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RELIABILITY ESTIMATE OF RC STRUCTURES IN A MARINE ENVIRONMENT USING MARKOV MODEL AND SEQUENTIAL MONTE CARLO SIMULATION

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ABSTRACT

Reinforced concrete (RC) structures in a marine environment deteriorate with time due to chloride-induced corrosion of reinforcing bars. In this paper, a methodology for quantifying life-cycle reliability of RC structures subjected to chloride attack is presented. The deterioration process of bridge components regarding the transition between the condition states is modeled as a Markov process. In addition, a procedure to obtain the transition probability matrix at time *t* after construction updated by the Sequential Monte Carlo Simulation (SMCS) is indicated. Transition probability matrix could be updated to be consistent with the observational information. The methodology is illustrated on an existing RC superstructures of wharves located in a splash zone. The effect of the updating on the life-cycle reliability estimate of RC wharves is discussed in this paper.

Keywords: Reliability, Markov model, SMCS, RC structure, Chloride attack.

1. INTRODUCTION

For structures located in a moderately or highly aggressive environment, multiple environmental and mechanical stressors lead to deterioration of structural performance. Such deterioration will reduce the service life of structures and increase the life-cycle cost of maintenance actions. Various environmental stressors affect the degradation mechanisms of structures. However, the effects of these stressors are difficult to predict, as these effects vary in time and space. Because of the presence of such uncertainties, long-term structural performance must be predicted based on probabilistic concepts and methods, and life-cycle reliability assessment methodologies must be established. (Ellingwood 2005; Frangopol 2011; Mori and Ellingwood 1993)

Markov chains are commonly used for performance assessment of deteriorating components and systems. The rate of transition from one state to another is constant over time for homogeneous

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Markov chains. PARI (2007), Kato (2009) and Furuya et al. (2012) applied the Markov model to the life-cycle reliability estimate of RC superstructures of wharves. Compared with reliability analysis of RC structures using hazard associated with airborne chloride and deterioration model of rebars (e.g. Akiyama et al. 2010), Markov model provides significant time efficiency in life-cycle reliability analysis where many iterations are required.

Failure probability of RC structures in an aggressive environment depends on many aleatory and epistemic uncertainties. The transition probability matrix used in Markov model depends on the evaluation of hazard assessment of airborne chloride, corrosion process of steel bars, and deterioration of structural performance. The model uncertainties associated with the estimation of the transition probability matrix could be very large if there are only information on design condition of RC structures analyzed. For existing structures it is possible to reduce epistemic uncertainties using inspection results (Akiyama et al. 2010; Yoshida 2009).

In this study, a computational procedure for estimating the life-cycle reliability of RC wharves located in a splash zone is presented. The Markov model and SMCS is used in conjunction with time-dependent condition state probability for RC structures. Illustrative example is presented.

2. LIFE-CYCLE RELIABILITY ESTIMATE BASED ON MARKOV MODEL

2.1. Markov model

The rate of transition from one state to another assumed to be constant over time for homogeneous Markov chains. The probability of moving from any given state k-1 to state k on the next time interval is called the transition probability p_k. Considering a bridge component with five condition states (i.e. k=0, 1, 2, 3, 4, and 5), the transition probability matrix that can be used in the prediction of the component performance is

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 & 0 & 0 & 0 \\ p_1 & 1-p_2 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 1-p_3 & 0 & 0 & 0 \\ 0 & 0 & p_3 & 1-p_4 & 0 & 0 \\ 0 & 0 & 0 & p_4 & 1-p_5 & 0 \\ 0 & 0 & 0 & 0 & p_5 & 1 \end{pmatrix}^t \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f_0(p_1,t) \\ f_1(p_1,p_2,t) \\ f_2(p_1,p_2,p_3,t) \\ f_3(p_1,p_2,p_3,p_4,t) \\ f_4(p_1,p_2,p_3,p_4,p_5,t) \\ f_5(p_1,p_2,p_3,p_4,p_5,t) \end{pmatrix} \quad (1)$$

In this study, the deterioration states of RC structures due to chloride-induced corrosion of

reinforcing bars are classified according to the criteria defined by JSCE (2001).

