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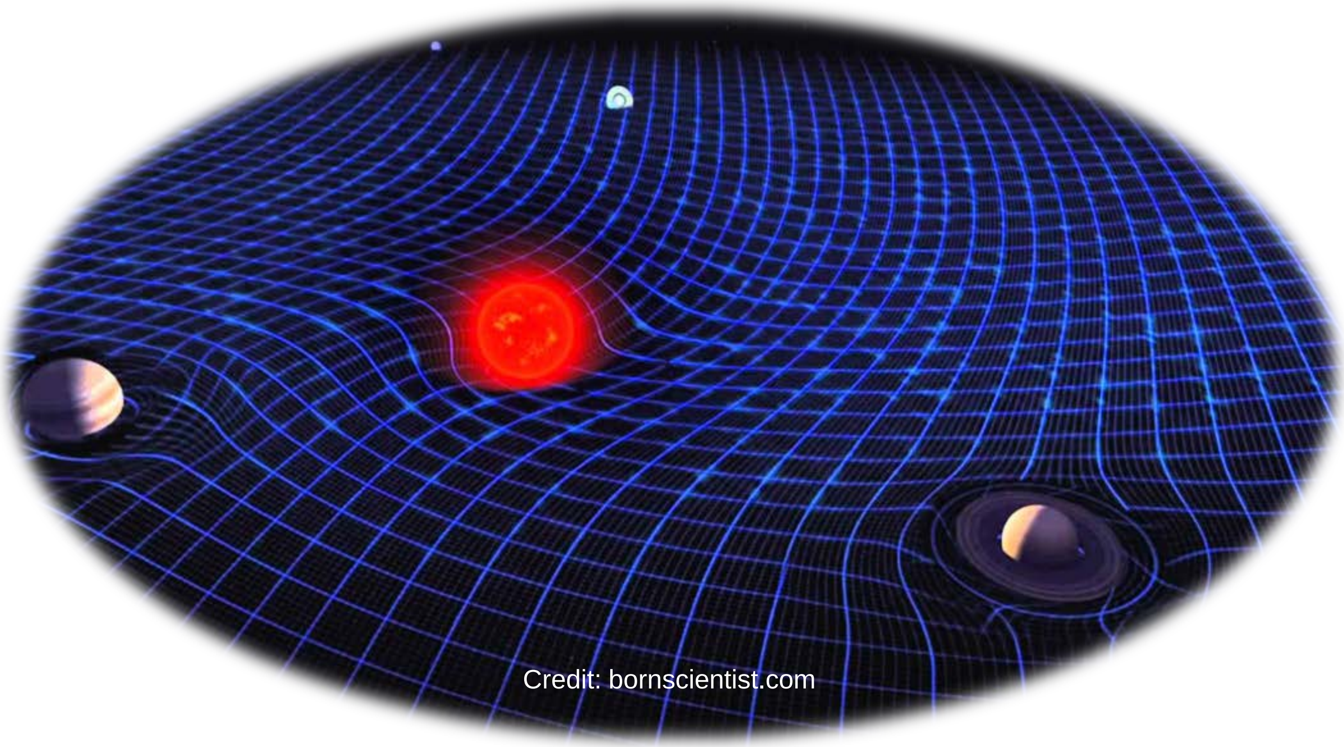
A material called spacetime



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The classical concept of space and time

Space:

Empty space
= three-dimensional “nothing”,
but can be filled with matter.

Time:

Describes the continued progress of existence and events.
Exists independently of anything else.
Progresses uniformly from $-\infty$ to $+\infty$.

Concept of absolute space and time works well in everyday life

Newtonian mechanics in general, celestial mechanics, continuum mechanics…

Prüfung - Technische Mechanik III

WS 11/12

TECHNISCHE UNIVERSITÄT DARMSTADT

FB 13, Festkörpermechanik
Prof. Dr.-Ing. F. Gruttmann

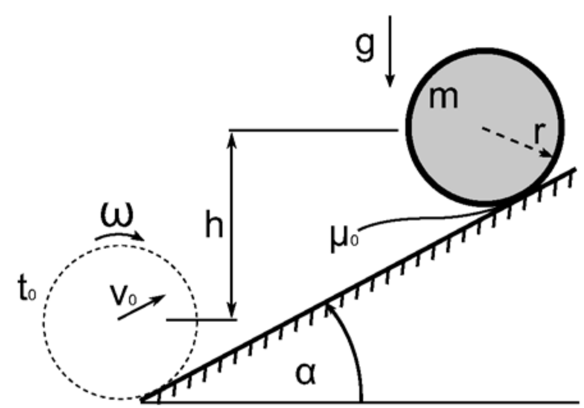
16. Februar 2012

Aufgabe 1 [23 Punkte]

