



HOKKAIDO UNIVERSITY

Title	Fracture Mechanics Based Fatigue Life Prediction Method for RC Slabs in a Punching Shear Failure Mode
Author(s)	鄧, 朋儒
Degree Grantor	北海道大学
Degree Name	博士(工学)
Dissertation Number	甲第12906号
Issue Date	2017-09-25
DOI	https://doi.org/10.14943/doctoral.k12906
Doc URL	https://hdl.handle.net/2115/67519
Type	doctoral thesis
File Information	Deng_Pengru.pdf



**FRACTURE MECHANICS BASED
FATIGUE LIFE PREDICTION METHOD FOR RC SLABS IN
A PUNCHING SHEAR FAILURE MODE**

(破壊力学に基づく鉄筋コンクリート床版の押し抜きせん断疲労寿命の予測手法)

Pengru DENG

Division of Engineering and Policy for Sustainable Environment
Graduate School of Engineering
Hokkaido University
Sapporo, Japan

September, 2017

FRACTURE MECHANICS BASED FATIGUE LIFE PREDICTION METHOD FOR RC SLABS IN A PUNCHING SHEAR FAILURE MODE

Pengru DENG

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY IN ENGINEERING

Examination Committee:

Prof. Takashi MATSUMOTO
Prof. Tamon UEDA
Prof. Hiroshi YOKOTA

Division of Engineering and Policy for Sustainable Environment
Graduate School of Engineering
Hokkaido University
Sapporo, Japan

September, 2017

ACKNOWLEDGMENT

First and foremost, I would like to express my deepest and heartfelt gratefulness to my supervisor, Professor Takashi Matsumoto, for his expert guidance, dearly encouragement and constructive criticisms throughout this research work. His advice on both research as well as on my way of thinking are priceless. His kindness and warm treatment are of great appreciation. It was a great experience having been working under his guidance.

I would also like to thank my committee members, Professor Tamon Ueda and Professor Hiroshi Yokota for their brilliant comments and suggestions.

My sincere gratitude is extended to Professor Toshiro Hayashikawa and Professor Xingwen He. Their valuable advice and comments are greatly appreciated. My sincere thankfulness is also extended to Professor Xuhui He who kept concerning about my research and life in Hokkaido University.

I also wish to thank the members of the Laboratory of Bridge and Structural Design Engineering. They have contributed immensely to my personal and professional time at Hokkaido University. The lab has been a source of friendships as well as good advice and collaboration.

I am grateful for my friends in Japan and my wonderful seniors. They made my life in Sapporo colorful. With their help, encouragement and concern, I have never felt lonely during these three years study.

I would like to acknowledge the Graduate School of Engineering, Hokkaido University, who accepted me as a graduate student and provided help whenever I encountered problems. I would also like to acknowledge my founding sources that allowed me to pursue my graduate studies: The China Scholarship Council.

Finally, I would like to dedicate my research work to my family who supported me during my time here. I would to express my greatest gratitude to my Mom and Dad for their constant love and support. I would also thank my elder sister who supported me and helped me taking care of my parents. Without your love and support, I cannot reach my present position.

ABSTRACT

In the last few decades, a punching shear failure model emerged as the main failure mode for RC bridge slabs subjected to repetitive moving load, which leads to an intensive study on life prediction of these RC bridge slabs. Considering that the reality load condition is too complicated to study, many studies have been conducted to predict the fatigue life of RC slab under a relatively simple cyclic moving load, because the punching shear failure mode and crack pattern of RC slabs in reality have been reproduced under this load condition in tests. However, most of the existing life prediction approaches are either empirical equations from fitting experimental data or time-consuming finite element method based numerical methods. Therefore, it is important to develop an efficient fatigue life prediction method which can account for and reflect internal degradation mechanisms.

In this thesis, a theoretical fatigue life prediction method for RC bridge slabs subjected to cyclic moving load is developed based on fracture mechanics. Compared with the existing researches, this theoretical method can not only account for and reflect the internal degradation mechanisms of RC structures under cyclic moving loads, but also save a lot of computing time.

The proposed life prediction method for RC slabs under cyclic moving load is introduced in detail as follows.

Firstly, according to the cracking process of an RC slab under cyclic moving load and the theoretical basis for the existing empirical life prediction equations, the problem of life prediction for the RC slab is simplified into the life prediction of a critical RC beam located in the midspan of the RC slab focusing on the punching shear cracks.

Secondly, since a crack width related concrete bridging degradation was confirmed as a primary degradation mechanism for RC slabs under cyclic moving load, a crack width determination method for RC beams is proposed based on a fracture mechanics based integral equation with a bond slip model.

In the integral equation, weight functions for the crack geometry are required. As the analytical weight functions for the critical beam with punching shear cracks are unavailable, the weight functions are determined from a finite element method based virtual crack extension technique in this study.

And then, accounting for the concrete bridging degradation and the bond slip degradation in the concrete/rebar interface, cracking states of the punching shear crack after every loading cycle are determined following the idea that the effects of sectional actions and reactions are equivalent to each other. With the evolving cracking states, the fatigue failure moment, i.e. fatigue life, is determined following some brittle shear failure modes.

Furthermore, based on the developed method, the fatigue life for several loading levels are calculated and then drawn together with the experimental fatigue life and results from several empirical fatigue life prediction equations in a double-logarithmic scale. The good agreement with fatigue life from other approaches verifies the reliability of this method.

Finally, the stresses of all materials in the entire life range are computed as well, from which a more confident and straight-forward image on the degradation mechanisms that should be included in fatigue analysis is figured out. The degradations of compressed concrete and tensioned rebars are concluded as negligible under service load conditions. As a further step, the sectional forces, moments and crack openings due to different components are calculated. It is found that the compressed concrete and tensioned rebars play dominant roles in resisting sectional rotation and crack opening which is coincident with common believes. However, the contribution from cracked concrete should be included as well for an accurate analysis.

Owing to the advantages of this method compared with existing researches, this method can be applied for many purposes. For example, due to the time-saving characteristic of this method, it is suitable for conducting parametric study on some design parameters, such as slab thickness, concrete modulus and reinforcement ratio, to identify their influences on fatigue life. This information can provide advice for structural design. The effect of environment related material and concrete/rebar interface deteriorations can be analyzed as well. Besides, for the existing RC slabs reinforced with certain measures, this method can also be employed to study the improvement owing to the measure. Obviously, all these researches are very meaningful.

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CHAPTER 1 INTRODUCTION

1.1. Background

The slab-on-girder superstructure is one of the most common structural systems used for highway bridges around the world. In Japan, more than 130,000 highway bridges are with this system. In this system, as the supported deck slab directly sustains repeated moving wheel load, it is one of the most bridge elements susceptible to fatigue failure. Consequently, fatigue performance is an important limit state that must be considered by designers of bridge decks.

However, provisions for fatigue safety evaluation for concrete structures have only recently been introduced into codes. These provisions rely on a narrow knowledge basis when compared to most other domains of concrete structures. As a result, the introducing of fatigue considerations has lead to significant changes to the design standards.

In addition, the employed theories for analysis and design of reinforced concrete slabs were based on some inappropriate assumptions. For example, according to the previous AASHTO provisions [1], concrete bridge decks are designed as transverse beams supported transversely on steel girders carrying the concentrated wheel loads entirely in flexure. The design code for reinforced concrete bridge deck slabs in Japan follows an allowable stress method based upon the thin elastic plate bending theory. All of these theories are focused on the flexural capacity of RC slabs regardless of the shear capacity of RC slabs, whereas the flexural capacity is generally higher than the shear capacity and punching shear failure is a possible failure mode for a concentrated wheel load applied on a RC slab. If the punching shear capacity is taken into consideration, RC slabs of the existing highway bridges were commonly designed to be lack of reinforcement rebar and slab thicknesses. Consequently, many instances of damage or collapse of bridge deck slabs due to punching shear fatigue failure have been reported in North American, Europe, Japan, etc [2-4]. To ensure safety of highway bridges, many projects on slab repairing or even replacement are held by both governments and companies, for example, a 378.9 billion Japanese yen slab repairing and replacement project is scheduled by the NEXCO East Group in Japan.

Therefore, it is of great significance conducting fatigue analysis of RC slab subjected to repeated wheel moving load to figure out the fatigue behavior, failure model and further fatigue life. These fatigue information are essential for strategic maintenance engineering of the structures. They can also provide references for deciding a suitable time and a suitable repair method for each structure, which are more important considering that insufficient consideration of fatigue degradation in the past theories and design codes.

1.2. Literature review

1.2.1. Punching shear failure mode of RC bridge slabs

Punching shear failure is referred to as a local shear failure that could occur around concentrated loads. This failure mode is commonly observed in reinforced concrete flat plates around columns because large bending and shear stresses developed and concentrated at these slab-column connections. For a thin bridge slab subjected to an increased wheel load, the

requirements for an occurrence of the punching shear failure are satisfied. That is why the punching shear failure occurs in bridge slabs.

Referencing to former design codes, RC bridge slabs were designed with small thickness, which makes the RC slabs too weak for sustaining the shear forces from concentrated wheel loads. As a result, a punching shear failure mode emerged as a main failure mode for RC bridge slabs under repetitive wheel loads, especially with the increasing of vehicle weight and traffic volume. Consequently, the fatigue life of RC slabs is drastically shortened, posing a great threat to the life and property safety of our society.

To establish theories on analysis the punching shear failure of RC bridge slabs, many research works have been conducted systematically. Obviously, a bridge slab which fails under reality load conditions is too complicated to be analyzed comprehensively to date. Researches on RC bridge slabs under relatively simple loading conditions have been conducted to figure out the failure modes of RC slabs.

Primarily, as the punching shear failure is the major failure mode of RC slab under a concentrated load on a small loading area, an equation of determined punching shear capacity was proposed in [5] for RC bridge slabs, where the punching shear acting surfaces were assumed as spreading from edges of the load area with a 45° respect to the slab bottom surface and the punching shear capacity was assumed from concrete in the compression zone and the dowel effect of rebars.

