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Title	A new approach to evaluate effective stress coefficient for strength in Kimachi sandstone
Author(s)	Dassanayake, A.B.N.; Fujii, Yoshiaki; Fukuda, Daisuke et al.
Citation	Journal of Petroleum Science and Engineering, 131, 70-79 https://doi.org/10.1016/j.petrol.2015.04.015
Issue Date	2015-07
Doc URL	https://hdl.handle.net/2115/66339
Rights(URL)	https://creativecommons.org/licenses/by-nc-nd/4.0/
Type	journal article
File Information	JPSE.131(2015).70-79.pdf



A new approach to evaluate effective stress coefficient for strength in Kimachi sandstone

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ABSTRACT

This paper is devoted to experimental investigation of effective stress coefficient for peak and residual strengths of saturated Kimachi sandstone. Authors have described the Modified Failure Envelope Method (MFEM), which can be used to obtain the effective stress coefficients for peak and residual strengths (α_{Peak} and α_{Residual}). The effective stress coefficients for intact and fractured Kimachi sandstone ($\alpha_{\text{Biot's}}$ and $\alpha_{\text{Fractured}}$) were also evaluated using conventional methods, and the data were compared with the coefficient values obtained by MFEM for the peak and residual strengths. The effective stress coefficient for intact rock, $\alpha_{\text{Biot's}}$ decreased with increasing confining pressure, and was in the range $1 > \alpha_{\text{Biot's}} > 0.8$. The effective stress coefficient for fractured rock, $\alpha_{\text{Fractured}}$ was larger than that for intact rock, and was close to unity. The effective stress coefficient calculated for peak strengths, α_{Peak} using both the single and multistage MFEMs decreased with increasing effective confining pressure and was in the range $0.8 > \alpha_{\text{Peak}} > 0.4$. For residual strength states, effective stress coefficient, α_{Residual} was between the peak strength value and that for intact rock. Based on the results, multistage MFEM is suitable for obtaining an effective stress coefficient for the peak strength, α_{Peak} . An equation to obtain the effective stress coefficient from total confining pressure and pore pressure, and a method to choose the coefficients for elastic stress analyses and failure evaluations for intact rock structures or structures in rock mass were proposed.

Key words: Effective stress coefficient, Modified Failure Envelope Method, peak strength, residual strength, bulk modulus

1. Introduction

The determination of effective stress is an important aspect of stability analysis, and has a number of engineering applications especially in geotechnical and petroleum engineering, including underground and surface rock structures, oil and gas production, and sequestration of carbon dioxide (Alam et al., 2009; Alam et al., 2010; Hu et al., 2010). Failure criteria are represented using the effective stress, the concept of which was first introduced by Terzaghi (1936) for soil, and is commonly known as Terzaghi's effective stress principle. It states that the effect of the total stress σ and the pore pressure P_p can be described using a single parameter, which is known as the effective stress σ' , and is defined as follows:

$$\sigma' = \sigma - P_p. \quad (1)$$

In general, Terzaghi's effective stress is not always valid for fluid-related rocks. Therefore, effective stress coefficient was proposed by Biot (1941) to modify the effective stress principle, giving

$$\sigma' = \sigma - \alpha P_p, \quad (2)$$

where α is the effective stress coefficient, a key parameter that quantifies the contribution of the pore pressure to the effective stress. For granular soil, the contact area between grains is typically very small; thus, it is possible to assume that a cross section will be occupied mostly by the fluid. For this reason, the corresponding effective stress coefficient can be approximated to $\alpha = 1$. In porous rock with significant cementation, the grain–grain contact will be considerably larger, and it is not possible to assume that the cross section is mostly occupied by the fluid. Consequently the corresponding effective stress coefficient will be less than 1 (Zhang et al., 2009).

Effective stress coefficient is usually calculated using experimentally measured data from the elastic region, based on poroelasticity theory (Biot, 1941, 1955). Effective stress coefficient values for the peak and residual strengths are important to evaluate rock failure; however, α obtained as above is not necessarily valid for these

strengths. There have been very few investigations into the effective stress coefficient for the peak strength (e.g. Franquet & Abass, 1999), and no investigations of how to determine α for the residual strength of rocks. In this paper, authors propose Modified Failure Envelope Method (MFEM), incorporating the results of triaxial compression tests to evaluate the effective stress coefficients for the peak and residual strengths of rocks ($\alpha_{\text{-Peak}}$ and $\alpha_{\text{-Residual}}$), based on the failure envelope method proposed by Franquet & Abass (1999), and apply it to Kimachi sandstone.

Multistage triaxial tests are used to reduce the number of specimens as well as the error caused by differences in the mechanical properties between specimens. The effective stress coefficients are also determined using a conventional method under hydrostatic stress states for intact and fractured rocks ($\alpha_{\text{-Biot's}}$ and $\alpha_{\text{-Fractured}}$), and are compared with the values of α for the strengths obtained using the single and multistage MFEMs. In addition, an equation to obtain the effective stress coefficient from total confining pressure and pore pressure, and a method to choose the coefficients for elastic stress analyses and failure evaluations, for intact rock structures or structures in rock mass are proposed.

2. Modified failure envelope method

The MFEM requires the construction of a failure envelope on the differential stress–effective confining pressure plane for saturated samples tested using a triaxial cell with zero pore pressure, followed by that for saturated samples with specific pore pressures (Fig. 1). In here, differential stress is the difference between axial stress and confining pressure.

$$\begin{aligned}\sigma_d &= \sigma' - P'_c \\ &= \sigma - \alpha P_p - (P_c - \alpha P_p) \\ &= \sigma - P_c\end{aligned}\tag{3}$$

where, σ_d is the differentials stress (MPa), σ is the axial stress (MPa), P_c is the confining pressure (MPa), σ' is effective axial stress (MPa), P'_c is effective confining pressure (MPa), α is the effective strength coefficient and P_p is Pore pressure (MPa).

Zero pore pressure means pore pressure was controlled to be zero.

From the first set of tests with zero pore pressure; Firstly, peak differential stresses are plotted as A and B, as illustrated in Fig. 1. For this case, effective confining pressure that equals the total confining pressure.

$$P'_c = P_c \quad (4)$$

Secondly, the peak differential stress from the second set of tests, where the confining pressure and pore pressure are varied can be plotted on the differential stress–effective confining pressure plane assuming $\alpha = 0$ (C in Fig. 1) and $\alpha = 1$ (D in Fig. 1). For this case,

$$P'_c = P_c - \alpha P_p \quad (5)$$

The data with non zero pore pressures can be moved to the left by increasing α from 0 (C) to 1 (D) and the crossing point (E in Fig. 1) to the failure envelope for zero pore pressure can be found. P'_c is the effective confining pressure value at E and α can be determined from P'_c as,

$$\alpha = \frac{P_c - P'_c}{P_p} \quad (6)$$

In the Failure Envelope Method (Franquet & Abass, 1999), they use tedious trial and error method and approximation by the linear Coulomb's failure criterion which induces errors during approximation by a straight line to evaluate effective stress coefficient. But in MFEM, the effective stress coefficient can be estimated without a trial and error method or assuming any failure criterion. Moreover, MFEM is the first such study, which considers evaluating effective stress coefficient for residual strength to date.

3. Materials and methods

3.1 Rock type

Kimachi sandstone which was sampled at the Shimane prefecture, Japan was used for the experiments. It is a relatively well-sorted medium-hard clastic rock with a

typical grain size in the range 0.4 – 1.0 mm. It consists mostly of rock fragments of andesite and crystal fragments of plagioclase, pyroxene, hornblende, biotite, and quartz, as well as calcium carbonate, iron oxides, and matrix zeolites (Dhakal et al., 2002).

3.2 Specimens and sample preparation

The P-wave velocities of the rock blocks were measured using 140-kHz sensors. Core boring was carried out in the direction of the lowest P-wave velocity to a diameter of 30 mm and cut to a length of 65 mm. The end faces of the specimens were ground to a length of 60 mm with a parallelism of 2/100.

The samples were fully saturated with deionized water under a vacuum. Then they were attached to stainless steel end pieces, which had central holes for water seepage. The samples with the end pieces were jacketed with heat-shrinkable tube, thus isolated from the confining water, and allowed to saturate for 24 hours.

