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THEORETICAL AND EMPIRICAL EXAMINATIONS OF
DECISION-MAKING UNDER DIFFERENT INSTITUTIONS

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Abstract

This dissertation studies, both theoretically and empirically, economic agents' decision-making when operating under differing forms of institutional environments. Two general forms of institutional environments are examined, one being an example of a non-market allocation environment, and the other an example of a market allocation environment. Each environment is studied in separate parts of the dissertation.

The non-market allocation environment is represented by the allocation methodology described by a contest structure, specifically modeled as a Tullock contest (Tullock 1980). Such contest forms are useful to study given that they have many applications, including R&D and patent contests, political elections and campaigns, labor market hiring and promotion, athletic competitions, war and other geopolitical conflicts, and lobbying and rent-seeking activities.

Much of the Tullock contest literature analyzes effort provision levels under the assumption of a single prize administrator. However, many important political, economic and social decisions are made in committees. Hence in this research, I allow a committee to decide the winner of the contest, and study the impact of committee size on effort provision levels. While a small literature has previously considered committee administration under probabilistic voting in contests, including Congleton (1984) and Amegashie (2002), I present an extension to such results by analyzing the impact on effort provision that is produced under the conditions of heterogeneity amongst contestants.

To begin with, I find that increasing the size of the committee produces an ambiguous impact on total effort provision, with the heterogeneity between the contestants acting as the primary factor determining a resultant increase or decrease in efforts. Further, I consider the impact of modifications to the institutional environment of the contest structure, allowing for variation in the committee voting rule determining the contest winner. Bias is introduced by way of modification to the standard simple majoritarian voting rule. By way of such institutional modifications, we are able to observe the way in which contestant heterogeneity is exacerbated or attenuated by bias in the contest structure. Overall, my results extend previous analyses of effort provision under committee structures, indicating that under heterogeneity, aggregate effort provision levels under committee administration can be larger than, equal to, or less than those generated when a single administrator awards prizes. Such results allow for a richer understanding of the degree of effort provision generated when committees are responsible for conferring prizes.

The second part of the dissertation concerns an empirical study of agents operating within a specific market-based allocation environment; specifically, the Shanghai Stock Exchange. While there are many examples of market institutions, one advantage of the particular setting I consider is that financial markets can serve as useful representations of real-world "ideal" markets, given their size, lack of market power, liquidity and information dissemination systems. In this setting, I study share clustering. Clustering is the phenomenon wherein which prices settle on certain values more frequently than others. Recent studies have posited that clustering in China has different—and less rational—characteristics than clustering that occurs in other markets. My study considers clustering in China with a more targeted and updated data set than previously studied in the literature, using decade-long pricing data on twenty of the top-50 companies traded on the Shanghai Stock Exchange. I confirm the presence of clustering, and while I find cultural factors do play a role in share clustering in China, I argue this behavior is primarily partaken when there is (relatively) low impact to the value of the trade action.

Additionally, the prevalence of clustering on the Shanghai Stock Exchange appears to be driven in large part by factors relating to the presence of informational and bargaining costs that are at least partly determined by the unique institutional structure of the exchange. Changes in the environment—in particular, the lifting in March 2010 of the ban on short sales—appear to have reduced these costs and created an incentive structure where agents are more likely to price equities less coarsely, characterized by a lower prevalence of clustering.

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PART I: Introduction

In this dissertation, I analyze, both theoretically and empirically, examples of decision-making by economic agents operating under differing forms of institutional environments. This dissertation consists of several essays, each of which is concerned in its own way with how economic agents come to make their decisions within certain institutional environments. In particular, I consider how agents maximize their expected utilities under two general forms of institutional environments: first, a theoretical treatment of a non-market contest-based allocative system, as described in Part II; and second, an empirical study of agents operating within a specific market-based system —the Shanghai Stock Exchange— as described in Part III.

1 Institutions in Economics

A key methodological approach of this study is the concept that the institutional framework within which economic agents exist helps to define the nature of their behavior and decision-making. As North (1994: 360) defines, "Institutions are the humanly-devised constraints that structure human interaction. They are made up of formal constraints (e.g., rules, laws, constitutions), informal constraints (e.g., norms of behavior, conventions, self-imposed codes of conduct), and their enforcement characteristics." Williamson (1996: 3) thusly declares that "institutions matter" in regard to which economic outcomes are chosen by economic actors and hence society at large. Institutions demarcate the incentive structures that are instrumental to the governance of economic exchange.

Adapting our models and reasoning to account for a variety of different institutional settings is important for several reasons. First, the range of economically-relevant institutional settings that operate across time and cultures is large, as has been described in detail by North & Thomas (1973), North (1990) and Acemoglu *et al.* (2005), among others. Even limiting one's analysis to "advanced modern market economies" would uncover a huge variety in terms of legal structures, private property rights protections, labor laws, environmental protections, and innumerable other features delineating the specific institutional environment of those economies. Second, within any given broad economic institutional structure, economic activities take place in a variety of smaller institutional settings. As Coase's (1937, 1960) early work helped establish, many transactions that occur within a market economy are performed not in a market setting, but rather within firms themselves. The dynamics governing within-firm transactions operate differently from those between consumers and suppliers in the marketplace. Likewise, even within the marketplace, the nature of the market setting can be salient to how transactions are conducted. (For instance, transactions on goods can occur within retail stores, or by utilization of the internet, and the nature of those transactions might well be different in each setting.) Hence, given that (i) there are many forms of institutional environments in existence, and (ii) that even within a specific institutional form, transactions take place in different sub-institutional forms, considering if and how economic agent decision-making is impacted by changes to the institutional environment furthers our understanding of the decision-making process itself.

Additionally, institutional frameworks are important to consider because they shape the incentive structures and types of trade-offs economic agents inherently face in making economic decisions. In both of the two general institutional settings described in this dissertation, the key to understanding the dynamics of agent decision-making centers on the

nature of the trade-offs they weigh, and the way in which changes to institutional parameters impact that trade-off. For instance, in the non-market environment, agents are faced with a trade-off between increasing the probability to win the contest by increasing effort provision, versus decreasing expected payoffs due to the additional cost outlays incurred in the course of effort provision. Likewise, in market environment, agents face a trade-off between suffering higher costs owing to obtaining information allowing for more precise pricing, versus the increased potential payoffs to be gained from higher pricing precision. They face a further trade-off when deciding if additional time spent on bargaining for the best price is worth the opportunity cost from not spending that time making additional trades. Again, the way in which agents decide to resolve such trade-offs depends on the specifics on the institutional structure, and I find that slight changes in that structure leads agents to change how they value the trade-off.

2 Selection of Institutional Settings

Given the prevalence and importance of both non-market and market economic structures to the functioning and determination of economic behavior by economic agents, I consider in my research dissertation an example of a non-market institutional setting, and an example of a market setting. There are many types of non-market institutional settings. In general, while goods are still valued by and distributed to economic agents, unlike in market settings, goods are not distributed via a price mechanism. There are many types of non-market distribution methodologies, including random lottery, one's need for goods, one's allegiance to a particular political party, or even brute force. However, I choose to consider the non-market allocation methodology of a contest setting, where goods are awarded based on the effort levels exerted by the economic agents competing for the good. Of the many forms and styles of non-market institutional structures, the contest structure is perhaps the most widely-applicable, describing several types of non-market institutional environments, including within-firm dynamics relating to hiring and promotion, competitions between firms to receive grants of property rights to an economic rent by government, political contests between candidates for the right to gain access to economically-beneficial resources granted by an electorate, and numerous other scenarios found within political economics. Hence, consideration of contest methodology can provide insights into the dynamics of decision-making across a wide variety of non-market forms.

Market institutions, in one form or another, are of key importance to the functioning of nearly all economies. As with non-market structures, there are many forms of market institutional settings, and my choice to consider a particular equity market represents just one of a set of possible examples. I consider the equity market setting as a useful example in

which to consider market dynamics for several reasons. To begin with, financial markets are useful to employ when considering market dynamics, for in many senses they embody the closest real-world manifestation of the "ideal" market as described in introductory economics textbooks. Modern financial markets are comprised of many thousands of agents, operating in a manner such that almost all market participants can be viewed as price takers; that is, any particular agent's buying or selling decisions regarding an asset do not impact the price level for that asset. Second, the relatively large scale of both transactions and market participants implies sufficient liquidity within the market to ensure that any agent wishing to either buy or sell an asset can do so with a minimal amount of search costs required. Market-clearing mechanisms further assist this process. Third, modern financial markets are comprised of atomized individual agents; hence, transactions take place amongst between anonymous buyers and sellers. Unlike situations in near-markets common in historical periods when personal relationships and family reputations weighed heavily on the transaction process, in modern markets buyers and sellers rarely are aware with whom they are transacting. Indeed, in the equity market I study the matching of buy and sell orders is done by computer. Finally, modern financial markets are characterized by the abundance of information; market structures themselves publish real-time information on pricing and sales figures, and multiple media outlets distribute pertinent information regarding market activity and economic data. Hence, while the consideration of equity markets represents just one example of many market structures, I believe it represents a particularly useful setting under which to consider decision-making behavior.

3 Structure of the Dissertation

The essays comprising this research dissertation examine the decision-making behavior of economic agents under different institutional settings. The focus of the research centers on analyzing how, as features of the institutional environment change, agents' decision choice sets, optimal decisions, and the aggregate equilibrium outcomes are affected. While this overall analysis methodology is consistent throughout the dissertation, the specifics of the topics and institutional environments considered vary significantly; as such, the results of the research examination comprising the dissertation are presented in two main parts, Part II and Part III. Each part deals with a separate general institutional environment and separate form of methodological instrumentation to analyze agent decision-making behavior. Each of the two parts is described in succession below.

3.1 Setting One: Non-Market Allocation-by-Contest Environment

The first general institutional environment in which the research analysis is conducted is a non-market environment, that of a competitive contest scenario, where players compete against one another to win a prize where allocation of a good (the contest prize) is determined by the winner of the contest. The contest format represents one of the most popular ways in which goods are allocated in a non-market setting. From a conceptual standpoint, contests are useful to characterize a multitude of political and economic activities, from a political election contest for political office, lobbying activities by economic actors competing to win property rights to an economic rent; or a promotion contest between employees within a firm setting. As in the common market setting, the good in question (or, in parlance of the contest literature, the "prize") can be valued differently by different actors, effectively setting the "demand" for

the prize. This variation of valuations on goods is allowed for in my model structure. Likewise as in market structures, under contests, economic agents look to exchange something of value for the good they desire. Unlike in the market setting, however, the good is not awarded via a price mechanism in a market process. Instead, the prize is awarded based (partly) on the effort levels of the actors competing for the good. Hence, economic agents "pay" costly effort in order to attempt to "buy" (win) the good (prize). One can conceptualize costly effort –in the most simplistic form– as money in the form of a bribe. More generally, however, effort can simply represent the usage of economic resources, all of which can be valued in the market via opportunity costs, and assigned some monetary value. At the core, then, as under markets, agents attempt to exchange money (effort) to receive a good (prize). Finally, a further dissimilarity between our contest setting and typical market environments is the role of uncertainty; while the dynamics of pure markets assume that an actor can capture a good if he/she is willing to pay (exert) the most money (costly effort), under Tullock contests the actor who exerts the most effort is not deterministically guaranteed access to the prize. Instead, there is a stochastic element to the determination of the winner of the contest, for agents who exert the most effort are simply more *likely* to win the contest, but do not do so with certainty. Hence the role of uncertainty plays a significant role in the behavior of the participants in this setting when they are deciding how much effort to exert in order to obtain the good.

While related to actual economic and political phenomena, the approach here is theoretical. Specifically, the analysis within Part II is based on my development of various modifications to the standard theoretical Tullock Contest model. The analysis is undertaken on a micro-level, designing a model wherein which relatively few agents interact. My main unit of analysis is the individual contestants within the contest, and I analyze their decision-

making in the context of how much effort they exert in equilibrium in order to attempt to win the contest's prize. As has been well-recognized in the literature since Tullock (1980) first introduced his contest model, of key importance to determining the expected effort provision level of a contestant is the behavior of other contestants. Each contestant's individual best choice decision depends on the best choice behavior of other contestants. In general, the more effort a contestant's opponent puts forth, the more effort a contestant will exert. My findings further collaborate much of what has been presented in previous literatures, that the degree of heterogeneity between contestants (modeled in my essays as the degree to which contestants differ according to their valuation of the prize, but conceptually and mathematically equivalent to differences in ability), is pivotal in determining the amount of effort provision. Contestants who are very similar to one another in terms of their probability to win the contest—that is, contestants who are involved in a very tight race—tend to choose in equilibrium relatively high levels of effort provision compared with contestants who are significantly dissimilar in their probabilities to win the contest.

Beyond these standard findings, my major contribution to the literature is to consider the way in which contestants' effort provision decisions are impacted by a variety of institutional factors. In particular, I consider the way in which contestant behavior is affected by the number of individuals determining the winner of the contest. While the majority of the literature has focused on contests where there is a single contest prize administrator, in the following set of papers I systematically analyze the impact of committee choosing the contest winner, allowing committee size to vary from the smallest possible (a single prize administrator) up to a large but finite sized committee. A second way in which I slightly modify the institutional environment of the contest is to analyze differences in effort provision resulting from different voting rules used to tabulate the voting decisions of the committee

members into an outcome which decides the contest winner. Finally, I also add to our understanding of how the interactions of changing environments and heterogeneity levels can impact the nature of effort provision. For nearly all institutional setting modifications (such as increasing the number of committee members), I find that contestant heterogeneity can either attenuate or exacerbate much of the impact of changes in the institutional environment.

While the main focus of the study is on contestant decision-making, inherently there is a simultaneous consideration of the behavior of the prize-awarders. The committee members—each individually vote (in a stochastic manner) according to which contestant directs the highest level of effort towards them—arrive at a collective decision to select the contest winner. Changes in the institutional environment impact not only the best response decisions of the contest participants, but also the collective decision outcome. When voting rules tabulating committee votes to choose a contest winner are modified, different contest winners emerge and different aggregate effort provision is exerted. While the importance of voting rules to contest outcomes is certainly not a new concept, my research provides another set of instances explicating the way in which voting rules can fundamentally alter the outcome of the contest, both in terms of its winner, and in the amount of effort contestants exert in competing to win the contest.

3.2 Setting Two: Market Environment

The second general institutional framework I utilize to study agent decision-making is a market institution. Markets are well-studied, and their importance to modern economic systems and hence economic agents' decision-making is evident. The specific financial market analyzed here is the market for common equity shares on the Shanghai Stock Exchange (SHSE). I chose to consider the SHSE for three main reasons. First, given that I consider to

study asset pricing along the lines of the presence of price clustering behavior, the SHSE's use of a consistent decimalized pricing system, along with a consistent minimum tick size across all price ranges, allows for a more straightforward data analysis. Second, the SHSE represents a huge equity market. Indeed, behind the Tokyo and Hong Kong stock markets, it is the third largest exchange in Asia. The large volumes of trades help ensure that in expectation, a random distribution of prices should be realized. Further, the SHSE's size implies wide participation amongst Chinese investors, with both institutional and individual actors well-represented. Third, the SHSE presents an interesting case study. As an emerging market, it has yet been not as well studied as some of the "developed" markets within the U.S., U.K., Europe or Japan. Further, it is a relatively new exchange, established slightly over twenty years ago at the start of China's transition phase from an ostensibly "socialist" regime to one capitalist. While economic agents in the non-market economy of pre-Deng China exhibited utility-maximizing behavior, it is conceivable that the market pricing mechanism was a concept about which they might lack an understanding, and we can consider whether the aftereffects of such an experience could still exhibit any impact on pricing behavior on the SHSE. Finally, some in the literature claim to have found evidence —Brown & Mitchell (2006) the most notable among them— that there appears to be certain pricing anomalies unique to the mainland Chinese stock markets that are attributable to "irrational" behavior by Chinese investors. A consideration of the SHSE allows us to consider the validity of this assertion.

Unlike the theoretical approach undertaken regarding the non-market allocation-by-contest environment described in Part II, here I undertake an empirical study when considering the market environment. The large number of transactional data available for the SHSE market easily affords itself to an empirical statistical analysis. Nonetheless, while the study itself is empirical in its methodology, the underlying rationale for this area of my research can

be found in a theoretical basis, utilizing theories of market behavior of individual agents operating within the institutional environment of a near-idealized market represented by modern equity markets. Thus, although the research utilizes a market-wide data series, fundamentally, my research represents a consideration of individual decision-making on asset price formation.

Unfortunately, given the nature of the available data set, I do not observe the individual decision-making process; that is, it is not possible to observe individuals' specific valuation and pricing decisions. Ideally, one would wish to be able to closely and constantly observe all market participants over a significant time period and correctly deduce all of their rationales regarding their valuation and buying/selling/not buying/not selling decisions on the set of equity assets available in the SHSE market. Note however that it is possible to partially overcome this inherent data limitation by the recognition that the aggregation of all of the individual equity transaction decisions is in fact the process that drives the overall market price formation mechanism.

Given that the distribution of market prices represents the aggregate shape of the sum of all market participants' pricing decisions, I compare the expected frequency of hundredths and tenths digits of share prices with the observed distribution of digits. Analysis of the differences between the expected and observed frequencies—and possible reasons for any such differences—provides insights into the individual decision-making processes that generate these aggregate pricing data.

I attempt to account for the observed differences between the expected and realized distributions by testing the validity of several potential explanations. First, I attempt to account for and analyze the role transaction and bargaining costs play in impacting the behavior of market participants and the nature of the price formation process occurring within the market

environment of the Shanghai Stock Exchange. Second, I consider the importance of the information dissemination process in the market. While true that modern financial markets, including the SHSE, are characterized by a relatively high degree of market information transmission, the acquisition of such information is nonetheless not costless. As such, a realistic analysis of the price formation process, which depends fundamentally on the access and dispersion of relevant information, must include a consideration of information costs. The more information an investor has, the better able is that investor to make an estimate regarding the true value of the equity share in question. Note that unlike in the contest setting, where good distribution is accomplished under uncertainty, markets distribute goods to agents via the price process where the market-clearing price values goods such that those agents who value it at least as much as that price can purchase the good. However, in financial markets, uncertainty does play an important role in regard to the price formation process. Given that an asset's price represents the expected returns of all its future cash flows, and given that future cash flows are not deterministic but stochastic, there exists uncertainty associated with their present value, and therefore, uncertainty regarding the true price of the asset. It is the existence of such uncertainty that helps to create the trade-off facing investors when considering the value of pursuing costly information necessary to overcome such uncertainty.

Note that in the theoretical model described in Part II, economic agents are modeled as able to calculate their expected return on potential effort provision levels to very exact precision levels in order to calculate their best responses. Importantly, they do so in an environment defined by complete information, with no information costs. Without the limitation of information costs, it is therefore highly worthwhile to calculate their maximization problem to very high precision. However, in the empirical finance world that I consider in Part III, while viewing this market dominated by rational agents looking to

calculate their maximization problem concerning their expected return on potential buy or sell decisions, they must do so while taking into account information costs—information, while potentially theoretically complete, is costly to obtain. Supposing that agents were able to costlessly possess all possible information, it would enable them to correctly and with great precision calculate exactly the expected value of all possible assets; in other words, such information would enable market participants to be highly precise in their valuation and pricing decisions. Given the costly nature of information, however, agents must decide whether it is worth the time and money costs necessary to gather the information allowing the correct calculation of share price valuation that extends to a degree of precision matching the smallest tick size. Hence they face a trade-off between the potential costs of information gathering, and the potential gain in profitability accruing from obtaining a given level of pricing precision. Ergo, for instance, if the returns to obtaining a hundredths or tenths level of pricing precision is in expectation to be less than the loss to be incurred from gathering the information allowing for that level of precision, then the agent will rationally choose not to gather the information, and instead to trade at a lower level of precision. Such a decision can lead therefore to observed deviations between expected and realized distributions in actual pricing data.

Ultimately, the argument presented in the essay concerning price clustering is that observed deviations from the expected distributions must not necessarily be attributed to irrational explanations such as bounded rationality; i.e., that agents lack the cognitive skills to correctly calculate their expected return. Rather, I argue that in fact, agents have the ability to correctly calculate their expected return at a high level of precision; however, given that information is costly, agents face a dilemma in that in order to get the degree of information necessary to exactly—out to very high precision levels—calculate expected values of assets,

they have to expend a great deal of cost. It seems apparent that in some instances, the result of this "cost-benefit analysis" undertaken to weigh whether expending those costs to obtain such information will be worth the potential return or loss in profits from buying or selling certain assets results in agents trading at price levels coarser than the minimum possible tick size.

3.3 Methodological Use of Different Institutional Environments

It is important to emphasize that the analysis of decision-making under differing institutional environments is undertaken more *within* each of the two general institutional environments, as opposed to across the two. In other words, the essays in this dissertation concern how decision-making is impacted when some of the specific features defining a general institutional environment are slightly modified. Hence, given the two general institutional frameworks—market and non-market—under consideration, the analysis of how decision-making by rational agents changes under the "different" institutional environments referenced in the dissertation title focuses on when there are slight modifications to the underlying general institutional settings, as opposed to any comparison between the two.

In the case of the non-market allocation-by-contest environment of Part II, the goal is to study the impact on agent decision-making (defined here as agents' choice in how much non-recoverable costly effort to expend in attempting to win the prize) when several of the parameters defining that environment change. Specifically, I analyze what happens to agent effort provision when the institutional format changes from that which the contest prize is administered by *one* judge, to one where there are multiple prize administrators. Additionally, under those scenarios where multiple judges (i.e., a committee) administer the prize, another vector along which I allow the institutional framework to vary is the voting rules used by a committee to decide the winner of the contest. Given the specific formulation of voting rules I

use to modify the standard simple majority rule, I am able to further consider the impact of introducing bias into the voting dynamic by changing voting rules that either advantage or disadvantage the stronger player. Finally, in all of these scenarios representing variations on our overall non-market allocation-by-contest institutional structure, I allow the degree to which the contestants competing for the prize exhibit heterogeneity in how they value the prize. I find repeatedly that the role of contestant heterogeneity is key in determining the degree to which changing the institutional rules shapes contest outcomes in terms of both the contest winner and the aggregate level of effort provision. Ultimately, I am concerned with analyzing how changes to the underlying institutional economic substructure affect agents' decision-making as measured by the equilibrium level of aggregate effort provision.

In the other institutional environment under consideration, the financial market environment, it must be recognized from the start that the analysis here stems from an empirical study, and as such it is not possible—as under the theoretical non-market scenario—to actively and purposively change features of the given institutional environment in order to analyze resultant changes in agent decision-making behavior. All empirical studies face the epistemological dilemma that a given data series represents the outcome of a unique temporally-specific set of circumstances, as well as importantly for the study's purposes, a unique institutional context. Unlike laboratory or theoretical studies, it is not possible to actively control specific features of the institutional environment, tweaking them in order to determine their marginal effects. Nevertheless, conceptually, in this study I attempt to capture a similar kind of analysis by looking to observe several small "natural experiments". That is to say, given that some features of the institutional environment of the SHSE change over time, it is possible to view comparisons across the different time frames as representing comparisons across institutional substructures.

My study of the Shanghai Stock Exchange is concerned with the level of share price precision employed by individual investors, as evidenced by their aggregate buying and selling behavior. I present the argument that in certain institutional frameworks, agents are more likely to use a higher level of precision than under other institutional frameworks, mainly due to the relative importance of information and transaction costs within varying institutional settings. Specifically, I consider two major changes to the institutional setting of the Shanghai Stock Exchange over the course of my data series. To begin with, the legal regulatory environment concerning short-selling was modified on the Shanghai Stock Exchange, with no short selling at all allowed prior to March 2010, and increasingly greater degree of short sales allowed since that time. Presuming that nothing else highly relevant to the price formation process occurred following the decision to allow short sales, it is possible to view the post March-2010 time frame as a "separate institutional sub-setting". As such, I analyze potential changes in the price formation process as indicated in the distribution of share prices following the lifting of the short-sales ban. Second, theories to explain deviations between the expected and realized price distributions depend in part on the degree of liquidity present in the marketplace, as well as the general level and pace of market activity. Using trade volume as a proxy for these concepts, I look to consider what happens when trade volume is at levels much higher than (high liquidity/trade activity periods) or much lower than (low liquidity/trade activity periods) normal average trade volume levels. Under such scenarios, the question I consider is whether the data indicate differences in the realized distributions in these high and low volume periods in ways suggested by the proposed explanatory theories. Hence, in essence, I look to use available "before" and "after" scenarios to simulate modifications to the general overall institutional environment, and study their potential effects on individual

decision making as represented by changes in the aggregate realized distributions contained within the data set of share prices on the Shanghai Stock Exchange.

4 Remarks

The essays comprising this dissertation research project are bound together by the way in which they consider, under two general institutional settings, how rational agents produce collective outcomes through the agents' individual utility-maximizing behavior. While the general institutional frameworks considered differ greatly (one a non-market setting, the other a market; one with relatively few agents, to one with many thousands; one with only one transaction, to one with hundreds of thousands), ultimately, the same fundamental issue is analyzed: how individual rational agents, operating independently from one another, produce collective outcomes that are impacted by the constraints and unique characteristics imposed by the institutional environments within which the individual agents operate.

Within each general institutional framework, I either explicitly change the parameters defining the specific institutional framework features (as in the case of the nonmarket scenario), or use a form of "natural experiments" to simulate changes in the underlying institutional structure by way of selecting sub-samples (as in the case of the financial market scenario). This methodological strategy allows for a consideration of how agent decision-making behavior can change in response to changes in the underlying institutional environment. Ultimately, this furthers understanding of the decision-making process, and a richer understanding of economic processes in general.

PART II: Agent Decision-making in Tullock Contests under Committee Administration with Contestant Heterogeneity*

1 Introduction

A contest is a game in which players expend non-recoverable effort to win a given prize. Under the Tullock contest model, the ratio of the level of effort expended by a given candidate, with respect to the effort expended by all candidates, determines the probability that a given contestant has to win the designated prize. The Tullock contest and its variants have been well-studied, and applied to a wide variety of applications in political economics and public choice.

In this section, I employ an extended Tullock contest model in order to study agent decisions on effort provision under the general institutional environment of a non-market allocation-by-contest. Under contest settings, a good (termed a prize) is allocated according to the winner of a competition between agents. While the literature has typically focused on homogeneous agents competing for a prize distributed by a single prize administrator, I study the impacts on aggregate contestant effort provision when a committee (i.e., multiple administrators) distributes the prize.

* Part II is an extension of my journal article "Tullock contests under committee administration", as published in *Economics Bulletin*, vol. 33(3), July 2013, pp. 1983-1990.

1.1 Tullock Contests, Heterogeneity, and Committees

Competition is perhaps one of the fundamental drivers of human behaviors. It is not surprising therefore that competition is a fundamental component of many aspects of modern social, political and economic systems. One of the most common structures by which competition is structured is within contests. Contests are defined by the way in which agents expend costly non-recoverable resources (money, time, effort) in order to secure a valuable prize. In the sub-sections below, I provide a brief description of the nature of the specific contest structure upon which I base the analysis contained within Part II of this dissertation.

Contest Theory

Numerous examples of contests abound in sociopolitical and economic life. People actively compete with others as described above –exerting money, time and effort– in order to win access to certain valuable "prizes". Examples abound, including, in the job market, a promotion or job offer; in politics, a political office; in sporting competition, a title; in geopolitical affairs, control over a resource such as land, fishing rights or a point-source resource. Contest theory has developed in order to gain better understanding of the incentives that drive individual behavior within contest systems, as well as the way in which the specific design and rules of the contest structure impact agent actions.

Since the seminal work of Tullock (1980) in presenting a formalized conceptualization of the contest format, there has been a tremendous production of literature describing and utilizing contests as a methodological tool to describe many forms of economic interactions. As such, presenting a complete review of the literature is challenging, although Corchón (2007), Konrad (2009), and Sisak (2009) make admirable attempts. Below, I provide a non-exhaustive account of the literature on contest theory and its applications.

The classic application of contest theory has been in the area of lobbying and rent-seeking competition. Tullock (1967) and Krueger (1974) are viewed as the originators of this branch of the literature, emphasizing the ways in which monopolies and other government-created rents impose market distortions that are not measured by the Harberger triangle alone, but also from the costs to society by the allocation of resources devoted to winning the rights to the rent; i.e., agents engage in socially wasteful (rent-seeking) activities causing resources to be expended, which could have otherwise been used productively, in competition for redistribution of political rents. Nitzan (1994) and Congelton, *et al.* (2008) provide surveys of this literature, which typically focuses on describing inefficiencies arising from government actions in the economy which causes rents, and measuring the degree of wastefulness arising from various forms of rent-seeking competition.

Another common strand of the contest literature is found in the analysis of political campaigns and elections. Brams & Davis (1973) provide a model showing that candidates running for public office will apportion campaign spending to electoral districts based on a "3/2" Rule, which indicates that given non-uniform district sizes, larger districts (and hence those with a large weight in the electoral outcome) garner effort greater than their proportional weight. Klumpp & Polborn (2006) also study allocation of campaign funding, although their primary focus is on the temporal nature of effort provision. In that respect, the authors find early momentum in a campaign to be rationale for the "New Hampshire effect" wherein which states holding early presidential primaries are allocated a share of campaign funds more than their proportional electoral weight would indicate. Amoros & Puy (2010, 2013) present a model wherein which two candidates strategically allocate campaign funds across policy issues, as opposed to districts, concluding that issue convergence between candidates is a function of each candidate's relative and absolute advantage in terms of the policy issues.

Contest models are also typically employed in the literature to model R&D contests, innovation tournaments and patent races (Nalebuff & Stiglitz 1983, Delbono & Denicolo 1991, Kamien *et al.* 1992, Che & Gale 2003). In such scenarios, players typically compete by expending effort on research in order to win a prize when they are first to develop a new technology. Baye & Hoppe (2003) provide support for the equivalence of innovation races with Tullock contest formulations. Recent literature (Clark & Konrad 2008) has focused on the role of property rights, emphasizing the way the institutional environment shapes the incentive structure facing the contestants in terms of inducing effort provision. Other applications of contests include sporting contests (surveyed by Szymanski 2003), military and geopolitical conflicts (surveyed by Garfinkel & Skaperdas 2007), and litigation (Baye *et al.* 2005, Robson & Skaperdas 2008).

Finally, an area of profound contribution to the contest literature has come from those focused on promotion contests and internal labor market tournaments. Tournament theory, as first advanced by Lazear & Rosen (1981), Nalebuff & Stiglitz (1983), O’Keeffe *et al.* (1984), and Rosen (1986), describes systems in which employees are remunerated according to their relative—as opposed to absolute—performance ranking. Worker performance is modeled as being (stochastically) related to effort provision, and using relative performance criteria can be beneficial when direct observation of effort is impractical or costly for the principal, and the stochastic shock on output is common across employees. Much of the goal of tournament theory focuses on how principals can alter the parameters of the institutional setting to extract optimal effort provision, for example by changing the number and heterogeneity of workers, how much performance feedback to provide workers, the number of rounds the competition should last before the prize is distributed, and the number of prizes. Extensive surveys of the literature on tournaments are provided by Lazear (1995) and Prendergast (1999).

Tullock Contest Architecture

While the concept of the contest in economic behavior had been in existence since at least Friedman's (1958) work on advertising, it was Tullock (1980) who first formalized a structured model of a rent-seeking contest, now termed the Tullock contest success function (CSF). Variations of the Tullock model are perhaps the most broadly used formulation of contests in the literature (Lockard & Tullock 2001 provide a non-extant collection). Indeed, the Tullock contest success function is the underlying CSF that I employ as the foundation of the analytical tools used in Part II of this dissertation.

One of the advantages of the Tullock model is its general intuitive construct, in which contestants compete by exerting a non-negative effort x_i to win a prize of fixed value V , and where contestant i 's probability of winning the contest, p_i , depends on the contestant's own effort and the sum of efforts exerted by all contestants; hence CSF payoff function has the following form:

$$p_i(x_1, \dots, x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \text{ if } \max \{x_1, \dots, x_n\} > 0; 1/n \text{ otherwise,}$$

where r is the "returns to scale", or efficiency, of the contest technology¹, and n denotes the number of contestants.

¹ r affects the degree to which players' efforts impact their winning probability. For $r = 1$, the winning probability is simply equal to the exact share of efforts exerted by a player in terms of total efforts exerted. At the lower bound, when $r = 0$, the CSF returns the value of $p_i = 1/n$ for any level of effort provision, thus signifying a pure lottery wherein all players have an equal chance to win the contest, regardless of effort. Conversely, in the upper limit case when $r \rightarrow \infty$, the CSF is more commonly known as the all-pay auction, in which no pure strategy equilibrium exists; as per Hillman & Riley (1989), in such cases only the two players with the highest prize valuation participate, with the player having the highest prize valuation winning the contest with certainty.

Heterogeneity in Contests

Tullock (1980) originally formulated his model as competition between symmetric players, and much of the literature has adopted this framework, advantageous in terms of its tractability. However, given that actual contests are frequently undertaken by agents possessing asymmetric qualities, a large branch of the contest literature has developed to incorporate such heterogeneity amongst contestants. Contestant heterogeneity plays an important role in terms of shaping the nature of effort provision within the contest framework, and in particular within the committee prize-awarding structure I consider.

