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Vibration suppression for mass-spring-damper systems with a tuned mass damper using IDA-PBC

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SUMMARY

This paper considers a vibration suppression control method using feedback for a mass-spring-damper system with a tuned mass damper. For vibration suppression, we adopt the interconnection and damping assignment passivity-based control, whereby the system is transformed to a system with a skyhook damper with an artificial structure matrix. The feedback law includes no accelerometer signal and uses only information on relative displacements and velocities. The proposed control method can simultaneously suppress the influences of the floor vibration and the disturbance force acting on the main body. A guideline for choosing parameters of the desired system is shown. The proposed method can be easily extended to nonlinear cases, which is demonstrated for a nonlinear-spring case. We also show the input-to-state stability property of closed-loop systems for linear cases and nonlinear cases. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: IDA-PBC, tuned mass damper, vibration suppression

1. INTRODUCTION

Passivity-based approaches [1–4] are widely used for stabilizing mechanical systems. These approaches also appear useful for vibration suppression control of mechanical systems that are subject to excitation by external disturbances. However, it is difficult to know whether passivity-based approaches are directly applicable to vibration isolation problems; e.g., designing suspension systems for vehicles and making buildings earthquake resistant.

To demonstrate this, let us consider a single-degree-of-freedom (SDOF) mass-spring-damper system (Figure 1). When the mass is subject to floor oscillation, we can suppress the vibration of the mass by decreasing the spring constant and tuning the damping constant according to the disturbance frequency. In contrast, when a force acts directly on the mass as a disturbance, its vibration can be controlled by increasing the spring and damping constants. Obviously, it is not possible to simultaneously suppress the effects of both disturbances by adjusting the coefficients. Even naive application of passivity-based control does not solve this problem, because the controller can observe only the relative displacement and velocity in many cases. This fact is thoroughly discussed in Section 2. There are two types of active vibration suppression control methods to avoid the abovementioned difficulty. One is the skyhook control method [5–10] and the other is the multiple-degree-of-freedom (MDOF) approach [11–19].

In the skyhook control method, the system is converted by feedback into a system where the sprung mass is hooked to a rigid ceiling by a virtual damper, and ideal control performance is

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realized. However, this method assumes the use of an absolute velocity with respect to a fixed point that is not affected by floor vibration [5–7]. In many cases, it is difficult to obtain precise information on the absolute displacement or velocity without expensive sensors. Therefore, many researchers have devised control methods that imitate the skyhook model under partial observation. There are methods that use an accelerometer for obtaining information on the absolute position and velocity [8–10]. However, the use of the information on acceleration in a static feedback control law results in the generation of an algebraic loop. Therefore, the use of an accelerometer should involve additional dynamics, and the extension of such a method to nonlinear cases is difficult.

Conversely, the MDOF approach utilizes the information on additional masses. Especially the 2-degree-of-freedom (2DOF) approach is well investigated. A passive additional mass-spring-damper itself, which is placed on the sprung mass, has a vibration suppression effect, and is called a tuned mass damper (TMD) or a dynamic vibration absorber [20, 21]. The passive TMD can reduce vibrations at the resonant frequency but requires a heavy damping mass. Moreover, a TMD may amplify vibrations at off-resonance frequencies. Active 2DOF (or generally MDOF) controllers can be designed by frequency domain approaches [17], optimal control theory [11–16], or H_∞ -control theory. However, these design procedures cannot be applied to nonlinear systems easily without solving Hamilton-Jacobi (H-J) partial differential equations.

The aim of this paper is to propose a theoretical design method for static controllers for the vibration suppression and isolation in nonlinear 2DOF spring-mass-damper systems, where the controller uses only information on relative displacement and velocity. We consider a mass-spring-damper system with a passive TMD as the controlled object, where the control force acts on the main mass. In this paper, we adopt the method of interconnection and damping assignment passivity-based control (IDA-PBC) [1, 2] for suppressing vibration, because this method is suitable for nonlinear systems and does not require solving H-J partial differential equations. Increasing the damping coefficients in the naive PBC method causes an adverse effect on the isolation of vibration. Hence, we choose a 2DOF skyhook system with an artificial structure matrix as the desired system in the IDA-PBC, which has an ideal vibration suppression performance. By the IDA-PBC feedback, the controlled object with 2DOF is transformed into the skyhook model, including the coefficients of disturbances. We can convert the controlled object into a skyhook system without information on absolute displacement or velocity, which is a major contribution of this paper. The damping coefficient of the virtual skyhook damper can be set to an arbitrarily large value when a negative damping coefficient is allowed for another skyhook damper that connects the TMD and the ceiling. The asymptotic stability of the zero-disturbance case is guaranteed by the nature of the port-Hamiltonian system. The input-to-state stability (ISS) [22] of the closed-loop system with nonlinear springs is also proved. From these results, we can show that the IDA-PBC method is effective for the disturbance attenuation problem as well as for asymptotic stabilization.

The rest of this paper is organized as follows: Section 2 states the problem background. Section 3 summarizes the IDA-PBC method. Section 4 describes the proposed method for linear cases. Section 5 presents the control for the system with nonlinear springs, and Section 6 shows the ISS property for the nonlinear system. Sections 5 and 6 give one example of the nonlinear extension of our method. However, the IDA-PBC method can be widely used for general Hamiltonian systems, and our result shows that the IDA-PBC is also useful for the vibration suppression/isolation control of nonlinear Hamiltonian systems. Finally, Section 7 states our conclusion.

2. TRADE-OFF IN THE VIBRATION SUPPRESSION PROBLEM

In this section, we explain the rationale behind our choosing a 2DOF model for the controlled object in this study, which is a mass-spring damper system with a tuned mass damper (TMD). Consider the simple mass-spring-damper system shown in Figure 1. The equation of motion for this system is

$$m_1 \ddot{z}_1 + c_1(\dot{z}_1 - \dot{z}_0) + k_1(z_1 - z_0) = F, \quad (1)$$

where m_1 denotes the mass of the controlled object, c_1 the damping coefficient, k_1 the elastic coefficient of the spring, F a disturbance force, z_0 the displacement of the floor, and z_1 the displacement of the mass. We regard z_0 and F as disturbances. Taking the Laplace transform of the above equation, we obtain

$$\begin{aligned}\mathcal{L}[z_1] &= G_{s1}(s)\mathcal{L}[F] + G_{s2}(s)\mathcal{L}[z_0] = \frac{1}{m_1 s^2 + c_1 s + k_1} \mathcal{L}[F] + \frac{c_1 s + k_1}{m_1 s^2 + c_1 s + k_1} \mathcal{L}[z_0] \\ &= \frac{1}{m_1(s^2 + 2\omega_n \zeta s + \omega_n^2)} \mathcal{L}[F] + \frac{2\omega_n \zeta s + \omega_n^2}{s^2 + 2\omega_n \zeta s + \omega_n^2} \mathcal{L}[z_0],\end{aligned}$$

where $G_{s1}(s)$ and $G_{s2}(s)$ indicate the transfer functions to z_1 from the external force F and the floor displacement z_0 , respectively, $\omega_n = \sqrt{k_1/m_1}$ denotes the natural angular frequency, and $\zeta = c_1/(2m_1\omega_n)$ denotes the damping ratio.

Figures 2 and 3 show the gain plots of $G_{s1}(s)$ and $G_{s2}(s)$, respectively, for various parameters. Large values of ω_n and ζ are effective for decreasing the gain of $G_{s1}(s)$; i.e., the values of c_1 and k_1 should be large. In contrast, for vibration isolation (i.e., to decrease the gain of $G_{s2}(s)$) the natural angular frequency ω_n should be small, and the value of ζ must be selected on the basis of the frequency of the oscillatory disturbance. There is a trade-off between two types of disturbances in the vibration suppression problem.

