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Fast computation method for a Fresnel hologram using three-dimensional affine transformations in real space

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Calculating computer-generated holograms takes a tremendous amount of computation time. We propose a fast method for calculating object lights for Fresnel holograms without the use of a Fourier transform. This method generates object lights of variously shaped patches from a basic object light for a fixed-shape patch by using three-dimensional affine transforms. It can thus calculate holograms that display complex objects including patches of various shapes. Computer simulations and optical experiments demonstrate the effectiveness of this method. The results show that it performs twice as fast as a method that uses a Fourier transform. © 2009 Optical Society of America

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1. Introduction

Three-dimensional (3D) display systems have recently become an active area of research. Holography is an ideal 3D display technology because it fulfills human visual performance requirements. Light from an object (object light) is recorded and reconstructed using holographic interference and diffraction. Recording materials that record interference patterns are called holograms. A computer-generated hologram (CGH) is made by computer simulation, and it shows a 3D virtual scene. The interference patterns created by object lights that are simulated using wave propagation theory from virtual objects and a reference light must be calculated. A CGH does not require real objects and optical systems for the recording process and can display a moving image. However, the computation time to make a CGH is tremendous. The purpose of our research is to find a way to calculate CGHs at high speed.

The two main types of calculation method for CGHs are the point light methods and methods based on Fourier transforms; the details are described in Section 2. It is easy to calculate object lights from complex objects using a point light method, but it takes a tremendous amount of computation time.

The methods based on Fourier transforms calculate the propagation between an object plane and a hologram plane. The calculation is faster than that with a point light method because the object plane, which includes many points, is calculated using one-time processing and a fast Fourier transform (FFT) algorithm to speed up the Fourier transform calculation [1]. Although the FFT algorithm is faster, it still takes a significant amount of time because it is a complex algorithm and is not suited for parallel computations. Normally, a patch model, which describes the shape of objects using patches, is used along with this method, and each patch requires a two-dimensional (2D) FFT [2]. Therefore, a complex object including many patches requires much calculation time. Various methods have been proposed to reduce the calculation time. One of the more

promising approaches is a precalculation method that calculates the object light of a primitive object and creates an all-inclusive object light by combining primitive object lights [3–6]. Although this approach performs the calculation faster, methods that use the patch model still require a FFT because transformation and combination of object lights are performed in frequency space [5].

Another approach is to use transformation in real space (i.e., without a FFT). One proposed method transforms a precalculated object light from a point light and calculates an object light from another point light at a different distance [4]. Another proposed method uses a patch model [7]. The object lights from various patches are obtained by transforming only one precalculated object light from a patch. This proposed method includes transformation of the translations along the x and y axes of the distance along the z axis of the rotations about the x , y , and z axes and of the scalings along the x and y axes in the Fraunhofer region. These transforms are incomplete for achieving 3D affine transformation, which means they lack skew transformations. Ordinarily, triangle patches are used for the patch model because they cover the object surface without any surface gaps. When 3D affine transformation is achieved, an arbitrary triangle patch is constructed from only one basic triangle patch. Therefore, it is important to perform the entire 3D transformation to generate a CGH of an arbitrary object. Moreover, transformations in the Fresnel region are required to achieve a wide viewing zone and a wide visual field. We describe the skew transformations in real space that are missing from the 3D affine transformation. In addition, all the transformations are developed in the Fresnel region, and we show that these transformations are effective in the Fresnel region.

2. Methods for Calculating Computer-Generated Holograms

The light propagating from an object to the hologram plane must be calculated. This calculation is derived by using the Fresnel–Kirchhoff diffraction integral. When the object's distribution is $g(x, y, z)$ and the object light distribution on the hologram plane is $u(x_h, y_h)$ (Fig. 1), the Fresnel–Kirchhoff diffraction integral is expressed as

$$u(x_h, y_h) = \frac{j}{\lambda} \int \int \int_{-\infty}^{\infty} g(x, y, z) \frac{\exp(-jkr)}{r} dx dy dz, \quad (1)$$

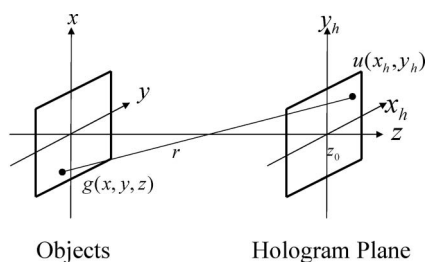


Fig. 1. Coordinate system for light propagation.

where $j(=\sqrt{-1})$ is the imaginary unit, $k(=2\pi/\lambda)$ is the wavenumber, r is the distance from a point on an object, $g(x, y, z)$, to a point on the hologram plane, $u(x_h, y_h)$. Therefore,

$$r = \{(x_h - x)^2 + (y_h - y)^2 + (z - z_0)^2\}^{1/2}. \quad (2)$$

The following two main computational methods, which are based on the Fresnel–Kirchhoff diffraction integral, are included in the calculation of CGHs.

