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Spin filter effects in an Aharonov–Bohm ring with double quantum dots under general Rashba spin–orbit interactions

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E-mail: kkondo@es.hokudai.ac.jp**Keywords:** spintronics, spin–orbit interaction, general Rashba effect, Aharonov–Bohm ring, spin filter

Abstract

Many researchers have reported on spin filters using linear Rashba spin–orbit interactions (SOI). However, spin filters using square and cubic Rashba SOIs have not yet been reported. We consider that this is because the Aharonov–Casher (AC) phases acquired under square and cubic Rashba SOIs are ambiguous. In this study, we try to derive the AC phases acquired under square and cubic Rashba SOIs from the viewpoint of non-Abelian SU(2) gauge theory. These AC phases can be derived successfully from the non-Abelian SU(2) gauge theory without the completing square methods. Using the results, we investigate the spin filtering in a double quantum dot (QD) Aharonov–Bohm (AB) ring under linear, square, and cubic Rashba SOIs. This AB ring consists of elongated QDs and quasi-one-dimensional quantum nanowires under an external magnetic field. The spin transport is investigated from the left nanowire to the right nanowire in the above structure within the tight-binding approximation. In particular, we focus on the difference of spin filtering among linear, square, and cubic Rashba SOIs. The calculation is performed for the spin polarization by changing the penetrating magnetic flux for the AB ring subject to linear, square, and cubic Rashba SOIs. It is found that perfect spin filtering is achieved for all of the Rashba SOIs. This result indicates that this AB ring under general Rashba SOIs can be a promising device for spin current generation. Moreover, the AB rings under general Rashba SOIs behave in totally different ways in response to penetrating magnetic flux, which is attributed to linear, square, and cubic behaviors in the in-plane momentum. This result enables us to make a clear distinction between linear, square, and cubic Rashba SOIs according to the peak position of the perfect spin filtering.

1. Introduction

Recently, many researchers have paid much attention to spintronics for its application to spin functional devices and quantum computing. One of the main purposes of spintronics is to make spin transistors and magnetoresistive devices by utilizing electron spin [1–7]. The generation of spin current is significant in order to realize these devices in the future. Moreover, spin current generation without ferromagnetic metals is a key issue for spintronics devices. The spin–orbit interaction (SOI) is expected to be a promising ingredient for this purpose. Since it has been confirmed that the strength of the Rashba SOI can be easily controlled by an external electric field [4], the theory of devices using SOI as a spin filter is very important in the field of spintronics. The spin-filtering devices can be used to generate and detect spin polarized currents [8, 9]. There exist spin filters using ring-type conductors or two path devices [10–16]. This opens up the field of spin-dependent Aharonov–Bohm (AB) physics. The AB ring-type spin-filter devices frequently utilize linear Rashba and/or Dresselhaus SOIs. Generally, the spin-splitting of electrons and light hole states is proportional to wavenumber k and the splitting of heavy hole states is proportional to k^3 [17]. Until now, only linear Rashba SOI was taken into consideration for electrons. However, cubic Rashba spin-splitting has recently been found in a quasi-two-dimensional electron system [18]. Also, the square Rashba effect has been found in bilayer graphene [19]. Therefore, we must take into consideration the square and cubic Rashba effects in spin-filtering devices.

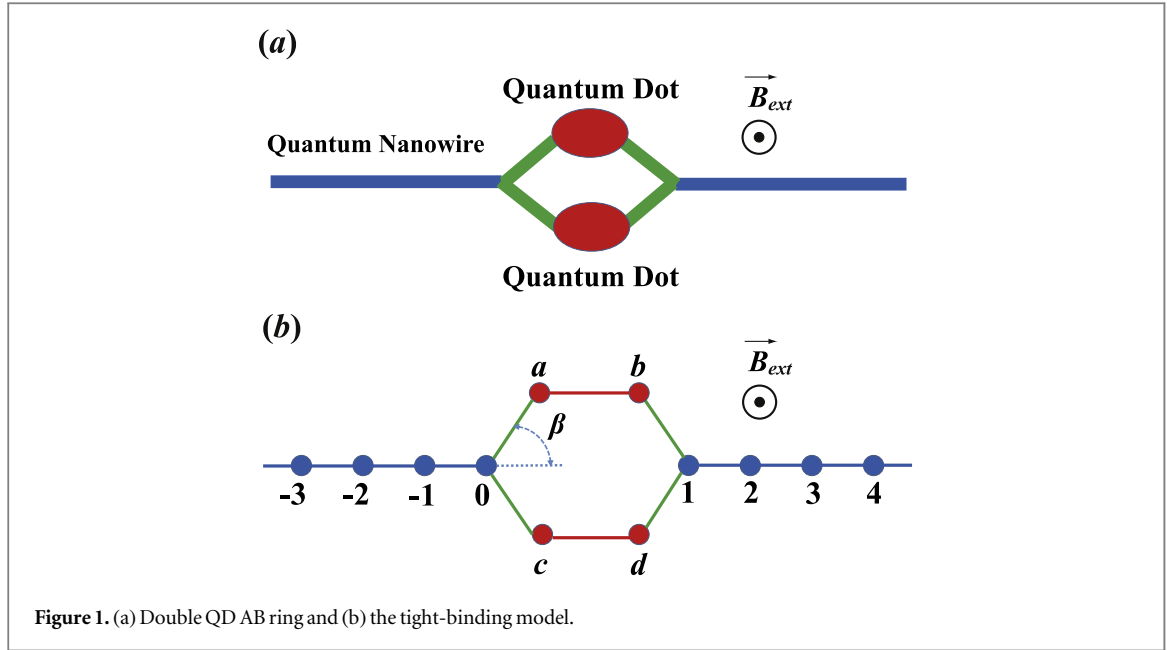


Figure 1. (a) Double QD AB ring and (b) the tight-binding model.

However, the Aharonov–Casher (AC) phases acquired under square and cubic Rashba SOIs are ambiguous. In this study, we try to derive the AC phases acquired under square and cubic Rashba SOIs from the viewpoint of non-Abelian SU(2) gauge theory without the completing square methods. Using the results, we investigate the spin filtering in a double quantum dot (QD) AB ring under general Rashba SOIs.

2. Theory

2.1. Modeling and tight-binding approximation

The double QD AB ring consists of two elongated QDs and quasi-one-dimensional quantum nanowires (QNs) under an external magnetic field $\vec{B}_{\text{ext}}$ as shown in figure 1(a). The two QDs and the four oblique QNs are subject to general Rashba SOI and the left and right straight QNs are free of Rashba SOI. The spin transport is investigated from the left nanowire to the right nanowire in this structure within the framework of tight-binding approximation as shown in figure 1(b).