3.3 Experimental procedure

3.3.1 Single-stage triaxial tests

The jacketed sample was placed in an ultra-compact triaxial cell (Alam et al., 2014). After completing the experimental set up as shown in Fig. 2, the axial stress, confining pressure, and pore pressure were incremented to the target values listed in Table 1 (Fig. 3a). Then the confining pressure and pore pressure were maintained constant, while axial compression was introduced at a constant strain rate of 10^{-5} s^{-1} (0.036 mm/min) until the axial strain reached 5% (3 mm). The effect of temperature on effective stress coefficient should be considered in future. However, the effect has not been considered in the present work and all the tests have been conducted under 295 K.

3.3.2 Multistage triaxial tests

In the multistage triaxial tests, only two samples were required. One sample was tested with zero pore pressure (Sample #1 in Fig. 4) using a modified multistage triaxial test (Youn and Tonon, 2010), which is a modification of the ISRM method (Kovari et al., 1983). The confining pressure was first increased up to the first target value (2 MPa), and an axial compression was introduced at a constant axial strain rate

of 10^{-5} s^{-1} (0.036 mm/min) until the sample reached the first failure point. Following this, the differential stress was released completely (Youn and Tonon, 2010) and the confining pressure was hydrostatically increased to the next level, 5 MPa, as shown in Fig. 3b. The differential stress was then increased to the second failure point, and so on. The failure point was defined by the region of the stress–axial strain curve where the tangent modulus becomes zero (Youn and Tonon, 2010). The confining pressure was increased stepwise to 2 MPa, 5 MPa, 10 MPa, and 15 MPa.

The second sample was tested by applying a non-zero pore pressure (Sample #2 in Fig. 4). In this test, the confining pressure and the pore pressure were introduced successively up to the first target values (15 MPa and 14 MPa, respectively). Axial compression was then introduced at the same strain rate until it reached the first failure point. The pore pressure was then reduced to the next level while maintaining a constant confining pressure of 15 MPa, and axial stress was introduced until the second failure point was reached, as shown in Fig. 3c. This procedure was repeated in 6 steps as the pore pressure was reduced to 12 MPa, 10 MPa, 8 MPa, 5 MPa, 1 MPa, and 0 MPa (see Table 2).

Due to the heterogeneity of samples, if Sample #2 is weaker than Sample #1 (Fig. 4a) α could not be obtained or could be underestimated. If Sample #2 is stronger than Sample #1 (Fig. 4b) α could be overestimated. Hence, prior to plotting the peak differential stresses for the Sample #2 (PDS_2), the stresses were corrected to reduce the variation in strength between specimens according to Eq. 7.

$$PDS'_2 = \frac{PDS_1^0}{PDS_2^0} \times PDS_2, \quad (7)$$

where PDS_i^0 is the peak differential stress of Specimen # i under zero pore pressure ($\#i$, $P_p = 0$ in Fig. 4).

Reducing the pore pressure to 0 MPa is important to enable this correction. Multistage tests were not applied to evaluate α for residual strength since the required strain range to stabilize the residual strength exceeded the capacity of the triaxial cell.

3.3.3 Hydrostatic tests for intact rock to evaluate α -Biot's

In the hydrostatic tests, specimens were exposed to an increased stress state with an isotropic stress field. Two tests were required: one to determine the bulk modulus of the rock, K , as a function of hydrostatic stress, and the other to determine the bulk modulus of the solid matrix (*i.e.*, mineral grains), K_s (Detournay and Cheng, 1993; Hu et al., 2010; Zimmerman, 1991).

The sample preparation process followed the same procedure as detailed in section 3.2, except that in the hydrostatic tests, two cross-type strain gauges were attached to the specimen to measure the axial and lateral strains. The bulk modulus of rock, K was evaluated using a drained hydrostatic compression test (Detournay and Cheng, 1993; Hu et al., 2010; Zimmerman, 1991). In this test, the hydrostatic stress was incremented up to 15 MPa under drained condition and with zero pore pressure to determine K , as shown in Fig. 5a.

The volumetric strain was calculated from the axial strain ε_a and the lateral strain ε_l using $\varepsilon_v = \varepsilon_a + 2\varepsilon_l$. The bulk modulus was calculated from the volumetric strain curve as a function of the hydrostatic stress (Fig. 6) using the following relation:

$$K = \left(\frac{\Delta P}{\Delta \varepsilon_v} \right)_{\Delta P_p = 0} \quad (8)$$

where ΔP is the hydrostatic stress increment, $\Delta \varepsilon_v$ is the volumetric strain increment, and ΔP_p is the pore pressure increment.

In the second hydrostatic test, the hydrostatic stress and pore pressure were simultaneously incremented up to 15 MPa using the same increments (*i.e.*, $\Delta P = \Delta P_p$), and maintaining $P_p = P - 1$ (MPa), as shown in Fig. 5b, to determine the bulk modulus of the solid matrix, K_s (Detournay and Cheng, 1993; Hu et al., 2010). After determining the volumetric strain, K_s was calculated from the linear region of the volumetric strain curve (Fig. 7) by,

$$K_s = \left(\frac{\Delta P}{\Delta \varepsilon_v} \right)_{\Delta P_p = \Delta P} \quad (9)$$

The initial part of the curve is not linear. This could be due to lack of contact between the strain gages and the specimen under small hydrostatic pressure.

According to the relationship between Biot's coefficient and the compressibility properties (Biot, and Willis, 1957; Detournay and Cheng, 1993; Geertsma, 1957; Makhnenko and Labuz, 2013; Nur and Byerlee, 1971), Biot's coefficient can be determined by,

$$\alpha_{\text{-Biot's}} = 1 - \frac{K}{K_s} \quad (10)$$

3.3.4 Hydrostatic tests for fractured rock to evaluate $\alpha_{\text{-Fractured}}$

Bulk modulus of solid matrix, K_s was measured in advance for intact rock specimen. To evaluate the effective stress coefficient for fractured rock, the bulk modulus of fractured rock must be determined.

Fractured rock specimens were produced from intact rock samples after fracturing them in standard triaxial compression tests with a confining pressure of 2 MPa and a pore pressure of 1 MPa. When the sample reached its residual state, the axial loading was terminated and the differential stress was released completely. The hydrostatic stress was then incremented up to 15 MPa under drained conditions with a constant pore pressure of 1 MPa to determine the bulk modulus of the fractured rock, K_f , as a function of the hydrostatic stress. The increment in the volumetric strain is given by,

$$\Delta \varepsilon_v = \frac{\Delta V}{V} + \frac{\Delta P}{K_s} \quad (11)$$

where ΔV is the water drainage increment and V is the volume of the specimen. The bulk modulus of the fractured rock and $\alpha_{\text{-Fractured}}$ are given by,

$$K_f = \left(\frac{\Delta P}{\Delta \varepsilon_v} \right)_{P_p \text{-constant}} . \quad (12)$$

$$\alpha_{\text{-Fractured}} = 1 - \frac{K_f}{K_s} \quad (13)$$

4. Experimental results

4.1. Deformation of rocks

It was confirmed that either peak or residual strength increased with increasing confining pressure and decreased with increasing pore pressure (Fig. 8). There is no obvious difference between the strengths obtained by single (Fig. 8a) and multistage (Fig. 8c) triaxial test. A transition from typical brittle to ductile behavior was clearly observed as the confining pressure increased, as shown in Figs 9 and 10a. The inverse happens when introducing pore pressure. There was a transition from ductile to brittle behavior as pore pressure increased (Fig. 10b).

With small confining pressures of 2 MPa and 5 MPa, the rock exhibited a nonlinear stress–strain relation at the initial stage of the stress–strain curve (see Fig. 10a), which is typically attributed to the closure or compaction of pores and/or pre-existing cracks (Sheng et al., 2012). However, at higher confining pressures of 10 MPa and 15 MPa, the phase corresponding to microcrack closure at the initial stage of the stress–strain curve was not distinct, which suggests that microcrack closure depended on the confining pressure. Following the pore closure phase, all the specimens deformed elastically. The departure from linear behavior was marked as the yield point (see Fig. 10a). The amount of the magnitude of permanent set, which is the phase in-between the maximum strength and yielding point, was increased with increasing confining pressure although it remains small.

