Detailed analysis of the fidelity of quantum teleportation using photons: Considering real experimental parameters

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In this paper we analyze the fidelity of the output states in the experimental quantum teleportation of photons, while taking into account the main experimental parameters. In particular, we consider the detector's ability to distinguish photon numbers, the quantum efficiency of the detectors, and the purity of entangled photon pairs. As a result, it is found that high-fidelity quantum teleportation, without the receiver feedback check, is realizable with multiphoton counters and conventional entangled pairs (visibility=90%) within a day.

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I. INTRODUCTION

Quantum teleportation, as proposed by Bennett *et al*. [1], has become indispensable to quantum communication and information processing. It has also been suggested that this technology is applicable for the demonstration of entangled swapping [2], quantum repeaters [3], and quantum computation [4].

One common way to assess the success of teleportation is fidelity. When the output state is exactly the same as the input state, the fidelity is equal to unity. However, if Alice transfers to Bob the quantum state of a single photon, which has any kind of polarization, by using any local measurement and classical communication, then the fidelity cannot be greater than 2/3. Therefore, a fidelity that exceeds 2/3 can be considered as a requirement for true ''quantum teleportation.''

In 1997, Bouwmeester *et al*. performed experimental quantum teleportation using photons [5]. In this experiment, the necessary entangled photon pairs were obtained by using spontaneous parametric fluorescence. High polarization fidelity was defined for the ensemble of photons that reached Bob. However, Braunstein and Kimble pointed out that teleportation fidelity was less than 2/3 when vacuum states in the output were taken into account [6]. The reason for this is, in order to perform the quantum teleportation; a single entangled photon pair has to be emitted from the source. However, the source sometimes generates not one pair in a pulse, but two. As a result, the output state becomes a mixture of vacuum states and successfully teleported states.

In our opinion, the definition of fidelity may change according to the experimental setup and the supposed usage of the output states. In this sense, we think that both definitions of fidelity are reasonable according to circumstances. Indeed, it may be said that Bouwmeester improved the fidelity using feedback from Bob's (receiver's) measurements. For some applications of quantum teleportation, we have to decrease the vacuum states in the output. In this sense, an experimental demonstration of ''high-fidelity quantum teleportation without receiver feedback'' is a challenging topic.

Braunstein and Kimble [6] suggested an improvement of the experiment performed by Bouwmeester *et al*. According to this suggestion, Kok *et al* analyzed teleportation fidelity [7]. In this analysis, the fidelity of the teleportation experiment was discussed just focusing on the multiphoton emission from the source in a pulse, and also the detector cascade strategy.

In the real experiments, however, additional experimental parameters such as the visibility of the sources, the dark counts of the detectors, and the detection principles of the detectors should also be taken into account. This is because these error sources may also have strong effects on the fidelity. The general analysis of the effect of such error sources, which seems to be important for future quantum information technologies using photons, is given in this paper.

In the analysis, we explored the usage of visible-light photon counters (VLPCs), that have both high quantum efficiencies and also multiphoton counting, in detail. As a result, we found that it is possible to perform a high-fidelity experiment using VLPCs and existing technologies even when all realistic experimental parameters are taken into account. We also hope that this paper provides some intersting viewpoints in the application of VLPCs, since the detector is now attracting attention in this field [8].

This paper is organized as follows. In Sec. II, we outline the claim of Bouwmeester *et al*., and Braunstein and Kimble. In Sec. III, we introduce a detailed general method to calculate the fidelity of teleportation. The fidelities are calculated using this method for the cases utilizing a VLPC, and entangled photon sources in pure, and then mixed states, in Secs. IV and V, respectively. Finally, we conclude this paper in Sec. VI.

