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# Quantum-Mechanical States as Ensembles of States in a Classical Phase Space

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# Quantum-Mechanical States as Ensembles of States in a Classical Phase Space

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ABSTRACT

Measuring processes of a single spin-1/2 object and of a pair of spin-1/2 objects in the EPR–Bohm state are modeled by systems of differential equations. These models are based on mainly the following two ideas: (i) as a result of the interaction with a measuring apparatus, the state of the object changes and approaches an attractor, in which the object possesses the measured value in the phase space of states of objects; (ii) a quantum-mechanical state corresponds to an ensemble of states in the corresponding attractor in the phase space. The latter model is a local model with hidden variables of the EPR–Bohm *gedanken* experiment. Although there is no dynamical interaction between the pair of spin-1/2 objects, the model reproduces approximately the quantum-mechanical correlations by using coincidence counting. Hence the Bell inequality is violated. This result supports the idea that the coincidence counting is the source of the apparent nonlocality in the EPR–Bohm gedankenexperiment.

CONTENTS

# 1. INTRODUCTION

The notion of probability is indispensable to quantum mechanics, because outcomes of measurements behave probabilistically in quantum phenomena. It is natural to consider that the probabilistic behavior of outcomes of measurements is due to the lack of our knowledge about the state of the object. This lack of our knowledge is compensated by finding variables that distinguish the states of the object that produce different outcomes. Since these variables have not been yet found experimentally, they are called hidden variables. The states of the object specified by values of the hidden variables are called hidden states. Since a state of the object is specified by sharp values of variables including hidden ones, we can say that states of the object are classical, and they constitute a classical phase space. Thus, in this viewpoint, *a quantum-mechanical state is an ensemble of states in the classical phase space.* Therefore a reinterpretation of quantum-mechanical probability based on this viewpoint will be called an *ensemble interpretation* of quantum-mechanical probability.

In the old days, there were attempts to interpret quantum-mechanical probability within the framework of usual probability theory, i.e., Kolmogorovian probability theory[(1)] which is based on measure theory. Since a probability measure represents an ensemble of hidden states, these attempts may be also called ensemble interpretations of quantum-mechanical probability. As is well known, these attempts, however, did not success,[(2,3)] because they postulated that the prepared object always produces an outcome directly, i.e., statistical properties of the object are calculated in a *single* probability space without conditioning. This postulate is a natural consequence in Kolmogorovian probability theory. Since quantum-mechanical probability is subject to a mathematical formalism based on Hilbert space theory, it is totally different from Kolmogorovian probability theory based on measure theory; this fact reflects the failure of these attempts. Unfortunately, the failure induced misunderstanding that no ensemble interpretations is possible. This misunderstanding leads to an idea that the probability is not secondary property but primary property of Nature. Moreover, it becomes to be considered that wave functions give the complete description of physical reality. The problem of this type of thought is that it needs the projection postulate,[(4)] i.e., reduction of wave function, which can be considered as a source of the Schrödinger cat paradox[(5)] and the de Broglie's paradox.[(6)]

The impossibility of reinterpretation of quantum-mechanical probability within the framework of Kolmogorovian probability theory means that *many* probability spaces are needed to describe quantum-mechanical probability. The choice of the probability spaces depends on what kind of physical quantity is to be measured. This dependence will be called *contextuality* of quantum-mechanical probability. This ensemble interpretation with many probability spaces are also called a contextual hidden-variable theory.[(7)] As shown by Gudder,[(8)] contextual hidden-variable theories are possible and contain no contradiction mathematically. But they have been regarded as just mathematical artifacts; there is space to discuss the meaning of the contextuality and to argue how quantum-mechanical probability emerges. In particular, what causes the contextuality has become an important problem in physics since the argument made by Bell.[(9)] Bell proves the so-called Bell inequality with assumption that a measurement of a spin does not influence a result of measurement of other spin far from the former spatially, where the pair of the two spins is in a singlet quantum-mechanical state. Since quantum-mechanical probability violates the Bell inequality, it is believed that the violation of the Bell inequality means existence of action at a distance. Does such action at a distance truly exist in Nature? Because the argument of Bell rests on Kolmogorovian probability theory, it should be considered that the problem of the violation of the Bell inequality is not the nonlocality but rather what causes the contextuality.

Since the contextuality is built in the mathematical formalism of quantum-mechanical probability theory, to discuss the cause of the contextuality it is necessary to see quantum-mechancial probability from another viewpoint, say ensemble interpretations. In this thesis, it is shown that there is a model based on ensemble interpretation of quantum-mechanical probability that the contextuality can be produced from another cause different from action at a distance. The aim of this thesis is to show that quantum-mechanical probability can be compatible to the traditional view of physics that there is no spooky action at a distance.

In the following, we will review the argument made by Bell briefly according to Ref. 10, and mention other attempts to save the physical locality.

Consider a pair of spin-1/2 particles in a singlet quantum-mechanical state, where the particles move to opposite directions. Measurements of components of the spins of the particles are performed at places far apart from each other. This is the EPR–Bohm *gedanken* experiment. We denote an element of reality which fixes all observables by $\lambda$. Its probability density is denoted by $\rho(\lambda)$. Let $A(\lambda;\mathbf{a})$ ($B(\lambda;\mathbf{b})$) be a random variable that represents outcomes of measurements of component of the spin of the one (other) particle along direction $\mathbf{a}$ ($\mathbf{b}$). $A(\lambda;\mathbf{a})$ assumes 1 ($-1$) if the outcome is spin-up (spin-down). It is similar for $B(\lambda;\mathbf{b})$. Here, the locality assumption is used which says that $A(\lambda;\mathbf{a})$ does not depend on $\mathbf{b}$ and $B(\lambda;\mathbf{b})$ does not depend on $\mathbf{a}$. Put

$$P(\mathbf{a},\mathbf{b}) := \int d\lambda \rho(\lambda) A(\lambda;\mathbf{a}) B(\lambda;\mathbf{b}).$$

Since $|A(\lambda;\mathbf{a})| = 1$,

$$\begin{aligned} |P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{b}')| &\leq \int d\lambda \rho(\lambda) |A(\lambda;\mathbf{a})(B(\lambda;\mathbf{b}) - B(\lambda;\mathbf{b}'))| \\ &= \int d\lambda \rho(\lambda) |B(\lambda;\mathbf{b}) - B(\lambda;\mathbf{b}')|. \end{aligned} \tag{1.1}$$

In the same way,

$$|P(\mathbf{a}',\mathbf{b}) + P(\mathbf{a}',\mathbf{b}')| \leq \int d\lambda \rho(\lambda) |B(\lambda;\mathbf{b}) + B(\lambda;\mathbf{b}')|. \tag{1.2}$$

While we can see that $|B(\lambda;\mathbf{b}) - B(\lambda;\mathbf{b}')| + |B(\lambda;\mathbf{b}) + B(\lambda;\mathbf{b}')| \equiv 2$. By this identity, (1.1), and (1.2),

$$\Delta(\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}') := |P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{b}')| + |P(\mathbf{a}',\mathbf{b}) + P(\mathbf{a}',\mathbf{b}')| \leq 2. \tag{1.3}$$

This is called the Bell inequality. Bell concludes that if the Bell inequality is violated, then the locality assumption is wrong; $A(\lambda;\mathbf{a})$ depends on $\mathbf{b}$, i.e., the setting up of the other measuring apparatus placed in the distance and so on.

Several EPR–Bohm type experiments have already been performed since then, and violations of the Bell type inequalities have been observed.(11) As a result, it is widely believed that quantum mechanics has a nonlocal character such as action at a distance.

