VIBRATION OF THERMO-ELASTICALLY DAMPED IMPERFECT RING GYRO

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ABSTRACT

Thermo-elastic damping of circular ring gyro with random point masses is investigated using in-extensional vibration of the ring model. Also, heat conduction equation is included to study the damping effects of the model. To determine the natural frequency of the structure, Rayleigh-Ritz method is adopted to consider the effect of concentrated point masses. Finally, the trends of Quality factor (Q-factor) variation are examined for the various irregularity of point masses distribution.

Keywords: Thermo-elastic damping, Point mass, Rayleigh-Ritz method, Q-factor.

1. INTRODUCTION

Sensitive devices such as resonators have been developed for rate sensors, such as gyroscopes using micro-electro-mechanical systems (MEMS) [1]. First of all, the system needs high Quality factors(Q-factors) which is defined as the ratio of the total kinetic and potential energy of the system to dissipation. Most of damping mechanisms can be excluded by using proper control, and it has been well known that one of the most important energy loss factors is thermo-elastic damping [2,3]. Moreover, the damping effect is one of the most important factors to estimate the quality of very small structures.

A number of works on circular ring has been performed by numerous researchers for the various applications. Lifshitz and Roukes [4] studied the thermo-elastic damping for the MEMS and nano-electro-mechanical system(NEMS). Wong et al. [5] analytically presented thermo-elastic damping effect using Q-factors for thin silicon ring.

General point of view, imperfections due to random point masses are inevitable for practical structures. Especially, the irregular masses may result in significant amount of the accuracy and performance of the gyroscopes. Using mechanical model of imperfect ring, frequency deviations

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were studied as follows. Fox [6] proposed simple analysis on frequency split for the ring with added masses and springs. Further, McWilliam et al. [7] considered random point masses and determined frequency splitting rule.

In this work, MEMS and NEMS are modeled as imperfect circular ring including thermo-elastic damping properties, and one-dimensional thermal conduction is considered. To get the natural frequency of the imperfect ring with random point masses, Rayleigh-Ritz method is applied. Furthermore, frequency split due to the anomalies of mass are investigated. Finally, the Q-factors are presented according to the various parameters of the structural model.

2. FORMULATIONS

As shown in Figure 1, thermo-elastic damping effect of circular ring with random point masses is considered to analyze the Q-factors of the model. Figure shows a global polar coordinate system, and depicts a thin ring model with rectangular cross-section shape.

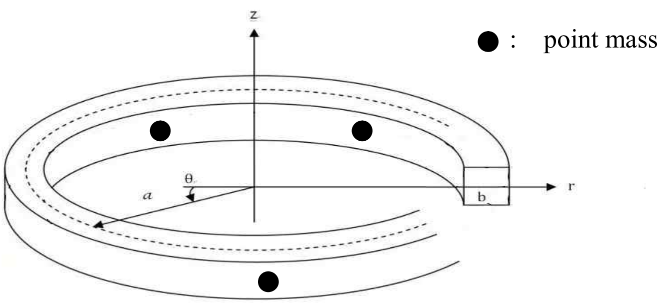

Figure 1: Global coordinate system of ring with point masses

During the in-plane flexural vibrations of the ring, the strain field ε_θ generates a variation in the temperature field $T(x,\theta,t)$. Then, the stresses(σ) and strains(ε) relation can be expressed as

$$\varepsilon_\theta = \frac{1}{E}\sigma_\theta + \alpha T, \qquad \varepsilon_r = \varepsilon_z = -\frac{\nu}{E}\sigma_\theta + \alpha T \tag{1.a, b}$$

In order to obtain the natural frequency of the imperfect ring, kinetic and strain energy expressions are evaluated at first. Thus, the kinetic energy of a ring with attached point masses as

$$K = \frac{1}{2}M\int_0^{2\pi}\left(\dot{u}^2 + \dot{v}^2\right)d\theta + \sum_{i=1}^{n}\int_0^{2\pi}\left[\frac{1}{2}m_i\left(\dot{u}_{m_i}{}^2 + \dot{v}_{m_i}{}^2\right)\delta\left(\theta - \theta_i\right)\right]d\theta \tag{2}$$

where M denotes mass of the ring. While, m_i means i -th concentrated mass, and $\delta(\theta - \theta_i)$ is the Dirac delta function at θ_i.

While, the variation of strain energy due to a slight imperfection is negligible, thus the strain energy of the ring with thermo-elastic damping can be expressed as

$$U = \frac{1}{2}ab\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_\theta\left(\varepsilon_\theta - \alpha T\right)drd\theta = \frac{1}{2}abE\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\varepsilon_\theta - \alpha T\right)^2 drd\theta \tag{3}$$

In order to use the kinetic and potential energies of the ring, in-plane vibration is to describe by radial and tangential displacements at circumferential location θ. Now, $u(\theta,t)$ and $v(\theta,t)$ represent the radial and tangential displacements at θ, and assuming harmonic vibration with frequency ω. Furthermore, the vibration is assumed to occur through the in-extentional motion.