Within the framework of the tight-binding approximation, the Schrödinger equation for the wave function $|\Psi_n\rangle$ at site n is described as follows [14]:

$$(\epsilon - \epsilon_n) |\Psi_n\rangle = \sum_{m \neq n} V_{nm} U_{nm} |\Psi_m\rangle = - \sum_{m \neq n} J_{nm} U_{nm} |\Psi_m\rangle, \quad (1)$$

where ϵ is the eigenenergy, ϵ_n is the energy of site n , V_{nm} is defined as $\langle \phi_n | v_n | \phi_m \rangle$, and J_{nm} is the hopping energy between site n and site m which is defined as $\langle \phi_n | \frac{-\hbar^2 \nabla^2}{2m_e} | \phi_m \rangle$. Here, m_e is the electron mass, $|\phi_n\rangle$ is the atomic wave function of site n without interaction, and v_n is the single-site atomic potential of site n . In (1), U_{nm} is the 2×2 unitary matrix describing the AB [20] and AC phase [21] that an electron acquires when it travels from site m to site n . The AB phase is attributed to the magnetic flux penetrating the ring and the AC phase is attributed to the SOI like Rashba effect. In order to know the AC phase acquired by the SOI, we need to know the gauge field. The method of completing the square of the Hamiltonian is only useful for linear Rashba and linear Dresselhaus SOIs [22]. This method is useless for square and cubic Rashba SOIs.

2.2. Effective fields for general Rashba SOIs

The Hamiltonians H_{R1} , H_{R2} , and H_{R3} for linear, square, and cubic Rashba SOIs have the following forms, respectively:

$$H_{R1} = \frac{\hbar^2 k_{R1}}{m_e} (k_y \sigma_x - k_x \sigma_y) = \frac{\hbar^2 k_{R1}}{m_e} \vec{\sigma} \cdot \vec{B}_{R1}(\vec{k}), \quad (2)$$

$$H_{R2} = i \frac{\hbar^2 k_{R2}}{m_e k_F} \left((k_-)^2 \sigma_+ - (k_+)^2 \sigma_- \right) = \frac{\hbar^2 k_{R2}}{m_e} \vec{\sigma} \cdot \vec{B}_{R2}(\vec{k}), \quad (3)$$

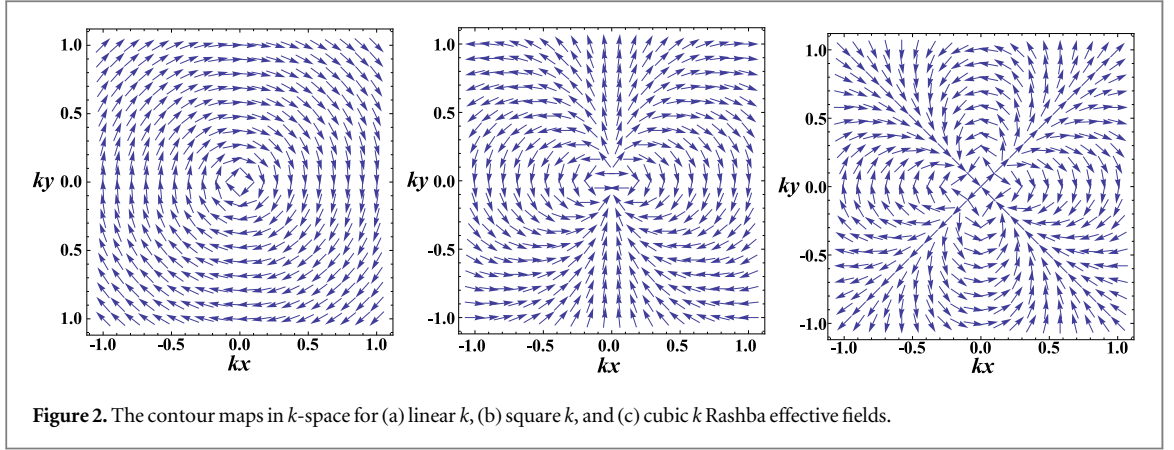


Figure 2. The contour maps in k -space for (a) linear k , (b) square k , and (c) cubic k Rashba effective fields.

$$H_{R3} = i \frac{\hbar^2 k_{R3}}{m_e k_F^2} \left((k_-)^3 \sigma_+ - (k_+)^3 \sigma_- \right) = \frac{\hbar^2 k_{R3}}{m_e} \vec{\sigma} \cdot \vec{B}_{R3}(\vec{k}), \quad (4)$$

where $\hbar$ is the Planck constant divided by 2π , k_{R1} , k_{R2} , and k_{R3} are the SOI strengths of the linear, square, and cubic Rashba effects, respectively, k_F is the Fermi wavenumber determined by the chemical potential, $\vec{B}_{R1}(\vec{k})$, $\vec{B}_{R2}(\vec{k})$, and $\vec{B}_{R3}(\vec{k})$ are effective magnetic fields for their SOIs, k_x and k_y are the wavenumbers in the x - and the y -directions, respectively, $k_{\pm} = k_x \pm ik_y$, and $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$. Here, σ_x and σ_y are the x - and y -components of Pauli matrix vector $\vec{\sigma}$. These SOIs affect the motion of electrons as effective magnetic fields as described in equations (2)–(4). These effective magnetic fields are explicitly written by the following equations:

$$\vec{B}_{R1}(\vec{k}) = (k_y, -k_x, 0), \quad (5)$$

$$\vec{B}_{R2}(\vec{k}) = (2k_x k_y, k_y^2 - k_x^2, 0), \quad (6)$$

$$\vec{B}_{R3}(\vec{k}) = (3k_x^2 k_y - k_y^3, 3k_x k_y^2 - k_x^3, 0). \quad (7)$$

The above effective magnetic fields are generated by non-Abelian gauge fields [23–25]. Figure 2 shows the contour maps of the above effective fields.

We find that the spin of the electron rotates once, twice, and three times around the origin in k -space under the linear, square, and cubic Rashba effects, respectively.

2.3. SU(2) non-Abelian gauge and AC phase

It is convenient to redefine the axes at each point $\vec{k}$ such that the reference k_z -axis is aligned to the direction of the local effective magnetic fields. In order to perform such a rotation, we need a unitary matrix $U_g(\vec{k})$ which satisfies $U_g(\vec{k})^\dagger (\vec{n}_i \cdot \vec{\sigma}) U_g(\vec{k}) = \sigma_z$, where $\vec{n}_i = \vec{B}_{Ri}(\vec{k})/|\vec{B}_{Ri}(\vec{k})|$, ($i = 1, 2, 3$). This rotation is a kind of local gauge transformation. Under the local gauge transformation $U_g(\vec{k})$, these Hamiltonians transform to the following equations,

$$H_{R1} = \frac{\hbar^2 k_{R1}}{m_e} \sigma_z |\vec{B}_{R1}(\vec{k})|, \quad (8)$$

$$H_{R2} = \frac{\hbar^2 k_{R2}}{m_e} \sigma_z |\vec{B}_{R2}(\vec{k})|, \quad (9)$$

$$H_{R3} = \frac{\hbar^2 k_{R3}}{m_e} \sigma_z |\vec{B}_{R3}(\vec{k})|. \quad (10)$$

In general, the local gauge transformation $U_g(\vec{k})$ is given by

$$U_g(\vec{k}) = \vec{m}(\vec{k}) \cdot \vec{\sigma}, \quad (11)$$

where $\vec{m}$ is a three component unit vector given as

$$\vec{m} = \left(\sin \frac{\theta(\vec{k})}{2} \cos \phi(\vec{k}), \sin \frac{\theta(\vec{k})}{2} \sin \phi(\vec{k}), \cos \frac{\theta(\vec{k})}{2} \right). \quad (12)$$