A static linear feedback controller using only relative values $z_1 - z_0$, $\dot{z}_1 - \dot{z}_0$ changes the values of c_1 and k_1 , but the equation of motion remains in the form of (1). Therefore, such a controller cannot simultaneously handle two types of disturbances. In many cases where the floor vibrates, real-time measurement of the absolute displacement in a wide frequency range is difficult without expensive sensors like a real-time kinematics global positioning system (RTK-GPS). To solve this problem, we may be able to use an accelerometer or an absolute displacement meter. However, in

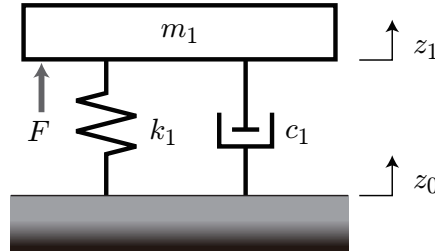


Figure 1. Simple mass-spring-damper system.

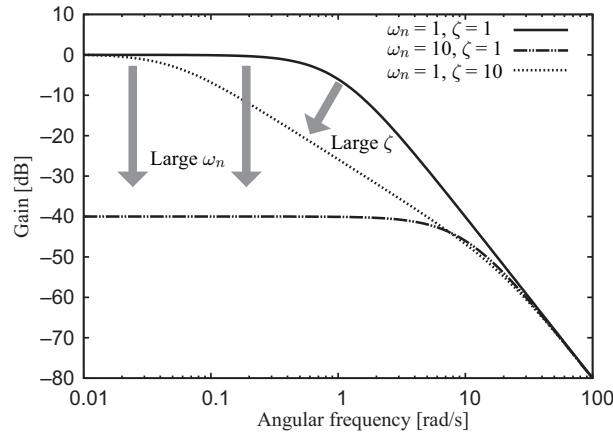


Figure 2. Gain plots of the system when the disturbance is the force acting directly on the main mass.

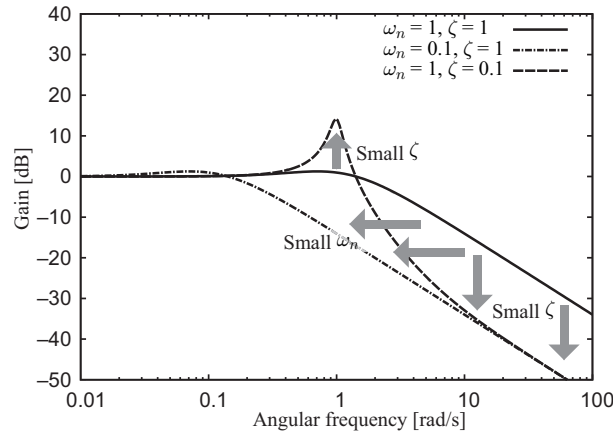


Figure 3. Gain plots of the system when the disturbance is floor vibration.

the static feedback case, utilizing the information from the accelerometer generates an algebraic loop, and the frequency range of the absolute displacement sensor is limited by its dynamics. To avoid the algebraic loop, Nagarajaiah et al. [9] introduced a time delay, but their method is based on the forward difference approximation. Priyandoko et al. [10] adopted the active force control (AFC) method [23, 24] with an accelerometer and a force sensor, where the transfer function of the actuator and a filter canceling the actuator dynamics were considered to eliminate the feed-through term. However, the AFC-based method is basically designed in the frequency domain and is not suitable as the theoretical method for vibration suppression of nonlinear systems.

In this paper, we use the displacement and velocity of a passive TMD [20, 21] attached to the main mass. Because the TMD is excited by the absolute movement of the main mass, we can indirectly obtain information related to the absolute vibration from the motion of the TMD. Primitive displacement meters and accelerometers have the same structure as the TMD. In other words, this work uses a TMD like an accelerometer (or a primitive displacement meter), and considers its dynamics to avoid an algebraic loop.

We adopt the IDA-PBC method to construct the feedback in expectation of applications to nonlinear cases. The state equation for (1) is represented as

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} 0 & -k_1/m_1 \\ 1 & -c_1/m_1 \end{bmatrix} \xi + \begin{pmatrix} 1/m_1 \\ 0 \end{pmatrix} F + \begin{pmatrix} k_1/m_1 \\ c_1/m_1 \end{pmatrix} z_0 \\ y = z_1 &= \begin{pmatrix} 0 & 1 \end{pmatrix} \xi \end{aligned} \quad (2)$$

where $\xi = (\dot{z}_1 + c_1/m_1(z_1 - z_0), z_1)^T$. The kinetic and potential energies cannot be expressed in terms of the state ξ alone. Therefore, the passivity-based control (PBC) method is not the most suited for a state space representation like (2). To avoid this problem, we define the state variables by regarding the floor displacement z_0 and the floor velocity $\dot{z}_0$ as independent variables in the controller design, while in the performance evaluation we use the trivial relationship $dz_0/dt = \dot{z}_0$.

3. IDA-PBC METHOD

The IDA-PBC method was proposed by Ortega et al. [1, 2]. The method changes not only the damping and the potential energy term in the Hamiltonian, as does the conventional PBC method, but also the inertia matrix and the structure matrix. Therefore, this method has a greater degree of freedom than the conventional PBC method in terms of managing the properties of closed-loop systems.

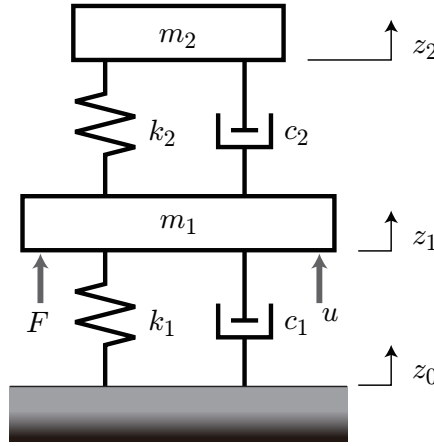


Figure 4. The controlled object is a mass-spring-damper system with a tuned mass damper.

In this section, we briefly explain the IDA-PBC method. Consider the port-Hamiltonian (pH) system (e.g., [3, 4])

$$\dot{x} = [J(x) - R(x)] \frac{\partial H(x)}{\partial x}^T + g(x, u) \quad (3)$$

and the desired system

$$\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d(x)}{\partial x}^T, \quad (4)$$

where the smooth functions $H(x)$ and $H_d(x)$ are the total stored energies, $J(x)$ is the skew-symmetric structure matrix, $J_d(x)$ is the artificial skew-symmetric structure matrix of the desired system, $R(x)$ and $R_d(x)$ are the positive semidefinite damping matrices, $g(x, u)$ is the vector function of an input term, and x is the state vector composed of the general positions and speeds. A part of the artificial structure matrix can be freely designed. The differences between the two systems are the changes in the potential and kinetic energies of the Hamiltonian as well as the changes in the damping matrix and structure matrix. The changes in the kinetic energy, potential energy, and damping matrix represent the virtual changes in the inertias, the elasticity coefficients or gravity terms, and the viscous resistance coefficients, respectively. The change in the structure matrix has no influence on the rate of decrease of the Hamiltonian. The systems (3) and (4) are equivalent if and only if there exists a feedback law $u = \alpha(x)$ such that

$$[J_d(x) - R_d(x)] \frac{\partial H_d(x)}{\partial x}^T = [J(x) - R(x)] \frac{\partial H(x)}{\partial x}^T + g(x, \alpha(x)). \quad (5)$$

In the classical PBC method, the structure matrix is unchanged; i.e. $J = J_d$. By considering the new artificial structure matrix we can also change the kinetic energy, which gives a big degree of freedom for choosing the desired Hamiltonian H_d .