It is little known, however, that several authors(12–16) showed, about ten years ago, that violation of the Bell inequality does not necessarily imply the existence of action at a distance, by making local models that violate the Bell type inequalities. According to the literature,(17) the Marshall–Santos–Selleri model(13) rests on the idea of a variable detection probability, namely, different photons behave differently, when interacting with photon detectors, so that the no-enhancement assumption(12) is not satisfied. This idea and other related models have been discussed extensively in Ref. 17; from it we can know subsequent development along these lines up to 1988. A simple example proposed by Ferrero *et al.*(18) in 1990 shows that if the efficiency of photon detectors is low, then the so-called *factorizability* is compatible with the quantum-mechanical prediction that is obtained by taking all the corrections for nonideal behavior into account. Further, Santos has shown(19,20) that no violation of the genuine Bell inequality is predicted by quantum mechanics, when *correctly* used, in experiments with correlated optical photon pairs, even if perfect polarizers and detectors were available. Reference 20 is worth reading, because the interpretation of the quantum formalism, the relation to realism, hidden variables, the Bell inequalities, etc. have been argued from the author's viewpoint as a scientist. On the other hand, Scarela,(14) Notarigo,(15) and Pascazio(16) investigated other possibilities in order to save the physical locality. They argue that the coincidence counting is a source of the apparent nonlocality. They make models only for photons, since most of the experiments were performed on pairs of photons. The attempts to save the physical locality have not yet disappeared. In 1993, Squires has argued the possibility of absence of the superluminal interaction in the EPR-like situation, using the Bohm hidden-variable model modified by the introduction of retarded positions into the wave function.(21)

Now, in this thesis, a local model for spin-1/2 objects, according to Bohm's version(22) of the EPR *gedanken* experiment, will be presented. The model violates the Bell inequality as a result of the coincidence counting. A hidden-variable model is suitable for the purpose of seeing clearly whether there exists action at a distance or not, because it refers to the change of a state of the object before and after a measurement, i.e., a processes of measurement. For this reason, the model comprises hidden variables. The purpose of this thesis is neither to explain why the value of spin is quantized nor to replace quantum mechanics by classical mechanics. The aim of this thesis is rather to find an example, at least in thinking, that shows that the coincidence counting is the source of the apparent nonlocality. In the model, in order to describe the time evolution of a measuring process, we shall use a system of differential equations that has attractors. Since the attractors are invariant under the flow, they are invariant in the measurement. From this fact, the attractors shall be related to corresponding quantum-mechanical eigenstates. This is a new feature of the model.

The organization of this thesis is as follows: In Sec. 2, we will construct a model of a measurement for a single spin-1/2 object. This section also contains preparations of the next section. In Sec. 3, we will make an extension of the model of Sec. 2 to the EPR–Bohm situation. The heart of this thesis is in this section. By taking an appropriate closing time in sampling data, the model approximately reproduces the correlations predicted by quantum mechanics. Section 4 is devoted to discussion and summary.

# 2. A HIDDEN-VARIABLE MODEL OF A MEASUREMENT OF A SINGLE SPIN-1/2 OBJECT

In this section, we will construct a model of measuring processes of a spin-1/2 object. A measuring apparatus changes a state of the object due to the interaction between them; this change is not instantaneous generally. To describe this change, we use a system of ordinary differential equations. For another setting of the measuring apparatus, time evolution of a state of the object is governed by another system of differential equations. Therefore the probability spaces of outcomes are different for different settings of the measuring apparatus. Thus our model becomes a contextual hidden-variable theory.

A spin-1/2 object is not a mere point-particle, but a system of many degrees of freedom, because the spin can be considered as those degrees of freedom that describe a rotation of the object about some axis. We denote these degrees of freedom by $\mathbf{S} = (S_x, S_y, S_z) \in \mathbf{R}^3$. For the sake of convenience, we use units in which $\hbar$=1, and put $j = 1/2$, $J = \sqrt{3/4}$. We denote the quantum-mechanical observables of the spin by a triple of operators $(\hat{S}_x, \hat{S}_y, \hat{S}_z)$. Then quantum mechanics gives us the following information about the spin: the spin-up(-down) eigenstate of $\hat{S}_z$ has the eigenvalue $+j(-j)$ for $\hat{S}_z$ and $J^2$ for $\hat{S}_x{}^2 + \hat{S}_y{}^2 + \hat{S}_z{}^2$, respectively. Hence the spin-up eigenstate of $\hat{S}_z$ may correspond to an ensemble of states whose members have such properties that $S_z = +j$ and $|\mathbf{S}| = J$. $\mathbf{S}$ is yet insufficient to describe a state of the spin-1/2 object, since the vector $\mathbf{S}$ possessing the above properties is not parallel to the $z$-axis. We must take account of other degrees of freedom that express whether a state is a member of an ensemble corresponding to the spin-up eigenstate. Let us denote them by $\mathbf{U} = (U_x, U_y, U_z)$, the value of which is parallel to the $z$-axis for any state in the ensemble corresponding to the spin-up eigenstate of $S_z$, and so on.

In actual experiments, it does not matter when the object enters the measuring apparatus and escapes from it. Hence we do not have to take account of such details of the motion of the object in the actual space. However, they become important for coincidence counting; we will take them into account in the next section. In this section, we shall see that the six degrees of freedom are sufficient to model the measuring process of a single spin-1/2 object.

We denote a six-dimensional space $\mathbf{R}^6$ by $\Gamma$ whose coordinates are given as $(\mathbf{S}, \mathbf{U}) = (S_x, S_y, S_z, U_x, U_y, U_z)$. A state of the object is represented by a point $(\mathbf{S}, \mathbf{U})$ in $\Gamma$. Let $\alpha_1$ be a subset of $\Gamma$ that is defined as

$$\alpha_1 = \{(\mathbf{S}, \mathbf{U}) \in \Gamma \mid S_z = +j, |\mathbf{S}|^2 = J^2, U_x = U_y = U_z - J = 0\}.$$

As stated before, we will identify an ensemble of states distributed uniformly in $\alpha_1$ with the spin-up eigenstate of $\hat{S}_z$. In the same way, let $\alpha_2$ be a subset of $\Gamma$ that is defined as

$$\alpha_2 = \{(\mathbf{S}, \mathbf{U}) \in \Gamma \mid S_z = -j, |\mathbf{S}|^2 = J^2, U_x = U_y = U_z + J = 0\},$$

and we will identify an ensemble of states distributed uniformly in $\alpha_2$ with the spin-down eigenstate of $\hat{S}_z$.

A good measuring apparatus for the $z$-component of the spin must effect a state of the object to approach to either $\alpha_1$ or $\alpha_2$. Therefore the measuring process of the $z$-component of the spin may be modeled by a system of differential equations for which $\alpha_1$ and $\alpha_2$ are attractors.(23) The simplest ones among such systems of differential equations may be the following:

$$\begin{cases} \dfrac{d\mathbf{S}}{dt} = \mathbf{U} \times \mathbf{S} - \epsilon_1 P_{xy} \dfrac{\partial \psi}{\partial \mathbf{S}} \psi - \epsilon_1 \{\theta(S_z - \beta)\phi_+ + \theta(-S_z + \beta)\phi_-\} \mathbf{e}_z, \\ \dfrac{d\mathbf{U}}{dt} = -\epsilon_2 P_{xy} \mathbf{U} - \epsilon_2 \{U_z - \mathrm{sign}(S_z - \beta) J\} \mathbf{e}_z - \epsilon_2 U_z \{|\mathbf{U}|^2 - J^2\} \mathbf{e}_z, \end{cases} \tag{2.1}$$

where $\epsilon_1 = 10.0$, $\epsilon_2 = 0.05$, and

$$\begin{aligned}
\psi(\mathbf{S}) &\equiv |\mathbf{S}|^2 - J^2, \\
\phi_\pm(\mathbf{S}) &\equiv S_z \mp j, \\
\omega(\mathbf{U}) &\equiv \cos^{-1}(U_z/|\mathbf{U}|), \\
\beta(\omega) &\equiv \{ j\cos\omega - \sqrt{J^2 - j^2}\cos(\frac{\pi}{2}(1-\cos\omega))\sin\omega \} \\
&\qquad \times \{ 0.98\theta(|\cos\omega| - 0.99) + \theta(0.99 - |\cos\omega|) \}, \\
\mathbf{e}_z &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \qquad P_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned}$$

Here $\theta(x)$ is the step function which is defined as $\theta(x) = 1$, if $x \geq 0$, $\theta(x) = 0$, otherwise; $\mathrm{sign}(x)$ is the sign function which is defined as $\mathrm{sign}(x) = 1$, if $x \geq 0$, $\mathrm{sign}(x) = -1$, otherwise. For simplicity's sake, the unit of time is chosen appropriately. Thus we assume that the evolution of a state $(\mathbf{S}, \mathbf{U})$ during *the measurement of* $S_z$, i.e., the $z$-component of the spin, is governed by Eq. (2.1).

