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Supplementary Information

The promotion of cooperation by the poor in dynamic chicken games

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Supporting Information

Derivation of the convergence value of each p^*

1. Derivation of Equation (11) for the hawk-dove game

Here, we calculate the convergence value of p^* (Equation (10)):

$$p^* = \frac{2 \log\left(\frac{V/2 + w}{V + w}\right)}{2 \log\left(\frac{V/2 + w}{w}\right) + \log\left(\frac{-C + w}{V + w}\right)} \quad (10)$$

Equation (10) is rewritten as follows:

$$\begin{aligned} p^* &= \frac{2 \log\left(\frac{V/2 + w}{w}\right) - 2 \log\left(\frac{V + w}{w}\right)}{2 \log\left(\frac{V/2 + w}{w}\right) + \log\left(\frac{-C + w}{w}\right) - \log\left(\frac{V + w}{w}\right)} \\ &= \frac{2 \log\left(\frac{V/2}{w} + 1\right) - 2 \log\left(\frac{V}{w} + 1\right)}{2 \log\left(\frac{V/2}{w} + 1\right) + \log\left(\frac{-C}{w} + 1\right) - \log\left(\frac{V}{w} + 1\right)} \quad (S1) \end{aligned}$$

Calculate the limit of p^* (Equation (S1)) as w approaches infinity:

$$\begin{aligned} \lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{2 \log\left(\frac{V/2}{w} + 1\right) - 2 \log\left(\frac{V}{w} + 1\right)}{2 \log\left(\frac{V/2}{w} + 1\right) + \log\left(\frac{-C}{w} + 1\right) - \log\left(\frac{V}{w} + 1\right)} \\ &= \frac{2 \log(0 + 1) - 2 \log(0 + 1)}{2 \log(0 + 1) + \log(0 + 1) - \log(0 + 1)} = \frac{0}{0} \quad (S2) \end{aligned}$$

The answer to Equation (S2) is an indeterminate form. Therefore, we apply l'Hôpital's rule to Equation (S1):

$$\begin{aligned}
\lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{2 \log\left(\frac{V/2 + w}{w}\right) - 2 \log\left(\frac{V + w}{w}\right)}{2 \log\left(\frac{V/2 + w}{w}\right) + \log\left(\frac{-C + w}{w}\right) - \log\left(\frac{V + w}{w}\right)} \\
&= \lim_{w \rightarrow \infty} \frac{\frac{d}{dw} \left\{ 2 \log\left(\frac{V/2 + w}{w}\right) - 2 \log\left(\frac{V + w}{w}\right) \right\}}{\frac{d}{dw} \left\{ 2 \log\left(\frac{V/2 + w}{w}\right) + \log\left(\frac{-C + w}{w}\right) - \log\left(\frac{V + w}{w}\right) \right\}} \\
&= \lim_{w \rightarrow \infty} \frac{\frac{-V}{V/2 + w} + \frac{2V}{V + w}}{\frac{-V}{V/2 + w} + \frac{C}{-C + w} + \frac{V}{V + w}} \quad (S3)
\end{aligned}$$

Multiply both the denominator and numerator of Equation (S3) by $(V/2+w)^2 (V+w)^2 (-C+w)$:

$$\begin{aligned}
\lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{\left(\frac{V}{2} + w\right)^2 (V + w)^2 (-C + w) \left(\frac{-V}{V/2 + w} + \frac{2V}{V + w}\right)}{\left(\frac{V}{2} + w\right)^2 (V + w)^2 (-C + w) \left(\frac{-V}{V/2 + w} + \frac{C}{-C + w} + \frac{V}{V + w}\right)} \\
&= \lim_{w \rightarrow \infty} \frac{-V \left(\frac{V}{2} + w\right) (V + w)^2 (-C + w) + 2V \left(\frac{V}{2} + w\right)^2 (V + w) (-C + w)}{-V \left(\frac{V}{2} + w\right) (V + w)^2 (-C + w) + C \left(\frac{V}{2} + w\right)^2 (V + w)^2 + V \left(\frac{V}{2} + w\right)^2 (V + w) (-C + w)} \quad (S4)
\end{aligned}$$