With low confining pressures of 2 MPa and 5 MPa, the Kimachi sandstone exhibited strain softening with clustering of localized shear damage (see Fig. 9). In contrast, at higher confining pressures of 10 MPa and 15 MPa, the rock exhibited strain hardening with compactive cataclastic flow (see Fig. 9). In other words, at lower effective confining pressures ($P_c = 2$ MPa and 5 MPa with $P_p = 1$ MPa) only a single rupture plane formed in the Kimachi sandstone (Fig. 9 b). Almost perfectly elasto-plastic deformation was observed in the Kimachi sandstone under high effective confining pressure and rupture plane was absent (Fig. 9 c). This occurred because large plastic deformation took place in the cementing materials.

4.2. Effective stress coefficients

In the drained hydrostatic compression test with zero pore pressure, the bulk modulus of the intact rock, K increased with the hydrostatic pressure, as shown in

Fig. 6. This was attributed to the closure of microcracks and elliptical pores. As a consequence, $\alpha_{\text{-Biot's}}$ decreased with increasing confining pressure, and was in the range $1 > \alpha_{\text{-Biot's}} > 0.8$, as shown in Fig. 12. The effective stress coefficients by hydrostatic tests for fractured rock, $\alpha_{\text{-Fractured}}$ were larger than those for intact rock $\alpha_{\text{-Biot's}}$ and were close to unity.

Effective stress coefficients for peak and residual strengths were obtained from peak differential stress and differential stress in the residual strength state, respectively (Fig. 11) through MFEM. In this analysis, stress data related to 1MPa pore pressure were omitted as the small P_p magnitude caused large errors in α value. When consider about the effective stress coefficient for strength, Jaeger et al. (2007) stated that effective stress coefficient for strength was unity. However, in our experiments, the effective stress coefficients for the peak strength, $\alpha_{\text{-Peak}}$ decreased with increasing effective confining pressure and was in the range $0.8 > \alpha_{\text{-Peak}} > 0.4$, under both single and multi stage MFEMs, as shown in Fig. 12. As described under the deformation of Kimachi sandstone, at lower confining pressures, only a single rupture plane formed. Almost perfectly elasto-plastic deformation was observed in the Kimachi sandstone under high confining pressure and rupture plane was absent (Fig. 9). This occurred because large plastic deformation took place in the cementing materials. This absence of rupture plane or decrease the width of the rupture plane leads to increase the grain to grain contact area and hence effective stress coefficient increased with increasing effective confining pressure. The coefficients $\alpha_{\text{-Peak}}$ were significantly lower than those for intact rock under hydrostatic conditions or $\alpha_{\text{-Biot's}}$. For residual strength, $\alpha_{\text{-Residual}}$, was almost constant and between $\alpha_{\text{-Peak}}$ and $\alpha_{\text{-Biot's}}$ (Fig. 12).

Assuming Eq. (14) for the effective stress coefficient, the constants A and B can be obtained as illustrated in Fig. 13 (Table 3). By altering Eq. (14), Eq. (15) can be obtained. Hence the effective stress coefficient can be calculated by substituting P_c , P_p , A and B . The low coefficient of correlation for $\alpha_{\text{-Fracture}}$ and $\alpha_{\text{-Residual}}$ may suggest that they are almost constant.

$$\alpha = A - BP'_c \quad (14)$$

$$\alpha = \frac{A - BP_c}{1 - BP_p} \quad (15)$$

5. Discussion

5.1 Applicability of the MFEM

MFEMs were successfully applied for the determination of effective stress coefficients for the peak and residual strengths (α_{Peak} and α_{Residual}) and multistage MFEM is recommended to obtain the effective stress coefficients for peak strength. The strength correction in the multistage MFEM is particularly effective in obtaining results with a small amount of data scattering.

Reproducibility of the MFEM was not directly investigated in present work. The coincident between α_{Peak} by single stage and multistage tests however indirectly suggests that reproducibility was fairly high.

5.2 Relationship between the coefficient for intact rock and strengths

The effective stress coefficients, both for peak strength and for the intact rock (α_{Peak} and $\alpha_{\text{Biot's}}$), can be well described using a linear fit, taking the effective normal stress on the rupture plane as the x-axis, as shown in Fig. 14 (assuming that the rupture plane is inclined by 30°). The coefficient values for residual strengths were somewhat scattered; however, they were located around the regression line. It follows that the rather tedious single-stage MFEM to evaluate the residual strength could be skipped in future.

5.3 Choice of effective stress coefficient

For elastic stress analysis of small structures in intact rock, the effective stress coefficient for intact rock, $\alpha_{\text{Biot's}}$, can be used. In here small structure refers to the considerably smaller excavations compared to the fracture spacing such as bore hole.

The value of the coefficient can be evaluated using the results of the hydrostatic tests for intact rock, as well as incorporating any other conventional method based on theory of poroelasticity.

To evaluate rock failure, however, the effective stress coefficient for the peak strength, α_{Peak} should be used. This coefficient can be evaluated using the multistage MFEM. The coefficient for the peak strength, α_{Peak} was considerably smaller than that for intact rock, $\alpha_{\text{Biot's}}$. Combining the coefficients for intact rock and the peak strength via an effective normal stress may allow a seamless analysis.

If Terzaghi's effective stress coefficient ($\alpha = 1$) is used in failure evaluations of rock, the effective strength of rock will be under-estimated (Fig. 15). In the case of α_{Peak}

$\alpha_{\text{Biot's}}$ which is greater than α_{Peak} for Kimachi sandstone, still the effective strength of rock will be under-estimated. This leads over design. If effective stress coefficient is assumed as zero, the effect of pore pressure to the effective stress is totally ignored and the structure is in danger under such conditions. However, if the effective stress coefficient for peak strength; α_{Peak} as evaluated in this research is used, the exact effective strength can be evaluated, thereby saving cost for the excavations ensuring safety which is reasonable than other two occasions.

For the elastic stress analysis of a structure in a rock mass, the coefficient for fractured rock, $\alpha_{\text{Fractured}}$ can be used, since rock mass is fractured. The coefficient can be evaluated using hydrostatic tests for fractured rock; however, in practice, this test can be skipped because the coefficient is larger than that of intact rock, and is close to unity. The differences in the scale and the origin between rupture planes in fractured rock specimens and fractures in a rock mass should be further investigated in future.

The effective stress coefficient for the residual strength, α_{Residual} should be used to evaluate rock mass failure, since rock mass is fractured. This coefficient can be evaluated using the single-stage MFEM and was found to lie between the coefficient for intact, rock, $\alpha_{\text{Biot's}}$ and the coefficient for peak strength α_{Peak} . The effective stress coefficient may be estimated using the effective normal stress; however, further investigation into the coefficient for the residual strength is required.

6. Conclusion

Authors have described the Modified Failure Envelope Method (MFEM), which can be used to obtain the effective stress coefficients for peak strength, α_{Peak} and residual strengths, α_{Residual} . The coefficients for intact and fractured Kimachi sandstone $\alpha_{\text{Biot's}}$ and $\alpha_{\text{Fractured}}$ were also evaluated using conventional methods, and the data were compared with the coefficient values obtained by MFEM for the peak and residual strengths.

The effective stress coefficient for intact rock decreased with increasing confining pressure, and was in the range $1 > \alpha_{\text{Biot's}} > 0.8$. The effective stress coefficient for fractured rock was larger than that for intact rock, and was close to unity. The effective stress coefficient calculated for peak strengths using both the single- and multistage MFEMs decreased with effective confining pressure and was in the range $0.8 > \alpha_{\text{Peak}} > 0.4$. For residual strength states, α was between the peak strength value and that for intact rock.

Based on these results, we can conclude that the multistage MFEM is suitable for obtaining an effective stress coefficient for the peak strength, α_{Peak} because it showed almost same α_{Peak} values as in the case of single stage MFEM. In addition, it requires only two samples and hence it reduces the error caused by differences in the mechanical properties between specimens. An equation to obtain the effective stress coefficient from total confining pressure and pore pressure, and a method to choose the coefficients for elastic stress analyses and failure evaluations for small rock structures or structures in rock mass were proposed.

The multistage MFEM should be modified to determine the coefficient for residual strength. Further data accumulation for various types of rocks and consideration of the differences in scale and the origin between rupture planes in fractured rock specimens and fractures in a rock mass are expected to contribute to future designs of rock structures. The effect of temperature on effective stress coefficient should be considered in future.

