# １９９４年度談話会・特別講演
# アブストラクト集

## Colloquium Lectures

北海道大学理学部数学教室

**Edited by M. Saito**

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# 1994年度談話会・特別講演アブストラクト

# 談話会・特別講演一覧（１９９４年度）

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| 3. | ４月２７日（水） | ＊ | 松 本 健 司 氏 | （北大・理） | 非線型力学系としてのタンパク質 |
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ABSTRACT

## EXTENSIONS OF FRAMES

Sung Sa HONG
(Department of Mathematics, Sogang University, Seoul, KOREA)

The purpose of this talk is to introduce a concept of convergence of filters in frames (= locales = complete Heyting algebras) by covers and then study extensions of frames based on the convergence theory in frames.

We define that a filter $F$ in a frame $L$ is convergent (clustered) if every cover $C$ of $L$ meets $F$ ($secF$, resp.). This clearly generalizes convergent filters or filters with cluster points in a topological space. Among others, we show that a regular frame $L$ is compact iff every maximal filter in $L$ is convergent. Moreover, a zero-dimensional frame $L$ is compact iff every maximal Boolean filter in $L$ is convergent.

Banaschewski has defined two extreme cases of extensions of topological spaces, namely simple and strict ones. Using simple extensions of frames and the right adjoint, we introduce a concept of strict extensions of frames as the subframe of the simple extension of $L$ generated by the image of $L$ under the right adjoint. Using these, we construct a zero-dimensional compactification of a zero-dimensional frame.

# 非線型力学系としてのタンパク質

松本 健司

タンパク質はアミノ酸が一次元状に繋がってできている高分子であるが、生体内でその機能を果すには特別の３次元構造をとる必要がある。この３次元構造は通常の生体内の環境では安定であるので、ポテンシャルエネルギーが最小の状態であると考えられる。

温度を上げる (保存系の言葉でいうと、運動エネルギーを上げてポテンシャル井戸から脱出させる)、pH を変える (原子のもつ電荷を変化させてポテンシャル自体を変える) などしてタンパク質の環境を生体内のものからずらすとポテンシャルエネルギー最小の３次元構造がほどけて、タンパク質としての機能を失なう (失活する)。

いくつかのタンパク質では、失活した後に環境を適当に回復させると元のポテンシャルエネルギー最小の３次元構造に戻ることが知られている。またタンパク質が生体内で作られるときにも他のタンパク質の助けを借りずに自力で３次元構造をとるものがあると考えられる。これをタンパク質のおりたたみという。

タンパク質のような複雑な系ではポテンシャルには非常な数のローカルミニマムが存在する。一方、上で述べたタンパク質の機能をもつ３次元構造はグローバルミニマムである。すると、自力で３次元構造をとるタンパク質は多くのローカルミニマムからグローバルミニマムを見つけるという難問を解決していることになる。しかも、それに要する時間はローカルミニマムを全部巡回するのに要する時間より数桁小さい。

ここには我々のまだ知らないメカニズムが存在していると思われる。

この講演は、このメカニズムの解明、特に保存系のカオスとの関連の解明を目指した試みの初期段階の報告である。

まずタンパク質のおりたたみが純粋に力学的な現象かどうかをみるために、分子動力学を使ってタンパク質のニュートン方程式をつくり、計算機シミュレーションでその軌道を計算した。現在、次のようなことがわかっている。

- 統計力学のエネルギー等分配の法則を使って運動エネルギーを温度に換算して室温程度の運動エネルギーでタンパク質の軌道はカオスになっている。

- 保存系であるにもかかわらず時間発展に伴なってポテンシャルエネルギーは減少の一途をたどる。つまり、おりたたみが起っているようである。

- あるローカルミニマムにしばらく滞在し、突然、そのミニマムからはずれ、別のミニマムに捕まり、ということをくりかえす現象がみられた。これは多自由度系のカオスによくみられ、カオス的遍歴と呼ばれている。このときポテンシャルエネルギーの時間変化をみていると、あるミニマムに滞在しているときは大体同じような値におちついているが、別のミニマムに移るときに大きく減少するのがみられる。未だかって、よりポテンシャルエネルギーの大きいミニマムに移るのは観測された例はない。

いまのところタンパク質のおりたたみ現象は純粋に力学的な現象であると思われる。

# 有限次元の力学系の微小ランダム摂動

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今、次のような常微分方程式を考えよう：

$$d\,X(t,x)/dt=b(X(t,x)),$$
$$X(0,x)=x..$$

ここで，b(x)は、ｄ次元ユークリッド空間$R^d$上のリプシッツ連続なベクトル値関数である。このX(t,x)を次のように摂動する:

$$X^\varepsilon(t,x)=x+\int_0^t b(X^\varepsilon(s,x))ds+\varepsilon W(t)$$

ここで，W(t)は，ｄ次元のウィナー過程とする。また、$X^\varepsilon(t,x)$は、$x+\varepsilon W(t)$から、一意的に決まる。

このX$^\varepsilon(t,x)$は、$\varepsilon$が０に行くときX(t,x)に任意の有限区間で一様に収束する。この意味で、$X^\varepsilon(t,x)$は、$\varepsilon\to 0$の時、X(t,x)の微小ランダム摂動と考えられる。

$X^\varepsilon(t,x)$に関する研究として、大まかに言って、

（１） ｂの性質に依存しない$X^\varepsilon(t,x)$の挙動を調べる：

$$X^\varepsilon(t,x)=\sum_{n=0}^{\infty}\varepsilon^n X^n(t,x)$$

となるような$X^n(t,x)$を求め、それらの性質を使って$X^\varepsilon(t,x)$の挙動を調べる

（２） ｂの性質に依存する$X^\varepsilon(t,x)$の挙動を調べる：

（あ）X(t,x)が、原点を唯一の漸近安定平衡点として持つときの$X^\varepsilon(t,x)$の定常確率測度の（$\varepsilon$が０に行くときの）漸近挙動を調べる

（い）Ｄをｄ次元ユークリッド空間の領域で、滑らかな境界$\partial D$を持つものとし、x∈Dが、次のいずれかを満たすものとする：

(a) あるs＜０が存在して、X(t,x)∈D(t>s), X(t,x)∉D(t<s), $X(t,x)\xrightarrow{t\to\infty}o$.

(b) あるs＞０が存在して、X(t,x)∈D(t<s), X(t,x)∉D(t<s),$X(t,x)\xrightarrow{t\to-\infty}o$.

(c) ある$s_1<0<s_2$が存在して、X(t,x)∈D($s_1<t<s_2$),X(t,x)∉D($s_1>t,t>s_2$)。

$\tau_D^\varepsilon=\inf\{t>0;X^\varepsilon(t,x)\notin D\}$を$X^\varepsilon(t,x)$のＤからの第一脱出時間とし

(i)$\tau_D^\varepsilon$の挙動を調べる

(ii)脱出点$X^\varepsilon(\tau_D^\varepsilon,x)$の挙動を調べる

(iii)次の経験分布を考える：

$$\mu^\varepsilon(dy)=\frac{1}{\tau_D^\varepsilon}\int_0^{\tau_D^\varepsilon}1_{\{dy\}}(X^\varepsilon(s,x))ds$$

(b)と(c)が、起り得るかどうかで、上の三つの問題は、著しく違った答えを持つ。

なお、偏微分方程式論との関係は、以下のようになっている：

（１） $R^d$で有界連続なfに対して$u_1^\varepsilon(t,x)=\int_{R^d}f(y)P(X^\varepsilon(t,x)\in dy)$は、次のPDEの解である

$$\partial u_1^\varepsilon(t,x)/\partial t=\varepsilon\Delta u_1^\varepsilon(t,x)/2+<b(x),\nabla u_1^\varepsilon(t,x)>(x\in R^d,t>0)$$
$$u_1^\varepsilon(0,x)=f(x)(x\in R^d)$$

（２） $\partial D$で連続なfに対して$u_2^\varepsilon(x)=\int_{\partial D}f(y)P(X^\varepsilon(\tau_D^\varepsilon,x)\in dy)$は、次のPDEの解である；

$$\varepsilon\Delta u_2^\varepsilon(t,x)/2+<b(x),\nabla u_2^\varepsilon(t,x)>=0(x\in D)$$
$$u_2^\varepsilon(x)=f(x)(x\in\partial D)$$

（３） $u_3^\varepsilon(t,x)=P_x(\tau_D^\varepsilon\le t)$は、次のPDEの解である。

$$\partial u_3^\varepsilon(t,x)/\partial t=\varepsilon\Delta u_3^\varepsilon(t,x)/2+<b(x),\nabla u_3^\varepsilon(t,x)>(x\in D,t>0)$$
$$u_3^\varepsilon(t,x)=1(x\in\partial D,t>0)$$
$$u_3^\varepsilon(0,x)=0(x\in D)$$

# Tate's Conjectures for $H^2$ of Shimura Varieties

Don Blasius

May 25, 1994

In this talk we describe recent progress concerning Tate's conjectures for $H^2$ of a Shimura variety $Sh(G,X)$. Here $G$ is a reductive group defined over $\mathbf{Q}$, $X$ is an additional datum which we need not specify, and $Sh(G,X)$ is a quasi-projective variety defined over a number field $E = E(G,X)$.

To state Tate's conjecture, let $Z^1(Y)$ be the rational vector space with basis the irreducible divisors on the projective variety $Y$, defined over $E$. Let $c_\ell^1 : Z^1(Y) \to [H_\ell^2(\overline{Y})(1)]^{G_E}$ be the $\ell$-adic Chern class map. Here $G_E = Gal(\overline{E}/E)$ is the Galois group of the algebraic closure $\overline{E}$ of $E$, $\overline{Y} = Y \times_E \overline{E}$, and $H_\ell^2(\overline{Y})(1)$ is as usual the $\ell$-adic etale cohomology group, Tate-twisted once. Tate conjectured that $Im(c_\ell^1)$ spans $[H_\ell^2(\overline{Y})(1)]^{G_E}$. This conjecture is proved for any variety which is birational to a product of curves, but remains open for most surfaces.

Over the 1980's, advances in the theory of automorphic forms made it reasonable to study this conjecture in the setting of Shimra varieties, i.e. of arithmetic quotients of bounded symmetric domains. The conjecture was proven for $GL_2 \times GL_2$ by Tunnell, for $G = R_{F/\mathbf{Q}}(GL_2)$,$[F : \mathbf{Q}] = 2$, by Harder, Langlands and Rapoport, as completed by Murty and Ramakrishnan, and separately, Klingenberg, for $G = GSp(4)$ by Weissauer, and for $G$ a group of unitary similitudes in 3 variables by Blasius and Rogawski.

In recent work with J.Schwermer, we have shown how the conjecture can be proved for most Shimura varieties. We use a number of techniques, building upon the above mentioned papers. In particular, it is often the case that one can prove a holomorphic Lefschetz theorem for $H^2$ using only Shimura varieties, and thus reduces the problem for $Sh(G,X)$ to related questions on these subvarieties $Sh(H,X_H)$.

Other cases require the trace formula (or at least its consequences) or the Eichler-Shimura congruence relations. Rather than constructing elements of $Z^1(Sh(G,X))$, we show simply that the dimension of the space of potential Tate classes is equal to that of the space of Hodge classes.

The main open cases are those associated to Shimura surfaces defined by quaternion algebras. However, Ramakrishnan and Murty have announced rather general results about these cases.

Generalized conformal structures in geometry and analysis.

Simon Gindikin.

Let $V$ be a conical algebraic set (real or imaginary) in $\mathbb{R}^n$. We will call the $V$-conformal structure on ~~the~~ a manifold $M$ a field of $V_x \subset T_x M$ where all $V_x$ are linearly equivalent to $V$.

Usual conformal structures correspond to the quadratical cone $V$, almost complex structures to ~~the~~ pairs of imaginary transversal planes of the dimension $n$ in $\mathbb{R}^{2n}$. There are several other examples of generalized conformal structures. A class of such structures is connected with symmetric manifolds: for each compact symmetric Hermitian manifold $M$ there is such a cone $V$ that on $M$ there is a local flat $V$-conformal structure that all its local automorphisms can be extended up to global automorphisms $M$. It gives a possibility to interpret compact symmetric spaces as generalized conformal compactifications of $\mathbb{R}^n$. (2

We have several curios local generalized ~~of~~ conformal isomorphisms for affine symmetric spaces (f.e. symplectic groups and spaces of symmetric matrices). These isomorphisms are ~~bir~~ birational.

~~There are~~ There are generalized conformal structures on manifolds of rational curves. The geometry of such structures gives a possibility to construct explicite solutions of self-dual Einstein equations.

# カオスと情報処理機能

奈良重俊

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現在、VLSI（大規模集積回路）技術によって社会には情報革命とも言える事態が進行しつつある。こうした発展は、ハードウェアは現代の半導体技術によって、ソフトウェアはノイマン型処理のアルゴリズムに基づく情報処理技術よって支えられていると言える。

こうした流れにも拘わらず、この巨大な技術にも不得手なものがあることが先鋭的に意識されつつある。それは「概念」を自分の行動に結びつける「知的情報処理」である。例えば「隣の部屋から灰皿を持って来て」という依頼を考えて頂きたい。こうした依頼をこなすことのできるコンピュータ搭載の家事ロボットを作ろうとして、「隣の部屋」とか「灰皿」とかいう概念を計算機に与えるプログラム言語で書き下そうとした場合、それは極めて難しい課題であることは容易に理解できるだろう。

人間は生活を維持する際、実に様々な状況下で外界から情報を受け取り、それを処理している。その中で重要なものの一つに言葉（記号）を操る機能がある。言葉は「概念」を表すものであるが、言葉を文字で表現した時、「概念」は「記号」で置き換えられる。記号化された概念は多種多様なものがあるが、その中で更に数値によって計量化できる場合がある。人間の持つ「記号処理」の機能の一部分及び「数値処理」の機能を工学的に再構成したものが「計算機」である。

しかしながら先に出てきた例のような高次情報処理機能を従来の情報処理技術の延長線上で実行させることは「人工知能」の計算機による実現の試みに代表されるように大きな努力が払われてはいるものの、現実的に言って極めて大きな難問であることは間違いのないところである。即ち、naive なノイマン型のアルゴリズム（ serial processing ）でそうした機能を実現させようとすると、

**1.** combinatorial explosion（場合の数の爆発）

**2.** program complexity（アルゴリズムの複雑さ）

の二者が発生して現実的な処理時間と現実的なプログラム長の範囲内では実行できないような事態が発生すると考えられるのである。しかしながらこうした言い方は物理学や数学の研究者の感覚では大変曖昧なものであるとの感が拭いきれない。「人間が計算するとはどういうことか？」という命題に極限まで考察を進めた Turing や Neuman らの偉大さを実感させられる。

ともあれ現代の情報処理技術が邁進する「高度な機構による高度な機能実現」の方向に対する不満や限界感が漠然としたものであれ感じられていることを背景にして、人間を含む動物の脳で起きる、あるいはもっと一般化して、生体の発生・成長を含め生命維持の活動の際に観察され、更には生体の行動に伴って観察される「柔軟かつ高次、あるいは知的」な情報処理機能あるいは制御機能の原理を探ろうとして生体の研究が活発化したのは当然と言える。その過程の中で大きな注目を集めているのは、生体において頻繁に観測される「カオス的ダイナミックス」である。歴史的なカオスの研究の中で特に系の持つ情報としての力学的エントロピーを議論する側面に加え、最近のこうした生体における多数の発見例をもとにカオスを「情報ある

いは情報処理と広い意味でのシステム制御」との関連において捉えようという問題意識を持った研究者が現れる段階に至っている。

こうした状況をもとに、我々（奈良 & P.Davis）の研究活動の動機は、

- 「カオスと生体の持つ複雑な情報処理機能や制御機能との関連」を物理学的に追求し、またそれに基づいた「カオスの Applicability 」を問う。そしてそれを新しい情報処理方式あるいは制御方式（「単純なルールによる複雑な機能の実現」という形での機能発現）として工学的に再構成することが可能か？

ということである。

こうした動機に基づく研究は、生物学、工学、物理学の三者に跨ったものであり、この三者を共通に貫くものが、「複雑なダイナミックス」であると考えるものである。そして考察を更に進め、当面の目標を、

- 「単純なルールで複雑な処理を行わせる原理」を「神経回路網におけるカオスを含む複雑なダイナミックス」の機能的側面に関する物理学的研究を通じて明らかにし、シミュレーションによって確認する。

と設定した。

具体的には、$20 \times 20 = 400$ 個のピクセルパターンで表されるような画像情報をリカレント型の神経回路網の興奮パターンとして取扱い、連想記憶、ノイズ除去、分類、などの機能特性を調べた。更にこの系のシステムパラメータを変化させることによりカオス的ダイナミックスを導入した。また「単純な学習ルール」を導入することによってカオスの動的構造の適応制御を試みた。そしてそれらを用いて「不良設定問題」としての

**(1)** 記憶のサーチ機能

**(2)** 記憶の合成機能

**(3)** モンタージュ機能

などの「複雑な機能の単純なルールによる実現」をシミュレーションによって試みた。

### Referecnes

**(1)** 神経回路網における複雑なダイナミックスの機能性

— カオス的及び遺伝子的ダイナミックス —

合原一幸編「ニューラルシステムにおけるカオス」第八章、285 頁－329 頁、東京電機大学出版局　1993 年

**(2)** カオスによるパターン発生とその適応制御

「システム・制御・情報」学会誌、第 37 巻、11 号、654 頁- 660 頁 (1993)

**(3)** カオスと記憶

数理科学特集「脳のモデル化最前線」 1994 年 7 月号、サイエンス社

## Degenerations of surfaces on Noether lines and their period maps

1 VI 94, at Hokkaidō University, Sampei Usui

Let $\varphi : \Delta^* \to D/\Gamma$ be a *period map*, i.e., a homolomorphic map with horizontal local liftings, from the punctured unit disc $\Delta^*$. Let $\mathfrak{h} \to \Delta^*$, $z \mapsto s = \exp(2\pi iz)$, be the universal cover, $\widetilde{\varphi} : \mathfrak{h} \to D$ a lifting of $\varphi$, $\gamma \in \Gamma$ an element satisfying $\widetilde{\varphi}(z+1) = \gamma\widetilde{\varphi}(z)$ for all $z \in \mathfrak{h}$, $N$ the logarithm of the unipotent part of $\gamma$, and $W = W(N)[-w]$ the monodromy weight filtration. We assume

$$(*) \qquad 0 = W_{w-2} \subset W_{w-1} \subset W_w \subset W_{w+1} := H_{\mathbf{Q}}, \quad \dim W_{w-1} = \begin{cases} 1 & \text{if } w \text{ is odd,} \\ 2 & \text{if } w \text{ is even.} \end{cases}$$

**Theorem.** *There exists a partial compactification $\overline{D/\Gamma}$ with the following properties:*
**(i)** *As point-sets $|\overline{D/\Gamma}| = |D/\Gamma| \cup \{\lim_{t\to 0} \varphi(t) \mid \varphi$ is a period map satisfying $(*)\}$.*
**(ii)** *$\overline{D/\Gamma}$ is a Hausdorff complex orbifold.*
**(iii)** *Any period map $\varphi : \Delta^* \to D/\Gamma$ has a holomorphic extension $\overline{\varphi} : \Delta \to \overline{D/\Gamma}$.*

We consider, as examples, 'tame' degenerations of surfaces of general type on the Noether lines, whose canonical images are rational ruled surfaces $\Sigma_d$ of degree $d$ (*i.e., type(d)* in the terminology in [H, I, p.363; II, p.127]). We denote by $S_0$ and $F$ the section of $\Sigma_d \to \mathbf{P}^1$ with $S_0^2 = -d$ and a fiber, respectively.

**(I)** Let $X$ be a minimal algebraic surface on the Noether line $c_1^2 = 2p_g - 4$, where $c_1 := c_1(X)$ is the first Chern class of $X$ and $p_g := p_g(X)$ is the geometric genus of $X$. We assume that $X$ is of type(d) in the above sense. Then, by [H, I, Theorem 1.6.iii], such a surface $X$ occurs, via the canonical map, as the minimal resolution of singularity of the double covering of $\Sigma_d$ branched along a curve $B \in |6S_0 + (p_g + 2 + 3d)F|$ with at most simple singularities, where $p_g \geq \max\{d+4,\, 2d-2\}$ and $p_g - d$ is even. The $p_g$ of such surfaces range over all integers $\geq 4$ and the fundamental groups $\pi_1$ are trivial [H, I, Theorem 3.4].

**(II)** Let $Y$ be a minimal algebraic surface on the Noether line $c_1^2 = 2p_g - 3$. We assume that $Y$ is of type(d). Then, by [H, II, Theorem (1.3.A)], such a surface $Y$ occurs, via the canonical map, as the contraction of a unique exceptional curve of the first kind on the minimal resolution of singularity of the double covering of $\Sigma_d$ branched along a curve $C \in |6S_0 + (p_g + 4 + 3d)F|$ where $p_g \geq 2d - 1$ and $p_g - d$ is even. $C$ has two quadruple points $x$, $y$, which may be infinitely near, on the same fiber other than simple singularities on the proper transform of $C$ to the blown-up of $\Sigma_d$ with center $x$ and $y$. The $p_g$ of such surfaces range over all integers $\geq 4$ and the fundamental groups $\pi_1$ are trivial [H, II, Theoren(4.8)].

Between these two series of surfaces, we cosider the following two types of degenerations of branch loci.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

**(II) → (I):** The $C_t \in |6S_0 + (p_g + 4 + 3d)F|$ $(t \in \Delta^*)$ on $\Sigma_d$, with two quadruple points on a fiber $F$ other than simple singularities, degenerate into $C_0 = B + 2F$, where $B \in |6S_0 + (p_g + 2 + 3d)F|$ has two ordinary double points on the fiber $F$ other than the simple singularities come from that on the $C_t$.

**(I) → (II):** The $B_t \in |6S_{0,d} + (p_g + 2 + 3d)F_d|$ $(t \in \Delta^*)$ on $\Sigma_d$ have one double point $P_t$ at which the two branches have contact 2 and two ordinary double points $A_t$, $A'_t$ and possibly other simple singularities. The three points $P_t$, $A_t$, $A'_t$ crash to make up one triple point $P$ on $B_0$. $P$ is apart from the minimal section $S_{0,d}$ and each pair of the three branches have contact 2 at $P$. $B_0$ is smooth at the other three intersection points with the fiber $F_d$ containing $P$. Blowing up $\Sigma_d$ at $P$ and contracting the proper transform of $F_d$, the total transform of $B_0$ becomes $C + 2F_{d-1}$ with $C \in |6S_{0,d-1} + ((p_g - 1) + 4 + 3(d-1))F_{d-1}|$ on $\Sigma_{d-1}$, which has two quadruple points on one fiber other than the simple singularities come from that on $B_0$.

According to these, we have two types of semi-stable degenerations of surfaces on the Noether lines.

**(II) → (I):** $f : \mathcal{X} \to \Delta$ is a semi-stable degeneration whose smooth fibers $X_t := f^{-1}(t)$ $(t \in \Delta^*)$ are minimal surfaces of type(d) with $c_1(X_t)^2 = 2p_g(X_t) - 3$, $p_g(X_t) \geq 4$. $X_0 := f^{-1}(0) = Y \cup V$, where $Y$ is a minimal surface of type(d) with $c_1(Y)^2 = c_1(X_t)^2 - 1 = 2p_g(Y) - 4$, $p_g(Y) = p_g(X_t) \geq 4$, and $V \simeq \mathbf{P}^2$ intersets with $Y$ along a smooth conic on $\mathbf{P}^2$ hence $Y \cap V$ has self-intersection $-4$ on $Y$. We need not extend the base in the semi-stable reduction in this case.

**(I) → (II):** $g : \mathcal{Y} \to \Delta$ is a semi-stable degeneration whose smooth fibers $Y_t := g^{-1}(t)$ $(t \in \Delta^*)$ are minimal surfaces of type(d) with $c_1(Y_t)^2 = 2p_g(Y_t) - 4$, $p_g(Y_t) \geq 5$. $Y_0 := g^{-1}(0) = X \cup V$, where $X$ is a minimal surface of type(d) with $c_1(X)^2 = c_1(Y_t)^2 - 1 = 2p_g(X) - 3$, $p_g(X) = p_g(Y_t) - 1 \geq 4$, $V$ is a rational surface, and $X \cap V$ is a smooth elliptic curve with self-intersection $-1$ on $X$ hence this is the exceptional curve of the minimal resolution of a simple elliptic singularity of type $\widetilde{E}_8$. We need to take a ramified double covering of the base in the semi-stable reduction in this case.

