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Title	CALCULATION FORMULAS OF DESIGN BENDING MOMENTS ON THE BOUNDARIES OF SLABS PART2: APPLICATION OF THE SAFETY-MARGIN FROM RC STANDARD TO OTHER SHAPED SLABS
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Description	The Thirteenth East Asia-Pacific Conference on Structural Engineering and Construction (EASEC-13), September 11-13, 2013, Sapporo, Japan.
Relation	Proceedings of the Thirteenth East Asia-Pacific Conference on Structural Engineering and Construction (EASEC-13), September 11-13, 2013, Sapporo, Japan, F-2-5.
Citation	Proceedings of the Thirteenth East Asia-Pacific Conference on Structural Engineering and Construction (EASEC-13), F-2-5
Issue Date	2013-09-12
Doc URL	https://hdl.handle.net/2115/54382
Type	conference paper
File Information	easec13-F-2-5.pdf



Calculation Formulas of Design Bending Moments on the Boundaries of Slabs

Part2: Application of the Safety-Margin from RC standard to Other Shaped Slabs

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ABSTRACT

The purpose of this study is to propose calculation formulas for design bending moments on boundary sides of a fixed end triangular slab and a fixed end tangential quadrilateral slab subjected to uniform load. With these formulas, you can calculate the design bending moment of these slabs more simply.

Keywords: Design bending moment, Triangular slab, tangential quadrilateral slab, Safety-margin

1. INTRODUCTION

The purpose of this study is to propose calculation formulas for design bending moments on boundary sides of a fixed end triangular slab and a fixed end tangential quadrilateral slab subjected to uniform load. In the Part1, the authors have assumed a certain width in which the average bending moment is equal to the design bending moment given by RC standard and we proposed the equation that gives the average value of bending moment in the mean-width as the calculation formula of design bending moment. In this Part2, the authors imagine the safety-margin against collapse, which RC standard formula is thought to give to the slab. Firstly, we investigate this safety-margin in RC standard, then we apply this safety-margin to the triangular slab and the tangential quadrilateral slab and show calculation formulas for design bending moment of these slabs.

2. THE SAFETY-MARGIN OF RC STANDARD FORMULA

In this chapter, we calculate the safety-margin against collapse, which RC standard formula gives a rectangular slab. This safety-margin μ can be calculated as the ratio of the ultimate uniform load $r_w u$ to the yield uniform load $r_w y$ of the rectangular slab. And, by assuming that all of the yielding bending moments in any directions on an upper or on a bottom surfaces are the same value M_y , we can calculate the yield uniform load $r_w y$ from RC formula, as the value of the bending moment equals to M_y . Note that, we assume that the collapse of the slab is caused only by bending so that we

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can get the ultimate uniform load $r_w u$ by yield line theory. In this theory, we need the ultimate bending moment M_0 , and we assume that the value is constant all over the slab, then we can write it with the yielding bending moment M_y and the positive constant C as follows.

$$M_0 = C \times M_y \quad (1)$$

2.1. Yield uniform load $r_w y$ in RC standard formula

RC standard says that the maximum bending moment in a rectangular slab is

$$M_{x1} = \frac{wl_x^2}{12} \times \frac{\lambda^4}{1 + \lambda^4} \quad (2)$$

where w is the value of uniform load, l_x is a length of the shorter side, λ is a ratio of long side to short side. So substituting M_y to M_{x1} , we can have the uniform load $r_w y$ that RC standard imagine as the yield uniform load as follows.

$$r_w y = \frac{12M_y}{l_x^2} \times \frac{1 + \lambda^4}{\lambda^4} \quad (3)$$

2.2. Ultimate uniform load $r_w u$ of the rectangular slab

The ultimate uniform load of the fixed end rectangular slab subjected to uniform load calculated by yield line theory can be referred in some papers. Figure 1 shows the collapse mechanism of rectangular slab. Solid lines in the figure show yield lines where positive bendings, which mean tension on the bottom and compression on the upper surface, occur, and dashed lines show yield lines where negative bendings occur. Angle ϕ can be calculated as shown in equation (4), and the ultimate uniform load $r_w u$ can be calculated as shown in equation (5).