Heterogeneity has been modeled along many lines in the literature. Most typical is to model heterogeneity amongst contestants along the lines of differences in ability or strength, their cost functions on effort provision, or their valuation of the prize.² Contestant heterogeneity is modeled in the literature as differences in ability in Fonseca (2009) and Siegel (2010), and on cost functions in Ryvkin (2007) and Hörtnagl *et al.* (2013). I model heterogeneity in this paper along differences in contestant valuations of the contest prize. Modeling intra-contestant heterogeneity as differences in prize valuation can also be found in Hillman & Riley (1989), Baik (1994), Nitzan (1994), Nti (1999), Klumpp & Polborn (2006), Parreiras & Rubinchik-Pessach (2006), Matros (2006), and Stracke *et al.* (2012). Baye *et al.* (1996) find these three different formulations of contestant heterogeneity to be isomorphic and hence able to be converted into each other by transformation of the utility function. They characterize the equilibrium of an all-pay auction with heterogeneous players under complete

² Other areas in which heterogeneity has been modeled that are not formally equivalent to heterogeneity in prize valuation include contestant differences in regard to information and timing. O'Keeffe *et al.* (1984) were among the first to introduce informational asymmetry into the tournament literature, endowing agents with private information regarding their asymmetric abilities. Timing takes on importance in non-simultaneous games, since the ability to act as first-mover is advantageous. Heterogeneity in terms of timing was first analyzed by Dixit (1987), who shows the incumbent is advantaged by possessing the ability to exert effort prior to other players his opponents.

information, while Nti (1999) and Stein (2002) extend the analysis to the Tullock contest form. Hence, as Baye *et al.* (1996: 292) note, differences in valuations are analogous to differences in ability and can be shown to be formally equivalent under conditions where contestants have identical prize valuations but differ in their linear cost functions or ability levels (since an increase in prize valuation and a decrease in cost of effort (or, vice versa) by the same factor induce the same change in the reaction function). As such, throughout Part II in regard to contestant heterogeneity, I refer to "stronger" and "weaker" candidates.

Committee Structures in Contests

As described above, the literature on contests is quite large. Much of the Tullock contest literature focuses on analyzing the impact on equilibrium effort production when various model parameters are modified, such as the number of contestants (Peng 2006, Fu *et al.* 2010), the number and degree of endogeneity of prizes (Chung 1996; Clark & Riis 1996, 1998; Szymanska & Valletti 2004; Epstein & Nitzan 2006), or contestant heterogeneity level, as referenced above. One area still relatively little-studied has been the structure of the prize distribution process. The vast majority of the literature on contests employ a single prize administrator to award the prize, with notable exceptions including Amegashie (1999, 2002), Klumpp & Polborn (2006), and Clark & Konrad (2007), who utilize a committee-analogous setting). I seek to add to the literature by providing a rigorous analysis of contest equilibrium conditions when prizes are distributed by multiple-person committees.

My consideration of a committee prize-granting format is based upon its widespread use in real-world settings. Many important political, economic and social decisions are made in committees. Whether it be the crafting of national legislation, setting central bank policy, the granting of tenure in universities, or even selecting the city to host an upcoming World Cup or

Olympics—all of these decisions are typically formulated within the committee framework. The prevalence of decision-making committees has been well-considered within the literature, for under the strict assumptions of homogeneity of rational committee members operating within a perfect information environment, there will be no difference in decision outcomes between a single decision-maker and multi-member committees. Hence, most attempts in the literature to explain the pervasiveness of committees as a decision-making apparatus have done so by relaxing the perfect information and homogeneity assumptions, and modeling information that is asymmetrically distributed and costly to obtain (Gersbach 1995, Gilligan & Krehbiel 1989). Under such a scenario, better decisions can be made if groups of decision-makers can sum various skill sets to optimize choices.³

Separate from the "information gathering and pooling" analysis of committee decision-making, an alternative rationale is advanced by Congleton (1984). Analyzing the level of rent-seeking efforts when prizes are awarded under a committee structure versus a single administrator, he finds aggregate rent-seeking efforts to be lower when committees award the rent-generating prize. Hence he argues that committees are more efficient than individual decision-makers vis-à-vis rent-seeking costs. Congleton (1984) suggests that the prevalence of committees as decision-making bodies is a function of this greater efficiency.

The supposition that the relative frequency of committees as a decision-making body derives from it generating *less* rent-seeking costs can be challenged when considering that interested parties often seek to generate the highest possible amount of rent-seeking efforts if they stand to capture the benefits of such efforts while not overly suffering from the inefficiencies created in the rent-seeking process. Further, subsequent research has suggested

³ This argument can be traced back to Condorcet (1785), who described committees as mechanisms that efficiently aggregated decentralized information. His argument, as outlined in his "jury theorem", claimed that increasing the number of committee members would lead to a concurrent increase in the probability that the committee would make the correct decision.

that aggregate rent-seeking efforts are not necessarily lower when committees award prizes as compared to when single administrators do so. Specifically, Amegashie (1999, 2002) demonstrates that under certain formulations, aggregate levels of effort provision can be at least as large under committees as under a single administrator, and might in fact be higher.

In this Part II of my dissertation, I allow a committee to decide the winner of the contest, and consider the impact committee size plays in regard to the level of effort expenditures. I find that depending on specific features between contestants, committees change the dynamic and incentive structure differently from the scenario under a single prize administrator. Further, as the size of committee varies, the pattern, direction and the degree of the changes on aggregate effort provision vary as well.

Committees and Voting Rules

Incentive structures facing players competing in a contest within a committee framework depend crucially upon the institutional rules directing how the committee tallies the votes and decides the contest winner. Most typically, committees operate under "majoritarian" voting rules.⁴ That is, each committee member gets one vote where all votes count equally (which is why committees are characteristically comprised of an odd-number of members), and the choice selected by the majority of the members becomes the "committee's decision". The limited literature on committee decision-making has primarily considered simple majoritarian decision-making voting rules.

The simple majoritarian framework is important to study precisely because it is the one most-often used in practice in committee decision-making processes, including in lawmaking

⁴ Majoritarian decision-making is unique in that pay-off functions under such a system are defined by their discontinuity—there exists a sharp threshold (i.e., 50 percent) above which positive pay-off is achieved, while pay-offs are strictly negative if only achieving a percentage of committee member support below 50 percent.

committees in legislative bodies elected under proportional representation systems. However, there exist numerous other ways committees can agree to formally arrive at their decisions. For instance, some committee decisions set the requirements for successful passage of a proposal to be higher than simply being strictly greater than 50 percent as under simple majority. Such super-majority voting rules impose stricter requirements on the threshold to arrive at a decision. The policy-setting Council of the European Union, for instance, requires unanimous votes to pass legislation on certain issues, such as taxation, social security, or the accession of applicant states to the European Union. Super-majority voting requirements are also found in academic settings, such as degree-granting dissertation committees or faculty hiring committees.

While alternative rules other than simple majority have been well-considered in the political economics literature⁵, there has been considerably less attention paid in contest structures with committee prize administration of efforts generated under differing voting rules. As such, my research also analyzes the effect on effort provision when the committee utilizes voting rules other than the simple majority rule, in order to better understand the interactions of committee size and heterogeneity as they play out under a variety of committee voting rules. Such a consideration also adds to the robustness of the analysis of committee size and effort expenditure.

Bias in Contests

The specification of the voting rules analyzed in this dissertation are structured to modify the simple majoritarian voting rule in such a way as to interpret these rule changes as

⁵ Levy (2007) looks at supermajorities in his analysis of the effect of transparency in the decision-making process in committees; Vermeule (2005) alternatively considers the role of sub-majoritarian voting rules in terms of enhancing transparency and greater accountability to minority concerns.

inducing bias into the contest architecture. Bias is represented as the condition when one of the contestants benefits from favorable odds-enhancing rules. Introducing the concept of bias into the contest via voting rule formulations is useful not only from the standpoint of analyzing aggregate effort provision levels under different institutional voting structures, but also because bias plays an important role in many observable contests. For instance, employment promotion contests that incorporate features of "Affirmative Action" programs (or other programs designed to bias selection towards certain socially-preferred candidates) can be considered under the voting rule specifications described in my model. The impact on effort provision under bias, especially in the presence of contestant heterogeneity, is an issue frequently studied within the "Affirmative Action" literature (Franke 2012). (Alternatively, it is possible to interpret the bias-inducing voting rules I formulate as if one of the candidates was to be given a "head start", as in Siegel (2011).⁶) Likewise, bias in contests can also be interpreted in a political economics framework; for instance, when one of the contestants possesses an incumbency advantage, and hence only needs to win a small percentage to maintain as incumbent (Besley & Preston 2007; Caselli *et al.* 2012). If considered from a lobbying or policy debate standpoint, then one of the policy options can be viewed as being more risky or unsure, and therefore requiring a greater level of support in order to be implemented or selected. Hence, I look to expand the literature by consideration of bias effects when comparing the impact of effort provision when committees or single administrators award the prize.

⁶ Assume that prior to the start of the formal contest, there exist committee member(s) who have strong preferences towards one of the candidates, such that these committee members will vote for that candidate with (near) certainty; all other committee members are all neutral voters who only vote according to the Tullock success function. Therefore contestants would not expend effort attempting to win support of such committee member(s).

1.2 Research Motivation and Structure of Part II

Models most closely related to the one developed in this paper are Amegashie (2002), Klumpp & Polborn (2006) and Clark & Konrad (2007), who also assume (or equivalently assume) multiple prize administrators; however, they do not explicitly analyze variations in committee size. Amegashie (2002) does compare rent-seeking expenditures under a 3-member committee versus under a single administrator, and finds rent-seeking expenditures to be higher under 3-member committees than under a single rent-giver.⁷ I consider the more general case by introducing heterogeneity between contestants and allowing variation in committee size. Allowing different levels of heterogeneity between contestants leads to different results when committee size varies. Klumpp & Polborn (2006) study campaign effort expended in different districts. In their model, the campaign winner is decided by simple majority rule, where each district can be interpreted as a committee member. As in Amegashie, their main focus is not on variation of committee size. Further, both papers assume homogeneous contestants, whereas I allow for heterogeneity. Somewhat related is Clark & Konrad (2007), who present a contest game with a single prize administrator but where two identical contestants compete in multiple tasks. Their focus is on designing an optimal rule in terms of how many tasks need to be considered to decide the winner in order to extract maximum efforts. As Clark & Konrad (2007) model the two players to be homogenous, the optimal decision rule is either to win at least a majority of the tasks if the number of tasks is an odd number ($k^*=(n+1)/2$); or win at least half of the tasks if the number of tasks is even number ($k^*=n/2$, except for that $k^*=2$ when $n=2$). In my research, I also examine which voting rules can generate highest possible effort provision under contestant heterogeneity. I obtain a

⁷ While Amegashie (2002) employs a modified majority voting rule, under only two contestants, it simplifies to the simple majority voting rule, creating a comparable setting with my model.

more general result, finding that under a given committee size, the optimal voting rule(s) is dependent upon the level of contestant heterogeneity. For the case encompassing homogenous contestants, I confirm the findings in Clark & Konrad (2007).

Part II of this dissertation analyzes the impact on aggregate effort expenditure that comes from concurrently varying the heterogeneity of contestants, the number of committee members awarding the contest prize, and the committee voting rules. I find that in general, increasing the committee size produces an ambiguous impact on total efforts. This derives from the fact that there are two counter-acting forces acting upon effort expenditure when the size of committee increases. First, as committee size grows, there are more committee members for contestants towards whom to direct efforts; thus there is an expected increase in aggregate efforts. Conversely, however, an increase in committee size means that a contestant must secure the support of a greater number of members in order to win the prize, decreasing the chance that the contestant's efforts towards a given committee member will produce a pivotal outcome. These counter-acting forces are such that the latter will tend to dominate the former as committee size increases. Hence, the significant findings of the research come out of the analysis of the interaction between heterogeneity and committee size, finding that the committee size at which this dominance happens is falling in the level of heterogeneity between contestants. Additionally, by modifying committee voting rules to impose bias in favor of (against) or against (in favor of) the stronger (weaker) player, also serves to affect effort provision by acting on the probability for a contestant to win the contest.

In the proceeding Section 2, I begin with the construction of a benchmark model that considers the smallest size of a committee structure that yields interesting results, that being a three-member committee. Some important insights can be obtained by comparing effort provision under the three-member committee, versus that under single prize administration.

Such analysis provides a fundamental basis for additional discussion of the impact of increasing committee size on influencing the direction and the degree of effort provision. I further allow modification to the underlying institutional environment by consideration of other voting rules within the three-member committee contest specification. Comparison of effort provision under differing committee voting rules enables for a more explicit analysis of the way in which contestant incentive dynamics are constrained by voting rules specifications. In Section 3, a generalized model is examined focusing on the impact of committee size on aggregate effort provision given different contestant heterogeneity levels. I perform such an analysis first under the simple majority committee voting rule, finding that effort provision follows similar patterns under given ranges of contestant heterogeneity, and different patterns when these ranges are expanded. Subsequently, I analyze the impact that variations in committee voting rules play upon effort provision in the more general model setting, and later provide a discussion of mixed strategy equilibrium when committee size converges to infinity. Finally, Section 4 summarizes the findings of Part II.

2 Aggregate Effort Provision under Three-Member Committees with Contestant Heterogeneity

2.1 Introduction

Several papers, including Congleton (1984) and Amegashie (2002), model effort provision exerted by contestants in a committee setting under probabilistic voting. Here I present an extension to these results by introducing heterogeneity amongst contestants, and analyzing its impact upon total contestant effort provision. Specifically, I explicitly compare the total effort provision levels under single prize administrator contests, with the total effort provision generated under contests administered by three-person committees. Theoretically, the direction and degree of the impact on aggregate effort provision generated when moving from a scenario where the contest prize is awarded by a single administrator, to one where the prize is awarded by a committee, are ambiguous. On the one hand, a greater number of prize administrators implies that contestants must exert effort towards a greater number of individuals, the effect of which is to *increase* aggregate effort provision. On the other hand, under single administration, all effort exerted is directed towards the administrator who is the determining voter (necessarily, since there is only one voter), and hence all effort exerted must impact a contestant's probability to win the contest. However, since under committee administration a particular administrator may or may not be the determining voter, increasing effort towards a particular prize administrator may or may not increase a contestant's probability to win the contest. In other words, the marginal return of additional effort provision is lower under committee administration compared to that under a single prize administrator. As such, there is less incentive for contestants to exert effort, which works to *decrease* aggregate efforts. The introduction of heterogeneity amongst contestants further complicates the outcome array.

Under the initial model formulations described below in the proceeding sub-section, 2.2, the three-person committee operates under a simple majority voting rule, meaning that a contestant must win the votes of at least two of the three committee members in order to win the contest and hence the prize. Under this scenario, probabilistic voting implies that, as the number of prize administrators increases, the probability of the weaker contestant winning the prize decreases, as now a contestant must win the support of not just one but at least two committee members in order to win the contest.

While true that many committee settings operate according to simple majority voting rules, consideration of other committee voting rules is relevant in order to gain insight into alternative methodologies that might increase or decrease contestant effort provision levels. Subsequently, in the following sub-section 2.3, I present a slight modification to the underlying institutional environment by allowing the contest structure to account for a wider array of voting rules within the three-member committee structure. Specifically, I allow the committee decision to be dictated by two additional voting rules besides simple majority—"biased unanimity" against the weaker player Contestant 2 (Contestant 2 can only win the contest when winning the votes of all three committee members), and "biased unanimity" against Contestant 1 (Contestant can only win the contest when winning the votes of all three committee members). Additional voting rules are introduced into the analysis specifically for the purpose of gaining a better sense of how the increase in committee size from one to three is affected by heterogeneity. The dynamics of heterogeneity among contestants and the number of prize administrators described in the preceding paragraph can be exacerbated or muted by changing the voting rules, given that these rules affect the incentive structure facing the contestants.

Overall, the results extend previous analyses of effort provision dynamics under committee structures, indicating that heterogeneity drives effort provision, allowing aggregate effort outlays under committee administration of prizes to be larger than, equal to, or less than those generated when a single administrator awards prizes.

2.2 Contest Model with Three-member Committee and Heterogeneous Contestants

Here I construct a model using the smallest size of committee that can yield interesting results, that being the three-member committee. Hence, consider a committee made up of $i = 3$ members (A, B, and C). There are two contestants (Contestant 1 and 2)⁸ competing to win a non-divisible prize, valued strictly positive. Each committee member votes for one of the contestants, with the committee decision being formulated by simultaneous vote under a simple majority voting rule. Note that with a three-member committee and two contestants, tie-breaking rules are not necessary.

Heterogeneity between contestants is modeled along differences in their individual valuations of the prize. Contestant 1 has a valuation of V_1 towards the value of the prize, and Contestant 2 has a valuation of V_2 . Without loss of generality, we assume $V_1 \geq V_2$. The heterogeneity level between the two contestants is represented by $t \equiv V_2/V_1 \leq 1$. Hence, $t \in (0,1]$, with $t = 1$ indicating the boundary condition of perfect homogeneity between contestants, and smaller values of t representing higher levels of heterogeneity.

The two risk-neutral contestants compete to win a prize. Contestants decide on their contributions to each committee member to maximize their expected payoff. We denote the

⁸ For simplicity of discussion, I refer to Contestant 1 as feminine, and Contestant 2 as masculine.

effort provision directed towards Committee Member i by Contestant 1 as x_i , and by Contestant 2 as y_i . Hence Contestant 1's bidding strategy is represented by vector $X = (x_A, x_B, x_C)$, and Contestant 2's bidding strategy vector is represented by $Y = (y_A, y_B, y_C)$.

Committee members vote in a probabilistic manner in response to contestants' effort provision levels. We designate q_i as the probability for Contestant 1 to win the vote from Committee Member i ; thus $1 - q_i$ denotes the probability for Contestant 2 to win the vote from Committee Member i . More specifically, Contestant 1's chance to win Committee Member i 's vote, $q_i(x_i, y_i)$, is a function that depends upon x_i (the effort directed towards Committee Member i provided by Contestant 1), and y_i (the effort directed towards Committee Member i provided by Contestant 2). As typical in much of the political economics literature, the functional form of the probabilistic voting model can be represented by the standard Tullock Contest Function, as first introduced in Tullock (1980), and provided its axiomatic specification in Skaperdas (1996). As such,

$$q_i(x_i, y_i) = \frac{x_i^r}{x_i^r + y_i^r}, \text{ given } x_i + y_i \neq 0; \text{ otherwise } q_i = \frac{1}{2},$$

where $r > 0$ represents Committee Member i 's sensitivity to contestants' efforts directed towards him, with all three members assumed to be homogeneous in regard to their effort sensitivity level r . The model specification indicates that the higher a contestant's effort directed towards Committee Member i compared to that of their opponent's, the higher probability he or she has to win the vote of the committee member. I define P to represent the probability for Contestant 1 to win the overall contest, and $1 - P$ the probability for Contestant 2 to win the contest. Under simple majority voting rules, a contestant must win at least two of the three committee members' votes in order to win the overall contest.

Contestants and committee members operate under a complete-information environment,

such that differences in prize valuations are common knowledge.⁹ I assume a linear cost structure on effort exerted, $c(x)$ and $c(y)$ for Contestants 1 and 2 respectively, to be the same across all players, and normalize costs such that $c(x) = x$, and $c(y) = y$. Thus, each contestant maximizes his or her expected utility with full knowledge of all contestants' prize valuations, with Contestant 1 maximizing her expected utility U_1 by solving $\max_{x_i \geq 0} E(U_1) = PV_1 - x_A - x_B - x_C$ (that is, the expected value of the prize minus the costs expended by directing effort to each of the committee members); likewise, Contestant 2 solves $\max_{y_i \geq 0} E(U_2) = (1 - P)V_2 - y_A - y_B - y_C$.

Contestants choose bidding strategies that maximize their utility; in doing so, they are concomitantly calculating $\partial P / \partial x_i$ and $\partial(1 - P) / \partial y_i$, respectively. In other words, they consider the marginal return on whether and how much the costly effort expended actually impacts the outcome of the contest. Hypothetically, a contestant could choose to direct the percentage of their total efforts across committee members in any of an innumerable set of bidding distributions, such as (50,50,0), $(33\frac{1}{3}, 33\frac{1}{3}, 33\frac{1}{3})$, or (0,0,100). Since effort provision is costly, however, a contestant will only wish to exert effort towards a committee member when doing so helps the contestant to win the contest. How much costly effort to expend, and how to distribute effort across committee members, depends upon two factors.

First, a contestant must consider the degree to which increasing effort towards a committee member will impact that committee member's decision to vote for the contestant; this is represented by r (as well as the actual levels of x and y exerted). If for example, in the extreme case where $r = 0$, then committee members would be immune to all effort provisions, and contestants would not exert any costly effort, since such effort would have no impact on

⁹ Private information environments are more suitable to considering the impact that uncertainty plays on effort provision; here, however, I seek to restrict the analysis to clearly identify the effects of heterogeneity.

the way in which committee members vote (in such an instance, the contest becomes a pure lottery). On the other hand, if committee members are highly sensitive to effort provision, it is more useful—all things equal—to direct effort towards them since such effort will impact voting behavior. In this section I assume $0 < r \leq 1$ to ensure the second-order condition is globally satisfied (Nti 1999).¹⁰

Second, contestants must consider whether it is worth vying for a particular committee member's support. Such consideration further depends on the particularity of the institutional framework, as defined by the rules of the contest; in this case, the contest winner is determined via majoritarian voting.¹¹ Under simple majority voting rules, to win the contest, a contestant needs at least two of the three members' support. Given that committee members vote simultaneously, and that voting behavior is probabilistic rather than deterministic, each contestant makes his or her bidding strategy decision as to whether to direct costly effort towards a given committee member based on the *probability* that the committee member is the pivotal voter. I define Q_i to be the probability that Committee Member i is the pivotal voter (meaning, the outcome of the contest depends on Committee Member i 's voting decision). Note that this only happens when exactly one of the two other committee members has voted for Contestant 1 and the other for Contestant 2. Further, it is clear that whenever a committee member is the pivotal voter in the contest for one player, that same committee member must also be the pivotal voter for the other player. Hence, the probability that Committee Member i is the pivotal voter for Contestant 1 must be the same as the probability that the same

¹⁰ Note that $r < 1$ represents decreasing returns to scale in terms of the effect of efforts on committee members. If $x_i > y_i$, then we can allow $y_i = \rho x_i$, where $\rho < 1$. It is clear then that the Tullock contest success function acts to soften the impact of effort provisions by assigning the probability to vote for Contestant 1 as $q_i(x_i, y_i) = 1/(1 + \rho^r)$. When $r < 1$, this value is necessarily smaller than when $r = 1$.

¹¹ I consider a wider array of voting rules in the following section.

Committee Member i is the pivotal voter for Contestant 2 (i.e., Q_i for Contestant 1 is the same as Q_i for Contestant 2).

Thus in solving their respective maximization problems, contestants simultaneously decide how to allocate effort levels across committee members A, B and C by considering whether exerting costly effort will sufficiently increase their probability to win the contest, determined by [i] Committee Member i 's probability to be pivotal (Q_i), and [ii] the degree to which effort provisions have an effect upon Committee Member i 's vote (simultaneously determined by r , x_i and y_i). (Note that Q_i is independent of the effort profile (x_i, y_i) directed towards Committee Member i .) Thus, Contestant 1's bidding choice x_i toward Committee Member i comes from

$$\frac{\partial P}{\partial x_i} = Q_i \cdot \frac{\partial q_i(x_i, y_i)}{\partial x_i} ;$$

likewise, Contestant 2's bidding choice y_i toward Committee Member i comes from

$$\frac{\partial (1-P)}{\partial y_i} = Q_i \cdot \left(-\frac{\partial q_i(x_i, y_i)}{\partial y_i} \right) .$$

To choose their respective optimal bidding strategies, Contestants 1 and 2 must maximize the payoffs that result from implementing these strategies, given by $E(U_1) = PV_1 - x_A - x_B - x_C$, and $E(U_2) = (1 - P)V_2 - y_A - y_B - y_C$, respectively. Solving each contestant's maximization problem will result in the first-order conditions (F.O.C.).

We first focus our analysis on contestants' bidding choices directed to a given committee member; for explication purposes, select Committee Member A. Then Contestant 1's F.O.C. corresponding to the effort level directed towards Committee Member A is

$$V_1 \cdot Q_A \cdot \frac{\partial q_A(x_A, y_A)}{\partial x_A} = 1 , \tag{1}$$

and Contestant 2's F.O.C. similarly becomes

$$V_2 \cdot Q_A \cdot \left(-\frac{\partial q_A(x_A, y_A)}{\partial y_A} \right) = 1 . \tag{2}$$

The symmetry owing to the fact that Q_A for Contestant 1 is the same as Q_A for Contestant 2 allows us to perform a nice simplifying algebraic procedure by dividing equation (1) by (2) to obtain

$$-\frac{V_1}{V_2} \cdot \frac{\partial q_A(x_A, y_A)/\partial x_A}{\partial q_A(x_A, y_A)/\partial y_A} = 1, \quad (3)$$

and since $\frac{\partial q_A(x_A, y_A)/\partial x_A}{\partial q_A(x_A, y_A)/\partial y_A} = -\frac{y_A}{x_A}$, equation (3) can be represented more succinctly as

$$\frac{y_A}{x_A} = \frac{V_2}{V_1} \equiv t \leq 1. \quad (4)$$

Importantly, I find that—*regardless the bidding profile directed towards other committee members*—the probability for Contestant 1 to win Committee Member A's vote is given by

$$q_A = \frac{1}{1+t^r} \geq \frac{1}{2}. \quad (5)$$

Contestant 2, therefore, wins Committee Member A's vote with probability $(1 - q_A)$.

Applying the same logic for effort directed towards other committee members, since $q_B = \frac{1}{1+t^r}$ and $q_C = \frac{1}{1+t^r}$, hence each is won with the same probability $q_i = q \equiv 1/(1 + t^r)$. Since $y_i/x_i = t$ and $q = 1/(1 + t^r)$ are properties of the pure strategy equilibrium, Q_i can be represented as $q + q - 2q^2 = 2q(1 - q)$ at equilibrium. Substituting these results back into the first order conditions, a unique solution for equations (1) and (2) can be obtained. This allows equilibrium efforts to be expressed as

$$x_i = x = 2V_1 \cdot \frac{r(t^r)^2}{(1+t^r)^4}. \quad (6)$$

Using equation (4), obtaining Contestant 2's equilibrium effort provision profile is trivial.

$$y_i = y = 2V_2 \cdot \frac{r(t^r)^2}{(1+t^r)^4} \quad (6')$$

These results allow us to obtain the following proposition.

Proposition 1: *Given the existence of pure strategy equilibrium (defined below), both contestants use uniform bidding strategies, $X = (x, x, x)$ and $Y = (y, y, y)$, where X and Y always satisfy the condition $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$. Since these uniform bidding strategies come from the unique solutions for Equations (1) and (2), respectively, they represent the unique pure strategy equilibrium of the contest game.*

To guarantee the existence of a pure strategy equilibrium, we need to ensure that neither contestant receives a negative payoff from playing the pure strategy equilibrium; otherwise they can always abstain from the contest altogether and be better off receiving a payoff of zero. At equilibrium, therefore, the participation constraints are such that the expected utility of Contestant 1 $[E(U_1) = PV_1 - 3x]$ and Contestant 2 $[E(U_2) = (1 - P)V_2 - 3y = t[(1 - P)V_1 - 3x]]$ must be non-negative. Since at equilibrium Contestant 1 (the strong player), always wins the support of each committee member with a higher probability ($q = \frac{1}{1+t^r} \geq \frac{1}{2}$) than Contestant 2 (the weak player), under a simple majority decision rule, the strong player always has an equal or higher chance to win the contest ($P \geq \frac{1}{2}$), guaranteeing that if the participation constraint holds for Contestant 2, it must also hold for Contestant 1. Therefore, the participation constraint can be expressed as $r \leq \frac{(1+t^r)(3+t^r)}{6}$. Note that if r is relatively small, or t relatively large, the participation constraint becomes easier to satisfy. In general, $r \leq 0.5$ guarantees that the participation constraint is met at any level of t .

Aggregate Effort Provision Analysis

I define $T(3)$ to represent aggregate effort provision levels when there are three committee members awarding the prize. Utilizing Proposition 1, it is easy to show that

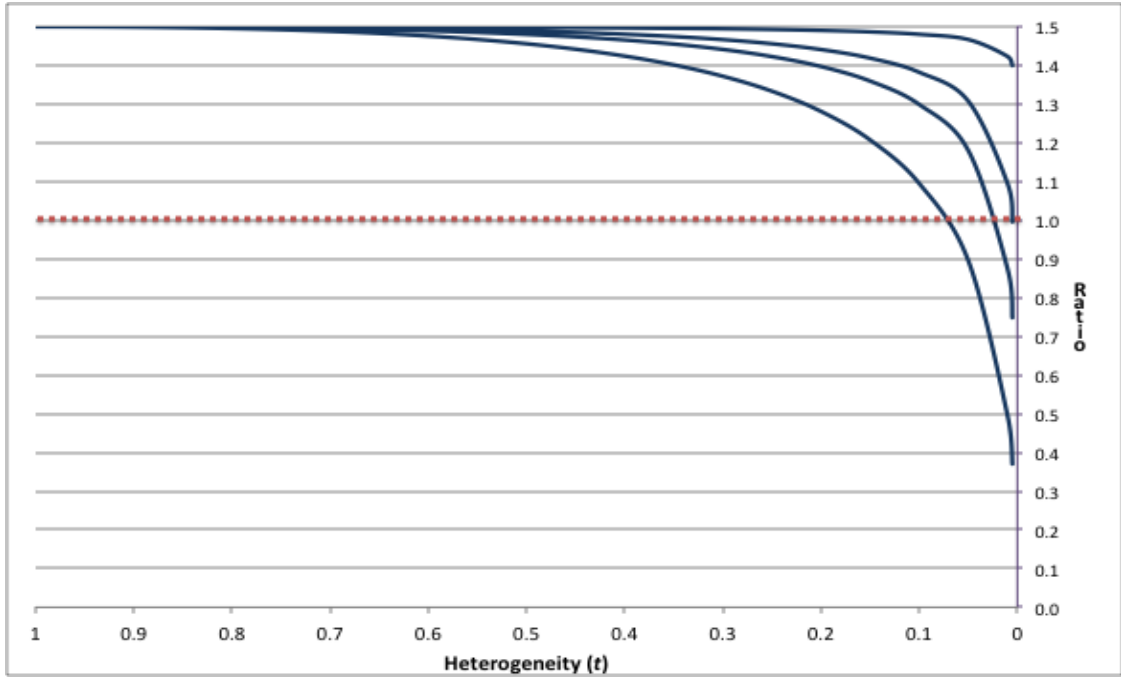
$$T(3) = 3 \left(2V_1 \cdot \frac{r(t^r)^2}{(1+t^r)^4} \right) + 3 \left(2V_2 \cdot \frac{r(t^r)^2}{(1+t^r)^4} \right) = 6 \left(\frac{r(t^r)^2}{(1+t^r)^4} \right) (V_1 + V_2) = 6V_1(1+t) \cdot \frac{r(t^r)^2}{(1+t^r)^4} \quad (7)$$

Likewise, I define $T(1)$ to represent the aggregate effort provision levels when there is a single prize administrator, such that

$$T(1) = V_1(1+t) \cdot \frac{r(t^r)}{(1+t^r)^2}. \quad (8)$$

I compare explicitly $T(3)$ and $T(1)$ through their ratio, as captured by Figure 2.2. It is clear that the ratio $T(3)/T(1)$ depends on the level of heterogeneity t . The far-left corner of Figure 2.2 is consequent to $t = 1$ (contestant homogeneity), which corresponds to the situation analyzed by Amegashie (2002), who argues that effort provision levels under committee administration are higher than under single administrator. While true for homogeneous contestants (aggregate effort provision is always one and a half times higher under the three-person committee than under a single administrator contest), the introduction of contestant heterogeneity induces situations where the reverse also prevails. Indicated on the graph by those parts of the lines below the ratio value 1.0, total effort provision can be lower under three-member committees compared to a single administrator. Hence, which format—single administrator or three-member committee—generates greater effort provision depends crucially on the degree of contestant heterogeneity. In general, the more heterogeneous are contestants, the more likely it is that they will exert less effort under a three-person committee, compared to a single prize administrator.

Figure 2.2: Aggregate Effort Provision ($T(3) / T(1)$)



Note: The curves on the graph represent the ratio of $T(3)/T(1)$ at varying levels of r , with the top-most curve representing $r = 0.1$, followed, in descending order, by $r = 0.25$, $r = 0.33$, and $r = 0.5$.

It is clear from equations (7) and (8) that for both single administrator and three-person committee contests, aggregate effort provision levels are monotonically decreasing in heterogeneity for a given level of r .¹² The shape of the lines in Figure 2.2, however, indicates that aggregate effort under three-member committees decreases more in response to greater heterogeneity than aggregate effort under single administration, as we see the ratio $T(3)/T(1)$ decreases as moving from the left to the right of the figure, implying the fact that moving from a single administrator to a three-person committee reinforces the effect of heterogeneity on reducing effort provision.

¹² Results for the single administrator setting match those first found by Nti (1999) and further described by Baik (2004). The fact that higher heterogeneity leads to lower efforts is well understood in the literature (Lazer & Rosen 1981, Gradstein 1995).

2.3 Contest Model with 3-member Committee and Heterogeneous Contestants, utilizing Three Voting Rules

In this sub-section, I consider a slight modification to the general contest institutional environment by introducing additional voting rules to tabulate committee members' votes in order to decide the contest winner. Specifically, I stipulate that the committee decision is formulated by simultaneous vote under a general fixed decision rule. The three decision rules under consideration will be denoted by $k \in \{1, 2, 3\}$. I define Contestant 1 as the winner of the contest if and only if she wins at least k votes amongst the three committee members; otherwise Contestant 2 is selected as the winner. Therefore, when $k=2$, the decision rule is a simple majority rule; that is, whoever wins two of the three votes is the contest winner. However, when $k=1$, the decision rule becomes a "biased unanimity" voting rule, in this case biased against the weaker player Contestant 2, since Contestant 2 can only win the contest when he wins the votes of all three committee members. Conversely, when $k=3$, the decision rule is again "biased unanimity", but now biased against Contestant 1 such that Contestant 1 will be awarded the prize only if she wins the votes of all three committee members. Note that under this formulation of voting rule specifications, tie-breaking rules are not necessary since there will always be one and only one winner of the contest.

As in 2.2, contestants solve their maximization problems by focusing on the marginal return of effort provision towards committee members, which depends crucially on whether the effort expended has significance in regard to the outcome of the contest. Following the same logic of pure-strategy equilibrium analysis as outlined in 2.2, it is straightforward to prove that efforts directed towards each individual committee member still satisfy $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$, and $q_i = \frac{1}{1+t^r} \equiv q$, at the equilibrium condition under the different decision rules.