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II. PREVIOUS EXPERIMENTAL PROBLEMS AND TELEPORTATION FIDELITY

In this section, we introduce the experiment of teleportation that was demonstrated by Bouwmeester *et al*. Next, us-

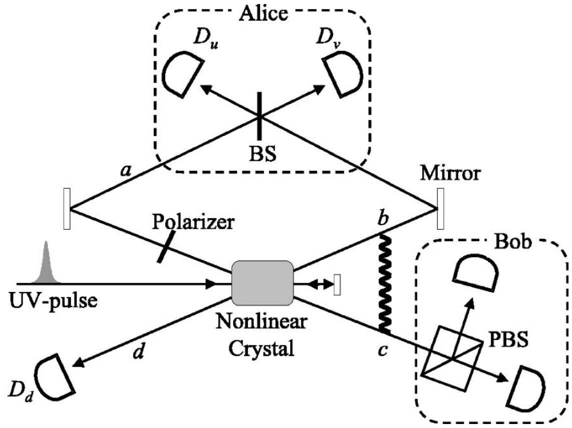

FIG. 1. Experimental teleportation system performed by Bouwmeester *et al.*

ing the fidelity we explain the point of view of Braunstein and Kimble.

Figure 1 shows the experimental system of Bouwmeester *et al*. Let us consider that the sender, Alice, wants to teleport the quantum state (polarization of a signal photon a). In the first stage, an entangled photon pair (photon b and c) has to be shared by Alice and the receiver, Bob. This photon pair is also called an Einstein-Podolsky-Rosen (EPR) pair. Then Alice performs a so-called "Bell measurement" with the signal photon a and the photon b. In an ideal case, the state of photon c is exactly the same as the state of original signal photon a. To analyze the polarization of the teleported output, photon c has to be detected. From the results of the measurement, Bob can define whether teleportation was successful or not. This analysis allows that there is always a photon in the output. However, Braunstein and Kimble pointed out that one should take account of vacuum states in the output. Fidelity can be used as a measure of the success of teleportation. When the output state is exactly the same as the input state, the fidelity is equal to unity. If Alice transports a single photon with any kind of polarization to Bob, by using only local measurements and classical communication, then the fidelity can be no more than 2/3. Therefore, according to Braunstein and Kimble, a fidelity that exceeds 2/3 is a requirement for true "quantum teleportation."

Braunstein and Kimble performed a rough estimation of the fidelity without the information of the detection event of photon c. In this case the fidelity was less than a half due to vacuum states in the output states. The cause of this result is that real sources generate photon pairs stochastically, i.e., sources sometimes emit not only one pair, but two pairs or even no pair. When two pairs are created in modes a and b, but no pairs in mode band c, signal photons may not reach the receiver. This can be the case even though the sender judged that the teleportation had been successful with a threefold coincidence signal. In the experiments, receiver feedback was used to ensure "high fidelity," and the output states of teleportation were destroyed. The fidelity calculated for the events where photons were detected by the receiver was as high as 80% [9].

In our opinion, the definition of fidelity may change according to the experimental setup and the supposed usage of the output state. For example, in quantum cryptography, the output state is always detected. On the other hand, in the logic circuits of a quantum computer, we cannot check an output state, because the information of a photon is destroyed by the measurement. For some applications of quantum teleportation, we have to decrease the vacuum states in the output. In this sense, to realize high fidelity of teleportation without receiver feedback is a challenge. In this paper, we aim to clarify the role of experimental parameters in achieving this goal.

III. A GENERAL METHOD TO CALCULATE THE FIDELITY OF QUANTUM TELEPORTATION WITH CONSIDERATION OF THE EXPERIMENTAL PARAMETERS

Here we will establish a method to calculate the fidelity of the quantum teleportation, whilst taking into consideration various experimental parameters. The following experimental conditions are considered: the purity of the entangled photon pairs, the detector's ability to distinguish photon numbers, the quantum efficiencies of detectors, and the probability of the generation of one entangled photon pair.