Perdikaris et. al. studied the effects of pulsating and traveling wheel loads on RC slabs designed with both orthotropic reinforcement and isotropic reinforcement [6, 7]. Experiments on the effects of traveling wheel fatigue on concrete bridge decks were also conducted in [8]. The observed shorten fatigue life of RC slabs in service was successfully reproduced through experiment, under cyclic moving load [5, 6]. The experimental moving load set-up is shown as **Figure 1.1**.

According to the failure crack pattern of slabs under traveling wheel load, an empirical S-N equation for life prediction of RC slabs under moving load was formulated as a function of applied load/punching shear capacity of an assumed beam in [8] through fitting a huge set of experiment data statistically.

The theory basis was then successfully employed for analyzing punching shear behavior of RC slab or retrofitted RC slabs under fatigue loading, frost damage in [9, 14]. The theories on fatigue characteristics of RC bridge slabs under static, fixed-point pulsation, traveling wheel load and load in reality were summarized in [15]. However, as the life prediction equations were derived statistically, inner mechanisms cannot be effectively reflected.

The above-mentioned works concluded that fatigue life of RC slab under traveling wheel load is determined by the punching shear cracks of a critical RC beam located in the midspan of the slab and vertical to the wheel moving load.

Moreover, taking advantage of finite element method, the punching shear failure model was reproduced through finite element analysis of RC slabs reinforced with plain bars in [16], where a numerical method based on the concrete bridging stress degradation concept is presented. This

literature also concluded that the bond slip effect should be efficiently included for accurate fatigue analysis. [17] successfully predicts the characteristic mode of failure (punching shear failure) under moving loads based on a direct path-integral scheme with fatigue constitutive models of concrete as well. Even though all the numerical approaches in finite element method were established on the basis of smeared crack model which requires less for convergence than another widely-employed model, discrete crack model, it was still very time-consuming for reaching the final convergence due to the widely-spread cracked element before failure. From the existing numerical analysis, it can be concluded that tension fatigue modeling for concrete plays an important role in predicting the fatigue behavior of RC slabs. In addition, the bond slip characteristic should be considered carefully.

However, existing researches possess their own disadvantages. For the empirical life prediction equations, since the equations were from statistically fitting experiment data, the inner degradation mechanism cannot be captured and reflected. In terms of the numerical approaches based on finite element method, convergence problem were commonly encountered owing the widely distributed crack before failure. In addition, the numerical analysis may take a long time even though a relatively time-saving modeling method, the smeared crack element model, is employed.

Therefore, innovation of an efficient and sophisticated life prediction method considering the dominant degradation mechanisms of RC bridge slabs under cyclic moving loads should be meaningful for structural design, maintenance and strengthening.

1.2.2. Mechanical degradations of RC structures under repetitive loads

In order to develop an effective method for predicting fatigue performances of RC bridge deck slabs, fatigue damage mechanisms are essential in an analytical modeling. The mechanism of structural fatigue can be explained by integrating fatigue damage evolution of materials over extremely small increments under different load conditions over a long timescale. As an RC slab under fatigue loads consists of two materials (concrete and steel rebar) under three stresses (compression, tension and shear) which are connected with each other with one interface (concrete/rebar interface), there are seven possible degradations in the RC slabs.

As for the concrete, it is a composite material composed of coarse aggregate bonded together with fluid cements that hardens over time, which is inherently full of flaws (such as pores, air voids, lenses of bleed water under coarse aggregates). Under uniaxial compression, damage (reduction of elasticity) is attributed to growth of distributed micro cracks. A comprehensive time-dependent constitutive model, which encompasses only near- and post-peak region in concrete compression, is proposed in [18] considering that the damage evolution was not seen under low stress states less than 30% of the specific uniaxial compression strength. These stress states are similar to stress levels of RC structures under service load conditions. Under uniaxial tension, the fatigue behaviors before and after cracking are tremendous different. Before cracking, it has been reported that plain concrete subjected to repeated uniaxial tension stresses exhibits no fatigue limit under 2×10^6 cycles [19, 20]. Hence there is no stress level below which the fatigue life of plain concrete will be infinite. After cracking, concrete behavior under tension fatigue is also dominated by crack propagation; early age micro-cracks in the cement matrix and at the interface between aggregates and the cement matrix propagate steadily and perpendicular to the load direction until the specimen fractures showing one discrete crack. The degradation of

cracked concrete is generally formulated as a concrete bridging degradation model in [21, 22]. The concrete bridging degradation has been concluded as the dominant degradation mechanism for concrete. Considering that cracking of a reinforced concrete member obeys the general laws of fracture mechanics, it is acceptable that the cracks appear in a direction perpendicular to that of principal tensile stresses in the concrete. Correspondingly, the concrete degradation due to shear stresses is not the primary factor.

Unlike the heterogeneous concrete, the steel rebar is an isotropic material. Therefore, the fatigue behaviors of rebars under different stress states are similar and can be analyzed at the same time. According to observations of fatigue behaviors of a steel under various levels of cyclic loads, many constitutive models have been proposed [23, 24]. Most of these models possess a common which is no degradation is assumed under stress lower than the yielding strength.

In terms of the bond between concrete and rebars, fatigue loading causes progressive deterioration to the interface even under relatively low load level. Therefore, the degradation mechanism of bond slip between concrete and rebar should always be included to figure out the degradation mechanisms of RC structures under fatigue loads.

Conclusively, for a RC structure under service loading level repetitive loads, at least two degradation mechanisms, i.e. concrete bridging degradation and bond slip degradation in the concrete/rebar interface, should be included for an accurate fatigue analysis.

1.3. Objectives

The main objective of this thesis is proposing a theoretical fatigue life prediction method for a RC slab failed in a punching shear failure model under cyclic moving loads based on nonlinear fracture mechanics for concrete cracking and dominant degradation mechanisms of RC structures under repetitive fatigue loads.



Figure 1.1 Experiment set-up for cyclic moving loads

1.4. Technical contributions

An original contribution of this research is that a theoretical life prediction method for a RC slab under a cyclic moving load is developed based on some simplifications according to fatigue behaviors of the RC slab, nonlinear fracture mechanics for concrete cracking and some dominant degradation mechanisms of RC structures under fatigue loads. Compared with the existing empirical life prediction equations from fitting experiment data statistically, the method can consider and reflect the internal mechanisms of RC slabs under cyclic moving load. In addition, this method takes about 1 hour for one calculation which is much shorter than existing finite element method based numerical approaches. The method is checked through comparing with experimental fatigue life and some empirical equations and found sufficiently accurate.

In this method, crack mouth opening displacements due to applied loads and crack bridging stress are calculated following a fracture mechanics based integral equation, where the weight functions for cracked geometries are required. Since analytical crack face weight functions for punching shear cracks are unavailable, the weight functions are determined through fitting and interpolating nodal weight functions obtained following a finite element method based virtual crack extension technique.

Since the method is focused on the propagation of the critical punching shear cracks, rebar stresses at crack locations should be determined. In this study, a method of determining rebar stresses at a crack is originally proposed through combining strengths of a widely employed smeared crack model and a bond slip model. The smeared crack model and the bond slip model are employed in determining average rebar stress and a relationship between peak rebar stress at cracks and average rebar stresses, respectively. With the obtained average stresses and peak stress/average stress relations, the peak rebar stresses are determined.

From this method, for materials along the punching shear crack cross-section, the stress states over an entire life range are computed. With the obtained material stresses, researches can figure out a more confident and straight-forward image on the degradations that should be included for an accurate analysis of RC slabs under cyclic moving loads. Furthermore, dominant sectional components for resisting sectional rotation and crack opening can be identified because all forces, moments and CMODs due to different components are obtainable using the corresponding material stresses.

1.5. Organization of the thesis

This thesis is organized into seven chapters.

Chapter 1 describes the background, objective, technique contributions of this research and the organization of this thesis.

Based on experimental observations and theoretical basis for existing researches, Chapter 2 simplifies a life prediction of a RC slab under a cyclic moving load into a life prediction of a critical RC beam cutting from the RC slab focusing on a propagation of punching shear cracks.

Since a crack width related concrete bridging degradation was confirmed as a dominant degradation mechanism for RC slabs under cyclic moving loads, Chapter 3 introduces an analytical approach of determining a crack width of a bridged crack in a RC structure, where a total crack width is decomposed into crack widths from applied loads, crack bridging stresses and bond slip in the concrete/rebar interface. For crack widths due to applied loads and crack bridging stresses, they are calculated following a fracture mechanics based integral equation. The crack width due to bond slip is calculated by introducing a bond slip model.

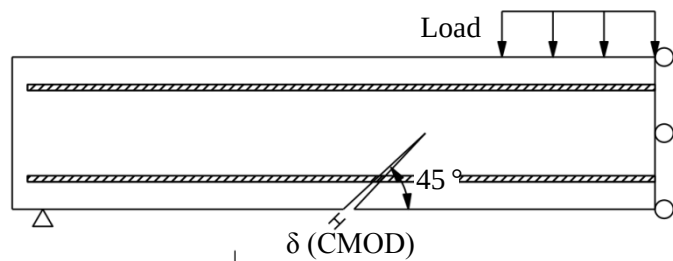
Considering that analytical weight functions, which are necessary for the integral equation, for a shear crack in a RC beam are unavailable, Chapter 4 presents a weight function determination method for shear cracks of RC beams based on finite element method.

In Chapter 5, a theoretical life prediction method of a RC slab subjected to cyclic moving load is proposed using the crack mouth opening displacement (CMOD) determination method and weight functions introduced in Chapter 3 and 4, respectively. Procedures for Chapter 5 are shown in **Figure 1.2**. In this method, structural degradation and crack propagation are regarded as a result of concrete bridging degradation and bond slip degradation in the concrete/slip interface, and the final failure moment is identified following some brittle shear failure criterion. This method is then verified by a good agreement in a fatigue life comparison with experimental results and some empirical equations.