The difference, however, is that the probability that a committee member is a pivotal voter is now dependent on the particular voting rule employed. Hence, I denote $Q_i(k)$ as the probability that Committee Member i is the pivotal voter under the decision rule k . At the equilibrium condition, we can obtain:

$$Q_i(1)=(1-q)(1-q); \quad Q_i(2)=2q(1-q); \quad Q_i(3)=q^2.$$

It proves trivial to substitute these results back into the first order conditions to obtain a unique solution for each of the k decision rules, as described in the following proposition.

***Proposition 2:** Given the existence of a unique pure strategy equilibrium (as defined below), both contestants use uniform bidding strategies, $X = (x, x, x)$ and $Y = (y, y, y)$, where effort bids to individual committee members always satisfy the condition $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$.*

Specifically, it can be shown that, under voting rules:

- (a) "simple majority" ($k=2$), $x = 2 V_1 \cdot \frac{r(t^r)^2}{(1+t^r)^4}$
- (b) "biased unanimity" against the strong contestant ($k=3$), $\tilde{x} = V_1 \cdot \frac{rt^r}{(1+t^r)^4}$
- (c) "biased unanimity" against the weak contestant ($k=1$), $\bar{x} = V_1 \cdot \frac{rt^{-r}}{(1+t^{-r})^4}$

It will be advantageous to subsequent calculations and analysis to denote a combined measure of effective heterogeneity as $\alpha \equiv t^r \in (0,1]$; since α increases in t and decreases in r , a higher value of α represents a closer competition between the two contestants.¹³ The aggregate effort provision levels under three-person committees for the three different decision rules can be expressed as follows:

¹³ Notice that for a given value of t , a decrease in r effectively reduces the difference between two contestants, since now committee members are less sensitive to efforts. As such, I use α to represent the level of "effective heterogeneity" between the contestants.

$$T(3) = 3(1+t)x = 6V_1(1+t) \cdot \frac{r(t^r)^2}{(1+t^r)^4} = 6V_1(1+t) \frac{r\alpha^2}{(1+\alpha)^4} ;$$

$$\tilde{T}(3) = 3(1+t)\tilde{x} = 3V_1(1+t) \cdot \frac{rt^r}{(1+t^r)^4} = 3V_1(1+t) \frac{r\alpha}{(1+\alpha)^4} ;$$

$$\bar{T}(3) = 3(1+t)\bar{x} = 3V_1(1+t) \cdot \frac{rt^{3r}}{(1+t^r)^4} = 3V_1(1+t) \frac{r\alpha^3}{(1+\alpha)^4} ,$$

where aggregate effort provision under each of the three voting rules of simple majority, biased unanimity against the strong contestant, and biased unanimity against the weak contestant is represented by T , $\tilde{T}$, and $\bar{T}$, respectively.

Note that under a single administrator, the three different decision rules $k \in \{1, 2, 3\}$ are identical to each other, since all decisions are decidedly "unanimously" given there is only one voter. Therefore, aggregate effort levels under a single administrator must also be equal for the three voting rules:

$$T(1) = \tilde{T}(1) = \bar{T}(1) = V_1(1+t) \cdot \frac{r\alpha}{(1+\alpha)^2}$$

To ensure the existence of the pure strategy equilibrium described above, we need to guarantee that neither of the contestants would receive a negative payoff from playing the pure strategy equilibrium. Under the simple majority rule, as discussed in Section 2, the PSNE condition of $k=2$ can be expressed as

$$r \leq \frac{(1+\alpha)(3+\alpha)}{6} . \quad (9)$$

Similarly under the decision rule of biased unanimity against Contestant 2 ($k=1$), the PSNE condition can be expressed as

$$r \leq \frac{(1+\alpha)}{3} . \quad (10)$$

Ensuring the participation constraint is met when contestants play under the decision rule of biased unanimity against Contestant 1 ($k=3$), however, is more complicated, since it is difficult to determine *a priori* which contestant has a higher probability to win the contest.

While Contestant 1 always has a higher probability than Contestant 2 to win the vote of any *individual* committee member, the biased decision rule handicaps her advantage to win the entire contest, since she needs to win the support of all three committee members in order to receive the prize. When the two contestants differ greatly (α approaches 0), Contestant 1's advantage over winning any particular committee member is relatively large. For such sufficiently low values of α (specifically, $\alpha < \sqrt[3]{2} - 1$), the reduction in Contestant 1's probability to win the contest that comes from requiring her to win the votes of all three committee members, compared to only two under simple majority rules, is not large enough to overcome the inherent advantage she has from winning any individual Committee Member i 's vote gives her in winning the entire contest. Hence she retains her overall advantage in terms of the probability to win the contest, such that $P > 1/2$. This implies that the weaker player (Contestant 2)'s participation constraint is more easily violated than that of the stronger contestant, and thus the logic of ensuring the participation constraint is met follows that previous discussed, and we obtain

$$\text{if } \alpha < \sqrt[3]{2} - 1, \quad r \leq \frac{(1+\alpha)(\alpha^2+3\alpha+3)}{3}. \quad (11)$$

On the other hand, when the degree of difference between the contestants is small, Contestant 1's probability to win the vote of a given committee member is not significantly larger than Contestant 2's. As such, introducing bias against the stronger player by requiring her to win all the votes of all three committee members in order to win the contest will cause her overall probability to win the contest to fall below fifty percent. In such an instance of sufficiently high values of α (specifically, $\alpha > \sqrt[3]{2} - 1$), the biased rule against the stronger Contestant 1 can "over-handicap" Contestant 1 ($P < 1/2$). In such an instance, it is easier for the stronger player (Contestant 1)'s participation constraint to be violated than the weaker player (Contestant 2)'s. In this case, it is required to validate that Contestant 1's participation

constraint is satisfied to ensure the participation constraint of the game is met. As such, the PSNE condition for biased unanimity voting rule against the strong contestant ($k=3$) can be expressed as:

$$\text{if } \sqrt[3]{2} - 1 \leq \alpha \leq 1, \quad r \leq \frac{(1+\alpha)}{3\alpha}. \quad (12)$$

Of the four participation constraint inequalities described in (9), (10), (11) and (12), the value of RHS of inequality (10) requires the smallest value to satisfy, indicating that r must be relatively small or t relatively large to satisfy (10). Satisfying equation (10) therefore also ensures (9) and (11) or (12) will also be satisfied. Given that the RHS of (10) will always be a value larger than $1/3$, then if r is less than or equal to $1/3$, all the participation constraints described by equations (9), (10), (11) and (12) will hold for any value of t indicating contestant heterogeneity.¹⁴

Aggregate Effort Provision Analysis

By restricting the value of the sensitivity to efforts of the prize administrators (r) to certain specified ranges, it is possible to ensure the participation constraints described above in equations (9), (10), (11) and (12) can be satisfied under all three voting rules. This allows us to compare aggregate effort provision under the three-member committee format with that under a single administrator format, for each of the decision-making rules. Specifically, we compare the relative magnitude of effort provision under each of the three voting rules by computing the following ratios:

$$\frac{T(3)}{T(1)} = \frac{6\alpha}{(1+\alpha)^2}, \quad \frac{\tilde{T}(3)}{\tilde{T}(1)} = \frac{3}{(1+\alpha)^2}, \quad \text{and} \quad \frac{\bar{T}(3)}{\bar{T}(1)} = \frac{3\alpha^2}{(1+\alpha)^2}.$$

¹⁴ Note that specifying values of r such that $r \leq 0.5$ guarantees the participation constraint described by (9) is satisfied for any given t ; $r \leq 1$ guarantees the participation constraint described by (11) will hold; and $r \leq 2/3$ guarantees the participation constraint described by (12) will hold.

First, as indicated above, given that $T(1) = \tilde{T}(1) = \bar{T}(1)$, our analysis technique effectively compares $T(3)$, $\tilde{T}(3)$, and $\bar{T}(3)$ by standardizing each by the same measure. Second, note that the ratio $\frac{\bar{T}(3)}{\bar{T}(1)} = \frac{3\alpha^2}{(1+\alpha)^2}$ is increasing in α , which implies that $\frac{\bar{T}(3)}{\bar{T}(1)} \leq \frac{3}{4}$, and thus $\bar{T}(3) < \bar{T}(1)$, for any value of α . This is understandable since when increasing the committee size from one to three, the decision rule of biased unanimity against the weak contestant places a handicap on the weaker contestant, making the competition even more asymmetrical compared to that under a single administrator. Similarly, since $\frac{T(3)}{T(1)}$ is increasing in α , and $\frac{\tilde{T}(3)}{\tilde{T}(1)}$ is decreasing in α , we obtain that $\frac{T(3)}{T(1)} \leq \frac{3}{2}$, and $\frac{3}{4} < \frac{\tilde{T}(3)}{\tilde{T}(1)} < 3$. Utilizing the result of $T(1) = \tilde{T}(1) = \bar{T}(1)$ under the single administration, the following two corollaries are presented.

Corollary 1: *Under the pure strategy equilibrium condition for the three-member committee setting, relative effort provision across voting rules is indicated by:*

(a) *Effort provision under the biased unanimity against the weak contestant voting rule is always no bigger than under either of the other two decision rules; i.e., $\bar{T}(3) \leq \tilde{T}(3)$, and $\bar{T}(3) < T(3)$, with the equality holding only when $t = \alpha = 1$.*

(b) *Effort provision under the simple majority compared to the biased unanimity against the strong contestant voting rules is dependent on contestant heterogeneity. If there is a sufficiently small degree of contestant heterogeneity, the simple majority rule generates higher effort provision; otherwise the opposite holds.*

Specifically, if $\frac{1}{2} < \alpha \leq 1$, $T(3) > \tilde{T}(3)$; if $0 < \alpha < \frac{1}{2}$, $T(3) < \tilde{T}(3)$;

if $\alpha = \frac{1}{2}$, $T(3) = \tilde{T}(3)$.

Corollary 2: Under pure strategy equilibrium conditions, relative effort provision between the three-member committee setting compared to the single administrator across voting rules is indicated by:

(a) Effort provision under the biased unanimity against the weak contestant rule is always lower under the three-person committee than single administrator; i.e., $\bar{T}(3) < \bar{T}(1)$ for any value of α .

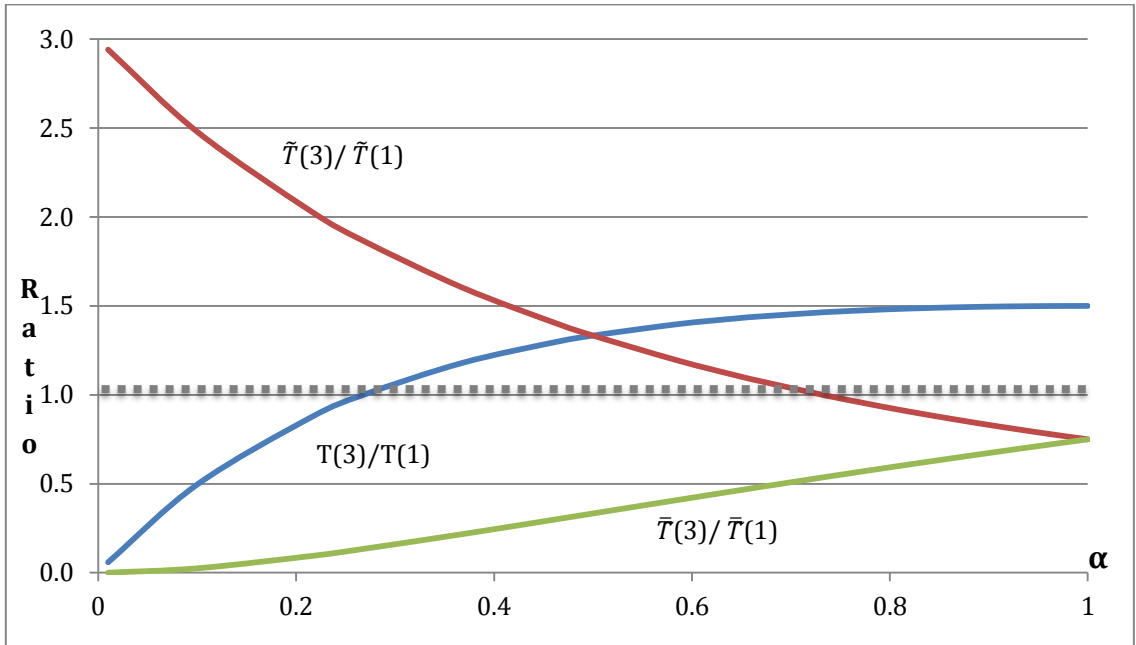
(b) Effort provision under the simple majority voting rule is dependent on contestant heterogeneity. If the degree of heterogeneity is sufficiently low, effort provision under a three-member committee is higher than under single administrator; otherwise the opposite holds.

Specifically, if $\alpha \in [2 - \sqrt{3}, 1]$, $T(3) \geq T(1)$; $T(3) < T(1)$ elsewhere, with the equality holding when $\alpha = 2 - \sqrt{3}$.

(c) Effort provision under the biased unanimity against the strong contestant rule is dependent on contestant heterogeneity. If the degree of heterogeneity is sufficiently high, effort provision under a three-member committee is higher than under single administrator; otherwise the opposite holds.

Specifically, if $\alpha \in (0, \sqrt{3} - 1)$, $\tilde{T}(3) > \tilde{T}(1)$; $\tilde{T}(3) \leq \tilde{T}(1)$ elsewhere, with the equality holding when $\alpha = \sqrt{3} - 1$.

Figure 2.3: Ratio of Effort Provision: 3-Member Committee and Single Administrator



Note: [1] The graph reflects sensitivity to effort provision of $r = 1/3$, selected to ensure the participation constraint holds for any voting rules at any t .
 [2] $T(1) = \tilde{T}(1) = \bar{T}(1)$

Figure 2.3 represents the graphical interpretation of Corollaries 1 and 2, allowing us to analyze the effect of the interplay between heterogeneity, different voting rules, and three-person committee versus single administration, on the provision of effort in two distinct ways. First, we can observe, for a given level of heterogeneity, which of the three rules provides the greatest and least contestant effort provision.¹⁵ For instance, for relatively high levels of heterogeneity, as indicated along the left-hand region of Figure 2.3, as well as by Corollary 1, the biased unanimity against strong contestant voting rule generates the highest level of effort provision; whereas under conditions of low heterogeneity, the simple majority rule elicits the greatest amount of effort provision in a three-member committee contest, as indicated on the right-hand side of the Figure 2.3. For any level of heterogeneity, the biased unanimity against

¹⁵ The difference between the ratios represented on the graph also simply represents the differences between the degrees of aggregate effort provision provided under the three-person committee setting for each of the voting rules.

weak contestant voting rule always generates the lowest level of effort provision. Second, it is possible to compare which setting—the single administrator or the three-member committee—generates higher effort provision *within* each rule. As indicated on Figure 2.3(1), the parts of the lines representing the value of the ratios above the ratio value 1 (shown by the dashed line) indicate at what levels of heterogeneity effort provision is greater under the three-member committee setting than under the single administrator. As is evident, for the biased unanimity against weak player voting rule, the single administrator setting will always generate higher effort provision, no matter what the heterogeneity level. For the biased unanimity against strong contestant voting rule, effort provision is greater in the committee setting except for relatively low levels of heterogeneity (high α), whereas for simple majority voting rule, effort provision is greater in the committee setting except for relatively high levels of heterogeneity (small α).

The intuition for these findings comes directly from the fact that changing the voting rule impacts the probability for each committee member to be the pivotal voter at equilibrium, which changes the effort provision incentive structure facing each contestant. Consider, for instance, the condition of high contestant heterogeneity, where each committee member will vote for Contestant 1 with probability near to one. Under the biased unanimity against strong player ($k=3$), $Q_i=q \cdot q$, the value of which is also close to one, meaning that for contestants, each committee member's vote is decisive, since it is more likely that the other two committee members both vote for Contestant 1, making each vote very important for each contestant. Conversely, under the simple majority voting rule, the fact that every committee member is likely to vote for Contestant 1 with a probability close to one, while Contestant 1 only needs two of the committee members' votes to win the contest, means that any particular committee member's vote is unlikely to be pivotal; both contestants, recognizing this situation, find the

incentive to fight for the final (and, by extension, any given) committee member's vote to be low. Therefore, the incentive for effort provision is much smaller under the simple majority rule compared to under biased unanimity against strong player, at high levels of heterogeneity. Conversely, as contestants become more similar, each committee member is more likely to vote to each candidate with same probability ($q \rightarrow \frac{1}{2}$), therefore increasing the probability that any given committee member will be pivotal under the simple majority rule, and hence inducing higher levels of effort provision than under the biased unanimity against the strong contestant. Finally, in regard to the biased unanimity against weak contestant ($k=1$) case, regardless of the contestant heterogeneity level, the probability for each committee member to be a pivotal voter is always smaller than under either the biased unanimity against strong contestant or simple majority voting rules, since the probability for the strong contestant to lose both of the votes of the other two committee members is low.¹⁶

Under any voting rule, note that as the number of committee members increase from one to three, the power of any given individual prize administrator's vote is less deterministic in deciding the outcome of the contest than it would be were that committee member acting as a single administrator. This implies a reduction in the incentive for contestants to exert efforts towards each individual prize administrator, the effect of which may not be compensated for by the increase in the number of committee members. The way these two counteracting factors interact with one other shapes the aggregate effort provision decision both through contestant heterogeneity and the voting rule determining how the contest winner is selected.

¹⁶ This can be observed directly by comparing the following ratios $\frac{Q(k=3)}{Q(k=1)} = \frac{q^2}{(1-q)^2} \geq 1$, and $\frac{Q(k=2)}{Q(k=1)} = \frac{2q(1-q)}{(1-q)^2} = \frac{2q}{1-q} > 1$.

2.4 Remarks

Comparing effort provision under the three-member committee with the single administrator has provided significant insights regarding the importance of the way in which the institutional environment impacts agents' decision-making. How the contest winner is selected (whether by a single administrator or a committee), and when by committee, the specific voting rules that determine the contest winner, are found to be crucial in determining the incentive structure guiding agents whose economic behaviors are constrained within the underlying institutional structure.

3 Generalized Model of Effort Provision under Committees with Contestant Heterogeneity

3.1 Contest Model with Committee Administration and Heterogeneous Contestants

I now describe a more generalized model of that analyzed in Section 2. Specifically, I allow for the committee size to be comprised of any odd-integer number of members. As in Section 2, contestants are involved in a contest defined by the Tullock CSF, and look to solve their maximization problems by focusing on the marginal return of effort provision towards committee members. The model set-up is as before except where specified. In particular, the contest prize will be awarded by a committee of $2m+1$ members, where m is any non-negative integer, and $m=0$ represents a single prize administrator. In this initial model formulation, in order to win the prize, a contestant must win the support of a simple majority ($m+1$ members) of the committee. Each contestant maximizes a more generalized expected utility function, such that Contestant 1 solves $\max_{x_i \geq 0} E(U_1) = PV_1 - \sum_{i=1}^{2m+1} x_i$, and Contestant 2 solves $\max_{y_i \geq 0} E(U_2) = (1 - P)V_2 - \sum_{i=1}^{2m+1} y_i$.

Contestant 1's bidding choice x_i is still reflected by the product of the probability for committee member i to be the pivotal prize administrator, times the marginal effect on the probability to win committee member i :

$$\frac{\partial P}{\partial x_i} = Q_i \cdot \frac{\partial q_i(x_i, y_i)}{\partial x_i},$$

where Q_i represents the probability that committee member i becomes the pivotal voter. Note that this now occurs when exactly m of the committee members (out of the remaining $2m$ committee members besides i) vote for Contestant 1.

The following discussion considers the pure-strategy equilibrium condition such that m satisfies $m \leq \bar{m}(r, t)$, which shall be defined below. As such, given any pure strategy profile (X, Y) , the probability for the committee member i to be the pivotal prize administrator must be same between the two contestants. The first order conditions for Contestants 1 and 2, respectively, are obtained as:

$$V_1 \cdot Q_i \cdot \frac{\partial q_i(x_i, y_i)}{\partial x_i} - 1 = 0 \quad (13)$$

$$V_2 \cdot Q_i \cdot \left(-\frac{\partial q_i(x_i, y_i)}{\partial y_i} \right) - 1 = 0 \quad (14)$$

When equation (13) is divided by equation (14), it is possible to identify the relationship of

$$-\frac{V_1}{V_2} \cdot \frac{q_x(x, y)}{q_y(x, y)} = 1. \text{ Since } q_x(x, y)/q_y(x, y) \text{ is equivalent to } -y/x, \text{ we find that } \frac{y_i}{x_i} = \frac{V_2}{V_1} \equiv t \leq$$

1; and thus $q_i = \frac{1}{1+t^r} \geq \frac{1}{2}$, regardless of the bidding profile directed towards other committee members. Since this reasoning can be applied to all committee members, each is won with the same probability, $q = 1/(1+t^r)$. Note that since $\frac{y_i}{x_i} = t$ and $q = 1/(1+t^r)$ are properties of the pure strategy equilibrium, Q_i can be represented as $C_{2m}^m q^m (1-q)^m$.¹⁷ Substituting these results back into the first order conditions attains unique solutions of equations (13) and (14); this leads to Proposition 3.

Proposition 3 *If the committee size is relatively small ($m \leq \bar{m}(r, t)$), there exists a unique pure strategy equilibrium, and in this pure strategy equilibrium, both contestants use uniform bidding strategies which satisfies $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$.*

¹⁷ In Proof Appendix A2, we show that in fact the PSNE bidding strategies of candidates must be uniform bidding strategies (offering same effort to each committee member), and that these equilibrium efforts satisfy the property of $y/x = t$.

The unique solution of equation (13) and (14) can be obtained as:

$$x(m) = V_1 \cdot C_{2m}^m \frac{r(t^r)^{m+1}}{(1+t^r)^{2m+2}}, \text{ and} \quad (15)$$

$$y(m) = V_2 \cdot C_{2m}^m \frac{r(t^r)^{m+1}}{(1+t^r)^{2m+2}}, \quad (16)$$

with total rent-seeking effort T defined as:

$$T(m) = (2m + 1)(1 + t) x(m). \quad (17)$$

Note that the above analysis is based on contestants playing the pure strategy equilibrium, the existence of which is dependent on contestants receiving non-negative payoffs under the pure strategy equilibrium (otherwise they could abstain playing and get zero payoffs). Since Contestant 1 always wins with probability more than one-half, we need only to guarantee the participation of the weak contestant. The participation constraint can be written as $r \leq \frac{1+\alpha}{(m+1)} \cdot [\sum_{i=0}^m \frac{C_{2m+1}^{m+1+i} \alpha^i}{C_{2m+1}^{m+1}}]$, where $\alpha \equiv t^r$.¹⁸ The smaller is t or the higher is r (the more effective heterogeneity exists between contestants), the more difficult it is for the participation constraint to hold; likewise, for a set of given values of r and t , as m grows, the participation constraint becomes more difficult to hold.¹⁹ This is because as committee size grows larger, it becomes more likely for the stronger contestant to win the contest, since each committee member always votes for the stronger contestant with a higher probability. When $m=0$, any $r \leq 1$ guarantees the existence of a pure strategy equilibrium; larger sizes of m , however, require smaller values of r for the participation constraint to hold. Hence $\bar{m}(r, t) \geq 0$ is defined as the largest value of m that satisfies this inequality. This ensures that the

¹⁸ Note that if $\alpha \rightarrow 1$, the RHS converges to $\frac{2^{2m+1}}{(m+1) \cdot C_{2m+1}^{m+1}}$, and if $\alpha \rightarrow 0$, the RHS converges to $\frac{1}{(m+1)}$.

¹⁹ A formal Proof is presented in Appendix A3.

participation constraint will always hold for values $m \leq \bar{m}$, and will fail to hold for values $m > \bar{m}$.²⁰

Analysis of Aggregate Effort Provision: Varying Committee Size

Equations (15), (16) and (17) are functions of m , enabling a consideration of the impact on effort expenditure as committee size varies. I first consider how committee size impacts the amount of effort contestants supply to *each* committee member. I compare the ratio of $x(m+1)/x(m)$ in the following corollary.

Corollary 3: *For a set of given values of t and r , the effort level supplied by each contestant to each individual committee member decreases in m .*

Proof: Define the function $f(\alpha) = \frac{\alpha}{(1+\alpha)^2}$. Then the ratio of the effort supplied by Contestant 1 when m increases by one is $\frac{x(m+1)}{x(m)} = \frac{C_{2m+2}^{m+1}}{C_{2m}^m} \cdot f(\alpha)$, which can be transformed²¹ into $\frac{x(m+1)}{x(m)} = f(\alpha) \cdot (4 - \frac{2}{m+1})$, which is increasing in m . Hence, $2f(\alpha) \leq \frac{x(m+1)}{x(m)} < 4f(\alpha)$. Since $f(\alpha)$ is increasing in α , $f(\alpha) \leq 1/4$. Therefore, under pure strategy equilibrium, $\frac{x(m+1)}{x(m)} < 1$, which implies $\frac{\partial x(m)}{\partial m} < 0$. Q.E.D.

²⁰ Table A1 in the Appendix presents numerical results describing the existence of pure strategy equilibrium conditions for various combinations of r and t .; these results indicate that $\frac{\partial \bar{m}(r,t)}{\partial t} \geq 0$, and $\frac{\partial \bar{m}(r,t)}{\partial r} \leq 0$.

²¹ Note since $C_{2m+2}^{m+1} = C_{2m+1}^m + C_{2m+1}^{m+1}$ and $C_{2m+1}^m = C_{2m+1}^{m+1}$, $C_{2m+2}^{m+1} = 2C_{2m+1}^m$; $C_{2m+1}^m = C_{2m}^{m-1} + C_{2m}^m$, and therefore $C_{2m+2}^{m+1} = 2C_{2m+1}^m = 2(C_{2m}^{m-1} + C_{2m}^m)$.

Corollary 3 implies that as m increases, the marginal benefit of additional effort directed toward a committee member is decreasing, since the probability for that committee member to be pivotal is decreasing; thus contestants have the incentive to reduce their effort provision.

While we have shown that as m increases effort-per-committee member decreases, note that it is possible for aggregate effort expenditure to increase, as was the case in the three-member committee scenario. As committee size becomes larger, contestants exert effort toward a greater number of committee members (albeit at lesser amounts to each committee member than when the committee size is smaller). Thus there are two counter-acting forces taking place: a lowering of effort being directed towards each individual committee member, yet a greater number of committee members towards which effort is being exerted. Which of these two counteracting effects is stronger depends on the level of heterogeneity as defined by t .

To understand which of these effects dominates, as above, we look to compare the ratio of total effort supplied at a given size of m , and a unit increase in m . Specifically, we have $\frac{T(m+1)}{T(m)} = \frac{2m+3}{2m+1} \cdot \frac{x(m+1)}{x(m)} = (4 + \frac{2}{m+1}) \cdot f(\alpha)$. This relationship is decreasing in m , so $T(m+1)/T(m)$ is bounded by $(4f(\alpha), 6f(\alpha)]$. It is clear then that the actual degree to which total efforts change depends on the value of t^r (or, α). Note that this was observed in Section 2; since $\frac{T(1)}{T(0)} = 6f(\alpha)$, it is clear that moving from single administration to the three-member committee generates either [i] larger aggregate rent-seeking efforts (for $f(\alpha) > 1/6$), [ii] the same aggregate efforts (for $f(\alpha) = 1/6$), or [iii] smaller efforts (for $f(\alpha) < 1/6$). In a similar manner, it is possible to analyze, for example, $\frac{T(2)}{T(1)} = 5f(\alpha)$. In other words, whether aggregate effort provision under a committee size $2m+3$ is larger than or smaller than the efforts under a committee size $2m+1$ depends on the value of α .

This finding deserves discussion as it adds to the understanding within the literature. When we allow for the more general case of heterogeneous contestants, we find that the aggregate efforts are not necessarily increasing when we increase m , instead depending on the value of both t and r , as they interact in terms of t^r . The specific nature of the relationship depends on the interaction between the two effects, which is discussed in the following Corollary.

Corollary 4: Given the existence of pure strategy equilibrium ($m \leq \bar{m}(r, t)$), we have the following properties:

- (a) *When contestants are identical (when $t=1$, or $\alpha \equiv t^r = 1$), the aggregate effort is increasing in committee size m , at a decreasing rate.*
- (b) *Under conditions of sufficiently high heterogeneity between contestants (i.e., when $0 < \alpha < 2 - \sqrt{3}$), aggregate effort is decreasing in committee size m , at an increasing rate.*
- (c) *Under conditions of weak or moderate heterogeneity between contestants (i.e., when $2 - \sqrt{3} \leq \alpha < 1$), aggregate effort is initially increasing (or non-decreasing) in committee size m , until some critical value $\tilde{m}(r, t)$, at which it reaches its maximum; thereafter, it is decreasing in committee size m , provided $\tilde{m}(r, t) < \bar{m}(r, t)$, else aggregate efforts are increasing in m .*

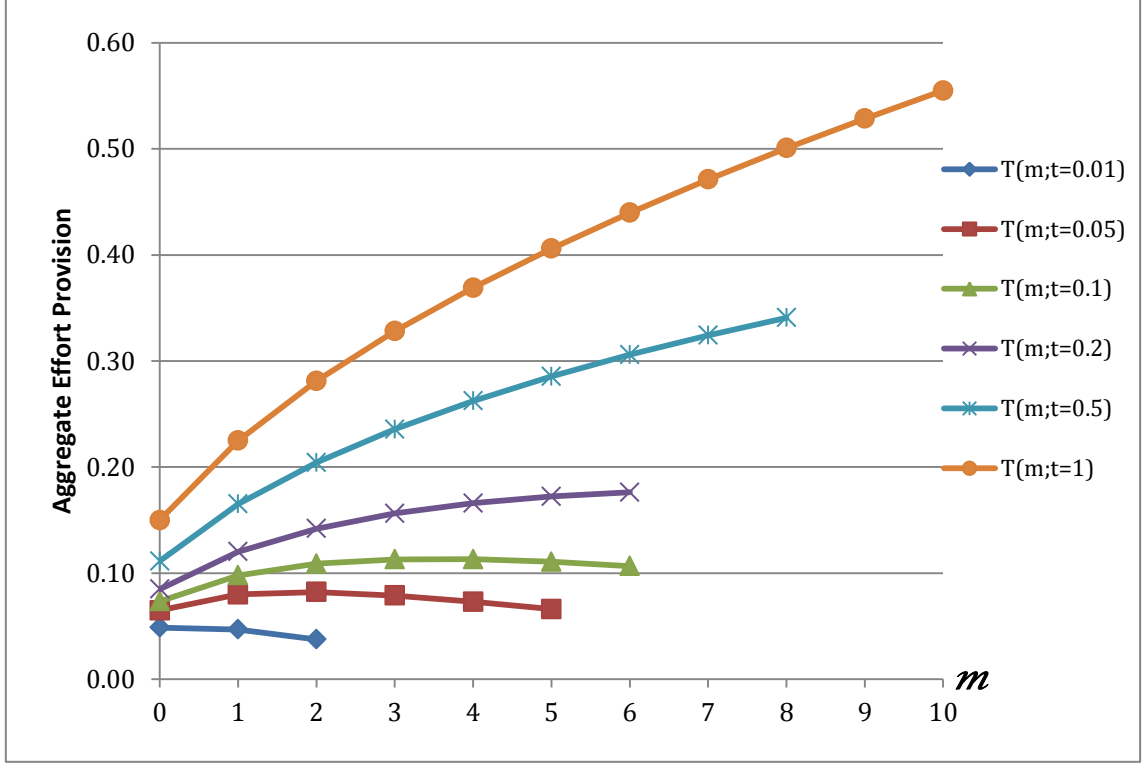
Proof:

- (a) When $t=1$, $f(\alpha) = \frac{1}{4}$, hence $\frac{T(m+1)}{T(m)} = 1 + \frac{1}{2m+2} > 1$;
- (b) When $0 < \alpha < 2 - \sqrt{3}$, $0 < f(\alpha) < \frac{1}{6}$, therefore $\frac{T(m+1)}{T(m)} \leq 6f(\alpha) < 1$;
- (c) When $2 - \sqrt{3} \leq \alpha < 1$, $\frac{1}{6} \leq f(\alpha) < \frac{1}{4}$, we have $\frac{T(1)}{T(0)} = 6f(\alpha) \geq 1$, since $\frac{T(m+1)}{T(m)}$ is decreasing in m , there exists a critical value $\tilde{m}(r, t)$ such that $\frac{T(m+1)}{T(m)} \geq 1$ for all

$m \leq \tilde{m}(r, t)$, and $\frac{T(m+1)}{T(m)} < 1$ for all $m > \tilde{m}(r, t)$. Suppose $\tilde{m}$ is the greatest value that satisfies the following inequality $\frac{T(m+1)}{T(m)} = (4 + \frac{2}{m+1}) \cdot f(\alpha) \geq 1$. Since $\frac{T(m+1)}{T(m)}$ is increasing in α , when we increase the value of α (which is equivalent either to an increase in t , or a decrease in r), the left hand side of the inequality becomes easier to hold; therefore, $\tilde{m}$ is non-decreasing in t , and non-increasing in r . The greater the heterogeneity between the two contestants is (the smaller t is, and hence the smaller is $\tilde{m}$), the faster the dominance will happen.

The way in which PSNE aggregate effort levels change with committee size, for given degrees of heterogeneity, is displayed in Figure 3.1. As can be seen, when contestants are very heterogeneous (i.e., when t is sufficiently low), aggregate efforts are strictly decreasing in m . This is evident from the (blue diamond) line where $t=0.01$. Conversely, when contestants are homogenous ($t=1$), effort levels are strictly increasing in m ; this can be observed from the (orange dotted) line representing $t=1$. Note, however, that for certain moderate values of heterogeneity, aggregate efforts are first increasing and then decreasing in m . This can be seen in the behavior of the lines representing $t=0.05$ (red squares) and $t=0.1$ (green triangles). Note that the dominance happens at a smaller committee size ($m=2$) for the case with higher heterogeneity ($t=0.05$), compared to the case with a lower level of heterogeneity ($t=0.1$), where dominance does not occur until $m=3$. Finally, the lines indicating when heterogeneity is at $t=0.2$ and $t=0.5$ are examples where the critical value of dominance $\tilde{m}$ is larger than the limitation of committee size $\bar{m}$: effort expenditures are increasing in m up through the limitation value of committee size meeting the requirement for the existence of pure strategy equilibrium.