In our analysis, we closely followed the analysis by Kok *et al*. However, in order to perform a high-fidelity teleportation experiment, we have to take into account certain experimental limitations. For example, in their analysis, only the case of pure state entangled photon pairs is considered. In real teleportation experiments, the visibility of the pairs is lower than 100%. Visibility is about 90% if entangled photon pairs are generated with a fs-pulsed pump beam [10]. Therefore, in order to estimate the other requirements for high-fidelity quantum teleportation, it is necessary to consider the purity of the photon pairs. Note that the mixed states should not be given by simple two-photon Werner states [11]. The states of entangled photon pairs should be mixtures of superposition of all states from vacuum to an infinite number of pairs. The explicit formula for these states will be given in Sec. V. Moreover, we added a phase shifter (quarter-wave plate) between x polarization and y polarization. In Kok's analysis, they considered only the case where the inputs were linearly polarized photons. In this case, teleportation fidelity had to satisfy the condition that $F>3/4$. However, with our approaches, we can prepare not only linear polarization but also circular polarization for the input state, and can decrease the critical value of the fidelity from 3/4 to 2/3 [7,12]. Further to this we desire to study the effective use of the multiphoton counter compared with conventional detectors. For this purpose, we analyzed the fidelity assuming different quantum efficiencies for each detector type. The effect of dark counts will be discussed in Sec. IV.

The schematic setup used in the analysis is given in Fig. 2. The outline of our analysis is as follows. First, let ρ_{source} be the quantum state of the photon emitted from the down-conversion processes in sources 1 and 2. Second, the ρ_{source} undergoes unitary transformations that correspond to optical components in the experimental setup. In order to derive the output state ρ_{out} of photon c, we perform positive operator valued measures (POVMs) that correspond to the detection

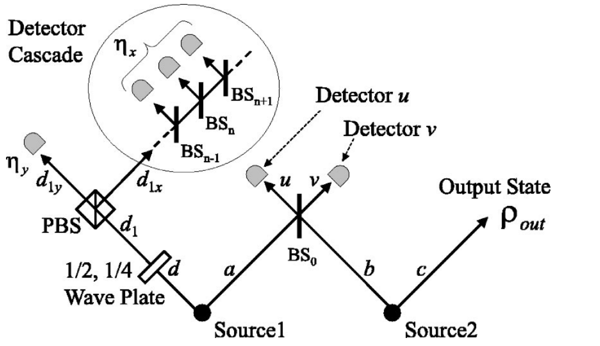

FIG. 2. The schematic for the experimental system of our analysis.

events of photons. From the obtained ρ_{out}, we can calculate the fidelity of the teleportation.

In Fig. 2, the state of the photons before the detectors is given by

$$|\Psi_{\theta,\varphi}\rangle\langle\Psi_{\theta,\varphi}| = U_{\text{cas}} U_{\theta,\varphi} U_{\text{BS}_0} \rho_{\text{source}} U^{\dagger}_{\text{BS}_0} U^{\dagger}_{\theta,\varphi} U^{\dagger}_{\text{cas}}, \tag{3.1}$$

where the density operator ρ_{source} describes the entangled photon emissions from sources 1 and 2. A detailed description of ρ_{source} will be given in Secs. IV and V.

U_{BS_0} is the unitary transformation of the beam splitter 0 (BS_0), and is given by

$$U_{\text{BS}_0} a^{\dagger}_j U^{\dagger}_{BS_0} = \sqrt{R} u^{\dagger}_j + \sqrt{1-R} v^{\dagger}_j,$$

$$U_{\text{BS}_0} b^{\dagger}_j U^{\dagger}_{BS_0} = \sqrt{1-R} u^{\dagger}_j + \sqrt{R} v^{\dagger}_j, \tag{3.2}$$

where $j \in \{x,y\}$, R is the reflectivity of the beam splitter, and $R=1/2$. The operators $a^{\dagger}_j$, $b^{\dagger}_j$, $u^{\dagger}_j$, and $v^{\dagger}_j$ are creation operators of a single photon in those modes with x or y polarization, indicated by the parameter $j \in \{x,y\}$. $U_{\theta,\varphi}$ is the unitary transformation of the $\lambda/2$ and $\lambda/4$ plates. This formula is given by

$$U_{\theta,\varphi} d^{\dagger}_x U^{\dagger}_{\theta,\varphi} = \cos\theta d^{\dagger}_{1x} + e^{i\varphi}\sin\theta d^{\dagger}_{1y},$$

$$U_{\theta,\varphi} d^{\dagger}_y U^{\dagger}_{\theta,\varphi} = -e^{i\varphi}\sin\theta d^{\dagger}_{1x} + \cos\theta d^{\dagger}_{1y}, \tag{3.3}$$

where θ is the rotation angle of the $\lambda/2$ plate and φ is determined by the $\lambda/4$ plate. U_{cas} are the unitary transformations of the detector cascade. This system is shown by Fig. 2 and consists of many beam splitters (BS_n). Several simple detector cascades with one, two, and three detectors are shown in Fig. 3.