In this paper, we use the above method for the vibration suppression problem of our system. In our problem, the disturbance terms are introduced into the system representations. Note that the choice of J_d affects the disturbance attenuation performance, because the change in the structure matrix generally leads to a change in the resonance frequency.

4. VIBRATION SUPPRESSION CONTROL FOR THE LINEAR CASE

4.1. Problem Setting

In this section, we consider the control of a system with a tuned mass damper, as shown in Figure 4, and which is excited by two types of disturbances. The main practical targets of this paper are

vehicle suspension controls [7, 10, 11] and vibration suppression controls for base-isolated buildings [9, 13–15, 19]. The control input u considered here is the force acting on the main mass, which is suitable for the targets of this paper. Our problem setting is different from that of the active tuned mass damper (ATMD) [16–18] where the control force acts on a TMD that has a comparatively large mass.

Let z_0 , z_1 , and z_2 denote the displacements of the floor, main mass, and TMD, respectively. The momenta are defined as $p_0 = m_0 \dot{z}_0$, $p_1 = m_1 \dot{z}_1$, and $p_2 = m_2 \dot{z}_2$, where m_0 , m_1 , and m_2 are the masses of the floor, main body, and TMD, respectively. We consider a temporary value of the floor mass for the convenience of explanation, and we will later derive an equation of motion that is independent of m_0 . The parameters, m_0 , m_1 , m_2 , k_1 , k_2 , c_1 , and c_2 are positive constants. The Hamiltonian of the system, which coincides with the total energy including the floor motion, is

$$\bar{H}_r(\bar{z}) = \frac{1}{2} \left\{ k_1(z_1 - z_0)^2 + k_2(z_2 - z_1)^2 + \frac{p_0^2}{m_0} + \frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} \right\},$$

where the state vector $\bar{z}$ is defined as $(z_0, z_1, z_2, p_0, p_1, p_2)^T$. As the dynamic equation of the system, including the dynamics of the floor motion, a port-Hamiltonian system is obtained:

$$\dot{\bar{z}} = (\bar{J} - \bar{R}) \frac{\partial \bar{H}_r}{\partial \bar{x}}^T + \bar{b}(u + F) + eE, \quad (6)$$

where u denotes the control input, F the force disturbance, E a virtual force acting on the floor, and

$$\begin{aligned} \bar{J} &= \begin{bmatrix} \mathbf{O}_{3 \times 3} & \mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{O}_{3 \times 3} \end{bmatrix}, \quad \bar{R} = \begin{bmatrix} \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} \\ \mathbf{O}_{3 \times 3} & \bar{C} \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 + c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix}, \quad \bar{b} = (0 \ 0 \ 0 \ 0 \ 1 \ 0)^T, \\ e &= (0 \ 0 \ 0 \ 1 \ 0 \ 0)^T. \end{aligned}$$

By eliminating the first and fourth components from (6) and replacing p_0 with $m_0 \dot{z}_0$, we obtain the following expression for a subsystem without the dynamics of the floor motion:

$$\dot{z} = (J - R) \frac{\partial \bar{H}}{\partial x}^T + \bar{d}\dot{z}_0 + b(u + F), \quad (7)$$

where

$$\bar{H}(z, z_0) = \frac{1}{2} \left\{ k_1(z_1 - z_0)^2 + k_2(z_2 - z_1)^2 + \frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} \right\}, \quad (8)$$

$$z = (z_1, z_2, p_1, p_2)^T,$$

$$J = \begin{bmatrix} \mathbf{O}_{2 \times 2} & \mathbf{I}_2 \\ -\mathbf{I}_2 & \mathbf{O}_{2 \times 2} \end{bmatrix}, \quad R = \begin{bmatrix} \mathbf{O}_{2 \times 2} & \mathbf{O}_{2 \times 2} \\ \mathbf{O}_{2 \times 2} & C \end{bmatrix}, \quad C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \quad (9)$$

$$\bar{d} = (0 \ 0 \ c_1 \ 0)^T, \quad b = (0 \ 0 \ 1 \ 0)^T.$$

In the representation (7), we regard z_0 and $\dot{z}_0$ as independent external disturbance signals.

Because the disturbance z_0 is included in the Hamiltonian $\bar{H}(z, z_0)$, we define the new state variables

$$x = (x_1, x_2, p_1, p_2)^T, \quad (10)$$

where $x_1 = z_1 - z_0$ and $x_2 = z_2 - z_0$ are the relative displacements. Using the new state vector, we can replace (7) with

$$\begin{aligned} \dot{x} &= (J - R) \frac{\partial H}{\partial x}^T + d\dot{z}_0 + b(u + F), \\ y &= z_1 = x_1 + z_0, \end{aligned} \quad (11)$$

where

$$H(\mathbf{x}) = \frac{1}{2} \left\{ k_1 x_1^2 + k_2 (x_2 - x_1)^2 + \frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} \right\} \quad (12)$$

$$\mathbf{d} = (-1 \quad -1 \quad c_1 \quad 0)^T. \quad (13)$$

Note that

$$\left. \frac{\partial H}{\partial x_i} \right|_{\substack{x_1=z_1-z_0 \\ x_2=z_2-z_0}} = \frac{\partial \bar{H}}{\partial z_i} \quad (i = 1, 2).$$

The Hamiltonian $H(\mathbf{x})$ has the same value as $\bar{H}(\mathbf{z}, z_0)$ but is a function of the new state $\mathbf{x}$ only. The relative displacements x_1 and x_2 have inconsistencies with the momenta p_1 and p_2 , because the p_i are the momenta for the absolute motions. These inconsistencies lead to a new disturbance term $(\mathbf{d} - \bar{\mathbf{d}})\dot{z}_0$. In (11), $\dot{z}_0$ is no longer a state variable, and can therefore be considered as an external signal. The output y is the displacement of the main body. However, this output is not the output in the sense of a pH system and it has no passivity property. The objective of the control is the approximate zeroing of the output y . This means that we aim for letting the state x_1 track the external (unknown) signal $-z_0$ approximately, rather than for a reduction in the value of x_1 . To achieve this objective, we use the IDA-PBC method, where it is assumed from an empirical knowledge that the desired system performs well.

We adopt the skyhook system (Figure 5) with an artificial structure matrix as the desired system with ideal properties. In a manner similar to that of (7), the desired system can be expressed as

$$\dot{\mathbf{x}} = (\mathbf{J}_d - \mathbf{R}_d) \frac{\partial H_d}{\partial \mathbf{x}}^T + \mathbf{b}F + \mathbf{d}_d \dot{z}_0 \quad (14)$$

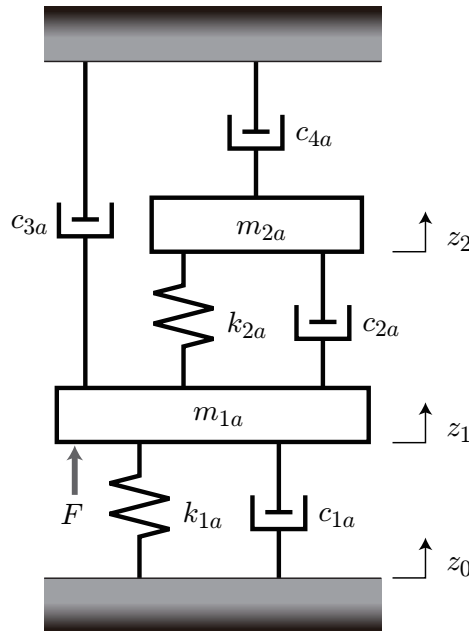


Figure 5. Desired skyhook system with a tuned mass damper.