$$\phi = \tan^{-1} \left(\frac{\sqrt{1 + 3\lambda^2} - 1}{\lambda} \right) \quad (4)$$

$$r_w u = \frac{48M_0}{l_x^2} \times \frac{\lambda^2}{(\sqrt{1 + 3\lambda^2} - 1)^2} \quad (5)$$

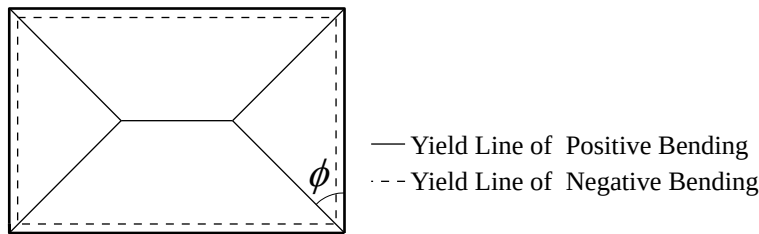


Figure 1: Collapse mechanism of the rectangular slab.

2.3. The safety-margin μ

We can have the safety-margin μ in RC standard as a ratio of the ultimate uniform load ${}_r w_u$ to the yield uniform load ${}_r w_y$, and we found it depends on the ratio of long side to short side λ .

$$\mu = \frac{{}_r w_u}{{}_r w_y} = \frac{4C\lambda^6}{(1+\lambda^4)(\sqrt{1+3\lambda^2}-1)^2} \quad (6)$$

3. A PROPOSITION OF CALCULATION FORMULAS FOR DESIGN BENDING MOMENTS

We assume that the calculation formulas for fixed end triangular or tangential quadrilateral slabs have the same safety-margin μ as RC standard formula has, and we apply the obtained safety-margin from RC standard to each slab. Consequently, the calculation formulas for design bending moments can be formulated as the division of the ultimate uniform load by the safety-margin μ . Note that, we assume that M_y and M_0 are constant all over these slabs.

3.1. Ultimate load of the triangular slab and tangential quadrilateral slab

The ultimate uniform load of each slab is calculated by yield line theory. And we need to find the collapse mechanism in which the ultimate uniform load is minimized. Figure 2 shows the collapse mechanism of the square slab calculated by equation (4). The yield lines in positive bending come to be bisectors from vertexes, and we recognized that these lines cross in the central point of the inscribed circle. The bisectors from vertexes of the triangular slab and the tangential quadrilateral slab cross also in the central point of the inscribed circle. And we can assume the collapse mechanism of each slab as shown in Figure 3. Note that, we had verified that the value of the ultimate uniform load obtained from each collapse mechanism was the smallest value by a numerical analysis, and the mechanism is correct.

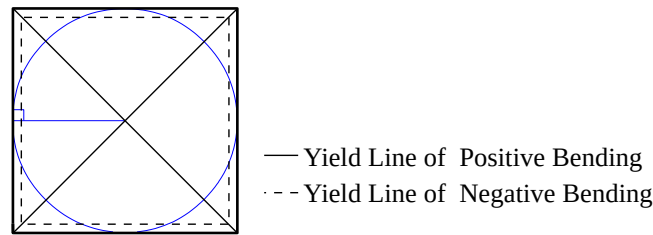


Figure 2: Collapse mechanism of the square slab.

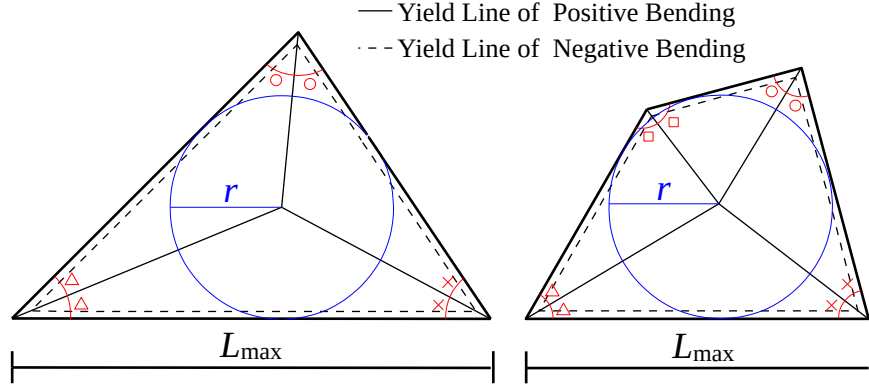


Figure 3: Collapse mechanism of the triangular slab and tangential quadrilateral slab.