In order to describe the measurement of photons by a polarization insensitive detector, we apply positive operator valued measures (POVMs). The POVM $E^{(0)}_k$ having no detection signal in mode k is given by [7]

$$E^{(0)}_k = \sum_{l,m} [1-\eta^{l+m}] |l,m\rangle_{k\,k}\langle l,m|, \tag{3.4}$$

and the POVM having a detection signal in mode k is

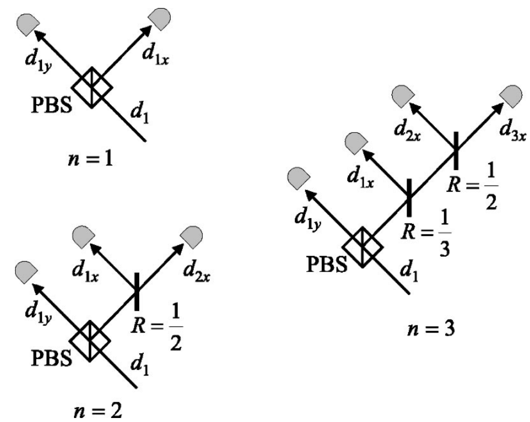

FIG. 3. Simple detector cascades.

$$E^{(1)}_k = \sum_{l,m} \eta^{l+m} |l,m\rangle_{k\,k}\langle l,m|, \tag{3.5}$$

where η is the quantum efficiency of the detectors, and l and m are the number of photons with x polarization and y polarization, respectively, in mode k. After the photons have passed the polarizing beam splitter, the POVMs are

$$E^{(0)}_{k_j} = \sum_{l} [1-\eta^l_j] |l\rangle_{k_j\,k_j}\langle l| \tag{3.6}$$

and

$$E^{(1)}_{k_j} = \sum_{l} \eta^l_j |l\rangle_{k_j\,k_j}\langle l|, \tag{3.7}$$

where $j \in \{x,y\}$.

Taking the partial trace over every pass except the mode c, the output state of teleportation is given by

$$\rho_{\text{out}} = \text{Tr}_{d_1,\ldots,d_n,u,v}\left[\sqrt{E_{\text{cas}} E^{(1)}_u E^{(1)}_v} |\Psi_{\theta,\varphi}\rangle \times \langle\Psi_{\theta,\varphi}| \sqrt{E^{\dagger}_{\text{cas}} E^{(1)\dagger}_u E^{(1)\dagger}_v}\right], \tag{3.8}$$

where E_{cas} is the POVM that corresponds to the detection of an x-polarized photon in the detector cascade and no y-polarized photon. For instance, with regard to the $n=2$ detector cascade, the E_{cas} is $E^{(0)}_{d_1 y}(E^{(1)}_{d_1 x} E^{(0)}_{d_2 x} + E^{(0)}_{d_1 x} E^{(1)}_{d_2 x})$.

$E^{(1)}_u$ and $E^{(1)}_v$ correspond to the single-photon detection by detectors u and v. From obtained output states, we calculate teleportation fidelities. In the next sections, we perform the fidelity calculation.

IV. TELEPORTATION FIDELITY USING VLPCS WITH EPR PAIRS IN PURE STATES

VLPCs can distinguish between a single-photon incidence and two-photon incidence [13] and have high quantum efficiencies [$(88.2\pm5)\%$, Ref. [14]). Here we calculate the fidelity of our future experiment using VLPCs, and compare it with those using other detector types (i.e., detector cascade or conventional detectors). In this section, we assume that the electron-paramagnetic-resonance (EPR) pairs generated from the sources are in pure states.

