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## Localization of two-dimensional electrons in a random magnetic field

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The localization properties of a two-dimensional electron system in the presence of a random magnetic field are numerically studied by calculating directly eigenstates of large systems. Analyzing eigenstates themselves gives precise information on the localization of wave functions compared with that obtained from transport properties. The finite-size scaling analysis for quasi-one-dimensional wave functions clearly shows that the single-parameter scaling hypothesis is valid for this system and all states are localized. Furthermore, an analysis based on the multifractality of a *localized* wave function also supports these results. [S0163-1829(96)05444-6]

The scaling theory of localization predicts that all electronic states in a disordered potential are localized in an infinite two-dimensional (2D) system.<sup>1</sup> In a strong and homogeneous magnetic field, however, extended states due to the broken time-reversal symmetry appear. This effect has been intensively studied in connection with the quantum Hall effect.<sup>2</sup> It is quite natural to ask whether extended states exist in a 2D system in which the time-reversal symmetry is *locally* and *randomly* broken by a random magnetic field (RMF). Although this is a stimulating and fundamental question and was proposed fifteen years ago,<sup>3</sup> there had been no systematic study on this problem for a long time because of the lack of corresponding realistic systems. In recent years, however, a great deal of effort has been concentrated on the localization properties of a noninteracting 2D charged fermion in the RMF,<sup>4-14</sup> since it has been suggested that the quantum states in this system are relevant to the physics of strongly correlated electron systems such as fractional quantum Hall systems<sup>10,15</sup> or high- $T_c$  superconductors.<sup>16</sup> In addition to this theoretical motivation, recent experimental works on direct realization of the 2D RMF system also encourage theoretical and numerical investigations.<sup>17,18</sup>

From the viewpoint of the nonlinear  $\sigma$  model, naively, all states in the RMF system would be localized because the topological term, proportional to the transverse conductivity  $\sigma_{xy}$ , in the effective Lagrangian vanishes due to no net magnetic field. However, Zhang and Arovas<sup>11</sup> have argued that the long-range logarithmic interaction between the topological densities leads to a Kosterlitz-Thouless transition from localized to extended states. In contrast to this theoretical argument, Aronov *et al.*<sup>12</sup> have shown that the localization problem of this system can be mapped onto a nonlinear  $\sigma$  model with unitary symmetry, which means that all states are localized. Another  $\sigma$  model for a multichannel random network system also suggests the absence of the mobility edge in the 2D RMF system.<sup>13</sup> Sugiyama and Nagaosa<sup>4</sup> have numerically studied this problem at the early stage. They concluded, from a finite-size scaling analysis, that all electronic states are localized. The same conclusion has been numerically presented by a two-channel random network model that is valid within the semiclassical limit.<sup>5</sup> However, many of recent numerical works show the existence of the mobility edge.<sup>6-10</sup> Thus the localization problem of this system is now controversial.

In most of the previous numerical works based on finite-size scaling analyses, the localization of a 2D electron in a RMF was examined through electron transport, such as a conductance or the Lyapunov exponent, instead of an eigenstate itself. In these numerical calculations, one must treat a quite long quasi-one-dimensional system. If the system length is not sufficient, the electron transmission occurs through several eigenstates whose eigenenergies are close each other. Therefore the transport properties reflect the superposition of these eigenstates and tend to lead to a conclusion of delocalization. In spite of the advantage that the direct calculation of eigenstates does not have this problem, there has been no finite-size scaling analysis by using eigenstates themselves due to the absence of a suitable diagonalization technique that satisfies the following conditions: (i) One can treat very large matrices, (ii) it is possible to calculate an eigenvector belonging to a specific eigenvalue that one needs, and (iii) calculated eigenvectors have great accuracy.

In this paper we compute electron eigenstates of large-scale 2D systems in the presence of a RMF by employing the forced oscillator method<sup>19,20</sup> and clarify whether the delocalization transition exists in this system. At first, eigenstates of quasi-one-dimensional systems are evaluated by a finite-size scaling argument and the localization lengths of infinite 2D system are estimated. The result shows that all states are localized. Furthermore, we perform an analysis based on the multifractality of a *localized* wave function. In this multifractal analysis, the mass exponents  $\tau(q)$  and multifractal spectra  $f(\alpha)$  of the eigenstates of systems with square geometry are calculated. The result of the multifractal analysis also supports the absence of the extended states.