A. Point Light Methods

In the point light methods, the faces of objects are considered to be covered with point light sources. The calculation order is $O(MN^2)$, where M is the number of point light sources and N is the number of pixels on each side of the hologram plane. It is easy to calculate object lights from complex objects by using the point light method. However, the number of point light sources is likely to be a significant amount because it takes several hundred point light sources per square millimeter to represent the object faces smoothly. Since the calculation time is proportional to the number of point light sources, it is a tremendous amount of time. The point light methods are simple and useful for parallel computation, so fast calculation methods using specialized hardware, a graphics processing unit (GPU), and a lookup table have been proposed [8–10].

B. Methods Based on Fast Fourier Transform

When an object is a plane and parallel to a hologram plane, a FFT can be used to calculate propagation as Fresnel diffraction or Fraunhofer diffraction. A FFT method can also be based on the angular spectrum of plane waves [11]. Here we used a method based on the angular spectrum of plane waves as the conventional method. Methods for calculating a plane not parallel to a hologram plane are described, and a patch model is used to model the objects.

1. Patch Model

A patch model is one of the more commonly used modeling methods. In this model, objects are covered with small surfaces without any gaps, and these small surfaces are called patches. In computer graphics, polygons are used as patches, and the model is called a polygon model. When curves are depicted by the patch model, a sufficient number of small patches is required. Here we use triangles as patches because their use makes it is easy to depict curves. The object light from a patch model is obtained as the sum of the object light from each patch. When the object light from a patch is u_i , the object light from patch model u is expressed as

$$u = \sum_{i=1}^P u_i, \quad (3)$$

where P is the number of patches. Use of the patch model prevents an increase in the calculation time by

increasing object complexity in comparison with the point light methods. These methods, however, require a 2D FFT, so the calculation time increases with the number of pixels. The calculation order of a 2D FFT is $O(N^2 \log N)$, so the calculation order of the propagation calculation is $O(PN^2 \log N)$. The details for Fresnel diffraction and Fraunhofer diffraction are described in Subsections 2.B.2 and 2.B.3.

2. Fresnel Diffraction

The diffraction when Fresnel approximation can be used is called Fresnel diffraction. Application of a Fresnel approximation changes Eq. (2) to

$$\begin{aligned} r &\approx z_0 + \frac{(x_h - x)^2}{2z_0} + \frac{(y_h - y)^2}{2z_0} \\ &= z_0 + \frac{x_h^2 + y_h^2}{2z_0} - \frac{x_h x + y_h y}{z_0} + \frac{x^2 + y^2}{2z_0}, \end{aligned} \quad (4)$$

where z_0 is the distance from an object plane to a hologram plane. When Eq. (4) is assigned to Eq. (1), the Fresnel diffraction can be expressed as

$$\begin{aligned} u(x_h, y_h) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \int \int_{-\infty}^{\infty} g(x, y) \\ &\quad \times \exp \left[-j \frac{\pi}{\lambda z_0} \{ (x_h - x)^2 \right. \\ &\quad \left. + (y_h - y)^2 \} \right] dx dy. \end{aligned} \quad (5)$$

3. Fraunhofer Diffraction

When the distance between a hologram plane and an object plane is much longer than the object size, Eq. (2) can be approximated as

$$r \approx z_0 + \frac{x_h^2 + y_h^2}{2z_0} - \frac{x_h x + y_h y}{z_0}. \quad (6)$$

In this case, Eq. (1) changes to

$$\begin{aligned} u(x_h, y_h) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \\ &\quad \times \exp \left(-jk \frac{x_h^2 + y_h^2}{2z_0} \right) \int \int_{-\infty}^{\infty} g(x, y) \\ &\quad \times \exp \left\{ j \frac{k}{z_0} (x_h x + y_h y) \right\} dx dy, \end{aligned} \quad (7)$$

which is the equation for Fraunhofer diffraction.

3. Overview of Our Method

We use the patch model as a modeling method for CGH calculation. In the patch model, objects consist of many patches, and the shapes of the patches differ. When the lights from these patches are calculated, a precalculated object light must be transformed

because object lights from individual patches also differ.

Our method comprises four steps.

1. Define a basic patch and calculate the object light from the patch (hereinafter referred to as basic object light).

2. Transform the basic object light in accordance with the translation, rotation, scale size, and skew of patches without a FFT, which is the 3D affine transformation in real space. When an object light from the i th patch is u_{hi} and the basic object light is u_b , u_{hi} is expressed as

$$u_{hi} = T_i[u_b], \quad (8)$$

where T_i is a transform function. The transform algorithms are described in Section 4.

