Grand Unification of Flavor Mixings

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An origin of flavor mixings in quark and lepton sectors is still a mystery, and a structure of the flavor mixings in lepton sector seems completely different from that of quark sector. In this letter, we point out that the flavor mixing angles in quark and lepton sectors could be unified at a high energy scale, when neutrinos are degenerate. It means that a minimal flavor violation at a high energy scale can induce a rich variety of flavor mixings in quark and lepton sectors at a low energy scale through quantum corrections.

An origin of flavor mixings is one of the most important mystery in the elementary particle physics. A structure of flavor mixings in the quark sector has been investigated as so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. On the other hand, neutrino oscillation experiments have revealed that the lepton sector has completely different flavor mixings, represented by Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [2], in which one large mixing angle θ_{12}, one nearly maximal mixing angle θ_{23} [3], and non-vanishing θ_{13} that is pointed out by recent long baseline and reactor experiments [4]. Anyhow, both flavor structures seem completely different from each other, and this situation motivates us to pursue an origin of flavor violation. In this letter, we will investigate a possibility that CKM and PMNS flavor mixing angles are unified at a high energy scale.[1] This is a kind of "grand unification of flavor mixings (GUFM)", where a minimal flavor violation at a high energy scale can induce a rich variety of flavor mixing structures in both quark and lepton sectors at a low energy scale. We will show that quantum corrections and degeneracy of neutrino masses play crucial roles for the realization of GUFM. As for the mass degeneracy, we should remind that only neutrinos can be degenerate among matter fermions.

We take a setup of minimal supersymmetric standard model (MSSM) with Weinberg operator [11], where Yukawa interactions are given by

$$\mathcal{L}_Y = -y_d \overline{Q_L} H_d d_R - y_u \overline{Q_L} H_u u_R - y_e \bar{L} H_d e_R$$
$$+\kappa (H_u L)(H_u L) + h.c.. \quad (1)$$

Here Q_L are the left-handed quarks, $u_R(d_R)$ are the right-handed up(down)-type quarks, e_R are the right-handed charged leptons, $H_u(H_d)$ is the up(down)-type Higgs,

and y_* ($* = u, d, e$) are the corresponding Yukawa couplings, respectively. The Weinberg operator can be obtained by integrating out a heavy particle(s), for example, right-handed neutrinos with masses of order 10^{14-16} GeV in type I seesaw mechanism [12]. Thus, an effective coupling κ is carrying mass dimension -1 as $(\mathcal{O}(10^{14-16})$ GeV$)^{-1}$. We also utilize PDG parametrization [3] for the CKM (V_{CKM}) and PMNS (V_{PMNS}) matrices as $V_{\text{CKM}} \equiv V_{uL}^\dagger V_{dL}$ and $V_{\text{PMNS}} \equiv V_{eL}^\dagger V_\nu D_p$, respectively, where V_{*L} are unitary matrices diagonalizing Yukawa coupling as $V_{*L}^\dagger y_* V_{*R} = y_*^{\text{diag}}$, and D_p is a diagonal phase matrix as $D_p \equiv \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$. The light (active) neutrinos are diagonalized as $V^T M_\nu V = M_\nu^{\text{diag}} \equiv \text{Diag}\{m_1, m_2, m_3\}$ with $M_\nu \equiv \kappa v_u^2$. Notice that two Majorana phases in D_p are included in the PMNS matrix.