where

$$\begin{aligned} \mathbf{J}_d &= \begin{bmatrix} \mathbf{O}_{2 \times 2} & \mathbf{M}^{-1} \mathbf{M}_d \\ -\mathbf{M}_d \mathbf{M}^{-1} & \mathbf{J}_2 \end{bmatrix}, \quad \mathbf{J}_2 = \begin{bmatrix} 0 & j_f \\ -j_f & 0 \end{bmatrix}, \\ \mathbf{M} &= \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad \mathbf{M}_d = \begin{bmatrix} m_{1a} & 0 \\ 0 & m_{2a} \end{bmatrix}, \\ \mathbf{R}_d &= \begin{bmatrix} \mathbf{O}_{2 \times 2} & \mathbf{O}_{2 \times 2} \\ \mathbf{O}_{2 \times 2} & \mathbf{C}_d \end{bmatrix}, \quad \mathbf{C}_d = \begin{bmatrix} c_{1a} + c_{2a} + c_{3a} & -c_{2a} \\ -c_{2a} & c_{2a} + c_{4a} \end{bmatrix}, \\ \mathbf{d}_d &= (-1 \quad -1 \quad c_{1a} \quad 0)^T, \quad \mathbf{b} = (0 \quad 0 \quad 1 \quad 0)^T \end{aligned} \quad (15)$$

and

$$H_d(\mathbf{x}) = \frac{1}{2} \left\{ k_{1a} x_1^2 + k_{2a} (x_2 - x_1)^2 + \frac{p_1^2}{m_{1a}} + \frac{p_2^2}{m_{2a}} \right\}. \quad (16)$$

We assume that m_{1a} , m_{2a} , k_{1a} , and k_{2a} are positive constants and $\mathbf{C}_d$ is positive definite. Owing to the skyhook damper c_{3a} , we expect the system (14) to have good vibration suppression for both disturbances. Note that $\mathbf{J}_d$ is an artificially determined structure matrix. Therefore, the pH system (14) is not exactly the same as the skyhook model shown in Figure 5.

The problem considered here is model matching with the system (14) through IDA-PBC, where the feedback should be expressed in terms of the relative quantities x_1 , x_2 , $\dot{x}_1$, and $\dot{x}_2$. In (11) and (14), the disturbance signal is $\dot{z}_0$ in place of z_0 , which does not affect the validity of the design procedure using IDA-PBC under the assumption of the differentiability of z_0 .

4.2. Application of IDA-PBC method

In the IDA-PBC method, the parameters of the desired model cannot be selected in an unrestricted manner. In what follows, we clarify the degree of freedom of the desired system. The equivalence condition (5) for the two systems is given as

$$[\mathbf{J}_d - \mathbf{R}_d] \frac{\partial H_d(\mathbf{x})}{\partial \mathbf{x}}^T = [\mathbf{J} - \mathbf{R}] \frac{\partial H(\mathbf{x})}{\partial \mathbf{x}}^T + (\mathbf{d} - \mathbf{d}_d) \dot{z}_0 + \mathbf{b}u. \quad (17)$$

Because (17) must hold for all $\mathbf{x}$ and $\dot{z}_0$, we obtain the constraints

$$k_{2a} = \frac{k_2}{r_2}, \quad c_{2a} = r_2 c_2 - c_{4a}, \quad j_f = -c_{4a} - (r_1 - r_2) c_2, \quad (18)$$

and the feedback law

$$\begin{aligned} u &= \left\{ k_1 + k_2 - r_1 \left(k_{1a} + \frac{k_2}{r_2} \right) \right\} x_1 + k_2 \left(\frac{r_1}{r_2} - 1 \right) x_2 + (c_{1a} - c_1) \dot{z}_0 \\ &+ \left(c_1 + c_2 - \frac{c_{1a} + r_2 c_2 + c_{3a} - c_{4a}}{r_1} \right) \frac{p_1}{m_1} \\ &+ \left\{ \left(1 - \frac{r_1}{r_2} \right) c_2 - \frac{2c_{4a}}{r_2} \right\} \frac{p_2}{m_2}, \end{aligned} \quad (19)$$

where $r_1 = m_{1a}/m_1$ and $r_2 = m_{2a}/m_2$. As stated in Section 2, the feedback (19) should be expressed as a function of the relative displacements and velocities. We substitute $p_2 = m_2 \dot{z}_2$ and $p_1 = m_1 \dot{z}_1$ into (19), and using (18) we rewrite (19) as

$$\begin{aligned} u &= k_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1) + (k_1 - r_1 k_{1a}) x_1 \\ &+ \left\{ \left(1 - \frac{r_1}{r_2} \right) c_2 - \frac{2c_{4a}}{r_2} \right\} (\dot{x}_2 - \dot{x}_1) + (c_1 - c_{1a} + \theta) \dot{x}_1 + \theta \dot{z}_0, \end{aligned} \quad (20)$$

where

$$\theta = c_{1a} + 2c_2 - \frac{c_{1a} + r_2 c_2 + c_{3a} - c_{4a}}{r_1} - \frac{2c_{4a} + r_1 c_2}{r_2}. \quad (21)$$

If θ is zero, we can express the feedback law (20) in terms of the relative displacements and velocities. Thus, by solving $\theta = 0$ for c_{3a} , we obtain an additional condition

$$c_{3a} = -\frac{(r_1 - r_2)^2}{r_2}c_2 + (r_1 - 1)c_{1a} - \left(\frac{2r_1}{r_2} - 1\right)c_{4a}. \quad (22)$$

For positive definiteness of the Hamiltonian H_d , it is necessary that m_{1a} , m_{2a} , k_{1a} and k_{2a} be positive. Considering (18), it is obvious that the Hamiltonian is positive definite if and only if $k_{1a} > 0$, $r_1 > 0$, and $r_2 > 0$. Moreover, for asymptotic stability of the desired system, the damping matrix C_d should be positive definite.

4.3. Guidelines for parameter selections

The equality constraints are summarized as

$$\begin{aligned} k_{2a} &= \frac{k_2}{r_2}, \quad c_{2a} = r_2 c_2 - c_{4a}, \quad j_f = c_{2a} - c_2 r_1, \\ c_{3a} &= -\frac{(r_1 - r_2)^2}{r_2}c_2 + (r_1 - 1)c_{1a} - \left(\frac{2r_1}{r_2} - 1\right)c_{4a}. \end{aligned} \quad (23)$$

The positive parameters m_1 , m_2 , k_1 , k_2 , c_1 , and c_2 are given a priori for the controlled object. Hence, there are nine parameters that we must design under the four equality constraints (23) along with the inequality constraints

$$k_{1a} > 0, \quad r_1 > 0, \quad r_2 > 0, \quad (24)$$

and $C_d > 0$. Therefore, we choose five constants c_{1a} , c_{4a} , r_1 , r_2 , and k_{1a} for parameterizing the desired system (14). The other parameters of (14) can be expressed in terms of these free parameters using (23).

