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Properties of Partially Coherent Light Using
the Mutual Intensity
and
the Ambiguity Function

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and

the Ambiguity Function

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Chapter 1

Introductory remarks

Recently, optical communication and optical measurement have been remarkably developed by virtue of the invention and the advancement of both the lasers as sources and the optical fibers as light transmission media.

The laser, which stands for light amplification by stimulated emission of radiation, was firstly realized in the form of a ruby laser by Maiman in 1960 [1]. Until today, various types of lasers have been studied and realized: solid-state lasers, liquid lasers, gas lasers, semiconductor lasers, X-ray lasers, free-running lasers, and so on. In particular, semiconductor lasers are now used in various fields of our life: optical communication, compact disk players, laser printers, and so on. Moreover, the semiconductor array lasers fabricated with many single-mode lasers on a monolithic substrate have been expected as the sources for optical multiplex communication systems and optical parallel processing in the future, because of both the ability of supplying these systems with high-power light and the simplicity of connections with micro-optics components.

On the other hand, optical fibers are the dielectric waveguides composed of the core surrounded by the cladding, whose refractive index is slightly smaller than that of the core in order to realize the total internal reflection. Since 1970 [2], the optical fibers with low transmission loss have been produced on the backgrounds of the clearness for the cause of the loss and the development of the production technology year after year. There are mainly two types of optical fibers: a single-mode type and a multimode one. For example, the single-mode type is now used in optical communications, and both types are used in measurement systems, such as an optical fiber interferometer.

Moreover, image fibers with the structure bundling many single-mode fibers have been developed as transmission media of images and, in particular, they have contributed to the field of medicine as instruments, such as an endoscopy.

In general, optical systems using a laser light are unavoidable from the speckle noise (i.e., the random interference effect of coherent optical waves) [3], because the structural fluctuations larger than the order of the wavelength of light are inherent in the systems, and because a laser light is highly coherent in time and space. However, the reduction of the speckle noise has been needed to realize the optical systems with high accuracy. The use of a partially coherent light instead of a coherent light in optical systems is considered as the effective method of achieving the reduction of the speckle noise. In the present study, the properties of the partially coherent quasi-monochromatic light are analyzed by using the mutual intensity (MI) and the ambiguity function (AF) in order to realize advanced optical systems.

The idea of the MI was firstly introduced by Zernike in 1938, as a time-averaged product of the disturbance at one point with the complex conjugate of the disturbance at the second point, in quasi-monochromatic optical fields produced from any types of sources (i.e., coherent, partially coherent and incoherent sources) [4]. In other words, the MI is equivalent to a quasi-monochromatic approximation form of the mutual coherence function defined by Wolf [5]–[10].

On the other hand, the idea of the AF was firstly introduced by Woodward in 1953 in the field of radar in order to evaluate the properties of the transmitted waveform concerning its target resolution, measurement accuracy, ambiguity and response to the clutter [11]. In practice, the AF was defined mathematically as a Fourier transform with respect to the time of a product of the signal at one time with the complex conjugate of the signal at the second time. The physical meaning of the AF corresponds to the response of a matched filter to the signal for which it is matched as well as to Doppler-frequency-shifted (mismatched) signals [12]. After about twenty years from the advent of the Woodward's AF, this function was firstly applied to coherent optics by Papoulis in 1974 in order to analyze the width of a coherent beam crossing a random

medium [13]. The AF in coherent optics was defined as a Fourier transform with respect to the spatial coordinates of a product of the optical field at one point with the complex conjugate of the optical field at the second point. Moreover, the AF was applied to partially coherent optics by Dutta *et al.* in 1977, and the importance of this function was pointed out for the partially coherent imaging [14]. The AF in partially coherent optics was defined as a Fourier transform of the MI with respect to the spatial coordinates. Moreover, the AF is defined as a Fourier transform of the cross-spectral density (CSD) [5], [7]–[10] with respect to the spatial coordinates under the condition of a quasi-monochromatic approximation. The AF is useful for analyses of partially coherent optics, because the propagation law of the AF is given by a simple form in comparison with that of the MI.

In Chapter 2 [15], the definitions and the propagation properties in the free space of the MI and the AF are described under a condition of a quasi-monochromatic approximation. In Chapter 3 [16, 17], the spatial coherence properties of the optical field in the Fraunhofer diffraction region produced from an optical fiber-end source are studied by using the MI in connection with the V number (i.e., the normalized frequency) of the fiber [18] and in comparison with those produced from an incoherent source obeying the van Cittert-Zernike theorem [4]–[10], [19]. Moreover, in Chapter 4 [20], the equi-AF ellipse (i.e., a trajectory of the AF at a constant value) of a Gaussian-Schell model (GSM) beam [21, 22] in the phase space is studied in connection with the beam parameters: the depth of the waist region of the beam, and the beam radius and the wave-front-curvature radius of the GSM beam. The GSM beam is the partially coherent light radiating from a GSM source having a Gaussian intensity profile and a Gaussian complex degree of spatial coherence with stationarity in space. In particular, the optical field produced from the optical fiber-end source, which is studied in the previous chapter, corresponds to an example of the GSM beam.

Next, in Chapter 5 [23], the spatial coherence properties of semiconductor array lasers are studied in connection with far-field patterns of fluctuating supermodes. Supermodes are the optical fields radiated from the array lasers, and they are derived on

the basis of the weakly-coupled mode theory [24]–[29]. In this chapter, the effects of the integrated intensity and the spatial correlation length of the fluctuating component of supermodes on the far-field patterns are analyzed by using the MI. Moreover, the effects of the order of supermodes and the number of emitters in the array laser on the far-field patterns are investigated simultaneously. In Chapter 6 [30], the Talbot effect [31, 32] under illumination of the partially coherent light is studied to manifest the properties of array sources. In this study, the effects of both the spatial coherence length of the illuminating light and the spatial correlation lengths of amplitude and phase fluctuations of the Ronchi grating on the Talbot images (i.e., the self-images) [31, 32] are analyzed by using the propagation law of the AF. In Chapter 7 [33], an array generator for the partially coherent light is studied; the method for generation of many spots with Gaussian intensity distributions from a GSM beam is manifested by using the propagation law of the AF. Finally, concluding remarks of the present study are described in Chapter 8.

Chapter 2

Mutual intensity and ambiguity function

In this chapter, basic ideas of the mutual intensity (MI) and the ambiguity function (AF) are introduced simply by referring to several literature [4]–[10], [13, 14], [34]–[37]. In the following, optical fields are assumed to be stationary and quasi-monochromatic. The quasi-monochromatic assumption means that the spectral bandwidth Δf of light is much smaller than the mean frequency f , i.e., $\Delta f \ll f$. Moreover, polarization effects of light are ignored.

2.1 Mutual intensity and its properties

2.1.1 Definition

Consider the space-dependent complex scalar optical fields, $V(\xi_1)$ and $V(\xi_2)$, at the two points, P_1 and P_2 , at the plane (ξ, η) as shown in Fig.2.1 where the position coordinates for P_1 and P_2 are denoted by the position vectors, ξ_1 and ξ_2 , respectively. In general, the statistical properties of the optical field at this plane are investigated in terms of the MI. The MI for $V(\xi_1)$ and $V(\xi_2)$ is defined as [4]–[10]

$$\Gamma(\xi, \Delta\xi) \triangleq \langle V(\xi + \Delta\xi/2) V^*(\xi - \Delta\xi/2) \rangle, \quad (2.1)$$

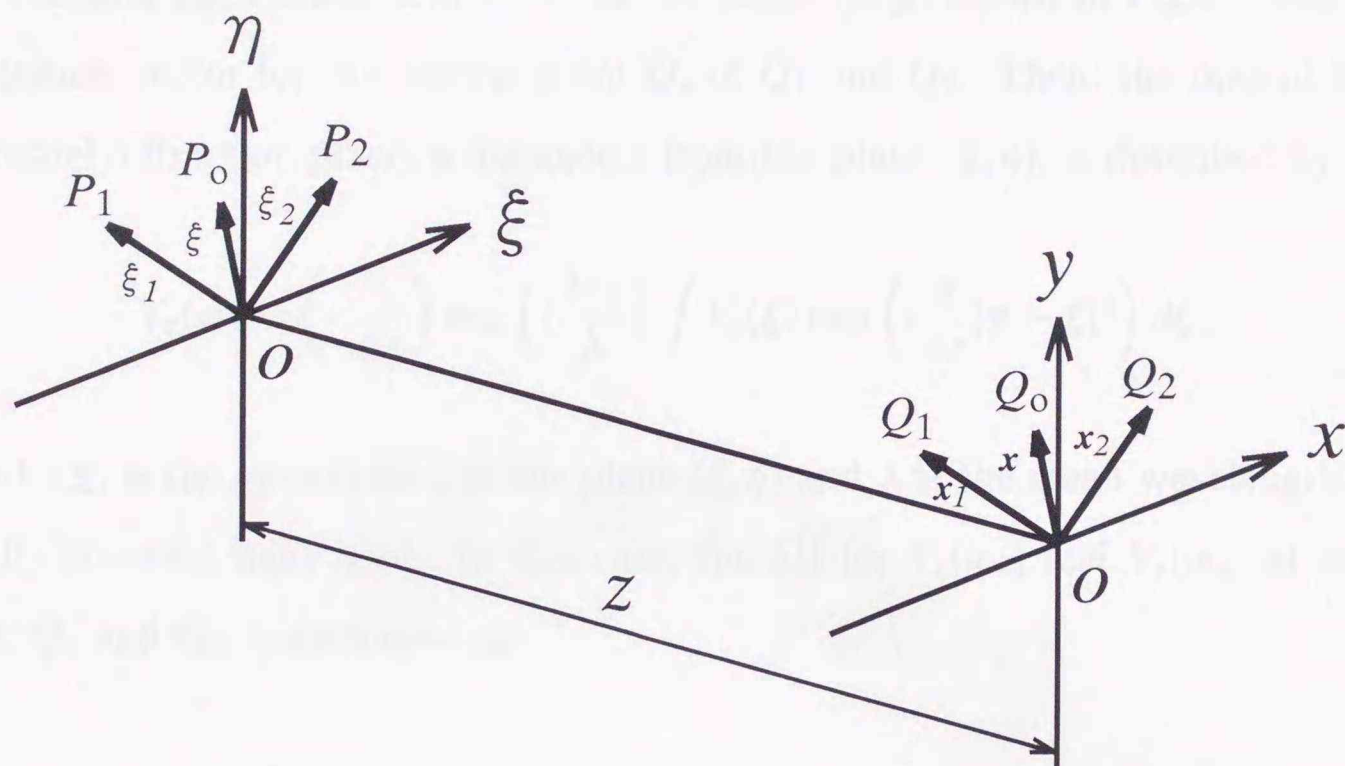


Figure 2.1 Optical coordinates for describing the mutual intensity.

where ξ is the position vector for the central point P_o of P_1 and P_2 , and $\Delta\xi$ is the spatial difference vector between P_1 and P_2 (i.e., $\xi = (\xi_1 + \xi_2)/2$ and $\Delta\xi = \xi_1 - \xi_2$). Moreover, $*$ and $\langle \rangle$ denote the complex conjugate and the ensemble average, respectively. In particular, $\langle \rangle$ represents simultaneously the time average under the assumption of an ergodicity for the optical field $V(\xi)$. Furthermore, the MI $\Gamma(\xi, \Delta\xi)$ is equivalent to the cross-spectral density (CSD) $W(\xi, \Delta\xi)$ [5], [7]–[10] under the condition of a quasi-monochromatic approximation.

In particular, when $\Delta\xi = \mathbf{o}$ in Eq.(2.1), the average intensity (AI) at the plane (ξ, η) is obtained as

$$\begin{aligned} \langle I(\xi) \rangle &= \Gamma(\xi, \Delta\xi = \mathbf{o}) \\ &= \langle V(\xi) V^*(\xi) \rangle. \end{aligned} \quad (2.2)$$

2.1.2 Propagation of the mutual intensity

Fresnel diffraction regime

Next, consider the optical field $V_z(\mathbf{x})$ at the plane (x, y) shown in Fig.2.1 where $\mathbf{x}$ is the position vector for the central point Q_o of Q_1 and Q_2 . Then, the optical field at the Fresnel diffraction plane, a distance z from the plane (ξ, η) , is described by [38]

$$V_z(\mathbf{x}) = \left(-\frac{i}{\lambda z}\right) \exp\left(i\frac{2\pi z}{\lambda}\right) \int V_o(\boldsymbol{\xi}) \exp\left(i\frac{\pi}{\lambda z}|\mathbf{x} - \boldsymbol{\xi}|^2\right) d\boldsymbol{\xi}, \quad (2.3)$$

where $V_o(\boldsymbol{\xi})$ is the optical field at the plane (ξ, η) and λ is the mean wavelength of the partially coherent light used. In this case, the MI for $V_z(\mathbf{x}_1)$ and $V_z(\mathbf{x}_2)$ at the two points, Q_1 and Q_2 , is expressed as

$$\begin{aligned} \Gamma_{near}(\mathbf{x}, \Delta\mathbf{x}) &\triangleq \langle V_z(\mathbf{x} + \Delta\mathbf{x}/2) V_z^*(\mathbf{x} - \Delta\mathbf{x}/2) \rangle \\ &= \left(\frac{1}{\lambda z}\right)^2 \iint \langle V_o(\boldsymbol{\xi}_1) V_o^*(\boldsymbol{\xi}_2) \rangle \\ &\quad \times \exp\left[i\frac{\pi}{\lambda z} \left(|\mathbf{x} + \Delta\mathbf{x}/2 - \boldsymbol{\xi}_1|^2 - |\mathbf{x} - \Delta\mathbf{x}/2 - \boldsymbol{\xi}_2|^2\right)\right] d\boldsymbol{\xi}_1 d\boldsymbol{\xi}_2. \end{aligned} \quad (2.4)$$

Moreover, a change of the variables as $\boldsymbol{\xi} = (\boldsymbol{\xi}_1 + \boldsymbol{\xi}_2)/2$ and $\Delta\boldsymbol{\xi} = \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2$ yields

$$\Gamma_{near}(\mathbf{x}, \Delta\mathbf{x}) = \left(\frac{1}{\lambda z}\right)^2 \iint \Gamma_o(\boldsymbol{\xi}, \Delta\boldsymbol{\xi}) \exp\left[i\frac{2\pi}{\lambda z}(\mathbf{x} - \boldsymbol{\xi})(\Delta\mathbf{x} - \Delta\boldsymbol{\xi})\right] d\boldsymbol{\xi} d\Delta\boldsymbol{\xi}. \quad (2.5)$$

In particular, when $\Delta\mathbf{x} = \mathbf{0}$ in Eq.(2.5), the AI in the Fresnel diffraction region is obtained as

$$\begin{aligned} \langle I_{near}(\mathbf{x}) \rangle &= \Gamma_{near}(\mathbf{x}, \Delta\mathbf{x} = \mathbf{0}) \\ &= \left(\frac{1}{\lambda z}\right)^2 \iint \Gamma_o(\boldsymbol{\xi}, \Delta\boldsymbol{\xi}) \exp\left[-i\frac{2\pi}{\lambda z}(\mathbf{x} - \boldsymbol{\xi})\Delta\boldsymbol{\xi}\right] d\boldsymbol{\xi} d\Delta\boldsymbol{\xi}. \end{aligned} \quad (2.6)$$

Fraunhofer diffraction regime

Finally, the MI of the optical field in the Fraunhofer diffraction region is investigated. The optical field at the Fraunhofer diffraction plane (i.e., at the plane (x, y) in Fig.2.1), a distance z from the plane (ξ, η) , is described by [38]

$$V_z(\mathbf{x}) = \left(-\frac{i}{\lambda z}\right) \exp\left(i\frac{2\pi z}{\lambda}\right) \exp\left(i\frac{\pi|\mathbf{x}|^2}{\lambda z}\right) \int V_o(\boldsymbol{\xi}) \exp\left(-i\frac{2\pi}{\lambda z}\mathbf{x}\boldsymbol{\xi}\right) d\boldsymbol{\xi}. \quad (2.7)$$

Therefore, the MI for $V_z(\mathbf{x}_1)$ and $V_z(\mathbf{x}_2)$ at the two points, Q_1 and Q_2 , is expressed as

$$\begin{aligned} \Gamma_{far}(\mathbf{x}, \Delta\mathbf{x}) &\triangleq \langle V_z(\mathbf{x} + \Delta\mathbf{x}/2) V_z^*(\mathbf{x} - \Delta\mathbf{x}/2) \rangle \\ &= \left(\frac{1}{\lambda z}\right)^2 \exp\left(i\frac{2\pi}{\lambda z}\mathbf{x}\Delta\mathbf{x}\right) \iint d\boldsymbol{\xi}_1 d\boldsymbol{\xi}_2 \langle V_o(\boldsymbol{\xi}_1) V_o^*(\boldsymbol{\xi}_2) \rangle \\ &\quad \times \exp\left[-i\frac{2\pi}{\lambda z}(\mathbf{x} + \Delta\mathbf{x}/2)\boldsymbol{\xi}_1\right] \exp\left[i\frac{2\pi}{\lambda z}(\mathbf{x} - \Delta\mathbf{x}/2)\boldsymbol{\xi}_2\right]. \end{aligned} \quad (2.8)$$

Moreover, a change of the variables as $\boldsymbol{\xi}=(\boldsymbol{\xi}_1+\boldsymbol{\xi}_2)/2$ and $\Delta\boldsymbol{\xi}=\boldsymbol{\xi}_1-\boldsymbol{\xi}_2$ yields

$$\begin{aligned} \Gamma_{far}(\mathbf{x}, \Delta\mathbf{x}) &= \left(\frac{1}{\lambda z}\right)^2 \exp\left(i\frac{2\pi}{\lambda z}\mathbf{x}\Delta\mathbf{x}\right) \\ &\quad \times \iint \Gamma_o(\boldsymbol{\xi}, \Delta\boldsymbol{\xi}) \exp\left[-i\frac{2\pi}{\lambda z}(\Delta\mathbf{x}\boldsymbol{\xi} + \mathbf{x}\Delta\boldsymbol{\xi})\right] d\boldsymbol{\xi} d\Delta\boldsymbol{\xi}. \end{aligned} \quad (2.9)$$

In particular, when $\Delta\mathbf{x}=\mathbf{0}$ in Eq.(2.9), the AI in the Fraunhofer diffraction region is obtained as

$$\begin{aligned} \langle I_{far}(\mathbf{x}) \rangle &= \Gamma_{far}(\mathbf{x}, \Delta\mathbf{x} = \mathbf{0}) \\ &= \left(\frac{1}{\lambda z}\right)^2 \iint \Gamma_o(\boldsymbol{\xi}, \Delta\boldsymbol{\xi}) \exp\left(-i\frac{2\pi}{\lambda z}\mathbf{x}\Delta\boldsymbol{\xi}\right) d\boldsymbol{\xi} d\Delta\boldsymbol{\xi}. \end{aligned} \quad (2.10)$$

2.2 Ambiguity function and its properties

2.2.1 Definition

The AF is defined as a Fourier transform of the MI with respect to ξ [14], [34]–[36]:

$$A(\Delta\nu, \Delta\xi) = \int \Gamma(\xi, \Delta\xi) \exp(-2\pi i \xi \Delta\nu) d\xi, \quad (2.11)$$

where $\Delta\nu = (\Delta\nu_\xi, \Delta\nu_\eta)$ is the spatial frequency-difference vector and the components correspond to the spatial frequency differences along the ξ and η (or the x and y) axes, respectively. Moreover, $\Gamma(\xi, \Delta\xi)$ denotes not only the MI but also the CSD $W(\xi, \Delta\xi)$ under the condition of a quasi-monochromatic approximation.

In particular, the MI $\Gamma(\xi, \Delta\xi)$ in the stationary random process can be written simply as $\Gamma(\Delta\xi)$, because it does not depend on ξ . In this situation, the AF of Eq.(2.11) becomes

$$A(\Delta\nu, \Delta\xi) = \Gamma(\Delta\xi) \delta(\Delta\nu), \quad (2.12)$$

where δ denotes the Dirac's δ function, and one of the definitions for the δ function is given by

$$\delta(\Delta\nu) = \int \exp(-2\pi i \xi \Delta\nu) d\xi. \quad (2.13)$$

2.2.2 Average intensity and mutual intensity near the optical axis

An inverse Fourier transform of the AF $A(\Delta\nu, \Delta\xi)$ in Eq.(2.11) with respect to $\Delta\nu$ yields the MI $\Gamma(\xi, \Delta\xi)$ in the form

$$\Gamma(\xi, \Delta\xi) = \int A(\Delta\nu, \Delta\xi) \exp(2\pi i \xi \Delta\nu) d\Delta\nu. \quad (2.14)$$

Therefore, the AI $\langle I(\xi) \rangle$ of the optical field of interest is expressed, in terms of the AF, as

$$\langle I(\xi) \rangle = \Gamma(\xi, \Delta\xi = \mathbf{o}) = \int A(\Delta\nu, \mathbf{o}) \exp(2\pi i \xi \Delta\nu) d\Delta\nu. \quad (2.15)$$

In this way, the AI can be derived from an inverse Fourier transform of $A(\Delta\nu, \mathbf{o})$ with respect to $\Delta\nu$.

Moreover, the MI near the optical axis z (i.e., $\xi = \mathbf{o}$) is given by

$$\Gamma(\xi = \mathbf{o}, \Delta\xi) = \int A(\Delta\nu, \Delta\xi) d\Delta\nu. \quad (2.16)$$

Equation(2.16) indicates that the MI near the optical axis z is obtained by integrating the AF over $\Delta\nu$.

2.2.3 Propagation of the ambiguity function

Fresnel diffraction regime

By substituting Eq.(2.5) into Eq.(2.11), the AF in the Fresnel diffraction region is obtained as

$$\begin{aligned} A_{near}(\Delta\nu, \Delta\mathbf{x}) &= \int \Gamma_{near}(\mathbf{x}, \Delta\mathbf{x}) \exp(-2\pi i \mathbf{x} \Delta\nu) d\mathbf{x} \\ &= \left(\frac{1}{\lambda z}\right)^2 \iiint \Gamma_o(\xi, \Delta\xi) \exp\left[i\frac{2\pi}{\lambda z}(\mathbf{x} - \xi)(\Delta\mathbf{x} - \Delta\xi)\right] \\ &\quad \times \exp(-2\pi i \mathbf{x} \Delta\nu) d\mathbf{x} d\xi d\Delta\xi \\ &= \left(\frac{1}{\lambda z}\right)^2 \iiint \Gamma_o(\xi, \Delta\xi) \exp\left[-i\frac{2\pi}{\lambda z}\xi(\Delta\mathbf{x} - \Delta\xi)\right] \\ &\quad \times \left\{ \int \exp\left[-i\frac{2\pi i}{\lambda z}(\lambda z \Delta\nu - \Delta\mathbf{x} + \Delta\xi)\mathbf{x}\right] d\mathbf{x} \right\} d\xi d\Delta\xi. \end{aligned} \quad (2.17)$$

Then, with the help of Eq.(2.13), the integration, involved in Eq.(2.17), with respect to x becomes

$$\int \exp \left[-i \frac{2\pi}{\lambda z} (\lambda z \Delta \nu - \Delta x + \Delta \xi) x \right] dx = (\lambda z)^2 \delta(\lambda z \Delta \nu - \Delta x + \Delta \xi). \quad (2.18)$$

Therefore, the following equation is obtained:

$$\begin{aligned} A_{near}(\Delta \nu, \Delta x) &= \int \Gamma_o(\xi, \Delta x - \lambda z \Delta \nu) \exp(-i 2\pi \xi \Delta \nu) d\xi \\ &= A_o(\Delta \nu, \Delta x - \lambda z \Delta \nu). \end{aligned} \quad (2.19)$$

In this way, the AF in the Fresnel diffraction region can be easily derived from that of the original field, i.e., $A_o(\Delta \nu, \Delta \xi)$, by changing the argument $\Delta \xi$ to $\Delta x - \lambda z \Delta \nu$. This fact means simultaneously that the MI in the Fresnel diffraction region is also obtained as an inverse Fourier transform of the resultant AF of Eq.(2.19) with respect to $\Delta \nu$.

Fraunhofer diffraction regime

In succession, the AF of the optical field in the Fraunhofer diffraction region is investigated. An inverse Fourier transform of Eq.(2.9) with respect to x yields the AF in the Fraunhofer diffraction region:

$$\begin{aligned} A_{far}(\Delta \nu, \Delta x) &= \int \Gamma_{far}(x, \Delta x) \exp(-2\pi i x \Delta \nu) dx \\ &= \left(\frac{1}{\lambda z} \right)^2 \iiint \exp \left(i \frac{2\pi}{\lambda z} x \Delta x \right) \Gamma_o(\xi, \Delta \xi) \\ &\quad \times \exp \left[-i 2\pi (\Delta \nu + \Delta \xi / \lambda z) x \right] \exp \left(-i \frac{2\pi}{\lambda z} \Delta x \xi \right) dx d\xi d\Delta \xi \\ &= \left(\frac{1}{\lambda z} \right)^2 \iint \Gamma_o(\xi, \Delta \xi) \exp \left(-i \frac{2\pi}{\lambda z} \Delta x \xi \right) \\ &\quad \times \left\{ \int \exp \left[-i \frac{2\pi}{\lambda z} (\lambda z \Delta \nu - \Delta x + \Delta \xi) x \right] dx \right\} d\xi d\Delta \xi. \end{aligned} \quad (2.20)$$

By using the result of the integration with respect to x given by Eq.(2.18), the AF in the Fraunhofer diffraction region is obtained as

$$\begin{aligned} A_{far}(\Delta\nu, \Delta x) &= \int \Gamma_o(\xi, \Delta x - \lambda z \Delta\nu) \exp\left(-i\frac{2\pi}{\lambda z} \Delta x \xi\right) d\xi \\ &= A_o(\Delta x/\lambda z, \Delta x - \lambda z \Delta\nu). \end{aligned} \quad (2.21)$$

In this way, the AF for the optical field in the Fraunhofer diffraction region can be obtained only by replacing the arguments of $A_o(\Delta\nu, \Delta\xi)$ as $\Delta\nu \rightarrow \Delta x/\lambda z$ and $\Delta\xi \rightarrow \Delta x - \lambda z \Delta\nu$.

Chapter 3

Spatial coherence properties of light from optical fibers

3.1 Introduction

Recently, the optical systems using optical fibers have been remarkably developed in various fields of optical communication, image transmission, optical measurement, and so on. In particular, the use of ends of the fibers as sources has become popular in the optical measurement. In this chapter, the spatial coherence properties of light emerging from optical fibers are studied by using the mutual intensity (MI) [4]–[10].

There are several studies [39]–[42] on the spatial coherence properties of optical fibers. Pask *et al.* [39] discussed the degree of coherence [4]–[10] of the optical field within an optical fiber by applying the concept of the Fraunhofer diffraction, when the incoherent quasi-monochromatic light was launched from the entrance. The speckle power spectrum of the optical field at the end of a multimode fiber, to which a coherent laser beam was launched from the entrance, was studied by Rawson *et al.* [40] on the basis of speckle theory [3] under the assumption that the field was space-invariant and spatially incoherent. Moreover, Imai *et al.* [41, 42] analyzed the modulus and the phase of the complex degree of spatial coherence at the exit of a coherently excited optical fiber. These studies, however, were performed with respect to the spatial coherence properties of the optical fields inside the fiber and at the end of it.

In this chapter, the spatial coherence properties of the optical field in the Fraunhofer diffraction region produced from an optical fiber under illumination of the coherent

light are studied in connection with the state of light waves propagating within the fiber. In general, the optical field produced by light emerging from the end of an optical fiber is observed as a speckle pattern, when a coherent laser beam is launched from the entrance. In the pattern, one spot appears for a single mode, but many spots distributed in a random fashion appear with the increasing number of modes. Moreover, with the increasing number of modes, the size of each spot corresponding to the speckle size becomes small and the whole extent of the speckle pattern becomes broad. Therefore, it may be noticed that the spatial coherence properties of light from the fibers depend strongly on the state of light propagating in them, because the mean size of speckles corresponds to the spatial coherence length.

An optical field at the end of the fiber, which takes the form of a speckle pattern, is expressed as an aggregation of the guided modes governed by the boundary conditions over the whole region in the structure of the fiber. It is, however, impossible to express exactly the field owing to various uncertain factors, such as random wall fluctuations at the boundary between the core and the cladding, the microscopic inhomogeneity of the refractive index, the bending of the fiber, and the mode conversion [43]. Therefore, the field at the end of the fiber may be expressed as a stochastic process caused by the uncertain factors mentioned [40, 44, 45], and a mathematical model must be appropriately chosen to describe the field [10]. In the present study, the optical field at the end of the fiber is expressed as an aggregation of the independent plane waves with random directions and phases, corresponding to stochastic guided modes. As a result, this field is called an optical fiber-end source model.

First, general expressions are given for the MI and the average intensity (AI) of the optical field in the Fraunhofer diffraction region from an optical fiber-end source. The analytical solutions for these expressions are obtained in the case where the amplitude distribution of plane waves at the source is given by a Gaussian profile. Then, the spatial coherence properties of the optical field are discussed in connection with the V number (i.e., the normalized frequency) of the fiber [18], by which the number of guided modes can be estimated. Moreover, the numerical analysis is performed in the case

where the amplitude distribution of the plane waves is given by a power-law profile.

The present study indicates that these expressions are applicable to quasi-single-mode fibers with small V numbers as well as to multimode fibers with large V numbers. In well-excited multimode fibers, the optical field in the Fraunhofer diffraction region is quasi-homogeneous in a statistical sense [46]–[48]. The resultant quasi-homogeneous optical field is characterized by the AI whose extent is finite and much larger than the spatial coherence length. On the other hand, in quasi-single-mode fibers, the optical field in the diffraction region is governed only by the diffraction pattern of plane waves emerging from the fiber.

The above optical fiber-end source is apparently different from the completely incoherent source consisting of the independent point sources from which optical waves radiate into space. The van Cittert-Zernike theorem [4]–[10], [19], which is well-known classically and describes the MI of the optical field from the incoherent source, is not applicable to the fiber-end source studied. However, it will be shown for well-excited multimode fibers that the spatial coherence properties of light emerging from the fibers coincide with those derived from the van Cittert-Zernike theorem. The difference between the optical fiber-end source and the incoherent source becomes noticeable as the situation approaches the single-mode fiber.

3.2 Optical fiber-end source model

As shown schematically in Fig.3.1, the end of an optical fiber to which a coherent beam is launched from the entrance is treated as a secondary source expressed in terms of the spatial coordinates ξ of one dimension. In the present study, the source is called an optical fiber-end source.

On the basis of ray optics, the fiber-end source radiates a number of rays with various directions to the optical axis (i.e., the z axis) within the angle restricted by the numerical aperture (NA) of the fiber. In this case, the optical waves propagating through the fiber are restricted to the guided modes governed by the boundary condi-

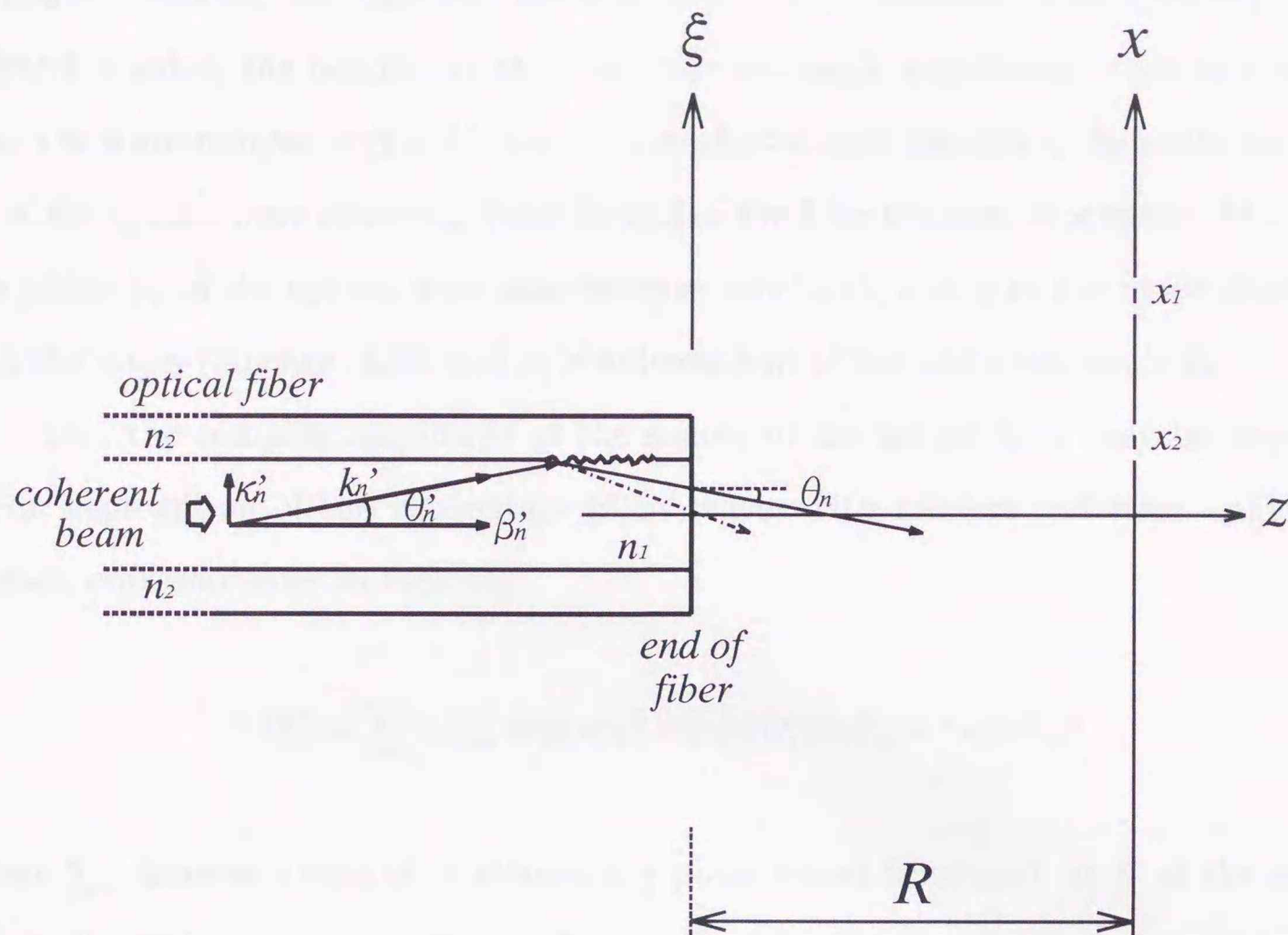


Figure 3.1 Optical coordinates at the optical fiber end and at the observation plane.

tions over the whole region in the structure of the fiber. Therefore, they propagate through the fiber with directions decided by the wave-number vectors $\mathbf{k}' = \{\mathbf{k}'_n\}_{n=1 \sim N}$ of guided modes. As shown in the figure, the wave-number vector $\mathbf{k}'_n$ of the n th mode can be divided into two components, β'_n and κ'_n , along the z and ξ axes, respectively. The magnitudes of β'_n and κ'_n are given by $\beta'_n = k' \cos \theta'_n$ and $\kappa'_n = k' \sin \theta'_n$, respectively, where θ'_n is the angle shown in the figure and $k' = |\mathbf{k}'| = 2\pi n_1 / \lambda$, n_1 and λ being the refractive index of the core and the wavelength of monochromatic light in air, respectively. Then, the optical wave emerges from the end of the fiber with the radiation angle θ_n given by $\sin^{-1}\{n_1 \sin \theta'_n\}$ in virtue of Snell's law. Therefore, it may be considered that the source radiates the optical waves with the radiation angles $\theta = \{\theta_n\}_{n=1 \sim N}$ governed by the wave-number vectors $\mathbf{k}'$ of guided modes.

It should now be pointed out that guided modes become not deterministic but stochastic owing to various uncertain factors, such as random wall fluctuations at the

boundary between the core and the cladding, the microscopic inhomogeneity of the refractive index, the bending of the fiber, and the mode conversion. This fact implies that the wave-number vector $\mathbf{k}'_n$ becomes stochastic and, therefore, the radiation angle θ_n of the optical wave emerging from the end of the fiber becomes stochastic. Moreover, the phase ϕ_n of the optical wave also becomes stochastic owing to the mode dispersion and the Goos-Hänchen shift, and it is independent of the radiation angle θ_n .

Thus, the complex amplitude at the source of the actual fiber may be expressed as an aggregation of the elementary plane waves with random radiation angles and phases, corresponding to each ray:

$$V(\xi) = \sum_n e_n(\xi) \exp(i\phi_n) \exp[ik(\xi \sin \theta_n + z \cos \theta_n)], \quad (3.1)$$

where $\sum_n$ denotes a sum of N elementary plane waves from $n=1$ to N at the source. These elementary plane waves are characterized by the amplitude distribution $e_n(\xi)$, the phase ϕ_n , and the radiation angle θ_n to the optical axis. In Eq.(3.1), k is the wave number given by $2\pi/\lambda$.

In order to analyze the spatial coherence properties of the optical field in the Fraunhofer diffraction region from the fiber-end source, the following conditions are taken into account:

- (i) The profile of the amplitude distribution $e_n(\xi)$ is same for all elementary plane waves.
- (ii) The radiation angle θ_n to the optical axis is a random variable and obeys a Gaussian distribution having the zero mean with the small standard deviation.
- (iii) The random phase ϕ_n has no correlation between the phases of different elementary plane waves.
- (iv) The radiation angle θ_n and the phase ϕ_n are statistically independent of each other.

Under the conditions of (i) and (ii) which yield $e_n(\xi)=e_o(\xi)$, $\sin \theta_n=\theta_n$, and $\cos \theta_n=1$, Eq.(3.1) reduces to

$$V_o(\xi) = e_o(\xi) \sum_n \exp(i\phi_n) \exp[ik(\xi\theta_n + z)]. \quad (3.2)$$

In Eq.(3.2), it is found that $e_o(\xi)$ is the amplitude distribution of the source as well as that of each elementary plane wave. As a result, the complex amplitude at the observation plane of the Fraunhofer diffraction region, a distance R from the fiber-end source, is obtained as

$$\begin{aligned} V(x) &= \int V_o(\xi) \exp(-ikx\xi/R) d\xi \\ &= \exp(ikz) \sum_n \int e_o(\xi) \exp(i\phi_n) \exp(ik\theta_n\xi) \exp(-ikx\xi/R) d\xi. \end{aligned} \quad (3.3)$$

This expression contains the random variables of ϕ_n and θ_n resulting from various uncertain factors mentioned previously. Therefore, the intensity $I(x)=|V(x)|^2$ at the observation plane is generally observed as a stochastic process.