Renormalization group equation (RGE) of κ is given by [13]

$$16\pi^2 \frac{d\kappa}{dt} = 6 \left[-\frac{1}{5} g_1^2 - g_2^2 + \text{Tr}\left(y_u^\dagger y_u\right) \right] \kappa$$
$$+ \left[\left(y_e y_e^\dagger\right) \kappa + \kappa \left(y_e y_e^\dagger\right)^T \right], \quad (2)$$

where g_is are gauge coupling constants. We can show the PMNS matrix at a high energy scale, Λ, through the neutrino mass matrix at Λ, $M_\nu(\Lambda) = \kappa(\Lambda) v_u^2$, by use of eq.(2). The $M_\nu(\Lambda)$ is given by $M_\nu(\Lambda) = I M_\nu(\Lambda_{\text{EW}}) I$ [14–17], where Λ_{EW} is a low energy (electroweak) scale and $I \equiv \text{Diag}\{\sqrt{I_e}, \sqrt{I_\mu}, \sqrt{I_\tau}\}$. Here, I_αs ($\alpha = e, \mu, \tau$) denote quantum corrections, which are defined by $I_\alpha \equiv \exp\left[\frac{1}{8\pi} \int_{t_\Lambda(\equiv \ln \Lambda)}^{t_{\text{EM}}(\equiv \ln \Lambda_{\text{EW}})} y_\alpha^2 dt\right]$. A dominant effect of the quantum corrections comes from y_τ, and we define small parameters as $\epsilon_{e(\mu)} \equiv 1 - \sqrt{\frac{I_\tau}{I_{e(\mu)}}}$. Typical values of $\epsilon_{e(\mu)}$ have been given in [16], and we take a region $10^{-3} \lesssim \epsilon_{e(\mu)} \lesssim 0.1$, which corresponds to $\mathcal{O}(10) \lesssim \tan\beta \lesssim \mathcal{O}(50)$ with $\kappa^{-1} \geq 10^{13}$ GeV. In the following analyses, we take a good approximation of $\epsilon \equiv \epsilon_e = \epsilon_\mu$. Then, the $M_\nu(\Lambda)$ is given by

$$M_\nu(\Lambda) = \begin{pmatrix} m_{11} & m_{12} & m_{13}(1+\epsilon) \\ m_{21} & m_{22} & m_{23}(1+\epsilon) \\ m_{31}(1+\epsilon) & m_{32}(1+\epsilon) & m_{33}(1+2\epsilon) \end{pmatrix}, (3)$$

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[1] Similar possibility has been studied in [5–7] (there are also related works in [8]). Such possibility in [5] has been realized by a radiative magnification [9]. The idea of radiative maginification has originally proposed for the neutrino mixing angles but not for an unification of CKM and PMNS mixing angles (see [10] for radiative magnification models.)
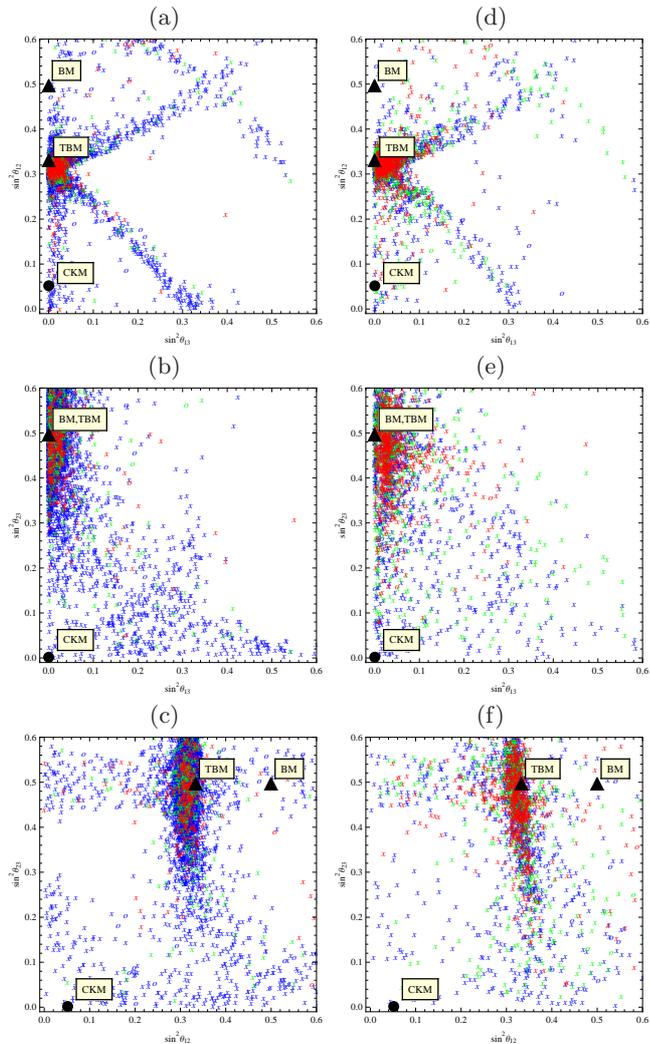
FIG. 1: The PMNS mixing angle at $\Lambda = 10^{14}$ GeV for the NH case. Red, green and blue plots show large hierarchy ($\sqrt{|\Delta m_{32}^2| + \Delta m_{21}} \leq m_3 < 0.1$ eV), weak degenerate (0.1 eV $\leq m_3 < 0.15$ eV) and strong degenerate (0.15 eV $\leq m_3 \leq 0.2$ eV) cases, respectively.