Here, θ and ϕ are defined as $\vec{B}_{Ri} = |\vec{B}_{Ri}(\vec{k})|(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ where $i = 1, 2, 3$. $|\vec{B}_{Ri}(\vec{k})|$ is the magnitude of the each effective magnetic field. Since this is the local gauge transformation, we can obtain the

next SU(2) non-Abelian gauge field $\mathbf{A}_{k_i}$ in reciprocal space.

$$\begin{aligned}
 \mathbf{A}_{k_i} &= -i\mathbf{U}_g(\vec{k})\partial_{k_i}\mathbf{U}_g(\vec{k}) \\
 &= -i(\vec{m}(\vec{k}) \cdot \vec{\sigma})\partial_{k_i}(\vec{m}(\vec{k}) \cdot \vec{\sigma}) \\
 &= -i(\vec{m}(\vec{k}) \cdot \vec{\sigma})(\partial_{k_i}\vec{m}(\vec{k}) \cdot \vec{\sigma} + \vec{m}(\vec{k}) \cdot \partial_{k_i}\vec{\sigma}) \\
 &= -i(\vec{m}(\vec{k}) \cdot \vec{\sigma})(\partial_{k_i}\vec{m}(\vec{k}) \cdot \vec{\sigma}) \\
 &= -i(\vec{m}(\vec{k}) \cdot \partial_{k_i}\vec{m}(\vec{k})) + \vec{\sigma}(\vec{m}(\vec{k}) \times \partial_{k_i}\vec{m}(\vec{k})) \\
 &= (\vec{m}(\vec{k}) \times \partial_{k_i}\vec{m}(\vec{k})) \cdot \vec{\sigma} \equiv \sum_{\alpha=x,y,z} A_{k_i}^{\alpha} \sigma_{\alpha},
 \end{aligned} \tag{13}$$

where $k_i = k_x, k_y$, and k_z . The gauge field is explicitly written in a vector notation with respect to the spin index α by the following equation:

$$\vec{A}_{k_i} = \frac{1}{2} \begin{pmatrix} -\partial_{k_i}\theta \sin \phi - \sin \theta \cos \phi \partial_{k_i}\phi \\ \partial_{k_i}\theta \cos \phi - \sin \theta \sin \phi \partial_{k_i}\phi \\ (1 - \cos \theta) \partial_{k_i}\phi \end{pmatrix}. \tag{14}$$

The corresponding field strength $\mathbf{F}_{\mu\nu}$ is defined by

$$\mathbf{F}_{\mu\nu} = (\partial_{\mu}A_{\nu}^{\alpha} - \partial_{\nu}A_{\mu}^{\alpha})\sigma_{\alpha} - \frac{i\tilde{e}}{\hbar} [A_{\mu}^{\alpha}\sigma_{\alpha}, A_{\nu}^{\beta}\sigma_{\beta}], \tag{15}$$

where $\tilde{e}$ is the coupling constant of the SU(2) gauge field, which contains the SOI strengths implicitly [22]. In fact, since the $\mathbf{A}_{k_i}$ is a pure gauge, the field strength equals zero identically. Therefore, we have to apply an approximation to the non-Abelian gauge in order to derive the finite effective magnetic field. Now, we impose the adiabatic approximation (AA), which corresponds to keeping only the diagonal terms of $\mathbf{A}_{k_i}$ mathematically [26]. As a result, we obtain an Abelian gauge field known as the spin Berry connection $\vec{A}^{\text{ad}}(\vec{k}, s)$, where $s = \pm 1$ corresponds to the majority and minority spins, respectively. The Abelian gauge field $\vec{A}^{\text{ad}}(\vec{k}, s)$ is explicitly given by

$$\vec{A}^{\text{ad}}(\vec{k}, s) = \frac{s}{2}(1 - \cos \theta)\nabla_{\vec{k}}\phi. \tag{16}$$

In this study, we can set $\theta = \frac{\pi}{2}$ since the Rashba effect works in-plane space. Then, equation (16) reduces to

$$\vec{A}^{\text{ad}}(\vec{k}, s) = \frac{s}{2}\nabla_{\vec{k}}\phi. \tag{17}$$

Using equation (17), the Abelian gauge fields of the linear, square, and cubic Rashba SOIs for the majority spin are given by the next equations, respectively,

$$\vec{A}_{\text{R1}}^{\text{ad}}(\vec{k}, s) = \frac{1}{2k_{\parallel}^2}(-k_y, k_x, 0), \tag{18}$$

$$\vec{A}_{\text{R2}}^{\text{ad}}(\vec{k}, s) = \frac{1}{2k_{\parallel}^2}(-2k_y, 2k_x, 0), \tag{19}$$

$$\vec{A}_{\text{R3}}^{\text{ad}}(\vec{k}, s) = \frac{1}{2k_{\parallel}^2}(-3k_y, 3k_x, 0), \tag{20}$$

where $k_{\parallel} = \sqrt{k_x^2 + k_y^2}$ is the magnitude of in-plane wavenumber. In reference [22], the method of completing the square was used in order to derive the AC phase using $\vec{A}^x$ and $\vec{A}^y$ components in spin-space. However, the technique of completing the square is not applicable for square and cubic Rashba SOIs. Therefore, we have to utilize the fact that $\vec{A}^z$ are related to $\vec{A}^x$ and $\vec{A}^y$ components in spin-space since the field strength of equation (15) vanishes identically:

$$\begin{aligned}
 (\partial_{\mu}A_{\nu}^z - \partial_{\nu}A_{\mu}^z)\sigma_z &= -(\partial_{\mu}A_{\nu}^x - \partial_{\nu}A_{\mu}^x)\sigma_x - (\partial_{\mu}A_{\nu}^y - \partial_{\nu}A_{\mu}^y)\sigma_y - \frac{2\tilde{e}}{\hbar}\epsilon_{\alpha\beta\gamma}A_{\mu}^{\alpha}A_{\nu}^{\beta}\sigma_{\gamma} \\
 &= -\left(\partial_{\mu}A_{\nu}^x - \partial_{\nu}A_{\mu}^x + \frac{2\tilde{e}}{\hbar}\epsilon_{\alpha\beta x}A_{\mu}^{\alpha}A_{\nu}^{\beta}\right)\sigma_x - \left(\partial_{\mu}A_{\nu}^y - \partial_{\nu}A_{\mu}^y + \frac{2\tilde{e}}{\hbar}\epsilon_{\alpha\beta y}A_{\mu}^{\alpha}A_{\nu}^{\beta}\right)\sigma_y \\
 &\quad - \frac{2\tilde{e}}{\hbar}\epsilon_{\alpha\beta z}A_{\mu}^{\alpha}A_{\nu}^{\beta}\sigma_z,
 \end{aligned} \tag{21}$$

where $\epsilon_{\alpha\beta\gamma}$ is the Levi–Civita symbol. This equation always holds in any space. Hereafter, we do not restrict μ and ν in k -space. Moreover, note that σ_z is the diagonal matrix and that both σ_x and σ_y do not have the diagonal elements. Therefore, this identity equation should result in the following equations:

$$\left(\partial_\mu A_\nu^x - \partial_\nu A_\mu^x + \frac{2\tilde{e}}{\hbar} \epsilon_{\alpha\beta x} A_\mu^\alpha A_\nu^\beta \right) \sigma_x + \left(\partial_\mu A_\nu^y - \partial_\nu A_\mu^y + \frac{2\tilde{e}}{\hbar} \epsilon_{\alpha\beta y} A_\mu^\alpha A_\nu^\beta \right) \sigma_y = 0 \quad (22)$$

$$\left(\partial_\mu A_\nu^z - \partial_\nu A_\mu^z \right) \sigma_z = - \frac{2\tilde{e}}{\hbar} \epsilon_{\alpha\beta z} A_\mu^\alpha A_\nu^\beta \sigma_z. \quad (23)$$

