generalized radiant emittance.

CHAPTER 4

Spatial Coherence of Light from Partially Coherent Planar Sources and Its Control Using a Source Filter

4.1 Introduction

There has been an enormous interest in spatial coherence of light generated from partially coherent sources. This is mainly because the concept of spatial coherence plays a significant role in various areas of optics such as optical metrology, optical communication and optical information processing. As is widely recognized, a speckle phenomenon observed in holographic systems occurs due to the highly spatial coherence of light used and, hence, its reduction can be achieved by decreasing the spatial coherence. The properties of imaging in microscopy or photolithography depend strongly on the state of spatial coherence of the illuminating light. These examples suggest that it is possible to change the characteristics of the system by actively controlling the spatial coherence of light used in the system. In fact, considerable attention has been given to the role of spatial coherence of light in practical applications.

An early attempt to elucidate the significance of controlling the spatial coherence was made by Hopkins in 1950's [163]. He established the theory of image formation on the basis of spatial coherence of light and determined the resolution limit in connection with the numerical aperture of a condenser. After his pioneering work, several researchers examined the relationship between the resolution and the intensity distribution across the condenser in imaging systems, and found an important consequence that the annular condenser was superior to the disc one in resolution limit [164, 165]. In these studies, the spatial coherence of light illuminating the object can be directly evaluated from the knowledge of the intensity distribution across the condenser by applying the van Cittert-Zernike theorem, due

to the assumption of the incoherent source as an illumination optics. It follows from the above consideration that the preceding investigations demonstrate implicitly a method of changing the imaging characteristics on the basis of the control of spatial coherence.

In modern optics, the significance of spatial coherence of light appears to have been recognized with close relation to lasers, as opposed to the early investigations. Since most of practical lasers tend to oscillate in several transverse modes and their mixture reduces the spatial coherence, they must be regarded as partially coherent sources whereas those under a single-mode oscillation provide theoretically the completely spatial coherent light [109]. Thus, the light generated from a multimode laser oscillating in many modes is no longer spatially completely coherent and, eventually, it may be approximately equivalent to the incoherent light provided the number of superposed modes becomes large enough. On the other hand, when only a few modes oscillate in the laser and hence the light becomes nearly coherent, non-stationarity in the sense that the spatial coherence depends on the position across the beam cannot be ignored at the same time. Completely spatial coherent light is only a mathematical idealization and cannot be realized at present by use of any practical lasers.

When using a laser under oscillating in one or a few modes, certain miscellaneous phenomena arising from its highly spatial coherence are often observed. The image intensity distribution of an object illuminated by the highly coherent light shows a ringing or fringing phenomenon and the position of edges appears to shift. In addition, speckles can be found in various systems, such as holographic systems and imaging systems, in which the source with the highly spatial coherence is used, and even in the case of light transmission through an optical fiber. Except for speckles, the above phenomena have been eliminated conventionally by the process of filtering over the aperture (it is called apodization). However, the equivalent consequences may also be achieved by properly controlling the spatial coherence of light in place of apodization. The control of spatial coherence for the purpose of changing the system characteristics is considered to be applicable especially when the apodization has no effect.

It has been observed in the above discussion that the spatial coherence of light is directly related to a characteristic of the system in various areas of optics.

In those areas, there are many cases in which the control of spatial coherence is required to realize the desired system. In principle, it is possible to obtain the prescribed performance in such systems, provided that the state of spatial coherence can be changed at will. Under these circumstances, a spatial filtering method over the source using a source filter is proposed in this chapter to control the spatial coherence of light generated from a partially coherent planar source. The source filter is a kind of a spatial filter which is placed just behind the source and changes the intensity distribution across the source.

From the mathematical point of view, the present analysis is analogous to the studies on diffraction by an aperture illuminated with the partially coherent light, which were thoroughly investigated several decades ago by a large number of researchers [166–179]. However, most of them dealt only with the diffraction patterns and little attention has been paid to the spatial coherence properties of light in the diffracted field, except for a few works [176–179]. Also in recent years, a similar formulation of the problem has been employed in the studies of radiometry with partially coherent sources (see Chapter 3). These studies have been proceeded mainly to obtain the radiometric quantities associated with a source of arbitrary state of spatial coherence and, as a consequence, have given a crucial fact that the directionality of the light which a source generates is closely related to the state of its spatial coherence. It has also become evident that the coherence properties of a source play a key role in determining its radiation characteristics including the spatial coherence of light in the radiated field. Nevertheless, most researchers examined only the intensity distribution of the resultant field and few consideration dealing with the spatial coherence of the field has been made as well as the preceding works on the diffraction phenomena.

This chapter is concerned with the spatial coherence of light generated from a partially coherent planar source covered with a source filter, as an extended continuation of the related investigations pertaining to the above two categories, i.e., the studies on both diffraction of partially coherent light and radiometry with partially coherent sources. Particular interest in this chapter is placed on the control of the spatial coherence using a source filter. In analysis, two kinds of the source filter are taken into consideration as a simple example for verifying the effect of the source filter. To provide a profound elucidation on the effects, the feature of the radiated

field from the source with no source filter is also examined in some detail. In the following, we begin by formulating mathematically the present problem and then show a number of illustrative examples.

4.2 Theoretical Background

Figure 4.1 shows an optical arrangement for analysis. The partially coherent planar source is placed at the source plane and provides the light at the observation plane located in arbitrary free space. In order to characterize the partially coherent field, the mutual intensity is properly chosen among several correlation functions. Analysis to be conducted below is formulated in accordance with the propagation law of the mutual intensity. The justification of this method for analysis (specifically, the reason for the adoption of the mutual intensity) is discussed in Section 4.2.3.

In what follows, we neglect the polarization effect and hence take the field to be represented by a complex scalar function. Furthermore, the quantum nature of light is also ignored since the present analysis is based on the classical theory of second-order coherence.

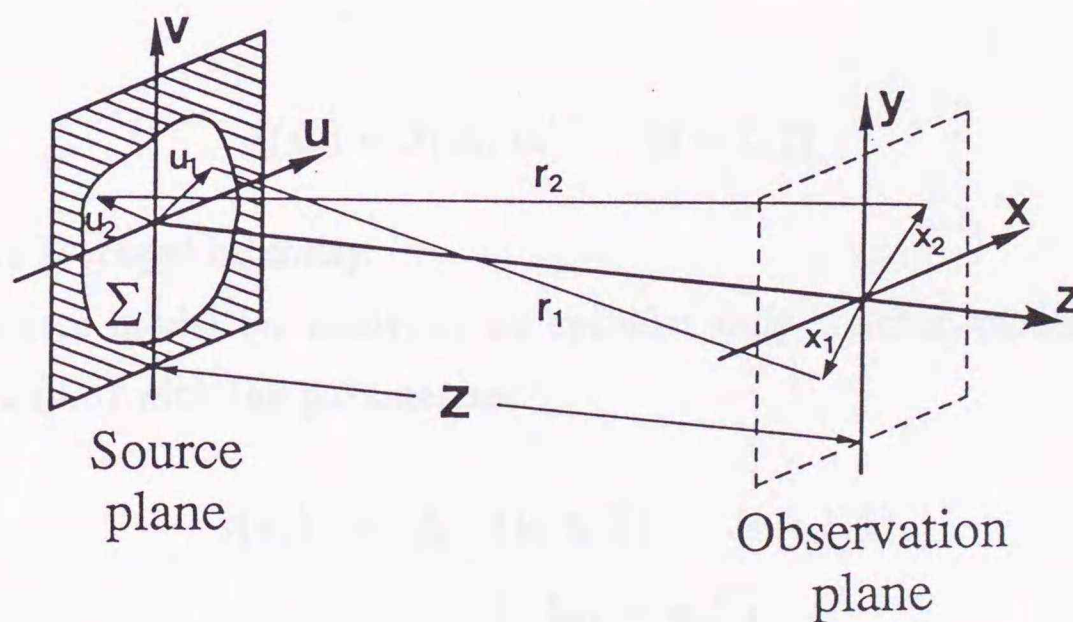


Figure 4.1. Optical arrangement for analysis. Position vectors describing the typical source points u_1 and u_2 are within the source region Σ and those describing the observation points x_1 and x_2 are within the observation plane parallel to the source.

4.2.1 Formulation of the problem

Let us consider a statistically stationary, quasi-monochromatic, partially coherent, planar source occupying a finite area Σ at the plane $z = 0$ and radiating into the half-space $z > 0$, as illustrated in Fig.4.1. The spatial coherence properties of light across the source are characterized by the mutual intensity defined as

$$J(\mathbf{u}_1, \mathbf{u}_2) = \langle V(\mathbf{u}_1, t) V^*(\mathbf{u}_2, t) \rangle, \quad (4.1)$$

where $V(\mathbf{u}_i, t)$ describes a complex analytic signal representing the field at a typical source point specified by a position vectors $\mathbf{u}_i$ at time t , the angular brackets denote either a time average or an ensemble average, and the asterisk denotes the complex conjugate. In terms of the complex degree of spatial coherence, $\gamma(\mathbf{u}_1, \mathbf{u}_2)$, defined by

$$\gamma(\mathbf{u}_1, \mathbf{u}_2) = \frac{J(\mathbf{u}_1, \mathbf{u}_2)}{\sqrt{I(\mathbf{u}_1)} \sqrt{I(\mathbf{u}_2)}}, \quad (4.2)$$

the mutual intensity of light across the source can be written in the form:

$$J(\mathbf{u}_1, \mathbf{u}_2) = \sqrt{I(\mathbf{u}_1)} \sqrt{I(\mathbf{u}_2)} \gamma(\mathbf{u}_1, \mathbf{u}_2), \quad (4.3)$$

where

$$I(\mathbf{u}_i) = J(\mathbf{u}_i, \mathbf{u}_i) \quad (i = 1, 2) \quad (4.4)$$

stands for the averaged intensity.

As a source model for analysis, we consider only a Schell-model source described by Eq.(4.3) with the parameters,

$$I(\mathbf{u}_i) = I_0 \quad (\mathbf{u}_i \in \Sigma) \quad (i = 1, 2), \quad (4.5)$$

$$\gamma(\mathbf{u}_1, \mathbf{u}_2) = \exp \left[-\frac{|\mathbf{u}_1 - \mathbf{u}_2|^2}{2\sigma^2} \right], \quad (4.6)$$

where I_0 and σ are positive constants: the intensity distribution across the source is uniform within the source region Σ and, in the same region, the complex degree of spatial coherence for two points takes a Gaussian form depending only on the mutual difference $|\mathbf{u}_1 - \mathbf{u}_2|$. The parameter σ contained in Eq.(4.6) determines the spatial

coherence condition over the source. The source filter, which plays an important role in the present analysis, is placed just behind the source to control the spatial coherence of light generated from the source and changes the intensity distribution across the source. Using Eqs.(4.3), (4.5) and (4.6), the mutual intensity of light just behind the source filter becomes

$$J_{sf}(u_1, u_2) = I_0 S(u_1)S(u_2) \exp \left[-\frac{|u_1 - u_2|^2}{2\sigma^2} \right], \quad (4.7)$$

where $S(u_i)$ implies the amplitude transmittance of the source filter. The source of this kind may be actually realized by the long propagation of light generated from an incoherent source having a Gaussian intensity distribution due to the van Cittert-Zernike theorem, or it is possibly produced by a certain statistically-averaged multimode laser.

We now describe the mutual intensity of light, $J_{ob}(x_1, x_2)$, at the observation plane in terms of the mutual intensity, $J_{sf}(u_1, u_2)$, across the source in order to examine the behavior of the field produced by the source with the source filter. According to the propagation law of the mutual intensity (see Section 2.5), the mutual intensity of light at the observation plane in arbitrary free space can be written in the form

$$J_{ob}(x_1, x_2) = \iint_{\Sigma} \iint_{\Sigma} J_{sf}(u_1, u_2) K(u_1, x_1) K^*(u_2, x_2) du_1 du_2, \quad (4.8)$$

where $K(u_i, x_i)$ is a propagation function given by

$$K(u_i, x_i) = \frac{e^{j\bar{k}r_i}}{j\bar{\lambda}r_i} \quad (i = 1, 2). \quad (4.9)$$

Here, r_i is the distance from the source point to the observation point shown in Fig.4.1, $\bar{\lambda}$ is the mean wavelength of light used, and $\bar{k} = 2\pi/\bar{\lambda} = 2\pi\bar{\nu}/c$ is the mean wavenumber, c being the speed of light in vacuum. When the observation plane is located in Fresnel and Fraunhofer diffraction fields, the propagation function given by Eq.(4.9) becomes, by applying the approximation of Fresnel and Fraunhofer

regimes,

$$K(u_i, x_i) = \frac{1}{j\bar{\lambda}z} \begin{cases} \exp \left[j \frac{\bar{k}}{2z} |u_i - x_i|^2 \right] & \text{:Fresnel diffraction field,} \\ \exp \left[-j \frac{\bar{k}}{z} u_i \cdot x_i \right] & \text{:Fraunhofer diffraction field,} \end{cases} \quad (4.10)$$

where z indicates the distance from the source to the observation plane (see Fig.4.1). Substituting Eqs.(4.7) and (4.10) into Eq.(4.8), we obtain the final expressions for the mutual intensity at the observation plane

$$\begin{aligned} J_{ob}(x_1, x_2) &= \frac{I_0}{(\bar{\lambda}z)^2} \iint_{\Sigma} \iint_{\Sigma} S(u_1) S(u_2) \exp \left[-\frac{|u_1 - u_2|^2}{2\sigma^2} \right] \\ &\times \exp \left[-j \frac{\bar{k}}{z} (u_1 \cdot x_1 - u_2 \cdot x_2) \right] \\ &\times \exp \left[j \frac{\bar{k}}{2z} (u_1^2 - u_2^2) \right] \exp \left[j \frac{\bar{k}}{2z} (x_1^2 - x_2^2) \right] du_1 du_2 \quad (4.11) \end{aligned}$$

in the Fresnel diffraction field, and

$$\begin{aligned} J_{ob}(x_1, x_2) &= \frac{I_0}{(\bar{\lambda}z)^2} \iint_{\Sigma} \iint_{\Sigma} S(u_1) S(u_2) \exp \left[-\frac{|u_1 - u_2|^2}{2\sigma^2} \right] \\ &\times \exp \left[-j \frac{\bar{k}}{z} (u_1 \cdot x_1 - u_2 \cdot x_2) \right] du_1 du_2 \quad (4.12) \end{aligned}$$

in the Fraunhofer diffraction field. By using Eq.(4.11) or Eq.(4.12), the intensity distribution at the observation plane is given by

$$I_{ob}(x) = J_{ob}(x, x), \quad (4.13)$$

and the complex degree of spatial coherence of light at the observation plane is given by

$$\gamma_{ob}(x_1, x_2) = \frac{J_{ob}(x_1, x_2)}{\sqrt{I_{ob}(x_1)} \sqrt{I_{ob}(x_2)}}. \quad (4.14)$$

Equations (4.13) and (4.14) are the basic quantities representing the characteristics of the resultant field.

Analytical solution for Eqs.(4.11) and (4.12) can be easily obtained if the source filter whose amplitude transmittance $S(u)$ takes a Gaussian form given by

$$S(u) = \exp\left(-\frac{u^2}{4\sigma_{sf}^2}\right) \quad (4.15)$$

σ_{sf} being a positive constant is covered with the infinite source. The source of this kind are called a Gaussian Shell-model source and has been frequently used in the previous studies of radiometry with partially coherent sources. However, mathematical difficulties are encountered when the other kind of source filter is introduced in general. In such a case, there are no analytical solutions for Eqs.(4.11) and (4.12) and, as a result, numerical evaluations using a computer are required. For this reason, the algebraic details for the evaluation of Eqs.(4.11) and (4.12) are given for one and two-dimensional cases in the next section.

4.2.2 Preliminaries for computer simulations

In this section, the four-dimensional integrations of Eqs.(4.11) and (4.12) are reduced to the two-dimensional integrations for computer simulations because the two-dimensional integration can be readily performed numerically using a computer. Since two-dimensional investigations do not often provide good perspective due to their complex mathematics, we deal not only with the mathematical derivation in two-dimensional cases, but also with that in one-dimensional cases. As a source specified by Σ in the previous section, for simplicity, slit and circular sources are assumed in one- and two-dimensional analyses, respectively. Furthermore, the source filter is taken to have a symmetrical transmittance about the optical axis (z -axis in Fig.4.1).

A. One-dimensional cases

Let us consider a geometry for analyzing in one dimension illustrated in Fig.4.2. Since the slit source with the width $2a$ is placed along the v -axis at the source plane, the diffraction pattern appears in a symmetrical form with respect to the y -axis at the observation plane.

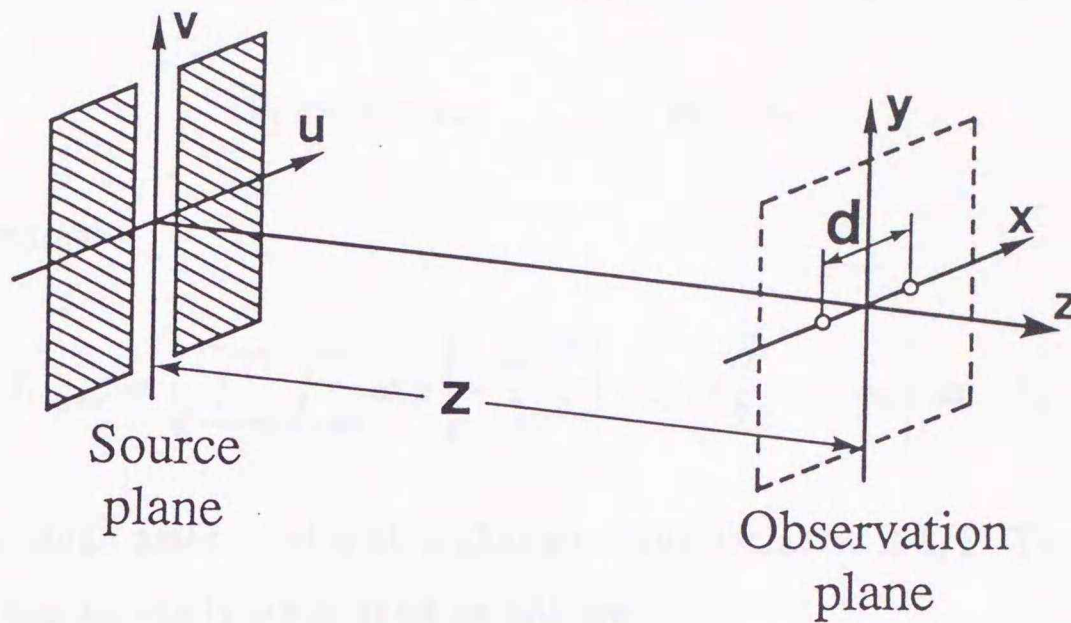


Figure 4.2. Geometry for the analysis in one dimension. Slit source with the width $2a$ is situated along the v -axis and the observation plane parallel to the source is located in the Fresnel or the Fraunhofer diffraction field of the source. Two points for measuring the spatial coherence (whose distance is d) are situated symmetrically about the z -axis.

Rewriting Eq.(4.11) in Cartesian coordinates, we obtain

$$\begin{aligned}
 J_{ob}(x_1, y_1; x_2, y_2) &= \frac{I_0}{(\bar{\lambda}z)^2} \int_{-a}^a du_1 \int_{-a}^a du_2 \int_{-\infty}^{\infty} dv_1 \int_{-\infty}^{\infty} dv_2 S(u_1)S(u_2) \\
 &\times \exp\left[-\frac{|u_1 - u_2|^2}{2\sigma^2}\right] \exp\left[-\frac{|v_1 - v_2|^2}{2\sigma^2}\right] \\
 &\times \exp\left[-j\frac{\bar{k}}{z}(u_1 \cdot x_1 - u_2 \cdot x_2)\right] \exp\left[-j\frac{\bar{k}}{z}(v_1 \cdot y_1 - v_2 \cdot y_2)\right] \\
 &\times \exp\left[j\frac{\bar{k}}{2z}(u_1^2 - u_2^2)\right] \exp\left[j\frac{\bar{k}}{2z}(v_1^2 - v_2^2)\right] \\
 &\times \exp\left[j\frac{\bar{k}}{2z}(x_1^2 - x_2^2)\right] \exp\left[j\frac{\bar{k}}{2z}(y_1^2 - y_2^2)\right]. \quad (4.16)
 \end{aligned}$$

Since the feature of the radiated field along the x -axis is only taken into account, we may set $y_1 = y_2 = 0$ in Eq.(4.16) without loss of generality. Let us now perform the integration of Eq.(4.16) with respect to v_1 and v_2 given by

$$I_{v_1, v_2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[-\frac{|v_1 - v_2|^2}{2\sigma^2}\right] \exp\left[j\frac{\bar{k}}{2z}(v_1^2 - v_2^2)\right] dv_1 dv_2. \quad (4.17)$$

By changing the variables of v_1 and v_2 in Eq.(4.17) with the relation of

$$v_1 - v_2 = v_-, \quad v_1 + v_2 = v_+, \quad (4.18)$$

Eq.(4.17) becomes

$$I_{v_+, v_-} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[-\frac{|v_-|^2}{2\sigma^2} \right] \exp \left[j \frac{\bar{k}}{2z} v_- \cdot v_+ \right] dv_- dv_+ \quad (4.19)$$

where the Jacobian associated with a change of the variables is $1/2$. The integration of Eq.(4.19) can be easily performed as follows:

$$I_{v_+, v_-} = \bar{\lambda} z. \quad (4.20)$$

Thus, corresponding to Eq.(4.11) or Eq.(4.16), the resultant mutual intensity produced by the slit source can be written in the form

$$\begin{aligned} J_{ob}(x_1, x_2) &= \frac{I_0}{\bar{\lambda} z} \int_{-a}^a \int_{-a}^a S(u_1) S(u_2) \exp \left[-\frac{|u_1 - u_2|^2}{2\sigma^2} \right] \\ &\times \exp \left[-j \frac{\bar{k}}{z} (u_1 \cdot x_1 - u_2 \cdot x_2) \right] \\ &\times \exp \left[j \frac{\bar{k}}{2z} (u_1^2 - u_2^2) \right] \exp \left[j \frac{\bar{k}}{2z} (x_1^2 - x_2^2) \right] du_1 du_2 \quad (4.21) \end{aligned}$$

for the Fresnel diffraction field. Likewise, the mutual intensity corresponding to Eq.(4.12) is given by

$$\begin{aligned} J_{ob}(x_1, x_2) &= \frac{I_0}{\bar{\lambda} z} \int_{-a}^a \int_{-a}^a S(u_1) S(u_2) \exp \left[-\frac{|u_1 - u_2|^2}{2\sigma^2} \right] \\ &\times \exp \left[-j \frac{\bar{k}}{z} (u_1 \cdot x_1 - u_2 \cdot x_2) \right] du_1 du_2 \quad (4.22) \end{aligned}$$

for the Fraunhofer diffraction field.

B. Two-dimensional cases

In two-dimensional cases, the circular source with the radius a is assumed in place

of the slit source in one-dimensional cases. For this purpose, it is convenient to introduce the polar coordinates as follows:

$$\begin{cases} u_i = r_i \cos \theta_i \\ v_i = r_i \sin \theta_i, \end{cases} \quad (i = 1, 2), \quad (4.23a)$$

$$\begin{cases} x_i = R_i \cos \phi_i \\ y_i = R_i \sin \phi_i, \end{cases} \quad (i = 1, 2). \quad (4.23b)$$

Using this relation, Eq.(4.11) becomes

$$\begin{aligned} J_{ob}(R_1, \phi_1; R_2, \phi_2) &= \frac{I_0}{(\bar{\lambda}z)^2} \int_0^{2\pi} \int_0^{2\pi} \int_0^a \int_0^a S(r_1)S(r_2) \\ &\times \exp \left[-\frac{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)}{2\sigma^2} \right] \\ &\times \exp \left[-j\frac{\bar{k}}{z} \{R_1r_1 \cos(\theta_1 - \phi_1) - R_2r_2 \cos(\theta_2 - \phi_2)\} \right] \\ &\times \exp \left[j\frac{\bar{k}}{2z} \{(r_1^2 - r_2^2) + (R_1^2 - R_2^2)\} \right] r_1r_2 dr_1 dr_2 d\theta_1 d\theta_2. \end{aligned} \quad (4.24)$$

Due to the assumption of the circular source and the circular symmetrical source filter in the analysis, diffraction patterns appear in a symmetrical form with respect to the normal axis passing through the center of the source. Thus, the angular coordinates ϕ_1 and ϕ_2 are taken as 0 and ϕ , respectively, without the loss of generality.

Here, we define the following symbols for simplicity,

$$u = \left(\frac{2}{\sigma}\right)^2, \quad w = u - j\left(\frac{4\bar{k}}{z}\right). \quad (4.25)$$

The integration with respect to θ_1 in Eq.(4.24) is performed as follows [180]:

$$\begin{aligned} I_{\theta_1} &= \int_0^{2\pi} \exp \left[\frac{r_1r_2}{\sigma^2} \cos(\theta_1 - \theta_2) \right] \exp \left[-j\bar{k} \frac{R_1r_1}{z} \cos \theta_1 \right] \\ &= 2\pi J_0 \left(jr_1 \sqrt{C - jD} \right) \end{aligned} \quad (4.26)$$

where J_0 is the Bessel function of zero order and the symbols C and D are given by

$$C = \left(\frac{r_2u}{4}\right)^2 - \left(\frac{\bar{k}R_1}{z}\right)^2,$$

$$D = \left(\frac{\bar{k} R_1 r_2 u}{2z} \right) \cos \theta_2. \quad (4.27)$$

Furthermore, integration with respect to r_1 given by

$$H(r_2, \theta_2) = \int_0^a S(r_1) \exp \left[-\frac{w}{8} r_1^2 \right] J_0 \left(j r_1 \sqrt{C - jD} \right) r_1 dr_1, \quad (4.28)$$

can be performed with the help of the Hankel transforms of zero order [181] if the transmittance of the source filter is given. As a result, Eq.(4.24) becomes

$$\begin{aligned} J_{ob}(R_1, 0; R_2, \phi) &= \frac{2\pi I_0}{(\bar{\lambda} z)^2} \int_0^{2\pi} \int_0^a S(r_2) H(r_2, \theta_2) \\ &\times \exp \left[-\frac{r_2^2}{2\sigma^2} \right] \exp \left[-j \frac{\bar{k}}{z} \left\{ \frac{1}{2} r_2^2 - R_2 r_2 \cos(\theta_2 - \phi) \right\} \right] \\ &\times \exp \left[j \frac{\bar{k}}{2z} (R_1^2 - R_2^2) \right] r_2 dr_2 d\theta_2 \end{aligned} \quad (4.29)$$

for the Fresnel diffraction field. In particular, for the source without a source filter corresponding to the case that transmittance of the source filter is unity within the source region, the Hankel transform of Eq.(4.28) becomes

$$\begin{aligned} H(r_2, \theta_2) &= \left(\frac{4}{jw} \right) \exp \left[-\frac{w}{8} a^2 \right] \\ &\times \left[U_1 \left(j \frac{w}{4} a^2, ja \sqrt{C - jD} \right) - j U_2 \left(j \frac{w}{4} a^2, ja \sqrt{C - jD} \right) \right], \end{aligned} \quad (4.30)$$

where U_1 and U_2 are Lommel's functions defined by

$$U_\nu(w, z) = \sum_{m=0}^{\infty} (-1)^m \left(\frac{w}{z} \right)^{(\nu+2m)} J_{\nu+2m}(z), \quad (4.31)$$

with $J_{\nu+2m}(z)$ being the Bessel function of the $(\nu + 2m)$ -th order. By inserting Eq.(4.30) into Eq.(4.29) and by replacing $S(r_2)$ contained in Eq.(4.29) by unity, the mutual intensity at the Fresnel diffraction field arising from the source having no source filter can be obtained.

When the observation plane is placed in the Fraunhofer diffraction field, the expression for the mutual intensity of light at the observation plane can be written,

corresponding to Eq.(4.29), in the form

$$J_{ob}(R_1, 0; R_2, \phi) = \frac{2\pi I_0}{(\bar{\lambda}z)^2} \int_0^{2\pi} \int_0^a S(r_2) H(r_2, \theta_2) \times \exp\left[-\frac{r_2^2}{2\sigma^2}\right] \exp\left[j\frac{\bar{k}}{z} R_2 r_2 \cos(\theta_2 - \phi)\right] r_2 dr_2 d\theta_2, \quad (4.32)$$

where

$$H(r_2, \theta_2) = \int_0^a S(r_1) \exp\left[-\frac{u}{8} r_1^2\right] J_0\left(jr_1\sqrt{C-jD}\right) r_1 dr_1. \quad (4.33)$$

If the transmittance of the source filter is unity within the range of integration, as well as Eq.(4.30), Eq.(4.33) can be easily evaluated and becomes

$$H(r_2, \theta_2) = \left(\frac{4}{ju}\right) \exp\left[-\frac{u}{8} a^2\right] \times \left[U_1\left(j\frac{u}{4} a^2, ja\sqrt{C-jD}\right) - jU_2\left(j\frac{u}{4} a^2, ja\sqrt{C-jD}\right)\right]. \quad (4.34)$$

By substituting Eq.(4.34) into Eq.(4.32) and eliminating the function $S(r_2)$ in Eq.(4.32), the mutual intensity in the Fraunhofer diffraction field arising from the source with no source filter can also be obtained.

4.2.3 Correlation functions in two domains

In this section, the implication for the adoption of the mutual intensity is briefly discussed to justify the present method of formulating the problem.

Let us recall that the present study is concerned with the field propagation from a partially coherent source in order to elucidate the effect of the spatial filtering method over the source, as an extended continuation of the studies of radiometry with partially coherent sources and the studies of diffraction by an aperture with the partially coherent light. The main aim of the former studies is to determine the radiometric quantities associated with a partially coherent source and hence the behavior of the energy flow in the far field of the source has been examined by means of the state of spatial coherence across the source. In these studies, the basic quantity to describe the field correlations and to formulate the theory is the correlation function in the space-frequency domain, i.e., the cross-spectral density function

$W(\mathbf{r}_1, \mathbf{r}_2; \omega)$, mainly because the spectral description of radiometric quantities was often convenient in conventional radiometry ([148], pp. 240–244). On the contrary, the correlation function in the space-time domain, i.e., the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2; \tau)$ or the mutual intensity $J(\mathbf{r}_1, \mathbf{r}_2)$, was used in the latter studies concerning diffractions with the partially coherent light. However, even in the former studies, there was an exceptional problem in which the cross-spectral density function was inadequate to describe the phenomena. As is well known, transverse modes of practical multimode lasers oscillate at slightly different frequencies and their mixture results in the reduction of their spatial coherence [109]. Since the Fourier component of the field vibrations with different frequencies are mutually uncorrelated [150], the process of the coherence reduction by mode superposition cannot be described precisely in terms of the correlation function in the space-frequency domain. Accordingly, Gori [182] analyzed the relationship between the Gaussian Shell-model source and the multimode lasers, using the mutual intensity under the quasi-monochromatic approximation in place of the cross-spectral density function. After his work, Wolf and Agarwal [118] dealt with the spatial coherence of laser resonator modes in the space-frequency domain on the basis of the orthogonal expansion of the cross-spectral density function established by Wolf [120], and they showed using the cross-spectral density function that the simultaneous excitation of several transverse modes reduces the spatial coherence. Although the main result of their analysis has no mathematical inconsistency, it appears to be one step further removed from the physics of interest because of its mathematical complexity. The problem of this kind can be well understood by introducing the mutual intensity as analyzed by Gori, rather than the cross-spectral density function.

In general, the phenomena described by the function in the space-time domain, rather than that in the space-frequency domain, seem to be well understood, primarily because the correlation function in the space-time domain is directly measurable through a simple experiment and hence it provides a good intuitive understanding of the phenomena. Although the spectral treatment of the problem can become a powerful tool for studying the phenomena having a strong frequency-dependence as mentioned in Section 2.2, the subject of this chapter does not belong to the category of this kind. In addition, recall here that the model source employed in this section is possibly synthesized also by a statistically-averaged multimode laser, as mentioned

in Section 4.2.1. To this end, the simplest correlation function in the space-time domain, i.e., the mutual intensity has been adopted under the quasi-monochromatic approximation, avoiding the mathematical complexity and emphasizing the physics of concern, in spite of the fact the study of radiometry with a partially coherent source is developed in the space-frequency domain.

Fortunately, the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ can be readily replaced by the mutual intensity $J(\mathbf{r}_1, \mathbf{r}_2)$, and *vice versa*, in any studies concerning the propagation of the partially coherent field because both functions propagate in accord with the same set of Helmholtz equations (see Section 2.5). The only difference is the expression of the frequency in these equations: mean frequency $\bar{\nu}$ (or mean wavelength $\bar{\lambda}$) is used in the mutual intensity while one-particular frequency ν (or one-particular wavelength λ) is used in the cross-spectral density function. Hence, the propagation law of the cross-spectral density function formulated, e.g., by Marchand and Wolf [156] in the study of radiometry with partially coherent sources can coincide with the propagation law of the mutual intensity appearing in many literatures [3, 147–149] provided $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is replaced by $J(\mathbf{r}_1, \mathbf{r}_2)$ and that the frequency ν (or the wavelength λ) is replaced by the mean frequency $\bar{\nu}$ (or the mean wavelength $\bar{\lambda}$). However, it should be noted that the physical meanings of those formulations in two different domains are quite different from each other even though each mathematical description is the same except for notation. From the spectral point of view, the mutual intensity is applicable only under the quasi-monochromatic approximation while the cross-spectral density function has no such limitation. Therefore, detailed knowledge of the spectral distribution of light used in the analysis is required to describe the feature of the field when using the correlation function in the space-frequency domain, i.e., $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$.

4.3 Results and Discussion

In this section, some illustrative results only in the case of one-dimensional analysis are presented for simplicity. To put it more precisely, the properties of the field produced by a partially coherent slit source are studied on the basis of Eqs.(4.21) and (4.22). From these formulas, the intensity distribution and the complex degree

of spatial coherence for two points are obtained by

$$I_{ob}(x) = J_{ob}(x, x) \quad (4.35)$$

$$\gamma_{ob}(x_1, x_2) = \frac{J_{ob}(x_1, x_2)}{\sqrt{I_{ob}(x_1)}\sqrt{I_{ob}(x_2)}}, \quad (4.36)$$

respectively. Since the two points for measuring the spatial coherence are assumed to be situated symmetrically about the optical axis at the observation plane (see Fig.4.2), Eq.(4.36) becomes

$$\gamma_{ob}(x, -x) = \frac{J_{ob}(x, -x)}{\sqrt{I_{ob}(x)}\sqrt{I_{ob}(-x)}} = \frac{J_{ob}(x, -x)}{I_{ob}(x)}, \quad (4.37)$$

where $I_{ob}(x) = I_{ob}(-x)$ because the resultant field appears in a symmetrical form with respect to the origin at the observation plane. Equations (4.35) and (4.37) were evaluated numerically on a computer using a subroutine program of two-dimensional integration by means of Simpson's method for various coherence conditions over the source and for some given source filters. To characterize the spatial coherence condition over the source, we make use of the normalized values of the coherence interval across the source defined by

$$\sigma_N = \frac{\sigma}{a} \quad (4.38)$$

where σ is a positive constant contained in Eq.(4.6) and a is the half-width of the slit source. By using Eqs.(4.6) and (4.38), the diagram of the correlation across the

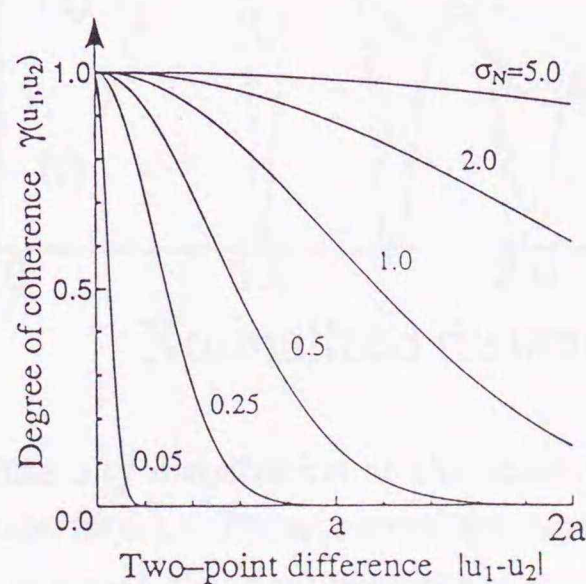


Figure 4.3. Gaussian form of correlation across the source for various values of σ_N .

source for various values of σ_N is depicted in Fig.4.3. In computations, the mean wavelength $\bar{\lambda} = 6.0 \times 10^{-7}[\text{m}]$ and the half-width of the source $a = 1.0 \times 10^{-3}[\text{m}]$ were used.

4.3.1 Radiation properties of the partially coherent planar source

Before investigating the effect of source filters on the spatial coherence of light in the radiated field, we first deal with the radiation properties of the partially coherent slit source having uniform intensity distribution, i.e., without the source filter. In this case, the transmittance of the source filter contained in Eqs.(4.21) and (4.22) is taken to be unity within the range of integrations.

Figure 4.4 shows the normalized intensity distributions at the observation plane located in the Fraunhofer diffraction field for various coherence conditions over the source σ_N . In drawing these curves, Eq.(4.35) was evaluated using Eq.(4.22). The

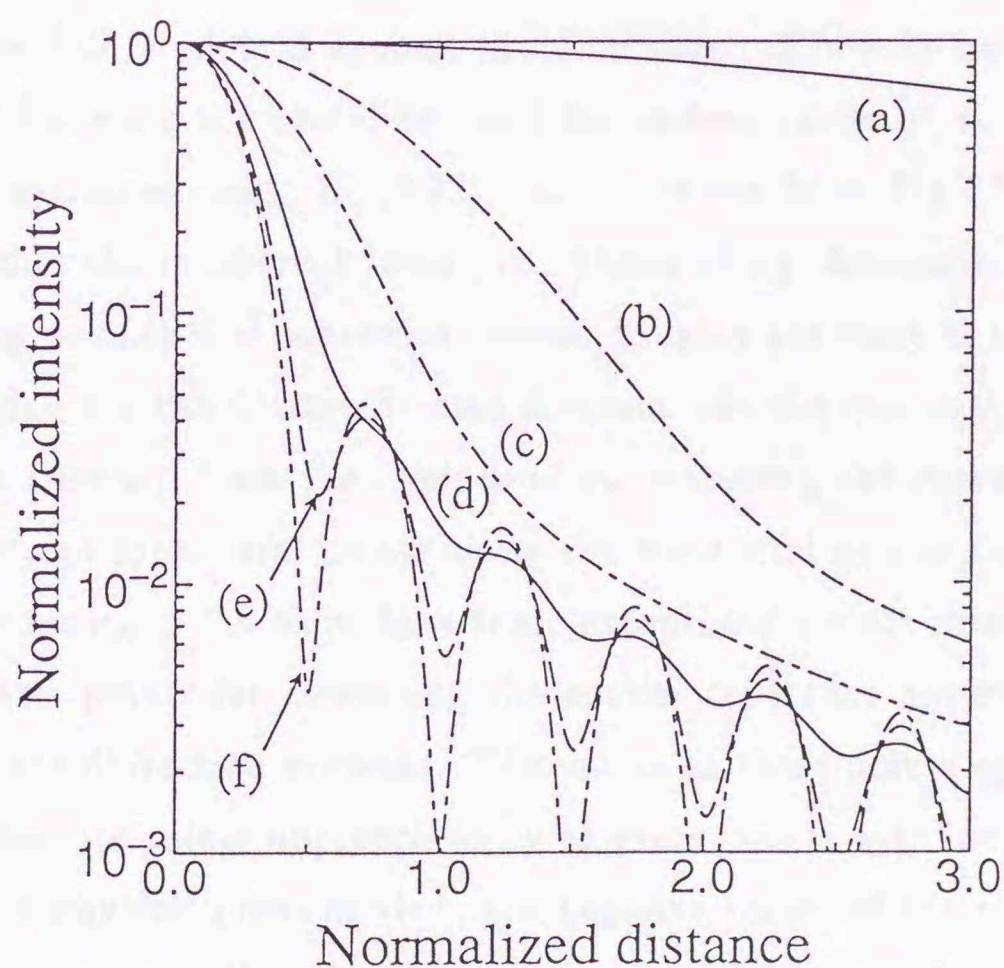


Figure 4.4. Normalized intensity distribution at the observation plane arising from the source itself (with no source filter). These curves are plotted against the normalized distance $ax/(\bar{\lambda}z)$ (where x is a real distance) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0 and (f) 5.0.

normalization of the intensity distribution was conducted by dividing the intensity at each point by the value at the origin. Curves of intensity are drawn in a log-normal scale against the normalized distance $ax/(\bar{\lambda}z)$ (where x is a real distance) from the optical axis at the observation plane in order to enhance the higher-order components of the intensity. It is seen from Fig.4.4 that, as the spatial coherence across the source increases (i.e., σ_N increases), the light tends to be collected around the optical axis at the observation plane. For small values of σ_N between 0.05 and 0.5, the spread of the intensity distribution varies sensitively by the values of σ_N as compared with the case of the values σ_N beyond 2.0. For large values of $\sigma_N > 2.0$, the qualitative features of the intensity distribution remain roughly the same for the case of completely coherent diffraction pattern, i.e., sinc^2 -form. Since detailed features of the intensity distribution produced by partially coherent sources of various kinds have already been studied thoroughly by a large number of researchers in the past [166–179], further discussion relating to the intensity is not presented here.

In Fig.4.5, the complex degree of spatial coherence for two points in the Fraunhofer diffraction field is plotted against the normalized difference between the two points $ad/(\bar{\lambda}z)$ (where d is a real difference) for various values of σ_N . In this case, Eq.(4.37) was evaluated using Eq.(4.22). It is obvious from Fig.4.5 that, as the source approaches the incoherent limit (i.e., values of σ_N decrease), the curves of the complex degree of spatial coherence become roughly the same as the curve evaluated by applying the van Cittert-Zernike theorem. On the contrary, as the source approaches the coherent limit (i.e., values of σ_N increase), the curves tend to approach unity except for several points along the horizontal axis in these coherence curves. Curves for $\sigma_N > 2.0$ show that these exceptional points correspond to the case that the two points for measuring the spatial coherence are situated at the dark region in the diffraction patterns. The values at these points approach -1.0 , so that their absolute values approach unity at every points with increasing values of σ_N . From the physical point of view, the negative values of the complex degree of spatial coherence reveal that the phase of the mutual intensity for the two points changes with π . In addition to the behavior of the coherence curve for large σ_N , an interesting phenomenon is observed in coherence curves also between the two coherent ($\sigma_N \rightarrow \infty$) and incoherent ($\sigma_N \rightarrow 0$) limits. Though the values of the complex degree of spatial coherence decrease with oscillations with an increase of

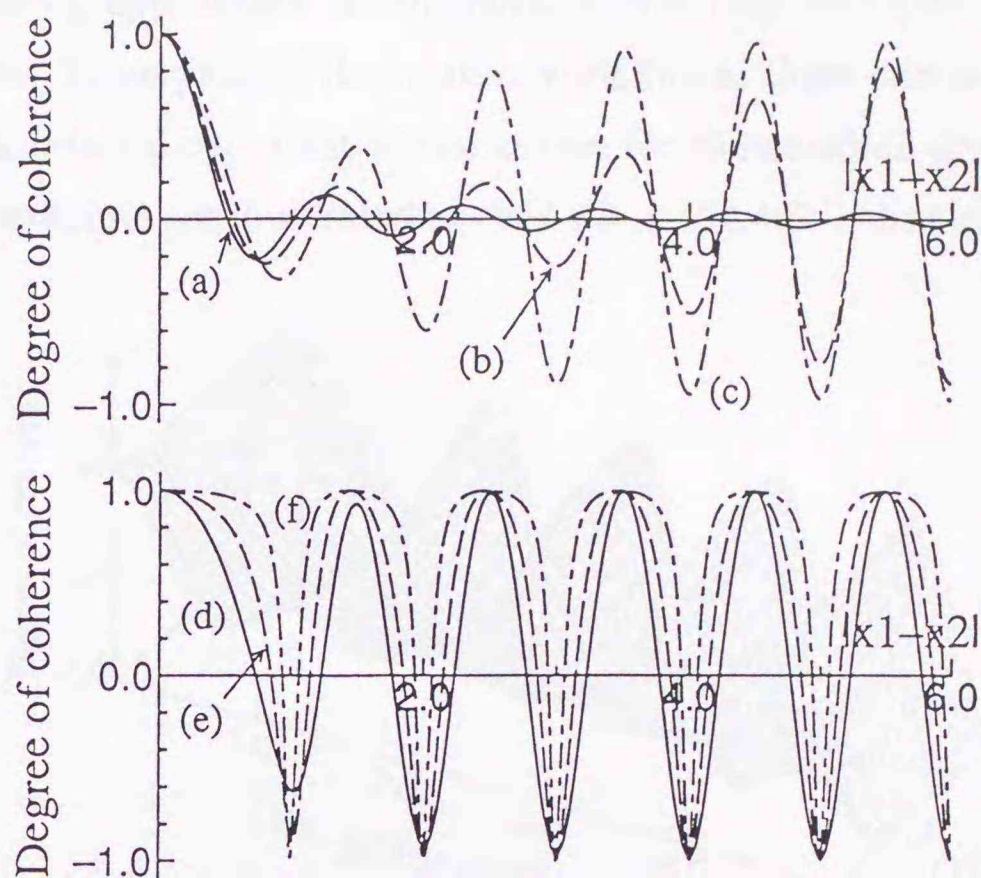


Figure 4.5. Complex degree of spatial coherence at the observation plane arising from the source itself (with no source filter). These curves are plotted against the normalized difference $ad/(\bar{\lambda}z)$ (where d is a real difference) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0 and (f) 5.0.

the two-point difference if the two points for measuring the spatial coherence are within the boundary at which the intensity drops about 80 per cent from its maximum, gradually increasing and oscillating performances appear in coherence curves as the difference becomes larger. This phenomenon can also be found in the past work [178] dealing with the far-field diffraction by a circular aperture illuminated with the partially coherent light. On the other hand, even in a similar study, this performance in coherence curves has never been observed in the far-field diffraction of a Gaussian Shell-model source [44]. Both studies were essentially concerned with the diffraction problem with the partially coherent light. Only the difference in the formulations in these studies is the intensity distribution across the diffracting aperture: uniform distribution over the aperture was assumed in the former study and the (infinite) Gaussian distribution was assumed in the latter study (the complex degree of spatial coherence over the aperture and the source was taken to have a Gaussian form in both studies). By comparing these two studies, it is understood

that this interesting appearance in coherence curves may be caused by the finite size of the source. To emphasize the relative variation of these curves as a function of the coherence interval σ_N , a set of the curves for the complex degree of spatial coherence for two points are illustrated against σ_N in Fig.4.6. It has become obvious

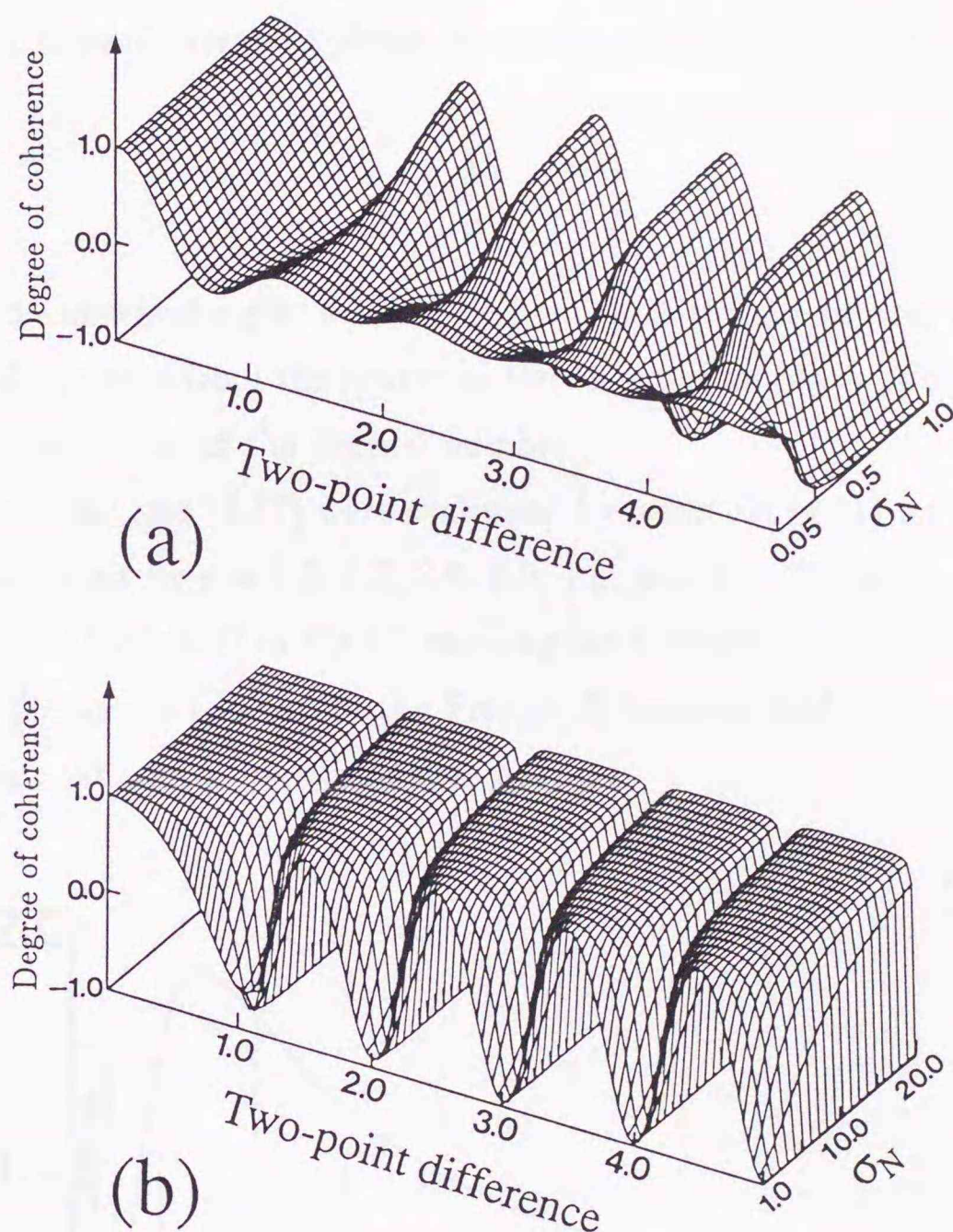


Figure 4.6. Complex degree of spatial coherence corresponding to Fig.4.5 for (a) small values of σ_N (from 0.05 to 1.0) and (b) large values of σ_N (from 1.0 to 20.0).

from this figure that the coherence properties over the radiated field is also largely affected by the coherence condition over the source σ_N .

4.3.2 Propagation of the partially coherent field

The previous section was concerned with the field properties in the Fraunhofer diffraction field arising from a partially coherent slit source. In practice, however, the

Fresnel diffraction field should also be investigated because the Fraunhofer diffraction is hardly realized in a strict sense in most applications of optics. For this purpose, we deal with in this section the field properties at various observation planes located especially in the transition region from the Fresnel to the Fraunhofer diffraction field.

To designate the observation planes, we make use of the propagation parameter p defined by

$$p = \frac{\bar{\lambda}z}{a^2}, \quad (4.39)$$

where $\bar{\lambda}$ is the mean wavelength, a is the half-width of the slit source, and z is the real longitudinal distance from the source to the observation plane. This parameter implies also the reciprocal of the Fresnel number.

Equations (4.35) and (4.37) were evaluated by using Eq.(4.21) for six observation planes, designated by $p = 1.0, 2.0, 3.0, 5.0, 7.0$, and 9.0 . These six observation planes are indicated by (a)-(f) in Fig.4.7 showing the intensity distribution along the normal axis to the source (z -axis) in the Fresnel diffraction field by the completely spatially coherent slit source for a reference purpose.

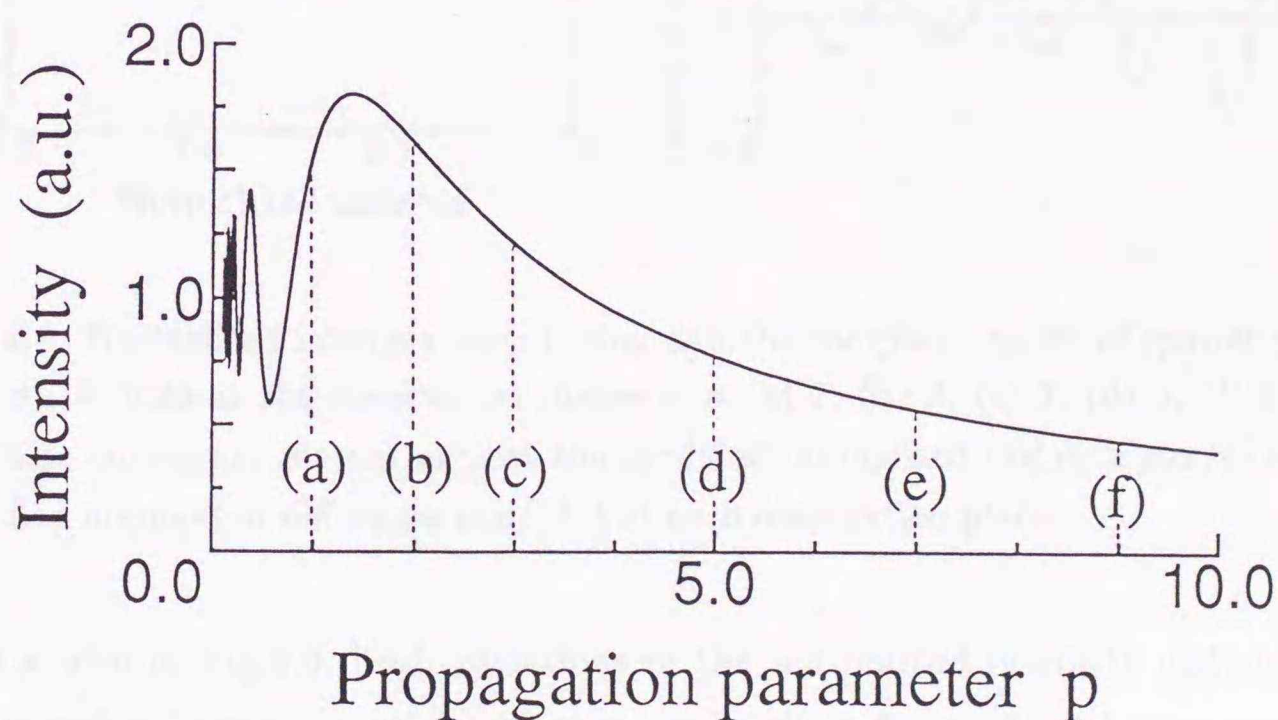


Figure 4.7. Axial intensity distribution in the Fresnel diffraction field of a coherent slit source. In the figure, six observation planes are indicated on which the variation of the normalized intensity and the complex degree of spatial coherence are investigated.

The normalized intensity distribution and the complex degree of spatial coherence at the six observation planes are shown in Figs.4.8-11 for several values of σ_N . By taking into account that the intensity distribution extends further out in the transverse direction at each observation plane with increasing distance from the source (i.e., the propagation parameter p increases), the intensity at the observation plane designated by the propagation parameter p is plotted against the modified normalized distance $pax/(\bar{\lambda}z)$ (where x is a real distance) from the optical axis at each plane. Similarly, the complex degree of spatial coherence for two points is plotted against the modified normalized difference for two points $pad/(\bar{\lambda}z)$ (where p is a propagation parameter and d is a real difference).

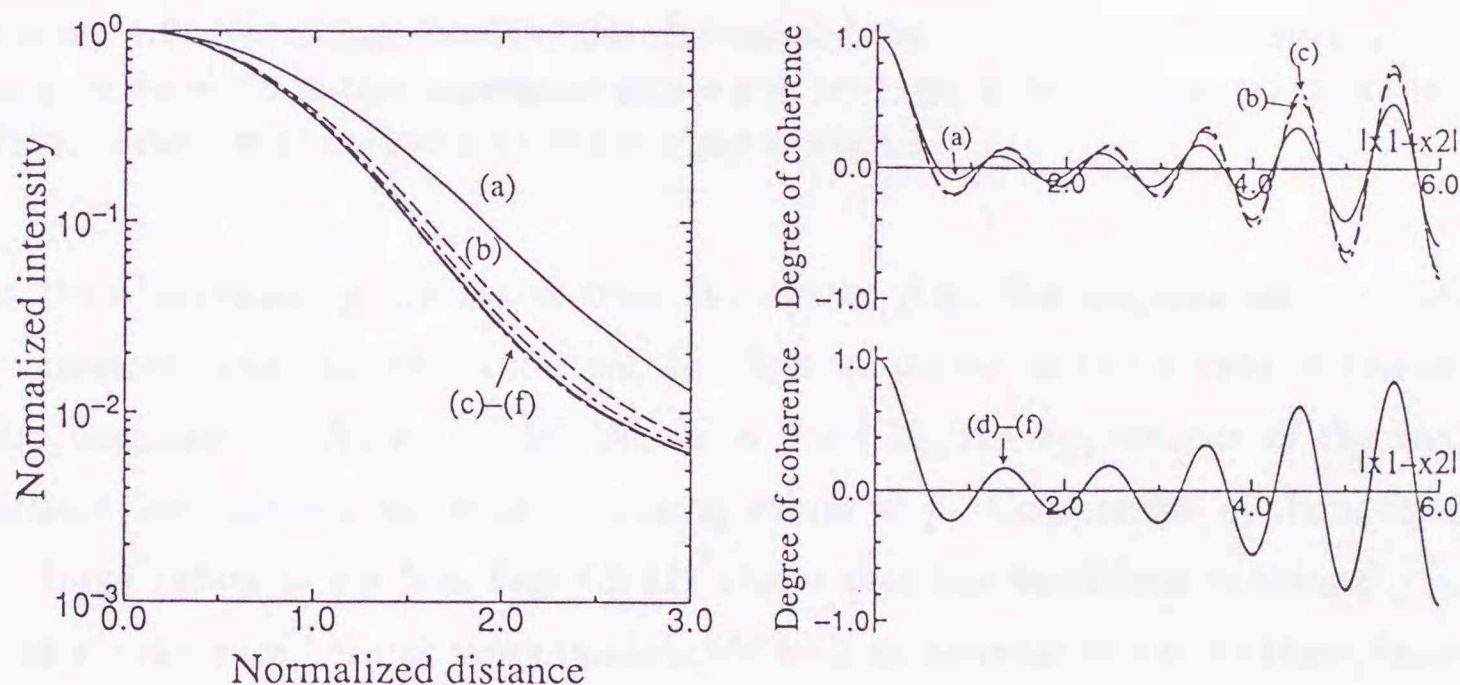


Figure 4.8. Normalized intensity distribution and the complex degrees of spatial coherence for $\sigma_N = 0.25$ at the observation planes $p =$ (a) 1, (b) 2, (c) 3, (d) 5, (f) 7, and (e) 9. These curves are plotted against the modified normalized distance $pax/(\bar{\lambda}z)$ and the modified normalized difference $pad/(\bar{\lambda}z)$ at each observation plane.