FIG. 2: The same figures as Fig.1 for the IH case. Red, green and blue plots show large hierarchy ($\sqrt{|\Delta m_{32}^2| + \Delta m_{21}} \leq m_2 < 0.1$ eV), weak degenerate (0.1 eV $\leq m_2 < 0.15$ eV) and strong degenerate (0.15 eV $\leq m_2 \leq 0.2$ eV) cases, respectively.

and

$$
\begin{aligned}
(M_\nu(\Lambda_{\rm EW}))_{ij} & \\
\equiv m_{ij} = (V_{\rm PMNS}^*(\Lambda_{\rm EW}) M_\nu^{\rm diag}(\Lambda_{\rm EW}) V_{\rm PMNS}^T(\Lambda_{\rm EW}))_{ij}. &
\end{aligned}
\tag{4}
$$

Note that we take a diagonal basis of charged lepton Yukawa matrix.

Now let us investigate effects of radiative corrections for the PMNS mixing angles. Numerical results are shown in Figs. 1 and 2. We have performed scatter plots with the following input parameters.

For the mass spectra of light neutrinos, we take two types of neutrino mass ordering, normal hierarchy (NH) $m_1 < m_2 < m_3$ and inverted hierarchy (IH) $m_3 < m_1 < m_2$, since the neutrino oscillation experiments determine only two mass squared differences, $\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2$
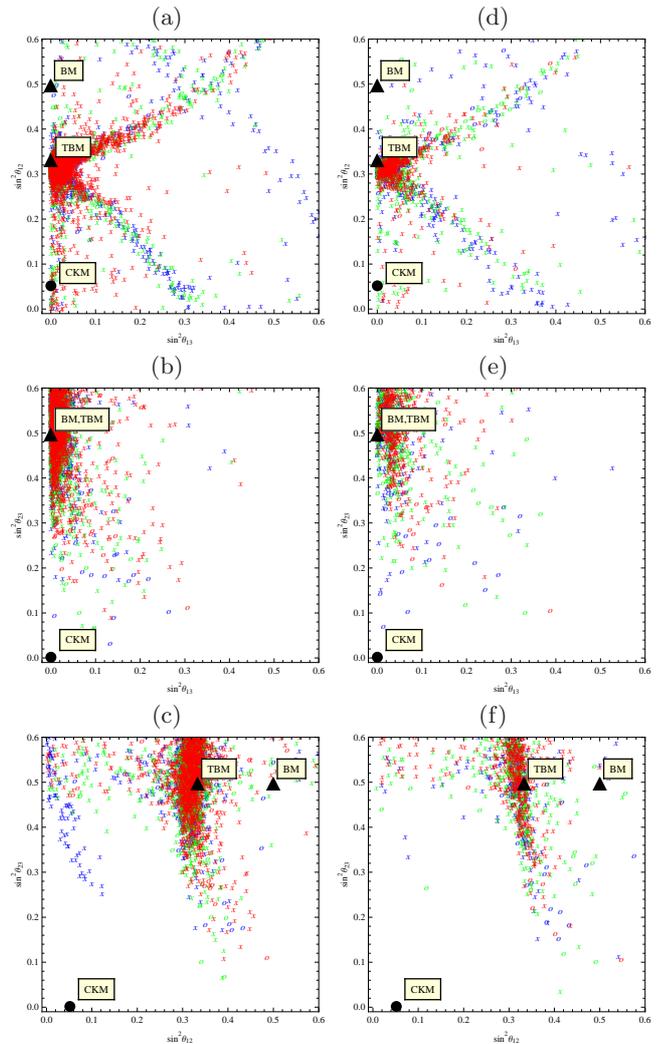
and $|\Delta m_{32}^2| \equiv ||m_3|^2 - |m_2|^2|$. At the $\Lambda_{\rm EW}$ scale, the NH case suggests