The detection principle of VLPCs is different from standard avalanche photodiodes operated in the Geiger mode. In the VLPC, only the electrons in the impurity band are excited (single-carrier avalanche) and it is restricted to a very small region (supposed to be about square of 10 μm) when it is compared to the whole region of the detector (1 mm in diameter). Therefore, the rest of the detector, which is a few thousand times larger than the single-avalanche region, is still sensitive for the additional incidence of photons. This means that we can consider a VLPC as a cascade of a few thousand detectors. In the light of this very high number, it is quite reasonable to describe a VLPC by an infinite cascade [15]. It is also reported that some degradation of quantum efficiency was observed when the number of incident photon was too large because of the local saturation effect [14]. However, the estimated degradation for a supposed experiment is much smaller than 1% [16], and we neglect it here.

The probability of one entangled photon pair creation at source 1 (2) is p_1 (p_2). With $p_1, p_2 \ll 1$, the state of entangled photon pairs ρ_{source} is given by

$$\begin{aligned}\rho_{\text{source}} = &(|0{,}0\rangle_b|0{,}0\rangle_c + \sqrt{p_2}|\psi^-\rangle_{bc} + p_2|\chi\rangle_{bc})\\ &\otimes(|0{,}0\rangle_a|0{,}0\rangle_d + \sqrt{p_1}|\psi^-\rangle_{ad} + p_1|\chi\rangle_{ad})\\ &\otimes({}_d\langle 0{,}0|\,{}_a\langle 0{,}0| + \sqrt{p_1}\,{}_{ad}\langle\psi^-| + p_1\,{}_{ad}\langle\chi|)\\ &\otimes({}_c\langle 0{,}0|\,{}_b\langle 0{,}0| + \sqrt{p_2}\,{}_{bc}\langle\psi^-| + p_2\,{}_{bc}\langle\chi|),\end{aligned} \tag{4.1}$$

where $|\psi^-\rangle_{ij} = (|0{,}1\rangle_i|1{,}0\rangle_j - |1{,}0\rangle_i|0{,}1\rangle_j)/\sqrt{2}$, $|\chi\rangle_{ij}$ is the state of two entangled photon pairs, where (i,j) should be (a,d) or (b,c). η_x and η_y are the quantum efficiencies of the detectors for x-polarized photons and y-polarized photons, respectively, on mode d. Putting Eq. (4.1) into Eq. (3.8) and using Eqs. (3.1)–(3.7), we have calculated the output state ρ_{out} up to order p^2 (i.e., p_1^2 or $p_1 p_2$) and found

$$\begin{aligned}\rho_{\text{out}} \propto\, & p_1^2\left[\frac{1}{n} + (1-\eta_y) + \left(2 - \frac{1}{n}\right)(1-\eta_x)\right]\\ &\times|0{,}0\rangle_{c\,c}\langle 0{,}0| + p_1 p_2|\phi\rangle_{c\,c}\langle\phi|,\end{aligned} \tag{4.2}$$

where $|\phi\rangle_c = \cos(\theta/2)|0{,}1\rangle_c + e^{i\varphi}\sin(\theta/2)|1{,}0\rangle_c$ is the initial state sent to Bob and n is the number of cascading detectors. Note that this result is different from the one in the previous analysis [17].

When an output state has a form like $\rho_{\text{out}} \propto |A|^2|0{,}0\rangle_{c\,c}\langle 0{,}0| + |B|^2|\phi\rangle_{c\,c}\langle\phi|$, then the quantum teleportation fidelity is $F = |B|^2/(|A|^2 + |B|^2)$. Therefore, the teleportation fidelity F for the output given in Eq. (4.2) is given by

$$F = \frac{1}{\dfrac{p_1}{p_2}\left[\dfrac{1}{n} + (1-\eta_y) + \left(2 - \dfrac{1}{n}\right)(1-\eta_x)\right] + 1}. \tag{4.3}$$

In Fig. 4, we plot the fidelity F as a function of the quantum efficiency of the detectors for $p_1 = p_2$. We found that it is possible to achieve a high-fidelity teleportation experiment without receiver feedback using the VLPCs. Note that the fidelity with an infinite detector cascade of conventional detectors (Perkin-Elmer SPCM-AQR series, $\eta = 70\%$) is below 2/3. When the number of detector cascade is 1 (the situation is shown in Fig. 3) and the quantum efficiency $\eta = 1$, the calculated fidelity shown in Fig. 4 is about 1/2. This means that even with a photon-number-indistinguishable detector that possesses unity quantum efficiency, we cannot exceed the critical fidelity of 2/3. These facts show that both high quantum efficiency and multiphoton distinguishability contribute to high fidelity.