As shown in Fig.4.8, both variations in the normalized intensity distribution and the complex degree of spatial coherence are hardly influenced by the propagation for small values of σ_N . On the other hand, it is found from Figs.4.9-11 that not only the intensity distribution but also the complex degree of spatial coherence are largely changed by the process of propagation for the values of σ_N approximately larger than 1.0. More precisely, the first minimum of the coherence curves decrease

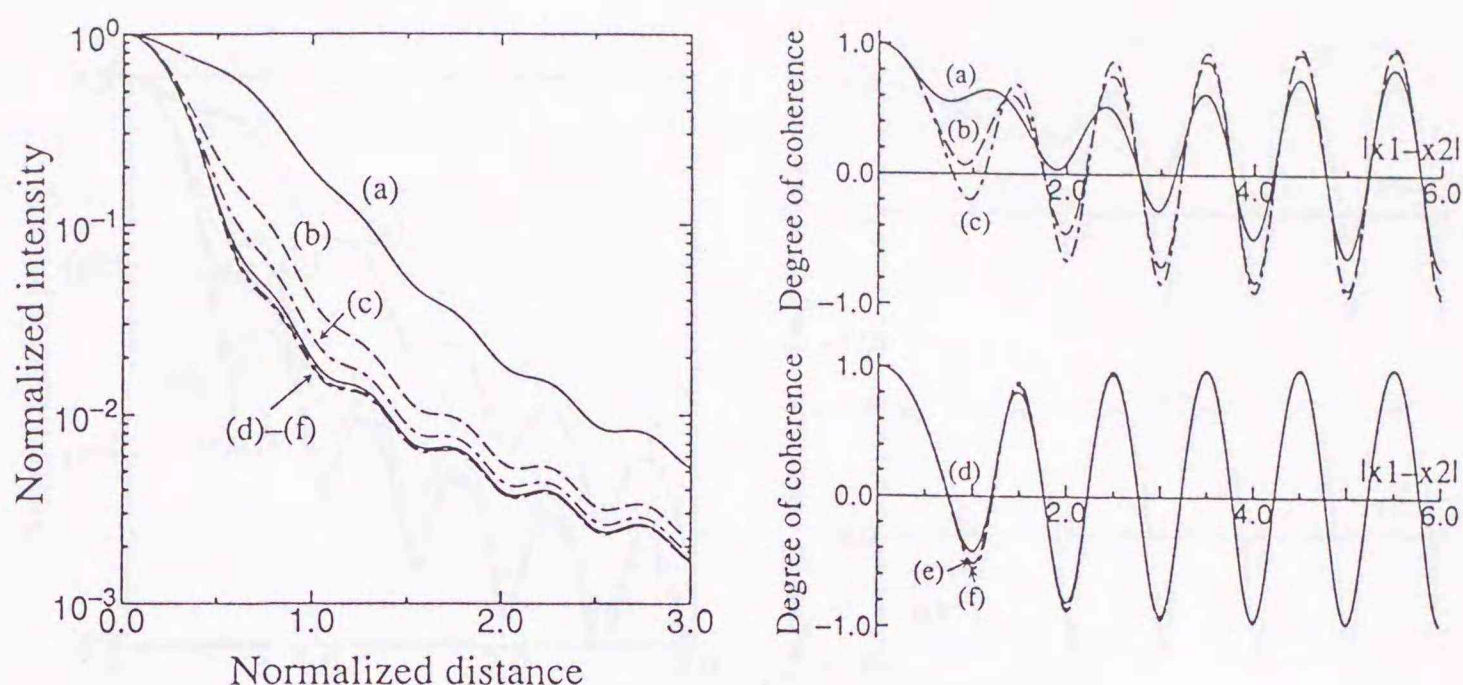


Figure 4.9. Normalized intensity distribution and the complex degrees of spatial coherence for $\sigma_N = 1.0$ at the observation planes $p =$ (a) 1, (b) 2, (c) 3, (d) 5, (f) 7, and (e) 9. These curves are plotted in a similar manner to Fig.4.8.

as the observation plane leaves from the source (i.e., the propagation parameter p increases) and, at the same time, the first minimum of the curves in intensity also decreases. At $\sigma_N = 5.0$ shown in Fig.4.11, the appearance of the fringe pattern becomes evident with increasing values of p . Comparison of these figures for large values of σ_N (see Figs.4.9-11) shows that the variations in intensity and spatial coherence become more remarkable with an increase of σ_N . In these figures, however, little variation is observed in intensity for the values of p larger than about 5.0 while the intensity changes gradually with p less than about 5.0. In other words, the Fraunhofer diffraction patterns are observed at each observation plane having the parameter p larger than about 5.0 and the behavior of spatial coherence is distinguished from each other even at these planes. In the limiting case of the completely spatially coherent source ($\sigma_N \rightarrow \infty$), no variation in spatial coherence with propagation may be found and the complex degree of spatial coherence is expected to be kept constant as unity with propagation, though the normalized intensity distribution is varied by the propagation in the Fresnel diffraction field near from the source.

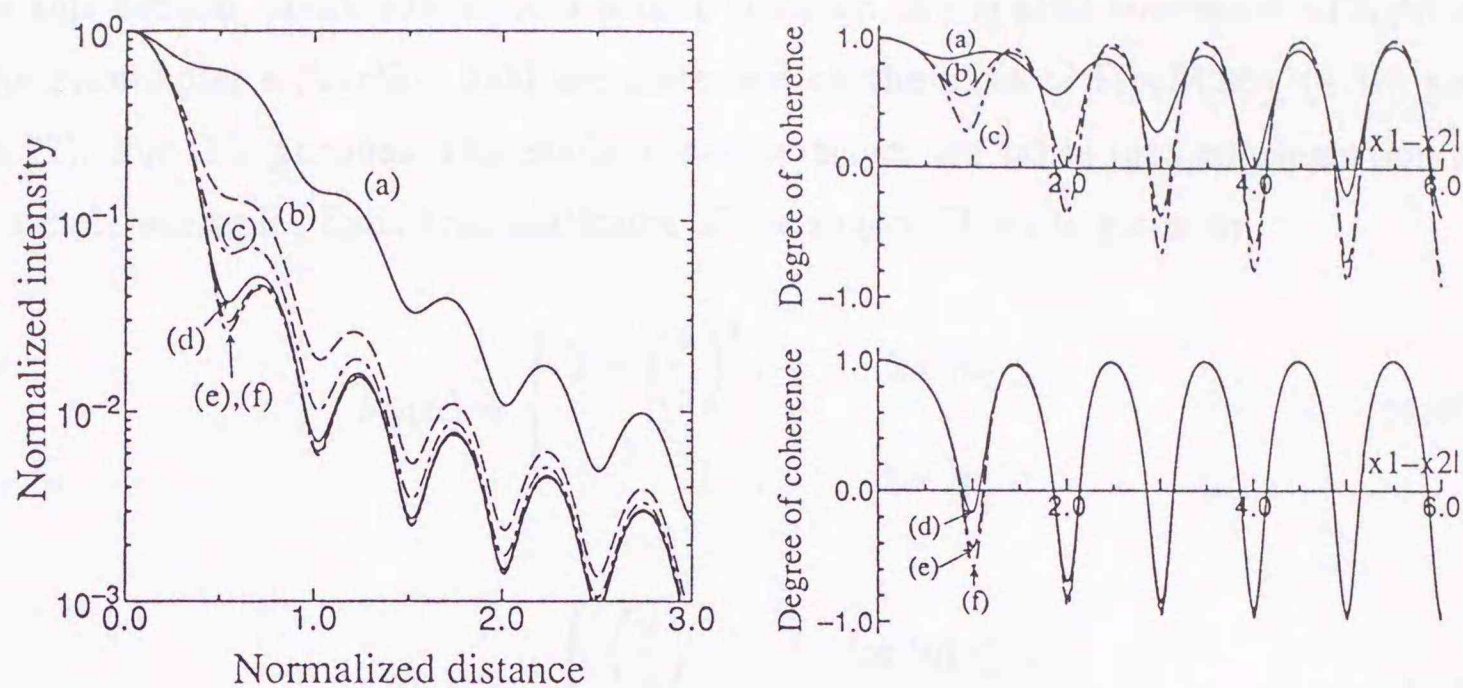


Figure 4.10. Normalized intensity distribution and the complex degrees of spatial coherence for $\sigma_N = 2.0$ at the observation planes $p = (a) 1, (b) 2, (c) 3, (d) 5, (f) 7$, and $(e) 9$. These curves are plotted in a similar manner to Fig.4.8.

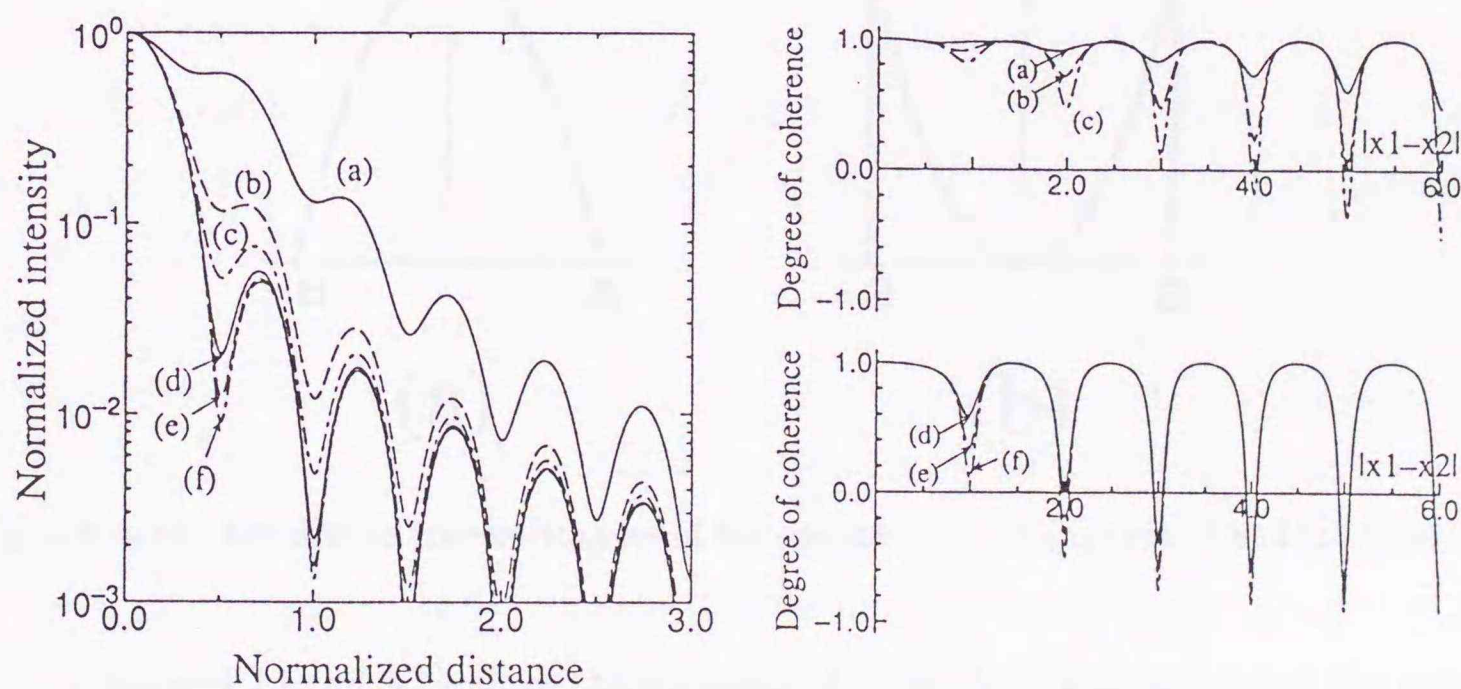


Figure 4.11. Normalized intensity distribution and the complex degrees of spatial coherence for $\sigma_N = 5.0$ at the observation planes $p = (a) 1, (b) 2, (c) 3, (d) 5, (f) 7$, and $(e) 9$. These curves are plotted in a similar manner to Fig.4.8.

4.3.3 Effects of source filters

In this section, some effects of a source filter on the spatial coherence of light in the Fraunhofer diffraction field are discussed on the basis of Eqs.(4.35), (4.37) and (4.22). For this purpose, two kinds of source filters are taken into consideration as a simple example. Each transmittance of the source filters is given by

$$S_B(u) = \begin{cases} 1 - \left(\frac{u}{a}\right)^2, & \text{for } |u| \leq a \\ 0, & \text{for } |u| > a, \end{cases} \quad (4.40)$$

$$S_C(u) = \begin{cases} \left(\frac{u}{a}\right)^2, & \text{for } |u| \leq a \\ 0, & \text{for } |u| > a, \end{cases} \quad (4.41)$$

where these source filters $S_B(u)$ and $S_C(u)$ are designated by type B and type C, respectively. The diagrams of these transmittances are illustrated in Fig.4.12.

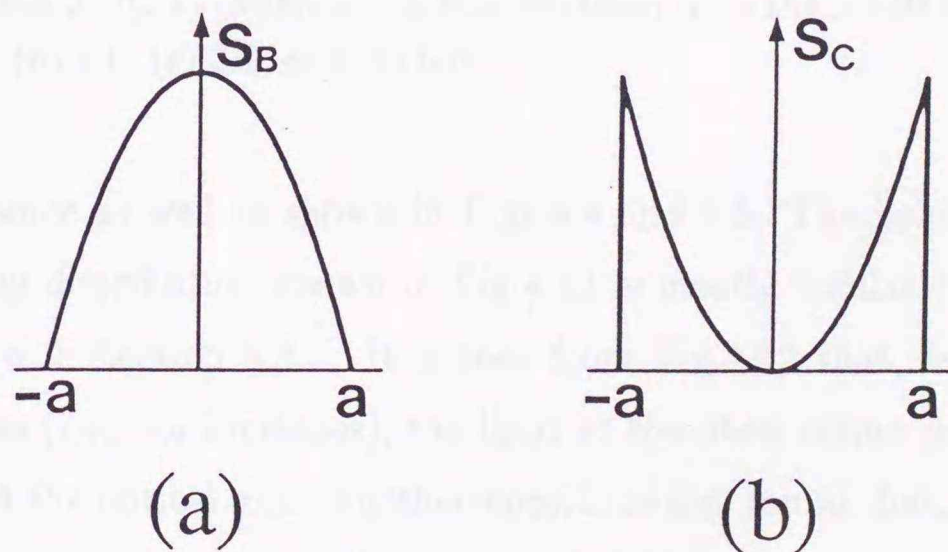


Figure 4.12. Amplitude transmittances of the source filters of (a) type B and (b) type C.

Figures 4.13 and 4.14 show the normalized intensity distribution and the complex degree of spatial coherence of light at the observation plane arising from the partially coherent source covered with the source filter of type B. These curves are also plotted against the normalized distance $ax/(\bar{\lambda}z)$ from the optical axis in the normalized intensity and the normalized difference $ad/(\bar{\lambda}z)$ in the complex degree

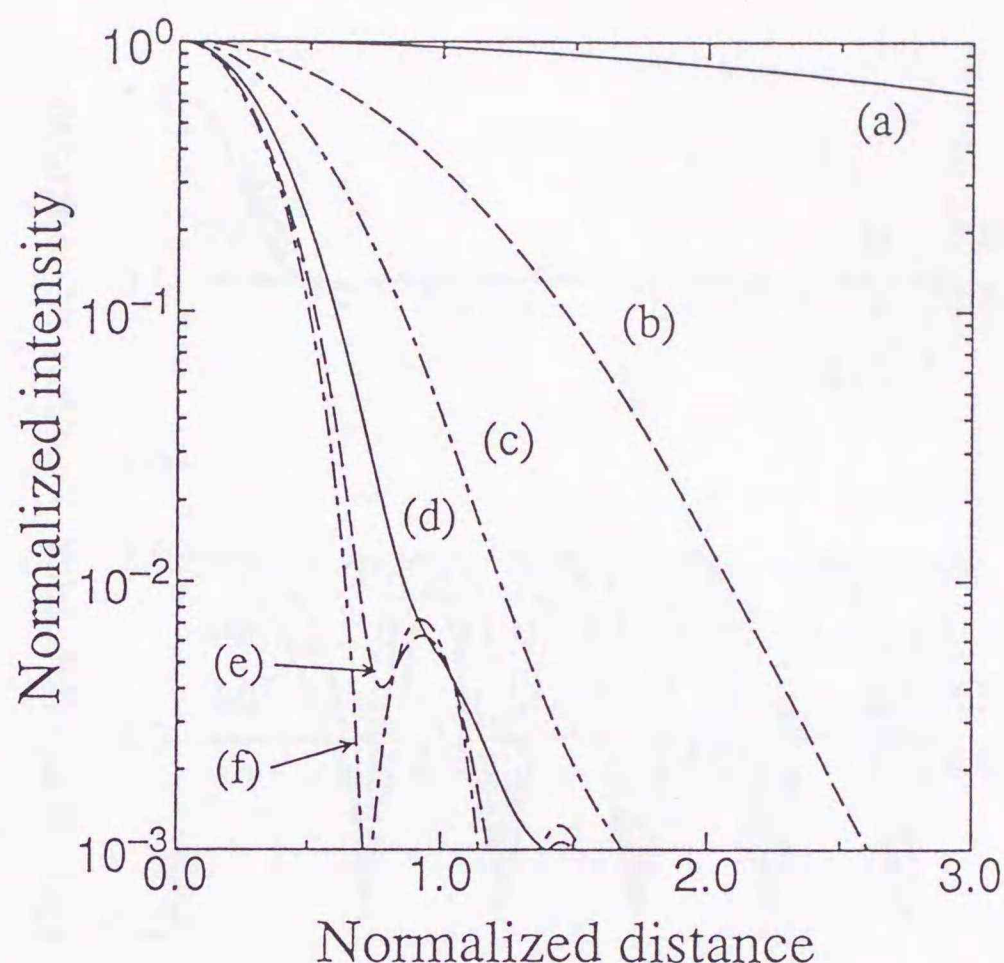


Figure 4.13. Normalized intensity distribution at the observation plane arising from the source covered with the source filter of type B. These curves are plotted against the normalized distance $ax/(\bar{\lambda}z)$ (where x is a real distance) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0, and (f) 5.0.

of spatial coherence as well as shown in Figs.4.4 and 4.5. The behavior of the normalized intensity distribution shown in Fig.4.13 is mostly similar to that shown in Fig.4.4 discussed in Section 4.3.1. It is seen from Fig.4.13 that, as the spatial coherence increases (i.e., σ_N increases), the light at the observation plane tends to be collected around the optical axis. Furthermore, it is also found that, with increasing values of σ_N , the intensity distribution varies sensitively for small values of σ_N while that for large values of σ_N remains almost stable in variation. However, detailed comparison of Figs.4.13 and 4.4 shows that secondary and higher-order maxima in intensity considerably decrease for large σ_N by use of the source filter of type B. This phenomenon implies that the source filter of this kind serves as a means of apodization even for a partially coherent source, as already pointed out by several researchers in the past works [168, 169]. On the other hand, the behavior in spatial coherence illustrated in Fig.4.14 is distinguished from that illustrated in Fig.4.5.

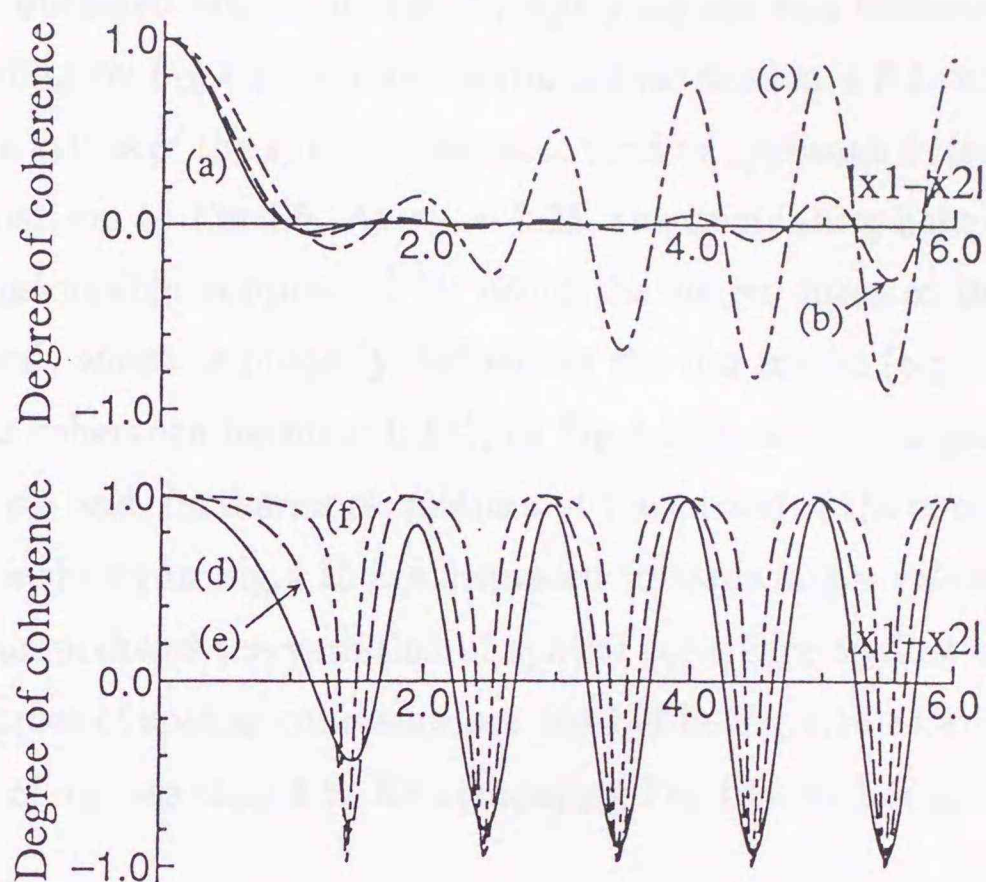


Figure 4.14. Complex degree of spatial coherence at the observation plane arising from the source covered with the source filter of type B. Those curves are plotted against the normalized difference $ad/(\bar{\lambda}z)$ (where d is a real difference) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0, and (f) 5.0.

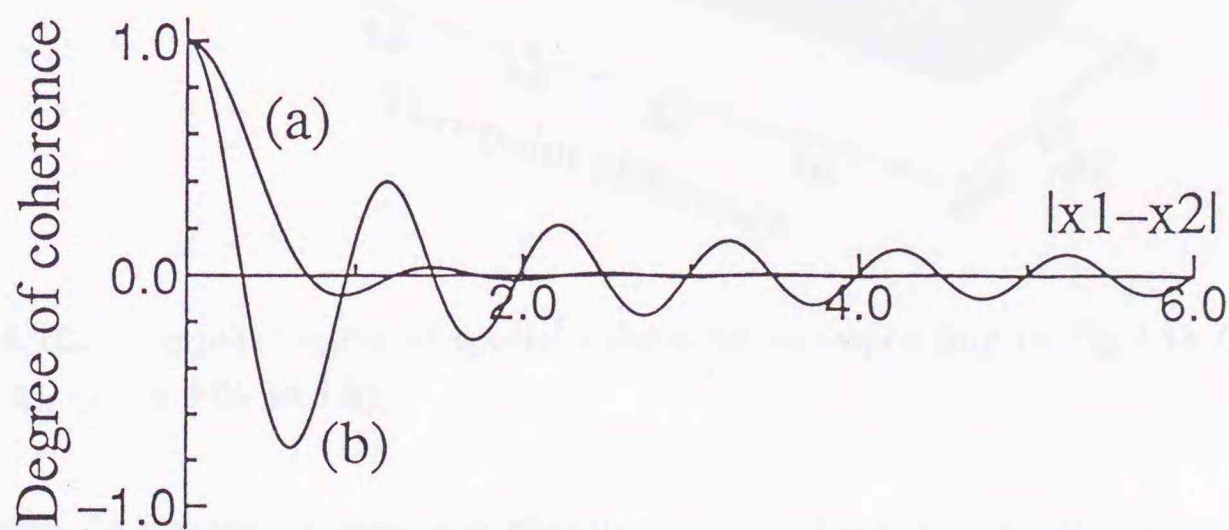


Figure 4.15. Complex degrees of spatial coherence of light generated from an incoherent slit source covered with the source filters of (a) type B and (b) type C. These curves are calculated by applying the van Cittert-Zernike theorem.

Although the curve of the complex degree of spatial coherence at $\sigma_N = 0.05$ in Fig.4.14 can be obtained approximately by applying the van Cittert-Zernike theorem (corresponding to Fig.4.15(a)) as mentioned in Section 4.3.1, with increasing values of σ_N , the values of the spatial coherence tend to approach unity in a different manner in comparison to Fig.4.5. At $\sigma_N = 0.25$, the fluctuating behavior in spatial coherence is considerably suppressed by using the source filter of this kind. The coherence interval, which is properly defined as the dimension (e.g., for which the degree of spatial coherence becomes 0.88), in Fig.4.14 is always larger than that in Fig.4.4 for each σ_N and, furthermore, minima of the curves of the complex degree of spatial coherence shown in Fig.4.15 are displaced towards larger values of two-point difference. To comprehend the variation of spatial coherence against the values σ_N , the complex degrees of spatial coherence are plotted in Fig.4.16 as a function of σ_N for small values of σ_N less than 1.0. By comparing Fig.4.14 with Fig.4.5, no distinct

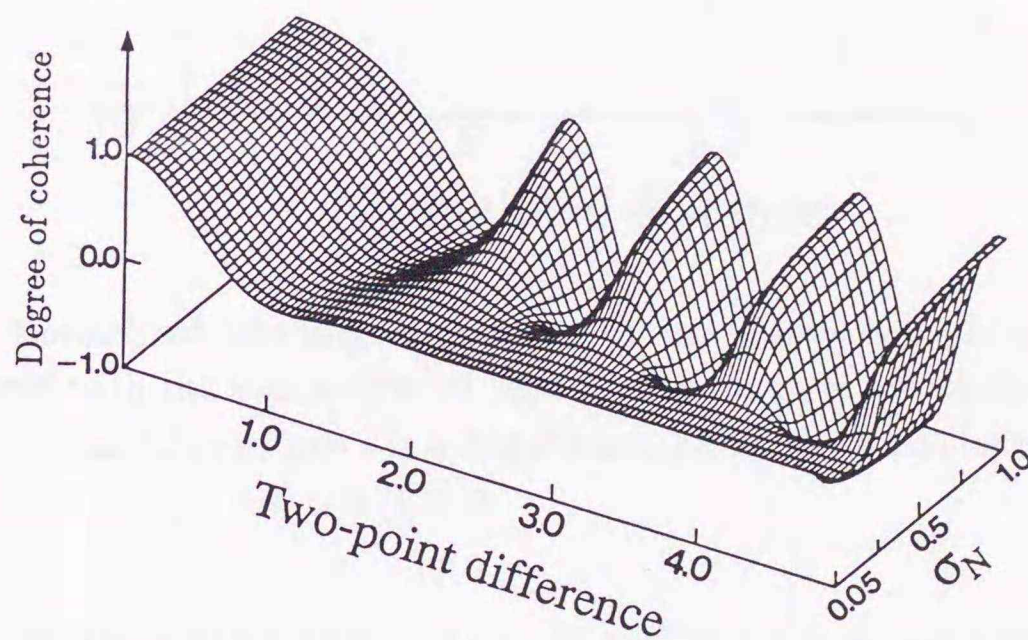


Figure 4.16. Complex degree of spatial coherence corresponding to Fig.4.14 for small values of σ_N (from 0.05 to 1.0).

performance in coherence curves is observed except for the positions of each minimum of the curves for large values of σ_N larger than 1.0. It has become obvious from Figs.4.14 and 4.16 that the source filter of type B providing the effect of apodization has the ability of growing the spatial coherence of light generated from a partially coherent source having uniform intensity distribution, from the standpoint of the coherence interval.

Figures 4.17 and 4.18 show the normalized intensity distribution and the complex degree of spatial coherence of light radiated from the source with the source filter of type C illustrated in Fig.4.12(b). Figure 4.19 shows a set of the complex de-

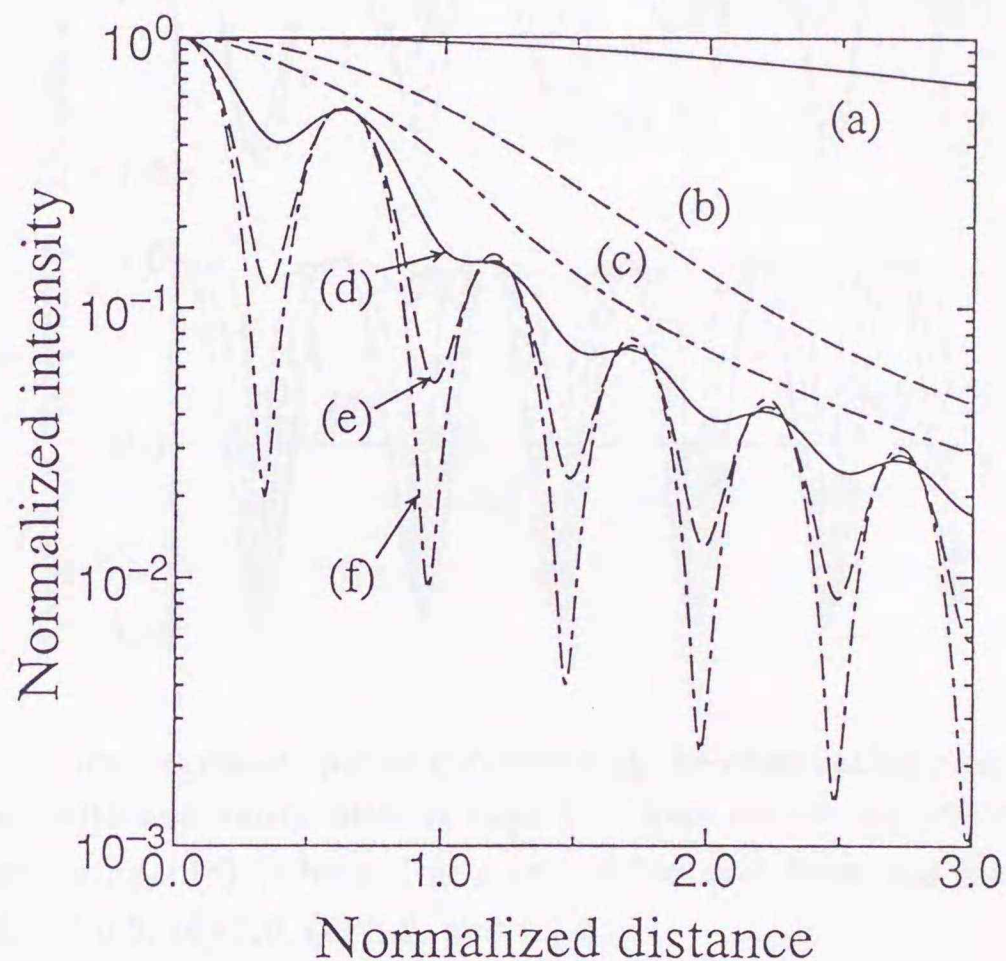


Figure 4.17. Normalized intensity distribution at the observation plane arising from the source covered with the source filter of type C. These curves are plotted against the normalized distance $ax/(\bar{\lambda}z)$ (where x is a real distance) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0, and (f) 5.0.

gree of spatial coherence as a function of σ_N . In addition, the spatial coherence curve calculated by use of the van Cittert-Zernike theorem is illustrated in Fig.4.15(b) for comparison. It is found in Fig.4.17 that, as the values of σ_N increase, the light shows the tendency to be collected around the optical axis at the observation plane and that the secondary and higher-order maxima in intensity appear. These maxima grow up with increasing values of σ_N and, at the same time, the appearance of fringes resulting from those maxima in the diffraction pattern becomes prominent. The normalized intensity shown in Fig.4.17, as opposed to that shown in Figs.4.4 and 4.13, varies sensitively with σ_N not only for small values (e.g., $\sigma_N=0.05$, 0.25 and 0.5), but for large values (e.g., $\sigma_N=1.0$, 2.0 and 5.0). As is seen from Fig.4.18, the

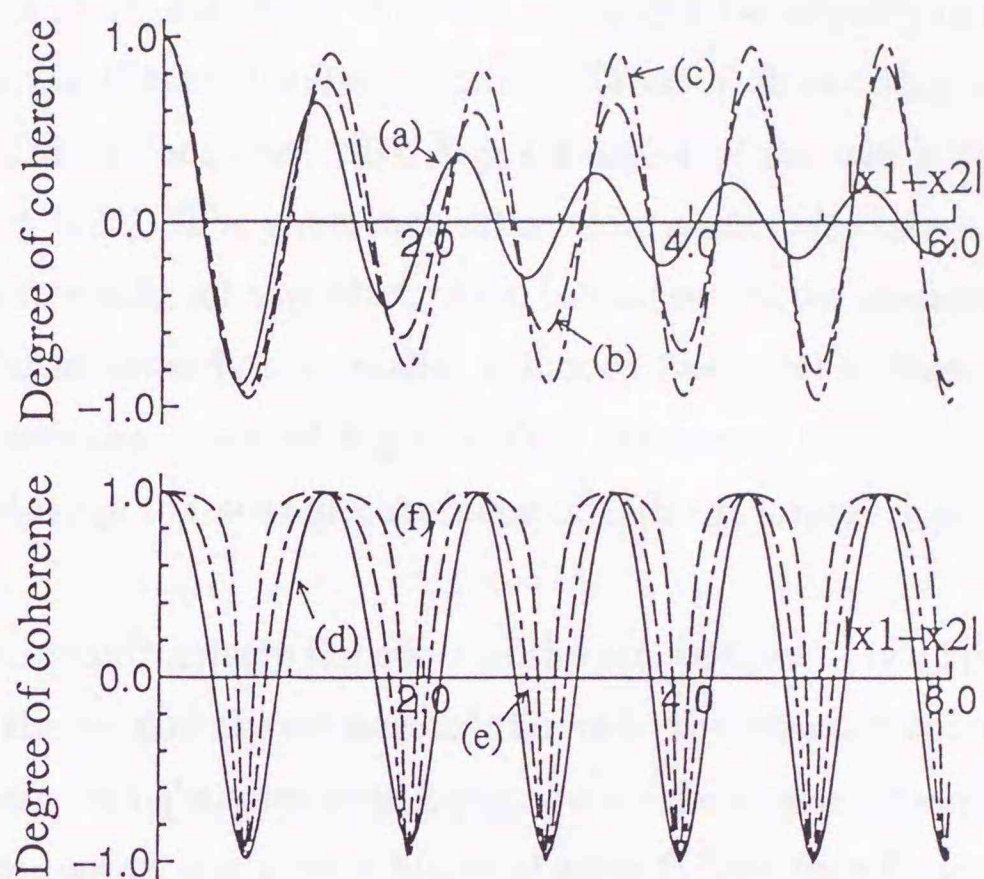


Figure 4.18. Complex degree of spatial coherence at the observation plane arising from the source covered with the source filter of type C. These curves are plotted against the normalized difference $ad/(\bar{\lambda}z)$ (where d is a real difference) from the z -axis for $\sigma_N =$ (a) 0.05, (b) 0.25, (c) 0.5, (d) 1.0, (e) 2.0, and (f) 5.0.

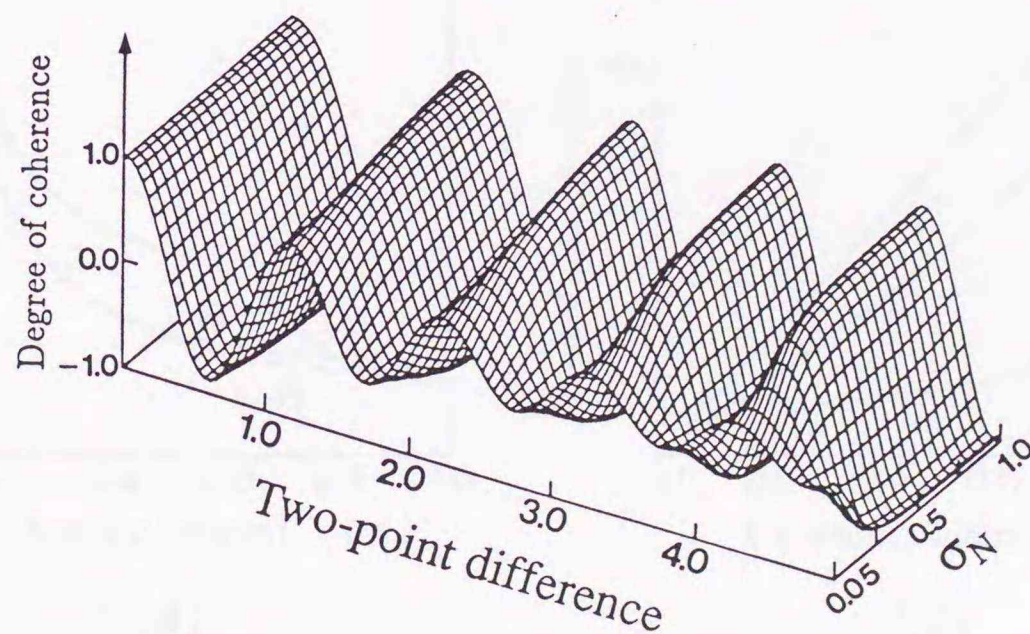


Figure 4.19. Complex degree of spatial coherence corresponding to Fig.4.18 for small values of σ_N (from 0.05 to 1.0).

trend in spatial coherence for very small σ_N (e.g., $\sigma_N=0.05$) is similar to that shown in Figs.4.5 and 4.14 in the sense that the curve can be approximately evaluated by applying the van Cittert-Zernike theorem. However, fluctuating performance is marked in Fig.4.18 as compared with Figs.4.5 and 4.14 for relatively small values of σ_N (e.g., $\sigma_N = 0.25$). The coherence interval for each value of σ_N is diminished by use of the source filter of this kind. As a consequence, the minima of coherence curves are displaced towards the smaller values of two-point differences. Hence, it is understood from the result of Fig.4.18 that the source filter of type C acts as a method of reducing the spatial coherence of light generated from the partially coherent source.

To evaluate quantitatively the effect of the source filter of two kinds, the intensity spread and the ratio of the coherence interval to the intensity spread are plotted in Fig.4.20 against the coherence condition σ_N over the source for each source filter. Curves corresponding to the source filters of type B and type C are indicated by

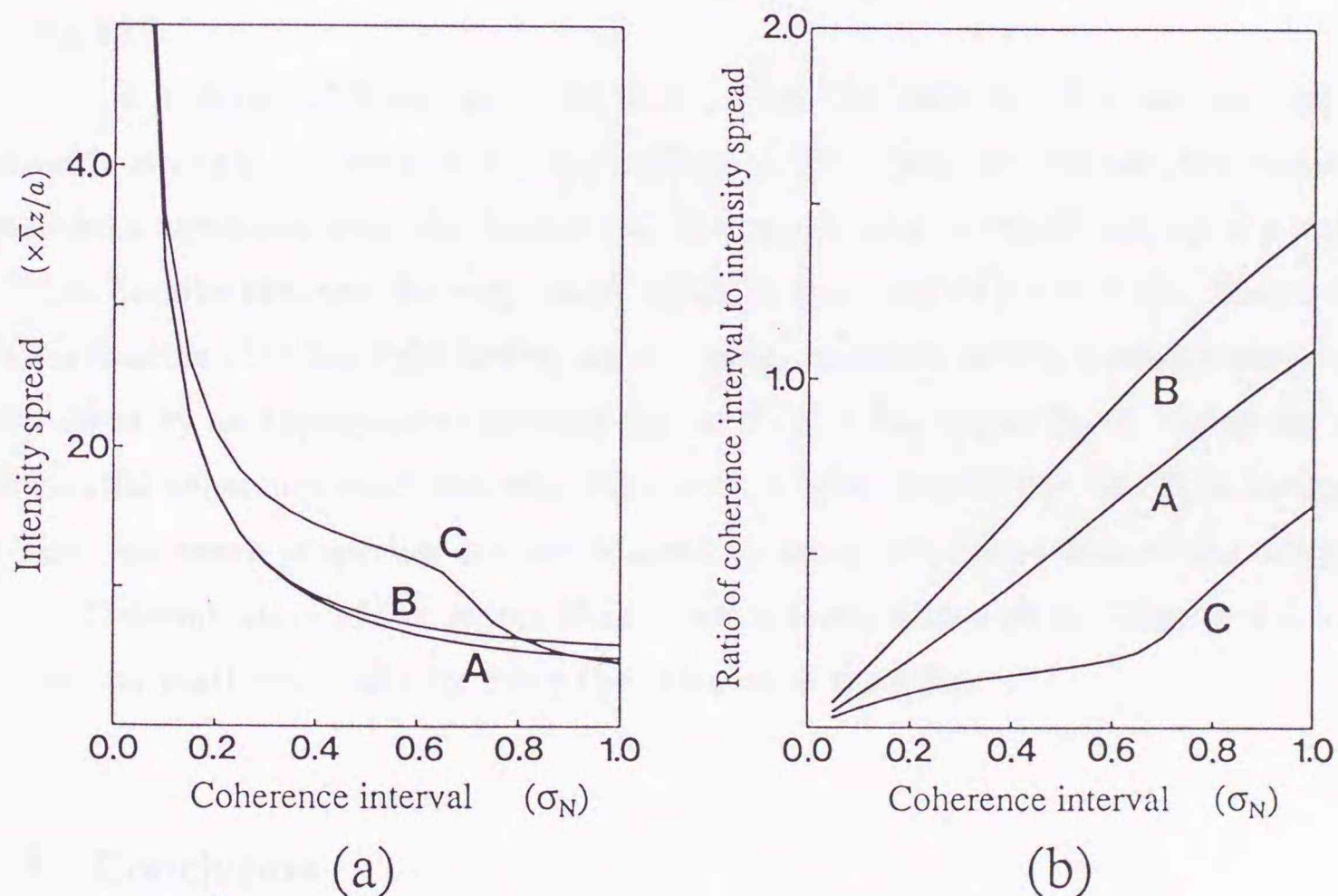


Figure 4.20. (a) Intensity spread and (b) the ratio of coherence interval to the intensity spread in the Fraunhofer diffraction field plotted against the coherence condition σ_N over the source for the source itself (indicated by A) and the source using the source filters of type B (indicated by B) and type C (indicated by C).

B and C in the figure, respectively. For comparison, curves corresponding to the non-filtered case (i.e., the source itself) are also added in the same figure and indicated by A. In the figure, the intensity spread is defined as the width for which the intensity decreases 40 per cent from its maximum value and the coherence interval is defined by the value $|x_2 - x_1|$ when the degree of spatial coherence $|\gamma(x_1, x_2)|$ comes to 0.5. It is seen from Figs.4.20(a) and (b) that, with an increase in the values of σ_N , the intensity spread decreases gradually and, conversely, the ratio of the coherence interval to the intensity spread increases monotonically. However, curve C in Fig.4.20(a) shows the tendency of dropping suddenly with σ_N between 0.6 and 0.7. After that, the values of curve C in intensity spread become the smallest. This phenomenon implies that the source filter of type C enlarges the intensity spread for small values of σ_N less than approximately 0.6 while it reduces the intensity spread for large values of σ_N larger than approximately 0.9. However, it should be noted that, even though such intensity distribution is realized for large σ_N , secondary and higher-order maxima appear in the diffraction pattern at the same time, as shown in Fig.4.17.

It is understood from the above results that the feature of the resultant field depends strongly not only on the type of the source filters, but also on the spatial coherence condition over the source σ_N , though it may be predicted by the van Cittert-Zernike theorem for very small values of σ_N such as $\sigma_N = 0.05$. Hence, it seems feasible that the light having an arbitrary condition of the spatial coherence is realized by an appropriate choice of the source and the source filter. Moreover, if the spatial coherence condition over the source is given in practice, the light having desired coherence properties can be obtained by an appropriate choice of the source filter. Determination of the source filter in accordance with a given purpose will be performed mathematically by using the calculus of variation.

4.4 Conclusion

In this chapter, a new concept of a source filter has been proposed to control the spatial coherence of light generated from a partially coherent planar source. As a simple example, two kinds of the source filter have been introduced and their perfor-

mances with relation to the spatial coherence of light in the Fraunhofer diffraction field of the source have been investigated together with the radiation properties from the source. Detailed formulations for analyzing the present chapter in one- and two-dimensional cases have also been developed. On the basis of those formulations, some numerical results are presented illustrating the effect of the source filters on the spatial coherence of the light field. A quantitative evaluation of the spatial coherence controlled by the source filters has been made by means of the ratio of the coherence interval to the intensity spread. It has been found from the evaluation that the ratio tends to be expanded by use of the source filter of type B and, on the contrary, diminished by use of that of type C for any state of spatial coherence σ_N over the source. In addition, it has also been observed that the varying behavior in coherence curves is strongly influenced by the type of the source filter. As a consequence, an arbitrary state of the spatial coherence is expected to be realized by an appropriate selection of the transmittance of the source filter for the given coherence condition over the source.

Finally, it is concluded throughout the whole analysis that the source filter can become a powerful tool for controlling the spatial coherence of light from a partially coherent planar source, with a view of obtaining the required system performance, in various fields of optics primarily due to its easy realization and secondarily due to its own ability.

CHAPTER 5

Radiation Efficiency of Partially Coherent Planar Sources

5.1 Introduction

The topic of the radiation from partially coherent sources has been a subject of great interest in the last twenty years [2, 19–21, 183]. From the course of such investigations, it follows that the fields produced by partially coherent sources are much influenced by the condition of the spatial coherence, together with the intensity distribution, across the source. In addition, these studies on so-called radiometry with partially coherent sources receive considerable attention especially in recent years due to the crucial discovery of a change in the spectrum of light on propagation induced by the spatial coherence property of the source [62].

Radiation efficiency of the sources to be considered in this chapter is defined as a ratio of the total radiant flux generated by the source to the source-integrated intensity, and acts as a measure for quantifying the feature of the radiation from partially coherent sources [21, 183]. It has been shown [21, 57, 183] that, depending on the characteristics of the source, radiation efficiency of any planar sources takes the value ranging from zero (corresponding to the totally non-radiating source) to unity (corresponding to the most *efficient* source).

In the past studies concerning radiation efficiency, either a planar Gaussian Schell-model source [57, 56] or a planar quasi-homogeneous source [24, 55, 56] has been frequently employed as a source model for analysis and the general relationship between the radiation efficiency and the spatial coherence condition over the source has been conclusively presented. However, owing to their adoption of the source model for analysis, these studies could provide little information about the dependence of the intensity distribution across the source on radiation efficiency. More specifically, their mathematical formulations of the problem cannot be directly ap-

plied to the source with an arbitrary intensity distribution. It is important to keep in mind here that there exist many kinds of the sources that do not satisfy the condition of quasi-homogeneity or cannot be approximated by the Gaussian Schell-model source in practical applications. In particular, the practical significance of such sources, i.e., the sources with various intensity distributions, may be recognized in connection with the spatial filtering method over the source using a source filter which has been proposed in Chapter 4 to control the spatial coherence of light produced by partially coherent sources. Therefore, it is necessarily required to take into account in these studies the variation of the intensity distribution across the source for a further generalization of the problem under consideration.

In this chapter, we investigate the radiation efficiency of partially coherent planar sources with various intensity distributions. As a source model for analysis, we consider an isotropic circular Schell-model source with uniform intensity distribution and introduce the source filter for changing the intensity distribution across the source. The coherence condition over the source is assumed to be specified by means of properly arranged Bessel functions. This chapter deals also with the radiation efficiency of quasi-homogeneous source as a limiting case of the present source model, though the ultimate purpose here is to evaluate the radiation efficiency of the source, not satisfying the condition of quasi-homogeneity, as a function of the source filter. Due to the adequate selection of the source model, a physical origin of the variation in radiation efficiency resulting from the source characteristics is satisfactorily manifested in terms of the spatial frequency spectrum of the source associated with evanescent waves. In the following, we confine our discussion to the case of planar sources while more recent studies on radiation efficiency have been concerned with the three-dimensional primary sources [59, 60]. Radiation efficiency of the three-dimensional sources are investigated in the next chapter.

5.2 Theory

5.2.1 General formulation of the problem

We consider an isotropic secondary planar Schell-model source occupying a finite area Σ at the plane $z = 0$ and radiating into the free half-space $z > 0$, as illustrated

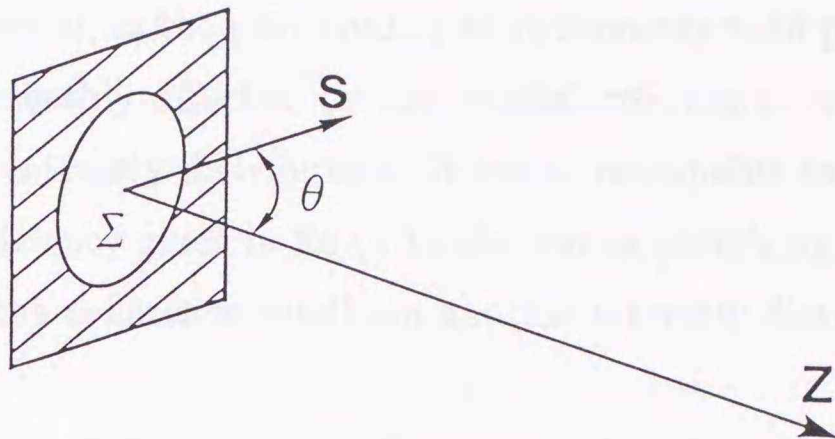


Figure 5.1. Schematic of the optical arrangement for analysis. The isotropic secondary Schell-model source is located at the plane $z = 0$ perpendicular to the z -axis and radiates into the free half-space $z > 0$. The symbol s implies the three-dimensional unit vector.

in Fig.5.1. Radiation efficiency R_ω of the source at frequency ω is defined by [21, 57, 183]

$$R_\omega = \frac{\Phi_\omega}{N_\omega}, \quad (5.1)$$

where Φ_ω implies the total flux emitted by the source into the half-space $z > 0$ and N_ω indicates the source-integrated intensity. In addition, the total flux Φ_ω in Eq.(5.1) is known to be expressible, in terms of the radiant intensity $J_\omega(s)$, in the form [2, 21, 183]

$$\Phi_\omega = \int_{(2\pi)} J_\omega(s) d\Omega, \quad (5.2)$$

where the integration extends over the 2π solid angle formed by all s -directions in the half-space $z > 0$ (s is the three-dimensional unit vector). It is shown [21, 183, 57] for any planar sources that the radiation efficiency R_ω defined by Eq.(5.1) satisfies the inequality

$$0 \leq R_\omega \leq 1 \quad (5.3)$$

and acts as a measure for quantifying the feature of the radiation from the source.

In Eq.(5.1), the source-integrated intensity N_ω depends only on the intensity distribution across the source and can be uniquely determined, irrespective of the coherence condition over the source, by the intensity distribution across the source.

On the contrary, the total flux Φ_ω may depend, in view of Eq.(5.2), not only on the source intensity but also on the source coherence on the ground that the radiant intensity has been proved, in the past studies on radiometry with partially coherent sources, to be considerably affected by the spatial coherence condition over the source including the intensity distribution. It seems reasonable to state, therefore, that the radiation efficiency given in Eq.(5.1) also varies, satisfying the inequality of Eq.(5.3), with both the coherence condition and the intensity distribution over the source.

The main purpose of the present study is to manifest the effect of the intensity distribution across the source on radiation efficiency, in contrast to the past studies concerning the radiation efficiency of partially coherent sources. A mathematical formulation of the problem must be trivially refined here, in comparison with the past studies, with a view of obtaining the specific expression of radiation efficiency for the sources with various intensity distributions. In the following analysis, we begin by characterizing the feature of the spatial coherence across the source in terms of the cross-spectral density function and, subsequently, establish the general expression for the radiation efficiency applicable to the partially coherent sources with arbitrary intensity distributions.

Before proceeding further, a few remarks should be made concerning the mathematical treatment for the formulation of the problem. Concurrently with the related investigations reviewed in Section 5.1 [24, 55–57, 59, 60], the present study is properly formulated by means of the correlation function in the space-frequency domain, i.e., the cross-spectral density, rather than that in the space-time domain, i.e., the mutual coherence function. This is mainly because these studies are essentially concerned with the behavior of the energy flow at one particular frequency. It should be borne in mind, however, that there are some cases in which the correlation function in the space-frequency domain is inadequate to describe the phenomena (see Section 4.2.3). In addition, it may be expedient to state that the correlation function in the space-time domain rather than that in the space-frequency domain plays an essential role with a view of providing the intuitive understanding for the phenomena because all measurements are practically performed only in the space-time domain [151]. As is widely known, the hypothesis of quasi-monochromaticity allows us to reduce the cumbersome difference of physics between these two domains in the

sense that both approaches (i.e., the formulation in the space-time domain and that in the space-frequency domain) are equivalent under such a condition. Accordingly, the adoption of the quasi-monochromatic hypothesis in the present study may also provide a profound comprehension for the phenomena. However, we do not specify the source spectrum in the subsequent analysis because the mathematical treatment of the problem to be stated below is unchanged independent of the spectral aspect of the source.

We now return to the main subject of the mathematical formulation of the present study. The cross-spectral density function of light across the isotropic Schell-model source is known to be expressible in the form:

$$W^{(0)}(\mathbf{r}_1, \mathbf{r}_2; \omega) = \sqrt{I^{(0)}(\mathbf{r}_1; \omega)} \sqrt{I^{(0)}(\mathbf{r}_2; \omega)} g(|\mathbf{r}_1 - \mathbf{r}_2|; \omega) \quad (5.4)$$

where

$$I^{(0)}(\mathbf{r}_i; \omega) = W^{(0)}(\mathbf{r}_i, \mathbf{r}_i; \omega) \quad (i = 1, 2) \quad (5.5)$$

stands for the spectral intensity at a position specified by the two-dimensional vector $\mathbf{r}_i$ and $g(|\mathbf{r}_1 - \mathbf{r}_2|; \omega)$ denotes the complex degree of spectral coherence. Here, we assume the source with uniform intensity distribution and introduce the source filter for changing the intensity distribution across the source. In general, there exists an optimum distribution of the source intensity in optical processing, depending on its own purpose. Thus, it is required to produce the source with various intensity distributions. It is, however, not a straightforward task to change the spatial distribution of the source intensity at will by controlling the source itself. Instead, as a matter of fact, the spatial filtering method over the source (i.e., the source filter) is widely adopted in practical applications of optics. For this reason, the subsequent analysis is proceeded explicitly in terms of the source filter. By referring to Eq.(5.4), the cross-spectral density of light just behind the source filter is then proved to be

$$W_{SF}^{(0)}(\mathbf{r}_1, \mathbf{r}_2) = I_0 S(\mathbf{r}_1) S(\mathbf{r}_2) g(|\mathbf{r}_1 - \mathbf{r}_2|), \quad (5.6)$$

where I_0 is a positive constant and $S(\mathbf{r}_i)$ is the amplitude transmittance of the source filter. Here and in the subsequent formulation, an explicit dependence on ω in each expression is omitted for brevity.

It is known that the radiant intensity radiated from the source specified by Eq.(5.6) is generally given by [19, 20, 23]

$$J_{\omega}(s) = I_0 \left(\frac{k}{2\pi} \right)^2 \cos^2 \theta \iint_{-\infty}^{\infty} C(x') g(|x'|) \exp[-jkx' \cdot s_{\perp}] d^2 x', \quad (5.7)$$

with

$$C(x') = \iint_{-\infty}^{\infty} S\left(x + \frac{1}{2}x'\right) S\left(x - \frac{1}{2}x'\right) d^2 x \quad (5.8)$$

being the autocorrelation function of the amplitude transmittance of the source filter. In Eq.(5.7), the symbol $s_{\perp}$ denotes the two-dimensional vector obtained by projecting the three-dimensional unit vector s onto the source plane and θ is the angle between the s direction and the normal to the source plane (see Fig.5.1). Thus, the position vector $s_{\perp}$ and the angle θ are found to be connected by the relation

$$|s_{\perp}|^2 + \cos^2 \theta = 1. \quad (5.9)$$

Using Eq.(5.9), the element $d\Omega$ of the solid angle contained in Eq.(5.2) can be represented in terms of the vector $s_{\perp}$ by the expression

$$d\Omega = \frac{d^2 s_{\perp}}{\sqrt{1 - s_{\perp}^2}}, \quad (5.10)$$

which is employed just below.

On substituting Eq.(5.7), together with Eqs.(5.9) and (5.10), into Eq.(5.2) and performing the integration with respect to $s_{\perp}$, the total flux Φ_{ω} becomes [24, 55]

$$\Phi_{\omega} = I_0 \frac{k^2}{2\sqrt{2\pi}} \iint_{-\infty}^{\infty} C(x') g(|x'|) \frac{J_{3/2}(k|x'|)}{(k|x'|)^{3/2}} d^2 x', \quad (5.11)$$

with

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right) \quad (5.12)$$

being the Bessel function of the first kind and order 3/2. In order to reduce Eq.(5.11) in a simple form, we introduce the polar coordinates in place of the Cartesian ones

and then perform the integration with respect to the angular component. Subsequently, by replacing the value $k|\mathbf{x}'|$ by x , the reduced expression for the total flux Φ_ω is obtained as

$$\Phi_\omega = I_0 \sqrt{\frac{\pi}{2}} \int_0^\infty x^{-\frac{1}{2}} C'(x) g'(x) J_{3/2}(x) dx. \quad (5.13)$$

This equation has been derived under the condition that the autocorrelation function $C(\mathbf{x}')$ given by Eq.(5.8) depends only on the absolute value of its argument, i.e., $C(\mathbf{x}')$ is a function of $|\mathbf{x}'|$. This condition is easily satisfied by assuming the circular symmetric transmittance of the source filter. It should also be noted that the functional forms of $C'(x)$ and $g'(x)$ contained in Eq.(5.13) are different from those of $C(|\mathbf{x}'|)$ and $g(|\mathbf{x}'|)$ contained in Eq.(5.11) because a change of the variable is conducted in deriving Eq.(5.13) from Eq.(5.11). Differences in these two functions are presented explicitly in the subsequent analysis.

On the other hand, the source-integrated intensity N_ω in Eq.(5.1) can be written, in terms of the amplitude transmittance of the source filter, in the form

$$N_\omega = I_0 \iint_{-\infty}^{\infty} [S(\mathbf{x})]^2 d^2\mathbf{x}. \quad (5.14)$$

Finally, substitution of Eqs.(5.13) and (5.14) into Eq.(5.1) leads to the required expression for radiation efficiency given by

$$R_\omega = \sqrt{\frac{\pi}{2}} \int_0^\infty x^{-\frac{1}{2}} C'_N(x) g'(x) J_{3/2}(x) dx. \quad (5.15)$$

In the above equation, $C'_N(x)$ stands for the normalized autocorrelation function of the amplitude transmittance of the source filter given by

$$C'_N(x) = \frac{\iint_{-\infty}^{\infty} S(\mathbf{t} + x/2k) S(\mathbf{t} - x/2k) d^2\mathbf{t}}{\iint_{-\infty}^{\infty} [S(\mathbf{t})]^2 d^2\mathbf{t}}, \quad (5.16)$$

where $\mathbf{t}$ and x denote the two-dimensional vector and the scalar quantity, respectively. The wavenumber k appears in Eq.(5.16) since a change of the variable was conducted in deriving Eq.(5.13) from Eq.(5.11). Aside from the wavenumber k ,

Eq.(5.16) represents the optical transfer function of an incoherent imaging system having a circular aperture with symmetric transmittance ([153], Sec. 6.3).