3.3 Mutual intensity in the Fraunhofer diffraction region

In order to analyze the spatial coherence properties of light at the observation plane, the MI of the complex amplitudes, $V(x_1)$ and $V(x_2)$, at x_1 and x_2 at the plane is introduced and given by

$$\begin{aligned} \Gamma(x_1, x_2) &= \langle V(x_1) V^*(x_2) \rangle \\ &= \left\langle \sum_n \sum_m \iint e_o(\xi_1) e_o^*(\xi_2) \exp[i(\phi_n - \phi_m)] \right. \\ &\quad \times \exp[ik(\theta_n \xi_1 - \theta_m \xi_2)] \exp\left[-\frac{ik(x_1 \xi_1 - x_2 \xi_2)}{R}\right] d\xi_1 d\xi_2 \Big\rangle, \end{aligned} \quad (3.4)$$

where $*$ and $\langle \rangle$ denote the complex conjugate and the ensemble average, respectively. Under the condition of (iv), Eq.(3.4) becomes

$$\begin{aligned} \Gamma(x_1, x_2) = & \sum_n \sum_m \iint e_o(\xi_1) e_o^*(\xi_2) \langle \exp[i(\phi_n - \phi_m)] \rangle \\ & \times \langle \exp[ik(\theta_n \xi_1 - \theta_m \xi_2)] \rangle \exp[-ik(x_1 \xi_1 - x_2 \xi_2)/R] d\xi_1 d\xi_2. \end{aligned} \quad (3.5)$$

Under the conditions of (ii) and (iii), the following two equations hold:

$$\langle \exp[i(\phi_n - \phi_m)] \rangle = \delta_{nm}, \quad (3.6)$$

$$\langle \exp[ik\theta_n(\xi_1 - \xi_2)] \rangle = \exp\left[-\frac{1}{2}k^2(\xi_1 - \xi_2)^2\langle\theta^2\rangle\right], \quad (3.7)$$

respectively, where δ_{nm} denotes the Kronecker's delta and $\langle\theta^2\rangle$ means the variance of radiation angles of elementary plane waves emerging from the fiber-end source. With the use of Eqs.(3.6) and (3.7), Eq.(3.5) reduces to

$$\begin{aligned} \Gamma(x_1, x_2) = & N \iint e_o(\xi_1) e_o^*(\xi_2) \exp\left[-\frac{1}{2}k^2(\xi_1 - \xi_2)^2\langle\theta^2\rangle\right] \\ & \times \exp[-ik(x_1 \xi_1 - x_2 \xi_2)/R] d\xi_1 d\xi_2. \end{aligned} \quad (3.8)$$

Finally, by changing the arguments of x_1 and x_2 in Eq.(3.8) into the forms,

$$x = x_1 \quad \text{and} \quad X = x_2 - x_1, \quad (3.9)$$

Eq.(3.8) becomes (see Sec.3.9)

$$\Gamma(x, x + X) = \frac{N\lambda}{\sqrt{2\pi\langle\theta^2\rangle}} \int E_o(u) E_o^*(u + X) \exp\left[-2(x - u)^2/q^2\right] du, \quad (3.10)$$

where $E_o(u)$ is a Fourier transform of the amplitude distribution $e_o(\xi)$ at the source:

$$E_o(u) = \int e_o(\xi) \exp(-iku\xi/R) d\xi \quad (3.11)$$

and

$$q = 2\sqrt{\langle\theta^2\rangle}R. \quad (3.12)$$

As shown in Eq.(3.10), the MI depends on the spatial coordinates x . This implies that the optical field of interest is not spatially stationary. Moreover, the AI of the optical field also depends on the spatial coordinates x and is given by

$$\langle I(x) \rangle = \Gamma(x, x) = C \int |E_o(u)|^2 \exp[-2(x-u)^2/q^2] du, \quad (3.13)$$

where $C = N\lambda/\sqrt{2\pi\langle\theta^2\rangle}$.

3.4 Analytical solution for a Gaussian amplitude distribution

Consider the situation that the amplitude distribution of the fiber-end source is given by a Gaussian profile:

$$e_o(\xi) = \sqrt{I_o} \exp(-\xi^2/a^2), \quad (3.14)$$

where I_o is a constant and a is the effective extent of the source. Then, Eq.(3.11) becomes

$$E_o(u) = \sqrt{\pi I_o} a \exp(-u^2/\Delta x^2), \quad (3.15)$$

where

$$\Delta x = \lambda R/\pi a. \quad (3.16)$$

Equation(3.15) physically describes the amplitude distribution in the Fraunhofer diffraction region produced by elementary plane waves emerging from the fiber-end source. Therefore, the intensity distribution due to these elementary plane waves is given by

$$I_o(u) = \pi a^2 I_o \exp \left(-2u^2 / \Delta x^2 \right), \quad (3.17)$$

where Δx is the extent of the Fraunhofer diffraction pattern. Moreover, Δx indicates the mean size of speckles appearing in the Fraunhofer diffraction region, when a diffuse object is illuminated by a coherent laser beam whose amplitude distribution is expressed by Eq.(3.14) [49].

By substituting Eq.(3.15) into Eq.(3.10), the MI at the observation plane is obtained as

$$\Gamma(x, x + X) = \Gamma(0, 0) \exp \left(-X^2 / 2\Delta x^2 \right) \exp \left[-2(x + X/2)^2 / (q^2 + \Delta x^2) \right], \quad (3.18)$$

where $\Gamma(0, 0)$ is a constant. In particular, in the neighborhood of the optical axis (i.e., $x=0$), Eq.(3.18) reduces to

$$\Gamma(0, X) = \Gamma(0, 0) \exp \left(-X^2 / 2\Delta X_c^2 \right), \quad (3.19)$$

where

$$\Delta X_c = \frac{\Delta x}{\sqrt{1 + 1/\{1 + (q/\Delta x)^2\}}}. \quad (3.20)$$

ΔX_c indicates the standard deviation of the MI distribution expressed by Eq.(3.19) and it refers to the spatial coherence length in the neighborhood of the optical axis. Moreover, the AI is obtained from Eq.(3.18) as

$$\langle I(x) \rangle = \Gamma(x, x) = \Gamma(0, 0) \exp \left(-2x^2 / \Delta Q^2 \right), \quad (3.21)$$

where

$$\Delta Q = \Delta x \sqrt{1 + (q/\Delta x)^2}, \quad (3.22)$$

which refers to the spatial extent of the AI distribution.

As a result, the spatial coherence length and the spatial extent of the AI distribution given, respectively, by Eqs.(3.20) and (3.22) are characterized by the parameter $q/\Delta x$. This parameter is related to the physical properties of optical fibers which will be discussed in the following sections.

3.5 Relation to the V number of the optical fiber

The propagation properties of guided modes within an optical fiber are decided by the core radius a and the refractive indices, n_1 and n_2 , in the core and cladding materials, respectively. For example, the maximum radiation angle of optical waves emerging from the end of the fiber is decided by the NA expressed as $n_1\sqrt{2\Delta}$, Δ being the relative difference of the refractive indices defined by $(n_1-n_2)/n_1$. Moreover, the V number (i.e., the normalized frequency), by which the number of guided modes can be estimated, is related to the NA and is given by [18]

$$V = ka(\text{NA}). \quad (3.23)$$

It may now be assumed that the NA corresponds well with the standard deviation of the radiation angles $\sqrt{\langle\theta^2\rangle}$ used in the present optical fiber-end source model. Then, the V number becomes

$$V = ka\sqrt{\langle\theta^2\rangle} = q/\Delta x, \quad (3.24)$$

where Eqs.(3.12) and (3.16) were used. By using the V number of Eq.(3.24), the spatial coherence length ΔX_c and the spatial extent ΔQ of the AI distribution given,

respectively, by Eqs.(3.20) and (3.22) are rewritten as

$$\Delta X_c = \frac{\Delta x}{\sqrt{1 + 1/(1 + V^2)}}, \quad (3.25)$$

$$\Delta Q = \Delta x \sqrt{1 + V^2}, \quad (3.26)$$

respectively.

The dependence of ΔX_c and ΔQ on the V number is shown in Figs.3.2 and 3.3, respectively. The spatial coherence length ΔX_c takes $\Delta x/\sqrt{2}$ ($=0.707\Delta x$) for sufficiently small V numbers (i.e., $V \ll 1$). This situation corresponds to quasi-single-mode fibers. Moreover, ΔX_c increases with the increasing V number and finally approaches Δx . This limiting situation corresponds to well-excited multimode fibers. On the other hand, the extent ΔQ of the AI distribution takes Δx for $V \ll 1$, and it increases steeply with the increasing V number.

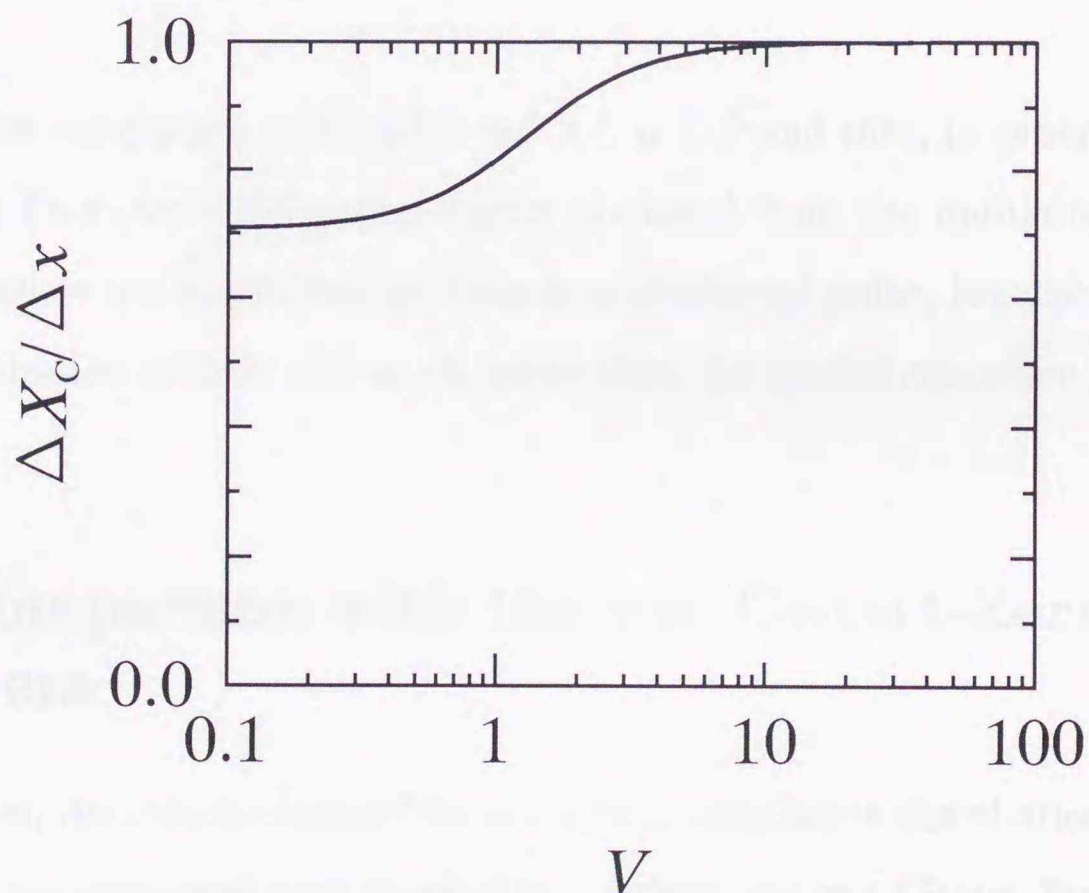


Figure 3.2 Dependence of the normalized spatial coherence length $\Delta X_c/\Delta x$ on the V number of the optical fiber for the Gaussian amplitude distribution.

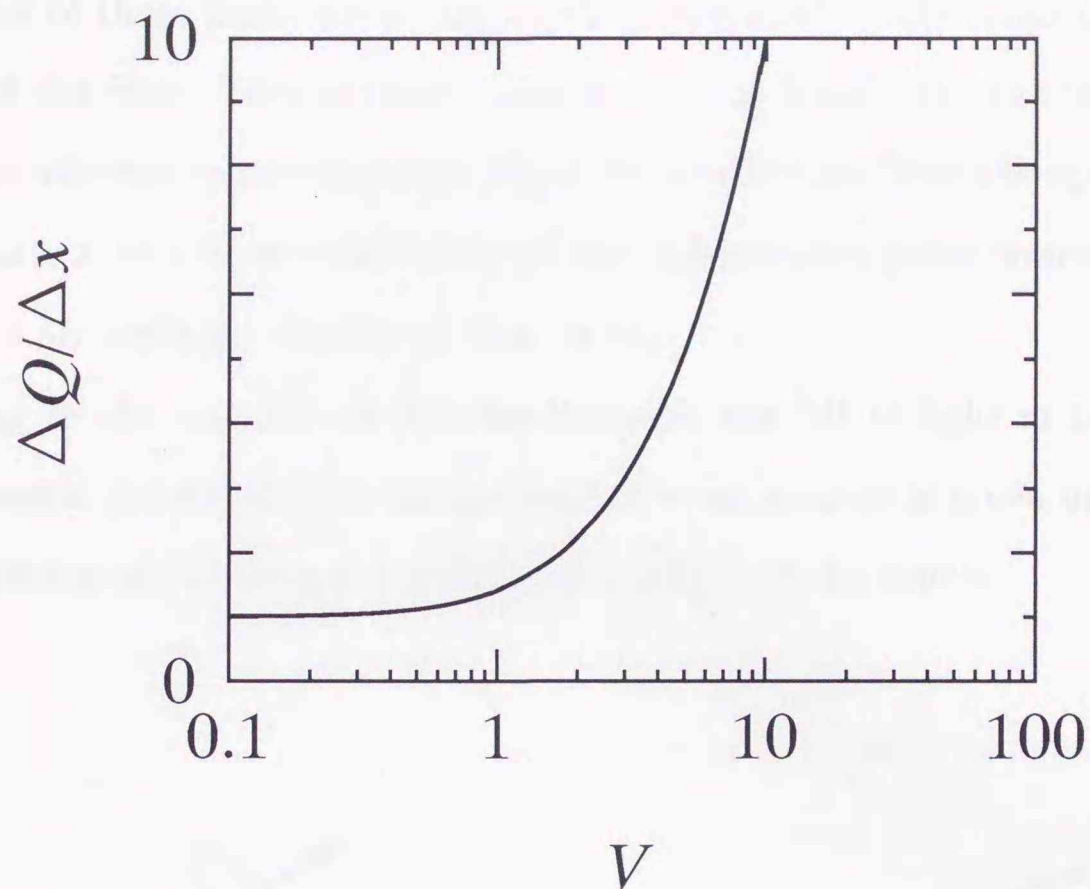


Figure 3.3 Dependence of the spatial extent ΔQ of the average intensity distribution on the V number for the Gaussian amplitude distribution.

From the comparison of Figs.3.2 and 3.3, it is found that, in general, the optical fields in the Fraunhofer diffraction region produced from the multimode fibers with large V numbers are quasi-homogeneous in a statistical sense, because the extent of the AI distribution is finite and much larger than the spatial coherence length.

3.6 Comparison with the van Cittert-Zernike theorem

In this section, the results obtained for a Gaussian amplitude distribution in the previous sections are compared with those obtained from the van Cittert-Zernike theorem. This discussion is useful for recognizing the difference between the optical fiber-end source and the incoherent source obeying the van Cittert-Zernike theorem. In the

previous discussion, the optical fiber-end source has been described as a source from which independent plane waves are radiated with random directions. However, the radiation angles of these plane waves are actually restricted to the angle decided by the V number of the fiber. This situation is shown in Fig.3.4(a). On the other hand, the completely incoherent source shown in Fig.3.4(b) is different from the optical fiber-end source, because it is a source consisting of the independent point sources from which optical waves are radiated to all over directions.

According to the van Cittert-Zernike theorem, the MI of light in the Fraunhofer diffraction region produced from the incoherent point sources is given in the form of a Fourier transform of the intensity distribution $I(\xi)$ over the source:

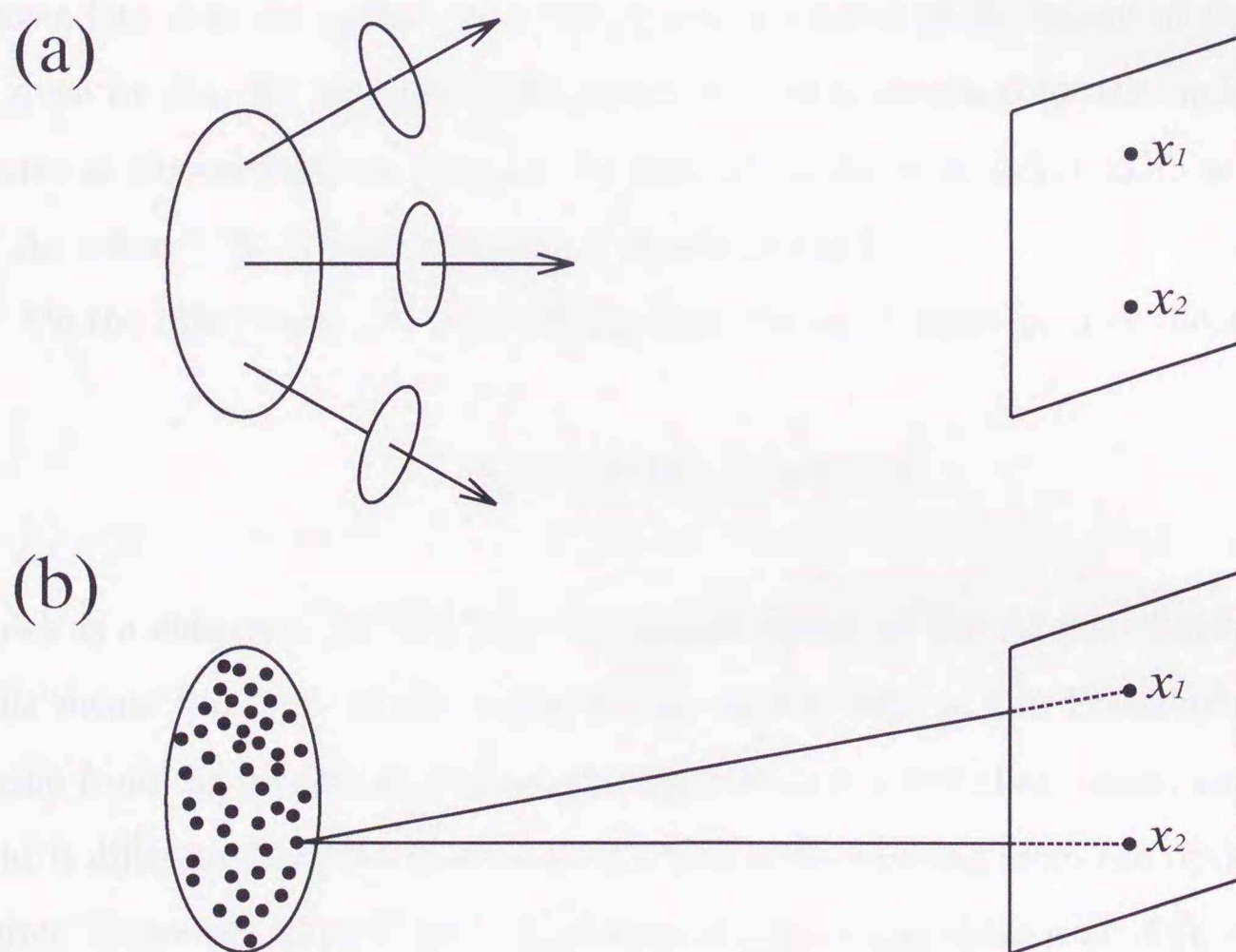


Figure 3.4 (a) Optical fiber-end source model (independent plane wave model) and (b) completely incoherent source model (independent point source model) obeying the van Cittert-Zernike theorem.

$$\Gamma(x_1, x_2) = \int I(\xi) \exp[-ik(x_1 - x_2)\xi/R] d\xi. \quad (3.27)$$

In particular, for the intensity distribution, resulting from the Gaussian amplitude distribution of Eq.(3.14), expressed by

$$I(\xi) = |e_o(\xi)|^2 = I_o \exp(-2\xi^2/a^2), \quad (3.28)$$

the MI becomes

$$\Gamma(x_1, x_2) = \Gamma(0, 0) \exp[-(x_1 - x_2)^2/2\Delta x^2], \quad (3.29)$$

where $\Gamma(0, 0)$ is a constant and Δx is given by Eq.(3.16). This result indicates the well-known fact that the spatial coherence length is related to the extent of the source and is given by Δx . By comparing this result with that obtained for the optical fiber-end source in the previous section (i.e., by comparing Δx with ΔX_c), ΔX_c is nearly equal to Δx when $V \gg 1$. This situation is shown in Fig.3.2.

On the other hand, the AI resulting from the van Cittert-Zernike theorem becomes

$$\langle I(x) \rangle = \Gamma(x, x) = \Gamma(0, 0) \quad (3.30)$$

which is a constant. In this case, the spatial extent of the AI distribution is infinite. This means that the whole region of the optical field in the Fraunhofer diffraction region from the incoherent source is homogeneous in a statistical sense, and the optical field is different from the quasi-homogeneous field resulting from the optical fiber-end source. However, when $V \gg 1$, ΔQ becomes infinite and there is no difference between the two sources. This situation is shown in Fig.3.3.

As a result, it is understood that, only when $V \gg 1$, the van Cittert-Zernike theorem, which is generally applicable to the incoherent source, gives the same result as the optical fiber-end source. In other words, for well-excited multimode fibers,

the optical fiber-end source is effectively equivalent to the incoherent source from the viewpoint of spatial coherence properties.

3.7 Numerical analysis for a power-law amplitude distribution

In the previous sections, the MI and the AI have been analytically evaluated for the amplitude distribution given by a Gaussian profile of Eq.(3.14). However, it is generally impossible to know exactly how the amplitude distribution is expressed for the optical fiber-end source. One requirement for the fiber-end source is that the amplitude distribution takes a finite extent, because the size of the fiber end is finite. Therefore, various functions satisfying this condition are applicable to the amplitude distribution over the fiber-end source. Here, a power-law profile is also considered as the amplitude distribution. The profile is described by

$$e_o(\xi) = \begin{cases} 1 - |\xi/a|^G, & (|\xi| \leq a), \\ 0, & (|\xi| > a), \end{cases} \quad (3.31)$$

where a and G denote the core radius of the fiber and the positive index number, respectively. By changing G , some different amplitude distributions are obtained and shown in the left side of Fig.3.5. In these cases, numerical evaluation is employed to investigate the spatial coherence properties of light resulting from the optical fiber-end source of Eq.(3.31), because analytical evaluation cannot be found.

In the numerical evaluation, the MI and AI distributions in the Fraunhofer diffraction region from the source are calculated according to Eqs.(3.10) and (3.13), respectively. Here, consider the MI of the optical field in the neighborhood of the optical axis (i.e., $x=0$). By changing the variable of u and the argument of X in Eq.(3.10) into the forms,

$$u' = u/R \quad \text{and} \quad X' = X/R, \quad (3.32)$$

respectively, the MI is rewritten as

$$\Gamma(0, X') = C \int E_o(u') E_o^*(u' + X') \exp \left[-2(u'R)^2 / (V \Delta x)^2 \right] du', \quad (3.33)$$

where $V = ka\sqrt{\langle \theta^2 \rangle}$ given by Eq.(3.24). Further, by changing similarly the variable of u in Eq.(3.13), the AI is rewritten as

$$\langle I(x') \rangle = C \int |E_o(u')|^2 \exp \left[-2\{(x' - u')R\}^2 / (V \Delta x)^2 \right] du', \quad (3.34)$$

where $x' = x/R$. To evaluate Eqs.(3.33) and (3.34), a Fourier transform $E_o(u)$ of $e_o(\xi)$ given by Eq.(3.31) is calculated for some positive index numbers G by performing the numerical integration.

3.7.1 Evaluation of the mutual intensity

The results of numerical evaluation of the MI distributions are shown in Fig.3.5 for the amplitude distributions of the source given by $G=2, 5$, and 100 in Eq.(3.31). The middle and right sides in the figure show the MI distributions for the cases of $V=0.1$ and 2 , respectively. In the figure, the solid lines (———) show the results given by Eq.(3.33) for the optical fiber-end source, while the broken lines (- - - -) show the results given by the van Cittert-Zernike theorem for the incoherent source.

When $V=0.1$, the difference of the distributions between the optical fiber-end source and the incoherent source becomes small for large G 's and, in particular, there is almost no difference for $G=100$. Moreover, for all G 's, the extent of the distribution corresponding to the spatial coherence length is smaller than that of the incoherent source.

On the other hand, when $V=2$, the difference becomes small for all G 's in comparison with the case of $V=0.1$. Moreover, for large G 's (i.e., $G=5$ and 100), the spatial coherence length is larger than that of the incoherent source. For the amplitude distribution given by the power-law profile, it is consequently found that the difference of

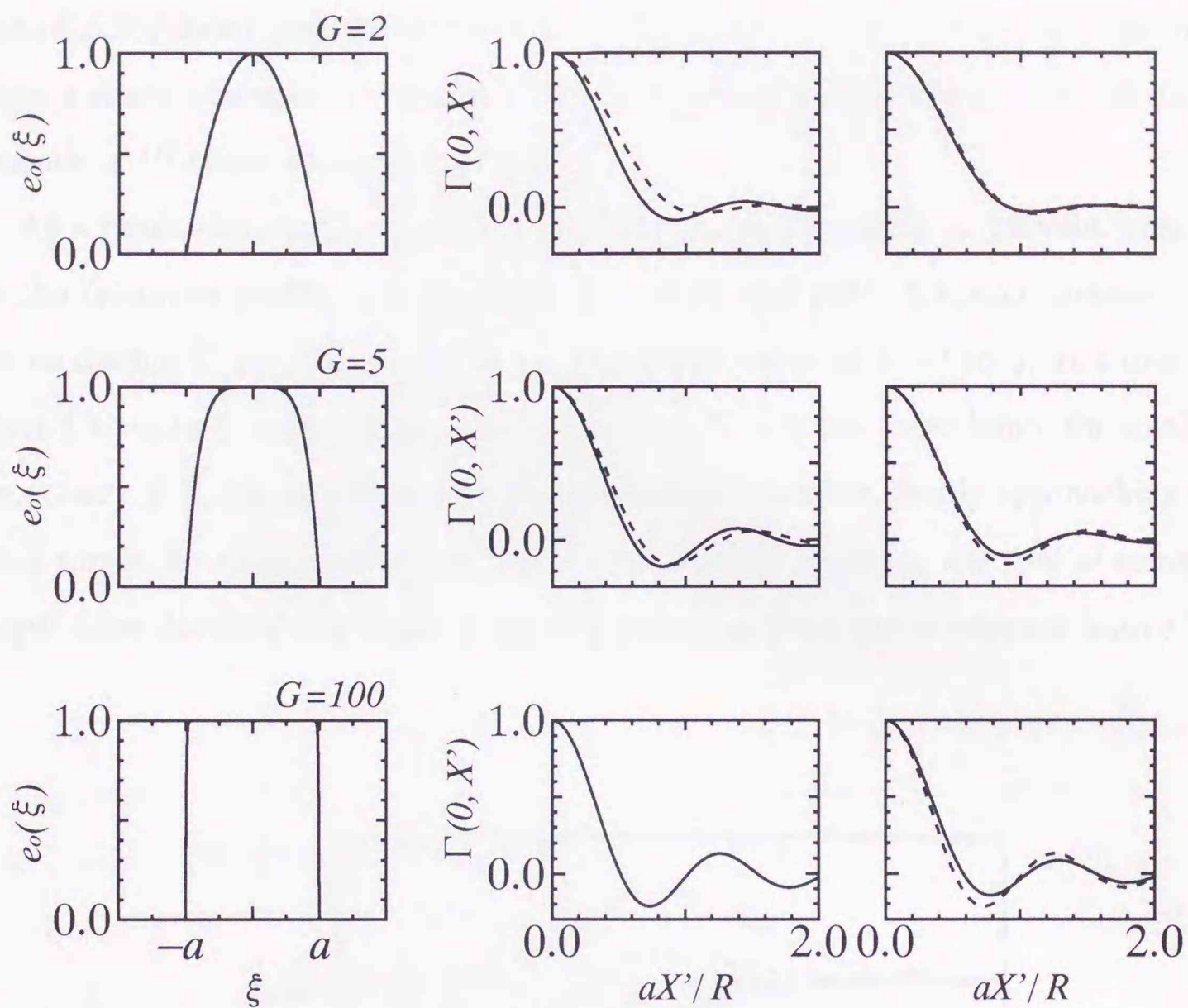


Figure 3.5 Mutual intensity distributions for the power-law amplitude distributions. The left side shows the amplitude distributions at the sources having $G=2, 5$ and 100 . The middle and the right sides show the mutual intensity distributions for $V=0.1$ and 2 , respectively. (—) shows the results for the optical fiber-end source, and (- - -) shows the results given by the van Cittert-Zernike theorem.

the MI distributions between the optical fiber-end source and the incoherent source is dependent strongly on the type of the source characterized by G .

Next, a variation of the spatial coherence length of light resulting from a fiber-end source of Eq.(3.31) is investigated as a function of the V number. The results are shown in Fig.3.6 together with the ones for the Gaussian profile. In the figure, the vertical axis expresses the normalized spatial coherence length $\Delta X_c/\Delta x$: it indicates

the spatial coherence length ΔX_c of the optical fiber-end source normalized by the spatial coherence length Δx which is given by the van Cittert-Zernike theorem. The case of $\Delta X_c/\Delta x=1$ means that there is no difference between the two sources. In the present study, the spatial coherence length is defined as the extent to which the MI becomes $e^{-1/2}$ times the maximum value.

As a result, the change of $\Delta X_c/\Delta x$ for the power-law profile is different from that for the Gaussian profile. For large G 's (i.e., $G=5$ and 100), $\Delta X_c/\Delta x$ increases with the increasing V number, becomes the maximum value at $V=2$ to 5 , and decreases beyond $V=2$ to 5 , approaching nearly 1 for $V \gg 5$. On the other hand, for small G 's (i.e., $G=2$), $\Delta X_c/\Delta x$ increases with the increasing V number, finally approaching 1. In other words, for quasi-single-mode fibers with small V numbers, the spatial coherence length from the fiber-end source is smaller than that from the incoherent source. In

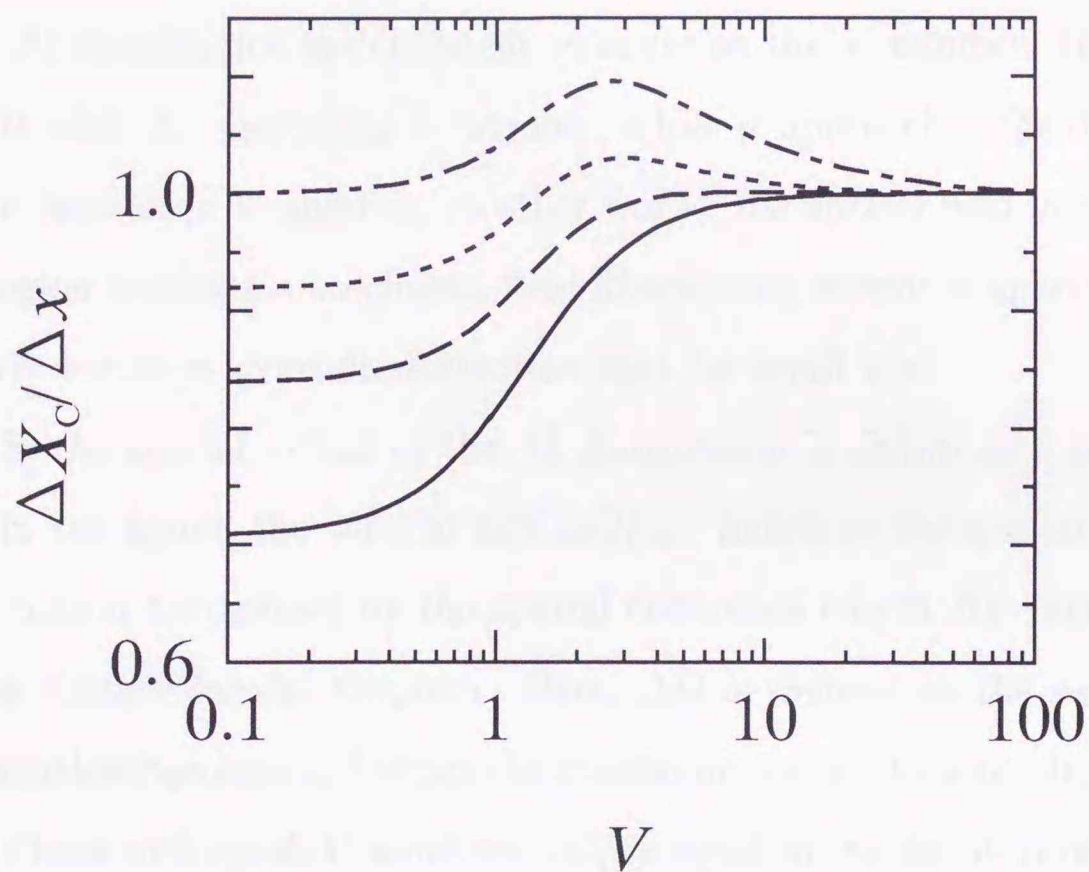


Figure 3.6 Dependence of the normalized spatial coherence length $\Delta X_c/\Delta x$ on the V number of the optical fiber for the power-law amplitude distribution. (—), (- - -), (- - - -) and (— · —) denote the results for Gaussian, and $G=2$, 5 and 100, respectively.

particular, the difference between the two is apparent for the Gaussian profile. Moreover, the difference increases as G becomes smaller for the power-law profile. However, for well-excited multimode fibers with very large V numbers, the spatial coherence length coincides with that from the incoherent source.

3.7.2 Evaluation of the average intensity

Next, the AI distribution and its spatial extent in the Fraunhofer diffraction region are quantitatively investigated from the numerical evaluation of Eq.(3.34). The results are shown in Figs.3.7 and 3.8.

In Fig.3.7, the AI distributions for $V=1$ to 5 are shown by the solid lines (—) for the power-law amplitude distributions having $G=2, 5$ and 100. On the other hand, the broken lines (- - -) show the diffraction patterns for the same amplitude distributions. As seen from the figure, there is almost no difference of the AI distributions among $G=2, 5$ and 100 for the same V number. However, for all amplitude distributions, the AI distribution is dependent strongly on the V number: the distribution becomes wide with the increasing V number, while it approaches the diffraction pattern with the decreasing V number. In other words, the optical field in the Fraunhofer diffraction region becomes a stochastic field distributed widely in space for large V 's, while it nearly becomes a certain diffraction field for small V 's.

In Fig.3.8, the spatial extent of the AI distribution is shown as a function of the V number. In the figure, the vertical axis $\Delta Q/\Delta x$ indicates the spatial extent ΔQ of the AI distribution normalized by the spatial coherence length Δx which is obtained from the van Cittert-Zernike theorem. Here, ΔQ is defined as the extent to which the AI distribution becomes e^{-2} times the maximum value. As a result, for the quasi-single-mode fibers with small V numbers, ΔQ is equal to Δx for all profiles. However, for well-excited multimode fibers with large V numbers, ΔQ is much larger than Δx . Moreover, an increasing rate of $\Delta Q/\Delta x$ for V becomes almost the same for all profiles, and the rate slightly becomes large with an increase of G for the power-law profile.

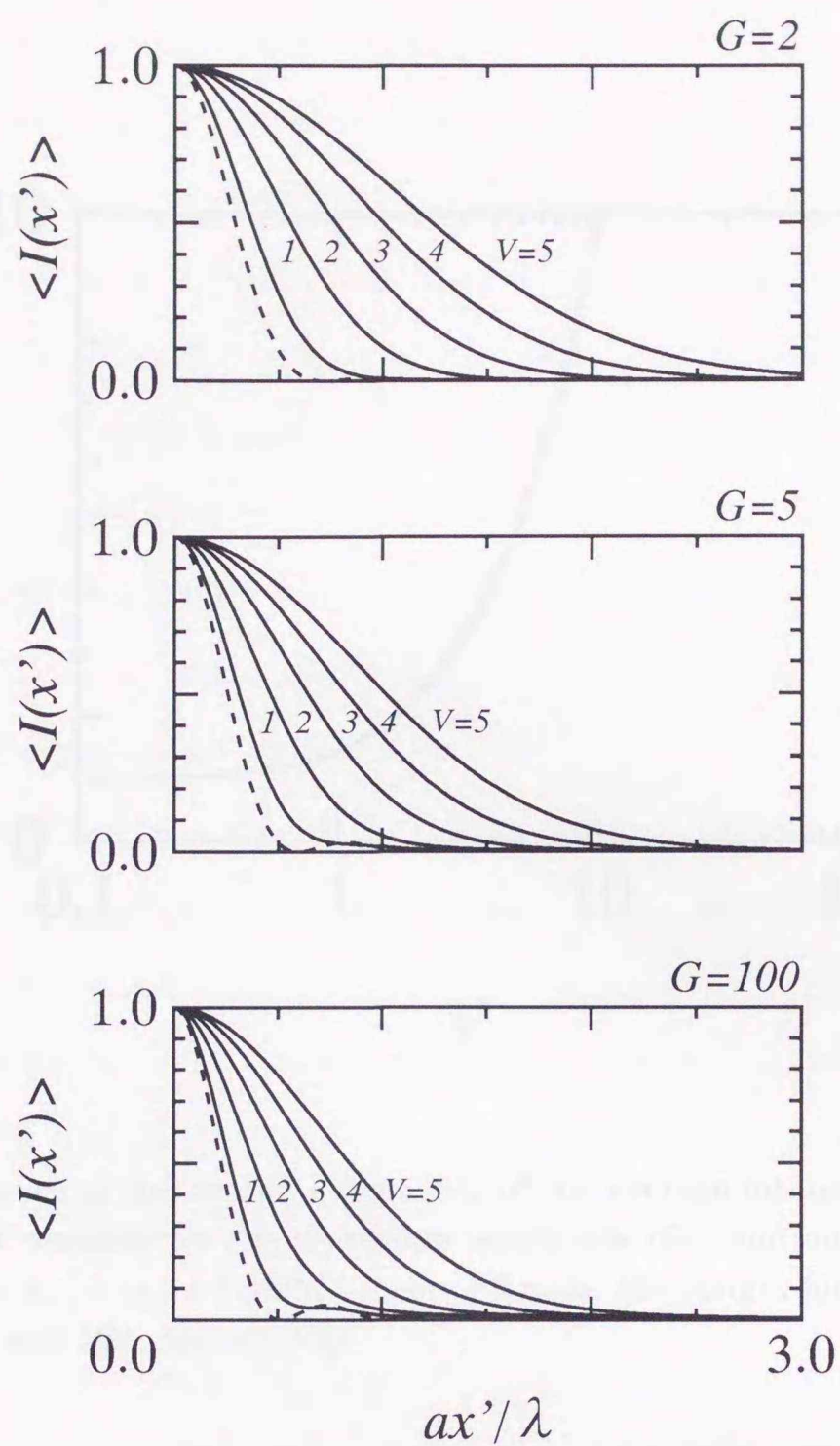


Figure 3.7 Average intensity distributions (———) and diffraction patterns (- - - -) for the power-law amplitude distributions.

From the comparison of Figs.3.6 and 3.8, it is found that the optical field in the Fraunhofer diffraction region from the multimode fiber becomes rapidly quasi-homogeneous in a statistical sense with the increasing V number.

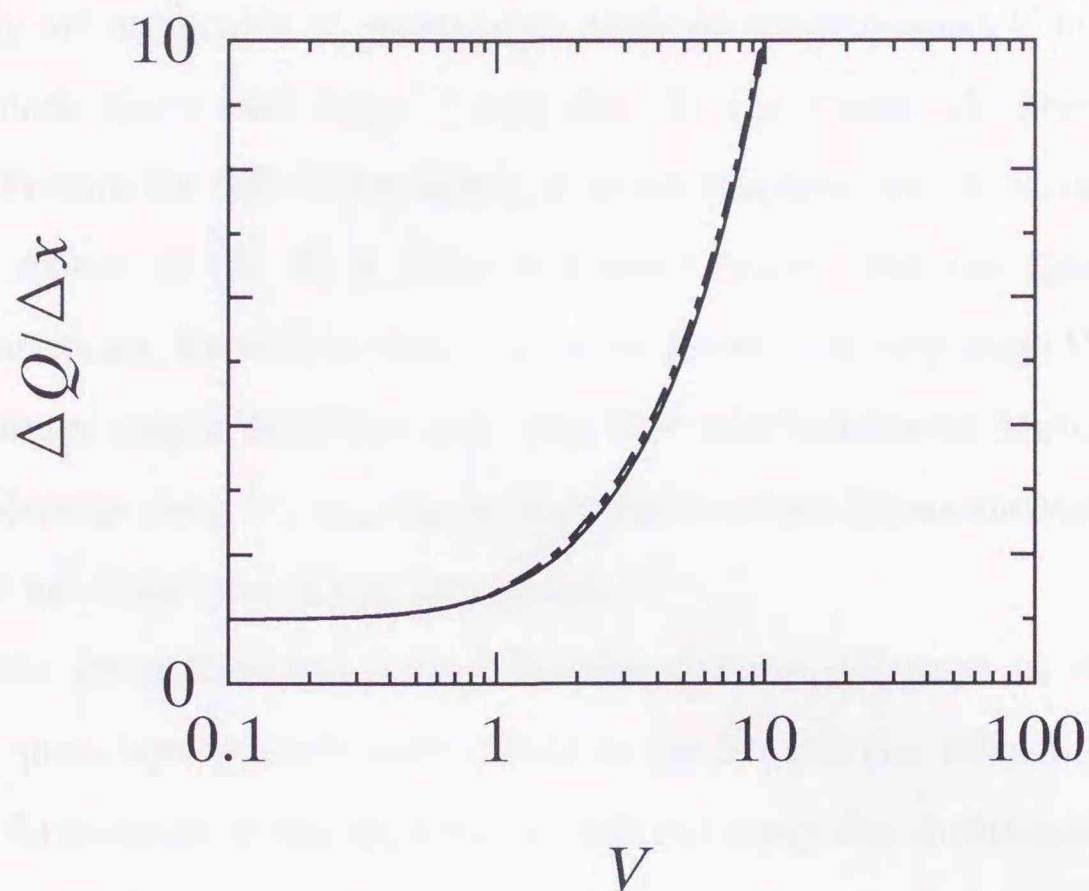


Figure 3.8 Dependence of the spatial extent ΔQ of the average intensity distribution on the V number for the power-law amplitude distribution. (—), (- - -), (- · -) and (· · ·) denote the results for Gaussian, and $G=2, 5$ and 100 , respectively.