We study the tight-binding model on a square lattice with nearest-neighbor hopping described by the Hamiltonian

$$H = - \sum_{\langle ij \rangle} |i\rangle \exp \left[ i \frac{2\pi}{\phi_0} a_{ij} \right] \langle j| + \text{H.c.}, \quad (1)$$

where  $i$  and  $j$  label the sites on the square lattice,  $\phi_0$  ( $=hc/e$ ) is the unit of flux quanta, and  $a_{ij}$  is the line integral of a random vector potential along the link  $(ij)$ . The magnetic flux per plaquette is given by  $\phi = \sum_{\square} a_{ij}$ , where the summation runs over four links around a plaquette. The

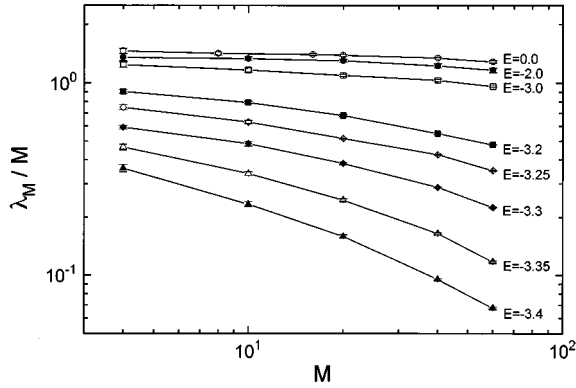


FIG. 1. Renormalized quasi-one-dimensional localization length  $\lambda_M/M$  as a function of the strip width  $M$  for various energies.

value of  $\phi$  for each plaquette is uniformly distributed between  $-\phi_0/2$  and  $\phi_0/2$ . Periodic boundary conditions have been considered here for both the  $x$  and  $y$  directions. The density of states of this system has a single band structure that is symmetric around the band center  $E_0=0$  due to a symmetry between bipartite sublattices, and there is no singularity in the density of states at the band center.

We have numerically calculated eigenenergies and corresponding wave functions by the forced oscillator method.<sup>19,20</sup> This numerical technique enables us to compute eigenvalues and eigenvectors of a very large matrix. Previously mentioned conditions for a suitable diagonalization algorithm [(i)–(iii)] are all satisfied by using this method. We calculate first wave functions of long-strip (quasi-one-dimensional) systems with finite widths  $M$  and lengths  $L$ . The values of  $L$  are chosen to be several tens of  $M$ . Wave functions are calculated at eight different energies in the region from the lower band edge  $-E_d$  ( $\approx -3.6$ ) to the band center  $E_0$  ( $=0$ ). We prepared about 15 wave functions in different magnetic flux configurations for the ensemble average. The accuracy of each wave function is as high as  $A_2/A_1 \approx 10^{-60}$ , where  $A_1$  is the amplitude of the eigenvector that we need and  $A_2$  is the largest amplitude of mixed modes.<sup>20</sup> The localization length  $\lambda_M(E)$  of the quasi-one-dimensional system has been calculated from the geometric mean of wave functions  $\psi(x, y_0)$  for fixed  $E$  and  $M$ , where  $y_0 = \int_0^M \int_0^L |\psi(x, y)|^2 dx dy$ . Figure 1 shows the values of  $\lambda_M(E)$  divided by the strip width  $M$  versus  $M$ . The ratio  $\lambda_M(E)/M$  is a monotonically decreasing function of  $M$  for every energy. This suggests that all states in a 2D RMF system are localized.