Equation (5.15) is the final expression for radiation efficiency of the source under consideration. Detailed knowledge for the source and the source filter is required to proceed further since Eq.(5.15) involves the unknown functions $g'(x)$ and $C'_N(x)$ associated with, respectively, the spatial coherence and the intensity distribution across the source. These two functions are specified in the next section.

5.2.2 Source model

In accordance with the preceding formulation of the problem, we assume a partially coherent circular source with uniform intensity distribution and introduce the source filter for varying the intensity distribution across the source. In this section, the source model for analysis including the source filter is treated in some detail.

We first specify the feature of the spatial coherence of light across the source employed here. As the complex degree of spectral coherence across the source, we assume the expression given by

$$g(|\mathbf{r}_1 - \mathbf{r}_2|) = \Gamma\left(1 + \frac{n}{2}\right) \frac{J_{n/2}(k|\mathbf{r}_1 - \mathbf{r}_2|/\alpha)}{(k|\mathbf{r}_1 - \mathbf{r}_2|/2\alpha)^{n/2}}, \quad (5.17)$$

where $\Gamma(\cdot)$ and $J_{n/2}(\cdot)$ denote the Euler's gamma function and the Bessel function of the first kind and order $n/2$, respectively, and k is the wavenumber of light across the source. Here, n and α can take any positive values (rigorously, $n \geq 0$ and $\alpha > 0$) and serve as parameters for changing the spatial coherence condition over the source. More specifically, the coherence interval, which is properly defined as the spatial distance for which the degree of coherence reaches a certain prescribed value, becomes larger with an increase of the value n for the fixed value of α . Similarly, the coherence interval increases with an increase of α for fixed values of n . In the following analysis, we consider for simplicity that α acts as a parameter for controlling the spatial coherence condition over the source and that n is fixed. While Eq.(5.17) is applicable to Eqs.(5.4), (5.6), (5.7) and (5.11), it cannot be directly substituted into Eqs.(5.13) and (5.15) because a change of the variable is made in deriving Eq.(5.13) from Eq.(5.11) as discussed in Section 5.2.1. The reduced

expression of Eq.(5.17) applicable to Eqs.(5.13) and (5.15) is given in the form:

$$g'(x) = \Gamma\left(1 + \frac{n}{2}\right) \frac{J_{n/2}(x/\alpha)}{(x/2\alpha)^{n/2}}. \quad (5.18)$$

To make clear the behavior in spatial coherence mentioned above, the complex degrees of spectral coherence are plotted in Fig.5.2 against the mutual difference x/α for some selected values of n on the basis of Eq.(5.18). In addition, the functional

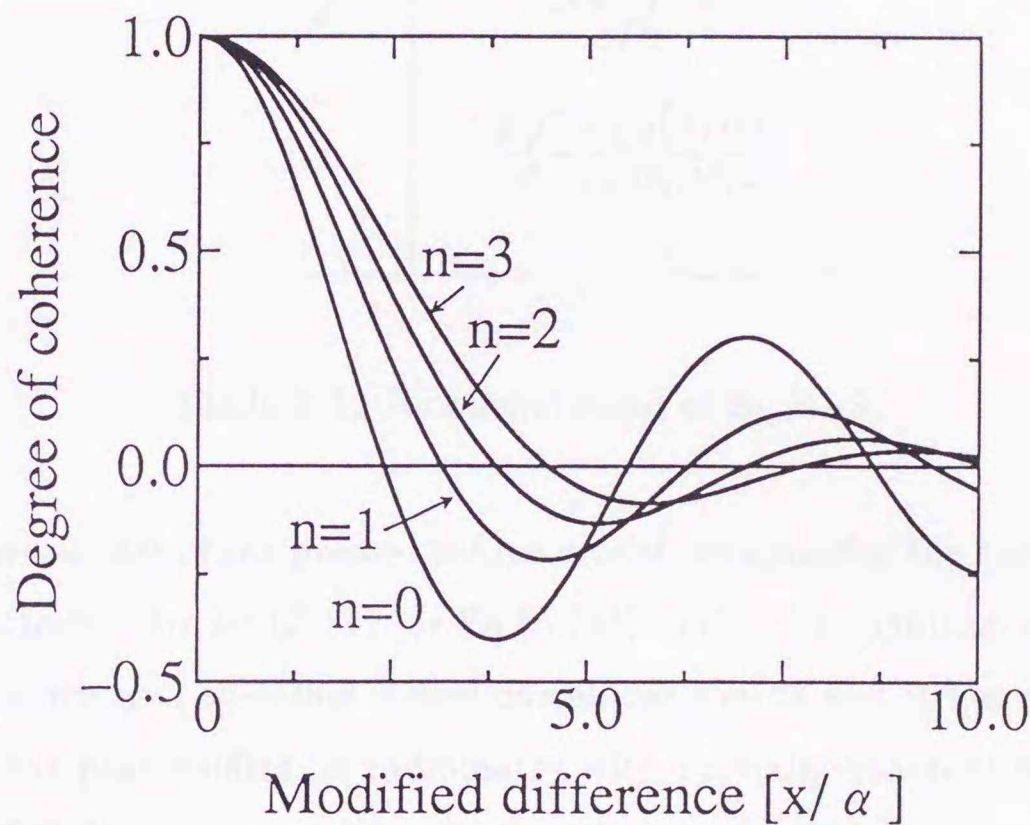


Figure 5.2. Complex degree of spectral coherence across the source. The curves are plotted against the modified difference x/α for some selected values of n .

forms of Eq.(5.18) corresponding to these curves are summarized in Table 5.1. For any values of n , the partially coherent source characterized by Eq.(5.18) approaches a spatially incoherent source for $\alpha \rightarrow 0$ in the sense that Eq.(5.18) is reduced to

$$g'(x) = \begin{cases} 1 & \text{for } x = 0, \\ 0 & \text{for } x \neq 0. \end{cases} \quad (5.19)$$

Conversely, it approaches a spatially coherent and cophasal source for $\alpha \rightarrow \infty$ in the sense that Eq.(5.18) is reduced to

$$g'(x) = 1 \quad \text{for all } x. \quad (5.20)$$

n	$g'(x)$
0	$J_0(x/\alpha)$
1	$\frac{\sin(x/\alpha)}{x/\alpha}$
2	$\frac{2J_1(x/\alpha)}{x/\alpha}$
3	$\frac{3\sqrt{\pi}}{4} \frac{J_{3/2}(x/\alpha)}{(x/2\alpha)^{3/2}}$

Table 5.1. Functional forms of Eq.(5.18).

As a special case of the present source model, we consider the partially coherent source characterized by Eq.(5.17) (or Eq.(5.18)) under the condition of $\alpha = 1$. Such a source is known as a so-called Bessel-correlated source and it has been favorably employed in the past studies on radiometry with partially coherent sources [19, 20, 23, 56, 183, 184]. It is important to note that the angular distribution of the intensity radiated from a quasi-homogeneous Bessel-correlated source obeys the $\cos^n \theta$ -law where θ is the angle formed between the direction of observation and the normal axis to the source. Thus the source of this kind is found to represent a lambertian source in the case of $n = 1$ and a completely incoherent (delta-correlated) source in the case of $n = 2$ [185]. In addition, a further point to be emphasized with regard to the quasi-homogeneous Bessel-correlated source is its spatial frequency spectrum defined by [19, 20, 184]

$$F(\mathbf{f}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} g(\mathbf{x}) \exp[-j\mathbf{f} \cdot \mathbf{x}] d^2\mathbf{x}, \quad (5.21)$$

where $\mathbf{f}$ and $\mathbf{x}$ are the two-dimensional vectors and $g(\mathbf{x})$ is the complex degree of spectral coherence across the source. It can be readily shown after some algebra that the quasi-homogeneous Bessel-correlated source has only the spatial frequency

component less than the wavenumber k (for short, we call it the *low-frequency* component). In other words, this reveals that such a source has no evanescent waves because the spatial frequency component above k (for short, we call it the *high-frequency* component) results in the evanescent waves that do not carry energy into the far field of the source.

The source model specified by Eq.(5.17) (or Eq.(5.18)) has an important parameter α , as opposed to the conventional Bessel-correlated source mentioned above. It is worth emphasizing that, due to the parameter α , the present source model has a distinct advantage especially in the study on radiation efficiency of the quasi-homogeneous source, as is the case also in the present study. To illustrate the significance of the present source model, we consider the spatial frequency spectrum of the source model in accordance with Eq.(5.21). By substituting Eq.(5.17) into Eq.(5.18) and performing the proper normalization, the spatial frequency spectrum of the source is found to be given by

$$F(f) \propto \begin{cases} \left[1 - \left(\frac{\alpha f}{k} \right)^2 \right]^{n/2 - 1}, & \text{for } 0 \leq f \leq k/\alpha \\ 0, & \text{for } f > k/\alpha \end{cases} \quad (5.22)$$

where f is the absolute value of the two-dimensional vector $\mathbf{f}$, i.e., $f = |\mathbf{f}|$. Equation (5.22) is plotted in Fig.5.3 as a function of the spatial frequency f for several values of n . The normalization of the spectrum was conducted by dividing the spectrum at each point by the value at the origin. This figure exhibits that the spatial frequency component of the source is influenced by the values of α . The source has only the *low-frequency* component of the spatial frequency spectrum, provided the value α is larger than unity. On the contrary, it involves not only the *low-frequency* component but also the *high-frequency* component, provided the value α is less than unity. Thus the source of this kind may possibly provide sufficient understanding for the behavior in radiation efficiency of the partially coherent source in connection with the spatial frequency spectrum of the source associated with the evanescent waves. This is a problem to be considered in Section 5.3.1.

We next determine the amplitude transmittance of a source filter employed here. As a simple example, we assume a circular source filter with circular symmetric

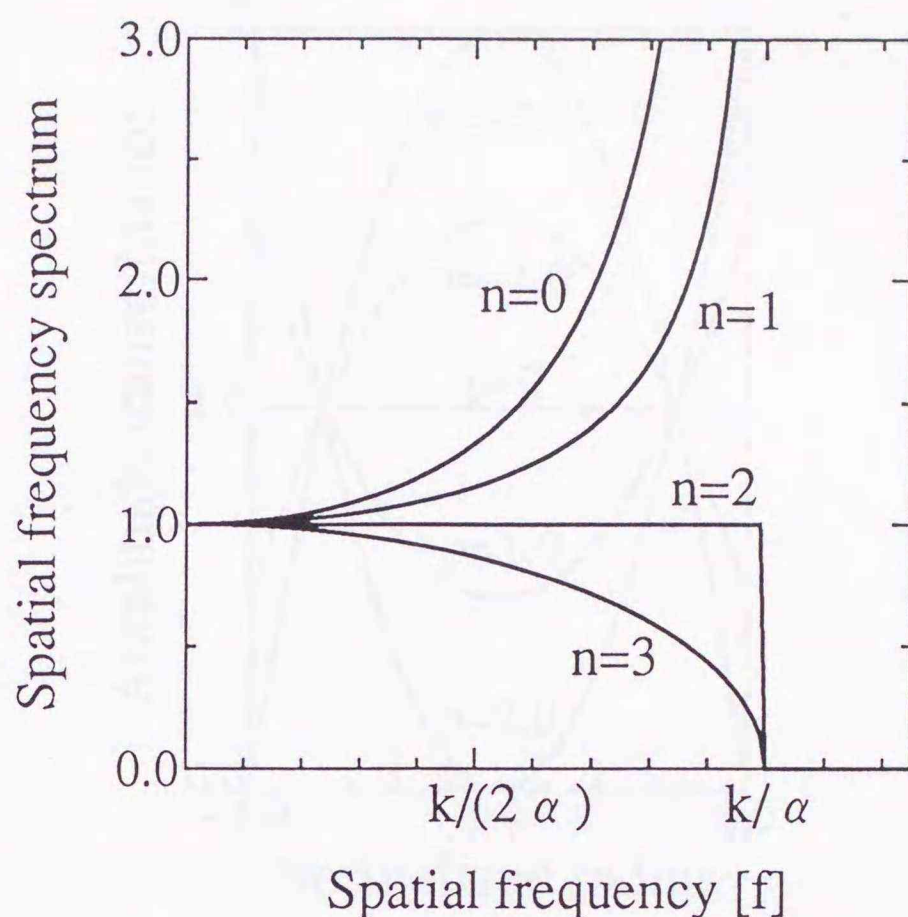


Figure 5.3. Normalized spatial frequency spectrum of the quasi-homogeneous source specified by the complex degree of spectral coherence illustrated in Fig.5.2. The curves are plotted against the normalized spatial frequency k/α for various values of n , where k is the wavenumber of light across the source.

amplitude transmittance given by

$$S(\mathbf{r}) = \begin{cases} p \left(\frac{|\mathbf{r}|}{a} \right)^2 + \left(1 - \frac{p}{2} \right), & \text{for } 0 \leq |\mathbf{r}| \leq a \\ 0, & \text{for } |\mathbf{r}| > a \end{cases} \quad (5.23)$$

where a represents the source radius and $\mathbf{r}$ is the two-dimensional position vector. The parameter p in Eq.(5.23) takes the values ranging from -2.0 to 2.0 and controls the appearance of the source filter. In particular, the uniform transmittance within the source region is realized in the case of $p = 0$. The amplitude transmittance of the source filter is illustrated in Fig.5.4 for several values of p . It is well understood from the figure that, in spite of the simple expression of Eq.(5.23), there exists a variety of the amplitude transmittance depending on the parameter p . When Eq.(5.23) is substituted into Eq.(5.16), the normalized autocorrelation function, which is required to evaluate the radiation efficiency in Eq.(5.15), is obtained and

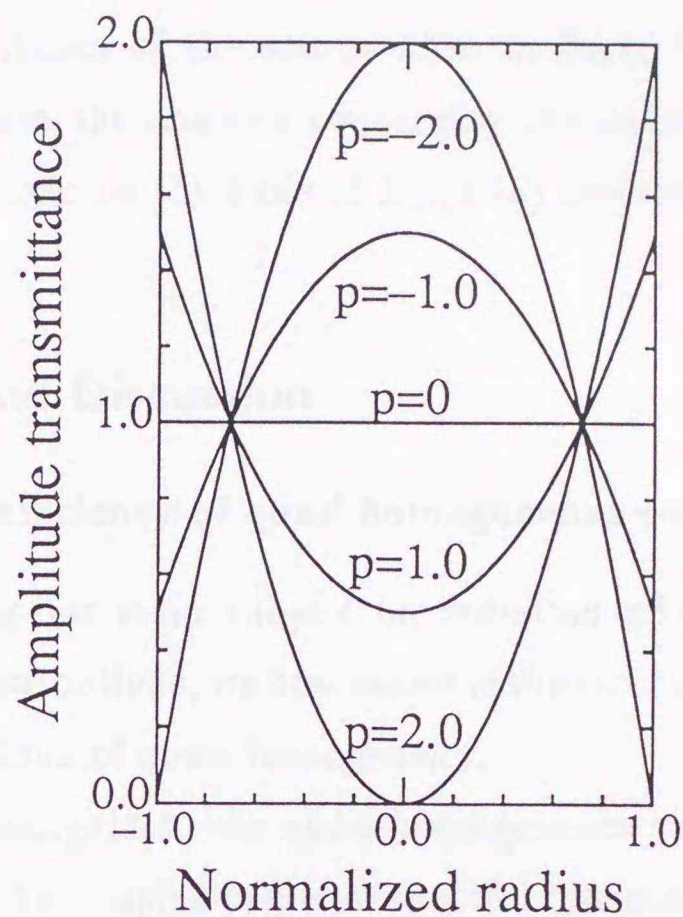


Figure 5.4. Amplitude transmittance of the source filter employed in the present analysis. The curves are plotted against the normalized radius for various values of p . Maximum values of the transmittance for each value of p , except for $p = 0$, are permitted to exceed unity because of the mathematical convenience in the present formulation of the problem.

becomes

$$C'_N(x) = \begin{cases} \pi^{-1} \left(\frac{1}{12} p^2 + 1 \right)^{-1} \\ \times \left[\left(\frac{x}{ka} \right) \left[-\frac{p^2}{36} \left\{ \left(\frac{x}{ka} \right)^4 - 7 \left(\frac{x}{ka} \right)^2 + 15 \right\} \right. \right. \\ \left. \left. - \frac{p}{2} \left\{ \left(\frac{x}{ka} \right)^2 + 4 \right\} - 1 \right] \sqrt{1 - \left(\frac{x}{2ka} \right)^2} \right. \\ \left. + \left\{ \frac{1}{6} (p^2 + 12) + 2p \left(\frac{x}{ka} \right)^2 \right\} \cos^{-1} \left(\frac{x}{2ka} \right) \right], & \text{for } 0 \leq x \leq 2ka \\ 0, & \text{for } x > 2ka. \end{cases} \quad (5.24)$$

In conclusion, determination of both the source model and the source filter for analysis is made in this section, yielding the complex degree $g'(x)$ of spectral

coherence in Eq.(5.18) and the normalized autocorrelation function $C'_N(x)$ of the amplitude transmittance of the source filter in Eq.(5.24). Thus, we are now in a position to perform the analysis concerning the radiation efficiency of partially coherent planar sources on the basis of Eq.(5.15) derived in the previous section.

5.3 Results and Discussion

5.3.1 Radiation efficiency of quasi-homogeneous sources

Before investigating our main subject on radiation efficiency of the sources with various intensity distributions, we first examine the radiation efficiency of the sources satisfying the condition of quasi-homogeneity.

As is widely recognized, the quasi-homogeneous sources are characterized by the properties that the complex degree of spectral coherence $g(\mathbf{r}')$ varies more rapidly with $\mathbf{r}'$ than a variation of the intensity distribution across the source $I(\mathbf{r})$ with $\mathbf{r}$. In the present formulation, the intensity distribution across the source is proportional to a square of the amplitude transmittance of the source filter, $[S(\mathbf{r})]^2$. Under this circumstance, the cross-spectral density function of light just behind the source filter is approximated by [24]

$$W_{SF}^{(0)}(\mathbf{r}_1, \mathbf{r}_2) = I_0 \left[S \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \right]^2 g(|\mathbf{r}_1 - \mathbf{r}_2|) \quad (5.25)$$

in place of Eq.(5.6) and, therefore, the autocorrelation function defined by Eq.(5.8) is proved to be

$$C(x') = \iint_{-\infty}^{\infty} [S(\mathbf{x})]^2 d^2\mathbf{x} = C, \quad (5.26)$$

where C is a constant. In particular, the normalized autocorrelation function $C'_N(x)$ in Eq.(5.16) becomes unity in view of the relation of Eq.(5.26). As a result, the radiation efficiency of the quasi-homogeneous source can be written in the form

$$R_\omega = \sqrt{\frac{\pi}{2}} \int_0^\infty x^{-\frac{1}{2}} g'(x) J_{3/2}(x) dx, \quad (5.27)$$

in accordance with Eq.(5.15). Equation (5.27) exhibits that an exact form of the intensity distribution across the source is of no importance in the present context insofar as the source fulfills the quasi-homogeneous condition.

n	$0 \leq \alpha < 1$	$\alpha = 1$	$\alpha > 1$
0	0	0	$\frac{\sqrt{\alpha^2-1}}{\alpha}$
1	$\frac{1}{2} \left[1 - \frac{1-\alpha^2}{2\alpha} \ln \left(\frac{1+\alpha}{1-\alpha} \right) \right]$	$\frac{1}{2}$	$\frac{1}{2} \left[1 + \frac{\alpha^2-1}{2\alpha} \ln \left(\frac{\alpha+1}{\alpha-1} \right) \right]$
2	$\frac{2}{3}\alpha^2$	$\frac{2}{3}$	$\frac{2}{3\alpha} \left[\alpha^3 - (\sqrt{\alpha^2-1})^3 \right]$
3	$\frac{3\alpha}{16} \left[2 - \left(\alpha^2 + \frac{1}{\alpha^2} \right) \right] \ln \left(\frac{1+\alpha}{1-\alpha} \right)$ $+ \frac{3}{8}(\alpha^2 + 1)$	$\frac{3}{4}$	$\frac{3\alpha}{16} \left[2 - \left(\alpha^2 + \frac{1}{\alpha^2} \right) \right] \ln \left(\frac{\alpha+1}{\alpha-1} \right)$ $+ \frac{3}{8}(\alpha^2 + 1)$

Table 5.2. Functional forms of Eq.(5.28).

When Eq.(5.18) is substituted into Eq.(5.27), the radiation efficiency of the present source model under the quasi-homogeneous approximation is obtained by ([180], 6.574, p. 692)

$$R_\omega = \begin{cases} \frac{n}{3}\alpha^2 F(1, 1 - \frac{n}{2}; \frac{5}{2}; \alpha^2), & \text{for } 0 \leq \alpha < 1 \\ \frac{n}{n+1}, & \text{for } \alpha = 1 \\ F(1, -\frac{1}{2}; 1 + \frac{n}{2}; \alpha^{-2}), & \text{for } \alpha > 1 \end{cases} \quad (5.28)$$

with F being hypergeometric functions. In spite of its cumbersome expression, Eq.(5.28) can be reduced to simple functional forms for some given values of n ([186], pp. 555–566) and, hence, their typical examples are summarized in Table 5.2. On the basis of the table, the radiation efficiencies for some values of n are illustrated in Fig.5.5 as a function of the parameter α . To provide the sufficient understanding for the behavior of radiation efficiency, it must be recalled here from Section 5.2.2 that the parameter α is closely related to both the coherence condition over the source and the spatial frequency spectrum of the source. It is clearly seen in Fig.5.5 that all the curves change with the parameter α in a similar fashion: the radiation

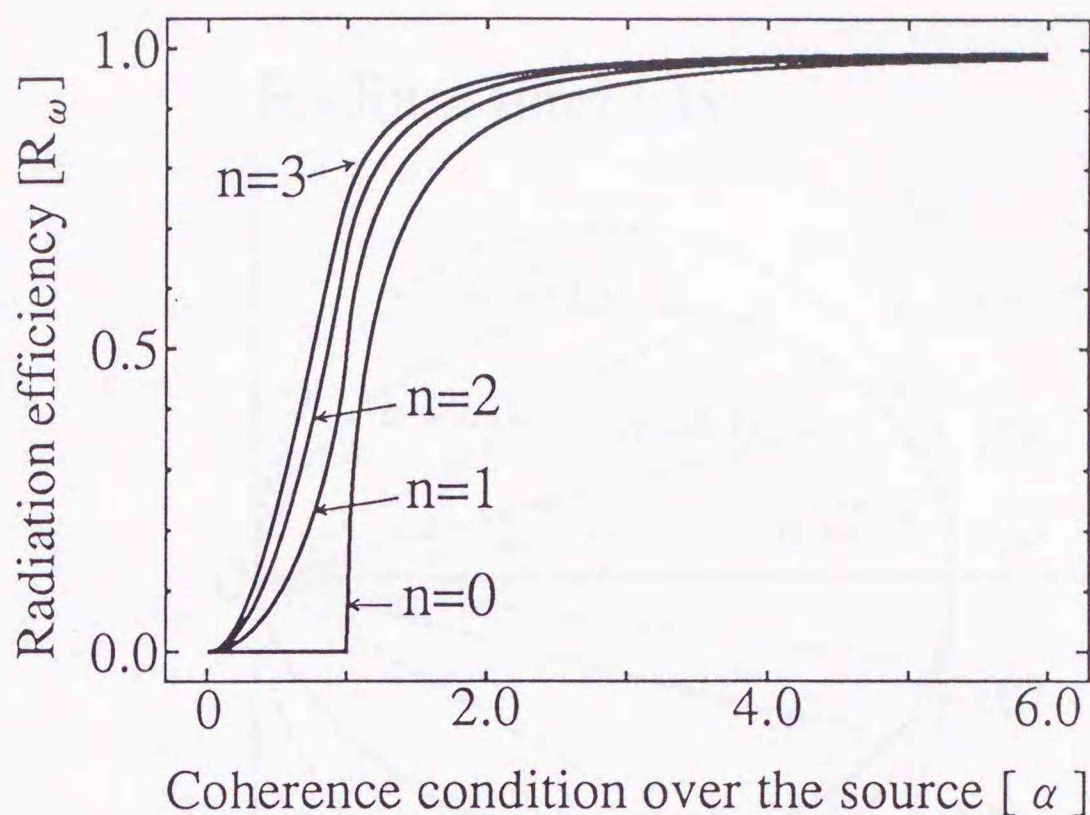


Figure 5.5. Radiation efficiency of the quasi-homogeneous source specified by the complex degree of spectral coherence illustrated in Fig.5.2. The curves are plotted against the coherence condition over the source α for various values of n .

efficiencies for any values of n rise suddenly from the value zero and, subsequently, they remain nearly stable just below the value unity with an increase of α . The value of $\alpha = 1.0$ appears, except for the case of $n = 0$, to be a boundary for the two categories in a variation of the radiation efficiency (i.e., sudden rise part and nearly stable part in the variation). From the viewpoint of the spatial coherence condition over the source, Fig.5.5 indicates that the radiation efficiency takes a minimum value of zero for a spatially incoherent source in the sense of Eq.(5.19) and that it takes a maximum value of unity for a coherent and cophasal source in the sense of Eq.(5.20). In addition, the reduction of radiation efficiency is obviously found to be caused by the reduction of the spatial coherence condition over the source since the spatial coherence of the source is reduced with a decrease in the value of α as already stated in Section 5.2.2. These facts are in agreement with the past works [20, 21, 24, 55, 56, 183]. For the reference purpose, the normalized radiant intensity of the source for $n = 2$ is illustrated in Fig.5.6 using Eq.(5.7). It is well understood from the figure that the radiated field tends to be collected around the forward direction as the value of α becomes larger, i.e., as the spatial coherence across the source

increases.

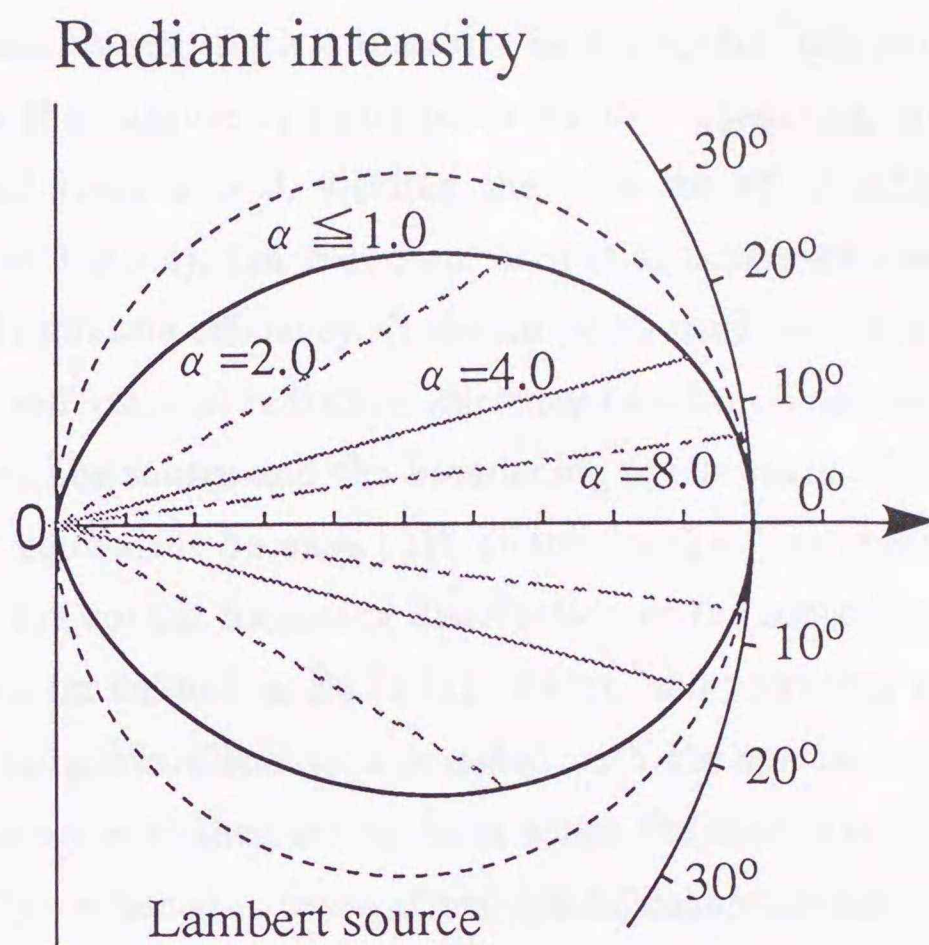


Figure 5.6. Radiant intensity distribution of the source corresponding to the case of $n = 2$ in the curves of the complex degree of spectral coherence illustrated in Fig.5.2. The curves are displayed in a polar diagram for various values of α . For the reference purpose, the dotted line indicates the radiant intensity of the lambertian source.

In order to elucidate further the physics of concern, we now investigate the behavior of radiation efficiency in connection with the spatial frequency spectrum of the source. As was discussed in Section 5.2.2, the spatial frequency spectrum of the present source is given by Eq.(5.22) and was illustrated in Fig.5.3. By recalling that the so-called *high-frequency* component of the source is associated with the evanescent waves, it follows from Fig.5.3 that the source has the evanescent waves for $\alpha < 1$ and has no evanescent waves for $\alpha \geq 1$. It seems therefore feasible, at the first glance, that the existence of evanescent waves results in the reduction of radiation efficiency, on the ground that a marked variation in radiation efficiency is observed in Fig.5.5 for $\alpha < 1$. However, detailed analysis of Fig.5.5 leads to the fact that the radiation efficiency of the source decreases considerably also in the region $\alpha \geq 1$ (there are no evanescent waves in this region) with a decrease of α . For this reason, it is expedient to state that the radiation efficiency is generally

reduced with the broadening in the spatial frequency spectrum of the source toward the higher frequency and that this trend in radiation efficiency is irrespective of the presence of evanescent waves. Such a behavior in the spatial frequency spectrum results intuitively in the mixture of many plane waves propagating toward various directions in the half space $z > 0$, yielding the wide spread of radiant intensity (e.g., as illustrated in Fig.5.6), the reduction of spatial coherence over the source and the reduction of radiation efficiency. It should be stressed here that the physical explanations for the reduction of radiation efficiency based on both the reduction of spatial coherence over the source and the broadening of the spatial frequency spectrum are essentially equivalent because both quantities (i.e., the spatial coherence over the source and the spatial frequency distribution of the source) are connected by the Fourier transform defined in Eq.(5.21). Hence, the preceding statement for the performance in radiation efficiency associated with the spatial frequency spectrum has the consistency with the past works in which the modification in radiation efficiency is physically explained in terms of the spatial coherence condition over the source.

A similar consequence has also been recognized in the past studies dealing with the radiation efficiency of the quasi-homogeneous source [20, 24, 55, 56, 183]. However, such studies could not provide sufficient information about the relationship between the radiation efficiency and the evanescent waves. This is primarily because these studies have dealt with either the Gaussian-correlated source or the Bessel-correlated source as a source model for analysis where the former source involves evanescent waves and the latter source does not involve them in any cases. Accordingly, by virtue of the appropriate selection of the source model as discussed in Section 5.2.2, the present study is the first attempt to explore quantitatively the behavior of radiation efficiency in connection with the presence of evanescent waves.

5.3.2 Effect of the intensity distribution across the source

On the basis of the formula (5.15), together with Eqs.(5.18) and (5.24), established in Section 5.2.1 (and 5.2.2), the radiation efficiency of the sources with various intensity distributions is quantitatively investigated in this section. In view of Eq.(5.24), numerical results of Eq.(5.15) are seen to depend not only on the parameter p governing the intensity distribution across the source (see Fig.5.4), but also on the

value ka where $k = 2\pi/\lambda$ is the wavenumber of light across the source and a is a radius of the source filter (or the source itself). To this end, we introduce a new parameter β defined by

$$\beta = \frac{a}{\lambda}, \quad (5.29)$$

corresponding to the value ka and refer to it simply as the source size by assuming the fixed value of the wavelength λ . In what follows, we confine our discussion for simplicity to the case of $n = 2$ in Eq.(5.18). In this case, the complex degree of spectral coherence of the source given by Eq.(5.18) reduces to

$$g'(x) = \frac{2J_1(x/\alpha)}{x/\alpha}, \quad (5.30)$$

according to Table 5.1 in Section 5.2.2 and, thus, the spatial coherence condition across the source depends only on the parameter α . To illustrate some numerical results, the integration of Eq.(5.15) was numerically evaluated by using a computer.

As a typical example, the radiation efficiency of the source under the condition of $\alpha = 1$ is plotted in Fig.5.7 against the parameter p for various source sizes β ranging from 25.0 to 1000.0. Here, the range of β is properly determined in accordance with the framework of physical optics ([3], Chap. 8; [147], Chap. 3; [153], Chap. 3) in which the linear dimension of the source is required to be much larger than the wavelength. In Fig.5.7, the parameter p controls the intensity distribution across the source as discussed in Section 5.2.2: the positive values of p give rise to the central dip in intensity across the source and, conversely, the negative values of p result in the sources having the soft edge (see Fig.5.4). The case of $p = 0$ corresponds to the circular source with uniform intensity distribution. It is clearly seen in the figure that the radiation efficiency, for any values of β , is attenuated as the value p increases. A range of the variation in radiation efficiency is reduced with an increase in the source size β and finally becomes negligible for the source size β being approximately larger than the value 500.0 in this figure.

From the physical point of view, the above behavior of radiation efficiency can be explained with relation to the spatial frequency spectrum of the source. In general, the spatial frequency spectrum of the source is known to be given, instead of Eq.(5.21) which is applicable only to the quasi-homogeneous sources, by the

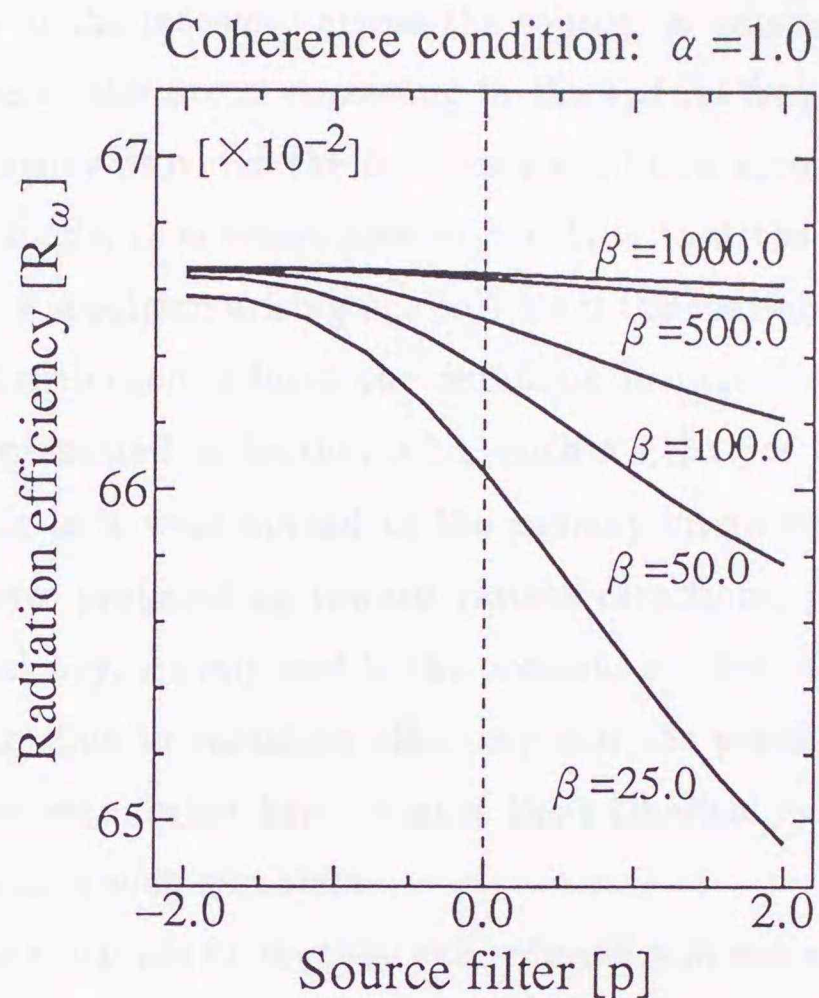


Figure 5.7. Radiation efficiency of the sources with various intensity distributions in the case of $\alpha = 1.0$. The curves are plotted as a function of the source filter p corresponding to Fig.5.4 for various source sizes β .

formula [19]:

$$F(f) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} C(x)g(x) \exp[-j\mathbf{f} \cdot \mathbf{x}] d^2x, \quad (5.31)$$

where $g(x)$ is the complex degree of spectral coherence of light across the source and $C(x)$ is the autocorrelation function of the amplitude distribution of the source (i.e., $C(x)$ is proportional to the autocorrelation function of the amplitude transmittance of the source filter in the present formulation). Equation (5.31) indicates that the spatial frequency spectrum of the source depends generally not only on the spatial coherence condition over the source, but also on the intensity distribution across the source. By recalling the performance in radiation efficiency that we have just discussed, an increase of p and a decrease of β , which yield essentially a rapid variation of the intensity distribution across the source, cause the reduction in radiation efficiency. On the other hand, it can be readily inferred in Eq.(5.31),

by considering some basic concepts of the standard Fourier analysis [187], that a rapid variation of the intensity across the source, in comparison with a slow variation, gives rise to the broad spreading in the spatial frequency spectrum toward the higher frequency provided the coherence condition across the source is fixed for brevity. Accordingly, it is reasonable to conclude that the broad spreading in the spatial frequency spectrum arising not only from the spatial coherence but also from the intensity distribution induces the reduction in radiation efficiency. As well as the discussion presented in Section 5.3.1, such a behavior in the spatial frequency spectrum results in a wide spread of the radiant intensity due to the mixture of many plane waves propagating toward various directions, producing the reduction of radiation efficiency. As opposed to the previous section, however, the relationship between the variation in radiation efficiency and the presence of evanescent waves cannot be easily manifested here because the numerical evaluation of Eq.(5.31) is required to examine such a subject.

In order to elucidate further the characteristics in radiation efficiency, the radiation efficiency of the source under the condition of $\alpha = 10.0$ is illustrated in Fig.5.8 as a function of the parameter p for various values of β . Evidently, Fig.5.8 bears a remarkable similarity to Fig.5.7 in the behavior of radiation efficiency. Detailed comparison of Fig.5.8 with Fig.5.7 reveals, however, that the *efficient* radiation in the sense of high radiation efficiency results from the high spatial coherence of the source (corresponding to the large α in Fig.5.8) rather than the low spatial coherence of the source (corresponding to the small α in Fig.5.7). Following the discussion presented in Section 5.3.1, a physical origin of such a tendency in variation of radiation efficiency is easily proved to be closely related to a variation of the spatial frequency spectrum stemming from the spatial coherence condition over the source.

In conclusion, it has become quantitatively obvious in this section that the variation in the spatial frequency spectrum arising from both the source coherence and the source intensity distribution affects the radiation efficiency.

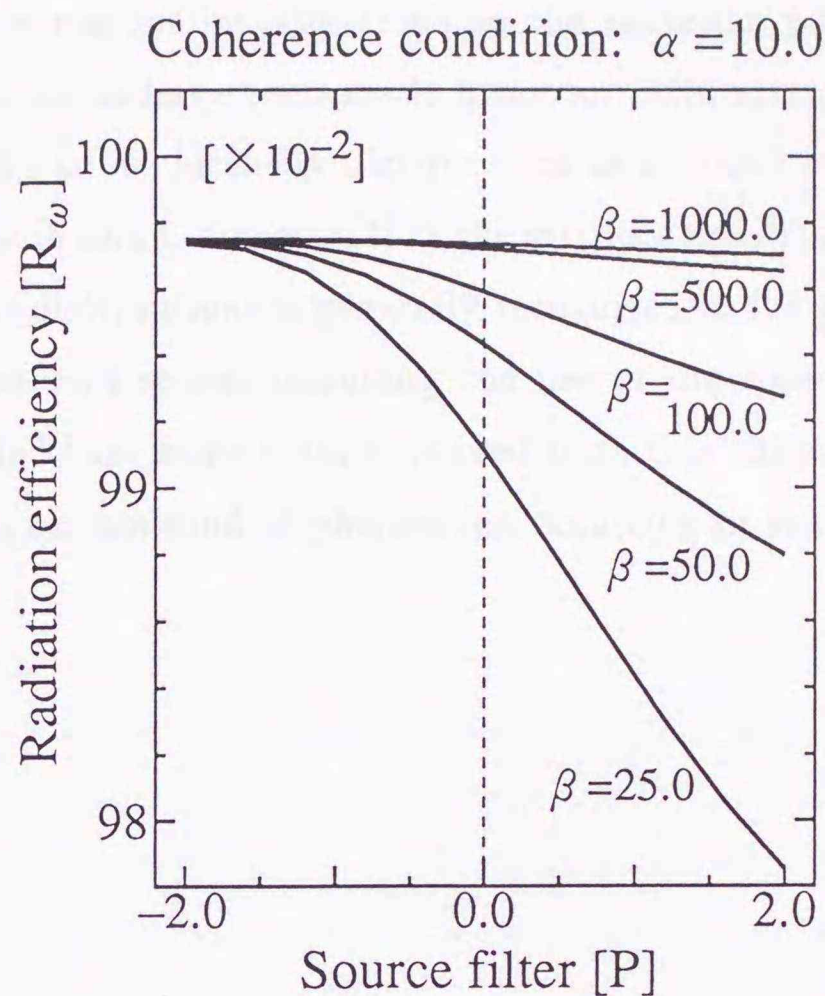


Figure 5.8. Radiation efficiency of the sources with various intensity distributions in the case of $\alpha = 10.0$. The curves are plotted as a function of the source filter p corresponding to Fig.5.4 for various source sizes β .

5.4 Conclusion

We have established a general formulation for radiation efficiency of the partially coherent planar sources with various intensity distributions. On the basis of the formulation, we have further investigated quantitatively the radiation efficiency of the source as a function of both the spatial coherence condition and the intensity distribution across the source. In particular, a physical origin of the variation in radiation efficiency has been manifested from the standpoint of the spatial frequency spectrum of the source associated with the evanescent waves. As a consequence, it has become obvious that the broadening in the spatial frequency spectrum of the source stemming from both the source coherence and the source intensity results in the reduction in radiation efficiency for the sources with various intensity distributions including the quasi-homogeneous ones and that this trend in radiation efficiency is irrespective of the presence of evanescent waves, at least, for the quasi-homogeneous

sources.

Although numerous investigations on the radiation properties from the partially coherent sources have been made hitherto, little attention has been given to the sources with various intensity distributions as a source model in their analyses. It should be kept in mind, however, that the intrinsic significance of the sources with various intensity distributions is generally recognized in the practical application of the partially coherent source including the use of the source filter. Therefore, it may be concluded that some results derived from this chapter serve as a means of discriminating a certain kind of phenomena occurring in practice.

CHAPTER 6

Radiation Efficiency of Partially Coherent Three-Dimensional Sources

6.1 Introduction

Numerous attempts have been made to elucidate the intimate relationship between spatially partially coherent sources and their radiated fields [2, 19–21, 183]. In the majority of investigations, partially coherent planar (or two-dimensional) sources have been frequently employed for the theoretical analyses in conjunction with the practical applications of optics and their radiation properties have been thoroughly demonstrated hitherto. In recent years, a great deal of attention has also been devoted to partially coherent three-dimensional (e.g., spherical) sources [38, 39, 59, 60, 84, 188–193], concurrently with planar ones.

Radiation efficiency of sources, which plays an important role in radiometry with partially coherent sources, was originally introduced as a measure for quantifying the features of radiation from quasi-homogeneous planar sources [24] and has been applied to various planar sources [55–57]. Here, radiation efficiency is defined as the total power radiated from a source normalized to a maximum value of unity. On the other hand, more recent investigations on radiation efficiency deal with partially coherent three-dimensional sources [59, 60] and lead to the significant consequence that the three-dimensional sources are in contrast to the planar ones in radiation efficiency: the maximum efficiency of radiation results from globally incoherent sources in the three-dimensional cases while the fully coherent condition over the source yields the maximum efficiency in the planar cases when the Gaussian Schell-model sources with a sufficiently large extent (in comparison with the wavelength) are employed for analyses in both cases. However, some characteristics in radiation efficiency of three-dimensional sources still remain obscure, mainly because emphasis has long been placed on the planar sources rather than the three-

dimensional ones.

In this chapter, radiation efficiency of three-dimensional sources is explored theoretically in some detail in order to supplement the preceding investigations. A partially coherent three-dimensional source to be treated here is assumed to be a primary one, such as an assembly of radiating atoms, within the formalism for scalar wavefields. To make the analysis as simple as possible, our discussion is confined to the homogeneous and isotropic source localized in a finite spherical domain. In addition, the spatial correlation of the source is assumed to be specified by means of properly arranged Bessel functions which take the same functional forms as in Chapter 5. Accordingly, a comparison of this chapter with the previous one (i.e., Chapter 5) may provide, in connection with the widely recognized characteristics of planar sources, comprehensive information about the features of radiation from partially coherent three-dimensional sources.

In the following, we first specify the source model for analysis and establish a specific expression for determining the radiation efficiency of the sources. On the basis of the expression, a number of representative curves of the radiation efficiency are illustrated as a function of the coherence condition of the sources. Finally, they are compared with the curves of the radiation efficiency for planar sources derived in Chapter 5.

6.2 Theoretical Preliminaries

We consider a partially coherent primary scalar source occupying a finite three-dimensional region. In accordance with the studies on primary sources [38, 39, 194], the source to be considered here is characterized by a correlation function of the true source distribution instead of that of the field within the source region. For brevity, we further consider a statistically homogeneous and isotropic source occupying a spherical region Σ of radius a with the uniform intensity distribution, which is illustrated in Fig.6.1. Such a model source appears to have physical inconsistency since no bounded source can be strictly homogeneous due to its edge effects. However, the radius a of the source is supposed here to be sufficiently large compared with the effective width of the source coherence in order that the boundary effects may

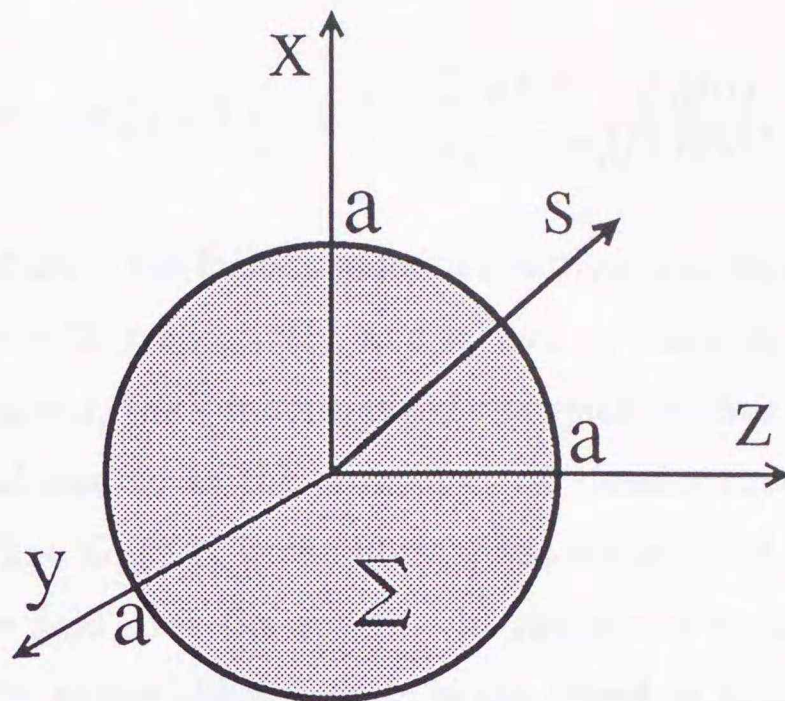


Figure 6.1. Illustration of the geometry for the present analysis. The homogeneous, isotropic and spherical source of radius a is located at the origin and radiates into the free whole-space. The symbol s denotes the three-dimensional unit vector.

be negligible [87]. Thus, the model source employed here can represent physically realizable sources as far as the above constraint holds between the source radius and the source coherence. The validity of this constraint is discussed in some detail in the next section.

An ultimate purpose of the present study is to examine the radiation efficiency (to be defined in the later section) of a three-dimensional source, especially under the consideration of a planar source. To make a bridge between the three-dimensional source and the planar one, the spatial coherence condition of the three-dimensional source employed here is specified by means of properly arranged Bessel functions that take the same functional form employed in Chapter 5 dealing with the radiation efficiency of planar sources. Hence, we begin by recalling succinctly the mathematical description of such sources and then determine the three-dimensional source model for the present analysis.

We consider a secondary planar Schell-model source occupying a finite area characterized by the normalized second-order correlation function of the field amplitude over the source in the space-frequency domain (i.e., the complex degree of

spectral coherence of the field over the source). It is given by the form

$$\mu_V(|\mathbf{r}_1 - \mathbf{r}_2|) = \Gamma\left(1 + \frac{n}{2}\right) \frac{J_{n/2}(k|\mathbf{r}_1 - \mathbf{r}_2|/\alpha)}{(k|\mathbf{r}_1 - \mathbf{r}_2|/2\alpha)^{n/2}}, \quad (6.1)$$

where $\Gamma(\cdot)$ and $J_{n/2}(\cdot)$ denote the Euler's gamma function and the Bessel function of the first kind and order $n/2$, respectively, and $\mathbf{r}_1$ and $\mathbf{r}_2$ stand for two-dimensional position vectors. In general, the (unnormalized) correlation function in the space-frequency domain is referred to as the cross-spectral density function. Thus, it is also possible to state that Eq.(6.1) is the normalized quantity of the cross-spectral density function of the field over the source. In the above equation, $k = \omega/c$ is the wavenumber of light across the source (c is the speed of light in vacuum) and the positive values of n and α (precisely, $n \geq 0$ and $\alpha > 0$) act as parameters for changing the spatial coherence condition of the field over the source. Specifically, the spatial coherence of the field over the source grows with an increase in both the value of α for the fixed value of n and the value of n for the fixed value of α . Here and in the following, the dependence on the frequency ω is not shown explicitly in each argument of equations for simplicity. Due to the initial postulate of the secondary source, Eq.(6.1) represents the correlation function of the field over the source region rather than that of the true source distribution.

As is widely known, the planar source whose spatial coherence is characterized by Eq.(6.1) in the case of $\alpha = 1$ has the property that the angular distribution of the radiant intensity obeys the law of $\cos^n \theta$ where θ is the angle formed between the direction of observation and the normal axis to the source when the source is homogeneous or quasi-homogeneous [19, 20, 183]. It is also known [195] that such a source can be characterized by means of the correlation function of the true source distribution taking a similar form as the correlation function of the field over the source. However, the source with the parameter of $\alpha > 1$ does not rigorously fulfill the $\cos^n \theta$ -law of the radiant intensity and radiates in a somewhat different manner (see Fig.5.6 in Chapter 5). By taking into account the relationship between the source correlation and the field correlation tabulated by Carter [195], the normalized second-order correlation function of the true source distribution in the space-frequency domain corresponding to Eq.(6.1) is readily found to be given

n	$\mu_V(x)$ of Eq.(6.1)	$\mu_Q(x)$ of Eq.(6.2)
0	$J_0(x/\alpha)$	$\frac{2J_1(x/\alpha)}{x/\alpha}$
1	$\frac{\sin(x/\alpha)}{x/\alpha}$	$\frac{3\sqrt{\pi}}{4} \frac{J_{3/2}(x/\alpha)}{(x/2\alpha)^{3/2}}$
2	$\frac{2J_1(x/\alpha)}{x/\alpha}$	$\frac{8J_2(x/\alpha)}{(x/\alpha)^2}$
3	$\frac{3\sqrt{\pi}}{4} \frac{J_{3/2}(x/\alpha)}{(x/2\alpha)^{3/2}}$	$\frac{15\sqrt{\pi}}{8} \frac{J_{5/2}(x/\alpha)}{(x/2\alpha)^{5/2}}$

Table 6.1. Explicit forms of Eqs.(6.1) and (6.2).

by

$$\mu_Q(|\mathbf{r}_1 - \mathbf{r}_2|) = \Gamma\left(2 + \frac{n}{2}\right) \frac{J_{n/2+1}(k|\mathbf{r}_1 - \mathbf{r}_2|/\alpha)}{(k|\mathbf{r}_1 - \mathbf{r}_2|/2\alpha)^{n/2+1}}, \quad (6.2)$$

provided that the source is homogeneous, where Eqs.(6.1) and (6.2) take the same form except for the parameter n : Eq.(6.2) becomes Eq.(6.1) when the value of n is replaced by $(n + 2)$ in Eq.(6.1). In this case, Eq.(6.2) is obviously the normalized quantity of the cross-spectral density function of the true source distribution [38, 194] and is termed, as is often the case, the complex degree of spectral coherence of the source. To emphasize the physical implication of these coherence functions, the explicit forms of Eqs.(6.1) and (6.2) are summarized in Table 6.1 for some selected values of n , where the argument $k|\mathbf{r}_1 - \mathbf{r}_2|$ in each equation is replaced by x for simplicity. It is evident from the table that the coherence functions for $\alpha = 1$ and $n = 1$ characterize a planar thermal source [185, 194] which pertains to an important class of actual sources.

We now return to the main subject. As mentioned at the beginning of this section, the present study is formulated by means of the correlation function of the true source distribution. To conduct a comparison study between the three-

dimensional source and the planar one, it seems to be expedient to connect the three-dimensional source to be employed here with the (primary) planar source specified in terms of the true source correlation. To do this, we assume that the complex degree of spectral coherence of the source for the present analysis is given, by referring to Eq.(6.2) rather than Eq.(6.1), in the form

$$\mu_Q(|\mathbf{x}_1 - \mathbf{x}_2|) = \Gamma\left(2 + \frac{n}{2}\right) \frac{J_{n/2+1}(k|\mathbf{x}_1 - \mathbf{x}_2|/\alpha)}{(k|\mathbf{x}_1 - \mathbf{x}_2|/2\alpha)^{n/2+1}}, \quad (6.3)$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ indicate three-dimensional position vectors. This implies, from the viewpoint of the source coherence, that an arbitrary cross-section of the three-dimensional source coincides with the preceding planar source to be employed for comparison. To make clear the behavior of the spatial coherence given by Eq.(6.3), the complex degree of spectral coherence of the source is illustrated in Fig.6.2 as a function of $k|\mathbf{x}_1 - \mathbf{x}_2|/\alpha$ for some values of n . It is worth noting here that the

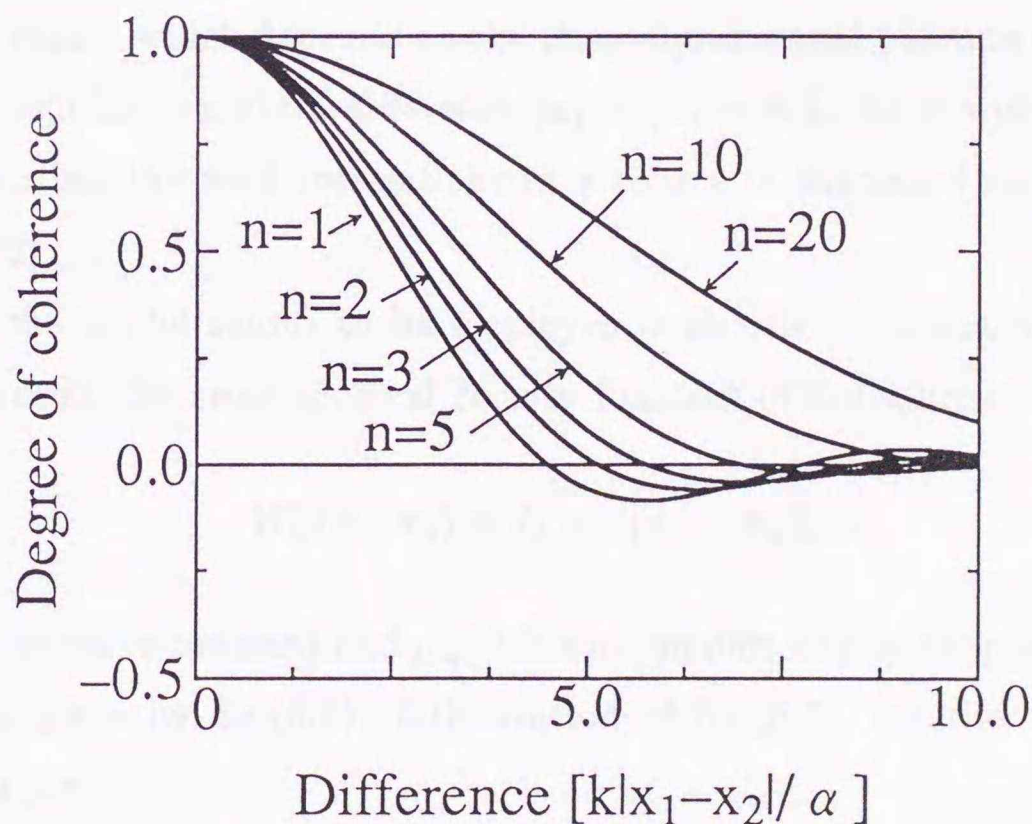


Figure 6.2. Complex degree of spectral coherence of the source.

physical significance of the source in question becomes obvious by considering the past investigation on the field correlations within a fluctuating homogeneous medium (which is essentially equivalent to the true source distribution). This connection is briefly shown below.

According to Nussenzveig *et al.* [196], the complex degree $\mu_V(|x_1 - x_2|)$ of spectral coherence of the field within the homogeneous and isotropic three-dimensional source is always given by the expression

$$\mu_V(|x_1 - x_2|) = j_0(k|x_1 - x_2|) \quad (6.4)$$

with x_1 and x_2 being three-dimensional position vectors, provided that the quantity

$$I_Q = \iiint_{-\infty}^{\infty} j_0(k|x|) W_Q(|x|) d^3x \quad (6.5)$$

takes a non-zero value (i.e., $I_Q \neq 0$) where

$$j_0(z) \equiv \frac{\sin z}{z} \quad (6.6)$$

stands for the spherical Bessel function of order zero and $W_Q(x)$ denotes the cross-spectral density function of the true source distribution for the corresponding three-dimensional source which depends on the three-dimensional position vectors x_1 and x_2 only through their mutual difference $|x| = |x_1 - x_2|$. As is widely recognized, Eq.(6.4) describes the field correlations in a source of thermal (blackbody) radiation [185, 197].