3.8 Conclusions

In this chapter, the spatial coherence properties of the optical field in the Fraunhofer diffraction region produced from an optical fiber have been analyzed by considering an optical fiber-end source over which the optical field is expressed as an aggregation of plane waves with random directions and phases. For a Gaussian amplitude distribution at the source, the spatial coherence length and the spatial extent of the AI distribution have been analytically obtained in connection with the V number of the fiber by which the total number of guided modes can be estimated. Moreover, for some power-law amplitude distributions at the source, the same properties have been

obtained by performing the numerical analysis.

As a result, the expressions for the spatial coherence properties obtained in the present study are applicable to quasi-single-mode fibers with small V numbers as well as to multimode fibers with large V numbers. In the multimode fibers, the optical field in the Fraunhofer diffraction region is quasi-homogeneous in a statistical sense, because the extent of the AI is finite and much larger than the spatial coherence length. In particular, for well-excited multimode fibers with very large V numbers, the spatial coherence length coincides with that from the incoherent source obeying the van Cittert-Zernike theorem, and the optical field becomes homogeneous over the wide region in the neighborhood of the optical axis.

Finally, the generalized van Cittert-Zernike theorem [10] must be referred to. In general, the quasi-homogeneous optical field in the Fraunhofer diffraction region from the source is formulated by this theorem. In order to apply this theorem to the fiber-end source, however, it is necessary to know beforehand the spatial coherence properties of the optical field at the end of the fiber. Therefore, if the theoretical analysis from the calculation of $\langle V(\xi_1)V^*(\xi_2) \rangle$ is started, the generalized van Cittert-Zernike theorem is applicable to the fiber-end source and the same results obtained in the present study will also be expected to be found. However, this fact is not so essential for the present study, where the optical fiber properties as well as the spatial coherence properties of the optical field in the Fraunhofer diffraction field from the optical fiber are discussed.

3.9 Appendix: Derivation of Eq.(3.10)

By replacement of $\alpha = k^2 \langle \theta^2 \rangle / 2 = 2(\pi/\lambda)^2 \langle \theta^2 \rangle$ and $\beta = 1/\lambda R$, Eq.(3.8) is rewritten in the form

$$\begin{aligned}\Gamma(x_1, x_2) &= N \iint e_o(\xi_1) e_o^*(\xi_2) \exp[-\alpha(\xi_1 - \xi_2)^2] \\ &\times \exp[-i2\pi\beta(x_1\xi_1 - x_2\xi_2)] d\xi_1 d\xi_2.\end{aligned}\quad (3A.1)$$

Moreover, by changing the arguments as $x_1=x$ and $x_2=x_1+X$ and by changing the variables of integration into the forms,

$$\xi_2 - \xi_1 = \xi_- \quad \text{and} \quad \xi_1 = \xi, \quad (3A.2)$$

Eq.(3A.1) becomes

$$\begin{aligned}\Gamma(x, x+X) &= N \int d\xi_- \exp(-\alpha\xi_-^2) \exp[2\pi i\beta(x+X)\xi_-] \\ &\times \left[\int e_o(\xi) e_o^*(\xi + \xi_-) \exp(2\pi i\beta X\xi) d\xi \right].\end{aligned}\quad (3A.3)$$

With the use of the Fourier integral for the function $e_o(\xi)$

$$e_o(\xi) = \int E_o(u) \exp(2\pi i\beta u\xi) du, \quad (3A.4)$$

Eq.(3A.3) becomes

$$\begin{aligned}\Gamma(x, x+X) &= N \int d\xi_- \exp(-\alpha\xi_-^2) \exp[2\pi i\beta(x+X)\xi_-] \\ &\times \left[\exp(-2\pi i\beta X\xi_-) \int E_o(u) E_o^*(u+X) \exp(-2\pi i\beta u\xi_-) du \right].\end{aligned}\quad (3A.5)$$

Finally, with the change of the order of integrations, Eq.(3A.5) becomes

$$\Gamma(x, x+X) = N\sqrt{\pi/\alpha} \int E_o(u) E_o^*(u+X) \exp\left[-\pi^2\beta^2(x-u)^2/\alpha\right] du, \quad (3A.6)$$

which is equivalent to Eq.(3.10).

Chapter 4

Equi-ambiguity-function ellipse of Gaussian Schell-model beams

4.1 Introduction

A Gaussian Schell-model (GSM) beam [21, 22] is the partially coherent light radiating from a GSM source having a Gaussian intensity profile and a Gaussian complex degree of spatial coherence with stationarity in space. In particular, a coherent Gaussian beam and a Gaussian quasi-homogeneous beam [50] are obtained as the limiting cases of the GSM beam. The beam radius and the wave-front-curvature radius of the GSM beam have been investigated in terms of the cross-spectral density (CSD) [5], [7]–[10] depending on the propagation distance of the beam and the spatial coherence properties of the source [21, 22].

In addition, the ambiguity function (AF) of the partially coherent light field has been investigated by many authors [14], [34]–[36]. The AF is a useful tool for analyzing the partially coherent light field, because one can easily obtain the propagation of the AF by changing only the argument without changing its functional form in the paraxial approximation [34]–[36]. However, the AF properties of the GSM beam have not yet been investigated.

In this chapter, the AF properties of the GSM beam are analyzed by changing the propagation distance of the beam and the spatial coherence properties of the source. The behavior of the equi-AF ellipse (i.e., a trajectory of the AF at a constant value) is actually investigated in the phase space of the spatial frequency difference and the

spatial difference between the two points of the GSM beam. In Sec.4.2, the AF's of both the GSM source and the GSM beam are derived theoretically. In Sec.4.3, the behavior of the ellipse is investigated with respect to the rotation angle, and the area and the extent of the ellipse are also studied in relation to the GSM beam parameters.

As a result, the ellipse inclines for a finite value of the propagation distance and approaches a canonical form with the increasing propagation distance. Moreover, the area of the ellipse is constant regardless of the propagation distance, though the area decreases with the decreasing spatial coherence length of the source. This fact is explained in connection with the ambiguity (uncertainty) and the homogeneity of the GSM beam in the phase space. Furthermore, it is shown that the width of the ellipse on the spatial difference axis corresponds to the extent of the autocorrelation function of the GSM beam and that the width is also equivalent to the characteristic distance by which both the beam radius and the wave-front-curvature radius of the GSM beam are uniquely determined as a function of the propagation distance.

4.2 Theory

4.2.1 Ambiguity function of the Gaussian Schell-model source

The CSD $W_o(x_1, x_2)$ of a GSM source radiating the partially coherent monochromatic light is given by [21]

$$W_o(x_1, x_2) = \sqrt{I_o(x_1)I_o(x_2)}\gamma_o(x_2 - x_1), \quad (4.1)$$

where $I_o(x)$ and $\gamma_o(x)$ denote the optical intensity distribution and the complex degree of spatial coherence of the source, respectively, which are given by

$$I_o(x) = I_s \exp(-x^2/a^2), \quad (4.2)$$

$$\gamma_o(x) = \exp(-x^2/2x_s^2), \quad (4.3)$$

respectively. In Eqs.(4.2) and (4.3), I_s , a and x_s are the central intensity, the effective extent and the spatial coherence length of the source, respectively.

By substituting Eqs.(4.2) and (4.3) into Eq.(4.1) and by changing the arguments as $x=(x_1+x_2)/2$ and $\Delta x=x_2-x_1$ (i.e., the spatial difference between the two points), Eq.(4.1) is rewritten in the form

$$W_o(x, \Delta x) = I_s \exp\left(-\frac{2x^2}{a^2}\right) \exp\left[-\frac{1}{2}\left(\frac{1}{a^2} + \frac{1}{x_s^2}\right) \Delta x^2\right]. \quad (4.4)$$

On the other hand, the AF $A(\Delta\nu, \Delta x)$ of the CSD $W(x, \Delta x)$ at an arbitrary plane in space has already been introduced in Eq.(2.11) [14], [34]–[36]:

$$A(\Delta\nu, \Delta x) = \int W(x, \Delta x) \exp(-2\pi i x \Delta\nu) dx, \quad (4.5)$$

where $\Delta\nu$ is the spatial frequency difference. Therefore, by substituting Eq.(4.4) into Eq.(4.5), the AF for the GSM source is obtained as

$$A_o(\Delta\nu, \Delta x) = A_o(0, 0) \exp\left[-\frac{(\pi a \Delta\nu)^2}{2}\right] \exp\left[-\frac{1}{2}\left(\frac{1}{a^2} + \frac{1}{x_s^2}\right) \Delta x^2\right], \quad (4.6)$$

where $A_o(0, 0)$ is the constant value given by

$$A_o(0, 0) = \sqrt{\pi/2} I_s a. \quad (4.7)$$

4.2.2 Ambiguity function of the Gaussian Schell-model beam

Next, consider the AF $A_z(\Delta\nu, \Delta x)$ at the observation plane of the Fresnel diffraction region, a distance z from the source. $A_z(\Delta\nu, \Delta x)$ is obtained in a paraxial approximation by changing only the argument of Eq.(4.6) without changing its functional form

[34]–[36] (see Eq.(2.19)):

$$A_z(\Delta\nu, \Delta x) = A_o(\Delta\nu, \Delta x - \lambda z \Delta\nu), \quad (4.8)$$

where λ is the optical wavelength. Substitution of Eq.(4.6) into Eq.(4.8) yields

$$A_z(\Delta\nu, \Delta x) = A_o(0, 0) \exp \left[-\frac{(\pi a \Delta\nu)^2}{2} \right] \exp \left[-\frac{1}{2a^2} \left(1 + \frac{a^2}{x_s^2} \right) (\Delta x - \lambda z \Delta\nu)^2 \right]. \quad (4.9)$$

With the replacement

$$\Delta p = a \Delta\nu, \quad (4.10)$$

$$\Delta q = \Delta x / a, \quad (4.11)$$

$$Z = z / z_o, \quad (4.12)$$

where

$$z_o = \pi a^2 / \lambda, \quad (4.13)$$

Eq.(4.9) becomes

$$A_z(\Delta p, \Delta q) = A_o(0, 0) \exp \left[-\frac{1}{2} \left\{ (\pi \Delta p)^2 + \left(1 + \frac{1}{\sigma^2} \right) (\Delta q - \pi Z \Delta p)^2 \right\} \right], \quad (4.14)$$

where the ratio σ of x_s to a , i.e.,

$$\sigma = x_s / a \quad (4.15)$$

is used as the normalized spatial coherence length of the source. The limiting cases of this parameter result in a coherent Gaussian beam for $\sigma \rightarrow \infty$ and a Gaussian quasi-homogeneous beam for $\sigma \ll 1$ [50].

In this way, Δp and Δq can be regarded as the normalized spatial frequency difference and the normalized spatial difference, respectively. Moreover, Z indicates the propagation distance of the beam normalized by z_o , which is known as a parameter designating the waist region of a coherent Gaussian beam [49]. These physical values will be used in place of the real physical values of $\Delta\nu$, Δx and z in order to evaluate the AF of Eq.(4.9) in the following.

4.2.3 Parameters of the Gaussian Schell-model beam

With reference to the study of Friberg *et al.* [21], the beam radius $w(Z)$ and the wave-front-curvature radius $R(Z)$ of the GSM beam are given by

$$w(Z) = \sqrt{1 + CZ^2}, \quad (4.16)$$

$$R(Z) = Z + 1/CZ, \quad (4.17)$$

respectively, where $w(Z)$ and $R(Z)$ are normalized by a and z_o , respectively. Moreover, the parameter C is defined as

$$C = 1 + 1/\sigma^2, \quad (4.18)$$

which depends only on the normalized spatial coherence length σ of the source.

In Eqs.(4.16) and (4.17), a characteristic distance Z_o at which the beam radius becomes $\sqrt{2}$ times that at the source and at which the wave-front-curvature radius $R(Z)$ of the beam takes the minimum value can be defined as

$$\begin{aligned} Z_o &= 1/\sqrt{C} \\ &= 1/\sqrt{1 + 1/\sigma^2}. \end{aligned} \quad (4.19)$$

Apparently, Z_o may be regarded physically as the distance characterizing the waist region of the GSM beam by its definition.

With the characteristic distance Z_o , Eqs.(4.16) and (4.17) are rewritten as

$$w(Z) = \sqrt{1 + (Z/Z_o)^2}, \quad (4.20)$$

$$R(Z) = Z \left\{ 1 + (Z_o/Z)^2 \right\}, \quad (4.21)$$

respectively, which are formally the same as the expressions for the beam radius and the wave-front-curvature radius of a coherent Gaussian beam.

Thus, the behavior of two parameters, $w(Z)$ and $R(Z)$, as a function of Z is uniquely determined by the characteristic distance Z_o with respect to the GSM beam. In this

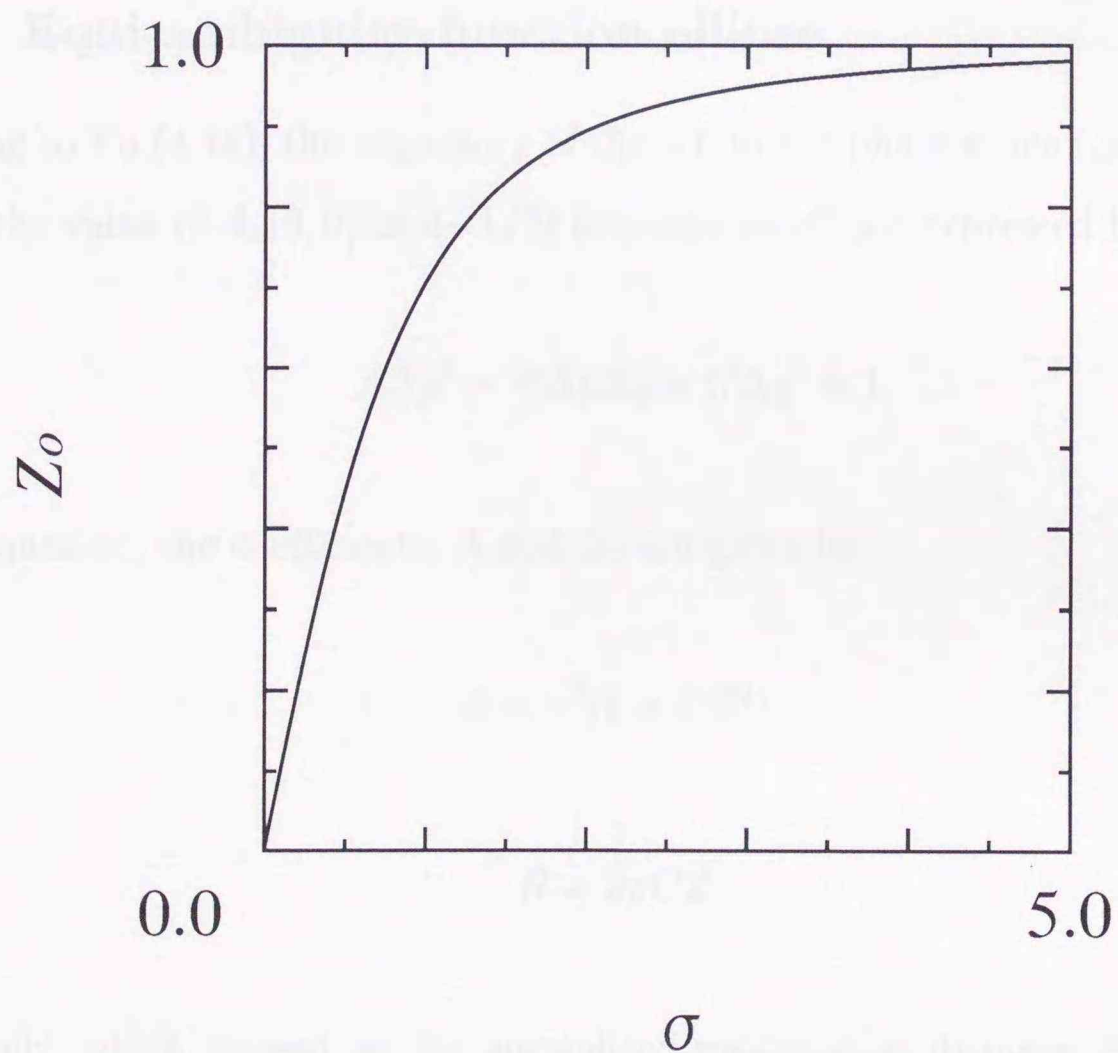


Figure 4.1 Dependence of the characteristic distance Z_o of the GSM beam on the normalized spatial coherence length σ of the GSM source.

case, as understood from Eq.(4.19), Z_o is related only to the normalized spatial coherence length σ of the source, and its dependence is shown in Fig.4.1. As seen in the figure, Z_o increases from zero with an increase of σ and finally approaches unity which actually corresponds to the real distance z_o . The behavior of Fig.4.1 implies physically that the waist region of the GSM beam is narrow for small σ 's (i.e., the case of low coherence) and becomes wide for large σ 's (i.e., the case of high coherence). In the following, the behavior of the equi-AF ellipse (which will be defined in Subsection 4.3.1) concerning the GSM beam will be investigated in detail in connection with the characteristic distance Z_o .

4.3 Properties of the equi-ambiguity-function ellipse

4.3.1 Equi-ambiguity-function ellipse

According to Eq.(4.14), the trajectory of the AF in the phase space $(\Delta p, \Delta q)$ on which it takes the value of $A_o(0,0) \exp(-1/2)$ becomes an ellipse expressed by

$$A\Delta p^2 - B\Delta p\Delta q + C\Delta q^2 = 1. \quad (4.22)$$

In this equation, the coefficients, A and B , are given by

$$A = \pi^2(1 + CZ^2), \quad (4.23)$$

$$B = 2\pi CZ, \quad (4.24)$$

respectively, which depend on the normalized propagation distance Z and the coefficient C given by Eq.(4.18).

4.3.2 Rotation of the ellipse

It is found from Eq.(4.22) that the equi-AF ellipse at the source plane always has a canonical form, because $B=0$ at $Z=0$. In this case, the axial lengths of the ellipse on the Δp and Δq axes are given by $1/\sqrt{A}$ and $1/\sqrt{C}$, respectively. Comparison of the parameters, A and C , at $Z=0$ indicates that the location of the major axis of the ellipse depends on the normalized spatial coherence length σ of the source in such a way that it is on the Δp axis for $A < C$ which means $\sigma < 1/\sqrt{\pi^2 - 1}$ and on the Δq axis for $A > C$ which means $\sigma > 1/\sqrt{\pi^2 - 1}$.

The rotation of the ellipse occurs for a finite value of the normalized propagation distance Z . The behavior of the ellipse accompanied with the rotation is shown in Fig.4.2 for $Z=0, 1.0$ and 2.0 , when the normalized spatial coherence length σ of the source is $1.0, 0.5$ or 0.2 . As seen in the figure, the ellipse rotates with the change of Z and the shape of the ellipse elongates noticeably along the major axis with an increase of σ . It is also recognized in Fig.4.2 that the area of the ellipse becomes small with a decrease of σ .

Such behavior of the ellipse characterizing the AF of the GSM beam is examined by transformation of the ellipse to a canonical form. In general, the canonical form of the ellipse given by Eq.(4.22) is obtained by the rotational transformation of coordinates as

$$A'\Delta p'^2 + C'\Delta q'^2 = 1, \quad (4.25)$$

where the rotation angle θ is chosen to be

$$\theta = \frac{1}{2} \tan^{-1}(s), \quad (4.26)$$

with the argument s given by

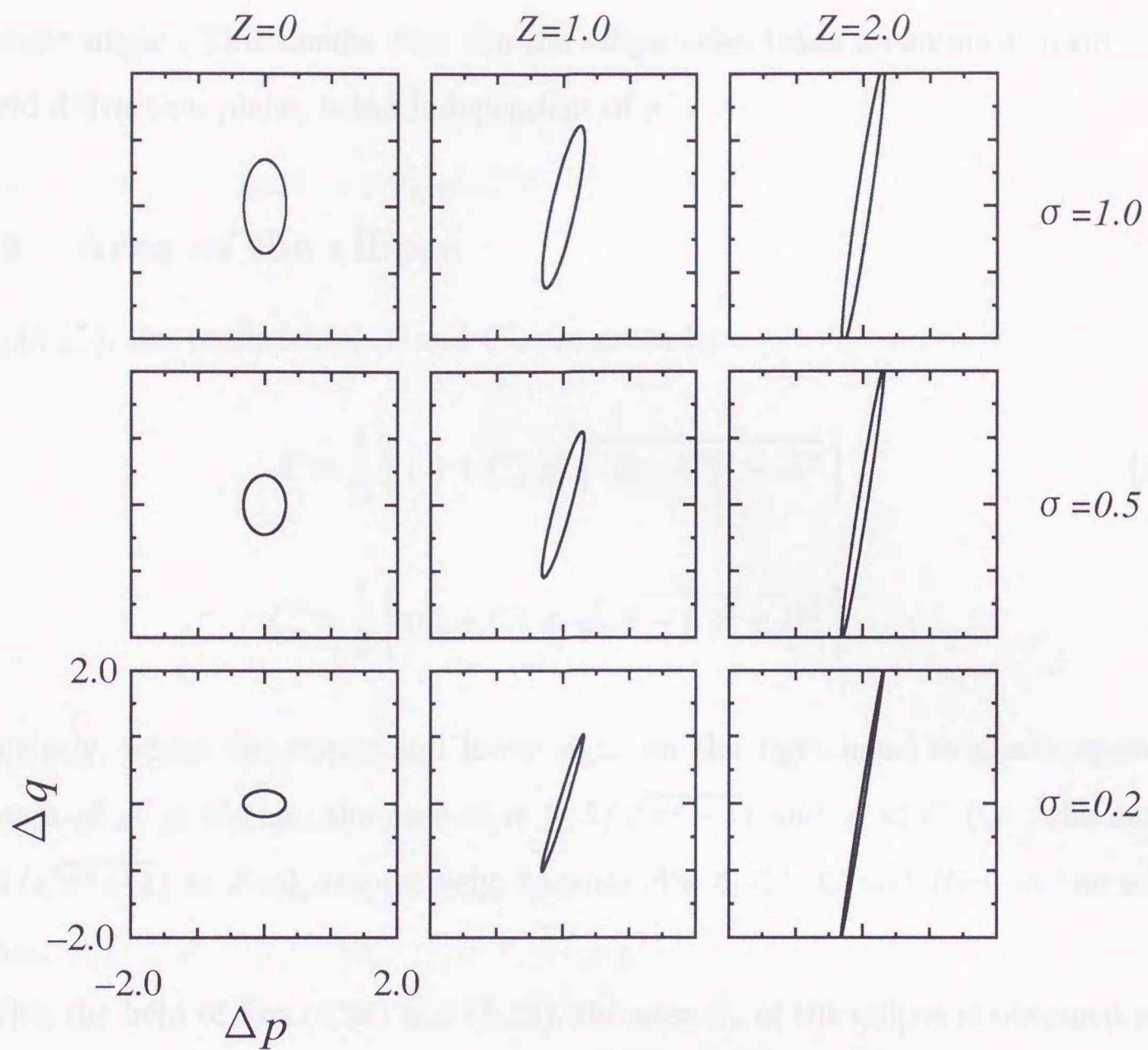


Figure 4.2 Behavior of the equi-AF ellipse in the coordinate systems of Δp and Δq for $Z=0, 1.0$ and, 2.0 (from the left-hand to the right-hand side) and for $\sigma=1.0, 0.5$ and, 0.2 (from top to bottom).

$$\begin{aligned}
 s &= -\frac{B}{A-C} \\
 &= -\left(\frac{2}{\pi}\right) \frac{Z}{Z^2 - (1/\pi^2 - 1/C)}. \quad (4.27)
 \end{aligned}$$

The rotation angle θ denotes an inclination angle of the ellipse on the basis of the canonical form for $Z=0$. From Eqs.(4.26) and (4.27), it is understood that, when Z is sufficiently large, the value of s approaches zero at which the rotation angle θ is zero or a right angle. This means that the ellipse also takes a canonical form in the far-field diffraction plane, being independent of σ .

4.3.3 Area of the ellipse

In Eq.(4.25), the coefficients, A' and C' , are given by

$$A' = \frac{1}{2} \left[(A + C) \pm \sqrt{(A - C)^2 + B^2} \right], \quad (4.28)$$

$$C' = \frac{1}{2} \left[(A + C) \mp \sqrt{(A - C)^2 + B^2} \right], \quad (4.29)$$

respectively, where the upper and lower signs on the right-hand side correspond to the cases of $A \geq C$ (i.e., the case of $\sigma \geq 1/\sqrt{\pi^2 - 1}$) and $A < C$ (i.e., the case of $\sigma < 1/\sqrt{\pi^2 - 1}$) at $Z=0$, respectively, because $A'=A$, $C'=C$ and $B=0$ at the source plane.

With the help of Eqs.(4.28) and (4.29), the area S_e of the ellipse is obtained as

$$\begin{aligned} S_e &= \pi / \sqrt{A'C'} \\ &= 1 / \sqrt{C} \\ &= 1 / \sqrt{1 + 1/\sigma^2}, \end{aligned} \quad (4.30)$$

where the relation of $4AC - B^2 = 4\pi^2 C$ is used. Thus, the area S_e of the ellipse in the phase space $(\Delta p, \Delta q)$ is numerically equal to the characteristic distance Z_o of the GSM beam given by Eq.(4.19). As a result, the area is independent of the normalized propagation distance Z and depends only on the normalized spatial coherence length σ of the source. As seen in Fig.4.1, the area S_e of the ellipse increases with an increase of σ

and approaches unity for the completely coherent beam. An increase in the ellipse area implies that the ambiguity (uncertainty) in the phase space $(\Delta p, \Delta q)$ increases with the increasing spatial coherence length of the source, because the region of homogeneity in the GSM beam becomes wide.

4.3.4 Extent and width of the ellipse

The extents of the ellipse (i.e., the maximum values of Δp and Δq shown in Fig.4.3) are found by the examination of the discriminants of Eq.(4.22), and are given by

$$|\Delta p_{max}| = 1/\pi, \quad (4.31)$$

$$|\Delta q_{max}| = \sqrt{Z^2 + 1/C}, \quad (4.32)$$

respectively. The extent of the ellipse in the direction of the Δq axis depends on both the normalized propagation distance Z and the normalized spatial coherence length σ of the source, while the extent in the direction of the Δp axis is fixed to be $1/\pi$.

If the characteristic distance Z_o is used in place of C , Eq.(4.32) is rewritten as

$$\begin{aligned} |\Delta q_{max}| &= Z_o \sqrt{1 + (Z/Z_o)^2} \\ &= Z_o w(Z), \end{aligned} \quad (4.33)$$

which is proportional to the beam radius $w(Z)$. Moreover, the widths, Δp_w and Δq_w , are defined in Fig.4.3 as the points on the ellipse passing across the Δp and Δq axes, respectively, and are obtained by setting $\Delta q=0$ and $\Delta p=0$ in Eq.(4.22), respectively:

$$\Delta p_w = 1/\pi w(Z), \quad (4.34)$$

$$\Delta q_w = Z_o. \quad (4.35)$$

These results are of particular importance, because two significant quantities, $w(Z)$ and Z_o of the GSM beam, are described by two widths, Δp_w and Δq_w , respectively, of the equi-AF ellipse. That is, the beam radius $w(Z)$ is given by the inverse value of $\pi\Delta p_w$, and the characteristic distance Z_o is given by Δq_w itself.

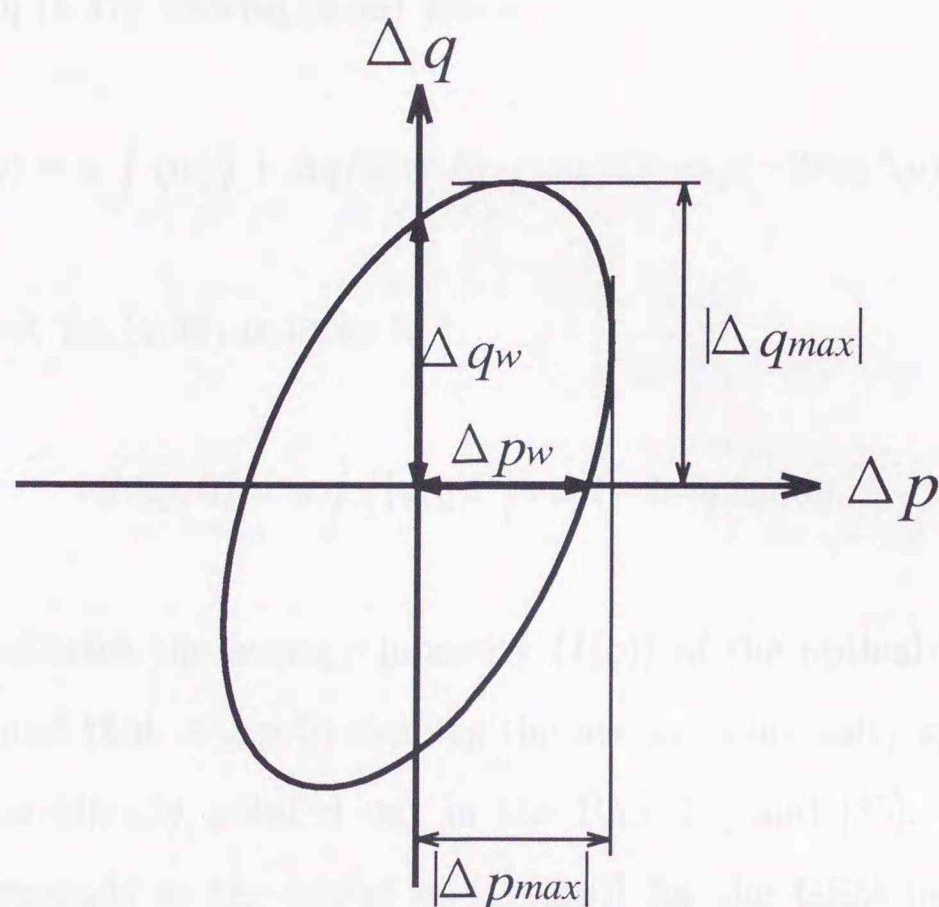


Figure 4.3 Extents, $|\Delta p_{max}|$ and $|\Delta q_{max}|$, of the ellipse in the directions of Δp and Δq , respectively, and widths, Δp_w and Δq_w , on the axes of Δp and Δq , respectively.

4.4 Discussion

In order to make clearer the fundamentals of the results of Eqs.(4.34) and (4.35), the AF of Eq.(4.5) is expressed by using the parameters of Δp in Eq.(4.10) and Δq in Eq.(4.11):

$$A(\Delta p, \Delta q) = a \int W(q, \Delta q) \exp(-2\pi i q \Delta p) dq, \quad (4.36)$$

where $q=x/a$. Moreover, the CSD $W(q, \Delta q)$ is expressed in terms of the optical field $u(q)$ and is given by [5], [7]–[10]

$$W(q, \Delta q) = \langle u(q + \Delta q/2) u^*(q - \Delta q/2) \rangle, \quad (4.37)$$

where $*$ and $\langle \rangle$ denote the complex conjugate and the ensemble average, respectively. Substitution of Eq.(4.37) into Eq.(4.36) yields

$$A(\Delta p, \Delta q) = a \int \langle u(q + \Delta q/2) u^*(q - \Delta q/2) \rangle \exp(-2\pi i q \Delta p) dq. \quad (4.38)$$

First, for $\Delta q=0$, Eq.(4.38) reduces to

$$A(\Delta p, 0) = a \int \langle |u(q)|^2 \rangle \exp(-2\pi i q \Delta p) dq, \quad (4.39)$$

where $\langle |u(q)|^2 \rangle$ indicates the average intensity $\langle I(q) \rangle$ of the optical field $u(q)$. From Eq.(4.39), it is found that $A(\Delta p, 0)$ denotes the averaged intensity spectrum $\langle \tilde{I}(\Delta p) \rangle$ of $u(q)$. This was already pointed out in the Refs.[34] and [35]. Therefore, Δp_w in Eq.(4.34) corresponds to the extent of $\langle \tilde{I}_Z(\Delta p) \rangle$ for the GSM beam located at a distance Z from the source. For this reason, it is natural that Δp_w should be inversely proportional to the beam radius $w(Z)$. As a result, Δp_w decreases with an increase of Z and the shape of the equi-AF ellipse becomes thin.

Next, for $\Delta p=0$, Eq.(4.38) becomes

$$A(0, \Delta q) = a \int \langle u(q + \Delta q/2) u^*(q - \Delta q/2) \rangle dq, \quad (4.40)$$

which is the autocorrelation function [36, 51] of $u(q)$. Therefore, Δq_w in Eq.(4.35) corresponds to the extent of the autocorrelation function of the GSM beam located at a distance Z from the source. As a result, it has been revealed that the extent Δq_w is identical to the characteristic distance Z_o and takes a constant value to be

determined only by the normalized spatial coherence length σ of the source, regardless of the normalized propagation distance Z .

In this way, the propagation properties of the partially coherent light radiating from the GSM source can be easily known through the behavior of equi-AF ellipses, for example, as shown in Fig.4.2.

4.5 Conclusions

In this chapter, the behavior of the equi-AF ellipse with respect to a GSM beam has been investigated in connection with the propagation properties of the partially coherent light radiating from a GSM source. In this case, the ellipses are formed in the phase space $(\Delta p, \Delta q)$ where the values of both coordinates are normalized by the effective extent a of the source in such a way that $\Delta p = a\Delta\nu$ and $\Delta q = \Delta x/a$ for the real spatial frequency difference $\Delta\nu$ and the real two-point distance Δx , respectively.

It has been found that the shape of the ellipses is determined by the characteristic distance Z_o by which the beam radius $w(Z)$ and the wave-front-curvature radius $R(Z)$ of the GSM beam are uniquely specified as a function of the normalized propagation distance Z of the beam. In this case, the characteristic distance Z_o is physically the depth of the waist region of the beam, and it is determined only by the spatial coherence length σ of the source normalized by a .

As a consequence, several interesting results with respect to the equi-AF ellipse have been obtained:

- (a) The ellipse always passes across the fixed points of $(0, Z_o)$ and $(0, -Z_o)$.
- (b) The area of the ellipses is constant, independent of Z , and numerically equivalent to Z_o .
- (c) The maximum extent in the direction of Δp is fixed to be $1/\pi$.
- (d) The maximum extent in the direction of Δq is proportional to the beam radius

$w(Z)$. Therefore, its extent increases with the increasing normalized propagation distance Z .

- (e) On the basis of the above results, the ellipse elongates toward the direction of the Δq axis and rotates with an increase of Z and, consequently, the width along the Δp axis becomes thin.
- (f) The point $(1/\pi w(Z), 0)$ lies on the ellipse. In other words, the beam radius $w(Z)$ is known from that point.

Among the above results, the first item (a) is of particular importance, because the fundamental parameter Z_0 of the GSM beam is firmly connected to the equi-AF ellipse.

Chapter 5

Far-field analysis of fluctuating supermodes radiated from phase-locked array lasers

5.1 Introduction

Semiconductor array lasers are widely expected as the sources for optical multiplex communication systems and optical parallel processing, because they are fabricated on a monolithic substrate and connected with micro-optics components. There are mainly three types of array lasers: the index-guided, gain-guided and surface-emitting ones. It has been known that the optical fields radiated from such sources are supermodes under the existing condition of the nearest-neighbor coupling between the emitters. In general, supermodes are derived from the weakly-coupled mode theory [24]–[29], and they are expressed as the coherent superposition of the single-transverse mode of each emitter without coupling. Moreover, the experimental verification has already been performed [52, 53].

On the other hand, the theoretical studies on far-field patterns of supermodes [24]–[29], [54]–[56] have also been conducted to judge whether or not the array lasers have the expected characteristics, because the far-field patterns can be easily observed by simple means. In these studies, it has been assumed that each emitter, which is isolated from its neighbors, radiates the same deterministic single-transverse mode. In practice, however, there are many uncertain factors which are physically unavoidable inside the

array lasers: the structural fluctuations of the waveguide of each emitter caused by the imperfection in the manufacturing process, the inhomogeneity of the gain distribution between the emitters caused by the instability of the injection current, and so on. Therefore, it is considered that the mode of each emitter is not deterministic but stochastic, and that supermodes radiated from the array lasers should be expressed as a stochastic process.

In this chapter, the effects of fluctuations of supermodes radiated from the array laser on the far-field patterns are studied by using the mutual intensity (MI) [4]–[10]. Expressions for the MI and the average intensity (AI) in the far field produced from supermodes with fluctuations are derived on the basis of the references [24]–[29], [54]–[56]. It is assumed that the supermodes are formed by the coherent superposition of the stochastic mode of each emitter; the stochastic mode is expressed as the summation of the deterministic and fluctuating components. Moreover, under the assumption that the fluctuating component of stochastic modes obeys Gaussian statistics, expressions for the MI and the AI are given in explicit forms. The final expression for the AI involves two statistical parameters, R and $\Delta\xi_f$, concerning fluctuations of supermodes. R denotes the ratio of the integrated AI of the fluctuating component to the total integrated AI of the stochastic mode of each emitter, and $\Delta\xi_f$ denotes the spatial correlation length for the fluctuating component of the stochastic mode of each emitter.

As a result, the far-field patterns are gradually deformed with increases of R and $\Delta\xi_f$ in comparison with those produced from supermodes without fluctuations, while the background intensities increase with an increase of $\Delta\xi_f$. However, the patterns approach those produced from supermodes without fluctuations, with the increasing N number of emitters in the array laser. Furthermore, there is almost no difference between the results of the effects of R , $\Delta\xi_f$ and N on the far-field patterns for the 1st- and highest-order supermodes.

5.2 Theory

5.2.1 Supermodes with fluctuations at the emitting plane

The analyzing optical system involving the index-guided array laser as a source is schematically shown in Fig.5.1(a). In this type of array laser, each emitter is formed by sandwiching the core (the plane area) between the claddings (the hatching areas), and only the single-transverse mode is excited. In the figure, the end of the array laser

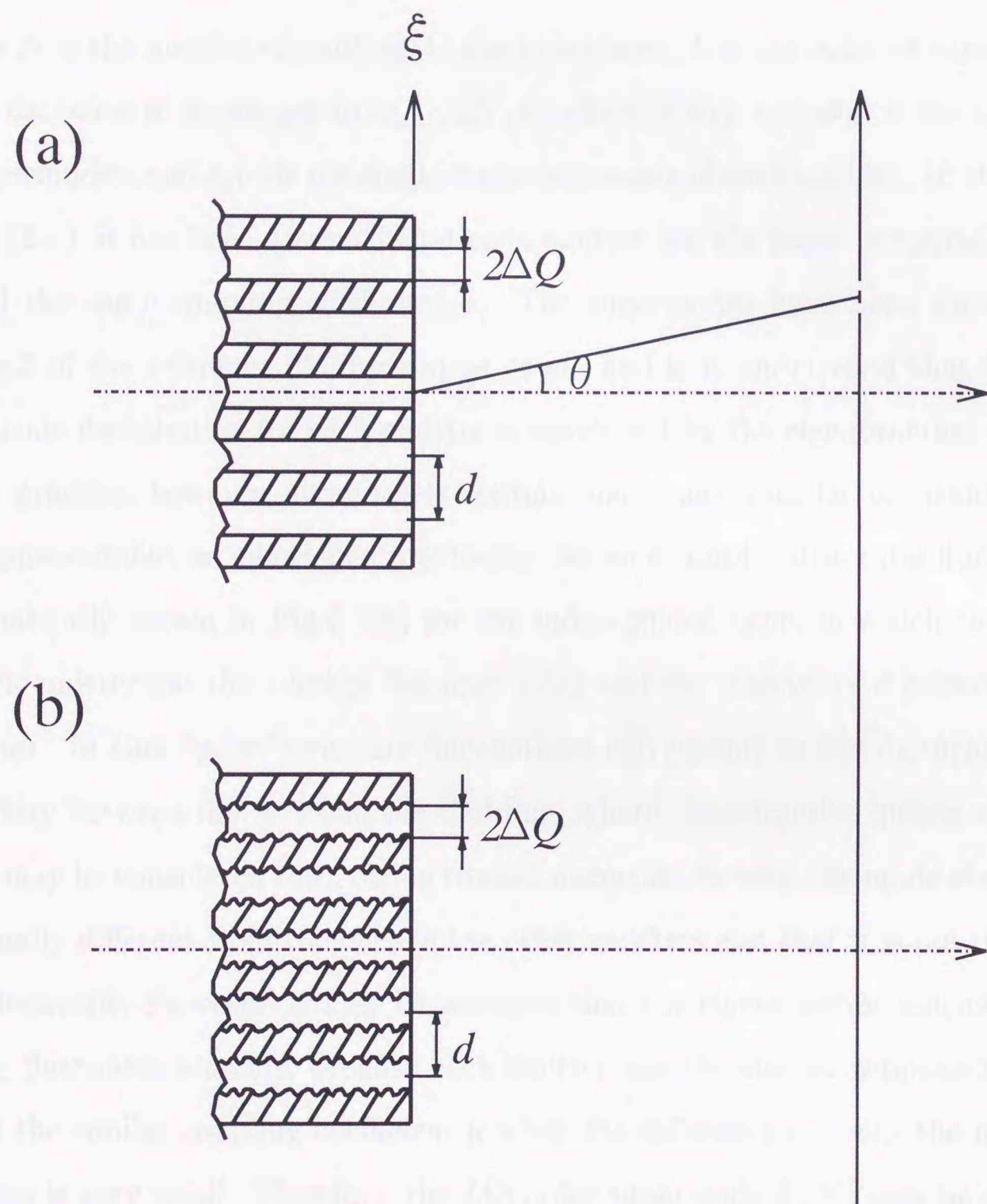


Figure 5.1 Index-guided array laser and optical coordinates: (a) the laser without structural fluctuations and (b) the one with the fluctuations.

is located at the plane ξ , and the emitters are placed at intervals of d , the aperture size of each emitter being $2\Delta Q$.