Thus two series of smooth families of surfaces with $(p_g, c_1^2)$ on the Noether lines in question are connected by the above 'tame' degenerations:

$$\begin{array}{cccccccccccc}
(II): & (4,5) & & (5,7) & & \cdots & & (p,2p-3) & & (p+1,2p-1) & & \cdots \\
 & \downarrow & \nwarrow & \downarrow & \nwarrow & \cdots & \nwarrow & \downarrow & \nwarrow & \downarrow & \nwarrow & \cdots \\
(I): & (4,4) & & (5,6) & & \cdots & & (p,2p-4) & & (p+1,2p-2) & & \cdots
\end{array}$$

**Remark(6.1).** Ashikaga and Konno [AK] showed that degenerations of the above type are observed widely in the geography of surfaces of general type.

For the above semi-stable degenerations, we observe the Clemens-Schmid sequences [C] and the Mayer-Vietoris sequences:

$$H^2(\mathcal{X}, \mathcal{X} - X_0) \to H^2(X_0) \to H^2(X_t) \xrightarrow{N} H^2(X_t),$$
$$H^1(Y \cap V) \to H^2(X_0) \to H^2(Y) \oplus H^2(V),$$
$$H^2(\mathcal{Y}, \mathcal{Y} - Y_0) \xrightarrow{\alpha} H^2(Y_0) \to H^2(Y_t) \xrightarrow{N} H^2(Y_t),$$
$$H^1(X) \oplus H^1(V) \to H^1(X \cap V) \xrightarrow{\beta} H^2(Y_0) \to H^2(X) \oplus H^2(V).$$

Since $H^1(Y \cap V) = 0$, we see that $H^2(X_0)$ carries a Hodge structure of pure weight 2 hence the monodromy weight filtration $W = W(N)[-2]$ is trivial, *i.e.*, $0 = W_1 \subset W_2 = H^2(X_t)$. As for the second family, since $H^1(X) \oplus H^1(V) = 0$ and $H^2(\mathcal{Y}, \mathcal{Y} - Y_0) \simeq H_4(Y_0) \xrightarrow{\sim} H_4(X) \oplus H_4(V)$, we see that $\beta$ is injective and $\operatorname{Im}\beta \cap \operatorname{Im}\alpha = 0$ in $H^2(Y_0)$ by the reason of weight. It follows that $W_0 = 0$ and $W_1 \xleftarrow{\sim} H^1(X \cap V)$. Hence $W$ satisfies the condition $(*)$ and we can apply Theorem to the period map $\varphi : \Delta^* \to D/\Gamma$ associated to the variation of Hodge structure of weight 2 arising from the smooth family $g : \mathcal{Y} - Y_0 \to \Delta^*$and obtain the holomorphic extension $\overline{\varphi} : \Delta \to \overline{D/\Gamma}$. Thus we can discuss about the differential $d\overline{\varphi}(0)$ of $\overline{\varphi}$ at $0 \in \Delta$.

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# Kummer Theory for Elliptic Curves

Ralph Greenberg

June 1, 1994

This lecture is related to a project of John Coates and myself concerning the "Selmer group" for an elliptic curve (and more generally for abelian varieties of arbitary dimension) over infinite Galois extensions $K$ of $\mathbf{Q}$ such that $Gal(K/\mathbf{Q})$ is a $p$-adic Lie group. The definition of the Selmer group involves the global and local Kummer theory for the elliptic curve. It was introduced originally to study the rank of the group of rational points $E(\mathbf{Q})$ on an elliptic curve $E$ defined over $\mathbf{Q}$.

Let $E[p^\infty]$ denote the $p$-power torsion points in $E(\overline{\mathbf{Q}})$. Let $G_{\mathbf{Q}}$ denote the absolute Galois group of $\mathbf{Q}$. With every element a in $E(\mathbf{Q}) \otimes (\mathbf{Q}_p/\mathbf{Z}_p)$ one can associate a cocycle class $k(a)$ in $H^1(G_{\mathbf{Q}}, E[p^\infty])$. The map $k$ (the Global Kummer homomorphism) is injective but far from surjective. The image is restricted by local obstructions arising from the local Kummer homomorphisms. For every prime $v$, the absolute Galois group of the $v$-adic numbers $\mathbf{Q}_v$ can be regarded as a subgroup of $G_{\mathbf{Q}}$: $G_{\mathbf{Q}_v} \subset G_{\mathbf{Q}}$. If $f \in Im(k)$, then $f|G_{\mathbf{Q}_v}$ must be in the image of each local Kummer homomorphism $k_v$. These local conditions define the Selmer group $S_E(\mathbf{Q})$. Thus $Im(k) \subset S_E(\mathbf{Q})$ and the quotient is the so-called Tate-Shafarevich group (or, more precisely, its $p$-primary subgroup). The Tate-Shafarevich group is conjecturally finite. We study the local Kummer homomorphisms. If $v \neq p$, then the result is quite simple: $Im(k_v) = 0$. If $v = p$, then $Im(k_v)$ is rather mysterious in general. But if $E$ has good, ordinary reduction at $p$, then there is a simple description of $Im(k_p)$.

Over infinite extensions $K$ of $\mathbf{Q}$ which are "deeply ramified" at all places of $K$ above $p$, it turns out that the image of the local Kummer homomorphism at places over $p$ has a very simple description (almost as simple as at places not dividing $p$, where the image is zero). More precisely, there is a subgroup $C$ of $E[p^\infty]$, which is invariant under the action of $G_{\mathbf{Q}_p}$. The description of $C$ is quite simple. For example, if $E$ has good reduction at $p$, then $C$ is just the points on $E[p^\infty]$ in the kernel of reduction. The image of the local Kummer homomorphism then turns out to be precisely the cocycle classes in $H^1(G_{K_v}, E[p^\infty])$ which contain a cocycle with values in $C$.

The main ingredient in the proof is a certain version of Hilbert's theorem 90 for formal groups. If $F$ is any commutative formal group law defined over

$\mathbf{Z}_p$ and if $\overline{\mathcal{M}}$ is the maximal ideal of $\overline{\mathbf{Q}_p}$, then $G_{\mathbf{Q}_p}$ ( and hence $G_{K_v}$ ) acts on the group $F(\overline{\mathcal{M}})$ of "points on the formal group" with coordinates in $\overline{\mathcal{M}}$. The result we prove is: $H^1(G_{K_v}, F(\overline{\mathcal{M}})) = 0$ when $K_v$ is a deeply ramified extension of $\mathbf{Q}_p$. This allows one to easily deduce the above result concerning the local Kummer homomorphism over $K_v$.

1994年6月22日

# 並行計算のモデルについて

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コンピュータ科学
・具体的対象はコンピュータ、ソフトウエア、ネットワーク、・・・
・その理論的基盤は数学、数理論理学と共通のものが多い：　カテゴリー　→図1 (次ページ)
・その成立はコンピュータに先行する：　λ計算　Church 1932/33
・その対象はコンピュータにとどまらない：　言語学
　　　　　　→　物質科学に対する広領域としての「形式科学」

・並行性をプロセス（エージェント）の通信による相互作用ととらえる：
C. A. R. Hoare, Communicating Sequential Processes (CSP)
R. Milner, Communication and Concurrency (A Calculus of Communicating Systems, CCS)
CCSの基礎　→　集中講義
CCSの発展（Milnerによるπ計算）を紹介する

**N**　name　　$x, y, z, \cdots$
**P**　term or agent　　$P, Q, R, \cdots$

prefix $\pi.P$　　　$\pi$ は $\bar{x}y$　または $x(y)$

$\bar{x}y.P$　$x$ という名前のリンクから名前 $y$ を送信した後　$P$ として振る舞う。
$x(y).P$　$x$ という名前のリンクから名前 $z$ を受信したら、$P$ における $y$ の自由な出現を $z$ でおきかえた $P\{z/y\}$ として振る舞う。

$P ::= \Sigma_{i \in I} \pi_i . P \quad | \quad 0 \quad | \quad P \mid Q \quad | \quad !P \quad | \ (y)P$
有限 summation　　composition　replication　restriction

$n(0) = \phi$
$n(\bar{x}y.P) = n(x(y).P) = \{x, y\} \cup n(P)$
$n((y)P) = \{y\} \cup n(P)$
$n(P \mid Q) = n(P) \cup n(Q)$

　$x \in n(P)$ のとき $x$ は $P$ に何回かあらわれるが、その場所を $x$ の出現という。項 $x(y).P$ や $(y)P$ における $y$ の出現は束縛されているといい、　$P$ を変数 $y$ のスコープという。束縛されていない出現を自由な出現という。$n(P)$ のうち束縛された出現をもつ変数全体を $bn(P)$、自由な出現をもつ変数全体を $fn(P)$ を記す。
$n(P) = bn(P) \cup fn(P)$ であるが、一般には $bn(P) \cap fn(P) \neq \phi$.

Structual congruence:
・$P \equiv Q$　if $P$ is $\alpha$-convertible to $Q$
　$\alpha$-conversion　束縛変数名のつけかえ（λ計算の用語）
　$x(y).P \equiv x(y').P\{y'/y\}$　　$y' \notin n(P) \cup \{x\}$
　$(x)P \equiv (x')P\{x'/x\}$　　$x' \notin n(P)$
・$P \mid 0 \equiv P$, $P \mid Q \equiv Q \mid P$, $P \mid (Q \mid R) \equiv (P \mid Q) \mid R$, $!P \equiv P \mid !P$
・$(x)0 \equiv 0$, $(x)(y)P \equiv (y)(x)P$
・$(x)(P \mid Q) \equiv P \mid (x)Q$　　$x \notin fn(P)$ の場合
**注**　$x \in fn(P)$、すなわち $x \in fn(P \mid (x)Q) \cap bn(P \mid (x)Q)$ なら $\alpha$-conversion により
　右辺 $\equiv P \mid (x')Q\{x'/x\}$　　$x' \notin n(P \mid Q)$
とした上でこの規則を適用する。

Reduction:
COM:　$\cdots + x(y).P + \cdots \mid \cdots + \bar{x}z.Q + \cdots \ \rightarrow \ P\{z/y\} \mid Q$

PAR:　$\dfrac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q}$

RES:　$\dfrac{P \rightarrow P'}{(y)P \rightarrow (y)P'}$

STRUCT:　$\dfrac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'}$

$x(y).x(z)$, $\bar{x}y.\bar{x}z$ を $x(y)(z)$, $\bar{x}yz$ と略記すると、reduction

$$\bar{x}uv.P' \mid x(yz).Q' \mid x(yz).R' \to P' \mid Q'\{u/y\}\{v/z\} \mid x(yz).R'$$
$$\bar{x}uv.P' \mid x(yz).Q' \mid x(yz).R' \to P' \mid x(yz).Q' \mid R'\{u/y\}\{v/z\}$$

とともに

$$\bar{x}uv.P' \mid x(yz).Q' \mid x(yz).R' \to P' \mid x(z).Q'\{u/y\} \mid x(z).R'\{v/y\}$$

も可能である。最後のreductionを許さないように記法を修正する：

$x(w).w(y_1).\cdots.w(y_n)$ を $x(y_1\cdots y_n)$ と記す。

$(w)\bar{x}w.\bar{w}y_1.\cdots.\bar{w}y_n$ を $\bar{x}y_1\cdots y_n$ と記す。

自動車電話

$$CAR(talk, switch) \overset{def}{=} talk.CAR(talk, switch) + switch(talk'\ switch').CAR(talk', switch')$$

name = (talk, switch, give, alert)

$$BASE(name) \overset{def}{=} \overline{talk}.BASE(name) + give(talk'\ switch').\overline{switch}talk'\ switch'.IDLEBASE(name)$$

$$IDLEBASE(name) \overset{def}{=} alert.BASE(name)$$

$name_i$ = $(talk_i, switch_i, give_i, alert_i)$ (i=1, 2)

N （集合としての） $name_1$, $name_2$の和集合

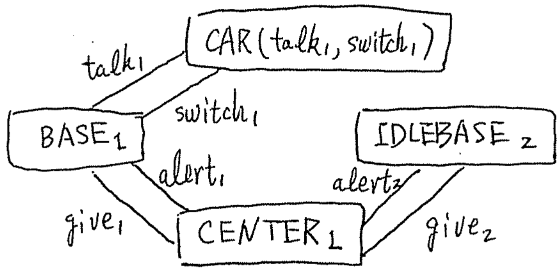
BASE($name_i$), IDLEBASE($name_i$) を $BASE_i$, $IDLEBASE_i$ と記す.

$$CENTER_1 \overset{def}{=} \overline{give_1}talk_2switch_2.\overline{alert_2}.CENTER_2$$
$$CENTER_2 \overset{def}{=} \overline{give_2}talk_1switch_1.\overline{alert_1}.CENTER_1$$

$$SYSTEM_1 \overset{def}{=} (N)(CAR(talk_1, switch_1) \mid BASE_1 \mid IDLEBASE_2 \mid CENTER_1)$$
$$SYSTEM_2 \overset{def}{=} (N)(CAR(talk_2, switch_2) \mid BASE_2 \mid IDLEBASE_1 \mid CENTER_2)$$

$$SYSTEM_1 \equiv (N)(CAR(talk_1, switch_1) \mid BASE_1 \mid IDLEBASE_2 \mid CENTER_1)$$
$$\to (N)(CAR(talk_1, switch_1) \mid \overline{switch_1}talk_2switch_2.IDLEBASE_1 \mid IDLEBASE_2 \mid \overline{alert_2}.CENTER_2)$$
$$\to (N)(CAR(talk_2, switch_2) \mid IDLEBASE_1 \mid IDLEBASE_2 \mid \overline{alert_2}.CENTER_2$$
$$\to (N)(CAR(talk_2, switch_2) \mid IDLEBASE_1 \mid BASE_2 \mid CENTER_2 \equiv SYSTEM_2$$

図 2




bit 1993年10月号

Milnerの Turing賞受賞記念講演

図 1

(集合) 非有基的集合

カテゴリー

(関数) λ計算

(位相) 領域理論

"A way to chaos from stable vector fields"

by

R. Labarca

Universidad de Santiago de Chile

Hokkaido University, Department of Mathematics, July 1994.

Let $M$ be a $C^\infty$, $m$-dimensional, compact, connected, boundaryless, Riemannian manifold. $\mathfrak{X}^r(M)$ will denote the set of $C^r$ vector fields on $M$ [ that is the set of $C^r$-sections of the fibre bundle $TM \to M$ ]

We endowed this set with the $C^r$-topology. For $X \in \mathfrak{X}^r(M)$, $X_t$ will denote its flow.

<u>Definition</u>: A cycle for the vector field $X$ is a compact, invariant set $\Gamma \subset M$ formed by:

(i) a finite number of singularities and periodic orbits $\Gamma_0 = \{\sigma_0, .., \sigma_m\}$;

(ii) The complement $\Gamma_1 = \Gamma \setminus \Gamma_0$ is a set of non-periodic regular trajectories, of the vector field $X$, which satisfies:

$(CC)_1$ For any trajectory $\gamma \subset \Gamma_1$ there exists $0 \le i \le n$ such that $\omega(\gamma) \subset \sigma_{(i+1) \bmod (n+1)}$ and $\alpha(\gamma) \subset \sigma_i$;

$(CC)_2$ Given $0 \le i \le n$ there exists one trajectory $\gamma \subset \Gamma_1$ such that $\omega(\gamma) \subset \sigma_{(i+1) \bmod (n+1)}$ and $\alpha(\gamma) \subset \sigma_i$.

Here $\omega(\gamma)$ $(\alpha(\gamma))$ denotes the $\omega$-limit set (resp. the $\alpha$-limit set) of the trajectory $\gamma$, that is

$\omega(\gamma) = \{ y \in M$ / there exists a sequence $(x_i) \subset \gamma$, $x_{i+1} = X_{t_i}(x_i)$, $t_i > 0$, $i \ge 0$ such that $x_i \to y \}$

(resp $\alpha(\gamma) = \{ y \in M$ / there exists a sequence $(x_i) \subset \gamma$, $x_{i+1} = X_{t_i}(x_i)$, $t_i < 0$; $i \ge 0$ such that $x_i \to y \}$ )

We will call the cycle $\Gamma \subset M$:

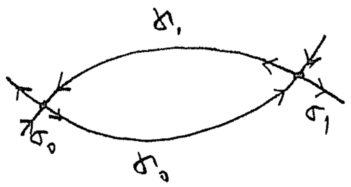
isolated: if there exists a neighborhood $U$, $\Gamma \subset U$ such that $\bigcap_t X_t(U) = \Gamma$;

hyperbolic: if all the critical elements in $\Gamma_0$ are hyperbolic (for a singularity $p$ this means that the linear isomorphism $DX(p)$ does not have eigenvalues on the imaginary axes; for a periodic orbit $\sigma$ this means that: for any point $q \in \sigma$ and any associated Poincare map $P_q$ the linear isomorphism $DP_q(q)$ does not have eigenvalues on the unitary circle);

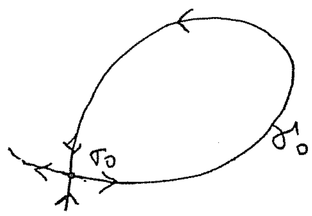
minimal: if no proper subset $S \subset \Gamma$ exist such that $S$ itself is a cycle;

Singular: if it contains a singularity.

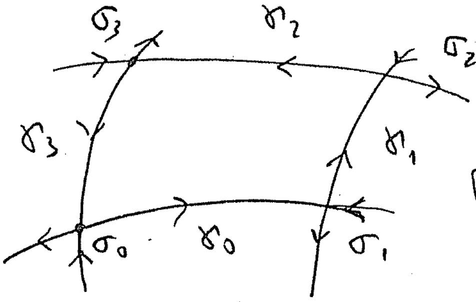
Examples of Cycles



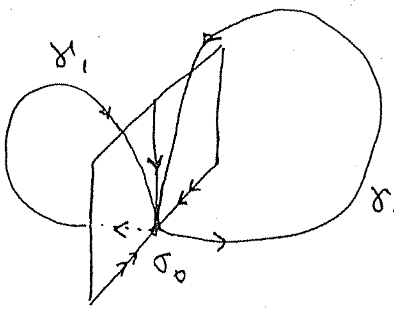

$\Gamma = \{\sigma_0, \gamma_0, \sigma_1, \gamma_1\}$



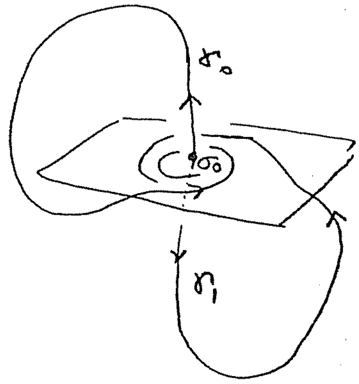

$\Gamma = \{\sigma_0, \gamma_0\}$

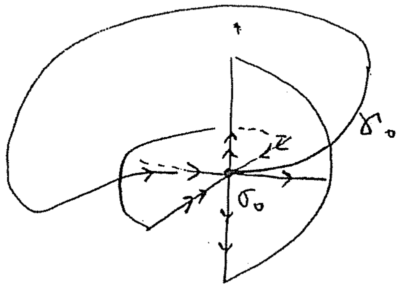



$\Gamma = \{\sigma_0, \gamma_0, \sigma_1, \gamma_1, \sigma_2, \gamma_2, \sigma_3, \gamma_3\}$

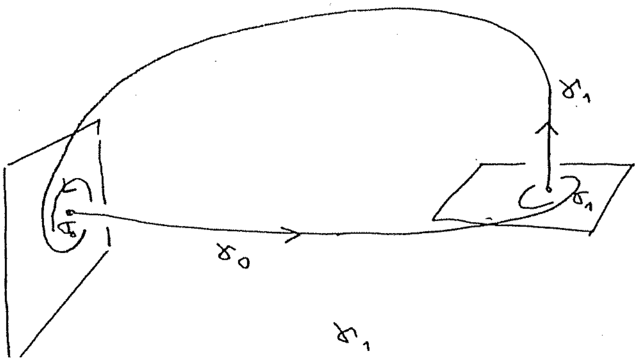



$\Gamma = \{\sigma_0, \gamma_0, \gamma_1\}$

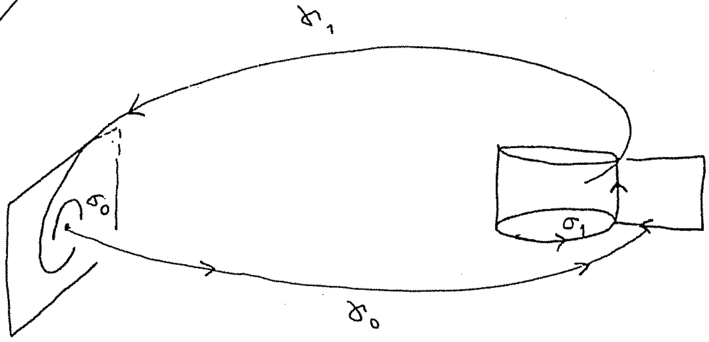


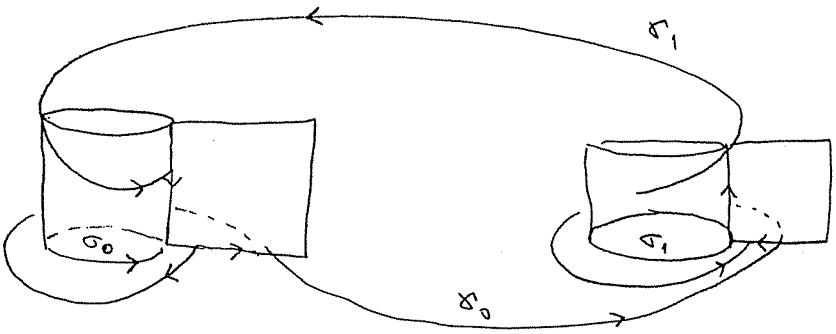

$\Gamma = \{\sigma_0, \gamma_0\}$




$\Gamma = \{\sigma_0, \gamma_0, \sigma_1, \gamma_1\}$




$\Gamma = \{\sigma_0, \gamma_0, \sigma_1, \gamma_1\}$




$\Gamma = \{\sigma_0, \gamma_0, \sigma_1, \gamma_1\}$

Definition We will say that the cycle $\Gamma \subset M$ is persistant if there exists a neighborhood $\mathcal{U}_X \subset \mathcal{X}^r(M)$ of the vector field $X$ such that any $Y \in \mathcal{U}_X$ has a cycle $\Gamma_Y$ which is obtained by analityc continuation of the cycle $\Gamma$.

Remark. A cycle $\Gamma \subset M$ is persistent if and only if
(i) it is hyperbolic ; (ii) it is non-singular and
(iii) any trajectory $\gamma \subset \Gamma_1 = (\Gamma \setminus \Gamma_0)$ is a trajectory of transversal intersection between $W^s(\omega(\gamma))$ and $W^u(\alpha(\gamma))$.
Moreover, a persistent cycle cannot be isolated.

Definition We will say that the cycle $\Gamma \subset M$ is codimension $k$ persistant if there is a neighborhood $\mathcal{U}_X \subset \mathcal{X}^r(M)$ of the vector field $X$ and a $C^1$- submersion $h: \mathcal{U}_X \to \mathbb{R}^k$ such that $X \in h^{-1}(0)$ and any $Y \in h^{-1}(0)$ has a cycle $\Gamma_Y$ which is obtained by analityc continuation of the cycle $\Gamma$.

Open problems

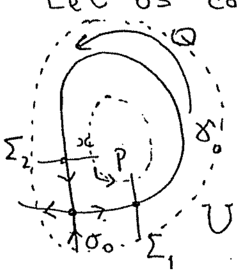
1) To characterize those cycles $\Gamma \subset M$ which are codimension $k$ persistant and isolated.
3) Let $\Gamma \subset M$ be a isolated, codimension $k$ persistant cycle for the vector field $X$. $U_0 \subset \mathbb{R}^k$ a small neighborhood of the origin and $\xi : U_0 \to \mathcal{U}_X$ a $C^1$-map transversal to $h^{-1}(0)$ such that $\xi(0) = X$
open probl: To study the bifurcation which appears in the family $\xi$ in the isolating neighborhood $U$

Remarks: 1) a codimension $k$ persistant, isolated cycle $\Gamma \subset M$ cannot contains more than $k$-singularities.

2) There are non-hyperbolic, codimension one, persistant, isolating cycles;

3) all the critical elements of the cycle must be of the saddle-type.

Example. Let us consider the cycle $\Gamma = \{\sigma_0, \gamma_0\}$ where $\sigma_0$ is a singularity which satisfies $\frac{\lambda_1}{\lambda_2} \neq 1$ where $-\lambda_1$ and $-\lambda_2$ are the eigenvalues associated to the linear isomorphism.

$\Gamma =$ 


By integrating this equation we can see that $P(x) = x^{\frac{\lambda_1}{\lambda_2}} H(x)$ where $H$ is a continuous map which satisfies $H(0) \neq 0$.

