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Production of a neutron-rich ${}^6_\Lambda\text{H}$ hypernucleus in the ${}^6\text{Li}(\pi^-, K^+)$ reaction

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We study phenomenologically the production of the neutron-rich hypernucleus ${}^6_\Lambda\text{H}$ in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c, using a distorted-wave impulse approximation in a one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by $\Sigma^- p \leftrightarrow \Lambda n$ coupling. The production cross section of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ is evaluated by a coupled $({}^5\text{H}-\Lambda) + ({}^5\text{He}-\Sigma^-)$ model with a spreading potential, in comparison with the data of the missing mass spectrum at the J-PARC E10 experiment. The result indicates that the Σ^- mixing probabilities in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ are $P_{\Sigma^-} < \sim 0.2\%$ both for s_Σ state and for p_Σ state in order to reproduce no significant peak in the Λ production data, so that the cross section of ${}^6_\Lambda\text{H}$ is less than on the order of 0.4 nb/sr. The sensitivity of the $\Sigma\Lambda$ coupling and Λ potentials to the near- Λ -threshold spectrum is discussed. The shape and magnitude of the spectrum provide valuable information on the $\Sigma\Lambda$ coupling in the production mechanism and also the nuclear structure of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$.

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I. INTRODUCTION

Recently, the J-PARC E10 collaboration [1,2] performed experimental measurements of the double-charge-exchange (DCX) reaction (π^-, K^+) on a ${}^6\text{Li}$ target at $p_{\pi^-} = 1.2$ GeV/c in order to confirm a neutron-rich hypernucleus ${}^6_\Lambda\text{H}$ in which an unbound ${}^5\text{H}$ nuclear core with neutron-proton excess ratio $(N - Z)/(N + Z) = 0.6$ is expected to be stable by Λ stabilization or glue [3,4]. No significant peak structure below and near the ${}^4_\Lambda\text{H} + 2n$ threshold was observed in missing mass spectra with K^+ forward-direction angles of $\theta_{\text{lab}} = 2^\circ - 14^\circ$. This is inconsistent with the observation by the ${}^6\text{Li}(K^-_{\text{stop}}, \pi^+)$ reaction in FINUDA experiments [5] which indicated evidence of ${}^6_\Lambda\text{H}$ with a binding energy of $B_\Lambda({}^6_\Lambda\text{H}) = 4.5 \pm 1.2$ MeV with respect to the ${}^5\text{H} + \Lambda$ threshold.

Dalitz and Levi-Setti [3] first discussed the Λ stabilization of the neutron-rich ${}^6_\Lambda\text{H}$ hypernucleus with the particle-unstable ${}^5\text{H}$ nuclear core beyond the neutron-drip line. Akaishi and Myint [6] paid attention to ${}^6_\Lambda\text{H}$ as a test ground for an attractive three-body ΛNN force caused by the $\Lambda N - \Sigma N$ coupling which may be more coherently enhanced in such neutron-excess environments [7,8]. Thus the 0^+ ground state of ${}^6_\Lambda\text{H}$ was predicted to have a large binding energy of $B_\Lambda({}^6_\Lambda\text{H}) = 5.8$ MeV with respect to the ${}^5\text{H} + \Lambda$ threshold due to rather large contribution of 1.4 MeV by the coherent $\Lambda\Sigma$ mixing [6]. Gal and Millener [9] showed that recent shell-model calculations including the $\Lambda\Sigma$ coupling give $B_\Lambda({}^6_\Lambda\text{H}) = 3.8 \pm 0.2$ MeV which seems to be in good agreement with $B_\Lambda({}^6_\Lambda\text{H}) = 4.5 \pm 1.2$ MeV reported in the FINUDA experiments [5,9]. Hiyaama *et al.* [10] suggested a less binding energy of $B_\Lambda({}^6_\Lambda\text{H}) = 2.47$ MeV corresponding to an unbound state with respect to the ${}^4_\Lambda\text{H} + 2n$ threshold in $tnn\Lambda$ four-body cluster-model calculations. The value of $B_\Lambda({}^6_\Lambda\text{H})$ is often calculated by the Λ -nucleus potential which strongly depends on the structure of the nuclear core as well as ΛN interaction involving the

$\Lambda\Sigma$ coupling. Therefore, it is very important to clarify the production and structure of ${}^6_\Lambda\text{H}$ which is strongly related to the structure of ${}^5\text{H}$ in nuclear physics.

The DCX (π^-, K^+) reaction is one of the most promising ways of searching for a bound state of the neutron-rich Λ hypernuclei with stabilized effects by Λ added. Indeed, Saha *et al.* [11] performed the first measurement of a significant yield for the ${}^{10}_\Lambda\text{Li}$ hypernucleus in (π^-, K^+) reactions on a ${}^{10}\text{B}$ target, whereas no clear peak has been observed with the lack of the experimental statistics. The data show that the absolute cross section for ${}^{10}_\Lambda\text{Li}$ at 1.20 GeV/c ($d\sigma/d\Omega \sim 11$ nb/sr) is twice larger than that at 1.05 GeV/c ($d\sigma/d\Omega \sim 6$ nb/sr). This incident-momentum dependence of $d\sigma/d\Omega$ exhibits a trend in the opposite direction for the theoretical prediction by Tretyakova and Lansky [12]. This might imply that the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling [13] is rather favored over the two-step mechanism, $\pi^- p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+ \Lambda$ (or $\pi^- p \rightarrow K^0 \Lambda$ followed by $K^0 p \rightarrow K^+ n$) in the production of neutron-rich Λ hypernuclear states, as pointed out in Ref. [11].

In this paper, we study phenomenologically the production of the neutron-rich ${}^6_\Lambda\text{H}$ hypernuclear states in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c. We demonstrate the calculated spectrum near the Λ threshold within a distorted-wave impulse approximation (DWIA) by using a coupled $({}^5\text{H}-\Lambda) + ({}^5\text{He}-\Sigma^-)$ model with a spreading potential [14]. Comparing the spectrum with the data of the J-PARC E10 experiment [1,2], we discuss the strengths of the $\Sigma\Lambda$ couplings related to the Σ -mixing probabilities and the strengths of the Λ - ${}^5\text{H}$ potentials which depend on the structure of the ${}^5\text{H}$ nuclear core in ${}^6_\Lambda\text{H}$.

II. CALCULATIONS

A. Distorted wave impulse approximation

The inclusive K^+ double-differential laboratory cross section of Λ production on a nuclear target in the DCX (π^-, K^+) reaction [15] is calculated by the Green's function method

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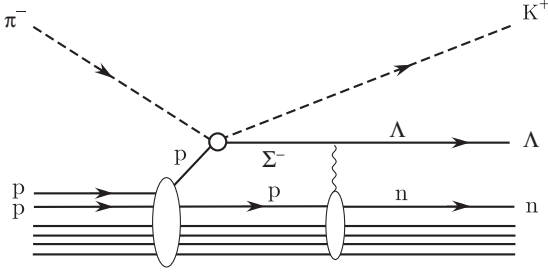


FIG. 1. Diagrams of a one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling, for production of Λ hypernuclear states by the DCX nuclear (π^- , K^+) reactions.

[16], assuming only the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \rightarrow \Lambda n$ conversion within the DWIA [13]. Figure 1 illustrates diagrams for the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways in the nuclear (π^- , K^+) reaction. The inclusive K^+ double-differential laboratory cross section on the nuclear target with a spin J_A and its z component M_A [15] is given by

$$\frac{d^2\sigma}{d\Omega dE} = \frac{1}{[J_A]} \sum_{M_A} S(E_B), \quad (1)$$

with $[J_A] = 2J_A + 1$, and the strength function $S(E_B)$ is written by

$$S(E_B) = -\frac{1}{\pi} \text{Im} \sum_{\alpha\alpha'} \int d\mathbf{r} d\mathbf{r}' F_{\Sigma}^{\alpha\dagger}(\mathbf{r}) G_{\Sigma}^{\alpha\alpha'}(E_B; \mathbf{r}, \mathbf{r}') F_{\Sigma}^{\alpha'}(\mathbf{r}') \quad (2)$$

as a function of the energy E_B for hypernuclear final states, where F_{Σ}^{α} is the Σ production amplitude defined by

$$F_{\Sigma}^{\alpha} = \beta^{\frac{1}{2}} \bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-} \chi_{p_K}^{(-)*} \chi_{p_{\pi}}^{(+)} \langle \alpha | \hat{\psi}_p | \Psi_A \rangle, \quad (3)$$

and $\langle \alpha | \hat{\psi}_p | \Psi_A \rangle$ is a hole-state wave function for a struck proton in the target; α denotes the complete set of eigenstates for the system. The energy and momentum transfer is $\omega = E_K - E_{\pi}$ and $\mathbf{q} = \mathbf{p}_K - \mathbf{p}_{\pi}$. The kinematical factor β denotes the translation from a two-body $\pi^- p$ laboratory system to a $\pi^- {}^6\text{Li}$ laboratory system. $\bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-}$ is a Fermi-averaged amplitude for the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction in nuclear medium [17].