In general, when there exists the nearest-neighbor coupling between the emitters, the L th order supermode $E_L(\xi)$ is derived, regardless of types of array lasers, from the weakly-coupled mode theory and is given by [24]–[29]

$$E_L(\xi) = \sum_{n=1}^N \sin(n\theta_L) e_o \left[\xi - \left(n - \frac{N+1}{2} \right) d \right], \quad (5.1)$$

where N is the number of emitters in the array laser, L is the order of supermodes and takes the value of an integer from 1 to N , $\theta_L = L\pi/(N+1)$, $\sin(n\theta_L)$ is the eigenfunction of supermodes, and $e_o(\)$ is the single-transverse mode of each emitter. In the derivation of Eq.(5.1), it has been assumed that each emitter has the same propagation constant β and the same coupling coefficient κ . The supermodes have been shown for $N=5$ in Fig.2 of the reference [26] by Kapon *et al.*, and it is understood that the complex amplitude distribution for each emitter is enveloped by the eigenfunction $\sin(n\theta_L)$.

In practice, however, array lasers contain many uncertain factors which are physically unavoidable, as mentioned previously. As an example, structural fluctuations are schematically shown in Fig.5.1(b) for the index-guided type, in which the waveguide of each emitter has the average diameter $2\Delta Q$ and the interval of d between the nearest ones. In this figure, structure fluctuations correspond to the disturbances at the boundary between the core and the cladding, where the refractive indices are different.

It may be considered that, owing to such uncertain factors, the mode of each emitter is actually different from the ones of the other emitters and that it is not deterministic but stochastic. However, it may be assumed that the eigenfunction $\sin(n\theta_L)$ of supermodes fluctuates scarcely, because each emitter has the similar propagation constant β and the similar coupling coefficient κ when the difference between the modes of the emitters is very small. Therefore, the L th order supermode $E_L(\xi)$ may be expressed as

$$E_L(\xi) = \sum_{n=1}^N \sin(n\theta_L) e_n \left[\xi - \left(n - \frac{N+1}{2} \right) d \right], \quad (5.2)$$

where $e_n(\cdot)$ is the mode of each emitter involving stochastic factors and may be expressed as

$$e_n(\xi) = r_{(D)}e_o(\xi) + r_{(F)}\Delta e_n(\xi). \quad (5.3)$$

Here, $e_o(\xi)$ denotes the mode of each emitter without fluctuations and is the deterministic component of $e_n(\xi)$. This component is assumed to be common in all emitters. On the other hand, $\Delta e_n(\xi)$ denotes the fluctuating component of $e_n(\xi)$. Furthermore, $r_{(D)}$ and $r_{(F)}$ are the weighting coefficients of the deterministic and fluctuating components, respectively, and will be determined in a way that the total integrated AI of the mode $e_n(\xi)$ of an emitter should be constant regardless of the degree of fluctuations. The actual values of these two parameters are given in the next subsection.

5.2.2 Mutual intensity at the emitting plane

In general, the MI $\Gamma_L(\xi, \Delta\xi)$ at the two points, ξ_1 and ξ_2 , at the emitting plane ξ is defined as

$$\Gamma_L(\xi, \Delta\xi) = \langle E_L(\xi + \Delta\xi/2) E_L^*(\xi - \Delta\xi/2) \rangle, \quad (5.4)$$

where $\xi = (\xi_1 + \xi_2)/2$ and $\Delta\xi = \xi_2 - \xi_1$. Moreover, $*$ and $\langle \rangle$ denote the complex conjugate and the ensemble average, respectively. In deriving $\Gamma_L(\xi, \Delta\xi)$ by substituting Eq.(5.2) into Eq.(5.4), the following assumptions are proposed:

- (i) $\langle \Delta e_n(\xi) \rangle = 0$.
- (ii) $\Delta e_n(\xi)$ and $\Delta e_m(\xi)$ for $n \neq m$ are statistically independent of each other.
- (iii) $e_o(\xi)$ and $\Delta e_n(\xi)$ are also statistically independent of each other.

Under these assumptions, the MI $\Gamma_L(\xi, \Delta\xi)$ of Eq.(5.4) is expressed as (see Subsection 5.5.1):

$$\Gamma_L(\xi, \Delta\xi) = \Gamma_{L(D)}(\xi, \Delta\xi) + \Gamma_{L(F)}(\xi, \Delta\xi), \quad (5.5)$$

where $\Gamma_{L(D)}(\xi, \Delta\xi)$ is the deterministic term given by

$$\begin{aligned} \Gamma_{L(D)}(\xi, \Delta\xi) &= r_{(D)}^2 \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) \\ &\times \Gamma_o \left[\xi - \frac{1}{2} \{ (n+m) - (N+1) \} d, \Delta\xi - (n-m)d \right], \end{aligned} \quad (5.6)$$

and $\Gamma_{L(F)}$ is the fluctuating term given by

$$\Gamma_{L(F)}(\xi, \Delta\xi) = r_{(F)}^2 \sum_{n=1}^N \sin^2(n\theta_L) \Gamma_{\Delta n} \left[\xi - \left(n - \frac{N+1}{2} \right) d, \Delta\xi \right]. \quad (5.7)$$

Moreover, $\Gamma_o(\xi, \Delta\xi)$ in Eq.(5.6) denotes the MI in the mode $e_o(\xi)$ of each emitter without fluctuations and is given by

$$\Gamma_o(\xi, \Delta\xi) = e_o(\xi + \Delta\xi/2) e_o^*(\xi - \Delta\xi/2). \quad (5.8)$$

When $e_o(\xi)$ is assumed to take a Gaussian form given by

$$e_o(\xi) = \exp(-\xi^2/\Delta Q^2), \quad (5.9)$$

the MI in Eq.(5.8) becomes

$$\begin{aligned} \Gamma_o(\xi, \Delta\xi) &= \exp(-2\xi^2/\Delta Q^2) \exp(-\Delta\xi^2/2\Delta Q^2) \\ &= i_o(\xi) \gamma_o(\Delta\xi), \end{aligned} \quad (5.10)$$

where $i_o(\xi)$ is the first Gaussian function denoting the intensity and $\gamma_o(\Delta\xi)$ is the second one denoting the complex degree of spatial coherence of $e_o(\xi)$ at $\xi=0$, respectively.

Furthermore, $\Gamma_{\Delta n}(\xi, \Delta\xi)$ in the expression Eq.(5.7) denotes the MI of the fluctuating component $\Delta e_n(\xi)$ and is defined as

$$\Gamma_{\Delta n}(\xi, \Delta\xi) = \langle \Delta e_n(\xi + \Delta\xi/2) \Delta e_n^*(\xi - \Delta\xi/2) \rangle. \quad (5.11)$$

For simplicity, it is assumed that $\Gamma_{\Delta n}(\xi, \Delta\xi)$ takes the same form as $\Gamma_{\Delta o}(\xi, \Delta\xi)$ for all values of n . Namely,

$$\begin{aligned}\Gamma_{\Delta n}(\xi, \Delta\xi) &= \Gamma_{\Delta o}(\xi, \Delta\xi) \\ &= \langle i_{\Delta o}(\xi) \rangle \gamma_{\Delta o}(\Delta\xi),\end{aligned}\tag{5.12}$$

where it has been assumed that $\Gamma_{\Delta o}(\xi, \Delta\xi)$ is given by the product of the AI $\langle i_{\Delta o}(\xi) \rangle$ and the complex degree of spatial coherence $\gamma_{\Delta o}(\Delta\xi)$ with respect to $\Delta e_n(\xi)$, and that they are given by

$$\langle i_{\Delta o}(\xi) \rangle = \exp(-2\xi^2/\Delta Q'^2),\tag{5.13}$$

$$\gamma_{\Delta o}(\Delta\xi) = \exp(-\Delta\xi^2/2\Delta\xi_f^2),\tag{5.14}$$

respectively, where $\Delta Q'$ is the spatial extent of the AI and $\Delta\xi_f$ is the spatial correlation length of $\Delta e_n(\xi)$.

Next, determine the parameters, $r_{(D)}$ and $r_{(F)}$, included in Eqs.(5.3), (5.6) and (5.7), by using the total integrated AI P_e of the mode $e_n(\xi)$ of an emitter which is isolated from its neighbors. Under the previous assumptions of (i)–(iii), the AI $\langle i_n(\xi) \rangle$ of $e_n(\xi)$ is expressed as

$$\begin{aligned}\langle i_n(\xi) \rangle &= \langle e_n(\xi) e_n^*(\xi) \rangle \\ &= r_{(D)}^2 i_o(\xi) + \gamma_{(F)}^2 \langle i_{\Delta o}(\xi) \rangle \\ &= r_{(D)}^2 \exp(-2\xi^2/\Delta Q^2) + r_{(F)}^2 \exp(-2\xi^2/\Delta Q'^2),\end{aligned}\tag{5.15}$$

where, in the right-hand side, the first and second terms result from the deterministic component $e_o(\xi)$ and the fluctuating one $\Delta e_n(\xi)$, respectively. The total integrated AI

P_e is expressed as follows:

$$\begin{aligned}
P_e &= \int \langle i_n(\xi) \rangle d\xi \\
&= r_{(D)}^2 \int i_o(\xi) d\xi + r_{(F)}^2 \int \langle i_{\Delta o}(\xi) \rangle d\xi \\
&= r_{(D)}^2 \sqrt{\pi/2\Delta Q} + r_{(F)}^2 \sqrt{\pi/2\Delta Q'} \\
&= P_{e(D)} + P_{e(F)},
\end{aligned} \tag{5.16}$$

where $P_{e(D)}$ and $P_{e(F)}$ are the integrated AI's for the deterministic and fluctuating components of $e_n(\xi)$, respectively. By assuming that the total integrated AI is conserved regardless of the magnitude of fluctuations, its value is obtained, for $r_{(D)}=1$ and $r_{(F)}=0$ (without fluctuations), as

$$P_e = \sqrt{\pi/2\Delta Q}. \tag{5.17}$$

Here, define a parameter R as

$$R = P_{e(F)}/P_e, \tag{5.18}$$

which indicates the ratio of the integrated AI of the fluctuating component to the total integrated AI of $e_n(\xi)$. Then, by using the parameter R , Eq.(5.16) is expressed as another form

$$\begin{aligned}
P_e &= P_{e(D)} + P_{e(F)} \\
&= (1 - R)P_e + RP_e \\
&= (1 - R)\sqrt{\pi/2\Delta Q} + R\sqrt{\pi/2\Delta Q}.
\end{aligned} \tag{5.19}$$

Therefore, comparison of Eq.(5.16) with Eq.(5.19) determines the parameters, $r_{(D)}$ and $r_{(F)}$, with the forms of

$$r_{(D)} = \sqrt{1 - R}, \quad (5.20)$$

$$r_{(F)} = \sqrt{R\Delta Q/\Delta Q'}, \quad (5.21)$$

respectively.

Finally, by substituting Eqs.(5.20) and (5.21) into Eqs.(5.6) and (5.7), the MI $\Gamma_L(\xi, \Delta\xi)$ at the emitting plane ξ is expressed as

$$\begin{aligned} \Gamma_L(\xi, \Delta\xi) &= \Gamma_{(D)}(\xi, \Delta\xi) + \Gamma_{(F)}(\xi, \Delta\xi) \\ &= (1 - R) \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) \\ &\quad \times \Gamma_o \left[\xi - \frac{1}{2} \{ (n + m) - (N + 1) \} d, \Delta\xi - (n - m)d \right] \\ &\quad + (R\Delta Q/\Delta Q') \sum_{n=1}^N \sin^2(n\theta_L) \Gamma_{\Delta o} \left[\xi - \left(n - \frac{N + 1}{2} \right) d, \Delta\xi \right]. \end{aligned} \quad (5.22)$$

5.2.3 Mutual and average intensities in the far field

In the present study, the coordinates in the far field are denoted by the angle θ between the optical axis and the line drawn from an arbitrary point of the far field to an origin of the plane ξ . Moreover, the operators of F_θ and $F_{\Delta\theta}$ are defined as

$$F_\theta [u(\Delta\xi)] = \int u(\Delta\xi) \exp(-ik\Delta\xi \sin \theta) d\Delta\xi, \quad (5.23)$$

$$F_{\Delta\theta} [u(\xi)] = \int u(\xi) \exp(-ik\xi \sin \Delta\theta) d\xi, \quad (5.24)$$

respectively, where $k=2\pi/\lambda$, λ being the free-space wavelength. These operators are useful to transform the MI from the space $(\xi, \Delta\xi)$ to the space $(\theta, \Delta\theta)$ (see Subsection 5.5.2). Then, the expression for the MI $\Gamma_L(\theta, \Delta\theta)$ at the two points, θ_1 and θ_2 , is given in a paraxial approximation by a Fourier transform of $\Gamma_L(\xi, \Delta\xi)$. Therefore, $\Gamma_L(\theta, \Delta\theta)$ is obtained as

$$\begin{aligned}\Gamma_L(\theta, \Delta\theta) &= F_\theta F_{\Delta\theta} [\Gamma_L(\xi, \Delta\xi)] \\ &= \Gamma_{L(D)}(\theta, \Delta\theta) + \Gamma_{L(F)}(\theta, \Delta\theta),\end{aligned}\quad (5.25)$$

where $\theta=(\theta_1+\theta_2)/2$ and $\Delta\theta=\theta_2-\theta_1$.

In Eq.(5.25), $\Gamma_{L(D)}(\theta, \Delta\theta)$ is the deterministic term given by

$$\Gamma_{L(D)}(\theta, \Delta\theta) = -\frac{1}{4}(-1)^L[A][B](1-R)\tilde{i}_o\left(\frac{\sin \Delta\theta}{\lambda}\right)\tilde{\gamma}_o\left(\frac{\sin \theta}{\lambda}\right), \quad (5.26)$$

where $\tilde{i}_o(\)$ and $\tilde{\gamma}_o(\)$ denote the intensity spectrum and the mutual spectrum, respectively, which are given by

$$\begin{aligned}\tilde{i}_o(\sin \Delta\theta/\lambda) &= F_{\Delta\theta} [i_o(\xi)] \\ &= \sqrt{\pi/2\Delta Q} \exp \left[-\frac{1}{2} \left(\frac{k\Delta Q \sin \Delta\theta}{2} \right)^2 \right],\end{aligned}\quad (5.27)$$

$$\begin{aligned}\tilde{\gamma}_o(\sin \theta/\lambda) &= F_\theta [\gamma_o(\Delta\xi)] \\ &= \sqrt{2\pi\Delta Q} \exp \left[-2 \left(\frac{k\Delta Q \sin \theta}{2} \right)^2 \right],\end{aligned}\quad (5.28)$$

respectively. Moreover, A and B in Eq.(5.26) are given by

$$A = \frac{\sin [N(P_+ + \theta_L)/2]}{\sin [(P_+ + \theta_L)/2]} - (-1)^L \frac{\sin [N(P_+ - \theta_L)/2]}{\sin [(P_+ - \theta_L)/2]}, \quad (5.29)$$

$$B = \frac{\sin [N(P_- + \theta_L)/2]}{\sin [(P_- + \theta_L)/2]} - (-1)^L \frac{\sin [N(P_- - \theta_L)/2]}{\sin [(P_- - \theta_L)/2]}, \quad (5.30)$$

respectively, where $\theta_L = L\pi/(N+1)$ and the abbreviations

$$P_+ = kd \left(\frac{\sin \Delta\theta}{2} + \sin \theta \right) \quad (5.31)$$

and

$$P_- = kd \left(\frac{\sin \Delta\theta}{2} - \sin \theta \right) \quad (5.32)$$

are used.

On the other hand, $\Gamma_{L(F)}(\theta, \Delta\theta)$ in Eq.(5.25) is the fluctuating term given by

$$\Gamma_{L(F)}(\theta, \Delta\theta) = \begin{cases} \frac{1}{2}[C](R\Delta Q/\Delta Q') \langle i_{\Delta o}(0) \rangle \gamma_{\Delta o} \left(\frac{\sin \theta}{\lambda} \right), & (\Delta\theta = 0), \\ \frac{1}{4}[D](R\Delta Q/\Delta Q') \langle i_{\Delta o} \left(\frac{\sin \Delta\theta}{\lambda} \right) \rangle \gamma_{\Delta o} \left(\frac{\sin \theta}{\lambda} \right), & (\Delta\theta \neq 0), \end{cases} \quad (5.33)$$

where $\langle i_{\Delta o}(\cdot) \rangle$ and $\gamma_{\Delta o}(\cdot)$ denote the intensity spectrum and the mutual spectrum, respectively, which are given by

$$\begin{aligned} \langle i_{\Delta o}(\sin \Delta\theta/\lambda) \rangle &= F_{\Delta\theta} [\langle i_{\Delta o}(\xi) \rangle] \\ &= \sqrt{\pi/2} \Delta Q' \exp \left[-\frac{1}{2} \left(\frac{k \Delta Q' \sin \Delta\theta}{2} \right)^2 \right], \end{aligned} \quad (5.34)$$

$$\begin{aligned} \gamma_{\Delta o}(\sin \theta/\lambda) &= F_{\theta} [\gamma_{\Delta o}(\Delta\xi)] \\ &= \sqrt{2\pi} \Delta\xi_f \exp \left[-2 \left(\frac{k \Delta\xi_f \sin \theta}{2} \right)^2 \right], \end{aligned} \quad (5.35)$$

respectively. Moreover, C and D in Eq.(5.33) are given by

$$C = N - (-1)^L \frac{\sin(N\theta_L)}{\sin\theta_L}, \quad (5.36)$$

$$D = 2 \frac{\sin(NP_{\Delta\theta})}{\sin P_{\Delta\theta}} - (-1)^L \left\{ \frac{\sin[N(P_{\Delta\theta} + \theta_L)]}{\sin(P_{\Delta\theta} + \theta_L)} + \frac{\sin[N(P_{\Delta\theta} - \theta_L)]}{\sin(P_{\Delta\theta} - \theta_L)} \right\}, \quad (5.37)$$

respectively, where

$$P_{\Delta\theta} = P_+(\theta = 0) = P_-(\theta = 0) = \frac{kd \sin \Delta\theta}{2}. \quad (5.38)$$

Finally, by substituting Eqs.(5.27), (5.28), (5.34) and (5.35) into Eqs.(5.26) and (5.33), the MI $\Gamma_L(\theta, \Delta\theta)$ in Eq.(5.25) is obtained in the form

$$\begin{aligned} \Gamma_L(\theta, \Delta\theta) = & -\frac{1}{4}(-1)^L[A][B]\pi(1-R)\Delta Q^2 \\ & \times \exp \left[-\frac{1}{2} \left(\frac{k\Delta Q \sin \Delta\theta}{2} \right)^2 \right] \exp \left[-2 \left(\frac{k\Delta Q \sin \theta}{2} \right)^2 \right] \\ & + \begin{cases} \frac{1}{2}[C]\pi R\Delta\xi_f\Delta Q \exp \left[-2 \left(\frac{k\Delta\xi_f \sin \theta}{2} \right)^2 \right], & (\Delta\theta = 0), \\ \frac{1}{4}[D]\pi R\Delta\xi_f\Delta Q \exp \left[-\frac{1}{2} \left(\frac{k\Delta Q' \sin \Delta\theta}{2} \right)^2 \right] \\ \quad \times \exp \left[-2 \left(\frac{k\Delta\xi_f \sin \theta}{2} \right)^2 \right], & (\Delta\theta \neq 0). \end{cases} \end{aligned} \quad (5.39)$$

Moreover, the AI $\langle I_L(\theta) \rangle$ in the far field is obtained, by setting $\Delta\theta=0$ in Eq.(5.39), as

$$\begin{aligned} \langle I_L(\theta) \rangle = & \frac{1}{4}[A_o]^2\pi(1-R)\Delta Q^2 \exp \left[-2 \left(\frac{k\Delta Q \sin \theta}{2} \right)^2 \right] \\ & + \frac{1}{2}[C]\pi R\Delta\xi_f\Delta Q \exp \left[-2 \left(\frac{k\Delta\xi_f \sin \theta}{2} \right)^2 \right], \end{aligned} \quad (5.40)$$

where

$$\begin{aligned}
 A_o &= A_{\Delta\theta=0} \\
 &= -(-1)^L B_{\Delta\theta=0} \\
 &= \frac{\sin [N(P_\theta + \theta_L)/2]}{\sin [(P_\theta + \theta_L)/2]} - (-1)^L \frac{\sin [N(P_\theta - \theta_L)/2]}{\sin [(P_\theta - \theta_L)/2]}, \quad (5.41)
 \end{aligned}$$

with the definition of

$$P_\theta = P_+(\Delta\theta = 0) = P_-(\Delta\theta = 0) = kd \sin \theta. \quad (5.42)$$

5.3 Numerical Analysis

In the previous section, the AI $\langle I_L(\theta) \rangle$ in the far field, produced from the L th order fluctuating supermode, has been derived. Equation(5.40) contains two parameters, R and $\Delta\xi_f$, resulting from fluctuations of supermodes. The parameter R denotes the ratio of the integrated AI of the fluctuating component to the total integrated AI of an emitter (see Eq.(5.18)). By definition, R takes a value from 0 to 1 and becomes large with increasing fluctuations. Moreover, the parameter $\Delta\xi_f$ denotes the spatial correlation length of fluctuations of supermodes (see Eq.(5.14)), and the area identified physically increases with an increase of $\Delta\xi_f$. In general, it is too complicated to treat directly the complex amplitude of fluctuations in an analytical form, because there exist many uncertain factors stemming from fluctuations of the optical field inside the array lasers, as mentioned previously. Therefore, in the present study, the fluctuations have been treated as a statistical quantity and, as a result, two statistical parameters, R and $\Delta\xi_f$, have been adopted to evaluate the effects of the fluctuations.

In the following, by changing the values of R and $\Delta\xi_f/\Delta Q$ (normalized by the aperture radius ΔQ of an emitter) in Eq.(5.40), the effects of fluctuations of supermodes

on the AI $\langle I_L(\theta) \rangle$ in the far field (i.e., the far-field patterns) are investigated. In particular, the analysis is performed under the conditions for the numbers of emitters with $N=5$ and 11, and the orders of supermodes with $L=1$ and N (corresponding to the 1st- and highest-order supermodes, respectively).

5.3.1 Effects of fluctuations on the far-field patterns for $N=5$

The 1st-order far-field patterns $\langle I_1(\theta) \rangle$ with $L=1$ are shown in Fig.5.2 for $\Delta\xi_f/\Delta Q =$ (a)0.2 and (b)1.0 in the case of $N=5$, $d=6(\mu m)$, $2\Delta Q=3(\mu m)$ (therefore, the array fills the factor $\equiv 2\Delta Q/d=0.5$) and $\lambda=0.88(\mu m)$. In the figure, the patterns for $R=0$, 0.2 and 0.5 are denoted by the solid line (———), the broken one (- - -) and the dotted one (· · · · ·), respectively. Similarly, the highest-order far-field patterns $\langle I_5(\theta) \rangle$ with $L=5$ are also shown in Fig.5.3 under the same conditions as in Fig.5.2. As a result, it is found that, with an increase of R (namely, the fluctuating component $\Delta e_n(\xi)$ becomes large), the profiles of $\langle I_1(\theta) \rangle$ and $\langle I_5(\theta) \rangle$ are gradually deformed in a way that the peak values decrease and the backgrounds increase, starting from the cases without fluctuations (i.e., $R=0$). Moreover, it is also found that the background intensity is large for large values of $\Delta\xi_f/\Delta Q$, because the spatial frequency bandwidth of the fluctuating component $\Delta e_n(\xi)$ becomes narrow and the background intensity around $\theta=0$ becomes high with an increase of $\Delta\xi_f/\Delta Q$.

In order to investigate quantitatively the effects of fluctuation parameters, R and $\Delta\xi_f/\Delta Q$, on $\langle I_L(\theta) \rangle$, the profile contrast C_L concerning the main peak is used. The contrast C_L is defined as

$$C_L = \frac{\langle I_{Lmax} \rangle - \langle I_{Lmin} \rangle}{\langle I_{Lmax} \rangle + \langle I_{Lmin} \rangle}, \quad (5.43)$$

where $\langle I_{Lmax} \rangle$ is the average height of the main peak and $\langle I_{Lmin} \rangle$ is the background level at the point that the far-field pattern $I_L(\theta)$ without fluctuations takes the first minimum. By using the contrast C_L , the degree of deformation of $\langle I_L(\theta) \rangle$ due to fluc-

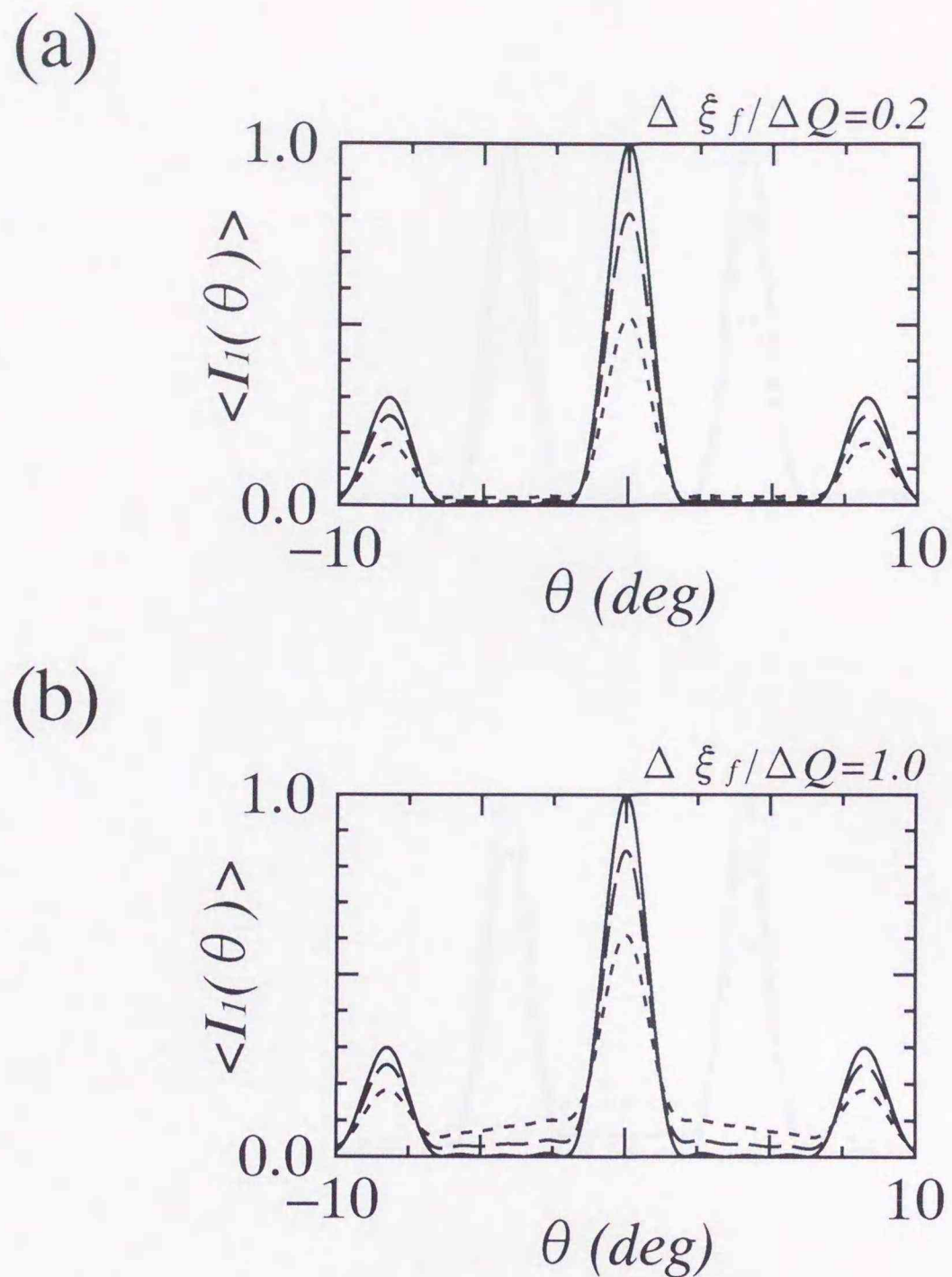


Figure 5.2 Far-field patterns $\langle I_1(\theta) \rangle$ for $\Delta \xi_f / \Delta Q =$ (a) 0.2 and (b) 1.0. (—), (- - -) and (- - - -) denote those for $R = 0, 0.2$ and 0.5 , respectively. $N = 5$, $d = 6(\mu m)$, $2\Delta Q = 3(\mu m)$ and $\lambda = 0.88(\mu m)$.

tuations of the emitters can be investigated quantitatively.

The contrast C_1 of $\langle I_1(\theta) \rangle$ as a function of the parameter R is shown in Fig. 5.4(a) for $\Delta \xi_f / \Delta Q = 0.2$ (—), 0.5 (- - -) and 1.0 (- - - -). As found from this figure, C_1 decreases with an increase of R and decreases similarly with an increase of $\Delta \xi_f / \Delta Q$, in a way that, for example, C_1 at the point of $R = 0.5$ becomes 0.92 , 0.83 and 0.72 for $\Delta \xi_f / \Delta Q = 0.2, 0.5$ and 1.0 , respectively.

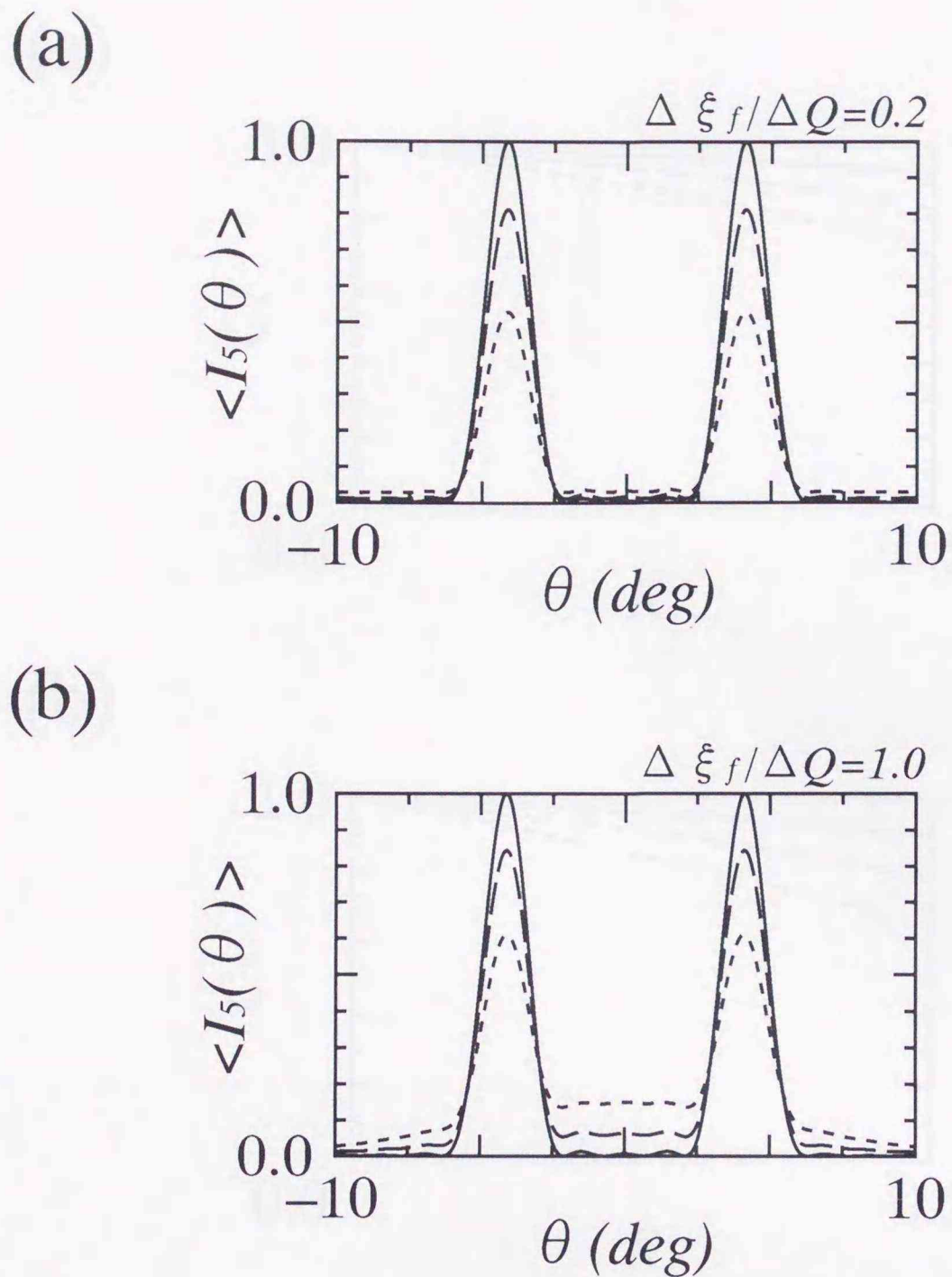
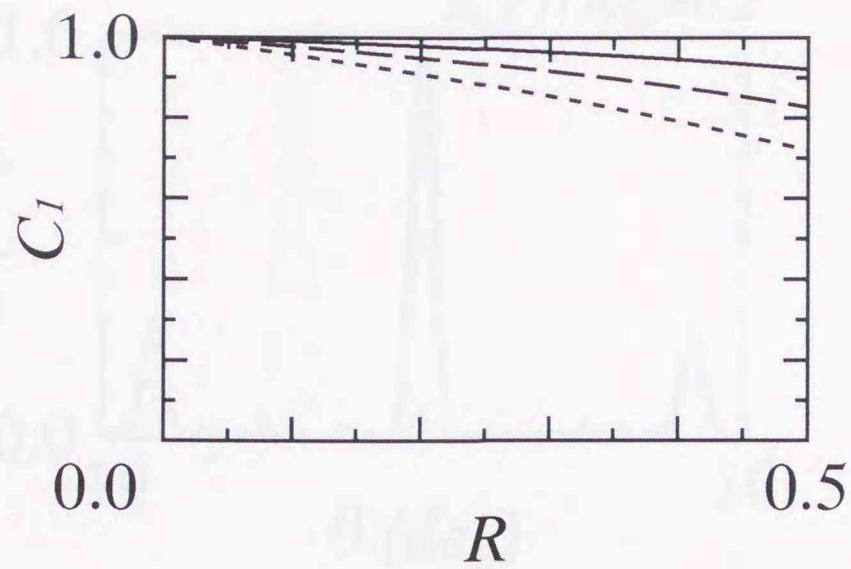


Figure 5.3 Far-field patterns $\langle I_5(\theta) \rangle$ for $\Delta \xi_f / \Delta Q =$ (a) 0.2 and (b) 1.0. (—), (- - -) and (- . - . -) denote those for $R=0, 0.2$ and 0.5 , respectively. $N, d, 2\Delta Q$ and λ are the same conditions as Fig. 5.2.

Moreover, the contrast C_5 of $\langle I_5(\theta) \rangle$ is shown in Fig. 5.4(b). From this figure, it is found that the tendency of C_5 is almost similar to that of C_1 . However, the decreasing rate of C_5 is somewhat larger than that of C_1 , because C_5 at the point of $R=0.5$ becomes 0.90, 0.78 and 0.64 for $\Delta \xi_f / \Delta Q = 0.2, 0.5$ and 1.0 , respectively. This means that the effects of fluctuations of supermodes on the far-field patterns are a little large for the highest-order supermode in comparison with the 1st-order one.

(a)



(b)

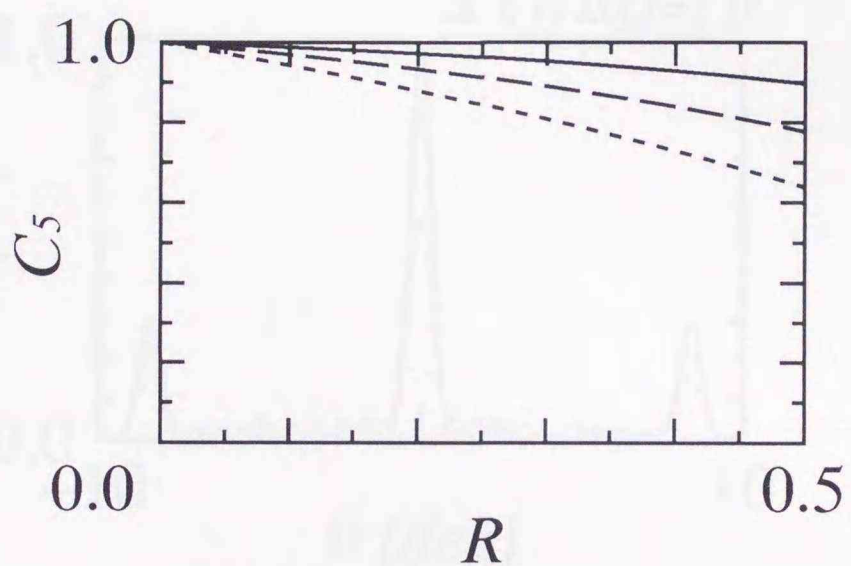
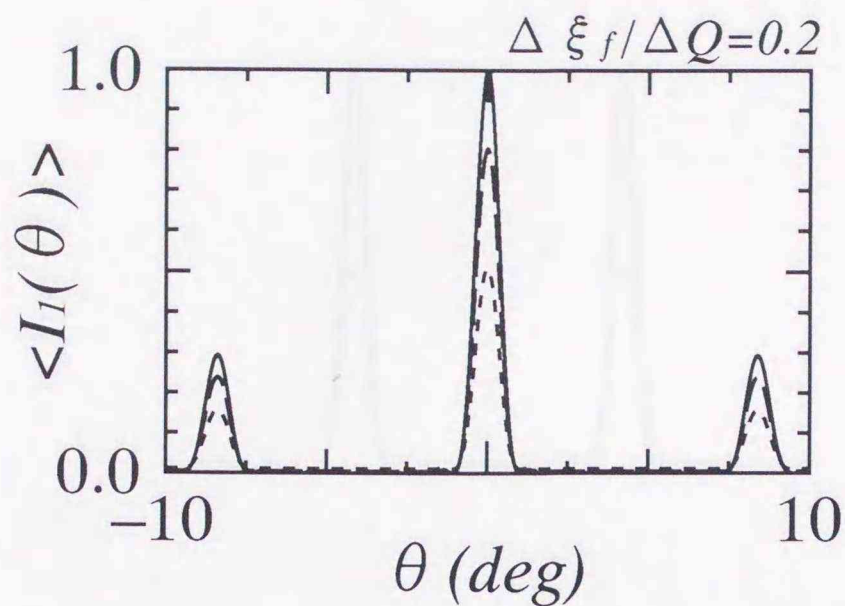


Figure 5.4 Contrast C_L of $\langle I_L(\theta) \rangle$ as a function of R : (a) the result for $L=1$ and (b) that for $L=5$. (—), (---) and (- - - -) denote those for $\Delta\xi_f/\Delta Q = 0.2, 0.5$ and 1.0 , respectively. Moreover, $N, d, 2\Delta Q$ and λ are the same conditions as Figs.5.2 and 5.3.