Figure 3.1: Effect of Committee Size on Aggregate Effort Provision



Notes: i) Only results satisfying the existence condition of pure strategy equilibrium are displayed.
ii) Calculated under assumptions $r=0.3$ and $V_1=1$.

Mixed strategies

The above analysis presents an examination of the pure strategy equilibrium. While the existence of such is assured when the committee size is relatively small; when the committee size m is greater than the critical value $\bar{m}(r, t)$, a pure strategy does not exist in equilibrium. To understand the intuition behind this, consider the case when the committee size approaches infinity. In this instance, the game where m approaches infinity is equivalent to the deterministic all-pay auction, since the law of large numbers indicates that the winner will

deterministically be the one offering higher bids to each of the committee members.²² If the two contestants were to utilize the pure strategy described above, then the strong contestant will be the winner with certainty, since each committee member always votes for the strong contestant with higher probability. However, such a condition cannot be a Nash equilibrium, since now the weak contestant—knowing he or she will lose the game with certainty—is made better off by abstaining from the game, implying also that the strong contestant can also be made better off by reducing his or her efforts to only exert ε (a value marginally above the zero effort exerted by the weak contestant). The weak contestant, knowing this, would therefore be better off to exert a very small amount γ above the ε exerted by the strong contestant. As is obvious, such a scenario is not stable, since each contestant can always be made better off by slightly adjusting his or her effort exertion. We can prove the existence of a mixed uniform bidding strategy equilibrium for committee size $m > \bar{m}(r, 1)$ ²³ by way of the following Proposition:

***Proposition 4:** If $m > \bar{m}(r, 1)$, there exists a mixed uniform bidding strategy $\vartheta^* = (\vartheta_1^*, \vartheta_2^*)$ such that in equilibrium, the expected payoff of Contestant 1 is $E(U_1) = V_1 - V_2$, with expected efforts $E(\vartheta_1^*) = V_2/2$, while the expected payoff of Contestant 2 is $E(U_2) = 0$, with expected efforts $E(\vartheta_2^*) = (V_2)^2/(2V_1)$.*

Proof: Define a symmetric game C^S , where both Contestants 1 and 2 value the prize equally, $\tilde{V}_1 = \tilde{V}_2 = \tilde{V}$. For comparability, set $\tilde{V} \equiv V_2$, Contestant 2's value of the prize under asymmetry. As per Proposition 2 in Klumpp & Polborn (2006: 1083), when $m > \bar{m}(r, 1)$,

²² Note that while Klumpp & Polborn (2006) also discuss an all-pay auction mixed strategy equilibrium for heterogeneous players under infinite committee size in their extensions, here I am able to show stronger results by proving the existence of a mixed strategy equilibrium even when the committee size is at some finite number.

²³ As per Klumpp & Polborn (2006), $\bar{m}(r, 1)$ represents the critical value where both contestants have identical valuations of the prize. Note that $\bar{m}(r, t) \leq \bar{m}(r, 1)$.

there exists a symmetric, uniform mixed strategy equilibrium in the symmetric game when both contestants have the same valuations. In this symmetric mixed strategy equilibrium, the rent is fully dissipated and both contestants get expected pay-offs of zero. Next define $\mu_1^* = \mu_2^* = \mu^*$ as the symmetric uniform mixed strategy equilibrium. Given that each contestant wins with probability of $\frac{1}{2}$, expected effort provisions are $E(\mu_1^*) = E(\mu_2^*) = V_2/2$. If we define an asymmetric game, C^A , where $V_1 \geq V_2$, then as per Lemma B.2 in Alcade & Dahm (2010: 6)²⁴, the mixed strategy profile $\vartheta^* = (\vartheta_1^*, \vartheta_2^*)$ will be a Nash equilibrium of C^A . This profile is defined such that Contestant 1 plays strategy $\vartheta_1^* = \mu_1^*$, where the domain of ϑ_1^* and μ_1^* are the same; and Contestant 2's strategy ϑ_2^* is to abstain (bid 0) with probability $(1 - V_2/V_1)$, and bid μ_2^* with probability V_2/V_1 .

We can show that neither contestant wants to deviate from $\vartheta^* = (\vartheta_1^*, \vartheta_2^*)$ by noting that when Contestant 1 plays mixed-strategy ϑ_1^* , the best response of Contestant 2 is ϑ_2^* . Since $\tilde{V} = V_2$, the value of the prize is the same for Contestant 2 in either C^S or C^A . This implies that any pure strategy employed by Contestant 2 in C^A yields an expected payoff of 0. Therefore Contestant 2 does not gain by choosing other strategies, and will be content to play ϑ_2^* . Facing ϑ_2^* , Contestant 1 must maximize his or her expected payoff of:

$$E^A(U_1) = V_1(1 - V_2/V_1) + V_1(V_2/V_1) \cdot E(P) - E(\vartheta_1) = (V_1 - V_2) + V_2 \cdot E(P) - E(\vartheta_1).$$

Given that Contestant 1's maximization problem of $E^A(U_1)$ is equivalent to the maximization problem in C^S of maximizing $V_2 \cdot E(P) - E(\vartheta_1)$, Contestant 1's best response is to play ϑ_1^* . Since $\vartheta_1^* = \mu_1^*$ is also the best response to ϑ_2^* , therefore it is not optimal either contestant to deviate from $\vartheta^* = (\vartheta_1^*, \vartheta_2^*)$. It is trivial to prove that $E(\vartheta_1^*) = E(\mu_1^*) = V_2/2$, and $E(\vartheta_2^*) = (V_2/V_1) \cdot E(\mu_1^*) = (V_2)^2/(2V_1)$. Q.E.D.

²⁴ Alcade & Dahm (2010) prove the existence of a mixed-strategy equilibrium in the Tullock contest game under a single administrator when $r \geq 2$.

The equilibrium result in mixed strategy mimics the results previously found to hold in all-pay auction games, that being where the weak contestant exerts efforts equal to an expected payoff fully dissipating the value of the prize, while the strong contestant gains strictly positive payoffs (Siegel 2009). Note that under mixed strategies, increasing committee size—after it has reached a sufficiently large number—does not impact aggregate effort provision. The aggregate effort can be expressed as $T = E(\vartheta_1^*) + E(\vartheta_2^*) = t(V_1 + V_2)/2$.

Remarks

As demonstrated, anticipating whether aggregate efforts will be larger or smaller as the size of the prize-awarding committee grows depends both upon the level of effort sensitivity of the committee members, and upon the degree of heterogeneity between the contestants, with heterogeneity exacerbating the degree to which aggregate effort is attenuated by increasing committee size. However, when committee size is sufficiently large, a pure strategy equilibrium does not exist, and instead a mixed uniform bidding strategy is obtained which is similar to the equilibrium result found in the all-pay auction framework. This has serious implications regarding the nature of the interaction between committee size and the provision of aggregate efforts, and it attenuates to some degree the findings of the pure strategy equilibrium analysis.

3.2 Contest Model with Committee Administration and Heterogeneous Contestants, utilizing Different Voting Rules

In this analysis, I perform a similar set of exercises to the previous model under committee voting procedures that modify the simple majority rule by requiring the winning contestant to secure a supermajority to win the contest. Placing the requirement on a particular contestant (the stronger) to obtain a supermajority in order to win the prize effectively biases the contest against that contestant. Formulating the voting rules as such thusly enables the impact that different supermajority voting rules have on effort provision to be observed by analyzing the influence of the favorable or disadvantageous bias in terms of its effect on contestant's winning probabilities and hence incentives to exert effort.

The extended model specification follows that of the benchmark model except where specified. The contest winner is decided by a committee of $(2m+1)$ members (since majority and super-majority voting rules only have meaning when the committee contains more than a single member, in this section we assume a committee size of $m \geq 1$), each committee member votes for one of the two contestants with the probability described by the Tullock CSF, as before. Contestant 1 is now required to win at least $k = m + d$ votes out of $(2m+1)$ committee members in order to win; otherwise Contestant 2 is selected as the winner ($V_1 \geq V_2$). Importantly, this voting rule formulation means the contest must have one and only unique winner awarded the prize. In order to obtain interesting results, we assume that d takes values from the following set:

$$d \in \{1, 2, \dots, m + 1\}.$$
²⁵

²⁵ Note that if we switch the valuations of the prize to make the Contestant 1 the weak player, the voting rules defined in this context are equivalent to rules where $k=m+d$ takes values from the low support set $\{1, 2, \dots, m + 1\}$, under the condition that $V_1 \geq V_2$. In this scenario, the weak contestant is handicapped by the voting rules, and the result is comparable to the case of the biased unanimity rule against the weak contestant under 3-member committee setting, as described in Section 2.3.

Hence, $k=m+1$ ($d=1$) represents the simple majority rule previously considered in 3.1; whereas $k \geq m+2$ ($d \geq 2$) means Contestant 1 must secure a supermajority in order to win the contest; $k=2m+1$ ($d=m+1$) implies a unanimity voting rule required for Contestant 1, and represents the maximum boundary value on d . In general, the higher is k , the more biased is the contest against Contestant 1.

The solution strategy in regard to the extended model proceeds as follows. First, utilizing the same logic of pure-strategy equilibrium analysis employed in 3.1 above, it is easy to prove that efforts directed towards each individual committee members still satisfy $\frac{y_i}{x_i} = \frac{V_2}{V_1} \equiv t \leq 1$ and $q_i = \frac{1}{1+tr} \equiv q$ at the equilibrium condition. While the structure of the first order conditions remains the same, the value of Q_i , the probability that Committee Member i is the pivotal voter, is affected differently depending on which voting rule specification is analyzed. Hence, to find the probability that a given committee member is the pivotal voter in general, we define $Q_i(m, d)$ as the probability that Committee Member i is the pivotal voter when the voting rule is $k=m+d$ and committee size $2m+1$. Then, at the equilibrium condition, we obtain:

$$Q_i(m, d) = C_{2m}^{m+d-1} q^{m+d-1} (1-q)^{m+1-d}$$

From this definition, a unique solution can be obtained from the first order conditions. Since for each committee member we can obtain same pair of first order conditions (one for each contestant), the same unique solution can be derived for each committee member, and hence for the game.

Proposition 5: Define a contest game $C(m, d)$ as a game where the committee size is $2m+1$, and the voting rule is $k = m + d$. Then, given the existence of the unique pure strategy equilibrium (discussed further below) of $C(m, d)$, both contestants use uniform bidding

strategies which satisfy $\frac{y}{x} = \frac{V_2}{V_1} \equiv t \leq 1$. Specifically, the equilibrium effort directed towards each individual committee members is defined by

$$x(m, d) = V_1 Q(m, d) \frac{r\alpha}{(1 + \alpha)^2} = V_1 C_{2m}^{m+d-1} \frac{r\alpha^{m+2-d}}{(1 + \alpha)^{2m+2}}$$

The aggregate effort provision can be written as

$$T(m, d) = (2m + 1)(1 + t)x(m, d)$$

A necessary condition for the existence of such pure strategy Nash equilibrium (PSNE) is that in equilibrium, both contestants will secure non-negative payoffs. Under the analysis contained within 3.1, we found that under the simple majority rule (which is analogous to $d = 1$) and contestant heterogeneity, since the weaker contestant always faced a lower probability to win the game, it would always be more difficult for the weaker contestant to meet the participation constraint. This shaped our strategy when calculating PSNE conditions for the simple majority game: finding the condition that satisfied the participation constraint for the weaker contestant implied that the participation constraint for the stronger contestant was necessarily simultaneously met.

However, under the broader array of voting rule formulations that correspond to when $d \geq 2$, the analysis to find the PSNE conditions becomes slightly more complex. The reason for this is that the bias against Contestant 1 (the stronger contestant) reduces her probability to win the game; in some instances, the bias can be so severe that Contestant 1's probability to win the contest falls below $\frac{1}{2}$. We define such situations as Contestant 1 being "over-handicapped". When Contestant 1 is "over-handicapped," the stronger contestant actually has a lower probability to win the contest than does the weaker contestant, and hence is the contestant for whom it is easier to obtain negative payoffs. Since determining the participation

constraint requires ensuring that the contestant with the lower probability to win the contest would receive non-negative payoffs, in instances that Contestant 1 is "over-handicapped", we must ensure that the stronger contestant's participation constraint is satisfied. Hence, our strategy to finding the participation constraints consists of two steps. First, we determine which of the two contestants has a lower probability to win the game. Second, we calculate the requirements to satisfy the participation of the contestant with the lower probability to win the game. (Note in either case, we only need to guarantee the participation of the contestant with the lower probability to win the game, since doing so will also necessarily ensure that the participation constraint of the contestant with a higher probability is simultaneously satisfied). We define our participation constraint calculation strategy as follows.

If α is relatively small compared to d (meaning the effective heterogeneity between contestants is relatively high) such that at the equilibrium condition the Contestant 1 still wins the overall contest with probability $P \geq \frac{1}{2}$, then the voting rule d does not "over-handicap" Contestant 1, and she maintains her advantage in winning the game vis-à-vis the weaker player Contestant 2. Note however that the asymmetry of the game represented by the level of contestant heterogeneity is reduced due to the bias against Contestant 1; that is, the bias against the stronger contestant helps to create a more even contest compared to the same level of heterogeneity when the game is played under simple majority rule. Note that for a given α and m , if d does not "over-handicap" Contestant 1, neither does any voting rule that is less biased against Contestant 1, such as $(d - 1)$, or $(d - 2)$, etc.

Similarly, if α is relatively large compared to d such that at the equilibrium condition Contestant 1 wins the overall contest with probability $P < \frac{1}{2}$, then the voting rule d "over-handicaps" Contestant 1, meaning it is easier for Contestant 1 to receive negative payoffs at equilibrium compared to Contestant 2. For a given α and m , if voting rule d "over-handicaps"

Contestant 1, so does any voting rule imposing greater bias against Contestant 1, such as $(d + 1), (d + 2)$, etc.

Hence, the necessary condition for the existence of pure strategy equilibrium of the contest game $C(m, d)$ is determined by the following conditions

$$\text{If } \alpha \leq \tilde{\alpha}(m, d) \quad r \leq \frac{(1+\alpha)(\sum_{i=0}^{m+d-1} C_{2m+1}^i \alpha^{2m+1-i})}{(2m+1)C_{2m}^{m+d-1} \alpha^{m+2-d}}; \quad (18)$$

$$\text{if } \alpha > \tilde{\alpha}(m, d) \quad r \leq \frac{(1+\alpha)(\sum_{i=0}^{m+1-d} C_{2m+1}^i \alpha^i)}{(2m+1)C_{2m}^{m+d-1} \alpha^{m+2-d}}. \quad (19)$$

Here, $\tilde{\alpha}(m, d)$ represents the value of effective heterogeneity at which the right hand side of equation (18) (RHS²) exactly equals the right hand side of equation (19) (RHS¹). When $\alpha = \tilde{\alpha}(m, d)$, the voting rule d can completely eliminate the difference in winning probabilities that is created by the level of contestant heterogeneity; in other words, $\tilde{\alpha}(m, d)$ represents the level of heterogeneity under voting rule $k(m, d)$ that exactly offsets the bias imposed by the voting rule wherein which each contestant wins the contest with probability $\frac{1}{2}$ at equilibrium. Notice that RHS² is increasing in α and RHS¹ is decreasing in α ,²⁶ and that RHS¹ approaches infinity when $\alpha \rightarrow 0$, while RHS² converges to $\frac{1}{m+2-d}$ when $\alpha \rightarrow 0$. Since both RHS¹ and RHS² are continuous in α and $\text{RHS}^1(1, d) \leq \text{RHS}^2(1, d)$, we can prove that there exists $\tilde{\alpha}(m, d) \in (0, 1]$ such that $\text{RHS}^1(\tilde{\alpha}, d) = \text{RHS}^2(\tilde{\alpha}, d)$.²⁷ Note that $\tilde{\alpha}(m, d)$ is decreasing in voting rule d .²⁸ Given different levels of α , different PSNE existence conditions can be obtained. However, it is possible to show that for any contest game $C(m, d)$, when r is

²⁶ To see this explicitly, we can rewrite RHS¹ and RHS² as:

$$\text{RHS}^1(\alpha, d) = \frac{1}{(2m+1)C_{2m}^{m+d-1}} [C_{2m+1}^{m+1-d} \left(\frac{1}{\alpha} + 1\right) + C_{2m+1}^{m-d} \left(\frac{1}{\alpha^2} + \frac{1}{\alpha}\right) + \dots + C_{2m+1}^0 \left(\frac{1}{\alpha^{m+2-d}} + \frac{1}{\alpha^{m+1-d}}\right),$$

$$\text{and, } \text{RHS}^2(\alpha, d) = \frac{1+\alpha}{(2m+1)C_{2m}^{m+d-1}} [C_{2m+1}^0 \alpha^{m-1+d} + C_{2m+1}^1 \alpha^{m-2+d} + \dots + C_{2m+1}^{m+d-1} \alpha^0].$$

It is trivial to prove that RHS¹ decreases in α , and RHS² increases in α .

²⁷ Note that when $d \geq 2$, $\text{RHS}^2(1, d) - \text{RHS}^1(1, d) = \frac{2}{(2m+1)C_{2m}^{m+d-1}} (C_{2m+1}^{m+2-d} + C_{2m+1}^{m+3-d} + \dots + C_{2m+1}^{m+d-1}) > 0$; and when $d = 1$, $\text{RHS}^2(1, d) = \text{RHS}^1(1, d)$. Therefore $\text{RHS}^1(1, d) \leq \text{RHS}^2(1, d)$

²⁸ A formal proof can be found in Appendix A4(3).

sufficiently small (specifically, $r \leq \min\{\frac{1}{m+2-d}, \text{RHS}^1(1)\}$), the PSNE existence condition will always hold for any heterogeneity level t .

However, it proves to be more difficult for us to obtain a consistent condition for the existence of PSNE when considering varies in d , meaning that, given contestant heterogeneity α , it is unclear whether increasing d will make the PSNE condition more or less difficult to hold. As discussed above, since the probability the stronger contestant has to win the contest is simultaneously determined by multiple mutable parameters, it is not known *a priori* whether the stronger contestant will be "over-handicapped" so as to face a winning probability less than $\frac{1}{2}$ as the size of the bias increases; all parameters t , r and m , as well as d , must be fixed before such a determination may be made, since the outcome is different depending on the specific value of d , and the way in which interacts with the other parameters to impact the contestants' winning probabilities.

As implied by the above discussion, the interaction between parameter values affecting the participation constraint of contest $C(m, d)$ is complex; a more detailed discussion is therefore provided in Appendix A4, where numerical examples and graphs are included to help illuminate the dynamics at play. In order to focus on factors that affect aggregate effort levels under pure strategy equilibrium, the subsequent analysis presented in the text of the paper will assume r to be sufficiently small so as to satisfy participation constraints.

Analysis of Aggregate Effort Provision: Different Voting Rules

In this proceeding, I fix the committee size to focus attention on comparing the way different voting rules impact aggregate effort provision. Focus is limited to the study of effort provision under PSNE (as previously noted, r is assumed to be sufficiently small such that the PSNE condition holds). We further assume $m \geq 1$ so that the rule of $d + 1 (\geq 2)$ has meaning,

which allows us to compare relative effort provision under different voting rules with the following ratio

$$\frac{x(m,d+1)}{x(m,d)} = \frac{C_{2m}^{m+d}}{C_{2m}^{m+d-1}} \cdot \frac{1}{\alpha} = \left(\frac{2m+1}{m+d} - 1\right) \frac{1}{\alpha},$$

which describes the relative level of effort directed by Contestant 1 towards each committee member (effort-per-committee member) as d increases by 1. Since $\frac{x(m,d+1)}{x(m,d)}$ is decreasing in d , if we stipulate $1 \leq d \leq m$ (to ensure the voting rule $d + 1$ has meaning), then the value $\frac{x(m,d+1)}{x(m,d)}$ is bounded by $[\frac{1}{2m\alpha}, \frac{m}{m+1}\frac{1}{\alpha}]$. Therefore, the key factor determining whether the ratio is greater or less than 1 is the heterogeneity level α .²⁹ Specifically, if the two contestants are homogeneous ($\alpha=1$), then any increase in bias will reduce efforts-per-committee member. If $\alpha < 1$, then the relative degree of heterogeneity α compared to the "bias level" of the chosen voting rule k determines whether a voting rule that increases the level of bias against the stronger contestant generates higher or lower efforts. We define $\bar{\alpha}$ as that specific value of heterogeneity α that equalizes two different voting rules in terms of their impact on effort provision. In other words, if the heterogeneity level of the two contestants is $\bar{\alpha} = \frac{2m+1}{m+d} - 1$, then voting rule d and $d+1$ extract same level of efforts, i.e. $\frac{x(m,d+1)}{x(m,d)} = 1$. Likewise, if $\alpha < \bar{\alpha}$, then a voting rule that increases the level of bias ($k = m + d + 1$) generates higher efforts; whereas if $\alpha > \bar{\alpha}$, then a voting rule that decreases the level of bias ($k = m + d$) generates higher efforts.

²⁹ Given that r is set to be a sufficiently (fixed) small number, α can be taken to represent the heterogeneity level between contestants.

Given committee size m , the optimal voting rule k^* that can extract maximum effort provision is decided by the effective heterogeneity level (α) between contestants. Specifically, we have the following:

If $\alpha > \frac{m}{m+1}$, then $\frac{x(m,d+1)}{x(m,d)} < 1$ for any $d \in [1, m]$, and aggregate effort is maximized

when the voting rule is the simple majority, where $k^* = m+1$.

If $\alpha < \frac{1}{2m}$, then $\frac{x(m,d+1)}{x(m,d)} > 1$ for any $d \in [1, m]$, and effort provision is maximized

when the voting rule is "biased unanimity" against the strong contestant where $k^* = 2m + 1$.

If $\frac{1}{2m} \leq \alpha \leq \frac{m}{m+1}$, then there exists a critical value of d , represented by $\bar{d}(\alpha)$ such that

for any $d > \bar{d}(\alpha)$, $\frac{x(m,d+1)}{x(m,d)} < 1$, and for any $d \leq \bar{d}(\alpha)$, $\frac{x(m,d+1)}{x(m,d)} \geq 1$; then efforts are

maximized when the voting rule is $k^* = m + \bar{d}(\alpha) + 1$ (and/or $k^* = m + \bar{d}(\alpha)$ if and only if $x(m, \bar{d}(\alpha)) = x(m, \bar{d}(\alpha) + 1)$) provided $\bar{d}(\alpha) \leq 2m$; otherwise, the voting rule maximizing

efforts is given by $k^* = 2m + 1$. Note that the critical value $\bar{d}(\alpha)$ is non-increasing in α . The more homogenous are the two contestants, the more likely that k^* approaches to simple majority rule $m+1$.

Figure 3.2: The effect of voting rules on aggregate effort provision

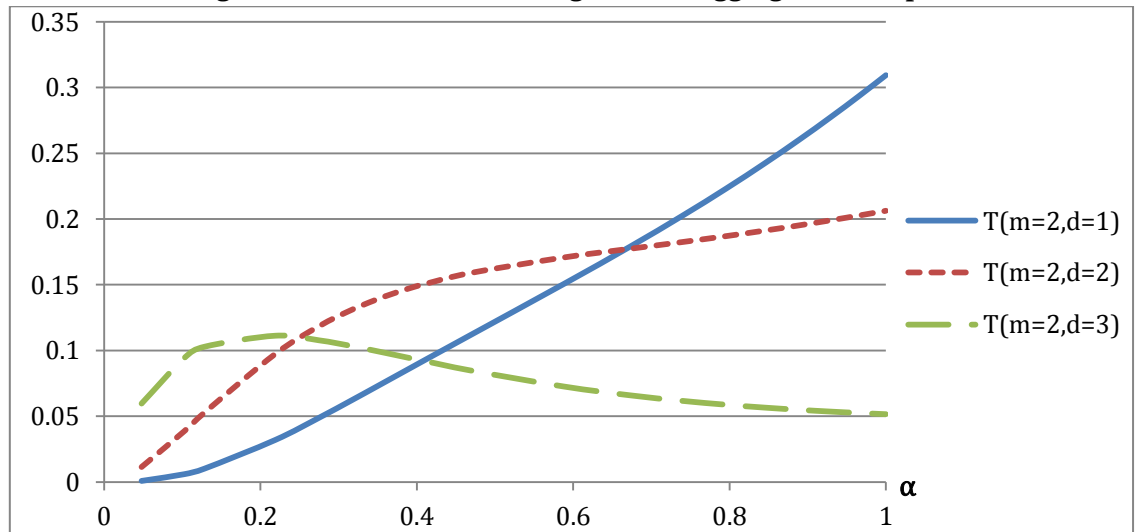


Figure 3.2 shows the aggregate effort level under different voting rules given a 5-member committee; $r=0.33$ is chosen to ensure the existence of PSNE. As shown, the optimal rule to generate maximum effort *changes* with heterogeneity level. When the heterogeneity level is relatively large (α is small), the biased unanimity ($d=3$) can extract the largest level of effort provision, compared to other voting rules. When the heterogeneity level is intermediate, the rule generating the largest aggregate effort is $d=2$. Finally, as heterogeneity becomes relatively small, then a simple majority voting rule ($d=1$) can extract the largest efforts.

Assuming that the objective of a committee is to extract maximum effort from the contestants, they can choose to select the optimal voting rule(s) corresponding to the specific contestant heterogeneity level. Indeed, in general, whether a voting rule leads to higher or lower effort provision under a given committee size is characterized by the level of contestant heterogeneity. Note that for a given value of heterogeneity level α , once the committee size changes, the relationship between different voting rules can also change. For example, for a given effective heterogeneity level $\alpha = 0.55$ ($t=0.3, r=0.5$), when the committee size is three, the simple majority rule $k=2$ will extract maximum effort provision; however, when the committee size increases to 5, then a super-majority voting rule corresponding to $k=4$ (instead of simple majority rule $k=3$) extracts the highest level of effort provision.³⁰

³⁰ In this example the participation constraint holds such that the contestants prefer to adopt the pure strategy equilibrium efforts.

3.3 Effort Provision under Committee Sizes Approaching Infinity: Mixed Strategies Solutions

Here I consider equilibrium strategies when the committee size approaches towards infinity. Notice that if the two contestants utilize uniform pure strategies and the voting rule is simple majority rule, then whoever bids more to committee members wins the contest. In this case, an equilibrium in pure strategies does not exist (further, any non-uniform pure strategy also cannot be the equilibrium condition, as has been proved in Appendix A2). Since the committee size is infinite, define the voting rule to be biased towards the weaker Contestant 2 if the stronger Contestant 1 has to win a supermajority proportion $\rho \in (\frac{1}{2}, 1)$ of the committee members in order to win the contest.³¹ Assume initially that contestants utilize uniform bidding strategies (I show a uniform bidding strategy equilibrium in mixed form exists subsequently). Then, aggregate effort provision by Contestant 1 and Contestant 2 can be expressed as $X = (2m + 1)x$, and $Y = (2m + 1)y$, respectively, where $m \rightarrow +\infty$. If Contestant 1 can win each vote with probability no smaller than ρ , then she would win at least a proportion of ρ support of the committee. Therefore, Contestant 1 will be selected as the winner if $\frac{x^r}{x^r + y^r} \geq \rho$, which is equivalent to $x \geq (\frac{1-\rho}{\rho})^{-1/r} y$, or $X \geq (\frac{1-\rho}{\rho})^{-1/r} Y$.

Further assume that at the equilibrium condition, contestants choose their total effort bids X and Y randomly with distribution functions $G_1(X)$ and $G_2(Y)$ on support $[0, \bar{b}_x]$ and $[0, \bar{b}_y]$. Then the density functions are $g_1(X)$ and $g_2(Y)$, respectively. Following arguments described in Baye *et al* (1996) in regard to all-pay auctions, the upper bound of support will

³¹ Klumpp & Polborn (2006) consider the case where one of the contestants has assured districts, which is equivalent to the voting rule of biasing one contestant in my model setting. The difference between their setting and mine is that they assume initially identical players, where I allow difference between them.

satisfy $\bar{b}_x = (\frac{1-\rho}{\rho})^{-1/r} \bar{b}_y$; this must be true in equilibrium, otherwise one of the contestants can always be better off by reducing his or her upper bound. For example, if $\bar{b}_x > (\frac{1-\rho}{\rho})^{-1/r} \bar{b}_y$, Contestant 1 does not gain anything by randomizing bids on interval $((\frac{1-\rho}{\rho})^{-1/r} \bar{b}_y, \bar{b}_x]$, but increases her cost since she can win with certainty by just bidding at $(\frac{1-\rho}{\rho})^{-1/r} \bar{b}_y$; a similar logic can be applied on $\bar{b}_y$. Note that since contestants will not bid higher than their valuations of prize, $\bar{b}_x \leq V_1$, and $\bar{b}_y \leq V_2$, the distribution functions $G_1(X)$ and $G_2(Y)$ will be continuous and strictly increasing on interval $(0, \bar{b}_x]$ and $(0, \bar{b}_y]$. The possible atom of the probability distribution will occur only at 0 for the (effectively) weaker contestant, who will get expected payoff of zero.

Given that Contestant 2 utilizes mixed strategy $G_2(Y)$, the expected payoff of Contestant 1 can be expressed as:

$$E(U_1(X)) = G_2\left(\left(\frac{1-\rho}{\rho}\right)^{1/r} X\right) \cdot V_1 - X$$

Since at the mixed strategy equilibrium, any pure strategy will lead to same level of payoff, differentiating $E(U_1(X))$ with respect to X should equal zero; hence, $g_2(Y) = \frac{1}{V_1} \left(\frac{1-\rho}{\rho}\right)^{-1/r}$.

Similarly, the expected payoff of Contestant 1 can be expressed as

$$E(U_2(Y)) = G_1\left(\left(\frac{1-\rho}{\rho}\right)^{-1/r} Y\right) \cdot V_2 - Y. \text{ Taking the first order condition produces } g_1(X) =$$

$$\frac{1}{V_2} \left(\frac{1-\rho}{\rho}\right)^{1/r}. \text{ Note that if the handicap placed on Contestant 1 is large enough}$$

(ρ is sufficiently high), then Contestant 1 can effectively become the weaker contestant; as such, she will play atom at 0. Conversely, if t is sufficiently small, Contestant 1 will maintain her advantage in winning the contest. The dynamics of the situation depend on specific value of t and ρ , which can be characterized in the following two cases:

[ii] When $(\frac{1-\rho}{\rho})^{-1/r} \cdot V_2 < V_1$; or, $t < (\frac{1-\rho}{\rho})^{1/r}$

Under these conditions, Contestant 1 remains the effectively-stronger contestant, as $\bar{b}_x = (\frac{1-\rho}{\rho})^{-1/r} V_2$, and $\bar{b}_y = V_2$. Contestant 1 does not play atom at 0; but Contestant 2 will play atom at 0 with probability $1 - \frac{V_2}{V_1} (\frac{1-\rho}{\rho})^{-1/r} > 0$. The expected payoff of Contestant 2 is $E(U_2(Y)) = 0$, and the expected payoff of Contestant 1 is $E(U_1(X)) = V_1 - V_2 (\frac{1-\rho}{\rho})^{-1/r} > 0$. Expected effort provision is therefore $E(X) = \int_0^{\bar{b}_x} X g_1(X) dx = \frac{V_2}{2} (\frac{1-\rho}{\rho})^{-1/r}$, and $E(Y) = \int_0^{\bar{b}_y} Y g_2(Y) dy = \frac{V_2^2}{2V_1} (\frac{1-\rho}{\rho})^{-1/r}$, while aggregate effort is given by $E(T) = E(X) + E(Y) = (\frac{1-\rho}{\rho})^{-1/r} \frac{V_2(1+t)}{2}$.

[iii] When $(\frac{1-\rho}{\rho})^{-1/r} \cdot V_2 > V_1$; or, $t > (\frac{1-\rho}{\rho})^{1/r}$

Under these conditions, Contestant 1 now becomes the effectively-weaker contestant, as $\bar{b}_x = V_1$, and $\bar{b}_y = (\frac{1-\rho}{\rho})^{1/r} V_1$. Hence, whereas Contestant 2 does not play atom at 0, Contestant 1 will play atom at 0 with probability $1 - \frac{V_1}{V_2} (\frac{1-\rho}{\rho})^{1/r} > 0$. The expected payoff of Contestant 1 is $E(U_1(X)) = 0$, and the expected payoff of Contestant 2 is $E(U_2(Y)) = V_2 - V_1 (\frac{1-\rho}{\rho})^{1/r} > 0$. Expected effort provision is given by $E(X) = \int_0^{\bar{b}_x} X g_1(X) dx = \frac{V_1^2}{2V_2} (\frac{1-\rho}{\rho})^{1/r}$, $E(Y) = \int_0^{\bar{b}_y} Y g_2(Y) dy = \frac{V_1}{2} (\frac{1-\rho}{\rho})^{1/r}$, with aggregate effort provision being $E(T) = E(X) + E(Y) = \left(\frac{1-\rho}{\rho}\right)^{\frac{1}{r}} \frac{V_1}{2} (1 + \frac{1}{t})$.

Given that when one of the contestants plays a mixed uniform bidding strategy according to the distribution function $G_1(X)$ or $G_2(Y)$, the other is indifferent with any (combinations of) pure strategies; as such, he or she would not want to deviate from the defined mixed strategy. Hence the mixed strategy profile defined above constructs a Nash

equilibrium. Notice that ratio of expected efforts still follows $E(X)/E(Y) = t$. While by giving a positive bias to the weaker Contestant 2, it is possible to eliminate some of the original difference between contestants —and thereby increase aggregate effort levels— if the bias is so strong as to completely overwhelm the original weakness, then a new asymmetry is produced, and aggregate effort levels will decrease in the level of the bias (ρ). If the level of ρ is set up such that the two contestants are symmetric to one another ($t = (\frac{1-\rho}{\rho})^{1/r}$), then the highest effort levels can be generated where $E(T) = (V_1 + V_2)/2$.