Acknowledgement

This work was partially supported by KAKENHI (26420840).

References

- [1]. Alam, A. A., Christensen, H. F., Fabricius, I. L., 2009. Effective stress coefficient and Biot's coefficient of chalk from the Valhall Field, North Sea. EUROPEC/EAGE Conference and Exhibition, 8–11 June 2009, Amsterdam, The Netherlands, SPE 121795-MS, pp. 1–8
- [2]. Alam, A. K. M. B, Niioka, M., Fujii, Y., Fukuda, D., Kodama, J., 2014. Effects of confining pressure on the permeability of three rock types under compression. *Int J Rock Mech Min Sci*; Vol. 65, pp. 49-61.
- [3]. Alam, M. M., Borre, M. K., Fabricius, I. L., Hedegaard, K., Røgen, B., Hossain, Z., Krogsbøll, A. S., 2010. Biot's coefficient as an indicator of strength and porosity reduction: calcareous sediments from Kerguelen Plateau. *Journal of Petroleum Science and Engineering* 70, 282–297
- [4]. Biot, M. A., 1941. General theory of three dimensional consolidation, *J Appl Phys*; 12(2):155–164.
- [5]. Biot, M. A., 1955. Theory of elasticity and consolidation for a porous anisotropic solid. *J Appl Phys*; 26:182–5.
- [6]. Biot, M. A., Willis, D. G., 1957. The elastic coefficients of the theory of consolidation. *ASME J Appl Mechs*, 24, 594–601
- [7]. Detournay E., Cheng A. H-D., 1993. Fundamentals of poroelasticity. In: Hudson JA, editor. *Comprehensive rock engineering*. Oxford: Pergamum, p. 113-171.
- [8]. Dhakal G., Yoneda T., Kato M., Kaneko K. 2002. Slake durability and mineralogical properties of some pyroclastic and sedimentary rocks. *Eng geol*; 65:31–45.
- [9]. Jaeger J.C., Cook N.G.W. and Zimmerman R.W., *Fundamentals of Rock Mechanics*, 2007, Blackwell Publishing, 4th Edition, 97-100.
- [10]. Franquet, J. A., Abass, H. H., 1999. Experimental evaluation of Biot's poroelastic parameter - Three different methods. 37th U.S. Symposium on Rock Mechanics (USRMS), June 7-9 Vail, CO, Balkema, Rotterdam, 349–355.
- [11]. Geertsma, J., 1957. The effect of fluid pressure decline on volumetric changes of porous rocks. *Petroleum Transactions, AIME*, 210, 331–340.

- [12]. Hu, D.W., Zhou, H., Zhang, F., Shao, J.F., 2010. Evolution of poroelastic properties and permeability in damaged sandstone. *Int J Rock Mech Min Sci*; 47 pp. 962–973
- [13]. Kovari, K., Tisa, A., Einstein, H.H., Franklin, J.A., 1983. Suggested methods for determining the strength of rock materials in triaxial compression: revised version. *Int J Rock Mech Min Sci*; 206:283–90.
- [14]. Makhnenko, R.M., Labuz, J.F., 2013. Unjacketed bulk compressibility of sandstone in laboratory experiments. *Proceedings of the 5th BIOT Conference on Poromechanics, Vienna, Austria, 10-12 July 2013*, paper No. 304
- [15]. Nur, A., Byerlee, J. D., 1971. An exact effective stress law for elastic deformation of rock with fluids. *Journal of Geophysical Research Solid Earth*, 76, 6414–6419
- [16]. Sheng, Q. Y., Hong, W. J., Shan, Y. W. 2012. Experimental Investigation on the Strength, Deformability, Failure Behavior and Acoustic Emission Locations of Red Sandstone Under Triaxial Compression. *Rock Mechanics and Rock Engineering*, vol. 45, no. 4, p. 583–606,
- [17]. Terzaghi, K., 1936. The shearing resistance of saturated soils and the angle between the planes of shear. *First Int. Conf. Soil Mech.*, Vol. 1, Harvard Univ., Cambridge, Mass.:54-56
- [18]. Youn, H., Tonon, F., 2010. Multi-stage triaxial test on brittle rock. *Int J Rock Mech Min Sci*; 47: 678–684
- [19]. Zhang, K., Zhou, H., Hu, D., Zhao, Y., Feng, X., 2009. Theoretical model of effective stress coefficient for rock/soil-like porous materials. *Acta Mechanica Solida Sinica* 22, 251–260.
- [20]. Zimmerman, R.W., 1991. *Compressibility of Sandstones*. Elsevier, New York. 173 pp.

Table 1. Target values of confining pressure and pore pressures

Table 2. Target values of confining pressure and pore pressures for multistage test with non zero pore pressure.

Table 3. Constant A and B in Eq.13 and Eq.14

- Fig. 1. A schematic diagram showing evaluation of α by the modified failure envelope method.
- Fig. 2. Experimental setup for triaxial and hydrostatic compression tests.
- Fig. 3. a) Schematic diagram showing the steps for reaching the desired confining pressure, pore pressure and compression phase of single stage triaxial test. b) Schematic diagram showing the steps for reaching the desired confining pressure, and compression phase of multi-stage triaxial test- with zero pore pressure c) Schematic diagram showing the steps for reaching the desired pore pressure and compression phase while maintaining a constant confining pressure, of multi-stage triaxial test- with pore pressure.
- Fig. 4. Correction of the peak differential stress for multistage tests.
- Fig. 5. Stress paths of hydrostatic compression tests a) with zero pore pressure: $P_p = 0$. b) with non-zero pore pressure: $\Delta P_c = \Delta P_p$.
- Fig. 6. Stress-strain curve in hydrostatic compression test for $P_p = 0$
- Fig. 7. Stress-strain curve in hydrostatic compression test with non zero pore pressure: $\Delta P = \Delta P_p$.
- Fig. 8. Peak strength with pore pressure for different confining pressures b) Residual strength with pore pressure for different confining pressures for single stage triaxial tests. c) Peak strength with pore pressure for multistage triaxial test.
- Fig. 9. Deformation of Kimachi sandstone samples tested in triaxial compression cell with confining pressure (a) Differential stress versus stroke based strain graph and blue resin-impregnated thin-section images for (b) 2 MPa confining pressure and 1 MPa pore pressure and (c) 15 MPa confining pressure and 1 MPa pore pressure
- Fig. 10. Differential stress versus stroke based strain for different confining pressures and pore pressures. a) Effect of confining pressure b) Effect of pore pressure.
- Fig. 11. a) Peak differential stresses in single stage MFEM b) Residual differential stresses in single stage MFEM and c) Peak differential stresses multistage MFEM
- Fig. 12. Summary of effective stress coefficients for Kimachi sandstone with effective confining pressure.
- Fig. 13. An example of approximation by Eq.14 (Multistage MFEM).
- Fig. 14. Effective normal stress vs α relationship for Kimachi sandstone.

Fig.15. Schematic diagram showing peak stress estimation based on different effective strength coefficients.

Table 1. Target values of confining pressure and pore pressures

Confining pressure (MPa)	Pore pressure (MPa)				
2	0	1	-	-	-
5	0	1	4	-	-
10	0	1	4	9	-
15	0	1	4	9	14

Table 2. Target values of confining pressure and pore pressures for multistage test with non zero pore pressure.