2.2. Transition probability

In this study, transition probability is determined based on survey results of existing RC structures in a marine environment. Komure et al. (2002) reported the relationship between transition probability and the distance from the sea level as shown in Figure 1. Assuming that transition probability is independent of conditional states (i.e. p_1=p_2=...=p_5), p_i is provided by

$$p_1 = p_2 = p_3 = p_4 = p_5 = 0.14 \cdot e^{-0.6 \cdot H} \tag{2}$$

where H = distance from the surface of the sea.

When model uncertainty associated with the estimation of transition probability p_i is taken into consideration, p_i is provided by

$$p_i = x_i \cdot 0.14 \cdot e^{-0.6 \cdot H} \tag{3}$$

where x_i = model uncertainties associated with the estimation of p_i. x_i is modeled as a lognormal variable.

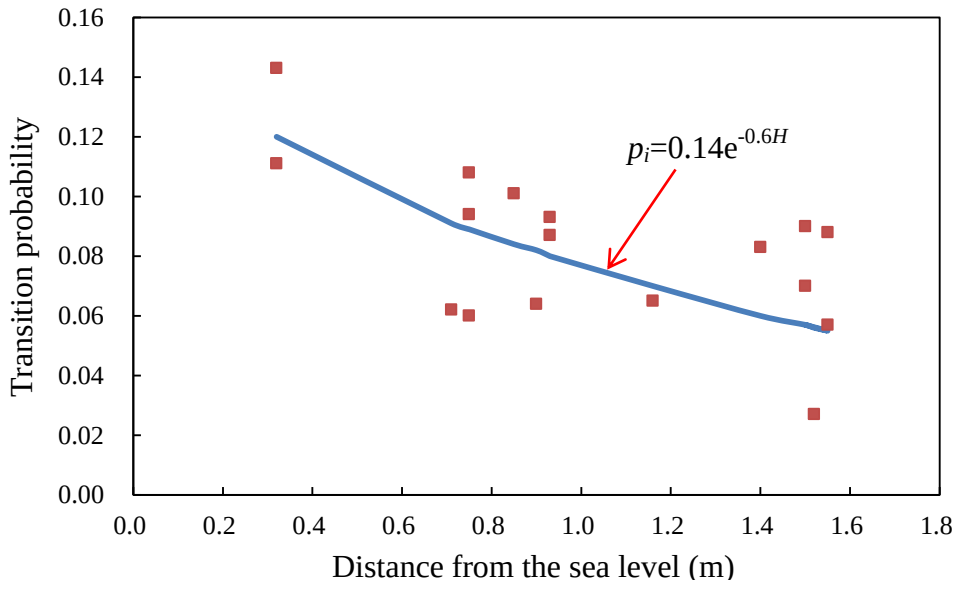


Figure 1: Relationship between transition probability and distance from the sea level

3. SEQUENTIAL MONTE CARLO SIMULATION (SMCS)

The state space model consists of two processes, the time updating process and the observation updating process. The time updating process is the one step ahead prediction based on the information at the (k-1)-th step. The predicted vector is

$$x_{k/k-1} = F(x_{k-1/k-1}, w_k) \tag{4}$$

where w_k is the system noise represented by the noise involved in the prediction process. It is assumed that observation information z_k is a function H of state vector $x_{k/k}$ and observation noise v_k as

$$z_k = H(x_{k/k}, v_k) \tag{5}$$

The PDF of these noises are assumed known and independent. The algorithm based on Monte Carlo Simulation (MCS) starts by assuming samples drawn from the distribution at (k-1)-th step

$$x_{k-1/k-1}^{(j)} \sim p(x_{k-1} \mid Z_{k-1}), \qquad j = 1,....n \tag{6}$$

$$Z_{k-1} = (z_1, z_2, ..., z_{k-1}) \tag{7}$$

The superscript (j) denotes the generated j-th sample realization. The PDF is approximately expressed by the samples with Dirac delta function δ as

$$p(x_{k-1} \mid Z_{k-1}) \cong \frac{1}{n}\sum_{j=1}^{n} \delta(x_{k-1} - x_{k-1/k-1}^{(j)}) \tag{8}$$

The above approximation form of PDF is called empirical PDF. The samples of the k-th step before observation updating are obtained by simply substituting them into state Eq. (4)

$$x_{k/k-1}{}^{(j)} = F(x_{k-1/k-1}{}^{(j)}, w_k{}^{(j)}) \tag{9}$$

The empirical PDF of the k-th step before updating is similarly estimated by the sample realization

$$p(x_k \mid Z_{k-1}) \cong \frac{1}{n}\sum_{j=1}^{n} \delta(x_k - x_{k/k-1}^{(j)}) \tag{10}$$