These equations are exact for a pure gauge. Although the AA keeps only the diagonal element ($\vec{A}^z$), it is found that the AA corresponds to keeping only the diagonal elements of the field strength $F_{\mu\nu}$ of equation (15) from the above equations. Namely, while keeping equation (22), it is only necessary to remove the left-hand side or the right-hand side of equation (23). Therefore, there exist two methods for the AA. The first method is to eliminate both $\vec{A}^x$ and $\vec{A}^y$ while keeping $\vec{A}^z$, which is a well known method. This method keeps equation (22) naturally. The second method is to eliminate $\vec{A}^z$ while keeping both $\vec{A}^x$ and $\vec{A}^y$ under the next condition. The condition is to keep equation (22), which imposes constraints on both $\vec{A}^x$ and $\vec{A}^y$. These are $\partial_\mu A_\nu^x - \partial_\nu A_\mu^x = 0$ and $\partial_\mu A_\nu^y - \partial_\nu A_\mu^y = 0$. As a result, the field strength $F_{\mu\nu}$ of equation (15) in the AA can be rewritten by the following equation:

$$F_{\mu\nu}^{\text{ad}} = \left(\partial_\mu A_\nu^z - \partial_\nu A_\mu^z \right) \sigma_z + \frac{2\tilde{e}}{\hbar} \epsilon_{\alpha\beta z} A_\mu^\alpha A_\nu^\beta \sigma_z. \quad (24)$$

Although the first term or the second term of equation (24) is zero in the AA, equation (24) is intentionally expressed in explicit form. Equation (24) is equal to zero identically by equation (23) if an approximation is not used. In the AA, the field strength $F_{\mu\nu}$ can obtain a finite value by eliminating $\vec{A}^x$ and $\vec{A}^y$ (the first method). Then, the field strength $F_{\mu\nu}^{\text{ad}}$ is given by the following equation:

$$F_{\mu\nu}^{\text{ad}} = \left(\partial_\mu A_\nu^z - \partial_\nu A_\mu^z \right) \sigma_z. \quad (25)$$

This is because the second term of equation (24) is set to zero. Conversely speaking, the same field strength $F_{\mu\nu}^{\text{ad}}$ in the AA can also be obtained by eliminating $\vec{A}^z$ while keeping both $\vec{A}^x$ and $\vec{A}^y$ under the above mentioned constraints (the second method), which means that the first term of equation (24) is set to zero. Then, the field strength $F_{\mu\nu}^{\text{ad}}$ is given by the following equation

$$F_{\mu\nu}^{\text{ad}} = \frac{2\tilde{e}}{\hbar} \epsilon_{\alpha\beta z} A_\mu^\alpha A_\nu^\beta \sigma_z = \frac{2\tilde{e}}{\hbar} (A_\mu^x A_\nu^y - A_\mu^y A_\nu^x) \sigma_z. \quad (26)$$

Consequently, the field strengths of equations (25) and (26) are exactly the same. However, this method is not applicable in k -space because the constraints ($\partial_\mu A_\nu^x - \partial_\nu A_\mu^x = 0$ and $\partial_\mu A_\nu^y - \partial_\nu A_\mu^y = 0$) do not hold. On the other hand, this situation happens in the case of the linear Rashba SOI in real space [22]. Namely, the scenario of the AA by $\vec{A}^x$ and $\vec{A}^y$ is realized naturally in the case of the linear Rashba SOI in real space.

Since the field strength $F_{\mu\nu}^{\text{ad}}$ is a 2-rank tensor, the rule for the field strength tensor transformation from some $\vec{b}$ -space into another $\vec{a}$ -space is generally as follows [26]:

$$F_{ij}^{\text{ad}}(\vec{a}) = \frac{\partial b_\mu}{\partial a_i} \frac{\partial b_\nu}{\partial a_j} F_{\mu\nu}^{\text{ad}}(\vec{b}). \quad (27)$$

When we regard $\vec{a}$ as $\vec{r}$ and regard $\vec{b}$ as $\vec{k}$, we can map the field strength $F_{\mu\nu}^{\text{ad}}(\vec{k})$ in k -space to the field strength $F_{ij}^{\text{ad}}(\vec{r})$ in real space. This means that we can obtain equation (26) in real space from the field strength $F_{\mu\nu}^{\text{ad}}(\vec{k})$ in k -space using the adequate transformation even though the AA of the second method happens naturally for the linear Rashba SOI in real space. As a result, we can conclude the following: the AA of the second method can be also realized for the square and cubic Rashba effects in real space using the same transformation as that for the linear Rashba effect, taking into consideration that $\vec{A}^{\text{ad}} (= \vec{A}^z)$ in k -space for the square and cubic Rashba effects are the same dependence on $\vec{k}$ as for the linear Rashba effect from equations (18)–(20).

In summary, in the AA, it can be found that the charge to which $\vec{A}^z (= \vec{A}^{\text{ad}})$ couples is exactly the same one to which $\vec{A}^x$ and $\vec{A}^y$ couples under the above constraints according to equations (23)–(26). Namely, the effective magnetic field generated by $\vec{A}^z$ is equivalent to the field which $\vec{A}^x$ and $\vec{A}^y$ induce under the above constraints.

Next, we would like to know the unitary matrices for AC effects induced by general Rashba effects. In real space, the unitary matrices U_{AC1} , U_{AC2} , and U_{AC3} for AC effects induced by linear, square, and cubic Rashba SOIs can be obtained by the following equations, respectively:

$$\begin{aligned}
U_{AC1} &= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R1}^{\text{ad}}(\vec{r}) \cdot d\vec{r} \sigma_z\right) \\
&= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \int_{-\infty}^{+\infty} \vec{A}_{R1}^{\text{ad}}(\vec{k}) \exp(i\vec{k} \cdot \vec{r}) d^2k \cdot d\vec{r} \sigma_z\right) \\
&= \exp(i\phi_{AC1} \sigma_z), \\
&\text{where } \phi_{AC1} = -\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R1}^{\text{ad}}(\vec{r}) \cdot d\vec{r},
\end{aligned} \tag{28}$$

$$\begin{aligned}
U_{AC2} &= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R2}^{\text{ad}}(\vec{r}) \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \int_{-\infty}^{+\infty} \vec{A}_{R2}^{\text{ad}}(\vec{k}) \exp(i\vec{k} \cdot \vec{r}) d^2k \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \int_{-\infty}^{+\infty} 2\vec{A}_{R1}^{\text{ad}}(\vec{k}) \exp(i\vec{k} \cdot \vec{r}) d^2k \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-2i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R1}^{\text{ad}}(\vec{r}) \cdot d\vec{r} \sigma_z\right) = \exp(i2\phi_{AC1} \sigma_z),
\end{aligned} \tag{29}$$