$$
m_1(\Lambda_{\rm EW}) = \sqrt{m_3^2(\Lambda_{\rm EW}) - |\Delta m_{32}^2| - \Delta m_{21}^2}, \tag{5}
$$

$$
m_2(\Lambda_{\rm EW}) = \sqrt{m_3^2(\Lambda_{\rm EW}) - |\Delta m_{32}^2|}, \tag{6}
$$

while the IH case does

$$
m_1(\Lambda_{\rm EW}) = \sqrt{m_2^2(\Lambda_{\rm EW}) - \Delta m_{21}^2}, \tag{7}
$$

$$
m_3(\Lambda_{\rm EW}) = \sqrt{m_2^2(\Lambda_{\rm EW}) - |\Delta m_{32}^2| - \Delta m_{21}^2}. \tag{8}
$$

We have taken $\sqrt{|\Delta m_{32}^2| + \Delta m_{21}^2} \leq m_{3(2)}(\Lambda_{\rm EW}) \leq 0.2$ eV for NH (IH) with

$$
\Delta m_{21}^2 = 7.62 \times 10^{-5} \text{ eV}^2, \tag{9}
$$

$$
|\Delta m_{32}^2| = 2.53(2.40) \times 10^{-3} \text{ eV}^2, \tag{10}
$$

which are the best fit values of experimentally observed neutrino mass squared differences [18]. The magnitude of 0.2 eV is consistent with cosmological bounds on sum of neutrino masses (see e.g. [19]). The mixing angles at $\Lambda_{\rm EW}$ are taken by

$$0.303 \leq \sin^2 \theta_{12} \leq 0.335, \tag{11}$$

$$0.44(0.46) \leq \sin^2 \theta_{23} \leq 0.57(0.58), \tag{12}$$

$$0.022(0.023) \leq \sin^2 \theta_{13} \leq 0.029(0.030), \tag{13}$$

from experimental results at 1σ level for the NH (IH) case [18]. Notice that $m_3(\Lambda_{\rm EW})$ $(m_2(\Lambda_{\rm EW}))$ is a free parameter in our analyses for the NH (IH) case, and it is related to the magnitude of degeneracy, i.e., a larger $m_3(\Lambda_{\rm EW})$ $(m_2(\Lambda_{\rm EW}))$ stands for a stronger degeneracy. When the degeneracy becomes stronger, the mixing angles can change drastically.

