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Strategic Reasoning in Extensive Games with Short Sight

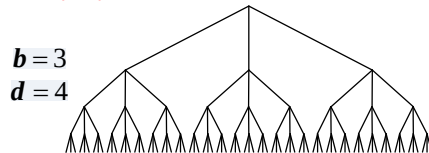
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A calculation

In a game like Chess, game tree's size grows exponentially with both its depth and its branching factor.

Time complexity: $O(b^d)$ (b for branching factor, d for depth)



Branching factor: $b \approx 35$, depth: $d \approx 100$.

Number of paths in the game tree: $35^{100} \approx 10^{135}$ - Much too big for a normal game tree search.

Comparison: Number of particles in the universe $\approx 10^{87}$

Outline

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- 3 Logic
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 - Axiomatization
- 4 Characterizing solutions concepts for games with short sight
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- Strong assumption:
Entire structure of a game is common knowledge to all players.
- Solution:
Grossi and Turrini proposed the concept of *games with short sight* (Grossi and Turrini, 2012), in which players can only see part of the game tree.



Figure: Short-sighted people

- Contribution:
A modal Logic system for reasoning about games with short sight.

Extensive game: an example

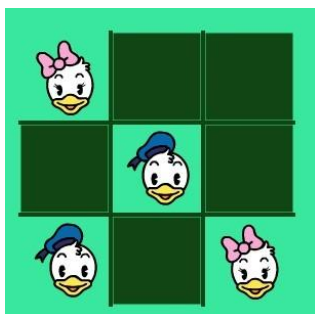


Figure: Tic-Tac-Toe game

Rule: Two players take turns to mark the spaces in a 3×3 grid. The player who succeeds in placing three respective marks in a horizontal, vertical, or diagonal row wins the game.

- Σ_i is a non-empty set of strategies. $\Sigma_i = \{\sigma_i\}$.
- σ_i is a strategy of player i , which is a function : $\{v \in V \setminus Z \mid t(v) = i\} \rightarrow V \setminus v$, assigning a successor v' of v to each non-terminal node v when it is i 's turn to move. (where $V \setminus v$ is the set of nodes extending v .)
- $\sigma = (\sigma_i)_{i \in N}$ represents a strategy profile which is a combination of strategies from all players and Σ represents the set of all strategy profiles.
- σ_{-i} denotes the collection of strategies in σ excluding those for player i .
- $O(\sigma)$ is the outcome if the strategy profile σ is followed by all players.
- $O(\sigma'_i, \sigma_{-i})$ is the outcome if player i use strategy σ'_i while all other players employ σ .
- $\succeq_i$ is a preference relation over V^2 for each player i .

Extensive game(Con'd)

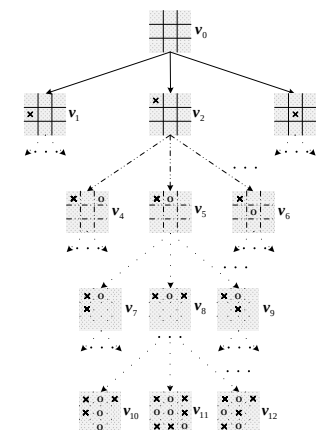
Definition

An extensive game is a tuple $G = (N, V, A, t, \Sigma_i, \succeq_i)$, where

- N is a non-empty set of the players,
- V a set of nodes or vertices including a root v_0
- $A \subseteq V^2$ a set of arcs. If $(v, v') \in A$, we call v' a successor of v . Leaves are the nodes that have no successors, denoted by Z .
- t is turn function assigning a member of N to each non-terminal node. $t(v) = i$;

Example

Two players: player 1($\times$) and player 2($\circ$).
Solid arrows: the moves of player 1,
dotted arrows: moves of player 2.
The initial state is v_0 .
 v_1, v_2, v_3 are all successors of v_0 .
 v_{10}, v_{11}, v_{12} are the terminal nodes (leaves).