The map $Q : V_1 \subset \Sigma_1 \to \Sigma_2$ is a diffeomorphism. So we have defined a map $Q \circ P : V_2 \subset \Sigma_2^+ \longrightarrow \Sigma_2$

Take $\mathcal{U}_X$ a small neighborhood of the vector field $X$. Any $Y \in \mathcal{U}_X$ will have a singularity $\sigma_0(Y)$, near $\sigma_0$. Let $p^s(Y)$ $(p^u(Y))$ be the intersection of $W^s(\sigma_0(Y))$ (resp. $W^u(\sigma_0(Y))$) with $\Sigma_2$. This intersection define a $C^r$-submersion $p^s : \mathcal{U}_X \to \Sigma_2$ (resp. $p^u : \mathcal{U}_X \to \Sigma$) and the condition $p^s(Y) = p^u(Y)$ defines a $C^r$-submanifold $N \subset \mathcal{U}_X$ of codimension one.

From this we get that $\Gamma$ is codimension one persistant. Let us assume $p^s(Y) = 0$ in $\Sigma_2$.

- Assume $\alpha = \frac{\lambda_1}{\lambda_2} < 1$. In this case any $Y \in \mathcal{U}_X$ such that $p^u(Y) < 0$ must have a repeling periodic orbit in $U$ ($U$ is the isolating neighborhood for $\Gamma$). For any $Y \in \mathcal{U}_X$ such that $p^u(Y) > 0$ we have $\bigcap_t Y_t(U) = \sigma_0(Y)$.

- For $\alpha = \frac{\lambda_1}{\lambda_2} > 1$. In this case any $Y \in \mathcal{U}_X$ such that $p^u(Y) > 0$ must have a contracting periodic orbit in $U$. For any $Y \in \mathcal{U}_X$ such that $p^u(Y) < 0$ we have

$$\bigcap_t Y_t(U) = \sigma_0(Y).$$

- Now let us consider a 3-dimensional, hyperbolic, singular cycle $\Gamma \subset M^3$ which contains a unique singularity $\sigma_0(X)$ and periodic orbits $\sigma_1(X), \ldots, \sigma_n(X)$, $n \geq 1$.

Let us assume the following regularity conditions:

(1) $\Gamma = \{\sigma_0(X), \gamma_0(X), \sigma_1(X), \gamma_1^1(X), \gamma_1^2(X), \ldots, \sigma_n(X), \gamma_n^1(X), \gamma_n^2(X)\}$ where $W_i^u = W^u(\sigma_i(X))$ transversally intersects $W^s_{(i+1)\bmod(n+1)} = W^s(\sigma_{(i+1)\bmod(n+1)}(X))$ along $\gamma_1^i(X) \cup \gamma_2^i(X)$ $i = 1, \ldots, n$.

Let $\mathcal{U}_X$ be a small neighborhood of $X$ in $\mathfrak{X}^r(M^3)$ with the usual $C^r$-topology.

(2) for any $Y \in \mathcal{U}_X$ the eigenvalues of $D_{\sigma_0(Y)}(Y): T_{\sigma_0(Y)}M^3 \circlearrowleft$ are real numbers $-\lambda_3(Y) < -\lambda_1(Y) < 0 < \lambda_2(Y)$ and satisfy a k-Stenberg condition, k big enough, to guarantee that we have $C^3$-linearizing coordinates, with $C^2$-depending on $Y \in \mathcal{U}_X$, in a neighborhood of $\sigma_0(Y)$;

(3) for every $p \in \gamma_0(X)$ and every invariant manifold $W(\sigma_0(X))$, of $X$, passing through $\sigma_0(X)$ (and $p$) and tangent (at $\sigma_0(X)$) to the space spanned by the eigenvectors associated to $-\lambda_1(X)$ and $\lambda_2(X)$ we have: $T_p(W(\sigma_0(X))) + T_p(W^s(\sigma_1(X))) = T_pM^3$;

(4) $\Gamma$ is isolated;

(5) Let $Q_i$ be a transversal section to $q_i(Y) \in \sigma_i(Y)$. Denote by $P_i(Y)$, $V_i \subset Q_i \to Q_i$ the first return map defined in a neighborhood of $q_i(Y)$. For any $Y \in \mathcal{U}_X$ we will assume that the eigenvalues of $D_{q_i}P_i : T_{q_i}V_i \to T_{q_i}Q_i$ are real numbers and satisfy a k-Stenberg condition, k big enough, to guarantee that we have $C^3$-linearizing coordinates, with $C^2$-depending on $Y \in \mathcal{U}_X$, in a neighborhood of $q_i(Y)$;

(6) The number $\alpha(Y) = \dfrac{\lambda_1(Y)}{\lambda_2(Y)}$ is different from 1 and

$$\beta(Y) = \frac{\lambda_3(Y)}{\lambda_2(Y)} \text{ satisfies } \beta(Y) > \alpha(Y) + 2$$

For the case $\alpha(X) < 1$ we will say that the cycle is expanding. In the case $\alpha(X) > 1$ we will call it contracting.

- Denote by $\Gamma(Y, U) \subset M$ the set $\bigcap_t Y_t(U)$ for $Y \in \mathcal{U}_X$

Remarks: It is easy to see that there exists a codimension one submanifold $N \subset \mathcal{U}_X$ containing $X$, such that:

(i) $Y \in N$ implies that $\Gamma(Y, U) = \{\sigma_0(Y), \gamma_0(Y), \ldots, \gamma_m^2(Y)\}$ that is $\Gamma$ is codimension one persistant;

(ii) $\mathcal{U}_X \setminus N$ has two connected components and one of them, that we shall denote $\mathcal{U}_-$, is such that $Y \in \mathcal{U}^-$ implies $\Gamma(Y, U) = \{\sigma_0(Y), \sigma_1(Y), .., \sigma_m(Y)\}$ and

(iii) Chaotic behavior and bifurcations for the maximal invariant set can only appears for $Y \in \mathcal{U}^+ = \mathcal{U}_X \setminus (N \cup \mathcal{U}^-)$.

Define, $\mathcal{U}^+_H$ as the set of $Y \in \mathcal{U}^+$ such that $\Gamma(Y, U)$ consists of $\sigma_0(Y)$ plus a transitive hyperbolic set and $\mathcal{U}^+_A$ as the set of $Y \in \mathcal{U}^+$ such that $\Gamma(Y, U)$ consists of $\sigma_0(Y)$, a transitive hyperbolic set and a hyperbolic attracting periodic orbit $\sigma(Y)$.

We have the following theorems:

<u>Theorem 1</u> 1) $\mathcal{U} \setminus (\mathcal{U}^+_H \cup \mathcal{U}^+_A)$ is laminated by codimension one, $C^1$-submanifolds such that; all elements in the same lamina have the same dynamics in the neighborhood $\bar{U}$ [ that is given a lamina $L \subset \mathcal{U}^+$ and $Y_1, Y_2 \in L$ then there exists a homeomorphism $h: U ⟲$ which is a topological equivalence between $Y_1|_{\bar{U}}$ and $Y_2|_{\bar{U}}$ ).

b) Any $Y \in \mathcal{U}^+_H \cup \mathcal{U}^+_A$ is structurally stable. and

c) For $Y \in (\mathcal{U} \setminus \mathcal{U}^+_H \cup \mathcal{U}^+_A)$ we have that $\Gamma(Y, U)$ is a chain recurrent expansive set.

Let now $\{X_\mu\} \subset \mathcal{U}_X$ be a one-parameter family of vector fields such that $X_{\mu=0} \in N$ and $\{X_\mu\}$ is transversal to $N$ at $\mu = 0$.

<u>Theorem 2</u> There exists $\nu = \nu(\{X_\mu\}) > 0$ such that

$$m(\{\mu ; 0 \leq \mu \leq \nu \text{ and } X_\mu \notin (\mathcal{U}^+_H \cup \mathcal{U}^+_A)\}) = 0$$

(Here $m(A)$ = Lebesgue measure of the set $A \subset \mathbb{R}$)

**Theorem 3** Let $\{Y_\mu\} \subset \mathcal{U}_{\bar{X}}$ be another one-parameter family transversal to $N$ at $\mu = 0$. There exists a homeomorphism $\rho : [0, \nu(\{X_\mu\})] \to [0, \nu(\{Y_\mu\})]$ and for each $\mu \in [0, \nu(\{X_\mu\})]$ a homeomorphism $h_\mu : U \circlearrowleft$ which is a topological equivalence between $X_\mu|_{\bar{U}}$ and $Y_{\rho(\mu)}|_{\bar{U}}$.

Moreover, for $\alpha(\bar{X}) < 1$ the map $\mu \mapsto h_\mu$ $[0, \nu] \to \mathrm{Homeo}(\bar{U})$, is continuous

**Theorem 4** For $\alpha(X) > 1$: the laminas which appears in Theorem 1 a), are of the following type:

1) Those laminas which presents saddle-node or flip bifurcations for periodic orbits;

2) those laminas which present a contracting singular cycle;

3) those laminas which present a homoclinic trajectory to $\sigma_0(X)$ and

4) those laminas which present a recurrent behavior for $\gamma_0(Y)$.

For $\alpha(X) < 1$: the laminas which appears in Theorem 1 a) are as in 2), 3) and 4) above. In 2) we change contacting" by expanding.

Theorems 1,2,3 for $\alpha(X)<1$ are contained in the paper " Explosion of Singular Cycles " by R. Bamón, R. Labarca, R. Mañé and M.J. Pacifico and will appear in Publ. Math. I.H.E.S.

Theorems 1,2,3,4) for $\alpha(X)>1$ are contained in "Bifurcations of contracting Singular cycles." by R. Labarca (preprint ICTP 94).

Abstract of the Colloquium Talk:

J. Tilouine

Since its setting by B. Mazur in 1987, the framework of deformations of Galois representations has proved very useful in many instances. We stress its link with geometric analogues as an introduction and present various refinements for which the representability of the deformation problem can be established. We discuss briefly at the end the question of constructing explicitely such deformations. More precisely, we explain first how to replace $GL_n$ by an arbitrary connected reductive group split at the prime under consideration. Then we define local problems. Some have geometric origin (like being cristalline at $p$, with filtration length less than $\frac{p-1}{2}$) other are partly originating from Algebraic Geometry and partly from combinatorics of reductive groups, like the notion of $\mathcal{P}$-nearly ordinarity. The relation between Algebraic Geometry and this combinatorics lies in the theory of automorphic forms and Shimura variety.

We show that the problem of deformations of a representation satisfying this kind of local conditions by representations of the same type is representable. We ~~formulate~~ speculate in some case ~~conjecture about~~ or the possible identity of ~~such~~ the universal ring and a local Hecke algebra involving automorphic forms on the dual group $\widehat{G}$ (in the split case).

Richard Pink

This is a short synopsis of my colloquium talk entitled

**Compactification of Drinfeld Modular Varieties: Local and Global Properties**

held at Hokkaido University, Sapporo, on Wednesday, July 13, 1994. It is a great pleasure to visit this university. Details and proofs of the statements below will appear elsewhere.

For overviews of Drinfeld modules and moduli schemes see Drinfeld refDrinfeld or Deligne-Husemoller [1]. Let $X$ be a smooth projective geometrically connected curve over a finite field $\mathbb{F}_q$ with $q$ elements. We fix a closed point $\infty \in X$ and abbreviate $U := X \smallsetminus \{\infty\}$ and $A := H^0(U, \mathcal{O}_U)$. Any non-zero ideal $I \subset A$ corresponds to a finite subscheme $D_I \subset U$ of degree $\deg(I) := \dim_{\mathbb{F}_q} A/I$. We abbreviate $U_I := U \smallsetminus D_I$. The degree of an element $0 \neq a \in A$ is defined as $\deg(a) := \deg((a))$.

For any commutative $\mathbb{F}_q$-algebra $R$ the ring of $\mathbb{F}_q$-linear endomorphisms of the additive group $\mathbb{G}_{a,R}$ consists of all polynomials of the form

$$u_0 X + u_1 X^q + u_2 X^{q^2} + \ldots \in R[X].$$

Composition of endomorphisms corresponds to substitution of polynomials. Thus the identity element is the polynomial $X$, and the endomorphism ring is generated over $R$ by the element $\tau := X^q$ subject to the relations $\tau u = u^q \tau$ for all $u \in R$. We denote this ring by $R\{\tau\}$.

Consider an $\mathbb{F}_q$-algebra homomorphism $\varphi : A \to R\{\tau\}$.

**Definition.** *$\varphi$ is called a* Drinfeld module of rank $r > 0$ *if and only if for all $0 \neq a \in A$*

$$\varphi(a) = \varphi_0(a) + \varphi_1(a) \cdot \tau + \ldots + \varphi_n(a) \cdot \tau^n$$

*with $n = r \deg(a)$ and $\varphi_n(a) \in R^*$.*

One sees easily that the coefficient $\varphi_0$ determines an algebra homomorphism $A \to R$, and therefore a morphism $\varphi_0^* : \operatorname{Spec} R \to U$.

For any non-zero ideal $I \subset A$ the intersection

$$\ker \varphi(I) := \bigcap_{0 \neq a \in I} \ker \varphi(a)$$

is a finite flat group scheme over $R$ of degree $q^{r \deg(I)}$. In the following we assume that it is étale; this is equivalent to the assertion that $\varphi_0^*$ factors through $U_I \subset U$. As usual, a level structure is a trivialization of this étale group scheme. Taking into account the natural action of $A$ (through $\varphi$) on $\ker \varphi(I)$, this leads to the following definition.

**Definition.** *A level structure of level $I$ is an $A$-linear homomorphism*

$$\lambda : V := (A/I)^r \longrightarrow R$$

*such that $\lambda(v) \in R^*$ for all $0 \neq v \in V$.*

Given a Drinfeld module together with a level structure $(\varphi, \lambda)$, we can obtain another one by conjugating with a unit $u \in R^*$, yielding $(u\varphi u^{-1}, u\lambda)$. We say that two pairs $(\varphi, \lambda)$ are isomorphic if and only if they can be obtained from each other by this process. The following fact is well-known:

**Theorem.** *If $I \neq A$, there exists an affine scheme $M^r_{A,I}$, smooth of relative dimension $r-1$ over $U_I$, and a bijection*

$$M^r_{A,I}(\operatorname{Spec} R) \;\cong\; \{(\varphi, \lambda) \text{ over } R \text{ up to isomorphy}\}$$

*that is functorial in $R$.*

The fact that this moduli scheme is in general not 'compact', i.e. not proper over $U_I$, can be seen easily by looking at the expansion

$$u\varphi(a)u^{-1} = \varphi_0(a) + u^{1-q}\varphi_1(a) + \ldots + u^{1-q^n}\varphi_n(a),$$

where the coefficients lie in the quotient field $K$ of a discrete valuation ring $R$. Assuming that the constant coefficient lies in $R$, we can certainly find $u \in K^*$ so that all remaining coefficients come to lie in $R$ as well. But to obtain an extension to a Drinfeld module over $R$ (i.e. one of constant rank) it is necessary that the highest coefficient be a unit in $R^*$, and in general (if $r \geq 2$) both conditions cannot be satisfied at the same time.

For certain applications to the Langlands-program for function fields, relating automorphic forms on the group $GL_r$ with $\ell$-adic representations of the absolute Galois group of the function field of $X$, it is very useful to dispose of good compactifications of this moduli space. One type of application is to the evaluation of the Lefschetz trace formula for Frobenius-Hecke-correspondences, in particular using a conjecture of Deligne. This conjecture had been proved before assuming the existence of a smooth toroidal compactification by the author [5] and independently by Shpiz [6] (with earlier results by Illusie and Zink), and has recently been proved in full generality by Fujiwara [3]. For other questions concerning local cohomology at the boundary it seems, however, still indispensable to have good explicit local and global information about a distinguished class of smooth toroidal compactifications of $M^r_{A,I}$. These matters will be explained elsewhere.

The main idea behind the following construction is to use the level structure as canonical global coordinates, with respect to which local charts for the desired compactifications can be defined and analyzed. In some ways this approach is similar to Mumford's theory of 'equations defining abelian varieties' [4] using algebraic theta-functions.

First we observe that the level structure alone determines a point of a certain moduli space, namely that of injective $\mathbb{F}_q$-linear homomorphisms from $V$ to the additive group, up to scalars. In explicit terms this moduli space is the closed subscheme

$$\Omega_V \subset T_V := \mathbb{G}_m^{V \setminus \{0\}} / \operatorname{diag}(\mathbb{G}_m)$$

given by the equations $x_{v+v'} = x_v + x_{v'}$ for all $v, v' \in V \setminus \{0\}$ such that $v + v' \neq 0$, and $x_{\alpha v} = \alpha x_v$ for all $v \in V \setminus \{0\}$ and $\alpha \in \mathbb{F}_q^*$. We shall use equivariant embeddings of the torus $T_V$.

**Definition.** *For any fan $\Sigma \subset Y_*(T_V)_{\mathbb{R}}$ we let $\Omega_{V,\Sigma}$ be the scheme-theoretic closure of $\Omega_V$ in the torus embedding $T_{V,\Sigma}$.*

Since families of points in $\Omega_V$ can approach the boundary of $T_{V,\Sigma}$ only in certain ways, it is natural to focus attention on fans which reflect this restriction. Suppose that $\lambda$ defines a point of $\Omega_V$ over the quotient field $K$ of a discrete valuation ring $R$, with normalized valuation ord. Then the behavior of this 'family' (equivalently, the degeneration behavior of the injective homomorphism $\lambda$) is determined by the cocharacter

$$y_\lambda := (\mathrm{ord}(\lambda_v))_v \in \mathbb{Z}^{V \setminus \{0\}} / \operatorname{diag}(\mathbb{Z}) = Y_*(T_V).$$

**Definition.** *An element $y \in Y_*(T_V)_{\mathbb{R}}$ is called* $\mathbb{F}_q$-adapted *if and only if, for every $v_0 \in V \setminus \{0\}$, the subset $\{0\} \cup \{ v \in V \setminus \{0\} \mid y_v \geq y_{v_0}\}$ is an $\mathbb{F}_q$-subspace of $V$. We let $C(V) \subset Y_*(T_V)_{\mathbb{R}}$ denote the set of all $\mathbb{F}_q$-adapted elements.*

It is easy to show that a cocharacter is $\mathbb{F}_q$-adapted if and only if it arises as $y_\lambda$ in the above way. The following result is elementary and not difficult.

**Theorem.** *(a) $C(V)$ is a finite union of convex rational polyhedral cones.*
*(b) For any smooth fan $\Sigma$ with support equal to $C(V)$, the scheme $\Omega_{V,\Sigma}$ is smooth and proper, and the complement $\Omega_{V,\Sigma} \setminus \Omega_V$ is a union of smooth divisors with at most normal crossings.*

The method for constructing smooth toroidal compactifications of Drinfeld moduli schemes follows the same pattern as for $\Omega_V$. The global coordinates are provided by the following proposition:

**Proposition.** *If $\deg(I) \gg 0$, then the morphism*

$$M^r_{A,I} \longrightarrow U_I \times \Omega_{V^r_I},\ (\varphi, \lambda) \mapsto (\varphi_0^*, \lambda)$$

*is a closed embedding.*

Next there is a notion of '$A$-adapted' elements of $Y_*(T_V)_{\mathbb{R}}$, whose definition is too cumbersome to explain here. Some of the important properties of the set $D(V)$ of all $A$-adapted elements (which also characterize it uniquely), are:

**Theorem.** *(a) $D(V)$ is a subset of $C(V)$.*
*(b) $D(V)$ is a finite union of convex rational polyhedral cones.*
*(c) A cocharacter is* $A$-adapted *if and only if some positive integral multiple arises from a point of $M^r_{A,I}$ over the quotient field of some discrete valuation ring.*

In analogy to the previous simpler situation we let $M^r_{A,I,\Sigma}$ denote the scheme-theoretic closure of $M^r_{A,I}$ in $U_I \times \Omega_{V^r_I,\Sigma}$, for any fan $\Sigma$. The central result is:

**Main Theorem.** *Suppose that $\Sigma$ is a fan with support equal to $D(V)$. Then*
*(a) $M^r_{A,I}$ is proper over $U_I$.*
*(b) If $\deg(I) \gg 0$ and $\Sigma$ is smooth, then the normalization of $M^r_{A,I,\Sigma}$ has only quotient singularities, and the complement $M^r_{A,I,\Sigma} \setminus M^r_{A,I}$ is the quotient by a finite group of a union of smooth divisors with at most normal crossings.*
*(c)* **Special case:** *Suppose that $A = \mathbb{F}_q[t]$. If $\deg(I) > 0$ and $\Sigma$ is smooth, then $M^r_{A,I,\Sigma}$ is smooth and the complement $M^r_{A,I,\Sigma} \setminus M^r_{A,I}$ is a union of smooth divisors with at most normal crossings.*

Since the construction is a priori global, the bulk of the proof of the Main Theorem consists of the local analysis of the resulting scheme. The necessary considerations are intimately related to those that lead to a uniformization theorem for degenerating Drinfeld modules over higher dimensional complete local rings. In another form, the ideas explained below had originally been conceived by Fujiwara in 1991.

Let $R$ be a normal noetherian complete local integral domain with quotient field $K$. A homomorphism $\varphi : A \to R\{\tau\}$ is called a *Drinfeld module in stable reduction form* if and only if its reduction modulo the maximal ideal $\mathfrak{m}$ of $R$ defines a Drinfeld module of positive rank. (In that case, it also induces a Drinfeld module over $K$, of rank at least that of the reduction.) Via the action of $R\{\tau\}$, the homomorphism $\varphi$ defines a structure of $A$-module on $K$.

**Definition.** *A* strict $\varphi$-lattice *is a finitely generated $A$-submodule $M \subset K$ with the properties*
*(a) $1/m \in R$ for all $0 \neq m \in M$,*
*(b) $m/m' \in R$ or $m'/m \in R$ for any $0 \neq m, m' \in M$, and*
*(c) for any integer $i \geq 0$ there are at most finitely many $0 \neq m \in M$ with $1/m \notin \mathfrak{m}^i$.*

Here (c) is a kind of discreteness condition, (a) says that the lattice should not be too dense, and condition (b) ensures that we can work with $M$ as if the ring $R$ were a valuation ring. For any strict $\varphi$-lattice $M$ the following product converges to an $F_q$-linear power series:

$$e_M(x) \;:=\; x \cdot \sum_{0 \neq m \in M} \left(1 - \frac{x}{m}\right) \;\in\; R\{\{\tau\}\} \;\subset\; R[[x]].$$

This is called the *exponential function* of $M$.

**Proposition.** *If $M$ is a strict $\varphi$-lattice, there exists a unique Drinfeld module in stable reduction form $\psi : A \to R\{\tau\}$ such that*

$$\psi(a) \circ e_M = e_M \circ \varphi(a)$$

*in $R\{\{\tau\}\}$ for all $a \in A$. Moreover the rank of $\psi$ over $K$ is equal to the rank of $\varphi$ over $K$ plus the generic rank of $M$.*

One can interpret the Drinfeld module $\psi$ as the 'quotient of $\varphi$ by $M$'. When $R$ is a valuation ring this is well-known and belongs under the heading 'Tate-uniformization'. Namely, in that case let $\hat{\bar{K}}$ denote the completion of an algebraic closure of $K$. Then $e_M$ defines an analytic function on all of $\hat{\bar{K}}$ which is known to be surjective. Since $M$ is, by assumption, an $A$-submodule, the defining property of $\psi$ can be expressed by the commutative diagram with exact rows

$$\begin{array}{ccccccccc} 0 & \longrightarrow & M & \longrightarrow & \hat{\bar{K}} & \longrightarrow & \hat{\bar{K}} & \longrightarrow & 0 \\ & & \downarrow \varphi(a) & & \downarrow \varphi(a) & & \downarrow \psi(a) & & \\ 0 & \longrightarrow & M & \longrightarrow & \hat{\bar{K}} & \longrightarrow & \hat{\bar{K}} & \longrightarrow & 0 \end{array}$$

Coming back to general $R$ observe that the definition of a strict $\varphi$-lattice applies to finite $A$-submodules and hence in particular to level structures. Thus a level structure $\lambda : V \to K$ of $\varphi$ over $K$ is called *strict* if and only if the image of $\lambda$ is a strict $\varphi$-lattice.

**Theorem.** *Let $\psi : A \to R\{\tau\}$ be a Drinfeld module in stable reduction form which has not good reduction, i.e. it does not define a Drinfeld module of constant rank over $R$. Suppose that $\psi$ possesses a strict level structure of level $I$, where $\deg(I) \gg 0$ (the bound depending on $A$ and the rank of $\psi$ over $K$). Then there exist a Drinfeld module in stable reduction form $\varphi : A \to R\{\tau\}$ and a strict $\varphi$-lattice $M$ such that*

$$\psi(a) \circ e_M = e_M \circ \varphi(a)$$

*for all $a \in A$ (i.e. $\psi$ is the quotient of $\varphi$ by $M$), and the rank of $\varphi$ over $K$ is strictly smaller than that of $\psi$. Moreover $\varphi$ again possesses a strict level structure of level $I$.*

Note that it is not asserted that $\varphi$ has good reduction; in fact, as Fujiwara observed, this cannot be achieved in general. But the last assertion of the theorem assures that it can be applied inductively, so that one always reaches a good reduction Drinfeld module in a finite number of steps.