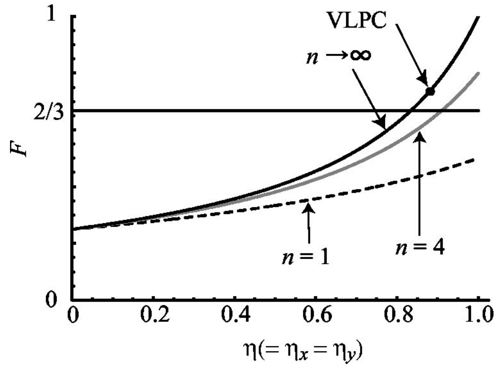

FIG. 4. Theoretical curves of the teleportation fidelity F of Eq. (4.3) for cascading detector of $n = 1,4$ and ∞, $p_1 = p_2$.

Let us estimate the effect of dark count rates of detectors. The reported intrinsic dark count rate of the VLPC is 10^4 cps when the detector is operated to have the highest quantum efficiency [14], however, we can decrease the effective dark count rate to 1/10 by using a gating circuit with the gate time of about 1 ns, when the repetition time of the femtosecond pump laser beam is 12.5 ns. On the other hand, the intrinsic dark counts of SPCMs are usually 100 cps (some are smaller than this). In real experiments, however, we may suffer dark counts caused by stray lights, which cannot be decreased by the gating method. Our experience suggests that this kind of dark count will be about 1000 cps. In summary, the dark count of VLPC is estimated to be about 2000 cps, and that of SPCMs 1000 cps, since the repetition rate is 82 MHz. The probability to have threefold coincidences is 3×10^{-7} cps. The rate is much smaller than teleportation events in the experiment under consideration [5]. In conclusion, the effect of dark counts is negligible in our analysis.

We also note that the fidelity can be increased by decreasing the ratio p_1/p_2. Braunstein and Kimble pointed out this technique. In the real experiments, however, this is limited by the stability of the whole experimental setup, as the decrease in p_1 requires longer experimental time.

V. TELEPORTATION FIDELITY USING VLPCS WITH EPR PAIRS IN MIXED STATES

In this section, we try to calculate the fidelity for the case where the EPR pairs are in mixed states. In the real experiments using femtosecond pumping lasers, the visibilities of the EPR sources were not 100%.

We consider the states that are simply a mixture of the pure EPR pair state and the state of the randomly polarized photons. The total state of the two sources, ρ_{source}, is given by

$$\rho_{\mathrm{source}} = \rho_1 \otimes \rho_2 . \tag{5.1}$$

Here ρ_1 and ρ_2 are the density matrices of the sources 1 and 2 in Fig. 2, respectively. When s is for the number of sources and i and j are for the modes (with order up to p^2), the density matrix of each pair is given as follows:

$$\begin{aligned}\rho_s = \gamma\rho_{\mathrm{EPR}} + (1-\gamma)\rho_{\mathrm{random}} = {}& \gamma(|0,0\rangle_i|0,0\rangle_j + \sqrt{p_s}|\psi^-\rangle_{ij} + p_s|\chi\rangle_{ij}) \otimes ({}_j\langle 0,0|\,{}_i\langle 0,0| + \sqrt{p_s}\,{}_{ij}\langle\psi^-| + p_s\,{}_{ij}\langle\chi|)\\ &+ (1-\gamma)(|0,0\rangle_i|0,0\rangle_j\,{}_j\langle 0,0|\,{}_i\langle 0,0| + p_s\tfrac{1}{4}\mathbb{1}_{ij} + p_s^2\rho_{ij}),\end{aligned} \tag{5.2}$$

