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Title	Interactive incompleteness for prediction/decision making in games
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Description	SOCREAL 2013 : 3rd International Workshop on Philosophy and Ethics of Social Reality 2013. Hokkaido University, Sapporo, Japan, 25-27 October 2013. Session 5 : Games, Knowledge, and Interaction
Citation	Proceedings of SOCREAL 2013 : 3rd International Workshop on Philosophy and Ethics of Social Reality 2013, 115-125
Issue Date	2013-10-25
Doc URL	https://hdl.handle.net/2115/54939
Type	conference paper
File Information	13Mamoru_Kaneko.pdf



Incompleteness arising from Prediction/Decision Making in Games

Tai-Wei Hu and Mamoru Kaneko, 27 October 2013

Prediction/decision making by a person in an **interactive situation** with the **other person**.

- **Ex Ante prediction/decision making in an interactive situation**

	a	b
a	5,5	1,6
b	6,1	3,3NE

Prisoner's Dilemma

	a	b
a	1,1NE	0,0
b	0,0	1,1NE

Coordination Problem

	a	b
a	2,1NE	0,0
b	0,0	1,2NE

Battle of the Sexes

- **Prediction/decision making -- Mental Activities:**
 - **Deductive Inferences** -- positive process (proof theory)
 - **Considerations of logical possibilities** -- negative process (semantics)
- **Independence of the other's mind from his own**

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Game Theory Traditions

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- Emphases on **mental states**, instead of **mental activities**;
 - A single *model* is considered, rather than the set of models
 - Logical inferences are all implicit (in the sense of logic)
- Set theory (probability theory) is the language
 - In an information partition model (extensive game);
 $P_i(\omega) \subseteq E$ is interpreted as meaning that player i knows event E .
- Independence of the other's mind is not treated, except for Nash ('51).
"Rationalizable strategies" has been well accepted among theorists.
- Common knowledge is emphasized, implying that thoughts are objectively true.

Emphases in this paper

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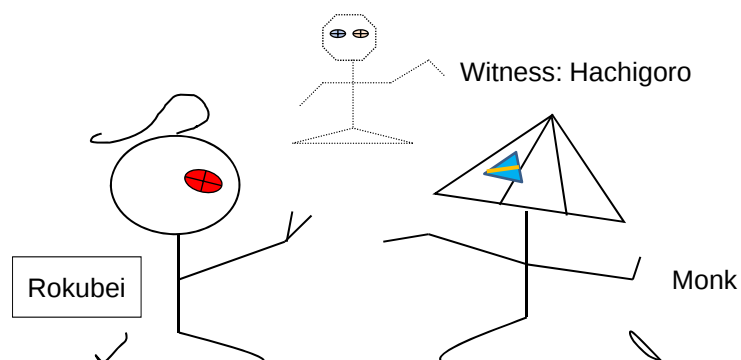
- Explicit treatments of logical inferences and logical possibilities
- **Independence of the other's mind from one's own**
- **Result:** Interactive Incompleteness - - indecisiveness occurs between the two minds

Agenda

- **Konnyaku Mondo** (Japanese comic story):
 - Symbolic natures of our thinking - interpretations
 - one cannot look into the other's mind
- **Infinite Regress Logic** (Fixed-Point Logic)
- **Axiomatic Approach to the Prediction/Decision Making**
- **Incompleteness Theorem from Interactive Dissonance**

Konnyaku Mondo (蒟蒻問答)

- Other's Mind and False Beliefs:
- CK as shared information, but
- Common Misunderstanding in interpretations of exchanged gestures



Konnyaku - - Devil's Tongue Jelly Product
 Konnyaku Mondo - a Japanese comic story in the Rakugo-style

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Konnyaku Mondo 1



There was a temple where no monks were living any longer. A devil's tongue jelly maker, named Rokubei, lived next door. He moved into the temple and started pretending to be a monk.

One day, a traveling Zen Buddhist monk passed by and challenged Rokubei to a debate on Buddhism. Rokubei had no knowledge on Buddhism and was not able to have a debate. He tried to refuse, but he couldn't escape and finally agreed.