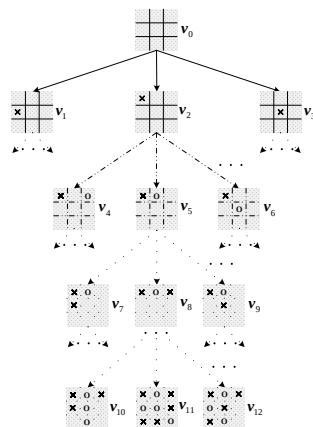
Example

Formally, $G = (N, V, A, t, \Sigma_i, \geq_i)$

- $N = \{1, 2\}$;
- $V = \{v_0, v_1, v_2, \dots\}$;
- $(v_0, v_1), (v_0, v_2), (v_0, v_3) \in A$;
- $v_{10}, v_{11}, v_{12} \in Z$;
- $t(v_0) = 1, t(v_2) = 2, j \dots$.
- a σ_1 such that $\sigma_1(v_0) = v_2, \sigma_1(v_5) = v_8, \dots$
a σ_2 such that $\sigma_2(v_2) = v_5, \dots$

Thus, a strategy profile $\sigma = (\sigma_1, \sigma_2)$ such that $O(\sigma) = v_{11}$.

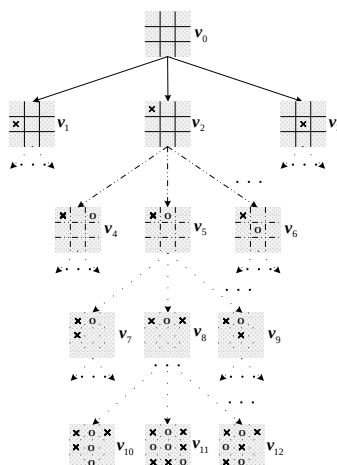
- $v_{12} \geq_1 v_{11} \geq_1 v_{10}$. (since player 1 **wins** the game in v_{12} , **loses** it in v_{10} , and gains a **draw** in v_{11}).



Sight function

Definition

(sight function). Let $G = (N, V, A, t, \Sigma_i, \geq_i)$ be an extensive game. A short sight function for G is a function $s : V \setminus Z \rightarrow 2^{V \setminus Z} \setminus \emptyset$, associating to each non-terminal node v a finite subset of all the available nodes at v .



- Two steps: $s(v_0) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}$

Definition

(Extensive game with short sight). An extensive game with short sight (Egss) is a tuple $S = (G, s)$ where G is a finite extensive game and s a sight function.

- Endowing an extensive game with a sight function.

Sight-filtrated extensive game

At each node v , players can see a subgame $S|_v$ of the whole game. This subgame is determined by their sight:

$$S|_v = (N|_v, V|_v, A|_v, t|_v, \Sigma_i|_v, \geq_i|_v)$$

- $N|_v = N$;
- $V|_v = s(v)$, which is the set of nodes within the sight of $s(v)$.
- $A|_v = A \cap (V|_v)^2$;
- $t|_v = V|_v \setminus Z|_v \rightarrow N$ so that $t|_v(v') = t(v')$;
- $\Sigma_i|_v$ is the set of strategies for each player available at v and restricted to $s(v)$. It consists of elements $\sigma_i|_v$ such that $\sigma_i|_v(v') = \sigma_i(v')$ for each $v' \in V|_v$ with $t|_v(v') = i$;
- $\geq_i|_v = \geq_i \cap (V|_v)^2$.

Example

- $N|_{v_0} = N$;
- $V|_{v_0} = \{v_0, v_1, v_2, v_3, v_4, v_5\}$;
- $Z|_{v_0} = \{v_1, v_3, v_4, v_5\}$;
- $A|_{v_0} = \{(v_0, v_1), (v_2, v_4), (v_2, v_5), \dots\}$;
- $\sigma|_{v_0} = (\sigma_1|_{v_0}, \sigma_2|_{v_0})$ such that $O|_{v_0}(\sigma|_{v_0}) = v_5$, with $\sigma_1|_{v_0}(v_0) = v_2$ and $\sigma_2|_{v_0}(v_2) = v_5$

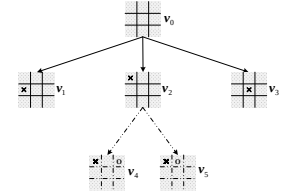
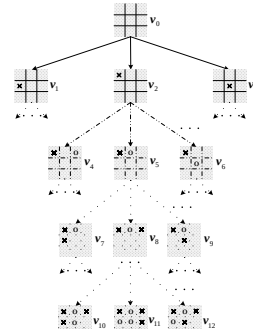