When the model source to be employed is strictly statistically homogeneous (i.e., unbounded), the cross-spectral density function of the source becomes

$$W_Q(x_1, x_2) = I_0 \mu_Q(|x_1 - x_2|), \quad (6.7)$$

where I_0 is a positive constant and $\mu_Q(\cdot)$ is the complex degree of spectral coherence of the source given by Eq.(6.3). Substitution of Eq.(6.7), together with Eq.(6.3), into Eq.(6.5) yields

$$I_Q = \begin{cases} \frac{8\pi^{3/2}I_0}{k^3} \frac{\Gamma(\frac{n}{2} + 2)}{\Gamma(\frac{n+1}{2})} \alpha^3 (1 - \alpha^2)^{\frac{n-1}{2}} \neq 0 & \text{for } 0 < \alpha < 1 \\ 0 & \text{for } \alpha > 1, \end{cases} \quad (6.8)$$

for all n being $n \geq 0$ ([180], 6.699, p. 747). In the case of $\alpha = 1$, I_Q takes zero for $n > 1$ and non-zero value for $0 \leq n \leq 1$. It follows from the preceding theory of

Nussenzveig *et al.* and Eq.(6.8) that the model source represents a three-dimensional thermal source, regardless of the values of α for $n \geq 0$, as long as the condition $0 < \alpha \leq 1$ is fulfilled where the equality holds for $0 \leq n \leq 1$. When such a source is bounded but sufficiently large as is assumed in the subsequent analysis, it may be regarded approximately (but, not strictly) as a spherical thermal source in the sense that the spatial correlations of the field within the source is expected to be approximately given by Eq.(6.4). The fact that all the model sources for this range of α are equally associated with one source (i.e. the thermal source) may be intuitively elucidated by considering that they radiate uniformly in the whole space irrespective of α (although the strength of the radiated field is varied with α), as will be seen in Eq.(6.23).

Finally, it may be noticed that, in spite of the same functional form for the complex degrees $\mu_Q(\cdot)$ of spectral coherence of the source given by Eqs.(6.2) and (6.3), the corresponding complex degrees $\mu_V(\cdot)$ of spectral coherence of the field are different from each other (i.e., Eqs.(6.1) and (6.4)). However, this result has no inconsistency. Such a fact stems from the difference of the source appearance: Eq.(6.2) is employed for the two-dimensional source while Eq.(6.3) is employed for the three-dimensional one.

6.3 Mathematical Framework for Deriving the Radiation Efficiency

In this section, we first introduce the concept of radiation efficiency of the three-dimensional source and then derive a specific expression for the radiation efficiency of the source.

The radiation efficiency of the three-dimensional source (at frequency ω) confined in a finite region is given by [60]

$$R_\omega = \frac{\Phi_\omega}{4\pi D N_\omega}, \quad (6.9)$$

where

$$\Phi_\omega = \int_{(4\pi)} J_\omega(s) d\Omega \quad (6.10)$$

indicates the total power radiated by the source into the whole space, D is the actual source volume, and

$$N_\omega = \iiint_\Sigma S_Q(\mathbf{x}) d^3x \quad (6.11)$$

stands for the integrated source intensity. In Eq.(6.10), $J_\omega(\mathbf{s})$ denotes the radiant intensity at frequency ω in the direction specified by a unit (three-dimensional) vector $\mathbf{s}$ and is shown, in terms of the cross-spectral density function $W_Q(\mathbf{x}_1, \mathbf{x}_2)$ of the true source distribution, to be given by the general expression [38]

$$J_\omega(\mathbf{s}) = \iiint_\Sigma \iiint_\Sigma W_Q(\mathbf{x}_1, \mathbf{x}_2) \exp[-j\mathbf{k}\mathbf{s} \cdot (\mathbf{x}_1 - \mathbf{x}_2)] d^3x_1 d^3x_2, \quad (6.12)$$

where $k = \omega/c$ denotes the wavenumber of light over the source. In Eq.(6.11), $S_Q(\mathbf{x})$ denotes the spectral intensity of the source at frequency ω and can be represented, in terms of $W_Q(\mathbf{x}_1, \mathbf{x}_2)$, by

$$S_Q(\mathbf{x}) = W_Q(\mathbf{x}, \mathbf{x}). \quad (6.13)$$

The radiation efficiency defined by Eq.(6.9) is shown to satisfy the inequality

$$0 \leq R_\omega \leq 1, \quad (6.14)$$

for any three-dimensional primary sources of the finite extent [60] and its value is proportional to the total power radiated by the source due to the definition. Thus, the radiation efficiency defined by Eq.(6.9) acts satisfactorily as a measure for quantifying the features of radiation from partially coherent three-dimensional sources.

Radiation efficiency of the present model source (which was specified in the previous section) is now evaluated on the basis of Eq.(6.9). To obtain a specific expression for radiation efficiency of the source, the radiant intensity $J_\omega(\mathbf{s})$ given by Eq.(6.12) is first dealt with. The cross-spectral density function $W_Q(\mathbf{x}_1, \mathbf{x}_2)$ to be considered here is given by Eq.(6.7) within the source region Σ , as well as for the strictly homogeneous source. Outside the source region Σ , the function $W_Q(\mathbf{x}_1, \mathbf{x}_2)$ becomes negligible in this case. When Eq.(6.7) is substituted into Eq.(6.12), the radiant intensity $J_\omega(\mathbf{s})$ becomes

$$J_\omega(\mathbf{s}) = I_0 \iiint_\Sigma \iiint_\Sigma \mu_Q(|\mathbf{x}_1 - \mathbf{x}_2|) \exp[-j\mathbf{k}\mathbf{s} \cdot (\mathbf{x}_1 - \mathbf{x}_2)] d^3x_1 d^3x_2, \quad (6.15)$$

where $\mu_Q(\cdot)$ is the complex degree of spectral coherence given by Eq.(6.3). By changing the variables of integrations from x_1 and x_2 to x and x' with

$$x = \frac{x_1 + x_2}{2}, \quad x' = x_1 - x_2, \quad (6.16)$$

and by assuming that the radius a of the source is much larger than the effective width Δ_Q of the degree $|\mu_Q(|x'|)|$ of spectral coherence to assure the hypothesis of the homogeneous source, i.e.,

$$\Delta_Q \ll a, \quad (6.17)$$

Eq.(6.15) may be approximately written as

$$J_\omega(s) = I_0 \iiint_{\Sigma} d^3x \iiint_{-\infty}^{\infty} \mu_Q(|x'|) \exp[-jks \cdot x'] d^3x' \quad (6.18)$$

with the range of integration Σ implying the sphere with radius a . Since the integration with respect to x in Eq.(6.18) results in the actual source volume

$$D = \frac{4}{3}\pi a^3, \quad (6.19)$$

Eq.(6.18) can be reduced to

$$J_\omega(s) = I_0 D \iiint_{-\infty}^{\infty} \mu_Q(|x'|) \exp[-jks \cdot x'] d^3x'. \quad (6.20)$$

Equation (6.20) reveals that the radiant intensity $J_\omega(s)$ of the source is given, aside from the constant factors I_0 and D , by means of the three-dimensional spatial Fourier transform of the complex degree $\mu_Q(|x'|)$ of spectral coherence of the source. By introducing the spherical coordinates and by performing the integrations with respect to the angular variables, Eq.(6.20) can be further reduced to ([187], pp. 251-253)

$$J_\omega(s) = 4\pi I_0 D \int_0^\infty \mu_Q(x) \left(\frac{\sin kx}{kx} \right) x^2 dx, \quad (6.21)$$

where $x = |x'|$. Substitution of Eq.(6.3) into Eq.(6.21) leads to

$$J_\omega(s) = 4\pi I_0 D \Gamma(2 + \frac{n}{2}) \int_0^\infty \frac{J_{n/2+1}(kx/\alpha)}{(kx/2\alpha)^{n/2+1}} \left(\frac{\sin kx}{kx} \right) x^2 dx, \quad (6.22)$$

yielding ([180], 6.699, p. 747)

$$J_{\omega}(s) = \begin{cases} \frac{8\pi^{3/2}I_0D}{k^3} \frac{\Gamma(\frac{n}{2}+2)}{\Gamma(\frac{n+1}{2})} \alpha^3 (1-\alpha^2)^{\frac{n-1}{2}} & \text{for } 0 < \alpha \leq 1 \\ 0 & \text{for } \alpha > 1. \end{cases} \quad (6.23)$$

Equation (6.23) is essentially the same as Eq.(6.8) except for the factor D although their physical meanings are different from each other. This fact stems from the profound relationship in mathematics between Eq.(6.5) (together with Eqs.(6.6) and (6.7)) and Eq.(6.21). In Eq.(6.23), it is worth noting that the radiant intensity of the source is independent of the direction s . Therefore, the total power Φ_{ω} radiated by the source defined by Eq.(6.10) is simply expressible in the form

$$\Phi_{\omega} = 4\pi J_{\omega}(s). \quad (6.24)$$

On the other hand, the integrated source intensity N defined by Eq.(6.11) is easily shown to become

$$N_{\omega} = I_0 D \quad (6.25)$$

with D implying the actual source volume given by Eq.(6.19). From Eqs.(6.9), (6.19), (6.23), (6.24) and (6.25), the radiation efficiency of the source becomes

$$R_{\omega} = \begin{cases} \frac{6\sqrt{\pi}}{(ka)^3} \frac{\Gamma(\frac{n}{2}+2)}{\Gamma(\frac{n+1}{2})} \alpha^3 (1-\alpha^2)^{\frac{n-1}{2}} & \text{for } 0 < \alpha \leq 1 \\ 0 & \text{for } \alpha > 1, \end{cases} \quad (6.26)$$

under the localized homogeneous source condition of Eq.(6.17), where a is the radius of the source and α is a parameter for changing the spatial coherence condition of the source.

Finally, the inequality of Eq.(6.17) is evaluated on the basis of the formula [198]

$$\Delta_Q = \left[\frac{\int_0^{\infty} x^2 |\mu_Q(x)|^2 dx}{\int_0^{\infty} |\mu_Q(x)|^2 dx} \right]^{1/2}, \quad (6.27)$$

implying the power-rms width of the complex degree $\mu_Q(x)$ of spectral coherence given by Eq.(6.3). This formula of determining the effective width of the function $\mu_Q(x)$ is appropriately adopted for the present purpose because it takes into account the behavior of the tail of the function due to its definition. By substituting Eq.(6.3) into Eq.(6.27), the inequality of Eq.(6.17) becomes ([180], 6.574, p. 692)

$$\left(\frac{\alpha}{2k}\right) \sqrt{\frac{(n+1)(2n+3)}{n}} \ll a, \quad \text{for } n > 0, \quad (6.28)$$

and it does not exist for $n = 0$ due to the diverging property of Δ_Q . In practice, the validity of the above inequality is questionable for $n \rightarrow 0$ (even if $n \neq 0$) unless the value of α becomes negligible, because the left-hand side of Eq.(6.28) approaches infinity. In this case, the function $\mu_Q(x)$ is expected to have a fluctuating long tail that hardly converges to zero. These facts seem to suggest that the model source with small n being nearly zero, regardless of the value α except for an extreme value being nearly zero, may not be homogeneous in the present sense mentioned in Section 6.2 if it is bounded in a finite region. Conversely, the source with the other range of n for any values of α can become a localized homogeneous one insofar as the condition of Eq.(6.28) holds.

Equation (6.26) with the above constraint is the final expression for the radiation efficiency of the source under consideration. On the basis of Eq.(6.26), some characteristics in radiation from the partially coherent three-dimensional source are investigated in the subsequent sections.

6.4 Radiation Efficiency of the Partially Coherent Three-Dimensional Source

As was mentioned in Section 6.2, the spatial coherence condition of the source varies generally with both values of n and α based on the complex degree of spectral coherence given by Eq.(6.3). However, we consider for simplicity that α acts as a parameter for controlling the spatial coherence condition of the source under the fixed condition of n . By referring to Fig.6.2 under the assumption of the homogeneous spherical source, two extremes of $\alpha \rightarrow 0$ and ∞ (in this context, the term ∞ is used as a finite value which is sufficiently larger than the wavelength of light over

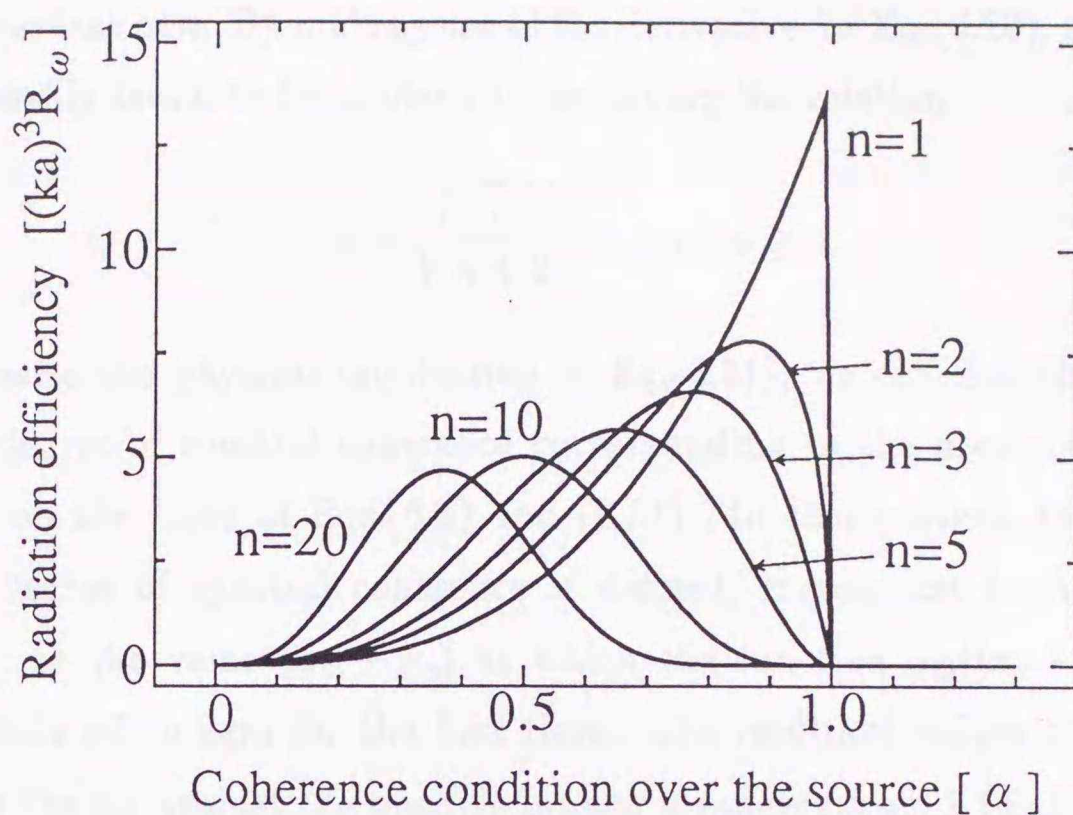


Figure 6.3. Radiation efficiency of the three-dimensional source illustrated in Fig.6.1. The spatial coherence condition of the source is characterized by Eq.(6.3). The curves are plotted against the coherence condition of the source α for various values of n .

the source) represent a spatially incoherent source and a *locally* spatially coherent cophasal one, respectively, in the sense that the complex degree of spectral coherence given by Eq.(6.3) is reduced, for any values of n , to

$$\mu_Q(|x_1 - x_2|) = \begin{cases} 1 & \text{for } x_1 = x_2 \\ 0 & \text{for } x_1 \neq x_2, \end{cases} \quad (6.29)$$

and

$$\mu_Q(|x_1 - x_2|) = 1 \quad \text{for all } x_1 \text{ and } x_2 \in S, \quad (6.30)$$

where S denotes a sufficiently small (but finite) area in comparison with the source area Σ . It is to be noticed, in either case, that the source must be *globally* spatially incoherent due to its homogeneous hypothesis given by Eq.(6.28).

On the basis of Eq.(6.26), the radiation efficiencies of the source for some values of n are illustrated in Fig.6.3 as a function of α . It is clearly seen in the figure that the radiation efficiency varies depending on the spatial coherence condition of the source through both values of n and α . In the range of $0 < \alpha < 1$ for $n \geq 1$, the

optimal values of α that maximize the radiation efficiency of the source are reduced with an increase of n . By making use of the derivative for Eq.(6.26), such an optimal state is readily found to be realized by satisfying the relation

$$\alpha = \sqrt{\frac{3}{n+2}} \quad \text{for } n \geq 1. \quad (6.31)$$

To emphasize the physical implication of Eq.(6.31), we examine the width of the complex degree of spectral coherence corresponding to the maximum efficiency of radiation on the basis of Eqs.(6.3) and (6.31). In this context, the width of the complex degree of spectral coherence is defined, in contrast to the definition of Eq.(6.27), as the value $|x_1 - x_2|$ at which the function $\mu_Q(|x_1 - x_2|)$ given by Eq.(6.3) falls off to zero for the first time. The resultant values of the width are plotted in Fig.6.4 against the positive integer n ranging from 1 to 10. It is apparent

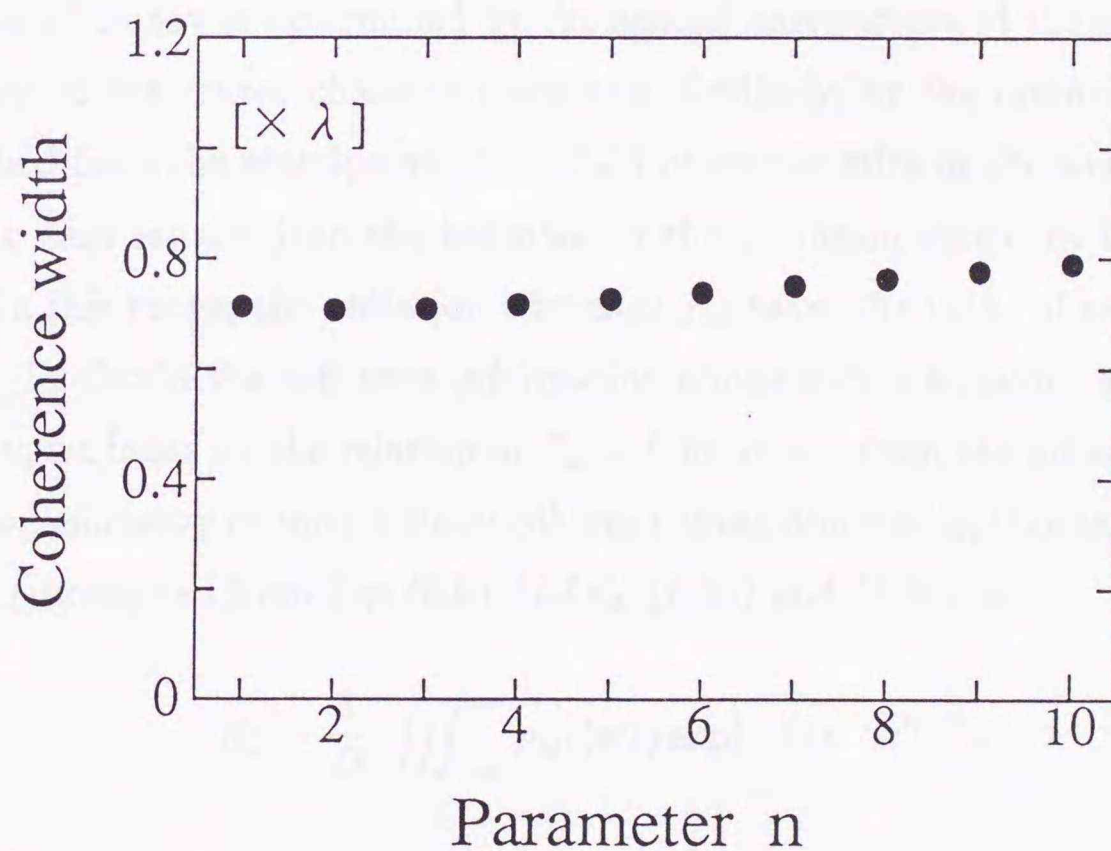


Figure 6.4. Width of the complex degree of spectral coherence that maximizes the radiation efficiency for various values of n .

from the figure that the width remains nearly constant at approximately 0.7λ , with λ denoting the wavelength of light of the source, for any values of n . This implies that the effective width of the source coherence that maximizes the radiation efficiency is given by about half a wavelength, in agreement with the past study [59] where

such a consequence is understood in connection with the interference effect in the source region.

It follows from the above discussion that the radiation efficiency of the homogeneous spherical source is controlled by the source correlations (i.e., the values of α), rather than the field correlations within the source, on the ground that all the sources with $0 < \alpha < 1$ for $n \geq 0$ (precisely, including the source with $\alpha = 1$ for $0 \leq n \leq 1$) represent equally a thermal source characterized by the field correlation function of Eq.(6.4), as was mentioned in Section 6.2. This is true in a sense, but more careful inspection is required to elucidate the phenomena: the intensity distribution (more specifically, the spectral density) of the field within the source region, rather than the field correlations, is expected to be varied with the source correlations by inferring from the related investigation [192] concerning the field properties within a sinc-correlated spherical source. Accordingly, it may be expedient to state, within the present treatment of the problem, that the behavior in radiation efficiency is determined by the spatial correlations of the source from the viewpoint of the source characteristics and, similarly, by the intensity distribution of the field from the standpoint of the field characteristics in the source.

We next inquire into the behavior of the radiation efficiency in the range of $\alpha > 1$. In this range, the radiation efficiency R_ω takes the value of zero as shown in Fig.6.3. To obtain the sufficient information about such a behavior of the radiation efficiency, we focus on the relation of $R_\omega = 0$ for $\alpha > 1$ from the different viewpoint. Radiation efficiency of the partially coherent three-dimensional homogeneous source may be represented from Eqs.(6.9), (6.24), (6.25) and (6.20) as

$$R_\omega = \frac{1}{D} \iiint_{-\infty}^{\infty} \mu_Q(|\mathbf{x}'|) \exp[-j\mathbf{k}\mathbf{s} \cdot \mathbf{x}'] d^3\mathbf{x}', \quad (6.32)$$

where D denotes the actual source volume in Eq.(6.19), $\mu_Q(\cdot)$ denotes the complex degree of spectral coherence in Eq.(6.3), and $\mathbf{s}$ is the three-dimensional unit vector. Thus, the condition of $R_\omega = 0$ may be uniquely expressible in the form

$$\tilde{\mu}_Q(K) \Big|_{|K|=k} = 0 \quad (6.33)$$

with

$$\tilde{\mu}_Q(\mathbf{K}) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} \mu_Q(\mathbf{r}) \exp[-j\mathbf{K} \cdot \mathbf{r}] d^3\mathbf{r} \quad (6.34)$$

being the three-dimensional spatial Fourier transform of the function $\mu_Q(\cdot)$, provided that the source is confined in a finite region. Equation (6.33) indicates that the source does not radiate (i.e., $R_w = 0$) if the three-dimensional spatial Fourier transform $\tilde{\mu}_Q(\mathbf{K})$ of its degree of spectral coherence vanishes when $|\mathbf{K}| = k$ with k being the wavelength of light over the source. It is equivalent to the condition for a localized, statistically stationary, partially coherent, three-dimensional scalar source to be non-radiating [188]. It follows from the above discussion that the source characterized by Eq.(6.3) for any values of n in the case of $\alpha > 1$ may be regarded as a realization of the non-radiating source.

Finally, a few remarks should be made on the source for $0 \leq n < 1$. It is found in Eq.(6.26) that the radiation efficiency of the source for $0 \leq n < 1$ diverges when α approaches the value of unity within the range of $0 < \alpha < 1$. Such a behavior in radiation efficiency contradicts the constraint of Eq.(6.14). However, this fact is not so surprising by considering that there exists no finite source in practice satisfying rigorously the condition of the homogeneity. The source with $\alpha = 1$ for $0 \leq n < 1$ may be only a mathematical idealization and may not exist in practice, as was also suggested in Section 6.3 using Eqs.(6.17), (6.27) and (6.28). Nevertheless, it is worth emphasizing that the present analysis serves as a means of exploring, at least qualitatively, some kinds of practical phenomena concerning the features of radiation from partially coherent three-dimensional sources.

6.5 Comparison Between the Three-Dimensional Source and the Planar Source

To find further the characteristics in radiation from the partially coherent three-dimensional source, we reexamine the results derived in the previous sections from the comparison with Chapter 5 dealing with the radiation efficiency of the partially coherent planar source. One of the sources employed in Chapter 5 is a partially coherent planar quasi-homogeneous source characterized by the complex degree of

spectral coherence given in the form of Eq.(6.1). Moreover, such a planar source is the same as the cross section of the three-dimensional spherical source that has been explored in this chapter. As a result, the comparison study may provide an intuitive understanding about the diversity of radiation between the planar source and the three-dimensional one, giving a sufficient elucidation on the features of radiation from the three-dimensional source. We start with recalling the main points of the previous chapter concerning the radiation efficiency of the planar source.

Radiation efficiency of the planar source at frequency ω is defined by

$$R_\omega = \frac{\Phi_\omega}{N_\omega} \quad (6.35)$$

where Φ_ω implies the total power radiated by the source into the free half-space and N_ω indicates the integrated source intensity. By comparing Eq.(6.35) with Eq.(6.9), the physics involved in the radiation efficiency of the planar source is found to be essentially equivalent to that of the three-dimensional one. Following Chapter 5, we assume the partially coherent planar source characterized by Eq.(6.1), satisfying the quasi-homogeneous condition. By making use of the definition in Eq.(6.35), radiation efficiency of such a source becomes

$$R_\omega = \begin{cases} \frac{n}{3}\alpha^2 F(1, 1 - \frac{n}{2}; \frac{5}{2}; \alpha^2) & \text{for } 0 < \alpha < 1 \\ \frac{n}{n+1} & \text{for } \alpha = 1 \\ F(1, -\frac{1}{2}; 1 + \frac{n}{2}; \alpha^{-2}) & \text{for } \alpha > 1 \end{cases} \quad (6.36)$$

with F being hypergeometric functions. In Eq.(6.36), n and α act as parameters for changing the spatial coherence condition over the source. Curves for the radiation efficiency given by Eq.(6.36) are illustrated in Fig.6.5 for some values of n . It is important to note that Eq.(6.36) is also valid for the homogeneous source owing to the mathematical analogy between the homogeneous source and the quasi-homogeneous one. Equation (6.36) and Fig.6.5 are the significant consequences in Chapter 5 and will be used in the subsequent analysis. We are now in a position to consider the subject for finding the difference in radiation between the three-dimensional source and the planar one.

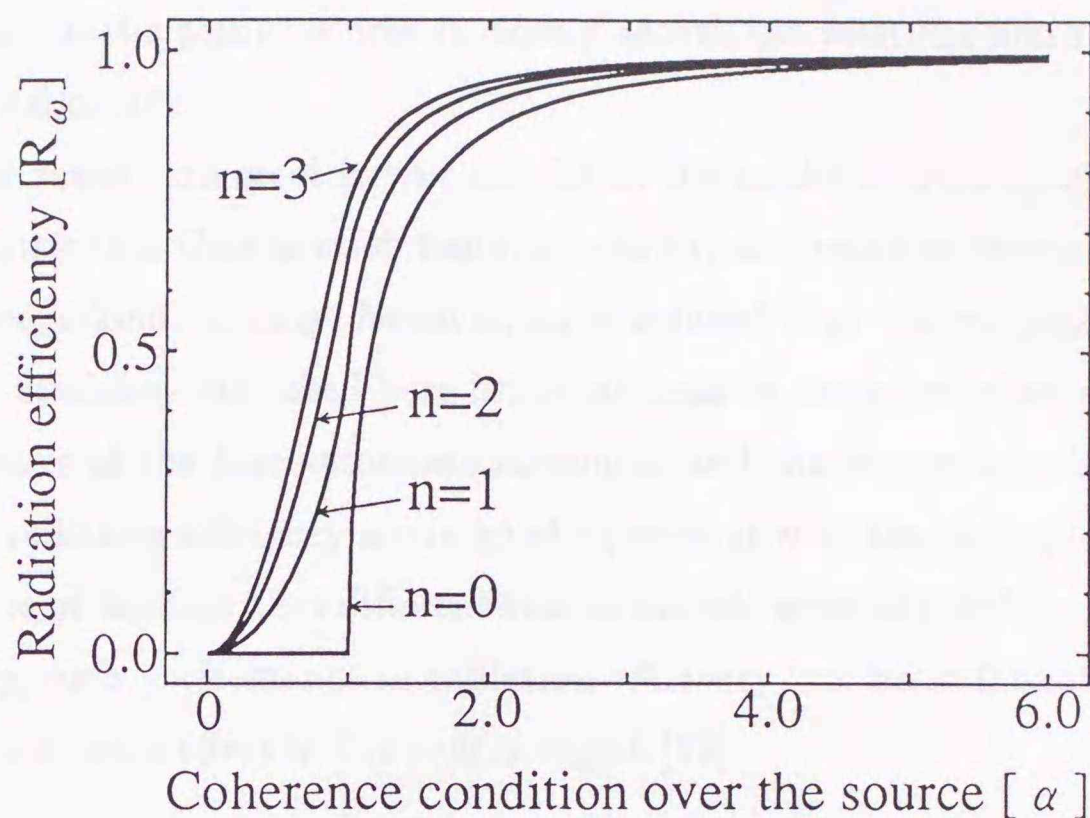


Figure 6.5. Radiation efficiency of the quasi-homogeneous source specified by the complex degree of spectral coherence in the form of Eq.(6.1). The curves are plotted against the coherence condition over the source α for various values of n . (This figure is taken from Chapter 5.)

When Fig.6.3 is compared with Fig.6.5, we notice a kind of the reciprocal behavior in radiation efficiency between the three-dimensional source and the planar one. In the region of $\alpha > 1$ for any values of n , the radiation efficiencies of the planar source approach their maximum possible value of unity though those of the three-dimensional source vanish. On the contrary, a sudden decrease of the curves in radiation efficiency of the planar source is observed with a decrease of α in the range of $0 < \alpha < 1$ for any values of n (rigorously, except for the case of $n = 0$) while maximum efficiencies of radiation from the three-dimensional source occur in the same range of α . It should be recalled that the parameter α is closely related to the spatial coherence condition over the source and, at the same time, to the presence of evanescent waves in the case of the planar source: there exist the evanescent waves for $0 < \alpha < 1$ and no evanescent waves for $\alpha \geq 1$. As is widely known, the evanescent waves do not carry the energy into the far field of the source. In other words, it does not radiate. Therefore, it seems reasonable to state that the non-radiating component of the planar source (i.e., the range of $0 < \alpha < 1$) contributes

to the radiation of the three-dimensional source and, conversely, that the radiating component of the planar source is closely associated with the non-radiating three-dimensional source.

The above statement is true insofar as the model sources specified in Section 6.2 are concerned. On the other hand, its validity still remains obscure for the source of a different kind. It may, however, be concluded that the reciprocal behavior in radiation efficiency discussed here holds, at least in the sense of an approximation, for any pairs of the three-dimensional source and the planar one, because general trends in radiation efficiency are in good agreement with the past studies [59, 60, 24, 57] in spite of the fact that different source models were adopted for their analyses. Physically, such performance in radiation efficiency can be well explained in terms of the interference effect in the source region [59].

6.6 Conclusion

Radiation efficiency of the partially coherent three-dimensional source has been investigated theoretically. The source model for analysis is assumed to be a primary, homogeneous, isotropic and spherical source and its spatial coherence property is specified in terms of properly arranged Bessel functions. It is found that the source of this kind represents physically either a thermal source or a non-radiating one by an appropriate selection of the source parameters. A number of representative curves of the radiation efficiency for the three-dimensional source have been illustrated as a function of the spatial coherence condition of the source and they have led to the fact that the radiation efficiency varies depending strongly on the source coherence. Specifically, the source radiates for $0 < \alpha < 1$ (corresponding to thermal sources) while it does not radiate for $\alpha > 1$ (corresponding to non-radiating sources) where α is a parameter for changing the spatial coherence condition of the source. The radiation property of the source with $\alpha = 1$ depends on the other parameter n for changing the spatial coherence of the source. The optimal coherence (source correlation) width that maximizes the radiation efficiency has also been examined and, as a consequence, its value has been found to be approximately 0.7λ in any cases. In addition, it has become obvious that the trends in variation of radiation

efficiency can be explained consistently not only in terms of the source characteristics, but also in terms of the field characteristics in the source. To provide an intuitive understanding about the features of radiation from the three-dimensional source, curves of the radiation efficiency for the three-dimensional source have been compared with those for the planar one whose spatial coherence is characterized by means of the same functional form of the Bessel functions. From the comparison, it has followed that there exists a kind of the reciprocal behavior in radiation efficiency between the three-dimensional source and the planar one. A relationship of the sort derived in the present analysis may serve as a means of exploring the newly developed (or unknown) three-dimensional source from the knowledge for the planar one and *vice versa*.

CHAPTER 7

Radiation from Three-Dimensional, Anisotropic Gaussian Schell-Model Sources

7.1 Introduction

There has long been an enormous interest in radiation from partially coherent sources [2, 19–21, 183]. In particular, considerable attention has recently been given to the subject of changes in spectra of radiation induced by spatial coherence properties of sources, which is often called the Wolf effect [62]. Several kinds of model sources have been introduced and their radiation properties have been, both theoretically and experimentally, explored in a series of such investigations. Gaussian Schell-model (GSM) sources are their typical examples and have played a significant role especially in the theoretical development of the problem (see, for example, Refs. [39, 44, 57, 69]). This is mainly because the model sources of this type represent many actual sources to a good approximation and, moreover, because they provide a comprehensive insight into the phenomena immediately after some simple operations in mathematics.

In this chapter, properties of radiation from three-dimensional, anisotropic Gaussian Schell-model (AGSM) sources are investigated within the framework of coherence theory established by Carter and Wolf [38]. Such model sources can represent, by an appropriate selection of source parameters, a wide variety of partially coherent sources: planar sources, spherically symmetric sources, and other sources with a certain anisotropy in intensity and/or in spatial coherence. Accordingly, the analysis to be conducted below enables us to elucidate also the difference in radiation between planar sources and three-dimensional sources, supplementing the related investigations regarding the radiation efficiency of partially coherent sources where planar sources have been compared with three-dimensional ones from the viewpoint of radiation efficiency (see Chapter 6 and Ref.[59]). In addition, it should

be noted that the present study is also a further extension of the past studies dealing with radiation from planar AGSM sources [41, 45, 46].

In the following, some theoretical preliminaries for the problem are first dealt with and, subsequently, specific expressions are derived for the radiant intensity and the far-field spectrum. On the basis of the spectrum expression, shifts of the spectral line on light propagation are investigated in some detail in connection with the Wolf effect. Emphasis is placed through the whole analysis on the phenomena arising from the three-dimensional anisotropy of the source.

7.2 Theoretical Preliminaries

We consider a primary Schell-model source occupying a finite three-dimensional region centered at the origin and radiating into the free whole-space, as illustrated in Fig.7.1. Such a source is characterized by the cross-spectral density function of

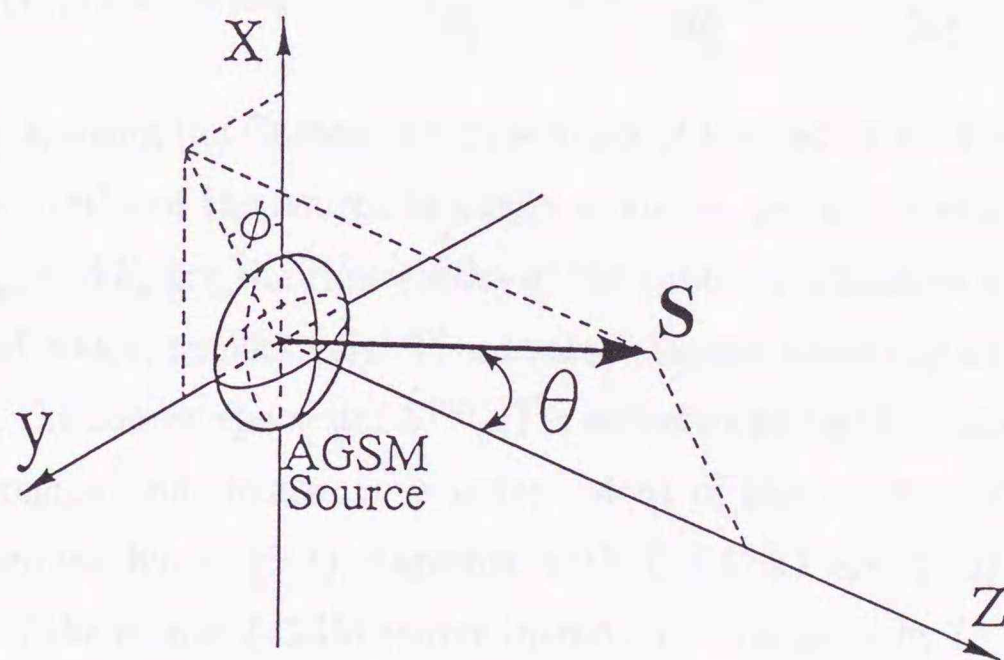


Figure 7.1. Illustration of the geometry. The three-dimensional, anisotropic Gaussian Schell-model (AGSM) source is located at the origin and radiates into the whole space. The symbol s denotes the three-dimensional unit vector having the components $s = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ in the Cartesian coordinate system and specifies the direction of the observation.

the true source distribution at frequency ω of the form [38, 39]

$$W_Q(r_1, r_2; \omega) = \sqrt{S_Q(r_1; \omega)} \sqrt{S_Q(r_2; \omega)} \mu_Q(r_1, r_2; \omega) \quad (7.1)$$

where

$$S_Q(\mathbf{r}_i; \omega) = W_Q(\mathbf{r}_i, \mathbf{r}_i; \omega) \quad (i = 1, 2) \quad (7.2)$$

stands for the spectral intensity at a position specified by the three-dimensional vector $\mathbf{r}_i$ and

$$\mu_Q(\mathbf{r}_1, \mathbf{r}_2; \omega) = g(\mathbf{r}_1 - \mathbf{r}_2; \omega) \quad (7.3)$$

with the condition of $g(0; \omega) = 1$ for all ω denotes the complex degree of spectral coherence which depends on $\mathbf{r}_1$ and $\mathbf{r}_2$ only through the difference $\mathbf{r}_1 - \mathbf{r}_2$. For an AGSM source with the three-dimensional extent, $S_Q(\mathbf{r}; \omega)$ and $\mu_Q(\mathbf{r}_1, \mathbf{r}_2; \omega)$ in Eq.(7.1) generally become

$$S_Q(\mathbf{r}; \omega) = S^{(0)}(\omega) \exp \left[-\frac{x^2}{2a_x^2} - \frac{y^2}{2a_y^2} - \frac{z^2}{2a_z^2} \right] \quad (7.4)$$

and

$$\mu_Q(\mathbf{r}_1, \mathbf{r}_2; \omega) = \exp \left[-\frac{(x_1 - x_2)^2}{2b_x^2} - \frac{(y_1 - y_2)^2}{2b_y^2} - \frac{(z_1 - z_2)^2}{2b_z^2} \right] \quad (7.5)$$

with x_i , y_i , and z_i being the Cartesian components of the vector $\mathbf{r}_i$. Here, a_x , a_y , and a_z are the rms widths of the source intensity along x -, y -, and z -axis, respectively. Likewise, b_x , b_y , and b_z are the rms widths of the spatial correlation over the source along x -, y -, and z -axis, respectively. These values depend generally on the frequency ω . In Eq.(7.4), the source spectrum $S^{(0)}(\omega)$ is assumed to be the same at all points in the source region and, hence, it is independent of the position $\mathbf{r}$. The model source characterized by Eq.(7.1), together with Eqs.(7.4) and (7.5), is a natural generalization of the planar AGSM source introduced originally by Li and Wolf [41].

In the present treatment of the problem, we confine our discussion to the case satisfying both conditions of $a_x = a_y (\equiv a_t, \text{ say})$ in Eq.(7.4) and $b_x = b_y (\equiv b_t, \text{ say})$ in Eq.(7.5), for brevity, under the assumption that the forward direction of the source coincides with the z -axis. This implies that the source is isotropic, both in intensity and in spatial correlation, across any cross-section of the source perpendicular to the z -axis (i.e., parallel to the x - y plane). Thus, the anisotropy of the source is governed by the following four parameters: the rms width a_t of the source intensity in the transverse directions, the rms width a_z of the source intensity along the z -axis, the

rms width b_t of the spatial correlation over the source in the transverse directions, and the rms width b_z of the spatial correlation over the source along the z -axis, where these values are assumed to be independent of the frequency ω . Specifically, the source intensity has spherically symmetric distribution for $a_t = a_z$, rotationally symmetric ellipsoidal distribution elongated along the z -axis for $a_t < a_z$, and that shortened along the same axis for $a_t > a_z$. In a similar fashion, the anisotropy of the spatial correlation over the source is controlled by the values of b_t and b_z . On the other hand, to simplify the subsequent analysis, the source spectrum in Eq.(7.4) is assumed to have a Gaussian profile of the form

$$S^{(0)}(\omega) = A \exp \left[-\frac{(\omega - \omega_0)^2}{2\sigma^2} \right], \quad (7.6)$$

where A and σ take positive constants, and ω_0 denotes the central (angular) frequency of the spectrum. In the above equation, σ characterizing the linewidth of the spectrum is assumed to fulfill the inequality $\sigma \ll \omega_0$. By substituting Eqs.(7.4), (7.5) and (7.6) into Eq.(7.1), and using the relations $a_x = a_y \equiv a_t$ and $b_x = b_y \equiv b_t$, the model source employed here is uniquely determined in terms of the cross-spectral density function of the source expressed by

$$\begin{aligned} W_Q(r_1, r_2; \omega) = & A \exp \left[-\frac{(\omega - \omega_0)^2}{2\sigma^2} \right] \\ & \times \exp \left[-\frac{x_1^2 + x_2^2 + y_1^2 + y_2^2}{4a_t^2} - \frac{z_1^2 + z_2^2}{4a_z^2} \right] \\ & \times \exp \left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2b_t^2} - \frac{(z_1 - z_2)^2}{2b_z^2} \right]. \end{aligned} \quad (7.7)$$

The subsequent analysis is mathematically developed in accordance with the theory established by Carter and Wolf [38]. The fundamental equation required below is the propagation law of the cross-spectral density function given by

$$\begin{aligned} W_V^{(\infty)}(r_1 s_1, r_2 s_2; \omega) = & \frac{\exp[jk(r_1 - r_2)]}{r_1 r_2} \iiint_{-\infty}^{\infty} \iiint_{-\infty}^{\infty} W_Q(r'_1, r'_2; \omega) \\ & \times \exp[-jk(s_1 \cdot r'_1 - s_2 \cdot r'_2)] d^3 r'_1 d^3 r'_2, \end{aligned} \quad (7.8)$$

where $W_V^{(\infty)}$ and W_Q represent the cross-spectral density function of the far field and that of the source distribution, respectively. In Eq.(7.8), s_i is the three-dimensional

unit vector specifying the observation direction, r_i is the distance between the origin and the observation point in the far zone, and $k = \omega/c$ is the wavenumber of light over the source (c is the speed of light in vacuum). By making use of Eq.(7.8), the general expressions for the radiant intensity ($J_\omega(\mathbf{s}) = r^2 W_V^{(\infty)}(r\mathbf{s}, r\mathbf{s}; \omega)$) and the far-field spectrum ($S_V^{(\infty)}(\mathbf{s}; \omega) = W_V^{(\infty)}(r\mathbf{s}, r\mathbf{s}; \omega)$) can be written in the form

$$J_\omega(\mathbf{s}) = \iiint_{-\infty}^{\infty} \iiint_{-\infty}^{\infty} W_Q(\mathbf{r}'_1, \mathbf{r}'_2; \omega) \exp[-jk\mathbf{s} \cdot (\mathbf{r}'_1 - \mathbf{r}'_2)] d^3\mathbf{r}'_1 d^3\mathbf{r}'_2 \quad (7.9)$$

and

$$S_V^{(\infty)}(\mathbf{s}; \omega) = \frac{1}{r^2} \iiint_{-\infty}^{\infty} \iiint_{-\infty}^{\infty} W_Q(\mathbf{r}'_1, \mathbf{r}'_2; \omega) \exp[-jk\mathbf{s} \cdot (\mathbf{r}'_1 - \mathbf{r}'_2)] d^3\mathbf{r}'_1 d^3\mathbf{r}'_2, \quad (7.10)$$

respectively. Finally, it is to be noticed that the radiant intensity $J_\omega(\mathbf{s})$ given by Eq.(7.9) is a function of solely the observation direction $\mathbf{s}$ due to its definition (e.g., Refs. [2, 21]) while the far-field spectrum $S_V^{(\infty)}(\mathbf{s}; \omega)$ given by Eq.(7.10) is a function of both $\mathbf{s}$ and ω .

7.3 Angular Distribution of Radiant Intensity from the Source

Before investigating the main subject concerning the spectral aspect of the propagating light, the radiant intensity is first treated in this section on the basis of Eqs.(7.7) and (7.9).

By substituting Eq.(7.7) into Eq.(7.9) and performing the six-dimensional integration, the radiant intensity $J_\omega(\mathbf{s})$ in the direction specified by a unit vector $\mathbf{s} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is obtained as

$$J_\omega(\theta) = (2\pi)^3 S^{(0)}(\omega) a_t^2 \gamma_t^2 a_z \gamma_z \exp \left[-\frac{k^2}{2} (\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta) \right] \quad (7.11)$$

where γ_t and γ_z are abbreviations given by

$$\frac{1}{\gamma_t^2} = \frac{1}{(2a_t)^2} + \frac{1}{b_t^2}, \quad \frac{1}{\gamma_z^2} = \frac{1}{(2a_z)^2} + \frac{1}{b_z^2}. \quad (7.12)$$

The symbol $S^{(0)}(\omega)$ is employed in Eq.(7.11) in place of the explicit form of the source spectrum given by Eq.(7.6) because the frequency ω should be regarded as

a constant here due to the definition of the radiant intensity. In addition, $J_\omega(s)$ is replaced by $J_\omega(\theta)$ in Eq.(7.11) on the grounds that the resultant expression for $J_\omega(s)$ depends on the direction s only through the angle θ (see, Fig.7.1). This exhibits, in other words, that the distribution of the radiant intensity from the source is rotationally symmetric with respect to the z -axis.

To illustrate conveniently the angular distribution of the radiant intensity, the normalization of the radiant intensity is conducted by dividing Eq.(7.11) by its maximum value, i.e., $J_\omega(0)$ for $\gamma_t \geq \gamma_z$ and $J_\omega(\pi/2)$ for $\gamma_t < \gamma_z$. The resultant expression, which is referred to as the normalized radiant intensity $J_{\omega,N}(\theta)$, can be written in the form

$$J_{\omega,N}(\theta) = \begin{cases} \exp\left(-\frac{1}{2}\alpha^2 \sin^2 \theta\right) & \text{for } \gamma_t \geq \gamma_z \\ \exp\left(-\frac{1}{2}\alpha^2 \cos^2 \theta\right) & \text{for } \gamma_t < \gamma_z \end{cases} \quad (7.13)$$

with α being

$$\alpha = k\sqrt{|\gamma_t^2 - \gamma_z^2|}. \quad (7.14)$$

The normalized radiant intensity is plotted in Figs.7.2(a) and (b) for several values of α using Eqs.(7.13) and (7.14). It is found from these figures that the radiant intensity from the source is independent of the direction θ provided $\alpha = 0$ (i.e., $\gamma_t = \gamma_z$). Namely, the source radiates uniformly into the whole space, yielding the spherically symmetric radiated field in the far zone. The difference of the values between γ_t and γ_z produces the explicit dependence of the radiant intensity on the observation angle θ . It is worth emphasizing, however, that the relationship between the angular distribution of the radiant intensity and the anisotropic properties of the source remains generally obscure in the sense that, in view of Eqs.(7.13), (7.14) and (7.12), the same angular dependence of radiation can be realized by various sources with different anisotropic properties. In particular, the conditions for the isotropic source, $a_t = a_z$ and $b_t = b_z$, do not necessarily hold true even when the source radiates uniformly into the whole space under the condition of $\gamma_t = \gamma_z$. Similar consequences have been recognized in the related investigation concerning the radiation from planar AGSM sources [41].

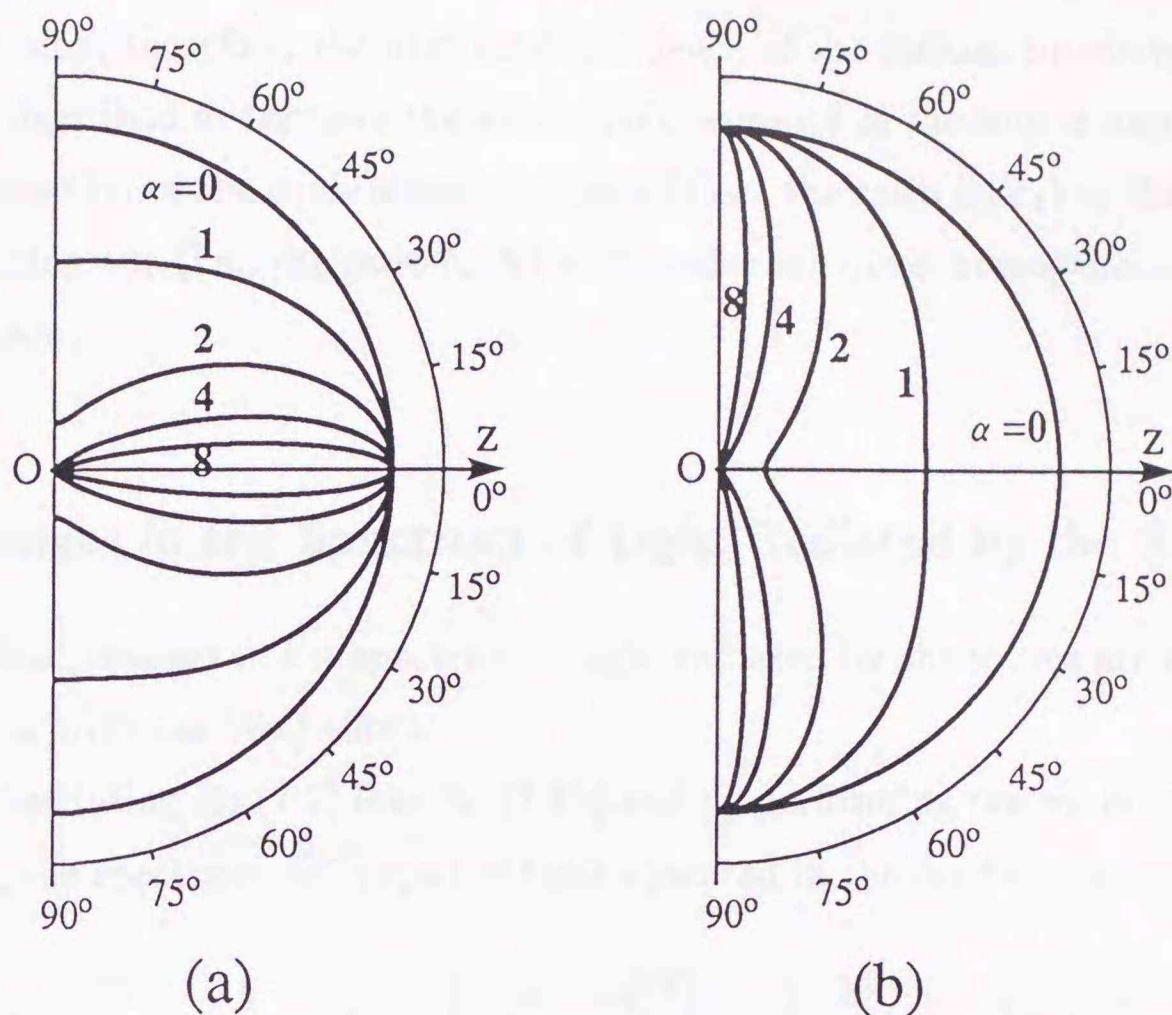


Figure 7.2. Angular distribution of the radiant intensity for several values of α in the cases of (a) $\gamma_t \geq \gamma_z$ and (b) $\gamma_t < \gamma_z$. The curves are plotted on the basis of Eqs.(7.13) and (7.14) only for the half-space $z \geq 0$.

With a view of obtaining a profound understanding for the phenomena, we next consider the source fulfilling the condition of the three-dimensional quasi-homogeneity [39] which implies that the effective widths, a_t and a_z , of the source intensity are sufficiently large in comparison with the effective widths, b_t and b_z , of the spatial correlation over the source (i.e., $a_t, a_z \gg b_t, b_z$). Under such circumstance, the parameters γ_t and γ_z may be replaced by b_t and b_z , respectively (see, Eq.(7.12)). As a result, as opposed to the preceding argument, the angular distribution of the radiant intensity is uniquely determined by means of the values of b_t and b_z , rather than those of γ_t and γ_z . More specifically, the quasi-homogeneous source with the isotropic property, $b_t = b_z$, of spatial coherence produces the spherically symmetric radiated field in the far field. On the contrary, the source is always isotropic in the sense that $b_t = b_z$ when it radiates uniformly into the whole space. As is clearly seen in Eqs.(7.13) and (7.14) with b_t and b_z being, respectively, replaced by γ_t and γ_z , the angular distribution of the radiant intensity depends on b_t and b_z through both

their mutual difference $|b_t^2 - b_z^2|$ and their relative magnitude for a given value of k . In conclusion, therefore, the angular dependence of the radiant intensity cannot be directly described in terms of the anisotropic property of the source implying either the distortion of the equi-intensity ellipsoid (i.e., the ratio a_z/a_t) or that of the equi-correlation one (i.e., the ratio b_z/b_t) even under the quasi-homogeneous source approximation.

7.4 Changes in the Spectrum of Light Radiated by the Source

In this section, changes in the spectrum of light radiated by the source are explored in connection with the Wolf effect.

By substituting Eq.(7.7) into Eq.(7.10) and by performing the six-dimensional integration, the spectrum $S_V^{(\infty)}(s; \omega)$ of light observed in the far field becomes

$$S_V^{(\infty)}(\theta; \omega) = \frac{(2\pi)^3}{r^2} a_t^2 \gamma_t^2 a_z \gamma_z A \exp \left[-\frac{(\omega - \omega_0)^2}{2\sigma^2} \right] \exp \left[-\frac{k^2}{2} (\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta) \right] \quad (7.15)$$

where γ_t and γ_z are given by Eq.(7.12) and $k = \omega/c$. In the above equation, the symbol $S_V^{(\infty)}(\theta; \omega)$ appears in place of $S_V^{(\infty)}(s; \omega)$ because the angular distribution of the far-field spectrum depends only on the observation angle θ as well as the radiant intensity treated in the previous section. Rearranging Eq.(7.15) with the help of a product theorem for Gaussian functions (appendix A of Ref. [101]), we obtain the final expression for $S_V^{(\infty)}(\theta; \omega)$ as follows:

$$S_V^{(\infty)}(\theta; \omega) = B \exp \left[-\frac{\omega_0^2}{2(\sigma^2 + \sigma'^2)} \right] \exp \left[-\frac{(\omega - \omega')^2}{2\sigma''^2} \right] \quad (7.16)$$

where

$$B = \frac{(2\pi)^3}{r^2} a_t^2 \gamma_t^2 a_z \gamma_z A, \quad (7.17)$$

$$\sigma' = \frac{c}{\sqrt{\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta}}, \quad (7.18)$$

$$\sigma'' = \frac{\sigma}{\sqrt{1 + (\sigma/c)^2 (\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta)}}, \quad (7.19)$$

$$\omega' = \frac{\omega_0}{1 + (\sigma/c)^2(\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta)}. \quad (7.20)$$

It is obviously seen from the second exponential function on the right-hand side of Eq.(7.16) that the resultant far-field spectrum takes a Gaussian form centered at the frequency ω' with its rms width being σ'' . The remaining factors on the right-hand side of Eq.(7.16) which are independent of the frequency ω determine the relative height of the resultant spectrum. In the present discussion, however, such factors are of no importance since the spectrum is normalized by its maximum value so as to examine the changes in the spectrum of light on propagation.

The formula (7.16), including Eqs.(7.19) and (7.20), clearly shows that the normalized spectrum of the far field generally differs from that of the source. In particular, the effective width and the central frequency of the resultant spectrum are varied on propagation with the values of γ_t and γ_z for given values of both the effective width σ of the source spectrum and the angle θ . Here, in view of Eq.(7.12), the values of γ_t and γ_z are strongly influenced by the spatial coherence condition over the source. Hence, it seems reasonable to state that the phenomena of changes in the spectrum under consideration pertain to the class of the recent investigations regarding the correlation-induced spectral changes predicted originally by Wolf [63].

As some typical analyses, we first examine in brief the spectral width of light on propagation. It is found from Eq.(7.19) that the rms width σ'' of the far-field spectrum and the rms width σ of the source spectrum are connected by the relation

$$\sigma'' \leq \sigma, \quad (7.21)$$

where the equality holds only when either $\sigma = 0$ or $\gamma_t = \gamma_z = 0$. Physically, this implies that the spectrum of light radiated by the source is generally narrowed on propagation except for the case that the source is either monochromatic ($\sigma = 0$) or completely incoherent ($b_t = b_z = 0$ in Eq.(7.12), i.e., $\gamma_t = \gamma_z = 0$). Although the condition of $a_t = a_z = 0$ in Eq.(7.12) also leads to the relation $\gamma_t = \gamma_z = 0$, it is not considered here because it cannot hold true insofar as the source does exist owing to the definition of the values a_t and a_z characterizing the effective widths of the source.

We next deal with the main subject of concern in the present investigation, namely, the shift of the spectral line on propagation. To specify the relative shift of

the spectrum, we employ the quantity z defined by

$$z = \frac{\lambda' - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega'}{\omega'}, \quad (7.22)$$

where $\omega_0 = 2\pi c/\lambda_0$ and $\omega' = 2\pi c/\lambda'$ are the central (angular) frequency of the source spectrum and that of the resulting far-field spectrum. Substituting Eq.(7.20) into Eq.(7.22), we obtain the expression

$$z = \left(\frac{\sigma}{c}\right)^2 (\gamma_t^2 \sin^2 \theta + \gamma_z^2 \cos^2 \theta) \quad (7.23)$$

with the value z satisfying

$$z \geq 0 \quad (7.24)$$

where the equality of Eq.(7.24) holds under the same circumstance as that of Eq.(7.21) holds. It follows from Eqs.(7.20) and (7.23) that the magnitude of the shift varies with the effective width of the source spectrum (σ), the spatial coherence condition over the source (γ_t, γ_z), and the observation angle (θ). In addition, Eq.(7.24) indicates that the far-field spectrum is mostly shifted toward lower frequencies (i.e., the red shift). However, the spectral shift produced in this way is restricted in magnitude to values that are effectively of the order of or smaller than the effective width σ of the source spectrum. It is important to note that the z -number given by Eq.(7.23) is generally frequency dependent due to the existence of σ in the expression. On the other hand, the z -number for the spectral shift arising from the Doppler effect is independent of the frequency. Hence, the spectral shift of light produced by the source under consideration does not imitate the Doppler effect and is clearly distinguished from such the effect.