Confining pressure (MPa)	Pore pressure (MPa)
15	14
15	12
15	10
15	8
15	5
15	1
15	0

Table 3. Constant A and B in Eq.14 and Eq.15

Effective stress coefficients	A	B (MPa ⁻¹)	Coefficient of correlation $ r $
α -Biot's	0.984	0.014	0.97
α -Fractured	0.974	0.001	0.36
α -Peak (singlestage MFEM)	0.834	0.035	0.98
α -Peak (multistage MFEM)	0.888	0.034	0.99
α -Residual (singlestage MFEM)	0.727	-0.001	0.09

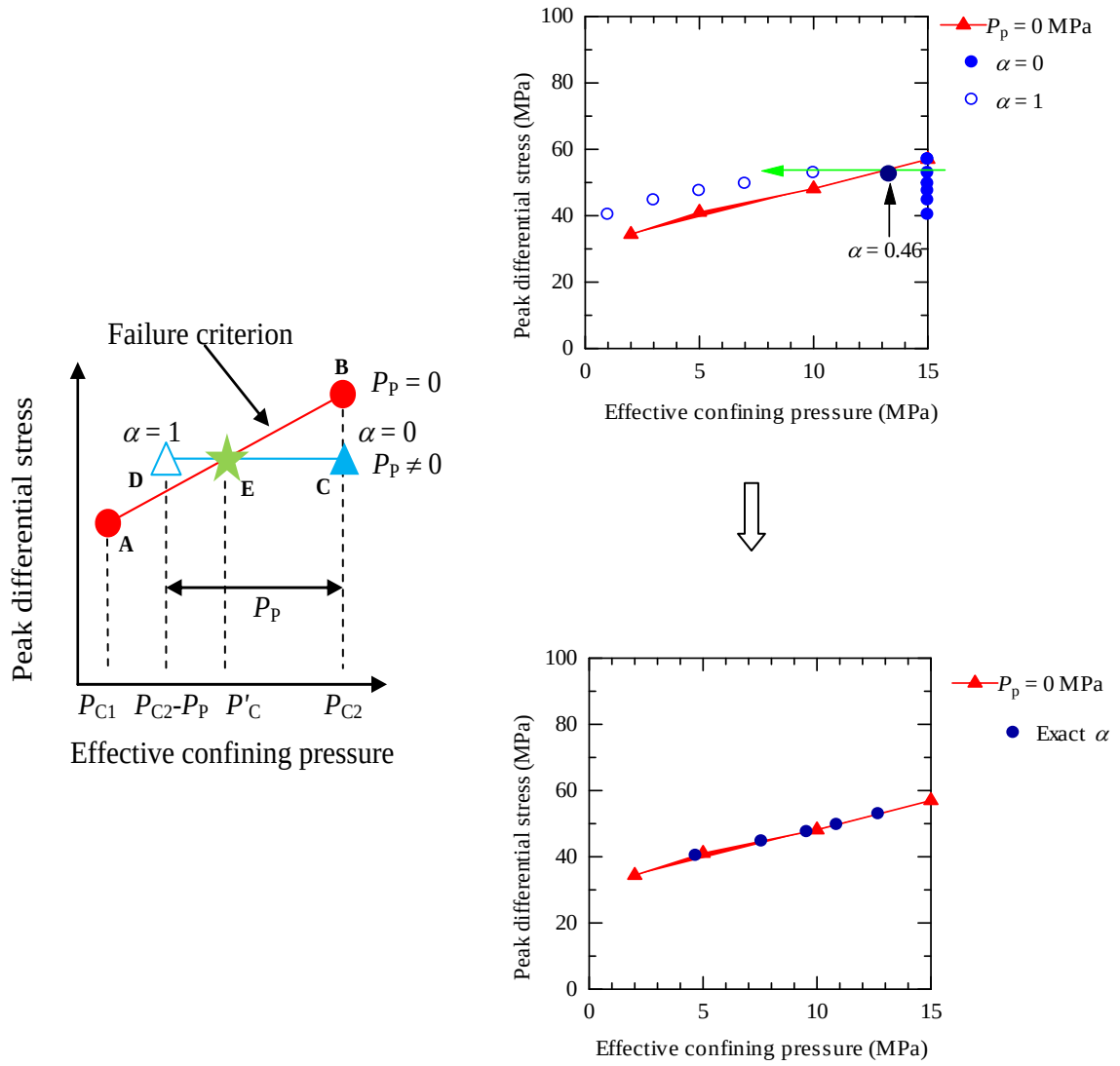


Fig. 1. A schematic diagram showing evaluation of α by the modified failure envelope method.

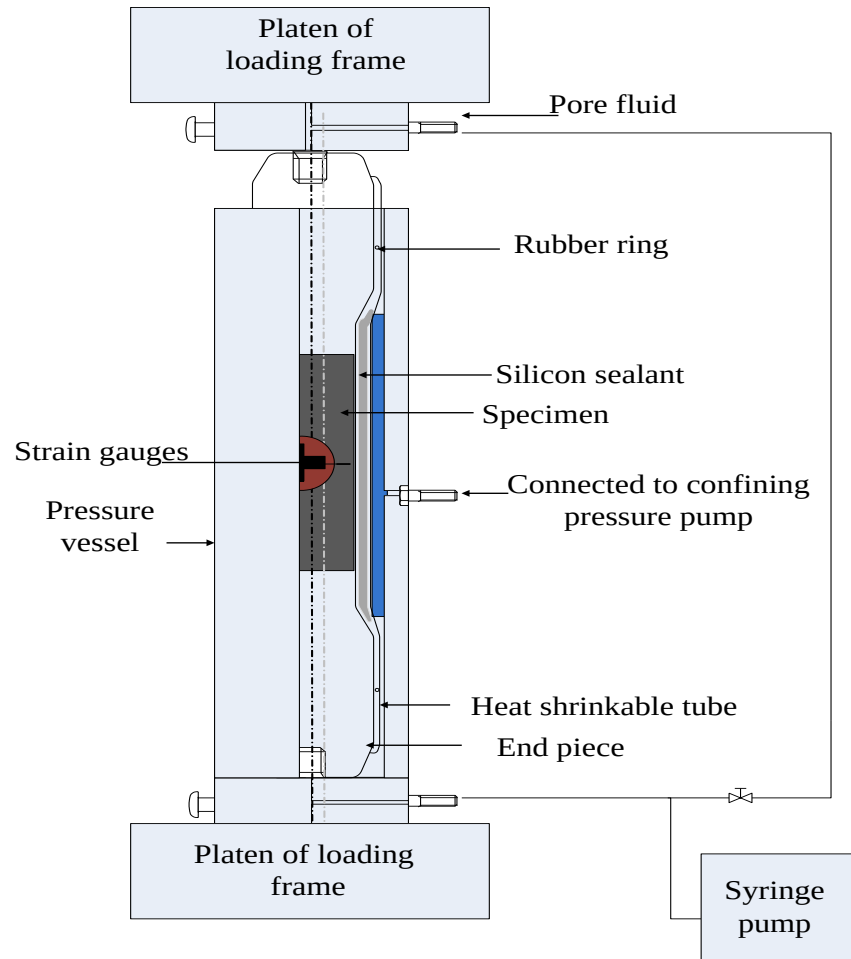


Fig. 2. Experimental setup for triaxial and hydrostatic compression tests.