This uniformization theorem leads to a good understanding of a certain class of degenerating Drinfeld modules over local rings of arbitrary dimension. The global construction of the compactification of $M^r_{A,I}$ is made such that the crucial strictness assumption holds locally on $M^r_{A,I,\Sigma}$. Thus, at least in heuristic sense, the uniformization theorem is the essential tool in the local analysis of our compactification.

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# $p$–adic $L$–functions and motives

**A.A.Panchishkin**

The subject of this colloqium is to describe some new general constructions of $p$–adic $L$–functions attached to arithmetical complex $L$–functions.

The starting point in the theory of zeta functions is the expansion of the Rieman zeta-function $\zeta(s)$ into the Euler product:

$$\zeta(s) = \prod_p (1-p^{-s})^{-1} \quad = \sum_{n=1}^{\infty} n^{-s} \quad (\mathrm{Re(s)} > 1)$$

The set of arguments $s$ for which $\zeta(s)$ is defined can be extended to all $s \in \mathbf{C}$, $s \neq 1$, and we may regard $\mathbf{C}$ as the group of all continuous quasicharacters $\mathbf{C} = \mathrm{Hom}(\mathbf{R}_+^\times, \mathbf{C}^\times)$, $\quad y \mapsto y^s$ of $\mathbf{R}_+^\times$. The special values $\zeta(1-k)$ at negative integers are rational numbers: $\zeta(1-k) = -\frac{B_k}{k}$, where $B_k$ are Bernoulli numbers and we know (by Sylvester – Lipschitz theorem) that $c \in \mathbf{Z}$ implies $c^k(c^k-1)\frac{B_k}{k} \in \mathbf{Z}$.

The theory of non-Archimedean zeta-functions originates in the work of Kubota and Leopoldt (1964) containing $p$–adic interpolation of these special values. Their construction turns out to be equivalent to classical Kummer congruences for the Bernoulli numbers, which we recall here in the following form. Let $p$ be a fixed prime number, $c > 1$ an integer prime to $p$. Put

$$\zeta_{(p)}^{(c)}(-k) = (1-p^k)(1-c^{k+1})\zeta(-k)$$

and let $h(x) = \sum_{i=0}^n \alpha_i\, x^i \in \mathbf{Z}_p[x]$ be a polynomial over the ring $\mathbf{Z}_p$ of $p$–adic integers such that $h(x) \in p^m\mathbf{Z}_p$ for all $x \in \mathbf{Z}_p$. Then we have that

$$\sum_{k=0}^{n} \alpha_i\, \zeta_{(p)}^{(c)}(-k) \in p^m\mathbf{Z}_p.$$

This property expresses the fact that the numbers $\zeta_{(p)}^{(c)}(-k)$ depend continuously on $k$ in the $p$–adic sense; it can be deduced from the known formula for the sum of $k$-th powers: $S_k(N) = \sum_{n=1}^{N-1} n^k = \frac{1}{k+1}[B_{k+1}(N) - B_{k+1}]$ in which $B_k(x) = (x+B)^k = \sum_{i=0}^k \binom{k}{i} B_i x^{k-i}$ denotes the Bernoulli polynomial. Indeed, all summands in $S_k(N)$ depend $p$–adic analytically on $k$, if we restrict ourselves to numbers $n$, prime to $p$, so that the desired congruence follows if we express the numbers $\zeta_{(p)}^{(c)}(-k)$ in terms of Bernoulli numbers.

The domain of definition of $p$–adic $L$–functions is the $p$–adic analytic Lie group $X_p = \mathrm{Hom}_{\mathrm{contin}}(\mathbf{Z}_p^\times, \mathbf{C}_p^\times)$ of all continuous $p$–adic characters of the profinite group $\mathbf{Z}_p^\times$, where $\mathbf{C}_p = \widehat{\overline{\mathbf{Q}}}_p$ denotes the Tate field (completion of an algebraic closure of the $p$–adic field $\mathbf{Q}_p$), so that all integers $k$ can be regarded as the characters $x_p^k : y \mapsto y^k$. The construction of Kubota and Leopoldt is equivalent to existence a $p$–adic analytic function $\zeta_p : X_p \to \mathbf{C}_p$ with a single pole at the point $x = x_p^{-1}$, which becomes a bounded holomorphic function on $X_p$ after multiplication by the elementary factor $(x_p\, x - 1)$ $(x \in X_p)$, and is *uniquely determined* by the condition $\zeta_p({x_p}^k) = (1 - p^k)\zeta(-k) \qquad (k \geq 1)$.

This result has a very natural interpretation in framework of the theory of non-Archimedean integration (due to Mazur): there exists a $p$–adic measure $\mu^{(c)}$ on $\mathbf{Z}_p^\times$ with values in $\mathbf{Z}_p$ such that $\int_{\mathbf{Z}_p^\times} x_p^k\, \mu^{(c)} = \zeta_{(p)}^{(c)}(-k)$.

The original construction of Kubota and Leopoldt was successesfully used by Iwasawa for the description of the class groups of cyclotomic fields. Since then the class of $L$–functions admitting $p$–adic analogues has gradually extended.

The major sources of such $L$–functions are:

1) *Galois representations* of $G_K = \mathrm{Gal}(\overline{K}/K)$ for algebraic number fields $K$ $r : G_K \to \mathrm{GL}(V)$, ($V$ a finite dimensional vector space), and one can attach to $r$ an Euler product $L(s, r)$ due to Artin.

2) *Algebrac varieties* $X$, defined over an algebraic number field $K$. In this case one can attach to $X/K$ its Hasse – Weil – zeta–functions $Z(X, s)$.

3) *Automorphic forms and automorphic representations.* In the classical case one associates to a modular form $f(z) = \sum_{n=1}^{\infty} a_n \exp(2\pi i n z)$ its Mellin transform $L(s, f) = \sum_{n=1}^{\infty} a_n n^{-s}$. In general one can attach an Euler product to each automorphic representation using the decomposition of such a representation into a tensor product indexed by all prime numbers $p$.

Conjecturally, all three types of $L$–functions can be related to each other using a general theory of motives. For a fixed prime number $p$ to the above complex $L$–functions one can attach in many cases a $p$–adic $L$–functions. These $p$–adic $L$–functions are certain analytic functions in a $p$–adic domain which are uniquely defined by interpolation of certain special values of the corresponding complex analytic $L$–functions. The existence of such an interpolation is equivalent to certain congruences of Kummer type for the above special values.

We describe a general criterium for the existence of a bounded $p$–adic $L$–function of motives using the notions of the Hodge polygon and the Newton polygon of a motive. Then we discuss the cases in which these $p$–adic $L$–functions can be actually constructed. Recent examples include Garrett triple products of modular forms and standard $L$–functions of Siegel modular forms.

**Refernces**

*A.A.Panchishkin,* Admissible Non-Archimedean standard zeta functions of Siegel modular forms, Proceedings of the Joint AMS Summer Conference on Motives, Seattle, July 20–August 2 1991, Seattle, Providence, R.I., 1993, vol.2, 251 – 292

# Minimal Hypersurfaces and Fluid Dynamics

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* on leave of absence from the Institute for Theoretical Physics, Karlsruhe University.

Concerning the problem of finding embeddings of 3-dimensional Lorentzian manifolds $M_3$ into 4-dimensional Minkowski-space for which the first variation (with respect to the embedding functions $X^\mu(\varphi^0, \varphi^1, \varphi^2)$, $\mu = 0,1,2,3$) of their volume $\int_{M_3} d^3\varphi \sqrt{G}$ vanishes, three different choices of parametrisation of $M_3$ are discussed, showing that locally the above problem is equivalent to

A) time-dependent 2-dimensional isentropic inviscid irrotational Kármán-Tsien gas dynamics, i.e. the equations

$$\begin{aligned} &\dot{q} + \vec{\nabla}(q \vec{\nabla} p) = 0 \\ &\dot{p} = \frac{1}{2}\left(\frac{1}{q^2} + (\vec{\nabla} p)^2\right) \end{aligned} \qquad (A)$$

where $\varrho = \varrho(t, x, y)$ is the mass-density of the gas,

$\phi = \phi(t, x, y)$ the velocity potential,

and the Pressure equals $-\frac{1}{\varrho}$;

B) the equations

$$\dot{\vec{x}} = \sqrt{1 - g/\rho^2}\,\vec{n} \qquad (B),$$

where $\vec{x}(t, \varphi^1, \varphi^2)$ describes the (time dependent) shape of a 2-dimensional surface $\Sigma_t$,

$\vec{n} = \vec{n}(t, \varphi^1, \varphi^2)$ is the surface-normal,

$g = (\partial_1 \vec{x} \times \partial_2 \vec{x})^2$ is the square of the surface area density, and

$\rho = \rho(\varphi^1, \varphi^2)$ is an (arbitrary, but fixed) density on $\Sigma_t$, independent of $t$;

c) 3 dimensional time-independent isentropic inviscid irrotational gas-dynamics, i.e. the equations

$$\vec{\nabla}(\gamma \vec{\nabla} t) = 0 \qquad (C)$$

where $\gamma = ((\nabla t)^2 - 1)^{-1/2}$ is the mass-density, and $t$ (= the time at which $\Sigma_t$ passes the point $\vec{x} = (x, y, z)$) the velocity-potential.

These results are based on:

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J. Hoppe; 'Surface Motions and Fluid Dynamics' (to appear in Phys. Lett. B),

and can be obtained by choosing:

$\varphi^0 = \frac{x^0 + x^3}{2} = t$, $\varphi^1 = x^1 = x$, $\varphi^2 = x^2 = y$ (for A)

$\varphi^0 = x^0$, $\dot{\vec{x}}\,\partial_1 \vec{x} = 0 = \dot{\vec{x}}\,\partial_2 \vec{x}$ (for B)

as in (B), but $t, \varphi^1, \varphi^2 \to x, y, z$ (for C).

**Note on the geometry of plane curves via Tschirnhausen resolution tower**

Mutsuo Oka

This is a joint work with N. A'Campo. Let $C$ be an irreducible germ of a plane curve at the origin. In the previous paper [26], the second author proved that a resolution tower of toric modifications

$$\mathcal{T} = \{X_k \xrightarrow{p_k} X_{k-1} \to \cdots \to X_1 \xrightarrow{p_1} X_0 = \mathbf{C}^2\}$$

with its weight vectors $\{P_i = {}^t(a_i, b_i)\}$ carries enough information to read important invariants like the Puiseux pairs, multiplicities and etc. However in the process of the inductive construction of a tower of toric modifications, we have to choose the modification local coordinates $(u_i, v_i)$ to construct the next modification $p_{i+1} : X_{i+1} \to X_i$. Here is an ambiguity which makes it difficult to study the equi-singularity problem of a given family of germs of plane curves. It is the purpose of this talk to show that there exists a canonical way to choose the modification local coordinates $(u_i, v_i)$. We assume that $C$ is defined by an analytic function $f(x, y)$ which is a monic polynomial of degree $n$ as a polynomial of $y$. Then by the irreducibility, we can write:

$$f(x, y) = (y^{a_1} + \xi_1 x^{b_1})^{A_2} + (\text{higher terms}), \quad a_1 \geq 2$$

where $\gcd(a_1, b_1) = 1$ and $n = a_1 A_2$. There is a unique factorization of $n = a_1 a_2 \cdots a_k$ with $a_i \geq 2$ which comes from a Tschirnhausen-good resolution tower and corresponds essentially to the Puiseux pairs of $C$ so that the following property is satisfied. Let $h_i(x, y)$ be the $A_{i+1}$-th Tschirnhausen approximate polynomial of $f(x, y)$ for $i = 1, \ldots, k$ where $A_{i+1} = a_{i+1} \cdots a_k$. Let $C_i = \{(x, y) \in U; h_i(x, y) = 0\}$ $(C_k = C)$. Then those Tschirnhausen approximate polynomials play a key role. We will prove that $\{C_i; i = 1, \ldots, k-1\}$ are also irreducible curves at the origin and there exists a tower of toric modification $\mathcal{T}$ such that the composition $\Phi_i := p_1 \circ \cdot \circ p_i : X_i \to X_0$ gives a good resolution of $C_i$ for each $i = 1, \ldots, k$. There is a unique way to choose the modification local coordinates $(u_i, v_i)$ so that $u_i = 0$ is the defining equation of the supporting exceptional divisor $E_i$ and the pull back of the $i$-th Tschirnhausen approximate polynomial is written as $\Phi_i^* h_i(u_i, v_i) = u_i^{m_i(h_i)} v_i$. Geometrically the series of the Tschirnhausen approximate curves $\{C_i; i = 1, \ldots, k\}$ $(C_k = C)$ corresponds to the series of compound torus knots. The important point here is that the Tschirnhausen approximate polynomials $h_i(x, y)$, $i = 1, \ldots, k-1$ depends only on the coefficients of $y^j$ with $j \geq n - a_1 \cdots a_k$.

The importance of the Tschirnhausen approximate polynomials was first observed by Abhyankar-Moh [2,3] and our work is very much influenced by them. However our result gives not only a geometric interpretation of [2,3] but also a new method to study the equi-singularity problem for a given family of germs of irreducible plane curves $f(x, y, t) = 0$ whose Tschirnhausen approximate polynomials $h_i(x, y)$, $i = 1, \ldots, k-1$ do not depend on $t$.

There exist many applications of the Tschirnhausen resolution tower. For example, we will give a new proof of the Suzuki-Abhyankar-Moh theorem from the viewpoint of the equi-singularity at infinity.

Date: 1994 9 6.

Speaker: C. Kearton, Maths Dept, University of Durham, Durham DH1 3LE, England.

Title: Branched cyclic covers of knots.

Abstract: This is an account of some joint work with S.M.J. Wilson. Let $k$ be an $n$-knot, by which I mean an oriented locally-flat PL pair $(S^{n+2}, S^n)$, and let $K_m \to S^{n+2}$ be the $m$-fold cyclic cover of $S^{n+2}$ branched over $S^n$. In general $K_m$ will not be a sphere, but if it is we obtain a knot $k^m$ which is the fixed point set of the $Z_m$-action on $S^{n+2} = K_m$ given by the covering transformations.

It has been known since work of C.McA. Gordon in 1972 that there are knots $k$ for which $K_m$ is a sphere for infinitely many values of $m$; but the knots $k^m$ are not distinct, there are in fact only finitely many different $k^m$. The main purpose of this talk is to give an example of a simple $(2q-1)$-knot $k$ such that $K_m$ is a sphere for infinitely many values of $m$ and for which the corresponding $k^m$ are all distinct.

# Extending local representations to global representations

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**Abstract**– It is a theorem of Deligne (and Deligne-Serre for weight 1) that for a cuspidal eigenform of the Hecke operators on the upper half plane which is of weight $k$, the eigenvalues of the Hecke operators $T_p$ are algebraic integers $a_p$ with $|a_p| \leq 2p^{(k-1)/2}$. In this note we pose a converse question to this, and analyse to what extent CM forms can be used to answer it. Analogous question is asked in the setting of Galois representations which can be thought of as the non-abelian analogue of the Grunewald-Wang theorem in Class Field Theory, and we answer it in one simple case.

The aim of this note is to pose the following question and provide an answer to it in some very particular cases.

**Question 1:** Suppose that we are given finitely many primes $p_1, \cdots, p_r$, and algebraic integers $\alpha_i$ for every $i, 1 \leq i \leq r$, which have the property that $\sigma(\alpha_i) \cdot \overline{\sigma(\alpha_i)} = p^{k-1}$ for some integer $k \geq 1$ and for every embedding $\sigma : \overline{\mathbf{Q}} \to \mathbf{C}$. Then does there exist a cusp form $f$ of weight $k$ which is an eigenform of all the Hecke operators such that the Euler factor at $p_i$ of the L-series of $f$, for every $i$, $1 \leq i \leq r$, is

$$L_{p_i}(f, s) = \frac{1}{(1 - \frac{\alpha_i}{p_i^s})(1 - \frac{\overline{\alpha}_i}{p_i^s})}?$$

Assuming the Shimura-Taniyama-Weil conjecture according to which all elliptic curves over $\mathbf{Q}$ are modular, this question can be settled rather easily in the affirmative in the case when $k = 2$ and $a_i = \alpha_i + \overline{\alpha}_i$ are rational integers as follows. By a theorem due to Honda and Tate, we can find an elliptic curves $E_i$ over the finite fields $\mathbf{F}_{p_i}$ with $p_i$ elements such that the cardinality of $E_i(\mathbf{F}_{p_i})$ is $1 + p_i - a_i$. If $E$ is any elliptic curve whose reduction modulo $p_i$ is the elliptic curve $E_i$ for every $i$, $1 \leq i \leq r$, then the L-function of $E$ is the Mellin transform of a desired modular form.

When $k = 2$ but $a_i$ are not integers, we can't imitate the above proof even assuming the generalised form of the Shimura-Taniyama-Weil conjecture according to which abelian varieties with real multiplication over $\mathbf{Q}$ also arise as factors of the Jacobians of the modular curves $X_0(N)$. The problem being that it is not clear if we can lift an abelian variety with real multiplication over the finite field $\mathbf{F}_{p_i}$ to one over $\mathbf{Q}$. There is then the problem of doing this for finitely many primes $p_1, \cdots, p_r$ simultaneously. We, however, don't even know if an abelian variety over $\mathbf{F}_p$ can be lifted to one over $\mathbf{Q}$!

In this note we analyse to what extent CM forms can be used to answer the question. Here is the main result of the note. All the numbers $\alpha_i$ appearing in the theorem below will have the property that $\sigma(\alpha_i) \cdot \overline{\sigma(\alpha_i)} = p^{k-1}$ for some integer $k \geq 2$ and for every embedding $\sigma : \overline{\mathbf{Q}} \rightarrow \mathbf{C}$.

**Theorem 1** *Assume that $a_i = \alpha_i + \overline{\alpha}_i$ is an integer such that $p_i$ does not divide $a_i$ for any $i$, $1 \leq i \leq r$. Then there is a CM cuspidal eigenform $f$ such that the Euler factor at $p_i$ of the L-series of $f$ is*

$$L_{p_i}(f, s) = \frac{1}{(1 - \frac{\alpha_i}{p_i^s})(1 - \frac{\overline{\alpha}_i}{p_i^s})}$$

*if and only if the quadratic imaginary fields $K_i = \mathbf{Q}(\sqrt{a_i^2 - 4p_i^{k-1}})$ are independent of $i$.*

**Remark 1 :** The weight 1 case of Question 1 can be completely answered using CM forms. One simply has to take a quadratic imaginary field in which the prime ideals $(p_i)$ split as $(p_i) = \pi_i \overline{\pi}_i$ and construct a finite order grossencharacter $\lambda$ on $L$ using the Grunewald-Wang theorem which is unramified at the primes $\pi_i$ and $\overline{\pi}_i$, and has the property that $\lambda(\pi_i) = \alpha_i$, and $\lambda(\overline{\pi}_i) = \overline{\alpha}_i$ for every $i$, $1 \leq i \leq r$.

**Remark 2 :** There is by now a well-known result for automorphic representations, cf. Rogawski [Ro], that there are automorphic representations whose local components are pre-assigned discrete series representations at finitely many places. However, in question 1 we want to construct automorphic representations whose local components are pre-assigned unramified principal series at finitely many finite places, and a discrete series at infinity when $k \geq 2$. It is unlikely that this question can be handled by techniques of harmonic analysis alone, as it is essential to specify the data which is used to define the unramified principal series at the finitely many local places, in the situation of question 1, to be of arithmetic kind.

Here is the non-abelian version of the Grunewald-Wang theorem, and is the Galois theoretic analogue of question 1 for weight 1.

**Question 2:** Suppose that we are given semi-simple matrices $A_1, \cdots, A_r$ in $GL(n, \mathbf{C})$ such that the eigenvalues of $A_i$ are roots of unity. Then is there a continuous irreducible representation $\Phi : \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to GL(n, \mathbf{C})$ which is unramified at the primes $p_i$ such that the conjugacy class of the image of the Frobenius at $p_i$ under the representation $\Phi$ contains $A_i$ for every $i$, $1 \leq i \leq r$?

**Proposition 1** *Let $G = S_n$, and suppose we are given $\rho_i : \mathrm{Gal}(\overline{\mathbf{Q}}_{p_i}/\mathbf{Q}_{p_i}) \to G$ for $1 \leq i \leq r$. Then there exists $\rho : \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to G$ such that the restriction of $\rho$ to $\mathrm{Gal}(\overline{\mathbf{Q}}_{p_i}/\mathbf{Q}_{p_i})$ is conjugate in $G$ to $\rho_i$ for every $i$.*

**Remark :** We don't know if the Proposition above is true even for $G = A_n$.

**Remark :** The problem of extending local representations to a global one is much subtler than the problem of constructing extensions of global fields with given local extensions. This is evident even in the case of a global cyclic extension in which case when the local field extension is unramified extension of the same degree, the local representation will be the additional data specifying which generator of the cyclic group the Frobenius corresponds to.

We end by noting that the questions raised in this note are more interesting than the fragmentary answers that we can provide and in writing this note it is partly our intention to draw attention to such questions.

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# Nonlinear nonelliptic Schrödinger equations

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The aim of this Lecture is to present a survey of results, including those of J.M. Ghidaglia and the author, concerning equations of the type

(1) $\quad i u_t + L u + F(u) = 0$ ,

where $u = u(x,t)$, $x \in \mathbb{R}^n$, $n \geq 2$, $t \in \mathbb{R}$ is a complex-valued function, $L$ is a constant coefficients differential operator with real symbol and $F$ a (possibly non local) nonlinear function.

Equations or systems such as (1) occur in various physical contexts: water-waves, plasma waves, nonlinear optics... Typically, the symbol of $L$ is given by the Hessian matrix of the linear dispersion. Classical examples ~~are for ins~~ involve the equation

(2) $\quad i u_t + u_{xx} - u_{yy} + |u|^2 u = 0$ or

the hyperbolic-elliptic and hyperbolic-hyperbolic Davey-Stewartson systems, or "higher order" Schrödinger equations.

We will ~~follow the plan~~ cover the following topics:

1. Linear estimates
2. The local Cauchy problem
3. The global Cauchy problem
4. Solitary waves

All of those points lead to interesting open problems. For instance it is not known whether the local solution of (2) blows up in finite time or not.

Hokkaido University, Sept 15, 1994

# Mathematical ~~Analysis~~ Theory of Some ~~Unconventional~~ Non-Standard Finite Elements

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**Abstract**

It is known that Zienkiewicz incomplete cubic element can only converge to the true solution for meshes generated by three sets of parallel lines. However, it can be proved that for the cross-diagonal mesh Zienkiewicz approximate solution may still have a limit which is the solution of another variational problem, slightly different from the original one. To remedy such a strange convergence behavior it is sufficient to modify the relevant variational formulation through the application of numerical integration to certain parts of the stiffness matrix. This results Agyris TRUNC element and a nine degree quasi-conforming element, both of which were initially based on some particularly mechanical considerations. A detailed mathematical analysis of convergence of these unconventional elements and the error estimates are established. The new elements impose no restriction on the meshes and need less computational costs given better results in comparison with many other plate elements including the original Zienkiewicz one. It is interesting to mention that the numerical integration may significantly improve the convergence property as well as the numerical accuracy of these elements.

# 単純環における付値論

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非可換環における付値論の研究はSchillingの本 [13] に始まる。Dをdivision ring, RをDのsubringとする。Rが次の２つの条件を満たすときDのinvariant valuation ring

と言われる。

(i) $D \ni x \neq 0 \Rightarrow x \in R$ or $x^{-1} \in R$ and

(ii) $D \ni x \neq 0 \quad xRx^{-1} = R$.