Distorted waves for outgoing K^+ and incoming π^- mesons, $\chi_{p_K}^{(-)}$ and $\chi_{p_{\pi}}^{(+)}$, are estimated with the help of the eikonal approximation in which total cross sections of $\sigma_{\pi} = 32$ mb for $\pi^- N$ and $\sigma_K = 12$ mb for $K^+ N$, and $\alpha_{\pi} = \alpha_K = 0$ are used as distortion parameters [17]. The recoil effects are taken into account in our calculations because an effective momentum transfer becomes $q_{\text{eff}} \simeq (1 - 1/A)q \simeq 0.83q$ for the light nuclear system with $A = 6$ due to large momentum transfer $q = 320\text{--}600$ MeV/ c in the (π^- , K^+) reaction.

Although the 1^+ ground state of ${}^6\text{Li}$ is well described as $\alpha + d$ clusters [18], wave functions for the ${}^6\text{Li}$ target are used in the single-particle (s.p.) description for simplicity. This s.p. description has also been used to study the Σ -nucleus potential for $A = 6$ by the missing-mass ${}^6\text{Li}(\pi^-, K^+)$ spectrum at the J-PARC E10 experiment [19].

Thus the s.p. wave functions for the proton in $1p_{3/2}$ and $1s_{1/2}$ are calculated by the Woods–Saxon (WS) potential with $a = 0.67$ fm, $R = 1.27A^{1/3} = 2.31$ fm [20]. The strength parameter of the potential is adjusted to be $V_0^N = -55.5$ MeV (-58.0 MeV) for the proton in the $p_{3/2}$ ($s_{1/2}$) state, and $V_{so}^N = -0.44V_0^N$, in order to reproduce the data of proton s.p. energies in ${}^6\text{Li}(p, 2p)$ reactions [21,22]. Thus the s.p. energies for $1p_{3/2}$ and $1s_{1/2}$ amount to -4.61 MeV and -21.48 MeV, respectively. The charge radius for ${}^6\text{Li}(1^+_{\text{g.s.}})$ becomes 2.48 fm of which value is slightly smaller than that of 2.56 ± 0.05 fm in electron elastic scatterings [23] due to the s.p. description. If we replace the s.p. wave function for the $1p_{3/2}$ ($1s_{1/2}$) state by a spectroscopic amplitude describing a $p_{3/2}$ ($s_{1/2}$) proton removal from ${}^6\text{Li}(1^+_{\text{g.s.}})$ within the $\alpha + d$ cluster model [24], we recognize that the calculated cross sections decrease by about 5%, in comparison with the results which will be discussed in Sec. III B. Thus our conclusion obtained in the s.p. description would be reliable.

B. Wave functions for ${}^6_{\Lambda}\text{H}$

To fully describe the one-step process, as shown in Fig. 1 and to estimate the production cross section of ${}^6_{\Lambda}\text{H}$, we perform Λ - Σ coupled-channel calculations [14] which reproduce the shape and magnitude of the data of the J-PARC E10 experiment in the Λ and Σ^- quasifree (QF) regions [19]. Here we employ a multichannel coupled wave function of the Λ - Σ nuclear state for a total spin J_B within a weak-coupling basis. It is written as

$$|\Psi_{J_B}({}^6_{\Lambda}\text{H})\rangle = \sum_{JJ''j_{\Lambda}j_{\Sigma}} [\Phi_{J''}({}^5\text{H}), \varphi_{j_{\Lambda}}^{(\Lambda)}(\mathbf{r}_{\Lambda})]_{J_B} + \sum_{JJ'j_pj_{\Sigma}} [\Phi_{J'}({}^5\text{He}), \varphi_{j_{\Sigma}}^{(\Sigma^-)}(\mathbf{r}_{\Sigma})]_{J_B}, \quad (4)$$

with

$$\begin{aligned} \Phi_{J''}({}^5\text{H}) &= \mathcal{A}[\Phi_J(s^3p), \varphi_{j_n}^{(n)}(\mathbf{r}_n)]_{J''}, \\ \Phi_{J'}({}^5\text{He}) &= \mathcal{A}[\Phi_J(s^3p), \varphi_{j_p}^{(p)}(\mathbf{r}_p)]_{J'}, \end{aligned} \quad (5)$$

where $\Phi_J(s^3p)$ is a wave function of the s^3p configuration state, $\mathcal{A}$ is the antisymmetrized operator for nucleons, and $\varphi_{j_{\Lambda}}^{(\Lambda)}$, $\varphi_{j_{\Sigma}}^{(\Sigma^-)}$, and $\varphi_{j_{n,p}}^{(n,p)}$ describe the relative wave functions of shell model states (that occupy j_{Λ} , j_{Σ} , and $j_{n,p}$ orbits) for the Λ , Σ^- , and neutron (proton), respectively; $\mathbf{r}_n$ ($\mathbf{r}_p$) denotes the relative coordinate between the s^3p nucleus and the neutron or proton, and $\mathbf{r}_{\Lambda}$ ($\mathbf{r}_{\Sigma}$) denotes the relative coordinate between the center of mass of the ${}^5\text{H}$ (${}^5\text{He}$) subsystem and the Λ (Σ^-). We take the ${}^5\text{H}$ core-nucleus state with $J^{\pi} = 1/2^+$ [ground state (g.s.)], and the ${}^5\text{He}$ core-nucleus states with $J^{\pi} = 3/2^-$ (g.s.), $1/2^-$, $3/2^+$, and $1/2^+$ that are given in $(1^+ \otimes p_{3/2,1/2})_{\frac{3}{2}^-, \frac{1}{2}^-}$ and $(1^+ \otimes s_{1/2})_{\frac{3}{2}^+, \frac{1}{2}^+}$ configurations formed by a proton-hole state on ${}^6\text{Li}(1^+_{\text{g.s.}})$. If the Λ component is dominant in a bound or resonant state, we can identify it as a state of the Λ hypernucleus ${}^6_{\Lambda}\text{H}$, in which the Σ^- -mixing probability can be estimated by

$$P_{\Sigma^-} = \sum_{j_{\Sigma}} \int_0^{\infty} \rho_{j_{\Sigma}}(r) r^2 dr, \quad (6)$$

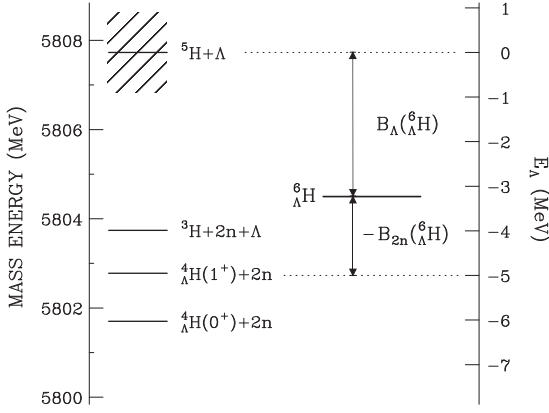


FIG. 2. Energy spectrum and decay threshold of the ${}^6_\Lambda\text{H}$ hypernucleus. Binding energies of $B_\Lambda({}^6_\Lambda\text{H})$ and $B_{2n}({}^6_\Lambda\text{H})$ are defined with respect to ${}^5\text{H} + \Lambda$ and ${}^4_\Lambda\text{H}(1^+) + 2n$ thresholds, respectively. The threshold-energy difference between ${}^5\text{H}(1/2^+_{\text{g.s.}})$ and the ${}^3\text{H} + 2n$ threshold is assumed to be 4.0 MeV.

where $\rho_{j_\Sigma}(r) = [\varphi_{j_\Sigma}^{(\Sigma^-)}(r)]^2$ denotes a Σ^- density distribution with the j_Σ shell under the normalization of

$$\sum_{j_\Lambda} \int_0^\infty \rho_{j_\Lambda}(r) r^2 dr + \sum_{j_\Sigma} \int_0^\infty \rho_{j_\Sigma}(r) r^2 dr = 1,$$

together with the Λ density distribution $\rho_{j_\Lambda}(r)$ with j_Λ . Because we assume the Σ^- doorway states that are selectively produced by non-spin-flip processes in the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction, we consider positive-parity (negative-parity) states with $J_\pi^+ = 1^+, 2^+, 3^+, \dots$, ($J_\pi^- = 0^-, 1^-, 2^-, 3^-, \dots$) for final states, which are populated on the nuclear ${}^6\text{Li}(1^+_{\text{g.s.}})$ targets; the 0^+ ground state of ${}^6_\Lambda\text{H}(0^+_{\text{g.s.}})$ is forbidden.