Ein homogener Zylinder mit der Masse m und dem Radius r rollt ohne zu rutschen eine schiefe Ebene (Steigungswinkel α) hinauf. Zum Zeitpunkt $t = 0$ habe sein Schwerpunkt die Geschwindigkeit v_0 parallel zur schiefen Ebene.

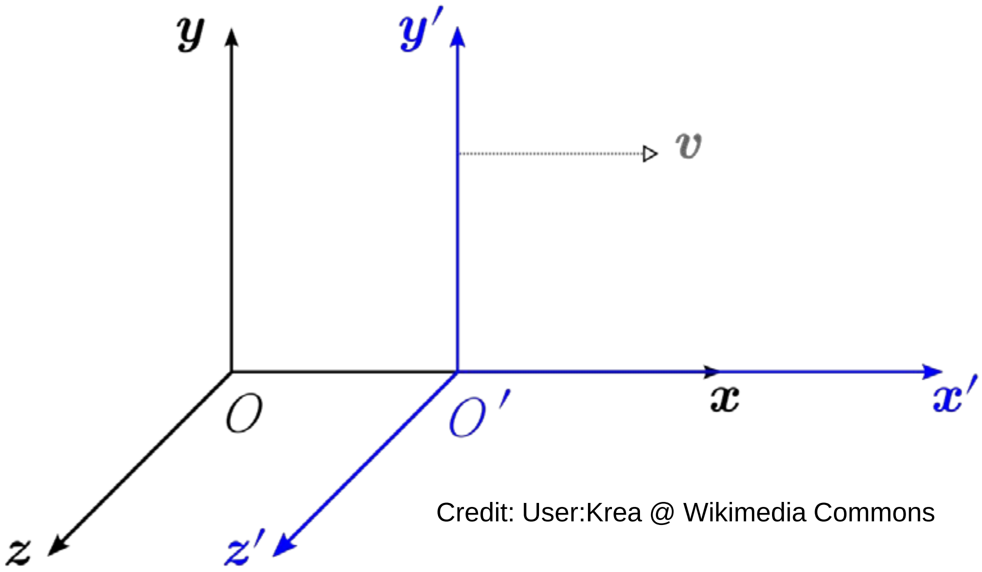
Gegeben: m, r, h, α, g

a) Berechnen Sie den minimalen Haftungskoeffizient μ_0, damit kein Rutschen auf der schiefen Ebene auftritt.



Galilei transformation

Frames of reference:



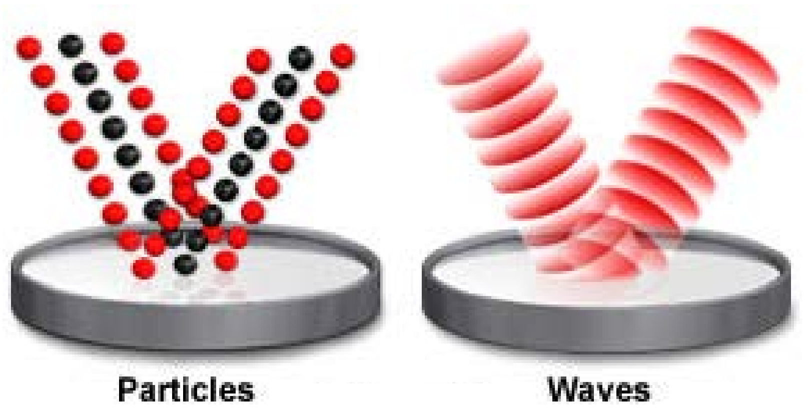
Transformation equations for any point (“event”) *P*:

$$t' = t \qquad y' = y$$

$$x' = x - vt \qquad z' = z$$

The nature of light

Particle theory vs. wave theory:



Credit: www.olympus-lifescience.com

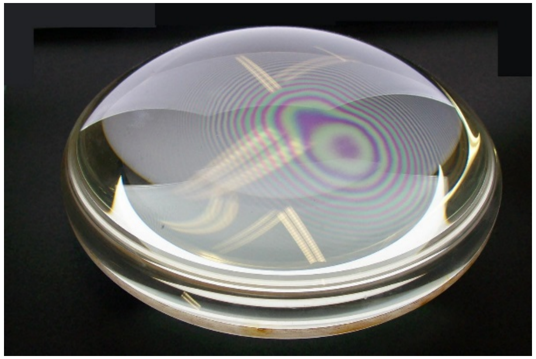
Wave theory triumphs:

Young, Fresnel (early 19th century):

Newton's rings, double-slit experiments

→ interference!

Light is a transverse wave.



Credit: U. Seifert @ Wikimedia Commons

Aether

Waves need a medium for propagation

→ **“aether”:**

Filling entire space.

Invisible, massless, no viscosity.

Much more rigid than steel to support the high frequencies (4–8 × 10^{14} Hz !) of light waves.

? ? ? Mystery ? ? ?

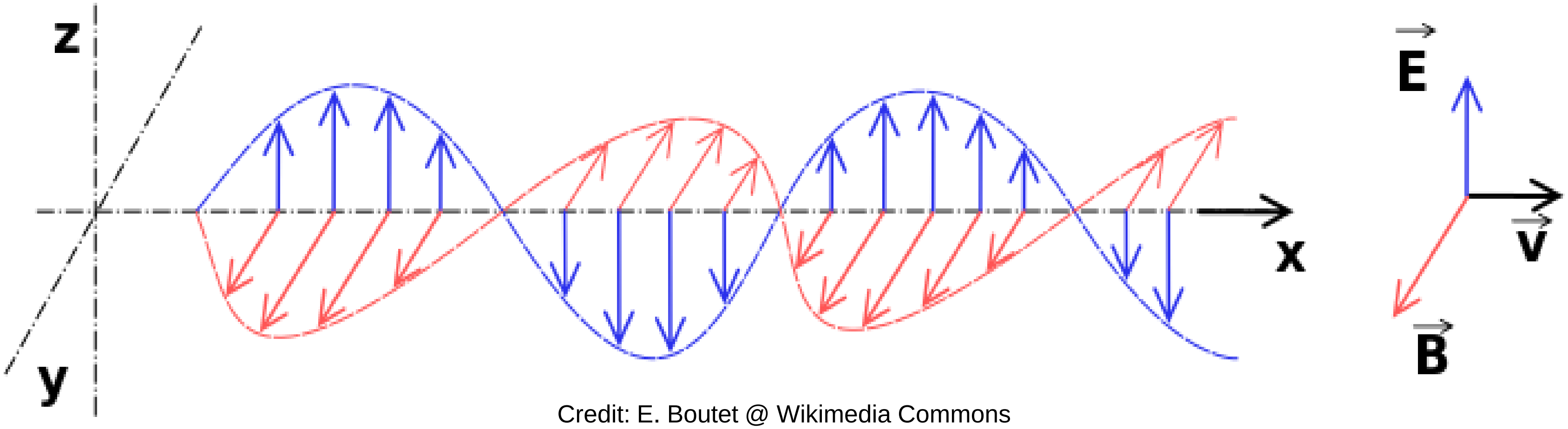
Electromagnetic theory

Faraday, Maxwell (mid-19th century):

Light is an electromagnetic wave.

Doesn't need a mechanical medium,

propagates through empty space.



Credit: E. Boutet @ Wikimedia Commons

Maxwell's (macroscopic) equations

Gauss's law: $$\nabla \cdot \mathbf{D} = \rho_\mathrm{f}$$

Gauss's law for magnetism: $$\nabla \cdot \mathbf{B} = 0$$

Faraday's law of induction: $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Ampère's law: $$\nabla \times \mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial \mathbf{D}}{\partial t}$$

E: electric field

D: electric displacement field

B: magnetic B-field (flux density)

H: magnetic H-field

ρ_f: free charge density

$\mathbf{J}_\mathrm{f}$: free current density

Closure requires constitutive equations

In vacuum (= empty space):

$$\mathbf{D} = \varepsilon_0 \mathbf{E}$$

$$\mathbf{H} = \mathbf{B}/\mu_0$$

ε_0: permittivity of free space
μ_0: permeability of free space

→ Wave equations:

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

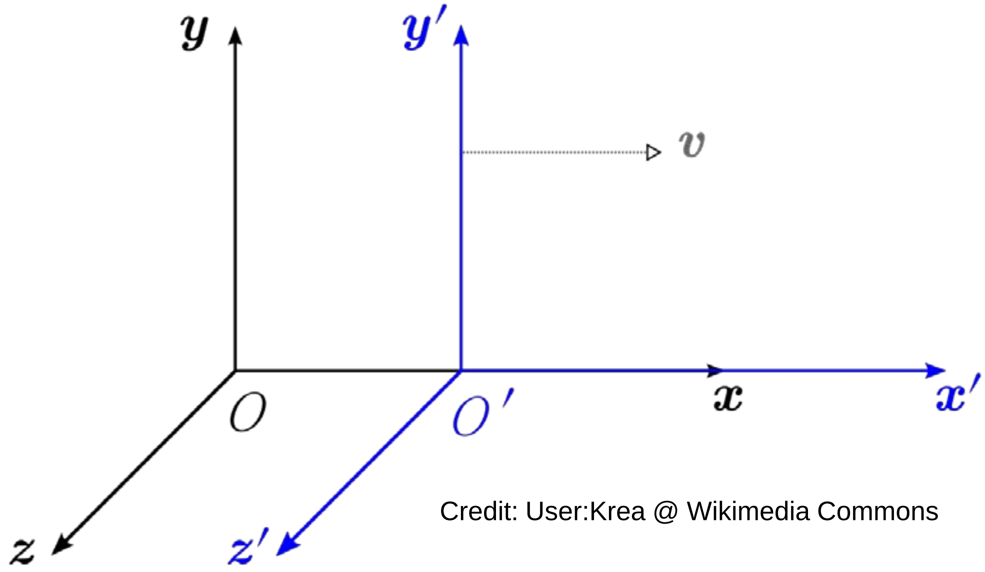
with $c = \dfrac{1}{\sqrt{\varepsilon_0\,\mu_0}}$

c: speed of light in free space
(c = 299,792,458 m/s)

Special relativity

Maxwell's equations are invariant in all inertial systems

In particular, *c* is the same for all observers



Credit: User:Krea @ Wikimedia Commons

Lorentz transformation:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad y' = y$$

$$x' = \gamma(x - vt) \qquad z' = z$$

$$\text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

~~Independent existence of space and time~~ → **spacetime**

Minkowski space

= flat 4D spacetime of special relativity

Four-position: $x^0 = ct\,, \quad x^1 = x\,, \quad x^2 = y\,, \quad x^3 = z$

Invariant line element:

$$\begin{aligned} \mathrm{d}s^2 &= c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \\ &= (\mathrm{d}x^0)^2 - (\mathrm{d}x^1)^2 - (\mathrm{d}x^2)^2 - (\mathrm{d}x^3)^2 \end{aligned}$$

$\rightarrow$ Metric tensor: $(g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ (time-positive)

General relativity

Einstein tensor:

$$G_{\mu\nu} = \mathrm{fct}(g_{\mu\nu} + \text{its derivatives})$$

(describes curvature of spacetime)

Energy–momentum (stress–energy) tensor:

$$(T^{\mu\nu}) = \begin{pmatrix} w & \frac{S^1}{c} & \frac{S^2}{c} & \frac{S^3}{c} \\ \frac{S^1}{c} & G^{11} & G^{12} & G^{13} \\ \frac{S^2}{c} & G^{12} & G^{22} & G^{23} \\ \frac{S^3}{c} & G^{13} & G^{23} & G^{33} \end{pmatrix}$$

w : energy density (often $\approx \rho c^2$)

S^i : energy flux density

$G^{ij} = \sigma^{ij} + \rho v^i v^j$: absolute stress tensor

("relative" stresses + advection)

Einstein field equations

Mass / energy / momentum act as source for curvature of spacetime:

$$G_{\mu\nu} \ \ (+\Lambda g_{\mu\nu}\ ) \ = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$G_{\mu\nu}$: Einstein tensor
$g_{\mu\nu}$: Metric tensor
$T_{\mu\nu}$: Energy–momentum (stress–energy) tensor
G : Newton's gravitational constant
(Λ : Cosmological constant)

Can be interpreted as a
constitutive equation of spacetime.