4 Conclusions

It is often the case that committees, rather than a single individual, award prizes in contests. In this part of my research dissertation, prior findings based on Congleton (1984), Amegashie (1999, 2002) and Lockard (2006) regarding the degree to which aggregate effort provision levels are higher or lower in contests judged by committees versus those by single administrators have been supplemented by a formal and rigorous approach that introduced contestant heterogeneity into the analysis. As demonstrated in this part of the dissertation, the key to whether aggregate effort provision levels are larger or smaller under committees as opposed to single administration of prizes depends fundamentally upon the degree of heterogeneity between the contestants. Further, the impacts on aggregate effort provision produced when allowing for changes in the committee size, as well as varying the specific voting rules employed in the committee setting—some of which introduce biases into the contest—have been shown to be exacerbated or attenuated by differing degrees of contestant heterogeneity. The nature of these multiple parameters' interactions determines their effects on the incentives of contestants—operating via the probability to win the contest—to exert effort, allowing the impact of heterogeneity to induce either greater or diminished aggregate effort provision under committee administration than under single administrator awarding of contest prizes.

In general, the more symmetric is a contest, the more competitive it is, and hence the higher are effort provisions. When contestants have initial heterogeneity in their abilities, the contest organizer can adjust the voting rules in order to extract the most (or least) effort provision. Further, when the committee size is relatively small, a unique pure strategy equilibrium exists, and variation in committee size can fluctuate aggregate effort level in a way

defined by the effective heterogeneity level between contestants ($\alpha \equiv t^r$). Meanwhile, the impact of voting rules on aggregate effort level is also dependent on the effective heterogeneity level between contestants. The dynamic between voting rules and effort level also changes as committee size varies. When the committee size approaches infinity, a uniform bidding strategy in mixed form exists, and voting rules can be applied as a way to reduce the asymmetry between two contestants. The mixed strategy equilibrium takes the similar form as in all-pay auctions, where the actual-disadvantaged contestant receives an expected payoff of zero, while the actual-advantaged contestant receives a positive payoff.

PART III: Share Price Clustering on the Shanghai Stock Exchange: Empirical Support for Rational and Cultural Explanations[§]

1 Introduction

The key concept under consideration in Part III of this research dissertation revolves the ways in which economic agents make (rational) pricing decisions in market institutions; in this case in particular, a financial market, that being the Shanghai Stock Exchange. The analysis starts from a theoretical basis in developing an understanding of how individual agents price equities in markets. Based on that understanding, aspects of the nature of the expected distribution of share prices are next considered; in particular, the focus is on the expected frequency of the final digit of share prices. We relate the individual decision-making process of economic agents to the distribution of market prices by noting that such prices represent the sum of all market participants' individual pricing decisions in the aggregate.

In theory, the frequencies at which values of the final digit occur are expected to be equal. Deviations from the expected uniform frequency are referred to in the literature as "price clustering". Share price clustering occurs when stock prices tend to "cluster" around certain prices such that the final digit of the minimum tick size is not distributed evenly across all minimum tick size values. As a result, price clustering is typically characterized as

[§] Part III of the dissertation is based on a co-authored work by Jae Harris and myself, currently a Working Paper entitled "Share Price Clustering on the Shanghai Stock Exchange – A Consideration of Rational and Cultural Explanations", March 2014. SSRN: <http://ssrn.com/abstract=2404826>

deviating from the concept of efficient markets. Since market efficiency presupposes share prices reflect the underlying value of expected future earnings of a given firm and can only be impacted by unanticipated (random) news announcements, over a large enough sample, prices should theoretically be distributed evenly in the final tick size of the share price. (For instance, in the case of a decimalized minimum tick size of Chinese Yuan 0.01 as on the Shanghai Stock Exchange, it is expected that the final digit of a random sampling of prices would be equally distributed amongst the values 0 through 9.)

The subsequent analysis undertaken in this paper is to compare the theoretical expected pricing frequencies with the actual pricing frequencies observed with the aggregate market data from the Shanghai Stock Exchange. Market data obtained for the Shanghai Stock Exchange indicate that extensive share price clustering exists in the observed prices. The focus of Part III of my dissertation is not, however, simply recording such deviations; more importantly, an attempt is made to provide explanations for why deviations between the expected and observed distributions exist. Specifically, the argument presented is that it is the institutional structure of stock markets in general that helps to create incentives that encourages behavior by market participants that leads to aggregate share price data indicating price clustering. The observed deviations from the expected distributions are not necessarily due to irrational explanations such as bounded rationality. I find evidence indicating that clustering results from behaviors by market participants that reflect rational responses to the specific institutional structure found within the Shanghai Stock Exchange, given its unique legal, regulatory, and cultural frameworks. As such, we are able to utilize several instances where some of the parameters defining the specific, unique institutional characteristics of the Shanghai Stock Exchange change, allowing for comparison tests to study if such changes might produce resultant changes in observed clustering outcomes, providing evidence as to (i)

the importance of such institutional factors, and (ii) the validity of various general theories regarding the drivers of the price clustering phenomenon. To conduct the analysis, frequency tests and multi-variate regression statistical analysis are employed on the pricing data set.

1.1 Asset Price Formation

Part III of this dissertation considers to what degree the collective behavior of Shanghai Stock Exchange market participants falls within the current understandings of stock market dynamics. Such a consideration is interesting for it provides us with a better and more formal understanding of the mechanisms determining market functioning of the Shanghai Stock Exchange, including its asset pricing mechanisms. One of the underlying assumptions of the study is that individual agents follow rational asset pricing behavior. As such, we first describe the general dynamics of equity price formation.

Asset Pricing Theory

Asset pricing models are designed to explain the pricing mechanism acting upon financial assets. In general, asset pricing models are based on equilibrium conditions holding within financial markets. At equilibrium, agents are assumed to hold an optimal set of assets. Combined with a market-clearing condition (which is that the aggregate of agents' preferred asset holdings must equal the aggregate "market portfolio" of all securities), these two conditions lead to the formation of equilibrium prices. The Capital Asset Pricing Model (CAPM), and its many derivative models, are all based on an equilibrium conceptualization of market asset pricing. As formalized by Sharpe (1964) and Lintner (1965), the CAPM

establishes the price of a specific asset within a general investment portfolio by finding the connection between risk and return. The CAPM holds under financial market equilibrium conditions, determining the risk level for each asset comprising the market portfolio.

As noted in Ferson (2003: 746), while there are numerous formulations of asset pricing models, nearly all of them represent modifications of the equation $P_t = E_t[m_{t+1}(P_{t+1} + X_{t+1})]$, where P_t is the price of the asset at time t ; X_{t+1} are payments generated by the asset (such as dividends or interest payments); and m_{t+1} is a random variable (referred to as the stochastic discount factor and usually conceptualized as the interest rate), such that $m \in (0,1)$. Asset prices are therefore derived from the discounted expected present value of all future expected payoffs.

Efficient Markets Hypothesis

The Efficient Market Hypothesis (EMH) underpins much of modern finance theory,³² providing a formal description of the process of asset price formation and change. The key concept is that changes in the price of an asset is driven only by the arrival of news relevant to the asset in question which updates the current information set held by investors. Since information generation is itself a random process, hence, asset price formation is a random process; ergo, prices cannot be predicted and are efficiently priced.

The development of the EMH is largely attributed to the contributions of the Nobel laureates Paul Samuelson and Eugene Fama. Samuelson (1965) in his article, "Proof That Properly Anticipated Prices Fluctuate Randomly", shows that if market participants can

³² While this remains true, it is also true that recent perceptions of the validity of the EMH have been trending negative, mainly in response to the recent global financial crisis. Indeed, in a recent talk by George Akerlof at the Berkley University Economics Lectures series entitled, "Was the recent financial crisis caused by the EMH?", he answered his presentation's question in the affirmative. The field behavioral finance in general is quite critical of the EMH over market pricing anomalies.

correctly anticipate future events using appropriate probabilities, then a market can be classified as "informationally-efficient." Under such conditions, it is not possible to forecast price changes, since current prices would inherently incorporate all of the current information and expectations of all of the market participants. As (Samuelson 1965: 41) notes, once a buyer or seller "could be sure that a price would rise, it would have already risen." Concurrently, Fama (1965) constructs a conceptual framework of increasing levels of efficiency operating in a market, based on the amount of information under consideration.³³ Later, Fama (1970) provides a more formal structure to EMH theory, based on the conceptualization of stock prices following a "random walk". A random walk describes the process of price movement such that successive single-period price changes are independent and identically distributed.

Thus the underlying foundations of the EMH regarding the asset price formation process are the assumptions that (i) stock prices fully absorb and reflect all available information, and (ii) the market immediately updates the available information set to incorporate new information. In this context, modeling stock price formation under the weak-form of the EMH is concerned with modeling the arrival of new information, and hence prices follow a martingale process. As such, the expected price of an asset can be represented by $E_t [P_{t+1} | \Omega_t] = P_t$, where Ω_t represents the information set, and thus, $E_t [P_{t+1} | P_t, P_{t-1}, P_{t-2}, \dots] = P_t$. Unless new information arrives outside of Ω_t , in expectation the price of the asset at time $t + 1$ should simply be the price of the asset at time t (i.e., remain unchanged). Hence, the expected distribution underlying the information-generating process—typically modeled following a normal distribution—will determine the distribution of prices.

³³ In particular, Fama describes three forms of the EMH, each defined by successively higher requirements on the information set: weak (information on past stock returns), semi-strong (all publically available information), and strong (all possible information).

Uniform Price Distribution and Tick Sizes

The underlying assumption commonly followed in the price clustering literature is that in expectation, the frequency at which the final digits of share prices take on each of the different values of the minimum tick size is uniform. I likewise adopt this assumption in the proceeding analysis of price clustering behavior. Support for this conceptualization is based upon the formal implications of the efficient market hypothesis, and the expectation that the movement stock prices approximate a random walk. Price changes are driven by the arrival of randomly-generated news. This random nature of the price formation process will generate a random series of changes to the final digits of share prices. Given that there is a finitely-limited number (10) of final digit values (0,1,2,...,9) through which the process can cycle, over sufficiently long trading periods the "law of large numbers" implies the frequency distribution describing these values approximates uniformity.

The minimum tick-size plays an important role in regard to transforming the stock price formation process into one of discrete movements. Given that information itself is not discrete, asset values are theoretically contained within the continuous set of positive real numbers. We assume that the underlying price formation process is transformable into a process modeled by discrete counterparts. The minimum tick size reflects an institutional framework imposing a particular *level* of discreteness upon stock prices in a given stock market. In principle, a smaller tick size (one allowing greater pricing precision) better approximates the continuous information process underlying stock price formation. Additionally, as discussed below, there are several theoretical and empirical issues concerning the effect that the minimum tick size has upon liquidity, price discovery, and the amount of information available to the market, all of which can impact the nature of the stock price formation process.

1.2 Share Price Clustering: Evidence and Theory

Although the discussion above indicates that theoretically, the frequency of the occurrence of the values of the final digit of stock prices representing the minimum tick size should be uniform, empirical evidence came to light in the 1960's suggesting that this basic assumption was being violated in stock pricing in practice. Osborne (1962, 1965) and Niederhoffer (1965, 1966) both found compelling evidence from empirical examinations of U.S. stock price data that share prices were *not* equally distributed on the minimum tick size. Instead, as coined by Niederhoffer, there tends to be "clustering" of share prices such that certain final digit values appear more frequently than others in the pricing data. Looking at data from the New York Stock Exchange, the studies by both authors found results similar to that reported by Osborne (1962: 370), that prices clustered on 8/8 most frequently, followed by 4/8, then 2/8 and 6/8, and finally 1/8, 3/8, 5/8, and 7/8.³⁴

These early studies led to a growing literature of empirical investigations of stock price clustering in a number of global equity markets, as well as other financial markets. Early studies subsequent to Osborne and Niederhoffer primarily considered relatively developed exchanges, principally focusing on the U.S. markets. Two significant studies followed, beginning with Harris (1991), who not only provided much of the theoretical backbone for price clustering theory, but also observed price clustering to vary temporally, between stocks and across differing market structures of the NYSE, AMEX and NASD. Soon thereafter Christie & Schultz (1994) and Godek (1996) found evidence of stock price clustering on the NASDAQ, especially on round numbers (i.e., whole integers). After the introduction of decimalization in 2001 on the U.S. equity exchanges, there was a new wave of price clustering

³⁴ As at that time, stock prices were quoted in eighths of a dollar, the decimal equivalents would be 8/8 represents .00; 4/8 represents .50; 2/8 and 6/8 represents .25 and .75, respectively; and 1/8, 3/8, 5/8, and 7/8 represent .125, .375, .625, and .775, respectively.

research, specifically concerned with the question as to whether more or less clustering took place as a result of the change in minimum tick size and pricing structure. These more recent studies continued to find such evidence of clustering behavior (Cooney *et al.* 2003, Ikenberry & Weston 2007). Cooney *et al.* (2003) use limit order data from the NYSE, and find similar patterns of clustering as in previous market order pricing data, that being price clustering distributed such that prices ending in 0 are the most numerous, followed by prices ending in 5, with other prices more or less uniformly distributed excluding those two.

Numerous empirical studies have been produced considering stock markets outside North America, including Grossman *et al.* (1997) and Huang & Stoll (2001), both of which look at clustering on the London Stock Exchange. Aitken *et al.* (1996) and Doucouliagos (2004) consider clustering on the Australian Stock Exchange, while Sonnemans (2003, 2006) looks at the Amsterdam Stock Exchange. Ruchardt & Vogt (2009) meanwhile compare the NYSE and the XETRA German stock markets, finding evidence of clustering in both. The Japanese stock markets have also recently attracted the attention of financial economists studying stock price clustering, with two significant papers produced by Ohta (2006) and Asçioglu & Comerton (2007). These and other studies confirmed the previously-observed pattern in stock price clustering that had been observed in the North American markets, where prices cluster especially on prices ending in 0, and lesser on prices ending in 5.

Recently, price clustering in emerging markets has also received attention, with Hameed & Terry (1998) providing some of the earliest evidence of price clustering outside the larger markets. Studying the Stock Exchange of Singapore, one of the first exchanges to adopt a decimalized stock pricing system and minimum tick size of 0.01, the authors used pricing data from 1980 through 1994, finding evidence of price clustering, again primarily on .00 and .x0, followed next by multiples of .05, next by even values (multiples of .02, .04., .06, .08), and

finally odd-ending values besides multiples of .05. Narayan & Smyth (2013) consider the relatively small stock market on Fiji, also confirming extensive price clustering to take place, with nearly fifty percent of all trades prices ending in values of 0 or 5. Further and similar evidence has also been put forth for the stock exchanges in Hong Kong (Ahn *et al.* 2005) and Taiwan (Chiao & Wang 2009, Jiang & Huang 2009, and Hsieh & Lin 2010).

Hence, share price clustering has been observed across a diverse set of countries and markets, with fairly consistent patterns in terms of the particular final digits that are clustered-upon. In addition, studies of other, non-equity financial markets have indicated that price clustering is evident in both futures (Chu & Lim 2001, studying the SIMEX; Chung *et al.* 2004, considering the NASDAQ100) and foreign exchange markets (Goodhart & Curcio 1991, Fischer 2004, Mitchell & Izan 2006), with similar clustering patterns emerging. Thus, as stated above, while fundamental financial theory suggests asset pricing behavior that would generate uniform distribution across the minimum tick size, actual empirical evidence—from a large sample of temporally and geographically diverse markets—has conclusively found evidence to the contrary in the data sets studied. An understanding as to why this sharp divergence between the expected and realized results exists is next considered.

Theoretical Understandings of Price Clustering Phenomenon

Ever since the phenomenon of share price clustering was first identified, the literature has sought to develop an understanding for the phenomenon. There are several general categories of theoretical explanations that range from strictly rational to more behavioral in type. In addition, and specifically in regard to price clustering in China, cultural explanations have also been put forth. A non-extant review is considered below.

Number Attraction

The attraction hypothesis has been put forward as one of the most straight-forward explanations for the clustering effect observed in equity pricing data. As advocated by Gottlieb & Kalay (1985), and later described in Goodhart & Curcio (1991), Aitken *et al.* (1996), and Asçıoğlu & Comerton (2007), the attraction hypothesis posits that certain numbers are more "salient" than others. This salience, presumably, causes investors to buy and sell stocks at prices which values end in the number they deem more salient more often than stocks whose prices end in less salient values. The hypothesis presupposes a structure on agents' preferences structure, such that their preferences for prices containing certain preferred values provides greater utility than receiving higher expected returns accruing from using buy and sell decision-making rules determined solely by expected return. Owing to the underlying observed data patterns, the hypothesis imposes (by assumption) a specific structure on the expected shape of (many) agents' preference distributions over numbers, being specifically that 0 has the strongest salience, followed by 5, followed by even numbers. However, numerous problems can be identified with the tractability of the attraction hypothesis. To begin with, it is not a testable hypothesis in its foundations, only in its realization; that is, at least from financial asset pricing data, it is not possible to test whether agents are purchasing or selling shares due to their own personal preferences for stocks ending in certain numbers. Further, the logic for the ordering of salience appears to be arbitrary and decided by assumption, again implying a lack of testability. Why, for instance, would 0 be more "salient" than other numbers; why are even numbers more salient than odd? Likewise, one can question why investors would only care about the last digit of a share price; were my preference structure such that I love the number 7, might I not also cluster on prices such as 17.43, or 8.72, in

addition to a price such as 14.57? Most important, the hypothesis lacks an economic rationale for its preference ordering.

Such problems aside, the literature has attempted to test for the attraction hypothesis. Harris (1991:35) finds no evidence that investors liked some values more than others *per se*, only that they clustered away from off-eighths to the same degree. Goodhart & Curcio (1991) claim to find support for the hypothesis in foreign exchange pricing data on the last digit (although not on the penultimate digit). Aitken *et al.* (1996) find little support for the hypothesis, as do Brown & Mitchell (2008).

Bounded Rationality

Among the earliest reasons proposed to explain price clustering were those of the behavioral and limited-cognitive skills nature. The contention is that economic actors possess limited cognitive ability and recall, which represents, in some ways, the bounded rationality viewpoint. Starting with the work of Simon (1955, 1974) and followed by Chase & Simon (1973) and Simon (1974), bounded rationality argues that individuals have constrained cognitive abilities, a finite amount of information they can access, and a finite amount of time available to make decisions; such limitations make being rational costly, and as a result, while economic agents partake in rational decision-making, they do so only subsequent to having applied rubrics that reduce the choice set and simplify their optimization problem. As such, human economic behavior can only approximate textbook rationality. Specifically, Simon (1955: 99) argues that we must "replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist." More recently, psychologists have attempted to

provide evidence supporting the concept of limited cognition. Cowan (2000), Gobert & Clarkson (2004), and Jones (2012) all research human cognitive ability, appearing to show that most individuals store and recall information in "chunks", and are able to easily recall only limited amounts of such chunks at a time. Applying such reasoning to the financial markets, Angel (1997: 667) argues such limitations can lead to observed price clustering behavior, since traders might find it "easier to remember the contents of an order book if they are concentrated on [for instance, only] seven different levels rather than 99. Eliminating economically insignificant information means that a trader's finite mental capacity can be used for more profitable activities, such as making markets in additional stocks." More recently, Kuo *et al.* (2012) present evidence that the proportion of limit orders submitted at round numbers is relational to an investor's cognitive ability.³⁵ In a related vein, Aitken *et al.* (1996: 298) propose that the way in which many societies educate children in mathematics—encouraging the use of rounding fractions to the whole digit for ease of calculation—contributes to the focus on round figures and, by extension, trading in share prices that end in a final digit of zero. The implication in Aitken *et al.* (1996) is that there is a cognitive ability argument to be made, namely that traders are only capable of correctly pricing to a limited set of digits, and that in many cases, the minimum tick size represents a region outside of that limitation. Hence, a reduction in possible pricing points that is implied by a coarser tick size works to decrease the amount of pricing information of which market participants must keep track.

³⁵ In their study on price clustering in Taiwan's stock market using limit order pricing data, the authors find that more cognitively-constrained traders suffer greater trading losses; they also find that past trading experience (as measured by the number of limit orders previously submitted) is directly inverse to the degree of round-number limit order submissions.

Culture

Similar in form to the Attraction hypothesis although based on behavioral response is a psychological framework that posits that humans are simply attracted to certain numbers more than others. Mitchell (2001) argues that certain numbers (such as 10,000) generate psychological responses and hence can explain observed pricing “barriers” in terms of stock indices approaching certain index values; or, by extension, individual stocks landing on round values. Why certain numbers would be favored over others—beyond issues pertaining to number attraction discussed above—is somewhat unclear in this theory, although “culture” is argued to play a strong role in affecting economic agents’ decision-making.³⁶ Mitchell & Izan (2006: 321) suggest that “symbolism, mysticism and even cultural convention may dictate some form of basic number preference.” This supposition will be explored in greater detail below in Section 1.3 in regard to price clustering in China specifically.

Collusion

Christie & Schultz (1994) published an intensely-debated article finding evidence of price clustering on the NASDAQ, which they argued to be due to collusion among traders in order to keep bid-ask spreads high and capture a greater share of profits from trading. Christie & Schultz argue that price collusion was the main factor driving share price clustering on the NASDAQ in their 1991 data set. Tellingly, they found transaction prices between market makers themselves to be priced in odd-eighths 22% of the time, while trades on the Small

³⁶ Boye (2005: 495), in his analysis of Ghanaian investors who chose to invest in a series of pyramid schemes which operated in the informal financial sector, claims that “a hidden cultural factor influenced this rather strange phenomenon.” Boye’s argument is that “standard” economic concepts (such as “optimization and maximization of utility by agents”) are Western concepts that are not universal across all cultural settings: “Western thinking is that investment decisions must be fully analytical... [and] the decision maker himself is analytical in his thinking, which is only true if considered in Western environments...this may not hold true in other societies.”

Trade System were priced in odd-eighths approximately only 5% of the time. While obviously not the only reason for clustering (given that market makers do not exist in every market), the sudden change in additional use of odd-eighths quotations after the publication of Christie & Schultz (1994) is cited by Christie *et al.* (1994) as evidence that collusion had in fact been occurring. Godek (1996), however, is among those who present evidence to the contrary regarding the collusion hypothesis.

Price Resolution and Negotiation

In response to behavioral explanations for price clustering, other theoretical work has attempted to explain much of the forces that drive price clustering from a more rational viewpoint by providing an economic justification for price clustering. These theories primarily revolve around informational failures. While a finer set of available pricing units permits more accurate pricing, as Grossman *et al.* (1997: 25) note, this presents “a mixed blessing”. Additional layers of fineness in pricing designation imply an increase in the time and effort necessary to obtain more precise asset valuations. Ball *et al.* (1985) were the first to present the “Price Resolution” theory, adapting the information argument of Grossman & Stiglitz (1980) in terms of trading activity. They contend that share price clustering is due to informational asymmetries and noise that lead to fundamental uncertainties regarding the true valuation of assets. Both (a) a lack of market information and (b) market volatility, make assigning precise valuations more difficult and less valuable, hence leading to the use of a coarser pricing set. They posit that agents will invest in acquiring costly information up until the point at which the marginal cost of additional information acquisition equals the marginal benefit from achieving a finer price resolution. Hence, the precision of traded prices is expected to be inversely related to information costs. Likewise, high uncertainty (sometimes

represented by higher volatility) is expected to increase clustering behavior. Sopranzetti & Datar (2002) find support for this supposition in the foreign exchange market, identifying a coarser price set used for trades involving "minor" currencies (which are traded less and subject to greater volatility and uncertainty), as opposed to finer pricing grid on trades between "major" currencies. Furthermore, Sopranzetti & Datar contend, market participants may cluster around particular values as a way to moderate search costs.

In a similar vein, the "Negotiation Hypothesis" (Harris 1991, Grossman *et al.* 1997) suggests that increasing the fineness of the units of pricing impacts the dynamics of trade negotiation and resolution for two reasons. First, since the possible valuations of the asset increase, so too does the probability that a potential buyer and seller will not initially match on the exact valuation. Logically, then, the expected time to achieve convergence on the negotiated price increases. Relatedly, Brown *et al.* (1991) construct a "prisoner's dilemma" model of equity trading, wherein which traders have incentives to overspend their time on negotiating, thus causing both parties to suffer high bargaining costs. The imposition of a minimum tick size helps to limit the amount of pricing possibilities on which traders can negotiate. These factors can increase the expected time to convergence on a negotiated price, meaning buyers and sellers are exposed to a greater degree of price risk during the negotiation process.

We must note here, importantly, that the basis of the argument that individuals will face longer time constraints by bargaining via the process of offer-counteroffer between one another over pricing differences is only relevant when human agents are the ones doing the bargaining. However, in many modern markets—including, importantly for our study, the Shanghai Stock Exchange—computers are employed to perform the matching of bid and offer prices automatically. As such, much of the older arguments pertaining to the costs of

negotiation owing to the back and forth process between two traders (or, market makers) no longer retain their full relevancy. Nonetheless, even in a computerized pricing-matching environment, market participants still seek ways to minimize outstanding price risk. One such way to reduce either negotiation time specifically, or simply the potential time to execution of a trade, is to reduce the possible number of offers and counter-offers, or potential unmatched bids or offers, by effectively increasing the minimum pricing unit (or effective tick size) on assets. This can be accomplished by choosing to cluster on round prices or values whose last digit ends in zero. Such a role of clustered values is well-recognized. Indeed, Grossman *et al.* (1997: 28) refer to a “customary unit of trade, which typically exceeds the minimum tick”, which is due to traders attempting to reduce their bargaining costs by choosing to discard all possible tradeable tick sizes and limit their bargaining costs by focusing on a smaller subset of asset values. Thus they are willing to lose a certain amount of expected return in order to complete a sale more quickly—the trade-off is losing potential fineness of price resolution while saving on bargaining costs. Harris (1991), Grossman *et al.* (1997), and Narayan & Smyth (2013) are among the studies providing support for the negotiation and price resolution hypothesis.

1.3 Share Price Clustering: China's Stock Markets

Beyond the standard explanations considering asset price formation and price clustering behavior in regard to most stock markets, recent studies of China's stock markets have identified some unique features in terms of their market structure and regulatory framework as impacting pricing behavior. In addition, the cultural background of market participants has also been considered as a potential explanatory factor in regard to understanding pricing

dynamics. Indeed, much of the price clustering literature pertaining to China's equity markets focuses on "cultural" factors unique to China.

China's equity markets present an interesting case, having been only relatively recently established two decades ago, and yet already rivaling in size some of the largest stock exchanges in the world. The Shanghai Stock Exchange, for instance, is one of the world's most technologically advanced bourses, operating as a purely electronic trading platform and possessing an automated auction price matching system that has removed the need for human market makers. These steps were undertaken explicitly to create as efficient a market as possible (Xu 2000).

The common description of the Chinese equity markets, however, is that they exhibit qualities far from efficiency. At the start of 2010, *The Financial Times* newspaper noted that in response to measures designed to improve market liquidity, "[n]ot everyone is convinced that the reforms will necessarily lead to more rational pricing in China's markets... Chris Ruffle, co-chairman of MC China, one of the largest foreign investors in mainland equities, says: "I wouldn't hold out too much hope in that regard [more rational pricing]..."³⁷. The academic literature has similarly taken a rather dim view of the domestic Chinese investor class. For example, Cheol Eun and Wei Huang, writing in the *Pacific Basic Finance Journal* in 2007, state that "individual investors, who have dominated stock trading, have been driven by speculative sentiments", and that China is home to "volatile stock markets, where long-term investors are scarce and speculators abound" (Eun & Huang 2007: 460-461).

However, there is also evidence to indicate that domestic investors are not any less rational than their foreign counterparts. Several studies of market efficiency using standard tests of the EMH have found mixed results when considering the presence of market efficiency

³⁷ *The Financial Times*, "Short Selling Opens Doors for China", 2010 January 12.

on Chinese stock exchanges, which is indeed similar to such studies of other and more developed markets. For instance, Several recent tests of China's stock markets have indicated support for Chinese stock markets to be at least weakly-form efficient. Early tests on China's markets resulted in mixed evidence; conducting simple unit root and variance tests, Liu *et al.* (1997) find the Shanghai and Shenzhen Stock Exchanges exhibit market efficiency, while Mookerjee & Yu (1999) and Darrat & Zhong (2000) find otherwise. A number of subsequent studies, however, which employ a series of more advanced parametric and non-parametric variance ratio tests— including Ma & Barnes (2001), Li (2003), Lima & Tabak (2004) and Fifield & Jetty (2008)— all find China's stock markets generally meet the criteria for weak-form efficiency in A-shares.

As suggested previously, share price clustering on Chinese stock markets appears to have different features than price clustering elsewhere. Specifically, Brown *et al.* (2002) and Brown & Mitchell (2008) posit that Chinese “culture and superstition” impact share price clustering by shaping the number preferences of market participants operating in the ethnic Chinese-dominated markets of Hong Kong, Singapore, Taiwan and the People’s Republic of China. Brown *et al.* (2002: 308) argue that Chinese investors might be inclined by “*feng shui* and Chinese superstition” to shy away from certain numbers while being attracted to others.³⁸ Brown & Mitchell (2008) and Na & Schneider (2010) repeat such claims, arguing, for example, that Chinese investors are attracted to the numbers 6, 8 and 9, while disliking 4 and 7. Such likes and dislikes, it is argued, help lead to share price clustering in China; prices cluster on numbers ending in “lucky” values and less on “unlucky” values. Both studies argue that

³⁸ As Yang (2011: 693) explains, most of the reasons why numbers are preferred or disliked are related to pronunciation considerations; for instance, in standard Mandarin Chinese, the number 4 (sì) is pronounced similarly to the word for death (sǐ). Complicating issues, however, is that the Chinese language is complex, with pronunciations differing widely across regions and dialects; the number 6, for instance, only sounds like “beneficial” in several central Chinese dialects, and not in standard Mandarin.

Chinese investors—particularly those located in mainland China—are more likely to buy/sell stocks ending in the digits 6, 8 or 9, and less likely to purchase shares ending in 4 or 7, thus inducing A-share price clustering on 6, 8 and 9, and away from 4 and 7. Na & Schneider (2010) argue for a similar phenomenon to occur specifically in share pricing of A-share IPOs in China. Hence, it is contended that domestic Chinese investors are so motivated by "cultural" factors (namely, a superstitious belief in the "luckiness" of certain numbers) that it affects their share price decision-making, and hence by extension, the efficiency and validity of the pricing mechanism in China's capital markets.

An interesting question comes out these studies, for the question of number preference by Chinese investors also proves to be slightly problematic. As in other markets, they find price clustering to occur on the final digits 0 and 5. However, while Brown *et al.* (2002) find little support of investors who are ethnically and culturally Chinese clustering on culturally "lucky" numbers in Taiwan, Hong Kong and Singapore, Brown & Mitchell (2008) and Na & Schneider (2010) provide evidence that mainland Chinese apparently do cluster on such values. Hence, we are faced with the puzzle as to why only mainland investors are impacted by Chinese numerology culture, while other ethnic Chinese are seemingly unaffected as it pertains to their investing decisions. Further, while Brown *et al.* (2002: 308) argue that Chinese investors are swayed to dislike the certain integers and prefer others, the full range of Chinese number preferences and dislikes have not been found to be equally acted-upon in terms of clustering behavior.

In response, we would argue that what is missing from the analysis provided by the literature thus far is a more nuanced and institutionalist approach. Specifically, Brown *et al.* (2002) and Brown & Mitchell (2008) appear to be arguing for a monolithic Chinese culture, across regions and time, in terms of the importance of numerology, and, in particular, its

importance in relation to the importance and understanding of market and pricing processes. The institutional framework affecting—and being affected-by—culture is constantly evolving. We expect that as different institutional structures change within Chinese society in general, and in those determining the pricing mechanisms of the Shanghai Stock Exchange in particular, historical trends in price clustering observed in China might also evolve over time. Our study attempts to analyze the way in which such institutional forms might impact the phenomenon of price clustering in China, both in regard to numbers considered important in Chinese culture, and in general.

1.4 Remarks and Structure of Part III

Since the 1960's (Osborne 1962, 1965; Niederhoffer 1965, 1966), it has been recognized through empirical examinations of stock price data that share prices are in fact not equally distributed on the minimum tick size, and in fact there tends to be "clustering" of share prices such that certain final digit values appear more frequently in the pricing data. As such, subsequent research has been undertaken to both document further examples of such clustering, and to attempt to provide explanations for such a phenomenon. While a multitude of theoretical explanations have been developed to address the empirical puzzle posed by observations of share price clustering, definitive evidence for or against any or all of these have been limited. Nonetheless, although multiple attempts have been made, there is as of yet a clear generalized answer for why clustering occurs in most markets. Many of the aforementioned studies present findings as to the reasoning for clustering that are at odds with one another. Ruchardt & Vogt (2009: 22) capture this when finding that "no one model... (the different explanations for stock price clustering such as ease of negotiation, convenience and

rounding, attraction, odd pricing, and aspiration level)... [is] able to capture all of our empirical observations."

The present study is an attempt to find a consistent rationale for the price clustering which has persisted on the Shanghai Stock Exchange over the past decade, and this study represents one such attempt to do so within a rational agent framework. Specifically, this research contributes three main findings to the literature on price clustering, and in particular price clustering in China. First, it considers price clustering in China with a more targeted and updated data set than previously studied in the literature, using decade-long pricing data on the twenty of the top-50 companies traded on the Shanghai Stock Exchange. Second, this is one of the few studies to expand the analysis of price clustering to consider not only the final digit but also potential clustering on the penultimate digit (in our case, the hundredths and tenths digits, respectively). Third, this study presents argumentation and evidence that complement previous research (e.g., Brown & Mitchell 2008) indicating that cultural factors in the form of "superstition" or "feng shui" drive some of the price clustering process in China. I perform a set of statistical analyses, in particular multivariate logit modeling, to see why there is share price clustering on the Shanghai Stock Exchange. The data and methodology used to do so are described in the proceeding section. The findings further confirm the presence of price clustering in China. While ultimately finding evidence for the existence of cultural factors playing a role in share price clustering in China, it is found that such behavior is primarily partaken when there is (relatively) low impact to the value of the trade action.

