ON SOME PROPERTIES OF HYPERSURFACES WITH CERTAIN CONTACT STRUCTURES

By

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§ 1. Introduction. Let E^{2n+2} be a $(2n+2)$-dimensional Euclidean space with cartesian coordinates. We put

$$F=(F_j^i)=\begin{pmatrix}0 & E_m\\ -E_m & 0\end{pmatrix},\quad g=(g_{ij})=\begin{pmatrix}E_m & 0\\ 0 & E_m\end{pmatrix},$$

where E_m $(m=n+1)$ is the unit matrix of dimension m. Then F is an almost complex structure in E^{2n+2} and the Euclidean metric g is a Hermitian metric with respect to the almost complex structure. Therefore we may consider E^{2n+2} as an almost Hermitian space. S. Sasaki and Y. Hatakeyama [5][1] showed that a hypersphere imbedded in E^{2n+2} is an example of a manifold admitting a normal contact metric structure. On the other hand, Y. Tashiro [7] proved that an orientable hypersurface in an almost complex space has an almost contact structure. These results gave us the problem to study the geometric properties of hypersurfaces in almost complex spaces.

In an almost complex space there always exists an affine connection, called F-connection, transposing the almost complex structure F parallely. Suitable restrictions for F-connection characterize the special class of almost complex space. In the paper [7], properties of hypersurfaces in almost complex spaces were discussed by making use of the relations between F-connection and its induced one. Concerning a Kählerian space Y. Tashiro [7] gave the geometric meaning of the condition in order that a hypersurface has a normal contact structure.

In an almost complex space there exists a Riemannian metric g and without loss of generality we may assume that g be an almost Hermitian metric [8]. The Kählerian space is characterized by that the Riemannian connection defined by g is an F-connection. In general, the Riemannian connection is not an F-connection and restrictions for covariant derivatives of F characterize the special class of an almost Hermitian space. In this paper we always assume that the treated almost complex structure is Hermitian and we shall use the

1) Numbers in brackets refer to the references at the end of the paper.

Riemannian connection and its induced one. The purpose of the present paper is to investigate some geometric properties of a hypersurface in a K-space. § 2 devoted to give the fundamental concepts of almost Hermitian structure and almost contact metric structure. In § 3 we shall give some relations satisfied for a hypersurface in an almost Hermitian space, which will be used in the following sections. In § 4 we shall give a certain property of a hypersurface of a K-space. §§ 5–6 devoted to give some properties of hypersurface in the special K-space. In § 5 we apply the result in § 4 to a hypersurface of a K-space with constant curvature. In the last section we shall obtain some properties of a hypersurface of a Kählerian space and finally we shall prove the theorem for a Kählerian space, which was obtained by Y. Tashiro in another way.

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§ 2. **Notations and terminologies.** Let M^{2n+2} be a $(2n+2)$-dimensional differentiable manifold with local coordinates x^i $(i=1, 2, \cdots, 2n+2)$. M^{2n+2} is called an almost Hermitian space if it admits a $(1, 1)$-tensor field F^i_j satisfying the relations

$$F^i_h F^h_j = -\delta^i_j , \quad g_{ij} F^i_h F^j_k = g_{hk} , \tag{2.1}$$

where g_{ij} is the Riemannian metric tensor in M^{2n+2}. From (2. 1) it follows that

$$F_{ij} = -F_{ji} . \quad (F_{ij} = g_{ih} F^h_j) \tag{2.2}$$

The $(1, 2)$-tensor N^i_{jh} defined by

$$N^i_{jh} = F^k_h(\partial_k F^i_j - \partial_j F^i_k) - F^k_j(\partial_k F^i_h - \partial_h F^i_k) \tag{2.3}$$

is called the Nijenhuis tensor, where ∂_i means partial differentiation with respect to the variable x^i.