5.3.2 Effects of fluctuations on the far-field patterns for $N=11$

Next, the effects of fluctuations of supermodes on the far-field patterns $\langle I_L(\theta) \rangle$ are investigated for the large number N of emitters. The 1st-order far-field patterns $\langle I_1(\theta) \rangle$ for $N=11$ are shown in Fig.5.5 where the conditions are the same as in Fig.5.2 except

(a)



(b)

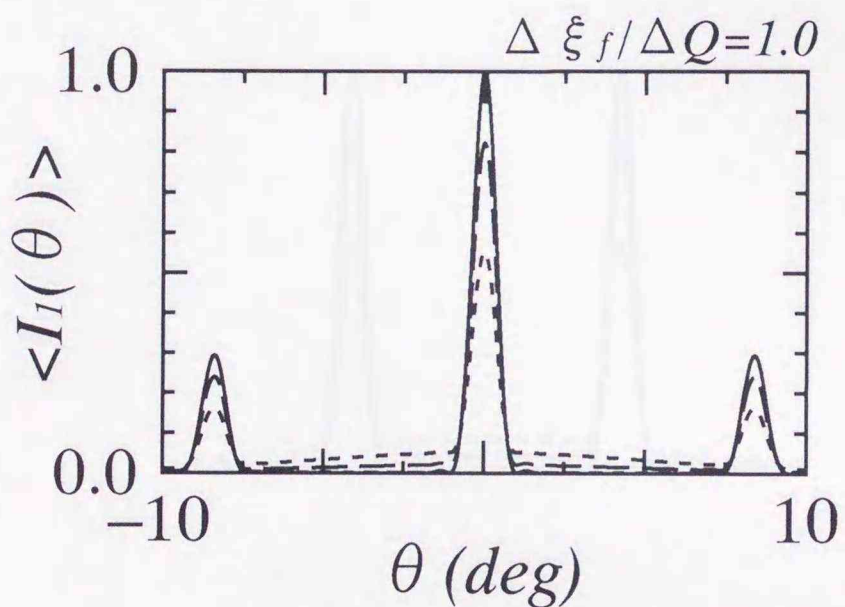


Figure 5.5 Far-field patterns $\langle I_1(\theta) \rangle$ for $\Delta \xi_f / \Delta Q =$ (a) 0.2 and (b) 1.0 under the conditions of $N=11$, $d=6(\mu m)$, $2\Delta Q=3(\mu m)$ and $\lambda=0.88(\mu m)$. (—), (- - -) and (- . - .) denote those for $R=0$, 0.2 and 0.5, respectively.

for $N=5$ (i.e., $d=6(\mu m)$, $2\Delta Q=3(\mu m)$, $\lambda=0.88(\mu m)$), and where (a) and (b) correspond to the patterns for $\Delta \xi_f / \Delta Q = 0.2$ and 1.0, respectively. Moreover, the highest-order far-field patterns $\langle I_{11}(\theta) \rangle$ with $L=11$ are also shown in Fig. 5.6 under the same conditions as in Fig. 5.5. Comparison of the profiles in Figs. 5.5 and 5.6 for $N=11$ with those in Figs. 5.2 and 5.3 for $N=5$ indicates that the effects of the fluctuations on $\langle I_L(\theta) \rangle$ for $N=11$ are not so large as those for $N=5$. In this case, the complex amplitude of the

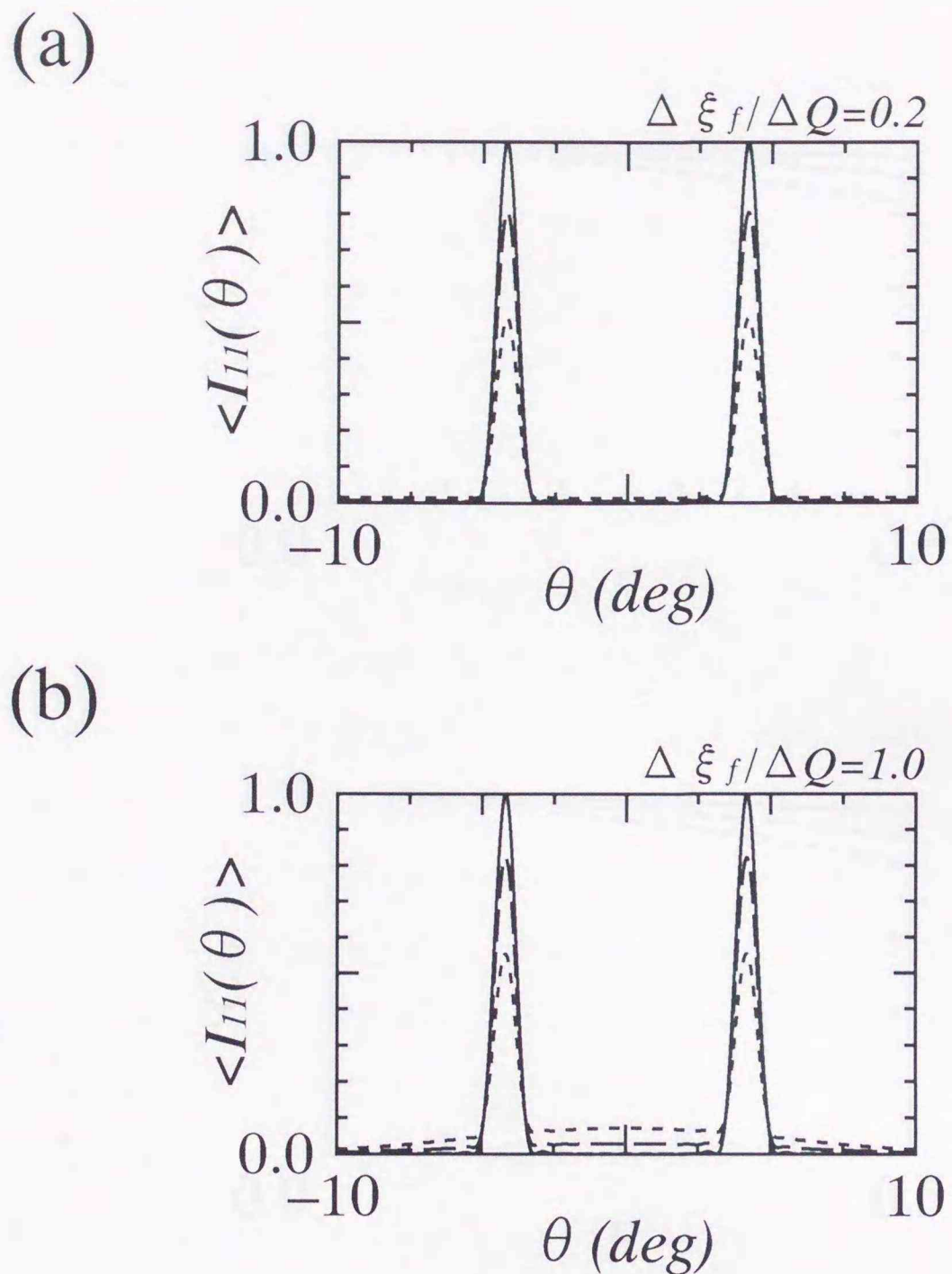
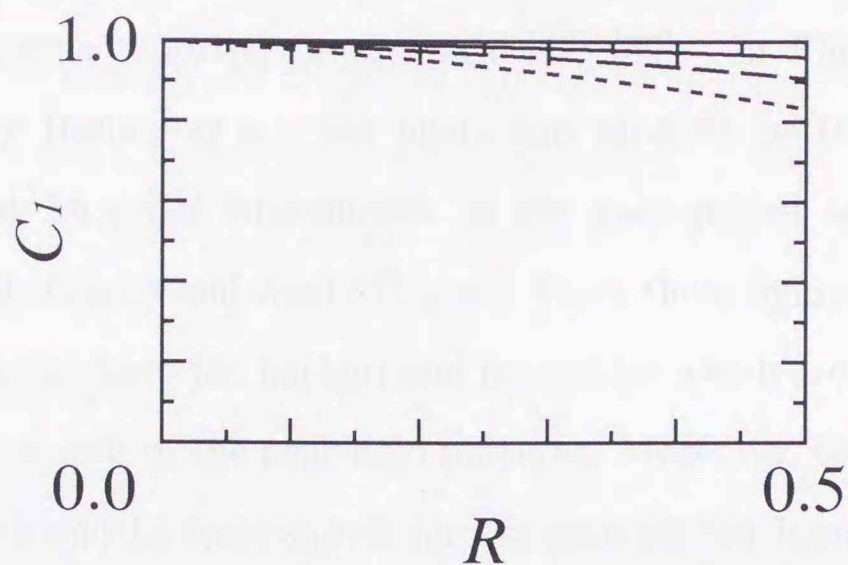


Figure 5.6 Far-field patterns $\langle I_{11}(\theta) \rangle$ for $\Delta \xi_f / \Delta Q =$ (a) 0.2 and (b) 1.0 under the same conditions of N , d , $2\Delta Q$ and λ as Fig. 5.5. (—), (---) and (- - - -) denote those for $R=0$, 0.2 and 0.5, respectively.

fluctuating component in the far field is produced by the N statistically independent superpositions of the spatial frequency distribution of $\Delta e_n(\xi)$. Therefore, with an increase of N , the effects of the fluctuations become small in a relative sense under the assumptions of (i) and (ii) in Subsection 5.2.2.

The results of the contrasts, C_1 and C_{11} , for $N=11$ are shown in Fig. 5.7. By comparing the behavior of the contrast in Fig. 5.7 with that in Fig. 5.4, it is found that

(a)



(b)

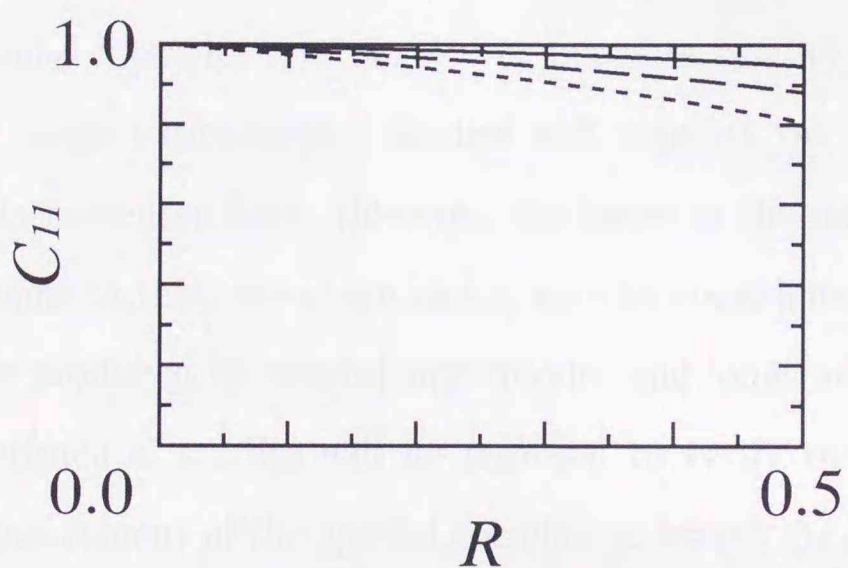


Figure 5.7 Contrast C_L of $\langle I_L(\theta) \rangle$ as a function of R for $N=11$: (a) the result for $L=1$ and (b) that for $L=11$. (—), (- - -) and (- . - . -) denote those for $\Delta\xi_f/\Delta Q=0.2$, 0.5 and 1.0 , respectively. Moreover, d , $2\Delta Q$ and λ are the same conditions as Figs.5.5 and 5.6.

the values of C_1 and C_{11} for $N=11$ are a little larger than those of C_1 and C_5 for $N=5$ under the same conditions of R and $\Delta\xi_f/\Delta Q$. Moreover, it is also found in Fig.5.7 that the tendency of C_{11} is almost the same as that of C_1 . This means that the effects of fluctuations of supermodes on the far-field patterns are almost the same for the 1st- and highest-order supermodes.

5.3.3 Validity for the present results

The present theoretical results are now discussed in comparison with the experimental far-field patterns reported in the past [57]–[59]. In Figs.4(a) and 6(a) of the reference [57] by Hadley *et al.*, the near- and far-field patterns were measured for the highest- and 1st-order supermodes in the gain-guided array laser with $N=10$, $d=10(\mu m)$, $2\Delta Q=6(\mu m)$ and $\lambda=0.815(\mu m)$. From these figures, it is understood that the far-field patterns have the background intensities which are caused by the fluctuating supermodes as seen in the near-field patterns. Moreover, the experimental far-field patterns for $N=5$ and 11 were shown for the gain-guided lasers in the references [58, 59] by Scifres *et al.* By comparing the pattern at $1.7I_{TH}$ in Fig.3 in the reference [58] with that at 50(mw) (corresponding to $1.6I_{TH}$) in Fig.3 in the reference [59], it is found that the background intensity level for $N=11$ becomes smaller than that for $N=5$.

In this way, these experimental studies will support, at least qualitatively, the theoretical results presented here. However, the lasers in the references [58, 59] are not completely the same in their structure and it may be considered that the observed far-field patterns are produced by several supermodes and longitudinal modes. Therefore, the further experimental studies will be required to verify the theoretical ones. For example, the measurement of the spatial correlation length $\Delta\xi_f$ at the output plane of the laser may also be needed to help the theoretical verification. The traditional method of this measurement can be done by the Young's interference experiment, because the contrast variations of Young's fringe patterns give the spatial correlation length $\Delta\xi_f$ of light at the plane of double pinholes used [5]. Although it may be extremely difficult to apply this method in a straight forward way to the tiny area (several μm area) of the laser exit, the modified method using the double-exposure specklegrams in the plane of double pinholes will be applicable to such light waves in the small area [60].

5.4 Conclusions

In this chapter, the effects of fluctuations of supermodes on the far-field patterns have been theoretically investigated by taking account of many uncertain factors inside the array lasers. In relation to fluctuations of supermodes on the AI $\langle I_L(\theta) \rangle$ in the far field, the effects of two statistical parameters, R and $\Delta\xi_f$, have been revealed by using the contrast C_L . Here, R and $\Delta\xi_f$ denote the parameter proportional to the integrated-intensity fluctuations and the spatial correlation length for the fluctuating component of supermodes, respectively. Moreover, the effects of fluctuations of supermodes on the far-field patterns $\langle I_L(\theta) \rangle$ have also been investigated by changing the number N of emitters in the array laser and the order L of supermodes.

As a result, it has been found that $\langle I_L(\theta) \rangle$ is gradually deformed with increases in fluctuation parameters of R and $\Delta\xi_f$, in comparison with those produced from the supermodes without fluctuations. In particular, the background intensity increases with increasing $\Delta\xi_f$ under the condition that R takes a constant value ($\neq 0$). However, the effects of R and $\Delta\xi_f$ on $\langle I_L(\theta) \rangle$ are more noticeable for small N 's. Furthermore, there is almost no difference between the results of the effects of $\Delta\xi_f$, R and N on $\langle I_L(\theta) \rangle$ for $L=1$ and N .

Finally, it is emphasized that the present analysis is applicable to all types of array lasers satisfying the nearest neighboring coupling inside them, even though the type becomes different like the index-guided, gain-guided or surface-emitting one.

5.5 Appendices

5.5.1 Derivation of Eq.(5.5)

Substitution of Eqs.(5.2) and (5.3) into Eq.(5.4) yields

$$\begin{aligned}
\Gamma_L(\xi, \Delta\xi) &= \langle E_L(\xi + \Delta\xi/2) E_L^*(\xi - \Delta\xi/2) \rangle \\
&= \left\langle \left\{ r_{(D)} \sum_{n=1}^N \sin(n\theta_L) e_o(\xi_1 - n'd) \right\} \left\{ r_{(D)} \sum_{m=1}^N \sin(m\theta_L) e_o^*(\xi_2 - m'd) \right\} \right. \\
&\quad + \left\{ r_{(D)} \sum_{n=1}^N \sin(n\theta_L) e_o(\xi_1 - n'd) \right\} \left\{ r_{(F)} \sum_{m=1}^N \sin(m\theta_L) \Delta e_m^*(\xi_2 - m'd) \right\} \\
&\quad + \left\{ r_{(F)} \sum_{n=1}^N \sin(n\theta_L) \Delta e_n(\xi_1 - n'd) \right\} \left\{ r_{(D)} \sum_{m=1}^N \sin(m\theta_L) e_o^*(\xi_2 - m'd) \right\} \\
&\quad \left. + \left\{ r_{(F)} \sum_{n=1}^N \sin(n\theta_L) \Delta e_n(\xi_1 - n'd) \right\} \left\{ r_{(F)} \sum_{m=1}^N \sin(m\theta_L) \Delta e_m^*(\xi_2 - m'd) \right\} \right\rangle, \tag{5A.1}
\end{aligned}$$

where $\xi_1 = \xi + \Delta\xi/2$, $\xi_2 = \xi - \Delta\xi/2$, $n' = n - (N+1)/2$ and $m' = m - (N+1)/2$.

Under the assumption of (iii), Eq.(5A.1) becomes

$$\begin{aligned}
\Gamma_L(\xi, \Delta\xi) &= r_{(D)}^2 \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) e_o(\xi_1 - n'd) e_o^*(\xi_2 - m'd) \\
&\quad + r_{(D)} r_{(F)} \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) e_o(\xi_1 - n'd) \langle \Delta e_m^*(\xi_2 - m'd) \rangle \\
&\quad + r_{(F)} r_{(D)} \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) \langle \Delta e_n(\xi_1 - n'd) \rangle e_o^*(\xi_2 - m'd) \\
&\quad + r_{(F)}^2 \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) \langle \Delta e_n(\xi_1 - n'd) \Delta e_m^*(\xi_2 - m'd) \rangle. \tag{5A.2}
\end{aligned}$$

Moreover, under the assumptions of (i) and (ii), $\Gamma_L(\xi, \Delta\xi)$ in Eq.(5A.2) is changed to

$$\begin{aligned}\Gamma_L(\xi, \Delta\xi) = & r_{(D)}^2 \sum_{n=1}^N \sum_{m=1}^N \sin(n\theta_L) \sin(m\theta_L) e_o(\xi_1 - n'd) e_o^*(\xi_2 - m'd) \\ & + r_{(F)}^2 \sum_{n=1}^N \sin^2(n\theta_L) \langle \Delta e_n(\xi_1 - n'd) \Delta e_n^*(\xi_2 - n'd) \rangle, \quad (5A.3)\end{aligned}$$

which is Eq.(5.5).

5.5.2 Mutual intensity in the far field

According to Eq.(13) in the reference [37] by Thompson *et al.*, the MI $\Gamma(x_1, x_2)$ at arbitrary points, x_1 and x_2 , in the far field from the plane ξ is given by

$$\Gamma(x_1, x_2) = \exp \left[\frac{ik(x_1^2 - x_2^2)}{2z} \right] \iint \Gamma(\xi_1, \xi_2) \exp \left(-\frac{ikx_1\xi_1}{z} \right) \exp \left(\frac{ikx_2\xi_2}{z} \right) d\xi_1 d\xi_2, \quad (5B.1)$$

where z is the distance between the planes of ξ and x .

By changing the arguments of x_1 and x_2 as $x_1 = x + \Delta x/2$ and $x_2 = x - \Delta x/2$, respectively, and by changing the variables of ξ_1 and ξ_2 as $\xi_1 = \xi + \Delta\xi/2$ and $\xi_2 = \xi - \Delta\xi/2$, respectively, Eq.(5B.1) is rewritten as

$$\Gamma(x, \Delta x) = \exp \left(\frac{ikx\Delta x}{z} \right) \iint \Gamma(\xi, \Delta\xi) \exp \left(-\frac{ik\Delta\xi x}{z} \right) \exp \left(-\frac{ik\xi\Delta x}{z} \right) d\Delta\xi d\xi. \quad (5B.2)$$

Since the arguments x/z and $\Delta x/z$ in Eq.(5B.2) are nearly equal to $\sin \theta$ and $\sin \Delta\theta$, respectively, and $\exp(ikx\Delta x/z) \simeq 1$ in a paraxial approximation, Eq.(5B.2) becomes

$$\begin{aligned}\Gamma(\theta, \Delta\theta) &= \iint \Gamma(\xi, \Delta\xi) \exp(-ik\Delta\xi \sin \theta) \exp(-ik\xi \sin \Delta\theta) d\Delta\xi d\xi \\ &= F_\theta F_{\Delta\theta} [\Gamma(\xi, \Delta\xi)], \quad (5B.3)\end{aligned}$$

where the operators of F_θ and $F_{\Delta\theta}$ are defined in Eqs.(5.23) and (5.24), respectively.

Chapter 6

Effects of amplitude and phase fluctuations of the Ronchi grating on Talbot images under illumination of the partially coherent light

6.1 Introduction

Recently, as shown in [32], the Talbot effect has been widely used in image processing, interferometry, optical metrology, and so on, because of the development of lasers. When a periodic object, such as a grating, is illuminated by a coherent monochromatic light, Talbot images (i.e., self-images) of the object appear at the observation planes with the distances of $Z_L = 2Ld^2/\lambda$ from the object, d , λ and L being the period of the object, the wavelength of the illuminating light and a positive integer, respectively. However, the self-images are not obtained in a perfect form, because the completely coherent monochromatic light and the periodic objects without amplitude and phase fluctuations are out of existence in practice.

Various studies have been conducted to investigate more practical cases concerning Talbot images of periodic objects. For example, the effects of amplitude fluctuations of the amplitude grating on the Talbot images have been studied [61]–[64]; the studies [62]–[64] investigated the case that each aperture of the grating had the same amplitude transmittance while its period had fluctuations obeying Gaussian statistics. Moreover,

the studies [65, 66] were concerned with the effects of the coherence properties of the illuminating light on the Talbot images. Furthermore, the relationship between the Talbot and Lau effects have also been studied [67, 68]. However, these studies were performed under the ideal conditions: the illumination was by the completely coherent monochromatic light [61]–[64] and the grating was strictly periodic [65]–[68].

In this chapter, the effects of not only amplitude but also phase fluctuations of the Ronchi grating (i.e., binary amplitude grating) on the Talbot images under illumination of the partially coherent quasi-monochromatic light are studied by using the propagation law of the ambiguity function (AF). Here, different types of fluctuations of the grating are treated in comparison with those in the previous studies [62]–[64]: each aperture of the periodic grating has random complex amplitude transmittance.

First, the general expression is derived in a paraxial approximation by the AF [14], [34]–[36] in order to reveal the average intensity (AI) of Talbot images of the Ronchi grating with fluctuations at the L th Talbot plane. In deriving the equation, the following conditions are taken into account:

- (i) The illuminating light is quasi-monochromatic and its mutual intensity (MI) is spatially stationary.
- (ii) The amplitude and phase fluctuations of the grating are also spatially stationary.
- (iii) The amplitude and phase fluctuations are statistically independent of each other.

As a result, it is shown that the AI at the L th Talbot plane is expressed by the convolution of the mutual spectrum of the illuminating light, the mutual spectra of amplitude and phase fluctuations of the grating, and the rectangular grating function.

Next, to investigate the effects of amplitude and phase fluctuations of the grating on the Talbot images, the contrast of the AI of the images at the first Talbot plane located at $Z_1=2d^2/\lambda$ from the grating is evaluated by analyzing numerically the derived equation for the AI by the computer. In the analysis, the complex degree of spatial coherence of the illuminating light and the normalized autocorrelation functions of the

amplitude and phase fluctuations are assumed to have Gaussian forms. As a result, it is shown that the contrast of the AI of the Talbot images is dependent on the spatial coherence length of the illuminating light and the spatial correlation lengths of the amplitude and phase fluctuations. The effect of the phase fluctuations is found to be most noticeable.

6.2 Average intensity of Talbot images due to the Ronchi grating with amplitude and phase fluctuations

Consider the situation that the Ronchi grating having apertures with amplitude and phase fluctuations is located at the plane ξ , and it is illuminated by the partially coherent quasi-monochromatic light $V_i(\xi)$ with the mean wavelength λ , as shown in Fig.6.1. In the present study, the absorption of light by the grating is treated as amplitude fluctuations. Then, by denoting the amplitude and phase components of the complex amplitude transmittance of fluctuations of the grating by $t_a(\xi)$ and $t_p(\xi)$, respectively, and by denoting the amplitude transmittance of the regular grating without fluctuations by $t_g(\xi)$, the optical field $V_o(\xi)$ emerging from the grating is expressed as

$$V_o(\xi) = V_i(\xi)t_a(\xi)t_p(\xi)t_g(\xi), \quad (6.1)$$

where the amplitude and phase components, $t_a(\xi)$ and $t_p(\xi)$, of fluctuations of the grating are assumed to be statistically independent of each other. Moreover, the amplitude transmittance of the regular grating $t_g(\xi)$ with the aperture size a of the opening and the period d is given by

$$\begin{aligned} t_g(\xi) &= \text{rect}(\xi/a) \otimes \sum_n \delta(\xi - nd) \\ &= \sum_n \text{rect}[(\xi - nd)/a], \end{aligned} \quad (6.2)$$

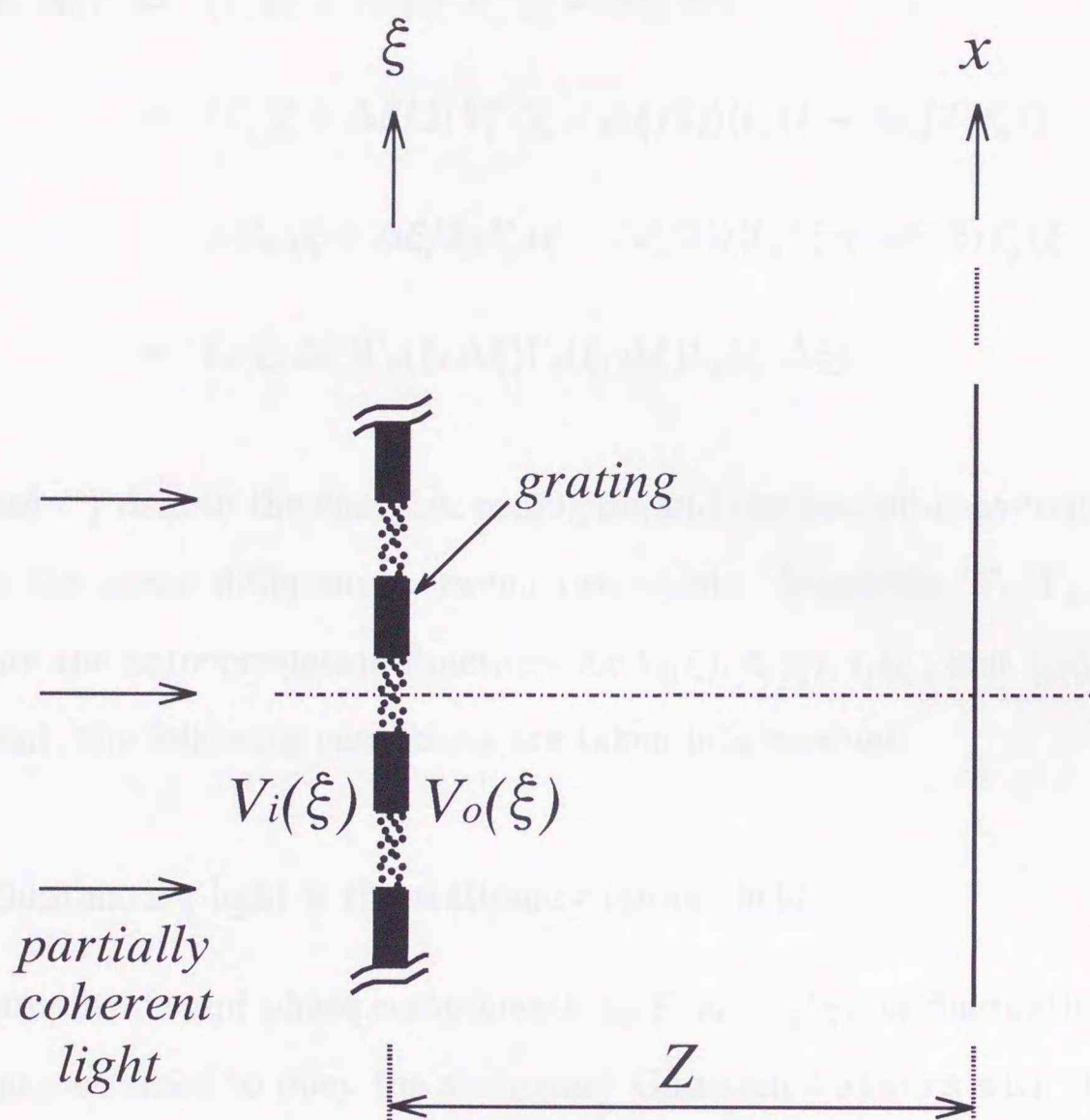


Figure 6.1 Analytical model of the Ronchi grating with amplitude and phase fluctuations and the optical coordinate systems for investigating the Talbot images.

where $\otimes$ denotes the convolution integral, n is an integer in the region extending from $-\infty$ to ∞ , and $\text{rect}(\)$ denotes the rectangular function described by

$$\text{rect}(\xi/a) = \begin{cases} 0, & (|\xi/a| > 1/2), \\ 1, & (|\xi/a| \leq 1/2). \end{cases} \quad (6.3)$$

Then, the space-dependent correlation function $\Gamma_o(\xi, \Delta\xi)$ for the optical field $V_o(\xi)$ emerging from the grating is expressed using Eq.(6.1) as follows:

$$\begin{aligned}\Gamma_o(\xi, \Delta\xi) &= \langle V_o(\xi + \Delta\xi/2) V_o^*(\xi - \Delta\xi/2) \rangle \\ &= \langle V_i(\xi + \Delta\xi/2) V_i^*(\xi - \Delta\xi/2) \rangle \langle t_a(\xi + \Delta\xi/2) t_a^*(\xi - \Delta\xi/2) \rangle \\ &\quad \times \langle t_p(\xi + \Delta\xi/2) t_p^*(\xi - \Delta\xi/2) \rangle \langle t_g(\xi + \Delta\xi/2) t_g^*(\xi - \Delta\xi/2) \rangle \\ &= \Gamma_i(\xi, \Delta\xi) \Gamma_a(\xi, \Delta\xi) \Gamma_p(\xi, \Delta\xi) \Gamma_g(\xi, \Delta\xi),\end{aligned}\quad (6.4)$$

where $*$ and $\langle \rangle$ denote the complex conjugate and the ensemble average, respectively, and $\Delta\xi$ is the space difference between two points. Moreover, Γ_i , Γ_a , Γ_p and Γ_g in Eq.(6.4) are the autocorrelation functions for $V_i(\xi)$, $t_a(\xi)$, $t_p(\xi)$ and $t_g(\xi)$, respectively. In this point, the following conditions are taken into account:

- (i) The illuminating light is the stationary optical field.
- (ii) The amplitude and phase components, $t_a(\xi)$ and $t_p(\xi)$, of fluctuations of the grating are assumed to obey the stationary Gaussian statistics with the zero mean.

Under these conditions, Eq.(6.4) reduces to

$$\Gamma_o(\xi, \Delta\xi) = \Gamma_i(\Delta\xi) \Gamma_a(\Delta\xi) \Gamma_p(\Delta\xi) \Gamma_g(\xi, \Delta\xi), \quad (6.5)$$

because $\Gamma_i(\xi, \Delta\xi)$, $\Gamma_a(\xi, \Delta\xi)$ and $\Gamma_p(\xi, \Delta\xi)$ are independent of ξ . Moreover, the AF $A_o(\Delta\nu, \Delta\xi)$ for $V_o(\xi)$ is defined as a Fourier transform of Eq.(6.5) with respect to ξ , and it is obtained as (see Eq.(2.11))

$$\begin{aligned}A_o(\Delta\nu, \Delta\xi) &= \int \Gamma_o(\xi, \Delta\xi) \exp(-2\pi i \xi \Delta\nu) d\xi \\ &= \Gamma_i(\Delta\xi) \Gamma_a(\Delta\xi) \Gamma_p(\Delta\xi) \int \Gamma_g(\xi, \Delta\xi) \exp(-2\pi i \xi \Delta\nu) d\xi\end{aligned}$$

$$= \Gamma_i(\Delta\xi)\Gamma_a(\Delta\xi)\Gamma_p(\Delta\xi)A_g(\Delta\nu, \Delta\xi), \quad (6.6)$$

where $\Delta\nu$ is the spatial frequency difference and $A_g(\Delta\nu, \Delta\xi)$ is the AF for $t_g(\xi)$.

Next, consider the AF of the optical field at the plane x of the Fresnel diffraction region, a distance Z from the grating (i.e., the ξ plane), where the Talbot effect occurs. The AF in the Fresnel diffraction region is obtained only by changing the argument of Eq.(6.6) without changing its functional form, and it is given by (see Eq.(2.19))

$$\begin{aligned} A_Z(\Delta\nu, \Delta x) &= A_o(\Delta\nu, \Delta x - \lambda Z \Delta\nu) \\ &= \Gamma_i(\Delta x - \lambda Z \Delta\nu)\Gamma_a(\Delta x - \lambda Z \Delta\nu)\Gamma_p(\Delta x - \lambda Z \Delta\nu) \\ &\quad \times A_g(\Delta\nu, \Delta x - \lambda Z \Delta\nu). \end{aligned} \quad (6.7)$$

Then, the AI in the Fresnel diffraction region is obtained as (see Eq.(2.15))

$$\begin{aligned} \langle I_Z(x) \rangle &= \Gamma_Z(x, \Delta x = 0) \\ &= \int A_Z(\Delta\nu, 0) \exp(2\pi i x \Delta\nu) d\Delta\nu \\ &= F^{-1}[\Gamma_i(-\lambda Z \Delta\nu)] \otimes F^{-1}[\Gamma_a(-\lambda Z \Delta\nu)] \otimes F^{-1}[\Gamma_p(-\lambda Z \Delta\nu)] \\ &\quad \otimes F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)], \end{aligned} \quad (6.8)$$

where $F^{-1}[\]$ indicates an inverse Fourier transform. Moreover, the explicit expression of $F^{-1}[\Gamma(-\lambda Z \Delta\nu)]$ is written as

$$\begin{aligned} F^{-1}[\Gamma(-\lambda Z \Delta\nu)] &= \int \Gamma(-\lambda Z \Delta\nu) \exp(2\pi i x \Delta\nu) d\Delta\nu \\ &= (1/\lambda Z) \int \Gamma(\beta) \exp(-2\pi i x \beta / \lambda Z) d\beta \\ &= (1/\lambda Z) \tilde{\Gamma}(x/\lambda Z), \end{aligned} \quad (6.9)$$

where $\tilde{\Gamma}(x/\lambda Z)$ means the mutual spectrum defined by

$$\tilde{\Gamma}(x/\lambda Z) \equiv \int \Gamma(\beta) \exp(-2\pi i x \beta / \lambda Z) d\beta. \quad (6.10)$$

By using Eq.(6.9), Eq.(6.8) is expressed as

$$\langle I_Z(x) \rangle = (1/\lambda Z)^3 \tilde{\Gamma}_i(x/\lambda Z) \otimes \tilde{\Gamma}_a(x/\lambda Z) \otimes \tilde{\Gamma}_p(x/\lambda Z) \otimes F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)]. \quad (6.11)$$

Furthermore, it can be found that the last factor $F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)]$ in Eq.(6.11) becomes equal to the grating function $t_g(x)$ in the case that the observation is conducted at the planes of Talbot images (see Subsection 6.6.1). Then, Eq.(6.11) reduces to

$$\langle I_{Z_L}(x) \rangle = (1/\lambda Z_L)^3 \tilde{\Gamma}_i(x/\lambda Z_L) \otimes \tilde{\Gamma}_a(x/\lambda Z_L) \otimes \tilde{\Gamma}_p(x/\lambda Z_L) \otimes t_g(x), \quad (6.12)$$

where Z_L is the distance to the plane of the L th Talbot images and is given by

$$Z_L = 2Ld^2/\lambda, \quad (L = 1, 2, 3, \dots). \quad (6.13)$$

As a result, it is found that the AI $\langle I_{Z_L}(x) \rangle$ of Talbot images of the Ronchi grating with amplitude and phase fluctuations under illumination of the partially coherent light is expressed by the convolution integral of the four functions, i.e., the mutual spectrum $\tilde{\Gamma}_i(x/\lambda Z_L)$ of the illuminating light, the mutual spectrum $\tilde{\Gamma}_a(x/\lambda Z_L)$ of the amplitude fluctuations, the mutual spectrum $\tilde{\Gamma}_p(x/\lambda Z_L)$ of the phase fluctuations, and the rectangular grating function $t_g(x)$.

6.3 Mutual spectra relating to the average intensity

To proceed with the discussion on the AI of Eq.(6.12), explicit forms of the mutual spectra, $\tilde{\Gamma}_i$, $\tilde{\Gamma}_a$ and $\tilde{\Gamma}_p$ are required.

6.3.1 Mutual spectrum of the illuminating light

Under the assumption of the spatial stationarity for the optical field of the illuminating light, consider the case that its field correlation (i.e., MI) is given by the following Gaussian form

$$\Gamma_i(\Delta\xi) = I_o \exp\left(-\Delta\xi^2/2\Delta\xi_i^2\right), \quad (6.14)$$

where I_o is a constant and $\Delta\xi_i$ is the spatial coherence length of the illuminating light. Then, with the help of Eq.(6.10), the mutual spectrum is obtained as

$$\tilde{\Gamma}_i(x/\lambda Z_L) = I_o \sqrt{2\pi} \Delta\xi_i \exp\left[-2(\pi \Delta\xi_i x/\lambda Z_L)^2\right]. \quad (6.15)$$

6.3.2 Mutual spectrum of the amplitude component of fluctuations

The transmittance of amplitude fluctuations of the grating may be written as

$$t_a(\xi) = t_o + r(\xi), \quad (6.16)$$

where t_o is a constant (i.e., bias component) and $r(\xi)$ is the component of random fluctuations. Then, the autocorrelation function of Eq.(6.16) is written as

$$\Gamma_a(\Delta\xi) = t_o^2 + \sigma_r^2 \gamma_r(\Delta\xi), \quad (6.17)$$

where the random component $r(\xi)$ has been assumed to obey the stationary Gaussian statistics with the zero mean [69]. In Eq.(6.17), σ_r and $\gamma_r(\Delta\xi)$ are the standard deviation and the normalized autocorrelation function of the amplitude fluctuations $r(\xi)$, respectively. Under the assumption that $\gamma_r(\Delta\xi)$ also takes the Gaussian form of $\exp(-\Delta\xi^2/2\Delta\xi_r^2)$ in which $\Delta\xi_r$ is the spatial correlation length of $r(\xi)$, the mutual spectrum resulting from Eq.(6.17) becomes

$$\tilde{\Gamma}_a(x/\lambda Z_L) = t_o^2 \delta(x/\lambda Z_L) + \sqrt{2\pi} \sigma_r^2 \Delta\xi_r \exp \left[-2(\pi \Delta\xi_r x / \lambda Z_L)^2 \right]. \quad (6.18)$$

6.3.3 Mutual spectrum of the phase component of fluctuations

The autocorrelation function of the phase component $t_p(\xi) = \exp[i\phi(\xi)]$ of fluctuations of the grating is given, under the assumption that the phase fluctuations $\phi(\xi)$ obey the Gaussian statistics with the zero mean, by

$$\Gamma_p(\Delta\xi) = \exp \left[-\sigma_\phi^2 \{1 - \gamma_\phi(\Delta\xi)\} \right], \quad (6.19)$$

where σ_ϕ and $\gamma_\phi(\Delta\xi)$ are the standard deviation and the normalized autocorrelation function of the phase fluctuations $\phi(\xi)$, respectively [69]. Unfortunately, the existing condition for a Fourier transform cannot be satisfied for the function of Eq.(6.19), because it converges not to zero but to $\exp(-\sigma_\phi^2)$ when $|\Delta\xi| \rightarrow \infty$. This difficulty is serious for small values of σ_ϕ . To avoid this problem, the following approximation form is used in place of Eq.(6.19) [70]:

$$\Gamma_p(\Delta\xi) = \exp(-\sigma_\phi^2) + [1 - \exp(-\sigma_\phi^2)] \exp(-\Delta\xi^2/2\Delta\xi_\phi^2), \quad (6.20)$$

where $\Delta\xi_\phi$ is the spatial correlation length of $\phi(\xi)$. The mutual spectrum resulting from Eq.(6.20) is expressed as

$$\begin{aligned}\tilde{\Gamma}_p(x/\lambda Z_L) = & \exp(-\sigma_\phi^2) \delta(x/\lambda Z_L) \\ & + \sqrt{2\pi} [1 - \exp(-\sigma_\phi^2)] \Delta\xi_\phi \exp[-2(\pi\Delta\xi_\phi x/\lambda Z_L)^2].\end{aligned}\quad (6.21)$$

The AI of Talbot images suffering some fluctuations can be evaluated by using the mutual spectra of Eqs.(6.15), (6.18) and (6.21) in Eq.(6.12).