The effects of quantum correction described by ϵ have been taken as $10^{-3} \leq \epsilon \leq 0.1$. In the figures, the "$o$" and "$\chi$" markers show relatively small ϵ ($10^{-3} \leq \epsilon < 0.01$) and large one ($0.01 \leq \epsilon \leq 0.1$), respectively. Note that ϵ is also a free parameter in our analyses, which is determined once values of $\tan\beta$ and Λ are fixed. The scatter plots in Figs. 1 and 2 denote the PMNS mixing angles for NH and IH cases in a typical high energy scale of $\Lambda = 10^{14}$ GeV, respectively. We analyse separately whether all CP-phases are relatively large $\pi/4 \leq |\delta^l|, |\rho|, |\sigma|$ ((a)-(c)) or small $0 \leq |\delta^l|, |\rho|, |\sigma| < \pi/4$ ((d)-(f)). The CKM mixing angles at $\Lambda = 10^{14}$ GeV [20] is shown in each figure by big black dot.

Now let us go back our starting point, "Can the GUFM be really possible?" For the NH case, there definitely exist regions of the GUFM, where all the PMNS mixing angles are equal to the corresponding CKM mixing angles, $\theta_{ij}^l = \theta_{ij}^q$ as shown by blue markers in Figs. 1. Notice also that the green and red markers cannot reach the CKM point (e.g. see Figs. 1 (c) and (f)). Large hierarchy and small degenerate cases cannot realize GUFM because of the stabilities of mixing angle θ_{23}. Therefore, the GUFM can be achieved in a case of strong degenerate neutrino mass spectrum through quantum corrections. As for CP-phases, the GUFM is easily realized when CP-phases are large. We can see it by comparing Figs. 1 (a)-(c) with Figs. 1 (d)-(f). As for the largeness of quantum corrections, the strong degenerate case (blue plot) can realize CKM angles even when the quantum effects are relatively small since blue "o" markers in Fig. 1 (b) or (c) really exist on CKM point. Numerically, we can see that $0.005 \lesssim \epsilon$, which corresponds to $10 \lesssim \tan\beta$, is enough for the realization of the GUFM in the NH degenerate case.

On the other hand, Figs. 2 show that the IH case cannot realize the GUFM. It is because θ_{23} becomes too large at $\Lambda = 10^{14}$ GeV. Thus, we can conclude that the strong degenerate NH mass spectrum can achieve the GUFM in a region of $0.002(0.005) \lesssim \epsilon$ with the large (small) CP-phases case, which corresponds to $\tan\beta \simeq 10(15)$ [16]. This situation is summarized by "CKM" in Tab. I. Addi-

	NH ($m_1 < m_2 < m_3$)	
	$\frac{\pi}{4} \leq \|\delta^l\|, \|\rho\|, \|\sigma\|$	$0 \leq \|\delta^l\|, \|\rho\|, \|\sigma\| < \frac{\pi}{4}$
CKM	○ 0.1 eV $\lesssim m_3$ $10 \lesssim \tan\beta$	○ 0.15 eV $\lesssim m_2$ $15 \lesssim \tan\beta$
BM	○ $8 \lesssim \tan\beta$	×
TBM	○	○
Figs.	Figs. 1 (a)-(c)	Figs. 1 (d)-(f)
	IH ($m_3 < m_1 < m_2$)	
	$\frac{\pi}{4} \leq \|\delta^l\|, \|\rho\|, \|\sigma\|$	$0 \leq \|\delta^l\|, \|\rho\|, \|\sigma\| < \frac{\pi}{4}$
CKM	×	×
BM	○ $10 \lesssim \tan\beta$	×
TBM	○	○
Figs.	Figs. 2 (a)-(c)	Figs. 2 (d)-(f)

TABLE I: This is the summary of main results. ○ (×) means that the corresponding mixing angles can (not) be realized at high energy scale.

tionally, "CKM" in Tab. II shows cases of different combinations of CP-phases, such as one of three is small (large) and others are large (small). In these cases, numerical results are not so changed, and we can conclude that the most important key for the realization of GUFM is not CP-phases but strong degeneracy.

