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学 位 論 文 内 容 の 要 旨
Abstract of Doctoral Thesis

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Title of Doctoral Thesis

The Lightcone Dualities for Submanifolds in the Sphere
(球面内の部分多様体の光錐的対称性)

In this thesis we study submanifolds in the unit sphere in the framework of the theory of Legendrian dualities between pseudo-spheres in Minkowski space. The subjects include the curves in the unit 2-sphere, the curves in the unit 3-sphere and the hypersurfaces in the unit n -sphere. The unit sphere can be naturally embedded in the lightcone and the de Sitter space. We respectively consider the Legendrian dualities between the lightcones, and between the lightcone and the de Sitter space. Then we obtained the lightcone dual hypersurface of a submanifold in the unit sphere embedded in the lightcone and the lightcone dual hypersurfaces of a submanifold in the unit sphere embedded in the de Sitter space. We studied the singular sets of these lightcone dual hypersurfaces which are called the focal surfaces respectively. We have the canonical projection from the lightcone to the unit sphere embedded in the lightcone. For a curve in the unit 2-sphere, the projections of the critical sets of the two kinds of lightcone dual hypersurfaces to the unit sphere in the lightcone are the same and they are equal to the image of the spherical evolute of the curve in unit 2-sphere in the Euclidean sense. For the curve in the unit 3-sphere, however, this fact does not hold. The projections of these focal surfaces to the unit 3-sphere in the lightcone are different surfaces. Moreover, the projections of the critical sets of lightcone focal surfaces are the same and equal to the spherical evolutes of the curve in the unit 3-sphere. Therefore, the situation of curves in the unit 3-sphere is quite different from that of curves in the unit 2-sphere.

In order to clarify the difference of the above two cases, we considered hypersurfaces in the unit n -sphere as a general case of curves in the unit 2-sphere. We have shown that the projections of lightcone focal sets of those two lightcone dual hypersurfaces are the same and they are equal to the image of the spherical evolute of the hypersurface in the unit n -sphere in the Euclidean sense. This means that the situation for the hypersurface case is the same as the case of curves in the unit 2-sphere. For the hypersurfaces in the unit n -sphere, we studied the contact of hypersurfaces with the parabolic $(n-1)$ -spheres and parabolic n -hyperquadric as an application of the theory of Legendrian singularities. As a consequence, we show that these contact types are uniquely determined by the hypersurface in the unit n -sphere. This is the geometric reason why the projection of the lightcone focal surfaces of the different lightcone duals are the same. We also gave the classifications of the singularities of these lightcone dual hypersurfaces for surfaces in the unit 3-sphere.