もしRが(i)のみを満たすとき、total valuation ringと言われる。Dがその中心F上で有限次元であり、VをFのvaluationとしたちき、これらのvaluation ringsに関しては次のことがわかっている：

（１）もしVがcomplete rank 1またはHenselian ringのときDのinvariant valuation ring Rで$R \cap F = V$となるRが存在する。

（２）$T = \{ x \in D \quad x$ : integral over $V\}$ としたとき、Tがringになる必要且つ十分条件はDのtotal valuation ring Rで$R \cap F = V$なるRが存在することである（[ ]）。

しかしこれろのvaluation ringsには次の２つの重大な欠点がある：

（ａ）$V \cap F = R$なるDの(invariant)total valuation ring Rが必ずしも存在しない。

（ｂ）非可換環で最も重要なsimple Artin ringsを含まない。

この２つの問題を同時にクリアした非可換valuation ringをDubrovinが定義し多くの重要な結果を出した（[4] [5]）。これ以後、invariant valuation rings and total valuation ringsを研究していた研究者がDubrovinの結果の重要性に気付き、最近、次々と成果をあげている。そのなかでいくつか重要な論文を下記にあげておく（[1]，[2]，[6]，[7]，[8]，[9]，[9]，[10]，[11]，[12] and [14]）。

特に、Dubrovin valuation ring RがそのcenterV上でintegralなとき、$v : Q \longrightarrow st(R)/U(R)$, $v(q) = \alpha U(R)$, ここで、QはRの商環、U(R)はRのunits群, $St(R) = \{ x \in Q \mid xR = Rx \}$ and $RqR = \alpha R = R\alpha$（[14]），

はvalued functionと呼ばれcrossed products, tensor productsに応用されている（[8] [12]）。最後に、

（1）Invariant valuation ringsの不存在に関しては[3]、

（2）Quaternion algebrasの中でのtotal valuation ringsの不存在に関しては[2]、

（3）Total vauation ringsの存在する環の例に関しては[1]

を参照のこと。

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# DIMENSION DEFECT FOR PSEUDOGROUP ACTIONS

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**Introduction.**

In this article, we define Hausdorff dimension $\dim_H \mathcal{G}$ of a finitely generated pseudogroup $\mathcal{G}$ acting on a compact metric space $X$. We show that $\dim_H \mathcal{G}$ does not exceed $\dim_H X$, the Hausdorff dimension of $X$, so one has a kind of dimension defect $d_H\mathcal{G} = \dim_H X - \dim_H \mathcal{G} \geq 0$. We show that Lipschitz equivalent pseudogroups have the same Hausdorff dimensions, so - in particular - the (transverse) dimension defect of a foliation $\mathcal{F}$ can be defined as that of its holonomy pseudogroup $\mathcal{H}$ acting on any compact complete transversal $T$. Several examples provided here show that the dimension defect $d_H\mathcal{G}$ is positive when there is enough of contraction (or, expansion) by elements of $\mathcal{G}$.

The motivation of this research comes from the following.

First, Hausdorff dimensions (and other related dimensions) occured to be useful in defining and studying fractals which appear often in the theory of (especially complex) dynamical systems as, for example, minimal invariant sets (see **[Ed]**, **[Fa]** and the references there). For some classes of sets (like quasi-circles which are defined as subsets of $\mathbb{R}^n$ homeomorphic to $S^1$ and satisfying some other natural conditions and which appear naturally in the study of Kleinian groups **[Bo]**), the equality $\dim_H X = \dim_X Y$ implies that $X$ and $Y$ are quasi-isometric **[FM]**, so $\dim_H$ becomes a good invariant to study dynamics of some systems.

Second, recent years brought wide interest in studying general dynamical systems like relations, group actions, pseudogroups, foliations, etc. The variety of problems and results in this area is pretty large. We list below just some of them (and give only some references) to introduce the reader into the area:

(1) invariant measures for general dynamics, ergodicity, amenability (**[Pl]**, **[Gar]**, **[Zi2]**, **[Zi3]**, etc.),
(2) entropy for relations, pseudogroups, foliations (**[GLW]**, **[LW1]**, **[LW2]**, **[Fr]**, **[Hu1]**, **[Hu2]**, **[Wi]**, **[Bi]**, etc.),
(3) other invariants measuring dynamics of general systems (**[Eg1]**, **[Eg2]**, **[LW3]**, etc.),
(4) rigidity of group actions (**[Gh]**, **[Hu3]**, **[Hu4]**, **[Zi1]**, etc.)
(5) geometry and dynamics of hyperbolic groups (**[Gr]**, **[GH1]**, **[GH2]**, **[CDP]**, **[GHV]**, **[C1]**, **[C2]**, **[Ch]**, etc.).

Finally, the ultimate impulse came from **[Le]**, where the author defined a measure-theoretic cost $l(\Phi)$ of generating measure-preserving relations (pseudogroups, in particular) given a finite generating set $\Phi$. $l(\Phi)$ has this property that it becomes

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larger if one wants to create more complicated dynamics. We realized that it should be possible to find a kind of topological cost of generating. Thinking about it we arrived to our dimension defect which has (to some extend) the similar property: If the dynamics of $\mathcal{G}$ is complicated, then $d_H\mathcal{G}$ is large. At the moment, we do not see clear relations between $d_H\mathcal{G}$ and the Levitt's cost $l(\Phi)$.

The article is organized as follows. Section 1 contains basis definitions, those of a pseudogroup, pseudogroup morphisms, holonomy pseudogroups of foliations, etc. In Section 2, we define the Hausdorff dimension and the dimension defect. We provide several examples and show basic properties related to morphisms, inavriant sets, subpseudogroups, etc. In Section 3, we show how to estimate $\dim_H \mathcal{G}$ and $d_H\mathcal{G}$ when $\mathcal{G}$ admits a good (called $s$-continuous there) invariant measure. Finally, Section 4 contains the proof of vanishing of the Hausdorff dimension of the pseudogroup generated by a non-elementary hyperbolic group $\Gamma$ on the boundary of its Cayley graph while some final remarks are collected in Section 5. Since our examples come from different fields (ergodic theory, dynamical systems, theory of foliations, theory of groups, Riemannian geometry) we tried to make the exposition clear to different readers, so we decided to include some backgroud material about holonomy of foliations (Section 1.3), invariant measures (Section 3.1), and hyperbolic spaces and groups (Section 4.1).

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# OPTIMAL AORTIC FLOW CURVES DETERMINED BY THE MINIMIZATION OF TIME DEPENDENT CHANGES IN VENTRICULAR EJECTION PRESSURE, AORTIC FLOW RATE AND WORK OF VENTRICLE.

## -- A THEORETICAL STUDY ----.

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Running head : Optimal aortic curves determined by the minimization of -----

Key word : Optimal control. Performance function. Aortic flow . External work. Ventricular ejection pressure.

ABSTRACT

An optimal control theory was applied to cardiovascular system to show the existence of the optimal control strategy for the production of physiological aortic flow curve. The performance function that minimized involved the time dependent change of ventricular ejection pressure, of aortic flow rate and absolute amount of work of ventricle. The work of ventricle is composed of the external work and potential energy of ventricle. Each term was timed with weighting coefficient so as to quantify the magnitude of minimization of target quantities. The arterial system was expressed by electrical equivalent Wind Kessel model and stroke volume was set to be constant as the integral type constraint. The arterial pressure, arterial flow rate and ventricular volume were converted into state variables and this biological problem was converted into mathematical problem for solving 6 simultaneous linear differential equations. The reconstructed aortic flow curves were similar with those of physiological ones. The reactive changes in calculated aortic flow curves with changes in system parameters were reasonable and in consistent with physiological changes. The changes in calculated aortic flow curves caused by changes in weighting coefficient relating with time dependent change in ejection pressure related with reduction of shear stress at arterial wall. They were reasonably and rationality explained from the stand point of optimality in the cardiovascular system. Present study showed that physiological aortic flow curve must be produced by the optimal principle with performance function we have proposed. Present theoretical investigation would be useful for the prediction of the optimal aortic flow pattern and evaluation of cardiac assistment devices.

# 一般化された Curve Shortening 方程式と退化非線形放物型方程式

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$t \geq 0$ をパラメータとして、$C(t) : \sigma \mapsto (x(\sigma,t), y(\sigma,t)) : S^1 \to \mathbb{R}^2$ を平面上の滑らかな曲線とする。一般化された C. S. 方程式

$$\frac{\partial C}{\partial t} = g(\kappa)\mathcal{N} \tag{1}$$

を考える。ここで、$\kappa$ は曲線 $C(t)$ の曲率、$\mathcal{N}$ はその内向き単位法線ベクトルとし、$g$ は $\mathbb{R}$ 上の非負値非減少関数である。

$C(t)$ を滑らかにはめ込まれた凸閉曲線とすると、方程式 (1) は

$$\frac{\partial \kappa}{\partial t} = \kappa^2 \left[ \frac{\partial^2 g(\kappa)}{\partial \theta^2} + g(\kappa) \right] \qquad (\theta \in \mathrm{T}_\nu, 0 \leq t < T) \tag{2}$$

と表すことが出来る。ここで、$\theta$ は曲線の接線と $x$ 軸とのなす角を表す。また、$\mathrm{T}_\nu = \mathbb{R}/2\nu\pi\mathbb{Z}$ であり、$\nu$は曲線の回転数である。

特に、$g(\kappa) = \mathrm{sign}\alpha|\kappa|^{\alpha-1}\kappa$ のときは、$u = \beta g(\kappa)$, $\beta = |\alpha|^{\alpha/(\alpha+1)}$ とおくことで、方程式 (2) は

$$u_t = u^\delta(u_{\theta\theta} + u), \qquad (\theta \in \mathrm{T}_\nu,\ 0 \leq t < T) \tag{3}$$

と書き直せる。方程式 (3) に関しては、初期値 $u_0$ が

(A1)　$u_0^{1-\delta} \in L^1(\mathrm{T}_\nu)\ (\delta \neq 1)$　or　$\log u_0 \in L^1(\mathrm{T}_\nu)\ (\delta = 1)$

を満たす場合、解は有限時間内に爆発することが言える。

更に、$\delta \neq 1$ のとき、$T$ を爆発時間とし、$S \subset \mathrm{T}_\nu$ を爆発点の集合、$t \in [0,T)$ に対して $M_t$ を $u(\theta,t)$ の極大点の集合、$M$ を

$$M = \{\theta \mid \exists\{(\theta(t_n), t_n)\} \in M_{t_n} \otimes [0,T) \quad \text{s.t.} \quad t_n \to T,\ \theta(t_n) \to \theta\}$$

とおくと、条件 (A1) の下で次が言える。

**Theorem 1** *For any* $\theta \in M$, $(\theta - \frac{\pi}{2}, \theta + \frac{\pi}{2}) \subset S$.

**Theorem 2** *Let* $\delta \geq 2$. *Suppose that* $S \neq [0, 2\nu\pi]$. *Then,*

$$S = \bigcup_{\theta \in M} \left[\theta - \frac{\pi}{2}, \theta + \frac{\pi}{2}\right].$$

**Theorem 3** *Suppose* $\delta \geq 2$ *and that* $S \neq [0, 2\nu\pi]$. *Let* $u$ *blow up at* $T$ *and* $\theta_0 \in M$. *Then,*

$$\frac{u(\theta, t)}{u(\theta_0, t)} \to \cos(\theta - \theta_0) \quad \text{for} \quad \theta \in (\theta_0 - \frac{\pi}{2}, \theta_0 + \frac{\pi}{2})$$

*uniformly as* $t \to T$. *Moreover, if* $\theta \notin \bigcup_{\theta \in M} \left[\theta - \frac{\pi}{2}, \theta + \frac{\pi}{2}\right]$, *then*

$$\frac{u(\theta, t)}{u(\theta_0, t)} \to 0.$$

**Theorem 4** *Suppose* $\delta \geq 2$ *and that* $S = [0, 2\nu\pi]$. *Let* $u$ *blow up at* $T$. *Then,*

$$u(\theta, t) \leq C(T - t)^{-1/\delta}, \qquad \forall \theta \in \mathrm{T}_\nu$$

*and*

$$(T - t)^{1/\delta} u(\theta, t) \to g(\theta), \qquad \forall \theta \in \mathrm{T}_\nu$$

*uniformly as* $t \to T$, *where* $g(\theta)$ *is a positive solution of the boundary value problem*

$$g_{\theta\theta} + g - \frac{1}{\delta g^{\delta - 1}} = 0, \qquad \forall \theta \in \mathrm{T}_\nu.$$

# Asymptotic connection between solutions of different nonlinear equations for large time

Ilia A. Shishmarev
Moscow State University, Russia

We established asymptotic connections as $t \to \infty$ between the solutions to the Cauchy problems for generalized Benjamin-Ono-Burgers equation, Burgers equation, and the Korteweg-de Vries-Burgers equation.

Further we consider asymptotic behaviour of solutions to the Cauchy problem for the Korteweg-de Vries-Burgers (KdVB) equation and find the first and the second principal terms of asymptotics as $t \to \infty$ in explicit form, moreover we obtain the estimate of remainder term of asymptotics in uniform metric. We established these results in two cases, when $t \to \infty$ and $|x|$ is bounded (the first case), and when $t \to \infty$ and $|x| \to \infty$ simultaneously (the second one).

Finally we considered the Cauchy problem for the generalized KdV equation

$$u_t + \frac{\partial}{\partial x}(u^n) + u_{xxx} = 0, \qquad u|_{t=0} = \bar{u}(x), n \geq 4 - \text{integer} \tag{1}$$

and obtained asymptotic behaviour as $t \to \infty$ for the solution of the problem (1). We produced the principal term of asymptotics in explicit form and get the estimate of the remainder term in uniform metric.

Asymptotic behaviour of solutions to the Navier-stokes and Magneto-Hydrodynamics equations

By Schonbek, maria E

Abstract for talk

I will consider the long time behaviour of solutions to the Navier Stokes equations, in $R^n$ $n = 2, 3, 4, 5$.

$$\text{(NS)} \quad u_t + u\nabla u + \nabla p = \Delta u, \qquad \operatorname{div} u = 0,$$

and the solutions to the Magneto Hydrodynamics equations in $R^n$ $n = 2, 3, 4$.

$$\text{MHD} \quad \begin{aligned} u_t + u\nabla u - B\nabla B + \nabla p &= \Delta u \\ B_t + u\nabla B - B\nabla u &= \Delta B \end{aligned}$$

$$\operatorname{div} u = \operatorname{div} B = 0$$

For solutions to the NS equations with data $u_0 \in H \cap L^1 \cap H^m$ we will show that the derivatives up to order $m$ decay in the $L^2$ norm with the same rate as solutions to the heat equations

$$\|D^\alpha u\|_{L^2}^2 \le C(t+1)^{-n/2-|\alpha|}$$

where $|\alpha| \le m$.

For solutions of the MHD equations we will obtain the same bound as above and we will also discuss the problem of lower bounds of rates of decay of the energy of the solutions.

# THE TOPOLOGY OF SPACES OF RATIONAL FUNCTIONS (ABSTRACT OF TALK)

KOHHEI YAMAGUCHI (NOVEMBER 16, 1994)

Department of Mathematics, The University of Electro-Communications.

Let $C^\infty(S^2, S^2)$ (resp. $C^\infty(S^2, S^2)_d$) denote the space consisting of all smooth maps $f : S^2 \to S^2$ (resp. with degree $d$) and consider the *energy function*

$$E : C^\infty(S^2, S^2) \to \mathbb{R}$$

which is given by

$$E(f) = \frac{1}{2}\int_{S^2} \|Df(x)\|^2 dvol$$

where the usual Riemannian structure is given on $S^2$.

Then it is well-known that $E$ has no the critical points apart from the holomorphic maps on which attain the absolute minimam. (Critical points of $E$ are called *harmonic maps.*)

Let $\mathrm{Hol}_d(S^2, S^2)$ (resp. $\mathrm{Hol}_d^*(S^2, S^2)$) denote the space consisting of all holomorphic maps $f : S^2 \to S^2$ of degree $d$ (resp. satisfying the condition $f(\infty) = 1$). (Here we consider $S^2 = \mathbb{C} \cup \{\infty\}$.)

The analogue of the usual Morse theory might lead one to hope that $\mathrm{Hol}_d(S^2, S^2)$ (resp. $\mathrm{Hol}_d^*(S^2, S^2)$) is a deformation retract of $C^\infty(S^2, S^2)_d \simeq \mathrm{Map}(S^2, S^2)_d$ (resp. $\Omega_d^2 S^2$). However, it is not correct, because the former space has the homotopy type of finite complexes and the latter has the that of infinite dimensional complexes.

But Segal ([Se2]) showed that it is correct if "$d \to \infty$". That is,

**Theorem (Segal).** *The inclusion maps $i_d$ : $Hol_d(S^2.S^2) \to Map(S^2, S^2)_d$ and $j_d$ : $Hol_d^*(S^2, S^2) \to \Omega_d^2 S^2$ are homotopy equivalence up to dimension $d$.* □

(Here the continuous map $f : X \to Y$ is called a *homotopy* (resp. *homology*) *equivalence up to dimension* $d$ when the induced homomorphism $f_* : \pi_k(X) \to \pi_k(Y)$ (resp. $f_* : H_k(X) \to H_k(Y)$) is bijective when $k < d$ and surjective when $k = d$.)

It is easy to see that $\mathrm{Hol}_d^*(S^2, S^2)$ is homeomorphic to the space of rational functions

$$f(z) = \frac{p(z)}{q(z)} = \frac{z^d + a_1 z^{d-1} + \cdots + a_d}{z^d + b_1 z^{d-1} + \cdots + b_d}$$

where $a_i, b_j \in \mathbb{C}$ and the polynomials $p(z)$ and $q(z)$ are coprime (i.e. there is no common factors). (Similar description can be described for $\mathrm{Hol}_d(S^2, S^2)$.) So we shall sometimes call the space $\mathrm{Hol}_d^*(S^2, S^2)$ (resp. $\mathrm{Hol}_d(S^2, S^2)$) as the *space of rational functions*.

From the above theorem, we may ask:

**Problem.** *Let $X \subset \mathbb{C}P^n$ be the complex projective variety with $H_2(X,\mathbb{Z}) = \mathbb{Z}$ and let $Hol_d^*(S^2, X)$ denote the space consisting of all holomorphic maps $f : S^2 \to X$ of degree $d$ with $f(\infty) = x_0$, where $x_0$ is a basepoint of $X$. Then is there non-negative integer $n(d, X)$ such that the inclusion map*

$$j_d : Hol_d^*(S^2, X) \to \Omega_d^2 X$$

*is a homology equivalence (or more generally homotopy equivalence) up to dimension $n(d, X)$ and $\lim_{d\to\infty} n(d, X) = \infty$??* □

Until now, the above problem is affirmative for the following cases:

(1) $X = \mathbb{C}P^n$. (2) $X =$ Granssmann manifolds. (3) $X =$ toric varieties. etc...

In this talk, we would like to consider generalizations of the result of Segal and the main result is as follows:

**Theorem.** *Let $n$ be a positive integer and let $X_n = \mathbb{C}P^n - \cup_{1\leq i<j\leq n} H_{i,j}$, where*

$$H_{i,j} = \{[z_0 : \cdots : z_n] \in \mathbb{C}P^n : z_i = z_j = 0\}.$$

*Then the natural inclusion map*

$$j_d : Hol_d^*(S^2, X_n) \to \Omega_d^2 X_n$$

*is a homotopy equivalence up to dimension $d$.* □

The above result is based on the joint work with M. Guest and A. Kozlowski ([GKY]). The essetial point of the proof is:

(1) First, we shall show the stabilizaed result, i.e. the limit of the stabilization map

$$\lim_{d\to\infty} \mathrm{Hol}_d^*(S^2, X_n) \xrightarrow[\simeq]{\lim\ s_d} \lim_{d\to\infty} \Omega_d^2 X_n \simeq \Omega_0^2 X_n$$

is a homotopy equivalence, where $s_d : \mathrm{Hol}_d^*(S^2, X_n) \to \mathrm{Hol}_{d+1}^*(S^2, X_n)$ is a stabilization map induced by adding the point from the infinity.

(2) Second is to show that $s_d$ is a homotopy equivalence up to dimension $d$.

In my talk we also would like to consider the homotopy types of $\mathrm{Hol}_d(S^2, S^2)$ and $\mathrm{Hol}_d^*(S^2, S^2)$ explicitely for small $d$.

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# III型 subfactor に対する T-set と S-set について

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幸崎秀樹

Hilbert空間の有界線型作用素のつくる $*$-algebraで作用素強位相に関して閉じたものは von Neumann 環と呼ばれる。特にその中心が1次元である場合には因子環(factor)と呼ばれる。Murray-von Neumann は1930年代に作用素環理論を創設し、因子環の $I_n$ ($n=1,2,\cdots,\infty$)，$II_1$，$II_\infty$，III型への分類、一般の von Neumann 環の因子環の直積分としての表示等理論の基礎を作った。1967年に発表された Modular 理論に基づき、Connes は T-set と S-set (それぞれ $\mathbb{R}$，$\mathbb{R}_+$ の部分群)と呼ばれる不変量を導入し、III型因子環を更に $III_0$，$III_\lambda$ $(0<\lambda<1)$，$III_1$ 型へと分類した。

多くの研究者の努力により有限次元環の増大列で近似可能(AFD)な因子環の分類は完成したが、Jones の指数理論は作用素環の研究に新しい研究課題を提供した。$II_1$型因子環とその部分因子環の対 $M \supseteq N$ に対する Jones index $[M:N]$ とは(作用素環論的意味での)MとNの「大きさの比」である。Jonesの定理は指数の取り得る値は $4\cos^2\frac{\pi}{n}$ $(n \geqq 3)$ 又は4以上であるというものであり、$4\cos^2\frac{\pi}{n}$ という離散的値は無限次元環の中に興味深い離散的構造がかくされている事を示している。Jonesの指数理論は部分因子環の構造解析の重要性を示しており、この方面の研究が活発に行なわれるようになった。たとえば AFD $II_1$型因子環の index 4以下の部分因子環は完全に分類されている。(Ocneanu, Popa)

一方、III₁型以外の一般の因子環に対する指数理論も整備されたが、特にIII型の場合が重要である。III型因子環の部分因子環に関する最近の研究を解説し、部分因子環こみの(relative) T-set及びS-setの概念、及びIII₁型の場合の部分因子環研究への応用を説明する。

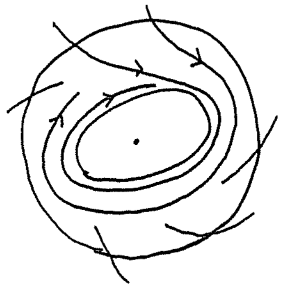
# 正則ベクトル場に対するPoincaré-Bendixson型定理とその応用

龍谷大学経済学部　伊藤敏和

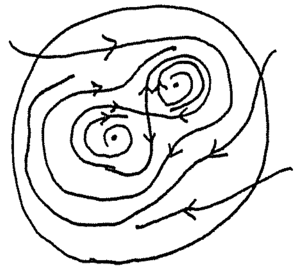
H. Poincaréによって研究がはじめられた微分方程式の解の大域的性質の研究の中で基本的性質の1つはPoincaré-Bendixson定理として知られている。　それは「$\mathbb{R}^2$上の微分方程式

$$\frac{dx}{dt} = f(x, y), \quad \frac{dy}{dt} = g(x, y)$$

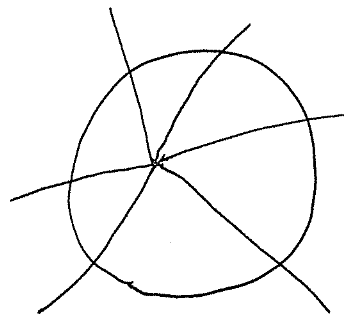
が原点を中心にもつ半径1の円 $S^1(1) = \partial \overline{D}^2(1)$ に横断的であれば，$S^1(1)$ と交わる解 $L$ は $\overline{D}^2(1)$ 内でどこに行きつくか」を考察するものである。　絵を書いてみる。



limit cycleにまきつく



Separatrixにまきつく

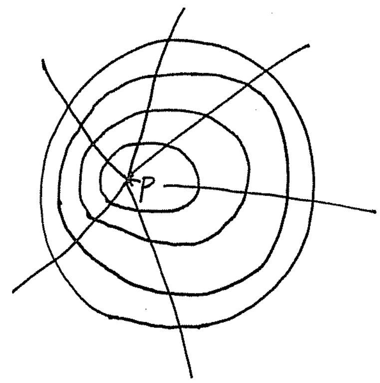


特異点に行きつく

我々は $\mathbb{C}^n$ 上の正則ベクトル場に対して，これと同じようなことを考察する。

## Poincaré-Bendixson type theorem for holomorphic vector fields

実2n次元closed disk $\overline{D}^{2n}(1) = \{ z \in \mathbb{C}^n \mid \|z\| \leqq 1 \} \subset \mathbb{C}^n$ の近傍上の正則ベクトル場 $Z$ が $\partial \overline{D}^{2n}(1) = S^{2n-1}(1)$ に横断的とする。　このとき，$\partial \overline{D}^{2n}(1)$ と交わる $Z$ の任意の解 $L$ は，



ただ1つの特異点Pにいきつく，i.e. $\overline{L} \ni P$。　さらに，$\overline{D}^{2n}(1)-\{P\}$ 上の葉層構造 $\mathcal{F}(Z)|_{\overline{D}^{2n}(1)-\{P\}}$ は $\mathcal{F}(Z)|_{\partial\overline{D}^{2n}(1)} \times (0,1]$ の構造（cone structure）になっている。

この定理の1つの美しい応用例として，Seifert予想が特別な場合に肯定的に解ける。

<u>系</u>　$\mathcal{F}(Z)$ を $S^{2n-1}(1)=\partial\overline{D}^{2n}(1)$ 上に制限した1次元foliation $\mathcal{F}(Z)|_{S^{2n-1}(1)}$ は少なくとも1つcompact leaf（$S^1$）をもつ。

特に，$n=2$ の場合はcompact leafの個数は1，2，∞ ケのどれかである。

さらに，この定理の帰結として，特異点のまわりでは横断的であるが，半径がしだいに大きくなるにしたがって横断性がこわれる点があらわれる。ここで何がおこっているのかをいくつかの例をみせながら考察する。

## Representation theory and harmonic analysis on reductive symmetric spaces

Shigeru SANO (Polytechnic University)

Let $G$ be a H-C class reductive Lie group. Let $\sigma$ be an involutive automorphism of $G$. The set of all fixed points of $\sigma$ is denoted by $G^\sigma$. Let $H$ be a closed subgroup of G such that $G_0^\sigma \subset H \subset G^\sigma$. We study harmonic analysis on the reductive symmetric space $G/H$.