A finite-size scaling analysis for  $\lambda_M(E)$  makes it possible to test the validity of the single-parameter scaling hypothesis,<sup>1,21</sup> i.e.,

$$\frac{\lambda_M(E)}{M} = f\left(\frac{\xi(E)}{M}\right), \quad (2)$$

where  $\xi(E)$  is the localization length for an infinite 2D system. The scaling function  $f(x)$  should behave as  $f(x) = x$  for  $x \ll 1$ . By fitting numerically the value of  $\xi$  for each energy and the form of the scaling function  $f$ , all data in Fig. 1 collapse to a single (solid) line shown in Fig. 2. This line approaches the straight line  $f(x) = x$  (shown by the dotted

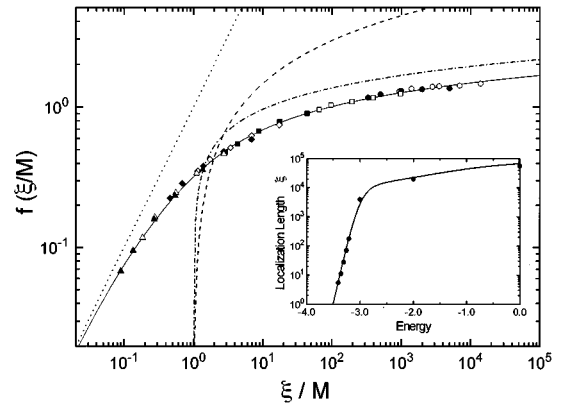


FIG. 2. Double logarithmic plot of the scaling function as a function of  $\xi/M$ . The scaling parameter  $\xi$  is chosen to fit all data in Fig. 1 onto a universal curve (solid line). The dotted straight line shows  $f(x) = x$ . The dashed and broken lines indicate  $f(x) = (2/\pi)\ln x$  and  $f(x) = (2/\pi)^{1/2}(\ln x)^{1/2}$ , which are asymptotic forms for  $x \gg 1$  predicted theoretically for the orthogonal and unitary classes, respectively. The energy dependence of the localization length  $\xi$  is shown in the inset.

line) for  $x \ll 1$ . The dashed and broken lines indicate the asymptotic forms of the scaling functions for  $x \gg 1$ ,  $f(x) = (2/\pi)\ln x$  and  $f(x) = (2/\pi)^{1/2}\sqrt{\ln x}$ , which are theoretically predicted if the system belongs to the orthogonal or unitary class, respectively.<sup>12,21</sup> The scaling function obtained numerically is close to the broken line, which suggests that the present system can be characterized by the unitary ensemble. The inset of Fig. 2 shows the energy dependence of the localization length  $\xi$  obtained in the least-squares fitting procedure. In the vicinity of the band edge,  $\xi$  increases as  $\xi = \xi_0 e^{(E - E_d)/\varepsilon}$ , where  $\xi_0 = 0.29$  and  $\varepsilon = 0.063$ . This agrees well with the result of Ref. 4. Approaching the band center, the localization length grows more slowly and takes a finite (but large) value at the band center. Consequently, these results by the finite-size scaling analysis show that all states in the 2D RMF system are localized.

In order to confirm the conclusion of the 2D electron localization in a RMF from another viewpoint, we perform a multifractal analysis of wave functions. It is supposed that a localized state with its localization length  $\xi$  exceeding the system size  $L$  cannot be distinguished from an extended state. These two states, however, should have some differences in distributions of amplitudes of the wave functions. The basic idea we use here is to separate localized states from extended ones by taking such differences into account. It is well known that if a system has a mobility edge  $E_c$  amplitudes of the wave function at  $E_c$  are distributed in a multifractal manner.<sup>22</sup> In analogy to the usual critical phenomena, it is plausible that the distribution of a wave function that is localized with a localization length  $\xi$  or extended with a correlation length  $\xi$  is also multifractal in a scale of  $L$  much smaller than  $\xi$ .<sup>23</sup> Furthermore, the multifractality does not depend on  $\xi$  or  $L$  as long as  $L \ll \xi$ . In fact, a recent theoretical study shows that a wave function localized in a 2D disordered system is multifractal if the system size  $L$  does not exceed the localization length  $\xi$ .<sup>24</sup>