The choices of the five free parameters are important to the vibration suppression performance. In this subsection, we provide guidelines for the parameter selections. From $C_d > 0$, an inequality constraint is obtained:

$$c_2 r_2 (2c_2 r_1 + c_{1a} r_1 + 2c_{4a} - c_2 r_2) > (c_2 r_1 + c_{4a})^2. \quad (25)$$

Conversely, if (25) and $r_2 > 0$ are satisfied, the damping matrix C_d is positive definite. Hence, the inequality constraints for this problems are (24) and (25). From the empirical knowledge for skyhook systems, we expect that a large value of the skyhook damper coefficient c_{3a} enlarges the vibration suppression / isolation effects. Increasing the values of r_1/r_2 and c_{1a} is effective for increasing c_{3a} . However, a large c_{1a} value amplifies the high-frequency gain from z_0 to x_1 , and there is an upper limit on r_1/r_2 for fixed c_{1a} and c_{4a} because of (25). To solve this trade-off problem, we set c_{4a} to a negative value as

$$c_{4a} = -c_2 r_1. \quad (26)$$

Lemma 1

Suppose that (23), (24), and (26) are satisfied. Then, $C_d > 0$ if and only if

$$\frac{r_2}{r_1} < \frac{c_{1a}}{c_2}. \quad (27)$$

Proof

Under the constraints (23) and (26), the (2, 2)-component of C_d is $r_2 c_2$, which is always positive due to (24). Hence, the necessary and sufficient condition for $C_d > 0$ is (25), which becomes $\det C_d = c_2 r_2 (c_{1a} r_1 - c_2 r_2) > 0$ under (26). Obviously, $\det C_d > 0$ if and only if (27) holds. $\square$

Positivity of c_{1a} follows from the inequalities (27) and (24). Under (26), the positive definiteness of the damping matrix C_d is ensured for large r_1/r_2 and positive c_{1a} . In addition, (22) is converted into $c_{3a} = c_{1a}(r_1 - 1) + c_2(r_1^2/r_2 + r_1 - r_2)$ by using (26). Increasing r_1 and decreasing r_2 can make c_{3a} large without any change in c_{1a} . The asymptotic stability of the origin of the closed-loop system with the null disturbances is also guaranteed under the parameter selection mentioned above.

Theorem 1

The feedback

$$u = k_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1) + (k_1 - r_1 k_{1a}) x_1 + \left\{ \left(1 - \frac{r_1}{r_2} \right) c_2 - \frac{2c_{4a}}{r_2} \right\} (\dot{x}_2 - \dot{x}_1) + (c_1 - c_{1a}) \dot{x}_1 \quad (28)$$

converts the system (11) into the desired system (14) with the equality constraints (23). Moreover, if (23), (24), (26), and (27) are satisfied, the origin of the system (14) with null disturbances ($z_0 = 0$ and $F = 0$) is globally asymptotically stable (GAS).

Proof

The first part of the theorem can easily be confirmed by simple substitution. To show the asymptotic stability, the Hamiltonian H_d is regarded as a Lyapunov function. For the system (14) with $z_0 = 0$ and $F = 0$, the time derivative of H_d is $\dot{H}_d = -(\partial H_d / \partial \mathbf{p}) \mathbf{C}_d (\partial H_d / \partial \mathbf{p})^T = -\mathbf{p}^T \mathbf{M}_d^{-1} \mathbf{C}_d \mathbf{M}_d^{-1} \mathbf{p} \leq 0$ where $\mathbf{p} = (p_1, p_2)^T$. Therefore, the origin of the system is globally stable and $\mathbf{p}$ converges to zero. If $\mathbf{p} = 0$ and $\dot{\mathbf{p}} = 0$, then $\partial H_d / \partial (x_1, x_2) = 0$ holds, which implies $x_1 = x_2 = 0$. From the invariance principle, it can be shown that the origin of the system is GAS. $\square$

To recapitulate the above discussion, taking r_1 large, r_2 small, c_{1a} positive, and c_{4a} equal to $-c_2 r_1$ are our recommendations for parameter selections. The value of $r_1 k_{1a}$ determines the low-frequency gain from F to z_1 . This parameter also affects the cutoff frequency, but the cutoff angular frequency is not dominated by $r_1 \sqrt{k_{1a}/m_{1a}} = \sqrt{r_1 k_{1a}/m_1}$ because of the effect of TMD movement. This fact is shown by the example in the next subsection. The parameter c_{1a} must be adequately chosen according to the frequency of the disturbance.

The advantage of the proposed method is that the model matching with the skyhook model can be performed by a static controller using only relative displacements and velocities. The skyhook damping coefficient can be set to an arbitrarily large value. Since the controller design is based on the IDA-PBC method, the proposed method can be applied to nonlinear cases, as shown in Section 5.

4.4. Case studies for confirmation of feedback effect

In this subsection, we verify the vibration suppression effect of the feedback (28). To simplify the discussion, we set k_{1a} as

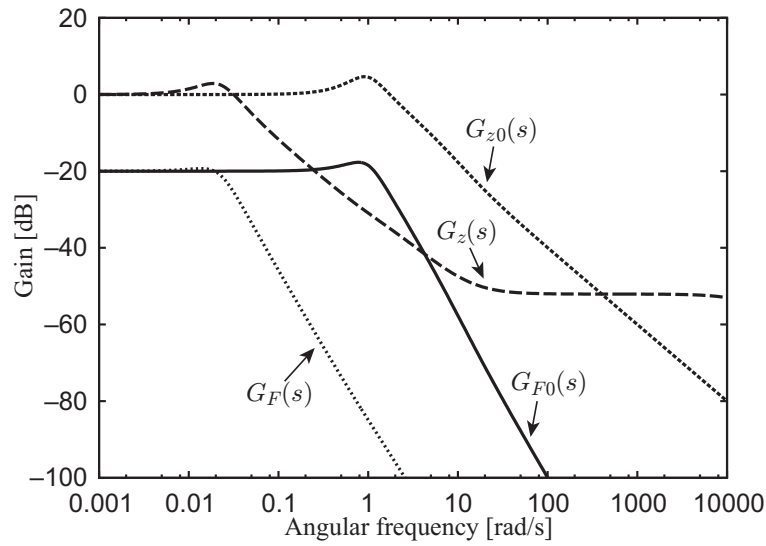
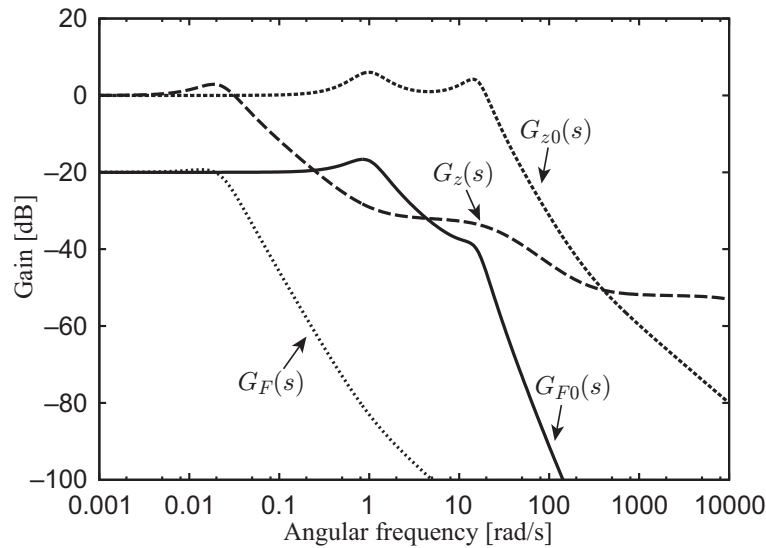
$$k_{1a} = \frac{k_1}{r_1}, \quad (29)$$

which maintains the low-frequency gain from F to z_1 at that of the open-loop system. Under the restrictions (23), (26), and (29), the free parameters describing the ideal closed-loop system are r_1 , r_2 , and c_{1a} . The frequency-domain expression for the closed-loop system then becomes

$$\begin{aligned} \mathcal{L}[y] &= G_F(s) \mathcal{L}[F] + G_z(s) \mathcal{L}[z_0] \\ &= \frac{m_2 s^2 + c_2 s + k_2}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \mathcal{L}[F] + \frac{(m_2 s^2 + c_2 s + k_2)(c_{1a} s + k_1)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \mathcal{L}[z_0], \end{aligned}$$

where

$$\begin{aligned} a_4 &= m_1 m_2, \quad a_3 = m_2 c_{1a} + m_1 c_2 + \left(2 + \frac{r_1}{r_2} \right) m_2 c_2, \\ a_2 &= c_{1a} c_2 + (m_1 + m_2) k_1 + \frac{r_1}{r_2} m_2 k_2, \quad a_1 = k_2 c_{1a} + k_1 c_2, \quad a_0 = k_1 k_2. \end{aligned}$$

Figure 6. Gain plots for the case with $k_2 = 3$.Figure 7. Gain plots for the case with $k_2 = 50$.