To find further properties of the phenomena, we deal with the dependence of the spectral shift on the direction of observation in some detail. To do this, the spectral shift z given by Eq.(7.23) is normalized by its maximum value, i.e., $z|_{\theta=\pi/2}$ for $\gamma_t \geq \gamma_z$ and $z|_{\theta=0}$ for $\gamma_t < \gamma_z$. As a result, the normalized quantity $z_N(\theta)$ of the spectral shift can be written in the form

$$z_N(\theta) = \begin{cases} \sin^2 \theta + R^2 \cos^2 \theta & \text{for } 0 \leq R \leq 1 \\ \frac{1}{R^2} \sin^2 \theta + \cos^2 \theta & \text{for } R > 1 \end{cases} \quad (7.25)$$

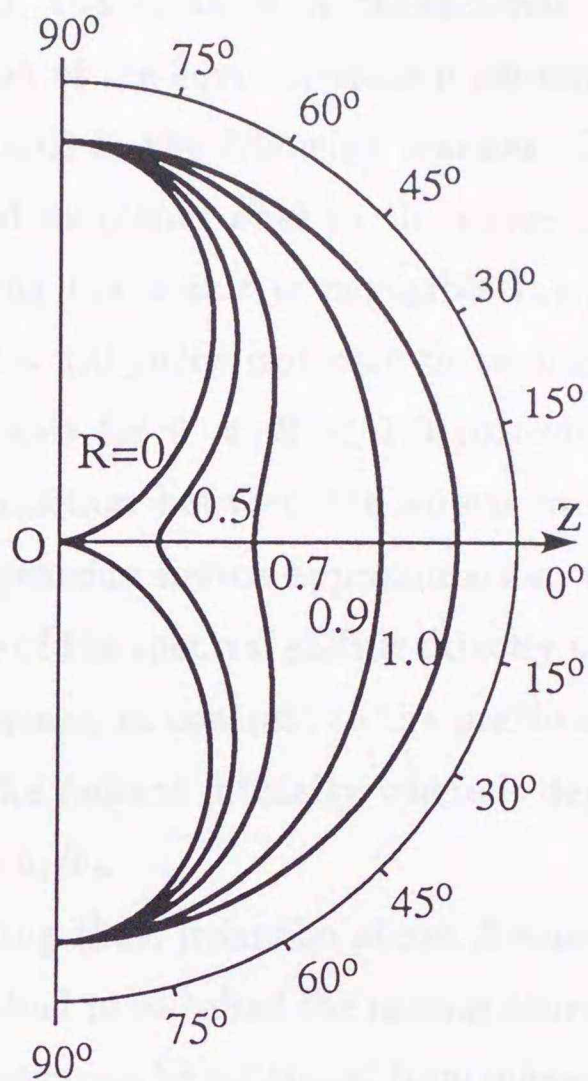


Figure 7.3. Angular distribution of the spectral shift for selected values of $R = \gamma_z/\gamma_t$. The value of R reduces to b_z/b_t in the quasi-homogeneous limit where b_t and b_z characterize the effective width of the spatial coherence over the source.

where R is given by

$$R = \frac{\gamma_z}{\gamma_t}. \quad (7.26)$$

On the basis of Eqs.(7.25) and (7.26), Fig.7.3 illustrates the magnitude of the spectral shift plotted in a polar diagram for some selected values of R . Evidently, this figure shows that the angular distribution of the shift depends strongly on the values of R . In particular, the magnitude of the shift remains constant with the value of θ in the case of $R = 1.0$ and, on the other hand, it increases from the value of zero with an increase of θ in the case of $R = 0$. As was also mentioned in the previous section, the physical implication of the parameter R given by Eq.(7.26) becomes apparent especially when the source satisfies the quasi-homogeneous condition. In this limit, Eq.(7.26) reduces to

$$R = \frac{b_z}{b_t}, \quad (7.27)$$

in view of Eq.(7.12). Then, the value of R characterizes the anisotropy of the source implying the distortion of the equi-correlation ellipsoid and serves to specify the spatial aspect of the source in the following manner. The sources having the value of $R = 0$ are regarded as planar ones in the sense that the effective width of the source coherence along the z -axis is negligible (i.e., $b_z = 0$ and $b_t \neq 0$). Conversely, the condition $R = 1.0$ yields isotropic three-dimensional sources in the sense that $b_t = b_z$. The cases for $0 < R < 1.0$ correspond to the anisotropic sources representing the transition between the above two limits. Therefore, at least under the quasi-homogeneous source approximation, it is expedient to state that the angular dependence of the spectral shift is directly connected by the spatial aspect of the source in that sense, in contrast to the previous discussion concerning the angular dependence of the radiant intensity where it depends on the difference $b_t^2 - b_z^2$ rather than the ratio b_z/b_t .

Finally, it is worth noting that, from the above discussion, the phenomena of the spectral shift may be applied to so-called the inverse source problems; the spatial aspect of the source in that sense can be extracted from measurements of the spectral shift (rather than the radiant intensity) as long as the source can be represented, to a good approximation, by a three-dimensional AGSM one. In fact, some related subjects have been investigated in recent years [97, 98]. However, detailed discussion on this problem is not presented because it leads to the digression from the main aim of the present investigation.

7.5 Conclusion

Radiant intensity and spectral changes of light generated from three-dimensional anisotropic Gaussian Schell-model (AGSM) sources have been investigated. Through the whole analysis, emphasis has been placed on the shift of the spectral line on light propagation under the consideration of recent topics regarding the correlation-induced spectral changes. Numerical examples have demonstrated that the angular distribution of the magnitude of the spectral shift depends strongly on the three-dimensional anisotropy of both the source intensity and the source coherence. In particular, under the quasi-homogeneous source approximation, it has become ob-

CHAPTER 8

Spatial Coherence of Light Passing Through Successive Apertures

8.1 Introduction

The phenomena resulting from wave propagation in periodic structures have been of great interest in various branches of physics. Among them, the periodic structure such as an iterated sequence of apertures has received considerable attention primarily with relation to the determination of laser resonator modes since the pioneering work of Fox and Li [111]. The spatial coherence of light propagating through such a structure has been, therefore, investigated from the viewpoint of exploring the spatial coherence of laser light.

The early contribution dealing with the spatial coherence in the laser resonator was made by Wolf [114] immediately after the paper of Fox and Li. Considering the periodic sequence of apertures equivalent to the laser resonator, he showed with the help of the theory of integral equations that the light field with any state of spatial coherence may become completely spatially coherent after a sufficient number of transits through the successive apertures. He ascribed this phenomenon to the effect of propagation and diffraction and succeeded in the theoretical explanation for the origin of the spatial coherence of light emerging from the laser resonator. On the basis of the mode theory, some numerical and analytical verifications of this theory have been conducted by Streifer [115] and Gori [116]. Streifer assumed the periodic sequence of apertures and that of lenses with small Fresnel numbers as analyzing models and investigated the rapidity with which the spatial coherence is built up in the periodic structure. On the other hand, Gori examined the similar subject by employing the periodic sequence of confocal lenses having Gaussian soft apertures at each focal plane for mathematical simplicity. As a result, the field characteristics after propagation through the periodic structure could be described mathematically

in a simple form and, hence, the quantitative evaluation for this phenomenon was easily performed.

Independent of these studies for the periodic structures associated with laser resonators, the significance of optical devices having certain periodicity has been recognized in recent years. A typical example is a dielectric planar waveguide corrugated along its surface by the medium with a slightly different refractive index, i.e., a waveguide diffraction grating [125, 124]. It serves in the field of integrated optics as a multistage beam-splitter, a spectral filter, a mode coupler, lens and so on [125] and, current interest in integrated optics [122] or microoptics [123] has broadened further the area of its application. Typical thickness of the planar waveguide is comparable to the wavelength of light guided and, hence, the light confined in the structure is highly spatially coherent since only few lower-order modes may survive in the structure. Thus, the study dealing with the spatial coherence of light in such waveguide may be somewhat meaningless. However, this is always not the case. When considering the planar waveguide with large thickness in comparison with the wavelength of light guided, i.e., multimode planar waveguide, for generalization, the features of spatial coherence of light can be varied due to the modal superposition and, therefore, they are worth investigating. Such structures as the multimode planar waveguide are expected to attract much attention due to the recent development of optical information processing [126] which explores the use of optical devices with various structures. Still, to the authors' knowledge, no study has been conducted on the spatial coherence properties of light propagating through the waveguides of these kinds. Though the waveguide with the periodicity is different in structure from the preceding periodic structure such as a sequence of apertures, both structures may be considered to be essentially the same at least in the sense that the propagating light is perturbed repeatedly by the structure. Therefore, it seems feasible that the study of Wolf and its continuations give some insights into the feature of spatial coherence of light propagating through the dielectric waveguide with the periodicity. However, different treatments must be properly adopted in order to examine rigorously the features of spatial coherence in the dielectric waveguide in question.

In this chapter, the spatial coherence of light passing through the periodic sequence of lenses separated by uniform apertures (i.e., successive apertures, as seen in the title of this chapter) is investigated as a further extension of the studies

of Streifer [115] and Gori [116]. Some restrictions encountered in their analyses are eliminated here for the sake of generalization: slit apertures in place of the Gaussian soft apertures are employed in the periodic structure and, furthermore, the periodic structure with large Fresnel numbers is also treated as well as that with small Fresnel numbers. This analysis may elucidate especially the effect of diffraction on the growing behavior of the spatial coherence because the slit aperture leads favorably to the diffraction phenomena in contrast to the Gaussian soft aperture. In addition, it may possibly provide a certain information to comprehend the spatial coherence properties of light propagating through the other kinds of periodic structures such as waveguide diffraction gratings, as mentioned above.

In the following, the initial light launched into the successive apertures is assumed to be incoherent with uniform intensity distribution under quasi-monochromatic approximation. The spatial coherence and the intensity distribution of light after passing through the structure are examined with the help of the mode theory in the space-time domain, in accordance with Streifer [115] and Gori [116]. The normal modes [110] of the present periodic structure are represented by the prolate spheroidal wave functions which form a complete orthonormal set of functions. As a result, the initial incoherent light can be decomposed by these normal modes and the variation of spatial coherence arising from propagation and diffraction of light through the structure can be written mathematically in a simple form by means of the normal modes and the corresponding eigenvalues. A method of extending the theory into the space-frequency domain is presented to complete the theoretical aspect of this chapter. Subsequently, some illustrative examples are given on the basis of the mathematical formulations in the space-time domain. Finally, results obtained in this way are compared with those of Gori [115] in order that the phenomena especially stemming from diffraction by sharp edges of the apertures are elucidated.

8.2 Theoretical Background

8.2.1 Propagation of the mutual intensity through the successive apertures

The successive apertures for the present analysis are constituted by an iterated sequence of a basic element illustrated in Fig.8.1(a). Three slit apertures with the

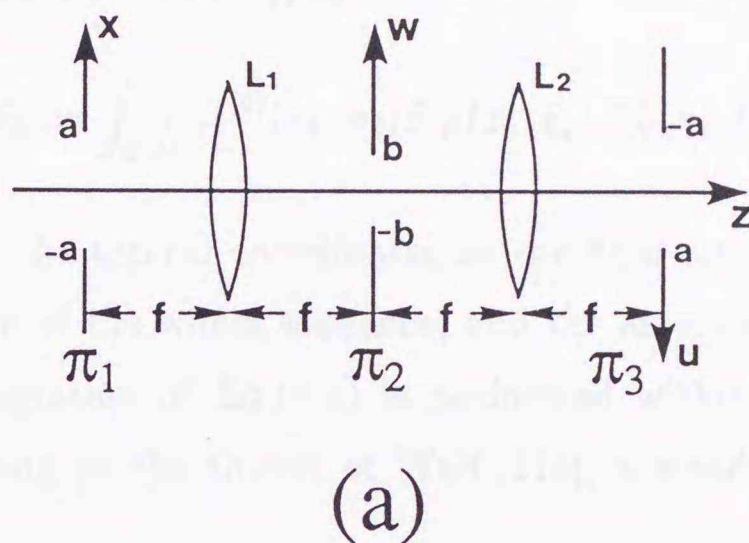


Figure 8.1. (a) Basic element for analysis and (b) its successive arrangement. Cylindrical lenses with focal lengths f are aligned at distance $2f$ and slit apertures having the widths $2a$ and $2b$ are located at each focal plane.

widths $2a$, $2b$ and $2a$ are located at each focal plane of two ideal cylindrical lenses (focal length f) arranged at distance $2f$. The N successive arrangement of the basic elements along the z -axis forms the whole periodic structure, as illustrated in Fig.8.1(b).

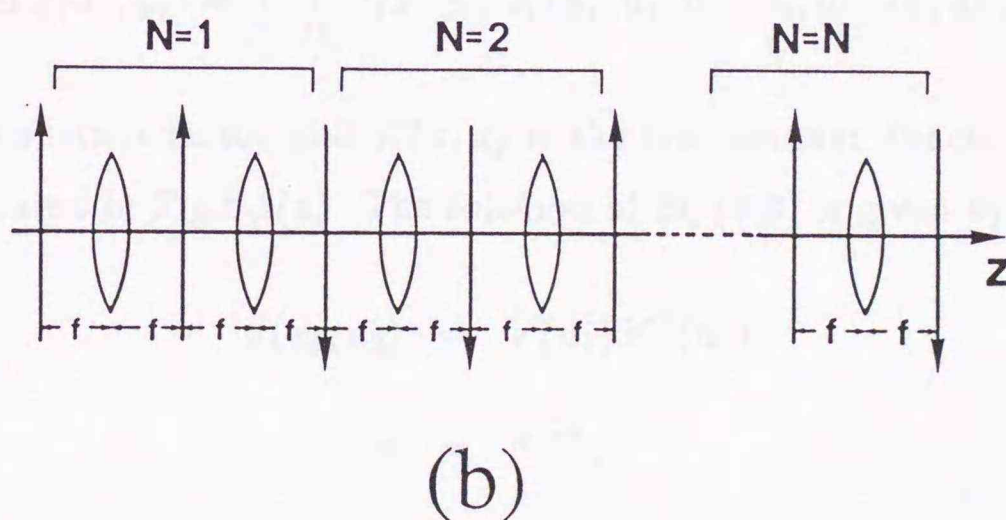


Figure 8.1. Cont.

Since the analysis of the main problem discussed here is fundamentally based on the study of Wolf [114], we begin by recalling briefly the main points of the

theory. We consider that a quasi-monochromatic spatially partially coherent light is incident on the first aperture of the sequence from the left. The spatial coherence properties of light are characterized by the mutual intensity $J^{(0)}(x_1, x_2)$, where x_1 and x_2 are the typical coordinates across the first aperture. The mutual intensity of the light emerging to the right from the final aperture is given, with the help of the propagation law of mutual intensity, by

$$J^{(N)}(\xi_1, \xi_2) = \int_{\Sigma} \int_{\Sigma} J^{(0)}(x_1, x_2) K_N(x_1, \xi_1) K_N^*(x_2, \xi_2) dx_1 dx_2, \quad (8.1)$$

where ξ_1 and ξ_2 are the typical coordinates at the final aperture, $K_N(x, \xi)$ is the transmission function of the whole sequence, and the asterisk denotes the complex conjugate. The integration of Eq.(8.1) is performed within the area of the first aperture Σ . According to the theory of Wolf [114], a steady state satisfying the relation

$$J^{(N+1)}(\xi_1, \xi_2) \propto J^{(N)}(\xi_1, \xi_2) \quad (8.2)$$

may be achieved after a sufficient number of transits. Here, this steady state implies that the intensity distribution over the final aperture is proportional to that over the first aperture and that the degrees of spatial coherence of light across both apertures are also the same. Such a steady state is obtained mathematically from the solutions of the integral equation given by

$$\alpha J(u_1, u_2) = \int_{\Sigma} \int_{\Sigma} J(x_1, x_2) K(x_1, u_1) K^*(x_2, u_2) dx_1 dx_2, \quad (8.3)$$

where α is a constant factor and $K(x, u)$ is the transmission function of the basic element illustrated in Fig.8.1(a). The solution of Eq.(8.3) is given by

$$J(u_1, u_2) = V(u_1) V^*(u_2), \quad (8.4)$$

$$\alpha = \beta \beta^*, \quad (8.5)$$

where the function $V(\cdot)$ and the constant factor β are the solutions of the simpler integral equation given by

$$\beta V(u) = \int_{\Sigma} V(x) K(x, u) dx. \quad (8.6)$$

As is widely known, Eq.(8.6) is the integral equation for determining the modes of the periodic structure. On the basis of the integral equation for the mutual intensity given by Eq.(8.3) and its solutions given by Eqs.(8.4) and (8.5), Wolf explained theoretically that the spatial coherence of light is gained by the process of light propagation through a periodic structure.

The main purpose of the present analysis is to examine the spatial coherence properties of light passing through the successive apertures directly in accordance with the method of the mode decomposition (in the space-time domain) conducted by Streifer [115] and Gori [116]. Thus, for a moment, we deal with the modes of successive apertures employed in the present analysis in some detail.

Suppose that a quasi-monochromatic, spatially coherent light field V_1 is incident on the first aperture π_1 of the basic element from the left (see Fig.8.1(a)). Since the planes π_1 and π_2 are placed at the front and back focal planes of the lens L_1 , respectively, the light field V_2 emerging from the plane π_2 can be obtained by means of a Fourier transform of the light field V_1 , aside from multiplicative phase factors, as:

$$V_2(w') = \frac{1}{\sqrt{\bar{\lambda}f}} \int_{-a}^a V_1(x') \exp\left(-j\frac{\bar{k}}{f}w'x'\right) dx', \quad (8.7)$$

where $\bar{k} = 2\pi/\bar{\lambda}$ is the mean wavenumber associated with the mean wavelength $\bar{\lambda}$ of light. In a similar fashion, the light field V_3 emerging to the right from the final aperture π_3 becomes

$$V_3(u') = \frac{\text{rect}(u'/2a)}{\sqrt{\bar{\lambda}f}} \int_{-b}^b V_2(w') \exp\left(j\frac{\bar{k}}{f}u'w'\right) dw', \quad (8.8)$$

where

$$\text{rect}(x) = \begin{cases} 1 & \text{for } |x| \leq \frac{1}{2} \\ 0 & \text{for } |x| > \frac{1}{2}. \end{cases} \quad (8.9)$$

Here, the Fourier kernel in Eq.(8.8) takes the positive index because of the opposed arrangement of the u -axis in the basic element. For the sake of generality, we introduce the normalized coordinates x , w and u defined by

$$x = \frac{x'}{a}, \quad w = \frac{w'}{b}, \quad u = \frac{u'}{a}, \quad (8.10)$$

where a and b is the half widths of slit apertures located at each focal plane in the basic element. By substituting of Eq.(8.7) into Eq.(8.8), by integrating with respect to w' , and using the normalized coordinates defined by Eq.(8.10), the light field V_3 at the final aperture π_3 can be directly described in terms of the initial light field V_1 in the form:

$$V_3(u) = \text{rect}\left(\frac{u}{2}\right) \int_{-1}^1 V_1(x) \frac{\sin\left[\frac{\bar{k}}{f}ab(u-x)\right]}{\pi(u-x)} dx. \quad (8.11)$$

As a result, the integral equation for determining the modes of the present periodic structure, corresponding to Eq.(8.6), becomes

$$\lambda_n(c) \Phi_n(c, u) = \int_{-1}^1 \Phi_n(c, x) K(c; x, u) dx, \quad (8.12)$$

where

$$K(c; x, u) = \frac{\sin[c(u-x)]}{\pi(u-x)}, \quad (8.13)$$

and

$$c = \frac{\bar{k}}{f}ab. \quad (8.14)$$

Equation (8.12) is a homogeneous Fredholm integral equation of the second kind. Since the kernel of this equation described in Eq.(8.13) has the properties of being continuous, symmetric and positive definite, the eigenfunctions $\Phi_n(c, \eta)$ form a complete orthonormal set of functions and the eigenvalues $\lambda_n(c)$ take positive values. The eigenfunctions $\Phi_n(c, \eta)$ and the eigenvalues $\lambda_n(c)$ represent the modes of the periodic structure and the loss per transit associated with each mode, respectively, and they depend strongly on the parameter c given by Eq.(8.14), where the parameter c corresponds to the Fresnel number of the present successive apertures. The integral equation of this kind has been thoroughly studied by Slepian and Pollak [199] and other researchers in 1960's, and its solutions have been shown to be

$$\Phi_n(c, \eta) \propto S_{0n}(c, \eta), \quad (8.15)$$

$$\lambda_n(c) = \frac{2c}{\pi} [R_{0n}^{(1)}(c, 1)]^2, \quad (8.16)$$

where $S_{0n}(c, \eta)$ and $R_{0n}^{(1)}(c, 1)$ denote, respectively, the angular and the radial prolate spheroidal wave functions in accordance with the Flammer's description [200]. In the present analysis, it is convenient to normalize the eigenfunctions so as to satisfy the following orthogonal relation:

$$\int_{-1}^1 \Phi_n(c, \eta) \Phi_m^*(c, \eta) d\eta = \begin{cases} 1 & \text{for } n = m \\ 0 & \text{for } n \neq m. \end{cases} \quad (8.17)$$

Thus, the eigenfunctions of the integral equation are determined uniquely and can be written in the form:

$$\Phi_n(c, \eta) = \frac{S_{0n}(c, \eta)}{\sqrt{u_n(c)}}, \quad (8.18)$$

where

$$u_n(c) = \int_{-1}^1 [S_{0n}(c, \eta)]^2 d\eta. \quad (8.19)$$

It has become clear from the above discussion that the modes of successive apertures under consideration are characterized by the prolate spheroidal wave functions. In particular, it is known that these functions can be approximated by the Hermite-Gauss functions in the case of $c \rightarrow \infty$ [112]. Further properties of the prolate spheroidal wave functions are briefly reviewed in Section 8.2.3.

By means of the eigenfunctions and the eigenvalues derived above, we now consider the propagation of mutual intensity through the successive apertures which is the main subject of the present analysis. As an incident light on the structure, we assume a quasi-monochromatic spatially incoherent light having the uniform intensity distribution within the aperture. The mutual intensity of such light is approximately given by ([147], Sec. 4.4)

$$J^{(0)}(x_1, x_2) = I_0 \operatorname{rect}\left(\frac{x_1}{2}\right) \operatorname{rect}\left(\frac{x_2}{2}\right) \delta(x_1 - x_2), \quad (8.20)$$

where $\delta(\cdot)$ stands for the one-dimensional Dirac delta function, I_0 is a positive constant and $\operatorname{rect}(\cdot)$ is defined by Eq.(8.9). Since the eigenfunctions defined by Eq.(8.18) form a complete orthonormal set of functions as already mentioned above, Eq.(8.20) can be described in the form:

$$J^{(0)}(x_1, x_2) = I_0 \operatorname{rect}\left(\frac{x_1}{2}\right) \operatorname{rect}\left(\frac{x_2}{2}\right) \sum_{n=0}^{\infty} \Phi_n(c, x_1) \Phi_n^*(c, x_2). \quad (8.21)$$

From the physical point of view, Eq.(8.21) expresses that the completely incoherent light is realized by the superposition of mutually uncorrelated modes. By substituting Eq.(8.21) into the right hand side in Eq.(8.3), the mutual intensity of light emerging from the exit aperture after propagation through the first basic element becomes

$$J^{(1)}(u_1, u_2) = I_0 \text{rect} \left(\frac{u_1}{2} \right) \text{rect} \left(\frac{u_2}{2} \right) \sum_{n=0}^{\infty} [\lambda_n(c)]^2 \Phi_n(c, u_1) \Phi_n^*(c, u_2). \quad (8.22)$$

By repetition of the same procedure, the mutual intensity of light emerging from the final aperture after passing through the N basic elements takes the form:

$$J^{(N)}(x_1, x_2) = I_0 \text{rect} \left(\frac{x_1}{2} \right) \text{rect} \left(\frac{x_2}{2} \right) \sum_{n=0}^{\infty} [\lambda_n(c)]^{2N} \Phi_n(c, x_1) \Phi_n^*(c, x_2), \quad (8.23)$$

where $\lambda_n(c)$ is the eigenvalues of the structure given by Eq.(8.16) and $\Phi_n(c, \eta)$ is the eigenfunctions characterizing the normal modes of the structure given by Eq.(8.18). Here and in the sequel, spatial coordinates across the exit aperture are specified by x_1 , x_2 and x . It is known that the values of $\lambda_n(c)$ are reduced from the value unity with an increase of the mode number n and that, finally, they fall off to zero. As a consequence, Eq.(8.23) reveals that higher order modes contained in initial incoherent light are diminished, associated with $\lambda_n(c)$, with an increase of N . The reduction of the higher order modes propagating through the successive apertures results in the emerging light to have the spatially high coherence [109].

To show some numerical results in the following sections, the intensity distribution and the complex degree of spatial coherence given by

$$I^{(N)}(x) = J^{(N)}(x, x), \quad (8.24)$$

$$\gamma^{(N)}(x_1, x_2) = \frac{J^{(N)}(x_1, x_2)}{\sqrt{I^{(N)}(x_1)} \sqrt{I^{(N)}(x_2)}}, \quad (8.25)$$

are evaluated by the use of Eq.(8.23).

8.2.2 Extension of the theory in the space-frequency domain

The present theory has been formulated by means of the mutual intensity which is the correlation function in the space-time domain. In recent years, however, most

studies concerning coherence theory tend to be formulated by means of the cross-spectral density which is the correlation function in the space-frequency domain. Hence, this section is briefly concerned with the interrelation between two formulations in both domains and, subsequently, it is shown that the preceding theory can be easily extended in the space-frequency domain.

From the historical point of view, the studies [114–116] based on the modal expansion of the correlation function has been formulated by means of the mutual intensity. This is mainly because these studies are closely related to laser resonator modes. As is well known, transverse modes of practical multimode lasers oscillate at slightly different frequencies and their superposition yields the reduction of their spatial coherence [109]. In addition, most of the practical laser light can be assumed to satisfy the quasi-monochromatic approximation. Accordingly, it appears to have been taken for granted hitherto that the mutual intensity, rather than the cross-spectral density, is adequate to describe the process of the coherence reduction of this kind [182]. In recent years, however, the new theory dealing with modal expansion of the cross-spectral density across the steady-state sources and fields was established by Wolf [120, 121] and this has induced us to extend the preceding studies associated with lasers and the present chapter to be conducted in the space-frequency domain. On the basis of the new theory of Wolf, the spatial coherence of laser resonator modes was also formulated in the space-frequency domain [118, 201], showing theoretically that the simultaneous excitation of several transverse modes reduces the spatial coherence. These recent works relating to the modal expansion of the partially coherent field provide the mathematical and physical foundation on a method of extending the theory into the space-frequency domain.

The mutual intensity can be readily replaced by the cross-spectral density because both functions propagate in accord with the same set of Helmholtz equations except for the difference of notations (see, Secs. 2.5 and 4.2.3). More specifically, the mutual intensity can be directly replaced by the cross-spectral density if the mean wavelength $\bar{\lambda}$ (or mean frequency $\bar{\nu}$) of light contained in the description of the mutual intensity is replaced by the particular wavelength λ (or particular frequency ν). In other words, the mutual intensity and the cross-spectral density are the same aside from the difference of notations although the physical meanings they possess are different from each other. In particular, two formulations in terms of the mutual

intensity and the cross-spectral density become essentially equivalent, provided that the quasi-monochromatic and other suitable approximations are satisfied. As a consequence, results of the present chapter can be directly applied to the study described by using the cross-spectral density under these approximations.

8.2.3 Some basic properties of the prolate spheroidal wave functions

In Section 8.2.1, the propagation of mutual intensity through the successive apertures employed here has been formulated by means of the prolate spheroidal wave functions. Numerical evaluation of these functions required to verify the preceding theory appears not to be a straightforward task because of their mathematical complexity. Therefore, some basic properties of these functions are briefly reviewed here and a method is presented for their evaluation adopted through the whole analysis. In the following, we make use of the mathematical description due to Flammer [200].

The prolate spheroidal wave functions, which characterize the normal modes of the successive apertures and the corresponding eigenvalues, are governed by the differential equation in the form:

$$\frac{d}{d\eta} \left[(1 - \eta^2) \frac{dU}{d\eta} \right] + \left[\chi_{mn}(c) - c^2 \eta^2 - \frac{m^2}{1 - \eta^2} \right] U = 0. \quad (8.26)$$

where the above equation is the scalar Helmholtz equation in the prolate spheroidal coordinate system. In Eq.(8.26), $\chi_{mn}(c)$ indicates the eigenvalues of the differential equation and the solutions are the angular prolate spheroidal wave functions $S_{mn}(c, \eta)$ of the first kind of order m and degree n . As is well known, Eq.(8.26) reduces to the differential equation for the associated Legendre functions when the parameter c is equal to zero. In general, the angular functions may be represented by means of the associated Legendre functions $P_{m+r}^m(\eta)$ in the form:

$$S_{mn}(c, \eta) = \sum_{r=0,1}^{\infty} d_r^{mn}(c) P_{m+r}^m(\eta). \quad (8.27)$$

Here, the prime over the summation symbol indicates that the summation is performed only over even values of r if $(n - m)$ is even and only over odd values of r if $(n - m)$ is odd. Since the present theory formulated in Section 8.2.1 is based on these functions of order zero as described in Eqs.(8.16) and (8.18), we confine the

following discussion to the case of $m = 0$. By substituting Eq.(8.27) into Eq.(8.26), we obtain the recursion formula for the expansion coefficients $d_r^{0n}(c)$ in the form:

$$A_1(r, c) \cdot d_{r+2}^{0n}(c) + A_2(r, c) \cdot d_r^{0n}(c) + A_3(r, c) \cdot d_{r-2}^{0n}(c) = 0, \quad (r \geq 0), \quad (8.28)$$

with

$$A_1(r, c) = \frac{(r+2)(r+1)c^2}{(2r+3)(2r+5)}, \quad (8.29)$$

$$A_2(r, c) = \left[r(r+1) - \chi_{0n}(c) + \frac{2r(r+1)c^2}{(2r-1)(2r+3)} \right], \quad (8.30)$$

$$A_3(r, c) = \frac{r(r-1)c^2}{(2r-3)(2r-1)}. \quad (8.31)$$

To determine these expansion coefficients uniquely, the normalization condition

$$S_{0n}(c, 0) = P_n(0) \quad (8.32)$$

is adopted, where $P_n(\cdot)$ stands for the Legendre function. As a result, the orthogonal relation of the function $S_{0n}(c, \eta)$ over the interval $(-1, 1)$ is found to be

$$\int_{-1}^1 S_{0n}(c, \eta) S_{0n'}(c, \eta) d\eta = \begin{cases} 2 \sum_{r=0,1}^{\infty} \frac{[d_r^{0n}(c)]^2}{2r+1} & \text{for } n = n' \\ 0 & \text{for } n \neq n'. \end{cases} \quad (8.33)$$

The radial prolate spheroidal wave functions, which correspond to the eigenvalues of the integral equation derived in the previous section, are also the solutions of the differential equation (8.26) because they differ from the angular functions by a real scale factor and hence they are also described generally by means of the series expansion. However, since only these functions of the first kind and argument $\eta = 1$, i.e., $R_{0n}^{(1)}(c, 1)$, appear in the present chapter as shown in Eq.(8.16), only the specific relation instead of the general expression is presented here as follows:

$$R_{0n}^{(1)}(c, 1) = \begin{cases} \frac{2^n \left(\frac{n}{2}!\right)^2 d_0^{0n}(c)}{n!} & \text{for } n: \text{ even} \\ \frac{2^n c \left(\frac{n-1}{2}!\right) \left(\frac{n+1}{2}!\right) d_1^{0n}(c)}{3(n+1)!} & \text{for } n: \text{ odd.} \end{cases} \quad (8.34)$$

In spite of cumbersome mathematics as stated above, the prolate spheroidal wave functions of order zero have played an important role in a wide variety of applied problems ([112]; [149], Sec. 6.1; [202]). In particular, the related integral equation given in the form

$$\frac{2c}{\pi} [R_{0n}^{(1)}(c, 1)]^2 S_{0n}(c, t) = \int_{-1}^1 \frac{\sin[c(t-s)]}{\pi(t-s)} S_{0n}(c, s) ds \quad (8.35)$$

appears in most of such problems. This equation coincides with Eq.(8.12) in the previous section. The integral equation of this kind has been thoroughly studied by several researchers particularly in relation to the eigenvalues $\lambda_n(c) = (2c/\pi)[R_{0n}^{(1)}(c, 1)]^2$. Consequently, it has become obvious that all the eigenvalues $\lambda_n(c)$ take real values less than unity and that they decrease rapidly with an increase of the value n once n exceeds $(2/\pi)c$ for large values of c . In addition, according to Slepian [203], these eigenvalues can be approximated by the formula

$$\hat{\lambda}_n = \frac{1}{(1 + e^{\pi \hat{b}})} \quad (8.36)$$

with

$$\hat{b} = \frac{n\frac{\pi}{2} - c + \frac{\pi}{4}}{0.2886 + 2 \ln 2 + \frac{1}{2} \ln c}, \quad (8.37)$$

for both large values of n and c . Rigorous values of the eigenvalues $\lambda_n(c)$ were tabulated by Slepian and Sonnenblick [204], also including the eigenvalues $\chi_{0n}(c)$ of the differential equation (8.26). Thus, the eigenvalues $\lambda_n(c)$ given by Eq.(8.16) are readily obtained from either the tables of Slepian and Sonnenblick or the approximation formula of Eq.(8.36). In practice, rigorous values tabulated by Slepian and Sonnenblick are applied to illustrate some numerical results in the present chapter.

The remaining problem required to examine the main subject of the present chapter formulated by Eq.(8.23) is the numerical evaluation of the angular functions defined by Eq.(8.27). By taking into consideration that the associated Legendre functions contained in Eq.(8.27) are easily evaluated by the use of an ordinary subroutine program in a personal computer, the determination of the expansion coefficient $d_r^{0n}(c)$ allows us to calculate the angular functions in question. By comparing Eqs.(8.16) and (8.34) with the tables of eigenvalues, the initial values of

the expansion coefficients $d_0^{0n}(c)$ and $d_1^{0n}(c)$ are obtained exactly for various values of c . Subsequently, higher-order coefficients can be determined by calculating the recursion formula Eq.(8.28) repeatedly using the fact that $d_r^{0n}(c) = 0$ for $r < 0$. Substitution of all the coefficients into Eq.(8.27) yields the angular prolate spheroidal wave functions. In fact, the infinite summation of Eq.(8.27) is reduced to the finite summation approximately by an appropriate selection of the range for the summation because $d_r^{0n}(c)$ gives its largest values when $r = n$ and it becomes zero when $r \rightarrow \infty$ and, as a result, the angular functions are easily calculated insofar as the eigenvalues are given in the tables.

8.3 Results and Discussion

8.3.1 Spatial coherence properties of light at the exit aperture

On the basis of the formulas (8.23), (8.24) and (8.25) derived in Section 8.2.1, the intensity distribution and the spatial coherence of light passing through the successive apertures are investigated in this section. To illustrate some numerical examples, we assumed through the whole analysis that $f = 5.0 \times 10^{-2}[\text{m}]$ and $\bar{\lambda} = 5.0 \times 10^{-7}[\text{m}]$. Emphasis is placed here especially on the feature of spatial coherence rather than that of intensity distribution.

We first examine the variations of the normalized intensity and the complex degree of spatial coherence of light at the exit aperture for various values of N showing the number of transits through the basic element depicted in Fig.8.1(a). Typical examples are shown in Figs.8.2(a)-(c) for three values of c , where the parameter c characterizes the width of the slit apertures employed in the basic element as defined in Eq.(8.14). In these figures, the normalization of the intensity distribution is conducted by dividing the intensity at each point by the value at the origin and the spatial coherence is evaluated for two points located symmetrically with respect to the optical axis at the aperture. All the curves are plotted against the normalized distance: the values of 1.0 and -1.0 in intensity correspond to the edges of the aperture and the value of 2.0 in spatial coherence implies that the two points for evaluating the spatial coherence are located at the right and the left side of the edges on the aperture. It is clearly seen in Figs.8.2(a) and (b) that the spread of the

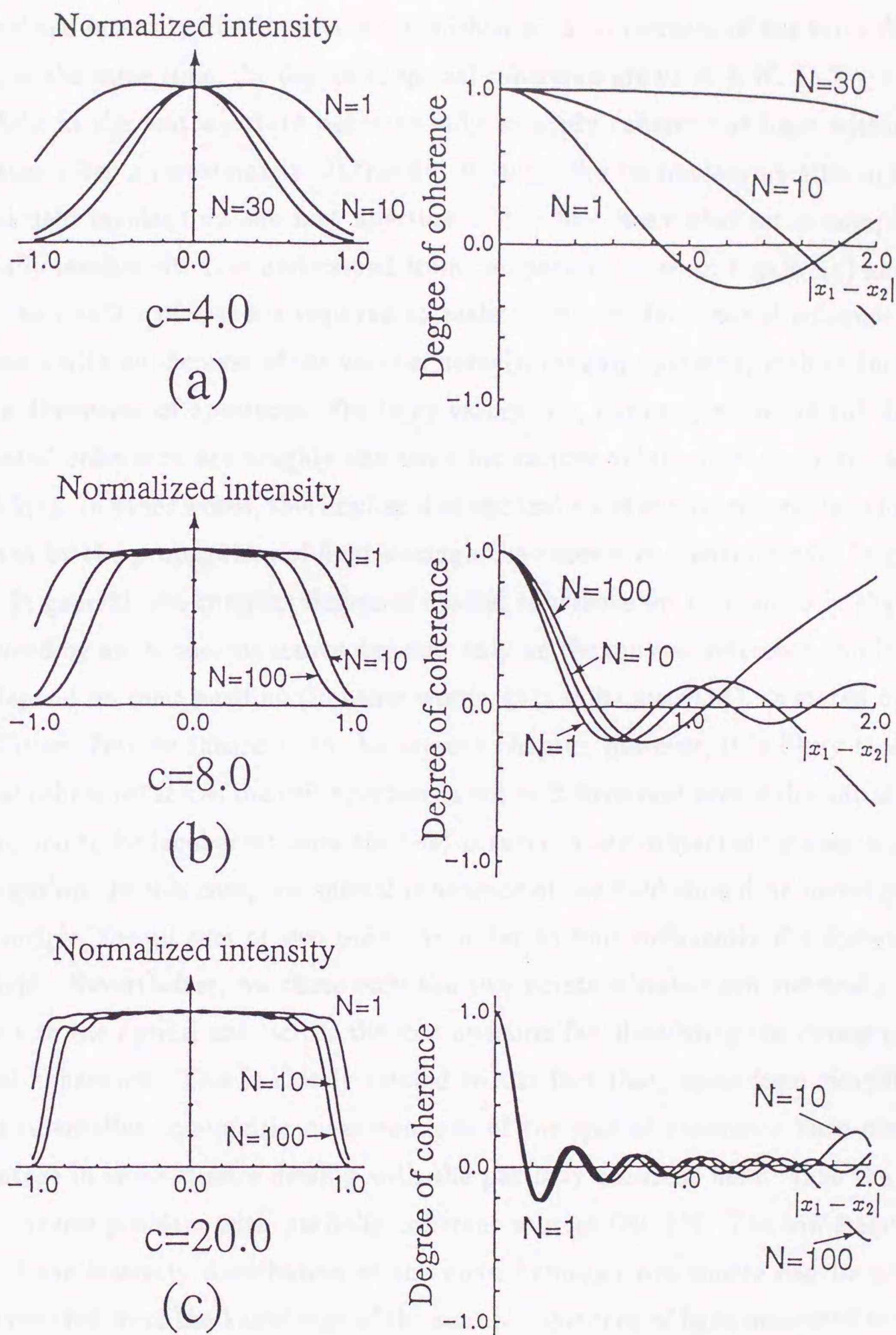


Figure 8.2. Normalized intensity distribution and the complex degree of spatial coherence at the exit aperture with $c =$ (a) 4.0, (b) 8.0 and (c) 20.0 for various values of N . The parameter c is given by $c = (\bar{k}/f)ab$ and, hence, it characterizes the width of the slit apertures employed in the present successive apertures. The parameter N corresponds to the number of transits through the basic element illustrated in Fig.8.1(a).

normalized intensity distribution is diminished with an increase of the value N and that, at the same time, the degree of spatial coherence grows with N . In Fig.8.2(a), the field at the exit aperture becomes fully spatially coherent at least within the aperture after approximately 30-transits through the basic element although the initial field incident on the first aperture of the first basic element is completely spatially incoherent. It is understood from comparison between Figs.8.2(a) and (b) that the number of transits required to realize the complete spatial coherent field increases with an increase of the value c , namely, roughly speaking, with an increase of the dimension of apertures. For large values of c , however, curves of the degree of spatial coherence are roughly the same for various values of N as illustrated in Fig.8.2(c). In other words, this implies that the feature of spatial coherence is hardly affected by the propagation of light through the successive apertures with large c .

In general, the complex degree of spatial coherence for two points in the field produced by an incoherent source depends only on the mutual difference and it does not depend on their position (in other words, it is shift-invariant), as stated by the van Cittert-Zernike theorem. In the present chapter, however, it is likely that the spatial coherence across the exit aperture is not shift-invariant even if the initial light is assumed to be incoherent since the field is successively diffracted by apertures on propagation. In this case, the spatial coherence of the field should be investigated, in principle, for all sets of two points in order to find sufficiently the features of the field. Nevertheless, we chose only the two points situated symmetrically with respect to the optical axis across the exit aperture for illustrating the curves of the spatial coherence. This is closely related to the fact that, aside from simplifying the mathematics, symmetric measurements of the spatial coherence have distinct advantage in some studies dealing with the partially coherent field. Take the case of an inverse problem with partially coherent sources [20, 52]. The low-frequency part of the intensity distribution of the quasi-homogeneous source can be simply reconstructed from the knowledge of the spatial coherence of light measured for two points situated symmetrically about the optical axis in the far field [20]. In addition, another example occurs also in the imaging system [205]. From this point of view, one may say that the results of the spatial coherence obtained by the method of this kind play generally an important role as typical quantities for deducing the feature of the whole field. As a consequence, we made use of symmetric measurements of the

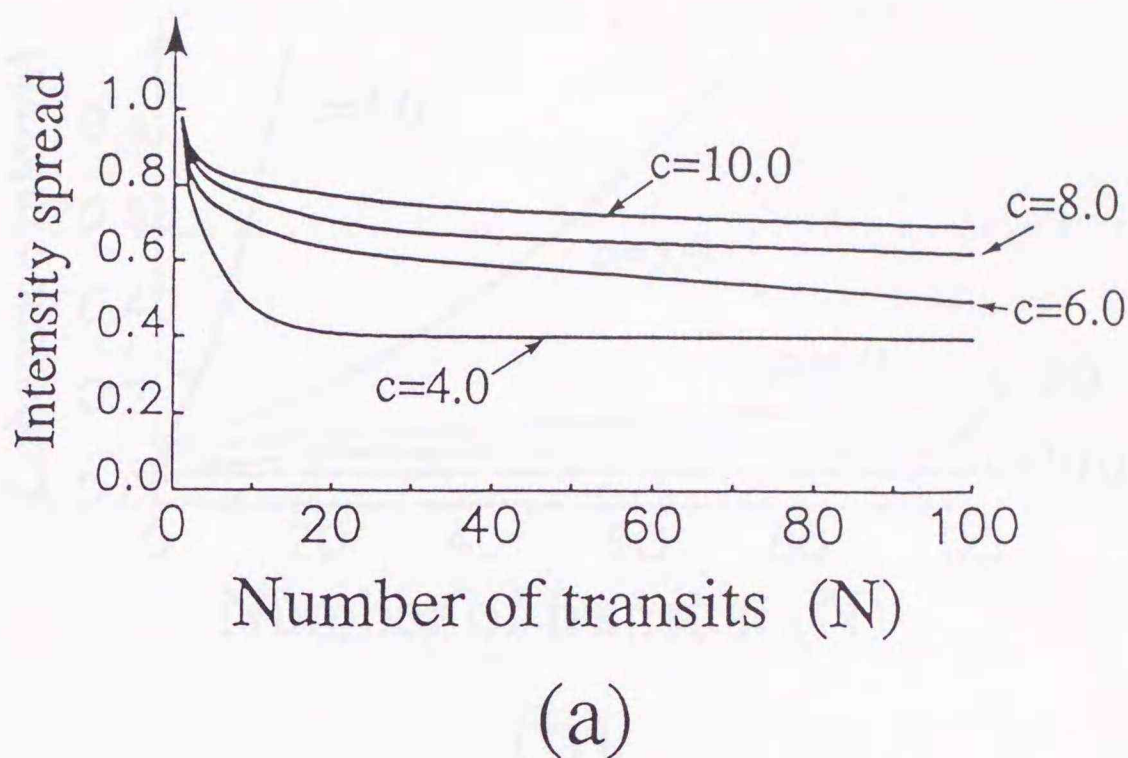
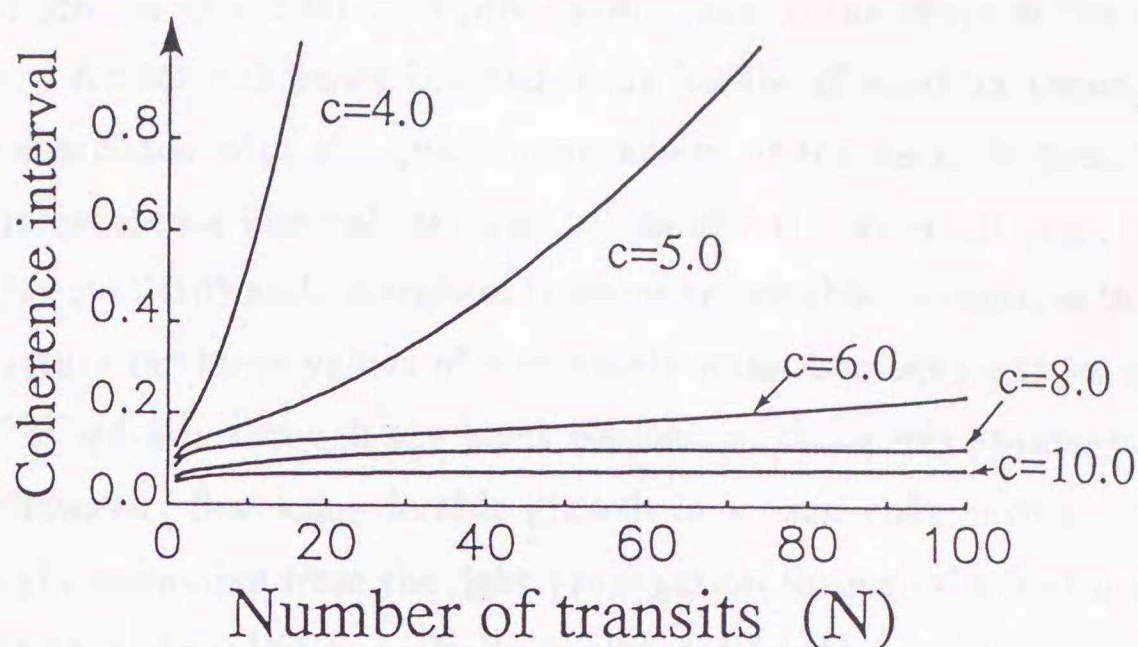


Figure 8.3. (a) Intensity spread and (b) coherence interval plotted against the values of N for various values of c .

spatial coherence to illustrate Fig.8.2 although the present study does not pertain to the classes of neither the inverse problem nor the imaging system.

In order to elucidate further the characteristics in intensity distribution and in spatial coherence, the intensity spread and the coherence interval are plotted in Fig.8.3 as a function of the number N of transits for various values of c . In the figures, the intensity spread is defined as the width for which the intensity decreases to $e^{-1/2}$ of its maximum value and the coherence interval is evaluated as the difference $|x_1 - x_2|$ normalized by the corresponding intensity spread, where the degree $\gamma(x_1, x_2)$ of spatial coherence takes the value 0.88. It is seen in Fig.8.3(a) that the intensity spread for small c drops suddenly and that, subsequently, it remains stable with an increase of the value N . The sudden variation in the curve of the intensity spread becomes ambiguous with increasing values of c and, as a consequence, gradually decreasing curves appear for relatively large values of c . On the other hand, the variation in coherence interval may be classified into two types depending on the values of c : the coherence interval rises rapidly with an increase of N for small values of c (e.g., $c = 4.0$ in Fig.8.3(b)) and, conversely, it remains nearly stable with an increase of N for large values of c (e.g., $c = 8.0$ and 10.0 in Fig.8.3(b)). The curve for $c = 5.0$ appears to be a boundary dividing into these two categories in



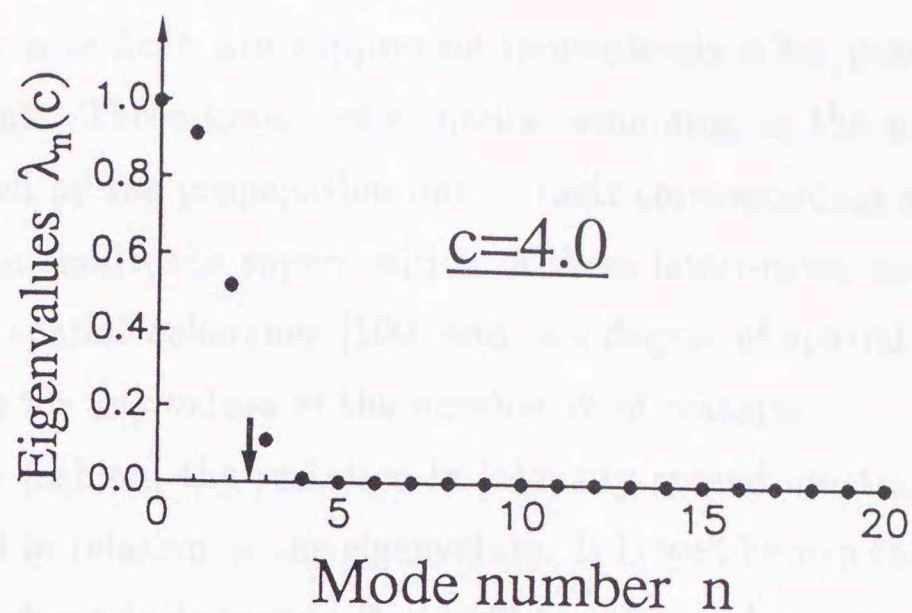
(b)

Figure 8.3. Cont.

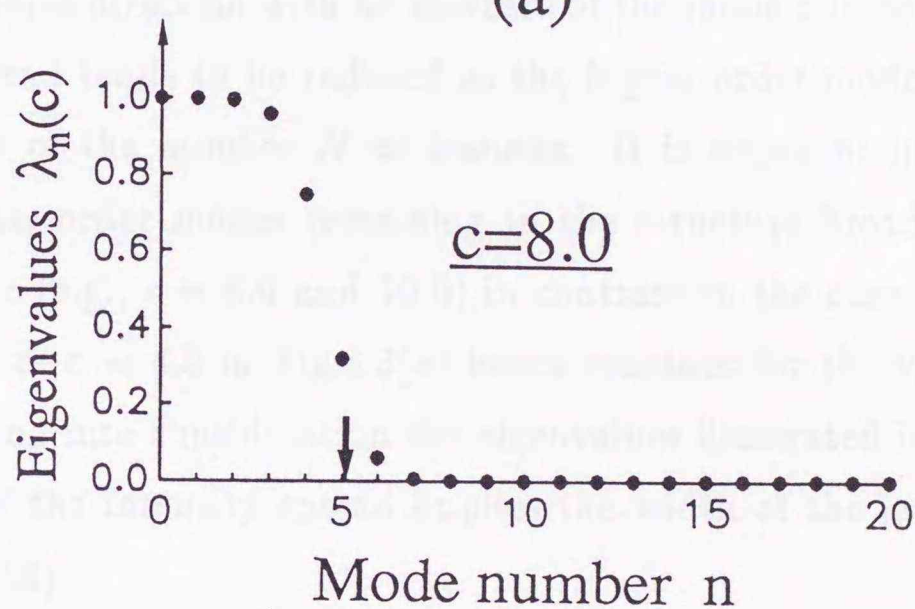
behavior of the coherence interval. In particular, the trend in the coherence interval for large values of c appears to be well understood by taking into account the series of studies dealing with the radiation properties from the quasi-homogeneous planar sources [21, 24, 33, 195, 206, 207]. The quasi-homogeneous source implies that the spatial coherence of light across the source depends only on the coordinate difference and, furthermore, that the source size and its spatial structure is much larger than the coherence interval (in this case, it is defined as the interval for which the degree of coherence decreases to a certain value, e.g., 0.88) across the source. Hence, such sources are always relatively incoherent in the global sense. The radiation properties from sources of this kind are found to be expressed by means of the source properties in quite simple manners [24]: the angular distribution of the radiant intensity and the complex degree of spatial coherence in the far field are, respectively, proportional to the two-dimensional Fourier transforms of the complex degree of spatial coherence and the intensity distribution across the source. Due to the reciprocal relationship of the pairs of the Fourier transform, the angular distribution of the radiant intensity extends further in comparison with the spread of the coherence function in the far field and, consequently, the resultant field in the far field is also relatively incoherent as well as the source itself. In the present study, the light propagation through the periodic sequence of lenses separated by apertures can be essentially considered as

an iterated process of the far field propagation due to the effect of the lens. Thus, the tendency for the coherence interval to be hardly affected on propagation may be closely associated with the quasi-homogeneity of the field. In fact, the typical value of the coherence interval for large values of c is very small (e.g., the value is below 0.1 for $c = 10.0$) and, therefore, it seems reasonable to suppose that the field in the structure for large values of c is nearly quasi-homogeneous in spite of the number N of transits through the basic element of the whole sequence. It should be noted, however, that considerable growth in spatial coherence is observed for any values of c stemming from the light propagation through the first basic element in the sense that the initially spatially incoherent (delta-correlated) light becomes spatially partially coherent with a certain degree of spatial coherence.

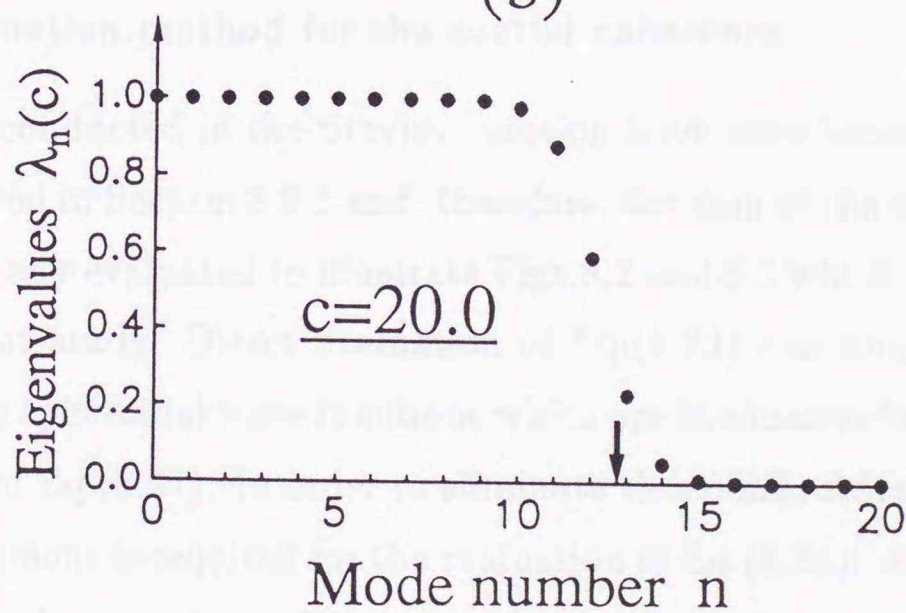
In general, the growth in spatial coherence and the convergence in intensity resulting from the light propagation through the periodic structure may be explained physically in association with the eigenvalues of the integral equation for determining the normal modes of the structure, as also mentioned in Section 8.2.1. Thus, in connection with the eigenvalues defined by Eq.(8.16), we next deal with the characteristics of the variation in spatial coherence illustrated in Fig.8.3(b). The eigenvalues $\lambda_n(c)$ are plotted in Fig.8.4 against the mode number n for several values of c using the tables of Slepian and Sonnenblick [204]. As have already been briefly discussed in Section 8.2.3, it is clearly seen in the figures that the values of $\lambda_n(c)$ for large values of c are nearly equal to unity for $n < 2c/\pi$, while they are attenuated very rapidly with an increase of n for $n > 2c/\pi$. For small values of c , as opposed to the case for large c , gradually decreasing dots are observed with increasing values of n (see, Fig.8.4(a)). The eigenvalues $\lambda_n(c)$ describes the loss of the modes in the periodic structure under consideration. More precisely, it may be better to express that each mode contained in the initial field (this is realized by the infinite mixture of the modes as mentioned in Section 8.2.1) is attenuated in accordance with the corresponding eigenvalue. It is obviously understood from Fig.8.4(a) that all the higher-order modes are reduced, corresponding to these eigenvalues, by the process of light propagation through the periodic structure and that finally only the lowest-order mode may survive in the structure, giving the highly spatially coherent field. As the values of c become larger, however, the number of low loss lower-order modes surviving in the periodic structure increases while the higher-order modes



(a)



(b)



(c)

Figure 8.4. Eigenvalues (given by Eq.(8.16)) of the integral equation for the normal modes of the successive apertures in the case of $c =$ (a) 4.0, (b) 8.0 and (c) 20.0. They are plotted by the use of the tables by Slepian and Sonnenblick. In the figures, the points $2c/\pi$ are indicated on the n -axis for the purpose of reference.

corresponding to $n > 2c/\pi$ are suppressed immediately after passing through the first basic element. These lower-order modes remaining in the periodic structure are hardly affected by the propagation due to their corresponding eigenvalues being nearly unity. As a result, the superposition of these lower-order modes gives rise to the reduction of spatial coherence [109] and the degree of spatial coherence keeps roughly the same for any values of the number N of transits.

In a similar fashion, the variation in intensity spread illustrated in Fig.8.4(a) may be explained in relation to the eigenvalues. It is well known that the amplitude distribution of each mode defined in Eq.(8.18) has the tendency of extending further out in the transverse direction with an increase of the mode number n [199]. Hence, the intensity spread tends to be reduced as the higher-order modes are suppressed with an increase of the number N of transits. It is apparent in Fig.8.3(a) that a number of lower-order modes remaining in the structure broaden the intensity spread for large c (e.g., $c = 8.0$ and 10.0) in contrast to the case for small c (e.g., $c = 4.0$). Curve of $c = 4.0$ in Fig.8.3(a) keeps constant for the values of N more than 20. By taking into consideration the eigenvalues illustrated in Fig.8.4(a), this constant value of the intensity spread implies the width of the lowest-order mode defined in Eq.(8.18).