Several theoretical calculations [22,25] predicted the ${}^5\text{H}$ ground state with $J^\pi = 1/2^+$ (g.s.), $T = 3/2$ as a continuum or unbound state, $E_r \simeq 1.6\text{--}3.0$ MeV, $\Gamma \simeq 1.5\text{--}4.0$ MeV with respect to the ${}^3\text{H} + 2n$ threshold in ${}^3\text{H} + n + n$ three-body calculations [25] and $E_r \simeq 3.0\text{--}4.5$ MeV in standard shell-model calculation with *spsd* space [26,27]. Since the structure of ${}^5\text{H}$ is still uncertain experimentally [10,22,28], we assume that the ${}^5\text{H}(1/2^+_{\text{g.s.}})$ nuclear core is a resonant state with $E_r = 4.0$ MeV [25] in the viewpoint of shell-model calculations, rather than that with $E_r = 1.7$ MeV in Ref. [29]. Thus the energy difference between ${}^5\text{He} + \Sigma^-$ and ${}^5\text{H} + \Lambda$ channels is $\Delta M = M({}^5\text{He}) + m_{\Sigma^-} - M({}^5\text{H}) - m_\Lambda = 57.6$ MeV, where $M({}^5\text{He})$, $M({}^5\text{H})$, m_{Σ^-} , and m_Λ are masses of ${}^5\text{He}$, ${}^5\text{H}$, Σ^- and Λ , respectively. Figure 2 illustrates the energy spectrum and decay threshold for the ${}^6_\Lambda\text{H}$ hypernucleus, where $B_\Lambda({}^6_\Lambda\text{H})$ and $B_{2n}({}^6_\Lambda\text{H})$ denote the binding energies with respect to ${}^5\text{H} + \Lambda$ and ${}^4_\Lambda\text{H}(1^+) + 2n$ thresholds, respectively.

C. Multichannel Green's functions

The Green's function method is one of the most powerful treatments in calculations for the spectrum [16]. The complete Green's function $\mathbf{G}(E)$ describes all information concerning $({}^5\text{H} \otimes \Lambda) + ({}^5\text{He} \otimes \Sigma^-)$ coupled-channel dynamics. We obtain it by solving the following equation with the hyperon-

nucleus potential U numerically:

$$\mathbf{G}(E) = \mathbf{G}^{(0)}(E) + \mathbf{G}^{(0)}(E) \mathbf{U} \mathbf{G}(E), \quad (7)$$

where

$$\mathbf{G}(E) = \begin{pmatrix} G_\Lambda(E) & G_X(E) \\ G_X(E) & G_\Sigma(E) \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} U_\Lambda & U_X \\ U_X & U_\Sigma \end{pmatrix}, \quad (8)$$

and the free Green's function $\mathbf{G}^{(0)}(E)$. The diagonal parts U_Λ (U_Σ) for $\mathbf{U}$ are the Λ -nucleus (Σ -nucleus) potentials, and the off-diagonal parts U_X are the $\Sigma\Lambda$ coupling potentials. Thus the inclusive Λ spectrum in Eq. (2) can be decomposed into different physical processes [14,16] by using the identity

$$\begin{aligned} \text{Im}(F_\Sigma^\dagger G_\Sigma(E) F_\Sigma) &= F_\Sigma^\dagger \Omega^{(-)\dagger} (\text{Im} G_\Lambda^{(0)}(E)) \Omega^{(-)} F_\Sigma \\ &+ F_\Sigma^\dagger G_X^\dagger(E) (\text{Im} U_\Lambda) G_X(E) F_\Sigma \\ &+ F_\Sigma^\dagger G_\Sigma^\dagger(E) (\text{Im} U_\Sigma) G_\Sigma(E) F_\Sigma, \end{aligned} \quad (9)$$

where $\Omega^{(-)}$ is the Möller wave operator and F_Σ is the production amplitude for Σ^- . The remarkable production of ${}^6_\Lambda\text{H}$ arises from the term of $F_\Sigma^\dagger G_X^\dagger (\text{Im} U_\Lambda) G_X F_\Sigma$.

The Y -nucleus (optical) potentials for $Y = \Lambda$ or Σ^- are given by the Woods-Saxon (WS) form:

$$U_Y(r) = [V_Y + iW_Y g(E_\Lambda)] f_Y(r), \quad (10)$$

where $f_Y(r) = \{1 + \exp[(r - R)/a]\}^{-1}$. For the ${}^5\text{H}-\Lambda$ channel, we use $a = 0.60$ fm, $r_0 = 1.080 + 0.395 A^{-2/3}$ fm and $R = r_0 A_{\text{core}}^{1/3} = 2.05$ fm [30]. Considering that the ${}^5\text{H}$ nuclear core may be an unbound state or a broad resonant state [10], the strength parameters of V_Λ should be adjusted appropriately to reproduce the experimental data. The spreading imaginary potential, $\text{Im} U_Y$, can represent complicated excited states for ${}^6_\Lambda\text{H}^*$; $g(E_\Lambda)$ is assumed to be an energy-dependent function which linearly increases from 0 at $E_\Lambda = 0$ MeV to 1 at $E_\Lambda = 60$ MeV with respect to the ${}^5\text{H} + \Lambda$ threshold, as often used in nuclear optical models. For the ${}^5\text{He}-\Sigma^-$ potential, we use the WS potential with $R = 1.1 A_{\text{core}}^{1/3} = 1.88$ fm and $a = 0.67$ fm, in comparison with the data of the J-PARC E10 experiment [2]. We take the strengths of $(V_\Sigma, W_\Sigma) = (+20 \text{ MeV}, -20 \text{ MeV})$ which can fully reproduce the data in Σ^- region, leading to the reduced χ^2 value of $\chi^2/N \simeq 0.97$ [19]. The spreading potential W_Σ expresses nuclear core breakup processes caused by the $\Sigma^- p \rightarrow \Lambda n$ conversion in the ${}^5\text{He}$ nucleus, and its effect is not involved in U_X which we will mention below.

D. $\Sigma\Lambda$ coupling potentials

The $\Sigma\Lambda$ coupling potential U_X in off-diagonal parts of $\mathbf{U}$ can be obtained by a two-body $\Sigma N-\Lambda N$ potential $v_{\Sigma N, \Lambda N}^S(\mathbf{r}', \mathbf{r})$ with the spin $S = 1, 0$ isospin $I = 1/2$ state. Here we use a zero-range interaction $v_{\Sigma N, \Lambda N}^S(\mathbf{r}', \mathbf{r}) = \tilde{v}_{\Sigma N, \Lambda N}^S \delta(\mathbf{r}' - \mathbf{r})$ in a real potential for simplicity, where $\tilde{v}_{\Sigma N, \Lambda N}^S$ is the strength parameter that should be connected with volume integral $\int v_{\Sigma N, \Lambda N}^S(\mathbf{r}) d\mathbf{r} = \tilde{v}_{\Sigma N, \Lambda N}^S$. Thus the matrix elements can be easily estimated by use of Racah

algebra [31,32]:

$$\begin{aligned}
 U_X(r) &= \langle [\Phi_{J'}(^5\text{He}) \otimes \mathcal{Y}_{j'\ell's}^{(\Sigma^-)}(\hat{r})]_{J_B} | \\
 &\quad \times \frac{1}{\sqrt{3}} \sum_i v_{\Sigma N, \Lambda N}^S(\mathbf{r}'_i, \mathbf{r}) \boldsymbol{\tau}_j \cdot \boldsymbol{\phi} \\
 &\quad \times [\Phi_{J'}(^5\text{H}) \otimes \mathcal{Y}_{j\ell s}^{(\Lambda)}(\hat{r})]_{J_B} \rangle \\
 &= \sum_{LSK} \tilde{v}_{\Sigma N, \Lambda N}^S C_{LSK}^{J_B}(J' J'') \mathcal{F}_{LSK}^{J' J''}(r), \quad (11)
 \end{aligned}$$

where $\boldsymbol{\tau}_j$ denotes the j th nucleon isospin operator and $\boldsymbol{\phi}$ is defined as $|\Sigma\rangle = \boldsymbol{\phi}|\Lambda\rangle$ in isospin space [33], and $\mathcal{Y}_{j\ell s} = [Y_\ell \otimes X_{\frac{1}{2}}]_j$ is a spin-orbit function and $C_{LSK}^{J_B}(J' J'')$ is a purely geometrical factor [31]; $\mathcal{F}_{LSK}^{J' J''}(r)$ is the nuclear form factor including a one-body transition density for the $A = 5$ shell model [26] and the center-of-mass correction of a factor $\sqrt{A/(A-1)}$ [34].