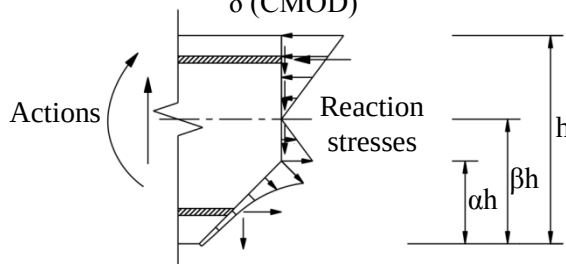
Chapter 6 presents stress states of all materials in the entire fatigue life, which can be referenced in determine dominant degradations. Furthermore, contributions from all components for resisting sectional rotation and crack opening sectional rotation are computed as well.

The final chapter, Chapter 7, contains a summary, overall conclusions and recommendations for future research.

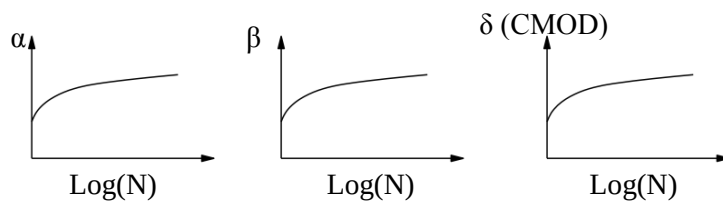
Step 1: Half of the critical RC beam with a punching shear crack.



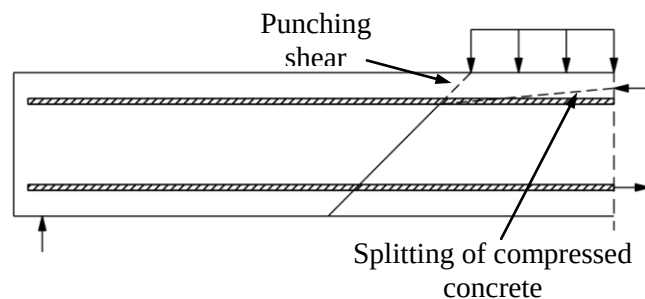
Step 2: Expresses all sectional stresses with α , β and δ after every loading cycle based on assumptions and material models considering degradations.



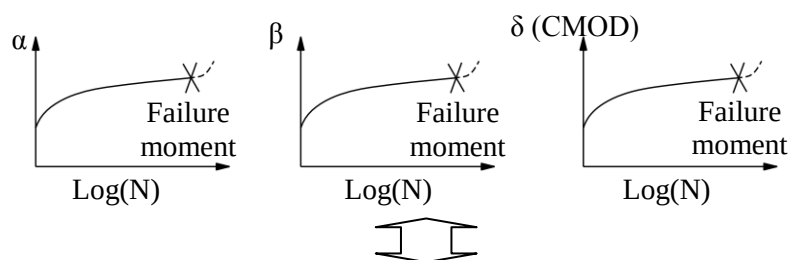
Step 3: Determine α , β and δ after every loading cycle through solving 3 independent equations.



Step 4: Select failure criterion referencing existing research and experimental observation



Step 5: Determine the failure moment, i.e. fatigue life.



Step 6: Method verification

- Comparison fatigue life with:
1. Experiment
 2. Empirical equations from research groups
 3. Empirical equations from codes.

Figure 1.2 Schematic diagram for fracture mechanic based life prediction method in Chapter 5

CHAPTER 2 PROBLEM SIMPLICITY

As fatigue behavior, failure mode and failure crack pattern for RC slabs under cyclic moving loads are close to those of slabs under load in reality, this study will focus on a fatigue analysis of RC bridge slabs under cyclic moving loads. However, a life prediction of RC slabs under cyclic moving is still too complicated. Therefore, before conducting fatigue analysis, a fatigue life prediction problem of RC slabs is simplified into the fatigue life prediction of a RC beam according to a cracking process of the RC slabs under cyclic moving loads and theoretical basis for existing empirical equation in this chapter.

2.1. Cracking process of RC slabs under cyclic moving load

For a RC slab under cyclic moving loads, a typical failure crack pattern is shown as **Figure 2.1** and the cracking process can be illustrated as following:

Step 1: Due to the existence of supporting girders, the drying shrinkage of concrete is constrained and correspondingly micro shrinkage cracks initiate along the direction vertical to the moving load direction as shown in **Figure 2.1**. Meanwhile, the moving load leads to the initiation and propagation of cracks along the same direction with the shrinkage cracks because of the generally smaller reinforcement ratio along the bridge axis direction for RC bridge slabs.

Step 2: With continuing cycles of moving load, a bidirectional anisotropy characteristics of the RC slab increases with the increasing number of cracks vertical to the moving load direction and propagations of the initiated cracks in Step 1. These parallel cracks, which can be observed from the b-b cross-section in **Figure 2.1**, divide the RC slab into several RC beams. An RC beam at midspan is the most dangerous beam and regarded as a critical beam. The width of the beam can be determined following [8].

Step 3: Due to a isolating effect of the parallel cracks, more bending moment is transferred along the direction vertical to the moving load and shear forces cannot be transferred effectively to the adjacent RC beams when the moving load moves onto one RC beam and also the critical beam. The load which is expected to be sustained by the whole slab is almost supported by the critical beam.

Step 4: In the critical beam, a large shear stress is concentrated around loading area. It is this shear stress that leads to propagations of a couple of punching shear crack symmetrical with respect to the moving load lane along the lines with almost 45° with respect to the bottom surface as shown in a-a cross section in **Figure 2.1**. These punching shear cracks lead to the final brittle failure.

2.2. Theoretical basis for existing empirical equations

According to the cracking process and failure crack pattern of RC slabs under a cyclic moving load, an empirical life prediction equation illustrating load levels versus number of loading cycles were formulated successfully using a punching shear capacity of the critical RC beam as a normalizing parameter in [8]. In this empirical equation, the punching shear capacity of the critical RC beam is the only parameter used except for applied loads and number of

loading cycles, which indicates that the fatigue life of the RC slab depends on the fatigue life of the critical RC beam. Following the same idea, several empirical life prediction equations have been derived by some other research teams, such as Abe research teams, and institutions, such as Japan Society of Civil Engineers (JSCE) and Public Work Research Institute (PWRI). The reported equations are listed as:

(1) Matsui equation

$$\log(P/P_{sx}) = -0.07835 \log N + \log 1.52. \quad (2.1)$$

(2) Abe equation

$$\log(P/P_{sx}) = -0.06417 \log N + \log 0.996. \quad (2.2)$$

(3) JSCE equation

$$\log(P/P_{sx}) = -0.04263 \log N + \log 0.790. \quad (2.3)$$

(4) PWRI equation

$$\log(P/P_{sx}) = -0.0545 \log N + \log 0.956. \quad (2.4)$$

where P and P_{sx} are applied load and a punching shear load capacity of the critical RC beam, respectively. All the equations are in a form as:

$$\log(P/P_{sx}) = k \log N + C \quad (2.5)$$

where k and C are constants. In is equation, the applied load is normalized with the punching shear load capacity.

It is found that relationships between the applied load (P) and applied number of loading cycles (N) are illustrated using the punching shear load capacity (P_{sx}) of the critical RC beam, which means the fatigue life of RC slabs under certain loading levels can be related directly to the fatigue life of the critical RC beams.

2.3. Problem simplification

According to the introduced cracking process and failure crack pattern, it is found that the fatigue failure of a RC slab subjected to a cyclic moving load is determined by the punching shear failure of some RC beams divided by a serial of parallel cracks perpendicular to the moving load direction. Among the RC beams, a RC beam located at the midspan is the critical one which determines the final failure. [8] proposed a simple method of determining the dimension of this critical RC beams, which is shown in **Figure 2.2**, where h and l are the depth and span of the RC beam, respectively; l_w and b are the length and width of the wheel/beam contact area; d_e is the effective depth for tensile rebar. Using the punching shear capacity of this RC beam as the only parameter, some empirical life prediction equations were successfully formulated.

Conclusively, the life prediction of a RC bridge slabs under a cyclic moving load can be simplified into the life prediction of a critical RC beam focusing on the propagation of punching shear cracks. For the critical RC beam as shown in **Figure 2.2**, the punching shear cracks are assumed to propagate along 45° lines symmetrical with respect to the moving load.

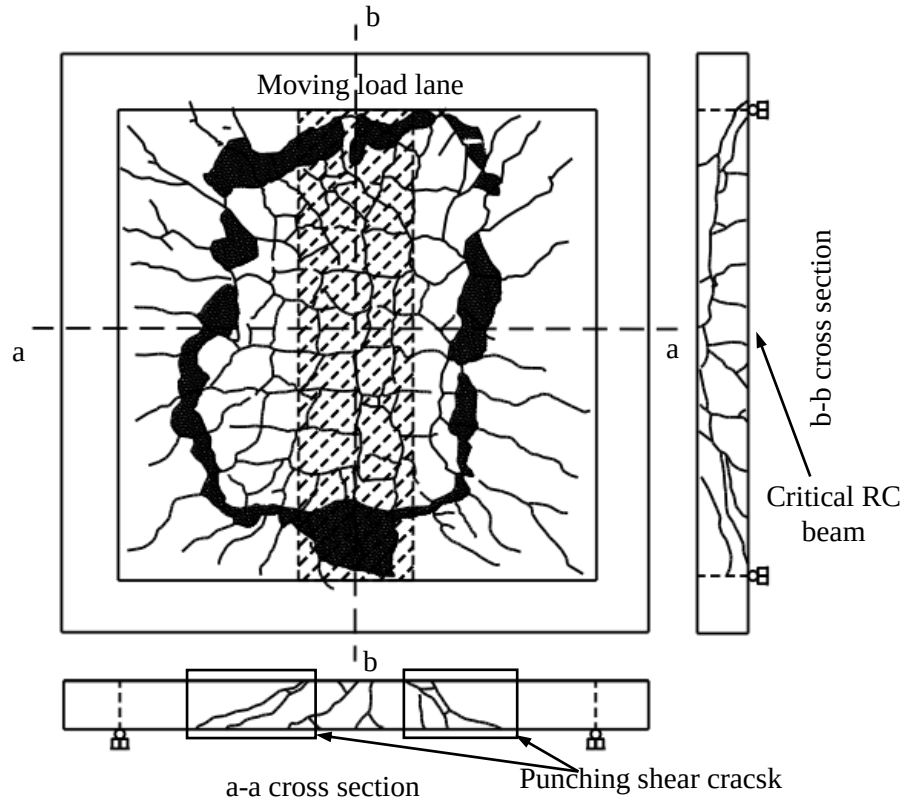


Figure 2.1 A typical failure crack pattern of a RC slab under cyclic moving loads