3. Sum up the object light from each patch calculated in step 2. When the object light from the entire patch model is $u_{h \text{ all}}$, $u_{h \text{ all}}$ is expressed as

$$u_{h \text{ all}} = \sum_{i=1}^P u_{hi}. \quad (9)$$

4. Calculate a reference light using the conventional method. When the interference pattern is I and the reference light is R , I is expressed as

$$I = |u_{h \text{ all}} + R|^2. \quad (10)$$

Figures 2 and 3 show the algorithms for making holograms using our method and the conventional method, respectively. The latter requires two 2D FFTs and an input object image has to be made for each patch. On the other hand, our method does not require a FFT, which requires a tremendous amount of calculation time, and also does not require an input object image for each patch.

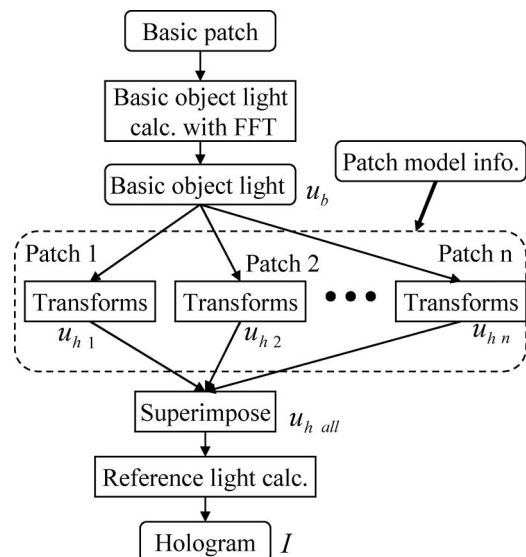


Fig. 2. Algorithm for our method.

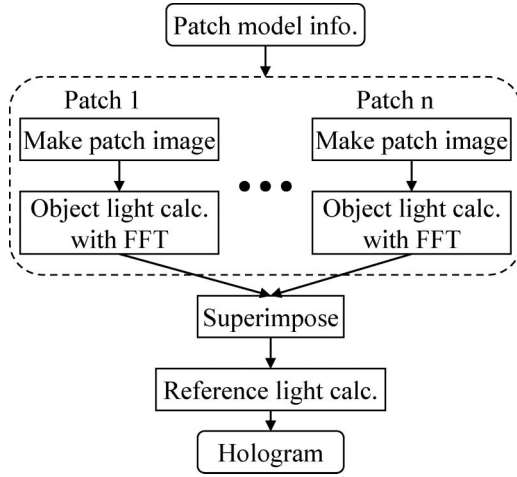


Fig. 3. Algorithm for the conventional method.

In our method, the calculation order of the transforms is $O(N^2)$, and the calculation order of all the calculations is $O(PN^2)$. Using our method should speed up the CGH calculation because the calculation order of the conventional method is $O(PN^2 \log N)$. If 3D affine transformation is achieved, the object light of the arbitrary triangle patches can be calculated from only one basic object light. When the basic object light is calculated beforehand, a FFT is not necessary to calculate a hologram.

4. Transforms

Here we discuss each transform of the 3D affine transformation in real space. First, each transform is derived in the Fraunhofer region except for the slide and rotation transform. Then the transforms are extended to the Fresnel region. In the Fraunhofer region, when the distance from a basic patch to a hologram is z_0 , the basic object light, $u_b(x_b, y_b)$, is derived by substituting $z = z_0$, $x_h = x_b$, and $y_h = y_b$ into Eq. (7). By the same token, in the Fresnel region $u_b(x_b, y_b)$ is derived by substituting $z = z_0$, $x_h = x_b$, and $y_h = y_b$ into Eq. (5). Equation (5) is then rewritten as

$$\begin{aligned}
 u_b(x_b, y_b) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \int \int_{-\infty}^{\infty} g(x, y) \\
 &\quad \times \exp \left[-j \frac{\pi}{\lambda z_0} \{ (x_b - x)^2 + (y_b - y)^2 \} \right] dx dy \\
 &= \frac{j}{\lambda z_0} \exp(-jkz_0) \exp \left\{ -jk \frac{x_b^2 + y_b^2}{2z_0} \right\} \\
 &\quad \times \int \int_{-\infty}^{\infty} g(x, y) \exp \left(-jk \frac{x^2 + y^2}{2z_0} \right) \\
 &\quad \times \exp \left\{ jk \frac{(x_b x + y_b y)}{z_0} \right\} dx dy. \quad (11)
 \end{aligned}$$

A. Slide and Rotation Transform

The slide and rotation transform represents the translations along the x and y axes and the rotation

about the z axis. When the objective patch is translated and rotated in a plane parallel to a basic patch, the transform is achieved by resampling the basic object light at the changed location. The transform is expressed as

$$\begin{aligned}
 u_h(x_h, y_h) &= u_b(x_h \cos \theta + y_h \sin \theta - \Delta x, y_h \cos \theta \\
 &\quad - x_h \sin \theta - \Delta y), \quad (12)
 \end{aligned}$$

where u_h is the object light from the objective patch and Δx , Δy , and θ are the transform amounts shown in Fig. 4.