It seems convenient to make some remarks on Eq. (2.1). The terms containing $\epsilon_i$'s, $i = 1, 2$, in Eq.(2.1) are crucial for the existence of the attractors $\alpha_1$ and $\alpha_2$. To see this, suppose that there are no terms on the right-hand sides of Eq. (2.1) except for $-\epsilon_1 P_{xy}(\partial\psi/\partial\mathbf{S})\psi$. Then $(d\psi^2/dt) = 2(\partial\psi/\partial\mathbf{S})\cdot(d\mathbf{S}/dt)\psi = -8\epsilon_1(S_x^2 + S_y^2)\psi^2 \leq 0$. As far as neither $S_x^2 + S_y^2$ nor $\psi$ vanishes, $\psi(\mathbf{S}(t))^2$ is strictly monotone decreasing as a function of time $t$. As $t \to \infty$, $\psi^2$ may vanish, i.e., $|\mathbf{S}|^2$ may approach $J^2$. This is similar for the other terms containing $\epsilon_1$ or $\epsilon_2$. The role of the function $\beta(\omega)$ is to give a border between $S_z \to +j$ and $S_z \to -j$. Thus, the terms containing $\epsilon_i$'s, $i = 1, 2$, effectively represent the influence of the measuring apparatus on a state of the object.

In the actual experiments, for example, the Stern-Gerlach magnet is used as a measuring apparatus of $S_z$. In this case, the $z$-component of the spin is not directly measured. The experimenter judges $S_z = +j$ or $-j$ according to the sign of the $z$-component of the velocity of the object gained eventually by the inhomogeneous magnetic field. In our model, if $\mathbf{S}$ is stabilized in a neighborhood of $\alpha_1(\alpha_2)$, in which $S_z > 0(< 0)$, then since $\mathbf{S}$ may behave as if it represented a magnetic dipole, the $z$-component of the velocity shall become positive(negative). From these considerations, let us regard the measurement of $S_z$ in our model as the following procedure: a state $(\mathbf{S}, \mathbf{U})$ of the object begins to evolve by Eq. (2.1), when the interaction between the object and the measuring apparatus is switched on; when the state in $\Gamma$ comes into an appropriate neighborhood $G(\alpha_1) \equiv \{(\mathbf{S}, \mathbf{U}) \in \Gamma : |S_z - j| < \delta\}$ of $\alpha_1$, $\delta = 0.01$, the measurement finishes and we obtain the outcome $+j$; otherwise, when the state in $\Gamma$ comes into an appropriate neighborhood $G(\alpha_2) \equiv \{(\mathbf{S}, \mathbf{U}) \in \Gamma : |S_z + j| < \delta\}$ of $\alpha_2$, the measurement finishes and we obtain the outcome $-j$. Rigorously speaking, the measured $z$-component of the spin is not represented by $S_z$; rather, it is represented by a slightly modified function $\check{S}_z$ defined on $\Gamma$ as

$$\check{S}_z(\rho) = \begin{cases} +j, & \text{if } \rho \in G(\alpha_1), \\ -j, & \text{if } \rho \in G(\alpha_2), \\ S_z(\rho), & \text{otherwise.} \end{cases}$$

For brevity, we will write that $S_z \Rightarrow j$ in place of $\check{S}_z = j$ and so on hereafter.

We have completed modeling the measuring process of $S_z$ here. Measuring processes of other components of the spin are modeled in the natural manner. It suffices to transform all the quantities of the model of the measurement of $S_z$ under the rotation that transform the $z$-direction into the direction along which the spin is measured.

To see whether our model goes well, we will study measurements of $S_z$ for several initial ensembles corresponding to the eigenstates of other components of the spin. First, each initial ensemble is defined in the following. Let $T_\theta$ be a rotation (matrix) in $\mathbf{R}^3$ through an angle $\theta$ about an axis lying in the $xy$-plane, say, the $y$-axis. When we perform a measurement of $\mathbf{S} \cdot T_\theta \mathbf{e}_z$, the evolution of a state is given by the system of differential equations that is induced from Eq. (2.1) by the rotation. Let $\beta_\theta$ be a subset of $\Gamma$ defined as

$\beta_\theta \equiv \{(T_\theta \mathbf{S}', T_\theta \mathbf{U}') \in \Gamma : (\mathbf{S}', \mathbf{U}') \in \alpha_1\}$, i.e., it is obtained by rotating $\alpha_1$. Then the $\beta_\theta$ is an attractor on which $\mathbf{S} \cdot T_\theta \mathbf{e}_z = +j$. An ensemble of states distributed uniformly in the $\beta_\theta$ is identified with the spin-up eigenstate of the quantum-mechanical observable corresponding to $\mathbf{S} \cdot T_\theta \mathbf{e}_z$. This is because that when we perform the measurements of $\mathbf{S} \cdot T_\theta \mathbf{e}_z$ on this ensemble, $\mathbf{S} \cdot T_\theta \mathbf{e}_z \Rightarrow j$ holds with certainty, since all the members of the ensemble are already in the neighborhood $G(\beta_\theta)$ of the attractor $\beta_\theta$.

Second, since we cannot solve Eq. (2.1) analytically, we must make numerical calculations. By solving Eq. (2.1) numerically for each member of the ensembles as an initial condition, the probabilities that the trajectories come into the neighborhoods $G(\alpha_1)$ of $\alpha_1$ and $G(\alpha_2)$ of $\alpha_2$ were calculated. In effect, 72 states distributed uniformly in $\beta_\theta$ ($\theta = 10n°$, $n = 0, \ldots, 36$) were taken as the initial conditions. Equation (2.1) was solved by the Euler method with two different step-sizes $\Delta t$=0.001, 0.0001. For each $\theta$, the probabilities were calculated as relative frequencies for the 72 samples. Since the results in the two step-sizes are almost the same, it can be said that the errors from the Euler method are very small in these numerical calculations. The maximum time when a state comes into one of the neighborhoods of the two attractors is short; it is 0.2447. This may be the reason why the errors are small, though the Euler method is used.

Figure 1 represents the probabilities of outcomes of the measurements of $S_z = \mathbf{S} \cdot \mathbf{e}_z$ being $+j$ with respect to the ensembles of states distributed uniformly in $\beta_\theta$ for several relative angles $\theta$ (calculated with $\Delta t = 0.0001$). The result fits with the prediction of quantum mechanics. In this sense, we can say that the model of this section is the one of measuring processes of a single spin-1/2 object.

Before closing this section, we note the following remark. We call the time when the interaction between the object and the measuring apparatus is switched on a *beginning time*. We call the time when a state comes into one of the neighborhoods of the two attractors a *finishing time*. Since we may put the beginning time for each state to be zero without loss of generality, we will do so hereafter.

Then the finishing time for each object depends both on what kinds of physical quantities is measured and on the initial states. We note that finishing times generally fluctuate due to the variation of initial states, even if the beginning times are the same. This fluctuation plays an important role when we use the coincidence counting as in the next section.

## 3. A LOCAL MODEL OF THE EPR–BOHM *GEDANKEN* EXPERIMENT

Now we will make an extension of the previous model to the EPR–Bohm *gedanken* experiment. We consider two spin-1/2 objects which are distinguished by being labeled as $A$ and as $B$. Let $\Gamma_A$ and $\Gamma_B$ be the phase spaces of the spin-1/2 objects $A$ and $B$, respectively. The whole phase space $\Gamma_{AB}$ is given by the direct product $\Gamma_A \times \Gamma_B$. Suppose that two measuring apparatuses are placed apart on the $y$-axis at equal distance from the origin. We will measure components of the spins along directions perpendicular to the $y$-axis.