Geodesic equation

Force-free motion along geodesics of the curved spacetime:

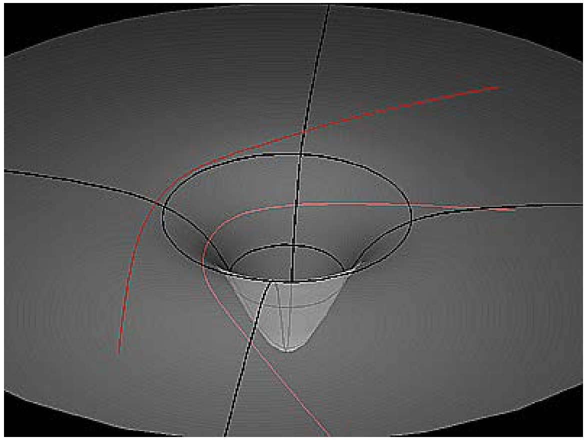
$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} = -\Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}$$

x^μ : Four-position (ct, x, y, z)

τ : Proper time (measured when following the motion)

$\Gamma^\mu{}_{\alpha\beta}$: Christoffel symbol [= fct($g_{\mu\nu}$ + derivatives)]

Gravity:
Not external force,
but consequence of
curved spacetime.



Credit: User:Loveless @ Wikimedia Commons

Gravitational waves

Einstein field equation [$G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$]
+ some simplifying assumptions
→ wave equation for the metric tensor.

Solutions: **Gravitational waves**

Ripples in the curvature of spacetime.

Travelling at the speed of light *c*.

Transverse waves
(like electromagnetic waves).

Radiated by accelerated masses
(not if motion is spherically of rotationally symmetric).

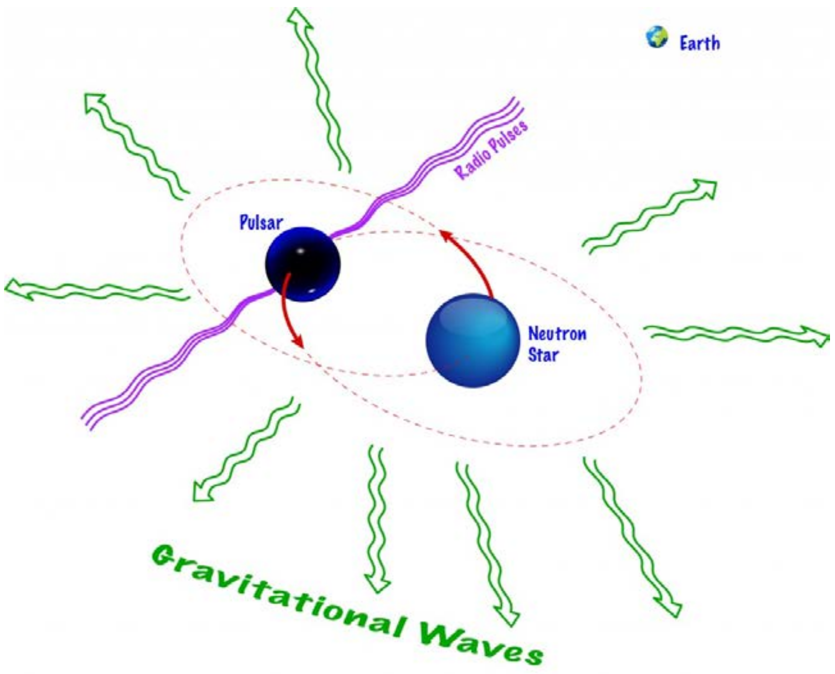
Indirect detection of gravitational waves

Hulse–Taylor binary

Binary pulsar,
discovered in 1974.

Gradual decay of its 7.75-hour orbit due to radiation of gravitational waves (power ~ 7×10^{24} W).