A $(2n+1)$-dimensional differentiable manifold M^{2n+1} with local coordinates u^α $(\alpha=1, 2, \cdots, 2n+1)$ is called an almost contact space, if it admits a $(1, 1)$-tensor field φ^α_β and two vector fields ξ^α and η_α satisfying the relations

$$\left\{ \begin{aligned} & \xi^\alpha \eta_\alpha = 1 , \\ & \operatorname{rank}(\varphi^\alpha_\beta) = 2n , \\ & \varphi^\alpha_\beta \xi^\beta = 0 , \\ & \varphi^\alpha_\beta \eta_\alpha = 0 , \\ & \varphi^\alpha_\beta \varphi^\beta_\gamma = -\delta^\alpha_\gamma + \xi^\alpha \eta_\gamma . \end{aligned} \right. \tag{2.4}$$

In every almost contact space, we can always find a Riemannian metric tensor $g_{\alpha\beta}$, called an associated Riemannian metric tensor, such that [3]

$$g_{\alpha\beta}\xi^{\alpha} = \eta_{\beta}\,, \quad g_{\alpha\beta}\varphi^{\alpha}_{\lambda}\varphi^{\beta}_{\mu} = g_{\lambda\mu} - \eta_{\lambda}\eta_{\mu}\,. \tag{2.5}$$

In this paper we shall say that four tensor fields φ^{α}_{β}, $g_{\alpha\beta}$, ξ^{α} and η_{α}, satisfying (2. 4) and (2. 5), define an almost contact metric structure for M^{2n+1}. By means of the last relation of (2. 4) and (2. 5), we can obtain

$$\varphi_{\alpha\beta} = -\varphi_{\beta\alpha}\,. \quad (\varphi_{\alpha\beta} = g_{\alpha\gamma}\varphi^{\gamma}_{\beta}) \tag{2.6}$$

Moreover, when the covariant tensor $\varphi_{\alpha\beta}$ satisfies the relation

$$\varphi_{\alpha\beta} = \frac{1}{2}(\partial_{\alpha}\eta_{\beta} - \partial_{\beta}\eta_{\alpha})\,, \tag{2.7}$$

we shall call M^{2n+1} the contact metric space. In every almost contact space there exists a (1, 2)-tensor $n^{\alpha}_{\beta\gamma}$ defined by

$$n^{\alpha}_{\beta\gamma} = \varphi^{\epsilon}_{\gamma}(\partial_{\epsilon}\varphi^{\alpha}_{\beta} - \partial_{\beta}\varphi^{\alpha}_{\epsilon}) - \varphi^{\epsilon}_{\beta}(\partial_{\epsilon}\varphi^{\alpha}_{\gamma} - \partial_{\gamma}\varphi^{\alpha}_{\epsilon}) + (\partial_{\beta}\xi^{\alpha})\eta_{\gamma} - (\partial_{\gamma}\xi^{\alpha})\eta_{\beta}\,. \tag{2.8}$$

When $n^{\alpha}_{\beta\gamma}=0$, we shall say that the almost contact structure is normal.

§ 3. Hypersurface in an almost Hermitian space. Let us consider that an orientable hypersurface M^{2n+1}, imbedded in an almost Hermitian space M^{2n+2}, be defined by $2n+2$ equations involving $2n+1$ independent parameters such that

$$x^{i} = x^{i}(u^{\alpha})\,. \quad (i = 1, 2, \cdots, 2n+2\,;\ \alpha = 1, 2, \cdots, 2n+1)^{2)} \tag{3.1}$$

The set of $2n+2$ linearly independent vectors (X^{i}_{α}, X^{i}) determines an ennuple at every point in M^{2n+1}, where $X^{i}_{\alpha} = \dfrac{\partial x^{i}}{\partial u^{\alpha}}$ and X^{i} denotes the contravariant component of the unit normal vector of the hypersurface. Now, we put

$$X^{\alpha}_{i} = g^{\alpha\beta}g_{ij}X^{j}_{\beta}\,, \quad X_{i} = g_{ij}X^{j}\,, \tag{3.2}$$

where $g^{\alpha\beta}$ is defined by $g^{\alpha\beta}g_{\beta\gamma}=\delta^{\alpha}_{\gamma}$ and $g_{\alpha\beta}$ is an induced Riemannian metric, i. e. $g_{\alpha\beta}=g_{ij}X^{i}_{\alpha}X^{j}_{\beta}$. Then, (X^{α}_{i}, X_{i}) is a conjugate ennuple with respect to the ennuple (X^{i}_{α}, X^{i}), and these quantities satisfy the following wellknown relations :

$$\begin{cases} X^{i}_{\alpha}X_{i} = 0\,, & X^{i}X^{\beta}_{i} = 0\,, \quad X^{i}X_{i} = 1\,, \\ X^{i}_{\alpha}X^{\beta}_{i} = \delta^{\beta}_{\alpha}\,, & X^{i}_{\alpha}X^{\alpha}_{j} + X^{i}X_{j} = \delta^{i}_{j}\,. \end{cases} \tag{3.3}$$