The PDF after updating is

$$\begin{aligned} p(x_k|Z_k) &= p(x_k|z_k, Z_{k-1}) \\ &= \frac{p(x_k, z_k|Z_{k-1})}{p(z_k|Z_{k-1})} \\ &= \frac{p(z_k|x_k, Z_{k-1}) \cdot p(x_k|Z_{k-1})}{\int p(z_k|x_k, Z_{k-1}) \cdot p(x_k|Z_{k-1}) \cdot dx_k} \end{aligned} \tag{11}$$

Substituting Eq. (10) into Eq. (11) and using the property of a delta function results in

$$p(x_k|Z_k) = \sum_{j=1}^{n}\left[\frac{q_k^{(j)}}{\sum_{i=1}^{n} q_k^{(j)}}\right]\cdot\delta(x_k - x_{k/k-1}^{(j)})$$
$$= \sum_{j=1}^{n} a_k^{(j)}\cdot\delta(x_k - x_{k/k-1}^{(j)}) \quad (12)$$

where

$$q_k^{(j)} = p(z_k \mid x_{k/k-1}^{(j)}) \quad (13)$$

$$a_k^{(j)} = \frac{q_k^{(j)}}{\sum_{i=1}^{n} q_k^{(i)}} \quad (14)$$

The term $a_k^{(j)}$ is the weight (likelihood ratio) of sample j. When a new observation is available, the weights are re-calculated and the approximate posterior PDF is sequentially updated.

4. ILLUSTRATIVE EXAMPLE

Taniguchi et al. (2004) reported the condition state rate of existing RC superstructures of wharves in a splash zone at 16 and 22 years after construction as listed in Table 1. To minimize the differences between estimated condition state probability using Eq. (1) and measured condition state rate in Table1, transition probability provided by Eq. (2) is determined.

In SMCS, the number of samples is set to 10,000, because computational results are almost the same if the number of samples is more than 10,000. The observation equation (i.e. Eq. (15)) based on the observation date is

Table 1: Inspection data of the existing RC structure (Taniguchi et al. 2004)

	Condition state rate					
	0	1	2	3	4	5
16 years	0.42	0.18	0.30	0.08	0.02	0.00
22 years	0.40	0.15	0.32	0.11	0.02	0.00

$$\begin{pmatrix} Z_0 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} + \begin{pmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix}$$

$$= \begin{pmatrix} f_0(p_1,t) \\ f_1(p_1,p_2,t) \\ f_2(p_1,p_2,p_3,t) \\ f_3(p_1,p_2,p_3,p_4,t) \\ f_4(p_1,p_2,p_3,p_4,p_5,t) \\ f_5(p_1,p_2,p_3,p_4,p_5,t) \end{pmatrix} + \begin{pmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix} \quad (15)$$

where Z_i is the inspection results of condition state i, X_i is the prediction of each condition state i and v_i is the observation noises of condition state i.

Figure 2 shows the comparisons of probability of being in condition state i (i=0, 1, 2, 3, 4, 5) provided by inspection results as shown in Table 1, Eq. (1) at t=22 years, and proposed model using SMCS at t=22 years. The difference between inspection results and predictions from proposed model using SMCS is much smaller than that between inspection results and Eq. (1) at t=22 years. Proposed model using SMCS could perform a more accurate life-cycle reliability estimate of RC superstructures of wharves. Figure 3 shows the effect of updating on the probabilistic density function (PDF) of model uncertainty associated with the estimation of transition probability. Coefficient of variation after updating is smaller than that without updating.