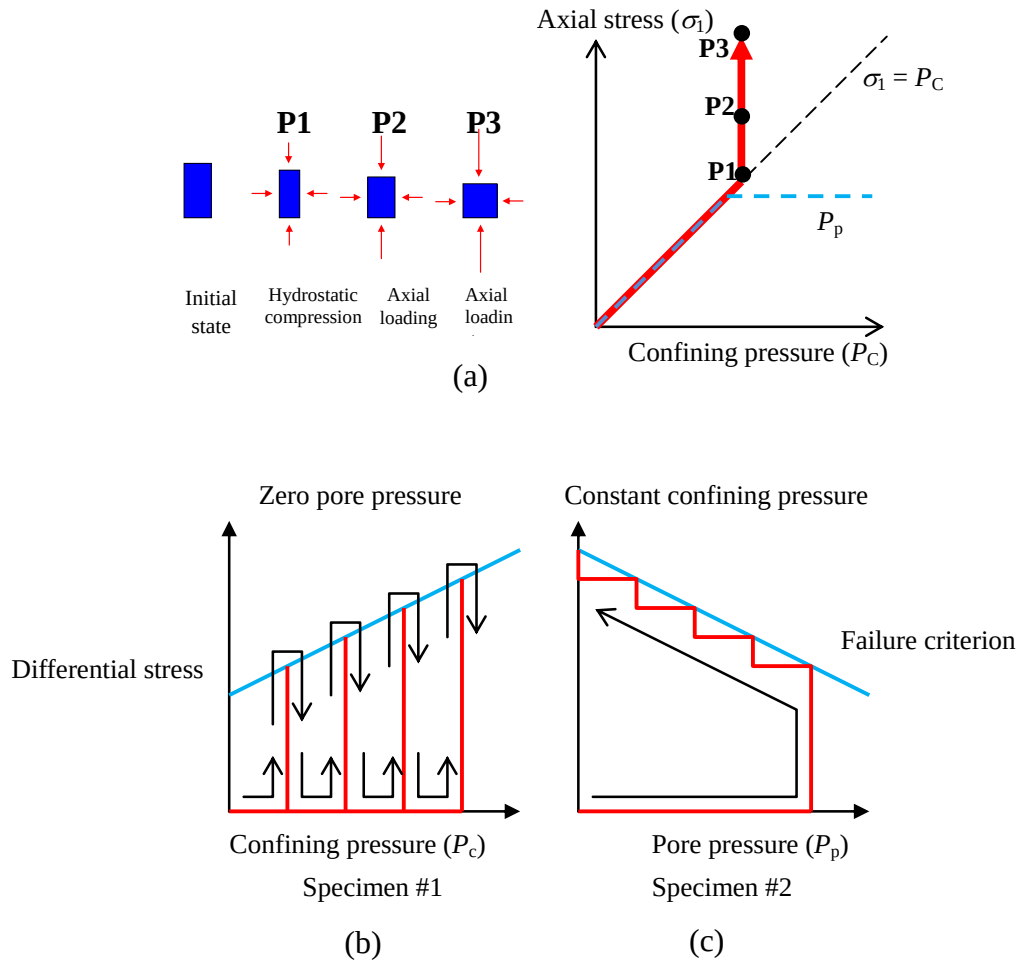


Fig. 3. a) Schematic diagram showing the steps for reaching the desired confining pressure, pore pressure and compression phase of single stage triaxial test. b) Schematic diagram showing the steps for reaching the desired confining pressure, and compression phase of multi-stage triaxial test- with zero pore pressure c) Schematic diagram showing the steps for reaching the desired pore pressure and compression phase while maintaining a constant confining pressure, of multi-stage triaxial test- with pore pressure.

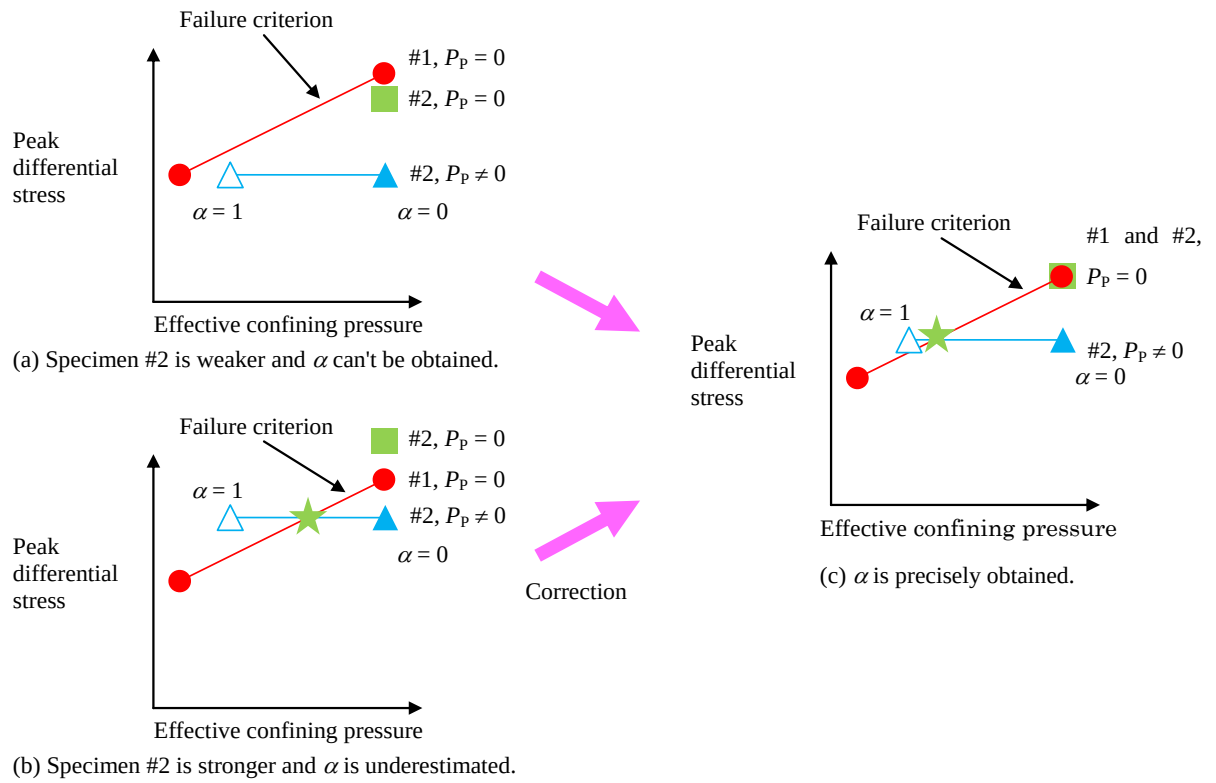


Fig. 4. Correction of the peak differential stress for multistage tests.

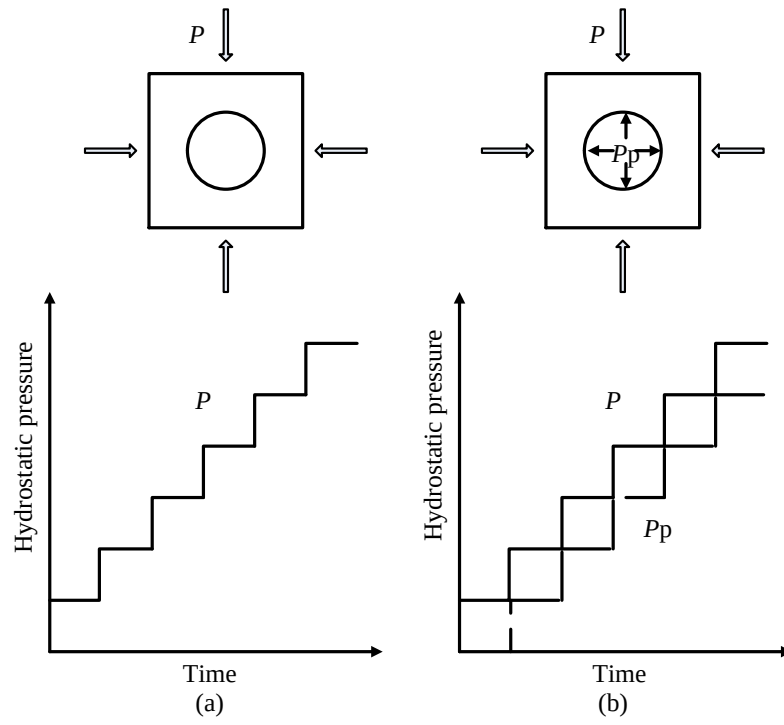


Fig. 5. Stress paths of hydrostatic compression tests a) with zero pore pressure: $P_p = 0$.
b) with non-zero pore pressure: $\Delta P = \Delta P_p$.

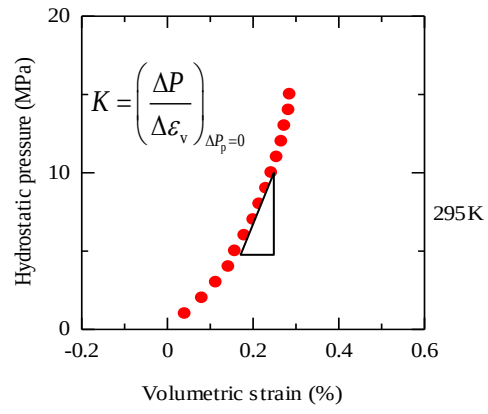


Fig. 6. Stress-strain curve in hydrostatic compression test for $P_p = 0$.