8.3.2 Approximation method for the spatial coherence

All the analyses conducted in the previous section have been based on the rigorous formulation derived in Section 8.2.1 and, therefore, the sum of the series in Eq.(8.23) has been numerically evaluated to illustrate Figs.8.2 and 8.3 which play a significant role in the present study. Direct evaluation of Eq.(8.23) was somewhat cumbersome due to the prolate spheroidal wave functions which are themselves to be evaluated by series expansion of Eq.(8.27). In order to eliminate these difficulties of mathematics, the different treatment is required for the evaluation of Eq.(8.23). An approximation method which works as a powerful tool to find readily the feature of the spatial coherence passing through the successive apertures is presented in this section. On the basis of this method, we provide a quantitative evaluation for the dependence of the width of apertures (specified by the parameter c) on the variation of spatial coherence.

The present attempt to obtain the approximation form of Eq.(8.23) is estab-

lished by the use of the Mercer's theorem in the theory of integral equations ([208], Sec. 3.12). Mercer's theorem states that the symmetric, continuous and positive definite kernel $K(x, u)$ of an integral equation may be expressed by

$$K(x, u) = \sum_{n=0}^{\infty} \lambda_n \Phi_n(x) \Phi_n^*(u), \quad (8.38)$$

where λ_n and Φ_n are the eigenvalue and the eigenfunction of the integral equation, respectively, and Φ_n forms a complete orthonormal set of functions. By comparing the right hand sides of Eqs.(8.38) and (8.23), the difference in description is found to be only the first term in the summation, i.e., the eigenvalues except for the multiplicative constants I_0 and $\text{rect}(\cdot)$. As a result, if the parameter c' is determined satisfying the relation

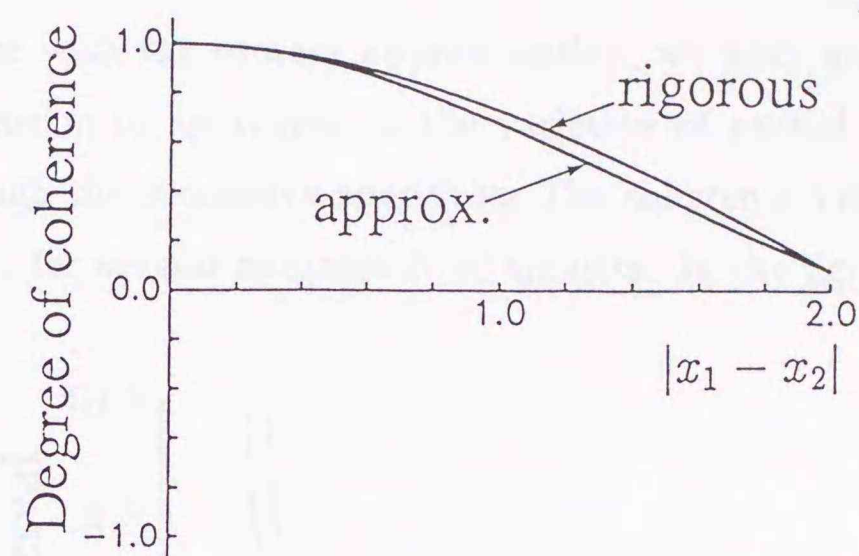
$$\lambda_n(c') = [\lambda_n(c)]^{2N} \quad (8.39)$$

for all values of n , Eq.(8.23) may be written, according to the Mercer's theorem, as

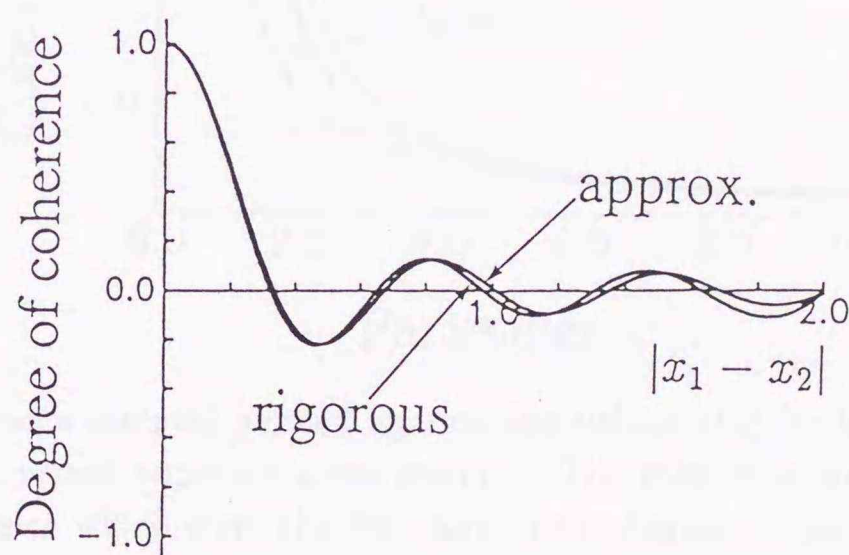
$$J^{(N)}(x_1, x_2) = I_0 \text{rect}\left(\frac{x_1}{2}\right) \text{rect}\left(\frac{x_2}{2}\right) \frac{\sin[c'(x_1 - x_2)]}{\pi(x_1 - x_2)}, \quad (8.40)$$

where the last term of the right hand side of the above equation is the kernel (which is symmetric, continuous and positive definite) of the integral equation given by Eq.(8.12). Since the values of c' satisfying exactly the relation of Eq.(8.39) for all values of n cannot be determined, we find these values of c' which fulfill approximately the relation of Eq.(8.39) for all values of n . By taking into account that the rectangular-like performance of the eigenvalues (see Fig.8.4), the determination of the approximation values of c' is conducted as follows: the values of c' satisfying Eq.(8.39) for $n = [2c/\pi]$ and for several values of n around $[2c/\pi]$ are first calculated and then they are averaged, where [...] implies the Gauss symbol and the mode number $n = [2c/\pi]$ gives the boundary value for which the eigenvalues drop suddenly. All the computations relating to the above procedure were easily performed because the analytical formula for the eigenvalues given in Eqs.(8.36) and (8.37) were utilized in the present approximation. The error resulting from the use of the formula of Eqs.(8.36) and (8.37) is negligible since it is shown to give the eigenvalues with high accuracy under a certain condition [204] which is applicable to the present

analysis. In confirmation of the accuracy for the present approximation, the complex degrees of spatial coherence evaluated by the use of the present approximation and of the rigorous formula Eq.(8.23) are plotted in Fig.8.5 for two values of c . In



(a)



(b)

Figure 8.5. Complex degree of spatial coherence at the exit aperture after the first transit through the basic element for $c =$ (a) 2.0 and (b) 10.0 evaluated on the basis of the present method for approximation. For comparison purpose, the curves evaluated in accordance with the rigorous formula Eq.(8.22) are added in these figures.

the figures, the number of transits through the basic element is assumed to be unity, i.e., $N = 1$. It is obvious from the figures that the curves calculated on the basis of the present method are in good agreement with those evaluated by the rigorous formula of Eq.(8.23).

By setting $x_1 = x_2 = x$ in Eq.(8.40), the intensity distribution is obtained and

it turns out to be constant over the aperture for each value of c' . This reveals that the present approximation is relevant to the comprehension of the feature in spatial coherence and that, however, it provides no information to specify the variation of the intensity distribution.

In accordance with the present approximation, we next examine the dependence of the dimension of apertures on the variation of spatial coherence by the light passing through the successive apertures. The coherence intervals are plotted in Fig.8.6 against c for several numbers N of transits. In the figure, the coherence

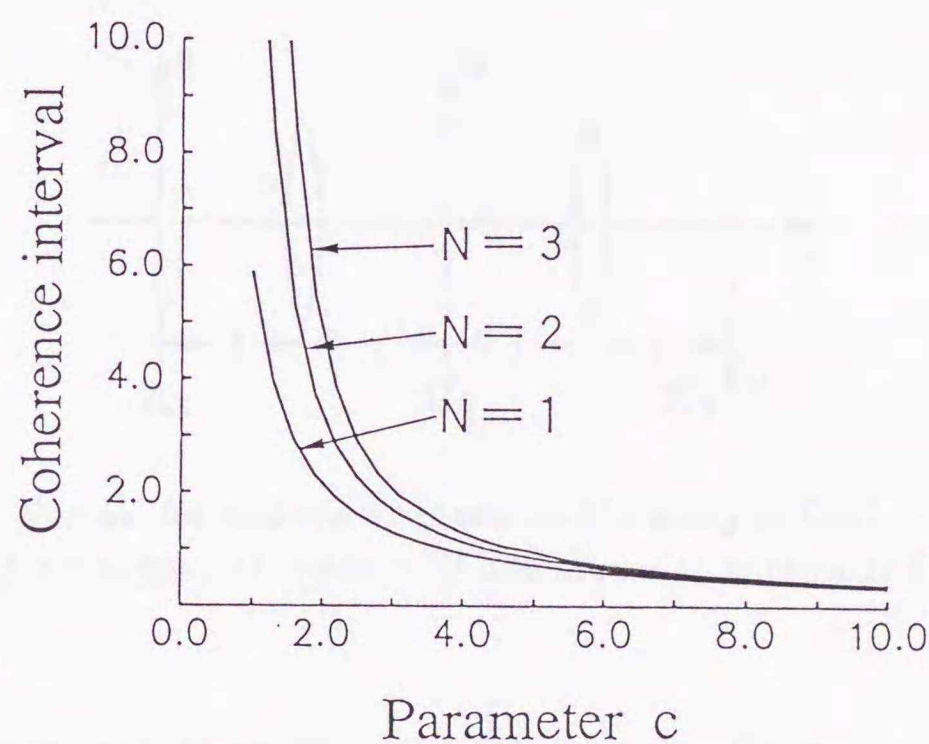


Figure 8.6. Coherence interval plotted against the values of c for various values of N on the basis of the present approximation method. The coherence interval is defined as the two-point difference which gives the first zero of the degree of spatial coherence. The value of 2.0 in the coherence interval corresponds to the full-width of the exit aperture.

interval is defined as the two-point difference which gives the first zero of the degree of spatial coherence for mathematical convenience, as opposed to the preceding definition of the coherence interval adopted to illustrate Fig.8.3(b). It is well understood quantitatively from the figure that the coherence interval grows considerably with an increase of the number N of transits for relatively small apertures (corresponding to small c) and that, conversely, it is hardly affected by the propagation for relatively large aperture (corresponding to large c). Finally, it is worth emphasizing that the quantitative evaluation of this kind could not be presented unless a drastic simplification such as the present method for approximation has been conducted.

8.3.3 Comparison with the study of Gori

In order to find further the characteristics of spatial coherence for light passing through the successive apertures, main points of the study conducted by Gori [116] are briefly reviewed in this section and, subsequently, they are compared with results obtained in this chapter. Emphasis is also placed on the spatial coherence rather than the intensity distribution.

The periodic structure employed by Gori is constituted by an iterated sequence of a basic element illustrated in Fig.8.7. Two ideal cylindrical lenses with both focal

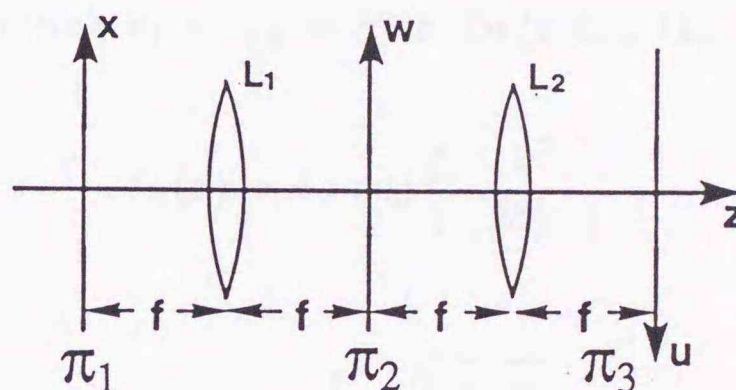


Figure 8.7. Basic element for analysis employed in the study of Gori. Cylindrical lenses with focal lengths f are aligned at distance $2f$ and Gaussian amplitude filters are located at each focal plane.

lengths f are arranged at interval $2f$ and one-dimensional Gaussian amplitude filters are located across each focal plane. The transmittances of these filters are given by

$$t(x) = \begin{cases} \exp\left(-\frac{x^2}{2\sigma^2}\right) & \text{for } \pi_1 \text{ and } \pi_3\text{-planes} \\ \exp\left(-\frac{x^2}{\sigma^2}\right) & \text{for } \pi_2\text{-plane,} \end{cases} \quad (8.41)$$

where σ takes a positive constant. Gori derived analytically the mutual intensity of light passing through the N basic elements with the help of Mercer's theorem, giving

$$J^{(N)}(x_1, x_2) = A_N \exp[-a_N(x_1^2 + x_2^2)] \exp[-b_N(x_1 - x_2)^2] \quad (8.42)$$

with

$$a_N = s \frac{1 - q_N}{1 + q_N}, \quad b_N = s \frac{2q_N}{1 - q_N^2}, \quad A_N = \gamma_N \sqrt{\frac{2s}{\pi(1 - q_N^2)}} \quad (8.43)$$

where

$$s = \frac{\sqrt{1+F^2}}{2\sigma^2}, \quad q_N = \left(\frac{F}{1+\sqrt{1+F^2}} \right)^{4N}, \quad \gamma_N = I_0 \sqrt{q_N} \quad (8.44)$$

and

$$F = \frac{2\pi\sigma^2}{\lambda f}. \quad (8.45)$$

In Eq.(8.44), I_0 and σ are both positive constants and σ is specified by Eq.(8.41). Here, Eq.(8.42) are formulated under the assumption that the first and the last filters of the iterated sequence of the basic element has the transmittances $t(x) = \exp(-x^2/2\sigma^2)$. By setting $x_1 = x_2 = x$ in Eq.(8.42), the intensity distribution becomes

$$I_N(x) = A_N \exp\left(-\frac{x^2}{2\sigma_{IN}^2}\right) \quad (8.46)$$

with

$$\sigma_{IN} = \frac{1}{2\sqrt{s}} \sqrt{\frac{1+q_N}{1-q_N}}. \quad (8.47)$$

On the other hand, the complex degree of spatial coherence is given by

$$\mu_N(x_1 - x_2) = \exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_{\mu N}^2}\right] \quad (8.48)$$

with

$$\sigma_{\mu N} = \frac{1}{2\sqrt{s}} \sqrt{\frac{1-q_N^2}{q_N}}. \quad (8.49)$$

The parameter A_N , q_N and s contained in Eqs.(8.46), (8.47) and (8.49) are given in Eqs.(8.43) and (8.44). In Eq.(8.48), the degree of spatial coherence depends on positions only through the difference $|x_1 - x_2|$ for any pairs of x_1 and x_2 . Mainly on the basis of Eqs.(8.46) and (8.48), Gori studied thoroughly the spatial coherence of light passing through the iterated sequence of Gaussian filters separated by cylindrical lenses.

We next deal with the comparison of the main results derived by Gori with those in the present study. In order to illustrate some numerical examples derived by the preceding theory of Gori, the parameter F given by Eq.(8.45) is used. The

value of F works as an important parameter characterizing the dimension of the Gaussian filters and corresponds to the Fresnel number of the periodic structure as well as the parameter c given by Eq.(8.14). By assuming that the width 2σ of the Gaussian amplitude filter placed at the π_2 -plane is equivalent to the width $2a$ of the uniform slit aperture employed here under the condition of $a = b$, the parameter F given by Eq.(8.45) can be replaced by the parameter c given by Eq.(8.14). The schematic of this "equivalent width" of the Gaussian filter is presented in Fig.8.8. In accordance with Eqs.(8.46) and (8.48), the intensity distribution and the complex

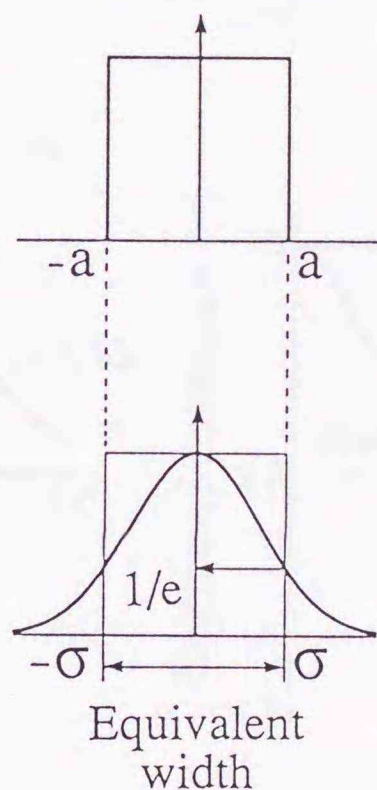
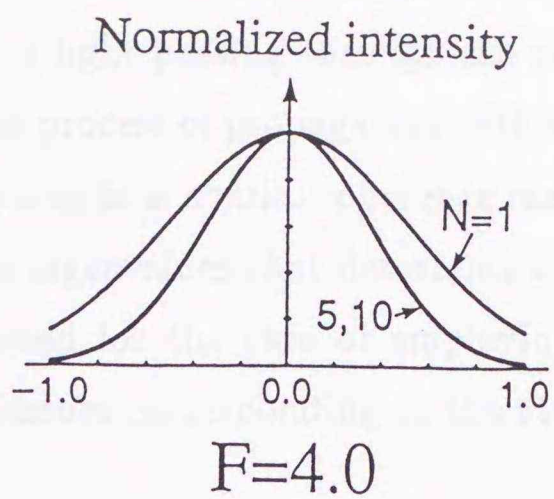
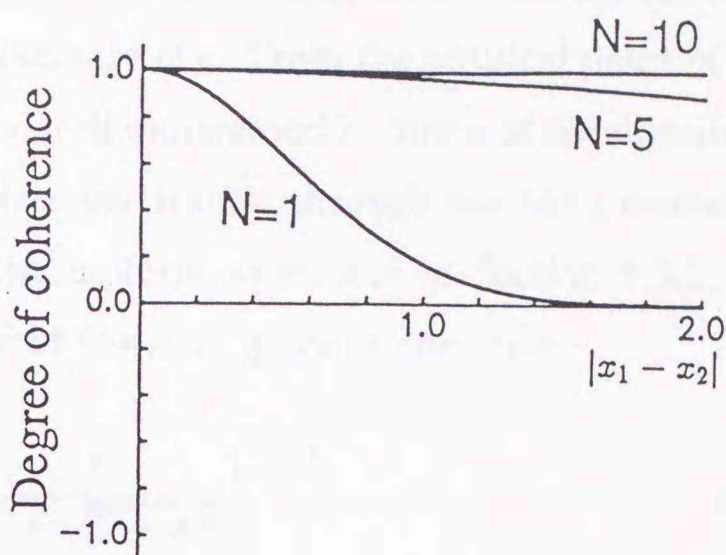
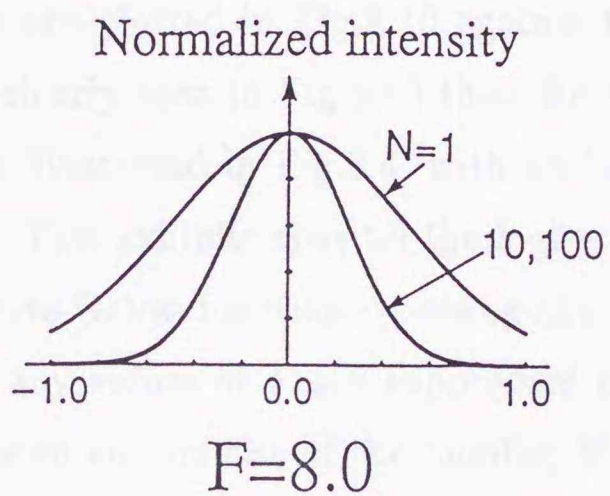


Figure 8.8. Schematic of the "equivalent width". The Gaussian amplitude filter whose transmittance is given by $t(x) = \exp(-x^2/\sigma^2)$ and the slit aperture having the width $2a$ are the same in the sense of the "equivalent width", provided that $\sigma = a$ under the condition of $a = b$.

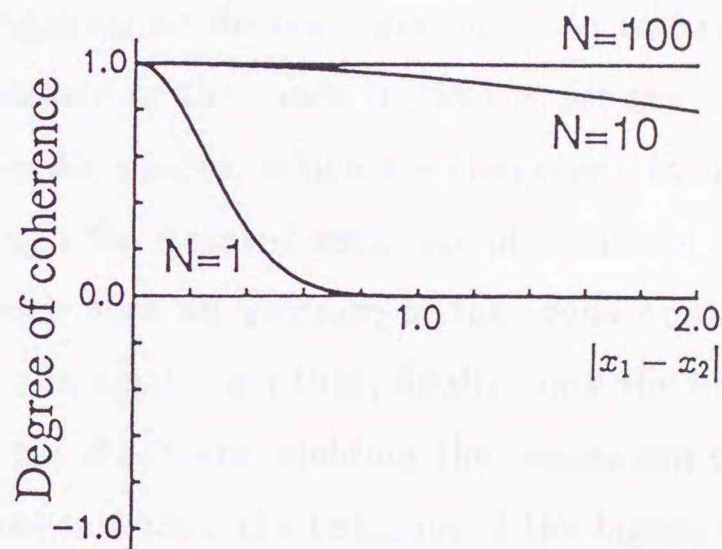
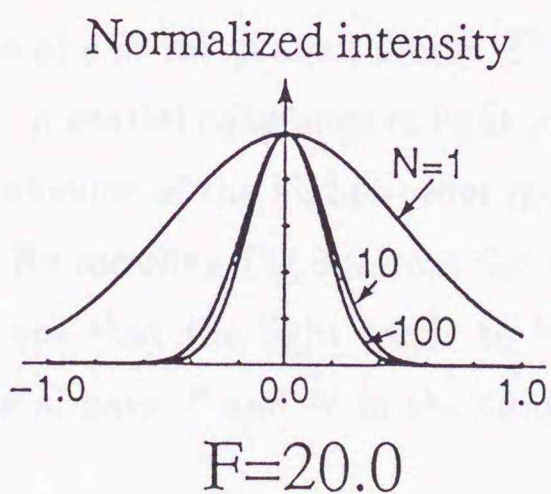
degrees of spatial coherence are illustrated in Fig.8.9 for several values of F . These figures are associated with Fig.8.2 on the grounds that the widths of the slit apertures and the Gaussian amplitude filters are the same in each pair in the sense of the "equivalent width". By comparison in each pair, i.e., Fig.8.9(a) with Fig.8.2(a), Fig.8.9(b) with Fig.8.2(b) and Fig.8.9(c) with Fig.8.2(c), it is understood that the complex degree of spatial coherence of light after the first transit ($N = 1$) is roughly the same for each pair of figures. However, the difference in variation of the spatial coherence is marked in each pair of figures as the number N of transits increases. The light passing through the iterated Gaussian filters may become fully spatially



(a)



(b)



(c)

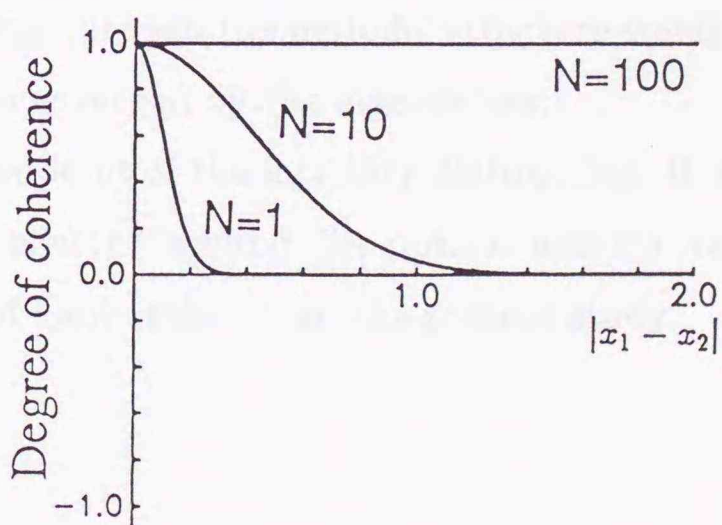


Figure 8.9. Normalized intensity distribution and the complex degree of spatial coherence of light across the output of the periodic structure for various values of N in the cases of $F =$ (a) 4.0, (b) 8.0 and (c) 20.0.

coherent after a certain number of transits even for large F while the spatial coherence of light passing through the successive uniform apertures is hardly influenced by the process of propagation with an increase of c . From the physical point of view, these trends in spatial coherence may be well understood in terms of the distribution of the eigenvalues that determine the loss per transit through the basic element, as discussed for the case of employing the uniform apertures in Section 8.3.1. The eigenvalues corresponding to the study of Gori are given in the form:

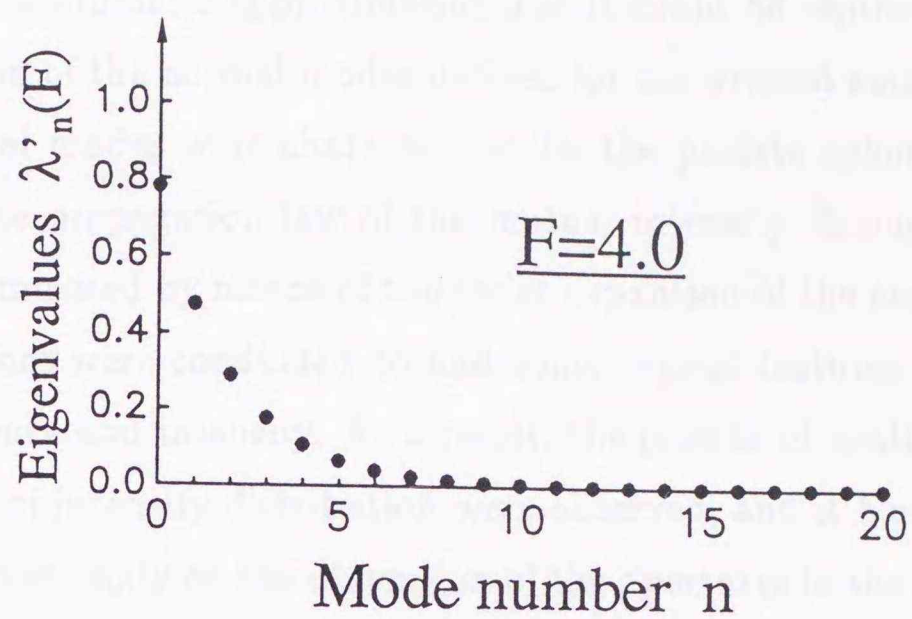
$$\lambda'_n(F) = \left(\frac{F}{1 + \sqrt{1 + F^2}} \right)^{2n+1}. \quad (8.50)$$

They are plotted in Fig.8.10 against the mode number n with relation to Fig.8.9. It is clearly seen in Fig.8.10 that the eigenvalues decrease gradually, in contrast to those illustrated in Fig.8.4, with an increase of the mode number n for any values of F . This exhibits that all the higher-order modes, which are characterized by the Hermite-Gauss functions, passing through the iterated sequence of Gaussian filters with any values of F are suppressed both with an increase of the mode number n and with an increase of the number N of transits and that, finally, only the lowest-order mode may necessarily survive in the structure, yielding the realization of the completely spatially coherent field. In other words, the behavior of the higher-order modes for any values of F in the study of Gori is roughly the same as that for small values of c in the present study. Therefore, it follows from the above study that the trend in spatial coherence of light passing through the periodic structure stems from the behavior of the higher-order modes governed by the eigenvalues.

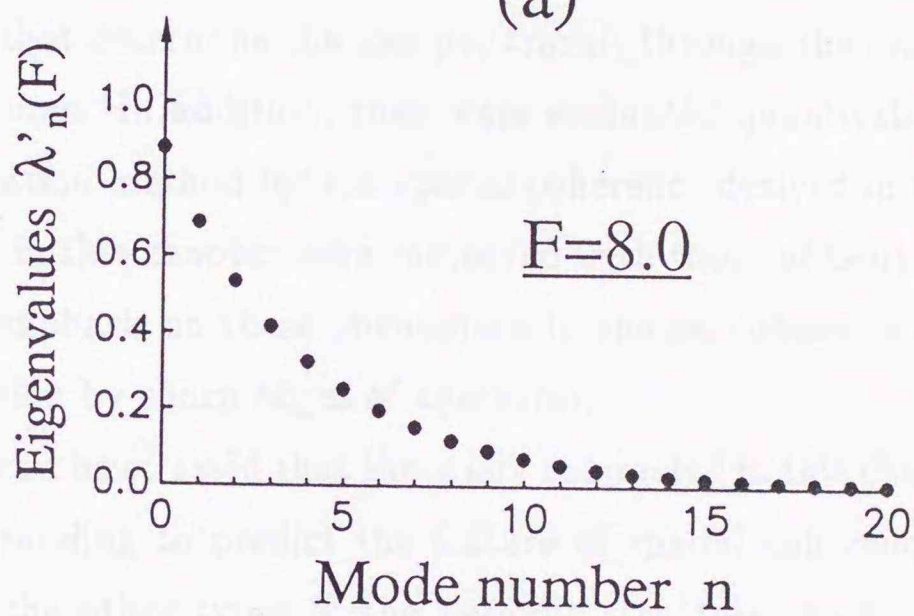
By recalling Fig.8.9 from the standpoint of the intensity distribution, it is also apparent that the light tends to be collected around the optical axis for various values of both F and N in the study of Gori rather than the present study.

8.4 Conclusion

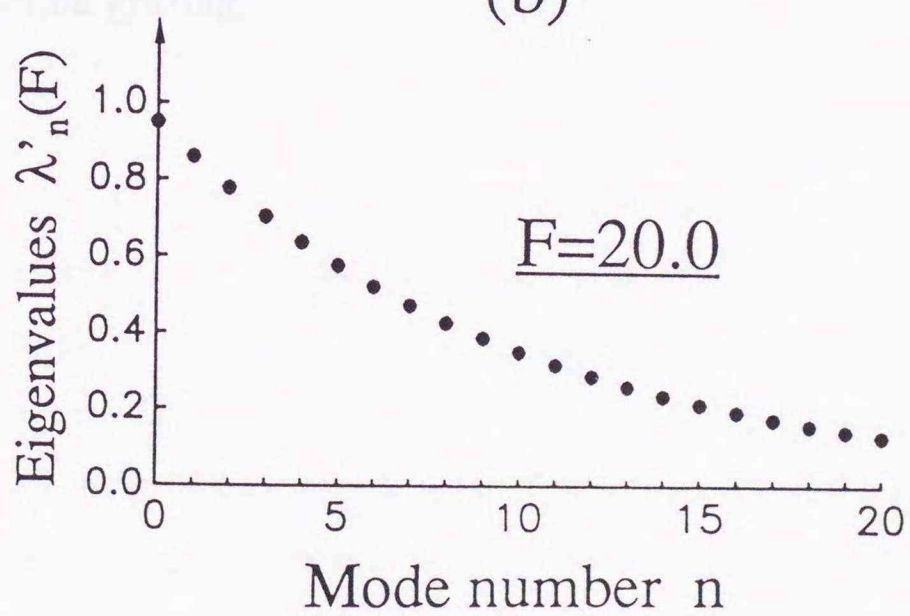
The spatial coherence of light passing through the successive apertures has been studied in association with the mode theory in the space-time domain. The initial light incident on the first aperture of the sequence is assumed to be incoherent



(a)



(b)



(c)

Figure 8.10. Eigenvalues (given by Eq.(8.50)) of the integral equation for the normal modes of the periodic structure constituted by Gaussian amplitude filters in the cases of $F =$ (a) 4.0, (b) 8.0 and (c) 20.0.

under quasi-monochromatic approximation and it could be represented by the infinite superposition of the normal modes derived for the present successive apertures, where the normal modes were characterized by the prolate spheroidal wave functions. Hence, the propagation law of the mutual intensity through the successive aperture was formulated by means of the series expansion of the normal modes. Numerical evaluations were conducted to find some typical features of the variations in spatial coherence and intensity. As a result, the growth of spatial coherence and the convergence of intensity distribution were observed, and it has become obvious that they depend strongly on the dimension of the apertures in the structure. These behaviors in spatial coherence and intensity were examined physically in terms of the eigenvalues that determine the loss per transit through the basic element of the successive apertures. In addition, they were evaluated quantitatively on the basis of the approximation method for the spatial coherence derived in this chapter. The results obtained in this chapter were compared with those of Gori and this comparison led to the emphasis on these phenomena in spatial coherence especially arising from the diffraction by sharp edges of apertures.

Finally, it can be stressed that the study conducted in this chapter may provide intuitive understanding to predict the feature of spatial coherence of light propagating through the other types of the periodic structure, such as the multimode waveguide diffraction grating.

CHAPTER 9

Spatial Coherence of Light Passing Through a Periodic Sequence of Lenses

9.1 Introduction

There has long been a great interest in the physical phenomena that occur on light propagation in a certain kind of medium or even in free space. Among them, the spatial coherence of light propagating through a periodic structure, such as an iterated sequence of apertures, has been investigated originally in connection with laser resonator modes.

The pioneering study was conducted by Wolf [114] in 1960's. Considering the periodic sequence of apertures equivalent to the laser cavity, he investigated theoretically the spatial coherence of laser resonator modes on the basis of an integral equation for the mutual intensity and showed that the quasi-monochromatic light that is initially partially coherent, or even incoherent, may become completely spatially coherent after a sufficient number of transits through the apertures. He ascribed this phenomenon to the effect of propagation and successive diffraction and, consequently, succeeded in the theoretical explanation for the origin of the spatial coherence of laser radiation. As a continuation of Wolf, the rapidity with which the spatial coherence builds up in the periodic systems of apertures and those of confocal lenses was investigated by Streifer [115]. In addition, Gori [116] dealt with a periodic sequence of confocal lenses separated by amplitude filters with a Gaussian transmittance for the reduction of mathematical difficulties in those studies. It should be noted that all of the studies mentioned above could not provide any information about certain effects arising from the spectral structure of the light since they were formulated in terms of the mutual intensity under the quasi-monochromatic approximation. To put it more precisely, modal expansion of the partially coherent field in the space-time domain was employed in those studies in place of that in the

space-frequency domain. In recent years, however, further extension of the theory has been conducted by Wolf and Agarwal [118, 201] in accordance with the new theory of Wolf [119–121, 201] dealing with the coherent-mode representation of the fields of any state of coherence in the space-frequency domain and, as a consequence, the restriction for the spectral structure has been satisfactorily eliminated.

Apart from the work of Wolf, its direct extension made by Streifer and Gori seems to raise the interest in the study of the periodic structure as a specific arrangement for varying the spatial coherence, rather than the study associated with lasers. At the same time, the modal approach for the study of the spatial coherence appears to have received somewhat renewed attention owing to the new theory of Wolf in recent years, yielding a large number of the studies concerning the modal expansion of the partially coherent field in the space-frequency domain [9, 10, 42, 43, 86, 209–218].

In practice, the significance of the periodic structure has been recently encountered especially in the fields of integrated optics [122] and microoptics [123], where various kinds of the structure have been newly proposed and adopted primarily for optical information processing. The typical examples are the waveguide diffraction grating [125] and the stacked planar microlens array [123]. The light confined in such devices may be expected to be highly spatially coherent due to their tiny structures. However, by taking into account what is called the “multimode” structures as a further generalization of the problem, the feature of the spatial coherence plays a significant role in such devices and they are worth investigating. In addition, although these devices may be quite different in structure from the preceding periodic structure such as an iterated sequence of apertures, both structures may be considered to be essentially the same at least in the sense that the propagating light experiences a repeated perturbation from the structure. As a result, it seems feasible that the study of Wolf [114] and its extension [115, 116] give a certain insight into the spatial coherence properties of light propagating through these devices. However, different treatments must be properly adopted for the rigorous evaluation of the spatial coherence properties of light in these devices.

In this chapter, the feature of the spatial coherence of light passing through a periodic structure is investigated in order to elucidate some typical phenomena arising from the successive propagation and diffraction, on the basis of the mode

theory in the space-frequency domain [118–121, 201]. As a periodic structure for analysis, a periodic sequence of confocal lenses separated by Gaussian amplitude filters is employed in accordance with Gori [116]. Since the initial light to be launched into the structure is assumed to be produced by a finite mixture of the Hermite-Gaussian functions which are the modes of the structure, it may be regarded as certain lasers under multimode oscillations and, therefore, it is spatially partially coherent [118, 219]. The source of this kind has been encountered in recent studies of the spatial coherence [86, 220] and it has proved to be well behaved in the discrimination of some effects stemming from the particular modes. This chapter can also be concerned with certain phenomena arising from the spectral structure of the light because it is formulated in terms of the cross-spectral density. However, we confine our discussion to the case of quasi-monochromatic light and the phenomena in relation with the spectral structure are assumed to be negligible for simplicity.

In the following, we examine only the variations of the spatial coherence and intensity distribution by the process of propagation through the periodic structure. More specifically, we show the various properties of the resultant field, including those of the initial field, i.e., source itself, especially in connection with the homogeneity of the spatial coherence characterizing the dependence of the degree of spatial coherence on positions across the field of interest. It is important to note that the present study may provide information to deduce the spatial coherence properties of light propagating through the other kind of periodic structures such as above-mentioned optical devices. Detailed study concerning the spectral aspect of light in the periodic structure is presented in Chapter 10.

9.2 Theory

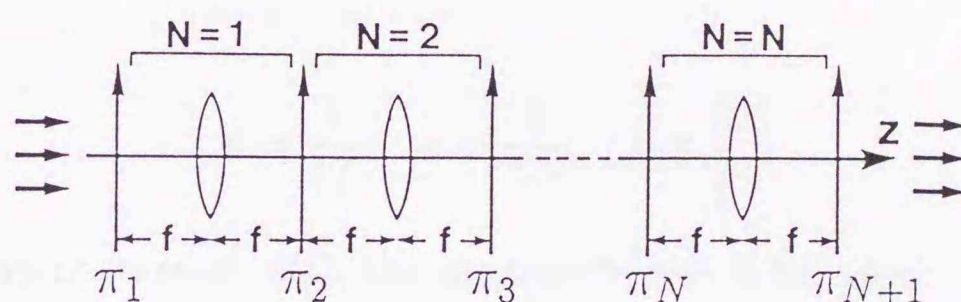
We consider the periodic structure constituted by perfectly aligned ideal cylindrical lenses and amplitude filters illustrated in Fig.9.1(a). The focal points of various lenses coincide since the lens spacing is twice the focal length f and one-dimensional Gaussian amplitude filters are located across each focal plane. The transmittances

of these filters are given by

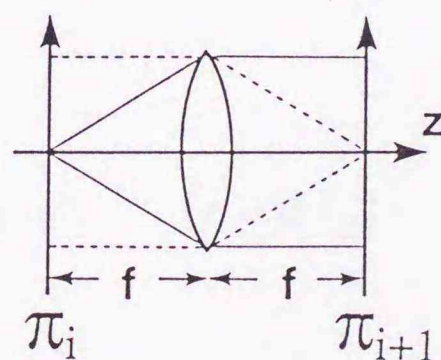
$$t(x) = \begin{cases} \exp\left(-\frac{x^2}{2\sigma^2}\right), & \text{for } \pi_1 \text{ and } \pi_{N+1} \text{ planes} \\ \exp\left(-\frac{x^2}{\sigma^2}\right), & \text{for } \pi_2, \pi_3, \dots, \pi_N \text{ planes} \end{cases} \quad (9.1)$$

where σ takes a positive constant.

The main aim of the present study is to investigate the feature of the spatial coherence of light propagating through the periodic structure in accordance with the modal decomposition of the partially coherent field. Hence, for a moment, we deal with the modes of the periodic structure under consideration in some detail.



(a)



(b)

Figure 9.1. (a) Periodic sequence of confocal lenses constituted by amplitude filters with Gaussian transmittance and (b) the basic unit for analysis. Across each focal plane the Gaussian amplitude filters are located.

In order to determine the modes of the structure with the help of the standard diffraction theory ([153], Sec. 5.2), let us consider a basic unit illustrated in

Fig.9.1(b), where successive sequence of the basic unit constitutes the whole sequence. By supposing that a monochromatic spatially coherent field $V^{(i)}(x)$ of wavelength λ is incident on the focal plane π_i from the left, the light distribution $V^{(i+1)}(u)$ emerging to the right of the focal plane π_{i+1} is given, aside from the multiplicative phase factors, by

$$V^{(i+1)}(u) = \frac{1}{\sqrt{\lambda f}} \int_{-\infty}^{\infty} V^{(i)}(x) \exp\left(-\frac{x^2 + u^2}{2\sigma^2}\right) \exp\left(-j\frac{k}{f}xu\right) dx, \quad (9.2)$$

where $k = 2\pi/\lambda$ is the wavenumber. Consequently, the eigenvalue equation for evaluating the modes of the lens sequence may be written as

$$\beta \Phi(u) = \sqrt{A} \int_{-\infty}^{\infty} \Phi(x) \exp[-a(x^2 + u^2)] \exp[-jbu x] dx, \quad (9.3)$$

where

$$A = \frac{1}{\lambda f}, \quad a = \frac{1}{2\sigma^2}, \quad b = \frac{k}{f}, \quad (9.4)$$

and β is a complex number with the magnitude less than unity. The eigenvalue equation (9.3) has the solutions

$$\Phi_n(x) = \left(\frac{2c}{\pi}\right)^{\frac{1}{4}} (2^n \cdot n!)^{-\frac{1}{2}} H_n(x\sqrt{2c}) \exp(-cx^2) \quad (9.5)$$

with the eigenvalues

$$\beta_n = \beta_0 \cdot q^n, \quad (n = 0, 1, 2, \dots, \infty) \quad (9.6)$$

where

$$c = \frac{\sqrt{b^2 + 4a^2}}{2}, \quad \beta_0 = \sqrt{\frac{A\pi}{a+c}}, \quad q = -j\frac{b}{2(a+c)}. \quad (9.7)$$

The functions (9.5) are Hermite-Gaussian polynomials. Thus, the modes of the structure are termed Hermite-Gaussian modes. They are normalized and orthogonal among each other forming an orthonormal set:

$$\int_{-\infty}^{+\infty} \Phi_n(x) \Phi_m^*(x) dx = \begin{cases} 1, & \text{for } n = m \\ 0, & \text{for } n \neq m. \end{cases} \quad (9.8)$$

Equation (9.3) has an infinite number of solutions, and every arbitrary field that can exist in the structure is described as a superposition of the solutions of the function (9.5).

As a source that provides the light incident on the periodic structure, we assume a partially coherent source synthesized by a finite superposition of equally-weighted Hermite-Gaussian modes $\Phi_n(x)$, which are spatially coherent but mutually uncorrelated, given by Eq.(9.5). As a result, the cross-spectral density function across the source, which characterizes the spatial coherence properties of the field in the space-frequency domain, becomes

$$W^{(0)}(x_1, x_2; \omega) = S^{(0)}(\omega) \sum_{m=0}^M \Phi_m(x_1) \Phi_m^*(x_2), \quad (9.9)$$

where $S^{(0)}(\omega)$ represents the spectrum of light across the source and is assumed to be independent of positions, and M is a positive integer showing an order of the highest-order modes contained in the source. Here, it is noteworthy that the modal expansion specified by Eq.(9.9) can be regarded as a coherent-mode representation of the partially coherent field established by Wolf [119–121]. In general, each mode $\Phi_n(x)$ contained in Eq.(9.9) also depends on the frequency ω of the light (or the wavelength $\lambda = 2\pi c/\omega$, c being the speed of light in vacuum) as well as the spectrum $S^{(0)}(\omega)$. In the present chapter, however, we confine our discussion to the case of the quasi-monochromatic light for simplicity. Hence, here and in what follows, we remove the explicit dependence on ω from the description of the cross-spectral density and spectrum $S^{(0)}(\omega)$ is replaced by the constant I_0 , yielding

$$W^{(0)}(x_1, x_2) = I_0 \sum_{m=0}^M \Phi_m(x_1) \Phi_m^*(x_2). \quad (9.10)$$

The partially coherent source specified by Eq.(9.10) has been encountered in some recent studies [86, 220] and it has been shown that such a source plays a significant role in the study dealing with the effect of certain modes in the partially coherent field. Furthermore, it has also mathematical advantage for the present purpose because the summation of Eq.(9.10) can be represented in a simple form using certain mathematical formula as will be seen in Section 9.3.1. By taking account of these facts, the source of this kind has been necessarily adopted here. It is obvious

from Eq.(9.10) that the source consists of the mixture of $(M + 1)$ modes owing to the summation from 0 to M . If the value M becomes infinity, Eq.(9.10) becomes the one-dimensional Dirac delta function, i.e.,

$$\sum_{m=0}^{\infty} \Phi_m(x_1) \Phi_m^*(x_2) = \delta(x_1 - x_2), \quad (9.11)$$

which represents physically an incoherent source having the uniform intensity distribution ([147], Sec. 4.4). On the contrary, if the value M is equal to 0, the source is spatially completely coherent with the Gaussian intensity distribution. This consequence shows that both the spatial coherence and the intensity distribution of the source are governed by the value of M .

Let us now consider the propagation of light in the periodic structure. The source specified by Eq.(9.10) is placed just in front of the first focal plane π_1 and provides the quasi-monochromatic, initially partially coherent light. We characterize the spatial coherence properties of light across the output plane of the n -th basic unit (i.e., π_{n+1} plane) in terms of the cross-spectral density $W^{(n)}(\eta_1, \eta_2)$, where η_1 and η_2 are typical points across the output plane. According to the propagation law of the cross-spectral density, the cross-spectral densities $W^{(n+1)}(\xi_1, \xi_2)$ and $W^{(n)}(\eta_1, \eta_2)$ are related by

$$W^{(n+1)}(\xi_1, \xi_2) = A \iint_{-\infty}^{\infty} W^{(n)}(\eta_1, \eta_2) K(\xi_1, \eta_1) K^*(\xi_2, \eta_2) d\eta_1 d\eta_2, \quad (9.12)$$

where

$$K(\xi, \eta) = \exp[-a(\xi^2 + \eta^2)] \exp[-jb\xi\eta], \quad (9.13)$$

and the parameters A , a and b are given in Eq.(9.4). By substituting of Eq.(9.10) into the right hand side of Eq.(9.12), the light emerging from the second focal plane π_2 becomes

$$W^{(1)}(x_1, x_2) = I_0 \sum_{m=0}^M |\beta_m|^2 \Phi_m(x_1) \Phi_m^*(x_2), \quad (9.14)$$

where β_m is given in Eq.(9.6) and is an eigenvalue of the integral equation for determining the modes of the periodic structure. Here and in the sequel, spatial coordinates across the output plane are assumed to be specified by x_1 , x_2 and x .

By repetition of the same procedure, the cross-spectral density function of light emerging from the output plane of the N -th basic unit (i.e., $(N + 1)$ -th focal plane π_{N+1}) turns out to be

$$W^{(N)}(x_1, x_2) = I_0 \sum_{m=0}^M |\beta_m|^{2N} \Phi_m(x_1) \Phi_m^*(x_2). \quad (9.15)$$

From Eqs.(9.10) and (9.15), the intensity distribution and the degree of spatial coherence, defined by

$$I^{(N)}(x) = W^{(N)}(x, x) \quad (9.16)$$

and

$$\mu^{(N)}(x_1, x_2) = \frac{W^{(N)}(x_1, x_2)}{\sqrt{I^{(N)}(x_1)} \sqrt{I^{(N)}(x_2)}} \quad (9.17)$$

are evaluated for both the source and the resultant field after propagation through the N -th basic unit.

9.3 Results and Discussion

To show some typical results derived from the present study, numerical computations were carried out under the relations of $f = 5.0 \times 10^{-2}[\text{m}]$ and $\lambda = 5.0 \times 10^{-7}[\text{m}]$.

9.3.1 Some properties of the incident light

We first examine various properties of the source producing the light incident on the structure. The spatial coherence of the source is characterized by the cross-spectral density expressed in Eq.(9.10). On substituting Eq.(9.5) into Eq.(9.10) and using the mathematical formula ([221], Eq.(11), p. 193), Eq.(9.10) becomes

$$\begin{aligned} W^{(0)}(x_1, x_2) = & \sqrt{\frac{2c}{\pi}} \frac{H_{M+1}(x_1\sqrt{2c})H_M(x_2\sqrt{2c}) - H_M(x_1\sqrt{2c})H_{M+1}(x_2\sqrt{2c})}{2^{M+1}M!(x_1 - x_2)\sqrt{2c}} \\ & \times \exp[-c(x_1^2 + x_2^2)]. \end{aligned} \quad (9.18)$$

In addition, substitution of Eq.(9.18) into Eq.(9.16) and use of another mathematical formula ([221], Eq.(35), p. 195) yield the intensity distribution across the source

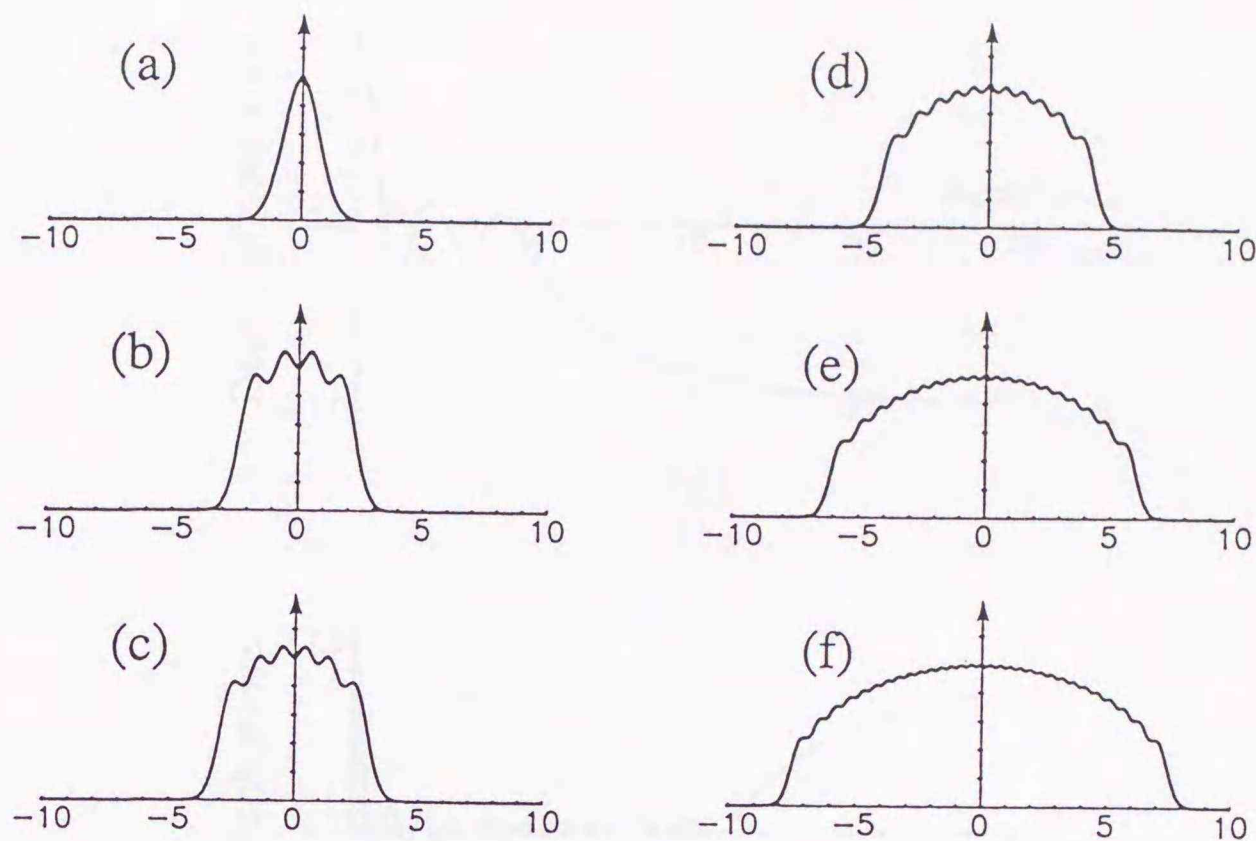


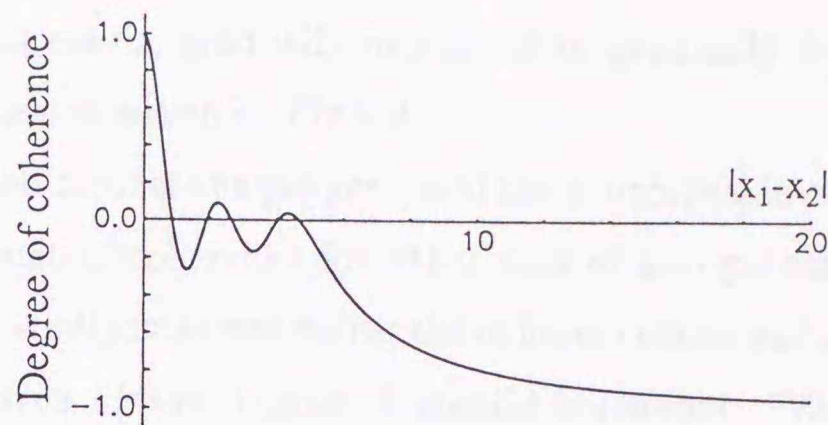
Figure 9.2. Normalized intensity distributions across the source for $M = 0$ (a), 3 (b), 5 (c), 10 (d), 20 (e), and 30 (f).

given by

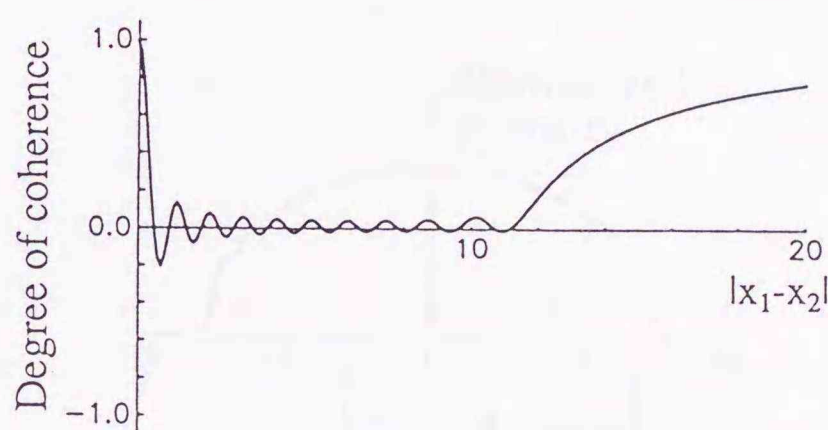
$$I^{(0)}(x) = \sqrt{\frac{2c}{\pi}} \frac{[H_{M+1}(x\sqrt{2c})]^2 - H_M(x\sqrt{2c})H_{M+2}(x\sqrt{2c})}{2^{M+1}M!} \exp(-2cx^2), \quad (9.19)$$

where the parameter c is expressed in Eq.(9.7). The parameter M contained in Eqs.(9.18) and (9.19) denotes the maximum order of the modes in the source corresponding to the number $(M + 1)$ of the superposed modes, as discussed in the previous section.

Figure 9.2 shows the variation of the normalized intensity distribution across the source for various values of M , on the basis of Eq.(9.19). It is clearly seen in this figure that the normalized intensity distribution extend further out in the transverse directions with an increase of the value M . This is because the amplitude distribution of the source consists of superposed Hermite-Gaussian functions whose spread increases with increasing values of M . The degree of spatial coherence across the source is plotted in Fig.9.3 for some values of M , where the degrees of spatial coherence are evaluated for two points situated on the source symmetrically about its center. The cross-spectral density corresponding to this case is obtained by setting



(a)



(b)

Figure 9.3. Degrees of spatial coherence across the source for two points situated symmetrically about the center for the cases of $M = 5$ (a) and 20 (b).

$x_1 = -x_2 = x$ in Eq.(9.18), yielding

$$W^{(0)}(x, -x) = \sqrt{\frac{2c}{\pi}} \frac{(-1)^M H_M(x\sqrt{2c}) H_{M+1}(x\sqrt{2c})}{2^{M+1} M! x \sqrt{2c}} \exp(-2cx^2). \quad (9.20)$$

As already mentioned in the previous section, not only the intensity distribution but also the spatial coherence are varied as a function of M . The coherence interval, which is defined as the two-point separation $|x_1 - x_2|$ for which the degree $\mu(x_1, x_2)$ of spatial coherence becomes the value 0.88, tends to reduce with an increase of the value M . This behavior in the spatial coherence indicates that multiple Hermite-Gaussian modes give rise to the reduction in the spatial coherence. However, even though many modes are superposed on the source, few lower-order modes play a significant role on the region far from the center across the source in transverse directions because the lower-order modes are collected around the center. In other words, the magnitude of the lower-order modes are so small that they are negligible

on the region. As a result, gradually increasing or gradually decreasing tails of the coherence curves are observed in Fig.9.3.

In order to find further the properties of the source, we investigate the variation of the degree of spatial coherence for other sets of two points on the source. To characterize the spatially coherent fields, the coherence interval is generally employed in place of the curve of the degree of spatial coherence. We examine thus the behavior of the coherence interval with respect to the source point P_1 . Schematic for this problem is illustrated in Fig.9.4(a). This figure shows the normalized intensity

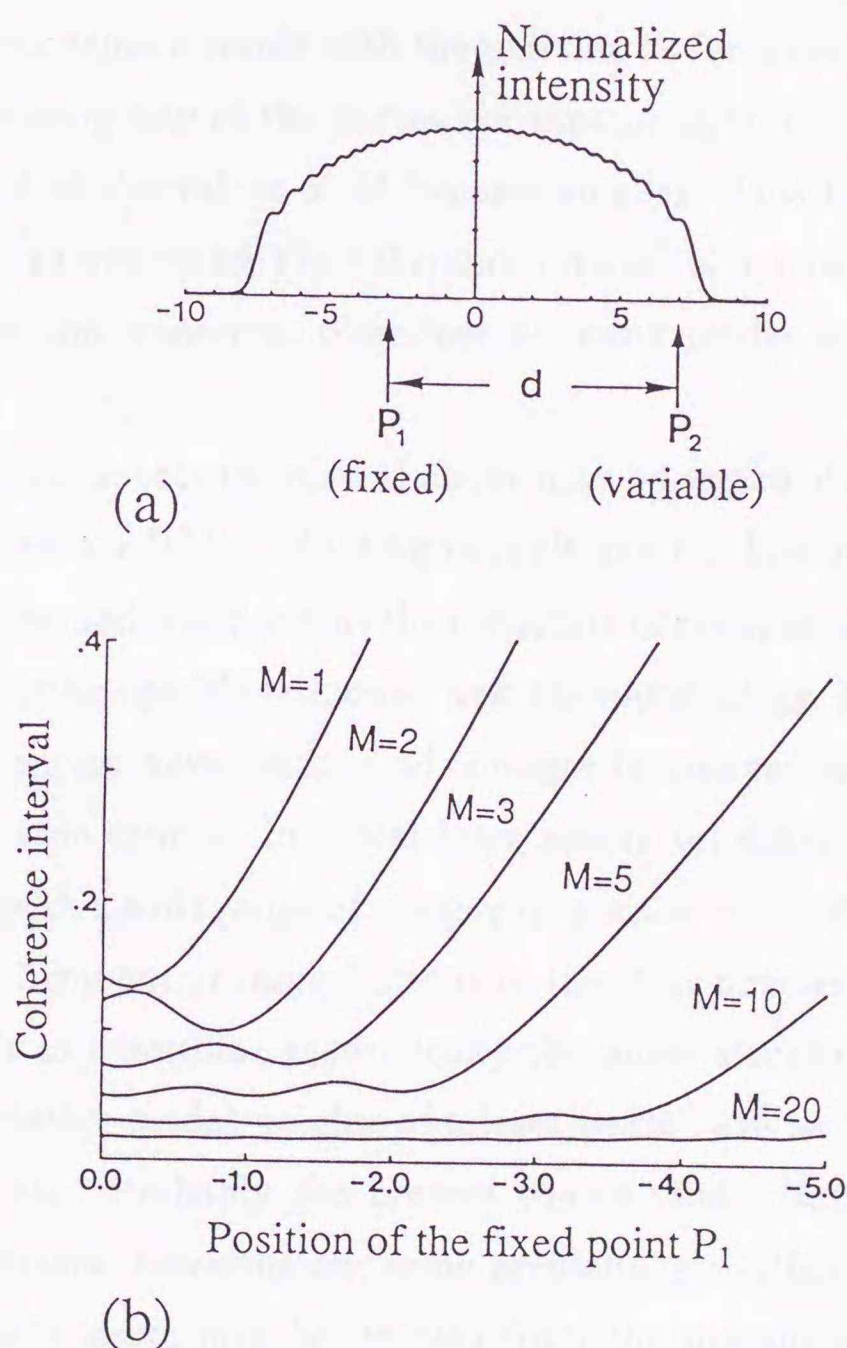


Figure 9.4. (a) Schematic of investigating the dependence of the position for the coherence intervals. The normalized intensity distribution having a certain value of M is shown. The point P_1 (specified by the position x_1) is fixed and the point P_2 (specified by the position x_2) is varied under the condition of $x_1 \leq x_2$. (b) Coherence intervals plotted as a function of the position x_1 for various values of M .

distribution of the source having a certain value of M . Assume that the point P_1 (specified by the position x_1) is fixed while the point P_2 (specified by the position x_2) is varied on the source under the condition of $x_1 \leq x_2$. The degree of spatial coherence decreases with increasing values of x_2 . When the degree $|\mu(x_1, x_2)|$ of spatial coherence becomes 0.88, the coherence interval is given by the value $(x_2 - x_1)$. In Fig.9.4(b), the coherence intervals, which are normalized by each intensity spread (which is defined as the width for which the intensity decreases to $e^{-1/2}$ of its maximum value), are plotted as a function of the position x_1 for various values of M . It is obvious from this figure that the curves showing the variation of the coherence intervals remain stable with the position x_1 for large values of M and that a gradually increasing way of the curves corresponding to relatively small values of M becomes rapid as the values of M become smaller. This fact indicates that the region across the source where the coherence interval is independent of the position extend further in the transverse directions as many modes are superposed on the source.