The balance of the study is organized as follows. Section 2 reviews the institutional structure of the Shanghai Stock Exchange, including its trading rules. Section 3 describes the data, and Section 4 reports the results and implications of the statistical analysis; finally, Section 5 concludes.

2 Overview of the Shanghai Stock Exchange

First established in 1990 as part of the People Republic of China's economic reform package moving away from centralized economic planning, the Shanghai Stock Exchange has now by 2013 become the third-largest stock market in Asia, ranking only behind the Tokyo Stock Exchange and the Hong Kong Stock Exchange in terms of volume transacted and value traded. The Shanghai Stock Exchange trades both debt and equity instruments, including central and local government bonds, provincial development bonds, enterprise bonds, banker acceptances, promissory notes, and commercial paper, as well as stocks, commodity futures, and warrants; as of 2010 March 31, short-sales in some stocks have been allowed. Starting with only eight listed companies when it was first established in 1990, the Shanghai Stock Exchange grew to listing 780 companies by 2003, and now has over 950 companies listed and traded on the bourse in 2013. Table 2.1 provides some key indicators for the Shanghai Stock Exchange over its two-decade history. The associated Figures 2.1a and 2.1b document the Shanghai Stock Exchange's significant growth over this period.

Table 2.1: Shanghai Stock Exchange Key Indicators, 1990 – 2013

Year	1990	1993	1997	2000	2003	2006	2009	2012
Listed Companies	8	106	383	572	780	842	870	954
Listed Securities	30	190	467	657	914	1126	1351	2098
Mkt Capitalization (RMB 100b)	0.1	22	92	269	298	716	1847	1587

source: Shanghai Stock Exchange, 2013 Yearbook

Figure 2.1a: Number of Listed Companies

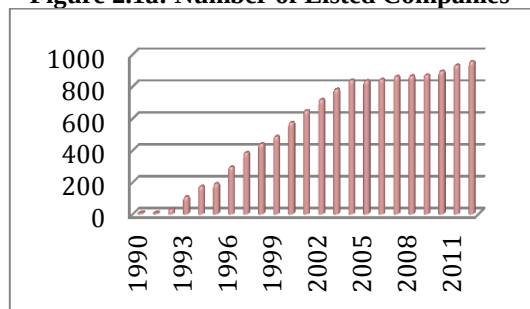
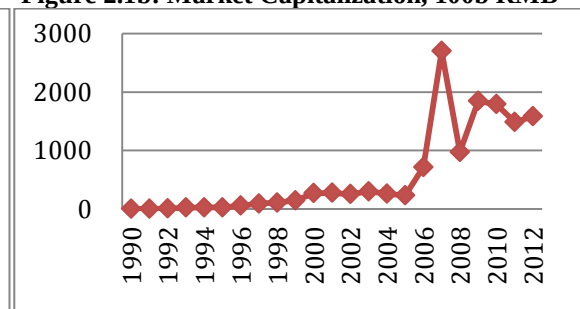


Figure 2.1b: Market Capitalization, 100b RMB



source: Shanghai Stock Exchange, 2013 Yearbook

Institutional Structure of the Shanghai Stock Exchange

The institutional rules governing the ownership of shares listed on the Shanghai Stock Exchange are somewhat unique, given that companies may list their shares on the exchange as either "A"-shares or "B"-shares, or both. A-shares are denominated in Chinese renminbi currency, and are primarily traded by domestic individuals and firms, whereas B-shares are denominated in U.S. dollars foreign currency, and can only be traded and held by foreign entities. Further complicating the landscape is that domestic shares may be classified into three sub-types: (a) state-owned shares, (b) legal entity shares, and (c) public shares. State-owned shares designate ownership rights in a company held by the central government, while legal entity shares are also government-controlled, frequently by other government entities (such as provincial governments, or national or provincial government agencies). Public shares represent the only category of "A"-shares that can be legally traded on China's stock exchanges.

The China Securities Regulatory Commission (CSRC) is the regulatory body governing all financial markets operating in China, including the Shanghai Stock Exchange. Charged with oversight of financial markets and the enforcement of the 1999 Securities Law regulating the issuance and trading of securities in China, the CSRC has seen its authority and independence increased, especially after revisions to the Securities Law were enacted in 2005. While technically forbidden practices such as insider trading were traditionally not well-monitored by the authorities (Lin *et al.* 2005: 7), recent measures have been undertaken to improve the CSRC's oversight abilities. In 2012, for instance, over 50 new rules and regulations were implemented, many of which focused on creating greater transparency, and tightening rules relating to how and when companies are delisted from the exchanges.³⁹

³⁹ *Bloomberg News*, "China Watchdog Embraces Risk After Everbright Fat Finger", 2013 October 1.

The trading procedure on the Shanghai Stock Exchange governing A-shares is determined by an automated, order-driven process. The trading day is comprised of three trading periods. First, from 9:15 to 9:25 opening prices are generated using a periodic auction that matches trades based on the Itoyase method. Then the morning session runs from 9:30 to 11:30, and the afternoon session from 13:00 to 15:00; price matching in both morning and afternoon sessions are operated as continuous discriminating auctions. Both market orders and limit orders are permitted. Xu (2000) looks at the microstructure of the Shanghai Stock Exchange (as well as the Shenzhen Stock Exchange). He provides a detailed analysis considering whether the specific structure of the stock market's rules might lead to an impact on the price formation process. Specifically, he finds that neither the periodic auction pricing methodology used at the market opening, nor the continuous auction used during the trading day, lead to persistence of stock prices over time. For our purposes, we note that for both of the two auction types utilized by the Shanghai Stock Exchange, prices are matched via a computer-driven automated process. The set of possible prices are what we are most concerned with analyzing, and which ultimately determine the set of agreed-upon prices that form our data set.

3 Data and Methodology

This study uses daily opening, closing, high and low pricing data on A-shares for the Shanghai Stock Exchange's largest stocks, as measured by market capitalization, as well as daily volume traded data for each company's stock included in the data set. The twenty stocks selected for the study's statistical analysis all have a continuous data series stretching back to January 2003 (i.e, companies which joined the Shanghai Stock Exchange as of January 2003 or earlier). This was done in order to ensure a consistent data set for each of the twenty stocks over the past decade. Furthermore, restricting analysis to the largest stocks on the Shanghai Stock Exchange means that I take a more micro-level approach to studying price clustering and examining its determinants on the Shanghai Stock Exchange. The approach helps direct the analysis to those firms most-heavily followed, researched, and traded by investors in China, with the highly liquid trading ensuring we can obtain non-stale pricing data for the entire series. Also, since only domestic individuals and institutions are able to trade in A-shares, limiting the pricing data to A-share prices allows us to focus on the domestic clustering process to better understand its dynamics.

Table 3.1 reports the summary statistics for the stock price data comprising our sample. The data encompass prices and volumes from 2003 January 2, up through 2013 July 5, slightly over a full decade of data points. The number of observations for each stock represents the number of prices obtained for that stock; hence, it is a number four times the number of dates prices were recorded.⁴⁰ As is evident, most stocks prices were typically priced in the RMB20 range over the course of the time frame. Average daily volume size in shares traded had a

⁴⁰ Note that the slight variation across stocks comes from missing data, or the occasion times trading was suspended on a stock for a given day due to the 10% up/down daily price limit exceeded on expected opening price, or CRSC concerns of stock price manipulation or other violations.

mean of approximately 37 million, although that number comprises a range from average daily volume only 2.2 million in stock 600111, up to 138.5 million done on average in stock 600348.

Table 3.1: Summary Statistics of Data Set (2003/01/02 — 2013/07/05)

Stock Number	Number of Obs (<i>n</i>)	Average Price				Range (Open Price)		Average Daily Volume
		Open	High	Low	Closing	Minimum	Maximum	
600000	9888	21.05	21.49	20.66	21.10	6.46	45.39	9144506
600010	9736	14.87	15.26	14.52	14.90	4.82	66.00	43857247
600015	9960	17.91	18.37	17.50	17.93	5.17	55.33	11801094
600016	10008	26.39	27.11	25.81	26.47	3.36	97.77	16526438
600019	9836	12.72	12.99	12.46	12.73	2.98	32.80	19221492
600028	9936	16.07	16.41	15.75	16.08	5.08	37.90	10850793
600030	9988	24.48	25.04	23.98	24.51	6.69	63.01	7313609
600031	9872	16.01	16.36	15.71	16.04	6.46	61.00	44960237
600036	9896	15.12	15.45	14.80	15.13	3.43	49.39	10625913
600048	9960	6.90	7.02	6.78	6.90	3.79	21.39	60045414
600050	9964	7.53	7.66	7.41	7.53	3.49	17.91	104110147
600058	9928	4.73	4.82	4.64	4.73	1.89	10.36	52610672
600104	9984	8.04	8.18	7.89	8.04	2.94	28.91	60258306
600111	9900	120.77	122.7	119.06	120.99	20.90	262.10	2214813
600123	9908	21.15	21.66	20.69	21.17	4.10	75.21	13245844
600188	9996	25.13	25.71	24.55	25.17	5.42	93.50	10717973
600256	9932	14.41	14.68	14.16	14.42	5.90	45.55	60166346
600348	10004	4.82	4.90	4.74	4.82	2.20	13.13	138523569
600362	9968	22.63	23.19	22.14	22.66	7.23	65.00	6393867
600489	9852	20.03	20.52	19.58	20.05	4.22	116.10	58591895
Totals:	198516	21.04	21.48	20.64	21.07	5.33	62.89	37059009

Data Analysis Methodologies

There are three main forms of data analysis in this study that I use to test for price clustering to exist on the Shanghai Stock Exchange. First I look at relative frequencies and test their statistical significance. In order to ascertain whether frequencies are significantly different from their expected values, z-test statistics are calculated. Further, following Grossman *et al.* (1997), Ikenberry & Weston (2008), and Palao & Pardo (2012), a statistical technique that captures the dispersion or concentration of values—the Hirshmann-Herfindahl

index (HHI) —is employed to measure the nature of the clustering of share prices.⁴¹ Given that random noise is likely to push values away from the expected value even under the condition of no price clustering, we use the χ^2 goodness-of-fit statistic to test if any observed deviations from the expected frequency of counts of final digits is statistically significant.⁴²

The second level of analysis conducted involves the generation of several sets of regression equations to observe the relationships between the frequency of a share price ending in a given digit, with trade price, daily trade volume, and a measure of intraday volatility defined below. The dependent variable in such regressions are binary, taking on the value of 1 when a given share price ends in the digit under consideration, and 0 if otherwise. It would be possible, therefore, to employ ordinary least squares regression to generate a linear probability model. Beck (2011) demonstrates that for most applications, the effective outperformance of logit compared to ordinary least squares under binary dependent variable conditions is small. Nonetheless, heteroscedasticity is an inherent problem within the linear probability model. Although Goldberger (1964) introduces a correction using a two-step, weighted estimator procedure to achieve unbiased estimators, even so, the linear probability model is necessarily linear in the explanatory variable, and given that $\sum \beta_k X_{ik}$ must be interpreted as a probability, this necessarily implies that estimated probabilities will outside the [0,1] interval. As such, owing to these inherent problems facing the linear probability model, I instead estimate a

⁴¹ The construction of the HHI involves summing the squared values of the frequency that trades are executed at prices ending in each of the possible digits 0 through 9. Hence, the index is formed as $HHI = \sum_{i=1 \dots n} (f_i)^2$, where f_i represents the frequency of trades executed at fraction i , with $i = 0, 1, \dots, 9$ (the possible minimum tick sizes). Given there are ten possible digits representing the minimum tick size, under the condition of no price clustering, the HHI should equal 10/100, or 0.10, which would represent a 10 percent frequency that each of the final digits 0 through 9 are observed. Values that are above 0.10 therefore represent deviations away from the no price-clustering condition.

⁴² The χ^2 statistic, with $n-1$ degrees of freedom, can be calculated as $\chi^2 = \sum_{i=1 \dots n} (A_i - E_i)^2 / E_i$, where A represents the actual observed count within the sample, and E represents the count expected given the expected distribution.

multivariate binomial logit model.⁴³ Utilizing a logit specification is useful when the dependent variable is binary. The estimated model, given the presence of price clustering, attempts to assist in understanding the key factors determining such clustering. I construct 10 separate dependent variables, each one corresponding to instances where stock prices contain the digit under consideration in the designated position. For example, when considering stock prices ending in the hundredth digit 5, the dependent variable $PCh = PC5$ (PriceClustering5) is assigned a value of 1 for all instances where prices fall on values of $x.05, x.15, \dots, x.95$; and a value 0 when prices are anything other than those ending in the digit 5. Separate iterations of the same model are run on each of the constructed dependent variables. Specifically, the logit model to be estimated is:

$$PCh_i = \alpha + \beta_1 \cdot \ln(PRICE_i) + \beta_2 \cdot \ln(VOLUME_i) + \beta_3 \cdot VOLATILITY_i + \varepsilon_i,$$

where i represents each iteration of a share price array based on a given trading day. As is standard in price clustering analysis, trade price (PRICE) is included under the supposition that as prices rise, the salience of the minimum tick size could fall, leading traders to "ignore" the final tick size and cluster onto a larger tick size. In such an instance, the expected relationship between PCh and the natural log of the share price should be positive. (Note that since we are regressing on every possible ending digit, "clustering" is understood to be present when the frequency distribution is significantly above or below the value of the expected frequency of 10%.) This expectation corresponds to both the Price Negotiation and Price Resolution hypotheses; in both instances, when the share price is high, the expected return that comes from trading on the minimum tick size is relatively low (that is, smaller than when trade price is low). As such, the expected benefits is lower from either (i) searching for additional information that would enable finer trade valuation (as per Price Resolution), or (ii) incurring

⁴³ As a robustness check, a multinomial probit model is also tested, the results of which are displayed in Appendix B.

additional bargaining—or, under the computerized bid-offer matching system used on the Shanghai Stock Exchange, additional potential bid-offer mismatches—costs (as per Price Negotiation).

Daily volume (VOLUME) has a more ambiguous interpretation in the model, for it is possible to conceptualize such trade activity for two possible proxies. The first could be termed as a measure of the degree of activity and hence "busy-ness" of trading activity. Periods of "quick action" would be those in which time becomes more valuable; when time is precious agents face higher opportunity costs from spending time engaging in other activities, such as bargaining or negotiating. According to the negotiation hypothesis, since agents have less time to negotiate to obtain their ideal price down to the finest available pricing unit, larger daily volumes would imply traders will be more likely to cluster in order to save on negotiation costs when facing greater time pressure and price risk. Hence we would expect a positive relationship between volume and clustering. Alternatively, it could be that daily volume proxies for liquidity. Liquidity is of key importance in regard to the market price formation, specifically via the price discovery process. Price discovery is the process via which a stock market incorporates new information about an asset's value into the asset's price. The more quickly a market is able to do this, the more it can be described as efficient. Price discovery is one of the primary functions of financial markets. Lehmann (2002: 259) describes price discovery as "the efficient and timely incorporation of the information implicit in investor trading into market prices"; i.e., prices themselves convey information—expectations about future activity—to market participants. Liquidity represents the opposite of thin-trading environments, where there is less information available on market participants' expectations of future because of fewer pricing points. Accordingly, Booth *et al.* (2000) provide evidence that thinly-traded markets are more likely to exhibit price clustering than

high-volume markets. In their study of the Helsinki Stock Exchange from 1993 through 1995, a very thin market at that time, they find a significant degree of price clustering behavior on ending values of 0 and 5. Hence, under the conceptualization of the price resolution hypothesis, if volume proxies for liquidity, which is itself fundamental to generating information to market participants, then an increase in volume means an increase in liquidity inferring more information available, hence lowering information costs. Under the price resolution hypothesis, lower information costs should lead to less price clustering; hence, we would posit a negative relationship between volume and clustering if volume is a proxy for liquidity.

Finally, I employ a measure of intraday volatility (VOLATILITY) as a proxy for uncertainty. Following Parkinson (1980), the intraday volatility⁴⁴ for a given stock i is calculated as:

$$(\log \text{HighPrice}_i - \log \text{LowPrice}_i)^2 / (4 \log 2).$$

The greater the volatility level, the more difficult, it can be argued, for market participants to precisely calculate asset valuation at the finest measurements; hence, increasing volatility can be viewed as equivalent to increasing uncertainty. Thus, according to the Price Resolution hypothesis, intraday volatility should be *positively* correlated with clustering. Likewise, large swings in prices also puts a premium on acting quickly in regard to one's buying and selling decisions (i.e., where waiting could mean an in-the-money trade quickly moves to the

⁴⁴ Note that Garman & Klass (1980) propose an alternative to the standard Parkinson (1980) estimator, by including information on the open and closing prices, and thereby defining intraday volatility as: $\frac{1}{2}(\log \text{HighPrice}_i - \log \text{LowPrice}_i)^2 - (2 \log 2 - 1) (\log \text{ClosingPrice}_i - \log \text{OpenPrice}_i)^2$. Christensen & Podolskij (2007), however, argue for the superiority of the Parkinson (1980) measure to produce the most efficient estimated of daily intraday volatility than other potential techniques in an idealized setting of continuous trading and no frictions. Martens & van Dijk (2006) extend this analysis to more realistic settings represented by trading frictions, informational asymmetries and microstructure noise. In any event, the use of this alternative measure of intraday volatility did not yield substantively different results, and so I only report the results using the more -commonly employed Parkinson (1980) technique.

negative). Similarly, therefore, the price negotiation hypothesis also expects a positive relationship between intraday volatility and clustering.

The third data analysis methodology I use is to consider a simulation of a set of "natural experiments". I do so in two ways. In the first instance, I utilize a change in the underlying institutional environment of the Shanghai Stock Exchange that occurred at the end of March 2010, when the CSRC introduced new regulations that lifted, for the first time on China's stock markets, its ban on short-selling. Initially allowing 90 stocks to be shorted (including, importantly for this study, all 50 stocks comprising the SSE 50 Index), this was later expanded in December 2011 to include 278 (including all those listed on the SSE 100 Index). Given this change in the underlying regulatory setting of the Shanghai Stock Exchange, I simulate a kind of "natural experiment" by regarding the time frame post-March 2010 as the "short sales" period. As such, we hope to find whether there are fundamental differences in realized price distributions indicated aggregate clustering behavior of the two samples.

Consideration of the impact of short-sales regulations can prove to be valuable in attempting to discern some of the forces driving the observed clustering behavior. As discussed, one of the theories under consideration in this study regarding the causes of price clustering is owing to the importance of information costs. As in the case discussed above in regard to the view of volume as proxying for liquidity, short-sales activity can also provide important information to market participants by aiding in the price discovery process. The argument that allowing short-selling behavior helps improve the price discovery process is well-known in the literature, including theoretical work by Miller (1977) and Diamond & Verrecchia (1987), and a growing amount of empirical work on various markets, including Chang & Yu (2004), Saffi & Sigurdsson (2010), and Boehmer & Wu (2012), all three of whom provide evidence that short-selling activity helps bring in additional investors whose

outlook is bearish, helping to correct the problem of upward bias in prices when short-selling is prohibited. Given the price negotiation hypothesis, we expect the lifting of short sales restrictions to increase market pricing information, thereby effectively reducing information costs, and hence reducing clustering activity; that is, to be negatively related to clustering.

The second instance of simulating a change in the institutional environment also utilizes daily volume data. As noted above, if volume proxies for the intensity of transactions, we would expect clustering and volume to be positively correlated, whereas if volume proxies for liquidity—and hence, information—clustering and volume would exhibit a negative relationship. *A priori*, it is unclear as to the nature of this relationship. Nonetheless, in either scenario, the presence of greater or less liquidity, or trade activity intensity, plays an important role in the price formation process in ways that should have significant impacts on observable price clustering. As such, I again break the complete pricing dataset, this time into two smaller sub-sets, one representative of periods of high market volume, and the other of low market volume. I then compare the degree of share price clustering in each of the volume environments.

4 Statistical Data Analysis and Discussion

4.1 Frequency Distribution Analysis

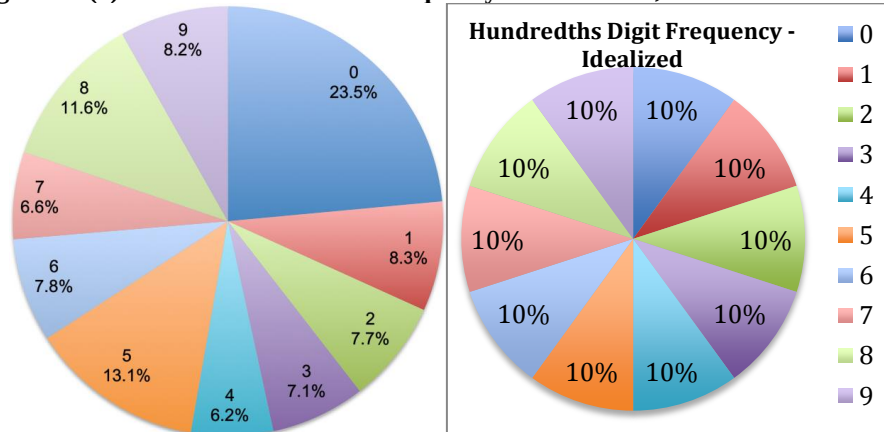
The first level of analysis centers upon determining whether in fact share price clustering exists in the Shanghai Stock Exchange, and if so in what manner and degree does it occur. I begin the analysis by first considering the degree to which price clustering takes place on the final digit of share price. Table 4.1(1) presents the summary of the distribution of final digits.

Table 4.1(1): Frequency Distribution of *hundredths* place digit, A-share daily prices

Digit	Actual		Expected (uniform)		Differences in Count		one-sample z-statistic
	Count	Frequency	Count	Frequency	Actual - Expected	Percentage	
0	46701	23.5%	19850	10.0%	26851	13.5%	200.9 *
1	16385	8.3%	19850	10.0%	-3465	-1.7%	-25.9 *
2	15320	7.7%	19850	10.0%	-4530	-2.3%	-33.9 *
3	14107	7.1%	19850	10.0%	-5743	-2.9%	-43.0 *
4	12241	6.2%	19850	10.0%	-7609	-3.8%	-56.9 *
5	25954	13.1%	19850	10.0%	6104	3.1%	45.7 *
6	15498	7.8%	19850	10.0%	-4352	-2.2%	-32.6 *
7	13032	6.6%	19850	10.0%	-6818	-3.4%	-51.0 *
8	23048	11.6%	19850	10.0%	3198	1.6%	23.9 *
9	16215	8.2%	19850	10.0%	-3635	-1.8%	-27.2 *
Totals	198501	100%	198501	100%	0.0	0.0%	--

HHI% 12.47% ; $\chi^2(9)$ 48892 (P<0.001)

Figure 4.1(1): Actual and Idealized Frequency Distributions, Hundredths Place Digit



From Table 4.1(1) and associated Figure 4.1(1), we can clearly observe the presence of price clustering, with several significant deviations from the expected (non-clustered) uniform distribution of 10% for each digit. As can be seen from the P-values on the z- statistic, all of the differences in frequencies from the expected distribution are significant. We find the 0-digit strongly over-represented at 23.5%. Likewise, share prices are more likely to end in the digit 5, at 13.1%. Finally, the digit 8 is over-represented at 11.6%, and 4 is the least represented at 6.2%. Clearly Chinese domestic investors appear to be "clustering" in the sense that there appears to be evidence that investors are executing trades ending in final digit values more or less frequently than others, based on some of the "cultural" norms described in the literature. Behavior regarding the "unlucky" number 7 is harder to discern, although the fact that it is the second-lowest appearing ending value at only 6.6% is certainly suggestive. On the other hand, in terms of other "lucky" values, such as 6 or 9, there does not seem to be signs of significant clustering.

A similar analytical technique performed above on the distribution of digits in share prices in the hundredths place, was subsequently performed for digits in the tenths place. Table 4.1(2) displays the outcome of such an analysis. The results are quite drastic when compared to the results from the distribution of digits in the hundredths place. To begin with, the overall measure of price dispersion via the HHI is very close to the value of 10% one would expect under a uniform distribution; meanwhile, the chi-square value, while still indicating a difference between the observed distribution and the expected distribution, is on the scale one hundred times lower than the value calculated for the hundredths place frequency distributions. Overall, the actual frequencies match more closely to the expected distribution of frequencies one expects from a uniform distribution of digit values in the tenths place. The one exception is the 0 digit; once again 0 is over-represented, although in a much smaller

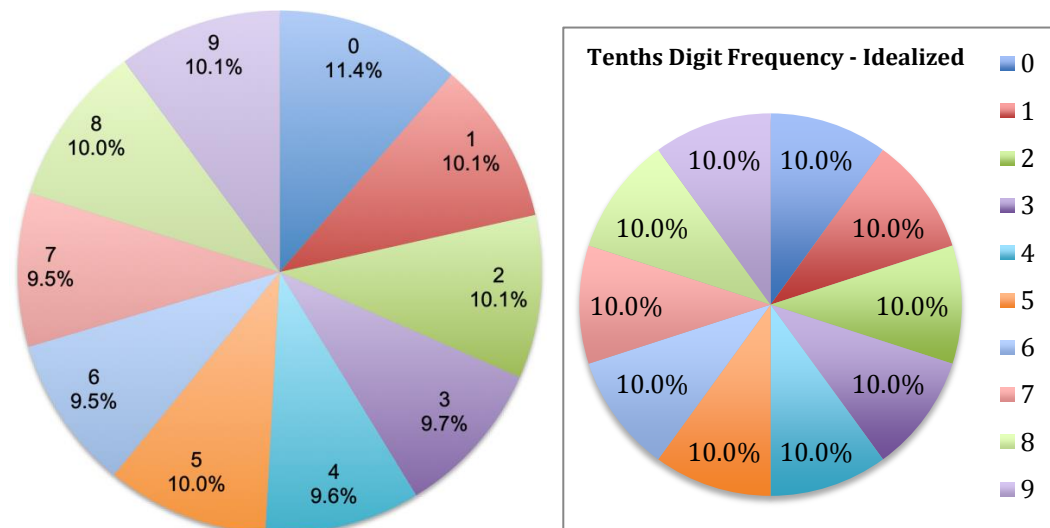
proportion that in the case of the hundredths place (11.4% versus 23.5%). This over-representation comes from clustering on pure round numbers, that is, the tendency for prices to fall on "x.00".

Table 4.1(2): Frequency Distribution of *tenths* place digit, A-share daily prices

Digit	Actual		Expected (uniform)		Differences in Count		one-sample z-statistic
	Count	Frequency	Count	Frequency	Actual - Expected	Percentage	
0	22586	11.4%	19850	10.0%	2736	1.4%	20.5 *
1	20104	10.1%	19850	10.0%	254	0.1%	1.9
2	20040	10.1%	19850	10.0%	190	0.1%	1.4
3	19292	9.7%	19850	10.0%	-558	-0.3%	-4.2 *
4	19035	9.6%	19850	10.0%	-815	-0.4%	-6.1 *
5	19860	10.0%	19850	10.0%	10	0.0%	0.1
6	18804	9.5%	19850	10.0%	-1046	-0.5%	-7.8 *
7	18920	9.5%	19850	10.0%	-930	-0.5%	-6.9 *
8	19901	10.0%	19850	10.0%	51	0.0%	0.4
9	19959	10.1%	19850	10.0%	109	0.1%	0.8
Total	198501	100%	198501	100%	0.0	0.0%	--

HHI% 10.03%
 $\chi^2(9)$ 430.7 (P<0.001)

Figure 4.1(2) Actual and Idealized Frequency Distributions , Tenths Place Digit



4.2 Multivariate Logit Regression Analysis

The next step in our analysis relate to the attempt to understand *why* we observe price clustering on the Shanghai Stock Exchange. In order to understand the dynamics of the factors impacting price clustering on the Shanghai Stock Exchange, I estimate a logit model to test the power of the explanatory model presented in Section 3. Table 4.2(1) presents the results of the logit model with 3 explanatory variables, run 10 times for each of the 10 dependent variables. Overall, the results tend to more strongly support the price resolution hypothesis, as first posited by Ball *et al.* (1985). Specifically, except for the digit 5, the natural log of the asset price $-\ln(\text{PRICE})$ is significant at the less than 0.01 level in all of the regressions. Except for prices ending in 0 or 8, the sign of the price variable is negative; that is, as a stock's price increases, the frequency of all digits besides 0 or 8 tends to fall. Conversely then, as a stock's price increases, it is more likely that the final digit of the price will end in 0 or 8. In terms of the digit 0, these results can be interpreted within the price resolution framework, in that as prices increase, obtaining more exact valuations at the smallest possible tick size becomes less cost-effective. In other words, determining if an asset's price is 1.33 or 1.34 represents a 0.75% price difference; however, determining an asset's correct valuation between 133.33 and 133.34 represents only a 0.0075% price difference. Depending on the cost of obtaining finer levels of valuation (usually due to a lack of easily-obtainable market information), such costs might not prove worthwhile to investors. As a result, investors choose to cluster on pricing levels of a higher magnitude, in particular 0.

Table 4.2(1): Logit model estimates of determinants of share price clustering, Hundredths-place digits, Daily A-share price data, Shanghai Stock Exchange Jan 2003–Jul 2013

Dependent Variable	Explanatory Variable			Model characteristics	
	ln(Price)	ln(Volume)	Volatility	McFadden R-squared	LR statistic
PC0	coefficient	0.548*	-0.091*	302.556*	
	std error	0.013	0.007	25.064	
	prob	0.000	0.000	0.000	
PC1	coefficient	-0.378*	0.066*	-150.796*	
	std error	0.025	0.012	52.117	
	prob	0.000	0.000	0.004	
PC2	coefficient	-0.374*	0.047*	-232.104*	
	std error	0.023	0.011	50.045	
	prob	0.000	0.000	0.000	
PC3	coefficient	-0.372*	0.027*	-230.864*	
	std error	0.023	0.011	51.386	
	prob	0.000	0.010	0.000	
PC4	coefficient	-0.311*	0.056*	-172.190*	
	std error	0.022	0.011	47.445	
	prob	0.000	0.000	0.000	
PC5	coefficient	-0.029	-0.027*	40.449	
	std error	0.016	0.008	31.475	
	prob	0.011	0.001	0.199	
PC6	coefficient	-0.165*	0.047*	-119.709*	
	std error	0.021	0.010	44.126	
	prob	0.000	0.000	0.007	
PC7	coefficient	-0.152*	0.045*	-174.367*	
	std error	0.021	0.010	45.890	
	prob	0.000	0.000	0.000	
PC8	coefficient	0.078*	-0.035*	104.250*	
	std error	0.016	0.008	31.576	
	prob	0.000	0.000	0.001	
PC9	coefficient	-0.089*	0.013	22.622	
	std error	0.018	0.010	36.498	
	prob	0.000	0.160	0.535	

Sample size N = 198516 (for all iterations)

* denotes significance at ≤ 0.01 level

N.B.: given its lack of economic meaning, the estimated constant was not included for the regression equations.

While the frequency data as seen in Table 4.1(1) show a general overall clustering on the digit 5, we nonetheless observe a *negative* coefficient on ln(PRICE) for the PC5 equation. Note that for our analysis purposes, we consider this result to be statistically significant, even though the *p*-value calculated from the standard error just barely grazes over our (admittedly arbitrary) 0.010 significance level at 0.011. To understand this, we must recognize that for some traders, the benefit from trading on finer prices is not worth the increase in information costs required to accurately price the equities on finer pricing points; as such, these traders are

inclined to price in values greater than the minimum tick size. Such traders, for instance, might be inclined to reduce the effective number of pricing options from 10 (0.01 tick size) to 2 (0.05 tick size), and hence cluster on digits 5 and 0, even at low pricing levels. For other traders, however, their information costs might be lower, enabling them to more readily utilize the entire set of pricing points offered by the minimum tick size, and hence chose not to engage in price clustering behavior. However, as per Ball *et al.* (1985), at higher price levels, the return to information-gathering is lower. As such, some traders who had clustered on 0 and 5 might now be inclined to reduce the effective number of pricing options from 2 (0.05 tick size) to 1 (0.1 tick size), and hence cluster only on digit 0. For these traders, high price levels would be associated with a tendency to move *away* from 5 and towards 0. Thus, there are two counteracting movements vis-à-vis the hundredths place 5 digit as prices rise—some investors moving towards 5, and others moving away. The net result apparently is such that the movement away from 5 slightly outweighs the movement towards 5. Further, note that while the sign in front of the coefficient is negative, the size effect relatively small at -0.029, in contrast to the scale of the coefficient values in front of $\ln(\text{PRICE})$ for all the other digits besides 5. Indeed, the explanatory power of the model for share prices ending in 5 is extremely low; the McFadden Adjusted R-Square value being only 0.0001, lower than any of the models run for the other digits, again suggesting counteracting investor behavior in response to changes in the explanatory variables.

The statistical relationship between price and the digit 8 presents a slight puzzle; as with digit 0, as price increases, there is a tendency for prices to end in 8 more frequently. Hence, this suggests there is a kind of "limit" as to when Chinese investors will engage in luck-related investing behavior. Overall, Chinese investors appear much more willing to consider using non-economic criteria (buying or selling when the price ends in 8 for good "luck", perhaps)

when the economic costs of doing so are relatively low. Hence we observe a positive relationship between the price level and the likelihood that a given share price will end in the digit 8.

Similar to the price variable, analysis of the relationship between the intraday volatility variable (VOLATILITY) and the relative frequency of occurrence of given digits in the hundredths place in share prices generally support the price resolution hypothesis. As seen from Table 4.2(1), the coefficient on VOLATILITY in the "PC0" model is large and statistically significant. Intraday volatility is typically taken as a proxy for uncertainty—when prices fluctuate greatly within the span of a single day, it implies significant (unexpected) news has been disseminated either about the specific stock in question, or perhaps about the state of the market in general, suggesting a period of market stress. In such circumstances, accurate market information can be more difficult and/or costly to obtain, leading to greater uncertainty on asset valuations. Such uncertainty implies less precision in price estimations, since the variance of the expected value has increased. As a result, traders are more likely to use valuations at levels greater than the minimum tick size, implying price clustering. All things equal, under the assumptions of the price resolution hypothesis, as volatility rises, we expect to see higher incidents of trades ending in 0, as traders cluster on asset price divisions larger than the minimum tick size. Since traders are moving towards clustering on 0 as volatility rises, they are simultaneously moving away from other values; as expected, we see the opposite sign on majority of other digits—the sign is negative and the coefficient significant for the models corresponding to digits 1, 2, 3, 4, 6, and 7. As volatility rises, prices are less likely to end on these values. Since traders are uncertain of exact (precise) values, clustering on higher tick sizes could be more cost-effective, so we observe movement towards the 0 digit, and away from digits 1, 2, 3, 4, 6, and 7. Likewise, the model representing the digit

5 is in keeping with a similar argument as with the digit 0, although the scale of the coefficient is considerably lower than that on 0 (40.5 compared to 302.6).