Figure: Sight-filtrated extensive game $S|_{v_0}$

Syntax and Semantics

$$\varphi ::= p \mid \neg\varphi \mid \varphi_0 \wedge \varphi_1 \mid \langle \leq_i \rangle \varphi \mid \langle \hat{\sigma} \rangle \varphi \mid \langle \hat{\sigma}_{-i} \rangle \varphi \mid \langle \hat{\sigma}_{-i}^s \rangle \varphi \mid \langle \hat{\sigma}_{-i}^s \rangle \varphi$$

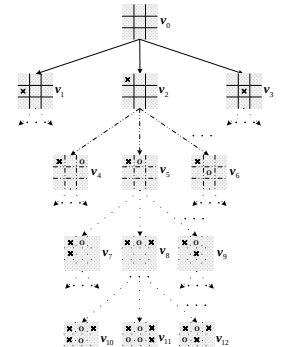
- The label $\leq_i$ denotes player i 's preference relation.
- The label $\hat{\sigma}$ stands for the outcomes of strategy profiles.
 $(v, v') \in R_{\hat{\sigma}}$ iff v' is the terminal node reached from v by following σ .
 $(v_0, v_{11}) \in R_{\hat{\sigma}}$

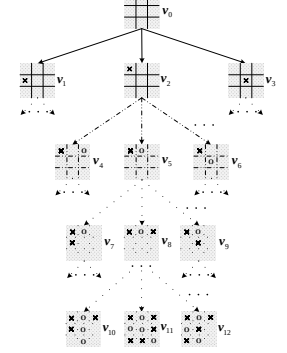
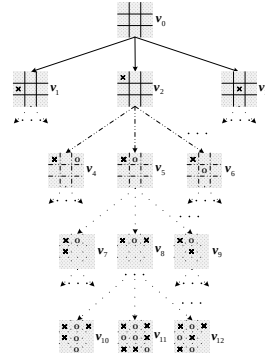


Syntax and Semantics

$$\varphi ::= p \mid \neg\varphi \mid \varphi_0 \wedge \varphi_1 \mid \langle \leq_i \rangle \varphi \mid \langle \hat{\sigma} \rangle \varphi \mid \langle \hat{\sigma}_{-i} \rangle \varphi \mid \langle \hat{\sigma}_{-i}^s \rangle \varphi \mid \langle \hat{\sigma}_{-i}^s \rangle \varphi$$

- $(v, v') \in R_{\hat{\sigma}_{-i}}$ iff v' is one of the leaf nodes extending v that player i can enforce provided that the other players strictly follow their strategies in σ .
 $O(\sigma_{-1}, \sigma'_1) = v_{12}$,
 $(v_0, v_{12}) \in R_{\hat{\sigma}_{-1}}$





$vR_{\leq_i} v'$	iff	$v' \geq_i v$
$vR_{\sigma} v'$	iff	$v' = O _v(\sigma _v)$
$vR_{\sigma_{-i}} v'$	iff	$v' \in O _v(\sigma_{-i} _v)$
$vR_{<} v'$	iff	$v' \in s(v)$
$vR_{\sigma^s} v'$	iff	$v' = O\uparrow_v(\sigma\uparrow_v)$
$vR_{\sigma^s_{-i}} v'$	iff	$v' \in O\uparrow_v(\sigma_{-i}\uparrow_v)$

$M, v \models \langle \leq_i \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\leq_i} u$.
$M, v \models \langle \hat{\sigma} \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\hat{\sigma}} u$.
$M, v \models \langle \hat{\sigma}_{-i} \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\hat{\sigma}_{-i}} u$.
$M, v \models \langle \prec \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\prec} u$.
$M, v \models \langle \hat{\sigma}^S \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\hat{\sigma}^S} u$.
$M, v \models \langle \hat{\sigma}_{-i}^S \rangle \varphi$	iff	$M, u \models \varphi$ for some $u \in V$ with $vR_{\hat{\sigma}_{-i}^S} u$.