Three parameters, $\tilde{v}_{\Sigma N, \Lambda N}^1$, $\tilde{v}_{\Sigma N, \Lambda N}^0$, and V_Λ , are very important for determining the Σ^- -mixing probability in ${}^6_\Lambda\text{H}$ and the production cross section of ${}^6_\Lambda\text{H}$ within the one-step mechanism [13]. These parameters are strongly connected each other for the shape of the spectrum and its magnitude. The effects of the ΣN - ΛN coupling can be evaluated by the volume integrals for ΣN - ΛN g matrices based on Nijmegen potentials [35–38], in which these values are model dependent; for example, -216.3 , -351.0 , -478.3 , and -826.6 MeV fm³ for $S = 1$ in ESC08c, ESC08a, ESC08b, and ESC08a'' potentials with $k_F = 1.0$ fm⁻¹, respectively [38]. Here we use the volume integrals calculated by the g matrix based on the D2' potential (D2'g) which can reproduce the binding energies of ${}^3_\Lambda\text{H}$, ${}^4_\Lambda\text{He}$, ${}^4_\Lambda\text{He}^*$, and ${}^5_\Lambda\text{He}$ [39], and we assume $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900$ MeV ($\tilde{v}_{\Sigma N, \Lambda N}^0 = 500$ MeV) as a standard, which corresponds to $\tilde{v}_{\Sigma N, \Lambda N}^1 = -941.2$ MeV ($\tilde{v}_{\Sigma N, \Lambda N}^0 = 513.6$ MeV) obtained by D2'g with $k_F = 1.05$ fm⁻¹. To see the dependence of the spectrum on the ΣN - ΛN coupling strength, we choose typical values of $\tilde{v}_{\Sigma N, \Lambda N}^1 = -450$, -900 , -1350 , and -1800 MeV ($\tilde{v}_{\Sigma N, \Lambda N}^0 = 250$, 500 , 750 , and 1000 MeV). Thus we attempt to determine important parameters of $\tilde{v}_{\Sigma N, \Lambda N}^S$ and V_Λ , demonstrating the calculated spectrum in comparison with the shape and magnitude of the experimental data, whereas no significant peak structure was observed near the ${}^4_\Lambda\text{H} + 2n$ threshold in the J-PARC E10 experiment.

III. RESULTS

Now let us examine the dependence of the shape and magnitude of the spectrum on $\tilde{v}_{\Sigma N, \Lambda N}^S$ and V_Λ , comparing the calculated inclusive Λ spectrum for ${}^6_\Lambda\text{H}$ with the data of the ${}^6\text{Li}(\pi^-, K^+)$ reaction at the J-PARC E10 experiment. In our calculation, we also took the energy-dependent Fermi-averaged t matrix for the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction which is essential to explain the Σ^- QF spectra of the (π^-, K^+) data on nuclear targets [17]. Therefore, it should be noticed that the following calculated spectra have reproduced the data in the Σ^- and Λ QF regions [19].

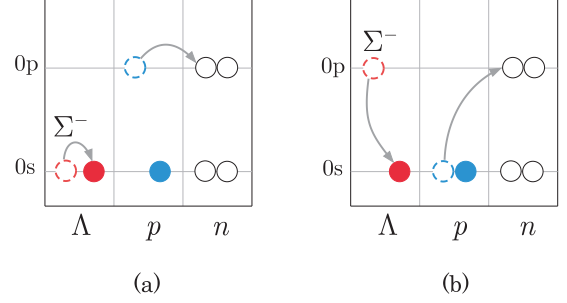


FIG. 3. Schematic illustration of shell-model configurations for (a) $p_p s_{\Sigma^-} \rightarrow p_n s_{\Lambda}$ transitions from $s_p^{-1} s_{\Sigma}$ components, and (b) $s_p p_{\Sigma^-} \rightarrow p_n s_{\Lambda}$ transitions from $p_p^{-1} p_{\Sigma}$ components in ${}^6_\Lambda\text{H}(1_{\text{exc}}^+)$.

A. Σ^- doorways

The nuclear (π^-, K^+) reaction can predominantly populate spin-stretched states of ${}^5\text{He} \otimes \Sigma^-$ doorways with $T = 3/2$ because the momentum transfer is very large ($q \simeq 359$ MeV/c around the Λ threshold) in the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction at 1.20 GeV/c [40]. It is also considered that non-spin-flip processes are dominant near the forward direction in this reaction [41]. Thus the 0^+ ground state of ${}^6_\Lambda\text{H}(0_{\text{g.s.}}^+)$ that is expected to have a large contribution by the coherent $\Lambda \Sigma$ mixing [6] is forbidden by spin-parity conservation when choosing ${}^6\text{Li}(1_{\text{g.s.}}^+)$ as a target, whereas the 1^+ excited state of ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ can be produced in the reaction. For ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ in the one-step mechanism via Σ^- doorways, we have

$$\begin{aligned}
 &{}^6\text{Li}(1_{\text{g.s.}}^+; T = 0) \\
 &\xrightarrow[\Delta L=0]{s_p \rightarrow s_{\Sigma^-}} [{}^5\text{He}(3/2_{\text{exc.}}^+, 1/2_{\text{exc.}}^+; T_c = 1/2) \otimes (s_{1/2})_{\Sigma^-}]_{1^+} \\
 &\rightleftharpoons [{}^5\text{H}(1/2_{\text{g.s.}}^+; T_c = 3/2) \otimes (s_{1/2})_{\Lambda}]_{1^+} \quad (12)
 \end{aligned}$$

in the $s_p^{-1} s_{\Sigma^-}$ configuration formed by the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction, and

$$\begin{aligned}
 &{}^6\text{Li}(1_{\text{g.s.}}^+; T = 0) \\
 &\xrightarrow[\Delta L=0,2]{p_p \rightarrow p_{\Sigma^-}} [{}^5\text{He}(3/2_{\text{g.s.}}^-, 1/2_{\text{exc.}}^-; T_c = 1/2) \otimes (p_{3/2,1/2})_{\Sigma^-}]_{1^+} \\
 &\rightleftharpoons [{}^5\text{H}(1/2_{\text{g.s.}}^+; T_c = 3/2) \otimes (s_{1/2})_{\Lambda}]_{1^+} \quad (13)
 \end{aligned}$$

in the $p_p^{-1} p_{\Sigma^-}$ configuration. Figure 3 illustrates these shell-model configurations in ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ schematically. The former process indicates the coherent $\Lambda \Sigma$ coupling with the $p_p s_{\Sigma^-} \rightarrow p_n s_{\Lambda}$ transition [7]. The latter process also contributes to ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ due to the $s_p p_{\Sigma^-} \rightarrow p_n s_{\Lambda}$ transition which induces nucleon-hole states with nuclear core-excitation in the Λ hypernucleus, as discussed in *ab initio* calculations for ${}^5_\Lambda\text{He}(1/2_{\text{g.s.}}^+)$ by Nemura *et al.* [42]. The type of this coupling is called as “incoherent” $\Lambda \Sigma$ coupling. We used single-particle wave functions for a proton in ${}^6\text{Li}(1_{\text{g.s.}}^+)$, reproducing the s -hole and p -hole energies in ${}^6\text{Li}(p, 2p)$ reactions [21].

B. ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$

Let us consider the $\Sigma \Lambda$ coupling potentials which determine the Σ^- mixing probabilities related to the production

TABLE I. Configurations for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$.

${}^5\text{H}(T = 3/2) \otimes \Lambda$		${}^5\text{He}(T = 1/2) \otimes \Sigma^-$	
J_C^π	$(\ell j)_\Lambda$	J_C^π	$(\ell j)_{\Sigma^-}$
$1/2^+$	$s_{1/2}$	$3/2^-, 1/2^-$ $1/2^+, 3/2^+$	$p_{3/2}, p_{1/2}, f_{5/2}$ $s_{1/2}, d_{5/2}, d_{3/2}$

cross section for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ in one-step mechanism. In Table I, we show configurations of the $[J_C^\pi \otimes (\ell j)_Y]_{1^+}$ state in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ composed by the $A = 5$ core nucleus with J_C^π and (ℓj) -shell hyperon. In Fig. 4, we display the calculated $\Sigma\Lambda$ coupling potentials $U_X(r)$ between $[{}^5\text{He}(J_C^\pi) \otimes (\ell j)_{\Sigma^-}]$ and $[{}^5\text{H}(1/2^+_{\text{g.s.}}) \otimes (s_{1/2})_\Lambda]$ states in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ as a function of a relative distance between ${}^5\text{H}$ (${}^5\text{He}$) and $Y = \Lambda$ (Σ^-), using the $\Sigma\Lambda$ coupling strengths of $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900$ MeV and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500$ MeV in Eq. (11); these coupling potentials are classified by the orbital angular momentum transfers $\Delta\ell$ to the hyperon in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$, where $\Delta\ell = |\ell_{\Sigma^-} - \ell_\Lambda|$. We find that the following coupling potentials are dominant:

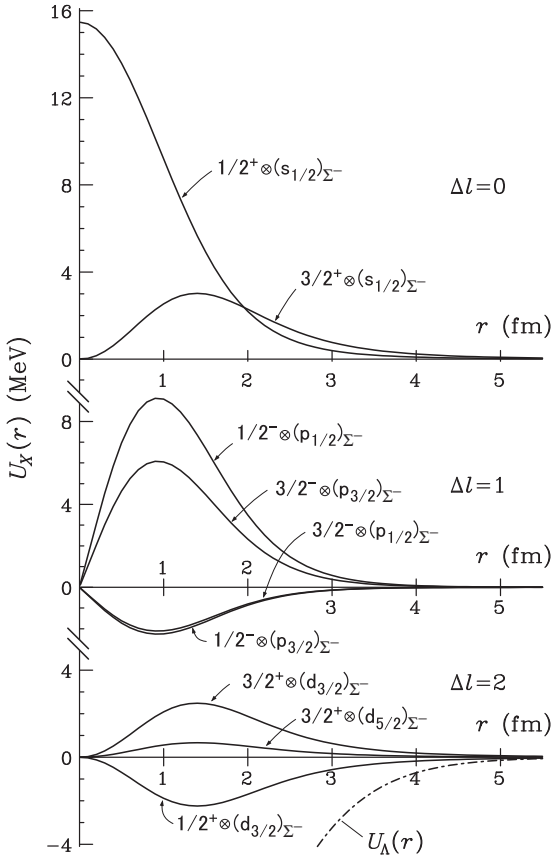


FIG. 4. Calculated $\Sigma\Lambda$ coupling potentials $U_X(r)$ between $[{}^5\text{He}(J_C^\pi) \otimes (\ell j)_{\Sigma^-}]$ and $[{}^5\text{H}(1/2^+_{\text{g.s.}}) \otimes (s_{1/2})_\Lambda]$ with $\Delta\ell = |\ell_{\Sigma^-} - \ell_\Lambda| = 0, 1, 2$ in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ at $E_\Lambda = 0$ MeV in Eq. (11), as a function of the relative distance between ${}^5\text{H}$ (${}^5\text{He}$) and Λ (Σ^-). $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900$ MeV and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500$ MeV are used. The dot-dashed curve denote the ${}^5\text{H}$ - Λ potential as a guide.