The gain plots of $G_F(s)$ and $G_z(s)$ are compared with the gain plots of $G_{F0}(s)$ and $G_{z0}(s)$, respectively, where

$$G_{F0}(s) = \frac{m_2 s^2 + c_2 s + k_2}{a_4 s^4 + \{c_1 m_2 + c_2(m_1 + m_2)\} s^3 + \{c_1 c_2 + k_1 m_2 + k_2(m_1 + m_2)\} s^2 + (c_2 k_1 + c_1 k_2) s + a_0}$$

denotes the transfer function from F to y for the open-loop system and $F_{z0}(s) = (c_1 s + k_1)G_{F0}(s)$ is the transfer function from z_0 to y for the open-loop system.

To verify the performance of the proposed method, we evaluate the gains for two cases. In the first case, the parameters of the controlled object are $m_1 = 10$, $m_2 = 0.2$, $k_1 = 10$, $k_2 = 3$, $c_1 = 10$, and

$c_2 = 2$, while the free parameters are chosen as $r_1 = 1$, $r_2 = 1/10000$, and $c_{1a} = 500$. The choice of the free parameters fulfills (24) and (27). In the second case, the value of k_2 is changed to 50, while retaining the other values from the first case. Figures 6 and 7 show the gain plots for the first ($k_2 = 3$) and second ($k_2 = 50$) cases, respectively.

Figure 6 shows that in the case of the open-loop system, the passive TMD has only a small vibration suppression effect because m_2 is small. However, the closed-loop system exhibits good vibration suppression effects for both types of disturbances, owing to its large c_{3a} value. In the high-frequency range ($\omega > 400$ rad/s), $|G_z(j\omega)|$ is larger than $|G_{z0}(j\omega)|$, but the gain of $G_z(s)$ is under -50 dB, which is sufficiently small. In the simple passive system (Figure 1), decreasing the cutoff frequency worsens the low-frequency gain from F to z_1 (see Figure 2). However, our method can maintain the low-frequency gain and still effect a decrease in the cutoff frequency of $G_F(s)$.

In the second case, the TMD in the open-loop system is underdamping, because the value of k_2 is large. As can be seen in Figure 7, the ill effects of the underdamping appear in the gain plots of the open-loop system as undesired peaks. However, the feedback diminishes the ill effects, and the undesired peaks in the gain plots of the closed-loop system are suppressed.

These results confirm that the proposed method can improve vibration suppression for the two disturbances (i.e., the direct force F and the floor displacement z_0).

5. VIBRATION SUPPRESSION CONTROL FOR A SYSTEM WITH NONLINEAR SPRINGS

In this section, we extend our method to the case of a controlled object with nonlinear springs. We assume that the spring forces shown in Figure 4 are expressed as cubic functions. This implies that the associated Hamiltonian is modified as

$$H(\mathbf{x}) = \frac{1}{4}\{\beta_1 x_1^4 + \beta_2 (x_2 - x_1)^4\} + \frac{1}{2}\left\{k_1 x_1^2 + k_2 (x_2 - x_1)^2 + \frac{p_1^2}{m_1} + \frac{p_2^2}{m_2}\right\}, \quad (30)$$

where $k_1, k_2, \beta_1, \beta_2, m_1$, and m_2 are positive constants, and the state is defined by (10). Except for the Hamiltonian modification, the pH system of the controlled object has the same representation as the linear case in (9), (11), and (13). Similarly, the desired model with nonlinear springs has the same configuration as that shown in Figure 5 and is expressed by (14) and (15) with a modified Hamiltonian

$$H_d(\mathbf{x}) = \frac{1}{4}\{\beta_{1a} x_1^4 + \beta_{2a} (x_2 - x_1)^4\} + \frac{1}{2}\left\{k_{1a} x_1^2 + k_{2a} (x_2 - x_1)^2 + \frac{p_1^2}{m_{1a}} + \frac{p_2^2}{m_{2a}}\right\}, \quad (31)$$

where $k_{1a}, k_{2a}, \beta_{1a}, \beta_{2a}, m_{1a}$, and m_{2a} are positive constants.

We apply the IDA-PBC method to this problem. Because (17) must hold for all $\mathbf{x}$, and $\dot{\mathbf{x}}_0$, we obtain the constraints

$$k_{2a} = \frac{k_2}{r_2}, \quad \beta_{2a} = \frac{\beta_2}{r_2}, \quad c_{2a} = r_2 c_2 - c_{4a}, \quad \dot{J}_f = -c_{4a} - (r_1 - r_2)c_2, \quad (32)$$

and the feedback law

$$u = \beta_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1)^3 - (r_1 \beta_{1a} - \beta_1) x_1^3 + k_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1) + (k_1 - r_1 k_{1a}) x_1 + \left\{ \left(1 - \frac{r_1}{r_2} \right) c_2 - \frac{2c_{4a}}{r_2} \right\} (\dot{x}_2 - \dot{x}_1) + (c_1 - c_{1a} + \theta) \dot{x}_1 + \theta \dot{z}_0, \quad (33)$$

where $r_1 = m_{1a}/m_1$, $r_2 = m_{2a}/m_2$, and θ is defined by (21). The condition to express (33) in terms of the relative displacements and velocities is the same as (22). We can choose the free parameters

as r_1 , r_2 , c_{1a} , c_{4a} , k_{1a} , and β_{1a} . The inequality constraints for these parameters are

$$r_1 > 0, \quad r_2 > 0, \quad k_{1a} > 0, \quad \beta_{1a} > 0, \quad (34)$$

$$c_2 r_2 (2c_2 r_1 + c_{1a} r_1 + 2c_{4a} - c_2 r_2) > (c_2 r_1 + c_{4a})^2, \quad (35)$$

which are the same as for the linear case in Section 4 except the condition on the coefficient β_{1a} of the high-order term. Because the quadratic approximation of the Hamiltonian (31) coincides with (16), the guideline of the parameter selections is same as the linear case; i.e., taking r_1 large, r_2 small, c_{1a} positive, and c_{4a} equal to $-c_2 r_1$.

Theorem 2

Suppose that (22), (26), (27), (32), and (34) hold. Then, the feedback

$$\begin{aligned} u = & \beta_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1)^3 - (r_1 \beta_{1a} - \beta_1) x_1^3 + k_2 \left(\frac{r_1}{r_2} - 1 \right) (x_2 - x_1) + (k_1 - r_1 k_{1a}) x_1 \\ & + \left\{ \left(1 - \frac{r_1}{r_2} \right) c_2 - \frac{2c_{4a}}{r_2} \right\} (\dot{x}_2 - \dot{x}_1) + (c_1 - c_{1a}) \dot{x}_1 \end{aligned}$$

converts the system (11) with the Hamiltonian (30) into the desired system (14) with (31). Moreover, the closed-loop system is GAS when the disturbances F and $\dot{z}_0$ are zero.