B. Distance Transform

The distance transform is the translation along the z axis. The distance from an objective patch to a hologram is $z_0 + \Delta z$, as shown in Fig. 5.

1. Fraunhofer Region

When $z = z_0 + \Delta z$ is assigned to Eq. (7), an object light on a hologram plane from an objective patch can be expressed as

$$\begin{aligned}
 u(x_h, y_h) &= \frac{j}{\lambda(z_0 + \Delta z)} \exp\{-jk(z_0 + \Delta z)\} \\
 &\quad \times \exp \left\{ -jk \frac{(x_h^2 + y_h^2)}{2(z_0 + \Delta z)} \right\} \int \int_{-\infty}^{\infty} g(x, y) \\
 &\quad \times \exp \left\{ j \frac{k}{z_0 + \Delta z} (x_h x + y_h y) \right\} dx dy \\
 &= \frac{j}{\lambda z_0} \frac{z_0}{z_0 + \Delta z} \exp(-jkz_0) \exp(-jk\Delta z) \\
 &\quad \times \exp \left\{ -jk \frac{(x_h^2 + y_h^2)}{2(z_0 + \Delta z)} \right\} \int \int_{-\infty}^{\infty} g(x, y) \\
 &\quad \times \exp \left[j \frac{k}{z_0} \left\{ \frac{z_0}{z_0 + \Delta z} (x_h x + y_h y) \right\} \right] dx dy \\
 &= \frac{z_0}{z_0 + \Delta z} \exp \left[-jk \left\{ \Delta z + \frac{x_h^2 + y_h^2}{2(z_0 + \Delta z)} \right. \right. \\
 &\quad \left. \left. - \frac{z_0(x_h^2 + y_h^2)}{2(z_0 + \Delta z)^2} \right\} \right] \frac{j}{\lambda z_0} \exp(-jkz_0) \\
 &\quad \times \exp \left(-jk \frac{x_b^2 + y_b^2}{2z_0} \right) \int \int_{-\infty}^{\infty} g(x, y) \\
 &\quad \times \exp \left\{ j \frac{k}{z_0} (x_b x + y_b y) \right\} dx dy \\
 &= C_1 \exp(-jkL_1) u_b(x_b, y_b), \quad (13)
 \end{aligned}$$

where

$$\begin{aligned}
 x_b &= \frac{z_0}{z + \Delta z} x_h, \quad y_b = \frac{z_0}{z + \Delta z} y_h, \\
 L_1 &= \Delta z + \frac{x_h^2 + y_h^2}{2(z_0 + \Delta z)} - \frac{z_0(x_h^2 + y_h^2)}{2(z_0 + \Delta z)^2},
 \end{aligned}$$

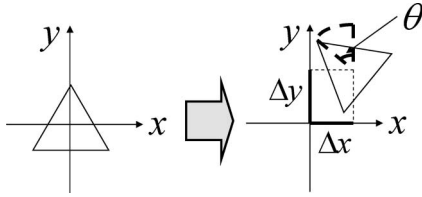


Fig. 4. Slide and rotation transform.

and L_1 is the approximate distance from (x_b, y_b) to (x_h, y_h) , as shown in Fig. 4. The distance transform represents the product of the phase at distance L_1 and $u_b(x_b, y_b)$.

2. Fresnel Region

When $z = z_0 + \Delta z$ is assigned to Eq. (5), an object light on a hologram plane from an objective patch can be expressed as

$$\begin{aligned}
 u(x_h, y_h) &= \frac{j}{\lambda(z_0 + \Delta z)} \exp\{-jk(z_0 + \Delta z)\} \\
 &\times \exp\left\{-jk \frac{(x_h^2 + y_h^2)}{2(z_0 + \Delta z)}\right\} \int \int_{-\infty}^{\infty} g(x, y) \\
 &\times \exp\left\{-jk \frac{x^2 + y^2}{2(z_0 + \Delta z)}\right\} \\
 &\times \exp\left\{j \frac{k}{z_0 + \Delta z} (x_h x + y_h y)\right\} dx dy \\
 &= \frac{z_0}{z_0 + \Delta z} \exp(-jkL_1) \frac{j}{\lambda z_0} \exp(-jkz_0) \\
 &\times \exp\left\{-jk \frac{x_b^2 + y_b^2}{2z_0}\right\} \int \int_{-\infty}^{\infty} g(x, y) \\
 &\times \exp\left\{jk \frac{x^2 + y^2}{2} \frac{\Delta z}{z_0(z_0 + \Delta z)}\right\} \exp\left(jk \frac{x^2 + y^2}{2}\right) \\
 &\times \exp\left\{j \frac{k}{z_0} (x_b x + y_b y)\right\} dx dy = \int \int_{-\infty}^{\infty} g'(x, y) \\
 &\times \exp\left(jk \frac{x^2 + y^2}{2}\right) \exp\left\{j \frac{k}{z_0} (x_b x + y_b y)\right\} dx dy,
 \end{aligned} \tag{14}$$

where

$$g'(x, y) = g(x, y) \text{error}_1(x, y),$$

$$\text{error}_1(x, y) = \exp\left\{jk \frac{x^2 + y^2}{2} \frac{\Delta z}{z_0(z_0 + \Delta z)}\right\}.$$