Once again, however, the coefficient in front of the digit 8 presents a result in line with the coefficient on 0, as there is a positive relationship between volatility and the likelihood for investors to buy or sell equity shares ending on the digit 8. This appears slightly puzzling, as the price resolution and negotiation theories would suggest that investors under uncertainty (as proxied by volatility) would move away from smaller tick sizes at the 0.01 level and towards larger tick sizes such as 0.05, or 0.1; hence the observed increased clustering on 5 and 0 as volatility increases. This would lead us to expect a negative sign on the coefficient in front of the volatility variable in the model representing digit 8. Perhaps, however, while some investors do engage in such "typical" clustering behavior, others—facing the increased uncertainty associated with higher volatility and market stress—in fact are more likely to turn to heuristics or psychologically-reassuring devices (such as identifying with "lucky" numbers) to help themselves cope with increased stress. Such a response, however, would fall outside the typical "rational" explanations of price clustering behavior. That said, it should be noted that the overall explanatory power of the estimated logit for the model on digit 8 is relatively low, with a McFadden Adjusted R-Square of only 0.002. We should also note that the coefficient for volatility in the regression equation representing clustering on the digit 9 lacks statistical significance. Interestingly as well, the overall explanatory power of the model representing the final digit 9 is quite low, with an adjusted R-Square value of only 0.001. We lack any particular economic rationale for such results, other than being caused by noise in the data series.

Finally, we turn to the results regarding the coefficients in front of the variable representing daily volume data $\ln(\text{VOLUME})$. Overall, these results can be perceived as

unexpected under an interpretation of volume as a proxy for increased speed of trade activity that would place a premium on fast trade resolution, as argued for by the price negotiation hypothesis. Conversely, however, if we interpret volume as a proxy for liquidity, then the logit regression results appear to nicely match those concepts associated with the price resolution hypothesis. As volume increases, the propensity to cluster onto the digits 0 and 5 falls, while the likelihood for share prices to end in digits 1, 2, 3, 4, 6 or 7 rises. Greater volume means greater information is available, implying lower information costs and a more efficient price discovery process, allowing traders to price equity shares at their minimum tick size more readily.

That said, as with the findings of the data on the digit 9 regarding intraday volatility, the likelihood for share prices to end in the digit 9 does not appear to have a statistically significant relationship with our measure of daily trade volume. In addition, and once again similar to the findings with both price level and intraday volatility, the sign of coefficient in front the daily volume variable for the model representing the digit 8 is opposite that of all the other typically "non-clustering" digits, and instead matches the negative sign in front of the typically clustered-upon digits of 0 and 5. Once again, we are confronted with a puzzle—in the face of *more* information, would Chinese investors appear to be less likely to buy or sell equity shares priced with a final digit 8.

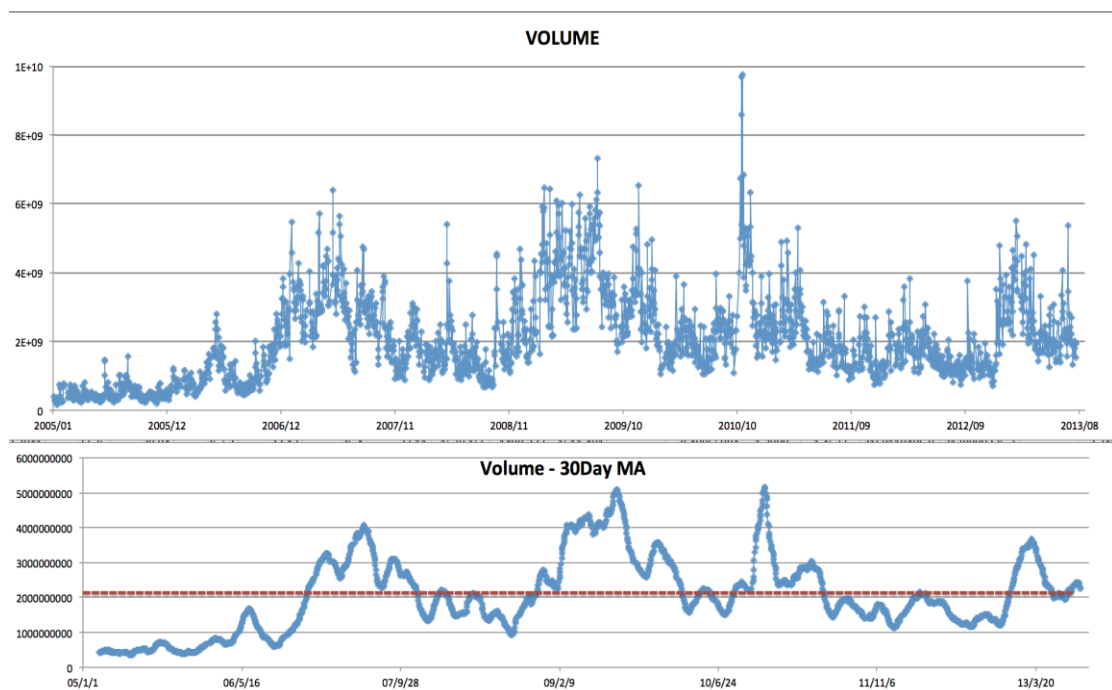
4.3 "Natural Experiments" Institutional Analysis

High/Low Volume Environments

In order to select the time periods to construct the sub-samples representing periods of high market volume and periods of low volume, I first need to identify periods of high and low market volume within the full data set. I construct the subsamples in an objective manner, by

first calculating the frequency distribution of volume data, and then calculating its standard deviation. I select those ranges that are more than one and half standard deviations higher or lower than the median value (I use the median rather than the mean given that the distribution is heavily skewed). This process can roughly be seen below in Figure 4.3(1), which graphs the daily volume of the Shanghai Stock Exchange for nearly the entire decade-long period under study. The top graph shows the raw daily volume data, while the bottom graph shows the same data but calculated at a 30-day moving average, in order to smooth the data series, with the dashed line representing the median value.

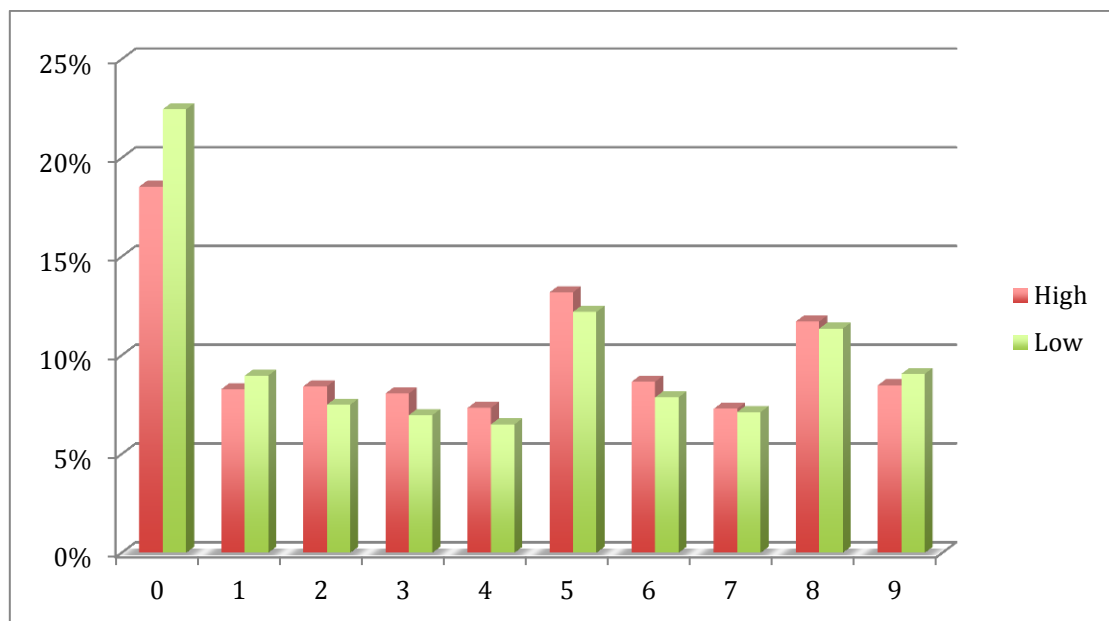
Figure 4.3(1): Daily Volume on SSE, 2005 – 2013



After obtaining the sub-samples representing periods of high and low market volume, I calculate separately the degree of price clustering in each of the two sub-samples. The results of the analysis are displayed below in Table 4.3(1) and Figure 4.3(2). Assuming that volume proxies for liquidity, which is similar to what the results of the logit regression analysis

indicate, then we can interpret the data as displaying relatively strong support for the price resolution hypothesis. As seen in the table and graph, during the high-volume period—when liquidity is high and hence information costs are relatively lower—the frequency of share price clustering on 0 is also lower: 18.5% versus 22.5%. This difference is significant at $p < 0.01$. Hence, although price clustering is slightly higher on the digit 5 (13.2% versus 12.2%), it is easy to argue that the overall occurrence of share price clustering is lower in the high-volume (higher liquidity, more information) period than in the low-volume (lower liquidity, less information) period. Overall, the respective χ^2 statistic for each of the two samples is 3093 for the high-volume period, while for the low-volume period it is 5129. While both values are therefore different from the theoretical uniform distribution at well more than $p < 0.001$, the low-volume period is noticeably more so. Overall, these results are in line with those expected under the price resolution hypothesis.

Figure 4.3(2): Comparison of High and Low-Volume Period Price Clustering



source: Author's calculations

Table 4.3(1): Comparison of High and Low-Volume Periods

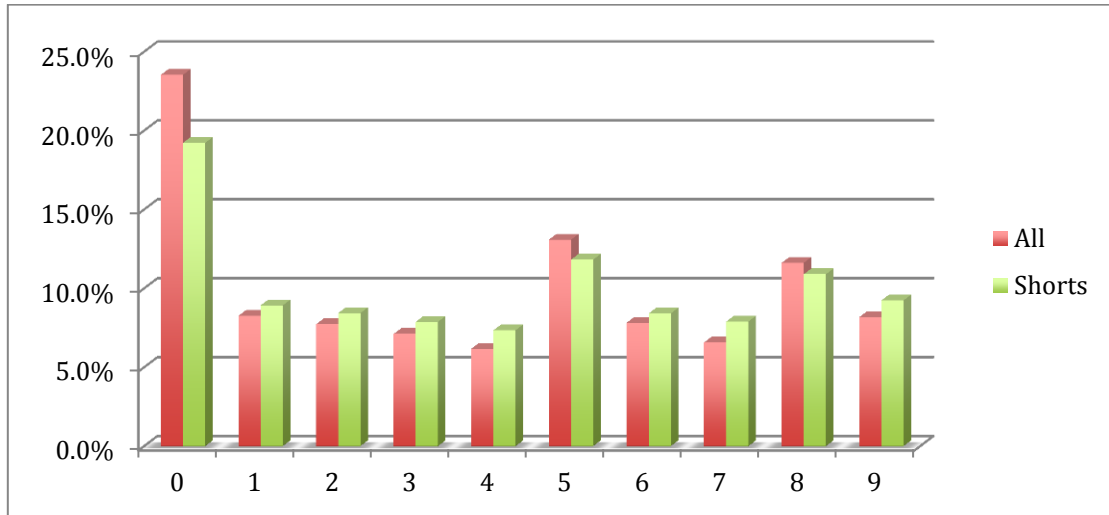
Digit	High Volume periods		Low-Volume periods		two sample t-statistic
	Count	Frequency	Count	Frequency	
0	5038	18.5%	5638	22.5%	213.0*
1	2251	8.3%	2252	9.0%	-36.0*
2	2290	8.4%	1882	7.5%	-50.76*
3	2191	8.1%	1750	7.0%	-59.52*
4	1992	7.3%	1632	6.5%	-45.3*
5	3584	13.2%	3062	12.2%	45.1*
6	2353	8.7%	1976	7.9%	-34.2*
7	1980	7.3%	1788	7.1%	9.2
8	3181	11.7%	2850	11.4%	19.2
9	2305	8.5%	2274	9.1%	-31.2*
Totals	27164	100%	25104	100%	--

* = p<0.01 level of significance

Short-Selling Environment

The growing literature described above suggesting that short sale restrictions limit the price discovery process implies that at the margin, we would expect to see more information production (and hence lower information costs, and therefore lower levels of clustering) in the sub-sample representing the period when short sales have been allowed on the Shanghai Stock Exchange, compared to when short sales are prohibited. Indeed, and again in support of the price resolution hypothesis, Table 4.3(2) and Figure 4.3(3) indicate that during the time period after the introduction of short selling was allowed on the stocks comprising the SSE 50 Index, the degree of share price clustering as observed on the stocks in our sub-sample is lower than in the entire time period.

Figure 4.3(3): Comparison of Price Clustering- Short-Sales Period vs. Entire Period



source: Author's calculations

Table 4.3(2): Comparison of Short-Sales Period and Entire Period

Digit	ALL		SHORT SALE		two sample t-statistic
	Count	Frequency	Count	Frequency	
0	46701	23.5%	11967	19.2%	234.5*
1	16385	8.3%	5552	8.9%	-36.0
2	15320	7.7%	5247	8.4%	-38.5*
3	14107	7.1%	4904	7.9%	-41.8*
4	12241	6.2%	4570	7.3%	-63.7
5	25954	13.1%	7356	11.8%	68.7*
6	15498	7.8%	5233	8.4%	-32.4*
7	13032	6.6%	4913	7.9%	-72.0*
8	23048	11.6%	6782	10.9%	39.1*
9	16215	8.2%	5748	9.2%	-57.7*
Totals	198501	100%	62272	100%	--

* = p<0.01 level of significance

The data indicate that during the time frame in which short sales were allowable, clustering behavior on the 0 digit in the hundredths place is only at 19.2%, as compared to 23.5% during the entire time frame. As indicated by the *t*-statistic, this difference is statistically significant. Clustering on other values has also fallen—down to 11.8% from 13.1

on the digit 5, and down to 10.9% from 11.6% on the digit 8; again, these comparisons are statistically significant at the $p < 0.01$ level. Hence, the greater availability of information in a short-sales institutional environment allows agents to, at the margin, more readily value and price equities at a finer price resolution. As from before, the χ^2 statistic for the entire sample is given by 48892, while that of the short-sale permitted subsample is 6967, suggesting the distribution of final digits in the latter better approaches that of the theoretical uniform distribution than the former. Overall, again, we find general support for the price resolution hypothesis.

5 Conclusions

The data and analysis presented make it clear that, at least for the largest listed firms, stock price clustering is present on the Shanghai Stock Exchange over the past decade. As has been found in nearly all studies on equity markets and price clustering, prices are most-heavily clustered around the prices whose final digit ends in values of 0, and next followed by those ending in 5. In the case of the Shanghai Stock Exchange, it is clear that the value of 0 clearly dominates.

However, also present in the Shanghai Stock Exchange is a more unique clustering phenomenon, that of the clustering on share prices ending in the digit 8. Given that 8 is viewed as "lucky" by many within Chinese society, many have posited that it is this "cultural" force driving the tendency for share prices to cluster on share prices ending in 8. As we have found, however, Chinese investors are only willing to turn to "luck" when it imposes relatively small economic costs; that is, when there is an 8 in the hundredths place, and not in the tenths. Nonetheless, the fact that clustering on 8 appears to be rising in the price of the stock remains puzzling.

The prevalence of clustering on the Shanghai Stock Exchange appears to be driven in large part by factors relating to the presence of informational and bargaining costs that are at least partly determining by the unique institutional structure of the exchange. Changes in the environment—in particular, the lifting of the short sale ban in March 2010—appears to have reduced these costs and created an incentive structure where agents are more likely to price equities more finely, and hence produce pricing distributions characterized by less prevalence of share price clustering.

Appendices

Appendix A1.

Table A1. Pure-strategy equilibrium condition: Critical value of Committee Size

	$\bar{m}(1, t)$	$\bar{m}(0.9, t)$	$\bar{m}(0.8, t)$	$\bar{m}(0.7, t)$	$\bar{m}(0.6, t)$	$\bar{m}(0.5, t)$	$\bar{m}(0.4, t)$	$\bar{m}(0.3, t)$	$\bar{m}(0.2, t)$	$\bar{m}(0.1, t)$
$\bar{m}(r, 1)$	2	3	4	5	7	11	18	34	77	313
$\bar{m}(r, 0.99)$	2	3	4	5	7	11	18	33	76	300
$\bar{m}(r, 0.9)$	1	2	3	4	6	9	14	27	64	282
$\bar{m}(r, 0.8)$	1	1	1	3	5	8	13	22	55	257
$\bar{m}(r, 0.7)$	1	1	1	3	4	6	10	19	46	212
$\bar{m}(r, 0.6)$	0	1	1	2	3	5	9	17	39	179
$\bar{m}(r, 0.5)$	0	1	1	2	3	4	8	14	34	143
$\bar{m}(r, 0.4)$	0	0	1	1	2	4	6	12	29	112
$\bar{m}(r, 0.3)$	0	0	0	1	2	3	5	10	24	100
$\bar{m}(r, 0.2)$	0	0	0	1	1	2	4	8	20	77
$\bar{m}(r, 0.1)$	0	0	0	0	1	2	3	6	15	62
$\bar{m}(r, 0.05)$	0	0	0	0	1	1	2	5	12	53
$\bar{m}(r, 0.01)$	0	0	0	0	0	1	2	3	9	36
$\bar{m}(r, 0.001)$	0	0	0	0	0	1	1	3	6	26
$\bar{m}(r, 0.000001)$	0	0	0	0	0	1	1	2	4	14

Note that $\bar{m}(r, t)$ is non-decreasing in t and non-increasing in r .

Appendix A2.

Properties of the Pure Strategy Nash Equilibrium

Proof: Any pure strategy equilibrium must be uniform bidding strategy and efforts directed towards each individual committee members satisfy $y/x=t$.

Given a pure strategy profile (X,Y) which constructs a Nash equilibrium, suppose (x_i, y_i) is the bidding strategy profile that directed to committee member i . Then as per the analysis in Part II, Section 3, the intersection of the best response functions of x_i and y_i only happens when $y_i/x_i = t$. Since the same reasoning can be applied to all committee members, the bidding efforts directed toward any committee member must satisfy $\frac{y_i}{x_i} = t$ for any $i \in [1, 2m+1]$, otherwise at least one of the players will deviate. Next we prove that any non-uniform pure strategy will not be equilibrium. Following similar arguments in the Lemma of Clark & Konrad (2007), we prove this by contradiction. Assume that $y_1 = tx_1 < y_2 = tx_2$ constructs a pure strategy equilibrium, given any strategy profile directed to the other $(2m-1)$ committee members ($i \neq 1, 2$). Let $\bar{p}(m)$ denote the probability that contestant 1 wins exactly m votes out of the rest of the $(2m-1)$ committee members, and let $\bar{p}(m-1)$ denote the probability that contestant 1 wins exactly $m-1$ votes out of the rest of the $(2m-1)$ committee members. Note that only under the two above scenarios, efforts directed to Committee Member 1 and 2 matters; otherwise one of the contestants wins (loses) with certainty, independent of the outcome of votes by the committee members 1 and 2. If $y_1 = tx_1 < y_2 = tx_2$ constructs any equilibrium, then any reallocation of effort directed towards committee members 1 and 2 will not increase contestant 1's probability of winning the contest, which means

$$\bar{P} \equiv \frac{x_1^r}{x_1^r + y_1^r} \cdot \frac{x_2^r}{x_2^r + y_2^r} \cdot \bar{p}(m-1) + \left(1 - \frac{y_1^r}{x_1^r + y_1^r} \cdot \frac{y_2^r}{x_2^r + y_2^r}\right) \cdot \bar{p}(m)$$

cannot be increased. We can see this by way of example, with a small increase in x_1 and small decrease in x_2 such that $dx_1 = -dx_2 > 0$ resulting in,

$$\begin{aligned} \Delta \bar{P} = & \left(\frac{rx_1^{r-1}y_1^r}{(x_1^r + y_1^r)^2} \cdot \frac{x_2^r}{x_2^r + y_2^r} - \frac{x_1^r}{x_1^r + y_1^r} \cdot \frac{rx_2^{r-1}y_2^r}{(x_2^r + y_2^r)^2} \right) \bar{p}(m-1) \\ & - \left(\frac{-rx_1^{r-1}y_1^r}{(x_1^r + y_1^r)^2} \cdot \frac{y_2^r}{x_2^r + y_2^r} + \frac{rx_2^{r-1}y_2^r}{(x_2^r + y_2^r)^2} \cdot \frac{y_1^r}{x_1^r + y_1^r} \right) \bar{p}(m) \end{aligned}$$

Evaluating $\Delta \bar{P}$ at $y_1 = tx_1$ and $y_2 = tx_2$ obtains

$$\Delta \bar{P} = \bar{p}(m-1) \frac{rt^r}{(1+t^r)^3} \left(\frac{1}{x_1} - \frac{1}{x_2} \right) + \bar{p}(m) \frac{rt^{2r}}{(1+t^r)^3} \left(\frac{1}{x_1} - \frac{1}{x_2} \right)$$

Therefore $\Delta \bar{P}$ is positive if $x_1 < x_2$, which contradicts the assumption that $y_1 = tx_1 < y_2 = tx_2$ constructs a pure strategy equilibrium, since now a small increase in x_1 and small decrease in x_2 can increase Contestant 1's probability of winning without imposing additional costs. Hence, any pure strategy equilibrium must be a uniform bidding strategy, and efforts directed towards each individual committee member satisfying $y/x=t$.

Appendix A3.

The impact of committee size on PSNE condition under simple majority voting

Proof: Given r and t , as m grows, the participation constraint becomes more difficult to hold under simple majority voting.

The pure strategy equilibrium condition is expressed as

$$r \leq \frac{1 + \alpha}{(m + 1)} \cdot \sum_{i=0}^m \frac{C_{2m+1}^{m+1+i} \alpha^i}{C_{2m+1}^{m+1}}$$

Define $\text{RHS}(m) \equiv \frac{1+\alpha}{(m+1)} \cdot \sum_{i=0}^m \frac{C_{2m+1}^{m+1+i} \alpha^i}{C_{2m+1}^{m+1}}$, therefore it is equivalent to prove that $\text{RHS}(m)$ is non-increasing in m , which is $\frac{\text{RHS}(m)}{\text{RHS}(m+1)} \geq 1$. Rewriting $\frac{\text{RHS}(m)}{\text{RHS}(m+1)}$, it is equivalent to prove

$$\frac{\text{RHS}(m)}{\text{RHS}(m+1)} = \frac{\sum_{i=0}^m C_{2m+1}^{m+1+i} \alpha^i}{\sum_{i=0}^{m+1} C_{2m+3}^{m+2+i} \alpha^i} \cdot \frac{(m+2)C_{2m+3}^{m+2}}{(m+1)C_{2m+1}^{m+1}} \geq 1$$

Reorganizing, we obtain

$$(4m+6) \sum_{i=0}^m C_{2m+1}^{m+1+i} \alpha^i - (m+1) \sum_{i=0}^{m+1} C_{2m+3}^{m+2+i} \alpha^i \geq 0$$

Denote $F(\alpha) \equiv (4m+6) \sum_{i=0}^m C_{2m+1}^{m+1+i} \alpha^i - (m+1) \sum_{i=0}^{m+1} C_{2m+3}^{m+2+i} \alpha^i$, such that

$$F(\alpha) = \sum_{i=0}^m [(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i}] \alpha^i - (m+1)\alpha^{m+1}$$

Note that the sign of $(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i} = \frac{(4m+6) \cdot (2m+1)! \cdot (m+1-i-i^2)}{(m+2+i)! \cdot (m+1-i)!}$ is determined by the component of $m+1-i-i^2$. For a given m , assume i_0 is the smallest value of i such that $m+1-i-i^2 < 0$; therefore, $m+1-i-i^2 \geq 0$, for $i \leq i_0 - 1$; and $m+1-i-i^2 < 0$, for $i \geq i_0$. Hence we can obtain the following:

$$(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i} \geq 0, \quad \text{if } i \leq i_0 - 1$$

$$(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i} < 0, \quad \text{if } i \geq i_0$$

Next, reorganize $F(\alpha)$ so that,

$$\begin{aligned} F(\alpha) &= \sum_{i=0}^{i=i_0-1} [(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i}] \alpha^i \\ &\quad - \sum_{i=i_0}^m [(m+1)C_{2m+3}^{m+2+i} - (4m+6)C_{2m+1}^{m+1+i}] \alpha^i - (m+1)\alpha^{m+1} \\ &\equiv A - B - (m+1)\alpha^{m+1} \end{aligned}$$

Here, A represents the first part of summation; B represents the second part of summation.

Note that $\alpha \in (0,1]$,

$$A \geq \alpha^{i_0-1} \sum_{i=0}^{i_0-1} [(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i}], \text{ and}$$

$$B \leq \alpha^{i_0} \sum_{i=i_0}^m [(m+1)C_{2m+3}^{m+2+i} - (4m+6)C_{2m+1}^{m+1+i}]$$

$$= \alpha^{i_0} [(m+1) (2^{2m+2} - 1 - \sum_{i=0}^{i_0-1} C_{2m+3}^{m+2+i}) - (4m+6)(2^{2m} - \sum_{i=0}^{i_0-1} C_{2m+1}^{m+1+i})]$$

Therefore,

$$F(\alpha) = A - B - (m+1)\alpha^{m+1}$$

$$\begin{aligned} &\geq (\alpha^{i_0-1} - \alpha^{i_0}) \sum_{i=0}^{i_0-1} [(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i}] + 2^{2m+1}\alpha^{i_0} \\ &\quad + (m+1)\alpha^{i_0} - (m+1)\alpha^{m+1} > 0 \end{aligned}$$

Recall that $(4m+6)C_{2m+1}^{m+1+i} - (m+1)C_{2m+3}^{m+2+i} \geq 0$, for $i \leq i_0 - 1$, and all the other components are non-negative, $(\alpha^{i_0-1} - \alpha^{i_0}) \geq 0$, and $(m+1)(\alpha^{i_0} - \alpha^{m+1}) \geq 0$.

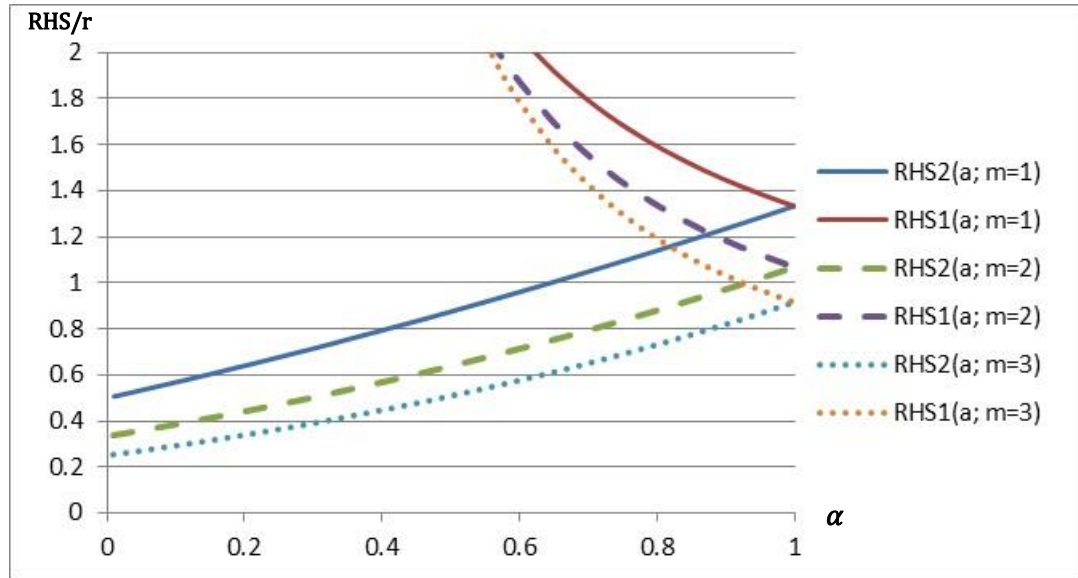
Hence, as m increases, $\text{RHS}(m)$ decreases, implying that it is more difficult for the pure strategy Nash equilibrium condition to hold.

Appendix A4.

PSNE existence condition

(1) PSNE existence condition under simple majority rule ($d=1$)

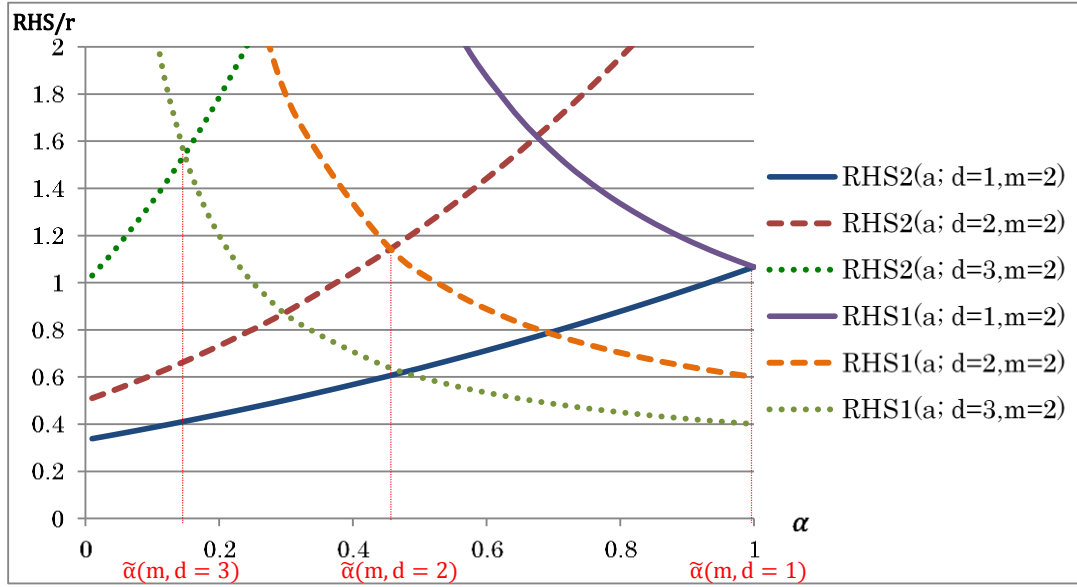
The PSNE existence condition for $d=1$ can be simply captured by $r \leq \text{RHS}^2(\alpha)$, since the participation constraints of Contestant 2 ($\text{RHS}^2(\alpha)$) are always lower than that of Contestant 1 ($\text{RHS}^1(\alpha)$). Reminding that both $\text{RHS}^1(\alpha)$ and $\text{RHS}^2(\alpha)$ are continuous in α , $\text{RHS}^1(\alpha)$ is decreasing in α , $\text{RHS}^2(\alpha)$ is increasing in α . Note the critical value of α such that the two contestants win with equal probability only happens when they are homogeneous, which is $\tilde{\alpha}(m, d) = 1$.



(2) PSNE existence condition under super-majority rule ($d \geq 2$)

There are two concerns we need to consider when calculating the PSNE existence condition. First, the combination of voting rule d and the level of contestant heterogeneity together determine which of the two contestants is the effectively-"weak" player; second, the PSNE existence condition requires that the effectively-"weak" player's participation constraint be satisfied. Note that the Right-hand-side of participation constraint of Contestant 2 ($\text{RHS}^2(\alpha)$) is increasing in α ; and the Right-hand-side of participation constraint of Contestant 1 ($\text{RHS}^1(\alpha)$) is decreasing in α , and that there exists a unique $\tilde{\alpha}(m, d) \in (0, 1]$ such that

$\text{RHS}^1(\tilde{\alpha}, d) = \text{RHS}^2(\tilde{\alpha}, d)$. Therefore, the PSNE condition can be captured by part of each of the curves of $\text{RHS}^2(\alpha)$ and $\text{RHS}^1(\alpha)$ that fall below their point of intersection.



(3) $\tilde{\alpha}$ decreases in d

As d increases, intuitively we would expect $\tilde{\alpha}$ to decrease. This can be proved through the following calculation. Suppose $\tilde{\alpha}$ is the critical value such that $\text{RHS}^2(\alpha; d) = \text{RHS}^1(\alpha; d)$. Define

$$C_1(\alpha) = C_{2m+1}^0 \alpha^{2m+1} + C_{2m+1}^1 \alpha^{2m} + \dots + C_{2m+1}^m \alpha^{m+1} + \dots + C_{2m+1}^{m+d-1} \alpha^{m+2-d}; \text{ and}$$

$$C_2(\alpha) = C_{2m+1}^0 \alpha^0 + C_{2m+1}^1 \alpha^1 + \dots + C_{2m+1}^{m+1-d} \alpha^{m+1-d}.$$

Therefore,

$$C_1(\tilde{\alpha}) = C_2(\tilde{\alpha})$$

When the voting rule d increases by 1, to $d+1$, we have

$$\begin{aligned} & \text{RHS}^2(\tilde{\alpha}; d+1) - \text{RHS}^1(\tilde{\alpha}; d+1) \\ &= \frac{(1 + \tilde{\alpha})}{(2m+1)C_{2m}^{m+d}\tilde{\alpha}^{m-d+1}} [C_1(\tilde{\alpha}) + C_{2m+1}^{m+d}\tilde{\alpha}^{m-d+1} \\ & \quad - (C_2(\tilde{\alpha}) - C_{2m+1}^{m+1-d}\tilde{\alpha}^{m+1-d})] \\ &= \frac{(1 + \tilde{\alpha})}{(2m+1)C_{2m}^{m+d}\tilde{\alpha}^{m-d+1}} (C_{2m+1}^{m+d}\tilde{\alpha}^{m-d+1} + C_{2m+1}^{m+1-d}\tilde{\alpha}^{m+1-d}) > 0 \end{aligned}$$

Since $\text{RHS}^2(\alpha; d+1) - \text{RHS}^1(\alpha; d+1)$ is a function increasing in α , $\tilde{\alpha}(m, d+1) < \tilde{\alpha}(m, d)$.