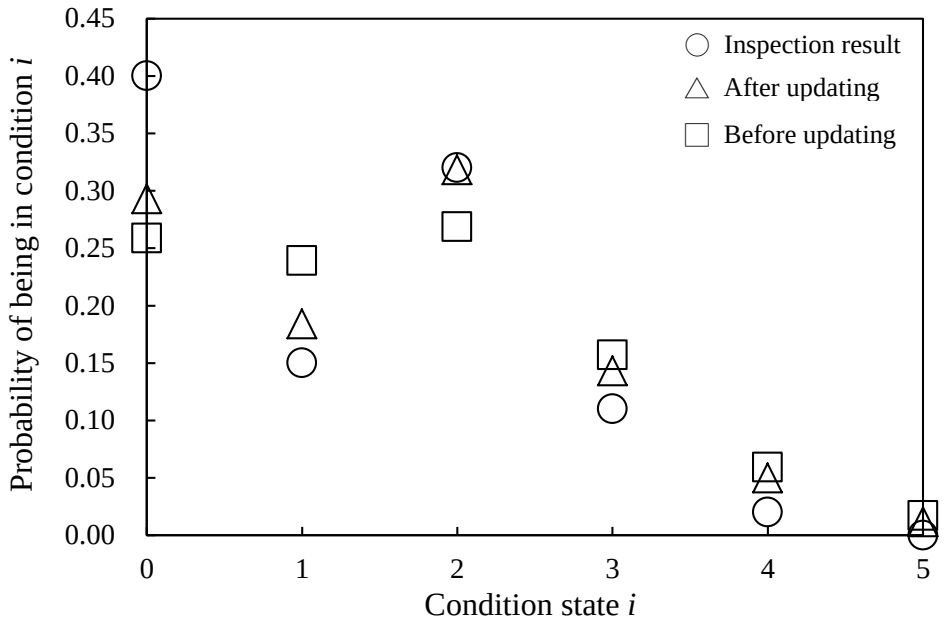


Figure 2: Predictions of condition state probability using proposed method

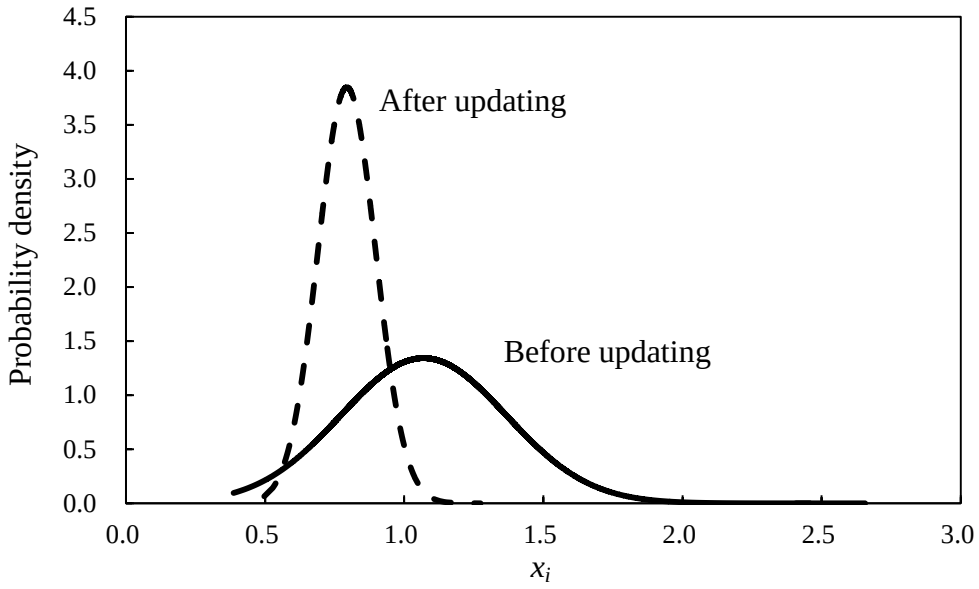


Figure 3: Effect of updating on the PDF

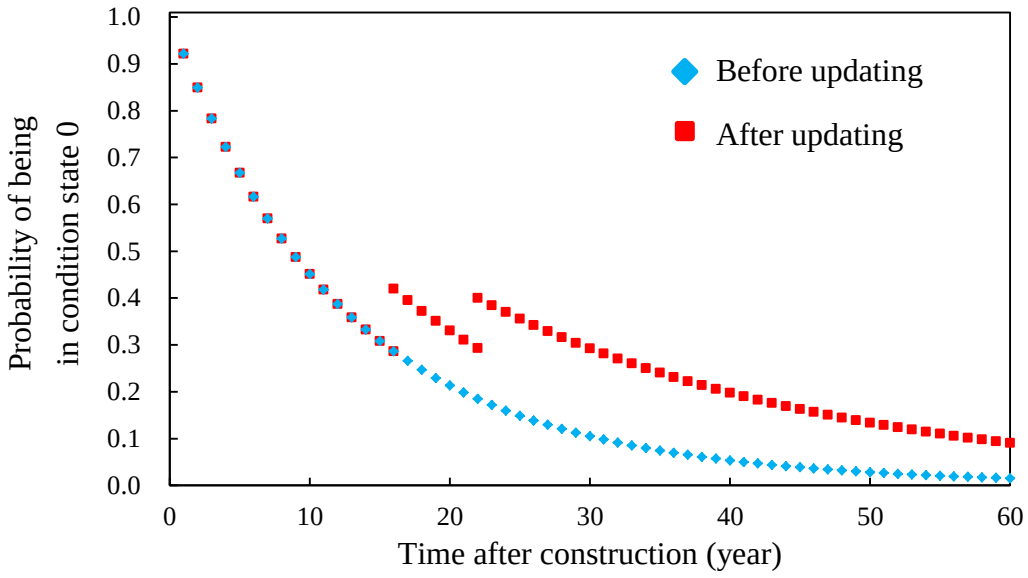


Figure 4 : Time-dependent probability of being in condition state 0 with and without updating

Figure 4 shows the time-dependent probability of being in condition state 0 with and without updating at 16 and 22 years. At 60 years, the difference of probability of being condition state 0 without and with updating is not negligible. It is important to update the conditional probabilities using inspection results for establishing the rational maintenance strategy.

5. CONCLUSIONS

In this study, life-cycle reliability estimation method using Markov chain and Sequential Monte Carlo Simulation is presented. The deterioration of components is modeled using condition state probabilities (i.e. Markov chain state probabilities) and transition probability matrix is updated by SMCS. In an illustrative example, it could be confirmed that the probability of being in conditional

state *i* is estimated by the proposed method. It is very important to have more inspection results for reducing the epistemic uncertainties, which can help in performing a more accurate life-cycle reliability estimate.

REFERENCES

Akiyama M, Frangopol D.M and Yoshida I (2010). Time-dependent reliability analysis of existing RC structures in a marine environment using hazard associated with airborne chlorides. Engineering Structures, 32(11), pp. 3768-3779.

Ellingwood B.R (2005). Risk-informed condition assessment of civil structure: State of practice and research issues. Structure and Infrastructure Engineering, 1(1), pp. 7-18.