Error₁ is the error term for the distance transform. The details of this error term and those of the following transforms are described in Subsection 4.F. If this error term is allowed, $u_h(x_h, y_h)$ in the Fresnel region can be expressed as

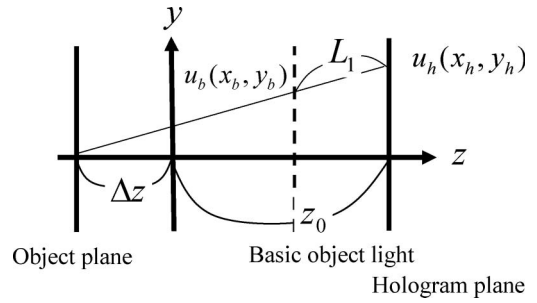


Fig. 5. Distance transform.

$$\begin{aligned}
 u_h(x_h, y_h) &\approx \frac{z_0}{z_0 + \Delta z} \exp(-jkL_1) \frac{j}{\lambda z_0} \exp(-jkz_0) \\
 &\times \exp\left\{-jk \frac{x_b^2 + y_b^2}{2z_0}\right\} \int \int_{-\infty}^{\infty} g(x, y) \\
 &\times \exp\left(jk \frac{x^2 + y^2}{2}\right) \\
 &\times \exp\left\{j \frac{k}{z_0} (x_b x + y_b y)\right\} dx dy \\
 &= C_1 \exp(-jkL_1) u_b(x_b, y_b).
 \end{aligned} \tag{15}$$

C. Tilt Transform

The tilt transform represents the rotations about the x and y axes. Here we derived the transforms (the tilt, scaling, and skew transforms) from one axis, but the transforms of another axis are derived in the same way. When an objective patch is rotated using angle ϕ about the y axis, the object distribution changes to $g(x \cos \phi, y, -x \sin \phi)$ and z changes to $z_0 + x \sin \phi$ in integral computation, as shown in Fig. 6. The reason z changes to $z_0 + x \sin \phi$ is that the distance between the object and the hologram plane changed. When $g(x \cos \phi, y, -x \sin \phi)$ and $z = z_0 + x \sin \phi$ in integral computation are assigned to Eq. (5), the object light from the objective patch can be expressed as

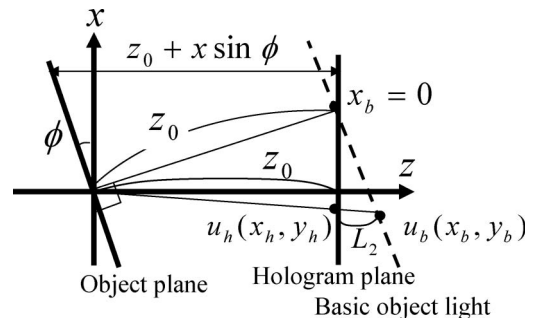


Fig. 6. Tilt transform.

$$\begin{aligned}
u_h(x_h, y_h) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \exp\left(-jk \frac{x_h^2 + y_h^2}{2z_0}\right) \\
&\times \int \int_{-\infty}^{\infty} g(x \cos \phi, y) \\
&\times \exp\left\{-jk \frac{x^2 + y^2}{2(z_0 + x \sin \phi)}\right\} \\
&\times \exp\left\{jk \frac{(x_h x + y_h y)}{z_0 + x \sin \phi}\right\} dx dy \\
&\approx C_2 \exp(-jkL_2) u_b(x_b, y_b),
\end{aligned} \quad (16)$$

where

$$\begin{aligned}
x_b &\approx \frac{x_h - z_0 \sin \phi}{2 - \cos \phi}, \quad y_b \approx \frac{x_0}{z_0(2 - \cos \phi)} y_h, \\
L_2 &\approx x \sin \phi + x_h^2 \left\{ 1 - \frac{1}{(2 - \cos \phi)^2} \right\} \\
&+ \frac{-2x_h + z_0 \sin \phi + z_0 \sin^2 \phi}{(2 - \cos \phi)^2} \\
&+ y_h^2 \left\{ 1 - \frac{z_0^2}{z_0^2(2 - \cos \phi)^2} \right\}, \\
\text{error}_2 &\approx \exp \left[-\frac{jk}{2z_0(z_0 + x \tan \phi)} \right. \\
&\times \left. \left\{ -x^3 + z_0 x^2 \left(\frac{1}{\cos^2 \phi} - 1 \right) - xy^2 \tan \phi \right\} \right].
\end{aligned}$$

In the same way as for the distance transform, L_2 is the approximate distance from (x_b, y_b) to (x_h, y_h) . The tilt transform represents the product of the phase at distance L_2 and $u_b(x_b, y_b)$. More precisely, u_h includes the error term for the tilt transform, error_2 .