The EPR–Bohm quantum-mechanical state is the singlet state. We note that it is rotationally invariant, and the pair of spins, in this quantum-mechanical state, has completely negative correlations. Let $s$ be a subset of $\Gamma_{AB}$ defined as

$$s \equiv \bigcup_{\phi=0}^{2\pi} \bigcup_{\theta=0}^{\pi} \Big\{ (\mathbf{S}_A, \mathbf{U}_A, \mathbf{S}_B, \mathbf{U}_B) \in \Gamma_{AB} \,\Big|\, \mathbf{S}_A + \mathbf{S}_B = \mathbf{0}, \mathbf{U}_A + \mathbf{U}_B = \mathbf{0}, \\ \big(R_y(-\theta) R_z(-\phi) \mathbf{S}_A, R_y(-\theta) R_z(-\phi) \mathbf{U}_A\big) \in \alpha_1 \Big\},$$

where $R_z(\phi)$ and $R_y(\theta)$ represent the rotation about the $z$-axis through an angle $\phi$ and the rotation about the $y$-axis through an angle $\theta$, respectively. An ensemble of states of pairs distributed uniformly in the subset $s$ has the above mentioned two features of the EPR–Bohm quantum-mechanical state. Hence let us take this ensemble as the initial condition just before measurements in the EPR–Bohm situation.

In the actual experiments, the coincidence counting is used in order to identify detected objects as a pair. Therefore the time when the object escapes from the measuring apparatus is important. Since the actual motion of the object in the measuring apparatus may be complicated, instead of modeling the details of the motion concretely, we just assume that there exists a threshold time $T$ such that if the finishing time is greater than $T$, then the value of the $y$-coordinate of the object becomes random as a result of the interaction with the measuring apparatus. We can also rephrase this assumption as follows: there is $T$ such that if the finishing time is greater than $T$, then the time when the object escapes from the measuring apparatus fluctuates. It should be emphasized that this assumption concerns itself with a single object ($A$ or $B$) and the measuring apparatus for it, so no action at a distance is stolen into our model by this assumption. It shall be shown at the end of this section that by the coincidence counting, a pair of spin-1/2 objects is taken into account as outcomes of measurements, only if their finishing times are less than $T$. Hence we call $T$ a *closing time* in sampling data.

Correlations of the spins of the objects $A$ and $B$ with a closing time $T$= 0.133 were calculated numerically as in the previous section. First, 3852 states distributed uniformly in $s$ were taken as initial conditions for the differential equations, and they were solved by the Euler method with step-size $\Delta t = 0.001$ with respect to various closing times $T$. It was found that $T$=0.133 is the best. Next, 15192 states distributed uniformly in $s$ were taken as initial conditions for the differential equations, and they were solved in the same way but with step-size $\Delta t = 0.0001$ with respect to closing times $T$=0.133. The correlations obtained from the two kinds of calculations with different sizes of samples and the step-sizes agree well; it can be said that the errors are very small in these numerical calculations.

The results are plotted in Fig. 2 and compared with the quantum-mechanical correlations. The correlations obtained from our model approximate the quantum-mechanical correlations. For comparison, the correlations without the closing time, i.e., without using the coincidence counting, were also calculated. The results are plotted in Fig. 3. In this case, since no closing time is instituted, the correlations are calculated essentially in a single probability space. Since our model has no action at a distance, as expected from the no-go theorems[2,3] of noncontextual hidden-variable theories, the result without the closing time does not agree with quantum mechanics.

Figure 4 represents the results of calculations of the quantity $F(\phi)$ that appears in Ref. 24. The Bell inequality implies that $F(\phi)$ does not exceed 2. The results with the closing time $T = 0.133$ agree with quantum mechanics approximately, and the Bell inequality is violated. The results without the closing time, on the other hand, satisfy the Bell inequality, and do not agree with quantum mechanics.

Our task is now to express the assumption for the motion of the position of the object more concretely, and to show that the coincidence counting leads to the institution of a closing time in sampling data. Suppose that the devices of the *gedanken* experiment are arranged as follows (see Fig. 5): the source of pairs of the objects is at the origin; measuring apparatuses, whose lengths in the $y$-direction are the same $W$, for the object $A$ and the object $B$ are placed apart on the $y$-axis at equal distance from the origin; a detector for the object $A$ is placed behind the measuring apparatus, say on $(0, -L, 0)$, viz., on the $y$-axis at the distance $L$ from the origin to the negative direction; in the same way, a detector for the object $B$ is placed on $(0, L, 0)$. Let $t_0$ be the earliest time when the object reaches the detector. Recall here that all the beginning times are zero. Let $v_0$ be the modulus of the $y$-component of the velocity of each object at the outside of the measuring apparatus. For the object whose finishing time is less than $T$, we denote the modulus of the $y$-component of the velocity at the inside of the measuring apparatus by $v$ (=const).

Then our assumption can be expressed concretely as follows: for an object with its finishing time $\tau$, if $\tau \leq T$, then the time when the object escapes from the measuring apparatus is $W/v$; if $\tau > T$, then the time when the object escapes from the measuring apparatus fluctuates uniformly in a time interval $[W/v, (W/v) + \tau]$ as a result of the interaction between the object and the measuring apparatus. Thus, the closing time $T$ characterizes the way of diffusion of the position of the object in the measuring process.

Let $\tau_A$ be a finishing time for the spin-1/2 object $A$. Then the probability of the object $A$ existing in an interval $[y, y + \Delta y]$ on the $y$-axis, $0 < \Delta y \ll 1$, at the time $t_0$ becomes approximately

$$\rho_A(y)\Delta y = \begin{cases} \chi_{[-L,-L+\Delta y]}(y), & \text{if } \tau_A \leq T, \\ \frac{1}{v_0\tau_A}\chi_{[-L,-L+v_0\tau_A]}(y)\Delta y, & \text{if } \tau_A > T. \end{cases}$$

Here we used the notation $\chi_E$ to represent the characteristic (defining) function of a subset $E$ of the real line which is defined as $\chi_E(x) = 1$, if $x \in E$; $\chi_E(x) = 0$, otherwise. In the same way, let $\tau_B$ be a finishing time for the spin-1/2 object $B$. The probability of detection of the object $B$ in an interval $[y, y+\Delta y]$, $0 < \Delta y \ll 1$, at the time $t_0$ becomes approximately

$$\rho_B(y)\Delta y = \begin{cases} \chi_{[L-\Delta y,L]}(y), & \text{if } \tau_B \leq T, \\ \frac{1}{v_0\tau_B}\chi_{[L-v_0\tau_B,L]}(y)\Delta y, & \text{if } \tau_B > T. \end{cases}$$

For the objects $A$ and $B$ whose finishing times are $\tau_A$ and $\tau_B$, respectively, we estimate the probability $p_c$ of coincidence detection. We partition the $y$-axis into intervals $[y_n, y_{n+1})$, $n \in \mathbf{Z}$, whose lengths are $\Delta y$, where $y_n = n\Delta y$. Since the coincidence detection is done not only at the time $t_0$ but also at delayed times, $p_c$ becomes, for $\Delta y \ll 1$,

$$\begin{aligned} p_c &= \sum_n \rho_A(-y_n)\Delta y \rho_B(y_n)\Delta y \\ &\approx \int dy \rho_A(-y)\rho_B(y)\Delta y \\ &= \begin{cases} 1, & \text{if } \tau_A, \tau_B \leq T, \\ O(\Delta y), & \text{otherwise.} \end{cases} \end{aligned}$$

This means that in the limit where the accuracy $\Delta y$ of position approaches zero, if at least one of the finishing times $\tau_A$ and $\tau_B$ is greater than $T$, then the probability $p_c$ of the objects $A$ and $B$ being detected at the same time vanishes. Therefore any objects $A$ and $B$ detected at coincidence have finishing times $\tau_A$ and $\tau_B$ both of which are less than $T$ with certainty.