With respect to the ennuple (X^{i}_{α}, X^{i}), components of the (1, 1)-tensor F^{i}_{j} are expressible as follows :

2) Throughout the present paper the Latin indices are supposed to run over the range 1, 2, $\cdots$, $2n+2$, and the Greek indices take the values 1, 2, $\cdots$, $2n+1$.

$$(3.4) \qquad F^i_j = \varphi^\alpha_\beta X^i_\alpha X^\beta_j + \varphi^\alpha X^i_\alpha X_j + \varphi_\beta X^i X^\beta_j + \varphi X^i X_j ,$$

where

$$(3.5) \quad \varphi^\alpha_\beta = F^i_j X^\alpha_i X^j_\beta , \quad \varphi^\alpha = F^i_j X^\alpha_i X^j , \quad \varphi_\beta = F^i_j X_i X^j_\beta , \quad \varphi = F^i_j X_i X^j .$$

However, by virtue of (2. 2) we have $\varphi = 0$. If we put

$$\varphi^\alpha = -\xi^\alpha , \qquad \varphi_\beta = \eta_\beta ,$$

in consequence of (2. 1) and (3. 3), we can easily verify that four tensor fields φ^α_β, $g_{\alpha\beta}$, ξ^α and η_α satisfy the relations (2. 4) and (2. 5), i. e. these quantities define an almost contact metric structure for the hypersurface M^{2n+1}. (2. 3) and (2. 8) are rewritten in

$$(3.6) \qquad N^i_{jh} = F^k_h (F^i_{j,k} - F^i_{k,j}) - F^k_j (F^i_{h,k} - F^i_{k,h}) ,$$

$$(3.7) \qquad n^\alpha_{\beta\gamma} = \varphi^\varepsilon_\gamma (\varphi^\alpha_{\beta;\varepsilon} - \varphi^\alpha_{\varepsilon;\beta}) - \varphi^\varepsilon_\beta (\varphi^\alpha_{\gamma;\varepsilon} - \varphi^\alpha_{\varepsilon;\gamma}) + \xi^\alpha_{;\beta}\eta_\gamma - \xi^\alpha_{;\gamma}\eta_\beta ,$$

where comma and semi-colon denote covariant differentiation with respect to the Riemannian connection defined by g_{ij} and its induced connection respectively. From (3. 5) it follows that

$$(3.8) \qquad \varphi^\alpha_{\beta;\gamma} = F^i_{j,k} X^\alpha_i X^j_\beta X^k_\gamma + F^i_j X^\alpha_{i;\gamma} X^j_\beta + F^i_j X^\alpha_i X^j_{\beta;\gamma} ,$$

$$(3.9) \qquad \xi^\alpha_{;\beta} = -F^i_{j,k} X^\alpha_i X^j X^k_\beta - F^i_j X^\alpha_{i;\beta} X^j - F^i_j X^\alpha_i X^j_{;\beta} .$$

On the other hand, covariant derivatives of the vectors X^i_α and X^i satisfy the following Gauss' equations :

$$(3.10) \quad \begin{cases} X^i_{\alpha;\beta} = H_{\alpha\beta} X^i , & X^\alpha_{i;\beta} = H^\alpha_\beta X_i , \\ X^i_{;\alpha} = -H^\gamma_\alpha X^i_\gamma , & X_{i;\alpha} = -H_{\alpha\gamma} X^\gamma_i , \end{cases} \qquad (H^\alpha_\beta = g^{\alpha\gamma} H_{\beta\gamma})$$

where $H_{\alpha\beta}$ is the covariant component of the second fundamental tensor. Substituting (3. 10) into the right hand side of (3. 8) and (3. 9) we find

$$(3.11) \qquad \varphi^\alpha_{\beta;\gamma} = F^i_{j,k} X^\alpha_i X^j_\beta X^k_\gamma + H^\alpha_\gamma \eta_\beta - H_{\beta\gamma} \xi^\alpha ,$$

$$(3.12) \qquad \xi^\alpha_{;\beta} = -F^i_{j,k} X^\alpha_i X^j X^k_\beta + H^\gamma_\beta \varphi^\alpha_\gamma .$$