The Buddhist dialogue started but Rokubei didn't know how to perform and he kept silent. The Buddhist monk tried to communicate to Rokubei in many ways.

After some time, Rokubei started responding with gestures to the body movements the monk made.

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Konnyaku Mondo 2



The monk took this as a style of dialogue and tried to answer in gestures, too. They exchanged gestures, and after some time, the monk told Rokubei, "your thoughts are profound and mine are of no comparison. I am very sorry to have bothered you".

After saying this, he left the temple.

Hachigoro, a neighbor of Rokubei, witnessed the whole debate, and followed the monk to ask what had happened.

The monk answered, "I'm not trained enough in Buddhist thoughts to compete with that master. Please convey to him my earnest apology for having left so abruptly". Almost as quickly as the words had left his lips, he ran away.

Hachigoro returned to the temple and asked Rokubei if he knew anything about Buddhism thoughts.

Rokubei answered, 'No, I have no idea about Buddhism, but the guy is, in fact, a beggar and he talked badly about my jelly products. That's why I gave him a lesson'.

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Implications to Game Theory and Epistemic Logic

• Two Independent Minds:

- What is the source for the individual basic beliefs?
- Experiential sources – Kaneko-Kline (2013)
- Implications of two independent minds to the subjective thinking for prediction/decision making ← Today

• Mutual misunderstanding - Different levels of common knowledge

- Correct common knowledge: Visual factors exchanged and their agreement of Rokubei's beating the Monk
- Interpretations and reasoning are different and incorrect: **One cannot look into the other's mind.**

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Language

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atomic preference formulae: $\text{Pr}_i(s; t)$ for $i = 1, 2$, and $s, t \in S = S_1 \times S_2$;

atomic decision/prediction formulae: $I_i(s_i)$ for $s_i \in S_i$, $i = 1, 2$.

In the scope of player i :

- $I_i(s_i)$ - - player i 's decision;
- $B_j(I_j(s_j))$ - - player i 's prediction about j 's decision.

logical connective symbols: $\neg$ (negation), $\supset$ (implication), $\wedge$ (conjunction), $\vee$ (disjunction)

unary belief operators: $B_1(\cdot)$, $B_2(\cdot)$;

binary infinite regress operators: $\text{Ir}_1(\cdot)$, $\text{Ir}_2(\cdot)$;

parentheses: $(,)$; *comma:* $,$; and *braces:* $\{, \}$.

In the scope of player i :

- $\text{Ir}_i(A_i; A_j)$ - an infinite regress derived from the pair of formulae (A_1, A_2) ;
- - the intended meaning - $\wedge \{A_i, B_j(A_j), B_j B_i(A_i), B_j B_i B_j(A_j), \dots\}$ in the scope of $B_i(\dots)$.
- In this paper, we adopt the fixed-point logic IR^2 .

Infinite Regress Logic IR^2

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- (L1) $A \supset (B \supset A)$;
 (L2) $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$;
 (L3) $(\neg A \supset \neg B) \supset ((\neg A \supset B) \supset A)$;
 (L4) $\wedge \Phi \supset A$ where $A \in \Phi$;
 (L5) $A \supset \vee \Phi$, where $A \in \Phi$;

$$\frac{A \supset B \quad A}{B} \text{ (MP)} \quad \frac{\{A \supset B : B \in \Phi\}}{A \supset \wedge \Phi} \text{ (\wedge-rule)} \quad \frac{\{B \supset A : B \in \Phi\}}{\vee \Phi \supset A} \text{ (\vee-rule)}.$$

- (K) $B_i(A \supset B) \supset (B_i(A) \supset B_i(B))$;
 (D) $\neg B_i(\neg A \wedge A)$;
- (Necessitation) $\frac{A}{B_i(A)}$.

KD²

Let $\mathbf{A} = (A_1, A_2)$, $\mathbf{B} = (B_1, B_2)$ be two pairs of formulae.