Divide both the denominator and numerator of Equation (S4) by w^4 :

$$\begin{aligned}
\lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{-V \left(\frac{V}{2w} + 1\right) \left(\frac{V}{w} + 1\right)^2 \left(\frac{-C}{w} + 1\right) + 2V \left(\frac{V}{2w} + 1\right)^2 \left(\frac{V}{w} + 1\right) \left(\frac{-C}{w} + 1\right)}{-V \left(\frac{V}{2w} + 1\right) \left(\frac{V}{w} + 1\right)^2 \left(\frac{-C}{w} + 1\right) + C \left(\frac{V}{2w} + 1\right)^2 \left(\frac{V}{w} + 1\right)^2 + V \left(\frac{V}{2w} + 1\right)^2 \left(\frac{V}{w} + 1\right) \left(\frac{-C}{w} + 1\right)} \\
&= \frac{-V + 2V}{-V + C + V} \\
&= \frac{V}{C} = p_{org}^* \quad (S5)
\end{aligned}$$

2. The snowdrift game (Eq. (15))

Here, we calculate the convergence value of p^* (Equation (14)):

$$p^* = \frac{\log\left(\frac{b+w}{b-\frac{c}{2}+w}\right)}{\log\left\{\frac{(b-c+w)(b+w)}{w(b-\frac{c}{2}+w)}\right\}} \quad (14)$$

Equation (14) is rewritten as follows:

$$\begin{aligned} p^* &= \frac{\log\left(\frac{b+w}{w}\right) - \log\left(\frac{b-\frac{c}{2}+w}{w}\right)}{\log\left(\frac{b-c+w}{w}\right) + \log\left(\frac{b+w}{w}\right) - \log\left(\frac{b-\frac{c}{2}+w}{w}\right)} \\ &= \frac{\log\left(\frac{b}{w} + 1\right) - \log\left(\frac{b-\frac{c}{2}}{w} + 1\right)}{\log\left(\frac{b-c}{w} + 1\right) + \log\left(\frac{b}{w} + 1\right) - \log\left(\frac{b-\frac{c}{2}}{w} + 1\right)} \quad (S6) \end{aligned}$$

Calculate the limit of p^* (Equation (S6)) as w approaches infinity:

$$\begin{aligned} \lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{\log\left(\frac{b}{w} + 1\right) - \log\left(\frac{b-\frac{c}{2}}{w} + 1\right)}{\log\left(\frac{b-c}{w} + 1\right) + \log\left(\frac{b}{w} + 1\right) - \log\left(\frac{b-\frac{c}{2}}{w} + 1\right)} \\ &= \frac{\log(0 + 1) - \log(0 + 1)}{\log(0 + 1) + \log(0 + 1) - \log(0 + 1)} = \frac{0}{0} \quad (S7) \end{aligned}$$

The answer to Equation (S7) is an indeterminate form. Therefore, we apply l'Hôpital's rule to Equation (S6):

$$\lim_{w \rightarrow \infty} p^* = \lim_{w \rightarrow \infty} \frac{\log\left(\frac{b+w}{w}\right) - \log\left(\frac{b-\frac{c}{2}+w}{w}\right)}{\log\left(\frac{b-c+w}{w}\right) + \log\left(\frac{b+w}{w}\right) - \log\left(\frac{b-\frac{c}{2}+w}{w}\right)}$$

$$\begin{aligned}
&= \lim_{w \rightarrow \infty} \frac{\frac{d}{dw} \left\{ \log \left(\frac{b+w}{w} \right) - \log \left(\frac{b - \frac{c}{2} + w}{w} \right) \right\}}{\frac{d}{dw} \left\{ \log \left(\frac{b-c+w}{w} \right) + \log \left(\frac{b+w}{w} \right) - \log \left(\frac{b - \frac{c}{2} + w}{w} \right) \right\}} \\
&= \lim_{w \rightarrow \infty} \frac{\frac{-b}{b+w} + \frac{b - \frac{c}{2}}{b - \frac{c}{2} + w}}{\frac{-b+c}{b-c+w} - \frac{b}{b+w} + \frac{b - \frac{c}{2}}{b - \frac{c}{2} + w}} \quad (S8)
\end{aligned}$$