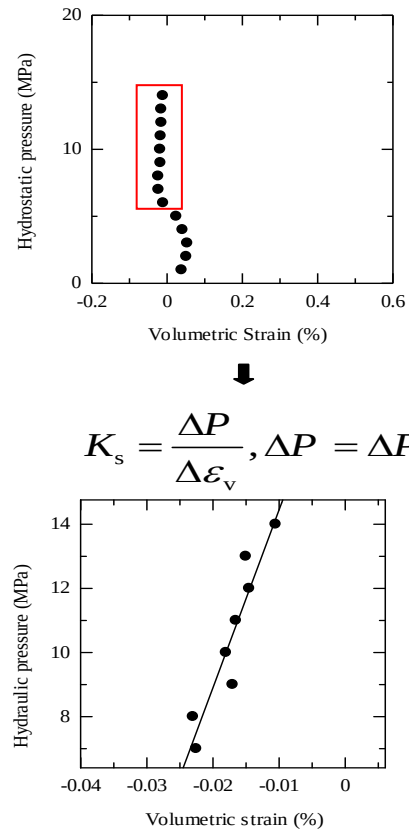


Fig. 7. Stress-strain curve in hydrostatic compression test with non zero pore pressure:

$$\Delta P = \Delta P_p$$

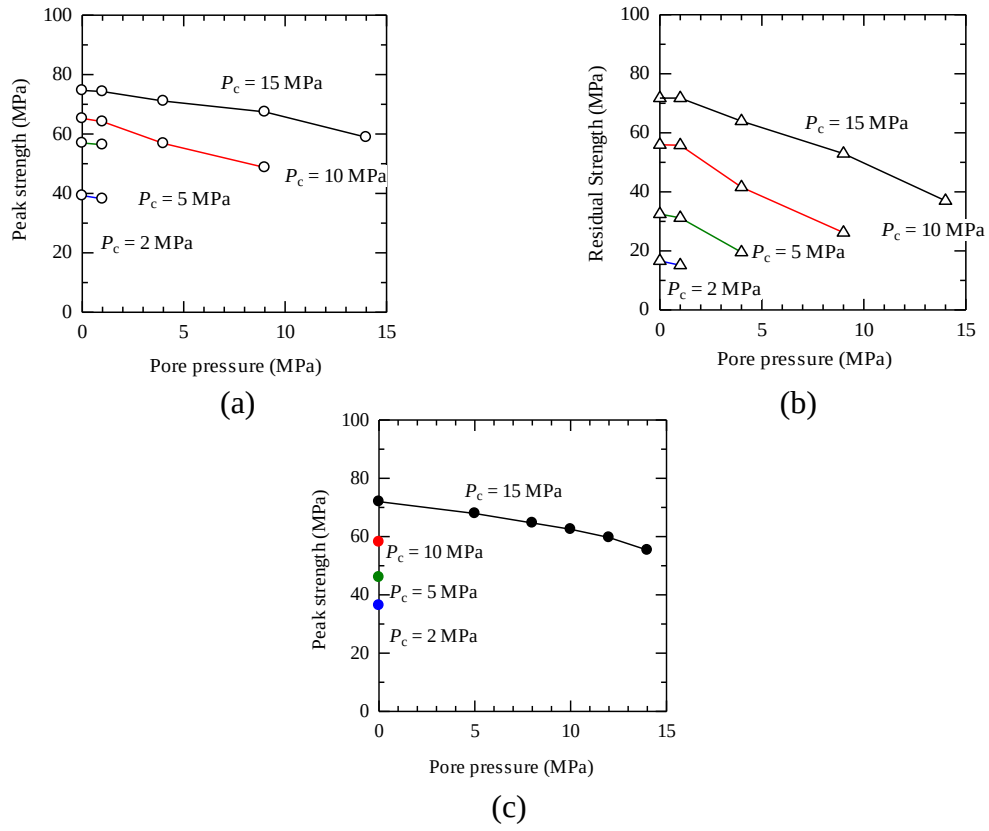


Fig. 8. Peak strength with pore pressure for different confining pressures b) Residual strength with pore pressure for different confining pressures for single stage triaxial tests. c) Peak strength with pore pressure for multistage triaxial test.

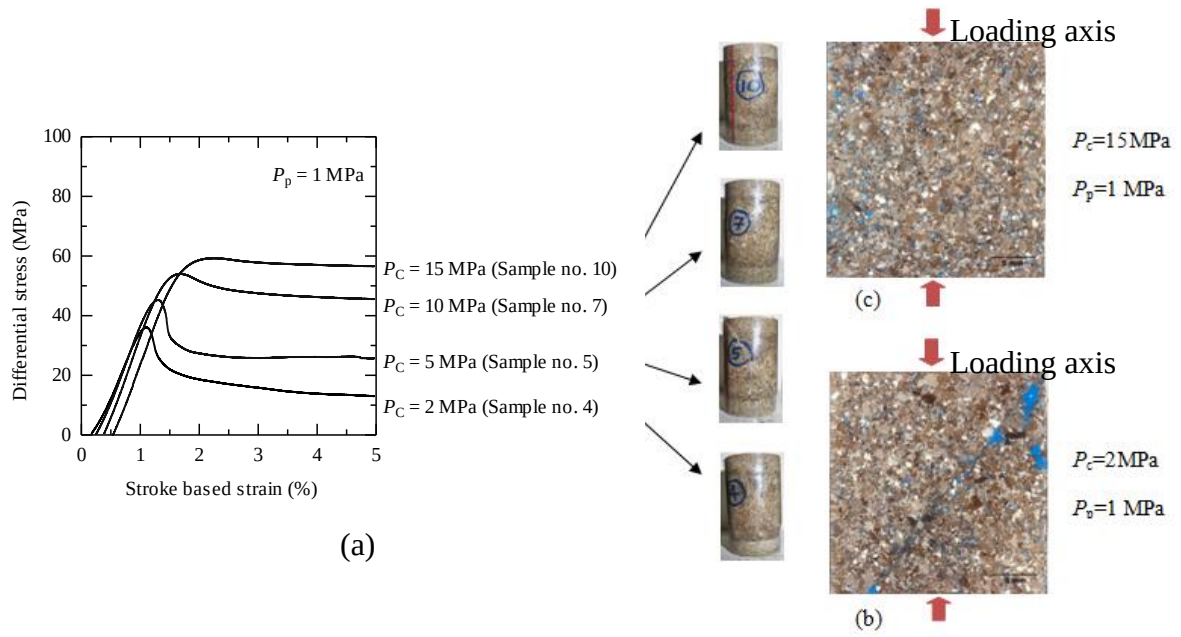
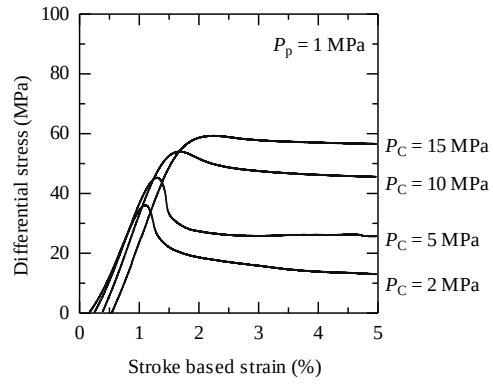
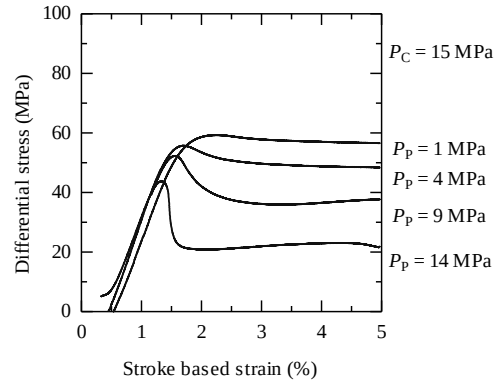


Fig. 9. Deformation of Kimachi sandstone samples tested in triaxial compression cell with confining pressure (a) Differential stress versus stroke based strain graph and blue resin-impregnated thin-section images for (b) 2 MPa confining pressure and 1 MPa pore pressure and (c) 15 MPa confining pressure and 1 MPa pore pressure.



(a)



(b)

Fig. 10. Differential stress versus stoke based strain for different confining pressures and pore pressures. a) Effect of confining pressure b) Effect of pore pressure.