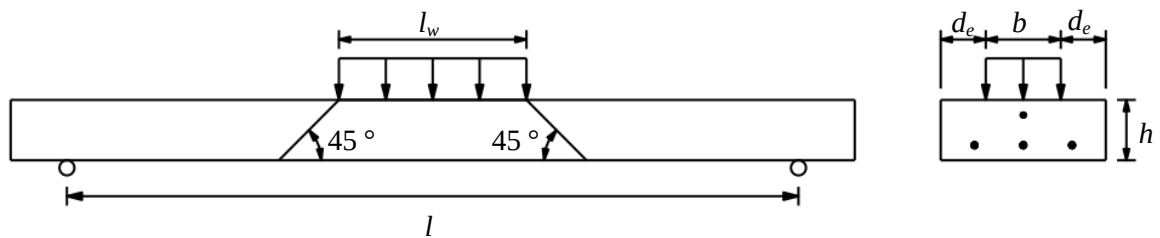


Figure 2.2 Geometry of the simplified RC beam and the punching shear cracks

2.4. Summary and conclusions

This chapter presented the cracking process and failure crack pattern of a RC slab subjected to cyclic moving loads, from which it was concluded that fatigue failure of RC slabs depends on a punching shear failure of a critical RC beam located in the midspan of the RC slabs and vertical to the cyclic moving load.

In addition, some empirical life prediction equations were formulated on the basis of a punching shear capacity of the critical RC beam, which indicated the fatigue punching shear failure of the critical RC beam determines the fatigue life of corresponding RC slabs.

According to existing researches and the fatigue behavior of RC slabs under cyclic moving loads, a life prediction the RC slabs under cyclic moving loads was simplified into a life prediction of the critical RC beam focusing on the punching shear cracks. The dimensions of this critical RC beam were determined following a widely-employed method introduced in [8]. Further analysis in this study is planned to be conducted on this RC beam.

CHAPTER 3 CRACK OPENING DISPLACEMENT DETERMINATION OF A RC STRUCTURE BASED ON FRACTURE MECHANICS

3.1. Introduction

In reinforced concrete (RC) structures, incorporating ductile steel rebars into brittle concrete matrix allows to improve several mechanical properties, such as cracking resistance, ductility, impact resistance and fatigue resistance. Consider a crack in an RC structure under external loads, the crack opening is governed by two mechanisms: the activation of bond forces at the rebar-concrete interface and the bridging effect of rebars crossing the crack. Over the last few decades, both mechanisms have been extensively studied in different fields and through different approaches.

For the crack opening due to externally applied loads and concrete bridging stresses, a integral transformation relating crack opening displacements (COD profile) with applied loads and crack bridging forces for various geometries and load conditions have been proposed in [25-30] based on fracture mechanics. By assuming rebar forces as a step function and following a weight function method for determining the stress intensity factor (SIF), a transformation between the rebar force crossing a crack under model I loading and the COD profile has been derived in [31].

In terms of the crack opening due to bond slip between concrete and rebars, many experimental and theoretical investigations have been carried out on rebar/concrete bond effect under various load conditions, such as monotonic load and cyclic load for various bond conditions. Several models are available in literatures [32-35]. By employing bond slip models for different loads and bond conditions, crack opening due to bond slip in the corresponding conditions can be obtained.

Therefore, theoretically the crack opening of RC structures can be analyzed based on fracture mechanics with an appropriate bond slip model.

3.2. An integral equation based on fracture mechanics

Specimens of embedded straight crack and single edge crack, shown in **Figure 3.1**, are acted on by remote tension field $\sigma_a(x)$. The fracture process is toughened by continuous bridging stress acting on both the crack surfaces. In order to calculate the COD at any location along the crack, a virtual line load P is applied at that location. Following Castigliano's theorem [36], the displacement at that position is given by

$$u(x) = \lim_{P \rightarrow 0} \frac{\partial W}{\partial P}, \quad (3.1)$$

where x is the distance between the target position and the bottom face of the beam as shown in **Figure 3.1**. W is the total strain energy of the system per unit width of crack front. Thus the derivation is valid only if the system is linear-elastic, apart from the nonlinearity in the action of

crack bridging stress. The strain energy may be written as an integral of the strain energy release rate per width of crack front, which is denoted as G

$$W = \int_0^a G da' = \int_0^a \frac{K_{tip}^2}{E'} da', \quad (3.2)$$

where a is the crack length, a' is the dummy variable for a . E' is a combination of elastic constants, which depends on whether the problem is in plane stress or plane strain conditions and also on whether the material is isotropic or orthotropic. For isotropic materials

$$E' = \begin{cases} E & \text{for plane stress} \\ \frac{E}{1-\nu^2} & \text{for plane strain} \end{cases}, \quad (3.3)$$

where ν is Posson's ratio of the material, E is the composite Young's modulus of the material given by

$$E = E_m(1 - f_r) + E_f f_r, \quad (3.4)$$

E_m and E_f are the Young's modulus of matrix and the embedded bridging material respectively and f_r is the volume ratio of the composite. For orthotropic cases, E should be determined from compliances along different directions, however, no orthotropic problem is solved in this work and those expressions are excluded from being stated which can be found in [28]. K_{tip} is the net crack tip stress intensity factor which is the superposed effect of applied loads, bridging forces and the virtual load P as

$$K_{tip} = K_a + K_b + K_P, \quad (3.5)$$

where K_a and K_b are SIF owing to applied loads and bridging forces, respectively. K_P is SIF due to the virtual load P .

To facilitate the derivation of a transformation between bridging force and COD, the stress intensity factors are determined according to the weight function method proposed by [37, 38] because once the weight function for a particular crack is determined, the stress intensity factor for any loading system can be calculated. Based on the weight function method, the SIF due to the virtual load P is

$$K_P = \begin{cases} G \cdot P & x < a \\ 0 & x \geq a \end{cases}, \quad (3.6)$$

where G is a weight function which depends on the crack geometry only. The weight functions for a variety of significant geometries can be found in handbooks for crack analysis [e.g. [39]].

For specified remote loading, the contribution K_a to the stress intensity factor can be easily calculated in terms of the field $\sigma_a(x)$ that would exist on the crack plane in the absence of the crack as

$$K_a = 2 \int_0^a G(x, a, b) \sigma_a(x) dx. \quad (3.7)$$

Similarly, the stress intensity factor due to the bridging stress is given by

$$K_b = -2 \int_0^a G(x, a, b) p(x) dx. \quad (3.8)$$

Differentiation of **Eq.(3.2)** with respect to P and substitution into **Eq.(3.1)** along with the use of **Eq.(3.5)** lead to the following expression for COD in terms of stress intensity factors as

$$u(x) = \frac{2}{E'} \int_x^a (K_a + K_b + K_p) G(x, a', b) da'. \quad (3.9)$$

By substituting **Eq.(3.7)** and **Eq.(3.8)** into **Eq.(3.9)**, the following integral transform is derived between crack bridging stress and COD where K_p vanishes due to the limit $P \rightarrow 0$

$$u(x) = \frac{4}{E'} \int_x^a \left\{ \int_0^s G(x', a', b) [\sigma(x') - p(x)] dx' \right\} G(x, a', b) da'. \quad (3.10)$$

3.3. A cracked RC beam model

In this study, to facilitate method verification through comparing with the experimental results in [34], both analytical and experimental study are conducted to simulate the crack opening of the reference beam. The static crack opening experiment is briefly summarized here. The dimensions, boundary and load conditions of the specimen are shown in **Figure 3.2**. A concrete cutter with a 1 mm thick cutter blade was used to create 3 mm wide notches of 1 cm depth at the middle of the bottom faces of the specimens. The notch ensures that the crack plane will be right at the middle. Basic material properties and specimen dimensions are listed in **Table 3.1**.