6.4 Contrast of the average intensity

In this section, the effects of amplitude and phase fluctuations of the Ronchi grating on the Talbot images observed at the Talbot planes are investigated. For this purpose, the contrast of the AI of the Talbot images is used as an evaluation parameter. The contrast is defined as

$$C_L = \frac{\langle I_{Z_L} \rangle_{\max} - \langle I_{Z_L} \rangle_{\min}}{\langle I_{Z_L} \rangle_{\max} + \langle I_{Z_L} \rangle_{\min}}, \quad (6.22)$$

where $\langle I_{Z_L} \rangle_{\max}$ and $\langle I_{Z_L} \rangle_{\min}$ denote the maximum and minimum values, respectively, of the AI $\langle I_{Z_L}(x) \rangle$ in Eq.(6.12). In the following, the evaluation of the image contrast is conducted to investigate the effects of the amplitude and phase fluctuations under illumination of the partially coherent light.

6.4.1 Effect of amplitude fluctuations

First, the effect of amplitude fluctuations of the Ronchi grating on the L th Talbot images is investigated under the condition of no phase fluctuations, i.e., $\sigma_\phi=0$. In this situation, the mutual spectrum of Eq.(6.21) concerning the phase fluctuations becomes $\tilde{\Gamma}_p(x/\lambda Z_L)=\delta(x/\lambda Z_L)$. By using this equation and Eqs.(6.15) and (6.18), the AI of the

Talbot images in Eq.(6.12) reduces to (see Subsection 6.6.2)

$$\langle I_{Z_L}(x) \rangle = \left[C_{a1} \exp \left(-\frac{2x^2}{\Delta x_i^2} \right) + C_{a2} \exp \left(-\frac{2x^2}{\Delta x_a^2} \right) \right] \otimes \sum_n \text{rect} \left(\frac{x - nd}{a} \right), \quad (6.23)$$

where

$$\Delta x_i = \lambda Z_L / \pi \Delta \xi_i, \quad (6.24)$$

$$\Delta x_a = (\lambda Z_L / \pi) \sqrt{1/\Delta \xi_i^2 + 1/\Delta \xi_r^2}, \quad (6.25)$$

and two coefficients, C_{a1} and C_{a2} , are dependent on Z_L and fluctuation parameters and are given by

$$C_{a1} = C_o t_o^2 \Delta \xi_i, \quad (6.26)$$

$$C_{a2} = C_o \sigma_r^2 \Delta \xi_i \Delta \xi_r / \sqrt{\Delta \xi_i^2 + \Delta \xi_r^2}, \quad (6.27)$$

respectively, where

$$C_o = I_o \sqrt{2\pi} / \lambda Z_L. \quad (6.28)$$

In the following, a variation of the contrast of the AI $\langle I_{Z_1}(x) \rangle$ at the first Talbot plane located at $Z_1 = 2d^2/\lambda$ from the grating is evaluated by analyzing numerically Eq.(6.23) by the computer. In this evaluation, the parameters of $\Delta \xi_i/d$ and $\Delta \xi_r/d$ normalized by the period d of the grating are used: $\Delta \xi_i/d$ is the normalized spatial coherence length of the illuminating light while $\Delta \xi_r/d$ is the normalized spatial correlation length of the amplitude fluctuations.

The resultant effects of the spatial coherence length $\Delta \xi_i$ of the illuminating light on the contrast C_1 of the AI $\langle I_{Z_1}(x) \rangle$ are shown in Fig.6.2. In the figure, the curved profiles indicate the AI distributions $\langle I_{Z_1}(x) \rangle$ for the cases of $\Delta \xi_i/d = (a)1.0$, (b)2.0

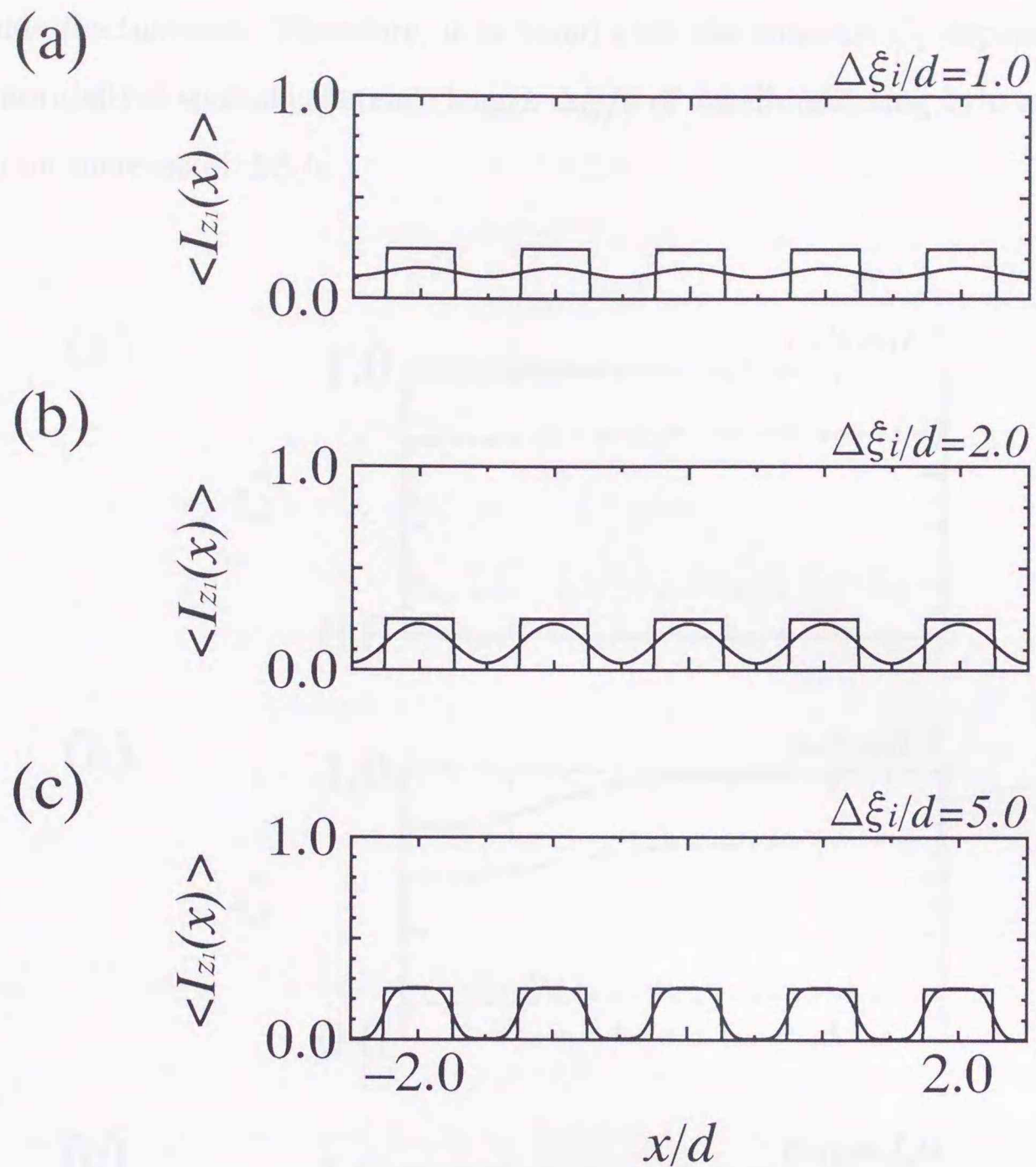


Figure 6.2 Average intensity distributions of Talbot images produced by the Ronchi grating with amplitude fluctuations having the normalized spatial correlation length $\Delta\xi_r/d=1.0$. The normalized spatial coherence length of the illuminating light is varied as $\Delta\xi_i/d=(a)1.0$, (b)2.0 and (c)5.0. The rectangular and the curved lines indicate the average intensity distributions of Talbot images without and with fluctuations, respectively. These were obtained for the case of $L=1$ and $\sigma_r/t_o=0.2$.

and (c) 5.0 under the conditions of $\sigma_r/t_o=0.2$ and $\Delta\xi_r/d=1.0$. On the other hand, the rectangular profiles indicate the self-image of the regular grating $t_g(x)$ under illumination of the coherent light. The contrast C_1 becomes 0.16, 0.73 and 0.97 for the above three cases in order: the contrast C_1 increases steeply between $\Delta\xi_i/d=1.0$ and

2.0, and it becomes nearly equal to 1.0 for $\Delta\xi_i/d=5.0$ irrespective of the existing amplitude fluctuations. Therefore, it is found that the contrast C_1 depends strongly on the normalized spatial coherence length $\Delta\xi_i/d$ of the illuminating light and it increases with an increase of $\Delta\xi_i/d$.

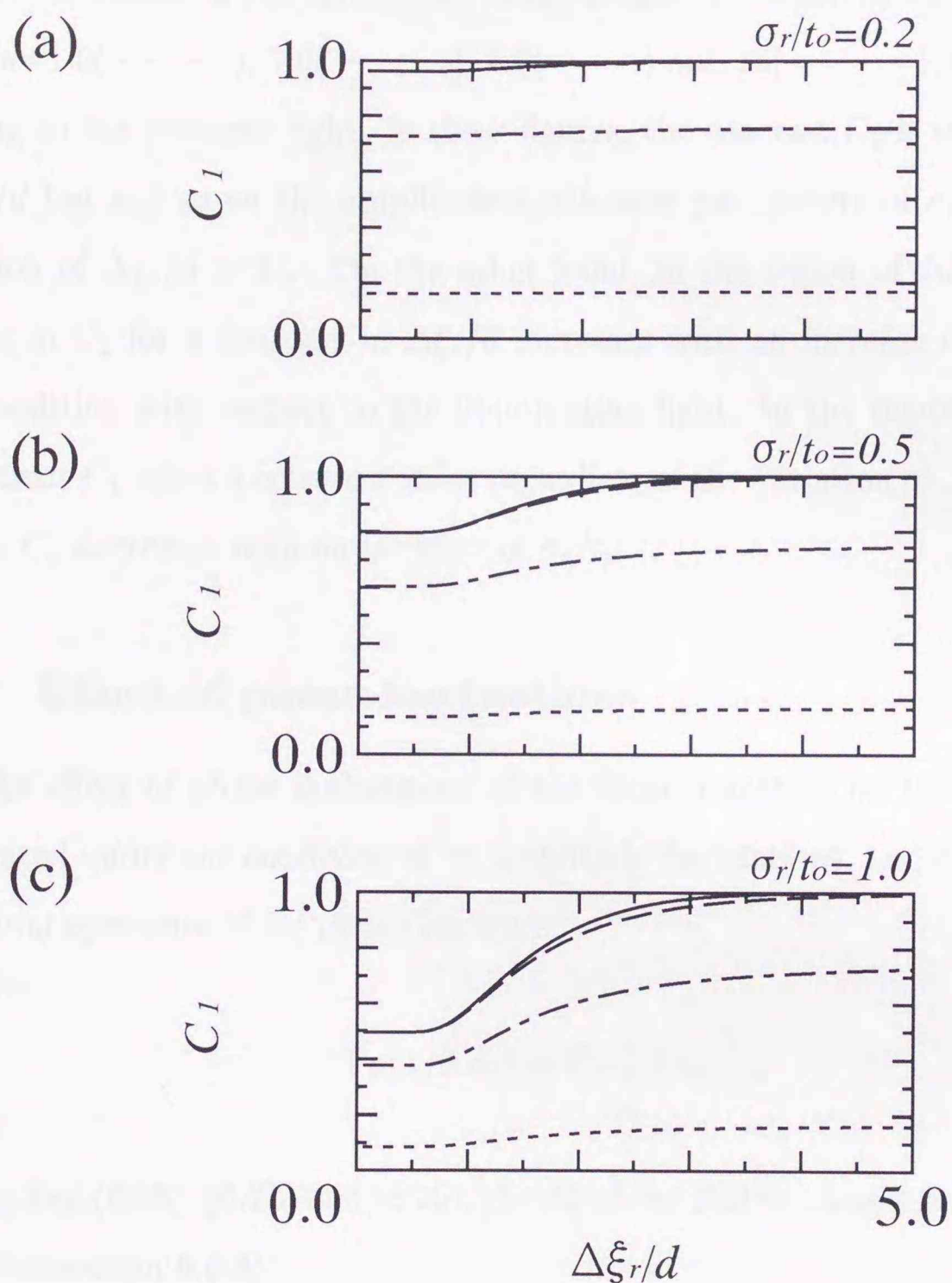


Figure 6.3 Contrast of the average intensity of Talbot images at the first Talbot plane as a function of the normalized spatial correlation length $\Delta\xi_r/d$ of amplitude fluctuations whose magnitude is set to take $\sigma_r/t_o=(a)0.2$, (b)0.5 and (c)1.0. In each figure, the different lines indicate the cases $\Delta\xi_i/d=1.0$ (- - - -), 2.0(— · —), 5.0(. . .) and ∞ (———) of the normalized spatial coherence length of the illuminating light.

By changing the parameters of σ_r/t_o and $\Delta\xi_i/d$, the effect of the amplitude fluctuations on the contrast C_1 is investigated in more detail. The contrast C_1 of the AI $\langle I_{Z_1}(x) \rangle$ is shown in Fig.6.3 as a function of the normalized spatial correlation length $\Delta\xi_r/d$ of the amplitude fluctuations. Figures 6.3 (a), (b) and (c) correspond to the results for the cases of $\sigma_r/t_o=0.2, 0.5$ and 1.0 , respectively. In the figure, the results for the different values of the spatial coherence length are depicted by the different lines as $\Delta\xi_i/d=1.0$ (- - - -), 2.0 (— · —), 5.0 (- - -) and ∞ (———), the last one corresponding to the coherent light. In these figures, the contrast C_1 is strongly dependent on $\Delta\xi_i/d$ but not so on the amplitude-fluctuation parameters of σ_r/t_o and $\Delta\xi_r/d$ in the region of $\Delta\xi_r/d > 3.0$. On the other hand, in the region of $\Delta\xi_r/d=1.0 \sim 3.0$, a decrease in C_1 for a decrease in $\Delta\xi_r/d$ increases with an increase of σ_r/t_o under the same condition with respect to the illuminating light. In the region of $\Delta\xi_r/d < 1.0$, the contrast C_1 takes a constant value regardless of the variation of $\Delta\xi_r/d$, though the contrast C_1 decreases with an increase of σ_r/t_o .

6.4.2 Effect of phase fluctuations

Next, the effect of phase fluctuations of the Ronchi grating on the Talbot images is investigated under the condition of no amplitude fluctuations, i.e., $\sigma_r=0$. In this case, the mutual spectrum of Eq.(6.18) becomes

$$\tilde{\Gamma}_a(x/\lambda Z_L) = t_o^2 \delta(x/\lambda Z_L). \quad (6.29)$$

By using Eqs.(6.15), (6.21) and (6.29), the AI of the Talbot images in Eq.(6.12) reduces to (see Subsection 6.6.3)

$$\langle I_{Z_L}(x) \rangle = \left[C_{p1} \exp\left(-\frac{2x^2}{\Delta x_i^2}\right) + C_{p2} \exp\left(-\frac{2x^2}{\Delta x_p^2}\right) \right] \otimes \sum_n \text{rect}\left(\frac{x - nd}{a}\right), \quad (6.30)$$

where

$$\Delta x_p = (\lambda Z_L/\pi) \sqrt{1/\Delta\xi_i^2 + 1/\Delta\xi_\phi^2}, \quad (6.31)$$

$$C_{p1} = C_o t_o^2 \exp(-\sigma_\phi^2) \Delta\xi_i, \quad (6.32)$$

$$C_{p2} = C_o t_o^2 [1 - \exp(-\sigma_\phi^2)] \Delta\xi_i \Delta\xi_\phi / \sqrt{\Delta\xi_i^2 + \Delta\xi_\phi^2}, \quad (6.33)$$

Δx_i and C_o are given by Eqs.(6.24) and (6.28), respectively. In the following, the AI $\langle I_{Z1}(x) \rangle$ is obtained from the numerical analysis of Eq.(6.30) by the computer, and a variation of the contrast of the AI resulting from the phase fluctuations is evaluated. The normalized spatial correlation length $\Delta\xi_\phi/d$ of the phase fluctuations is used here as a parameter for the evaluation.

The resultant effects of the spatial coherence length $\Delta\xi_i$ of the illuminating light on the contrast C_1 of the AI $\langle I_{Z1}(x) \rangle$ are shown in Fig.6.4. In the figure, the curved profiles indicate the AI distributions $\langle I_{Z1}(x) \rangle$ for the cases of $\Delta\xi_i/d=(a)1.0$, (b)2.0 and (c)5.0 under the conditions of $t_o=0.5$, $\sigma_\phi=0.2\pi$ and $\Delta\xi_\phi/d=1.0$. On the other hand, the rectangular profiles indicate the self-image of the regular grating $t_g(x)$ under illumination of the coherent light. The contrast C_1 becomes 0.12, 0.54 and 0.71 for the above three cases in order. As is clear from Fig.6.4, the contrast C_1 depends strongly on the normalized spatial coherence length $\Delta\xi_i/d$ of the illuminating light in such a way that it increases with an increase of $\Delta\xi_i/d$. This result is similar to the case of the amplitude fluctuations shown in Fig.6.2. However, it is pointed out that, under the same conditions with respect to the illuminating light, the contrast C_1 resulting from the phase fluctuations is lower than that resulting from the amplitude fluctuations. In other words, an increase in C_1 with an increase of $\Delta\xi_i/d$ for the case of the phase fluctuations is smaller than that for the case of the amplitude fluctuations.

By changing the parameters of σ_ϕ and $\Delta\xi_i/d$, the effect of the phase fluctuations on the contrast C_1 is finally investigated in detail. The contrast C_1 of the AI $\langle I_{Z1}(x) \rangle$ is shown in Fig.6.5 as a function of the normalized spatial correlation length $\Delta\xi_\phi/d$ of the phase fluctuations. Figures 6.5 (a), (b) and (c) correspond to the results for the cases of $\sigma_\phi=0.2\pi$, 0.5π and π , respectively. In the figure, the results are shown for $\Delta\xi_i/d=1.0$ (- - - -), 2.0(— · —), 5.0(- - -) and ∞ (———). From these figures,

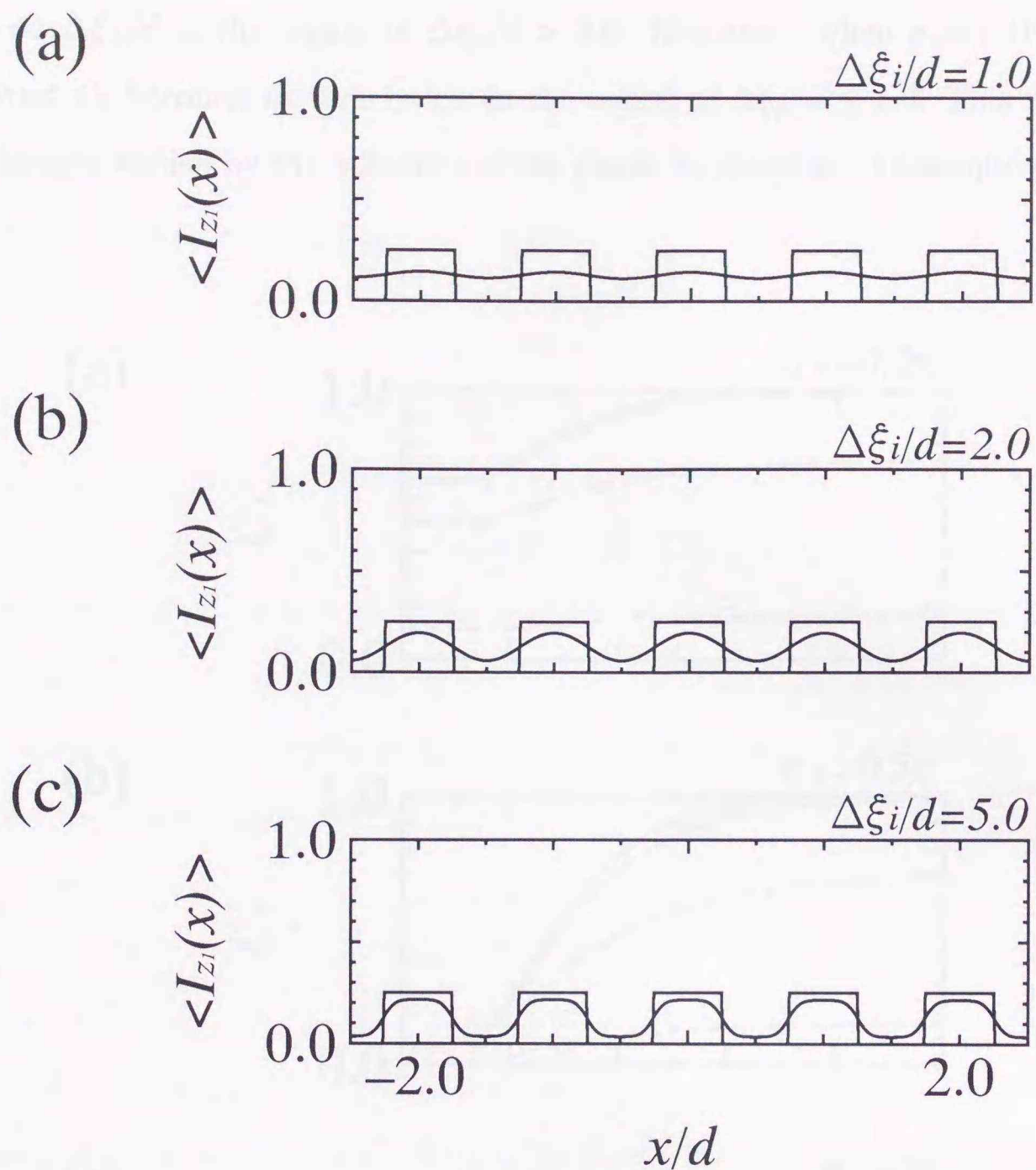


Figure 6.4 Average intensity distributions of Talbot images produced by the Ronchi grating with phase fluctuations having the normalized spatial correlation length $\Delta \xi_\phi/d=1.0$. The normalized spatial coherence length of the illuminating light is varied as $\Delta \xi_i/d=(a)1.0$, (b)2.0 and (c) 5.0. The rectangular and the curved lines indicate the average intensity distributions of Talbot images without and with fluctuations, respectively. These were obtained for the case of $L=1$, $t_o=0.5$ and $\sigma_\phi=0.2\pi$.

it is found that, when σ_ϕ is 0.2π (Fig.6.5 (a)), the variation of C_1 for $\Delta \xi_\phi/d$ is regarded as the same type in comparison with that of the amplitude fluctuations as already shown in Fig.6.3. On the other hand, with an increase of σ_ϕ (0.5π or π), the variation of C_1 for $\Delta \xi_\phi$ becomes different from that for the amplitude fluctuations. Namely,

when $\sigma_\phi=0.5\pi$ (Fig.6.5(b)), the contrast C_1 depends strongly not only on $\Delta\xi_i/d$ but also on $\Delta\xi_\phi/d$ in the region of $\Delta\xi_\phi/d > 3.0$. Moreover, when $\sigma_\phi=\pi$ (Fig.6.5(c)), the contrast C_1 becomes extremely low in the region of $\Delta\xi_\phi/d \leq 1.0$. This means that the self-images vanish by the influence of the phase fluctuations. Consequently, it is found

6.6 Conclusions

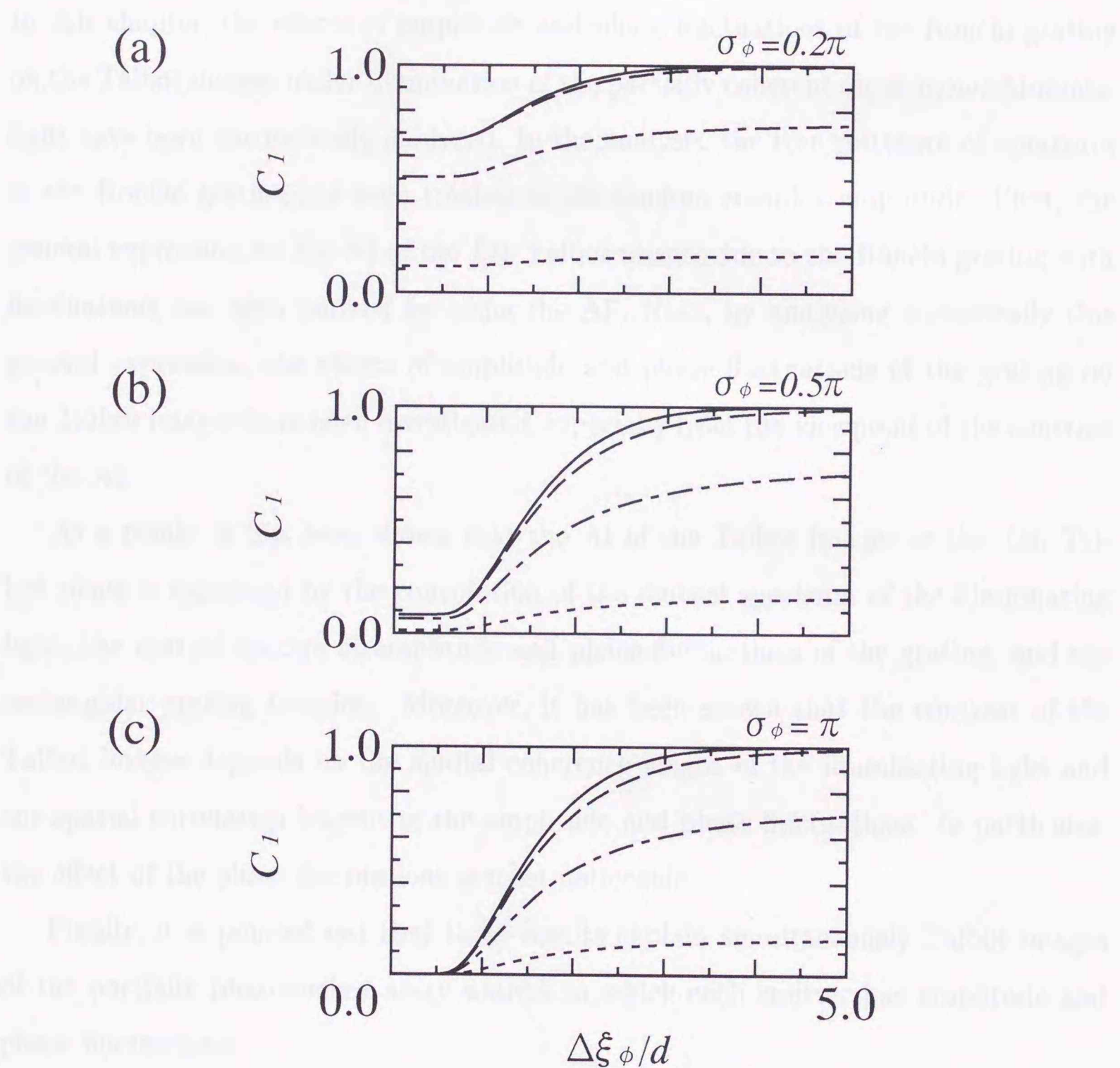


Figure 6.5 Contrast of the average intensity of Talbot images at the first Talbot plane as a function of the normalized spatial correlation length $\Delta\xi_\phi/d$ of phase fluctuations whose magnitude is set to take $\sigma_\phi=(a)0.2\pi$, (b) 0.5π and (c) π . In each figure, the different lines indicate the cases $\Delta\xi_i/d=1.0$ (- - - -), 2.0 (— · —), 5.0(· · ·) and ∞ (———) of the normalized spatial coherence length of the illuminating light.

that the phase fluctuations have great influence on the contrast C_1 in comparison with the amplitude fluctuations.

6.5 Conclusions

In this chapter, the effects of amplitude and phase fluctuations of the Ronchi grating on the Talbot images under illumination of the partially coherent quasi-monochromatic light have been theoretically analyzed. In the analysis, the transmittance of apertures in the Ronchi grating has been treated as the random complex amplitude. First, the general expression for the AI of the L th Talbot images due to the Ronchi grating with fluctuations has been derived by using the AF. Next, by analyzing numerically this general expression, the effects of amplitude and phase fluctuations of the grating on the Talbot images have been investigated, especially from the viewpoint of the contrast of the AI.

As a result, it has been shown that the AI of the Talbot images at the L th Talbot plane is expressed by the convolution of the mutual spectrum of the illuminating light, the mutual spectra of amplitude and phase fluctuations of the grating, and the rectangular grating function. Moreover, it has been shown that the contrast of the Talbot images depends on the spatial coherence length of the illuminating light and the spatial correlation lengths of the amplitude and phase fluctuations. In particular, the effect of the phase fluctuations is most noticeable.

Finally, it is pointed out that these results explain simultaneously Talbot images of the partially phase-locked array sources in which each emitter has amplitude and phase fluctuations.

6.6 Appendices

6.6.1 Derivation of Eq.(6.12)

The amplitude transmittance $t_g(\xi)$ with the period d is expressed in a form

$$t_g(\xi) = \sum_n G(n\nu_o) \exp(2\pi i n \nu_o \xi), \quad (6A.1)$$

where $\nu_o = 1/d$ is the fundamental spatial frequency and $G(n\nu_o)$ is a Fourier expansion component of $t_g(\xi)$. Then, the correlation function $\Gamma_g(\xi, -\lambda Z \Delta\nu)$ of $t_g(\xi)$ is obtained as

$$\begin{aligned} \Gamma_g(\xi, -\lambda Z \Delta\nu) &= \langle t_g(\xi + \lambda Z \Delta\nu/2) t_g^*(\xi - \lambda Z \Delta\nu/2) \rangle \\ &= \sum_n \sum_m G(n\nu_o) G^*(m\nu_o) \exp[i\pi(n+m)\lambda Z \nu_o \Delta\nu] \\ &\quad \times \exp[2\pi i(n-m)\nu_o \xi]. \end{aligned} \quad (6A.2)$$

In this case, the averaging operation is not required, because no random variables are involved. Moreover, the AF A_g is defined as a Fourier transform of Eq.(6A.2) with respect to ξ , and it is given by

$$\begin{aligned} A_g(\Delta\nu, -\lambda Z \Delta\nu) &= \int \Gamma_g(\xi, -\lambda Z \Delta\nu) \exp(-2\pi i \xi \Delta\nu) d\xi \\ &= \sum_n \sum_m G(n\nu_o) G^*(m\nu_o) \exp[i\pi(n+m)\lambda Z \nu_o \Delta\nu] \\ &\quad \times \delta[\Delta\nu - (n-m)\nu_o]. \end{aligned} \quad (6A.3)$$

Then, an inverse Fourier transform $F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)]$ with respect to $\Delta\nu$ becomes

$$\begin{aligned} F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)] &= \int A_g(\Delta\nu, -\lambda Z \Delta\nu) \exp(2\pi i x \Delta\nu) d\Delta\nu \\ &= \sum_n \sum_m G(n\nu_o) G^*(m\nu_o) \exp[i\pi(n^2 - m^2)\lambda Z \nu_o^2] \end{aligned}$$

$$\times \exp[2\pi i(n-m)\nu_o x]. \quad (6A.4)$$

In particular, in the L th Talbot plane of $Z_L = L(2d^2/\lambda) = L(2/\lambda\nu_o^2)$ (L is a positive integer), Eq.(6A.4) reduces to

$$\begin{aligned} F^{-1}[A_g(\Delta\nu, -\lambda Z \Delta\nu)] &= \left| \sum_n G(n\nu_o) \exp(2\pi i n \nu_o x) \right|^2 \\ &= |t_g(x)|^2, \end{aligned} \quad (6A.5)$$

which is equivalent to $t_g(x)$ when the amplitude transmittance is given by a binary profile, such as a Ronchi grating.

6.6.2 Derivation of Eq.(6.23)

By using $\tilde{\Gamma}_p(x/\lambda Z_L) = \delta(x/\lambda Z_L)$ and Eqs.(6.15) and (6.18), Eq.(6.12) becomes

$$\begin{aligned} \langle I_{Z_L}(x) \rangle &= (1/\lambda Z_L)^3 \tilde{\Gamma}_i(x/\lambda Z_L) \otimes \tilde{\Gamma}_a(x/\lambda Z_L) \otimes \delta(x/\lambda Z_L) \otimes t_g(x) \\ &= (1/\lambda Z_L)^2 \tilde{\Gamma}_i(x/\lambda Z_L) \otimes \tilde{\Gamma}_a(x/\lambda Z_L) \otimes t_g(x) \\ &= \left\{ I_o \sqrt{2\pi} \Delta\xi_i / (\lambda Z_L)^2 \right\} \exp \left[-2 (\pi \Delta\xi_i x / \lambda Z_L)^2 \right] \\ &\quad \otimes \left\{ t_o^2 \delta(x/\lambda Z_L) + \sqrt{2\pi} \sigma_r^2 \Delta\xi_r \exp \left[-2 (\pi \Delta\xi_r x / \lambda Z_L)^2 \right] \right\} \otimes t_g(x) \\ &= \left\{ I_o \sqrt{2\pi} t_o^2 \Delta\xi_i / (\lambda Z_L)^2 \right\} \exp \left[-2 (\pi \Delta\xi_i x / \lambda Z_L)^2 \right] \otimes \delta(x/\lambda Z_L) \otimes t_g(x) \\ &\quad + \left\{ I_o (2\pi) \sigma_r^2 \Delta\xi_i \Delta\xi_r / (\lambda Z_L)^2 \right\} \exp \left[-2 (\pi \Delta\xi_i x / \lambda Z_L)^2 \right] \\ &\quad \otimes \exp \left[-2 (\pi \Delta\xi_r x / \lambda Z_L)^2 \right] \otimes t_g(x) \\ &= \left\{ I_o \sqrt{2\pi} t_o^2 \Delta\xi_i / \lambda Z_L \right\} \exp \left[-2 (\pi \Delta\xi_i x / \lambda Z_L)^2 \right] \otimes t_g(x) \\ &\quad + \left\{ I_o \sqrt{2\pi} \sigma_r^2 \Delta\xi_i \Delta\xi_r / \lambda Z_L \sqrt{\Delta\xi_i^2 + \Delta\xi_r^2} \right\} \\ &\quad \times \exp \left[-2 (\pi x / \lambda Z_L)^2 / (1/\Delta\xi_i^2 + 1/\Delta\xi_r^2) \right] \otimes t_g(x). \end{aligned} \quad (6B.1)$$

Therefore, the AI of Talbot images is expressed as

$$\langle I_{Z_L}(x) \rangle = [C_{a1} \exp(-2x^2/\Delta x_i^2) + C_{a2} \exp(-2x^2/\Delta x_a^2)] \otimes t_g(x), \quad (6B.2)$$

where

$$\Delta x_i = \lambda Z_L / \pi \Delta \xi_i, \quad (6B.3)$$

$$\Delta x_a = (\lambda Z_L / \pi) \sqrt{1/\Delta \xi_i^2 + 1/\Delta \xi_r^2}, \quad (6B.4)$$

and two coefficients, C_{a1} and C_{a2} , are dependent on Z_L and fluctuation parameters, and are given by

$$C_{a1} = C_o t_o^2 \Delta \xi_i, \quad (6B.5)$$

$$C_{a2} = C_o \sigma_r^2 \Delta \xi_i \Delta \xi_r / \sqrt{\Delta \xi_i^2 + \Delta \xi_r^2}, \quad (6B.6)$$

respectively, where

$$C_o = I_o \sqrt{2\pi} / \lambda Z_L. \quad (6B.7)$$

6.6.3 Derivation of Eq.(6.30)

By substituting Eq.(6.29) into Eq.(6.12), the following equation is obtained:

$$\langle I_{Z_L}(x) \rangle = (1/\lambda Z_L)^3 \tilde{\Gamma}_i(x/\lambda Z_L) \otimes t_o^2 \delta(x/\lambda Z_L) \otimes \tilde{\Gamma}_p(x/\lambda Z_L) \otimes t_g(x). \quad (6C.1)$$

By using the property of the δ function and Eqs.(6.15) and (6.21), Eq.(6C.1) becomes

$$\begin{aligned}
\langle I_{Z_L}(x) \rangle &= \left\{ I_o \sqrt{2\pi} t_o^2 \Delta \xi_i / (\lambda Z_L)^3 \right\} \exp \left[-2 (\pi \Delta \xi_i x / \lambda Z_L)^2 \right] \otimes \delta(x / \lambda Z_L) \\
&\quad \otimes \left\{ \exp \left(-\sigma_\phi^2 \right) \delta(x / \lambda Z_L) + \sqrt{2\pi} \left[1 - \exp \left(-\sigma_\phi^2 \right) \right] \Delta \xi_\phi \right. \\
&\quad \times \exp \left[-2 (\pi \Delta \xi_\phi x / \lambda Z_L)^2 \right] \left. \right\} \otimes t_g(x) \\
&= \left\{ I_o \sqrt{2\pi} t_o^2 \exp \left(-\sigma_\phi^2 \right) \Delta \xi_i / \lambda Z_L \right\} \exp \left[-2 (\pi \Delta \xi_i x / \lambda Z_L)^2 \right] \otimes t_g(x) \\
&\quad + \left\{ I_o (2\pi) t_o^2 \left[1 - \exp \left(-\sigma_\phi^2 \right) \right] \Delta \xi_i \Delta \xi_\phi / (\lambda Z_L)^2 \right\} \exp \left[-2 (\pi \Delta \xi_i x / \lambda Z_L)^2 \right] \\
&\quad \otimes \exp \left[-2 (\pi \Delta \xi_\phi x / \lambda Z_L)^2 \right] \otimes t_g(x) \\
&= \left\{ I_o \sqrt{2\pi} t_o^2 \exp \left(-\sigma_\phi^2 \right) \Delta \xi_i / \lambda Z_L \right\} \exp \left[-2 (\pi \Delta \xi_i x / \lambda Z_L)^2 \right] \otimes t_g(x) \\
&\quad + \left\{ I_o \sqrt{2\pi} t_o^2 \left[1 - \exp \left(-\sigma_\phi^2 \right) \right] \Delta \xi_i \Delta \xi_\phi / \lambda Z_L \sqrt{\Delta \xi_i^2 + \Delta \xi_\phi^2} \right\} \\
&\quad \exp \left[-2 (\pi x / \lambda Z_L)^2 / (1 / \Delta \xi_i^2 + 1 / \Delta \xi_\phi^2) \right] \otimes t_g(x). \tag{6C.2}
\end{aligned}$$

Therefore, Eq.(6C.2) is rewritten as

$$\langle I_{Z_L}(x) \rangle = \left[C_{p1} \exp \left(-2x^2 / \Delta x_i^2 \right) + C_{p2} \exp \left(-2x^2 / \Delta x_p^2 \right) \right] \otimes t_g(x), \tag{6C.3}$$

where

$$\Delta x_p = (\lambda Z_L / \pi) \sqrt{1 / \Delta \xi_i^2 + 1 / \Delta \xi_\phi^2}, \tag{6C.4}$$

$$C_{p1} = C_o t_o^2 \exp \left(-\sigma_\phi^2 \right) \Delta \xi_i, \tag{6C.5}$$

$$C_{p2} = C_o t_o^2 \left[1 - \exp \left(-\sigma_\phi^2 \right) \right] \Delta \xi_i \Delta \xi_\phi / \sqrt{\Delta \xi_i^2 + \Delta \xi_\phi^2}, \tag{6C.6}$$

with Δx_i and C_o given by Eqs.(6B.3) and (6B.7).