D. Scaling Transform

The scaling transform represents the scalings along the x and y axes. When an objective patch is scaled by scaling factor R_x , as shown in Fig. 7, the object distribution changes to $g(\frac{x}{R_x}, y)$. When $g(\frac{x}{R_x}, y)$ is assigned to Eq. (5), the object light from the objective patch can be expressed as

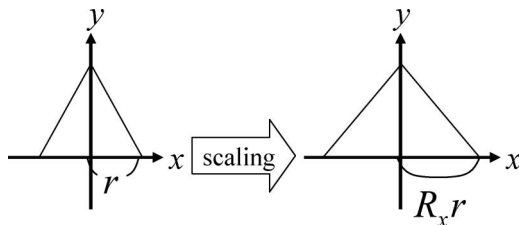


Fig. 7. Scaling transform.

$$\begin{aligned}
u_h(x_h, y_h) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \exp\left\{-jk \frac{x_h^2 + y_h^2}{2z_0}\right\} \\
&\times \int \int_{-\infty}^{\infty} g\left(\frac{x}{R_x}, y\right) \exp\left(-jk \frac{x^2 + y^2}{2z_0}\right) \\
&\times \exp\left\{jk \frac{(x_h x + y_h y)}{z_0}\right\} dx dy \\
&\approx C_3 \exp(-jkL_3) u_b(x_b, y_b),
\end{aligned} \quad (17)$$

where

$$\begin{aligned}
x_b &= R_x x_h, \quad y_b = y_h, \quad L_3 = \frac{x_h^2(1 - R_x^2)}{2z_0}, \\
\text{error}_2 &= \exp\left\{jk \frac{x^2}{2z_0} (1 - R_x^2)\right\},
\end{aligned}$$

and u_h includes the error term for scaling transform error_3 .

E. Skew Transform

The skew transform represents the skew along the x and y axes. When an objective patch is skewed by skew factor S_x , as shown in Fig. 8, the object distribution changes to $g(x - yS_x, y)$. When $g(x - yS_x, y)$ is assigned to Eq. (7), the object light from the objective patch can be expressed as

$$\begin{aligned}
u_h(x_h, y_h) &= \frac{j}{\lambda z_0} \exp(-jkz_0) \exp\left\{-jk \frac{x_h^2 + y_h^2}{2z_0}\right\} \\
&\times \int \int_{-\infty}^{\infty} g(x - S_x y, y) \exp\left(-jk \frac{x^2 + y^2}{2z_0}\right) \\
&\times \exp\left\{jk \frac{(x_h x + y_h y)}{z_0}\right\} dx dy \\
&\approx C_4 \exp(-jkL_4) u_b(x_b, y_b),
\end{aligned} \quad (18)$$

where

$$\begin{aligned}
x_b &= x_h, \quad y_b = y_h + S_x x_h, \\
L_4 &= -\frac{S_x x_h (S_x x_h + 2y_h)}{2z_0}, \\
\text{error}_4 &= \exp\left\{-jk \frac{S_x y (S_x y + 2x)}{2z_0}\right\}.
\end{aligned}$$

In the same way, u_h includes the error term for skew transform error_4 .

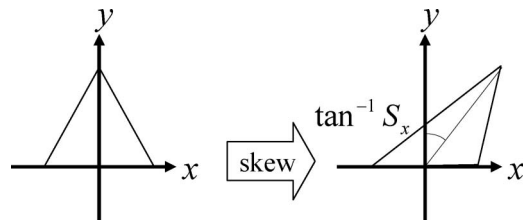


Fig. 8. Skew transform.

Table 1. Parameters for the Experiments

Basic Object Light	
Basic patch image	Equilateral triangle, 500 pixels a side
No. of pixels	4096 × 4096
Distance	$z_0 = 0.15$ (m)
Hologram	
No. of pixels	2048 × 2048
Common Parameters	
Sampling pitch	6.35×6.35 (μm)
Wavelength	632 (nm)

F. Transform Error Terms

The error terms mean that a phase is added to the basic patch. In other words, they do not affect the location and shape of the basic patch; they affect the phase terms of the basic patch, which means that our transforms change the reflectance property of the basic patch. Here we calculate the basic patch as a perfectly diffuse surface, so, in theory, error terms are allowed in the Fresnel and Fraunhofer regions.

G. Combinations of Transforms

These transforms are used in combination by using one-time processing, so various transforms are achieved at high speed. When a triangle is used as a basic patch, the object light from an arbitrary patch is calculated by using only one basic object light.