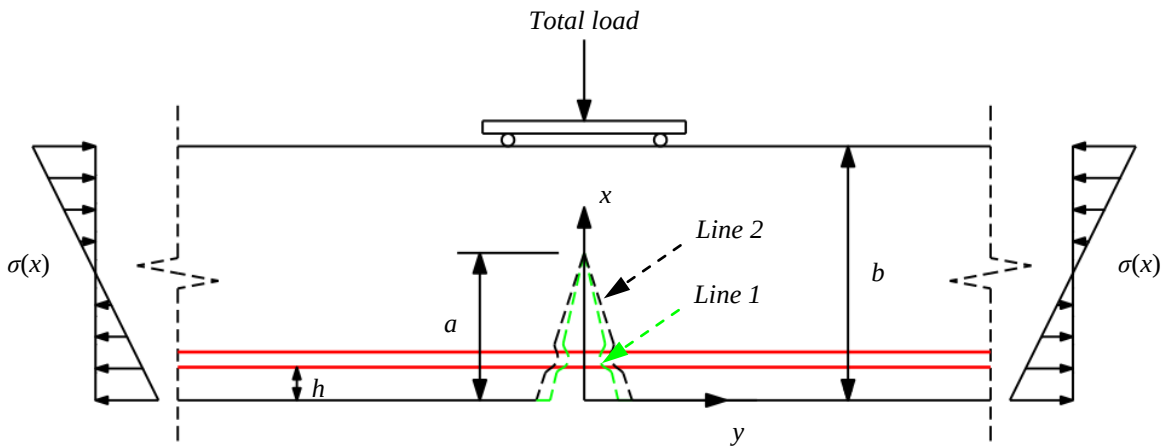
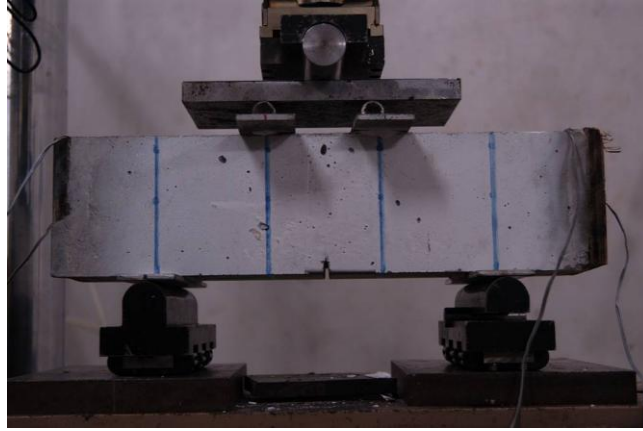
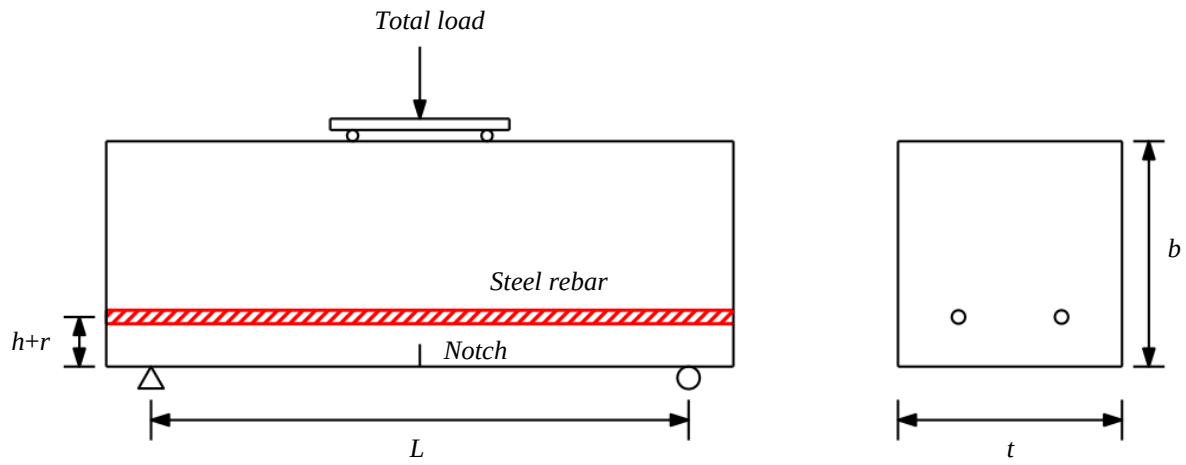


Figure 3.1 Schematic diagram for the understanding of cracking process



(a) Experimental set-up



(b) Specimen details

Figure 3.2 Experimental set-up and specimen details

Table 3.1 Details of the tested beam

Parameters	Value
Concrete cylinder strength	30 MPa
Critical stress intensity factor K_{IC}	10 N/mm ^{3/2}
Yield strength of steel	345 MPa
Young's modulus of steel	200 GPa
Young's modulus of concrete	28 GPa
Possion's ratio of concrete	0.2
Possion's ratio of steel	0.3
Beam total depth (b)	10 cm
Beam width (t)	10 cm
Clear cover (h)	32 mm
Steel rebar radius (r)	3 mm
Beam span (L)	24 cm

3.4. Crack opening displacement determination of the cracked RC beam

For the tested RC beam, the concrete bridging stress in the crack process zone is neglected considering following reasons: (1) In the experiment on the RC beam, a notch was set in to middle span to ensure that a wide major crack initiates and propagates in the pure bending range under model I loading condition, whereas the concrete bridging stress is a function of crack opening width. (2) The depth for the model I crack in the RC beam is determined following the criterion for crack advance in bridged crack model under monotonic loading, which is the crack tip stress intensity factor due to applied load and bridging elements equaling to the fracture toughness of reinforced concrete from experiment. This means the bridging effect of concrete is included implicitly.

Mostly, the CODs along the crack are almost impossible to be measured with high accuracy for a crack in an existing RC structure, which means the calculated CODs based on fracture mechanics with a bond slip model is difficult to be verified with experimental data. This study is focus on a more easily measureable parameter, crack mouth opening displacement (CMOD). Thus, the complicated process of measuring and optimizing COD profiles is obviated. To better understand the concrete crack opening process, this process is interpreted as consisting of two steps. Step 1: Perfect bond is assumed. The crack opens under applied loads and rebar bridging forces. The resulting crack profile is shown in **Figure 3.1** marked as Line 1. Step 2: Bond slip occurs at rebar location and then transfers to the other positions along the crack. Correspondingly, the COD profile shifts to Line 2 in **Figure 3.1**. Therefore, CMODs of RC beams are calculated following as: (1) For RC beams, the sectional tension contribution of concrete is generally negligible compared with that of rebars. Neglecting the tensile strength of concrete, the rebar force is calculated based on the cracked RC beam section analysis; (2) With the rebar force, the crack depth and CMODs due to applied loads and rebar bridging stress are calculated based on fracture mechanics; (3) Calculating the CMOD due to bond slip through introducing a local bond slip model. The summation of CMODs due to applied loads, rebar bridging stress and bond slip is the analytical CMOD; (4) Verifying the proposed method of concrete cracking of a reinforced concrete structure based on fracture mechanics through comparing analytical CMOD and experimental CMOD.

3.4.1. CMOD due to applied load and rebar bridging forces

For the crack geometry shown in **Figure 3.1**, such a single dominate crack at the midspan resembles a single edge notch (SEN) fracture specimen, and a zero or negligible shear at the crack plane ensures Mode I fracture. A two-dimensional bridged crack model is exploited for the through-the-thickness cracked RC beam. Therefore, the relevant quantities are applicable for unit thickness of the beam.

Linear elastic behavior of rebar and concrete is assumed after the crack has passed the rebars. For the cracked RC beam, the CMOD due to applied loads and rebar bridging forces is calculated following the integral equation for a bridged crack in an elastic medium based on fracture mechanics employing weight functions for the single edge notch specimen.

For the SEN specimen under Mode I load conditions, the weight function is given as

$$G(x, a, b) = \frac{1}{\sqrt{\pi a}} \frac{h_1(x/a, a/b)}{(1 - x^2/a^2)^{1/2}}, \quad (3.11)$$

where b is the beam depth, and

$$h_1(x/a, a/b) = \frac{g(x/a, a/b)}{(1 - a/b)^{3/2}}, \quad (3.12)$$

assuming $a/b = \xi$, then

$$g(x/a, a/b) = g(x/a, \xi), \quad (3.13)$$

and

$$g(x/a, \xi) = g_1(\xi) + \frac{x}{a} g_2(\xi) + \frac{x^2}{a^2} g_3(\xi) + \frac{x^3}{a^3} g_4(\xi), \quad (3.14a)$$

$$g_1(\xi) = 0.46 + 3.06\xi + 0.84(1 - \xi)^5 + 0.66\xi^2(1 - \xi)^2, \quad (3.14b)$$

$$g_2(\xi) = -3.52\xi^2, \quad (3.14c)$$

$$g_3(\xi) = 6.17 - 28.22\xi + 34.54\xi^2 - 14.39\xi^3 - (1 - \xi)^{3/2} - 5.88(1 - \xi)^5 - 2.64\xi^2(1 - \xi)^2, \quad (3.14d)$$

$$g_4(\xi) = -6.63 + 25.16\xi - 31.04\xi^2 + 14.41\xi^3 + 2(1 - \xi)^{3/2} + 5.04(1 - \xi)^5 + 1.98\xi^2(1 - \xi)^2. \quad (3.14e)$$

Then, as introduced in Section 3.2, the COD profile can be calculated as

$$u(x) = \frac{4}{E'} \int_x^a \left[\int_0^{a'} G(x', a', b) [\sigma(x') - f(x')] dx' \right] G(x, a', b) da', \quad (3.15)$$

where $\sigma(x)$ is the stress that would exist on the crack faces in the absence of a crack and $f(x)$ is the stress on crack faces due to rebar force per unit length along the crack. x' is the dummy variable of x . Correspondingly, defining $u_a(x)$ and $u_b(x)$ as the crack opening and crack closing due to applied load and rebar bridging, separately, then the complete forms of these two components of CODs are

$$u_a(x) = \frac{4}{E'} \int_x^a \left[\int_0^{a'} G(x', a', b) \sigma(x') dx' \right] G(x, a', b) da', \quad (3.16)$$

$$u_b(x) = -\frac{4}{E'} \int_x^a \left[\int_0^{a'} G(x', a', b) f(x') dx' \right] G(x, a', b) da'. \quad (3.17)$$

For the cracked RC beam under pure bending loads as shown in **Figure 3.1**, linearly distributed bending stresses would exist on crack faces in the absence of the crack according to the widely-approved plane cross-section assumption for a linear elastic system. Thus, a stress distribution due to applied loads should be:

$$\sigma(x) = \sigma \left(1 - \frac{2x}{b} \right), \quad (3.18)$$

where σ is the maximum stress at the extreme concrete fibers.

The linear elastic behavior is assumed until rebar yielding. Post-yielding behaviors are not considered.

The term $f(x)$ is assumed as a step function following [34].

$$F = \frac{M}{jd}, \quad (3.19)$$

$$f(x) = f \cdot [H(x-h) - H(x-h-d_b)], \quad (3.20)$$

where M is the acting bending moment, jd is the internal lever arm between the tension force of rebars and the resultant compression force of concrete, where d is the effective beam depth. h and d_b are clear distance of rebar from the bottom face and rebar diameter. H is Unit Step Function. $f(x)$ in kN/mm is the force acting on per unit length along both crack faces due to rebar force, which is obtained according to the principle that the integration of $f(x)$ over the cracked domain is equal to the total rebar force F . Correspondingly, $f=F/d_b$ is the value of $f(x)$.