In practice, the source mentioned above may be regarded as a laser constituted by a confocal resonator [112] oscillating in multimodes. The dominant effect of the multiple transverse modes appears as the reduction of the spatial coherence. In many applications including speckle reduction and micrometrology, these partially coherent multimode sources have distinct advantages in comparison with a completely coherent single-mode source. In a real laser resonator, different transverse modes tend to oscillate with a wide range of relative intensities which depend on several factors such as misalignment or imperfections in the laser mirrors [109]. Consequently it is very difficult to determine theoretically the mode structure of real multimode lasers and, the relative modal weights of a laser beam have to be measured through certain experiments. Probably the present source model thus does not represent exactly any real lasers. Nevertheless, some predictions relating to the general properties of multimode lasers may be derived from the present study. On the other hand, it is also very important to note that the source of this kind pertains to the class of the quasi-homogeneous sources [21, 24, 33, 206], provided that the number of superposed modes specified by M is sufficiently large. The quasi-homogeneous source means that the spatial coherence of light across the source depends only on the coordinate difference and, furthermore, that the source size and its spatial

structure is much larger than the coherence interval (in this case, it is defined as the interval for which the degree of coherence decreases to a certain value, e.g., 0.88) across the source. Thus, such sources are always relatively incoherent in the global sense. The radiation properties from sources of this kind are found to be expressed by means of the source properties in quite simple manners [24]: the angular distribution of the radiant intensity and the complex degree of spatial coherence in the far field are, respectively, proportional to the two-dimensional Fourier transforms of the complex degree of spatial coherence and the intensity distribution across the source. Although the above discussion concerning the radiation properties cannot be directly applied to the present analysis even in the case of large M , it may play a significant role in another application of the source adopted in the present study.

9.3.2 Propagation of the partially coherent field

In accordance with Eqs.(9.15), (9.16) and (9.17), we next examine the variations of the normalized intensity distribution and the degree of spatial coherence as a function of N corresponding to the number of transits through the basic unit. Typical examples are illustrated in Figs.9.5 and 9.6 for two values of W_f under the condition of $M = 20$, where W_f implies the ratio of the width of the Gaussian amplitude filter located at each focal plane to that of the lowest-order mode (it also takes a Gaussian form) contained in the source. From these figures, the spread of the normalized intensity distribution is seen to decrease and finally approach a constant with an increase of the value N . This phenomenon implies that the higher-order modes contained in the source are suppressed by the process of propagation through the periodic structure and, as a consequence, only the lowest-order mode survives in the structure. On the other hand, the spatial coherence is improved with an increase of the value N and the field emerging from the structure becomes fully spatially coherent after a sufficient number of transits. Comparison of Figs.9.5 and 9.6 reveals that W_f , whose value is practically proportional to the aperture size of the structure, works as an important parameter to determine the rapidity in the variation of the normalized intensity distribution and the spatial coherence against the value of N . In Fig.9.7, the behaviors of the coherence interval and the intensity spread are shown as a function of N for various values of W_f under the condition of $M = 20$. It is understood from this figure that the variation of the intensity

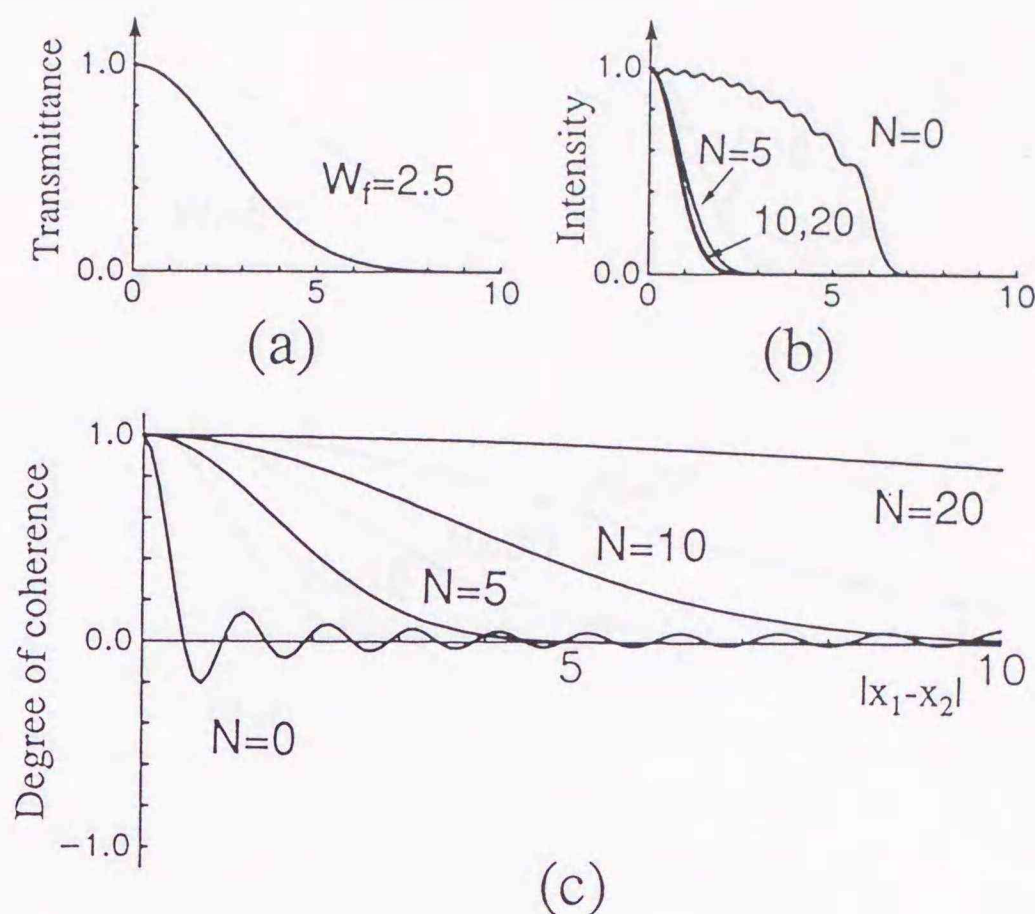


Figure 9.5. (a) Transmittance of the Gaussian amplitude filter for $W_f = 2.5$. (b) Normalized intensity distribution emerging from the periodic structure for various values of N . (c) Degree of spatial coherence for various values of N . The parameter N corresponds to the number of transits through the basic unit illustrated in Fig.9.1(b). The case $N = 0$ corresponds to the initial light, i.e., the source.

spread and the coherence interval against N become rapid as the aperture size of the structure becomes smaller.

In order to characterize the spatial coherence properties of the light field emerging from the structure, we finally investigate the variation of the coherence interval of the field with respect to the position x_1 (see Fig.9.4(a)). The coherence intervals, which are normalized in a similar manner already discussed, are plotted in Figs.9.8(a) and (b) against the position x_1 for various values of N . These figures are distinguished by the values of M : Figs.9.8(a) and (b) correspond to the cases of $M = 1$ and $M = 5$, respectively. The normalized intensity distribution relating to Figs.9.8(a) and (b) are also illustrated in Figs.9.9(a) and (b), respectively, for various values of N .

The curves of the coherence interval illustrated in Fig.9.8(a) vary as a function of the position x_1 , and its variation tend to reduce as the value N become larger. On the other hand, the curves illustrated in Fig.9.8(b) become constant with the

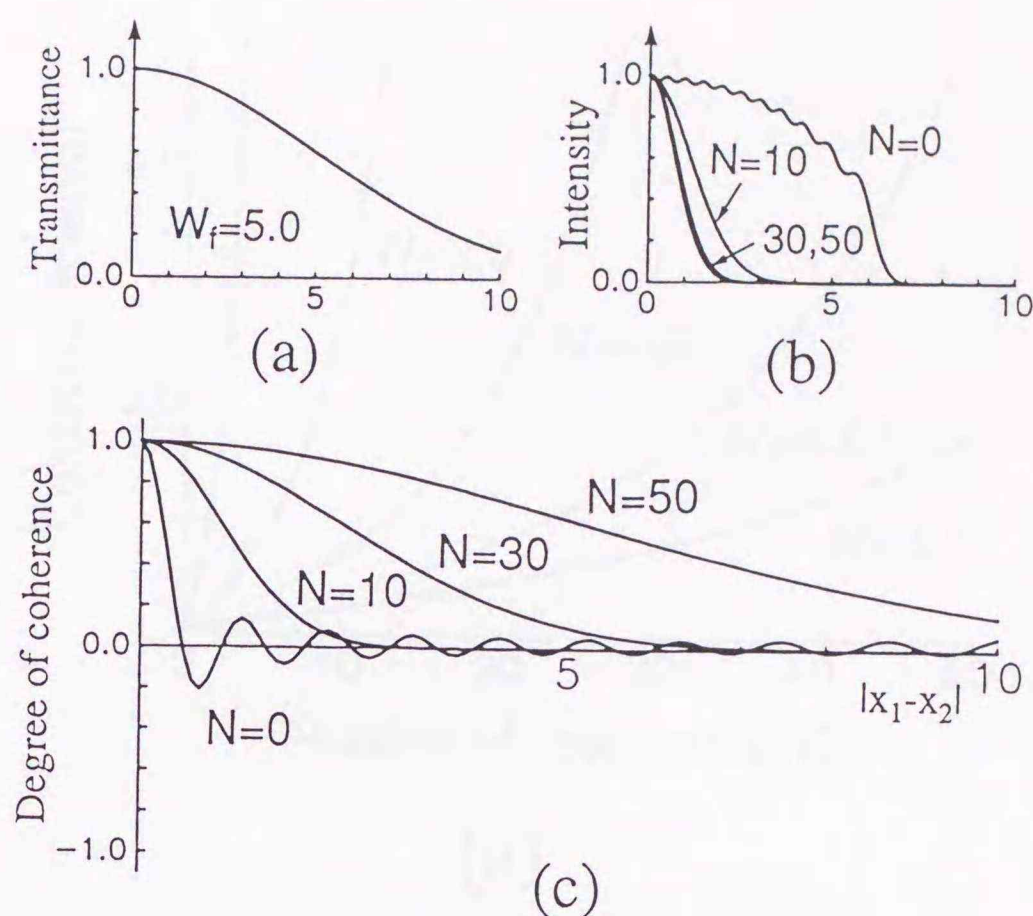
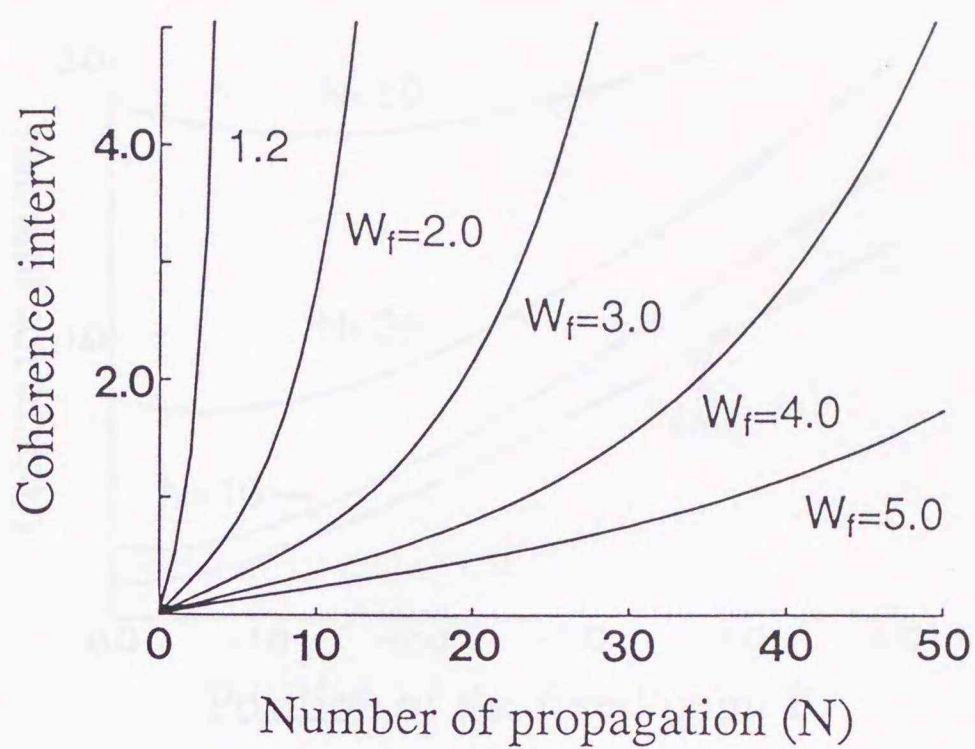
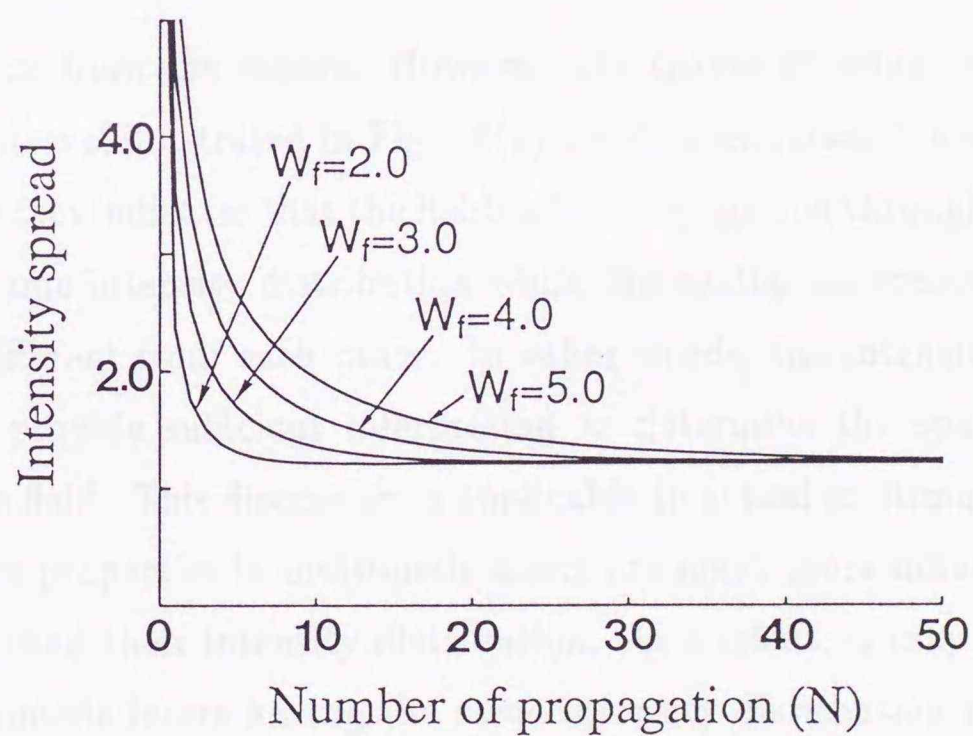


Figure 9.6. (a) Transmittance of the Gaussian amplitude filter for $W_f = 5.0$. (b) Normalized intensity distribution emerging from the periodic structure for various values of N . (c) Degree of spatial coherence for various values of N .

position x_1 for large values of N . This implies that, even though the initial field to be launched into the structure is inhomogeneous in the sense that the coherence interval depends on the position, the field becomes homogeneous in the sense that the coherence interval is independent of the position after transits and that, in addition, the number of transits required to reach the homogeneous field in that sense depends on the number of superposed modes on the source. As the number of modes contained in the source becomes larger, the homogeneous field in that sense can be realized by the processes of a small number of the transits. Conversely, it can be realized only after the sufficient number of transits if the number of modes contained in the source is extremely small. Therefore, it may be concluded that the existence of many modes in the source is desired to produce rapidly the homogeneous field from a multimode laser radiation using the periodic structure. In order to elucidate further this performance in the variation of the coherence interval, we consider the variation of the normalized intensity distribution for various values of N , as illustrated in Figs.9.9(a) and (b). In these figures, the intensity distribution for the values N more than 10 approximately takes a Gaussian form and, consequently, it is very difficult



(a)



(b)

Figure 9.7. (a) Coherence intervals and (b) intensity spread plotted against the values of N for various values of W_f in the case of $M = 20$.

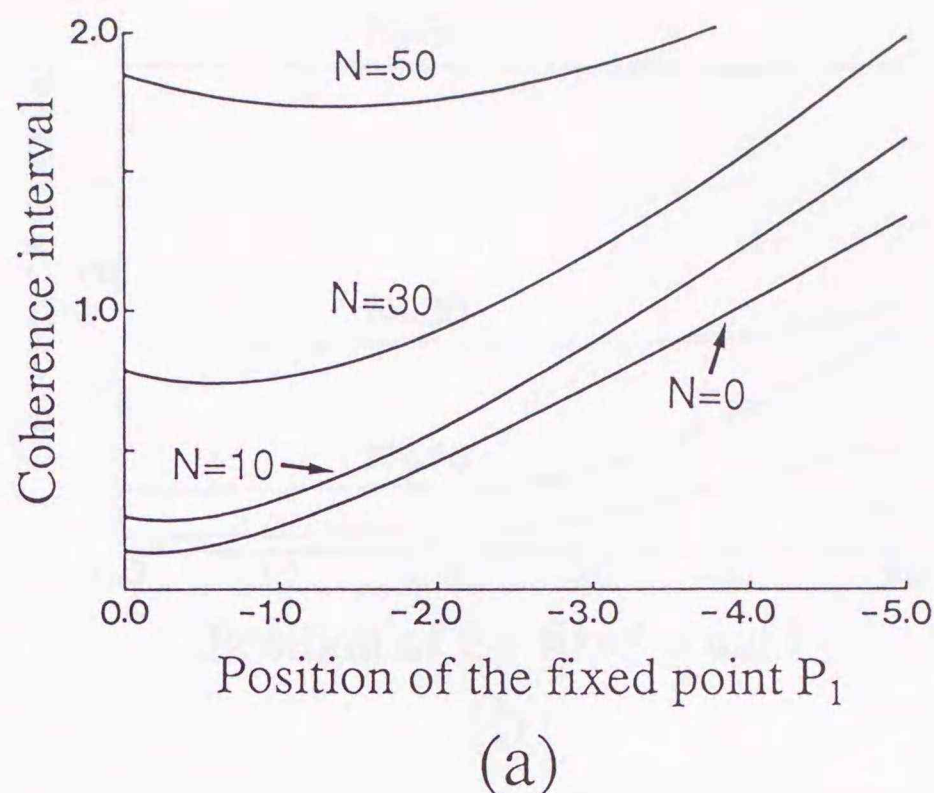


Figure 9.8. Coherence intervals across the output of the periodic structure plotted against the position x_1 for various values of N in the case of $M = 1$ (a) and 5 (b). The parameter W_f is fixed at the value 5.0.

to distinguish one from the others. However, the curves showing the variation of the coherence interval illustrated in Fig.9.8(a) are different from those illustrated in Fig.9.8(b). This fact indicates that the fields after propagation through the structure exist with the same intensity distribution while the spatial coherence properties of the fields are different from each other. In other words, the intensity distribution alone does not provide sufficient information to determine the spatial coherence properties of the field. This discussion is applicable to actual multimode lasers. The spatial coherence properties in multimode lasers are much more influenced by their mode structure than their intensity distribution. As a result, it may be possible to make such multimode lasers having the same intensity distribution in each output aperture but with the different spatial coherence properties.

The trends in the spatial coherence and the intensity distribution on propagation mentioned above may be well understood by taking into consideration the Gaussian Schell-model (GSM) sources [20, 42–44, 182]. Due to their mathematical convenient Gaussian distributions of both the intensity and the degree of spatial coherence, the GSM sources have been frequently adopted and played an important role in recent studies especially dealing with radiometry [20, 44]. Various properties

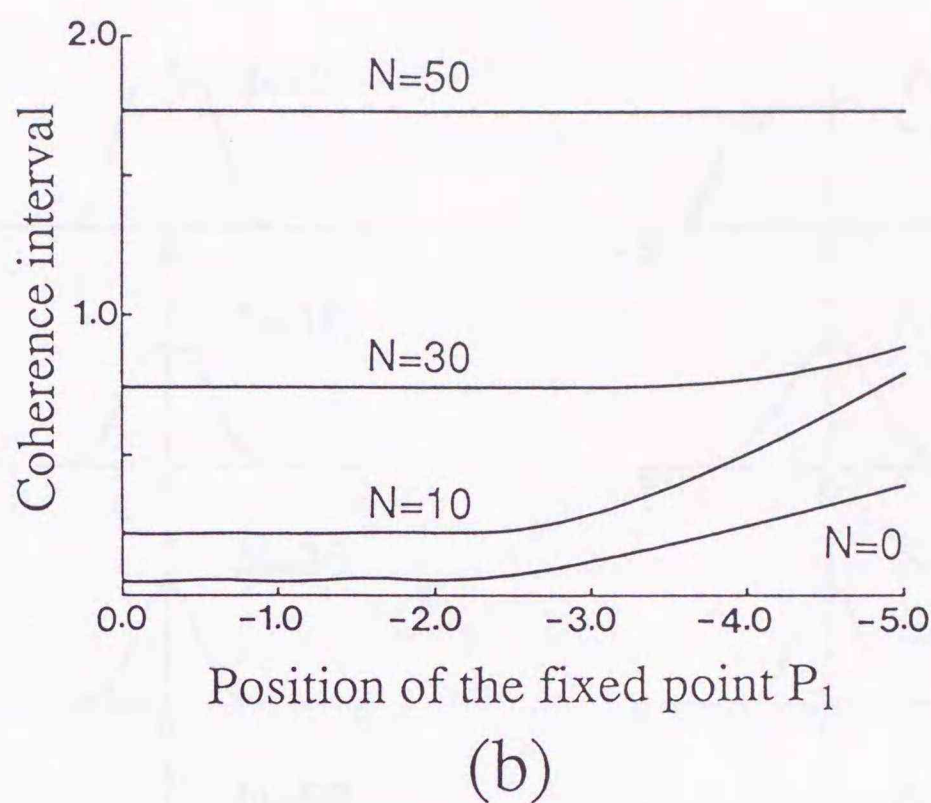


Figure 9.8. Cont.

of the sources and their radiated field have been shown in detail including the fact that the GSM sources can be realized by an infinite superposition of the properly weighted Hermite-Gaussian functions [42, 43, 118, 182]. In the present study, the field emerging from the output plane after N -transits through the structure belongs strictly to the class of the GSM sources, provided that the number of superposed modes becomes infinity (i.e., the initial light launched into the structure is completely spatially incoherent). In this case, the behavior in the intensity and the degree of spatial coherence (both distributions take Gaussian forms with different widths) can be described mathematically in a simple form [118] and, moreover, it has been found that the degree of spatial coherence of the light depends always only on the coordinate difference for any number N of transits. It has become obvious from the above discussion that higher-order modes contained in the GSM source are intimately related to the homogeneity of the spatial coherence adopted in the present investigation. Accordingly, it is also possible to assert that the present analysis is a further extension of the study conducted by Gori [43] dealing with the effect of the finite number of the modes in the GSM source.

As regards the latter half discussion of the previous paragraph, it is not surprising that there exist the partially coherent fields with the same intensity distributions but with different spatial coherence properties, by taking into account that the GSM

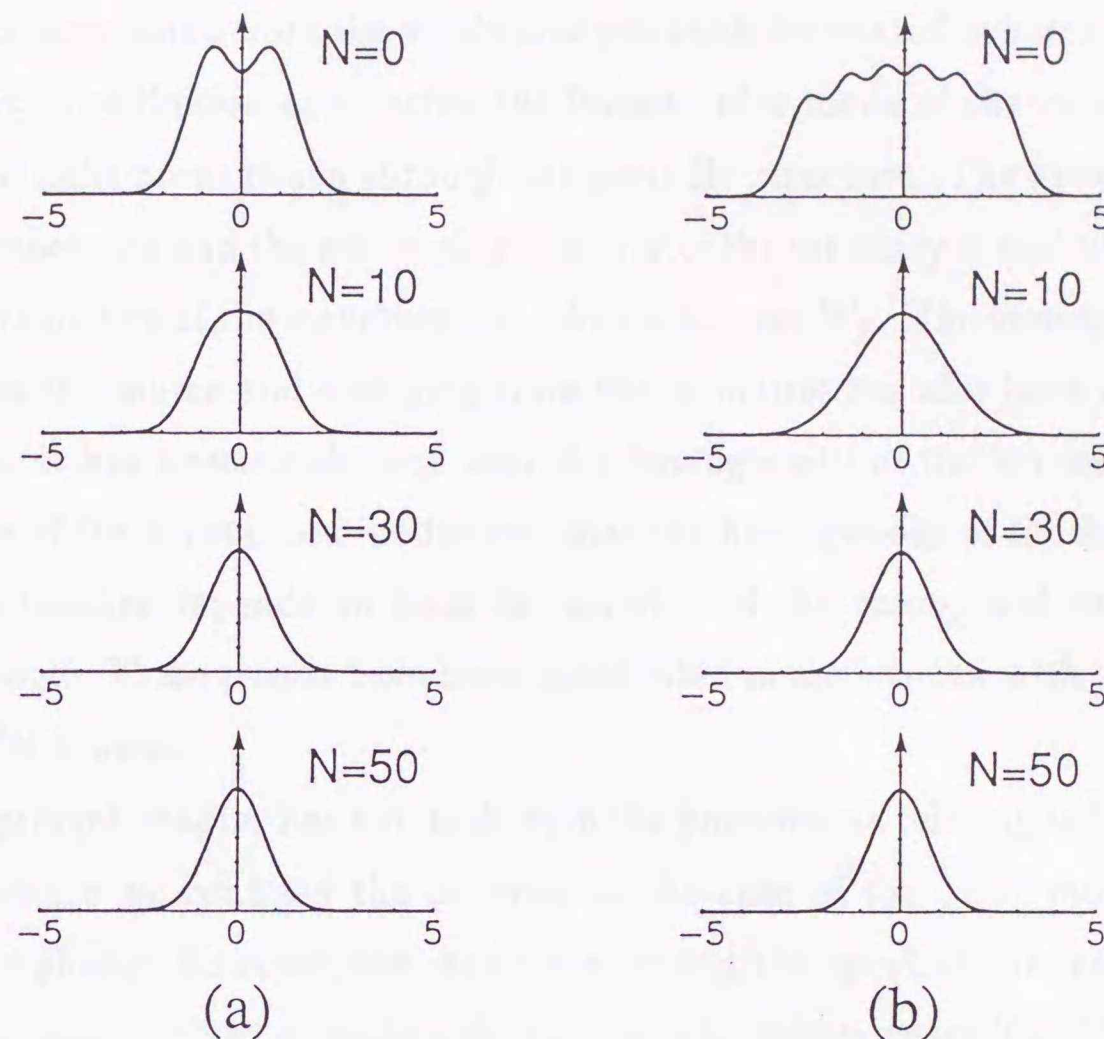


Figure 9.9. Normalized intensity distribution across the output of the periodic structure plotted against the position x_1 for various values of N in the case of $M = 1$ (a) and 5 (b). The parameter W_f is fixed at the value 5.0.

sources have also the same characteristics.

9.4 Conclusion

The periodic sequence of confocal lenses separated by Gaussian amplitude filters are studied in relation to the spatial coherence, on the basis of the mode theory in the space-frequency domain. Since the source which provides the light incident on the periodic structure is assumed to be synthesized by a superposition of the Hermite-Gaussian modes, which are the modes of the present periodic structure, under the quasi-monochromatic approximation, it can be regarded as a certain practical laser under multimode oscillations. By adopting this source model for analysis, the present study may serve to clarify the spatial coherence properties of a certain kind of multimode lasers.

It has been found from the whole analysis that the spatial coherence grows and the intensity distribution approaches the lowest-order mode of the structure by the processes of light propagation through the periodic structure. The growing rapidity of spatial coherence and the converging rapidity of the intensity distribution depends on the aperture size of the structure, i.e., the parameter W_f . The homogeneity of the fields across the source and emerging from the structure has also been investigated. As a result, it has become obvious that the homogeneity of the source depends on the number of the superposed modes and that the homogeneity of the field emerging from the structure depends on both the number of the superposed mode and the structure itself. These trends have been manifested in association with the Gaussian Schell-model sources.

The present chapter has not dealt with the phenomena relating to the spectrum of light because we confined the analysis to the case of the quasi-monochromatic light for simplicity. However, the study concerning the spectral changes of light on propagation has received considerable attention in recent years [62, 63, 65]. More recently, such phenomena appear to have been manifested especially in connection with the medium in which light propagates [93, 107, 222]. Therefore, it seems reasonable to investigate the feature of the spectrum of light propagating through the present periodic structure. The sequel of the analysis concerning the spectral aspect is given in the next chapter.

CHAPTER 10

Spectral Aspect of Light Passing Through a Periodic Sequence of Lenses

10.1 Introduction

The phenomena that occur on the propagation of light through a periodic structure have long been the subject of great interest. By considering a periodic structure like an iterated sequence of apertures equivalent to a laser resonator, numerous investigations have been conducted on the laser resonator modes in parallel with the advent of lasers [113]. In a similar manner, the physical origin of spatial coherence in the laser radiation has also been theoretically explored within the framework of coherence theory in the space-time domain [114–116]. On the other hand, the practical significance of the periodic structure appears to have been reencountered in recent years, irrespective of laser resonators, especially in the fields of integrated optics and microoptics [122, 123]. One of the representative configurations may be a corrugated periodic waveguide acting, for example, as a lens, a coupler and a spectral filter [125]. Recent advances in optical information processing have broadened the possibility of the periodic structure and the area of its applications. Although such devices with certain periodicity are in contrast in their structure with the periodic system of apertures associated with lasers, both can be considered to be essentially equivalent at least in the sense that the propagating light in each case suffers repeated perturbation from the structure.

The previous chapter dealt theoretically with the feature of spatial coherence of light passing through a periodic sequence of confocal lenses separated by Gaussian amplitude filters, in connection with both the lasers and the periodic devices to be employed for certain optical processing. The mathematical treatment of the problem was, as opposed to the early attempts [114–116], based on the coherence theory in the space-frequency domain that can directly provide information about the spatial

coherence properties of light including its spectral aspect. However, the phenomena relating to the spectrum of light passing through the periodic structure in question have not yet been investigated. As is widely recognized, the spectrum of light is generally influenced by the propagation process through a certain kind of medium or even in free space. Accordingly, the subject about the spectral modification due to the light propagation may be of considerable interest and worth investigating as far as a practical application of optics is taken into account where even the quasi-monochromatic light may not be necessarily available. Moreover, the particular interest in this problem stems also from the recent works on the correlation-induced spectral changes which are often called the Wolf effect [62].

This chapter is concerned with the spectral aspect of light passing through a periodic structure comprised of both lenses and spatial filters with Gaussian transmittances, as a continuation of Chapters 8 and 9 in which emphasis has been placed on the spatial coherence properties of the propagating light in the structure. Specifically, trends in the spectrum of light passing through the periodic structure are theoretically investigated by making use of the modal decomposition of the partially coherent field within the framework of coherence theory in the space-frequency domain. The mode employed in the subsequent analysis, which is derived by means of the conventional method for determining the laser resonator mode, can be regarded as the coherent mode originally established by Wolf [120, 121] after the theory of Wolf and Agarwal [118]. The initial light launched into the structure is assumed to be a polychromatic, spatially incoherent light having a single spectral line with the Lorentzian profile. It has been shown in Chapters 8 and 9 that, in spite of the initially incoherent light, the propagating light in the structure becomes spatially partially coherent due to propagation and successive diffraction, its property of spatial coherence being varied on propagation. This fact enables us to examine the phenomena concerning the spectral modifications in the propagating light under the consideration of the Wolf effect. Therefore, the analysis to be conducted here proceeds mainly from the viewpoint of the Wolf effect.

In the following, preliminary considerations for the present problem are first dealt with and, subsequently, some mathematical formulas are derived for the spectrum of light passing through the periodic structure. On the basis of these formulas, two types of the reduced arrangements for analysis, which correspond basically to

the Fourier transform and the imaging geometries, are particularly taken into account in place of the general configuration for the periodic structure. As illustrative numerical examples, the shift of the resulting spectrum is given as a function of the number of transits through the structure for various observation points. Likewise, the changes in the spectrum of each mode are also examined numerically. These examples may serve to elucidate the phenomena pertaining to the Wolf effect from the viewpoint of the coherent-mode representation of the field. The present study is also motivated by the recent investigation of Gamliel [86] regarding the modal analysis of the Wolf effect.

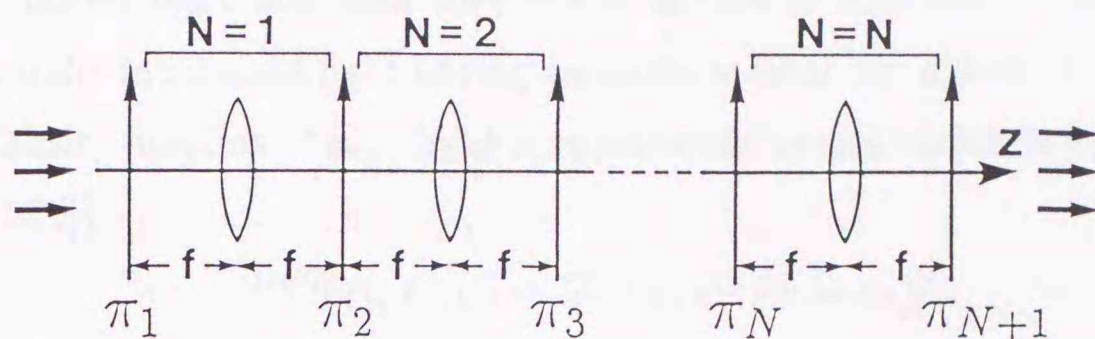
10.2 Some Preliminaries

The ultimate purpose of the present analysis is to examine the modifications in the spectrum of light passing through a periodic structure. As a preliminary consideration, the underlying circumstances required for the subsequent analysis are summarized.

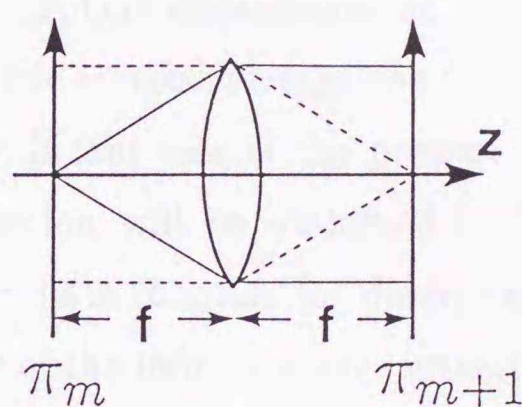
We first specify an optical arrangement for analysis characterizing the periodic structure. Consider the periodic sequence of lenses separated by spatial filters illustrated in Fig.10.1(a). The ideal cylindrical lenses of focal length f are perfectly aligned with separation distance $2f$ and the spatial filters are located at each focal plane. The amplitude transmittances of these filters take one-dimensional Gaussian forms given by

$$t(x) = \begin{cases} \exp\left(-\frac{x^2}{2\sigma^2}\right), & \text{for } \pi_1 \text{ and } \pi_{N+1} \text{ planes} \\ \exp\left(-\frac{x^2}{\sigma^2}\right), & \text{for } \pi_2, \pi_3, \dots, \pi_N \text{ planes} \end{cases} \quad (10.1)$$

where σ is a positive constant indicating the effective transverse width of the periodic structure. For the determination of the modes to be conducted in the next section, it is convenient to consider that the periodic structure depicted in Fig.10.1(a) consists of N successive arrangements of the basic unit depicted in Fig.10.1(b); the basic unit itself consists of one cylindrical lens and two spatial filters with the same transmittances of $t(x) = \exp(-x^2/2\sigma^2)$. In arranging the whole sequence, the second



(a)



(b)

Figure 10.1. (a) Periodic sequence of cylindrical lenses separated by spatial filters with one-dimensional Gaussian amplitude transmittances and (b) the basic unit for analysis. The spatial filters are located at each focal plane.

filter (which is placed at the π_{m+1} plane) of the m -th basic unit is attached to the first filter (which is placed at the π_{m+1} plane as well) of the $(m+1)$ -th basic unit so that the total transmittance across the π_{m+1} plane becomes $t(x) = \exp(-x^2/\sigma^2)$ for $m = 1, 2, \dots, (N-1)$, whereas the transmittance across the π_1 and π_{N+1} planes is given by $t(x) = \exp(-x^2/2\sigma^2)$. In this structure, the initial light is incident on the π_1 plane from the left and, as a consequence, it emerges from the π_{N+1} plane after passing through the N sequence of the basic unit. This type of the optical arrangement has frequently been employed in previous studies dealing with the modes of a laser resonator [113] and the spatial coherence properties of light passing through a periodic structure [116]. In the present treatment, the model structure depicted in Fig.10.1(a) is regarded as a typical configuration having certain periodicity [122, 123, 125], together with an example of a laser resonator.

The initial light launched into the structure is assumed to be a polychromatic, spatially incoherent light having a uniform intensity distribution. The cross-spectral density function of such light is approximately expressible in the form ([147], Sec. 4.4; [210])

$$W^{(0)}(x_1, x_2; \omega) = S^{(0)}(\omega) \delta[k(x_1 - x_2)], \quad (10.2)$$

where $S^{(0)}(\omega)$ denotes the spectrum of the light (to be specified below) and $\delta(\cdot)$ stands for the one-dimensional Dirac delta function. Here, x_1 and x_2 imply the spatial coordinates across the plane just behind the π_1 plane and $k = \omega/c$ is the wavenumber of light. The explicit dependence of the delta function on k , which originates from a more realistic correlation function for characterizing the incoherent light [147, 210], plays a significant role in the present subject associated with the Wolf effect and this connection will be discussed in Section 10.4. As is widely known, the use of the Dirac delta function for describing the incoherent light leads to a physical inconsequence of the infinite source intensity. Yet, from Eq.(10.2), the intensity distribution across the source is assumed to be given by

$$S^{(0)}(x; \omega) = W^{(0)}(x, x; \omega) = S^{(0)}(\omega), \quad (10.3)$$

in accordance with the conventional implication for the delta-correlated light. To simplify the theoretical development, the source spectrum $S^{(0)}(\omega)$ encountered in the above equations is assumed to be the same at all points in the source region. In other words, the source spectrum is independent of the position across the source. This assumption has already been incorporated in Eqs.(10.2) and (10.3) and, thus, $S^{(0)}(\omega)$ is represented as a function of only the frequency ω . In addition, the source spectrum is assumed to consist of a single spectral line with the Lorentzian profile given by

$$S^{(0)}(\omega) = S_0 \frac{\Gamma_0^2}{(\omega - \omega_0)^2 + \Gamma_0^2}, \quad (\Gamma_0/\omega_0 \ll 1) \quad (10.4)$$

where ω_0 denotes the central (angular) frequency, Γ_0 takes a positive constant characterizing the linewidth of the spectrum, and S_0 is a positive constant. This assumption may serve to provide a clear understanding for the numerical confirmation of the present analysis to be conducted in Section 10.5. By substituting Eq.(10.4) into Eq.(10.2), the spatial coherence property of the initial light, including the spectral

structure, is uniquely determined by means of the cross-spectral density function in Eq.(10.2).

Finally, a few remarks are made on the Wolf effect since the analysis of the present problem proceeds from the standpoint of the Wolf effect. It has been shown theoretically and experimentally that the spectrum of light radiated from a partially coherent source is generally varied on propagation, depending not only on the source spectrum but also on the spatial coherence condition over the source [62]. This phenomenon was predicted originally by Wolf [63, 65] and, hence, it is often called the Wolf effect. Due to the intensive investigations on this phenomenon, the effect that occurs on the propagation of light in free space and in a weakly scattering medium seems to have been thoroughly elucidated hitherto. In recent years, considerable attention has been devoted to the related effect arising from the propagation of light through a certain kind of medium, such as an optical fiber [90], a strong scattering medium [103], and a dielectric slab [93]. As a further extension of a series of such investigations, the main problem of this chapter may be the first attempt to consider the spectral aspect of light in the periodic structure on the basis of the Wolf effect.

10.3 Mathematical Formulation of the Problem

We consider that the incoherent light specified in the previous section is incident on the π_1 plane of the periodic structure illustrated in Fig.10.1(a). In this section, some mathematical expressions for the spectrum of light after passing through the periodic structure are derived mainly in accordance with the previous study [116] and Chapter 9.

To obtain the specific expression for the spectrum of light across the π_{N+1} plane for an arbitrary N , we consider the propagation of a cross-spectral density function through the structure. To do this, the basic unit depicted in Fig.10.1(b), rather than the whole sequence depicted in Fig.10.1(a), is first taken into account. By making use of the propagation law of the cross-spectral density function, the function $W^{(m+1)}(\xi_1, \xi_2; \omega)$ of light across the π_{m+1} plane at frequency ω can be

written, in terms of $W^{(m)}(\eta_1, \eta_2; \omega)$ across the π_m plane, in the form:

$$W^{(m+1)}(\xi_1, \xi_2; \omega) = A \iint_{-\infty}^{\infty} W^{(m)}(\eta_1, \eta_2; \omega) K(\xi_1, \eta_1; \omega) K^*(\xi_2, \eta_2; \omega) d\eta_1 d\eta_2 \quad (10.5)$$

where

$$K(\xi, \eta; \omega) = \exp[-a(\xi^2 + \eta^2)] \exp(-jb\xi\eta) \quad (10.6)$$

and

$$A = \frac{\omega}{2\pi cf}, \quad a = \frac{1}{2\sigma^2}, \quad b = \frac{\omega}{cf} \quad (10.7)$$

with c being the velocity of light in vacuum. Hence, the eigenvalue equation for determining the modes of the periodic structure becomes [201]

$$\beta \Phi(\xi; \omega) = \sqrt{A} \int_{-\infty}^{\infty} \Phi(\eta; \omega) K(\xi, \eta; \omega) d\eta, \quad (10.8)$$

where β is a complex value with the magnitude less than unity and the parameter A and the function $K(\xi, \eta; \omega)$ are given in Eq.(10.7). It is known that the eigenvalue equation (10.8) has the solutions

$$\Phi_n(\xi; \omega) = \left(\frac{2d}{\pi}\right)^{\frac{1}{4}} (2^n \cdot n!)^{-\frac{1}{2}} H_n(\xi\sqrt{2d}) \exp(-d\xi^2) \quad (10.9)$$

with the eigenvalues

$$\beta_n = \beta_0 \cdot q^n \quad (10.10)$$

where

$$d = \frac{\sqrt{4a^2 + b^2}}{2}, \quad \beta_0 = \sqrt{\frac{A\pi}{a+d}}, \quad q = -j \frac{b}{2(a+d)} \quad (10.11)$$

and n takes non-negative integer values. Equation (10.9) indicates that the modes of the structure in question are represented by means of Hermite-Gaussian functions and, hence, are termed Hermite-Gaussian modes. These functions are known to fulfill the orthonormal relation:

$$\int_{-\infty}^{+\infty} \Phi_n(\xi; \omega) \Phi_m^*(\xi; \omega) d\xi = \begin{cases} 1, & \text{for } n = m \\ 0, & \text{for } n \neq m \end{cases} \quad (10.12)$$

and form a complete basis. Such properties of Hermite-Gaussian functions serve to develop the subsequent analysis concerning the modal expansion of the initial light.

From a mathematical point of view, the modes given by Eq.(10.9) are derived by making use of the conventional method for determining the laser resonator modes, namely, on the basis of the integral equation given by Eq.(10.8). On first inspection, thus, the modes introduced here seem to be distinct from the recent concept of coherent-modes established by Wolf [120, 121]. This is not true, however. It has been shown in the theory of Wolf and Agarwal [118] that both modes are strictly equivalent. Therefore, the modes employed here are considered as coherent modes and will be used throughout the analysis. The particular significance of the coherent-mode representation has become evident in numerous investigations (see, for example, Refs. [9, 10, 42, 43, 86, 209–218]).

Here, we recall that the cross-spectral density function of the initial light launched into the structure is given by Eq.(10.2). By making use of both the property of the delta function ([187], p. 76) and the equality $k = \omega/c$, Eq.(10.2) may also be written in the form

$$W^{(0)}(x_1, x_2; \omega) = S^{(0)}(\omega) \left(\frac{c}{\omega} \right) \delta(x_1 - x_2). \quad (10.13)$$

As a result, it becomes

$$W^{(0)}(x_1, x_2; \omega) = S^{(0)}(\omega) \left(\frac{c}{\omega} \right) \sum_{n=0}^{\infty} \Phi_n(x_1; \omega) \Phi_n^*(x_2; \omega), \quad (10.14)$$

due to the complete orthonormality of the Hermite-Gaussian functions where $\Phi_n(x; \omega)$ is given by Eq.(10.9). Equation (10.14) implies physically that the spatially incoherent light with the uniform intensity distribution is produced by the infinite superposition of equally-weighted Hermite-Gaussian modes.

By substituting Eq.(10.14) into the right hand side of Eq.(10.5) and by using Eq.(10.8), the light emerging from the π_2 plane becomes

$$W^{(1)}(x_1, x_2; \omega) = S^{(0)}(\omega) \left(\frac{c}{\omega} \right) \sum_{n=0}^{\infty} |\beta_n|^2 \Phi_n(x_1; \omega) \Phi_n^*(x_2; \omega) \quad (10.15)$$

where β_n is given by Eq.(10.10). By repetition of the same procedure, the cross-spectral density function of light across the π_{N+1} plane after passing through the N

basic units takes the form

$$W^{(N)}(x_1, x_2; \omega) = S^{(0)}(\omega) \left(\frac{c}{\omega} \right) \sum_{n=0}^{\infty} |\beta_n|^{2N} \Phi_n(x_1; \omega) \Phi_n^*(x_2; \omega). \quad (10.16)$$

In the above equations and in the following, spatial coordinates across the exit plane are specified by x_1 , x_2 and x . From Eq.(10.16), the spectrum of light across the π_{N+1} plane is given by

$$S^{(N)}(x; \omega) = \sum_{n=0}^{\infty} S_n^{(N)}(x; \omega), \quad (10.17)$$

where

$$S_n^{(N)}(x; \omega) = S^{(0)}(\omega) M_n^{(N)}(x; \omega) \quad (10.18)$$

and

$$M_n^{(N)}(x; \omega) = \left(\frac{c}{\omega} \right) |\beta_n|^{2N} |\Phi_n(x; \omega)|^2. \quad (10.19)$$

The function $M_n^{(N)}(x; \omega)$ in Eqs.(10.18) and (10.19) characterizes the contribution of each mode to the spectral change by the process of propagation and, accordingly, we refer it as an “elemental modifier” of the spectrum. It is known from a previous study [116] that the values of $|\beta_n|$ are reduced from the value unity with an increase of the mode number n and, eventually, become negligible. As a result, it is seen in Eqs.(10.17), (10.18) and (10.19) that higher-order modes which are initially equally weighted are diminished and that, finally, only the lowest-order mode survives in the structure with an increase of N . Obviously, such behavior of the modes in the structure results in the emerging light having the spatially high coherence, as was pointed out previously [114–116]. In other words, the spatial coherence properties of light are determined by means of the relative strength of each mode, i.e., the distribution of the eigenvalues $|\beta_n|^{2N}$.

Equation (10.17), together with Eqs.(10.18) and (10.19), is one of the required expressions for the spectrum of light across the π_{N+1} plane. On the other hand, the resulting spectrum can also be reduced to a simple closed form in place of the infinite summation of the modes given by Eq.(10.17). This will be shown next.

For mathematical simplicity in the subsequent analysis, we introduce the new

parameter F_ω defined by

$$F_\omega = \frac{\omega\sigma^2}{cf} = \frac{b}{2a}, \quad (10.20)$$

where a and b are given in Eq.(10.7). The value of F_ω is proportional to the Fresnel number encountered in the theory of a laser resonator, provided that f and σ are regarded as the longitudinal length and the transverse width of a resonator, respectively. By making use of F_ω , all the parameters given in Eq.(10.11) can be rewritten in the form

$$d = a\sqrt{1 + F_\omega^2}, \quad \beta_0 = \left(\frac{F_\omega}{1 + \sqrt{1 + F_\omega^2}} \right)^{1/2}, \quad q = -j \frac{F_\omega}{1 + \sqrt{1 + F_\omega^2}} \quad (10.21)$$

and, moreover, $|\beta_n|^{2N}$ contained in Eqs.(10.16) and (10.19) becomes

$$|\beta_n|^{2N} = q_N^{n+1/2} \quad (10.22)$$

with

$$q_N = \left(\frac{F_\omega}{1 + \sqrt{1 + F_\omega^2}} \right)^{2N}. \quad (10.23)$$

By substituting Eqs.(10.9) and (10.22) into Eq.(10.16) and by using the generating function for Hermite polynomials $H_n(x)$ given by ([221], Eq.(22), p. 194)

$$\sum_{n=0}^{\infty} \frac{z^n}{2^n \cdot n!} H_n(x) H_n(y) = \frac{1}{\sqrt{1 - z^2}} \exp \left[\frac{2xyz - (x^2 + y^2)z^2}{1 - z^2} \right], \quad (10.24)$$

we obtain, in place of the modal representation in Eq.(10.16), the expression

$$W^{(N)}(x_1, x_2; \omega) = A_N \exp \left[-\frac{d}{1 - q_N^2} \left\{ (1 + q_N^2)(x_1^2 + x_2^2) - 4q_N x_1 x_2 \right\} \right], \quad (10.25)$$

where

$$A_N = S^{(0)}(\omega) \left(\frac{c}{\omega} \right) \sqrt{\frac{2dq_N}{\pi(1 - q_N^2)}}. \quad (10.26)$$

By using Eqs.(10.25) and (10.26), the spectrum of light across the π_{N+1} plane can be written in the simple form

$$S^{(N)}(x; \omega) = S^{(0)}(\omega) M^{(N)}(x; \omega) \quad (10.27)$$

with

$$M^{(N)}(x; \omega) = \left(\frac{c}{\omega}\right) \sqrt{\frac{2dq_N}{\pi(1-q_N^2)}} \exp\left[-2d\frac{1-q_N}{1+q_N}x^2\right], \quad (10.28)$$

where we refer to $M^{(N)}(x; \omega)$, corresponding to Eq.(10.19), as a “total modifier” of the spectrum. It is clearly seen in Eqs.(10.27) and (10.28) that the spatial distribution of the spectrum takes a Gaussian form whose rms width is given by

$$\sigma_I^{(N)}(\omega) = \frac{1}{2} \sqrt{\frac{1}{d} \frac{1+q_N}{1-q_N}}. \quad (10.29)$$

It should be noted here that Eq.(10.28) is related to Eq.(10.19) through the formula

$$M^{(N)}(x; \omega) = \sum_{n=0}^{\infty} M_n^{(N)}(x; \omega) \quad (10.30)$$

because Eq.(10.17) is essentially equivalent to Eq.(10.27) aside from the difference of parameters encountered in each expression. Equation (10.27), together with Eq.(10.28), is the main expression required in the subsequent analysis dealing with the spectral modification of light passing through the periodic structure, as well as Eqs.(10.17), (10.18) and (10.19).

We next consider the complex degree of spectral coherence of light across the π_{N+1} plane given by

$$\mu^{(N)}(x_1, x_2; \omega) = \frac{W^{(N)}(x_1, x_2; \omega)}{\sqrt{S^{(N)}(x_1; \omega)} \sqrt{S^{(N)}(x_2; \omega)}}. \quad (10.31)$$

By substituting Eqs.(10.25) and (10.27) into Eq.(10.31) and by using Eqs.(10.26) and (10.28), Eq.(10.31) becomes

$$\mu^{(N)}(x_1, x_2; \omega) = \exp\left[-2d\frac{q_N}{1-q_N^2}(x_1 - x_2)^2\right], \quad (10.32)$$

where $\mu^{(N)}(x_1, x_2; \omega)$ takes a Gaussian form, as in the case of the spatial distribution of the spectrum given in Eqs.(10.27) and (10.28), and depends on the spatial coordinates x_1 and x_2 only through their mutual difference $|x_1 - x_2|$. The rms width

of the spatial coherence is given by

$$\sigma_{\mu}^{(N)}(\omega) = \frac{1}{2} \sqrt{\frac{1}{d} \frac{1 - q_N^2}{q_N}}. \quad (10.33)$$

As a measure for characterizing the spatial coherence condition of light across the π_{N+1} plane, we introduce the global coherence defined by

$$\sigma_R^{(N)}(\omega) = \frac{\sigma_{\mu}^{(N)}(\omega)}{\sigma_I^{(N)}(\omega)}. \quad (10.34)$$

Substitution of Eqs.(10.29) and (10.33) into Eq.(10.34) yields

$$\sigma_R^{(N)}(\omega) = \frac{1 - q_N}{\sqrt{q_N}} \quad (10.35)$$

where q_N is given by Eq.(10.23). The global coherence given by Eq.(10.35) plays an important role in examining a physical origin of the spectral changes on the light propagation through the periodic structure in connection with the Wolf effect.

Finally, it is worth noting that the light across the π_{N+1} plane, with N being an arbitrary value, is exactly the same as a Gaussian Schell-model (GSM) source [183] due to the Gaussian forms of both the intensity distribution and the spatial coherence. Accordingly, the modal decomposition of the field across each plane characterized by Eq.(10.16), together with Eqs.(10.17), (10.18) and (10.19), is also the coherent-mode representation of the GSM source [42].

10.4 Special Cases

We now consider two special cases where the periodic structure consists of either one basic unit (i.e., $N = 1$) or two basic units (i.e., $N = 2$).

We first deal with the simplest arrangement corresponding to the case of $N = 1$, namely, the basic unit itself illustrated in Fig.10.1(b). In this case, the initial light is launched on the π_1 plane and emerges from the π_2 plane after passing through one cylindrical lens. This is basically an one-dimensional Fourier transforming configuration. The spectrum of the emerging light can be readily obtained

by referring to Eqs.(10.27) and (10.28) in the previous section and is expressed by

$$S^{(1)}(x; \omega) = S^{(0)}(\omega) M^{(1)}(x; \omega), \quad (10.36)$$

where the total modifier given by Eq.(10.28) reduces to the simple form

$$M^{(1)}(x; \omega) = \frac{1}{2f\sqrt{2\pi a}} \exp(-2ax^2) \quad (10.37)$$

using Eqs.(10.7), (10.20), (10.21) and (10.23). It follows from Eqs.(10.36) and (10.37) that the normalized spectrum across the exit π_2 plane is the same as the normalized initial spectrum across the π_1 plane since the total modifier of Eq.(10.37) is independent of the frequency ω , where the normalization of the spectrum is made by the formula:

$$s(\omega) = \frac{S(\omega)}{\int_0^\infty S(\omega) d\omega}. \quad (10.38)$$

In other words, the normalized spectrum remains unchanged by the propagation of light through one basic unit. To elucidate further the phenomenon, such a behavior in the spectrum is examined under the consideration of the pioneering work by Wolf [63] concerning the conditions for the invariance of the spectrum on propagation. According to Wolf [63], the normalized spectrum of light radiated by a planar quasi-homogeneous secondary source remains invariant on propagation provided that the degree of spatial coherence obeys a scaling law in the sense

$$\mu^{(0)}(x_1, x_2; \omega) = h[k(x_2 - x_1)], \quad (k = \omega/c) \quad (10.39)$$

with $h(\cdot)$ being an arbitrary function and that the source spectrum is independent of the position across the source. In the present analysis, the above conditions are all satisfied if the π_1 plane is regarded as a secondary source. For this reason, the phenomenon in question is satisfactorily explained in terms of the Wolf theory. We next consider the spatial coherence properties of the resultant field. By setting $N = 1$ in Eq.(10.32) and by using Eqs.(10.7), (10.20), (10.21) and (10.23), the degree of spatial coherence across the π_2 plane becomes

$$\mu^{(1)}(x_1, x_2; \omega) = \exp \left[-\frac{1}{8af^2} \left(\frac{\omega}{c} \right)^2 (x_1 - x_2)^2 \right]. \quad (10.40)$$

It is clearly seen from the above equation that the spatial coherence property of light across the π_2 plane obeys the scaling law in the sense of Eq.(10.39). This fact allows us to manifest the changes in the spectrum of light across the π_3 plane on the basis of the Wolf theory. It is to be noted here, however, that the light across the π_{N+1} plane does not generally satisfy, except for $N = 1$, the scaling law.