Making use of (3. 6), (3. 11) and (3. 12) it is easy to see that

$$(3.13) \qquad \begin{aligned} n^\alpha_{\beta\gamma} = {} & N^i_{jk} X^\alpha_i X^j_\beta X^k_\gamma + F^i_{j,k} X^k X^\alpha_i X^j_\gamma \eta_\beta - F^i_{j,k} X^k X^\alpha_i X^j_\beta \eta_\gamma \\ & + H^\varepsilon_\beta \varphi^\alpha_\varepsilon \eta_\gamma - H^\varepsilon_\gamma \varphi^\alpha_\varepsilon \eta_\beta + H^\alpha_\varepsilon \varphi^\varepsilon_\gamma \eta_\beta - H^\alpha_\varepsilon \varphi^\varepsilon_\beta \eta_\gamma . \end{aligned}$$

§ 4. A certain property of a hypersurface of a K-space. Let us consider that M^{2n+2} be a K-space, then we have the following condition [6] :

$$F^i_{j,k} + F^i_{k,j} = 0 ,$$

and from this condition we have

$$\varphi_{\alpha\beta;\gamma}+\varphi_{\alpha\gamma;\beta}=H_{\alpha\gamma}\eta_\beta+H_{\alpha\beta}\eta_\gamma-2H_{\beta\gamma}\eta_\alpha \tag{4.1}$$

by virtue of (3.11). On the other hand, in a manifold with normal contact metric structure the relation

$$\varphi_{\alpha\beta;\gamma}=\eta_\alpha g_{\beta\gamma}-\eta_\beta g_{\alpha\gamma} \tag{4.2}$$

holds good [5]. Then if the hypersurface of a K-space has the normal contact metric structure, by means of (4.1) and (4.2) we get

$$-\eta_\beta g_{\alpha\gamma}-\eta_\gamma g_{\alpha\beta}+2\eta_\alpha g_{\beta\gamma}=\eta_\beta H_{\alpha\gamma}+\eta_\gamma H_{\alpha\beta}-2\eta_\alpha H_{\beta\gamma}\,, \tag{4.3}$$

$$-\eta_\gamma g_{\beta\alpha}-\eta_\alpha g_{\beta\gamma}+2\eta_\beta g_{\gamma\alpha}=\eta_\gamma H_{\beta\alpha}+\eta_\alpha H_{\beta\gamma}-2\eta_\beta H_{\gamma\alpha}\,. \tag{4.3'}$$

Subtracting (4.3′) from (4.3) it follows that

$$(H_{\alpha\gamma}+g_{\alpha\gamma})\eta_\beta=(H_{\beta\gamma}+g_{\beta\gamma})\eta_\alpha\,. \tag{4.4}$$

This equation means that expressions in parentheses for each value of γ are proportional to η_α. Then we may put

$$H_{\alpha\gamma}=-g_{\alpha\gamma}+\eta_\alpha\rho_\gamma\,, \tag{4.5}$$

where ρ_γ being proportional factor. If (4.4) be multiplied by $\xi^\alpha\xi^\gamma$ and summed for α and γ, we get

$$(H_{\alpha\gamma}+g_{\alpha\gamma})\xi^\alpha\xi^\gamma\eta_\beta=H_{\beta\gamma}\xi^\gamma+\eta_\beta\,. \tag{4.6}$$

Also, multiplying ξ^α to (4.5) and summed for α we have

$$H_{\alpha\gamma}\xi^\alpha+\eta_\gamma=\rho_\gamma\,. \tag{4.7}$$

In consequence of (4.6) and (4.7) we can write $\rho_\alpha=\phi\eta_\alpha$, where ϕ is a scalar function. Therefore the second fundamental tensor has the form

$$H_{\alpha\beta}=-g_{\alpha\beta}+\phi\eta_\alpha\eta_\beta\,. \tag{4.8}$$

Then we have the following

Theorem 4.1. *If a hypersurface of a K-space has the normal contact metric structure, the second fundamental tensor has the form* (4.8).