$$\begin{aligned}
U_{AC3} &= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R3}^{\text{ad}}(\vec{r}) \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \int_{-\infty}^{+\infty} \vec{A}_{R3}^{\text{ad}}(\vec{k}) \exp(i\vec{k} \cdot \vec{r}) d^2k \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \int_{-\infty}^{+\infty} 3\vec{A}_{R1}^{\text{ad}}(\vec{k}) \exp(i\vec{k} \cdot \vec{r}) d^2k \cdot d\vec{r} \sigma_z\right), \\
&= \exp\left(-3i\frac{\tilde{e}}{\hbar} \int_{r_1}^{r_2} \vec{A}_{R1}^{\text{ad}}(\vec{r}) \cdot d\vec{r} \sigma_z\right) = \exp(i3\phi_{AC1} \sigma_z),
\end{aligned} \tag{30}$$

where $d^2k (= dk_x dk_y)$ is the two-dimensional k -space differential element. Here, equations (18)–(20) are used. Also, we assumed that the SOI strengths k_{R1} , k_{R2} , and k_{R3} are equal and that the rotation axis of the spin is in the z -direction, which do not lose generality. As a result, it is found that we can derive the unitary matrices for AC effects in real space under square and cubic Rashba effects easily once we know the unitary matrix for AC effects under the linear Rashba effect in real space. Then, it is well known that the linear Rashba SOI in real space has the charge $\theta = 2k_{R1}$ with respect to the non-Abelian gauge in the notation of equation (2) [22] and that the unitary matrix U_{AC1} for the AC effect induced by linear Rashba SOI in real space can be expressed by the following equation [14, 22]:

$$U_{AC1} = \exp(i\vec{\beta}_{AC1} \cdot \vec{\sigma}), \tag{31}$$

$$\vec{\beta}_{AC1} = k_{R1}D(-d_y, d_x, 0), \tag{32}$$

where D is the length of the path in real space, and d_x and d_y are the x - and y -components of the unit vector $\vec{d}$ in the direction of the path. This equation describes the rotation about the z -axis in real space, which can be proved using the Baker–Campbell–Hausdorff formula [22]. Also, double and triple spin–orbit strengths correspond to the double and triple rotation angles. Therefore, taking into consideration equations (28)–(32) and different SOI strengths, we can obtain the following 2×2 unitary matrix U_{AC} for AC effects induced by general Rashba SOIs in real space:

$$U_{AC} = \exp(i\vec{\beta}_{AC} \cdot \vec{\sigma}), \tag{33}$$

where each $\vec{\beta}_{AC}$ is defined by the following equation for linear, square, and cubic Rashba SOIs, respectively.

$$\vec{\beta}_{AC} = k_{R1}L(-l_y, l_x, 0) = \alpha_1(-l_y, l_x, 0), \text{ for linear Rashba,} \tag{34}$$

$$\vec{\beta}_{AC} = k_{R2}L(-2l_y, 2l_x, 0) = \alpha_2(-2l_y, 2l_x, 0), \text{ for square Rashba,} \tag{35}$$

$$\vec{\beta}_{AC} = k_{R3}L(-3l_y, 3l_x, 0) = \alpha_3(-3l_y, 3l_x, 0) \text{ for cubic Rashba.} \tag{36}$$

Here, $\alpha_1 = k_{R1}L$, $\alpha_2 = k_{R2}L$, and $\alpha_3 = k_{R3}L$. Also, l_x and l_y are the x - and y -components of the unit vector $\vec{l}$ in the direction of the path which an electron propagates from site m to site n . L is the length of the path. Therefore,

the unitary matrix U_{nm} for both the AB and AC effects in real space is written by

$$U_{nm} = \exp\left(-i\frac{2\pi}{\Phi_0} \int_m^n \vec{A}_{\text{ext}} \cdot d\vec{l}\right) \exp\left(i\vec{\beta}_{\text{AC}} \cdot \vec{\sigma}\right), \quad (37)$$

where $\vec{A}_{\text{ext}}$ is the vector potential of the external magnetic field $\vec{B}_{\text{ext}}$, $\Phi_0 = h/e$ is the flux quantum, e is the elementary charge, and $d\vec{l}$ is the line element vector. As a result, the wave function acquires an AB phase $\phi = 2\pi\Phi/\Phi_0$, where Φ is the magnetic flux penetrating the AB ring, when an electron goes around the AB ring shown in figure 1.

Finally, we discuss the validity of the AA in a realistic situation. The AA used in deriving the finite field strength of the Rashba effects means that the interband transition does not occur between the two split bands with opposite spins. The interband transition induces the rapid spin-flip, which violates the AA. However, the process can be neglected for the following reason. We consider that the interband transition can be neglected if the following criterion is satisfied:

$$\Delta E \gg k_B T, \quad (38)$$

where ΔE is the interband gap for the general Rashba effect, k_B is the Boltzmann constant, and T is the absolute temperature. The interband gaps ΔE for general Rashba effects are reported to be about 0.1 meV $\sim$ 30 meV by many experiments [4, 18, 27, 28]. In a realistic situation, the present experimental technique enables us to measure the physical properties at a very low temperature of about 40 mK (3.4 μ eV). Equation (38) is completely cleared. As a result, we consider that interband transition can be neglected in most experiments. Moreover, the possibility of the spin-flip in the same band has to be considered. The spin relaxation mechanism consists mainly of Elliot–Yafet and D’yakonov–Perel (DP) mechanisms [29, 30]. In particular, the DP mechanism is dominant in the III–V semiconductors used in making quantum rings and QNs [29–31]. The spin relaxation time τ_S in the DP mechanism is given by the equation [30]

$$\frac{1}{\tau_S} = Q\alpha^2 \frac{(k_B T)^3}{\hbar^2 E_g} \tau_m, \quad (39)$$

where Q is the dimensionless factor ranging from 0.8 to 2.7, α is the spin–orbit coupling constant, E_g is the band gap of the materials, and τ_m is the momentum relaxation time. Equation (39) indicates that the spin relaxation time τ_S becomes longer as both the temperature T and the momentum relaxation time τ_m decrease. In realistic experiments, the spin relaxation time τ_S ranges from 0.3 to 4 ns [31]. On the other hand, the momentum relaxation time τ_m is at most 38 ps at a high electron mobility of $10^2 \text{ m}^2 (\text{V} \cdot \text{s})^{-1}$ in highly doped ($N_s = 2.25 \times 10^{15} \text{ m}^{-2}$) GaAs–GaAlAs heterostructures [32]. Then, the Fermi velocity v_F is $2.05 \times 10^5 \text{ m s}^{-1}$ and the mean free path $l_F (= v_F \tau_m)$ is about 7.8 μm . Therefore, the spin of the electron seldom relaxes in the propagation of the ring in our system because the spin relaxation time is much longer than the momentum relaxation time. Therefore, the AA is guaranteed. As a result, we can expect that the following numerical results can be observed in realistic systems.