$$\mathbb{1}_{ij} = |1,0\rangle_i|1,0\rangle_j\,{}_j\langle 1,0|\,{}_i\langle 1,0| + |1,0\rangle_i|0,1\rangle_j\,{}_j\langle 0,1|\,{}_i\langle 1,0| + |0,1\rangle_i|1,0\rangle_j\,{}_j\langle 1,0|\,{}_i\langle 0,1| + |0,1\rangle_i|0,1\rangle_j\,{}_j\langle 0,1|\,{}_i\langle 0,1|, \tag{5.3}$$

$$\begin{aligned}\rho_{ij} = {}& \tfrac{1}{16}|2,0\rangle_i|2,0\rangle_j\,{}_j\langle 2,0|\,{}_i\langle 2,0| + \tfrac{1}{8}|2,0\rangle_i|1,1\rangle_j\,{}_j\langle 1,1|\,{}_i\langle 2,0| + \tfrac{1}{8}|1,1\rangle_i|2,0\rangle_j\,{}_j\langle 2,0|\,{}_i\langle 1,1|\\ &+ \tfrac{1}{4}|1,1\rangle_i|1,1\rangle_j\,{}_j\langle 1,1|\,{}_i\langle 1,1| + \tfrac{1}{16}|2,0\rangle_i|0,2\rangle_j\,{}_j\langle 0,2|\,{}_i\langle 2,0| + \tfrac{1}{8}|1,1\rangle_i|0,2\rangle_j\,{}_j\langle 0,2|\,{}_i\langle 1,1|\\ &+ \tfrac{1}{16}|0,2\rangle_i|2,0\rangle_j\,{}_j\langle 2,0|\,{}_i\langle 0,2| + \tfrac{1}{8}|0,2\rangle_i|1,1\rangle_j\,{}_j\langle 1,1|\,{}_i\langle 0,2| + \tfrac{1}{16}|0,2\rangle_i|0,2\rangle_j\,{}_j\langle 0,2|\,{}_i\langle 0,2|,\end{aligned} \tag{5.4}$$

where $\{s,i,j\}$ should be $\{1,a,d\}$ or $\{2,b,c\}$ and γ is a mixing parameter.

When $p_1, p_2 \ll 1$, γ is equivalent to the visibility of the entangled photon sources. ρ_1 and ρ_2 are a kind of direct extension of the Werner state. Again, as in the preceding section, we put Eq. (5.1) into Eq. (3.8), and by using Eqs. (3.1)–(3.7), the density matrices of the output state ρ'_{out} were calculated as follows:

$$\rho'_{\mathrm{out}} \propto P_V|0,0\rangle_c\,{}_c\left\langle 0,0\right| + P_R\frac{1}{2}\mathbb{1}_c + P_I\left|\phi\right\rangle_{c\;c}\langle\phi|. \tag{5.5}$$

Here

$$P_V = \left[\frac{1}{n} + (1-\eta_y) + \left(2 - \frac{1}{n}\right)(1-\eta_x)\right]2p_1(3-6\gamma+4\gamma^2), \tag{5.6}$$

$$P_R = 2p_2(1-\gamma^2), \tag{5.7}$$

$$P_I = 2p_2\gamma^2, \tag{5.8}$$

and

$$\mathbb{1}_c = |1,0\rangle_c\,{}_c\langle 1,0| + |0,1\rangle_c\,{}_c\langle 0,1|. \tag{5.9}$$

These correspond to the ratios of vacuum states, the randomly polarized states, and the successfully teleported states, respectively. Therefore, the teleportation fidelity is calculated as follows.

$$F' = \frac{1}{P_V+P_R+P_I}\left(P_R\frac{1}{2} + P_I 1\right) = \frac{(1+\gamma^2)/2}{\dfrac{p_1}{p_2}A(3-6\gamma+4\gamma^2)+1}, \tag{5.10}$$

where

$$A = \frac{1}{n} + (1-\eta_y) + \left(2 - \frac{1}{n}\right)(1-\eta_x). \tag{5.11}$$