Multiply both the denominator and numerator of Equation (S8) by $(b+w)^2 (b-c/2+w)^2 (b-c+w)$:

$$\begin{aligned}
\lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{(b+w)^2 \left(b - \frac{c}{2} + w\right)^2 (b-c+w) \left(\frac{-b}{b+w} + \frac{b - \frac{c}{2}}{b - \frac{c}{2} + w} \right)}{(b+w)^2 \left(b - \frac{c}{2} + w\right)^2 (b-c+w) \left(\frac{-b+c}{b-c+w} - \frac{b}{b+w} + \frac{b - \frac{c}{2}}{b - \frac{c}{2} + w} \right)} \\
&= \lim_{w \rightarrow \infty} \frac{-b(b+w) \left(b - \frac{c}{2} + w\right)^2 (b-c+w) + (b - \frac{c}{2})(b+w)^2 \left(b - \frac{c}{2} + w\right) (b-c+w)}{(-b+c)(b+w)^2 \left(b - \frac{c}{2} + w\right)^2 - b(b+w) \left(b - \frac{c}{2} + w\right)^2 (b-c+w) + (b - \frac{c}{2})(b+w)^2 \left(b - \frac{c}{2} + w\right) (b-c+w)} \quad (S9)
\end{aligned}$$

Divide both the denominator and numerator of Equation (S9) by w^4 :

$$\begin{aligned}
&\lim_{w \rightarrow \infty} p^* \\
&= \lim_{w \rightarrow \infty} \frac{-b \left(\frac{b}{w} + 1 \right) \left(\frac{b - \frac{c}{2}}{w} + 1 \right)^2 \left(\frac{b-c}{w} + 1 \right) + (b - \frac{c}{2}) \left(\frac{b}{w} + 1 \right)^2 \left(\frac{b - \frac{c}{2}}{w} + 1 \right) \left(\frac{b-c}{w} + 1 \right)}{(-b+c) \left(\frac{b}{w} + 1 \right)^2 \left(\frac{b - \frac{c}{2}}{w} + 1 \right)^2 - b \left(\frac{b}{w} + 1 \right) \left(\frac{b - \frac{c}{2}}{w} + 1 \right)^2 \left(\frac{b-c}{w} + 1 \right) + (b - \frac{c}{2}) \left(\frac{b}{w} + 1 \right)^2 \left(\frac{b - \frac{c}{2}}{w} + 1 \right) \left(\frac{b-c}{w} + 1 \right)} \\
&= \frac{-b + b - \frac{c}{2}}{-b + c - b + b - \frac{c}{2}} \\
&= \frac{c}{2b - c} = p_{org}^* \quad (S10)
\end{aligned}$$

3. The prisoner's dilemma and stag hunt games (Equations (19) and (23))

Here, we calculate the convergence value of p^* (Equations (18) and (22)):

$$p^* = \frac{\log\left(\frac{d+w}{b+w}\right)}{\log\left[\frac{(a+w)(d+w)}{(b+w)(c+w)}\right]} \quad (18) \text{ and } (22)$$

Equations (18) and (22) are rewritten as follows:

$$\begin{aligned} p^* &= \frac{\log\left(\frac{d+w}{w}\right) - \log\left(\frac{b+w}{w}\right)}{\log\left(\frac{a+w}{w}\right) - \log\left(\frac{b+w}{w}\right) - \log\left(\frac{c+w}{w}\right) + \log\left(\frac{d+w}{w}\right)} \\ &= \frac{\log\left(\frac{d}{w} + 1\right) - \log\left(\frac{b}{w} + 1\right)}{\log\left(\frac{a}{w} + 1\right) - \log\left(\frac{b}{w} + 1\right) - \log\left(\frac{c}{w} + 1\right) + \log\left(\frac{d}{w} + 1\right)} \end{aligned} \quad (S11)$$