Chapter 7

Realization of multiple images due to the partially coherent light from a Gaussian Schell-model source

7.1 Introduction

Array sources are demanded to realize optical multiplex communication systems and optical parallel processing in the future. As a method of realizing array sources, semiconductor array lasers [53, 57, 71] fabricated on a monolithic substrate and connected with micro-optic components are widely expected. Moreover, array generators (array illuminators) [72]–[74] are also expected as a simple method of splitting an incoming beam of light into an array of many light spots. However, these methods are applied only to the coherent light under which, in general, the optical systems cannot avoid the speckle noise [3]. Therefore, array sources for the partially coherent light are expected to achieve the low speckle noise and to supply the high-power light.

In this chapter, a method of realizing the multiple images with the spots similar to a Gaussian Schell-model (GSM) source [21, 22] is introduced by using the grating having the apertures of the opening with the Gaussian amplitude transmittance. The GSM source has a Gaussian intensity profile and a Gaussian complex degree of spatial coherence with stationarity [5] and radiates the partially coherent light which is called as a GSM beam [21, 22]. Moreover, a coherent Gaussian laser source and a Gaussian quasi-homogeneous source [48] are regarded as the limiting cases of the GSM source.

In Sec.7.2, the average intensity (AI) at the observation plane of the Fresnel diffraction region from the GSM source is derived, under illumination of the GSM beam over the grating with an arbitrary complex amplitude transmittance, by the propagation law of the ambiguity function (AF) [34]–[36]. The AF is a powerful tool for analyzing the partially coherent light field, because the propagation of the AF is easily described only by changing the argument without changing its functional form in the paraxial approximation [35]–[36]. That is, when the AF $A_{z=z_1}(\Delta\nu, \Delta x)$ at the plane of $z=z_1$ is known, $A_{z=z_2}(\Delta\nu, \Delta x)$ at the plane of $z=z_2$ in the Fresnel diffraction region is given by (see Eq.(2.19))

$$A_{z=z_2}(\Delta\nu, \Delta x) = A_{z=z_1}[\Delta\nu, \Delta x - \lambda(z_2 - z_1)\Delta\nu], \quad (7.1)$$

where λ is the wavelength, and $\Delta\nu$ and Δx denote the spatial frequency difference and the two-point distance, respectively. In addition, the AI $\langle I_z(x) \rangle$ at the z plane is obtained by an inverse Fourier transform of $A_z(\Delta\nu, \Delta x=0)$ with respect to $\Delta\nu$ in the form (see Eq.(2.15))

$$\langle I_z(x) \rangle = \int A_z(\Delta\nu, \Delta x = 0) \exp(2\pi i \Delta\nu x) d\Delta\nu. \quad (7.2)$$

In Sec.7.3, the condition for the locations of both the grating and the observation plane to realize the multiple images with the spots similar to the GSM source is considered. Moreover, for the grating having the apertures of the opening with the Gaussian amplitude transmittance, the multiple images are evaluated by changing the source size, the spatial coherence length of the source, and the fill factor (the opening ratio) of the grating. Finally, conclusions are described in Sec.7.4.

7.2 Average intensity at the observation plane

Consider the situation in Fig.7.1 that the grating with an arbitrary complex amplitude transmittance is illuminated by a GSM beam, i.e., the partially coherent light radiated from a GSM source, and is placed in the Fresnel diffraction region at a distance z_1 from the GSM source. Moreover, the observation plane is placed in the Fresnel diffraction region at a distance z_2 from the grating. For simplicity, the coordinates x is commonly used for the GSM source, the grating, and the observation plane.

In the following, the AI at the observation plane is derived by the propagation law of the AF which is valid in the paraxial domain.

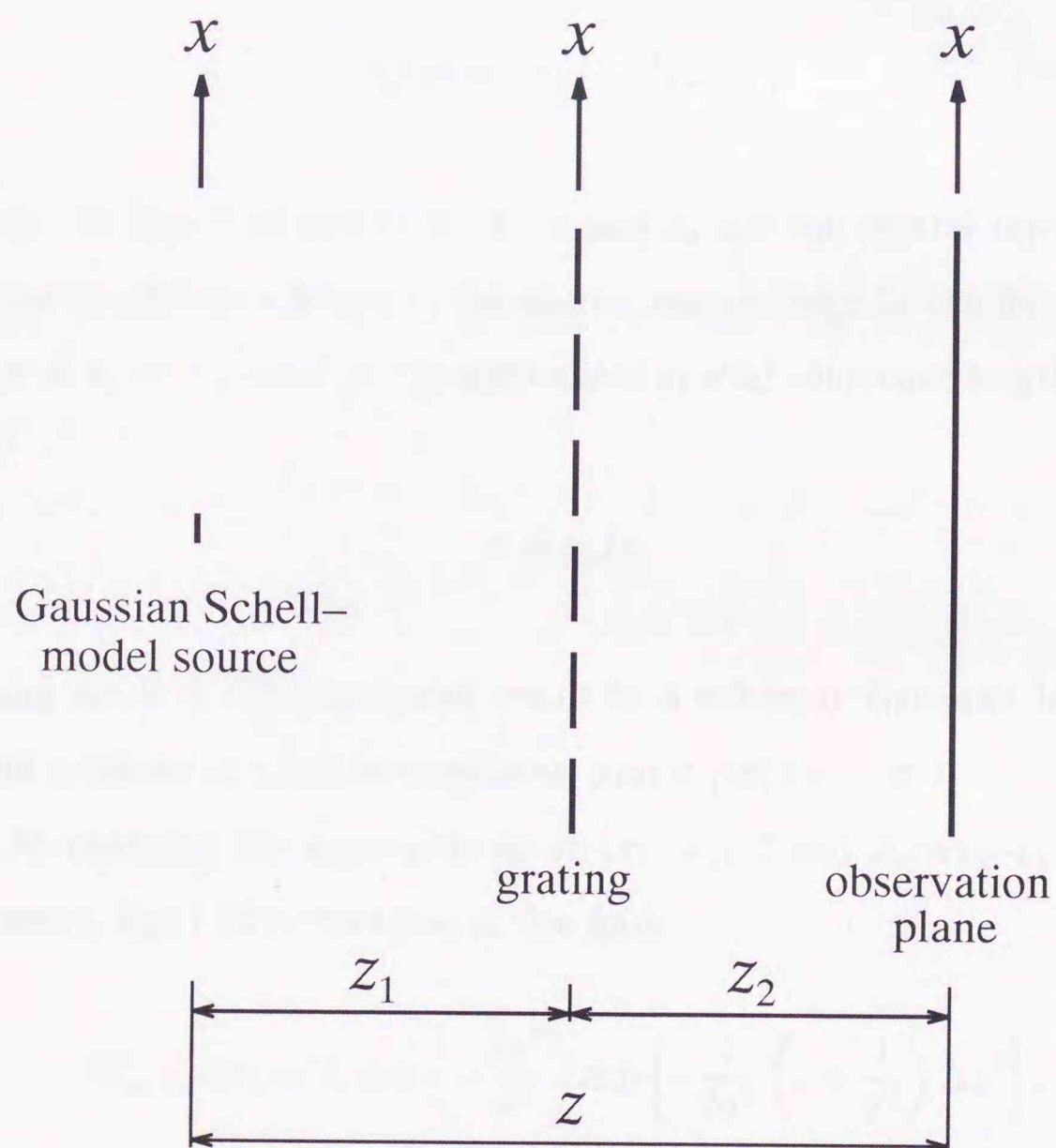


Figure 7.1 Optical coordinates at the GSM source, the grating, and the observation plane.

7.2.1 Ambiguity function of the Gaussian Schell-model source

The cross-spectral density (CSD) $W_o(x_1, x_2)$ [5], [7]–[10] of a GSM source radiating the partially coherent monochromatic light is given by [21]

$$W_o(x_1, x_2) = \sqrt{I_o(x_1)I_o(x_2)}\gamma_o(x_2 - x_1), \quad (7.3)$$

where $I_o(x)$ and $\gamma_o(x)$ denote the optical intensity distribution and the complex degree of spatial coherence of the source, respectively. In the present study, $I_o(x)$ and $\gamma_o(x)$ are given by

$$I_o(x) = I_s \exp(-2x^2/a^2), \quad (7.4)$$

$$\gamma_o(x) = \exp(-x^2/2x_s^2), \quad (7.5)$$

respectively. In Eqs.(7.4) and (7.5), I_s , a and x_s are the central intensity, the size, and the spatial coherence length of the source, respectively. In the following analysis, the ratio σ of x_s to a is used as the normalized spatial coherence length of the source defined by

$$\sigma = x_s/a. \quad (7.6)$$

The limiting cases of this parameter result in a coherent Gaussian laser source for $\sigma \rightarrow \infty$ and a Gaussian quasi-homogeneous source [48] for $\sigma \ll 1$.

Then, by changing the arguments as $x=(x_1+x_2)/2$ and $\Delta x=x_2-x_1$ (i.e., the two-point distance), Eq.(7.3) is rewritten in the form

$$W_o(x, \Delta x) = I_s \exp\left(-\frac{2x^2}{a^2}\right) \exp\left[-\frac{1}{2a^2}\left(1 + \frac{1}{\sigma^2}\right)\Delta x^2\right]. \quad (7.7)$$

On the other hand, the AF $A(\Delta\nu, \Delta x)$ of the CSD $W(x, \Delta x)$ has already been intro-

duced in Eq.(2.11) [14], [34]–[36]:

$$A(\Delta\nu, \Delta x) = \int W(x, \Delta x) \exp(-2\pi i x \Delta\nu) dx, \quad (7.8)$$

where $\Delta\nu$ is the spatial frequency difference. By substituting Eq.(7.7) into Eq.(7.8), the AF for the GSM source is obtained as

$$A_o(\Delta\nu, \Delta x) = A_o(0, 0) \exp\left[-\frac{(\pi a \Delta\nu)^2}{2}\right] \exp\left[-\frac{1}{2a^2} \left(1 + \frac{1}{\sigma^2}\right) \Delta x^2\right], \quad (7.9)$$

where $A_o(0, 0)$ is the constant value given by

$$A_o(0, 0) = \sqrt{\pi/2} I_s a. \quad (7.10)$$

7.2.2 Ambiguity function just before the grating

Next, consider the AF $A_{z1-}(\Delta\nu, \Delta x)$ just before the grating placed in the Fresnel diffraction region at a distance z_1 from the source. In this case, by applying Eq.(7.1) to Eq.(7.9), $A_{z1-}(\Delta\nu, \Delta x)$ is easily obtained as

$$\begin{aligned} A_{z1-}(\Delta\nu, \Delta x) &= A_o(\Delta\nu, \Delta x - \lambda z_1 \Delta\nu) \\ &= A_o(0, 0) \exp\left[-\frac{(\pi a \Delta\nu)^2}{2}\right] \\ &\quad \times \exp\left[-\frac{1}{2a^2} \left(1 + \frac{1}{\sigma^2}\right) (\Delta x - \lambda z_1 \Delta\nu)^2\right]. \end{aligned} \quad (7.11)$$

The physical meaning of Eq.(7.11) has already been investigated in detail in connection with the GSM beam parameters (given lately by Eqs.(7.18) and (7.19)), by using the equi-AF ellipse (i.e., a trajectory of the AF at a constant value) (see Chapter 4).

7.2.3 Ambiguity function just behind the grating

In general, the periodic amplitude transmittance $t_g(x)$ of a grating can be described by a Fourier expansion as

$$t_g(x) = \sum_n a_n \exp\left(i2\pi n \frac{x}{d}\right), \quad (7.12)$$

where n is an integer in the region extending from $-\infty$ to ∞ . Moreover, a_n and d denote a Fourier expansion coefficient of $t_g(x)$ and a period of the grating, respectively. In addition, when $t_g(x)$ is deterministic, the CSD $W_g(x, \Delta x)$ of $t_g(x)$ is written as

$$\begin{aligned} W_g(x, \Delta x) &= t_g(x + \Delta x/2) t_g^*(x - \Delta x/2) \\ &= \sum_n a_n \exp\left[i2\pi n \frac{(x + \Delta x/2)}{d}\right] \sum_m a_m^* \exp\left[-i2\pi m \frac{(x - \Delta x/2)}{d}\right] \\ &= \sum_n \sum_m a_n a_m^* \exp\left[i\pi(n + m) \frac{\Delta x}{d}\right] \exp\left[i2\pi(n - m) \frac{x}{d}\right]. \end{aligned} \quad (7.13)$$

Then, with the help of Eq.(7.8), the AF $A_g(\Delta\nu, \Delta x)$ of the grating is obtained as

$$A_g(\Delta\nu, \Delta x) = \sum_n \sum_m a_n a_m^* \exp\left[i\pi(n + m) \frac{\Delta x}{d}\right] \delta\left(\Delta\nu - \frac{n - m}{d}\right). \quad (7.14)$$

The AF $A_{z1+}(\Delta\nu, \Delta x)$ just behind the grating is obtained by performing the convolution integral of $A_{z1-}(\Delta\nu, \Delta x)$ and $A_g(\Delta\nu, \Delta x)$ (see Chapter 6), and it is given by

$$\begin{aligned} A_{z1+}(\Delta\nu, \Delta x) &= A_{z1-}(\Delta\nu, \Delta x) \otimes A_g(\Delta\nu, \Delta x) \\ &= A_o(0, 0) \sum_n \sum_m a_n a_m^* \exp\left[i\pi(n + m) \frac{\Delta x}{d}\right] \\ &\quad \times \exp\left[-\frac{(\pi a)^2}{2} \left(\Delta\nu - \frac{n - m}{d}\right)^2\right] \\ &\quad \times \exp\left[-\frac{1}{2a^2} \left(1 + \frac{1}{\sigma^2}\right) \left\{\Delta x - \lambda z_1 \left(\Delta\nu - \frac{n - m}{d}\right)\right\}^2\right], \end{aligned} \quad (7.15)$$

where $\otimes$ denotes the convolution integral.

7.2.4 Average intensity at the observation plane

Finally, the AF $A_z(\Delta\nu, \Delta x)$ is derived at the observation plane of the Fresnel diffraction region, a distance $z(=z_1+z_2)$ from the source. Use of Eq.(7.1) in Eq.(7.15) yields

$$\begin{aligned} A_z(\Delta\nu, \Delta x) &= A_{z1+}(\Delta\nu, \Delta x - \lambda z_2 \Delta\nu) \\ &= A_o(0, 0) \sum_n \sum_m a_n a_m^* \exp \left[i\pi(n+m) \frac{\Delta x - \lambda z_2 \Delta\nu}{d} \right] \\ &\quad \times \exp \left[-\frac{(\pi a)^2}{2} \left(\Delta\nu - \frac{n-m}{d} \right)^2 \right] \\ &\quad \times \exp \left[-\frac{1}{2a^2} \left(1 + \frac{1}{\sigma^2} \right) \left(\Delta x - \lambda z \Delta\nu + \lambda z_1 \frac{n-m}{d} \right)^2 \right]. \end{aligned} \quad (7.16)$$

Successively, by substituting Eq.(7.16) into Eq.(7.2), the AI $\langle I_z(x) \rangle$ is obtained as (see Subsection 7.6.1)

$$\begin{aligned} \langle I_z(x) \rangle &= \int A_z(\Delta\nu, \Delta x = 0) \exp(2\pi i \Delta\nu x) d\Delta\nu \\ &= I_z(0) \sum_n \sum_m a_n a_m^* \exp \left[-\frac{2}{w_z^2} \left\{ \frac{z_2}{z_T} (n-m)d \right\}^2 \left(1 + \frac{1}{\sigma^2} \right) \right] \\ &\quad \times \exp \left[-\frac{2}{w_z^2} \left\{ x - \frac{z_2}{z_T} (n+m)d \right\}^2 \right] \\ &\quad \times \exp \left[i2\pi x \frac{n-m}{d} \left(\frac{R_z - z_2}{R_z} \right) \right] \\ &\quad \times \exp \left[-i2\pi(n^2 - m^2) \frac{z_2}{z_T} \left(\frac{R_z - z_2}{R_z} \right) \right], \end{aligned} \quad (7.17)$$

where w_z and R_z are the beam radius and the wave-front-curvature radius of the GSM beam at the observation plane, a distance z from the source in the free-space

propagation. These are given, as a function of z , by [21]

$$w_z = a \sqrt{1 + \left(\frac{\lambda z}{\pi a^2} \right)^2 \left(1 + \frac{1}{\sigma^2} \right)}, \quad (7.18)$$

$$R_z = z \left\{ 1 + \frac{1}{(\lambda z / \pi a^2)^2 (1 + 1/\sigma^2)} \right\}, \quad (7.19)$$

respectively. Moreover, z_T in Eq.(7.17) is the Talbot distance resulting from the grating and is given by [32]

$$z_T = 2d^2/\lambda. \quad (7.20)$$

In Eq.(7.17), $I_z(0)$ is the constant value given by

$$I_z(0) = I_s a / w_z. \quad (7.21)$$

7.3 Multiple images with spots similar to the GSM source

7.3.1 Conditions

In Eq.(7.17), consider the condition of

$$\frac{z_2}{z_T} \left(\frac{R_z - z_2}{R_z} \right) = N, \quad (N = 1, 2, 3, \dots), \quad (7.22)$$

where N is a positive integer. This condition is the same as that to realize Talbot images (i.e., self-images) [75] of the periodic object under illumination of a coherent Gaussian laser beam except that the left term is multiplied by 1/2 (compare Eq.(7.22) with Eq.(25) in Ref.[75]). The difference is caused by the fact that the AI is treated here

while the amplitude is treated in Ref.[75]. Further, impose on Eq.(7.17) the condition of

$$z_2 = Mz_T, \quad (M = N + 1, N + 2, \dots, 2N), \quad (7.23)$$

where M is a positive integer between $N+1$ and $2N$ (see Subsection 7.6.2). In particular, for the case of $M=2N$, z_1 and z_2 are obtained from Eqs.(7.22) and (7.23) as (see Subsection 7.6.2)

$$z_1 = 2Nz_T \sqrt{1 - \left(\frac{\pi}{4N}\right)^2 \left(\frac{a}{d}\right)^4 \frac{\sigma^2}{1 + \sigma^2}}, \quad (7.24)$$

$$z_2 = 2Nz_T, \quad (7.25)$$

respectively.

In Eq.(7.24), a/d denotes the ratio of the source size a to the period d of the grating. The necessary condition for the existence of z_1 with respect to a/d is decided by the condition that the content of the root in Eq.(7.24) must take a value of 0 or more. The severest case of this condition is $\sigma=\infty$ and $N=1$ in Eq.(7.24). In this case, the allowable range of a/d is obtained as

$$0 < a/d < \sqrt{4/\pi}. \quad (7.26)$$

It is easily understood in Eq.(7.24) that small values of a/d result in $z_1 \approx 2Nz_T$, irrespective of N and σ . For example, z_1 becomes $1.96Nz_T$ for the case of $a/d=0.5$, $\sigma=\infty$ and $N=1$. In this way, $z_1 \approx 2Nz_T$ in $0 < a/d \leq 0.5$ (i.e., the source size a is less than one half of the grating period d) for any values of σ and N .

Under these conditions, the AI $\langle I_z(x) \rangle$ of Eq.(7.17) becomes

$$\begin{aligned} \langle I_z(x) \rangle = & I_z(0) \sum_n \sum_m a_n a_m^* \exp \left[-\frac{2}{w_z^2} \{2(n-m)Nd\}^2 \left(1 + \frac{1}{\sigma^2}\right) \right] \\ & \times \exp \left[-\frac{2}{w_z^2} \{x - 2(n+m)Nd\}^2 \right] \exp \left(i2\pi x \frac{n-m}{2d} \right). \end{aligned} \quad (7.27)$$

In Eq.(7.27), w_z is defined as

$$w_z = \frac{8Nd^2}{\pi a} \sqrt{1 + \frac{1}{\sigma^2}}, \quad (7.28)$$

under the condition of $z_1 \approx 2Nz_T$ for $0 < a/d \leq 0.5$.

7.3.2 Multiple images due to the grating with the Gaussian amplitude transmittance

In the case that each aperture of the grating has the opening with the Gaussian amplitude transmittance, the amplitude transmittance $t_g(x)$ is expressed as

$$t_g(x) = \exp(-x^2/a_g^2) \otimes \sum_n \delta(x - nd), \quad (7.29)$$

where a_g is each aperture size of the grating. In particular, when the fill factor (the opening ratio) f of the grating defined by

$$f = 2a_g/d \quad (7.30)$$

takes a small value (for example, $0 < f \leq 0.5$), a Fourier expansion coefficient a_n of the grating is obtained as

$$\begin{aligned} a_n &= \frac{1}{d} \int_{-d/2}^{d/2} \exp(-x^2/a_g^2) \exp(-i2\pi n \frac{x}{d}) dx \\ &\approx \frac{1}{d} \int_{-\infty}^{\infty} \exp(-x^2/a_g^2) \exp(-i2\pi n \frac{x}{d}) dx \\ &= \left(\sqrt{\pi f/2} \right) \exp[-(n\pi f/2)^2]. \end{aligned} \quad (7.31)$$

By substituting Eq.(7.31) into Eq.(7.27), the normalized AI $\langle i_z(x) \rangle$ is obtained, after tedious calculations, as (see Subsection 7.6.3)

$$\langle i_z(x) \rangle = \exp\left(-\frac{2x^2}{w_{env}^2}\right) \left\{ \exp\left(-\frac{2x^2}{w_{spot}^2}\right) \otimes \sum_n \delta(x + 2nd) \right\}, \quad (7.32)$$

where w_{env} and w_{spot} denote the extent of an envelop and the spot size of the multiple images, respectively, which are given by

$$w_{env} = \frac{8Nd}{\pi} \sqrt{\frac{1 + 1/\sigma^2}{(a/d)^2} + \frac{1}{f^2}}, \quad (7.33)$$

$$w_{spot} = a \sqrt{1 + \left(\frac{2a_g}{a}\right)^2}, \quad (7.34)$$

respectively.

As a result, Eq.(7.32) implies that the spots with the Gaussian intensity distribution are realized at the period of $2d$ at the observation plane. That is, $\langle i_z(x) \rangle$ represents the multiple images with the spots similar to the GSM source. It is understood from Eq.(7.33) that the extent w_{env} of the multiple images is proportional to N given by Eq.(7.22) and increases with decreases in a/d , σ and f . On the other hand, the spot size w_{spot} given by Eq.(7.34) is dependent only on a_g/a , in such a way that it approaches the source size a with a decrease of a_g/a .

7.4 Evaluation of multiple images

In this section, the multiple images $\langle i_z(x) \rangle$ expressed by Eq.(7.32) are evaluated quantitatively by changing the parameters a/d , σ and f , under the conditions of $0 < a/d \leq 0.5$ and $0 < f \leq 0.5$. In this evaluation, N is fixed to 1, because the effect of N on the extent w_{env} of the envelop over the multiple images is apparent as mentioned previously. Furthermore, the number of the spots involved within w_{env} is introduced as the

parameter N_s . Such spots are specified to have the maximum values being $\exp(-2)$ or more.

7.4.1 Effect of the source size

First, the effect of the source size a on the multiple images $\langle i_z(x) \rangle$ was investigated by changing a/d . The AI distributions $\langle i_z(x) \rangle$ of the multiple images are shown in Fig.7.2

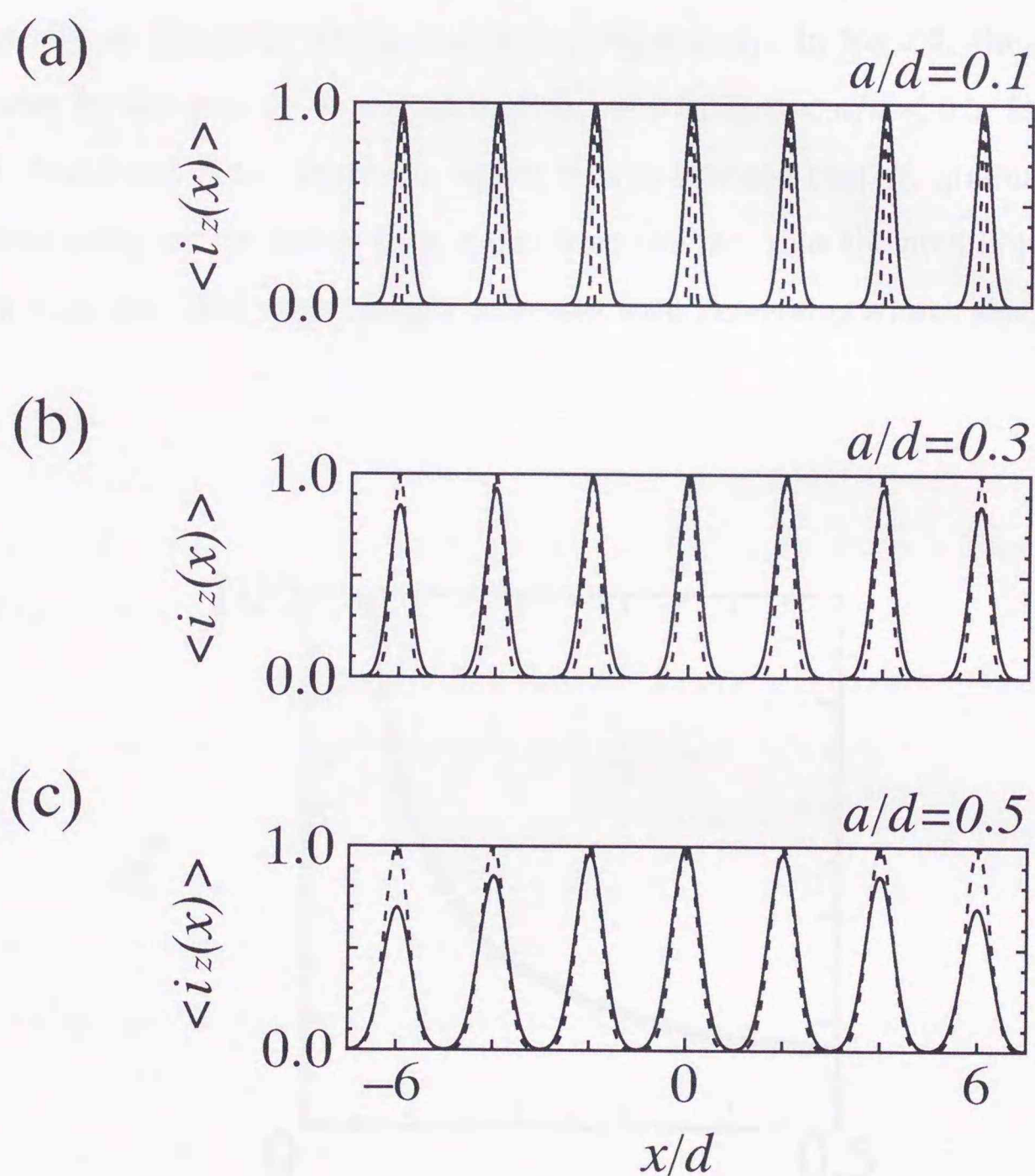


Figure 7.2 Multiple images $\langle i_z(x) \rangle$ (—) for the case of $\sigma=0.5$, $f=0.3$ and $N=1$. (a), (b) and (c) correspond to the results for $a/d=0.1$, 0.3 and 0.5 , respectively. Moreover, (- - -) denotes the normalized intensity distributions of GSM sources placed at intervals of $2d$.

by the solid lines (———) for the case of $\sigma=0.5$, $f=0.3$ and $N=1$. Figures 7.2(a), (b) and (c) correspond to the results for $a/d=(a)0.1$, (b)0.3 and (c)0.5, respectively. It can be understood from the figure that the extent of the envelop over the multiple images becomes narrow with an increase of a/d . In the figure, the broken lines (- - - -) denote the normalized intensity distributions of GSM sources placed at intervals of $2d$. By comparing the solid lines with the broken ones, as understood from Eq.(7.34), the spot size of the multiple images approaches the source size with an increase of a/d .

For each case, the number N_s of the spots in the multiple images was obtained numerically as $N_s=(a)58$, (b)21 and (c)15, respectively. In Fig.7.3, the values of N_s are shown by the dots (• • • •) at intervals of 0.01 in $0 < a/d \leq 0.5$, for the case of $\sigma=0.5$, $f=0.3$ and $N=1$. From this figure, it is understood that N_s gradually increases with decreasing source size a . This comes from the fact that the area illuminating the grating with the GSM beam steeply increases with decreasing source size, for a given

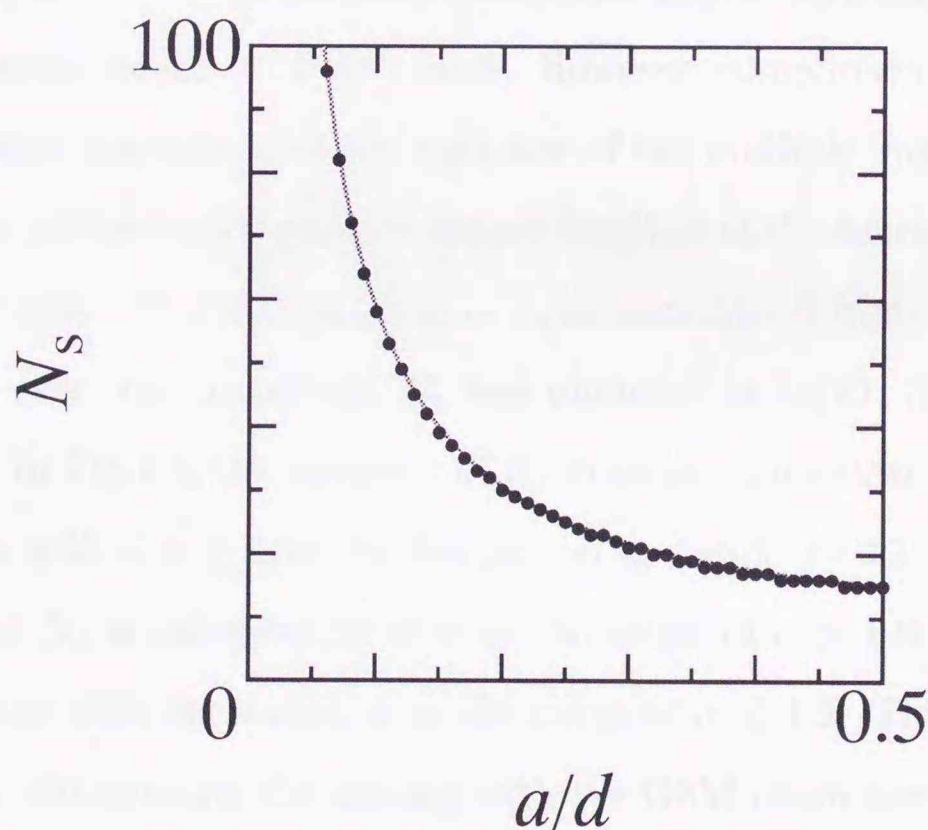


Figure 7.3 Dependence of N_s (• • • •) on a/d at intervals of 0.01 in $0 < a/d \leq 0.5$, for the case of $\sigma=0.5$, $f=0.3$ and $N=1$.

spatial coherence length of the source.

As a result, it was shown for the source size being small that the spots of the multiple images are distributed in a wide region and that the spot size is larger than the source size. On the contrary, for the source size being large, the distribution of the spots becomes narrow but the spot size approaches the source size. In this way, the distribution of the spots and the spot size of the multiple images are controllable by changing the source size.

7.4.2 Effect of the spatial coherence length

Next, the effect of the spatial coherence length of the source on the multiple images $\langle i_z(x) \rangle$ was investigated. The AI distributions $\langle i_z(x) \rangle$ of the multiple images are shown by the solid lines (———) for the case of $a/d=0.3$, $f=0.3$ and $N=1$. Figures 7.4 (a), (b), (c) and (d) correspond to the results for $\sigma=(a)0.2$, (b)0.5, (c)1.0 and (d) ∞ , respectively. In the figure, the broken lines (- - - -) denote the normalized intensity distributions of GSM sources placed at intervals of $2d$. This figure shows that the extent of the envelop over the multiple images becomes narrow with the increasing normalized spatial coherence length σ of the source. However, comparison of the solid lines with the broken ones indicates that the spot size of the multiple images is unchanged by a change in the normalized spatial coherence length σ of the source. This is owing to the fact that the spot size is independent of σ , as understood from Eq.(7.34).

For each case, the parameter N_s was obtained as (a)45, (b)21, (c)15 and (d)13, respectively. In Fig.7.5, the behavior of N_s analyzed numerically is shown by the dots (• • • •) in $0.01 \leq \sigma \leq 100$, for the case of $a/d=0.3$, $f=0.3$ and $N=1$. This figure indicates that N_s is independent of σ in the range of $\sigma > 1.0$. However, it becomes extremely large with decreasing σ in the range of $\sigma \leq 1.0$. This comes from the fact that the area illuminating the grating with the GSM beam steeply increases with the decreasing spatial coherence length of the source.

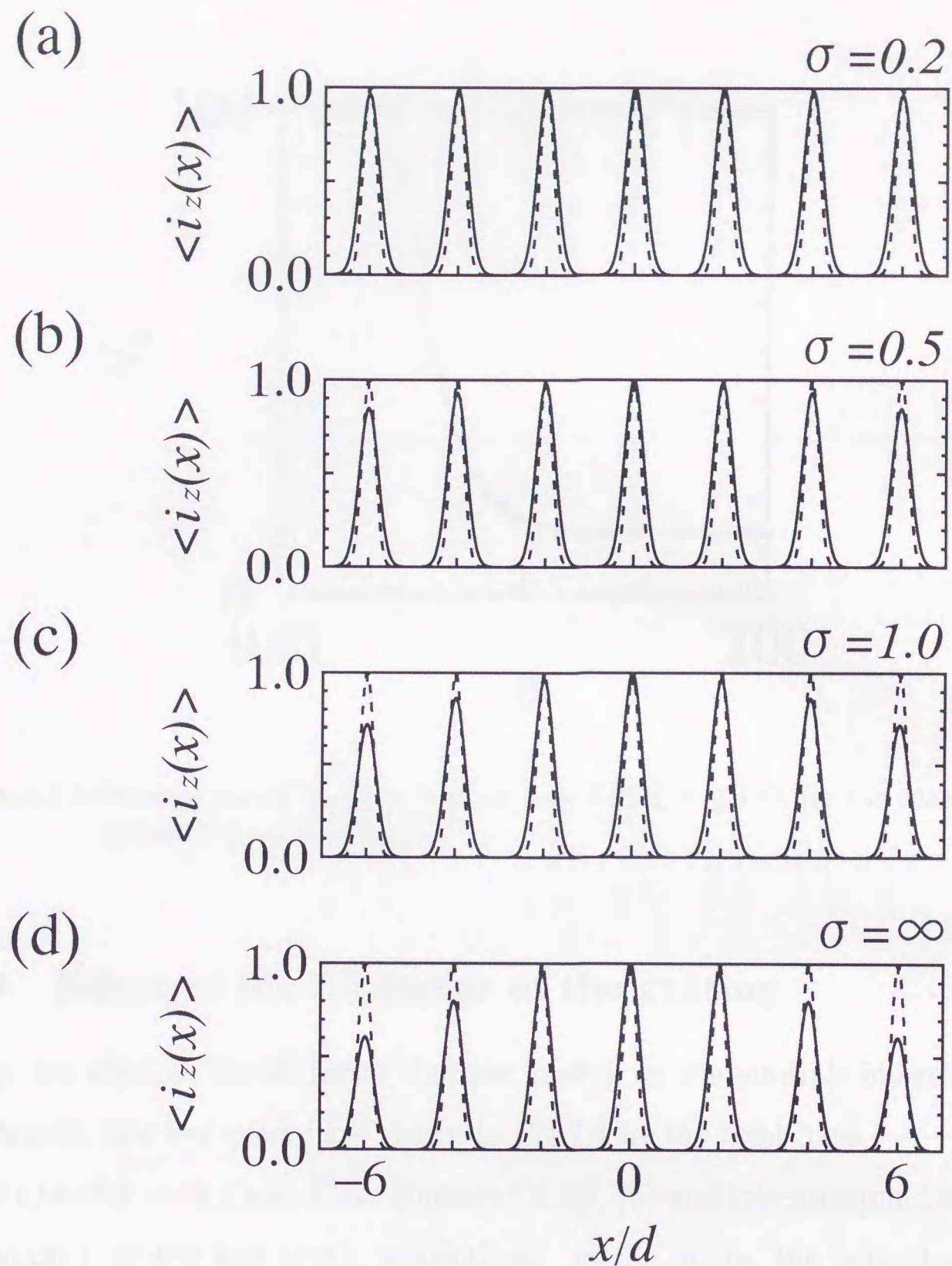


Figure 7.4 Multiple images $\langle i_z(x) \rangle$ (—) for the case of $a/d=0.3$, $f=0.3$ and $N = 1$.

(a), (b), (c) and (d) correspond to the results for $\sigma=0.2$, 0.5 , 1.0 and ∞ , respectively. Moreover, (- - -) denotes the normalized intensity distributions of GSM sources placed at intervals of $2d$.

As a result, the distribution of the spots in the multiple images due to the GSM beam becomes wider as the source approaches a spatially incoherent state. On the contrary, it becomes narrower as the source approaches a coherent state.

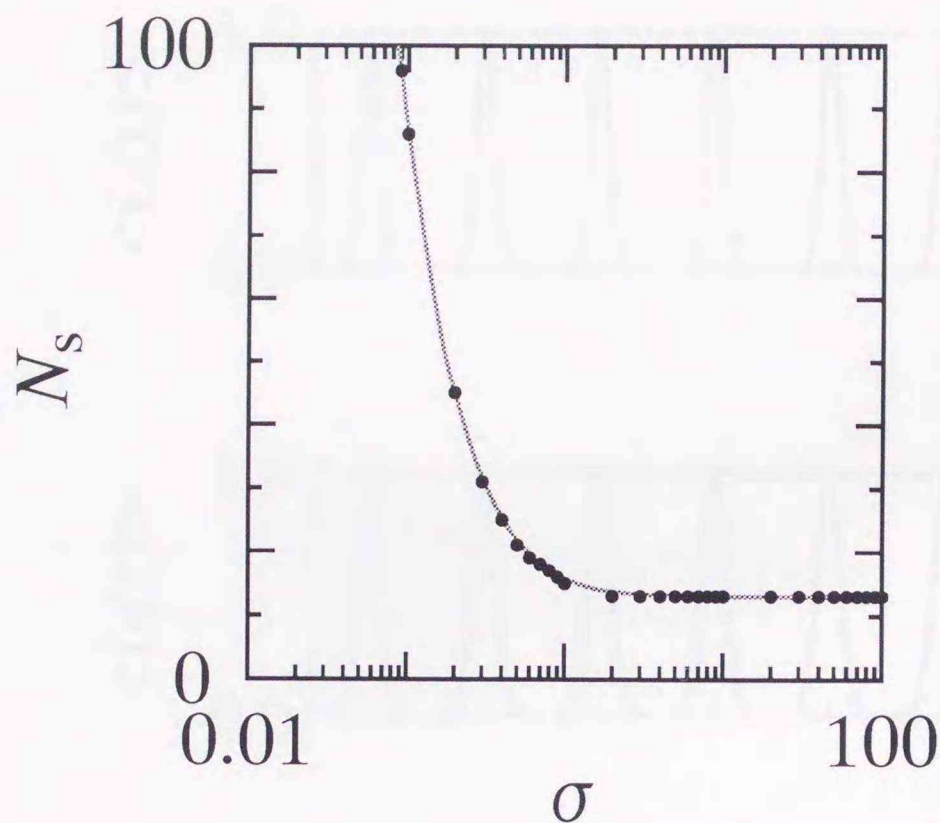


Figure 7.5 Dependence of N_s (• • • •) on σ in $0.01 \leq \sigma \leq 100$, for the case of $a/d=0.3$, $f=0.3$ and $N=1$.

7.4.3 Effect of the fill factor of the grating

Finally, the effect of the fill factor f of the grating on the multiple images $\langle i_z(x) \rangle$ was investigated, and the results are shown in Fig.7.6 by the solid lines (———) for the case of $a/d=0.3$, $\sigma=0.5$ and $N=1$. Figures 7.6 (a), (b) and (c) correspond to the results for $f=(a)0.1$, (b)0.3 and (c)0.5, respectively. In the figure, the normalized intensity distributions of GSM sources placed at intervals of $2d$ are also shown by the broken lines (- - - -). It is seen from the figure that the extent of the envelope in the multiple images becomes narrow with increasing f . On the other hand, it is understood by comparing the solid lines with the broken ones that, with an increase of f , the spot size of the multiple images becomes larger than the source size.