The ultimate uniform load of each collapse mechanism is calculated by principle of virtual work. In spite of the fact that their figures are different, the ultimate uniform load can be expressed in the same form, if you use the radius of the inscribed circle as follows.

$${}_a w_u = \frac{12CM_y}{r^2} \quad (7)$$

Here, r is the radius.

3.2. The safety-margin that apply to the triangular slab and tangential quadrilateral slab

As expressed in equation (6), the safety-margin μ in RC standard is changed by the ratio λ . As described in Part1, the ratio μ might be 1.0. And we have the safety-margin μ_d for the triangular or the tangential quadrilateral slab by substituting $\mu=1$ to equation (6) as follows.

$$\mu_d = 2C \quad (8)$$

3.3. Design bending moments and the decreasing rate D_m

As described before, calculation formulas for design bending moments can be calculated as the division of the ultimate uniform load ${}_a w_u$ by the safety-margin μ_d .

$${}_d w_y = \frac{{}_a w_u}{\mu_d} = \frac{6M_y}{r^2} \quad (9)$$

In this paper, we had assumed that M_0 is can be written in the form of multiplication of C and M_y , but we found out that we don't need the C value to calculated the uniform load ${}_d w_y$ as shown in equation (9).

Solving equation (9) for M_y , and letting ${}_d w_y$ and M_y to be a design uniform load w and a design bending moment M_{dm} gives the following the calculation formula for design bending moment.

$$M_{dm} = \frac{1}{6} wr^2 \quad (10)$$

From here, we estimate a decreasing ratio which is given by the division of the value of the equation (10) by the maximum bending moments which occurs in the slabs. Note that, calculation formulas for maximum bending moments on boundary sides of the triangular slab and the tangential quadrilateral slab had been proposed in previous paper as follows

$${}_tM_i = 0.198\left(\frac{L_i}{r}\right)^{0.175} \times wr^2, {}_qM_i = 0.190\left(\frac{L_i}{r}\right)^{0.175} \times wr^2 \quad (11)$$

Here, ${}_tM_i$ is a maximum bending moment on the i -th boundary side of the triangular slab, ${}_qM_i$ is a maximum bending moment on the i -th boundary side of the tangential quadrilateral slab, L_i is a length of the i -th boundary side.

As we can understand from these equations, the maximum bending moment in the triangular slab and in the tangential quadrilateral slab occurs on the longest boundary side in each slab. Letting the maximum bending moment ${}_tM_{\max}$ in the triangular slab, ${}_qM_{\max}$ in the tangential quadrilateral slab, and longest boundary side $L_{\max}$ gives the following equations.

$${}_tM_{\max} = 0.198\left(\frac{L_{\max}}{r}\right)^{0.175} \times wr^2, {}_qM_{\max} = 0.190\left(\frac{L_{\max}}{r}\right)^{0.175} \times wr^2 \quad (12)$$

So we have the decreasing ratios ${}_tD_m$ for the triangular slab and ${}_qD_m$ for the tangential quadrilateral slab are the ratios of equation (11) to equation (12). They can be calculated as shown in equation (13).

$${}_tD_m = \frac{M_{dm}}{{}_tM_{\max}} = \frac{1}{6} \frac{1}{0.198\left(\frac{L_{\max}}{r}\right)^{0.175}}, {}_qD_m = \frac{M_{dm}}{{}_qM_{\max}} = \frac{1}{6} \frac{1}{0.190\left(\frac{L_{\max}}{r}\right)^{0.175}} \quad (13)$$

Figure 4 shows the values of decreasing ratios ${}_tD_m$ and ${}_qD_m$. A horizontal axis in the figure shows $L_{\max}/r$. A solid line in the figure expresses ${}_tD_m$, and a dashed line expresses ${}_qD_m$. Range of $L_{\max}/r$ is about 3.46 to 7.46 in the triangular slab and about 2.00 to 4.73 in the tangential quadrilateral slab. These ranges come from the assumed slab figure. The decreasing ratio is around 60% to 70% for the triangular slab and around 65% to 80% for the tangential quadrilateral slab. The value of maximum bending moment can be further decreased, as $L_{\max}/r$ is increased.