Accordingly, as far as we use the coincidence counting, our local model violates the Bell inequality.

## 4. DISCUSSION AND CONCLUSION

As we have seen in the previous section, there exists a local model that violates the Bell inequality even for spin-1/2 objects. This fact supports the idea that the coincidence counting is the source of the apparent nonlocality, and that there exists no action at a distance in the EPR–Bohm situation.

The model is an example that shows that interaction between the object and the measuring apparatus is not the unique reason of the contextuality, i.e., the probability spaces of outcomes of measurements are different according to what are measured. In fact, by introducing the closing time $T$, the model of Sec. 3 produces different probability spaces of outcomes according to the choice of different settings of the measuring apparatuses. Thus this local model becomes a contextual hidden-variable theory. In the model, the sample space changes according to the choice of $\theta_{\mathbf{ab}}$, which is the angle between the directions $\mathbf{a}$ and $\mathbf{b}$ of the spins of the objects $A$ and $B$ to be measured. Nevertheless, as shown in Fig. 6, the variation of the number of samples is small (less than 10%). This results seem to be consistent with the results of the experiment of Aspect *et al.*,[25] though it was done for photons, in which the number of samples is found constant when the setting of the measuring apparatuses is changed.

In Sec. 2, we identified the ensemble of the states of the object distributed uniformly in each attractors with the corresponding eigenstates. In Sec. 3, we have identified the ensemble distributed in the subset $s$ of $\Gamma_{AB}$ with the EPR–Bohm quantum-mechanical state. One may ask what ensemble corresponds to a given quantum-mechanical state. The question is beyond the scope of this thesis, because it is almost equivalent to understanding the superposition principle of quantum states. In fact, in order to find the answer, we must understand the meaning of a phase of a quantum-mechanical state vector. The phenomenological model in this thesis lacks this information of the phase.

However, it is interesting and important to find the answer, because this has deep connection with whether our human reason can understand things existing in the external world or not. It is known that propositions for quantum phenomena are subject to some non-Boolean logic.[26] These propositions are concerned with outcomes of measurements. But, since the human reason is subject to the Boolean logic, the outcomes of the measurements contradict the human reason. Accordingly, in order to understand the things behind the quantum phenomena by the human reason, we cannot help assuming something that is subject to the Boolean logic. Thus, to understand the things in the external world necessarily means introduction of some hidden variables. In addition, we must also comprehend how the Boolean object characterized by the hidden variables produces such non-Boolean phenomena as quantum phenomena. Although the model in this thesis is so restricted that it may have less connection with the things existing in the external world, it gives an example such that a Boolean object leads to non-Boolean phenomena. In this sense, the model is instructive. The model also suggests that the things existing in the external world would be local.

The results of this thesis are summarized in the following. A local hidden-variable model of spin-1/2 objects in the EPR–Bohm *gedanken* experiment is constructed. By instituting the appropriate closing time in sampling data, the correlations that are calculated by the model approximate the quantum-mechanical correlations. Consequently, as far as the coincidence counting is used, the local model violates the Bell inequality with no action at a distance. Therefore quantum-mechanical probability can be compatible to the traditional view of physics that there is no action at a distance.

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# FIGURES

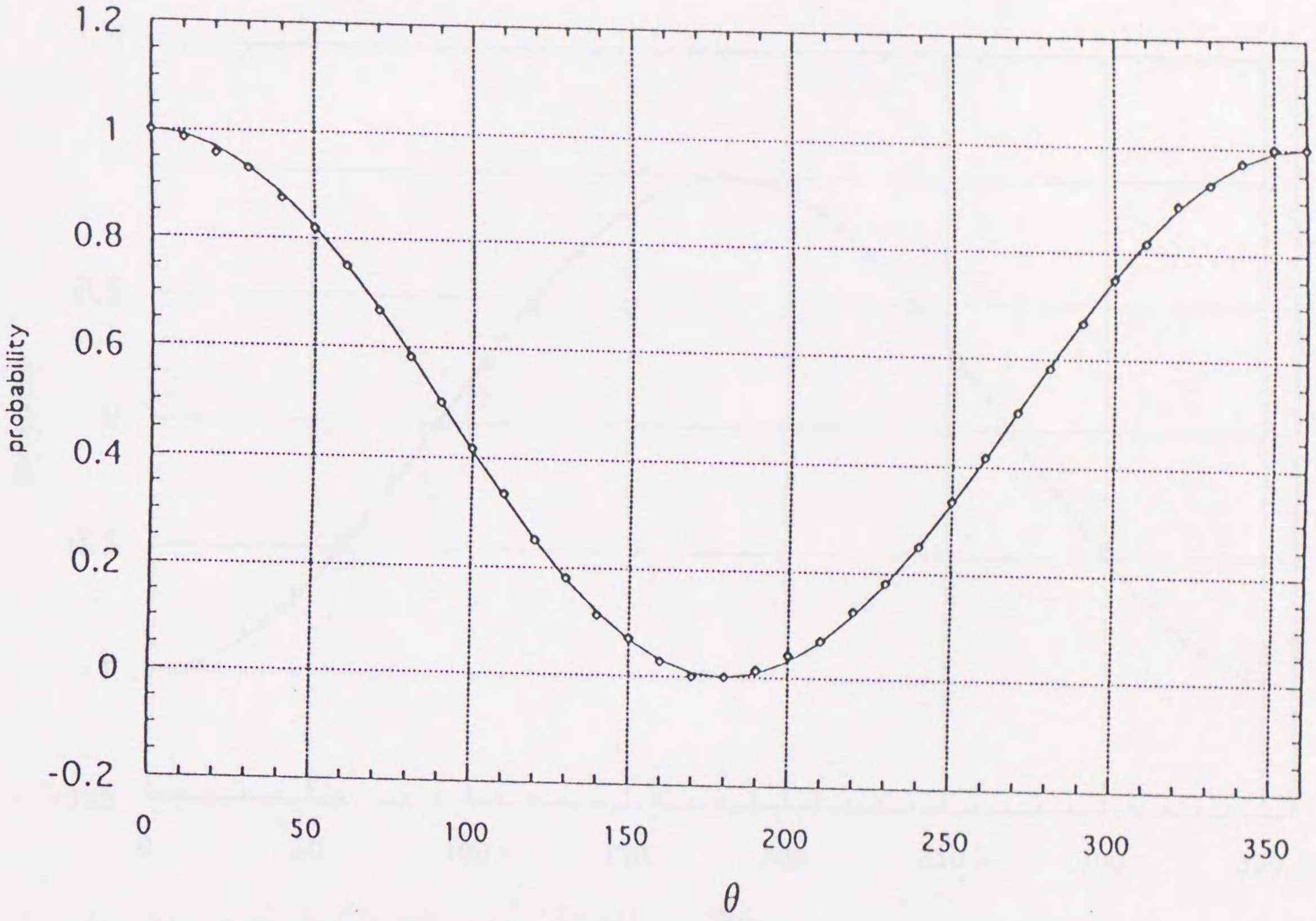



Fig. 1. The probability of $S_z$ being $+j$ for the ensemble of states distributed uniformly in $\beta_\theta$ plotted versus the relative angle $\theta$. $\diamond$ represents the results of our model, and the solid curve is the corresponding results of quantum mechanics.