§5. Some properties of η-umbilical hypersurface in the space with constant curvature. In the following, when the second fundamental tensor has the form (4.8) we shall call the hypersurface *"η-umbilical hypersurface"*. A totally umbilical hypersurface of a constant curvature space is also a constant curvature space [1]. Now, we shall seek the analogous problem in the case of η-umbilical hypersurface.

When M^{2n+2} is a space of constant curvature, the equation of Gauss-Codazzi may be reduced to

$$(5.1) \qquad R_{\alpha\beta\gamma\delta} = K(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + H_{\alpha\gamma}H_{\beta\delta} - H_{\alpha\delta}H_{\beta\gamma}\,.$$

Substituting (4.8) into (5.1) we have

$$(5.2) \qquad \begin{aligned} R_{\alpha\beta\gamma\delta} &= (K+1)\,(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) \\ &\quad + \phi(-g_{\alpha\gamma}\eta_\beta\eta_\delta - g_{\beta\delta}\eta_\alpha\eta_\gamma + g_{\alpha\delta}\eta_\beta\eta_\gamma + g_{\beta\gamma}\eta_\alpha\eta_\delta)\,. \end{aligned}$$

Then, if an η-umbilical hypersurface is also a constant curvature space, we may put

$$(5.3) \qquad -g_{\alpha\gamma}\eta_\beta\eta_\delta - g_{\beta\delta}\eta_\alpha\eta_\gamma + g_{\alpha\delta}\eta_\beta\eta_\gamma + g_{\beta\gamma}\eta_\alpha\eta_\delta = \psi(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})\,,$$

where ψ being a scalar function. If (5.3) be multiplied by $g^{\alpha\delta}$ and summed for α and δ, we get

$$(2n-1)\eta_\beta\eta_\gamma = -(2n\psi+1)g_{\beta\gamma}\,.$$

The last relation gives the fact that the rank of the matrix $(g_{\alpha\beta})$ be less than $2n+1$. This result contradicts to the fact that $g_{\alpha\beta}$ is an induced metric tensor of the hypersurface. Therefore we have

Theorem 5.1. *An η-umbilical hypersurface of a constant curvature space is not a constant curvature space.*

If (5.1) be multiplied by $g^{\alpha\delta}$ and summed for α and δ we get

$$(5.4) \qquad R_{\beta\gamma} = -2nKg_{\beta\gamma} + H_{\beta\delta}H^\delta_\gamma - (2n+1)HH_{\beta\gamma}\,,$$

where $H = \dfrac{1}{2n+1}H_{\alpha\beta}g^{\alpha\beta}$. If we substitute (4.8) into (5.4) it follows that

$$(5.5) \qquad R_{\beta\gamma} = \{(2n+1)H - 2nK + 1\}\,g_{\beta\gamma} + \{\phi^2 - [(2n+1)H + 2]\phi\}\,\eta_\beta\eta_\gamma\,.$$

Then we have

Theorem 5.2. *An η-umbilical hypersurface of a constant curvature space is an η-Einstein space.*

According to the theorem obtained by M. Okumura [2], if a normal contact space in an η-Einstein space, i. e. the Ricci tensor has the form $R_{\alpha\beta} = ag_{\alpha\beta} + b\eta_\alpha\eta_\beta$, then a and b must be constants. Then, in consequence of Theorem 4.1 and Theorem 5.2, if a hypersurface of a K-space has normal contact metric structure we get from (5.5)

$$(2n+1)H - 2nK + 1 = \text{const}\,.$$

As it follows that

$$K = \text{const}\,.$$

from the theorem of Schur, we obtain $H=$const. Then by means of (4.8) we also have $\phi=$const. Therefore we have

Theorem 5.3. *If a hypersurface of a K-space with constant curvature has a normal contact metric structure then*

$$H_{\alpha\beta}=-g_{\alpha\beta}+\phi\eta_\alpha\eta_\beta\,,$$

where $\phi=$const. and the hypersurface has constant mean curvature.