- i. $[1/2^+ \otimes (s_{1/2})_{\Sigma^-}] - [1/2^+ \otimes (s_{1/2})_\Lambda]$ for $\Delta\ell = 0$;
- ii. $[1/2^- \otimes (p_{1/2})_{\Sigma^-}] - [1/2^+ \otimes (s_{1/2})_\Lambda]$ for $\Delta\ell = 1$;
- iii. $[3/2^- \otimes (p_{3/2})_{\Sigma^-}] - [1/2^+ \otimes (s_{1/2})_\Lambda]$ for $\Delta\ell = 1$.

This nature may originate from the fact that a significant $\pi + \rho$ meson exchange related with the SU(3) coupling constant generates a $(\sigma_N \cdot \sigma_Y)(\tau_N \cdot \phi_Y)$ component in $\Lambda N - \Sigma N$ potentials, and that the nuclear form factors $\mathcal{F}_{LSK}^{JJ'}(r)$ in Eq. (11) have the collectivity of nuclear core excitations in microscopic $A = 5$ shell-model calculations. We recognize that the $s_p p_{\Sigma^-} \rightarrow p_n s_\Lambda$ transitions are significant to describe $\Lambda - \Sigma$ dynamics in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ as well as the $p_p s_{\Sigma^-} \rightarrow p_n s_\Lambda$ transitions caused by coherent $\Lambda\Sigma$ couplings, as discussed by Akaishi *et al.* [7].

To see the dependence of the spectrum on $\tilde{v}_{\Sigma N, \Lambda N}^S$, here we take $V_\Lambda = -19$ MeV for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$, whose potential gives the binding energy of $B_\Lambda({}^6_\Lambda\text{H}) = 1.492$ MeV when omitting $\tilde{v}_{\Sigma N, \Lambda N}^S$. This value of B_Λ is moderately larger than that of $B_\Lambda({}^4_\Lambda\text{H}) = 0.96 \pm 0.04$ MeV for the ${}^4_\Lambda\text{H}(1^+)$ subsystem in ${}^6_\Lambda\text{H}$. We consider single-particle wave functions for Λ , n in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ as well as those for ${}^6_\Lambda\text{H}(0^+_{\text{g.s.}})$ in which the s_Λ state has the root-mean-square radius of $\langle r^2 \rangle^{1/2} = 3.35$ fm, in comparison with $\langle r^2 \rangle^{1/2} = 4.01$ fm for valence neutrons in ${}^6_\Lambda\text{H}$. Thus the Λ, n distributions in ${}^6_\Lambda\text{H}$ simulate a similar structure to the layer distributions of single-particle t , Λ , and n densities obtained by the $tnn\Lambda$ four-body calculations [10].

1. Binding energies and Σ^- -mixing probabilities

In Table II, we show the results of the binding energies and Σ^- -mixing probabilities in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$. When we take $\tilde{v}_{\Sigma N, \Lambda N}^1 = -450, -900, -1350$, and -1800 MeV ($\tilde{v}_{\Sigma N, \Lambda N}^0 = 250, 500, 750$, and 1000 MeV), we find the Σ^- mixing probabilities of $P_{\Sigma^-} = 0.07\%, 0.32\%, 0.79\%$, and 1.58% , respectively. We stress that there appear not only s_Σ components but also p_Σ components in the Σ^- -mixing probabilities; the value of $P_{\Sigma^-}(p_\Sigma) = 0.04\% - 0.82\%$ is larger than that of $P_{\Sigma^-}(s_\Sigma) = 0.03\% - 0.68\%$. The d_Σ components are relatively small. The corresponding energy positions of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ are shifted downward by the $\Sigma\Lambda$ coupling. We obtain the energy-level shift ΔE_Λ caused by the $p_p s_{\Sigma^-} \leftrightarrow p_n s_\Lambda$ coupling in Eq. (12), e.g., $\Delta E_\Lambda \simeq -148$ keV when $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900$ MeV and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500$ MeV. This value is slightly smaller than that of ${}^9, 10_\Lambda\text{Li}$ in several microscopic shell-model calculations [43,44]. For ΔE_Λ caused by the $s_p p_{\Sigma^-} \leftrightarrow p_n s_\Lambda$ coupling in Eq. (13), we estimate $\Delta E_\Lambda \simeq -201$ keV. This effect may be often eliminated in the model space by g -matrix description, and it is not taken into account explicitly in standard calculations [43,44].

In Fig. 5, we display the density distribution of $\rho_{\alpha Y}(r)$ for $Y = \Lambda, \Sigma^-$ with $\alpha = \{n\ell j\}$ in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ when we use the $\Sigma\Lambda$ coupling potential given in Fig. 4. Thus we have $P_{\Sigma^-}(s_\Sigma) = 0.13\%$ and $P_{\Sigma^-}(p_\Sigma) = 0.17\%$, as seen in Table II. We find that the Σ^- components are located near the center of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$, e.g., the renormalized root-mean-square radius of $\langle r^2 \rangle^{1/2} = 1.47$ (1.70) fm for s_Σ (p_Σ) states, respectively, in comparison with those of $\langle r^2 \rangle^{1/2} = 1.98$ (3.03) fm for s_p (p_p) states in ${}^6_\Lambda\text{H}(1^+_{\text{g.s.}})$. This compactness of these Σ^- distributions

TABLE II. Calculated production cross sections of $d\sigma/d\Omega$ for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ by one-step mechanism in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c (7°), depending on the $\Sigma\Lambda$ coupling parameters of $\tilde{v}^S_{\Sigma N, \Lambda N}$. P_{Σ^-} is the Σ^- -mixing probability, and $B_\Lambda({}^6_\Lambda\text{H})$ and $B_{2n}({}^6_\Lambda\text{H})$ are binding energies of Λ and $2n$, respectively. $V_\Lambda = -19$ MeV is used.

$\tilde{v}^S_{\Sigma N, \Lambda N}$ (MeV)		$B_\Lambda({}^6_\Lambda\text{H})$	$B_{2n}({}^6_\Lambda\text{H})$	P_{Σ^-} (%)				$d\sigma/d\Omega$ (nb/sr)		
$S = 1$	$S = 0$	(MeV)	(MeV)	s_Σ	p_Σ	d_Σ	Total	s_p^{-1}	p_p^{-1}	Total
0	0	1.492	-3.508	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-450	250	1.576	-3.424	0.03	0.04	0.00	0.07	0.03	0.01	0.04
-900	500	1.841	-3.159	0.13	0.17	0.02	0.32	0.16	0.06	0.22
-1350	750	2.328	-2.672	0.34	0.41	0.04	0.79	0.44	0.15	0.59
-1800	1000	3.100	-1.900	0.68	0.82	0.08	1.58	1.00	0.32	1.32

may originate from the short-range nature of the $\Sigma\Lambda$ coupling potentials obtained in Eq. (11), and this nature is already seen in the *ab initio* calculation by Ref. [42].