Let $\mathfrak{g}$ be the Lie algebra of $G$. We denote by the same letter $\sigma$ the corresponding automorphism of $\mathfrak{g}$. Let $\mathfrak{g} = \mathfrak{h} + \mathfrak{q}$ the eigenspace decomposition of $\mathfrak{g}$ for $\sigma$. Let $\theta$ be the Cartan involution of $\mathfrak{g}$ commuting to the involution $\sigma$ and $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ the eigenspace decomposition of $\mathfrak{g}$.

Example $SL(n,\mathbf{R})/S0(n)$, $SL(n,\mathbf{R})/SO(n-p,p)$, $SL(2n,\mathbf{R})/Sp(n,\mathbf{R})$,

We consider the Hilbert space $L^2(G/H), dgH$. The regular representation $R$ of $G$ on $L^2(G/H)$ is defined by $[R_y f](xH) = f(y^{-1}xH)$. We consider the problem to give the irrreducible representation decomposition of the regular representation. For group manifolds and Riemmanian symmetric spaces, the decompositions are well known. For the problem, we give a structure decompsition of the symmetric space $G/H$ which depend on the principal series representation of $G$ for the symmetric space $G/H$. Let $\mathfrak{a}_\mathfrak{q}$ be a Cartan subspace of $\mathfrak{q}$. Let $\mathfrak{j}$ be the Cartan subspce of $\mathfrak{g}$ such that it contains the space $\mathfrak{a}_\mathfrak{q}$ and the dimension of its split part is maximal. We put $\mathfrak{a}_\mathfrak{p} = \mathfrak{j} \cap \mathfrak{p}, L = Z_G(\mathfrak{a}_\mathfrak{p})$ and $X(L) = \mathrm{Hom}(L, \mathbf{R}^\times)$. We define $M = \bigcap_{\chi \in X(L)} Ker \mid \chi \mid$, $A_\mathfrak{p} = \exp \mathfrak{a}_\mathfrak{p}$. Then $L = MA_\mathfrak{p}$, $M \cap A_\mathfrak{p} = \{e\}$.

We take a $\mathfrak{h}$-compatible order of the root system $\Sigma(\mathfrak{a}_\mathfrak{p}, \mathfrak{g})$ and put $\mathfrak{n} = \sum_{\alpha>0} \mathfrak{g}^\alpha, N = \exp \mathfrak{n}$. We have a parabolic subgroup $P = MA_\mathfrak{p}N$ of $G$. According to the subspace $\mathfrak{a} = \mathfrak{a}_\mathfrak{q} \cap \mathfrak{p}$, Weyl groups $W$ and $W_H$ are defined by $W = N_K(\mathfrak{a})/Z_K(\mathfrak{a})$, $W_H = N_{K\cap H}(\mathfrak{a})/Z_{K\cap H}(\mathfrak{a})$.

Let $\mathfrak{a}_\mathfrak{q}^1, \mathfrak{a}_\mathfrak{q}^2, ..., \mathfrak{a}_\mathfrak{q}^n$ be a maximal system of Cartan subspces of $\mathfrak{q}$ which

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

are not conjugate each other under the actions of $K_H = K \cap H$. According to $\mathfrak{a}_{\mathfrak{q}}^k$, we take the parabolic subgroup $P^k$ of $G$ and Weyl groups $W^k$ and $W_H^k$. We set $W_k^* = W^k/W_H^k$.

Proposition 1

(1) $\bigcup_k \bigcup_{w \in W_k^*} (K_H w P^k) H = G/H$

(2) If $j \neq k$, then $\dim L^{j,k}/(L^{j,k} \cap H) < \dim G/H$, where $L^{j,k} = (K_H w P^j) \cap (K_H w' P^k)$.

For the Langlands decomposition $P^k = M_k A_{\mathfrak{p}}^k N_k$, we define Fourier transforms on $G/H$ using above decompositions. Since rank $M_k$ = rank $M_k \cap K$, there exist discrete series representations of $M_k$. Let $\mathcal{E}d(M_k/M_k \cap H)$ be the set of all equivalence classes of discrete series representations with $M_k \cap H$ fixed vectors. For any $\omega$ in $\mathcal{E}d(M_k/M_k \cap H)$, there exists a G-invariant closed subspace $V_\omega$ of $L^2(M_k/M_k \cap H)$. Let $\sigma_\omega$ be the projective spherical distribution $\sigma_\omega : L^2(M_k/M_k \cap H) \longrightarrow V_\omega$.

Irreducible unitary representations of $A_{\mathfrak{p}}^k$ with $A_{\mathfrak{p}}^k \cap H$-fixed vectors are given by $\nu \in \sqrt{-1}\,\mathfrak{a}_k^*$. We put $A_k = \exp \mathfrak{a}_k$. For any $\sigma_\omega, \nu$ and $k$, we define a Fourier transform

$\sum_k < \mathcal{F}^{\omega,\nu,k}, f >$

$$= \sum_{k=1}^{n} \int_{K_H} \int_{M_k^* \times A_k \times N_k^*} f(kmanH)\sigma_\omega(m) \exp(-\nu \log a) dk dm^* da dn^*.$$

Theorem 2 There exsist measures $\mu(\sigma_\omega, \nu)$ such that for any $f$ in $C_c^\infty(G/H)$, we have

$$f(eH) = \sum_{k=1}^{n} \sum_{\omega \in \mathcal{E}d(M_k/M_k \cap H)} \int_{\nu \in \sqrt{-1}\mathfrak{a}_k^*} < \mathcal{F}^{\omega,\nu,k}, f > \mu(\sigma_\omega, \nu) d\nu.$$

## MacPherson-Chern Classes and Topological Radon Transformations

Toru Ohmoto, Department of Math., Kagoshima University
December 8, 1994, at Sapporo

The purpose of this talk is to give an outline of a joint work with *Shoji Yokura* ( Kagoshima University ) and *Lars Ernström* ( Ohio State University ) [2].

Let $X$, $Y$ and $Z$ be smooth algebraic varieties, and consider a *divergent diagram* of algebraic maps $f$ and $g$ : $X \overset{f}{\leftarrow} Z \overset{g}{\rightarrow} Y$. Associated to this divergent diagram, we define a *topological Radon transformations of constructible functions* by the composition of the push-foward and the pull-buck

$$F^{Rad} := g_* \circ f^* : F(X) \to F(Y).$$

In his thesis [1], Lars Ernström studied the Radon transformation $F^{Rad}$ in a special diagram $P^N \overset{p}{\leftarrow} I_k \overset{q}{\rightarrow} Gr_k(P^N)$ where $Z = I_k$ is the point-$k$-plane correspondance and $p$, $q$ are natural projections. He gave an explicit description of the image of *the local Euler obstraction* ( constructible function ) $Eu_V \in F(P^N)$ of a reduced subvariety $V$ via the transformation $F^{Rad}$, which involves the affirmative answer to following *Viro's conjecture* (cf. [4] ) as his formula in the case of $k = N-1$:

$$F^{Rad}(Eu_V) = e1_{P^{*N}} + (-1)^{N-1}Eu_{V^*}$$

where $e$ means $\chi(V \cap L)$ for a generic hypersurface $L$.

On the other hand, as an analogy to the Chern character in the K-theory, MacPherson gave the following theorem, which was conjectured by Deligne and Grothendieck :

**Theorem 1 ([3]).** *There exists a natural transformation from the functor $F$ of constructible functions to the functor $H_*$ of singular homology groups, $c_* : F \to H_*$, which, on a non-singular algebraic variety $X$, assigns to the constant constructible function $1_X$ the Poincaré dual of the total Chern class of $X$.*

Usually $c_*$ is called *the MacPherson-Schwarz-Chern transformation* . We note that by MacPherson's construction of $c_*$, for an algebraic variety $X$, $c_*(Eu_X)$ is given by *the total Mather-Chern class* $C_M(X)$ of $X$.

It hence seems to be natural to consider the following problems :

How can we define "Radon-type transformations" on the level of homology groups appropriately? Futhermore, how can we describe the relationship between the Mather-Chern homology class of a projective variety $V$ and the one of the dual variety $V^*$ ( or the $k$-th dual variety ) ?

For a divergent diagram of smooth varieties, we introduce *the homological Verdier-Radon transformation*

$$V_H^{Rad} := g_* \circ f^{!!} : H_*(X) \to H_*(Y).$$

Here $f^{!!} : H_*(X) \to H_*(Z)$ is defined by $f^{!!}(\xi) := c(T_f) \cap f^!(\xi)$, where $T_f$ is the virtual relative tangent bundle $TZ - f^*TX$ and $f^!$ denotes the Gysin homomorphism of the homology groups.

Then, using "Verdier-type Riemann-Roch", we can show

**Proposition 1.** *For any smooth divergent diagram $X \overset{f}{\leftarrow} Z \overset{g}{\rightarrow} Y$, the above two Radon transformations commute together with the MacPherson's transformation, $c_* \circ F^{Rad} = V_H^{Rad} \circ c_*$.*

**Corollary 1.** *For a reduced subvariety $V$ of $P^N$, it holds that*

$$j_*(C_M(V^*)) = (-1)^{N-1}\{q_*(c(T_p) \cap p^! i_* C_M(V)) + e\, c(P^{*N}) \cap [P^{*N}]\},$$

*where $i$ ( resp. $j$ ) is the natural inclusion of $V$ in $P^N$ ( resp. $V^*$ in $P^{*N}$ ).*

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# Riemannian metrics with prescribed Ricci curvature

Dennis DeTurck and Hubert Goldschmidt

Let $R$ be a given symmetric 2-form on a real-analytic manifold $X$ of dimension $n \geq 3$; we seek (locally) a Riemannian metric $g$ whose Ricci curvature $\mathrm{Ric}(g)$ is equal to $R$, that is, we wish to solve the equation

$$\mathrm{Ric}(g) = R \tag{1}$$

for the metric $g$. When $R$ is non-degenerate, it is known that a local solution always exists both in the $C^\infty$-case and in the real-analytic category. Here we are interested in the case when $R$ is degenerate, but still has constant rank. We also impose an additional condition on $R$: the kernel $K$ of the morphism $R^\flat : T \to T^*$ determined by $R$, where $T$ and $T^*$ are the tangent and cotangent bundles of $X$, is an integrable sub-bundle of $T$. Our main result asserts that, if $R$ is real-analytic and if the 1-jet of $R$ at a point $x \in X$ satisfies certain appropriate conditions, the equation (1) admits local real-analytic solutions in a neighborhood of $x$. In particular, it implies the following:

Theorem. *Let $n \geq 3$ and $0 \leq m \leq n$ be given integers. Let $R = (R_{ij})$ be a real-analytic symmetric 2-form on $\mathbb{R}^n$ defined in a neighborhood $U$ of the origin satisfying $R_{ij} = 0$, for all $1 \leq i \leq n$ and $1 \leq j \leq m$, and*

$$R_{ij}(x) = R^0_{ij} + \sum_{k=1}^{m} Q^k_{ij} x^k + \sum_{l=m+1}^{n} R^l_{ij} x^l + O(x^2),$$

*for all $m+1 \leq i, j \leq n$ and $x \in U$, where*

$$R^0 = (R^0_{ij})_{m+1 \leq i,j \leq n}, \qquad Q^k = (Q^k_{ij})_{m+1 \leq i,j \leq n}$$

*are $(n-m) \times (n-m)$ symmetric matrices which possess the following three properties:*

(i) *the matrix $R^0$ is invertible;*
(ii) *the matrices $Q^1, \ldots, Q^m$ are linearly independent;*
(iii) *the matrices $Q^1, \ldots, Q^m$ are traceless.*

*If $\nu$ is the dimension of the common null-space (in $\mathbb{R}^{n-m}$) of the matrices $Q^1, \ldots, Q^m$, suppose that the following conditions do not hold:*

(iv) *we have $m = 2$ and $n = 4$;*
(v) *we have $m = 2$, $n = 5$ and $\nu = 1$;*
(vi) *we have $m = 3$ and $n = 6$.*

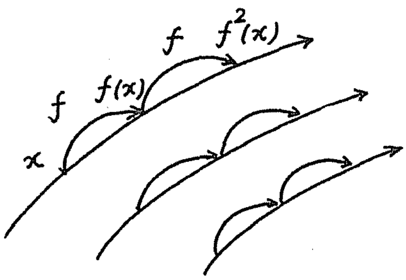
*Then there exists a real-analytic Riemannian metric $g$ on a neighborhood of the origin in $\mathbb{R}^n$ which is a solution of the equation (1).*

We have also found a necessary condition, which $R$ must satisfy in order that our equation have local solutions. This condition arises from the Bianchi identity for the Ricci curvature and only occurs when $R$ is degenerate. It leads to examples of degenerate symmetric 2-forms on $\mathbb{R}^n$ which are not Ricci tensors of any Riemannian metric near the origin.

# 平面の同相写像における流体化可能性

広島大・総科　中山 裕道

いま，$X$ を位相空間とし，$f: X \to X$ を $X$ の同相写像とする．力学系理論では，点 $x$ を通る軌道 $O_f(x) = \{f^n(x) ; n \in \mathbb{Z}\}$ の漸近的な性質を調べることを目標とする．一般に軌道は ばらばらな点であるから，それ自体を図示しにくい．そこで，軌道を連続的につないで下図のように表現できれば便利である．




このことは，$X$ の $\mathbb{R}$ 作用 $\varphi: \mathbb{R} \times X \to X$ で，$\varphi(1,x) = f(x)$ となるものが存在するかという問題になる．このとき，$\varphi(n,x) = f^n(x)$ $(n \in \mathbb{Z})$ より，軌道 $O_f(x)$ は $\{\varphi(t,x) ; t \in \mathbb{R}\}$ によりつながれている．力学系理論では，$\mathbb{R}$ 作用のことを flow（流）と呼び，$\varphi(1,x) = f(x)$ となる flow $\varphi$ が存在するとき，$f$ を flowable（流体化可能）という．

そこで，与えられた同相写像がいつ flowable になるかという問題を考える．これについては，歴史的に次の問題が研究されてきた．

**問題**　向きを保つ $\mathbb{R}^2$ の同相写像で固定点 ($f(x) = x$ となる点) を持たないものは flowable か？

結論から述べると，次図の図形を保つ同相写像は flowable にならない (Brown 1985)．この同相写像が flowable でないことを示すのに，いくつかの道具が作られた．ここでは軌道空間を考えることにする．各軌道 $O_f(x)$ を 1 点につぶしてできる空間を軌道空間という．商写像を

$$\pi : X \to X/f$$

とし，$X/f$ に $\pi$ による商位相をいれるとき，$X/f$ は一般に Hausdorff にならない．例えば，Figure 2 の Reeb 成分を保つ同相写像については，$X/f$ が Hausdorff にならない．

$X/f$ 上の Hausdorff にならない点全体を $S$ とするとき，$\pi^{-1}(S)$ を非ハウスドルフ集合という．例えば，Reeb 成分を保つ同相写像では 2 本の直線になるし，Brown の例では枝付きの 2 本の棒になる．flowable な同相写像では，非ハウスドルフ集合が flow に関する軌

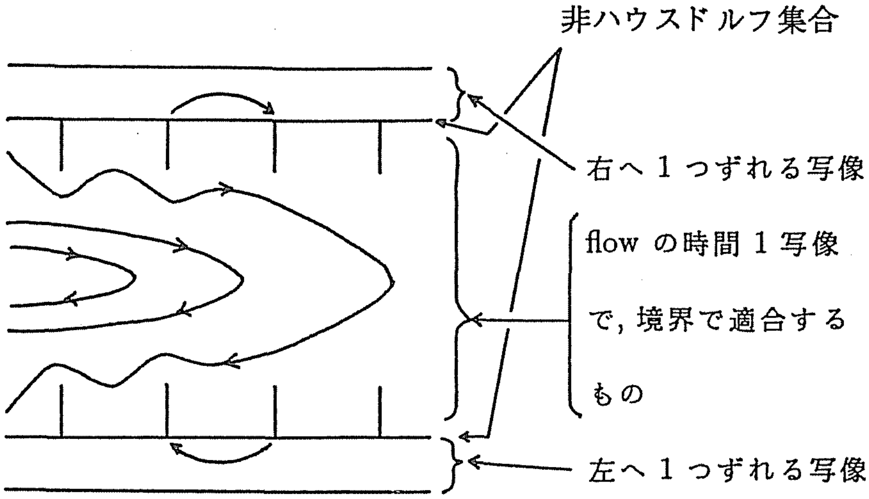


Figure 1: Brown の例

道 $\{\varphi(t,x)\ ;\ t \in \mathbb{R}\}$ の和とならなければならないことが容易に示される．Brown の例の場合，分岐点が障害となり flowable にならないのである．このように非ハウスドルフ集合は非コンパクト空間上の力学系を調べる上で非常に有用である．

そこで，向きを保ち固定点を持たない $\mathbb{R}^2$ の同相写像について，非ハウスドルフ集合の位相的性質を調べ，次の結果を得た．

**定理 1.** 非ハウスドルフ集合は，空で無いならば次元が 1 である．

**定理 2.** 非ハウスドルフ集合が分岐点を持たないのに，flowable でない $\mathbb{R}^2$ の同相写像が存在する．

最近では非特異流について非ハウスドルフ集合の応用を研究している．特に，$\mathbb{R}^3$ の非特異流で non-wandering set が空になるときを調べている．これまで，minimal set が必ず存在すること (Schweitzer の問題 34)，更に Hausdorff ならば flow が product になることを示した．しかし，Marin の例 (1994) 等もあり，一般の場合は何もわかっていないのが現状である．

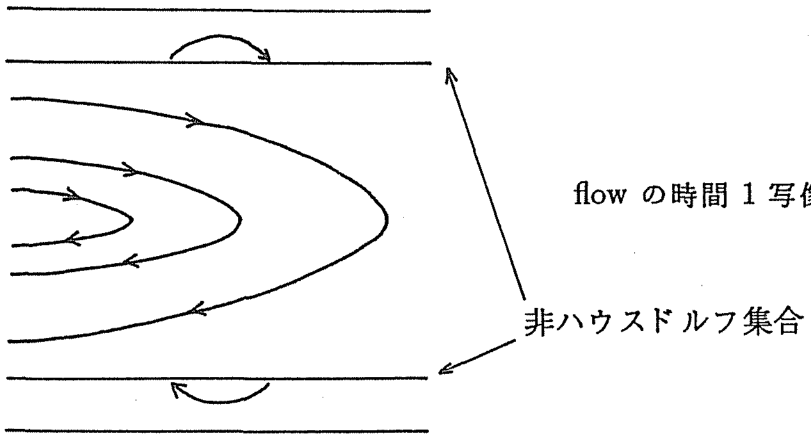


Figure 2: Reeb 成分を保つ同相写像

# Symbolic dynamics and rational subgroups of hyperbolic groups

京大理 加藤毅

(1)Hyperbolic groups　ここでは有限生成離散群 $\Gamma$ の幾何を扱う。一般にその生成元の集合 $A$ を決めると、一次元単体複体、距離空間であるケイリイグラフ $G(\Gamma,A)$ と呼ばれるものが自然に作られる。その頂点は群の元と対応している。特に $\Gamma \subset G(\Gamma,A)$ で $\Gamma$ に距離が $d_A$ が入る。さて、$\Gamma$ がコンパクトリーマン多様体の基本群 $\pi_1(M)$ であったとしよう。普遍被覆空間 $\tilde{M}$ の一点 $x \in \tilde{M}$ を取れば、$\Gamma x \subset \tilde{M}$ にリーマン計量から導かれる距離 $d_M$ が入る。$(\Gamma, d_A)$ と $(\Gamma, d_M)$ は Lipschitz equivalent の関係にある。即ちある $C \geq 1$ があって $1/C \leq d_A/d_M \leq C$ を満たす。そこで $\Gamma$ の距離を使った性質で Lipschitz equivalence で不変なものを研究したい。特に $M$ $(\tilde{M})$ 上の距離を使った幾何から見つけた $\Gamma = \pi(M)$ の 性質が、Lipschitz equivalence で不変な時、その性質を群 $\Gamma$ だけの言葉で formulate できる場合がある。それがうまくいった例が Gromov hyperbolic group である。負曲率閉多様体 $M$ の普遍被覆空間 $\tilde{M}$ の中の三角形はやせていることは良く知られている。リーマン幾何学の基本的な事実である。この性質は、Lipschitz eqivalence で不変であることが分かり、従って $\Gamma = \pi(M)$ の生成元を決めた時のケイリイグラフの中の三角形もやせている。

定義 離散群 $\Gamma$ が hyperbolic とは、任意に生成元を決めた時、そのケイリイグラフの中の三角形がやせている時をいう。Refs は

(1)M.Gromov, Hyperbolic groups , in Essays in Group Theory, MSRI publ.8, Springer.
(2)E.Ghys, P. de la Harpe eds, Sur les Groupes Hyperboliques dáprés Mikhael Gromov.

Gromov hyperbolic group はいくつかの重要な性質をみたすが、ここでは群の boundary について扱う。

定義 hyperbolic group $\Gamma$ のバウンダリイ $\partial\Gamma$ とは、ケイリイグラフ上、一点からの測地線全体を同値類で割ったものである。コンパクトで、$\Gamma$ が自然に作用する。

(2)Symbolic dynamics　$\Gamma = \mathbb{Z}$ の場合、それが作用する空間の力学系を調べることはよくなされてきている。特に $S$ を有限集合とした時、$\sum(S) = \{\mathbb{Z}$ から $S$ への写像全体$\}$ とおけば、自然に $\mathbb{Z}$ が $\sum(S)$ に作用する。$\sum(S)$ は Cantor set である。$\sum \equiv \sum(\Gamma, S)$ を $\Gamma$ から $S$ への写像全体とすれば、上の事実はこの場合にもいえる。

さて、$\Gamma$ が hyperbolic group の時、$(\partial\Gamma, \Gamma)$ はどんな力学系であろうか。それは、$(\sum, \Gamma)$ とどれくらい違うものであろうか。例えば、$\Gamma$ が $n$ 次元負曲率閉多様体の基本群であったら、$\partial\Gamma = S^{n-1}$ であり、自由群ならば、Cantor set である。hyperbolic group は様々なバウンダリイを持ち得ることが想像される。しかし、これらも、共通に持つ幾何学的な性質がある。それを知る為に、以下で定義をする。

定義 A subshift of $\sum = \sum(\Gamma, S)$ とは、closed , $\Gamma$ invariant subset of $\sum$ のことである。

$A \subset \sum$, $F \subset \Gamma$ を有限集合とする時、$C = \{\sigma \in \sum \mid \sigma|_F \in A\}$ を cylinder と呼ぶ。

定義 $C$ を cylinder とする時、$\Phi = \cap_{\gamma \in \Gamma} \gamma C$ を $\sum$ の subshift of finite type と呼ぶ。

さて $\omega$ をコンパクト空間で $\Gamma$ が連続に作用しているとする。

定義 $(\omega, \Gamma)$ が finite type とは、ある subshift of finite type $\Phi \subset \sum(\Gamma, S)$ と連続、全射、$\Gamma$equivariant な写像 $\pi : \Phi \mapsto \omega$ が存在する時をいう。

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

定義 $(\omega,\Gamma)$ が finitely presented とは、ある subshift of finite type$\Phi$ に対して finite type であり、$R(\pi)=\{(\phi_1,\phi_2)\in\Phi\times\Phi|\pi(\phi_1)=\pi(\phi_2)\}$ とおいた時、$R(\pi)$ も finite type である時をいう。

定理（グロモフ） $\Gamma$ が hyperbolic group の時、$(\partial\Gamma,\Gamma)$ は finitely presented である。

この定理は、M.Coornaert and A.Papadopoulos, Symbolic dynamics and Hyperbolic Groups, Lecture Notes in Maths,1539, Springer に詳しく載っている。

(3)Rational subgroups and hyperbolic groups.