Figure 5 shows the required parameters to perform the high-fidelity "receiver-feedback-free" quantum teleportation with EPR pairs in mixed states. When we calculated the fidelities for cases where the VLPCs [18] were used for the modes d_{1x} and d_{1y} in Fig. 2, the fidelities exceeded 2/3 in the weakly shaded area. The fidelity was 1 when $\gamma=1$ and $p_1/p_2=0$, and gradually decreased as γ decreased or p_1/p_2 increased. It should be noted that even when we use entangled photon sources emitting mixed states ($\gamma=0.90$), we can perform the receiver-feedback-free experiment without increasing the required experimental time ($p_1=p_2$). The dark area also shows the cases where quantum efficiency is decreased to 80% by optical losses and misalignment. In this case we found that a receiver-feedback-free experiment is possible for $\gamma=0.90$ and $\eta=80\%$, when p_1/p_2 is smaller than 0.71. This means that the experimental time should be increased by about 1.4. When we use a similar experimental

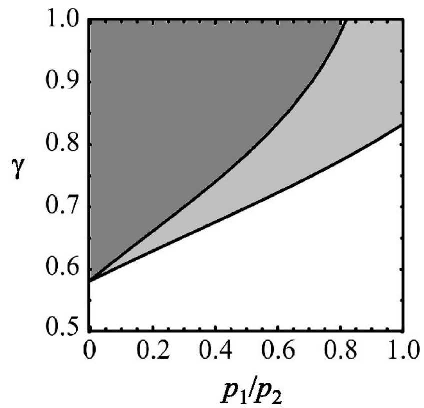

FIG. 5. The required experimental parameters p_1/p_2 and γ for the teleportation fidelity $F'>2/3$. p_1/p_2 is the rate of the probability of one entangled photon pair creation at sources. γ is the mixing parameter of the two EPR sources in Fig. 2. The weekly shaded area shows the region where VLPC is fully utilized ($\eta=88\%$). The dark area shows the region where the effective efficiency of VLPC is reduced to 80%.

setup to the previous one [5], in which 17 h is required, we will need about one day to perform the receiver-feedback-free experiment.

VI. CONCLUSION

In this paper, we calculated the teleportation fidelity, whilst taking account of all relevant experimental parameters, namely, the purity of entangled photon pairs, the ability of detectors to distinguish photon numbers, the quantum efficiency of detectors, and experimental times. As a result, we find that it is possible to perform high-fidelity teleportation in a day, using conventional EPR sources ($\gamma=90\%$) and VLPCs in which quantum efficiency is decreased due to the loss of optical component ($<10\%$).

The detailed analysis shown in this paper is important to clarify the real obstacles to decrease vacuum states in the output states. Ideas to utilize quantum teleportation for complicated quantum information processing have been proposed [4,19,20] however, to our knowledge, the effect of these vacuum outputs has not been sufficiently investigated.

An alternative method to understand the fidelity of quantum teleportation has recently been proposed, which connects the decrease in fidelity to the leaked information in the teleportation process [21]. Together with this method, we hope that the results presented in this paper will contribute to a deeper understanding of fidelity in quantum teleportation, and its application to the field of quantum information technology.

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[15] In Ref. [13], $\eta_1^2=\eta_2$, where η_1 and η_2 are efficiencies for detecting one photon and two photons, respectively. Note that η_1 was reduced from 88% to 70% by optical misalignment.
[16] When the number of incident photons was smaller than 2×10^5 cps, the degradation of quantum efficiency was smaller than 1% [13]. The estimated single count rates are less than 2×10^4 cps, according to the existing experiments.
[17] When $\eta_x=\eta_y=\eta$, the coefficient term of the state $|0,0\rangle\langle0,0|$ in Eq. (4.2) is given by $p_1^2(1+(3n-1)(1-\eta))/n$. However, the result given in the previous analysis is $p_1^2[1+(5n-3)(1-\eta)]/n$. ($\eta=\eta_c^2$ in the previous analysis.) In the previous result, we found a decrease of fidelity with increasing cascade number, however, this may not be physically understandable. According to our analysis, this kind of error can be made by a mistake in the order of calculating Eq. (3.8)
[18] A VLPC is regarded as an infinite cascade of detectors, as is explained in Sec. IV.
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