With the COD calculation equations derived in this section, CMODs due to applied load, um_a , and rebar bridging force, um_b , are obtained if the virtual load P acts on the crack mouth. Thus

$$um_a = u_a(0) = \frac{4}{E'} \int_0^a \left[\int_0^{a'} G(x', a', b) \sigma \left(1 - \frac{2x'}{b} \right) dx' \right] G(x, a', b) da', \quad (3.21)$$

$$\begin{aligned} um_b &= u_b(0) \\ &= -\frac{4}{E'} \int_0^a \left[\int_0^{a'} G(x', a', b) f [H(x' - h) - H(x' - h - d_b)] dx' \right] G(x, a', b) da' . \\ &= -f \cdot um_{bunit} \end{aligned} \quad (3.22)$$

where um_{bunit} is CMOD due to unit linear rebar force.

3.4.2. CMOD due to bond slip

The bond slip relation is very complicated because it is influenced by many factors, such as the concrete strength, rebar surface characteristics and embedment length. In this paper, the method is employed in analyzing a beam in [34], where a newly cast beam was cracked under the monotonic load. Therefore, a bond slip model for non-deteriorated conditions should be employed. To ensure a crack under Mode I loading, a notch was set in the midspan of the RC

beam in [34]. As a result, a major crack was formed in the midspan as shown in **Figure 3.1** and long embedment condition was satisfied. For long embedment condition, the local bond slip relations can be well predicted too by the following simple equation in [40] as:

$$\tau = 0.9 f_c^{2/3} \left(1 - e^{-40s^{0.6}} \right), \quad (3.23)$$

where $s=S/d_b$ is the normalized slip referred to the rebar diameter. This model is employed in this study because the boundary condition of the model and the employed test identify with each other.

In addition, there are some general relations among bond stress, slip and rebar strain. As illustrated in **Figure 3.3**, defining a point on rebar where both rebar strain and bond slip are equaling zero as the origin of z coordinate, the slip at the point (z) is an integration of rebar strain from z to the origin. With the concept, the bond slip is defined as the displacement of the rebar at the point concerned measured relative to a fixed point in the concrete, and also the relative displacement between the rebar and concrete as for the long embedment condition. L_s is debonding length. This relation is simply expressed as

$$S(z) = \int_0^z \varepsilon(t) dt, \quad (3.24)$$

where $\varepsilon(z)$ is rebar strain at any point and t is the dummy variable for the integration.

The local bond stress at any location along a rebar is proportional to the slope of the strain distribution curve at that point. **Figure 3.4** shows the stresses acting on dz rebar element. The equilibrium equation of the rebar element is as

$$\tau \cdot \pi d_b \cdot dz = \frac{d\sigma_s}{dz} \cdot dz \cdot \frac{\pi d_b^2}{4}, \quad (3.25)$$

thus

$$\tau = \frac{E_s \cdot d_b}{4} \frac{d\varepsilon}{dz} = \frac{E_s \cdot d_b}{4} \frac{d^2 S}{dz^2}, \quad (3.26)$$

where E_s is the elastic modulus of rebar, $\sigma_s(z)$ is rebar stress at any point. $d\varepsilon/dz$ is the slope of strain distribution curve. The relations between rebar strain, bond stress and bond slip are shown in **Figure 3.5**.

Eq.(3.24) and **Eq.(3.26)** are valid under the following assumptions:

- (1) Rebar has a linear elastic constitutive law in the longitudinal direction;
- (2) For any point along the rebar in the debonding region, the concrete strain is negligible compared with the rebar strain.

Substituting **Eq.(3.26)** into **Eq.(3.23)**, differential equation of the normalized slip with respect to location is obtained.

$$\frac{E_s \cdot d_b^2}{4} \frac{d^2 s}{dz^2} = 0.9 f_c^{2/3} (1 - e^{-40s^{0.6}}). \quad (3.27)$$

For long enough embedment condition, boundary conditions (**Figure 3.3**) are:

$$z = L_s \Rightarrow \begin{cases} s = S_s / d_b \\ \sigma_s = f \cdot d_b / A \\ \varepsilon = \sigma_s / E_s \end{cases} \quad z = 0 \Rightarrow \begin{cases} s = 0 \\ \sigma_s = 0 \\ \varepsilon = 0 \end{cases}, \quad (3.28)$$

where A is rebar sectional area; L_s is the debonding length which is unknown. Obviously, **Eq.(3.27)** has no theoretical solution and should be solved numerically. The procedure of numerical solving is:

Step 1: Calculate $\varepsilon(L_s) = f \cdot d_b / E_s \cdot A$ and assume $S(L_s) = S_s$ arbitrarily;

Step 2: Substitute $S(L_s)$ into **Eq.(3.22)**, $\tau(L_s)$ is obtained. For the region from $z = L_s$ to $z = L_s - \Delta L_s$, bond stress τ can be considered as equaling $\tau(L_s)$ if ΔL_s is small enough. Following **Eq.(3.25)** and the relations illustrated in **Figure 3.5**, formulas for both $\varepsilon(L_s - \Delta L_s)$ and $S(L_s - \Delta L_s)$ calculations are obtained, which are expressed as:

$$\varepsilon(L_s - \Delta L_s) = \varepsilon(L_s) - \frac{\tau(L_s)}{E_s} \cdot \Delta L_s, \quad (3.29)$$

$$s(L_s - \Delta L_s) = s(L_s) - \frac{\varepsilon(L_s) + \varepsilon(L_s - \Delta L_s)}{2} \cdot \Delta L_s. \quad (3.30)$$

Step 3: Repeating Step 2, slip and rebar strain curves are obtained. And then check whether the boundary conditions at $z=0$ are satisfied or not.

Step 4: Adjusting S_s and repeating Step 2 and Step 3 to ensure boundary conditions at $z=0$ satisfied. Consequently, a numerical result of strain to slip relation is obtained, and the debonding length L_s is obtained simultaneously.

Taking $\varepsilon(L_s) = 0.0035$ as example and employing parameter values of the RC beam in [34],

(1) $S(L_s) = 0.250\text{mm}$ is assumed;

(2) Following the Step 2, the slip and rebar strain curves corresponding to $S(L_s) = 0.250\text{mm}$ are obtained, which are shown in **Figure 3.6**. It is found that the boundary condition, slip and rebar strain equaling zero at a same point, cannot be satisfied.

(3) Adjusting $S(L_s)$ to ensure that the boundary condition can be satisfied at a certain point. This point is the origin of z coordinate. It is found that the boundary condition is satisfied when

$S(L_s)$ equals 0.278mm. The rebar strain and slip distribution curves corresponding to $S(L_s)=0.278\text{mm}$ are shown in **Figure 3.6** as well.

To facilitate application, the numerical result of strain to slip relation is fitted by a polynomial function. The fitting result of the strain to slip relation is simply expressed as

$$S(z) = S(\varepsilon(z)) \quad (3.31)$$

Both the numerical and fitting strain to slip relations are shown in **Figure 3.7**.

Regarding the slip as the COD of the center point of rebars, the CMOD can be obtained by assuming the COD due to bond slip increasing linearly from rebars to the crack mouth with a slope of

$$\psi = \frac{a}{a - h - r}, \quad (3.32)$$

where r is the rebar radius. Then the CMOD due to bond slip, $um_s(\varepsilon_s)$, is given as

$$um_s(\varepsilon_s) = S(\varepsilon_s) \cdot \psi, \quad (3.33)$$

where $\varepsilon_s = f \cdot d_b / E_s \cdot A$ is rebar strain at crack location.