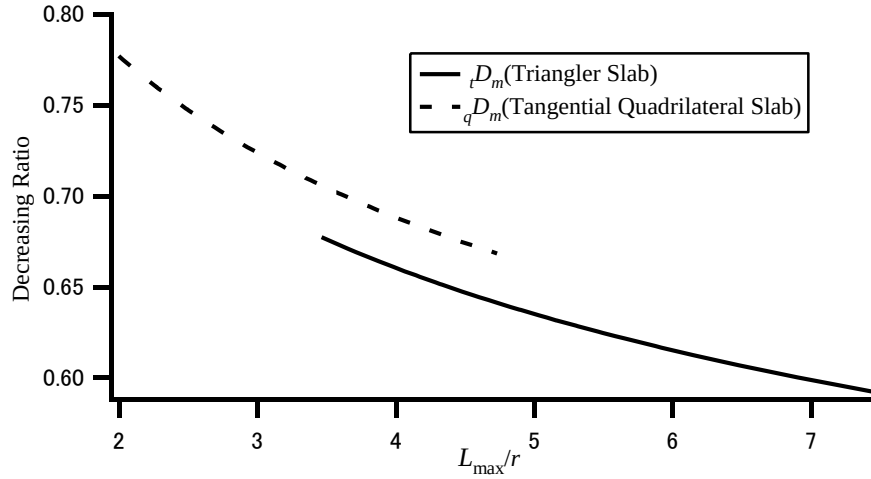


Figure 4: Decreasing ratio ιD_m and ${}_q D_m$.

4. DIFFERENCE OF CALCULATION FORMULAS FOR DESIGN BENDING MOMENTS

In this paper, the idea of the safety-margin of rectangular slabs against the collapse applied to the other figured slabs gives the calculation formulas for design bending moment. In the Part1, the idea of the mean-width gives the other calculation formulas. In this chapter, we describe difference of these formulas are described.

Figure 5 shows the decreasing ratios calculated from these formulas. A horizontal axis in the figure shows L_{max}/r . Solid lines in the figure show the decreasing ratio obtained from the mean-width, and dashed lines show the decreasing ratio obtained from the safety-margin. Their trends are similar to each other. Its maximum difference is around 4%, and calculation formula for design bending moment obtained from the mean-width gives the safer value that obtained from the safety-margin.

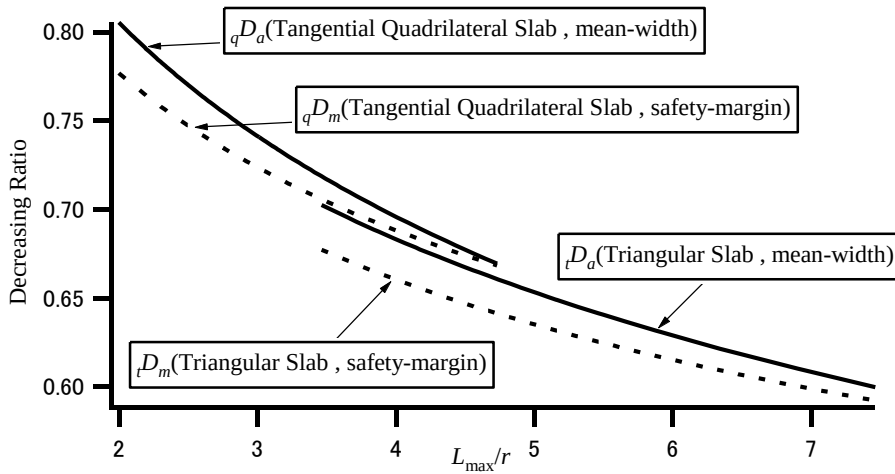


Figure 5: Comparison of decreasing ratio.

5. CONCLUSION

In this paper, we proposed the calculation formulas of design bending moment from of the point of view of the safety-margin against the collapse. And the differences between this idea and that in Part1 are described. The arrangement of reinforcement in concrete will be a future problem.

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