Calculate the limit of p^* (Equation (S11)) as w approaches infinity:

$$\begin{aligned} \lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{\log\left(\frac{d}{w} + 1\right) - \log\left(\frac{b}{w} + 1\right)}{\log\left(\frac{a}{w} + 1\right) - \log\left(\frac{b}{w} + 1\right) - \log\left(\frac{c}{w} + 1\right) + \log\left(\frac{d}{w} + 1\right)} \\ &= \frac{\log(0 + 1) - \log(0 + 1)}{\log(0 + 1) - \log(0 + 1) - \log(0 + 1) + \log(0 + 1)} = \frac{0}{0} \end{aligned} \quad (S12)$$

The answer to Equation (S12) is an indeterminate form. Therefore, we apply l'Hôpital's rule to Equation (S11):

$$\begin{aligned} \lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{\log\left(\frac{d+w}{w}\right) - \log\left(\frac{b+w}{w}\right)}{\log\left(\frac{a+w}{w}\right) - \log\left(\frac{b+w}{w}\right) - \log\left(\frac{c+w}{w}\right) + \log\left(\frac{d+w}{w}\right)} \\ &= \lim_{w \rightarrow \infty} \frac{\frac{d}{dw} \left\{ \log\left(\frac{d+w}{w}\right) - \log\left(\frac{b+w}{w}\right) \right\}}{\frac{d}{dw} \left\{ \log\left(\frac{a+w}{w}\right) - \log\left(\frac{b+w}{w}\right) - \log\left(\frac{c+w}{w}\right) + \log\left(\frac{d+w}{w}\right) \right\}} \end{aligned}$$

$$= \lim_{w \rightarrow \infty} \frac{\frac{-d}{d+w} + \frac{b}{b+w}}{\frac{-a}{a+w} + \frac{b}{b+w} + \frac{c}{c+w} - \frac{d}{d+w}} \quad (S13)$$

Multiply both the denominator and numerator of Equation (S13) by $(a+w)(b+w)(c+w)(d+w)$:

$$\begin{aligned} \lim_{w \rightarrow \infty} p^* &= \lim_{w \rightarrow \infty} \frac{(a+w)(b+w)(c+w)(d+w) \left(\frac{-d}{d+w} + \frac{b}{b+w} \right)}{(a+w)(b+w)(c+w)(d+w) \left(\frac{-a}{a+w} + \frac{b}{b+w} + \frac{c}{c+w} - \frac{d}{d+w} \right)} \\ &= \lim_{w \rightarrow \infty} \frac{-d(a+w)(b+w)(c+w) + b(a+w)(c+w)(d+w)}{-a(b+w)(c+w)(d+w) + b(a+w)(c+w)(d+w) + c(a+w)(b+w)(d+w) - d(a+w)(b+w)(c+w)} \quad (S14) \end{aligned}$$

Divide both the denominator and numerator of Equation (S14) by w^3 :

$$\begin{aligned} &\lim_{w \rightarrow \infty} p^* \\ &= \lim_{w \rightarrow \infty} \frac{-d \left(\frac{a}{w} + 1 \right) \left(\frac{b}{w} + 1 \right) \left(\frac{c}{w} + 1 \right) + b \left(\frac{a}{w} + 1 \right) \left(\frac{c}{w} + 1 \right) \left(\frac{d}{w} + 1 \right)}{-a \left(\frac{b}{w} + 1 \right) \left(\frac{c}{w} + 1 \right) \left(\frac{d}{w} + 1 \right) + b \left(\frac{a}{w} + 1 \right) \left(\frac{c}{w} + 1 \right) \left(\frac{d}{w} + 1 \right) + c \left(\frac{a}{w} + 1 \right) \left(\frac{b}{w} + 1 \right) \left(\frac{d}{w} + 1 \right) - d \left(\frac{a}{w} + 1 \right) \left(\frac{b}{w} + 1 \right) \left(\frac{c}{w} + 1 \right)} \\ &= \frac{-d + b}{-a + b + c - d} \\ &= \frac{-b + d}{a - b - c + d} = p_{org}^* \quad (S15) \end{aligned}$$