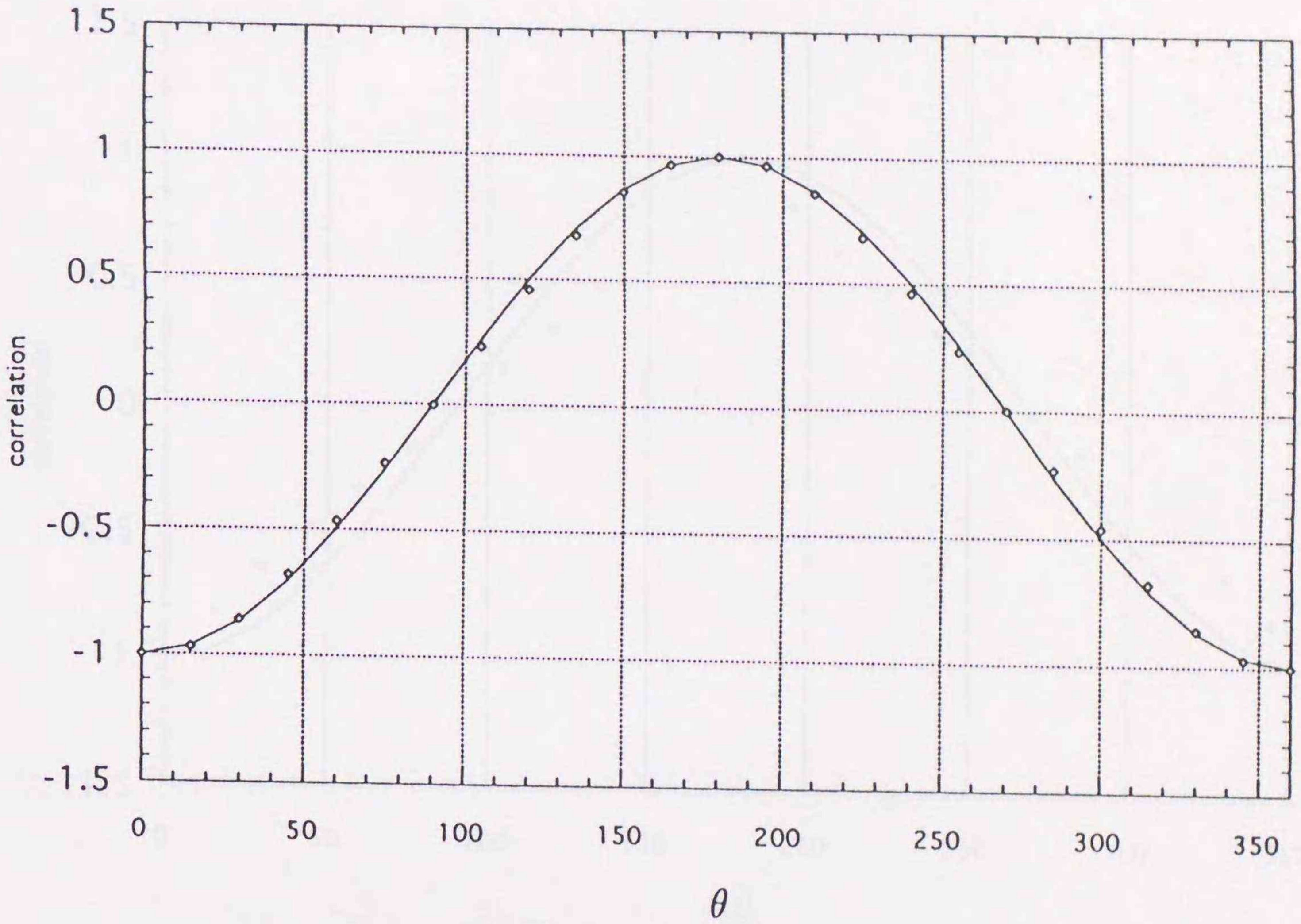



Fig. 2. The correlation of $\mathbf{S}_A \cdot \mathbf{e}_z$ and $\mathbf{S}_B \cdot T_\theta \mathbf{e}_z$ for the ensemble of states distributed uniformly in $s$ plotted versus the relative angle $\theta$. $\diamond$ represents the results of our model, and the solid curve is the corresponding results of quantum mechanics. The value of the closing time $T$ in sampling data is 0.133.

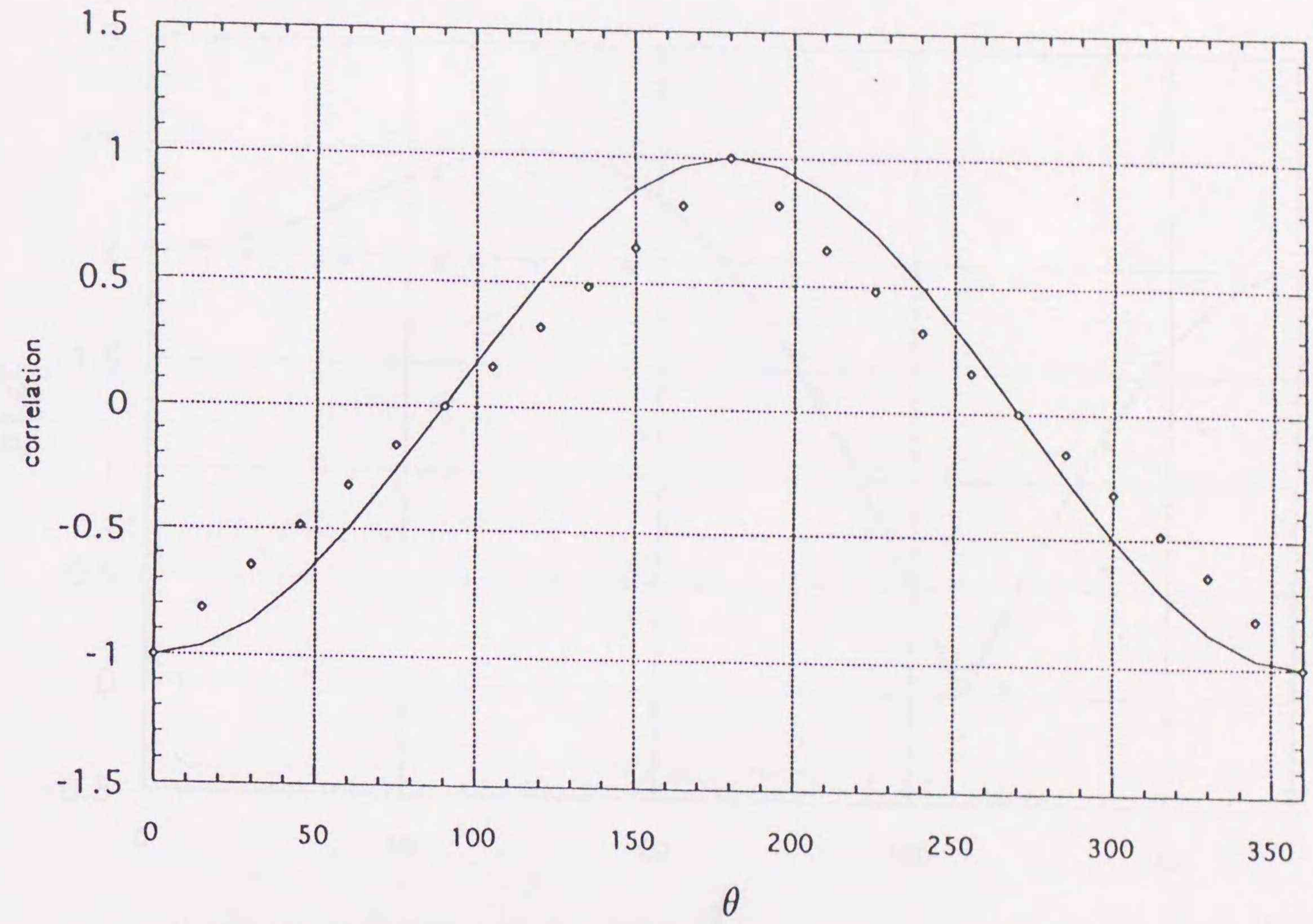



Fig. 3. The correlation of $\mathbf{S}_A \cdot \mathbf{e}_z$ and $\mathbf{S}_B \cdot T_\theta \mathbf{e}_z$, without the closing time, for the ensemble of states distributed uniformly in $s$ plotted versus the relative angle $\theta$. $\diamond$ represents the results of our model, and the solid curve is the corresponding results of quantum mechanics.