IRA_i: $\text{Ir}_i(\mathbf{A}) \supset A_i \wedge B_j(A_j) \wedge B_j B_i(\text{Ir}_i(\mathbf{A}))$;

Axiom

IRI_i: $\frac{B_i \supset A_i \wedge B_j(A_j) \wedge B_j B_i(B_i)}{B_i \supset \text{Ir}_i(\mathbf{A})}$.

Inference rule

Kripke Semantics

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(V6) $(M, w) \models \text{Ir}_i(A_1, A_2)$ if and only if (i) $(M, w) \models A_i$, and (ii) for any v that is alternatingly reachable from w with $(i_1, \dots, i_\nu)$, $(M, v) \models A_{i_\nu}$.

Theorem 2.1 (Soundness and Completeness). *Let $A \in \wp$. Then, $\vdash A$ in IR^2 if and only if $M \models A$ for all serial models M .*

In the scope of player i :

- $\text{Ir}_i(A_i; A_j)$ - an infinite regress derived from the pair of formulae (A_1, A_2) ;
 - - the intended meaning of $\text{Ir}_i(A_i; A_j)$
 - - $\wedge \{A_i, B_j(A_j), B_j B_i(A_i), B_j B_i B_j(A_j), \dots\}$ in the scope of $B_i(\dots)$.

Lemma 2.2 (Change of Scope). *The following three statements are equivalent (both with and without axiom T): (1): $\vdash A \supset C$; (2): $\vdash B_i[A \supset C]$; (3): $\vdash B_i(A) \supset B_i(C)$.*

Theorem: For any A_1, A_2 , Axiom T $\vdash \text{Ir}_i(A_i; A_j) \equiv C(A_1 \wedge A_2)$.

2-person games and the Nash Noncooperative Solution theory

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- $G = (N, \{S_i\}_{i \in N}, \{h_i\}_{i \in N})$ is a finite game, where
 - $N = \{1, 2\}$ is the set of players;
 - each S_i is a finite set of pure strategies;
 - Each $h_i: S_1 \times S_2 \rightarrow \mathbb{R}$ is the payoff function for player i .
- A pair of strategies $s = (s_1, s_2)$ is a Nash equilibrium (NE) iff
 - for $i = 1, 2$, $h_i(s_i; s_j) \geq h_i(t_i; s_j)$ for all $t_i \in S_i$.
- A set of NE's E satisfies **Interchangeability** iff
 - (*) if $s = (s_1, s_2), t = (t_1, t_2) \in E$, then $(s_1, t_2), (t_1, s_2) \in E$.
- A game G is **solvable** iff it has the nonempty set $E(G)$ of NE's and $E(G)$ satisfies (*). For example, the PD game is solvable.
- A nonempty set of NE's E is a **subsolution** iff (*) holds and it is maximal w.r.t. the set-inclusion relation. For example, the CP game has two subsolutions.

	a	b
a	5,5	1,6
b	6,1	3,3

Prisoner's Dilemma

	a	B
a	1,1	0,0
b	0,0	1,1

Coordination Problem

	a	b
a	1,1	0,1
b	1,0	0,0

Intersection

Prediction/Decision Criterion:

$N1^o$: PL1 chooses a strategy to maximize his payoff under the prediction based on $N2^o$;

$N2^o$: PL2 chooses a strategy to maximize his payoff under the prediction based on $N1^o$.



Consider a pair of strategies subsets (E_1, E_2) of (S_1, S_2) and the following translations of $N1^o$ and $N2^o$

$N1$: for any $s_1 \in E_1$, $Best_1(s_1; t_2)$ holds for **all** $t_2 \in E_2$;

$N2$: for any $s_2 \in E_2$, $Best_2(s_2; t_1)$ holds for **all** $t_1 \in E_1$.

$Best_1(s_1, t_2)$ means " $h_1(s_1, t_2) \geq h_1(s'_1, t_2)$ for all $s'_1 \in S_1$ ".

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Nash Noncooperative Solutions

Theorem 3.2 (The Nash Noncooperative Solutions): (0): G has a Nash equilibrium if and only if there is a nonempty pair (E_1, E_2) satisfying N1-N2.