Then,

$$u(\theta,t)=U_0\sin(n\theta)e^{\mathrm{i}\omega_n t},\quad v(\theta,t)=\frac{U_0}{n}\cos(n\theta)e^{\mathrm{i}\omega_n t} \tag{4}$$

Referring to the reference, the displacement functions for point masses can be represented as

$$u_{m_i}=U_0\sin\left(n\left(\theta_i-\zeta\right)\right)e^{\mathrm{i}\omega_n t},\qquad v_{m_i}=\frac{U_0}{n}\cos\left(n\left(\theta_i-\zeta\right)\right)e^{\mathrm{i}\omega_n t} \tag{5}$$

where ζ denotes the shift angle for the orientation of corresponding mode shape.

Applying Eq.(2), the maximum kinetic energy can be expressed as

$$K_{\max}=\frac{1}{2}M\pi\omega^2\frac{\left(n^2+1\right)}{n^2}U_0^2+\frac{1}{2}\sum_{i=1}^{p}2\pi\omega^2U_0^2m_i\left(\sin^2\left(n\left(\theta_i-\zeta\right)\right)+\frac{1}{n^2}\cos^2\left(n\left(\theta_i-\zeta\right)\right)\right) \tag{6}$$

where subscript i denotes the i- th point mass out of total p point masses.

While, potential energy includes the mechanical and thermo-elastic energies, simultaneously. To get the temperature profile, heat conduction equation is to solve

$$\frac{\partial T}{\partial t}-\chi\nabla^2T=-\frac{E\alpha T_a}{C_v(1-2\zeta)}\frac{\partial\varepsilon}{\partial t} \tag{7}$$

where ∇^2, χ, C_v and T_a denote the Laplacian operator, the thermal diffusivity of the material, the heat capacity per unit volume and the ambient temperature in Kelvin, respectively.

Now, distribution of T in the thickness is to refer the previous results in Ref. [5], and then the circumferential strain ε_θ due to in-plane bending can be assumed to be proportional to the distance from the neutral plane, such that:

$$\varepsilon_\theta=-\frac{x}{a^2}\left(\frac{\partial^2u}{\partial\theta^2}+u\right) \tag{8}$$

Using Eq.(4) and inserting Eq. (8) into Eq. (3), the total energy is

$$U_{\max}=\frac{1}{2}abE\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{x}{a^2}(1-n^2)U_0\sin\left(n\theta\right)+\alpha T\right]^2drd\theta \tag{9}$$

Therefore, the natural frequency and shift angles are as follows

$$\omega_n^2=\omega_{0n}^2\frac{1}{1+\varepsilon_{Kn}}=\omega_{0n}^2\left[1+\Delta_E\left(1+f(\omega)\right)\right] \tag{10.a}$$

$$\tan(2n\zeta) = \frac{\sum_{i=1}^{p} m_i \sin(2n\theta_i)}{\sum_{i=1}^{p} m_i \cos(2n\theta_i)} \tag{10.b}$$

where, $\omega_{0n}^{-2} = \left[(n^2+1) + \sum_{i=1}^{p} m_i \left((n^2+1) \pm (n^2-1)\cos\left(2n(\theta_i - \zeta)\right) \right) \delta(\theta - \theta_i)/M \right] / (n^2+1)$

and $f(\omega) = \frac{24}{h^3k^3}\left(\frac{kh}{2} - \tan\left(\frac{kh}{2}\right)\right)$. Also, $\Delta_E = \frac{E\alpha^2 T_a}{C_v}$ is the relaxation strength of the Young's modulus and k is complex value defined as $k = (1+i)\sqrt{\frac{\omega}{2\chi}}$.

3. RESULTS AND DISCUSSION

As a numerical model, a silicon ring with random point masses is used to analyze according to the various parameters. Natural frequencies are discussed with and without imperfection of the ring. In this work, the thermo-elastic damping effect is included, because Q-factor is important for the performance of MEMS and NEMS.

3.1. Verification

Primarily, the frequency formula is exactly the same for the in-extensional vibration for a ring without considering the thermo-elastic damping effect and attached masses. Also, only the thermo-elastic damping effect in the silicon ring is considered, then the natural frequencies is obtained as the results in Ref. [5].

Thus, $Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega_n)}{\mathrm{Re}(\omega_n)}\right|$ (11)

Table 1 shows the result compared with the previous data for the perfect thermo-elastic ring to check the validity of the Q-factors for the n=2 mode. For each radius of the ring model, relative difference with respect to Ref.5 is sufficiently small as indicated.

Table 1: Verification of Q-factors for n=2 modes

Radius(mm)	Radial thickness (μm)		Errors (%)
	Ref. 5	Present study	
1	12.64	12.62	0.12
3	91.96	91.81	0.16
5	254.8	254.38	0.17

In the following section, imperfection effect is considered in the ring model.