定義 $\Gamma$ を離散群とし、その生成元を決め、$G(\Gamma)$ をそのケイリイグラフとする。$H\subset\Gamma$ : subgroup が rational とはある $N\geq 0$ があって、任意の $x,y\in H$ に対し、$l$ を $G(\Gamma)$ の中の $x$ と $y$ をつなぐ任意の測地線とすると、$d(l,H)\leq N$ をみたす。

$H\subset\Gamma$ が有限指数ならば rational である。

Heisenberg group は $\Gamma=<\alpha,\beta,\gamma|[\alpha,\beta]\gamma^{-1},[\alpha,\gamma],[\beta,\gamma]>$ と表示されるが, $H\equiv\{\gamma^i|i\in\mathbb{Z}\}\subset\Gamma$ とおくと、$H$ は rational でない。一方、次の定理がある。

定理（W.Neumman） $\Gamma$ を hyperbolic group とし、$\phi$ を $\Gamma$ の quasi isometric な自己同型群とする。その時、$\phi$ の固定点は $\Gamma$ の rational subgroup である。

主定理 $H$ を hyperbolic group$\Gamma$ の rational subgroup とする。その時、ある cylinder $C$ があって、$\Phi\equiv\cap_{\gamma\in H}\gamma C$ とおくと、ある連続、全射、$H$equivariant な写像 $a_H:\Phi\mapsto\partial H$ が存在する。

Remark(1) hyperbolic group より広いクラスで幾何学的に良い性質をもつものを見つけることは易しくない。例えば、良い性質を持つ群とその部分群から新しい群を幾何学的に作ることはできないであろうか。それの環版が作用素環論で成功している。

(2) 離散群論では特に無限遠の様子が大事だが、hyperbolic group でないと、無限遠のコンパクト化をどうしてよいかわからない。

(3) どのような空間が hyperbolic group のバウンダリイになり得るか、またはなり得ないか調べるのはおもしろい問題である。

(4) 離散群の無限遠のトポロジーを反映させたコホモロジー理論をつくって、その応用として、$\Gamma$ が hyperbolic group の時、closed$K(\Gamma,1)$manifold には、positive scalar curvature が入らないことを J.Roe が示している。

Dec. 16, 1994

# Infinitesimal symmetries and local invariants of isometric embeddings of Riemannian manifolds

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We consider a submanifold of dimension $n$ of $\mathbb{R}^{n+d}$, locally given by $u^{n+i} = h^i(u^1 \cdots u^n)$, $i = 1, \cdots, d$.

A local invariant of order $m$ is a smooth function of the $m$-th jet of $h = (h^1 \cdots h^d)$ which is invariant under isometries.

A system of real valued function $u = (u^1, \cdots, u^{n+d})$ of a Riemannias manifold $(M, g)$ is an isometric embedding if $u$ satisfies

$$\sum_{\alpha=1}^{n+d} \frac{\partial u^\alpha}{\partial x^i} \frac{\partial u^\alpha}{\partial x^j} = g_{ij}(x) \quad , \quad \text{for each } i, j = 1, \cdots, n,$$

where $x = (x^1, \cdots, x^n)$ is a local coordinate system of $M$.

In this paper, we describe the connection between the symmetry of the embedding equations and the local invariants of submanifolds, and compute the local invariants of order 2 and 3 as examples. Actual calculation envolves prolongation of the isometric embedding equations, which was done by using Mathematica R.

# Gillet-Soulé Adams operation using Nielsen's Schur complex

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## 1 Gillet-Soulé Adams operation

J.-P. Serre は [10] の中で, 次のような予想を提出している。

**予想 1.1** $X$ が連結な regular scheme で $Y$, $Z$ が $X$ の既約被約な閉部分スキーム, $W$ は $X \cap Y$ の既約成分とする。この時,

**1** $\operatorname{codim}_X W \leq \operatorname{codim}_X Y + \operatorname{codim}_X Z$.

**2** $\operatorname{codim}_X W < \operatorname{codim}_X Y + \operatorname{codim}_X Z$ ならば,

$$\sum_i (-1)^i l_{\mathcal{O}_{W,X}}(\operatorname{Tor}_i^{\mathcal{O}_{W,X}}(\mathcal{O}_{W,Y}, \mathcal{O}_{Z,X})) = 0.$$

**3** $\operatorname{codim}_X W = \operatorname{codim}_X Y + \operatorname{codim}_X Z$ ならば,

$$\sum_i (-1)^i l_{\mathcal{O}_{W,X}}(\operatorname{Tor}_i^{\mathcal{O}_{W,X}}(\mathcal{O}_{W,Y}, \mathcal{O}_{Z,X})) > 0.$$

Serre は [10] の中で, **1** に証明を与え, **2** と **3** については, $\mathcal{O}_{W,X}$ が不分岐な時に証明している。**3** の一般の場合は未解決だが, **2** は, P. Roberts [7, 8] と Gillet-Soulé [2, 3] によって独立に証明された。

Gillet と Soulé の証明では, support を持つ代数的 $K$ ー理論に$\lambda$環の構造を導入して Adams 作用素を定義することが鍵となる。また, この Adams 作用素は, S. Dutta, P. Roberts によって導入された Dutta multiplicity を自然に記述するのにも役立つのではないかと蔵野和彦氏によって予想され, ごく最近 P. Roberts がその証明を与えた。

詳しくは [3] または [11] を参照して頂きたいが, $\lambda$構造と Adams operation が何かについてまず述べたい。スキームはすべて有限次元のネータ分離的なものとする。

スキーム $X$ に対して, $\mathrm{sub}(X)$ によって, $X$ の closed subset の全体が, $\bigcap$ によってなす monoid を表す。$Y \in \mathrm{sub}(X)$ に対して, $X$ 上の locally free sheaves (of finite rank) のなす bounded complex

$$\mathbb{F} : 0 \to F_h \xrightarrow{\partial_h} F_{h-1} \to \cdots \to F_1 \xrightarrow{\partial_1} F_0 \to 0$$

で, $\mathbb{F}|_{X-Y}$ が完全であるようなもの全体が(チェイン写像を射として)なす圏を $C_Y(X)$ で表す。$C_Y(X)$ の対象の同型類を $\bar{C}_Y(X)$ で表すと, $\bar{C}_Y(X)$ は$\oplus$ を和として additive monoid になる。$\mathbb{F}$ が $C_Y(X)$ の対象の時, $\mathbb{F}$ の $\bar{C}_Y(X)$ における類を $[\mathbb{F}]$ で表そう。$\bar{C}_Y(X)$ に, 次の 2 つの関係で生成される同値関係 $\sim$ を考える。

**1** $0 \to \mathbb{G} \to \mathbb{F} \to \mathbb{H} \to 0$ が $C_Y(X)$ の exact sequence の時, $[\mathbb{F}] \sim [\mathbb{G} \oplus \mathbb{H}]$.

**2** $\mathbb{F}, \mathbb{G}$ が $C_Y(X)$ の対象, $\mathbb{F}$ が完全の時, $[\mathbb{F} \oplus \mathbb{G}] \sim [\mathbb{G}]$.

容易に分かるように, $K_0^Y(X) := \bar{C}_Y(X)/\sim$ には, $\oplus$ を和とする加群の構造が誘導される。さらに, $K_0^\sigma(X) := \bigoplus_{Y \in \mathrm{sub}(X)} K_0^Y(X)$ とおいて, $[\mathbb{F}] \in K_0^Y(X)$, $[\mathbb{G}] \in K_0^Z(X)$ に対して, 積を $[\mathbb{F}] \cup [\mathbb{G}] := [\mathbb{F} \otimes \mathbb{G}] \in K_0^{Y \cap Z}(X)$ と定めることによって, $K_0^\sigma(X)$ は $\mathrm{sub}(X)$-graded な可換環になる。

$X$ 上の普通のバンドルの $K$ー群 $K_0(X)$ は $K_0^X(X)$ と同型である。ところで, $X$ 上のバンドルには, $\oplus$, $\otimes$ (それぞれ和と積に対応) に加え, symmetric power, exterior power などの多重線形的な operation がある。和, 積に加えて, exterior power $\bigwedge^i$ に対応する新しい一項演算 $\lambda^i$ $(i = 0, 1, \ldots)$ を $\lambda^i[E] := [\bigwedge^i E]$ で定義すると, well-defined で, これによって $K_0(X)$ が $\lambda$環になることはそれほど難しくない。ここに, 可換環 $R$ が $\lambda$ 環であるとは, $i \geq 0$ に対して$\lambda^i : R \to R$ が与えられ,

**1** $\lambda^0 = 1$, $\lambda^1(x) = x$ $(x \in R)$, $\lambda^k(1) = 0$ $(k > 1)$.

**2** $\lambda^k(x+y) = \sum_{i=0}^k \lambda^i(x)\lambda^{k-i}(y)$.

**3** $\lambda^k(xy) = P_k(\lambda^1(x), \ldots, \lambda^k(x); \lambda^1(y), \ldots, \lambda^k(y))$, ここに, $P_k \in \mathbb{Z}[X_1, \ldots, X_k; Y_1, \ldots, Y_k]$ は, 変数 $\xi_1, \ldots, \xi_k; \eta_1, \ldots, \eta_k$ に対して

$$e_k(\xi_1\eta_1, \ldots, \xi_k\eta_k) = P_k(e_1(\xi), \ldots, e_k(\xi); e_1(\eta), \ldots, e_k(\eta))$$

($e_i$ は $i$ 番目の elementary symmetric function) で一意的に定まる多項式。

**4** $\lambda^k(\lambda^l(x)) = P_{k,l}(\lambda^1(x), \ldots, \lambda^{kl}(x))$. ここに, $P_{k,l} \in \mathbb{Z}[X_1, \ldots, X_{kl}]$ は

$$e_k \circ e_l = P_{k,l}(e_1, \ldots, e_{kl})$$

で一意的に定まる多項式 ($\circ$ は symmetric function の plethysm, [5] 参照)。

$\lambda$環 $R$ と $\alpha \in R$ に対して,

$$E(t)(\alpha) = \sum_{r \geq 0} \lambda^r(\alpha)t^r, \qquad E'(t)(\alpha) = \sum_{r \geq 0} r\lambda^r(\alpha)t^{r-1}$$

とおき,

$$\sum_{r\geq 0}(-1)^r\psi^{r+1}(\alpha)t^r = E'(t)(\alpha)/E(t)(\alpha)$$

によって $\psi^k : R \to R$ $(k \geq 1)$ を定める。$\psi^k$ を $R$ の $k$番目の Adams operation と呼ぶ。$\psi^k : R \to R$ は環の自己準同型である。

$X$ が regular の時, $K_0^Y(X)_{\mathbb{Q}} = K_0^Y(X) \otimes_{\mathbb{Z}} \mathbb{Q}$ は自然に $Y$ 上の coherent sheaves の $K$-group に $\mathbb{Q}$ を tensor したものと同型であり, Grothendieck Riemann-Roch theorem によって, これは $Y$ の rational Chow-group $CH(Y)_{\mathbb{Q}}$ と同型である。$CH(Y)_{\mathbb{Q}}$ は, $X$ における codimension によって, 明らかに直和分解しているが, 対応する $K_0^Y(X)$ の直和分解の意味は, 一見しては明らかでない。$K_0^Y(X)$ には, ホモロジーの support の $X$ における codimension によって, 自然な filtration が入るが, 直和分解は与えない。実は, $CH(Y)_{\mathbb{Q}}$ の codimension $i$ の component $CH_X^i(Y)_{\mathbb{Q}}$ に対応するのは, $K_0^Y(X)_{\mathbb{Q}}$ の, $\psi^k$ に対する, 固有値 $k^i$ に対応した固有空間 (実は $k$ に depend しない) である。特に, $K_0^Y(X)_{\mathbb{Q}}$ は $X$ が regular なら, $\psi^k$ の固有空間の直和である。この事実が, Serre の予想の 2 の証明に本質的に利いている。

# 2 $\mathbb{Q}$-scheme の場合の construction

このような $\lambda$構造で自然なものを $K_0^\sigma(X)$ に導入するには, $X$ 上の coherent sheaves の複体の圏と, simplicial な coherent sheaves の圏とが同値であることを利用して, 一旦 $C_Y(X)$ の object を simplicial sheaf と見てから (項別に) exterior power を作用させ, 得られた simplicial sheaf をもう一度複体だと思い直すことによって得られる。しかし, この構成は, それほど単純とはいい難く, 与えられた $\mathbb{F}$ に対して, $\lambda^i[\mathbb{F}]$ を代表する複体を書き下すことは容易ではない。

そこで, (恐らく誰でも思いつくことと思われるが), $\lambda^i$ は, $\bigwedge^i$ から得られるべきもので, $\bigwedge^i\mathbb{F}$ は, $(\mathbb{F})^{\otimes i}$ に $i$ 次対称群が交代的に作用した時の invariant subcomplex として, もっと簡単に得られるのではないかという疑問がわく。この考えは, $\mathbb{Q}$-scheme の場合には正しい。$(\mathbb{F})^{\otimes i}$ には, $i$ 次対称群が作用している。単純にテンサーの順番を入れ換えるだけではチェイン写像にならないので, 役に立たないが, 多少の符号を考えると, チェイン写像で作用している。この作用を使い, 一般線形群の既約表現である Schur 加群を複体の場合に ($\mathbb{Q}$ 上の仮定をおいて) Nielsen [6] が一般化している。$\bigwedge^i\mathbb{F}$ はその特別な場合として, 自然に得られる $(\mathbb{F})^{\otimes i}$ の直和因子である。残念ながら, 標数一般の場合には, この手の construction は, どうやっても完全列を完全列に写すようには出来ず, well-defined な $\lambda^i$ を $K_0^Y(X)$ に induce させるものにはならないと思われる。しかしながら, $\mathbb{Q}$ 上の場合には, この construction でうまく$\lambda$構造が定義できる。これを check することは, 大して難しいことではない。

細かい定義をしていないので, statement としては挙げないが, 得られた結果は, Gillet-Soulé の construction と, 上の $\mathbb{Q}$-scheme に限って得られた上の Nielsen の Schur complex を用いた construction が同じ $\lambda$-structure を与えているということである。

Schur complex は, local ring 上の bounded free complex の minimality を保つ。Gillet-Soulé の construction では, minimality は保たれない。

尚, 標数 $p>0$ の体上では, $p$ 番目の Adams operation は, Frobenius によって与えられる。

# 3 Dutta 重複度

$(A,\mathfrak{m},k)$ を $d$ 次元ネータ完備局所環とする。$\mathbb{F}$ は finite free complex で, homology の length は有限とする。この時, $\mathbb{F}$ の Dutta 重複度 $D_A(\mathbb{F})$ が標数 $p>0$ の場合, 次のように定義される。

$$D_A(\mathbb{F}) := \lim_{n\to\infty} \frac{1}{p^{de}} \sum_{i\geq 0}(-1)^i l_A(H_i({}^e\mathbb{F})).$$

ここに, ${}^e\mathbb{F}$ は, $\mathbb{F}$ の $e$ 回の Frobenius, つまり, $\mathbb{F}$ の boundary map の行列の各成分を $p^e$ 乗したものである。標数一般の場合は, local Chern character (Fulton [1] 参照) を用いて $D_A(\mathbb{F})$ は定義される [4]. Dutta multiplicity について, 次の予想がある。

**予想 3.1** $\mathbb{F}$ が長さ $d$ で, $H_0(\mathbb{F})\neq 0$ の時, $D_A(\mathbb{F})>0$.

この予想が正しければ, Serre の予想の positivity part **3** が, $Y$ が Cohen-Macaulay の時に正しくなる。$\dim X\leq 5$ の時にも正しくなる。$A$ が complete intersection なら, この予想は正しい。また, $A$ でこの予想が正しければ, $A$ の非零因子 $y$ について, $A/yA$ でもこの予想は正しい)。これは, $\mathbb{F}$ が長さ $d-1$ の $A/yA$ 上のホモロジーの長さ有限の non-exact な finite free complex とすると, 長さ $d$ の $A$-free complex で, $\mathbb{F}$ と quasi-isomorphic なものがとれる (吉野の $L$-complex) ことによる。

また, $A$ が標数 $p>0$ でも正しい。この事実は, New intersection conjecture の解決 [9] に有効に使われた。

次の定理は, 蔵野によって予想され, 最近 P. Roberts によって解決された。

**定理 3.2** 任意の $k>1$ に対して, $D_A(\mathbb{F})=\lim_{k\to\infty}\frac{1}{k^d}\chi_A(\psi^k[\mathbb{F}])$, ここに, $\chi_A$ はオイラー標数 (ホモロジーの長さの交代和) である。

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# 線形写像の空間の種々の order と norm

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この講演は私が北大を停年退職するにあたっての「最終講義」となったもので，単純ではあるがまだ完全な回答までには到達していないある問題へ向けての研究の現在までの状況の概観である.（しかしこのレジメでは問題を正確に定式化することにとどめる.）

$\mathbb{M}_n \overset{def}{=} \{n \times n$ （複素）行列の全体 $\}$ とする．Matrix unit $E_{ij}$ を使って各行列 $X \in \mathbb{M}_n$ を以下のように表示する：

$$X = [\xi_{ij}] = \sum_{i,j=1}^{n} \xi_{ij} E_{ij}.$$

考察の対象を $\mathbb{M}_n \longmapsto \mathbb{M}_m$ の線形写像 $\Psi$ 及び線形写像の全体のなす空間 $\mathcal{L}(\mathbb{M}_n, \mathbb{M}_m)$ に限る．" 線形性" より $\Psi$ に関する情報は $\Psi(E_{ij})$ $(i, j = 1, 2, \ldots, n)$ ですべてわかるので：

$$[\Psi] \overset{def}{=} [\Psi(E_{ij})] \in \mathbb{M}_{nm}$$

を考える．対応 $\Psi \longleftrightarrow [\Psi]$ で $\mathcal{L}(\mathbb{M}_n, \mathbb{M}_m)$ と $\mathbb{M}_{nm}$ は同一視されるが，自然な対応で $\mathbb{M}_{nm}$ はテンソル積 $\mathbb{M}_n \otimes \mathbb{M}_m$ と同一視される．また $\mathbb{M}_{nm}$ は $\mathbb{C}^{nm}$ の線形写像の空間であるが，$\mathbb{C}^{nm}$ は自然な対応により $\mathbb{M}_{n,m} \overset{def}{=} \{n \times m$ 行列の全体 $\}$ と同一視される.

写像 $\Psi \in \mathcal{L}(\mathbb{M}_n, \mathbb{M}_m)$ が Hermitian であることを $\Psi(X^*) = \Psi(X)^*$ for all $X \in \mathbb{M}_n$ で定義するのは当然であるが，$\Psi$ の positivity として 3 種類のものを考える.

$\Psi$ が **positive** $\overset{def}{\Longleftrightarrow}$ $\Psi(X) \geq 0$ whenever $X \geq 0$.

$\Psi$ が **completely positive** $\overset{def}{\Longleftrightarrow}$ $Y_k \in \mathbb{M}_{m,n}$ $(k = 1, 2, \ldots, N)$ があり

$$\Psi(X) = \sum_{k=1}^{N} Y_k X Y_k^* \quad \text{for all } X \in \mathbb{M}_n.$$

$\Psi$ が **strongly positive** $\overset{def}{\Longleftrightarrow}$ $0 \leq D_k \in \mathbb{M}_n, 0 \leq F_k \in \mathbb{M}_m$ $(k = 1, 2, \ldots, N)$ があり

$$\Psi(X) = \sum_{k}^{N} tr(X D_k) F_k \quad \text{for all } X \in \mathbb{M}_n.$$

以下は容易に判る：

(1) $\Psi$ positive $\Longleftrightarrow [\langle \Psi(E_{ij}) u, u \rangle] \geq 0$ in $\mathbb{M}_n$ for all $u \in \mathbb{C}^m$

$$\Longleftrightarrow tr([\Psi] \cdot (X \otimes X^*)) \geq 0 \text{ for all rank-one } X \in \mathbb{M}_{n,m}.$$

(2) $\Psi$ completely positive $\Longleftrightarrow [\Psi] \geq 0$ in $\mathbb{M}_{nm}$

$$\Longleftrightarrow tr([\Psi]\cdot(X\otimes X^*)) \geq 0 \text{ for all } X \in \mathbb{M}_{n,m}.$$

(3) $\Psi$ strongly positive $\Longleftrightarrow [\Psi] = \sum_{k=1}^{N} D_k \otimes F_k$

for some $0 \leq D_k \in \mathbb{M}_n, 0 \leq F_k \in \mathbb{M}_m$.

これらの positivity に基づいて $\mathcal{L}(\mathbb{M}_n, \mathbb{M}_m)$ の上で3つの凸錘

$$\mathcal{P}_- \overset{def}{=} \{\text{positive 写像の全体}\}, \quad \mathcal{P}_0 \overset{def}{=} \{\text{completely positive 写像の全体}\},$$

$$\mathcal{P}_+ \overset{def}{=} \{\text{completely positive 写像の全体}\}$$

を考える.

以下は容易に判る：

(1) $\mathcal{P}_-$ の双対錘 (dual cone) $= \mathcal{P}_+$,

(2) $\mathcal{P}_0$ の双対錘 = 自分自身 $\mathcal{P}_0$,

(3) $\mathcal{P}_+$ の双対錘 $= \mathcal{P}_-$.

$\Phi \in \mathcal{P}_0$ が与えられた (Hermitian) $\Psi$ の **completely positive majorant** とは $\Phi + \Psi$ 及び $\Phi - \Psi$ が共に completely positive であることである. このとき

$$\Phi_1 \overset{def}{=} \frac{\Phi + \Psi}{2}, \quad \Phi_2 \overset{def}{=} \frac{\Phi - \Psi}{2}$$

とおくと, $\Phi_1, \Phi_2$ は共に completely positive で

$$\Phi = \Phi_1 + \Phi_2, \quad \Psi = \Phi_1 - \Phi_2 \quad \text{(Jordan 型分解)}$$

となる. 同様に **strongly positive majorant** 及び **positive majorant** も定義される.

$\alpha, \beta$ が $1, \infty$ のどちらかのとき, 線形写像 $\Psi$ の norm $\|\Psi\|_{(\alpha,\beta)}$ を通常のように

$$\|\Psi\|_{(\alpha,\beta)} \overset{def}{=} \max_X \frac{\|\Psi(X)\|_\beta}{\|X\|_\alpha}$$

で定義する.

**Problem.** $\alpha, \beta$ が $1, \infty$ のどちらかとする.

$$\sup_\Psi \inf_\Phi \{\frac{\|\Phi\|_{(\alpha,\beta)}}{\|\Psi\|_{(\alpha,\beta)}} \,;\, \Phi \pm \Psi \in \mathcal{P}_0\}$$

を求めよ. 同様なことを $\mathcal{P}_+, \mathcal{P}_-$ にも考えよ.