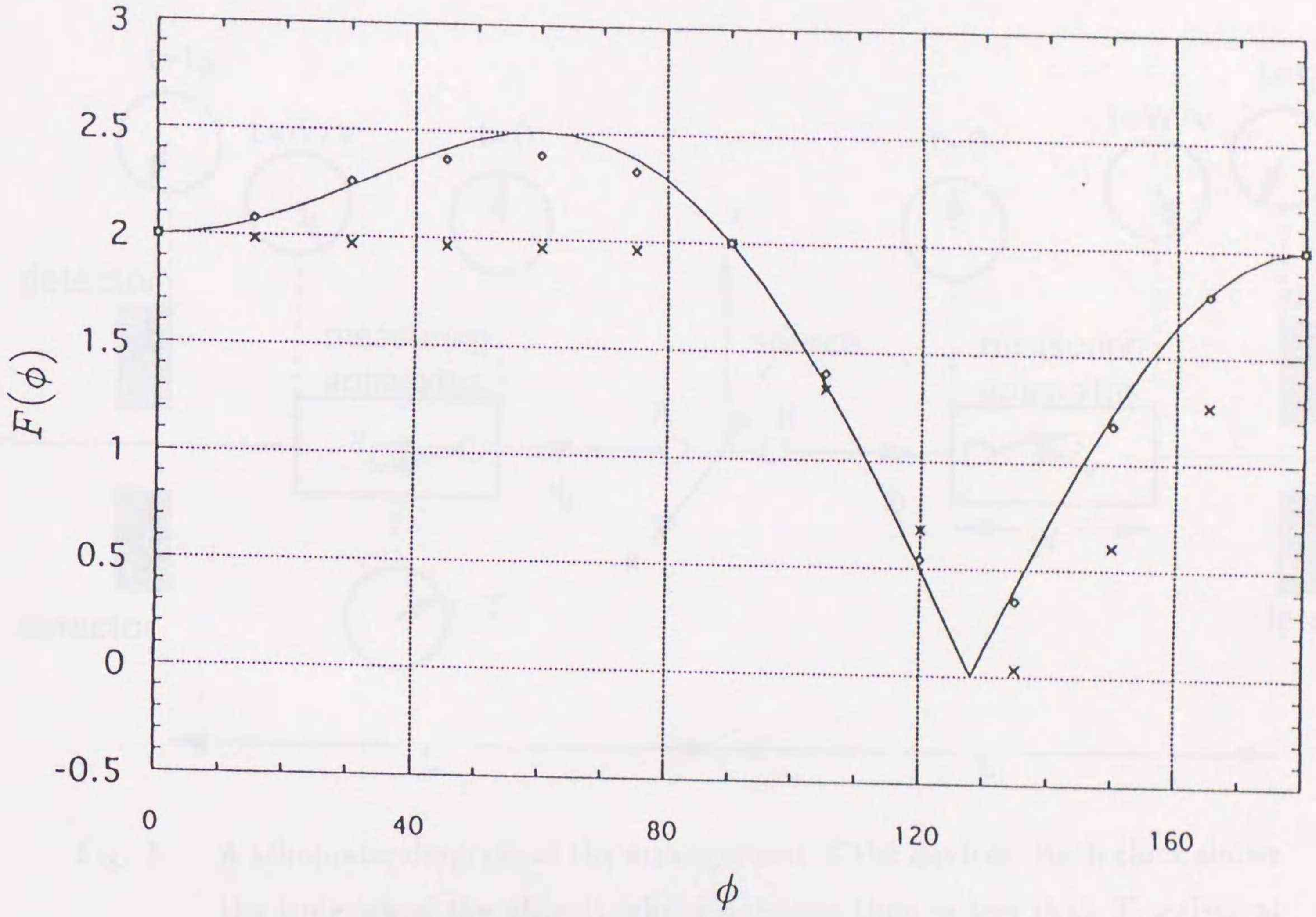



Fig. 4. Graph of $F(\phi)$ given in Ref. 22 against the relative angle $\phi$. $\diamond$ represents the results of our model with the closing time $T$=0.133. $\times$ represents the results of our model without the closing time. The solid curve is the corresponding results of quantum mechanics.

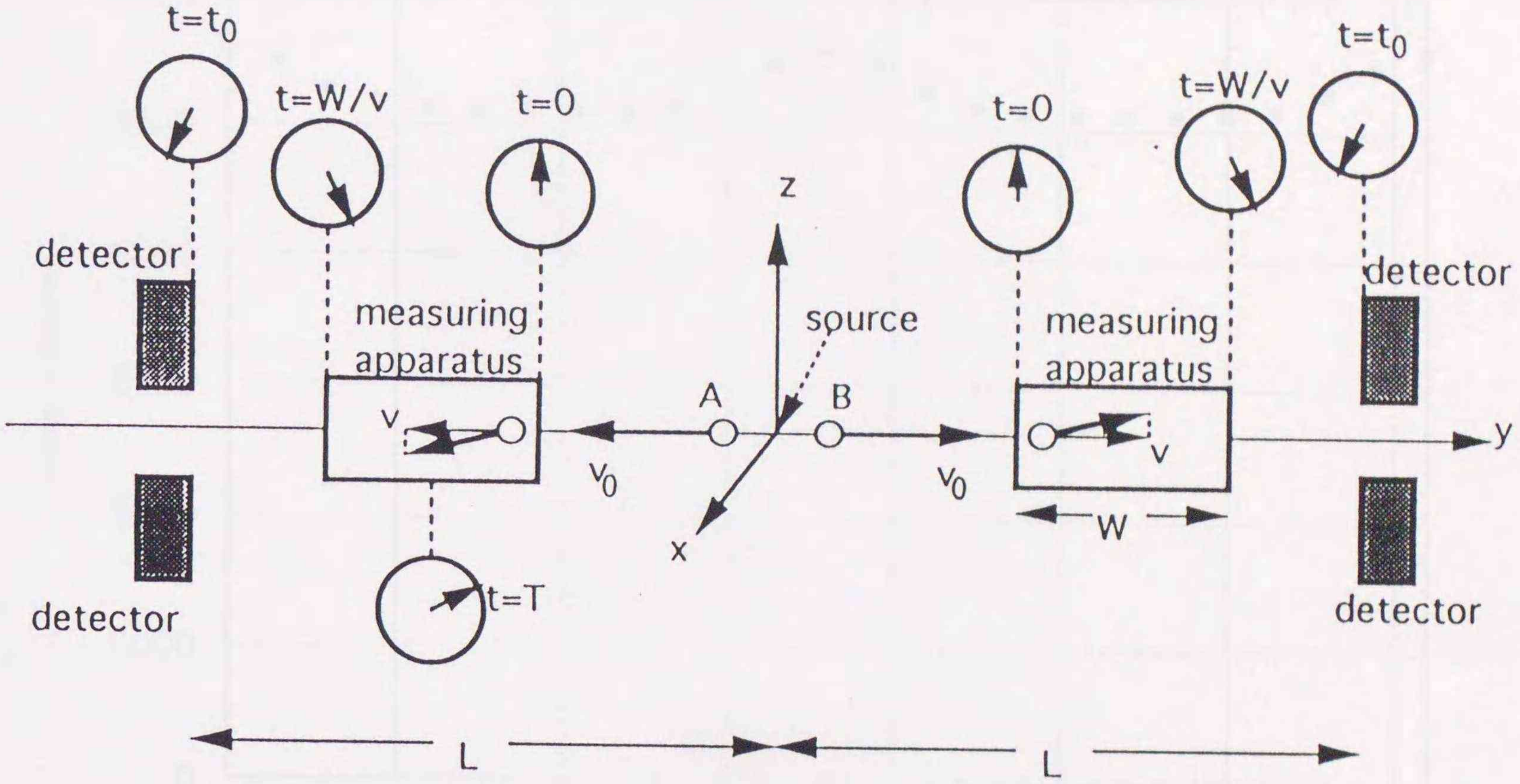



Fig. 5. A schematic diagram of the arrangement of the devices. Each clock shows the time when the object, whose finishing time is less than $T$, exists at each indicated place.