(Solar system only ~ 5 kW)



Credit: Shane L. Larson, NASA/GSFC

1993 Nobel Prize in Physics

Direct detection of gravitational waves

Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) – 2 observatories in the US

LIGO Livingston (LA) Observatory. Credit: www.ligo.caltech.edu

Direct detection of gravitational waves

Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

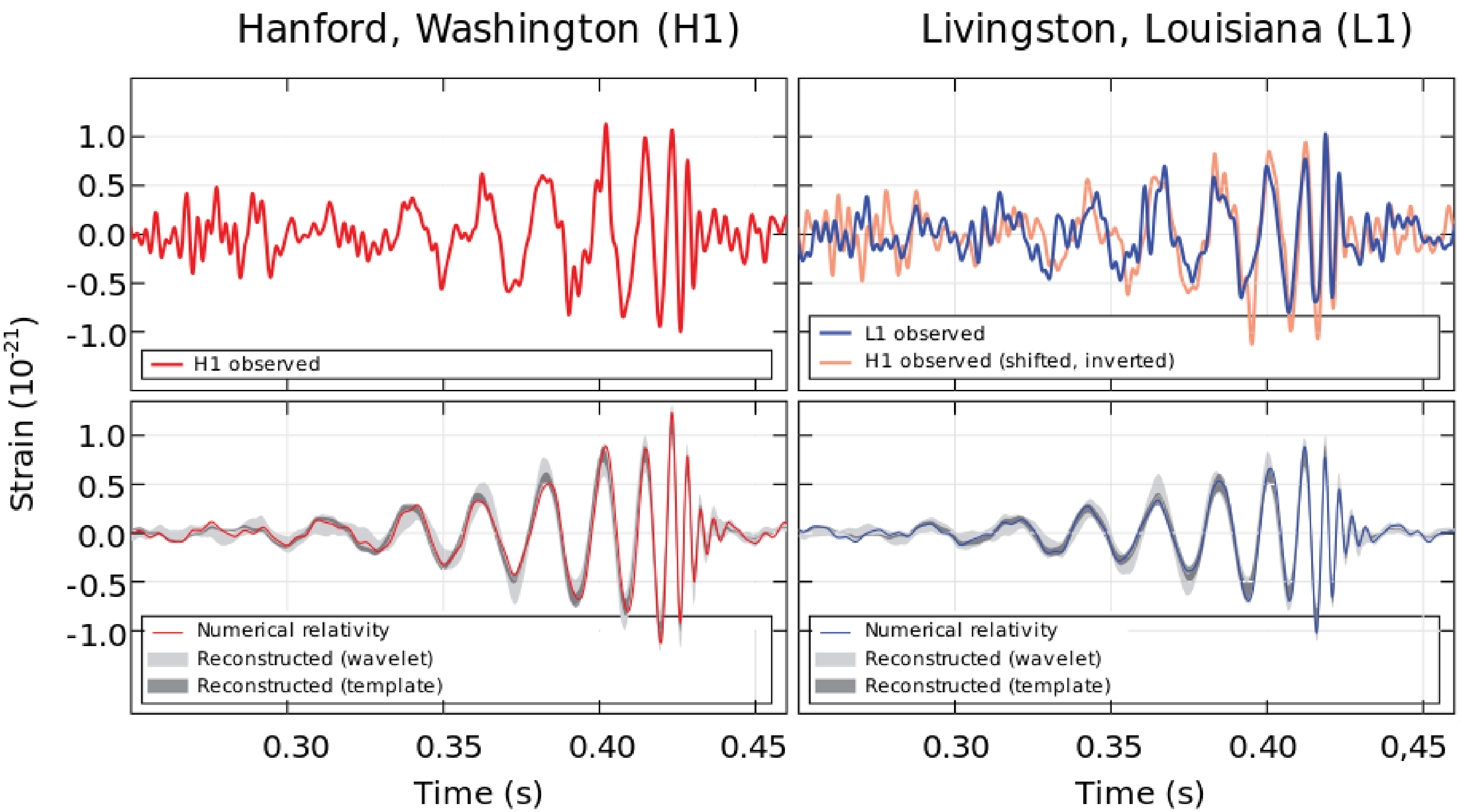
Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

Direct detection of gravitational waves



20xx Nobel Prize in Physics ?

Summary

Spacetime itself can be considered a material:

physical properties, constitutive equations.

Alternative interpretation:

spacetime = aether.

Open questions, speculations

Physics of black holes & Big Bang:

General relativity + quantum mechanics.

Other spacetimes

(or something completely different)?

THANK YOU

Credit: dansalisbury.com