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Validities

N	Modality	Schema	Property
T	$[\leq_i]$	$[\leq_i]\varphi \rightarrow \varphi$	reflexivity
	$[\prec_i]$	$[\prec_i]\varphi \rightarrow \varphi$	
4	$[\leq_i]$	$[\leq_i]\varphi \rightarrow [\leq_i][\leq_i]\varphi$	transitivity
D	$[\hat{\sigma}]$	$[\hat{\sigma}]\varphi \leftrightarrow \langle \hat{\sigma} \rangle \varphi$	determinism
	$[\hat{\sigma}^s]$	$[\cdot]\varphi \leftrightarrow \langle \cdot \rangle \varphi$	
I	$([\hat{\sigma}], [\hat{\sigma}_{-i}])$	$[\hat{\sigma}_{-i}]\varphi \rightarrow [\hat{\sigma}]\varphi$	inclusiveness
	$([\hat{\sigma}^s], [\hat{\sigma}_{-i}^s])$	$[\hat{\sigma}_{-i}^s]\varphi \rightarrow [\hat{\sigma}^s]\varphi$	
M	$[\hat{\sigma}]$	$[\hat{\sigma}](\langle \hat{\sigma}' \rangle \varphi \leftrightarrow \varphi)$	terminating
	$[\hat{\sigma}_{-i}]$	$[\hat{\sigma}_{-i}](\langle \hat{\sigma}'_{-i} \rangle \varphi \leftrightarrow \varphi)$	
γ	$([\prec], [\hat{\sigma}^s])$	$[\prec]\varphi \rightarrow [\hat{\sigma}^s]\varphi$	visibility
	$([\prec], [\hat{\sigma}_{-i}^s])$	$[\prec]\varphi \rightarrow [\hat{\sigma}_{-i}^s]\varphi$	

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Soundness and Completeness

LEGS is sound and complete w.r.t. the class of all games with short sight.

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Solution concepts for traditional extensive games

Definition

(Best response and Nash equilibrium)

A best response for player i of an extensive game is a strategy profile σ^* such that $O(\sigma_i^*, \sigma_{-i}^*) \geq_i O(\sigma_i, \sigma_{-i}^*)$ for every strategy σ_i of player i . A strategy profile σ^* is a Nash equilibrium of an extensive game if it is a best response for every player i .

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Solution concepts for extensive games with short sight

Definition

(Sight-compatible best response and Nash equilibrium).

Let $S = (G, s)$ be an Egss and $S \upharpoonright_v$ be the sight-filtrated extensive game at v . A strategy profile σ^* is a sight-compatible best response for i if for every nonterminal node v , it holds that $O \upharpoonright_v(\sigma_i^* \upharpoonright_v, \sigma_{-i}^* \upharpoonright_v) \geq_i \upharpoonright_v O \upharpoonright_v(\sigma_i \upharpoonright_v, \sigma_{-i}^* \upharpoonright_v)$ for any strategy $\sigma_i \upharpoonright_v$ available to i .

σ^* is a *sight-compatible Nash equilibrium*(SCNE) of S if it is a sight-compatible best response for every player $i \in N$.

Characterize solution concepts

Theorem

Let S be an Egss given by $(N, V, A, t, \Sigma_i, \geq_i, s)$. Then for any player i , any strategy profiles σ in S and any formulas φ of $\mathcal{LS}$:

- (a) σ is a sight-compatible best response (SCBR) of S for i iff $\mathcal{F}_S \models [\hat{\sigma}^s] \varphi \rightarrow [\hat{\sigma}_{-i}^s] \langle \leq_i \rangle \varphi$.
- (b) σ is a sight-compatible Nash equilibrium (SCNE) of S iff $\mathcal{F}_S \models \bigwedge_{i \in N} ([\hat{\sigma}^s] \varphi \rightarrow [\hat{\sigma}_{-i}^s] \langle \leq_i \rangle \varphi)$.
- (c) σ is a subgame perfect equilibrium (SPE) of $S \upharpoonright v$ iff for any $u \in V \upharpoonright_v \setminus Z \upharpoonright_v$, $\mathcal{F}_{S \upharpoonright_v}, u \models \bigwedge_{i \in N} ([\hat{\sigma}] \varphi \rightarrow [\hat{\sigma}_{-i}] \langle \leq_i \rangle \varphi)$.
- (d) A strategy profile σ is a sight-compatible SPE of S iff for all $v \in V \setminus Z$, $\mathcal{F}_{S \upharpoonright_v}, v \models [\prec] (\bigwedge_{i \in N} ([\hat{\sigma}] \varphi \rightarrow [\hat{\sigma}_{-i}] \langle \leq_i \rangle \varphi))$.

Thank you!



Davide Grossi and Paolo Turrini.

Short sight in extensive games.

In *AAMAS*, pages 805–812, 2012.