2. Inclusive Λ spectra and cross sections

In Fig. 6, we show the calculated inclusive Λ spectrum of the ${}^6\text{Li}(\pi^-, K^+)$ reaction at $p_\pi = 1.20$ GeV/c and $\theta_{\text{lab}} = 7^\circ$, together with the data for the average cross section $\bar{\sigma}_{2^\circ-14^\circ}$, taken into account a detector resolution of 3.2 MeV FWHM. We find that the calculated spectrum below the ${}^5\text{H} + \Lambda$ threshold is rather sensitive to $\tilde{v}^S_{\Sigma N, \Lambda N}$ in the one-step mechanism, where ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ is particle unstable above the ${}^4\text{H} + 2n$ threshold. The integrated cross sections for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ account for $d\sigma/d\Omega = 0.04$ – 1.32 nb/sr for $\tilde{v}^1_{\Sigma N, \Lambda N} = (-450)$ – (-1800) MeV and $\tilde{v}^0_{\Sigma N, \Lambda N} = 250$ – 1000 MeV, as listed in Table II. We display the values of $d\sigma/d\Omega$ for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ as a bin with a finite width of 1 MeV for particle decay channels at $M_x \simeq 5806.16$ – 5804.63 MeV/ c^2 , as also shown in Fig. 6. It is remarkable that the Λ production spectra are composed of proton-hole states, s_p^{-1} and p_p^{-1} , populated by the (π^-, K^+) reactions. The value of

$d\sigma(p_p^{-1})/d\Omega = 0.01$ – 0.32 nb/sr is considerably smaller than that of $d\sigma(s_p^{-1})/d\Omega = 0.03$ – 1.00 nb/sr, whereas $P_{\Sigma^-}(p_\Sigma) = 0.04\%$ – 0.82% is larger than $P_{\Sigma^-}(s_\Sigma) = 0.03\%$ – 0.68% , as mentioned above.

IV. DISCUSSION

A. s -hole proton vs p -hole proton

To see the feasibility of producing the neutron-rich Λ hypernucleus in the one-step mechanism, we consider the contribution of the inclusive spectra via Σ^- doorways from the proton p^{-1} (s^{-1}) state on the ${}^6\text{Li}$ target. The integrated laboratory cross section may be roughly written as

$$\frac{d\sigma(j_p^{-1})}{d\Omega_L} \approx \beta |\bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-}|^2 \times S_p(j_p) |F_{\Delta L}^{(j_p \rightarrow j_\Sigma)}(q)|^2 P_{\Sigma^-}(j_\Sigma), \quad (14)$$

where $S_p(j_p)$ is a spectroscopic factor for j_p -shell proton, and $\bar{f}_{\pi^- p \rightarrow K^+ \Sigma^-}$ is a Fermi-averaged amplitude for the $\pi^- p \rightarrow K^+ \Sigma^-$ reactions. Thus we recognize the behavior of the form factor $F_{\Delta L}^{(j_p \rightarrow j_\Sigma)}(q)$ for the $j_p \rightarrow j_\Sigma$ transition with angular-momentum transfer ΔL , depending on the momentum transfer q in the (π^-, K^+) reactions. Using a harmonic-oscillator model in the plane-wave approximation [40], we can estimate

$$\frac{S_p(p_p) |F_{\Delta L=0,2}^{(p_p \rightarrow p_\Sigma)}(q)|^2}{S_p(s_p) |F_{\Delta L=0}^{(s_p \rightarrow s_\Sigma)}(q)|^2} \approx \frac{1}{2} \left[1 - \frac{1}{3}(\tilde{b}q)^2 + \frac{7}{180}(\tilde{b}q)^4 \right] \simeq 0.20 \quad (15)$$

for $q \simeq 360$ MeV/c corresponding to the Λ threshold at 1.2 GeV/c. Here we adopted $S_p(p_p)/S_p(s_p) \simeq 1/2$ for ${}^6\text{Li}$ and the oscillator radius parameter $\tilde{b} = 1.38$ fm. This $\tilde{b}$ value indicates that the Σ^- components are distributed near the center of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$. As a result, we confirm that the value of $d\sigma(p_p^{-1})/d\Omega$ is considerably smaller than that of $d\sigma(s_p^{-1})/d\Omega$, whereas $P_{\Sigma^-}(p_\Sigma)$ and $P_{\Sigma^-}(s_\Sigma)$ have almost the same value.

B. $\tilde{v}^S_{\Sigma N, \Lambda N}$ strengths

As far as $\tilde{v}^1_{\Sigma N, \Lambda N} = (0.0)$ – (-900) MeV and $\tilde{v}^0_{\Sigma N, \Lambda N} = 0.0$ – 500 MeV leading to $P_{\Sigma^-}(s_\Sigma) = 0.0\%$ – 0.13% and $P_{\Sigma^-}(p_\Sigma) = 0.0\%$ – 0.17% , therefore, the calculated spectra can fairly explain the data of the J-PARC E10 experiment. No peak structure of ${}^6_\Lambda\text{H}$ originates from the small $\Sigma\Lambda$ coupling

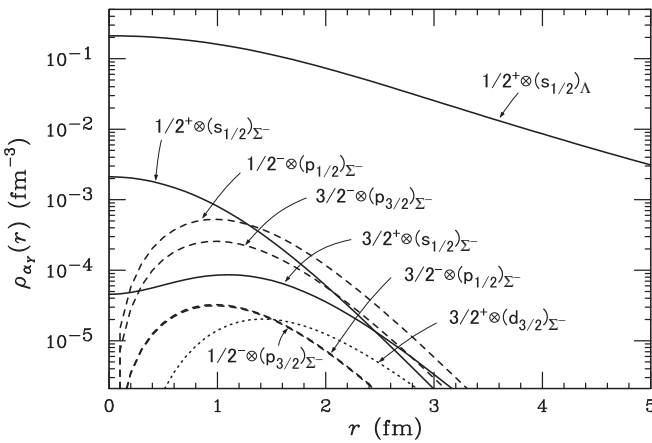


FIG. 5. Single-particle density distributions of Λ and Σ^- in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$, $\rho_{\alpha Y}(r)$ with $\alpha = \{n\ell j\}$ and $Y = \Lambda, \Sigma^-$, as a function of the relative distance between ${}^5\text{H}$ (${}^5\text{He}$) and Λ (Σ^-). The $\Sigma\Lambda$ coupling potentials given in Fig. 4 are used. Solid, dashed, and dotted curves denote the components of the hyperon densities in $(s_{1/2})_Y$, $(p_{3/2, 1/2})_Y$, and $(d_{3/2})_Y$ orbits, respectively.

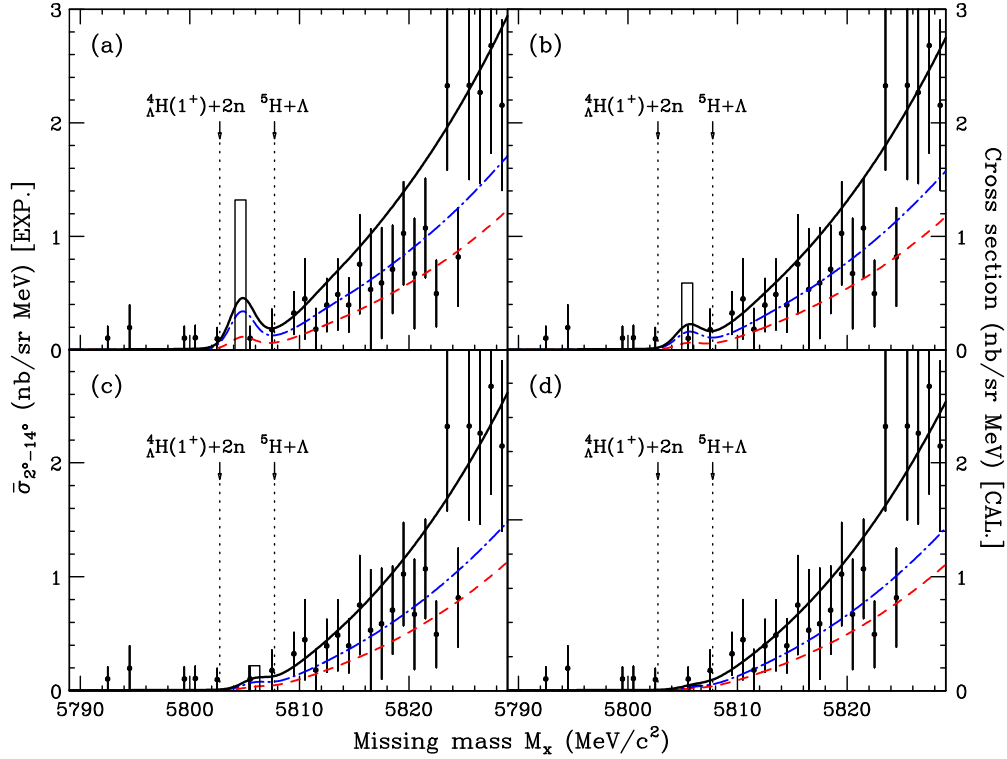


FIG. 6. Calculated missing mass spectra of the ${}^6\text{Li}(\pi^-, K^+)$ reactions near the Λ threshold at $1.2\text{ GeV}/c$ and $\theta_{\text{lab}} = 7^\circ$, with a detector resolution of 3.2 MeV FWHM . The $\Sigma\Lambda$ coupling strengths of $\tilde{v}_{\Sigma N, \Lambda N}^1 =$ (a) -1800 , (b) -1350 , (c) -900 , and (d) -450 MeV [$\tilde{v}_{\Sigma N, \Lambda N}^0 =$ (a) 1000 , (b) 750 , (c) 500 , and (d) 250 MeV] are used, together with $V_\Lambda = -19\text{ MeV}$ for the Λ - ${}^5\text{H}$ potential. Solid, dashed and dot-dashed curves denote contribution of total, p -hole, and s -hole spectra, respectively. The data are taken from Ref. [1]. The bins with a finite width of 1 MeV denote the cross sections for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ which is located in particle decay channels.

strength and also the loosely resonant Λ state in the ${}^5\text{H}$ nuclear core. Although the Σ^- -mixing probabilities for ${}^6_\Lambda\text{H}$ are very small, the sensitivity of the spectrum below the ${}^5\text{H} + \Lambda$ threshold on $\tilde{v}_{\Sigma N, \Lambda N}^s$ indicates the possibility to extract the precise Σ^- components in wave functions for ${}^6_\Lambda\text{H}$ in the nuclear (π^-, K^+) reactions. We confirm that the $\Sigma\Lambda$ coupling potential plays an essential role in the formation of the Λ hypernuclear state near the Λ threshold. Consequently, the calculated spectrum seems to be in good agreement with that of the ${}^6\text{Li}(\pi^-, K^+)$ data when we use the $\Sigma\Lambda$ coupling strengths of $\tilde{v}_{\Sigma N, \Lambda N}^1 \simeq -900\text{ MeV}$ and $\tilde{v}_{\Sigma N, \Lambda N}^0 \simeq 500\text{ MeV}$, whose values correspond to those of the volume integrals for the D2'g potential [39].