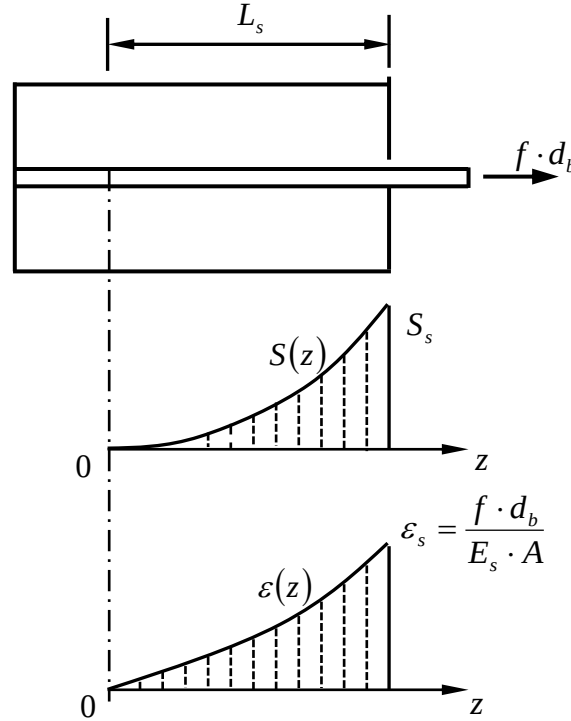


Figure 3.3 Bond slip and rebar strain along rebar

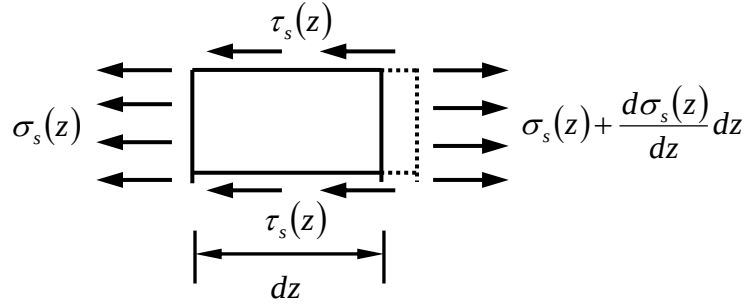


Figure 3.4 Stresses acting on rebar element

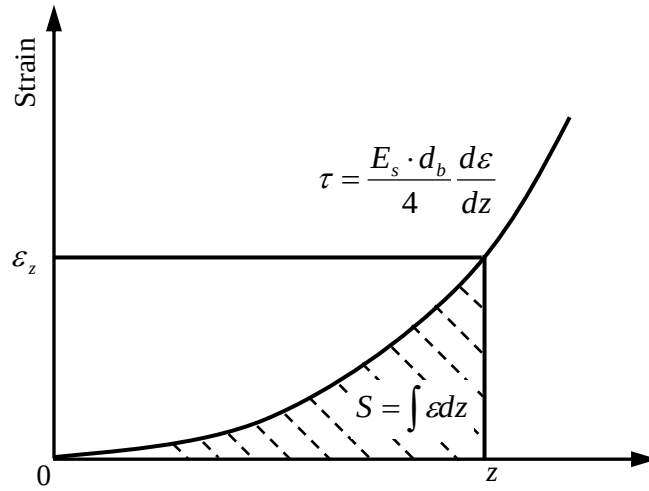


Figure 3.5 Relation between rebar strain, bond stress and bond slip

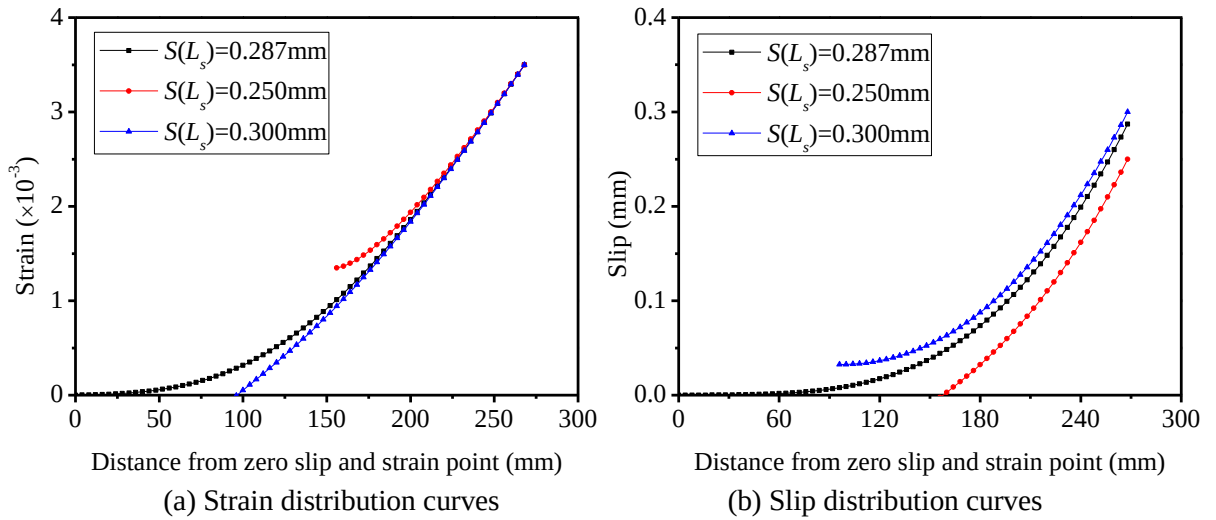


Figure 3.6 Strain and slip distribution curves

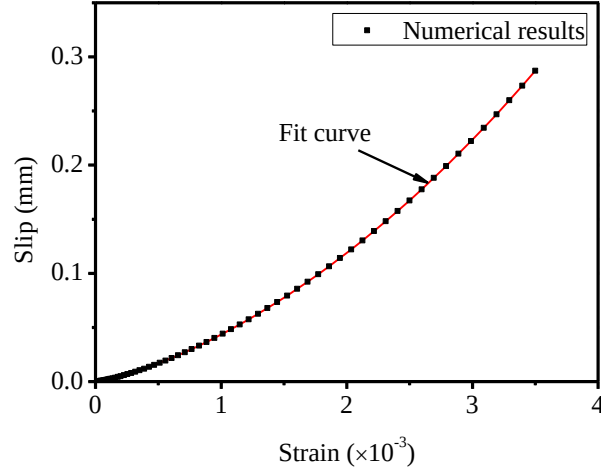


Figure 3.7 Strain vs slip relation

3.5. Method verification with experimental results

For RC beams, the sectional tension contribution of concrete is generally negligible compared with that of rebars. Neglecting the tensile strength of concrete, a rebar force is calculated based on the cracked RC beam section analysis using **Eq.(3.18)**. Thus, the rebar force from **Eq.(3.18)** is on the safe side. With this rebar forces from **Eq.(3.18)**, a crack length, a , is determined following the criterion for crack advance in the bridged crack model under monotonic loading in the current analysis, which is

$$K_{tip} = K_a + K_b = K_{IC} \quad (3.34)$$

where K_a and K_b are stress intensity factors due to the external loads and the rebar forces, respectively, and K_{tip} is net stress intensity factor combining both effects, KIC is the fracture toughness of reinforced concrete (see [41] for details).

Then, adding COD profiles from **Eq.(3.16)** and **Eq.(3.17)** yields analytical COD profiles excluding COD due to bond slip. Substituting the rebar force into **Eq.(3.33)**, the bond slip related COD at rebar location can be obtained. Assuming that this COD increases from rebars towards the crack mouth and decreases from rebars towards the crack tip with the slope determined by **Eq.(3.35)**, analytical COD profiles including slip is obtained.

$$\psi = \frac{a}{a - h - r} \quad (3.35)$$

The analytical COD profiles excluding and including bond slips under different load conditions are shown as **Figure 3.8**. COD profiles naming as slip included in **Figure 3.8** are summations of COD profiles due to applied loads, crack bridging stresses and bond slips. Due to the existence of rebars, each COD profile experiences a depression in the region of rebar, which is from the value or the horizontal axis equaling 32mm to 38mm. These depressions implicitly exhibit a influence of rebars on a local deformation. It is also found from **Figure 3.8** that the COD due to bond slip takes up a great proportion of total CODs in all cases, even before steel

yielding, which means the COD due to bond slip is not negligible. With the analytical COD profiles, the analytical CMODs excluding and including bond slip under different load conditions are listed in **Table 3.2**.

In the experiment, crack width and depth data at different load levels were collected by a laboratory apparatus consisting of a microscopic digital camera and a three-axis controlled system as described in [34]. Since the focus of this literature was the Mode I fracture problem, a notch was set at the mid-span on the bottom face of the beam to ensure the major crack initiates from this position. As a result, under four-point bending load, the major crack of the RC beam propagated vertically and almost no shear force was transferred across the crack because the crack remained in the pure bending region all the time.

The COD data were collected at points with 1 mm spacing along the crack. A typical experimental COD profiles are drawn by collecting the isolated COD data points with straight line as shown in **Figure 3.9**. Random fluctuations are observed, which is due to both inherent toughness of the fracture surface, such as aggregates, impurities and voids, and errors in COD measurement.

In this study, firstly, the maximum COD, COD_{max} , is treated as the experimental CMOD. This treatment seems to be more reasonable when the adopted assumption that is the tensile stresses in the concrete are negligible comparing with that of rebars. However, due to the existence of the notch, the COD_{max} should be still smaller than the actual experimental CMOD. Thus, another experimental COD denoted by COD_{ext} is regarded as experimental CMOD as well. The COD_{ext} is determined by extending the COD profile to the bottom surface of the beams following the linear polynomial fitting function of the experimental COD profile. The reasons for using the linear polynomial function are, firstly, to reduce the influence of the fitting curve slope in the immediate vicinity of the notch, because the slope is determined by the COD data closing to the notch; secondly, a strong linear relation is observed in crack faces for all conditions, which is basically abide to a generally accepted assumption for RC beam behavior under bending, plane cross-section assumption. Fitting curves and functions for the experimental COD profiles under different load conditions are shown in **Figure 3.9**.

Both of these two kinds of CMODs under different load conditions are listed in **Table 3.2** together with the two kinds of analytical CMODs. It is found that the analytical CMODs including bond slip achieve a good accuracy compared with the corresponding experimental CMODs and the CMOD due to bond slip takes up a great proportion of the total CMOD in all cases, even before steel yielding. Therefore, to determine the opening of a crack in a reinforced concrete structure, not only the effect of crack bridging stresses but also the crack opening due to bond slip in the concrete/rebar interface should be considered comprehensively.