5. Experimental Results

We carried out computer simulations and optical experiments to determine the effectiveness of our transforms. The parameters of these experiments are listed in Table 1. For the optical experiments, the holograms were printed on a transparent sheet with a black-and-white binary image. A red light-emitting diode was used to reconstruct the holograms, and all the viewpoints were located across the hologram plane from the objects. The basic object light was calculated as a perfectly diffuse surface.

A. Limitation of Transforms

In theory, transforms in the Fresnel and Fraunhofer regions are allowed, as discussed in Subsection 4.F. The transforms, however, are restricted by the size of the basic object light. In other words, the basic object

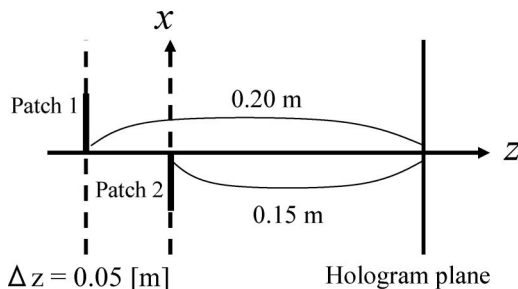


Fig. 9. Object locations from slide and rotation transform and distance transform.

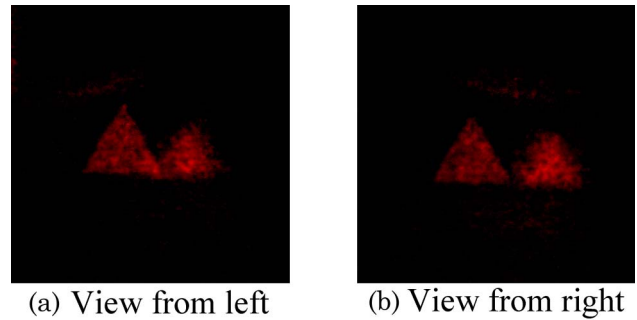


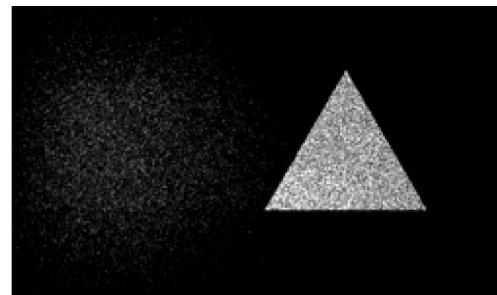
Fig. 10. (Color online) Results of distance transform.

light must be larger than the hologram because, when an objective patch is transformed, the object light on the hologram plane might refer to the basic object light data outside the precalculated area. If the basic object light does not cover the area referred to by the hologram, regions of the hologram data will disappear. To prevent such disappearance, the transform parameters must be limited.

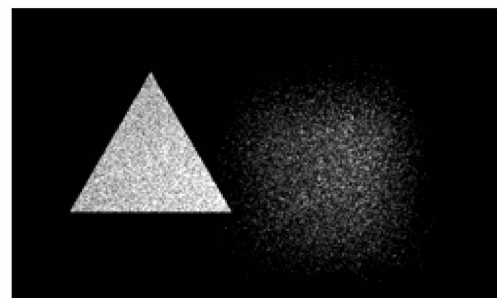
In the experiments described below, the basic object light was twice as large as the hologram on a side, and thus four times larger than the area. When the parameters were set as shown in Table 1, we completely avoid disappearing data by using the range of parameters described below:

$$-0.0065_{[m]} \leq \Delta x \leq 0.0065_{[m]},$$

$$-0.0065_{[m]} \leq \Delta y \leq 0.0065_{[m]},$$



(a) Propagation distance was 0.20 m



(b) Propagation distance was 0.15 m

Fig. 11. Computer-simulated distance transform.

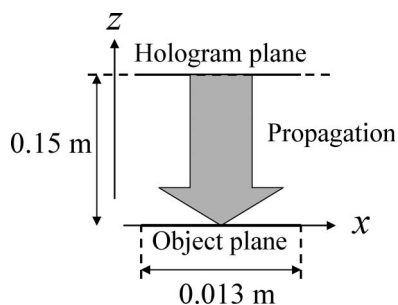


Fig. 12. Propagation simulation.

$$-0.075_{[m]} \leq \Delta z,$$

$$-2.4^\circ \leq \phi \leq 2.4^\circ,$$

$$0.0 < R_x \leq 2.0,$$

$$-1.0 \leq S_x \leq 1.0.$$

When the parameters that meet the ranges are used, (x_b, y_b) of the transform expressions do not exceed the size of the basic object light. In actuality, a larger range of parameters is allowed because the hologram does not require the object light for the whole hologram plane. However, when the transforms are combined, the range of parameters is narrowed.