We study the localization properties in terms of the energy dependence of the mass exponent  $\tau(q)$  characterizing the multifractality of wave functions. This quantity  $\tau(q)$  is defined by the relation<sup>22</sup>

$$Z(q, l) \equiv \sum_b \left( \sum_{i \in b(l)} |\psi_i|^2 \right)^q \propto l^{\tau(q)}, \quad (3)$$

where  $\psi_i$  is the amplitude of the wave function at the  $i$ th site and is normalized as  $\sum_i |\psi_i|^2 = 1$  (the summation is taken over all sites in the whole system). The first summation in Eq. (3) is taken over boxes of a linear size  $l$  into which one divides the whole system and the second summation over sites in the box. For  $q=2$ ,  $\tau(2)$  indicates the system-size dependence of the inverse participation ratio  $P$  as  $P \propto L^{-\tau(2)}$ . Let us consider the energy dependence of  $\tau(2)$  for a finite system possessing localized states whose localization lengths increase monotonically with the energy approaching the band center  $E_0$  ( $=0$ ). In the vicinity of the band edge, the value of  $\tau(2)$  is close to zero because the localization length  $\xi$  is much smaller than the system size  $L$ . Approaching the band center,  $\tau(2)$  increases until  $\xi$  reaches the scale  $L$ . For  $\xi \gg L$ , the value of  $\tau(2)$  takes a specific value  $\tau^*$  less than 2 which is the Euclidean dimension of the system. If all states are localized,  $\tau(2)$  keeps this value  $\tau^*$  up to the band center. In contrast, if the system has a mobility edge  $E_c$ ,  $\tau(2)$  could again depend on the energy near the band center. Since the localization (or correlation) length  $\xi$  of a wave function is a function of  $|E - E_c|$ ,  $\xi$  exceeds the system size  $L$  for  $|E - E_c| < \varepsilon_L$ , where  $\varepsilon_L$  is the energy satisfying the relation  $\xi(E_c \pm \varepsilon_L) = L$ . In this energy region  $|E - E_c| < \varepsilon_L$ ,  $\tau(2)$  should take the value  $\tau^*$ . In the lower half band, if  $E_c + \varepsilon_L < E_0$ , the mass exponent  $\tau(2)$  increases again with energy toward  $\tau(2) = 2$  in the region  $E_c + \varepsilon_L < E < E_0$ . Therefore it is possible to distinguish a localized wave function with the localization length much longer than the system size from truly extended states.

We have calculated eigenstates of 2D RMF systems with square geometry ( $64 \times 64$ ,  $128 \times 128$ , and  $256 \times 256$ ) by means of the forced oscillator method.<sup>19,20</sup> For each size of systems, 20 (for  $64 \times 64$  and  $128 \times 128$ ) and 10 (for  $256 \times 256$ ) wave functions were prepared for ensemble averages. Figure 3 shows the calculated  $\tau(2)$  as a function of energy. The mass exponent  $\tau(2)$  takes a small value near the band edge and increases with energy for every system size. The  $\tau(2)$ 's reach a constant value  $\tau^*$  at  $E \approx -2.0$  ( $\equiv -E_{64}$ ),  $-1.5$  ( $\equiv -E_{128}$ ), and  $-1.0$  ( $\equiv -E_{256}$ ) for the systems of  $L = 64$ , 128, and 256, respectively, and keep this value up to the vicinity of the band center. The value of  $\tau^*$  is  $1.79 \pm 0.01$  independently of the system size. The fact that  $\tau(2)$  does not depend on the energy for  $-E_L \leq E < 0$  implies the following two possibilities. (a) All electronic states are localized, whereas localization lengths are longer than the system size  $L$  for  $-E_L < E < E_L$ . (b) Extended states exist, but the correlation lengths  $\xi$  exceed the size  $L$  in all the metallic region  $|E| < E_c$ . We cannot exclude possibility (b) by our multifractal analysis. However, even in case (b), the mobility edge  $E_c$  should be less than  $E_L/2$  because the length  $\xi$  is a function of  $|E - E_c|$ . Our numerical result for  $L = 256$  gives  $E_{256} = 1.0$ , which means that the mobility edge would exist within the range  $0 \leq E_c \leq 0.5$ , even if  $E_c$  exists.