In view of the above discussion about some properties of light across the π_2 plane, we now consider the second case of the light propagation through the two basic units. This arrangement exhibits essentially an imaging geometry, provided that the amplitude filters located at each focal plane are removed. In a similar fashion, the spectrum of light across the π_3 plane is given by

$$S^{(2)}(x; \omega) = S^{(0)}(\omega) M^{(2)}(x; \omega) \quad (10.41)$$

with

$$M^{(2)}(x; \omega) = \frac{1}{2} \left(\frac{c}{\omega} \right) F_{\omega}^2 \sqrt{\frac{a}{\pi(2 + F_{\omega}^2)}} \exp \left(-4a \frac{1 + F_{\omega}^2}{2 + F_{\omega}^2} x^2 \right), \quad (10.42)$$

and the corresponding degree of spatial coherence is obtained by

$$\mu^{(2)}(x_1, x_2; \omega) = \exp \left[-\frac{a}{4} \frac{F_{\omega}^4}{2 + F_{\omega}^2} (x_1 - x_2)^2 \right]. \quad (10.43)$$

As opposed to the preceding case, the normalized spectrum of light across the π_3 plane is different from that across the π_2 plane since Eq.(10.42) is no longer independent of the frequency ω . In other words, the spectrum of light is changed on propagation through the second basic unit. From the Wolf theory, this variation of the spectrum appears to stem from the violation of quasi-homogeneity across the π_2 plane, by considering that the spatial coherence of light across the π_2 plane obeys the scaling law (see, Eq.(10.40)) and that the normalized spectrum of light across the π_2 plane in the sense of Eq.(10.38) is independent of the position (see, Eqs.(10.36) and (10.37)). In fact, when the global coherence $\sigma_R^{(2)}(\omega)$ across the π_2 plane defined by Eq.(10.35) is sufficiently small in comparison to unity, i.e.,

$$\sigma_R^{(2)}(\omega) \ll 1, \quad (10.44)$$

(roughly speaking, this corresponds to the limit of $F_\omega \rightarrow \infty$), the light across the π_2 plane can be approximated by a quasi-homogeneous source and, as a result, the total modifier given by Eq.(10.42) reduces to

$$M^{(2)}(x; \omega) = \frac{1}{4f\sqrt{\pi a}} \exp(-4ax^2). \quad (10.45)$$

Obviously, Eq.(10.45) is independent of the frequency ω and, hence, the normalized spectrum turns out to keep invariant on propagation in this limit.

It is interesting to note that the optical arrangements treated in this section are similar to those for realizing the secondary sources with controlled spatial coherence proposed for experimental verification of the Wolf effect [64].

10.5 Numerical Examples

As some numerical examples, the spectral modifications of light passing through the periodic structure are quantitatively investigated chiefly on the basis of the two formulas derived in Section 10.3: the modal description of the spectrum given by Eqs.(10.18) and (10.19), and the resultant total spectrum in terms of a simple closed form given by Eqs.(10.27) and (10.28). In general, the spectrum of light may be distorted and shifted by the process of propagation through a certain kind of medium, like the periodic structure in question. As a measure for evaluating the spectral modifications, however, we consider only the shift of the peak spectrum defined by

$$\Delta\omega = \frac{\omega' - \omega_0}{\omega_0}, \quad (10.46)$$

where ω_0 and ω' are the peak frequencies of the initial spectrum (given by Eq.(10.4)) and the resultant spectrum, respectively. Thus, the distortion of the spectrum is not taken into account for simplicity in the subsequent analysis. For convenience, the change in the spectrum is termed the blue shift when the peak spectrum of the resultant light is shifted toward higher frequencies (i.e., $\Delta\omega > 0$ in Eq.(10.46)) and, conversely, the opposite change is termed the red shift (i.e., $\Delta\omega < 0$ in Eq.(10.46)). In the numerical evaluation of Eq.(10.46), the full width at the half maximum (FWHM)

of the initial spectrum is assumed throughout the analysis to have a fixed value of $0.1\omega_0$ by setting $\Gamma_0/\omega_0 = 0.05$ in Eq.(10.4).

We first examine the behavior of the spectrum observed on the optical axis across the π_{N+1} plane. In this case, a functional form of the spectrum is obtained by substituting Eqs.(10.4) and (10.28) into Eq.(10.27) under the condition $x = 0$. On the basis of the expression for the spectrum and by using Eq.(10.46), the spectral shift of light across the π_{N+1} plane is numerically evaluated and illustrated in Fig.10.2 as a function of N for some values of F_0 , where the parameter F_0 , which

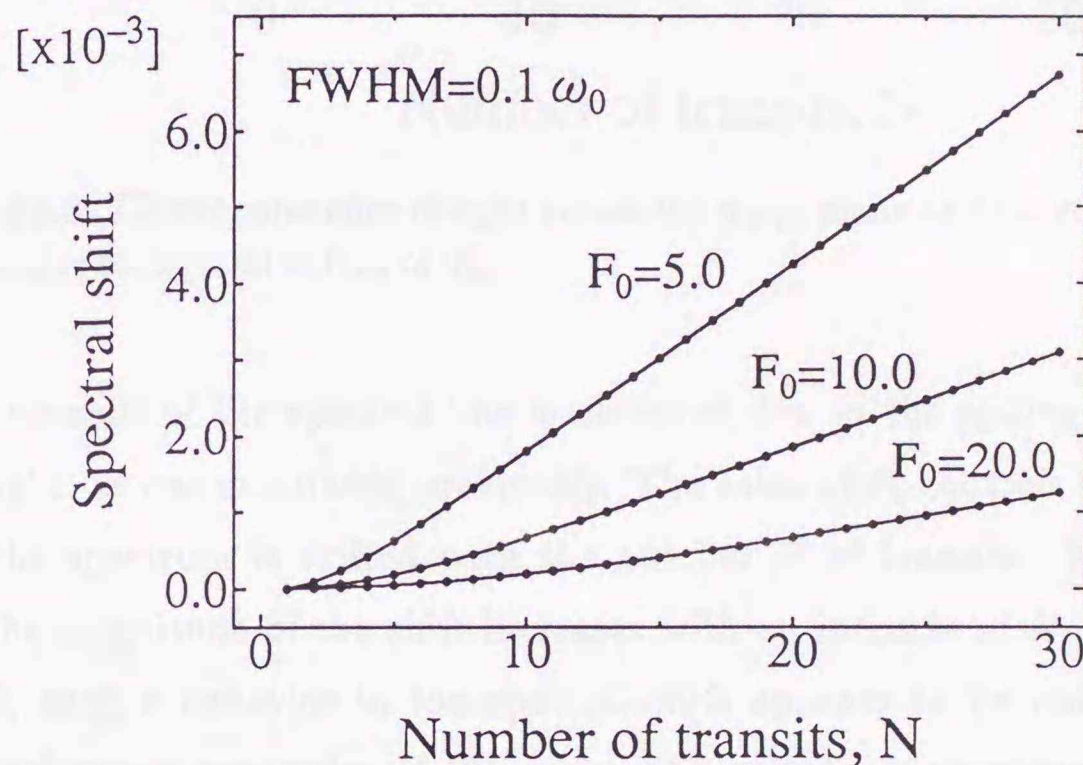


Figure 10.2. Spectral shift of light across the π_{N+1} plane, observed at the on-axis point, as a function of the number N of transits for several values of F_0 . The value of F_0 characterizes the effective width of the spatial filter located at each focal plane.

is obtained by setting $\omega = \omega_0$ in Eq.(10.20), characterizes the effective width of the Gaussian amplitude filter situated at each focal plane and is independent of the frequency ω . In a similar fashion, the global coherence of light across the π_{N+1} plane is plotted in Fig.10.3 against N using Eq.(10.35). In Figs.10.2 and 10.3, the value of N specifying the observation plane indicates the number of transits through the basic unit experienced by the propagating light because the light across the π_{N+1} plane is detected after passing through the N sequence of the basic unit. It is found in Fig.10.2 that the spectral line observed at the π_{N+1} plane is always, except for the case of $N = 1$, shifted toward the blue side. For $N = 1$ (i.e., across the π_2

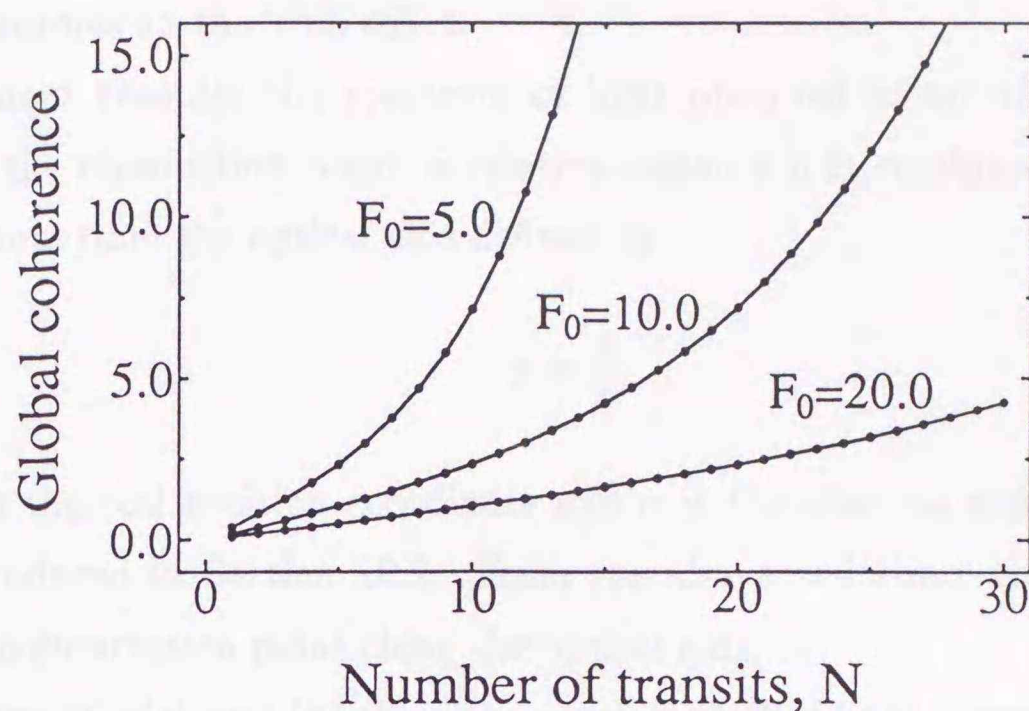


Figure 10.3. Global coherence of light across the π_{N+1} plane as a function of the number N of transits for several values of F_0 .

plane), no shift of the spectral line is observed due to the scaling-law form of the initial light, as was mentioned previously. The value of F_0 controls the rapidity with which the spectrum is shifted with the number N of transits. For a fixed value of F_0 , the magnitude of the shift increases with an increase of N . Compared with Fig.10.3, such a behavior in the spectral shift appears to be independent of the spatial coherence properties of the propagating light. A monotonically increasing manner of the spectral shift remains unchanged with an increase of N although the spatial coherence condition is varied with the value of N . Similar behaviors of the spectrum have been reported in past studies concerning the Wolf effect [68, 69, 80]. More specifically, it has been shown in these studies that the spectrum of light radiated by a partially coherent planar source, when observed at an on-axis point, shifts generally towards the blue side and that the magnitude of the shift is irrespective of the spatial coherence condition of the source. This kind of behavior in the spectrum has been proven, e.g., in the study of Dačić and Wolf [68], to be originated from the frequency-dependent factor ($1/\lambda = \omega/2\pi c$) in front of the standard diffraction integral formulated on the basis of the Huygens-Fresnel principle. This implication is still true in the present phenomena concerning the on-axis spectrum of light passing through the periodic structure because the present

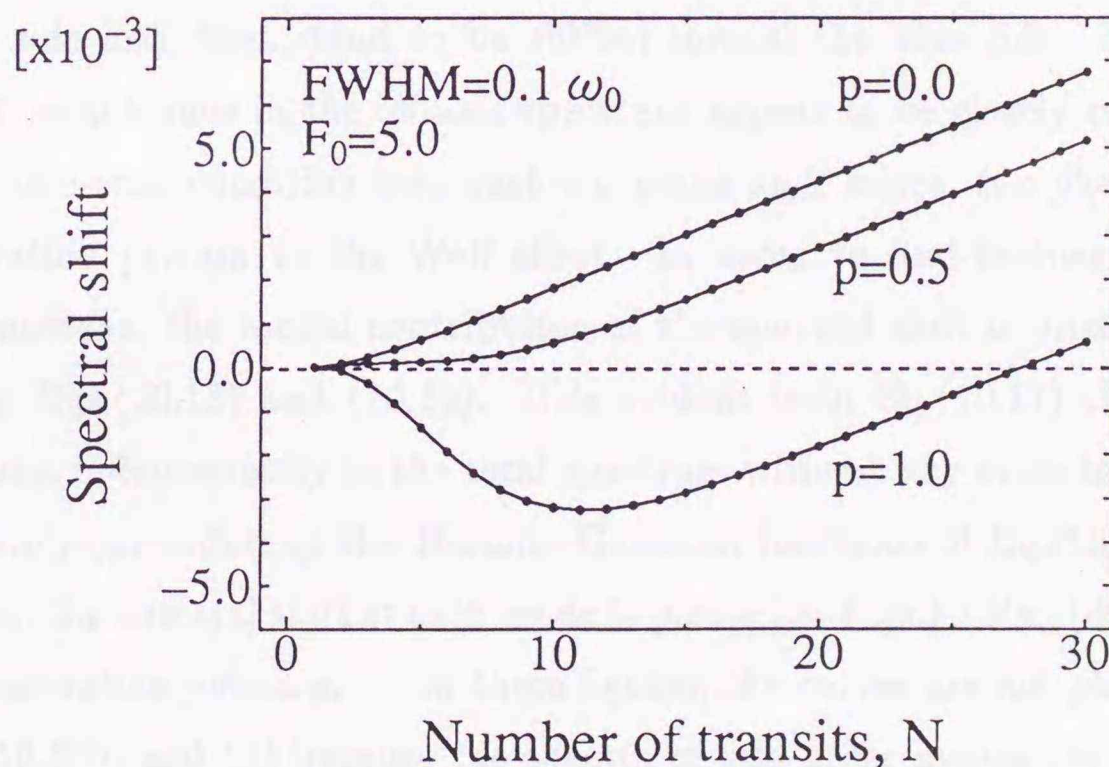
study is basically established by means of the above diffraction formula, as well as in all the studies on the Wolf effect.

We next consider the spectrum of light observed at an off-axis point. To designate the observation point, a relative measure p is employed to characterize the departure from the optical axis defined by

$$p = \frac{x}{\sigma}, \quad (10.47)$$

where x is the real position coordinate and σ is the effective width of the spatial filters introduced in Section 10.2. From the above definition, the case of $p = 0$ implies the observation point along the optical axis.

Figures 10.4(a) and (b) show the spectral shift of light across the π_{N+1} plane as a function of N for some observation points p in the cases of $F_0 = 5.0$ and 10.0, respectively. It is seen from these figures that the spectra observed at the off-axis



(a)

Figure 10.4. Spectral shift of light across the π_{N+1} plane, observed at the on-axis and the off-axis point, as a function of the number N of transits for several observation points p under the conditions of $F_0 = 5.0$ (a) and 10.0 (b). The value of p indicates the distance between the observation point and the optical axis and, hence, the case of $p = 0$ corresponds to the on-axis observation point.

points, as opposed to the case for the on-axis observation, are first shifted toward

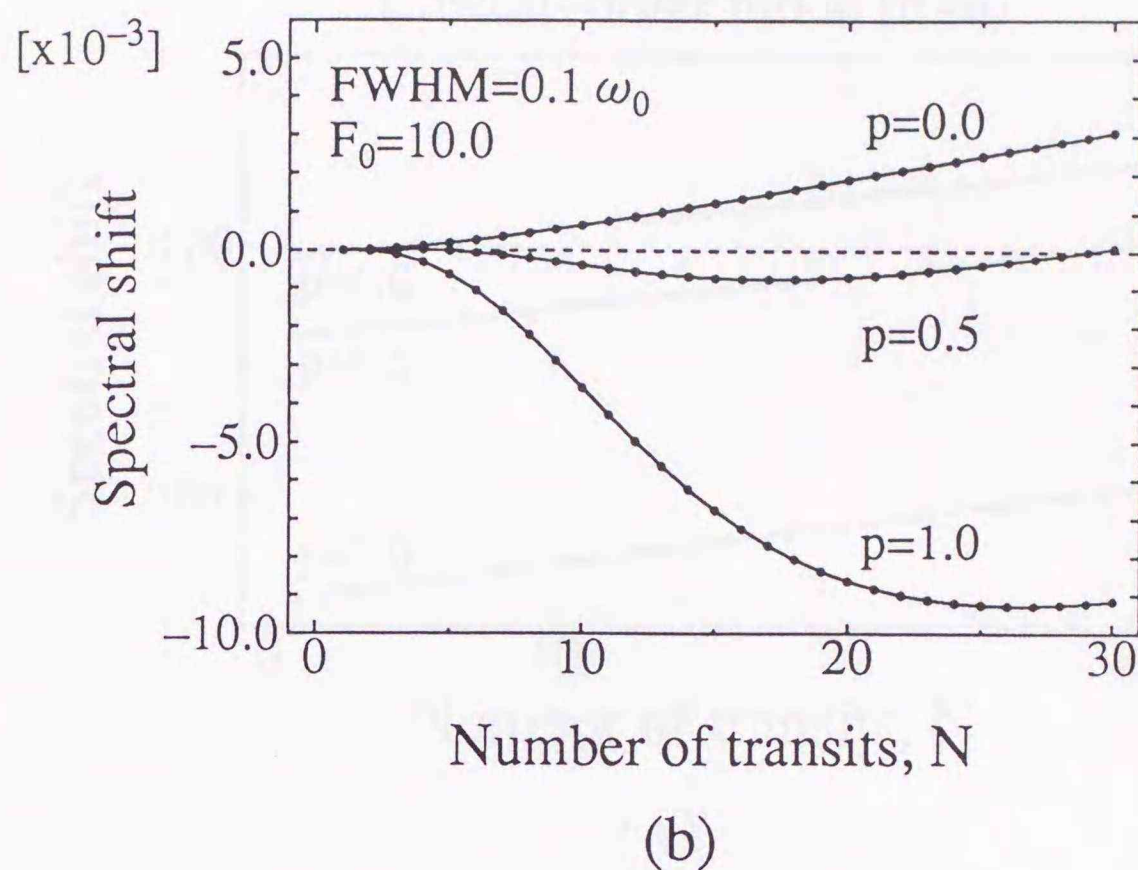


Figure 10.4. Cont.

the red side and, then, tend to be shifted toward the blue side. By referring to Fig.10.3, such trends in the off-axis spectrum appear to be closely related with the spatial coherence condition over each π_N plane and, hence, the phenomena under consideration pertain to the Wolf effect. In order to find further properties for the phenomena, the modal contribution of the spectral shift is briefly investigated by using Eqs.(10.18) and (10.19). It is evident from Eq.(10.17) that each mode contributes independently to the total spectrum without any cross-terms. Here, the modes are represented by the Hermite-Gaussian functions of Eq.(10.9). As typical examples, the spectral shift of each mode is plotted in Figs.10.5(a)-(d) against N for some observation points p . In these figures, the curves are not plotted for $p = 0$ in Figs.10.5(b) and (d) because the spectra of odd-order modes are always zero at the observation point $p = 0$ due to the property of Hermite polynomials. It is clearly seen in Figs.10.5(a)-(d) that all the curves change with the number N of transits in a similar fashion: the magnitude of the shift increases with an increase of N . However, the absolute value of the magnitude depends strongly on the order n , the observation point p , and the number N of transits. It is to be noticed here that the spectral shift occurring for a single mode results simply from the effect of diffraction because each mode is spatially completely coherent, but mutually uncorrelated [86]. By recalling

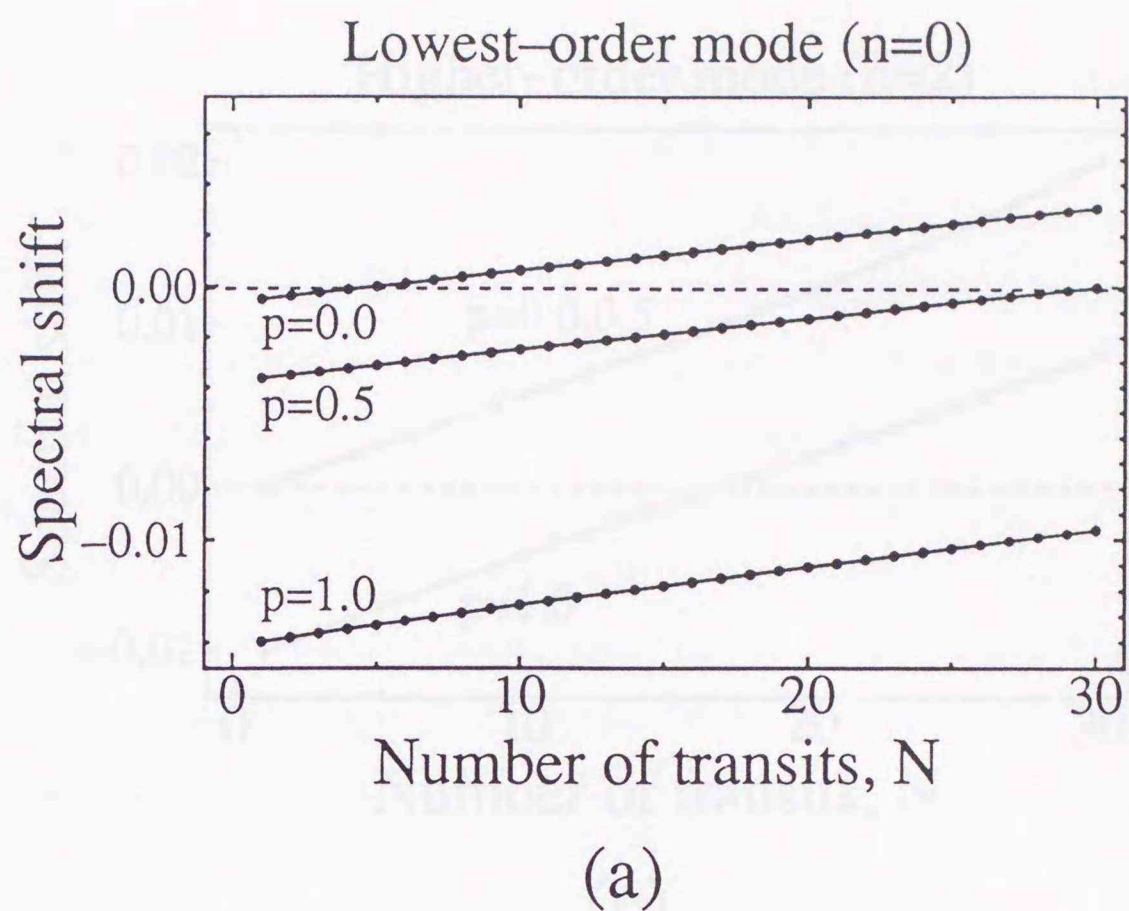


Figure 10.5. Spectral shift of each mode across the π_{N+1} plane as a function of the number N of transits for several observation points p in the cases of the mode number, $n = 0$ (a), 1 (b), 2 (c), and 3 (d).

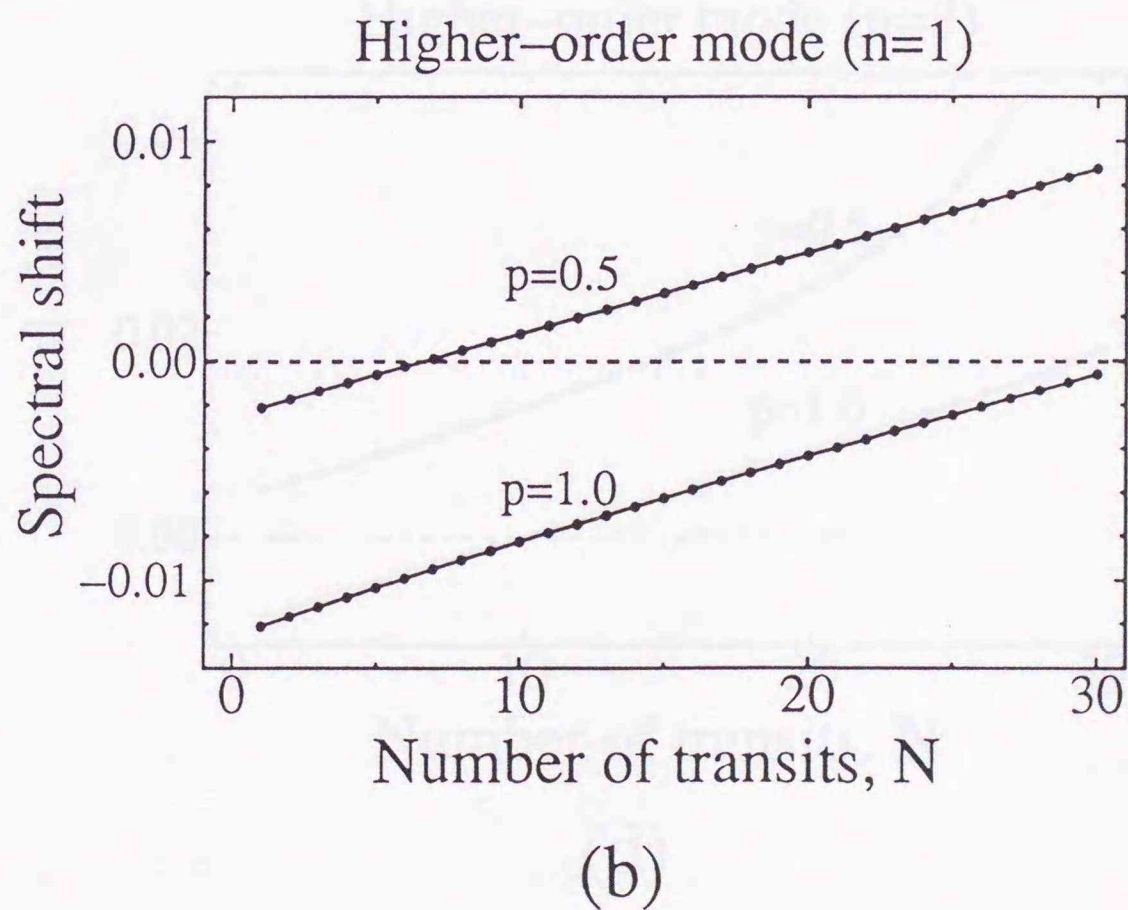
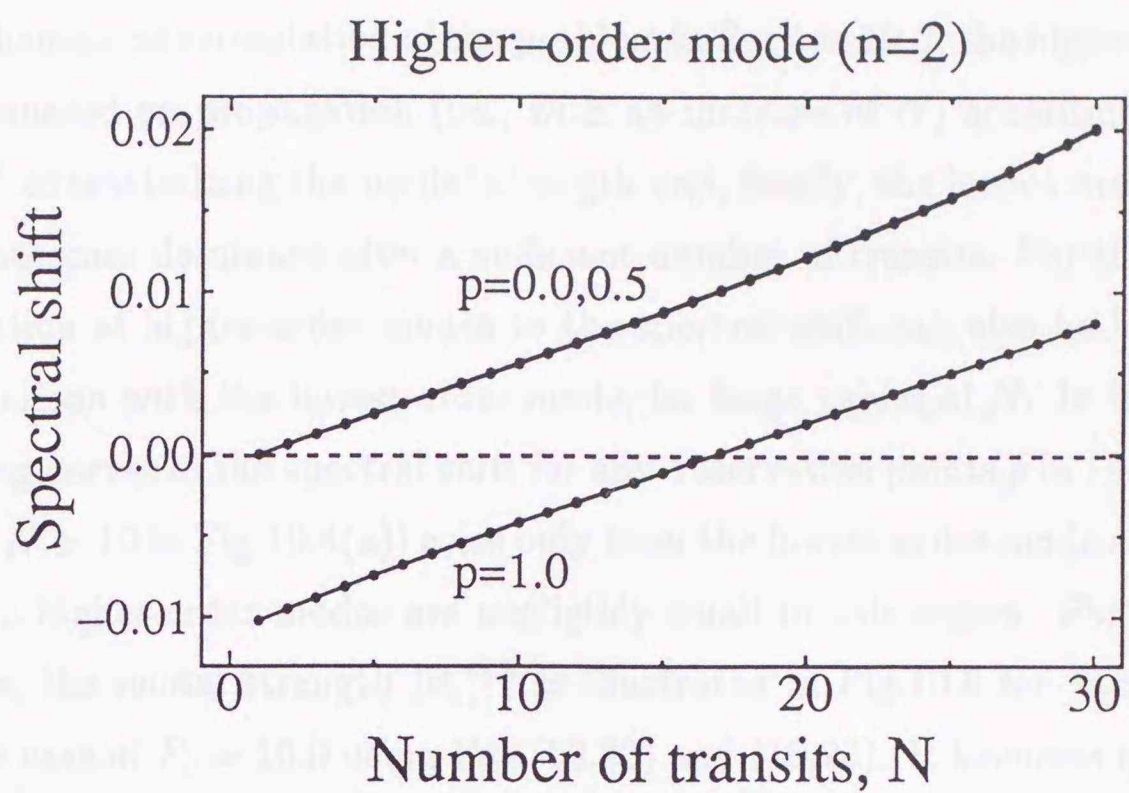
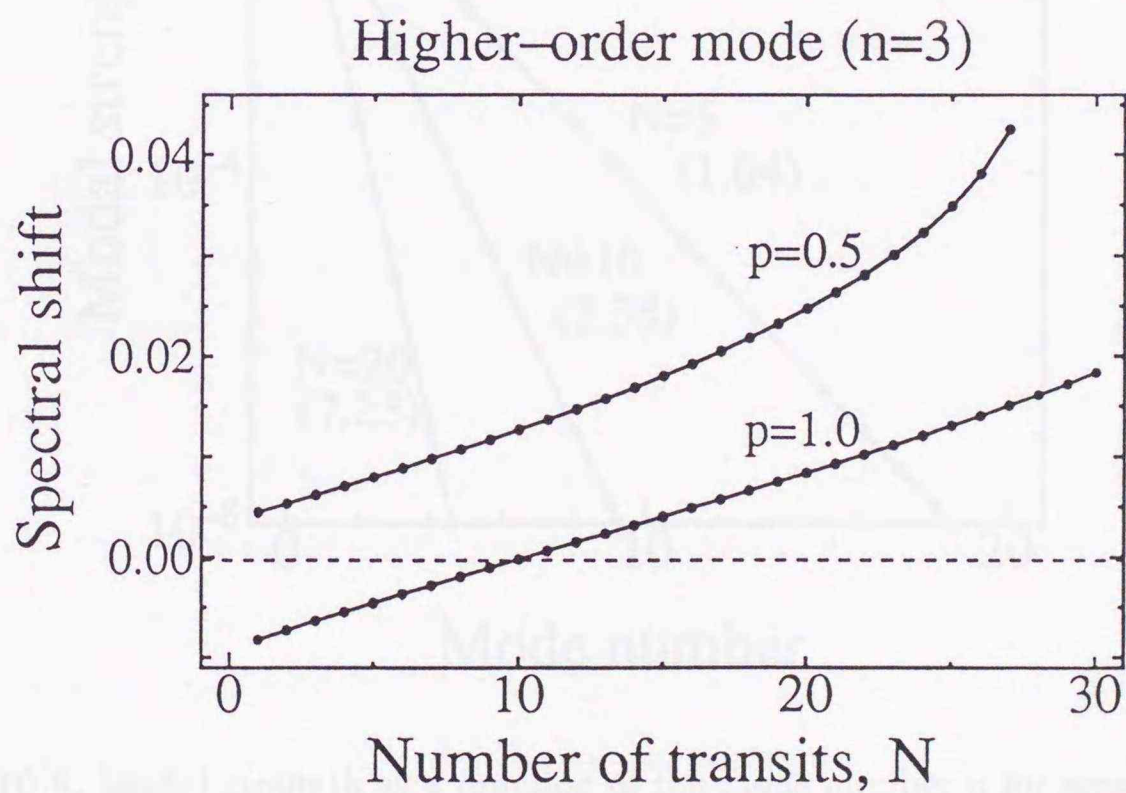


Figure 10.5. Cont.



(c)

Figure 10.5. Cont.



(d)

Figure 10.5. Cont.

the mathematical formulation of the problem in Section 10.3, the higher-order modes are attenuated on propagation (i.e., with an increase of N) according to the value of $|\beta_n|^{2N}$ characterizing the modal strength and, finally, the lowest-order mode (i.e., $n = 0$) becomes dominant after a sufficient number of transits. For this reason, the contribution of higher-order modes to the spectral shift can also be less dominant, in comparison with the lowest-order mode, for large values of N . In fact, gradually increasing curves of the spectral shift for any observation points p in the case of large N (e.g., $N > 10$ in Fig.10.4(a)) arise only from the lowest-order mode and, therefore, all of the higher-order modes are negligibly small in this region. For the reference purposes, the modal strength $|\beta_n|^{2N}$ is illustrated in Fig.10.6 for various values of N in the case of $F_0 = 10.0$ using Eqs.(10.22) and (10.23). It becomes quantitatively

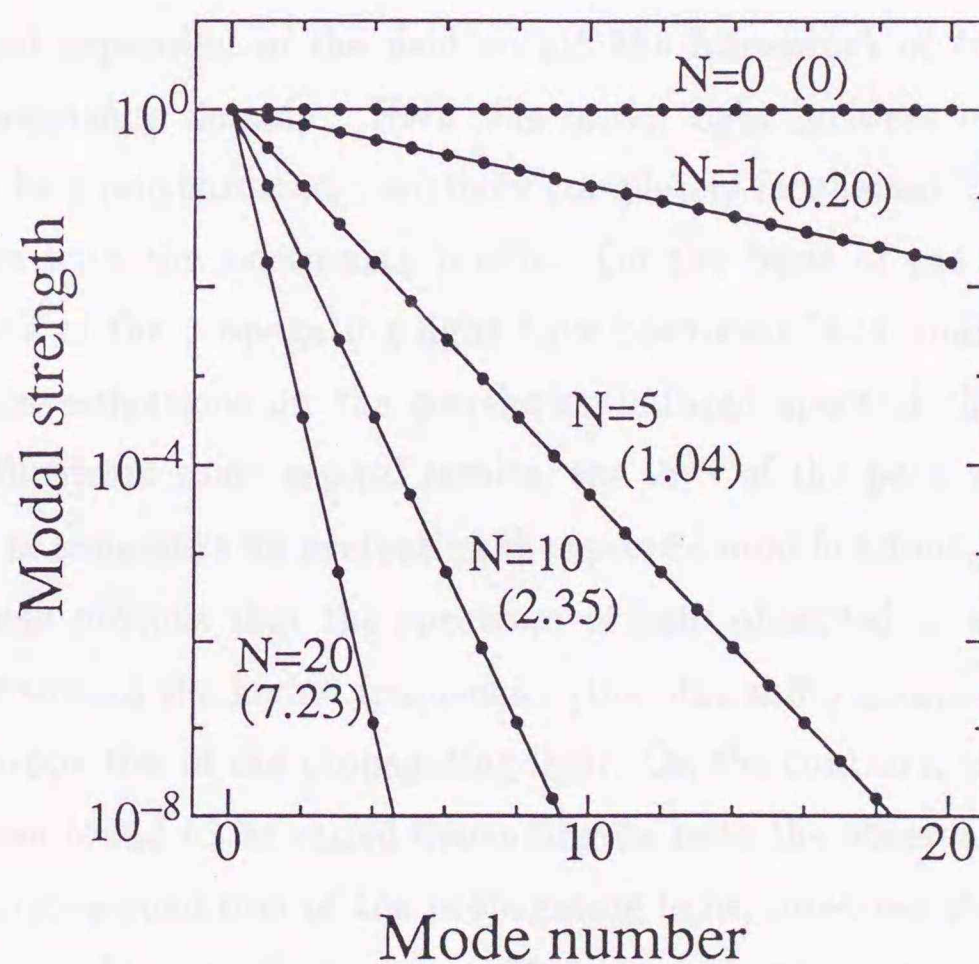


Figure 10.6. Modal strength as a function of the mode number n for some values of the number N of transits in the case of $F_0 = 10.0$. Each value of the modal strength, which is calculated from $|\beta_n|^{2N}$ given in Eqs.(10.22) and (10.23), is normalized by the value for the lowest-order mode $n = 0$. The values inside the parentheses are the corresponding global coherence evaluated on the basis of Eq.(10.35).

obvious from this figure that the higher-order modes are diminished on propagation,

i.e., with an increase of the value N and that, at the same time, the light becomes more coherent due to the reduction of higher-order modes. In addition, this figure clearly exhibits that the coherence properties of light are uniquely determined by the distribution of the modal strength, as was also pointed out in Section 10.3. In this connection, it is worth emphasizing that the spectral shift connected with the Wolf effect should be examined in terms of the modal aspect rather than the spatial coherence properties of the light.

10.6 Conclusion

General formulas have been derived for the spectrum of light passing through a periodic sequence of lenses separated by Gaussian amplitude filters by making use of the modal expansion of the field within the framework of coherence theory in the space-frequency domain. Here, the initial light incident on the structure is assumed to be a polychromatic, spatially completely incoherent light having a single spectral line with the Lorentzian profile. On the basis of the formulas, spectral modifications of the propagating light have been examined mainly by referring to the recent investigations on the correlation-induced spectral changes of the Wolf effect. To illustrate some typical results, the shift of the peak spectrum has been introduced as a measure for evaluating the spectral modifications. As a consequence, it has become obvious that the spectrum of light observed at an on-axis point is shifted only toward the higher frequencies (the blue shift) irrespective of the spatial coherence properties of the propagating light. On the contrary, the spectral shift of light has been found to be varied depending on both the observation point and the spatial coherence condition of the propagating light, provided that the observation point is located in the off-axis area. The trends in the spectral modifications of this kind have been satisfactorily elucidated under the consideration of the Wolf effect. In addition, the spectral aspect of each mode on propagation has also been illustrated by some numerical examples. It follows from the present theory that the resulting spectrum, together with the feature of spatial coherence, is completely determined by the distribution of the relative strength of modes involved. To this end, curves for the distribution of the modal strength plotted for reference purposes

have served to manifest the contribution of each mode to the total spectral changes in connection with the Wolf effect.

Concluding Remarks

The present dissertation has been concerned theoretically with reflection and propagation of the partially coherent light within the framework of optical coherence theory. Throughout this dissertation has been followed mainly on the phenomena bearing the close relation to the spatial coherence properties of light. More specifically the reflection and propagation of a wide variety of spatially partially coherent waves have been dealt with in the former half part (Chapters 1-7) of this dissertation. In the latter half part (Chapters 8-10) the propagation characteristics of light in a periodic structure have been explained through the analysis of the spatial coherence property of the propagating light. Besides obtained in these two chapters, the results have been deeply extended in an appendix with the reference to the experimental findings.

In Chapter 2, some basic concepts of optical coherence theory were briefly reviewed in order to establish the theoretical foundation of the present dissertation. The physical importance of the correlation functions and the fluctuation of light fields was first put forward. Then, three kinds of coherence functions were introduced to characterize the correlation and their physical interpretations were presented. The basic notion of the spatial coherence, together with that of the temporal coherence, was succinctly discussed through particular interest in this dissertation was the phenomena associated with the spatial coherence rather than the temporal coherence. Particular attention on the subject of spatial coherence was made by considering the simplest geometry for an interference experiment, i.e., as in Young's interference experiment. The analysis demonstrated that the spatial coherence of light (more precisely, the complex degree of spatial coherence) can be readily observed in such an experiment. It is known that, in general, not only the spatial coherence but also the temporal coherence can be observed in some experiments and that, thus, the optical coherence theory is well based on the basis of the interference experi-

CHAPTER 11

Concluding Remarks

The present dissertation has been concerned theoretically with radiation and propagation of the partially coherent light within the framework of optical coherence theory. Attention throughout this dissertation has been focussed mainly on the phenomena bearing the close relation to the spatial coherence properties of light. More specifically, the properties of radiation from a wide variety of spatially partially coherent sources have been dealt with in the former half part (Chapters 4–7) of this dissertation. In the latter half part (Chapters 8–10), the propagation characteristics of light in a periodic structure have been explored through the analysis of the spatial coherence properties of the propagating light. Results obtained in these investigations have been deeply examined so as to provide us with the practical discernment on the theoretical findings.

In Chapter 2, some basic elements of optical coherence theory were briefly reviewed in order to establish the theoretical foundations of the present dissertation. The physical importance of the correlations between the fluctuations of light waves was first put forward. Then, three kinds of coherence functions were introduced to characterize the correlations and their physical implications were summarized. The basic notion of the spatial coherence, together with that of the temporal coherence, was succinctly discussed although particular interest in this dissertation was the phenomena associated with the spatial coherence rather than the temporal coherence. Further elucidation on the concept of spatial coherence was made by considering the simplest geometry for an interference experiment, known as a Young's interference experiment. The analysis demonstrated that the spatial coherence of light (more precisely, the complex degree of spatial coherence) can be readily observed in such an experiment. It is known that, in general, not only the spatial coherence but also the temporal coherence can be measured in some simple experiments and that, thus, the optical coherence theory is established on the basis of the measurable quanti-

ties only. This is an attractive feature of this theory and is in contrast with the elementary theory of optics, where the basic quantity is the light amplitude that is not measurable in practice due to its rapid oscillation. The remainder of this chapter afforded a basis on the propagation laws of the second-order coherence that became a nucleus of all the analyses carried out in the subsequent chapters. It was shown that all the coherence functions introduced in this chapter obey a pair of wave equations. Explicit formulas for describing the propagation of such functions were also presented.

Chapter 3 was devoted to the fundamentals of conventional radiometry and those of generalized radiometry, as a preliminary consideration for the subsequent analyses conducted especially in the former half part of this dissertation. Here, radiometry is a science concerning the detection of light and has been developed over the past few centuries. In an early stage of the studies, the transport of energy by optical radiation was formulated on the basis of geometrical optics (conventional radiometry). On the other hand, in recent years, the same has been reformulated on the basis of optical coherence theory (generalized radiometry). Such a modern treatment of the problem was intimately related to the central subjects of this dissertation and thus served to proceed the subsequent investigations. In this brief chapter, the basic radiometric quantities and their interrelationships were focussed on in the context of both conventional and generalized radiometries. Consequently, it was stated that some generalized radiometric quantities do not possess all the attributes of the conventional radiometric quantities. In other words, this implies that such generalized quantities have physically no meanings. It was shown that, nevertheless, not only they play a significant role in the generalized radiometry, but also they provide a great deal of insight into the behavior of energy from partially coherent sources.

In Chapter 4, the features of radiation from spatially partially coherent planar sources were investigated. As a source model for analysis, we introduced a quasi-monochromatic, Gaussian-correlated source occupying a finite planar domain with an uniform intensity distribution. In analysis, the spatial coherence properties of the radiated field were explored in some detail, rather than the resultant field intensity. Curves of the complex degree of spatial coherence, including those of the intensity distribution, were systematically presented as a function of both the source

coherence and the propagation distance. As a result, it was observed in these figures that, under certain circumstances, the spatial correlations between the fields at two points in the radiated field are rather high when the two points are located at a relatively large distance, in comparison with the case when the two points are located at a relatively short distance. By the close examination, such an extraordinary phenomenon was shown to be due to the sharp boundary (i.e., the edge) of the source. In conjunction with the above analyses, a spatial filtering method over the source using a source filter was also discussed with a view of controlling the spatial coherence of the radiated field. Here, the source filter is a kind of the spatial filter placed just behind the source and changes the intensity distribution across the source. To verify its effect, two kinds of the source filter were introduced as simple examples. It was demonstrated in the analysis that the spatial coherence properties of the radiated field are varied, depending strongly on the amplitude transmittances of the source filters. As a result, it was concluded that an arbitrary state of the spatial coherence may be realized by an appropriate selection of the transmittance of the source filter.

Chapters 5 and 6 were devoted to the studies on radiation efficiency of spatially partially coherent sources. Radiation efficiency is defined as the total power radiated from a source normalized to a maximum value of unity, and works as a measure for quantifying the features of radiation from a source. As a source model for analysis, Chapter 5 adopted partially coherent planar (two-dimensional) sources while Chapter 6 adopted partially coherent three-dimensional sources. The spatial coherence properties of the source were characterized, in both cases, by properly arranged Bessel functions. In analysis of Chapter 5, the radiation efficiency of the source was examined quantitatively as a function of both the spatial coherence condition and the intensity distribution over the source. As a limiting case of the present model source, the source under the quasi-homogeneous condition was also examined from the viewpoint of radiation efficiency. Then, a physical origin of the variation in radiation efficiency was analyzed by introducing the spatial frequency spectrum of the source that has the close connection with the evanescent waves. Consequently it was found that the broadening in the spatial frequency spectrum arising from both the source coherence and the source intensity causes the reduction in radiation efficiency and that such a behavior is rather independent of the presence

of the evanescent waves in the source region.

On the other hand, in Chapter 6, the radiation efficiency of three-dimensional sources was examined quantitatively as a function of the spatial coherence condition over the source. Since this chapter was focussed only on the specific source model that fulfills the (localized) homogeneous condition, the intensity distribution over the source had no importance in this analysis. In addition to the general trends in radiation efficiency, the optimal width of the source coherence that maximizes the radiation efficiency was evaluated. As a result, its value was found to be approximately 0.7λ in any cases (with λ being the wavelength of light), and such a consequence was shown to be satisfactorily elucidated by means of the interference effect of the light waves in the source region. To obtain an intuitive understanding about the features of radiation from the three-dimensional source, curves of the radiation efficiency for the three-dimensional source derived in this chapter were compared with those for the planar source derived in Chapter 5. It followed from such comparison that there exists a kind of the reciprocal behavior in radiation efficiency between the three-dimensional source and the planar one. It should be noted here that the model sources employed in Chapters 5 and 6 were newly proposed in the course of the present investigation and were shown to serve to study the diversity of radiation from various kinds of partially coherent sources.

Chapter 7 was concerned with the radiation from three-dimensional, anisotropic Gaussian Schell-model sources. By a proper adoption of the source parameters, such sources can represent a wide variety of partially coherent sources, e.g., planar sources, spherically symmetric sources, and other types of sources with a certain anisotropy in intensity and/or in spatial coherence. Thus, the analysis of this chapter served to elucidate the difference in radiation between planar sources and three-dimensional sources, supplementing the comparison study conducted in a part of Chapter 6. In analysis, the radiant intensity and the far-field spectrum of light radiated by the source were investigated for various source parameters. As one of the representative results, it was found that the isotropic source both in intensity and in spatial coherence radiates uniformly in the whole space. It was also shown, however, that the isotropic condition over the source is only the necessary condition, but is not the sufficient condition for the source radiating uniformly. In other words, the isotropic radiation can be produced by a certain source with anisotropy.

This led to an important consequence that sufficient information about the source cannot be extracted from measurements of the radiant intensity. On the other hand, changes in the spectrum of light produced by the source were examined in connection with the correlation-induced spectral changes which are often referred to as the Wolf effect. Consequently, it was proved that the shift of the spectral line on light propagation is described exactly by the source parameters. Detailed examination of this phenomenon revealed that the angular distribution of the spectral shift is determined uniquely by the source appearance. Accordingly, it became clear that, in contrast to the case of the radiant intensity, the source information can be extracted satisfactorily from measurements of the spectral shift.

Chapter 8 dealt with the spatial coherence of light passing through the successive apertures with uniform transmittances separated by lenses on the basis of the mode theory in the space-time domain. In this context, the system of successive apertures mimics not only the essential aspect of the laser resonator, but also the typical structure with certain periodicity encountered recently in the fields of integrated optics and microoptics (e.g., the waveguide diffraction grating). Various features of spatial coherence and intensity distribution of the propagating light were examined under the condition that the initial light launched into the structure was quasi-monochromatic and completely spatially incoherent. As a result, the growth in spatial coherence and the convergence in intensity were observed and these behaviors were proved to be influenced strongly by the width of the aperture employed in the structure. Moreover, the variation in spatial coherence was examined physically with relation to the eigenvalues of the integral equation for determining the modes of the periodic structure, yielding a consequence that the trends in the spatial coherence of the propagating light are likely to be affected by the behavior of the higher-order modes whose strengths are governed by the eigenvalues. The results obtained in this chapter were also compared with those obtained in the previous investigation of Gori, and this analysis made clear the phenomena in spatial coherence especially arising from the diffraction by sharp edges of apertures.

In Chapter 9, the spatial coherence properties of the propagating light through the periodic sequence of confocal lenses separated by Gaussian amplitude filters were investigated on the basis of the mode theory in the space-frequency domain. As was in the case of the previous chapter, the periodic structure introduced here

was associated with both the laser resonator and the optical device with certain periodicity. However, as opposed to the previous treatment, the initial light incident on the structure was assumed to be synthesized by a finite superposition of the modes of the periodic structure and thus it was spatially partially coherent. Due to the model source of this kind, this chapter made clear also the spatial coherence properties of a certain kind of multimode lasers and, furthermore, gave certain insight into the effects of the multimode incidence on the periodic devices with periodicity. Various characteristics of the source itself and those of the propagating light were analyzed mainly by introducing a new measure of the homogeneity of the field that characterizes the dependence of the spatial coherence condition on positions across the field. As a result, it was found that the homogeneity of the source depends strongly on the number of the superposed modes and that the homogeneity of the propagating light depends on both the number of the superposed mode and the structure itself. In particular, it was shown that the propagating light can readily become the homogeneous field by the process of light propagation through the periodic structure even though it is initially inhomogeneous, provided the number of superposed modes are large enough. In contrast, it was also shown that the inhomogeneous field may not be easily refined by the light propagation through the periodic structure when the number of superposed modes are sufficiently small.

In Chapter 10, changes in the spectrum of light passing through the same periodic structure as in Chapter 9 were investigated on the basis of the modal expansion of the field in the space-frequency domain. The initial light incident on the structure was assumed to be a polychromatic, spatially incoherent light with a single spectral line. By taking into account the results obtained in Chapters 8 and 9, the trends in spectral changes due to the light propagation in the periodic structure turned out to be satisfactorily manifested from the standpoint of correlation-induced spectral changes (i.e., the Wolf effect). In addition, it was demonstrated that the propagating light in the structure is exactly the same as a Gaussian Schell-model source and that, thus, the mathematical treatment of the problem in this chapter can also be regarded as the coherent-mode representation of the Gaussian Schell-model source. Illustrative examples were presented for the shift of the peak spectrum. They showed that the spectrum of light observed at an on-axis point is shifted only toward the higher frequencies, irrespective of the spatial coherence properties of the propagat-

ing light. On the other hand, the spectral shift of light was varied, depending on both the observation point and the spatial coherence condition of the propagating light, provided that the observation point was located in the off-axis area. Effects of each mode on the spectral changes were also examined. Consequently, it followed that the modal aspect rather than the spatial coherence properties of light plays an important role to understand the trends in the shift of the spectrum arising from the propagation through the periodic structure.

Throughout the present dissertation, various phenomena that occur in radiation and propagation of light waves have been investigated theoretically. All of the above analyses have been proceeded to elucidate the intimate relationship that may exist between the physical origin of such phenomena and the intrinsic fluctuations of light waves, i.e., coherence properties of light. Indeed, this dissertation has proved that the concept of spatial coherence plays a key role in a wide class of optical problems associated with radiation and propagation of light waves.

In recent years, there have been major advances in optics, especially in the fields of what is called *engineering optics*. This is mainly responsible for the advent of lasers in 1960's. Recent development of laser technology, together with that of low-loss optical fibers, semiconductor optical devices and other fabrication technologies, enables us to introduce optical processing in a wide range of engineering such as metrology, communications and information processing. In these fields, vast amount of its capabilities has been offered: nondestructive optical sensings, high-bandwidth optical fiber transmissions, high-speed and high-density optical memories, sophisticated operations using a simple optical element such as Fourier transforms and correlations, and other novel techniques. At the present time, in order to exploit further potentials of the technology towards new generations of engineering optics, it is desirable to understand profoundly the behaviors of light waves in various circumstances and, eventually, to actively control them at will. In this connection, we are convinced that the present dissertation gives some new insights into these fields of current research interest, i.e., engineering optics, even though it deals only with fairly basic problems regarding radiation and propagation of light waves. Finally, it is worth noting that the concept of spatial coherence will make a significant contribution even to these modern fields of optics since it governs almost all the behaviors of light waves in any cases as seen in the course of the present dissertation.

Acknowledgements

I would like to express my sincere appreciation to Professor T. Asakura for his continuous guidance and encouragement throughout the whole course of the present work, without which this dissertation would not have been possible. In particular, it is my heartfelt pleasure that I have learned from him much about optics, together with his own philosophy on optical coherence theory. I also wish to express my thanks to Professors T. Mishima, M. Koshiha, Y. Ohtsuka and I. Suemune for their suggestions and guidance.

I would like to extend my gratitude to Dr. J. Uozumi for his valuable comments and helpful suggestions on a wide variety of problems regarding fundamentals of statistical optics. I am also indebted to Dr. T. Okamoto for providing me with some bases of laser speckle that have been partly relevant to the present work. Mr. Y. Harada deserves acknowledgement for some stimulating advices on several points in this dissertation. My hearty thanks are also extended to Mr. Y. Kawase and Mr. T. Ushizaka for their encouragement and fruitful discussion on some related subjects.

Cordial appreciation should be made to Dr. T. Iwai who has given a number of interesting comments on the interrelationship between coherence theory and scattering theory. I also gratefully acknowledge helpful discussions with Dr. Y. Aizu at Muroran Institute of Technology on several points in this dissertation, as well as the other points of great importance that have really helped me with the present work. My special thanks are also due to all the students and guest researchers at Laboratory of Optical Systems Engineering, Research Institute for Electronic Science, Hokkaido University, where I have been associated as a graduate student in the Doctor's course.

By virtue of the above supports, the present dissertation has been successfully accomplished.

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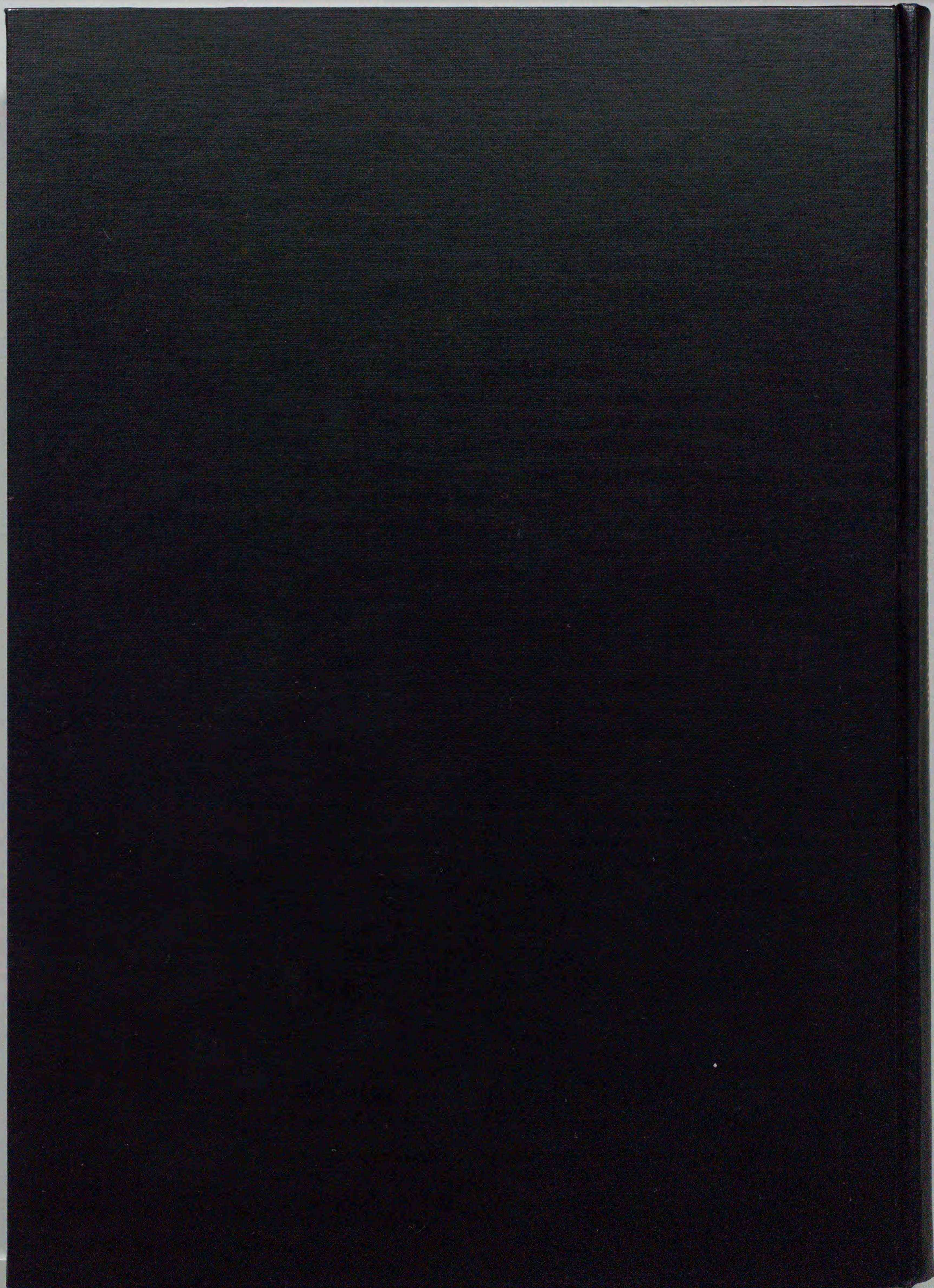
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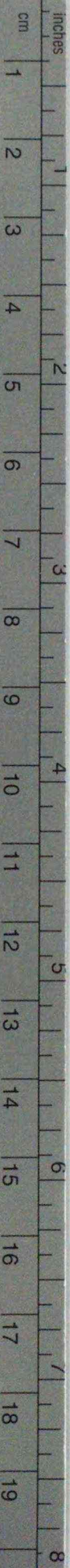
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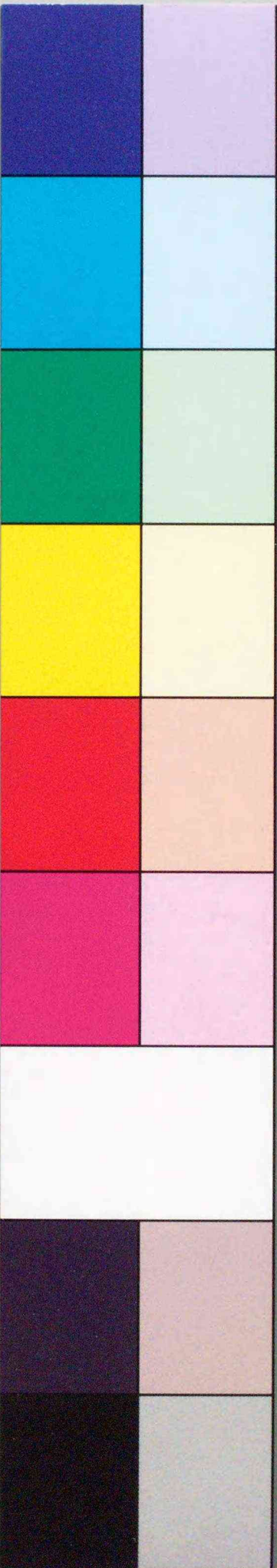




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