B. Results of Slide and Rotation Transform and Distance Transform

First, we carried out an optical experiment to determine the effectiveness of the slide and rotation transform and the distance transform. Patch 1, which was transformed by the slide and rotation transform, and patch 2, which was transformed by the slide and rotation transform and the distance transform, were arranged as shown in Fig. 9. Figure 10 shows the results of the optical experiments that focused on patch 2. When viewed from the left [see Fig. 10(a)], the two patches overlapped. In contrast, when viewed from the right [Fig. 10(b)], they did not overlap. This change in the view with the change in viewpoints demonstrates the effectiveness of the transforms.

We conducted computer simulations to confirm the correctness of the changed distance. The object lights

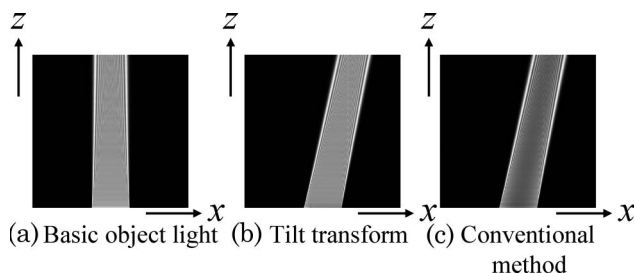


Fig. 13. Results of tilt transform by propagation simulation as in Fig. 12.

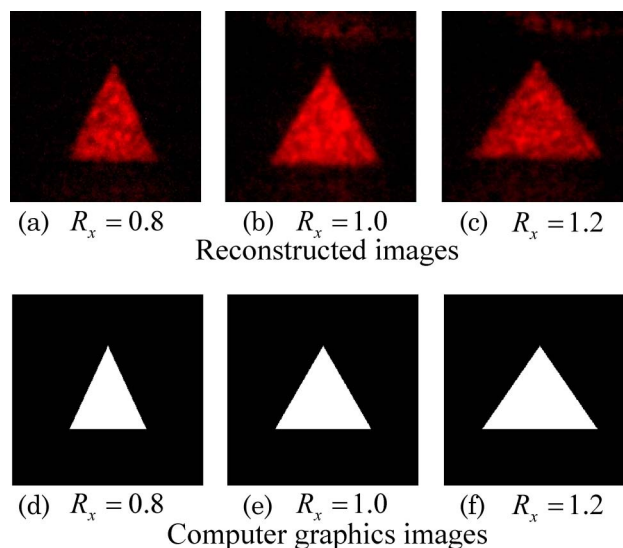


Fig. 14. (Color online) Results of scaling transform.

on the hologram plane were propagated back to the positions of patch 1 and patch 2, and the amplitude images were shown. Figure 11(a) shows the result of propagating back to the position of patch 1 (i.e., the propagation distance was 0.20 m), and Fig. 11(b) shows the result of propagating back to the position of patch 2 (i.e., the propagation distance was 0.15 m). In Fig. 11(a), patch 1 is in focus and patch 2 is not. In contrast, only patch 2 is in focus in Fig. 11(b), meaning that the changed distance was correct.

C. Results of Tilt Transform

Computer simulations were conducted to determine the effectiveness of the tilt transform. The propagation of the object light was simulated as in Fig. 12. The reason for carrying out computer simulations is that it is difficult to determine the effectiveness of the tilt transform using optical experimentation.

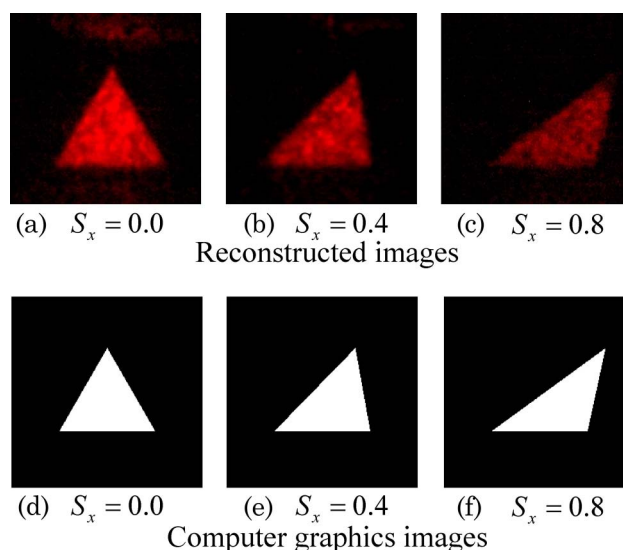


Fig. 15. (Color online) Results of skew transform.

the transformations by conducting optical experiments and computer simulations. Our method does not require a FFT for each patch because all the transformations are achieved in real space, and it performs twice as fast as the conventional method. All the transformations in our method are simple and useful for parallel computation. Speed can be further increased by using specialized hardware such as a GPU and a lookup table. Our method will be more effective for future display devices with more pixels. We plan to calculate a CGH from complex objects including tilted planes.

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