3.2. Imperfect ring model

Imperfect ring model with attached masses is investigated. In this work, two attached masses are used, and the weight of each mass is 2.5% of total mass of the perfect ring. Figure 2 shows the variation of the natural frequencies as a function of radial thickness and angle between the two masses. Case 1 is the perfect ring model, and case 2 to 6 are the angles of two masses between 0 to 90 degrees with increment to be 15 degrees. As thickness of the ring is increased, the frequency is decreased. The frequency difference due to the change of angle between masses does not occur, while the difference is appeared for the perfect ring.

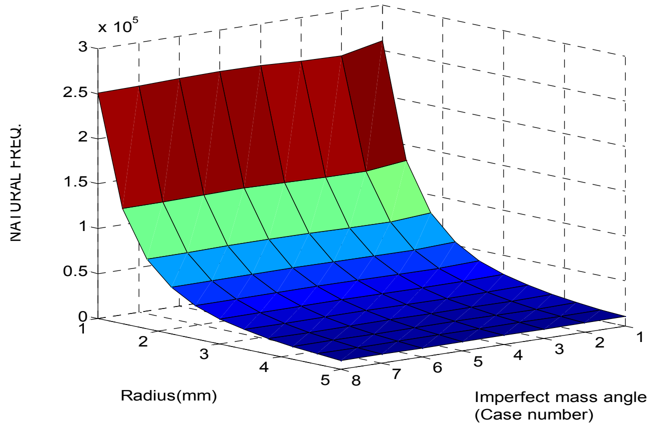

Figure 2: Variation of the natural frequencies due to the radial thickness and angle

Figure 3 shows the variation of the frequencies as a function of radial thickness and imperfect mass ratios. Case 1 to 9 are the ratio of total mass between -10 to 10 % with 2.5% increment. Radial thickness of the ring is increased, the frequency is decreased, and the frequency difference due to the change of the ratio of total mass does not occur.

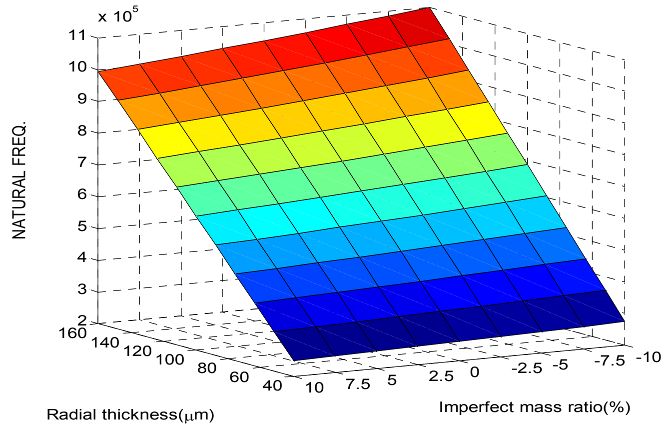

Figure 3: Variation of the natural frequencies due to the radial thickness and imperfect mass ratio

Figure 4 shows the variation of frequency split due to the increase of attached masses ratio for total mass. In case of perfect ring, frequency split does not occur. While, the ratio is increased, the gap between higher and lower modes is increased. This means that the frequency split is increased.

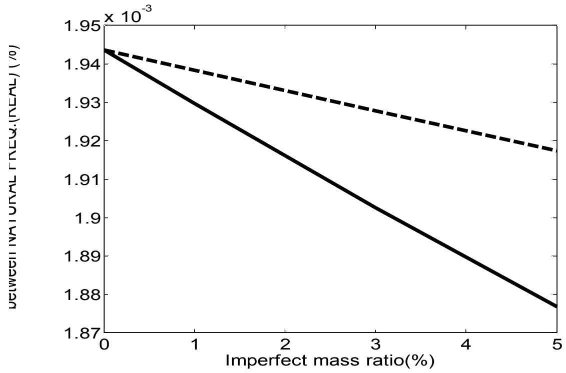

Figure 4: Variation of frequency split due to the increase of attached masses ratio

4. CONCLUSIONS

Analytical model for imperfect circular ring with thermo-elastic damping is investigated using in-extensional ring model such as MEMS and NANO structures. Due to the damping, the total energy of the ring dissipates, and then the Q-factor is the essential role to evaluate the performance of the model. Moreover, the random distribution of the masses can represent uneven mass of practical ring. Also, this imbalance affects to whole system and results in split of natural frequencies and increase of the thermo-elastic damping effect. Generally, a resonator will have different mode shape orientations and split frequencies due to slight imperfection. And then, the thermo-elastic damping changes the magnitude of split frequencies. In this work, effects of added or deleted point masses on the structures are investigated. For a future research, trimming of unbalanced mass distribution will be studied.

5. ACKNOWLEDGEMENT

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