Proof

The first part of the theorem can easily be confirmed by simple substitution. When the disturbances z_0 and F are zero, $\dot{H}_d = -\mathbf{p}^T \mathbf{M}_d^{-1} \mathbf{C}_d \mathbf{M}_d^{-1} \mathbf{p} \leq 0$ for the system (14) with (31). Therefore the closed-loop system is globally stable, and $\mathbf{p}$ converges to zero. The restriction $\mathbf{p} \equiv 0$ implies

$$\begin{aligned} \frac{\partial H_d}{\partial x_1} &= \beta_{1a} x_1^3 + \beta_{2a} (x_1 - x_2)^3 + k_{1a} x_1 + k_{2a} (x_1 - x_2) = 0 \\ \frac{\partial H_d}{\partial x_2} &= -\beta_{2a} (x_1 - x_2)^3 - k_{2a} (x_1 - x_2) = 0. \end{aligned}$$

Hence, from the invariance principle, we conclude that the origin of the desired system (14) with (31) is GAS for null disturbances. $\square$

The result of this section is only for nonlinear spring cases. However, the IDA-PBC method is commonly used for the control problem of nonlinear Hamiltonian systems. The contribution of this paper is to show the IDA-PBC method is also useful for vibration suppression and isolation problems, because the disturbance attenuation performance around the origin is dominated by that of the linearly approximated desired system. In other words, it is expected that, even for nonlinear cases, it is useful to set a negative skyhook damping coefficient for the additional mass in the desired system. As long as the desired system is chosen under this guideline, the difference between the IDA-PBC method and the proposed method is the restriction (22). This fact suggests that the applicability of the proposed method to a wide class of nonlinear systems.

In conclusion, the model matching with the skyhook model by the IDA-PBC method is also effective in the nonlinear cases, where the nonlinear controller obtained uses only relative displacements and velocities.

6. INPUT TO STATE STABILITY OF THE NONLINEAR SYSTEM

For global vibration suppression of a nonlinear system, its input-to-state stability (ISS) [22] should be ensured. Asymptotically stable linear systems are always input-to-state stable, but GAS does not imply ISS for nonlinear cases. In this section, we investigate the ISS property of the desired nonlinear system (14) with (31) for the disturbances z_0 and F . In a Hamiltonian system, the Hamiltonian is not a strict Lyapunov function; i.e., the time derivative of the Hamiltonian is not

negative definite, although it is negative semidefinite. Hence, the Hamiltonian is not able to become an ISS Lyapunov function. In this paper, an ISS Lyapunov function is constructed by perturbing the Hamiltonian $H_d(x)$ in a way similar to that of Romero et al. [25].

In this section, we assume that the parameters of the desired system are selected as (22), (26), (27), (32), and (34). We consider the ISS Lyapunov function candidate

$$H_p(x) = \frac{1}{4}(\beta_{1a}x_1^4 + \beta_{2a}x_2^4) + \frac{1}{2} \left\{ k_{1a}x_1^2 + k_{2a}(x_2 - x_1)^2 + \frac{(p_1 + \epsilon k_{1a}m_1x_1)^2}{m_{1a}} + \frac{(p_2 + \epsilon k_{1a}m_2x_2)^2}{m_{2a}} \right\}, \quad (36)$$

which is formed by adding perturbations to the Hamiltonian (31), where ϵ is a small positive constant. If $\epsilon = 0$, then $H_p(x)$ coincides with the Hamiltonian $H_d(x)$ in (31).

The system (14) with (31) can be converted into

$$\dot{x} = (J_p - R_p) \frac{\partial H_{p0}(x)}{\partial x}^T + \eta(x) + d\dot{z}_0 + bF, \quad (37)$$

where the quadratic approximation of $H_p(x)$ is

$$H_{p0}(x) = \frac{1}{2} \left\{ k_{1a}x_1^2 + k_{2a}(x_2 - x_1)^2 + \frac{(p_1 + \epsilon k_{1a}m_1x_1)^2}{m_{1a}} + \frac{(p_2 + \epsilon k_{1a}m_2x_2)^2}{m_{2a}} \right\},$$

the high-order term is

$$\eta(x) = (0, 0, -\beta_{1a}r_1x_1^3, -\beta_{2a}r_2x_2^3)^T,$$

and

$$J_p = \begin{bmatrix} 0 & 0 & j_1 & j_2 \\ 0 & 0 & j_3 & j_4 \\ -j_1 & -j_3 & 0 & j_5 \\ -j_2 & -j_4 & -j_5 & 0 \end{bmatrix}, \quad j_1 = r_1 - \frac{\epsilon}{2}c_{1a} + \frac{\epsilon^2}{2}m_1k_{1a}, \quad j_2 = \frac{\epsilon^2}{2}k_{1a}m_2, \quad (38)$$

$$j_3 = \frac{\epsilon}{2} \left\{ -c_{1a} + c_2 \frac{k_{1a}}{k_2} (r_1 + 2r_2) \right\} + \frac{\epsilon^2}{2}m_1k_{1a},$$

$$j_4 = r_2 - \frac{\epsilon}{2}c_2 \frac{k_{1a}}{k_2} r_2 + \frac{\epsilon^2}{2}m_2k_{1a} \left\{ 1 + \frac{k_{1a}}{k_2} r_2 \right\},$$

$$j_5 = c_2r_2 + \frac{\epsilon^2}{2}m_2 \left\{ -c_{1a}k_{1a} + c_2 \frac{k_{1a}^2}{k_2} (r_1 + 2r_2) \right\},$$

$$R_p = \begin{bmatrix} \epsilon B_1 & \epsilon B_2(\epsilon) \\ \epsilon B_2(\epsilon)^T & B_3(\epsilon) \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 + k_{1a}r_2/k_2 \end{bmatrix},$$

$$B_2(\epsilon) = \begin{bmatrix} -(c_{1a} + \epsilon m_1k_{1a})/2 & -\epsilon m_2k_{1a}/2 \\ -\{c_{1a} - c_2(r_1 + 2r_2)/k_2 + \epsilon m_1k_{1a}\}/2 & -k_{1a}\{c_2r_2 + \epsilon m_2(k_2 + k_{1a}r_2)\}/k_2 \end{bmatrix},$$

$$B_3(\epsilon) = \begin{bmatrix} \gamma_1(\epsilon) & \gamma_2(\epsilon) \\ \gamma_2(\epsilon) & \gamma_3(\epsilon) \end{bmatrix}, \quad \gamma_1(\epsilon) = r_1 \left\{ c_{1a} + c_2 \left(2 + \frac{r_1}{r_2} \right) \right\} - \epsilon m_1k_{1a}(r_1 - \epsilon c_{1a}) \quad (39)$$

$$\gamma_2(\epsilon) = -c_2(r_1 + r_2) + \frac{\epsilon^2}{2}m_2 \frac{k_{1a}}{k_2} \{c_{1a}k_2 - c_2k_{1a}(r_1 + 2r_2)\}$$

$$\gamma_3(\epsilon) = c_2r_2 - \epsilon \frac{k_{1a}}{k_2} m_2r_2(k_2 - \epsilon c_2k_{1a}).$$

For negative definiteness of $\dot{H}_p(x)$ with null disturbances, the new damping matrix R_p should be positive definite.

Lemma 2

Assume that (22), (26), (27), (32), and (34) hold. Then, there exists a positive value of ϵ such that the damping matrix R_p , defined by (39), is positive definite.

Proof

From the Schur complement, the necessary and sufficient conditions for $\mathbf{R}_p > 0$ are $\epsilon \mathbf{B}_1 > 0$ and $\mathbf{S}(\epsilon) = \mathbf{B}_3(\epsilon) - \epsilon \mathbf{B}_2(\epsilon)^T \mathbf{B}_1^{-1} \mathbf{B}_2(\epsilon) > 0$. Obviously, $\mathbf{B}_1$ is positive definite. Because $\mathbf{S}(\epsilon)$ consists of polynomials with respect to ϵ , it is continuous with respect to ϵ . Note that $\mathbf{S}(0)$ coincides with the positive definite matrix $\mathbf{C}_d$. Hence, there exists a small positive ϵ such that $\mathbf{S}(\epsilon) > 0$. Thus, the proof is complete. $\square$

The following is the result for the ISS property.

Theorem 3

Suppose that (22), (26), (27), (32), and (34) are satisfied. Then, the desired nonlinear system (14) with (31) is ISS with respect to the disturbances z_0 and F .