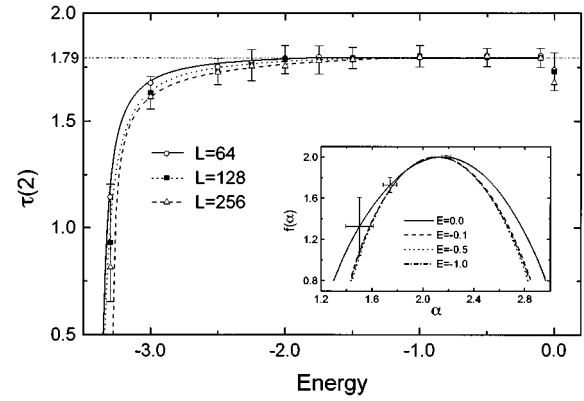


FIG. 3. Mass exponent  $\tau(2)$  as a function of the energy for systems with square geometry ( $\circ$ ,  $64 \times 64$ ;  $\blacksquare$ ,  $128 \times 128$ ;  $\triangle$ ,  $256 \times 256$ ). Error bars only for  $L = 128$  are indicated. The inset shows the multifractal spectra  $f(\alpha)$  for the system of  $L = 256$ . Error bars show typical errors at three representative points.

The mobility edges predicted by previous numerical works take values around 3.0.<sup>6-9</sup> The smallest value of them, as far as we know, is  $E_c = 2.4$ .<sup>7</sup> Even our largest possible mobility edge ( $E_c = 0.5$ ) is much less than this value. Considering this situation and our result by the finite-size scaling, we believe that the energy dependence of  $\tau(2)$  shows case (a), namely, all states are localized. These behaviors of the mass exponent have been observed also for other values of  $q$ .

In Fig. 3,  $\tau(2)$  only at the band center takes a value slightly less than  $\tau^*$ . This implies that the wave function at the band center has a fractality different from other energy states. In order to demonstrate this more clearly, we calculated multifractal spectra  $f(\alpha)$  obtained from  $\tau(q)$  through the relation  $f(\alpha) = q\alpha - \tau(q)$ , where  $\alpha = \partial\tau(q)/\partial q$ . We found, as shown in the inset of Fig. 3, that the multifractal spectra  $f(\alpha)$  of wave functions at energies other than the band center collapse to a single function  $f^*(\alpha)$ , whereas  $f(\alpha)$  at the band center has a curve different from  $f^*(\alpha)$ . This fact clearly shows that the distribution of the wave function at the band center is qualitatively different from those of other states. We guess that the origin of the different fractality is due to the symmetry of the bipartite lattices.

In conclusion, we studied the localization properties of noninteracting 2D electrons in a RMF by means of direct calculations of eigenstates. The results by both the finite-size scaling and the multifractal analyses show that all states are localized. Kalmeyer and Zhang<sup>10</sup> have shown that the inverse participation ratio  $P$  of the 2D RMF system depends on the system size  $L$  as  $P \propto L^{-1.5}$  and they have concluded the existence of extended states. This result, meaning  $\tau(2) = 1.5$ , is qualitatively consistent with our result of  $\tau^* = 1.79$ . We, however, consider that the fact that  $\tau^* < 2$  is evidence of the localization. Kawarabayashi and Ohtsuki<sup>14</sup> have recently analyzed electronic states of the RMF system by simulating quantum diffusion processes. They found that the exponent  $\alpha$  characterizing an autocorrelation function  $C(t)$  of the diffusion as  $C(t) \propto t^{-\alpha}$  takes a value of 0.9 regardless of energies in the vicinity of the band center. Since  $\tau(2) = 2\alpha$ ,<sup>25</sup> their result shows that  $\tau(2)$  ( $= 1.8$ ) does not depend on the energy, which agrees quite well with results

by our multifractal analysis. They claim from this result that all states near the band center are critical. This conclusion, however, conflicts with our finite-size scaling analysis and with the fact that the energy  $E_L$  at which  $\tau(2)$  starts to converge to  $\tau^*$  depends on the system size  $L$ . It is natural to interpret that localized states are observed on a scale much less than the localization length. All these numerical facts suggest localized states in the whole band.

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