C. V_Λ strengths

On the other hand, another important parameter V_Λ for the ${}^5\text{H}$ - Λ potential also affects the binding energies and the production cross sections for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$. The energy position of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ is shifted downward by the attraction of V_Λ . We find that, when $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900\text{ MeV}$ and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500\text{ MeV}$, the binding energies are $B_\Lambda({}^6_\Lambda\text{H}) = 0.050, 1.841, 3.726$ and 5.493 MeV for $V_\Lambda = -11, -19, -24$, and -28 MeV , respectively, so that the Σ^- -mixing probabilities amount to $P_{\Sigma^-} = 0.07\%$, 0.32% , 0.38% , and 0.40% . In Fig. 7, we show the dependence of the inclusive Λ spectrum for the ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ production on these values of V_Λ when $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900\text{ MeV}$ and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500\text{ MeV}$.

500 MeV. We show that the calculated spectrum for ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ is considerably changed by the value of V_Λ , where the integrated cross sections of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ become $d\sigma/d\Omega = 0.04, 0.22, 0.34$

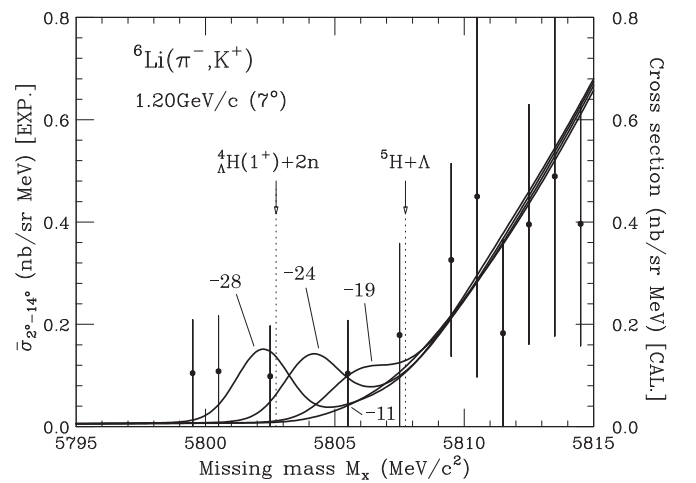


FIG. 7. Dependence of the calculated inclusive Λ spectrum in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at $p_{\pi^-} = 1.2\text{ GeV}/c$ (7°) on various strengths of V_Λ , together with the experimental data [1]. Solid curves denote the spectra by $V_\Lambda = -28, -24, -19$, and -11 MeV when $\tilde{v}_{\Sigma N, \Lambda N}^1 = -900\text{ MeV}$ and $\tilde{v}_{\Sigma N, \Lambda N}^0 = 500\text{ MeV}$ with a detector resolution of 3.2 MeV FWHM .

and 0.41 nb/sr for $V_\Lambda = -11, -19, -24$, and -28 MeV, respectively. The calculated spectra with $V_\Lambda = (-24)-(-28)$ MeV seem to disagree with the data of no peak structure below the ${}^5\text{H} + \Lambda$ threshold. This fact may indicate that the ${}^5\text{H}-\Lambda$ potential is quite shallow in comparison with the Λ -nucleus potentials which are well known as $V_\Lambda \simeq -28$ MeV in ordinary nuclei [30], and the neutron-rich nuclear core ${}^5\text{H}$ should be an unbound or broad resonant state.

D. V_Σ and W_Σ strengths

As discussed above, we recognize that the calculated spectrum is in good agreement with that of the ${}^6\text{Li}(\pi^-, K^+)$ data when we use the $\Sigma\Lambda$ coupling strengths of $\tilde{v}_{\Sigma N, \Lambda N}^1 \simeq -900$ MeV and $\tilde{v}_{\Sigma N, \Lambda N}^0 \simeq 500$ MeV, together with $V_\Sigma \simeq (+20)-(+30)$ MeV and $W_\Sigma \simeq -20$ MeV for the ${}^5\text{He}-\Sigma^-$ potential [19]. The nature of the repulsive component in this potential is consistent with that in the Σ -nucleus potential obtained on heavier targets [17]. The calculated spectrum fully explains the data in the Σ^- and Λ QF regions by the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling.

E. ${}^5\text{H}(1/2_{\text{g.s.}}^+)$ resonant state

Current experiments have reported that the ${}^5\text{H}$ ground state is located at $E_r = 1.7 \pm 0.3$ MeV with $\Gamma = 1.9 \pm 0.4$ MeV above the ${}^3\text{H} + 2n$ threshold [29], or at $E_r = 5.5 \pm 0.2$ MeV with $\Gamma = 5.4 \pm 0.6$ MeV [45]. The problem of whether the ${}^5\text{H}(1/2_{\text{g.s.}}^+)$ ground state exists as a narrow resonant state with $E_r = 1.7$ MeV and $\Gamma = 1.9$ MeV may still be unsettled [22,28]. Several theoretical investigations [22,25] suggest the energy of the ${}^5\text{H}$ ground state with $E_r \simeq 1.6-3.0$ MeV, $\Gamma \simeq 1.5-4.0$ MeV in tnn three-body calculations [25] and $E_r \simeq 3.0-4.5$ MeV in the shell-model calculations with $spsd$ space [26,27]. It is expected that the $\Sigma\Lambda$ coupling matrix elements work reasonably within the shell-model description. In the viewpoint of shell-model calculations, we assume that the ${}^5\text{H}(1/2_{\text{g.s.}}^+)$ nuclear core is a resonant state with $E_r = 4.0$ MeV, rather than that with $E_r = 1.7$ MeV; if we have $E_r = 1.7$ MeV in the shell models, we would need to artificially add an extreme attraction to the ${}^5\text{H}$ system, e.g., by three-nucleon forces [10]. To see effects of the energy of the ${}^5\text{H} + \Lambda$ threshold on ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$, we calculate the inclusive spectrum near the Λ threshold, changing the energy of the ${}^5\text{H}(1/2_{\text{g.s.}}^+)$ resonant state. In Fig. 8, we show the dependence of the inclusive Λ spectrum for ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ near the Λ threshold, using $E_r = 4.0$ MeV and 1.7 MeV which determine the position of the ${}^5\text{H} + \Lambda$ threshold. We recognize that the shape of the calculated spectrum for ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ is considerably changed by the value of E_r which depends on whether ${}^5\text{H}$ is a narrow resonant state. The structure of ${}^5\text{H}$ may influence the scenario of production of ${}^6_\Lambda\text{H}$ at FINUDA [5]. Thus the spectrum near the Λ threshold provides the ability to study the structure of the ${}^5\text{H}$ core nucleus in detailed comparison with the precise data, as well as the structure of ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$.

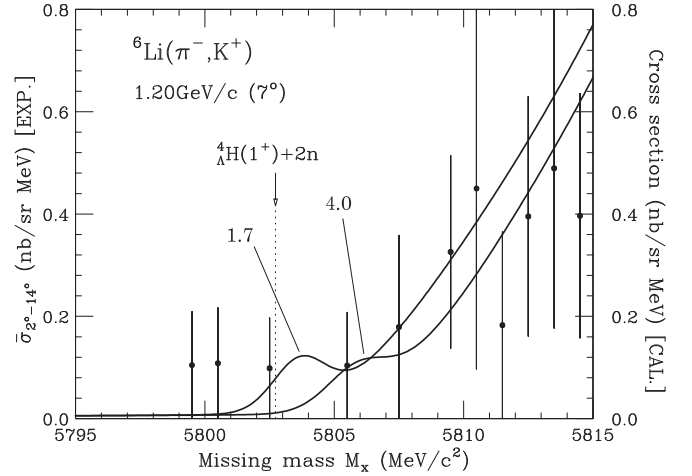


FIG. 8. Comparison between the calculated inclusive Λ spectrum of $E_r = 4.0$ MeV and that of $E_r = 1.7$ MeV for the energy of ${}^5\text{H}(1/2_{\text{g.s.}}^+)$ in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at $p_\pi = 1.2$ GeV/c (7°). $V_\Lambda = -19$ MeV is used. See also the caption in Fig. 7.