# 離散的勾配流の補間

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ある関数（または写像）のクラス $\mathcal{K}$ で定義された汎関数 $J$ に付随する勾配流の方程式

$$(1)\qquad \begin{cases} u_t = -\dfrac{1}{2}\mathrm{grad}J(u) \quad (t>0), \\ u(0) = u_0 \in \mathcal{K} \end{cases}$$

を考える。ここで、$\mathcal{K}$ は $\mathcal{K} \subset L^2(\Omega)$（$\Omega$ は $\mathbf{R}^m$ 内の有界領域）とし、$J$ がその上で Gâteaux 微分可能であるとする。grad$J$ は Gâteaux 微分（第一変分）を表す。すなわち、

$$\langle \mathrm{grad}J(u), \varphi\rangle = \frac{d}{d\varepsilon}J(u+\varepsilon\varphi)\bigg|_{\varepsilon=0}$$

が存在するとする。$J$ が Dirichlet 積分

$$J(u) = \int_\Omega |\nabla u|^2 dx$$

のとき、(1) は熱方程式になる。(1) は発展方程式の一つであるが、ここではそれを変分学的に解く方法を考えよう。$h>0$ を固定し、汎関数

$$J_n^h(u) = \int_\Omega \frac{|u-u_{n-1}^h|^2}{h}dx + J(u)$$

の $\mathcal{K}$ における minimizer として $u_n^h$ を帰納的に定義する。上の式の右辺第１項の第一変分を求めると、

$$\frac{d}{d\varepsilon}\int_\Omega \frac{|u+\varepsilon\varphi-u_{n-1}^h|^2}{h}dx\bigg|_{\varepsilon=0} = \int_\Omega \frac{2(u-u_{n-1}^h)\varphi}{h}dx = \left\langle \frac{2(u-u_{n-1}^h)}{h}, \varphi\right\rangle_{L^2(\Omega)}$$

となる。従って、$u_n^h$ が満たす方程式は

$$(2)\qquad \frac{u_n^h - u_{n-1}^h}{h} = -\frac{1}{2}\mathrm{grad}J(u_n^h)$$

となり、(1) を時間方向に差分化したものとなる。このことから minimizer の列 $\{u_n^h\}$ を離散的勾配流 (discrete Morse semiflow) と呼ぶことにする。

離散的勾配流 $\{u_n^h\}$ を用いて、半無限区間 $(0,\infty)$ 上の関数 $\bar{u}^h$ と $u^h$ を次の様に定義する。$t \in ((n-1)h, nh]$ に対し、

$$(3)\qquad \begin{cases} \bar{u}^h(t) = u_n^h, \\ u^h(t) = \dfrac{t-(n-1)h}{h}u_n^h + \dfrac{nh-t}{h}u_{n-1}^h \end{cases}$$

とおく。$\mathcal{K}$ や $\mathrm{grad}J(\cdot)$ に適当な仮定が有れば、これらの関数は $h\downarrow 0$ のとき収束し、(1) の解を与える。

(3) は、離散的勾配流 $\{u_n^h\}$ の補間である。しかし、$\bar{u}^h$ は補間として粗いものであるし、もともとの変分問題が非線形条件の下で考えなくてはならない場合、$u^h$ は同じ条件を満たず、都合が悪い。このことから、もっと、変分学的な補間を考える必要が出てくる。新しい補間 $\tilde{u}^h$ は、

- $h\downarrow 0$ のとき $\bar{u}^h$ や $u^h$ の収束先と同じ関数に収束する。
- $J(\tilde{u}^h(t))$ が非増加関数になる。

を満たすように作るべきであろう（第 2 の要請は、(1) の解は形式的には、

$$\frac{d}{dt}J(u) = \langle \mathrm{grad}J(u), u_t\rangle = -2\int_\Omega |u_t|^2 dx \le 0 \tag{4}$$

を満たす事による)。

そこで、$0<\theta\le 1$ に対し、

$$J_n^{\theta h}(u) = \int_\Omega \frac{|u-u_{n-1}^h|^2}{\theta h}dx + J(u)$$

の $\mathcal{K}$ における minimizer として $u_n^{\theta h}$ を定義する。$(0,\infty)$ 上の関数 $\tilde{u}^h$ を

$$\tilde{u}^h((n-1+\theta)h) = u_n^{\theta h}$$

で定義する。但し、$n\in\mathrm{N}$, $0<\theta\le 1$ とする。$\tilde{u}^h(nh)=u_n^h$ なので、$\{u_n^h\}$ の補間になっている。

**定理 1.** 補間 $\tilde{u}^h$ は、上の 2 つの要請を満たす。

しかし、上の 2 つの要請は $\bar{u}^h$ も満たしており、これだけでは $\tilde{u}^h$ の有効性を主張することはできない。そこで、第 3 の要請を与える。

- 定常流などの特別な場合を除き、$J(\tilde{u}^h(t))$ は狭義減少関数になる。

この要請は (4) より妥当と思われるし、$J(\bar{u}^h(t))$ は満たさない。

**定理 2.** 補間 $\tilde{u}^h$ は、第 3 の要請をある意味で満たす。

調和写像に対する勾配流を考えても分かるように、(1) の弱解に対して $J(u(t))$ は一般に連続にならず、第 1 種の不連続性があらわれる。従って、$J(\tilde{u}^h(t))$ が連続になるように補間しようとする試みはあまり意味がない。むしろ不連続性があらわれて自然であり、その解析が重要である。上の補間でも、一般に第 1 種の不連続性があらわれる。しかし、不連続性は変分学的に特徴づけることができる。これから特に、$J$ が凸であれば、$J(\tilde{u}^h(t))$ は連続になることが分かる。

不連続性に関する定理は紙面の制約上割愛した。定理 1, 2 の主張も正確さを欠いている事をお詫びする。

## Dirac Operators with Unbounded Potentials at Infinity

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Let us consider the Dirac operator

$$H = \sum_{j=1}^{3} \alpha_j D_j + \beta + Q(x), \quad x \in \mathbf{R}^3, \quad D_j = -i\frac{\partial}{\partial x_j},$$

on $[L^2(\mathbf{R}^3)]^4$, where $\alpha_j\,(1 \le j \le 3)$ and $\alpha_4 = \beta$ are $4 \times 4$ Hermitian symmetric constant matrices satisfying the anti-commutation property

$$\alpha_j \alpha_k + \alpha_k \alpha_j = 2\,\delta_{jk}\, I \qquad (1 \le j, k \le 4),$$

and $Q(x)$ is an Hermitian symmetric matrix valued function (,where $I$ is the $4 \times 4$ unit matrix). Recently, the spectral theory of the Dirac operator satisfying

$$\lim_{|x| \to \infty} |q_{jk}(x)| = +\infty$$

for some components $q_{jk}(x)$ of $Q(x)$, has been extensively investigated (Hachem [2], [3], Helffer–Nourrigat–Wang [4], Suzuki [10]). Our interest here is to study the spectrum of the Dirac operator with $Q(x) = p(x)\beta + q(x) I$, where $p(x)$ and $q(x)$ are real valued smooth functions satisfying the following ;

Case 1

$$\lim_{|x| \to \infty} q(x) = +\infty \quad \text{and} \quad p(x) \equiv 0,$$

Case 2

$$\lim_{|x| \to \infty} p(x) = +\infty \quad \text{and} \quad q(x) \equiv 0,$$

Case 3

$$p(x) \equiv q(x) \quad \text{and} \quad \lim_{|x| \to \infty} p(x) = +\infty.$$

**Case 1**

If $q(x)$ is a spherically symmetric potential satisfying

$$\int_R^{+\infty} \frac{|q'(r)|}{q(r)^2} < \infty \quad \text{for some} \quad R > 0,$$

then it is shown by Titchmarsh [13] and Erdélyi [1] that

$$\sigma_{ac}(H) = \mathbf{R} \text{ and } \sigma_{sc}(H) = \emptyset .$$

For more general potentials (not spherically symmetric), it is well known that $\sigma(H) = \mathbf{R}$ under some additional conditions (see, e.g., Thaller [12, p.131]). At a glance it is conjectured that the absolute continuity of the spectrum will be discussed by using the Mourre estimate (Mourre [8]). The application of the Mourre estimate, however, is not an easy task to me at present but the non-existence or the discreteness of eigenvalues can be shown for a class of potentials satisfying

$$q(x) = O(|x|) \text{ as } |x| \longrightarrow +\infty$$

in view of Kalf [7], Tamura [11] aand Uchiyama–Yamada [14].

**Case 2**

Some works on the Case 2 and Case 3 are found in physical articles (see, e.g., Ikhdair–Mustafa–Sever [5], Jena–Tripati [6], Ram–Halasa [9]). Their potentials are of type $p(x) = C\,r^{\alpha}$ $(0 < \alpha \leq 2)$ or $C \log r$ ($C$ is a positive constant). It seems that there is not written the spectral structure of $H$, although the numerical analysis of the eigenvalues is studied.

*Proposition 1.* If $p(x)$ satisfies

$$\lim_{|x| \to \infty} p(x) = +\infty , \quad \frac{\partial p}{\partial r} = O(p(x)) \text{ as } |x| \longrightarrow \infty ,$$

then $\sigma(H) = \sigma_d(H)$ (a discrete set of eigenvalues of $H$ with finite multiplicity), which is unbounded above and below.

**Case 3**

In this case we may state that the property of Case 2 appears in the positive line, while the property of Case 1 appears in the negative line if $p(x) = O(r^2)$ as $|x| \longrightarrow \infty$.

*Proposition 2.* Let $p(x) \equiv q(x)$. Then the following properties hold ;

(1) Let $p(x)$ be a positive homogeneous function of degree $\alpha > 0$. Then we have

$$\sigma(H) \cap (0, +\infty) = \sigma_d(H),$$

which is unbounded above,

(2) Let $p(x)$ be a positive homogeneous function of degree $\alpha$ such that $0 < \alpha \leq 2$. Then we have

$$(-\infty, 0] \subset \sigma(H) \text{ and } \sigma_p(H) \cap (-\infty, 0] = \emptyset .$$

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# Recent topics on the Ginzburg-Landau equation

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**Abstract**

Recent papers [7], [8] and [10] studies the Ginzburg-Landau equation $\Delta\Phi + \lambda(1 - |\Phi|^2)\Phi = 0, \Phi = u_1 + iu_2$ in a bounded domain $\Omega \subset \mathbb{R}^n$ with the homogeneous Neumann boundary condition. Those works revealed the instability of non-constant solutions in any convex domain and the existance of stable non-constant solutions in topologically non-trivial domains.

In the field of superconductivity the Ginzburg-Landau (GL) equation has been playing an important role for the understanding of macroscopic superconducting phenomena. This equation was originally proposed in [4], where the magnetic effect casued by the current of superconducting electrons is taken account into the equation. Here we are concerned the simple version with no magnetic effect. Then GL equation with non-dimensional form is written as

$$\Delta\Phi + \lambda(1 - |\Phi|^2)\Phi = 0$$

where $\Delta$ denotes the Laplacian, $\Phi$ is a complex valued function $\Phi = u_1 + iu_2$ and $\lambda$ is a positive parameter. Hereafter the set of complex values $\mathbb{C}$ is identified with the one of 2-dimensional vectors $\mathbb{R}^2$, so the above equation is also written in real vector form:

$$\Delta u + \lambda(1 - |u|^2)u = 0, \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad |u|^2 = u_1^2 + u_2^2.$$

In the Ginzburg-Landau theory $\Phi$ denotes the macrowave function describing a superconducting state and $|\Phi|^2$ is the desnsity of superconducting electrons. Therefore $|\Phi| = 0$ corresponds to the normal state and a solution with zeros physically represents a mixed state of superconducting and normal ones. Then the zero of $\Phi$ is called a vortex. Here we are concerned with GL equation in a bounded domain $\Omega \subset \mathbb{R}^n (n \geq 2)$ subject to the homogeneous Neumann boundary condition, that is,

$$\begin{cases} \Delta\Phi + \lambda(1 - |\Phi|^2)\Phi = 0 & \text{in } \Omega \\ \frac{\partial \Phi}{\partial \nu} = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\partial/\partial\nu$ denotes the outer normal derivatives on the smooth boundary $\partial\Omega$. We remark that Equation ( 1 ) is an Euler equation of the following energy functional (called the Ginzburg-Landau energy)

$$\mathcal{E}(\Phi) := \frac{1}{2}\int_\Omega \{|\nabla\Phi|^2 + \frac{\lambda}{2}(1 - |\Phi|^2)^2\}dx \tag{2}$$

or the stationary equation of the evolutionary GL equation

$$\begin{cases} \frac{\partial \Phi}{\partial t} = \Delta\Phi + \lambda(1 - |\Phi|^2)\Phi & \text{in } \Omega \\ \frac{\partial \Phi}{\partial \nu} = 0 & \text{on } \partial\Omega \\ \Phi(0, x) = \Phi_0(x) \end{cases} \tag{3}$$

namely a solution to ( 1 ) is given by an equlibrium solution to ( 2 ). Indeed Equation ( 3 ) is the gradient equation for the energy functional ( 2') and equilibrium solutions are only allowed as the asymptotic state as $t \to \infty$ of ( 3 ) (see [6]). Here, as a state space, we take a fuction space $C(\bar{\Omega} : \mathbb{C})$ of continuous functions of $\bar{\Omega}$ into $\mathbb{C}$ with sup-norm, where $\bar{\Omega}$ denotes the closure of $\Omega$. One easily sees that the functional ( 2 ) has a family of global minimizers $\{\Phi(x) \equiv a : |a| = 1\}$ and that those are stable constant equilibrium solutions to ( 3 ), so we are interested in the existence of stable non-constant solutions to ( 1 ) (or non-constant local minimizers of ( ').) Then "stable solutions" to ( 1 ) are meant by stable equilibrium solutions to ( 3 ) and "stable" is used in the sense of Lyapunov. We summarize some of main results given in the works [7], [8] and [10] in relevence to the existence of stable non-constant solutions to ( 1 ). In the work [7] it was revealed that any convex domain $\Omega$ never admits stable non-constant solutions, that is, the constant solutions, $\Phi \equiv a, |a| = 1$, are only stable solutions in all convex domains. On the other hand we see from [8], [9] and [10] that for a domain $\Omega$ which is topologically "non-trivial" in some sense, there exist stable non-constant solutions if $\lambda$ is sufficiently large. Here "non-trivial" means "not simply connected" provided that $n = 2$ or 3 (for the precise definition, see [10]). For the non-trivial domain there are infinitely many homotopy classes in the space $C(\bar{\Omega}; S^1), S^1 = \{|z| = 1\}$. It can be proved that given any homotopy class $\gamma$ there exists a stable solution $\Phi_\lambda$ with $\Phi_\lambda/|\Phi_l| \in \gamma$ if $\lambda$ is sufficiently large. We remark that all the stable solutions constructed in [7], [8] and [10] don't vanish in the domains. Hence we have the natural query: is there a stable solution with zeros? Or is there a stable non-constant solution in a simply connected domain? For this problem see [3] and [9].

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# Asymptotic behavior of the transition density for jump type processes in small time

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Consider the stochastic differential equation (S.D.E.):

$$x_t(x) = x + \sum_{s \le t} \gamma(x_{s-}(x), \Delta z(s)),$$

where $z(t)$ denotes an $\mathbf{R}^d$-valued Lévy process (semimartingale) of pure jump type with the Lévy measure $h(d\zeta)$, $\Delta z(t) = z(t) - z(t-)$ and $\gamma(x,\zeta)$ denotes a non-degenerate bounded function from $\mathbf{R}^d \times \mathbf{R}^d$ to $\mathbf{R}^d$. It is known that the process $x_t(x)$ has a generator $L$ of the corresponding semigroup which is of the form

$$Lf(x) = \int_{\mathbf{R}^d \setminus \{x\}} [f(z) - f(x)]\ g(x, dz),$$

for a function $f$ in a certain class. Here $g(x,A) = \int_{\mathbf{R}^d} 1_{A\setminus\{x\}}\ (x+\gamma(x,\zeta))h(d\zeta)$ is the *Lévy measure* of $x_t(x)$.

Under certain conditions (including $g(x,dz) = g(x,z)dz$), Léandre (1987) studied the asymptotic behavior of the transition density $p_t(x,y)$ of this process, and showed that

$$p_t(x,y) \sim g(x,y)\ t \quad \text{as} \quad t \to 0 \quad \text{if} \quad g(x,y) \neq 0.$$

Here we note that the condition "$g(x,y) \neq 0$" implies that the process can reach $y$ from $x$ by a single jump. The object of this paper is to give a refinement of Léandre's result in the following form :

$$p_t(x,y) \sim C(x,y,\alpha(x,y))\ t^{\alpha(x,y)} \qquad \text{as} \quad t \to 0,$$

where $\alpha(x,y)$ can be interpreted as, roughly speaking, the minimum number of jumps by which the trajectory can reach $y$ from $x$ $(y \neq x)$.

# 固体ヘリウムの結晶成長

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固体ヘリウム ($^4He$) の結晶成長の研究は、極低温が必要で難しいにも関わらず幾つかの興味深い実験が行なわれてきた。これは 1.5K 以下の固体ヘリウムの結晶成長に於いては、(1) 潜熱が伴わないこと、(2) 液相が超流動相であり有効熱伝導率が非常に大きいこと、の 2 つの理由の為に結晶成長に伴う熱の流れを無視できるという特徴があるからである。また、このことは荒れた面では結晶成長係数が非常に大きく、また容易に結晶の平衡形を観察できることを意味する。

結晶の平衡形は、体積一定のもとでその界面エネルギーを最小にする図形 (Wulff's construction) により決定される。固体ヘリウムの界面張力は高温ではほぼ等方的であるが、低温では異方性が増加する。これに伴い平衡形は球形から、1.23K で (0001) 面、0.95K で (1010) 面、0.35K で (1011) 面がファセットとなり六角柱状な多面体へと変化する。

固体ヘリウムの結晶成長では、ファセットの面とそれ以外の荒れた面での振舞いは著しく異なることが知られている。

まず、荒れた面に於いては界面の成長速度 $v$ は、

$$v = K\Delta\mu$$

と表わされることが実験的に確認されている。ここで、$\Delta\mu$ は液相と固相の化学ポテンシャルの差、(等温的な条件では $\Delta\mu = \Delta p_L/\rho_L - \Delta p_S/\rho_S$ となる。また $\Delta p_S = \Delta p_L + \gamma/R$ という関係も満たす必要がある。) $K$ は結晶成長係数 (kinetic growth coefficent) と呼ばれる界面の易動度に相当する量である。また $K$ は結晶の方位にも依存する。ここで興味深いのは、高温で界面張力がほぼ等方的な場合 (結晶の平衡形は球形) でも $K$ は等方的ではなく大きな異方性を持っているという点である。つまり、体積一定の容器のなかに外部からヘリウムを送り込むことにより結晶を成長させるとき、その成長過程の形は平衡形から大きくずれ、そして、ヘリウムを送り込むことをやめ成長が止まると (体積一定の条件で) また結晶の平衡形にもどる運動を行う。

また、低温での結晶成長係数の実測値は非常に大きいことより、通常の結晶では見られない現象が現われる。この代表的な例が結晶化波 (crystallization wave) と呼ばれる界面波で液相の超流動体の流れを慣性、界面張力を復元力とするものである。この結晶化波の分散式は

$$\omega^2 = \gamma\frac{\rho_L}{(\rho_S - \rho_L)^2}k^3$$

で与えられることが知られており、光学的方法によりその存在が確認されている。

一方、ファセットの面の結晶成長の振舞いは、荒れた面のときのように $v = K\Delta\mu$ のように $\Delta\mu$ に対して比例関係では表わされず $\Delta\mu$ が小さい範囲では界面の成長速度は非常に小さく 0 と考えてよい範囲があることが実験的に知られている。

固体ヘリウムの結晶成長の実験による研究は、低温を用いることもありまだ残された問題も多く、これからの研究が期待されている。

# An explicit formula for the Fourier coefficients of Siegel-Eisenstein series of degree 3

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Let $k$ be an even integer such that $k \geq 6$ and

$$E_k(Z) = \sum_{\{C,D\}} \mid CZ + D \mid^{-k}$$

Siegel Eisenstein series of degree 3 and of weight $k$. where $\{C, D\}$ runs over all representatives of the equivalence classes of coprime symmetric pairs of degree 3. Then $E_k(Z)$ has the following Fourier expansion:

$$E_k(Z) = \sum_C c_k(C) exp(2\pi i Tr(CZ)),$$

where $C$ runs over all semi-positive definite half-integral matrices of degree 3 over $\mathbf{Z}$, and $Tr$ denotes the trace. In this talk, we give an explicit formula for $c_k(C)$ for any positive definite half-integral matrix of degree 3. To state our main result explicitly, let $\mathbf{Z}_p$ be the ring of $p$-adic integers, and $\mathbf{Z}_p^*$ the group of $p$-adic units. When $p \neq 2$, for $a = p^r c \in \mathbf{Z}_p$ with $r \in \mathbf{Z}, c \in \mathbf{Z}_p^*$ define

$$\chi_p(a) = (\frac{c}{p}) \text{ or } 0$$

according as $r$ is even or odd. Here $(\frac{}{p})$ is the quadratic residue symbol modulo $p$. When $p = 2$, for $a = p^r c \in \mathbf{Z}_p$ with $r \in \mathbf{Z}, c \in \mathbf{Z}_p^*$ put

$$\tilde{\chi}_2(c) = \begin{cases} +1 & \text{if } r \equiv 0, \ c \equiv 1 \bmod 8 \\ -1 & \text{if } r \equiv 0, \ c \equiv 5 \bmod 8 \\ 0 & \text{otherwise.} \end{cases}$$

For a non-degenerate symmetric matrix $B = (b_{ij})$ with entries in the field $\mathbf{Q}_p$ of $p$-adic numbers of degree 3 put

$$m_1 = m_{1,p}(B) = \min(\mathrm{ord}(b_{11}), \mathrm{ord}(b_{22}), \mathrm{ord}(b_{33}), \mathrm{ord}(2b_{12}), \mathrm{ord}(2b_{13}), \mathrm{ord}(2b_{23})),$$

$$m_2 = m_{2,p}(B) = \min(\mathrm{ord}(4B_{11}), \mathrm{ord}(4B_{22}), \mathrm{ord}(4B_{33}), \mathrm{ord}(8B_{12}), \mathrm{ord}(8B_{13}), \mathrm{ord}(8B_{23})),$$

$$m_3 = m_{3,p}(B) = \mathrm{ord}(4\det B),$$

where ord denotes the normalized $p$-adic order of $\mathbf{Q}_p$ and $B_{ij}$ is the $(i,j)$-th cofactor of $\det B$. We put

$$\eta = \eta_p(B) = -s_p(B) \text{ or } s_p(B)$$

according as $p = 2$ or not, where $s_p(B)$ is the Hasse invariant of $B$. Further we define $\xi_p(B)$ and $\zeta_p(B)$ as follows:

$$\xi_p(B) = \begin{cases} \chi_p(-B_{i_0j_0}) & \text{if } m_3 - 2m_2 + m_1 \geq -\delta_{2p} + 1 \\ 1 & \text{otherwise} \end{cases},$$

$$\zeta_p(B) = \begin{cases} \chi_p(-B_{i_0j_0})^2 & \text{if } m_3 - 2m_2 + m_1 \geq -\delta_{2p} \\ 1 & \text{otherwise} \end{cases},$$

where $B_{i_0j_0}$ is a cofactor of $\det B$ such that $\mathrm{ord}(2^{3-\delta_{i_0j_0}} B_{i_0j_0}) = m_2$. Now let $B$ be a non-degenerate half-integral matrix over $\mathbf{Z}$ of degree 3. Then for each prime number $p$ and integer $k$ we define $F_{p,k}(B)$ by

$$\begin{aligned} &F_{p,k}(B) \\ &= \sum_{i=0}^{m_1}\Big(\sum_{j=0}^{[(m_2-\delta_{2p}-1)/2]-i} p^{(5-2k)j}\Big)p^{(3-k)i} \\ &+ \eta(B)p^{(2-k)m_3-(3-2k)([m_2/2]-\delta_{2p})}\sum_{i=0}^{m_1}\Big(\sum_{j=n'}^{[m_2/2]-\delta_{2p}-i} p^{(3-2k)j}\Big)p^{(2-k)i} \\ &+ p^{(5-2k)[m_2/2])-(2-k)m_1}\zeta_p(B)\sum_{i=0}^{m_3-2m_2+m_1}(p^{(2-k)}\xi_p(B))^i\sum_{j=0}^{m_1}p^{(2-k)j}, \end{aligned}$$

where $n' = n'_p(B)$ is the number defined by

$$n'_p(B) = \begin{cases} 1 & \text{if } p \neq 2 \text{ and } m_2 \equiv 0 \bmod 2, \text{ or } p = 2, m_3 - 2m_2 + m_1 = -4, \text{ and } m_2 \equiv 0 \bmod 2 \\ 0 & \text{otherwise} \end{cases}.$$

Remark that the set $\{F_{p,k}(B)\}_p$ is genus invariant. Further remark that $F_{p,k}(B)$ is expressed explicitly as a polynomial of $p^{-k}$ of degree $m_{3,p}(B)$, and in particular it is 1 for almost all $p$. Then our main result in talk is

**Theorem 1.1.** *Let $B$ be a positive definite half-integral matrix over $\mathbf{Z}$ of degree 3. Then we have*

$$c_k(B) = \frac{(-1)^{k/2} 2^{5-k} k(k-1) \mid 2B \mid^{k-2}}{B_{k/2} B_{k-1}} \prod_{p \| 2B |} F_{p,k}(B),$$

*where $B_i$ is the $i$-th Bernoulli number.*

# Areas and Volumes in Complex Analysis

H. Alexander

The following is due to Alexander, Taylor and Ullman, [1972]. Here $\mathbb{D}$ is the open unit disk.

THEOREM. Let $f \in H^2$ with $f(0) = 0$. Then

$$\pi \int_0^{2\pi} |f^*(e^{i\theta})|^2 \, d\left(\frac{\theta}{2\pi}\right) \leq \text{area}\,(f(\mathbb{D})).$$

We shall discuss some extensions and applications of the theorem. It should be noted that the area is taken <u>without</u> multiplicity.

(1) The theorem extends to uniform algerbras, where $f(\mathbb{D})$ is replaced by the spectrum of a member of the algebra.

(2) There is a direct generalization to mappings from the unit ball in $\mathbb{C}^n$ to $\mathbb{C}^n$

(3) The theorem can be applied to obtain lower bounds for volumes of analytic subvarieties of $\mathbb{C}^n$

(4) The theorem gives a connection between BMO and area. (Axler and Shapiro)

(5) Combined with an isoperimetric inequality for polynomial hulls, one obtains the following, which answers a question of Stout.

Let $X$ be compact in $\mathbb{C}^n$, let $p \in \hat{X}$, the polynomial hull of $X$, and let $r$ be the distance from $p$ to $X$. Then

$$\mathcal{H}^1(X) \geq 2\pi r$$

(Here, $\mathcal{H}^1$ is one-dimensional Hausdorff measure)