In Fig.7.7, the behavior of the parameter N_s as a function of the fill factor f of the grating is shown by the dots (• • • •) at intervals of 0.01 in $0 < f \leq 0.5$, for the case of $a/d=0.3$, $\sigma=0.5$ and $N=1$. From this figure, N_s is found to increase gradually with

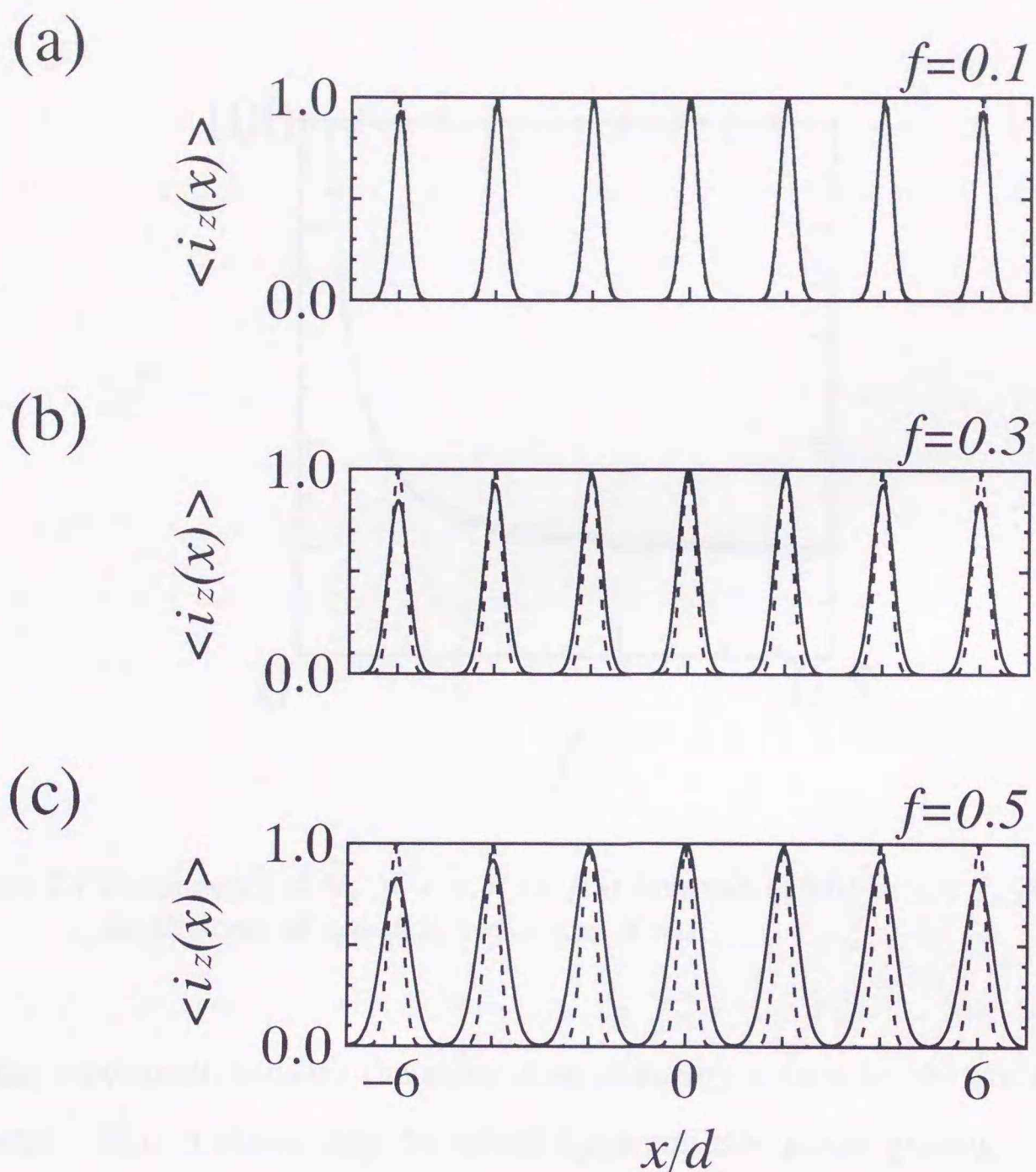


Figure 7.6 Multiple images $\langle i_z(x) \rangle$ (—) for the case of $a/d=0.3$, $\sigma=0.5$ and $N = 1$. (a), (b) and (c) correspond to the results for $f=0.1$, 0.3 and 0.5, respectively. Moreover, (- - -) denotes the normalized intensity distributions of GSM sources placed at intervals of $2d$.

f decreasing approximately up to $f=0.1$, and then to increase steeply. This stems from the fact that the light passing through the grating consisting of small apertures is diffracted to wide directions, irrespective of the spatial coherence length of the source.

As a result, for the case of the fill factor of the grating being small, the spots of the multiple images are distributed in a wide region and the spot size becomes almost equal to the source size. However, the ratio of energy in the resultant multiple images to that in the original GSM source becomes very low under the condition of the fill fac-

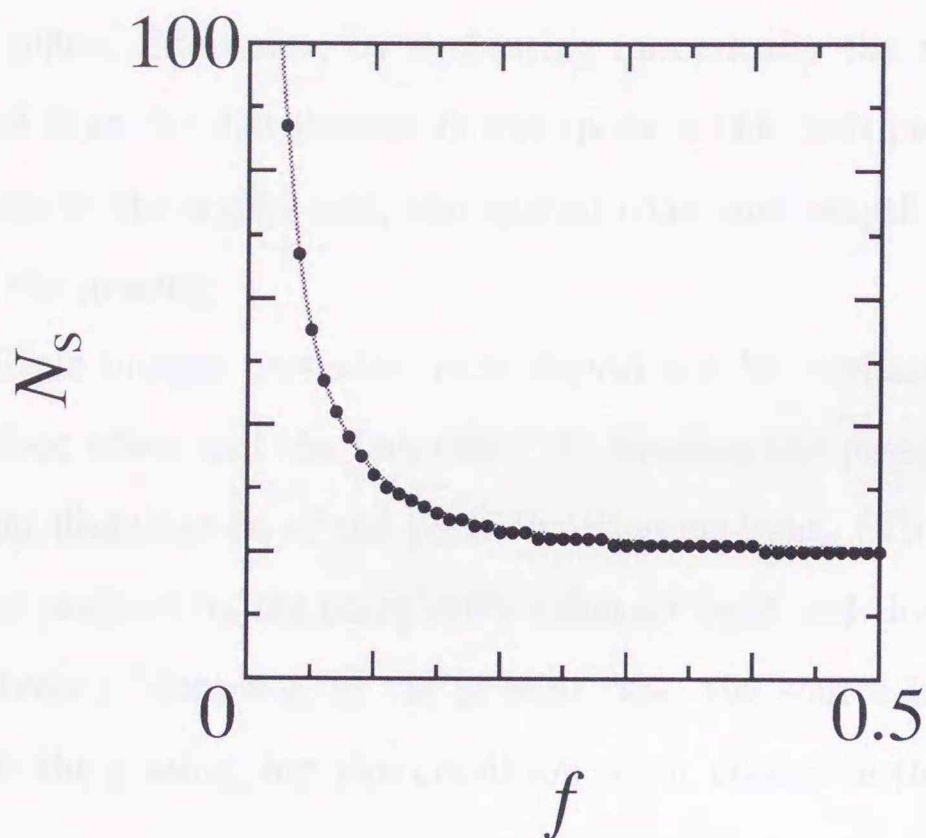


Figure 7.7 Dependence of N_s (• • • •) on f at intervals of 0.01 in $0 < f \leq 0.5$, for the case of $a/d=0.3$, $\sigma=0.5$ and $N=1$.

tor being very small, because the absorption of energy caused by the grating becomes very large. This problem may be solved by a suitable phase grating instead of an amplitude one.

7.5 Conclusions

In this chapter, the formation of the multiple images due to the grating under illumination of a GSM beam has been investigated. In the analysis, the grating consisting of the apertures with the Gaussian amplitude transmittance and with the period of d is used. The analysis is based on the propagation law of the AF, which is valid in the paraxial approximation and from which the AI is derived.

As a result, the multiple images with the spots similar to the GSM source are realized with the period of $2d$ at the observation plane of the Fresnel diffraction region

at a distance $4Nz_T$ (N : a positive integer, z_T : Talbot distance $=2d^2/\lambda$) from the GSM source, when the grating is placed in the middle between the GSM source and the observation plane. Moreover, by evaluating numerically the multiple images, it has been revealed that the distribution of the spots in the multiple images becomes wide with decreases in the source size, the spatial coherence length of the source, and the fill factor of the grating.

The multiple images presented here should not be confused with those resulting from the Talbot effect and the Lau effect [7], because the present multiple images are realized under illumination of the partially coherent light. (The Talbot effect and the Lau effect are realized by the completely coherent light and the completely incoherent one, respectively.) Moreover, in the present case, the source is located to be a finite distance from the grating, but this condition is not needed in the Talbot effect and the Lau effect.

In this way, the present method of realizing the multiple images with the spots similar to the GSM source can be applied to the optical elements such as partially coherent array lasers and array generators (array illuminators) for the partially coherent light. Moreover, in another standpoint, our method can be applied to the method of imaging a mask, for example, in the field of optical lithography.

Finally, it is pointed out that the results obtained in this chapter will also be obtained by performing the analysis using the Wigner distribution function [35] which has become popular in optics in recent years.

7.6 Appendices

7.6.1 Derivation of Eq.(7.17)

By substituting Eq.(7.16) into Eq.(7.2), the AI $\langle I_z(x) \rangle$ is obtained as

$$\begin{aligned}
\langle I_z(x) \rangle &= A_o(0,0) \sum_n \sum_m a_n a_m^* \int \exp \left[-\frac{(\pi a)^2}{2} \left(\Delta\nu - \frac{n-m}{d} \right)^2 \right] \\
&\quad \times \exp \left[-\frac{1}{2} \left(\frac{\lambda z}{a} \right)^2 \left(1 + \frac{1}{\sigma^2} \right) \left\{ \Delta\nu - \frac{z_1(n-m)}{zd} \right\}^2 \right] \\
&\quad \times \exp \left[i2\pi \Delta\nu \left\{ x - \frac{\lambda z_2(n+m)}{2d} \right\} \right] d\Delta\nu \\
&= A_o(0,0) \sum_n \sum_m a_n a_m^* \int \exp \left[-A(\Delta\nu - B)^2 \right] \\
&\quad \times \exp \left[-C(\Delta\nu - D)^2 \right] \exp \left[i2\pi \Delta\nu(x - E) \right] d\Delta\nu, \quad (7A.1)
\end{aligned}$$

where

$$A = \frac{(\pi a)^2}{2}, \quad (7A.2)$$

$$B = \frac{n-m}{d}, \quad (7A.3)$$

$$C = \frac{1}{2} \left(\frac{\lambda z}{a} \right)^2 \left(1 + \frac{1}{\sigma^2} \right), \quad (7A.4)$$

$$D = \frac{z_1(n-m)}{zd}, \quad (7A.5)$$

and

$$E = \frac{\lambda z_2(n+m)}{2d}. \quad (7A.6)$$

Then, a part of the integral in Eq.(7A.1) is calculated analytically as

$$\sqrt{\frac{\pi}{A+C}} \exp \left[-\frac{AC(B-D)^2}{A+C} \right] \exp \left[-\frac{\{\pi(x-E)\}^2}{A+C} \right] \exp \left[i2\pi(x-E) \frac{AB+CD}{A+C} \right]. \quad (7A.7)$$

Therefore, substitution of Eqs.(7A.2) to (7A.6) into Eq.(7A.7) yields

$$\sqrt{\frac{2}{\pi}} \frac{1}{w_z} \exp \left[-\frac{2}{w_z^2} \left\{ \frac{z_2}{z_T} (n-m)d \right\}^2 \left(1 + \frac{1}{\sigma^2} \right) \right] \exp \left[-\frac{2}{w_z^2} \left\{ x - \frac{z_2}{z_T} (n+m)d \right\}^2 \right] \\ \times \exp \left[i2\pi x \frac{n-m}{d} \left(\frac{R_z - z_2}{R_z} \right) \right] \exp \left[-i2\pi (n^2 - m^2) \frac{z_2}{z_T} \left(\frac{R_z - z_2}{R_z} \right) \right], \quad (7A.8)$$

where w_z , R_z and z_T are given by Eqs.(7.18), (7.19) and (7.20), respectively.

Finally, Eq.(7.17) is derived by substituting Eq.(7A.8) into Eq.(7A.1).

7.6.2 Condition for M in Eq.(7.23) and derivation of Eq.(7.24)

By substituting Eq.(7.23) into Eq.(7.22), the following equation is obtained:

$$\frac{R_z - Mz_T}{R_z} = \frac{N}{M}. \quad (7B.1)$$

Since the left side part of Eq.(7B.1) takes a value between 0 and 1, M must take a value of $N+1$ or more:

$$M \geq N + 1, \quad (7B.2)$$

by which the lower limit of M is decided.

On the other hand, by using the parameters of z_o and C defined by

$$z_o = \pi a^2 / \lambda, \quad (7B.3)$$

$$C = \sqrt{1 + \frac{1}{\sigma^2}}, \quad (7B.4)$$

respectively, R_z in Eq.(7.19) is rewritten as

$$R_z = z + \frac{1}{z} \left(\frac{z_o}{C} \right)^2. \quad (7B.5)$$

Moreover, since $z = z_1 + z_2$ and $z_2 = Mz_T$ in Eq.(7.23), Eq.(7B.5) becomes

$$R_z = z_1 + Mz_T + \frac{1}{z_1 + Mz_T} \left(\frac{z_o}{C} \right)^2. \quad (7B.6)$$

Then, substitution of Eq.(7B.6) into Eq.(7B.1) produces

$$(M - N)C^2 z_1^2 + M(M - 2N)C^2 z_T z_1 + \{(M - N)z_o^2 - C^2 M^2 N z_T^2\} = 0, \quad (7B.7)$$

and z_1 is calculated analytically as

$$z_1 = -\frac{M(M - 2N)}{2(M - N)} z_T \pm \frac{\sqrt{(M^2 C z_T)^2 - \{2(M - N)z_o\}^2}}{2(M - N)C}. \quad (7B.8)$$

In order for at least one positive real solution to exist, the following two conditions must be satisfied simultaneously:

$$M - 2N \leq 0 \quad (7B.9)$$

and

$$\frac{M^2}{M - N} \frac{C z_T}{2 z_o} \geq 1. \quad (7B.10)$$

Equation(7B.9) corresponds to the condition that the axial equation concerning the quadratic function of the left side part in Eq.(7B.7) must become 0 or more, and Eq.(7B.10) corresponds to the condition that the discriminant for Eq.(7B.7) must become 0 or more.

Therefore, from Eqs.(7B.2) and (7B.9), the condition for M is obtained as

$$N + 1 \leq M \leq 2N. \quad (7B.11)$$

In particular, under the condition of

$$M = 2N, \quad (7B.12)$$

$z_1 (> 0)$ becomes a simple form of

$$z_1 = 2Nz_T \sqrt{1 - \left(\frac{z_o}{2NCz_T} \right)^2}. \quad (7B.13)$$

Finally, Eq.(7.24) is obtained by substituting Eqs.(7.20), (7B.3) and (7B.4) into a part of the root in Eq.(7B.13).

7.6.3 Derivation of Eq.(7.32)

Substitution of Eq.(7.31) into Eq.(7.27) yields

$$\begin{aligned} \langle i_z(x) \rangle = & \exp \left(-\frac{2x^2}{w_z^2} \right) \sum_m \exp(-\alpha m^2) \exp(\beta m) \exp \left(-i2\pi x \frac{m}{2d} \right) \\ & \times \sum_n \exp(-\alpha n^2) \exp[(\beta + \gamma m)n] \exp \left(i2\pi x \frac{n}{2d} \right), \end{aligned} \quad (7C.1)$$

where

$$\alpha = \left(\frac{\pi f}{2} \right)^2 + \gamma \left(\sigma^2 + \frac{1}{2} \right), \quad (7C.2)$$

$$\beta = \frac{8Nd}{w_z^2} x \quad (7C.3)$$

and

$$\gamma = \left(\frac{4Nd}{w_z \sigma} \right)^2. \quad (7C.4)$$

In general, the periodic function $t(x)$ with the period d is expressed by the following

two forms

$$t(x) = A(x) \otimes \sum_n \delta(x - nd) \quad (7C.5a)$$

$$= \sum_n a_n \exp\left(i2\pi n \frac{x}{d}\right), \quad (7C.5b)$$

where n is an integer in the region extending from $-\infty$ to ∞ , $\otimes$ denotes the convolution integral, δ denotes the Dirac's delta function, and $A(x)$ denotes the complex amplitude distribution in the range of $n - d/2 \sim n + d/2$. Moreover, a_n is a Fourier expansion component of the periodic function $t(x)$ defined by

$$\begin{aligned} a_n &= \frac{1}{d} \int_{-d/2}^{d/2} A(x) \exp\left(-i2\pi n \frac{x}{d}\right) dx \\ &\approx \frac{1}{d} \int_{-\infty}^{\infty} A(x) \exp\left(-i2\pi n \frac{x}{d}\right) dx, \end{aligned} \quad (7C.6)$$

where the approximate expression may be applied in the case that the extent of $A(x)$ is smaller than $d/4$ (for example, see Eq.(7.31)). Then, it is understood that a form of Eq.(7C.5b) is rewritten as that of Eq.(7C.5a), by performing an inverse Fourier transform of a_n with respect to n/d .

From this relation, a part of $\sum_n$ in Eq.(7C.1) is rewritten as

$$\begin{aligned} \sum_n \cdots &= \exp\left(\frac{\beta^2}{4\alpha}\right) \exp\left(\frac{\beta\gamma}{2\alpha}m\right) \exp\left(\frac{\gamma^2}{4\alpha}m^2\right) \\ &\times \left[\sum_n \left\{ \exp\left[-\frac{x^2}{\alpha(2d/\pi)^2}\right] \exp\left(i2\pi x \frac{\beta + \gamma m}{4\alpha d}\right) \right\} \otimes \delta(x + 2nd) \right], \end{aligned} \quad (7C.7)$$

where a constant value is omitted. Then, by substituting Eq.(7C.7) into Eq.(7C.1), $\langle i_z(x) \rangle$ becomes

$$\begin{aligned}
\langle i_z(x) \rangle &= \exp\left(-\frac{2x^2}{w_z^2}\right) \exp\left(\frac{\beta^2}{4\alpha}\right) \exp\left(i2\pi x \frac{\beta}{4\alpha d}\right) \\
&\times \sum_n \exp\left[-\frac{(x+2nd)^2}{\alpha(2d/\pi)^2}\right] \exp\left(i2\pi \frac{\beta}{2\alpha} n\right) \\
&\times \sum_m \exp\left[-\left(\alpha - \frac{\gamma^2}{4\alpha}\right) m^2\right] \exp\left[\beta \left(1 + \frac{\gamma}{2\alpha}\right) m\right] \\
&\times \exp\left[-i2\pi \left\{\left(1 - \frac{\gamma}{2\alpha}\right) x - \frac{\gamma d}{\alpha} n\right\} \frac{m}{2d}\right]. \quad (7C.8)
\end{aligned}$$

Moreover, by using again the relation of Eqs.(7C.5a) and (7C.5b), a part of $\sum_m$ in Eq.(7C.8) is rewritten as

$$\begin{aligned}
\sum_m \cdots &= \exp\left[\frac{\beta^2(2\alpha + \gamma)}{4\alpha(2\alpha - \gamma)}\right] \\
&\times \left[\sum_m \left\{ \exp\left[-\frac{\pi^2\alpha}{d^2(2\alpha + \gamma)(2\alpha - \gamma)} \left(\frac{2\alpha - \gamma}{2\alpha}x - \frac{\gamma d}{\alpha}n\right)^2\right] \right. \right. \\
&\quad \times \exp\left[-i2\pi \left(\frac{2\alpha - \gamma}{2\alpha}x - \frac{\gamma d}{\alpha}n\right) \frac{\beta}{2d(2\alpha - \gamma)}\right] \Big\} \\
&\quad \left. \otimes \delta\left[\left(\frac{2\alpha - \gamma}{2\alpha}x - \frac{\gamma d}{\alpha}n\right) - 2md\right] \right], \quad (7C.9)
\end{aligned}$$

where a constant value is omitted. Then, by substituting Eq.(7C.9) into Eq.(7C.8), $\langle i_z(x) \rangle$ becomes

$$\begin{aligned}
\langle i_z(x) \rangle &= \exp\left(\frac{\beta^2}{2\alpha - \gamma}\right) \exp\left(-\frac{2x^2}{w_z^2}\right) \\
&\times \sum_n \sum_m \exp\left[-\frac{1}{(d/\pi)^2(2\alpha + \gamma)} \{x + (n - m)d\}^2\right] \\
&\times \exp\left[-\frac{\pi^2}{2\alpha - \gamma} (n + m)^2\right] \exp\left[i2\pi \frac{\beta}{2\alpha - \gamma} (n + m)\right]. \quad (7C.10)
\end{aligned}$$

Furthermore, by substituting Eqs.(7.28), (7C.2), (7C.3) and (7C.4) into Eq.(7C.10),

$\langle i_z(x) \rangle$ becomes

$$\begin{aligned} \langle i_z(x) \rangle = & \exp\left(-\frac{2x^2}{w_{env}^2}\right) \sum_n \sum_m \exp\left[-\frac{2}{w_{spot}^2}\{x + (n-m)d\}^2\right] \\ & \times \exp\left[-\frac{2}{f^2 + (a/d)^2\sigma^2/(1+\sigma^2)}(n+m)^2\right] \\ & \times \exp\left[i2\pi(n+m)\frac{x}{4Nd}\frac{1}{\{f/(a/d)\}^2(1+1/\sigma^2)+1}\right], \quad (7C.11) \end{aligned}$$

where w_{env} and w_{spot} are given by Eqs.(7.33) and (7.34), respectively.

In particular, the third term of $\exp(\)$ in Eq.(7C.11) is regarded as a constant weighting term for the second term of $\exp(\)$ (i.e., the spots of the multiple images). However, the contribution of the third term is very small for arbitrary values of n and m except $n+m=0$. (Under the condition of $n+m=0$, the third term becomes 1 for any values of a/d , σ and f . However, under the condition of $|n+m|=1$, the maximum value of the third term becomes $\exp(-4) \approx 2 \times 10^{-2}$, for the case of $a/d=0.5$, $\sigma=\infty$ and $f=0.5$. Furthermore, under the condition of $|n+m|=2$, the maximum value of the third term becomes $\exp(-16) \approx 10^{-7}$, for the case of $a/d=0.5$, $\sigma=\infty$ and $f=0.5$.) Therefore, by substituting $n+m=0$ into Eq.(7C.11), $\langle i_z(x) \rangle$ is finally given by Eq.(7.32).

Chapter 8

Concluding remarks

In the present study, the properties of the partially coherent quasi-monochromatic light have been studied by using the mutual intensity (MI) and the ambiguity function (AF).

In Chapter 3, the spatial coherence properties of the optical field in the Fraunhofer diffraction region produced from an optical fiber-end source have been studied by using the MI. As a result, the optical fiber-end source radiating the independent plane waves with random directions and phases is apparently different from the incoherent source consisting of independent point sources. Moreover, the spatial coherence properties of light from the optical fiber-end source are dependent strongly on the number of guided modes of the optical fiber and, in particular, approaches those from the incoherent source with the increasing number of guided modes. Therefore, it has been found that optical fiber-end sources are the partially coherent sources by which the spatial coherence properties can be controlled.

In Chapter 4, the equi-AF ellipse of a GSM beam in the phase space has been studied in connection with the beam parameters. As a result, the area of the ellipse is constant regardless of the propagation distance of the beam, and it increases with the increasing spatial coherence length of the GSM source. An increase in area means an increase in uncertainty of the beam in the phase space. Moreover, the width of the ellipse on the spatial difference axis is equivalent to the extent of the autocorrelation of the beam, and it is also equivalent to the depth of the waist region of the beam. Therefore, it has been found that the unknown spatial coherence length of the source can be measured from the equi-AF ellipse, and this fact has suggested a new method

of measurements of the spatial coherence length.

In Chapter 5, the spatial coherence properties of semiconductor array lasers have been studied by using the MI in connection with far-field patterns of fluctuating supermodes. As a result, the contrast of the far-field patterns becomes low with increases in the integrated intensity and the spatial correlation length of the fluctuating component of supermodes. Moreover, the contrast is independent of the order of supermodes, and it becomes high with the increasing number of emitters in the array laser. From this study, the coherence properties of array lasers have been clearly related to far-field patterns of supermodes.

In Chapter 6, the Talbot effect under illumination of the partially coherent light has been studied by using the propagation law of the AF. As a result, the contrast of the Talbot images becomes low with decreases in parameters: the spatial coherence length of the illuminating light, and the spatial correlation lengths of amplitude and phase fluctuations of the Ronchi grating. In particular, the effect of the spatial correlation length of the phase fluctuations on the contrast is most noticeable. These results have explained Talbot images of the partially phase-locked array sources where each emitter has amplitude and phase fluctuations.

In Chapter 7, an array generator for the partially coherent light has been studied by using the propagation law of the AF. As a result, multiple images with Gaussian intensity spots are realized at the observation plane, a distance $4Nz_T$ (N : a positive integer, z_T : Talbot distance) from a GSM source, when the grating of the aperture with a Gaussian amplitude transmittance is placed in the middle between the GSM source and the observation plane. In particular, the number of spots of the multiple images increases with decreases in the source size, the spatial coherence length of the source, and the aperture size of the grating.

In conclusion, it is expected to apply the results of the present study to the optical systems using the partially coherent light in the future.

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Bibliography

- [1] T. H. Maiman: "Stimulated optical radiation in ruby," *Nature* **187**, 493–494 (1960).
- [2] F. P. Kapron, D. B. Keck and R. D. Maurer: "Radiation losses in glass optical waveguides," *Appl. Phys. Lett.* **17**, 423–425 (1970).
- [3] J. W. Goodman: "Statistical Properties of Laser Speckle Patterns," in *Laser Speckle and Related Phenomena*, edited by J. C. Dainty, pp. 9–76 (Springer-Verlag, Berlin, 1975).
- [4] F. Zernike: "The concept of degree of coherence and its application to optical problems," *Physica* **5**, 785–795 (1938).
- [5] M. Born and E. Wolf: *Principles of Optics*, 6th edition, pp. 491–555 (Pergamon, Oxford, 1980).
- [6] E. Wolf: "A macroscopic theory of interference and diffraction of light from finite sources II. Fields with a spectral range of arbitrary width," *Proc. Roy. Soc. London (A)* **230**, 246–265 (1955).
- [7] M. J. Beran and G. B. Parrent, Jr.: *Theory of Partial Coherence*, pp. 1–44 (Prentice-Hall, New Jersey, 1964).
- [8] J. Peřina: *Coherence of Light*, pp. 1–46 (Van Nostrand Reinhold, London, 1972).
- [9] A. S. Marathay: *Elements of Optical Coherence Theory*, pp. 27–181 (John Wiley & Sons, New York, 1982).
- [10] J. W. Goodman: *Statistical Optics*, pp. 157–236 (John Wiley & Sons, New York,

1985).

- [11] P. M. Woodward: *Probability and Information Theory with Applications to Radar*, pp. 115–125 (Pergamon, London, 1953).
- [12] M. I. Skolnik: *Introduction to Radar Systems*, 2nd edition, pp. 411–420 (McGraw-Hill, Singapore, 1980).
- [13] A. Papoulis: "Ambiguity function in Fourier optics," *J. Opt. Soc. Am.* **64**, 779–788 (1974).
- [14] K. Dutta and J. W. Goodman: "Reconstructions of images of partially coherent objects from samples of mutual intensity," *J. Opt. Soc. Am.* **67**, 796–803 (1977).
- [15] N. Takai, H. Yoshimura and T. Asakura: "Propagation of ambiguity function of partially coherent light emerging from quasi-homogeneous sources," *Bull. Fac. Eng. Hokkai-Gakuen Univ.* **20**, 187–202 (1993).
- [16] N. Takai, H. Yoshimura and T. Asakura: "Spatial coherence properties of light from optical fibers," *Proc. SPIE* **976**, 143–149 (1988).
- [17] H. Yoshimura, N. Takai and T. Asakura: "Spatial coherence properties of light from optical fibres," *Opt. Quantum Electron.* **24**, 631–646 (1992).
- [18] D. Marcuse: *Principles of Optical Fiber Measurements*, pp. 1–10 (Academic, New York, 1981).
- [19] P. H. van Cittert: "Die wahrscheinliche schwingungsverteilung in einer von einer lichtquelle direkt oder mittels einer linse beleuchteten ebene," *Physica* **1**, 201–210 (1934).
- [20] H. Yoshimura, N. Takai and T. Asakura: "Equi-ambiguity-function ellipse of Gaussian-Schell model beams," *J. Opt. Soc. Am. A* **11**, No.3 (1994) (to be published).
- [21] A. T. Friberg and R. J. Sudol: "Propagation parameters of Gaussian Schell-model beams," *Opt. Commun.* **41**, 383–387 (1982).

- [22] A. T. Friberg and R. J. Sudol: "The spatial coherence properties of Gaussian Schell-model beams," *Opt. Acta* **30**, 1075–1097 (1983).
- [23] H. Yoshimura, N. Takai and T. Asakura: "Far-field analysis of fluctuating supermodes radiated from phase-locked array lasers," *IEEE J. Quantum Electron.* **30**, No.2 (1994) (to be published).
- [24] J. K. Butler, D. E. Ackley and D. Botez: "Coupled-mode analysis of phase-locked injection laser arrays," *Appl. Phys. Lett.* **44**, 293–295 (1984).
- [25] J. K. Butler, D. E. Ackley and D. Botez: "Coupled-mode analysis of phase-locked injection laser arrays – Erratum," *Appl. Phys. Lett.* **44**, 935 (1984).
- [26] E. Kapon, J. Katz and A. Yariv: "Supermode analysis of phase-locked arrays of semiconductor lasers," *Opt. Lett.* **10**, 125–127 (1984).
- [27] E. Kapon, J. Katz and A. Yariv: "Supermode analysis of phase-locked arrays of semiconductor lasers – Erratum," *Opt. Lett.* **10**, 318 (1984).
- [28] J. K. Butler, D. E. Ackley and M. Ettenberg: "Coupled-mode analysis of gain and wavelength oscillation characteristics of diode laser phased arrays," *IEEE J. Quantum Electron.* **QE-21**, 458–464 (1985).
- [29] H. -J. Yoo, J. R. Hayes, E. G. Paek, A. Scherer and Y. -S. Kwon: "Array mode analysis of two-dimensional phased arrays of vertical cavity surface emitting lasers," *IEEE J. Quantum Electron.* **26**, 1039–1051 (1990).
- [30] H. Yoshimura, N. Takai and T. Asakura: "Effects of amplitude and phase fluctuations of the Ronchi grating on Talbot images under illumination of the partially coherent light," *J. Mod. Opt.* **40**, 825–839 (1993).
- [31] H. F. Talbot: "Facts relating to optical science. No.IV," *Phil. Mag.* **9**, 401–407 (1836).
- [32] K. Patorski: "The self-imaging phenomenon and its applications," in *Progress in*

Optics, vol. **XXVII**, edited by E. Wolf, pp. 1–108 (Elsevier, Amsterdam, 1989).

- [33] H. Yoshimura, N. Takai and T. Asakura: "Realization of multiple images due to the partially coherent light from a Gaussian-Schell model source," *J. Opt. Soc. Am. A* **11** (1994) (to be published).
- [34] J. -P. Guigay: "The ambiguity function in diffraction and isoplanatic imaging by partially coherent beams," *Opt. Commun.* **26**, 136–138 (1978).
- [35] K. -H. Brenner and J. Ojeda-Castañeda: "Ambiguity function and Wigner distribution function applied to partially coherent imagery," *Opt. Acta* **31**, 213–223 (1984).
- [36] R. H. Clarke: "Ambiguity function for time-varying random diffracted fields," *J. Opt. Soc. Am. A* **3**, 1224–1226 (1986).
- [37] B. J. Thompson and C. Roychoudhuri: "On the propagation of coherent and partially coherent light," *Opt. Acta* **26**, 21–34 (1979).
- [38] J. W. Goodman: *Introduction to Fourier Optics*, pp. 57–76 (McGraw-Hill, San Francisco, 1968).
- [39] C. Pask and A. W. Snyder: "The van Cittert-Zernike theorem for optical fibres," *Opt. Commun.* **9**, 95–97 (1973).
- [40] E. G. Rawson, J. W. Goodman and R. E. Norton: "Frequency dependence of modal noise in multimode optical fibers," *J. Opt. Soc. Am.* **70**, 968–976 (1980).
- [41] M. Imai, S. Satoh and Y. Ohtsuka: "Complex degree of spatial coherence in an optical fiber: theory and experiment," *J. Opt. Soc. Am. A* **3**, 86–93 (1986).
- [42] M. Imai, Y. Ohtsuka and S. Satoh: "Spatial coherence analysis of light propagation in optical fibers by interferometric methods," *J. Opt. Soc. Am. A* **3**, 1059–1064 (1986).
- [43] D. Marcuse: *Light Transmission Optics*, 2nd edition, pp. 348–414 (Van Nostrand

Reinhold, New York, 1982).

- [44] R. E. Epworth: "Modal noise – causes and cures," *Laser Focus* **17**, 109–115 (1981).
- [45] J. W. Goodman and E. G. Rawson: "Statistics of modal noise in fibers: a case of constrained speckle," *Opt. Lett.* **6**, 324–326 (1981).
- [46] N. Takai and T. Asakura: "Statistical properties of laser speckles produced under illumination from a multimode optical fiber," *J. Opt. Soc. Am. A* **2**, 1282–1290 (1985).
- [47] N. Takai and T. Asakura: "Laser speckles produced by a diffuse object under illumination from a multimode optical fiber: an experimental study," *Appl. Opt.* **27**, 557–562 (1988).
- [48] W. H. Carter and E. Wolf: "Coherence and radiometry with quasihomogeneous planar sources," *J. Opt. Soc. Am.* **67**, 785–796 (1977).
- [49] T. Asakura and N. Takai: "Dynamic laser speckles and their application to velocity measurements of the diffuse objects," *Appl. Phys.* **25**, 179–194 (1981).
- [50] E. Collett and E. Wolf: "Beams generated by Gaussian quasi-homogeneous sources," *Opt. Commun.* **32**, 27–31 (1980).
- [51] J. A. Ratcliffe: "Some aspects of diffraction theory and their application to the ionosphere," *Rep. Prog. Phys.* **19**, 188–267 (1956).
- [52] T. L. Paoli, W. Streifer and R. D. Burnham: "Observation of supermodes in a phase-locked diode laser array," *Appl. Phys. Lett.* **45**, 217–219 (1984).
- [53] P. L. Gourley, M. E. Warren, G. R. Hadley, G. A. Vawter, T. M. Brennan and B. E. Hammons: "Coherent beams from high efficiency two-dimensional surface-emitting semiconductor laser arrays," *Appl. Phys. Lett.* **58**, 890–892 (1991).
- [54] D. Botez: "Array-mode far-field patterns for phase-locked diode-laser arrays: coupled-mode theory versus simple diffraction theory," *IEEE J. Quantum Electron.*

QE-21, 1752–1755 (1985).

- [55] D. Botez: "Author's reply," *IEEE J. Quantum Electron.* **QE-22**, 376 (1986).
- [56] D. E. Novoseller and D. Botez: "Phase-locked laser array far-field normalization," *IEEE J. Quantum Electron.* **25**, 1179–1181 (1989).
- [57] G. R. Hadley, J. P. Hohimer and A. Owyong: "Free-running modes for gain-guided diode laser arrays," *IEEE J. Quantum Electron.* **QE-23**, 765–774 (1987).
- [58] D. R. Scifres, R. D. Burnham and W. Streifer: "Phase-locked semiconductor laser array," *Appl. Phys. Lett.* **33**, 1015–1017 (1978).
- [59] D. R. Scifres, R. D. Burnham, W. Streifer and M. Bernstein: "Lateral beam collimation of a phased array semiconductor laser," *Appl. Phys. Lett.* **41**, 614–616 (1982).
- [60] N. Takai, H. Ambar and T. Asakura: "Spatial coherence measurements of quasi-monochromatic thermal light using double-exposure specklegrams," *Opt. Commun.* **60**, 123–127 (1986).
- [61] B. Smolińska and A. Kalestyński: "Autoidolon of quasi-periodic optical objects," *Opt. Acta* **25**, 257–263 (1978).
- [62] A. P. Smirnov and A. D. Galpern: "Effect of errors of a periodic transparency on Fresnel images," *Opt. Spektrosk.* **48**, 589–592 (*Opt. Spectrosc.* **48**, 324–326) (1980).
- [63] R. Uma Maheswari, N. Takai and T. Asakura: "Talbot images by a spatially fluctuating grating," *Opt. Commun.* **81**, 259–266 (1991).
- [64] R. Uma Maheswari, T. Asakura and N. Takai: "Talbot images by a spatially randomized array of dots," *Optik* **87**, 163–169 (1991).
- [65] H. Fujiwara: "Effects of spatial coherence on Fourier imaging of a periodic object," *Opt. Acta* **21**, 861–869 (1974).

- [66] A. P. Smirnov: "Effect of partial spatial coherence on the contrast of self-reproduced images of periodic transparencies," *Opt. Spektrosk.* **61**, 618–623 (*Opt. Spectrosc.* **61**, 386–389) (1986).
- [67] R. Sudol and B. J. Thompson: "Lau effect: theory and experiment," *Appl. Opt.* **20**, 1107–1116 (1981).
- [68] L. Liu: "Partially coherent diffraction effect between Lau and Talbot effects," *J. Opt. Soc. Am. A* **5**, 1709–1716 (1988).
- [69] J. W. Goodman: *Statistical Optics*, pp. 361–384 (John Wiley & Sons, New York 1985).
- [70] N. Takai: "Relation between statistical properties of surface roughness and the averaged speckle intensity in the diffraction field," *Opt. Commun.* **14**, 24–29 (1975).
- [71] D. Botez: "High-power monolithic phase-locked arrays of antiguided semiconductor diode lasers," *IEE Proc. J* **139**, 14–23 (1992).
- [72] N. Streibl: "Beam shaping with optical array generators," *J. Mod. Opt.* **36**, 1559–1573 (1989).
- [73] P. Szwaykowski and V. Arrizon: "Talbot array illuminator with multilevel phase gratings," *Appl. Opt.* **32**, 1109–1114 (1993).
- [74] M. T. Gale, M. Rossi, H. Schütz, P. Ehbets, H. P. Herzig and D. Prongué: "Continuous-relief diffractive optical elements for two-dimensional array generation," *Appl. Opt.* **32**, 2526–2533 (1993).
- [75] S. Szapiel and K. Patorski: "Fresnel diffraction images of periodic objects under gaussian beam illumination," *Opt. Acta* **26**, 439–446 (1979).

List of papers by the author

1. N. Takai, H. Yoshimura and T. Asakura: "Spatial coherence properties of light from optical fibers," *Proc. SPIE* **976**, 143–149 (1988).
2. H. Yoshimura, N. Takai and T. Asakura: "Spatial coherence properties of light from optical fibres," *Opt. Quantum Electron.* **24**, 631–646 (1992).
3. N. Takai, H. Yoshimura and T. Asakura: "Propagation of ambiguity function of partially coherent light emerging from quasi-homogeneous sources," *Bull. Fac. Eng. Hokkai-Gakuen Univ.* **20**, 187–202 (1993).
4. H. Yoshimura, N. Takai and T. Asakura: "Effects of amplitude and phase fluctuations of the Ronchi grating on Talbot images under illumination of the partially coherent light," *J. Mod. Opt.* **40**, 825–839 (1993).
5. H. Yoshimura, N. Takai and T. Asakura: "Far-field analysis of fluctuating supermodes radiated from phase-locked array lasers," *IEEE J. Quantum Electron.* **30**, No.2 (1994) (to be published).
6. H. Yoshimura, N. Takai and T. Asakura: "Equi-ambiguity-function ellipse of Gaussian-Schell model beams," *J. Opt. Soc. Am. A* **11**, No.3 (1994) (to be published).
7. H. Yoshimura, N. Takai and T. Asakura: "Realization of multiple images due to the partially coherent light from a Gaussian-Schell model source," *J. Opt. Soc. Am. A* **11** (1994) (to be published).