F. Finite range

To clarify the one-step mechanism for production of the neutron-rich Λ hypernucleus, we obtained the $\Sigma\Lambda$ coupling potential constructed by the zero-range two-body interaction for simplicity, using the WS form for diagonal potentials in ${}^5\text{H} + \Lambda$ and ${}^5\text{He} + \Sigma^-$ channels. On the other hand, it is known that a finite range of the two-body interaction provides modified nuclear potentials [31]. To see effects of the finite range of the interaction, we have a Gaussian shape, $v_{\Sigma N, \Lambda N}^S(\mathbf{r}', \mathbf{r}) = v_{\Sigma N, \Lambda N}^S(\text{FR}) \exp(-|\mathbf{r}' - \mathbf{r}|^2/\beta^2)$, where β is a range parameter. Here we choose $v_{\Sigma N, \Lambda N}^S(\text{FR}) = -369.4$ MeV and $v_{\Sigma N, \Lambda N}^0(\text{FR}) = 205.2$ MeV for $\beta = 0.8$ fm; these strength parameters correspond to a spin-averaged ΛN strength of $\bar{v}_{\Lambda N}(\text{FR}) = -105.9$ MeV with $\beta = 0.8$ fm, which reproduce the Λ binding energies for light p -shell nuclei. In the folding potential model, we realize that the radial shape of the $\Sigma\Lambda$ coupling potential $U_X(r)$ is more smoothly behaved and the range of $U_X(r)$ becomes slightly extended. Thus we find that the Σ^- -mixing probabilities for ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ account for $P_{\Sigma^-}(s_\Sigma) = 0.13\%$ and $P_{\Sigma^-}(p_\Sigma) = 0.11\%$ in comparison with 0.13% and 0.17% shown in Table II. The integrated cross section for ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ is $d\sigma/d\Omega = 0.17$ nb/sr and the (π^-, K^+) spectrum is not so modified. It seems that a value of $P_{\Sigma^-}(p_\Sigma)$ is relatively reduced whereas $P_{\Sigma^-}(s_\Sigma)$ is not changed. This modification may depend on nuclear structures of the ${}^5\text{H}$ and ${}^5\text{He}$ core states as well as properties of the two-body ΛN , ΣN and $\Lambda N-\Sigma N$ effective interactions. Therefore, more investigation is needed to qualitatively clarify nuclear dynamics by sophisticated microscopic calculations.

G. Two-step processes of $\pi^- p \rightarrow K^0 \Lambda$ followed by $K^0 p \rightarrow K^+ n$

Finally we discuss the integrated laboratory cross sections of $d\sigma/d\Omega$ for ${}^6_\Lambda\text{H}(1_{\text{exc.}}^+)$ by the two-step mechanism, $\pi^- p \rightarrow K^0 \Lambda$ followed by $K^0 p \rightarrow K^+ n$ or $\pi^- p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+ \Lambda$ in the DCX ${}^6\text{Li}(\pi^-, K^+)$ reaction for production of the neutron-rich Λ hypernuclei [13]. We

roughly estimate the contribution of the two-step processes for $\pi^- p \rightarrow K^0 n$ followed by $K^0 p \rightarrow K^+ \Lambda$, which are expected to be a main component, rather than those for $\pi^- p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+ \Lambda$. The sum of the cross sections by a harmonic oscillator model [46] for ${}^6\text{Li}$ targets at $p_{\pi^-} = 1.2$ GeV/c (0°) is given as

$$\sum_f \left(\frac{d\sigma_f^{(2)}}{d\Omega_L} \right)_{0^\circ} \approx \frac{2\pi\xi}{p_K^2} \left\langle \frac{1}{r^2} \right\rangle \left(\alpha \frac{d\sigma}{d\Omega_L} \right)_{0^\circ}^{\pi^- p \rightarrow K^0 \Lambda} \times \left(\alpha \frac{d\sigma}{d\Omega_L} \right)_{0^\circ}^{K^0 p \rightarrow K^+ n} N_{\text{eff}}^{pp}, \quad (16)$$

where $\xi = 0.0370$ mb $^{-1}$ is the factor integrated over angle $\theta_{\text{lab}}^{(K^0)}$ for $\pi^- p \rightarrow K^0 \Lambda$ with $-\theta_{\text{lab}}^{(K^+)}$ for $K^0 p \rightarrow K^+ n$ to restore $\theta_{\text{lab}} = 0^\circ$ in the angular distributions of the two elementary processes, $p_K \simeq 0.842$ GeV/c is the intermediate kaon momentum, and $\langle 1/r^2 \rangle \simeq 0.0280$ mb $^{-1}$ is the mean inverse-square radial separation of the proton pair. $N_{\text{eff}}^{pp} \simeq 1$ is the effective number of proton pairs including the nuclear distortion effects. The elementary laboratory cross section $(\alpha d\sigma/d\Omega_L)_{0^\circ}$ is estimated to be ~ 0.35 mb/sr for $\pi^- p \rightarrow K^0 \Lambda$ or ~ 1.96 mb/sr for $K^0 p \rightarrow K^+ n$, depending on the nuclear medium corrections. The results show $\sum_f (d\sigma_f^{(2)}/d\Omega_L)_{0^\circ} \simeq 1.4\text{--}1.9$ $\mu\text{b/sr}$ for $\pi^- p \rightarrow K^0 n$ followed by $K^0 p \rightarrow K^+ \Lambda$, and also $0.20\text{--}0.34$ $\mu\text{b/sr}$ for $\pi^- p \rightarrow \pi^0 n$ followed by $\pi^0 p \rightarrow K^+ \Lambda$. Considering the large momentum transfer $q \simeq 360$ MeV/c in the (π^-, K^+) reactions, we expect that the production probabilities for loosely bound or resonant Λ states do not exceed $10^{-3}\%$ in the quasielastic Λn production, so that the cross section of ${}^6_\Lambda\text{H}$ in the two-step mechanism may be on the order of 10^{-2} nb/sr at $\theta_{\text{lab}} = 7^\circ$. This result suggests that the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling is rather favored than the two-step mechanism.

V. SUMMARY AND CONCLUSION

We studied phenomenologically the production of a neutron-rich hypernucleus ${}^6_\Lambda\text{H}$ in the ${}^6\text{Li}(\pi^-, K^+)$ reaction at 1.2 GeV/c, considering the DWIA in the one-step mechanism, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways caused by $\Sigma^- p \leftrightarrow \Lambda n$ coupling. We evaluated the production cross section of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ by using the coupled $({}^5\text{H}-\Lambda) + ({}^5\text{He}-\Sigma^-)$ model

with a spreading potential and compared it with the data of the missing mass spectrum at the J-PARC E10 experiment. The results are summarized as follows:

- (i) The Σ^- -mixing probabilities in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ are $P_{\Sigma^-} \lesssim 0.2\%$ both for s_Σ state and for p_Σ state in order to reproduce no significant peak in the Λ production data, so that the cross section of ${}^6_\Lambda\text{H}$ is less than on the order of 0.4 nb/sr.
- (ii) The shape and magnitude of the near- Λ -threshold spectrum significantly depend on the $\Sigma\Lambda$ coupling and Λ potentials.
- (iii) The cross section of ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ is also sensitive to the structure of the ${}^5\text{H}$ core nucleus independent of whether the ${}^5\text{H}(1/2^+_{\text{g.s.}})$ ground state exists as a resonant state bound with a narrow width.
- (iv) The one-step mechanism via Σ^- doorways seems to be rather favored over the two-step mechanism because the cross section of ${}^6_\Lambda\text{H}$ in the two-step mechanism may be on the order of 10^{-2} nb/sr at $\theta_{\text{lab}} = 7^\circ$ by the harmonic-oscillator model.

In conclusion, the calculated spectrum of the ${}^6_\Lambda\text{H}$ hypernucleus by the one-step mechanism via Σ^- doorways can evaluate the near- Λ -threshold data of the DCX ${}^6\text{Li}(\pi^-, K^+)$ reaction at 1.20 GeV/c. The result shows that the Σ^- -mixing probabilities in ${}^6_\Lambda\text{H}(1^+_{\text{exc.}})$ are $P_{\Sigma^-} \lesssim 0.2\%$ both for s_Σ state and for p_Σ state in order to explain no significant peak in the Λ production spectrum obtained at the J-PARC E10 experiment. The sensitivity to the potential parameters implies that the nuclear (π^-, K^+) reactions with much less background experimentally provide the high ability to study precise wave functions for Λ , Σ^- and the ${}^5\text{H}$ nuclear core in the neutron-rich Λ hypernucleus. Systematic analysis based on microscopic calculations is required for the extended J-PARC E10 experiment [47]. This investigation is in progress.

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