2.4. Application of the AC phase of general Rashba SOIs to an AB-ring

Now the expression of U_{nm} has been obtained explicitly and we are going to solve equation (1) after the example of reference [14]. Only the hopping energy between nearest neighbors is considered to solve equation (1). Except for the sites ‘0, 1, a , b , c , d ’, the hopping energy between the other sites on the leads is a constant energy of V_0 , which means $J_{12} = J_{21} = J_{0-1} = J_{-10} = V_0$. In this study, we set the other sites energies on the leads to be zero. Using the Bloch theorem and equation (1), the energy dispersion of the leads results in $\epsilon = -2V_0 \cos(ka)$, where a is the lattice constant of the leads. The Schrödinger equations for the six sites 0, 1, a , b , c , d are described as follows:

$$\begin{aligned} (\epsilon - \epsilon_0) |\Psi_0\rangle &= -J_{0a} U_{0a} |\Psi_a\rangle - J_{0c} U_{0c} |\Psi_c\rangle - J_{0-1} |\Psi_{-1}\rangle, \\ (\epsilon - \epsilon_a) |\Psi_a\rangle &= -J_{a0} U_{a0} |\Psi_0\rangle - J_{ab} U_{ab} |\Psi_b\rangle, \\ (\epsilon - \epsilon_b) |\Psi_b\rangle &= -J_{b1} U_{b1} |\Psi_1\rangle - J_{ba} U_{ba} |\Psi_a\rangle, \\ (\epsilon - \epsilon_c) |\Psi_c\rangle &= -J_{cd} U_{cd} |\Psi_d\rangle - J_{c0} U_{c0} |\Psi_0\rangle, \\ (\epsilon - \epsilon_d) |\Psi_d\rangle &= -J_{dc} U_{dc} |\Psi_c\rangle - J_{d1} U_{d1} |\Psi_1\rangle, \\ (\epsilon - \epsilon_1) |\Psi_1\rangle &= -J_{1b} U_{1b} |\Psi_b\rangle - J_{1d} U_{1d} |\Psi_d\rangle - J_{12} |\Psi_2\rangle. \end{aligned} \quad (40)$$

After tedious manipulation to remove the wavefunctions $|\Psi_a\rangle$, $|\Psi_b\rangle$, $|\Psi_c\rangle$, and $|\Psi_d\rangle$, we can finally obtain the following system of equations:

$$\begin{aligned}(\epsilon - E_0)|\Psi_0\rangle &= \mathbf{Y}|\Psi_1\rangle - V_0|\Psi_{-1}\rangle, \\(\epsilon - E_1)|\Psi_1\rangle &= \mathbf{Y}^\dagger|\Psi_0\rangle - V_0|\Psi_2\rangle,\end{aligned}\quad (41)$$

where $\mathbf{Y}$ is a 2×2 matrix, and E_0 and E_1 are renormalized site energies of site 0 and 1, respectively. They are explicitly written as

$$E_0 = \epsilon_0 + \frac{|J_{0a}|^2}{(\epsilon - \epsilon_a) - \frac{|J_{ab}|^2}{\epsilon - \epsilon_b}} + \frac{|J_{0c}|^2}{(\epsilon - \epsilon_c) - \frac{|J_{cd}|^2}{\epsilon - \epsilon_d}}, \quad (42)$$

$$E_1 = \epsilon_1 + \frac{|J_{b1}|^2}{(\epsilon - \epsilon_b) - \frac{|J_{ab}|^2}{\epsilon - \epsilon_a}} + \frac{|J_{d1}|^2}{(\epsilon - \epsilon_d) - \frac{|J_{cd}|^2}{\epsilon - \epsilon_c}}, \quad (43)$$

$$\begin{aligned}\mathbf{Y} &= -\frac{J_{0a}J_{ab}J_{b1}}{(\epsilon - \epsilon_a)(\epsilon - \epsilon_b) - |J_{ab}|^2}\mathbf{U}_{0a}\mathbf{U}_{ab}\mathbf{U}_{b1} \\&\quad - \frac{J_{0c}J_{cd}J_{d1}}{(\epsilon - \epsilon_c)(\epsilon - \epsilon_d) - |J_{cd}|^2}\mathbf{U}_{0c}\mathbf{U}_{cd}\mathbf{U}_{d1}.\end{aligned}\quad (44)$$

Here, we assume the following solutions for the site n for incoming electrons from the left lead using the Bloch theorem:

$$|\Psi_n\rangle = |\eta_{\text{in}}\rangle \exp(inka) + \mathbf{R}|\eta_{\text{in}}\rangle \exp(-inka), \quad n \leq 0, \quad (45)$$

$$|\Psi_n\rangle = \mathbf{T}|\eta_{\text{in}}\rangle \exp(i(n-1)ka), \quad n \geq 1, \quad (46)$$

where $\mathbf{R}$ is the 2×2 reflection coefficient matrix, $\mathbf{T}$ is the 2×2 transmission coefficient matrix, and $|\eta_{\text{in}}\rangle$ is the spin components of incoming electrons. When equations (45) and (46) are inserted into the system of equations (41), $\mathbf{R}$ and $\mathbf{T}$ can be explicitly obtained as follows:

$$\begin{aligned}\mathbf{R} &= -\mathbf{I} - \frac{i2V_0 \sin(ka)(E_1 + V_1 \exp(-ika))}{(E_0 + V_0 \exp(-ika))(E_1 + V_1 \exp(-ika))\mathbf{I} - \mathbf{Y}\mathbf{Y}^\dagger}, \\ \mathbf{T} &= \frac{i2V_0 \sin(ka)\mathbf{Y}^\dagger}{(E_0 + V_0 \exp(-ika))(E_1 + V_1 \exp(-ika))\mathbf{I} - \mathbf{Y}\mathbf{Y}^\dagger},\end{aligned}\quad (47)$$

where $\mathbf{I}$ is the 2×2 identity matrix. Provided that $|\pm \vec{n}\rangle$ are the incoming normalized spinors which are eigenvectors of the spin along the unit vector $\pm \vec{n}$, the outgoing spinor $|\pm \eta_{\text{out}}\rangle$ is determined by

$$t_\pm |\pm \eta_{\text{out}}\rangle = \mathbf{T}|\pm \vec{n}\rangle, \quad (48)$$

where $t_\pm$ are the transmission amplitudes for polarized electrons in the direction of the spin axis of the outgoing normalized spinor $|\pm \eta_{\text{out}}\rangle$, respectively. Taking the complex conjugate of equation (48) and making the inner product with the original one, the following equation is obtained:

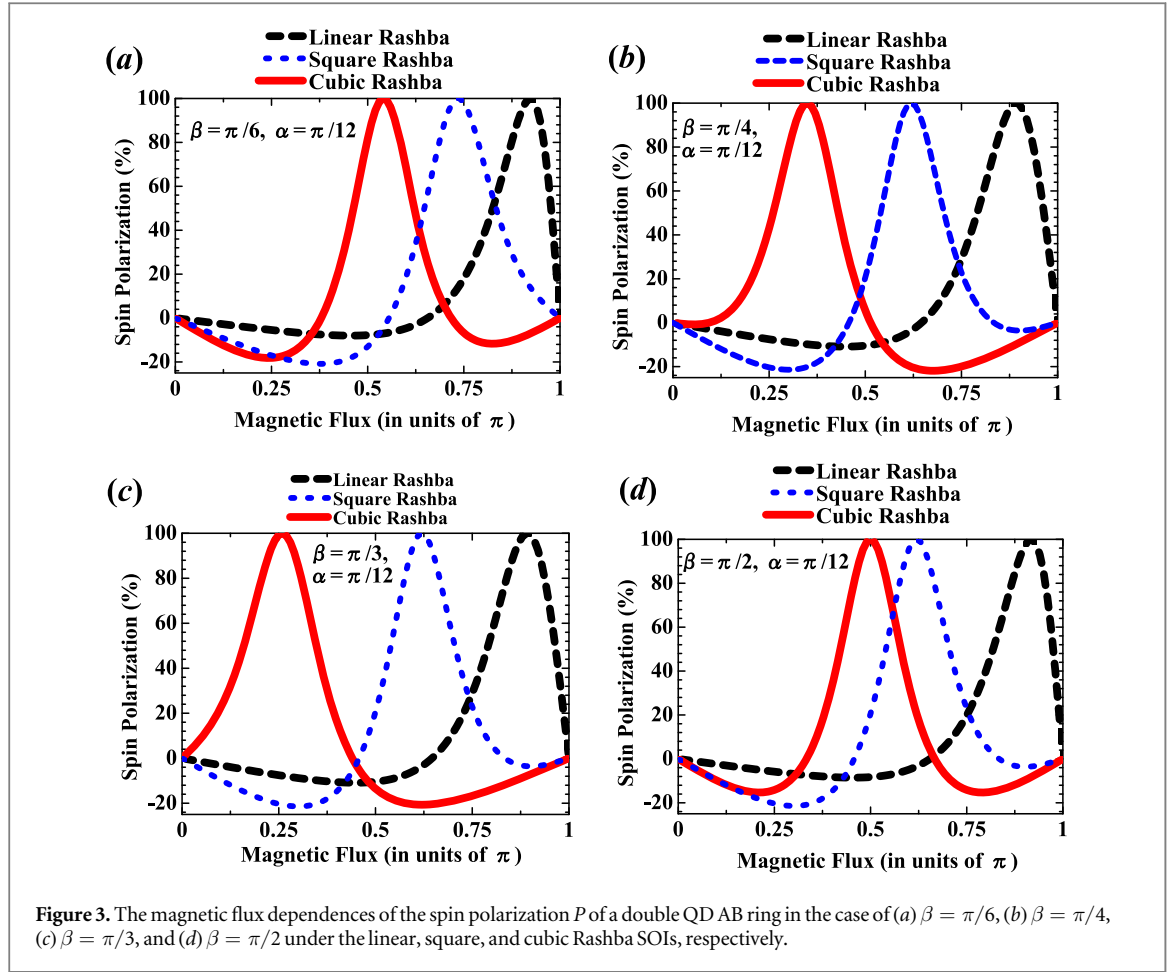
$$|t_\pm|^2 = \langle \pm n | \frac{4V_0^2 \sin^2(ka) \mathbf{Y}\mathbf{Y}^\dagger}{[(E_0 + V_0 \exp(-ika))(E_1 + V_1 \exp(-ika))\mathbf{I} - \mathbf{Y}\mathbf{Y}^\dagger]^2} | \pm n \rangle. \quad (49)$$

If you choose the incoming normalized spinors which are eigenvectors of $\mathbf{Y}\mathbf{Y}^\dagger$, the above equation reduces to

$$|t_\pm| = \frac{2V_0 \sin(ka) \sqrt{\rho_\pm}}{[(E_0 + V_0 \exp(-ika))(E_1 + V_1 \exp(-ika))\mathbf{I} - \rho_\pm]}, \quad (50)$$

where $\rho_\pm$ are eigenvalues of $\mathbf{Y}\mathbf{Y}^\dagger$. Therefore, the spin polarization P of the spin filter is defined by

$$P = \frac{|t_+|^2 - |t_-|^2}{|t_+|^2 + |t_-|^2}. \quad (51)$$



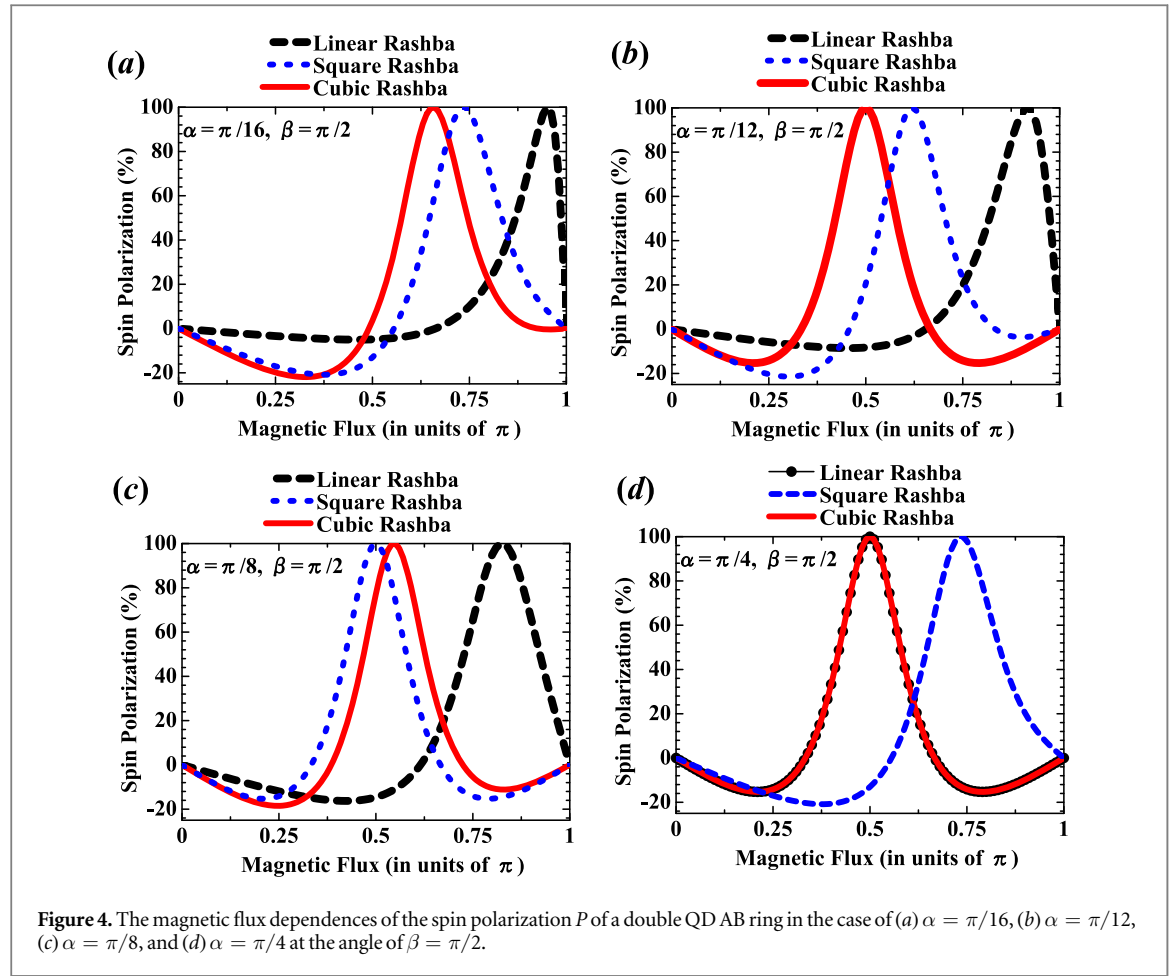
3. Numerical results

In this calculation, all the energies can be normalized by the hopping energy V_0 and we set the energy ϵ to be zero, which means $ka = \pi/2$. Also, the next condition is imposed in order to symmetrize the system and to simplify the equation of Y

$$\frac{J_{0a}J_{ab}J_{b1}}{(\epsilon - \epsilon_a)(\epsilon - \epsilon_b) - |J_{ab}|^2} = \frac{J_{0c}J_{cd}J_{d1}}{(\epsilon - \epsilon_c)(\epsilon - \epsilon_d) - |J_{cd}|^2}. \quad (52)$$