§6. Some properties of a hypersurface of a Kählerian space. In this section we shall assume that M^{2n+2} be a Kählerian space. Then, we have from (3.12) and (3.13)

$$n^\alpha_{\beta\gamma}=(H^\epsilon_\beta\varphi^\alpha_\epsilon-H^\alpha_\epsilon\varphi^\epsilon_\beta)\eta_\gamma-(H^\epsilon_\gamma\varphi^\alpha_\epsilon-H^\alpha_\epsilon\varphi^\epsilon_\gamma)\eta_\beta\,, \tag{6.1}$$

$$\eta_{\beta;\alpha}-\eta_{\alpha;\beta}=\varphi^\epsilon_\alpha H_{\beta\epsilon}-\varphi^\epsilon_\beta H_{\alpha\epsilon}\,. \tag{6.2}$$

In an almost contact metric space there exists a (1, 1)-tensor n^α_β defined by

$$n^\alpha_\beta=\xi^\gamma(\varphi^\alpha_{\beta;\gamma}-\varphi^\alpha_{\gamma;\beta})-\varphi^\gamma_\beta\xi^\alpha_{;\gamma}\,. \tag{6.3}$$

By means of (3.11) and (3.12), we have the following expression of the tensor n^α_β:

$$n^\alpha_\beta=-H^\alpha_\beta-H^\gamma_\epsilon\varphi^\alpha_\gamma\varphi^\epsilon_\beta+H^\alpha_\epsilon\xi^\epsilon\eta_\beta\,. \tag{6.4}$$

Since the vanishment of $n^\alpha_{\beta\gamma}$ implies the vanishment of n^α_β [4], if an induced almost contact metric structure is normal contact, the second fundamental tensor must satisfy the relation

$$H^\alpha_\beta+H^\gamma_\epsilon\varphi^\alpha_\gamma\varphi^\epsilon_\beta-H^\alpha_\epsilon\xi^\epsilon\eta_\beta=0\,. \tag{6.5}$$

Conversely if (6.5) be multiplied by φ^β_σ and summed for β, it follows that

$$\varphi^\beta_\sigma(H^\alpha_\beta+H^\gamma_\epsilon\varphi^\alpha_\gamma\varphi^\epsilon_\beta)=0\,. \tag{6.6}$$

Making use of the last relation of (2.4), the above relation reduces to

$$H^\gamma_\sigma\varphi^\alpha_\gamma-H^\alpha_\beta\varphi^\beta_\sigma=H^\gamma_\epsilon\xi^\epsilon\varphi^\alpha_\gamma\eta_\sigma\,. \tag{6.7}$$

Substituting the right hand side of (6.7) into (6.1), we find that $n^\alpha_{\beta\gamma}=0$. Then we have

Theorem 6.1. *In order that an induced contact metric structure of a hypersurface in a Kählerian space is normal, it is necessary and sufficient that the second fundamental tensor $H_{\alpha\beta}$ satisfy the relation* (6.5).

Corollary. *With respect to an induced contact metric structure of a hypersurface in a Kählerian space, relations $n^\alpha_{\beta\gamma}=0$ and $n^\alpha_\beta=0$ are equivalent.*

Next, we shall consider the case when the induced almost contact metric structure of a hypersurface in a Kählerian space reduces to a normal contact metric structure.

In consequence of Theorem 4. 1, the second fundamental tensor $H_{\alpha\beta}$ has the form (4. 8), as a Kählerian space is the special class of a K-space. Conversely from (4. 8) and (6. 1), we have $n^{\alpha}_{\beta\gamma}=0$, i. e. the induced structure is normal. Also, substituting (4. 8) into the right hand side of (6. 2), it follows that

$$\frac{1}{2}(\varphi^{\gamma}_{\alpha} H_{\beta\gamma} - \varphi^{\varepsilon}_{\beta} H_{\alpha\varepsilon}) = \varphi_{\alpha\beta} .$$

Then, we have

$$\frac{1}{2}(\eta_{\beta;\alpha} - \eta_{\alpha;\beta}) = \varphi_{\alpha\beta} ,$$

i. e. the induced structure is contact. Therefore we have

Theorem 6. 2[3]**.** *In order that an induced almost contact metric structure of a hypersurface in a Kählerian space is normal contact metric structure, it is necessary and sufficient that the second fundamental tensor has the form* (4. 8).

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3) This result was obtained by Y. TASHIRO and in the paper [7] it is proved by another way.