(1): Suppose that G is solvable. Then E is the Nash solution $E(G)$ if and only if the greatest pair (E_1, E_2) satisfying N1-N2 exists and $E = E_1 \times E_2$.

(2): Suppose that G has a Nash equilibrium but is unsolvable. Then E is a Nash sub-solution if and only if (E_1, E_2) is a nonempty maximal pair satisfying N1-N2.

- If the quantifier “for all” is replaced by “for some” in N1-N2, then the result characterizes “**rationalizable strategies**”.
- However, we cannot make this replacement when we follow the assumptions of independent minds.

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Rationalizable Strategies

$R1$: for any $s_1 \in E_1$, $Best_1(s_1; t_2)$ holds for **some** $t_2 \in E_2$;

$R2$: for any $s_2 \in E_2$, $Best_2(s_2; t_1)$ holds for **some** $t_1 \in E_1$.

Theorem 3.5 (Rationalizability): (R_1, R_2) is the greatest pair satisfying R1-R2.

Implication: “rationalizability” is an incorrect concept, relative to “independence” of the other’s mind

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Formulation in Infinite-Regress Logic IR²

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D0_i (Optimization against all predictions):

$$\bigwedge_{s_i \in S_i} \{I_i(s_i) \supset \bigwedge_{s_j \in S_j} [B_j(I_j(s_j)) \supset \text{best}_i(s_i; s_j)]\}.$$

D1_i (Predictability): $\bigwedge_{s_i \in S_i} [I_i(s_i) \supset B_j B_i(I_i(s_i))]$.

D2_i (Necessity of predictions): $\bigwedge_{s_i \in S_i} [I_i(s_i) \supset \bigvee_{s_j \in S_j} (B_j(I_j(s_j)))]$.

For each $i = 1, 2$, let $D_i = \bigwedge \{D0_i, D1_i, D2_i\}$, and let $D = (D_1, D_2)$.

Adopt the infinite regress: $IR_1(D_1, D_2)$ as an axiom for prediction/decision making;

- The intended meaning: $D_1, B_2(D_2), B_2 B_1(D_1), B_2 B_1 B_2(D_2), \dots$

We adopt the infinite regress of the payoff functions: $IR_1(g_1, g_2)$; and

the axiom of choice of the weakest formulae satisfying $IR_1(D_1, D_2)$: $IR_1(W_1, W_2)$

Solvable Games

Candidate formulae: $A^*(s_i) := \bigvee_{s_j \in S_j} Ir_i[\text{best}_i(s_i; s_j); \text{best}_j(s_j; s_i)]$.

Theorem 4.1 (Characterization of the Nash Solution under Solvability). *Let G be a solvable finite two-person game and let $g = (g_1, g_2)$ be its associated payoff formula. Then, for all $s \in S$,*

$$Ir_i(g), Ir_i(D), Ir_i(WF) \vdash Ir_i[I_1(s_1) \equiv A_1^*(s_1), I_2(s_2) \equiv A_2^*(s_2)]. \quad (9)$$

Corollary 4.1 (Playability for Solvable Games). *Let G be a solvable finite two-person game and let $g = (g_1, g_2)$ be its associated payoff formula. Then, for all $s = (s_1, s_2) \in E(G)$,*

$$Ir_i(g), Ir_i(D), Ir_i(WF) \vdash Ir_i(I_1(s_1), I_2(s_2)), \quad (10)$$

and for all $s = (s_1, s_2) \in \tilde{E}(G) = (S_1 - E_1(G)) \times (S_2 - E_2(G))$

$$Ir_i(g), Ir_i(D), Ir_i(WF) \vdash Ir_i(\neg I_1(s_1), \neg I_2(s_2)). \quad (11)$$

	a	b
a	5,5	1,6
b	6,1	3,3NE

Prisoner's Dilemma

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Unsolvable Games and Incompleteness