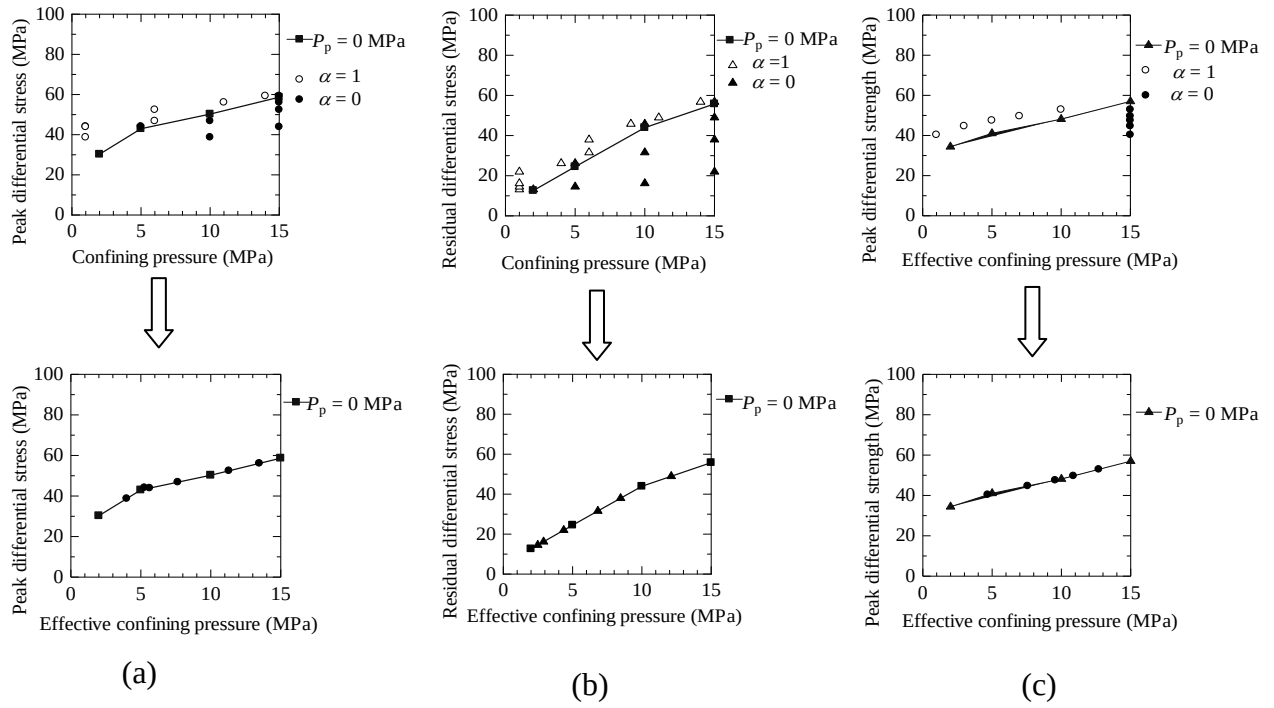


Fig. 11. a) Peak differential stresses in single stage MFEM b) Residual differential stresses in single stage MFEM and c) Peak differential stresses multistage MFEM

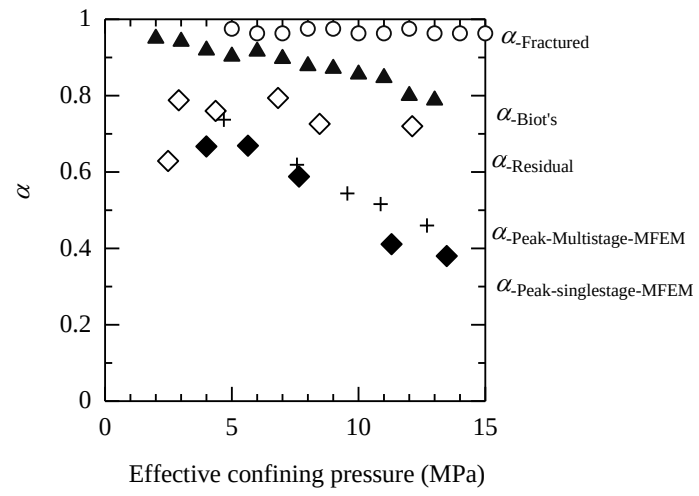


Fig. 12. Summary of effective stress coefficients for Kimachi sandstone with effective confining pressure.

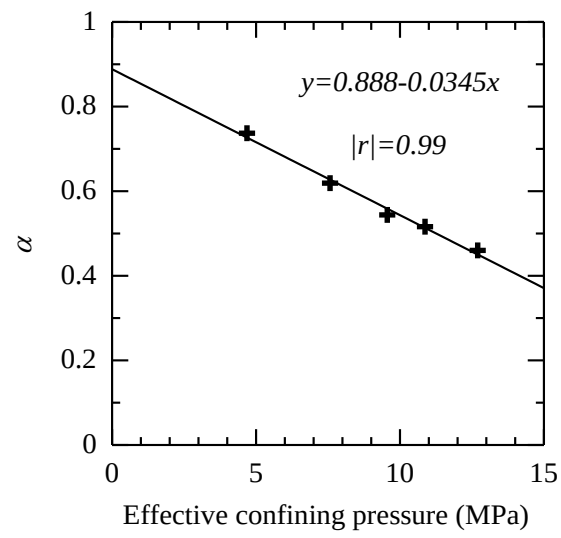


Fig. 13. An example of approximation by Eq.14 (Multistage MFEM).

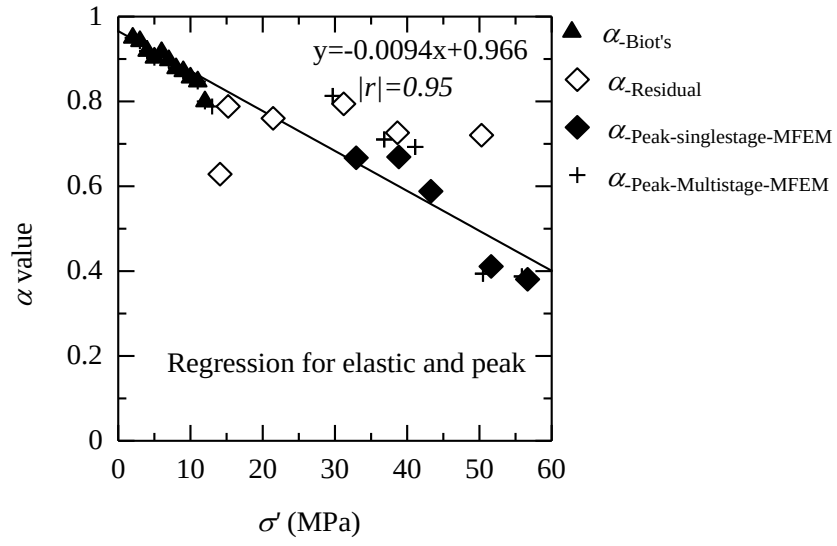


Fig. 14. Effective normal stress vs α relationship for Kimachi sandstone.

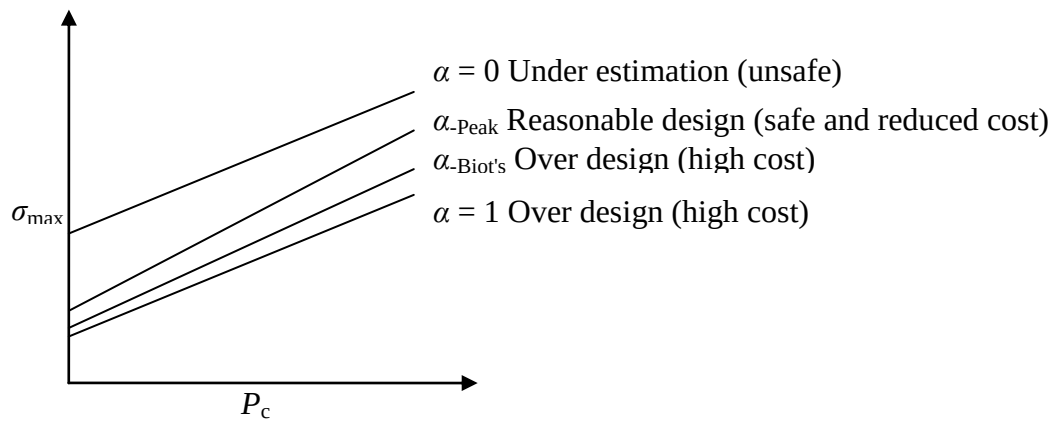


Fig.15. Schematic diagram showing peak stress estimation based on different effective strength coefficients.