We give some comments on our the results. First, our results are consistent with ones of [5]. The work of [5] utilized relatively large $m_3(\geq 0.17)$ and $\tan\beta(=55)$, which are favor for the realization of GUFM as we have shown. These values of m_3 and $\tan\beta$ are inputs in [5] while we have scanned over m_3 and $\tan\beta$, and we have successfully obtained lower bounds on m_3 and $\tan\beta$ for the GUFM in this work. We have also shown that the GUFM cannot be realized in the IH case. Second, there generically exist threshold effects for neutrino masses [21]. We did not take care of such effects but it was shown in [6] that size of the effects is sufficient to have concordance between the GUFM model and experimental results of neutrino oscillation.

We also comment on a correlative mixing pattern, $\theta_{12} + \theta_{23} + \theta_{13} = \pi/2$, at a low energy scale. Even when we change value of θ_{13} as keeping the relation $\theta_{12} + \theta_{23} + \theta_{13} = \pi/2$ within the experimentally allowed values, the main results given in Tab. I are not changed.

Finally, although it is nothing to do with the GUFM, we comment on bi-maximal ($\sin^2 \theta_{12} = \sin^2 \theta_{23} = 1/2$ and $\sin^2 \theta_{13} = 0$) and tri-bimaximal ($\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$, and $\sin^2 \theta_{13} = 0$) mixing angles just for reference, which are shown by big black triangles in Figs. 1 and 2. We can find regions where all the PMNS mixing angles at $\Lambda = 10^{14}$ GeV are close to the bi-maximal and tri-bimaximal [22] points in Figs. 1 (a)-(c) and Figs. 2 (a)-(c). For the BM mixing, $0.0015(0.002) \lesssim \epsilon$ is required for NH (IH) case, which corresponds to

	CKM	BM	TBM
$\frac{\pi}{4} \leq |\delta^l|, |\rho|, \ 0 \leq |\sigma| < \frac{\pi}{4}$	◯ (×)	×	◯
$\frac{\pi}{4} \leq |\delta^l|, |\sigma|, \ 0 \leq |\rho| < \frac{\pi}{4}$	◯ (×)	×	◯
$\frac{\pi}{4} \leq |\rho|, |\sigma|, \ 0 \leq |\delta^l| < \frac{\pi}{4}$	◯ (×)	◯	◯
$\frac{\pi}{4} \leq |\delta^l|, \qquad 0 \leq |\rho|, |\sigma| < \frac{\pi}{4}$	◯ (×)	×	◯
$\frac{\pi}{4} \leq |\rho|, \qquad 0 \leq |\delta^l|, |\sigma| < \frac{\pi}{4}$	◯ (×)	×	◯
$\frac{\pi}{4} \leq |\sigma|, \qquad 0 \leq |\delta^l|, |\rho| < \frac{\pi}{4}$	◯ (×)	×	◯

TABLE II: The realizations of CKM, BM, and TBM in cases of various combinations of CP-phases for the NH (IH) case.

$\tan \beta \simeq 8(10)$ [16]. And, the BM mixing cannot be realized in small CP-phases cases both for NH and IH cases. In different combinations of CP-phases, the BM cannot be realized unless $\pi/4 \leq |\rho|, |\sigma|$ for both NH and IH cases. These mean the largeness of $|\rho|$ and $|\sigma|$ is important for the realization of BM at the high energy scale (see Tab. II). On the other hand, the TBM mixing angles at the high energy are allowed for all cases (NH/IH and large/small CP-phases (see Tab. II)). All figures show

that the TBM is easy to be realized at high energy scale with relatively small quantum effects ("o" marker), since the TBM fits the PMNS mixing angles well at the low energy scale.

We have investigated whether the GUFM is possible or not in the framework of the MSSM. We have found the GUFM is really possible when neutrino has degenerated NH spectrum with 0.1 eV $\lesssim m_3$ through the quantum corrections, $0.005 \lesssim \epsilon$ ($10 \lesssim \tan \beta$). We have also investigated the possibility that BM and TBM mixing angles are realized at the high energy scale.

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