Frangopol D.M (2011). Life-cycle performance, management, and optimization of structural systems under uncertainty: Accomplishments and challenges. Structure and Infrastructure Engineering, 7(6), pp. 389–413.

Furuya K, Yokota H and Hashimoto K (2012). Reliability of Deterioration prediction based on Markov model for Mooring Facilities. Journal of Society of Civil Engineers, 67(4), pp. 159-168.

Japan Society of Civil Engineers (JSCE) (2001). Guidelines for Concrete No.4 Standard Specifictions For Concrete Strucures. JSCE Concrete Committee [in Japanese].

Kato E, Iwanami M and Yokota H (2009). Development of Life Cycle Management System for Open-type Wharf. Report of PARI, 48(2), pp. 3-36 [in Japanese].

Komure K, Hamada H, Yokota H and Yamaji T (2002). Development of a Model on Deterioration Progress for RC Deck of Open Type Wharf. Report of PARI, 41(4), pp. 3-38 [in Japanese].

Mori Y and Ellingwood B.R (1993). Reliability-based service life assessment of aging concrete structures. Journal of Structural Engineering, 119(5), pp. 1600–1621.

Port and Airport Research Institute (PARI) (2007). Technical manual on maintenance and rehabilitation for port and harbor facilities. Coastal Development Institute of Technologies, Tokyo [in Japanese].

Ristic B, Arulampalam S and Gordon N (2004). Beyond the Kalman filter, particle filters for tracking applications. Artech House.

Taniguchi O, Tamura T, Sano K and Hamada H (2004). Condition state estimate of deteriorated RC superstructure of wharves using inspection results. Proceedings of Japan Concrete Institute, 26(1), pp. 2049-2054 [in Japanese].

Yoshida I (2009). Data assimilation and reliability estimation of existing structure. COMPDYN 2009, CD281. Rhodes, Greece.