Moreover, the site energies of a, b, c, d are assumed to be the same and we set $J_{0a} = J_{0c} = 2.0$, $J_{ab} = J_{cd} = 5.0$, and $J_{b1} = J_{d1} = 2.0$ in units of V_0 . Figure 3 shows the magnetic flux dependences of the spin polarization P of a double QD AB ring with changing angle β under the specific value of $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = \pi/12$ for each Rashba SOI. Here, the angle β is shown in figure 1(b) and determines the shape of the double QD AB ring. From figure 3, it is found that the perfect spin filtering is achieved for all the Rashba SOIs. This result indicates that this AB ring under general Rashba SOIs can be a promising device for spin current generation. Moreover, they behave in totally different ways in response to penetrating magnetic flux, which is attributed to linear, square, and cubic behaviors in the in-plane momentum.

Figure 4 shows the magnetic flux dependences of the spin polarization P of a double QD AB ring with changing SOI coupling constant $\alpha = \alpha_1 = \alpha_2 = \alpha_3$ under the specific value of the angle $\beta = \pi/2$. It is found that the polarization curve becomes the same form for the adequate pairs of $\alpha = \pi/4$ and $\beta = \pi/2$ for linear and cubic Rashba SOIs. This is due to the following reason: the spin rotates by $3\pi/4$ under the cubic Rashba SOI when the spin rotates by $\pi/4$ under the linear Rashba SOI. Since the spin polarization P is a periodic function with period π , the rotation angle of $\pi/4$ is equivalent to the rotation angle of $3\pi/4$ with respect to the phase. Generally, the different α generate the different behaviors and the different peaks of the spin polarization P even at the same angle β . These results enable us to make a clear distinction between linear, square, and cubic Rashba SOIs according to the peak position of the perfect spin filtering. This method will be very useful for determining the k -dependence of Rashba SOIs using experiments.



4. Conclusion

In this study, we have obtained the correct expression of AC phases for square and cubic Rashba SOIs, which were obstacles to investigating spin filters using square and cubic Rashba SOIs. Then, the spin filtering was investigated in a double QD AB ring under linear, square, and cubic Rashba SOIs by applying the AC phase to the system. As a result, it was found that the perfect spin filtering is achieved for all the Rashba SOIs at the specific penetrating magnetic flux. This means that this AB ring under general Rashba SOIs can be a promising device for spin current generation. Also, the peak positions for linear, square, and cubic Rashba SOIs behave in completely different ways. This result enables us to make a clear distinction among linear, square, and cubic Rashba SOIs according to the peak position of the perfect spin filtering. As a result, it will be easy to determine the k -dependence of Rashba SOIs using experiments.

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References

- [1] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
- [2] Sugahara S and Tanaka M 2004 *Appl. Phys. Lett.* **84** 2307
- [3] Yuasa S, Nagahama T, Fukushima A, Suzuki Y and Ando K 2004 *Nat. Mater.* **3** 868
- [4] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 *Phys. Rev. Lett.* **78** 1335
- [5] Chen J, Jalil M B A and Tan S G 2011 *J. Appl. Phys.* **109** 07C722
- [6] Kondo K 2010 *J. Appl. Phys.* **107** 09C709
- [7] Kondo K 2014 *J. Appl. Phys.* **115** 17C701
- [8] Koga T, Nitta J, Takayanagi H and Datta S 2002 *Phys. Rev. Lett.* **88** 126601

- [9] Kondo K 2012 *J. Appl. Phys.* **111** 07C713
- [10] Nitta J, Meijer F E and Takayanagi H 1999 *Appl. Phys. Lett.* **75** 695
- [11] Souma S and Nikolić B K 2004 *Phys. Rev. B* **70** 195346
- [12] Wu B H and Cao J C 2006 *Phys. Rev. B* **74** 115313
- [13] Cohen G, Hod O and Rabani E 2007 *Phys. Rev. B* **76** 235120
- [14] Aharony A, Tokura Y, Cohen G Z, Entin-Wohlman O and Katsumoto S 2011 *Phys. Rev. B* **84** 035323
- [15] Tang H Z, Zhai L X, Shen M and Liu J J 2014 *Phys. Lett. A* **378** 2790
- [16] Ying Y, Jin G and Ma Y 2009 *J. Phys.: Condens. Matter* **21** 275801
- [17] Winkler R 2003 *Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems* (Springer Tracts in Modern Physics vol 191) (Berlin: Springer)
- [18] Nakamura H, Koga T and Kimura T 2012 *Phys. Rev. Lett.* **108** 206601
- [19] McCann E and Fal’ko V I 2006 *Phys. Rev. Lett.* **96** 086805
- [20] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
- [21] Aharonov Y and Casher A 1984 *Phys. Rev. Lett.* **53** 319
- [22] Hatano N, Shirasaki R and Nakamura H 2007 *Phys. Rev. A* **75** 032107
- [23] Murakami S, Nagaosa N and Zhang S C 2003 *Science* **301** 1348
- [24] Fujita T, Jalil M B A and Tan S G 2010 *New J. Phys.* **12** 013016
- [25] Fujita T, Jalil M B A, Tan S G and Murakami S 2011 *J. Appl. Phys.* **110** 121301
- [26] Bliokh K Y and Bliokh Y P 2005 *Ann. Phys., NY* **319** 13
- [27] Akabori M, Guzenko V A, Sato T, Schäpers T, Suzuki T and Yamada S 2008 *Phys. Rev. B* **77** 205320
- [28] Hinz J, Buhmann H, Schäfer M, Hock V, Becker C R and Molenkamp L W 2006 *Semicond. Sci. Technol.* **21** 501
- [29] Zutić I, Fabian J and Das Sarma S 2004 *Mod. Phys. Rev.* **76** 323
- [30] Meier F and Zakharchenya B (ed) 1984 *Optical Orientation* (Modern Problems in Condensed Matter Sciences vol 8) (Amsterdam: North Holland)
- [31] Dzhioev R I, Kavokin K V, Korenev V L, Lazarev M V, Meltser B Y, Stepanova M N, Zakharchenya B P, Gammon D and Katzer D S 2002 *Phys. Rev. B* **66** 245204
- [32] Mendez E E, Price P J and Heiblum M 1984 *Appl. Phys. Lett.* **45** 294