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Theorem 5.2 (Interactive incompleteness). *Let G be a finite two-person unsolvable game with $K > 1$ subsolutions $F_1, \dots, F_K$, and let g be its associated formula. Then, for all $s_i \in \text{Proj}_i(E(G) - \bigcap_{k=1}^K F_k)$,*

$$\text{Ir}_i(g), \text{Ir}_i(D), \text{Ir}_i(\text{WF}) \not\models I_i(s_i)$$

and

$$\text{Ir}_i(g), \text{Ir}_i(D), \text{Ir}_i(\text{WF}) \not\models \neg I_i(s_i).$$

Theorem 5.3 (No formula property). *Let G be a finite two-person unsolvable game with $K > 1$ subsolutions $F_1, \dots, F_K$, and let g be its associated formula. Let $s_i \in \text{Proj}_i(E(G) - \bigcap_{k=1}^K F_k)$. Then, there is no preference formula A such that*

$$\text{Ir}_i(g), \text{Ir}_i(D), \text{Ir}_i(\text{WF}) \vdash I_i(s_i) \equiv A.$$

	a	b
a	1,1	0,0
b	0,0	1,1

Coordination Problem

	a	b
a	2,1	0,0
b	0,0	1,2

Battle of the Sexes

Semantic Counterpart

Theorem 5.1 (Semantic characterization of Nash (sub)solutions). *Let G be a finite two-person unsolvable game and let g be its associated formula. Suppose that $E(G) \neq \emptyset$.*

(1) *For any model $M = (\langle W; R_1, R_2 \rangle, \tau)$ and any $w \in W$, if $(M, w) \models \text{Ir}_i(g) \wedge \text{Ir}_i(D)$ and if $(M, w) \models \text{Ir}_i(\text{WF}(A))$ for all A , then there exists a subsolution $E_1 \times E_2$ of G such that for all $s \in E$,*

$$(M, w) \models \text{Ir}_i[I_1(s_1), I_2(s_2)], \quad (12)$$

and for all $s \in \tilde{E} = (S_1 - E_1) \times (S_2 - E_2)$,

$$(M, w) \models \text{Ir}_i[\neg I_1(s_1), \neg I_2(s_2)]. \quad (13)$$

(2) *For any subsolution $E_1 \times E_2$ of G , there exists a model M such that $M \models \text{Ir}_i(g) \wedge \text{Ir}_i(D)$, $M \models \text{Ir}_i(\text{WF}(A))$ for any A , and (12) and (13) hold for $E_1 \times E_2$.*

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- **Ex ante prediction/decision making**
- **Independence** of the mind of one person from the other's.
 - **Each player cannot look into the other's mind**
 - **"for all" toward one's prediction causes incompleteness result.**
- **An Infinite regress caused by mutual reciprocity is not the cause for incompleteness.**

	a	b
a	1,c	0,c
b	0,c	1,c

- PL2 is indifferent between his a and b.
 - He can make either choice.
- PL1's decision depends upon 2's choice.
 - PL1 cannot predict which PL2 would choose, and his choice is impossible.

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Interactive Incompleteness

- **Interactive Incompleteness**
 - A player cannot decide whether a given strategy is be a decision.
 - This is not necessarily caused by infinite regresses.
 - It is simply caused by the **incapability of reading the other's mind.**
- **This is different from Kaneko-Nagashima's (1996) incompleteness theorem.**
 - It holds form a 3-person game with a unique NE in mixed strategies
 - **The existence of a NE is common knowledge, but no player can identify where the NE is.**
 - **It is caused by the lack of a concept of the radical expression $\sqrt{\cdot}$.**

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How to be solved in reality ?

- We have concentrated on the *Ex Ante* prediction/decision making.
- It is one-shot!
- In the repeated situation, we can have “regular behavior”
 - Lewis’s (1969) convention.
- Inductive game theory has been developed in Kaneko-Kline (2008, 2013), based on similar ideas to Lewis’s convention, but it’s focus is the emergence of player’s understanding of the game situation from experiences.
- In IGT, a player may face *ex ante* decision making, while the regular behavior may help him to behave in such a situation.
- It is open to have explicit connections between the epistemic logic approach and IGT.

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