Proof

In what follows, we choose the value of ϵ such that $\mathbf{R}_p > 0$. From (36) and (37), the time-derivative of the ISS Lyapunov function candidate $H_p(\mathbf{x})$ is obtained as

$$\begin{aligned} \dot{H}_p = & -\frac{\partial H_{p0}}{\partial \mathbf{x}} \mathbf{R}_p \frac{\partial H_{p0}}{\partial \mathbf{x}}^T \\ & - \epsilon k_{1a} (\beta_{1a} x_1^4 + \beta_{2a} x_2^4) \\ & + \mathbf{x}^T \mathbf{w}_f F + \{\mathbf{x}^T \mathbf{w}_z - (\beta_{1a} x_1^3 + \beta_{2a} x_2^3)\} \dot{z}_0, \end{aligned} \quad (40)$$

where

$$\begin{aligned} \mathbf{w}_f &= \begin{pmatrix} \epsilon \frac{k_{1a}}{r_1} & 0 & \frac{1}{m_1 r_1} & 0 \end{pmatrix}^T \\ \mathbf{w}_z &= \begin{pmatrix} -k_{1a} \left\{ 1 - \frac{\epsilon}{r_1} (c_{1a} - \epsilon m_1 k_{1a}) \right\} \\ -\epsilon^2 \frac{m_2 k_{1a}^2}{r_2} \\ \frac{c_{1a}}{m_1 r_1} - \epsilon \frac{k_{1a}}{r_1} \\ -\epsilon \frac{k_{1a}}{r_2} \end{pmatrix}. \end{aligned}$$

Note that $\partial H_{p0} / \partial \mathbf{x}$ is linear with respect to $\mathbf{x}$; that is,

$$\frac{\partial H_{p0}}{\partial \mathbf{x}}^T = \mathbf{A} \mathbf{x},$$

where the regular matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} k_{1a} + k_{2a} + \epsilon^2 m_1 k_{1a}^2 / r_1 & -k_{2a} & \epsilon k_{1a} / r_1 & 0 \\ -k_{2a} & k_{2a} + \epsilon^2 m_2 k_{1a}^2 / r_2 & 0 & \epsilon k_{1a} / r_2 \\ \epsilon k_{1a} / r_1 & 0 & 1 / (m_1 r_1) & 0 \\ 0 & \epsilon k_{1a} / r_2 & 0 & 1 / (m_2 r_2) \end{bmatrix}.$$

Since the coefficients of the disturbances in (40) have third-order terms with respect to the state, our problem is slightly different from that of Romero et al. [25]. By completing the squares, (40) is

evaluated as

$$\begin{aligned}
\dot{H}_p &= -\frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{R}_p \mathbf{A} \mathbf{x} \\
&\quad - \frac{1}{4}\{\mathbf{x} - 2(\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f F\}^T \mathbf{A} \mathbf{R}_p \mathbf{A} \{\mathbf{x} - 2(\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f F\} \\
&\quad - \frac{1}{4}\{\mathbf{x} - 2(\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z \dot{z}_0\}^T \mathbf{A} \mathbf{R}_p \mathbf{A} \{\mathbf{x} - 2(\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z \dot{z}_0\} \\
&\quad + \{\mathbf{w}_f^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f\} F^2 + \{\mathbf{w}_z^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z\} \dot{z}_0^2 \\
&\quad + \frac{\beta_{1a} x_1^2}{2} \left\{ -\epsilon k_{1a} x_1^2 - \epsilon k_{1a} \left(x_1 + \frac{\dot{z}_0}{\epsilon k_{1a}} \right)^2 + \frac{\dot{z}_0^2}{\epsilon k_{1a}} \right\} \\
&\quad + \frac{\beta_{2a} x_2^2}{2} \left\{ -\epsilon k_{1a} x_2^2 - \epsilon k_{1a} \left(x_2 + \frac{\dot{z}_0}{\epsilon k_{1a}} \right)^2 + \frac{\dot{z}_0^2}{\epsilon k_{1a}} \right\} \\
&\leq -\frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{R}_p \mathbf{A} \mathbf{x} \\
&\quad + \{\mathbf{w}_f^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f\} F^2 + \{\mathbf{w}_z^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z\} \dot{z}_0^2 \\
&\quad + \beta_{1a} \left\{ -\frac{\epsilon k_{1a}}{2} \left(x_1^2 - \frac{\dot{z}_0^2}{2\epsilon^2 k_{1a}^2} \right)^2 + \frac{\dot{z}_0^4}{8\epsilon^3 k_{1a}^3} \right\} \\
&\quad + \beta_{2a} \left\{ -\frac{\epsilon k_{1a}}{2} \left(x_2^2 - \frac{\dot{z}_0^2}{2\epsilon^2 k_{1a}^2} \right)^2 + \frac{\dot{z}_0^4}{8\epsilon^3 k_{1a}^3} \right\} \\
&\leq -\frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{R}_p \mathbf{A} \mathbf{x} \\
&\quad + \{\mathbf{w}_f^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f\} F^2 + \{\mathbf{w}_z^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z\} \dot{z}_0^2 + \frac{\beta_{1a} + \beta_{2a}}{8\epsilon^3 k_{1a}^3} \dot{z}_0^4.
\end{aligned} \tag{41}$$

Thus, from (41) we can obtain the class- $\mathcal{K}^\infty$ functions

$$\begin{aligned}
\gamma(\|\mathbf{x}\|) &= \frac{1}{2} \lambda_{\min}(\mathbf{A} \mathbf{R}_p \mathbf{A}) \|\mathbf{x}\|^2 \\
\delta_1(|F|) &= \{\mathbf{w}_f^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_f\} |F|^2 \\
\delta_2(|\dot{z}_0|) &= \{\mathbf{w}_z^T (\mathbf{A} \mathbf{R}_p \mathbf{A})^{-1} \mathbf{w}_z\} |\dot{z}_0|^2 + \frac{\beta_{1a} + \beta_{2a}}{8\epsilon^3 k_{1a}^3} |\dot{z}_0|^4
\end{aligned}$$

satisfying

$$\dot{H}_p \leq -\gamma(\|\mathbf{x}\|) + \delta_1(|F|) + \delta_2(|\dot{z}_0|),$$

where $\|\cdot\|$ denotes a 2-norm and $\lambda_{\min}(\cdot)$ is the minimum eigenvalue. Therefore, $H_p(\mathbf{x})$ becomes an ISS Lyapunov function [26], and so the ISS property has been shown. $\square$

7. CONCLUSION

This paper proposed a feedback vibration suppression method without information on absolute displacements, velocities, and accelerations. The feedback was designed using the IDA-PBC technique, whereby a 2DOF system with a tuned mass damper (TMD) was converted into a skyhook model. This method can suppress the two disturbances, namely, the floor vibration and the direct force to the main body. The effectiveness of the proposed method is confirmed by the gain plots for typical cases. In the case study, despite the small mass of the TMD, the gains of the resulting closed-loop system from two disturbances to the displacement of the main mass are smaller than those of the open-loop system. Since the proposed method is based on the IDA-PBC, it is expected that we can extend this method to nonlinear cases. In fact, we showed that the proposed method can

be applied to nonlinear-spring case. The disturbance-attenuation performance of the closed-loop system in nonlinear cases obeys that of the linearly approximated system for small disturbances. The closed-loop system for the nonlinear-spring case was shown to be ISS by using a skewed Hamiltonian as an ISS Lyapunov function.

The proposed method is a promising approach for the nonlinear disturbance suppression control, which theoretically guarantees the stability of the closed-loop system. The design procedure of the proposed method is almost same as the IDA-PBC control except the restriction (22) and the selection guide of the desired system described in Section 4.3. Consequently, we can show that passivity-based control is also effective for the disturbance suppression problem.

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