(4) Impact of d on PSNE condition

The graph below shows the difficulty in identifying the movements of $RHS^2(\alpha; d)$ and $RHS^1(\alpha; d)$ as d varies; nonetheless, we can prove that if α is relatively large, increasing the bias level d can help the participation constraint (PC) of Contestant 2 to hold. In other words, $RHS^2(\alpha; d)$ increases in d when α is relatively large. If α is relatively small, increasing the bias level d worsens the PC of the Contestant 1, i.e., $RHS^1(\alpha; d)$ decreases in d at relatively low values of α .

$$\begin{aligned} RHS^2(\alpha; d) - RHS^2(\alpha; d + 1) \\ = \frac{(1 + \alpha)}{(2m + 1)(m + d)C_{2m}^{m-d}\alpha^{m-d+1}} \left[\left(\frac{m - d + 1}{\alpha} - m - d \right) \cdot C_1 \right. \\ \left. - (m + d)C_{2m+1}^{m+d}\alpha^{m+d-1} \right] \end{aligned}$$

Therefore if $\alpha \geq \frac{m+1-d}{m+d}$, $\frac{m-d+1}{\alpha} - m - d \leq 0$, we find

$$RHS^2(\alpha; d) - RHS^2(\alpha; d + 1) < 0$$

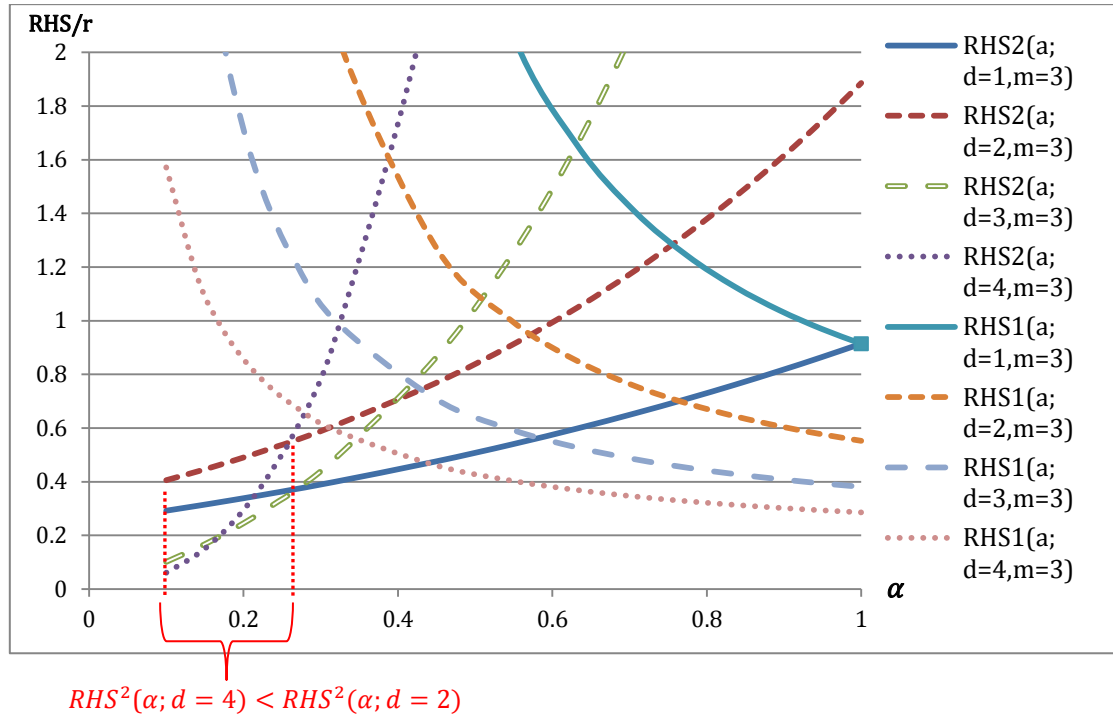
Similarly, we can obtain the following

$$\begin{aligned} RHS^1(\alpha; d) - RHS^1(\alpha; d + 1) \\ = \frac{(1 + \alpha)}{(2m + 1)(m + d)C_{2m}^{m-d}\alpha^{m-d+2}} \left[((m + 1 - d - \alpha(m + d)) \cdot C_2 \right. \\ \left. + (m + d)C_{2m+1}^{m+1-d}\alpha^{m+2-d} \right] \end{aligned}$$

Therefore if $\alpha \leq \frac{m+1-d}{m+d}$, $RHS^1(\alpha; d) - RHS^1(\alpha; d + 1) > 0$.

However, when α is not in the above defined range, it proves difficult to identify how varying voting rule d influences the PCs. The intuition is that although increasing the bias level can increase Contestant 2's probability of winning, it may also make the contest more competitive (when $\alpha < \tilde{\alpha}(m, d)$) such that both contestants increase their costly efforts. Whether the benefit from the increase in probability of winning outweighs the increasing effort is ambiguous. As we can see from the following example ($m=3$), although the biased rule $d = 4$ gives more advantage to the weak contestant 2 compared to the biased rule $d = 2$, when α is relatively small, the extra advantage is outweighed by the increase in efforts. Therefore we observe that $RHS^2(\alpha; 4) < RHS^2(\alpha; 2)$ when α takes relatively small values, as highlighted (in red text) in the graph below. The PSNE condition when $d \geq 2$ can be observed by the part of curves below the intersection point of RHS^2 and RHS^1 . For a given value of α , when voting rules changes, the PSNE condition can become either stricter or more relaxed.

In addition, the graph shows that $\tilde{\alpha}(m, d)$, which is the heterogeneity level at which RHS^1 and RHS^2 intersect, decreases in d . Note that if r is larger than the level of the intersection, the PSNE condition does not hold for any t .



Appendix B: Output for Multinomial Probit model

Table B1: Multinomial Model, 10-category (digits 0 – 9) Dependent Variable

Dependent Variable values	Explanatory Variables		
	ln(Price)	ln(Volume)	Volatility
0	(base outcome)		
1	coefficient	-0.4920*	0.0831*
	prob	0.000	0.000
2	coefficient	-0.4982*	0.0748*
	prob	0.000	0.000
3	coefficient	-0.4961*	0.0646*
	prob	0.000	0.000
4	coefficient	-0.4656*	0.0785*
	prob	0.000	0.000
5	coefficient	-0.3101*	0.0337*
	prob	0.000	0.000
6	coefficient	-0.3872*	0.0738*
	prob	0.000	0.000
7	coefficient	-0.3799*	0.0726*
	prob	0.000	0.000
8	coefficient	-0.2506*	0.0290*
	prob	0.000	0.000
9	coefficient	-0.2470*	0.0551*
	prob	0.000	0.000

* denotes significance at the <0. 001 level

Table B2: Multinomial Probit model: Marginal effects

Dependent Variable values	Explanatory Variables			
		ln(Price)	ln(Volume)	Volatility
0	coefficient	0.0897*	-0.0151*	49.226*
	prob	0.000	0.000	0.000
1	coefficient	-0.0204*	0.0039*	-8.157*
	prob	0.000	0.000	0.0004
2	coefficient	-0.0248*	0.0034*	-14.957*
	prob	0.000	0.000	0.000
3	coefficient	-0.0237*	0.0020*	-13.867*
	prob	0.000	0.004	0.000
4	coefficient	-0.0211*	0.0040*	-12.369*
	prob	0.000	0.000	0.000
5	coefficient	-0.0005	-0.0033*	5.807
	prob	0.797	0.001	0.134
6	coefficient	-0.0112*	0.0036*	-8.422*
	prob	0.000	0.000	0.008
7	coefficient	-0.0101*	0.0034*	-12.174*
	prob	0.000	0.000	0.000
8	coefficient	0.0118*	-0.0042*	12.403*
	prob	0.000	0.000	0.001
9	coefficient	0.0104	0.0011	2.509
	prob	0.029	0.195	0.448

* denotes significance at the <0. 001 level

Bibliography

- Acemoglu, D., Johnson, S., and Robinson, J. (2005). The Rise of Europe: Atlantic Trade, Institutional Change and Economic Growth, *American Economic Review*, 95(3), pp. 546-579.
- Ahn, K., Cai, J. and Cheung, Y. (2005). Price clustering on the limit-order book: Evidence from the Stock Exchange of Hong Kong, *Journal of Financial Markets*, vol. 8, pp.421-451.
- Aitken, M., Brown, P., Buckland, C., Izan, H., and Walter, T. (1996). Price clustering on the Australian Stock Exchange, *Pacific Basin Finance Journal*, vol. 4, pp. 297-314.
- Alcalde, J. and Dahm, M. (2010). Rent seeking and rent dissipation: A neutrality result, *Journal of Public Economics*, vol. 94, pp. 1-7.
- Allard, R. (1988). Rent seeking with non-identical players, *Public Choice*, vol. 57, pp. 3-14.
- Amegashie, J. (1999). The Number of Rent-Seekers and Aggregate Rent-Seeking Expenditures: An Unpleasant Result, *Public Choice*, vol. 99, pp. 57-62.
- Amegashie, J. (2002). Committees and rent-seeking effort under probabilistic voting, *Public Choice*, vol. 112, pp. 345-350.
- Amoros, P. and Puy, M. (2010). Indicators of electoral victory, *Public Choice*, vol. 144(1), pp. 239-251.
- Amoros, P. and Puy, M. (2013). Issue convergence or issue divergence in a political campaign?, *Public Choice*, vol. 155(3), pp. 355-371.
- Angel, J. (1997). Tick Size, Share Prices, and Stock Splits, *Journal of Finance*, vol. 52(2), pp.655-681.
- Ascioglu, A. and Comerton, F. (2007). Price clustering on the Tokyo stock exchange, *Financial Review*, vol. 42(2), pp. 289-301.
- Bagehot, W. (1971). The only game in town, *Financial Analysts Journal*, vol. 27(2), pp.12-14.
- Baik, K. (1994). Effort Levels in Contests with Two Asymmetric Players, *Southern Economic Journal*, vol. 61, pp. 367-378.
- Baik, K. (2004). Two-Player Asymmetric Contests with Ratio-Form Contest Success Functions, *Economic Inquiry*, vol. 42(4), pp. 679-689.
- Ball, C., Torous, W. and Tschoegl, A. (1985). The degree of price resolution: the case of the gold market, *Journal of Futures Markets*, vol. 5, pp. 29-43.
- Baye, M. and Hoppe, C. (2003). The strategic equivalence of rent-seeking, innovation, and patent-race games, *Games and Economic Behavior*, vol. 44(2), pp. 217-226.

- Baye, M., Kovenock D. and de-Hries, C. (1996). The all-pay auction with complete information, *Economic Theory*, vol. 8, pp.1-25.
- Baye, M., Kovenock, D., and de-Vries, C. (2005). Comparative analysis of litigation systems: an auction-theoretic approach, *Economic Journal*, vol. 115, pp. 583-601.
- Beck, N. (2011). Is OLS with a binary dependent variable really OK?: Estimating (mostly) TSCS models with binary dependent variables and fixed effects, Working Paper, NYU.
- Besley, T. and Preston, I. (2007). Electoral Bias and Policy Choice: Theory and Evidence, *Quarterly Journal of Economics*, vol. 122(4), pp. 1473-1510.
- Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing, *Journal of Business*, vol. 45(3), pp.444-455.
- Black, F., Jensen, M., and Scholes, M. (1972). The Capital Asset Pricing Model: Some empirical tests, in Jensen, M. (ed.), Studies in the Theory of Capital Markets. New York: Praeger.
- Boehmer, E. and Wu, J. (2012). Short selling and the price discovery process, *Review of Financial Studies*, advance online access.
- Booth, G., Kallunki, J., Lin, J., and Martikainen, T. (2000). Internationalization and Stock Price Clustering: Finnish Evidence, *Journal of International Money and Finance*, vol. 19, pp. 737-751.
- Boye, S. (2005). Cultural Factors that Shape Investment Decisions, *Pakistan Journal of Social Sciences*, vol. 3, pp. 495-502.
- Brams, S. and Davis, M. (1973). Models of Resource Allocation in Presidential Campaigning: Implications for Democratic Representation, *Annals of the New York Academy of Sciences*, vol. 219, pp. 105-123.
- Brown, P., Chua, A., and Mitchell, J. (2002). The influence of cultural factors on price clustering: evidence from Asia-Pacific stock markets, *Pacific Basin Finance Journal*, vol. 10, pp. 307-332.
- Brown, P. and Mitchell, J. (2008). Culture and stock price clustering-Evidence from The Peoples' Republic of China, *Pacific-Basin Finance Journal*, vol. 16, pp. 95-120.
- Brown, S., Laux, P., and Schachter, B. (1991). On the Existence of an Optimal Tick Size, *Review of Futures Markets*, vol. 10, pp. 50-72.
- Caselli, F., Cunningham, T., Morelli, M., and Barreda, I. (2012). Signalling, Incumbency Advantage, and Optimal Reelection Rules, CEP Discussion Papers dp1122, Centre for Economic Performance, LSE.

- Chang, E. and Yu, Y. (2007). Short-Sales Constraints and Price Discovery: Evidence from the Hong Kong market, *Journal of Finance*, vol. 62(5), pp. 2097-2121.
- Chase, W and Simon, H. (1973). Perception in chess, *Cognitive Psychology*, vol. 4, pp. 55-81.
- Che, Y. and Gale, I. (2003). Optimal Design of Research Contests, *American Economic Review*, vol. 93(3), pp. 646-671.
- Chiao, C. and Wang, Z. (2009). Price Clustering: Evidence Using Comprehensive Limit-Order Data, *The Financial Review*, vol. 44, pp. 1-29.
- Christensen, K. and Podolskij, M. (2007). Realised range-based estimation of integrated variance, *Journal of Econometrics*, vol. 141, pp. 323-349.
- Christie, W., Harris, J., and Schultz, P. (1994). Why Did NASDAQ Market Makers Stop Avoiding Odd-Eighth Quotes?, *Journal of Finance*, vol. 49(5), pp. 1841-1860.
- Christie, W. and Schultz, P. (1994). Why do NASDAQ market makers avoid odd-eighth quotes?, *Journal of Finance*, vol. 49, pp. 1813-1840.
- Chu, S. and Lim, G. (2001). The Proposed Merger of GK Goh and Vickers Ballas, *Asian Case Research Journal*, vol. 5(2), pp. 141-165.
- Chung, K., Chuwonganant, C., and McCormick, D. (2004). Order preferencing and market quality on NASDAQ before and after decimalization, *Journal of Financial Economics*, vol. 71 (3), pp. 581-612.
- Chung, T. (1996). Rent-seeking contest when the prize increases with aggregate efforts, *Public Choice*, vol. 87, pp. 55-66.
- Clark, D. and Konrad, K. (2007). Contests with multi-tasking, *Scandinavian Journal of Economics*, vol. 109(2), pp. 303-319.
- Clark, D. and Konrad, K. (2008). Fragmented Property Rights and Incentives for Rand D, *Management Science*, vol. 54(5), pp. 969-981.
- Clark, D. and Riis, C. (1996). A multi-winner nested rent-seeking contest, *Public Choice*, vol. 87, pp. 177-184.
- Clark, D. and Riis, C. (1998). Competition over more than one prize, *American Economic Review*, vol. 88, pp. 276-289.
- Coase, R. (1937). The Nature of the Firm, *Economica*, vol. 4, no. 16, pp. 386-405.
- Coase, R. (1960). The Problem of Social Cost, *Journal of Law and Economics*, vol. 3(1), pp. 1-44.

- Condorcet, J. and Marquis de, N. (1785). Essai sur l'application de l'analyse à la probabilité des décisions: rendues à la pluralité des voix. Paris: L'imprimerieroyale.
- Congleton, R. (1984). Committees and rent-seeking effort, *Journal of Public Economics*, vol. 25, pp. 197-209.
- Congleton, R., Hillman, A., and Konrad, K. (Eds.) (2008). 40 Years of Research on Rent Seeking, Volumes I and II. Berlin: Springer.
- Cooney, J., Van Ness, B., and Van Ness, R. (2003). Do investors prefer eight-even prices? *Journal of Banking and Finance*, vol. 27, pp. 719-748.
- Corchón, L. (2007). The theory of contests: a survey, *Review of Economic Design*, vol. 11(2), pp. 69-100.
- Cowan N. (2000). The magical number 4 in short-term memory: a reconsideration of mental storage capacity, *Behavioral and Brain Sciences*, vol. 24, pp. 87-185.
- Darrat, A. and Zhong, M. (2000). On Testing the Random-Walk Hypothesis: A Model-Comparison Approach, *The Financial Review*, vol. 35, pp. 105-124.
- Delbono, F. and Denicolo, V. (1991). Incentives to Innovate in a Cournot Oligopoly, *Quarterly Journal of Economics*, vol. 106(3), pp.951-961.
- Diamond, D. and Verrecchia, R. (1987). Constraints on short-selling and asset price adjustment to private information, *Journal of Financial Economics*, vol. 18, pp. 277-311.
- Dixit, A. (1987). Strategic Behavior in Contests, *American Economic Review*, vol. 77(5), pp. 891-898.
- Doucouliafos, H. (2004). Number preference in Australian stocks, *Applied Financial Economics*, vol. 14(1), pp. 43-54.
- Epstein, G., and Nitzan, S. (2006). Reduced prizes and increased effort in contests, *Social Choice and Welfare*, vol. 26 (3), pp. 447-453.
- Eun, C. and Huang, W. (2007). Asset pricing in China's domestic stock markets: Is there a logic?, *Pacific-Basin Finance Journal*, vol. 15, pp. 452-480.
- Fama, E. (1965). The behavior of stock-market prices, *Journal of Business*, vol. 38(1), pp. 34-105.
- Fama, E. (1970). Efficient capital markets: A review of theory and empirical work, *Journal of Finance*, vol. 25, pp.383-417.
- Fama, E. (1991). Efficient Capital Markets: II, *Journal of Finance*, vol. 46(5), pp. 1575-1617.

- Ferson W. (2003). Tests of Multifactor Pricing Models, Volatility Bounds and Portfolio Performance. in G.M. Constantinides, M. Harris and R. Stulz (eds.), Handbook of the Economics of Finance, Amsterdam: Elsevier, pp. 743-793.
- Fifield, S. and Jetty, J. (2008). Further evidence on the efficiency of the Chinese stock market: A note, *Research in International Business and Finance*, vol. 22, pp. 351-361.
- Fischer, A. (2004). Price Clustering in the FX Market: A Disaggregate Analysis using Central Bank Interventions, Working Paper no. 04.04, Swiss National Bank, Zurich, SWI.
- Fonseca, M. (2009). An Experimental Investigation of Asymmetric Contests, *International Journal of Industrial Organization*, vol. 27, pp. 582-591.
- Franke, J. (2012). Affirmative action in contest games, *European Journal of Political Economy*, vol. 28 (1), pp. 105-118.
- Friedman, L. (1958). Game-Theory Models in the Allocation of Advertising Expenditures, *Operations Research*, vol. 6(5), pp. 699-709.
- Fu, Q., Jiao, Q., and Lu, J. (2010). Contest Design with Endogenous Entry: Disclosure of Number of Contestants and Choice of Contest Technology, *Economic Inquiry*, vol. 48(1), pp. 80-88.
- Garfinkel, M. and Skaperdas, S. (2007). Economics of Conflict: An Overview, in T. Sandler and K. Hartley (eds.), Handbook of Defense Economics, Volume 2. Amsterdam: Elsevier, pp. 650-705.
- Gersbach, H. (1995). Information Efficiency and Majority Decisions, *Social Choice and Welfare*, vol. 12, pp. 363-370.
- Gilligan, T.W. and Krehbiel, K. (1989). Asymmetric information and legislative rules with a heterogeneous committee, *American Journal of Political Science*, vol. 33, pp. 459-490.
- Gobet F. and Clarkson, G. (2004). Chunks in expert memory: evidence for the magical number four or is it two?, *Memory*, vol. 12, pp. 732-747.
- Godek, P. (1996). Why NASDAQ market makers avoid odd-eighth quotes, *Journal of Financial Economics*, vol. 41, pp. 465-474.
- Goldberger, A. (1964). Econometric Theory. New York: John Wiley and Sons.
- Goodhart, C. and Curcio, R. (1991). The clustering of bid/ask prices and the spread in the foreign exchange market, Discussion paper no. 110, Financial Markets Group Discussion Paper Series (LSE, London).
- Gottlieb, G. and Kalay, A. (1985). Implications of the Discreteness of Observed Stock Prices, *The Journal of Finance*, vol. 40(1), pp. 135-153.

- Gradstein, M. (1995). Intensity of competition, entry and entry deterrence in rent seeking contest, *Economics and Politics*, vol. 7, pp. 79-91.
- Grossman, S., Miller, M., Cone, K., Fischel, D. and Ross, D. (1997). Clustering and competition in asset markets, *Journal of Law and Economics*, vol. 40, pp. 23-60.
- Grossman, S. and Stiglitz, J (1980). On the impossibility of informationally efficient markets, *American Economic Review*, vol. 70(3), pp. 393-408.
- Hameed, A. and Terry, E. (1998). The Effect of Tick Size on Price Clustering and Trading Volume, *Journal of Business Finance and Accounting*, vol. 25, pp. 849-867.
- Harris, L. (1991). Stock price clustering and discreteness, *Review Financial Studies*, vol. 4, pp. 389-415.
- Hillman, A. and Riley, J. (1989). Politically contestable rents and transfers, *Economics and Politics*, vol. 1, pp. 17-39.
- Hörtnagl, T., Kerschbamer, R., Stracke, R. and Sunde, U. (2013). Heterogeneity in rent-seeking contests with multiple stages: Theory and experimental evidence, CESifo Working Paper no. 4435.
- Hsieh, T. and Lin, C. (2010). Impact of Tick-Size Reduction on the Price Clustering: Evidence from the Emerging Order-Driven Market, *Asia Pacific Management Review*, vol. 15(4), pp. 517-532.
- Huang, R. and Stoll, H. (2001). Tick Size, Bid-Ask Spreads, and Market Structure, *Journal of Financial and Quantitative Analysis*, vol. 36(4), pp. 503-522.
- Ikenberry, D. and Weston, J. (2008). Clustering in U.S. stock prices after decimalization, *European Financial Management*, vol. 14(1), pp. 30-54.
- Jiang, C. and Huang, Y. (2009). Price clustering at the opening and closing in a call market: Evidence from the Taiwan Stock Exchange, *International Research Journal of Finance and Economics*, vol. 31, pp. 16-27.
- Jones, G. (2012). Why Chunking Should be Considered as an Explanation for Developmental Change before Short-Term Memory Capacity and Processing Speed, *Frontiers in Psychology*, vol. 3, pp. 167.
- Kamien, M., Muller, E. and Zang, I. (1992). Research Joint Ventures and RandD Cartels, *American Economic Review*, vol. 82(5), pp. 1293-1306.
- Katz, E. and Tokatlidu, J. (1996). Group competition for rents, *European Journal of Political Economy*, vol. 12, no. 4, pp. 599-607.
- Klumpp, T. and Polborn, M. (2006). Primaries and the New Hampshire Effect, *Journal of Public Economics*, vol. 90, pp. 1073-1114.

- Konrad, K. (2009). Strategy and dynamics in contests. London: Oxford University Press.
- Krueger, A. (1974). The Political Economy of the Rent-Seeking Society, *American Economic Review*, vol. 64(3), pp. 291-303.
- Kuo, W., Lin, T., and Zhao, J. (2014). Cognitive Limitation and Investment Performance: Evidence from Limit Order Clustering, *Review of Financial Studies*, forthcoming.
- Kyle, A. (1985). Continuous Auctions and Insider Trading, *Econometrica*, vol. 53, pp. 1315-1335.
- Lazear, E. (1995). A Jobs-Based Analysis of Labor Markets, *American Economic Review*, vol. 85(2), pp. 260-65.
- Lazear, E. and Rosen, S. (1981). Rank-Order Tournaments as Optimum Labor Contracts, *Journal of Political Economy*, vol. 89(5), pp. 841-864.
- Lehmann, B. (2002). Some desiderata for the measurement of price discovery across markets, *Journal of Financial Markets*, vol. 5, pp. 259-276.
- Levy, G. (2007). Decision making in committees: transparency, reputation and voting rules, *American Economic Review*, vol. 97 (1), pp. 150-168.
- Li, S. (2013). Tullock contests under committee administration, *Economics Bulletin*, vol. 33(3), pp. 1983-1990.
- Li, X. (2003). Time-varying Informational Efficiency in China's A-share and B-share Markets, *Journal of Chinese Economic and Business Studies*, vol. 1, pp. 33-56.
- Lima, E. and Tabak, B. (2004). Tests of the random walk hypothesis for equity markets: evidence from China, Hong Kong and Singapore, *Applied Economics Letters*, vol. 11(4), pp. 255-258.
- Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, vol. 47(1), pp. 13-37.
- Liu, X., Song, H., and Romilly, P. (1997). Are Chinese Stock Markets Efficient? A Cointegration and Causality Analysis, *Applied Economics Letters*, vol. 4, pp. 511-515.
- Lockard, A. (2006). Note on Rent-Seeking and Committees Using a Proportionate-Sharing Rule, *Public Choice*, vol.129, pp. 315-319.
- Lockard, A. and Tullock, G. (2001). Efficient Rent-Seeking: Chronicle of an Intellectual Quagmire. New York: Springer-Verlag.

- Ma, S. and Barnes, M. (2001). Are China's Stock markets really Weak-form Efficient?, Discussion Paper 0119, Centre for International Economic Studies, University of Adelaide, Australia.
- Madhavan, A. (2000). Market microstructure: A survey, *Journal of Financial Markets*, vol. 3(3), pp. 205-258.
- Martens, M. and van Dijk, D. (2006). Measuring volatility with the realized range, Erasmus University Rotterdam Econometric Institute Report 2006-10.
- Matros, A. (2006). Rent-seeking with asymmetric valuations: Addition or deletion of a player, *Public Choice*, vol. 129(3), pp. 369-380.
- Miller, E. (1977). Risk, Uncertainty, and Divergence of Opinion, *Journal of Finance*, vol. 32, pp. 1151-1168.
- Mitchell, J. (2001). Clustering and Psychological Barriers: The Importance of Numbers, *Journal of Futures Markets*, vol. 21, pp. 395-428.
- Mitchell, J. and Izan, H. (2006). Clustering and psychological barriers in exchange rates, *Journal of International Financial Markets, Institutions and Money*, vol. 16(4), pp. 318-344.
- Mookerjee, R. and Yu, Q. (1999). Seasonality in returns on the Chinese stock markets: The case of Shanghai and Shenzhen, *Global Finance Journal*, vol. 10 (1), pp. 93-105.
- Na, S. and Schneider, T. (2010). The use of 'lucky' numbers in the pricing of Chinese A-share initial public offerings, SSRN Working Paper 1588352.
- Nalebuff, B. and Stiglitz, J. (1983). Prices and Incentives: Towards a General Theory of Compensation and Competition, *Bell Journal of Economics*, vol. 14(1), pp. 21-43.
- Narayan, P. and Smyth, R. (2013). Has Political Instability Contributed to Price Clustering on Fiji's Stock Market?, *Journal of Asian Economics*, vol. 28(C), pp. 125-130.
- Niederhoffer, V. (1965). Clustering of stock prices, *Operations Research*, vol. 13, pp. 258-265.
- Niederhoffer, V. (1966). A new look at clustering of market prices, *Journal of Business*, vol. 39, pp. 309-313.
- Nitzan, S. (1994). Modeling Rent-Seeking Contests, *European Journal of Political Economy*, vol. 10, pp. 41-60.
- North, D. (1990). Institutions, Institutional Change and Economic Performance. London: Cambridge University Press.
- North, D. (1994). Economic Performance through Time, *American Economic Review*, vol. 84(3), pp. 359-363.

- North, D. and Thomas, R. (1973). The rise of the Western world: A new economic history. London: Cambridge University Press.
- Nti, K. (1999). Rent-seeking with asymmetric valuations, *Public Choice*, vol. 98, pp. 415-430.
- O'Keefe, M., Viscusi, W., and Zeckhauser, R. (1984). Economic Contests: Comparative Reward Schemes, *Journal of Labor Economics*, vol. 2(1), pp. 27-56.
- Ohta, W. (2006). An analysis of intraday patterns in price clustering on the Tokyo Stock Exchange, *Journal of Banking and Finance*, vol. 30(3), pp. 1023-1039.
- Osborne, M. (1962). Periodic structure in the Brownian motion of stock prices, *Operations Research*, vol. 10, pp. 345-379.
- Osborne, M. (1965). The dynamics of stock trading, *Econometrica*, vol. 33, pp. 88-111.
- Palao, F. and Pardo, A. (2012). When size matters: Clustering in the European carbon market, Working Paper Series, Instituto Valenciano de Investigaciones Económicas, S.A. (Ivie).
- Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return, *Journal of Business*, vol. 53, pp. 61-65.
- Parreiras, S. and Rubinchik-Pessach, A. (2006). Contests with heterogeneous agents, CORE Discussion Papers 2006004, Université catholique de Louvain.
- Pearson, K. (1905). On the General Theory of Skew Correlation and Non-linear Regression. London: Dulau & Co.
- Peng, B. (2006). On the Number of Contestants and Equilibrium Individual Effort, *Topics in Theoretical Economics*, vol. 6(1), pp. 1-13.
- Prendergast, C. (1999). The Provision of Incentives in Firms, *Journal of Economic Literature*, vol. 37(1), pp. 7-63.
- Robson, A. and Skaperdas, S. (2008). Costly enforcement of property rights and the Coase theorem, *Economic Theory*, vol. 36(1), pp. 109-128.
- Rosen, S. (1986). Prizes and Incentives in Elimination Tournaments, *American Economic Review*, vol. 76(4), pp. 701-715.
- Rüchardt, K. and Vogt, B. (2009). Comparison of the Stock Price Clustering of stocks which are traded in the US and Germany – Is XETRA more efficient than the NYSE?, FEMM Working Paper no. 16.
- Ryvkin, D. (2007). Tullock contests of weakly heterogeneous players, *Public Choice*, vol. 132(1), pp. 49-64.

- Saffi, P. and Sigurdsson, K. (2011). Price Efficiency and Short Selling, *Review of Financial Studies*, vol. 24(3), pp. 821-852.
- Samuelson, P. (1965). Proof that properly anticipated prices fluctuate randomly, *Industrial Management Review*, vol. 6(2), pp. 41-49.
- Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk, *The Journal of Finance*, vol. 19(3), pp. 425-442.
- Siegel, R. (2009). All-Pay Contests, *Econometrica*, vol. 77(1), pp. 71-92.
- Siegel, R. (2010). Asymmetric contests with conditional investments, *American Economic Review*, vol. 100(5), pp. 2230-2260.
- Siegel, R. (2011). Head starts in contests, Working Paper, Northwestern University.
- Simon, H. (1955). A Behavioral Model of Rational Choice, *Quarterly Journal of Economics*, vol. 69(1), pp. 99-118.
- Simon, H. (1974). How Big Is a Chunk? By combining data from several experiments, a basic human memory unit can be identified and measured, *Science*, vol. 183(4124), pp. 482-488.
- Sisak, D. (2009). Multiple-Prize Contests-The Optimal Allocation of Prizes, *Journal of Economic Surveys*, vol. 23(1), pp. 82-114.
- Skaperdas, S. (1996). Contest success functions, *Economic Theory*, vol. 7(2), pp. 283-290.
- Sonnemans, J. (2003). Price Clustering and Natural Resistance Points in the Dutch Stock Market, Tinbergen Institute Discussion Papers 03-043/1, Tinbergen Institute.
- Sonnemans, J. (2006). Price clustering and natural resistance points in the Danish stock market: a natural experiment, *European Economic Review*, vol. 50, pp. 1937-1950.
- Sopranzetti, B. and Datar, V. (2002). Price clustering in foreign exchange spot markets, *Journal of Financial Markets*, vol. 5, pp. 411-417.
- Stein, W. (2002). Asymmetric rent-seeking with more than two contestants, *Public Choice*, vol. 113, pp. 325-336.
- Stracke, R., Kerschbamer, R., and Sunde, U. (2012). Two-Stage Elimination Contests with Endogenous Continuation Values: An Analytical Solution, Economics Working Paper Series 1220, University of St. Gallen, School of Economics and Political Science.
- Szymanski, S. (2003). The Economic Design of Sporting Contests, *Journal of Economic Literature*, vol. 41(4), pp. 1137-1187.
- Szymanski, S. and Valletti, T. (2004). Incentive effects of second prizes, Discussion Papers: TBS/DP04/3, Tanaka Business School.

Tullock, G. (1980). Efficient rent seeking, in Buchanan, J., Tollison, R., and Tullock, G. (eds.), Toward a theory of the rent-seeking society. College Station: Texas A&M University Press, pp. 97-112.

Tullock, G. (1967). The Welfare Costs of Tariffs, Monopolies, and Theft, *Western Economic Journal*, vol. 5 (3), pp. 224-232.

Vermeule, A. (2005). Submajority rules: Forcing accountability upon majorities, *Journal of Political Philosophy*, vol. 13(1), pp. 74-98.

Williamson, O. (1981). The Economics of Organization: The Transaction Cost Approach. *American Journal of Sociology*, vol. 87, pp. 548-577.

Williamson, O. (1996). The Mechanisms of Governance. New York: Oxford University Press.

Xu, C. (2000). The microstructure of the Chinese stock market, *China Economic Review*, vol. 11, pp. 79-97.

Yang, Z. (2011). 'Lucky' numbers, unlucky consumers, *Journal of Socio-Economics*, vol. 40, pp. 692-699.