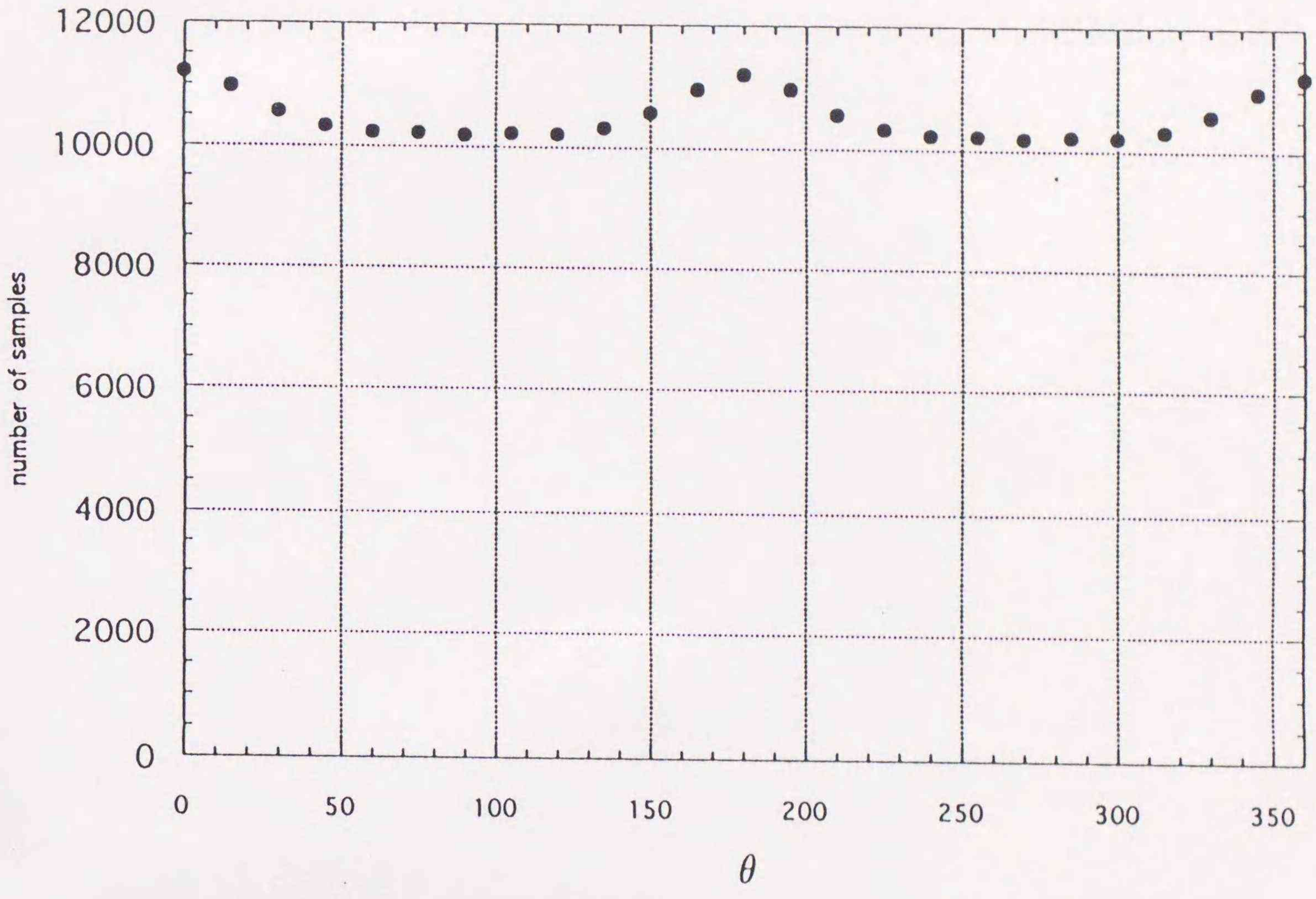
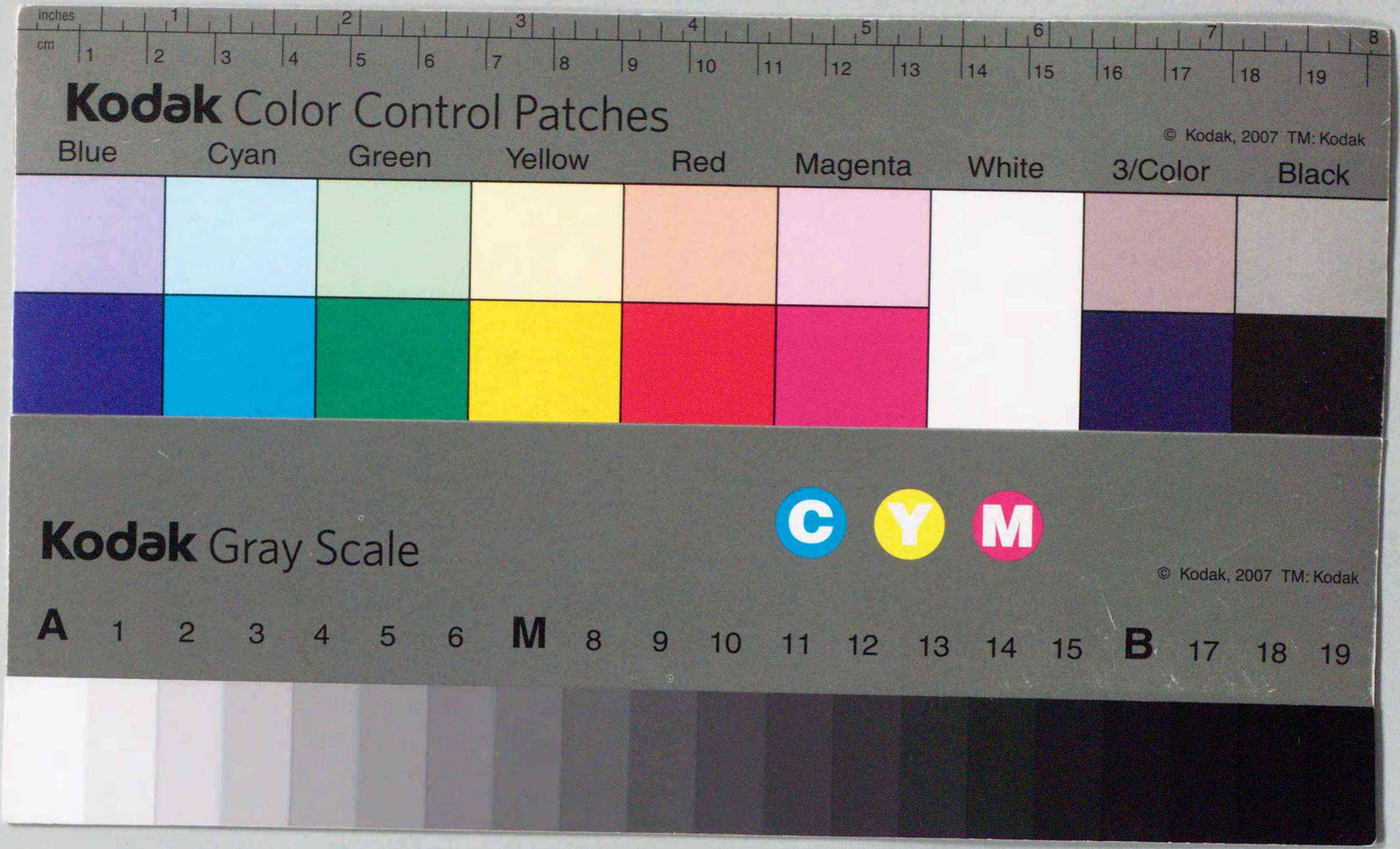



Fig. 6. Graph of the number of samples in the calculations of the correlations for the ensemble of states distributed uniformly in $s$ against the relative angle $\theta$. The value of the closing time $T$ is 0.133.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cm
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White
3/Color
Black
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