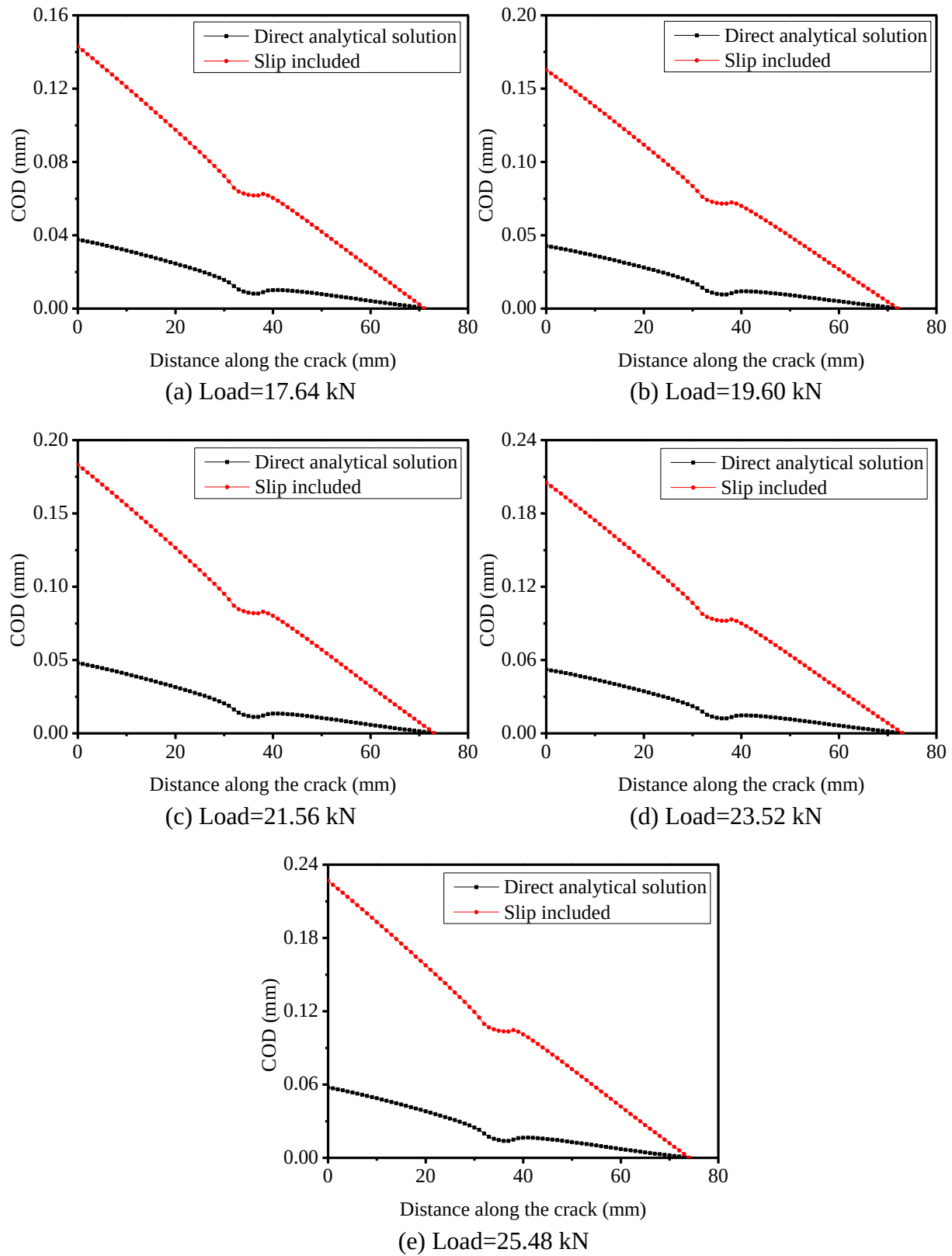


Figure 3.8 Analytical COD profiles of an RC beam under different load conditions

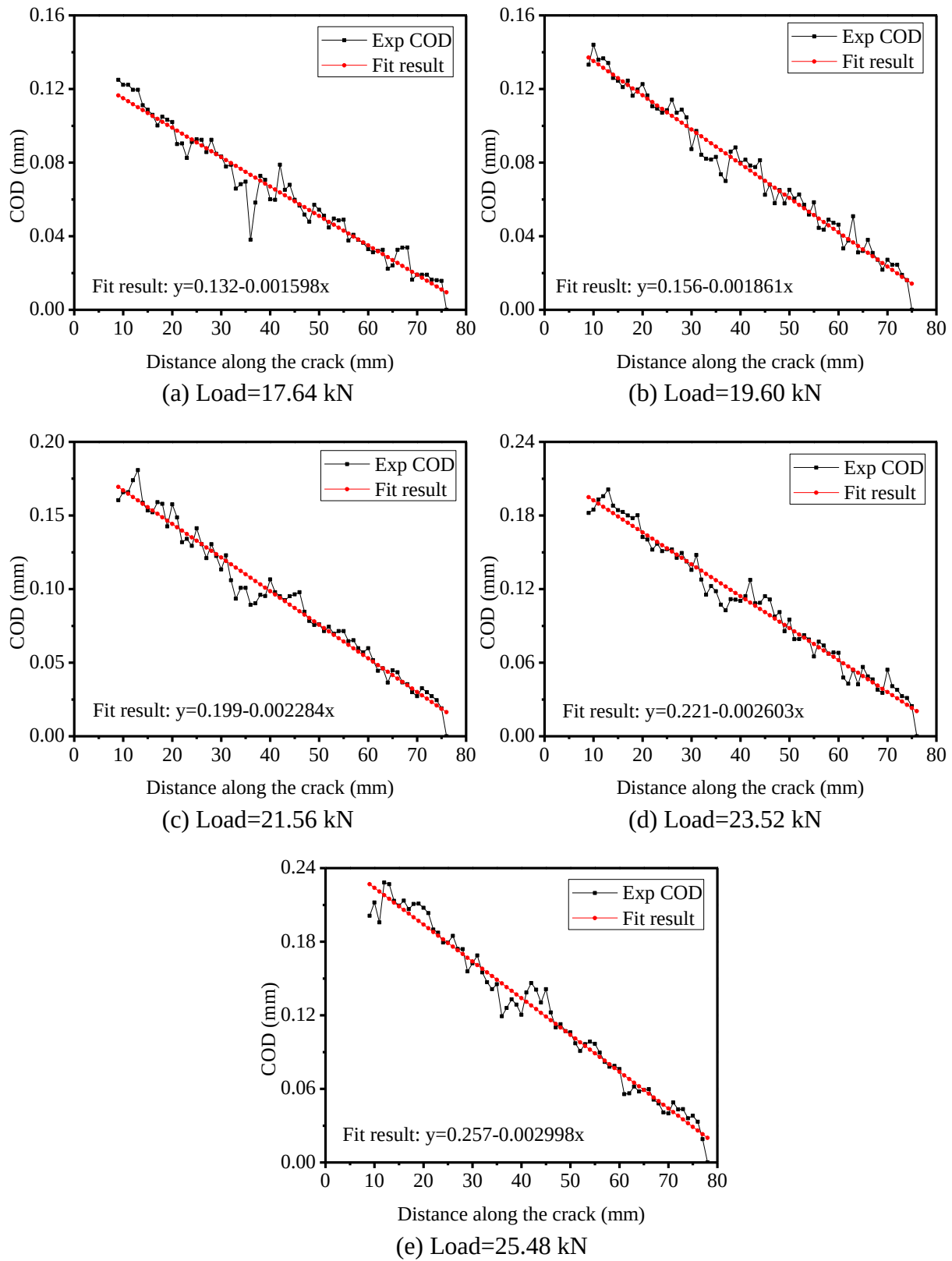


Figure 3.9 Experiment COD profiles of an RC beam under static loads

Table 3.2 CMODs from different approaches

Load (kN)	CMOD _{max}	CMOD _{ext}	CMOD _{ana} excluding slip	CMOD _{ana} including slip
17.64	0.125	0.132	0.038	0.143
19.60	0.144	0.156	0.043	0.155
21.56	0.181	0.192	0.048	0.183
23.52	0.201	0.221	0.052	0.205
25.48	0.228	0.257	0.058	0.224

Note: CMOD_{max} is the maximum crack opening width at the top of crack tip
 CMOD_{ext} is the crack opening

3.6. Summary and conclusions

Following the mechanisms of concrete cracking of RC structures, a crack mouth opening displacement (CMOD) determination method for cracks in an RC beam was developed based on the fracture mechanics with a bond slip model. In the method, a total CMOD was decomposed into CMODs due to applied loads and rebar stresses and CMOD due to bond slip.

The CMODs due to applied loads and rebar stresses were calculated following a fracture mechanics based integral equation, whereas the CMOD due to bond slip was determined through introducing a local bond slip model.

The proposed method was employed successfully in determine the CMOD of a crack in a RC beam under four-point bending tests. Through comparing with the experimental CMODs, the good coincidence verified the applicability and reliability of the method.

In addition, it was found that the CMOD due to bond slip takes a large proportion of the total CMOD, which means the concrete cracking of an RC structure cannot be conducted accurately if the crack opening due to bond slip is not considered comprehensively.

CHAPTER 4 WEIGHT FUNCTION DETERMINATIONS FOR SHEAR CRACKS IN RC BEAMS BASED ON FINITE ELEMENT METHOD

4.1. Introduction

As introduced in Chapter 3, to determine crack opening of a shear crack including the punching shear crack in an RC based on fracture mechanics, weight functions for corresponding crack geometry should be determined beforehand. A weight function concept which was firstly proposed by [37, 38] processes very strong advantages because the stress intensity factors for any arbitrary state of loading can be determined if the weight function of a given crack geometry is evaluated from a (perhaps simple) reference state of loading owing to load-independent characteristics of weight functions. With the weight functions, a crack opening displacement for the shear crack can be calculated following the integral equation introduced in Chapter 3.

Unfortunately, to date, even though weight functions for many important crack geometries have been summarized in [39], the number of fracture problems with close form analytical solution of weight functions is very limited. Because of that, different numerical methods suitable for the determination of weight functions have been presented, one of which is based on Finite Element Method (FEM). Virtual Crack Extension (VCE) technique, as suggested by [41, 43], provides an efficient finite element calculation of stress intensity factors and nodal weight functions. This technique has been employed in determining a 2-D Mode I weight functions in [44] and extended to 2-D Mixed Mode fracture problems in [45] through the use of symmetric mesh in the vicinity of crack-tip. Obviously, an oblique shear crack in RC beams can be simply considered as 2-D Mixed Mode fracture problems.

4.2. Formulation for mixed fracture mode

Exploiting VCE technique in finite element method, nodal weight functions for Mode I 2-D crack problems were represented in a displacement differentiation form according to the physical meaning of weight function, which is normalized rate of change of displacements due to a unit change in the crack length for a reference state of loading, in [38]. This computationally efficient finite element methodology for Mode I cracks was extended to mixed mode cracks, with combined tension and shear loading conditions, in [45] through a use of symmetric mesh in the crack-tip neighborhood. The symmetric mesh provides the decoupling characteristic of the stress, strain, displacement and traction field parameters into Mode I and Mode II components with respect to xt axis in the crack-tip neighborhood as shown in **Figure 4.1**, as a result, the stress intensity factors and nodal weight functions are separated into Mode I and Mode II components. The decoupled nodal weight functions for Mode I and Mode II at i 's nodal location (x_{t_i}, y_{t_i}) with crack length (a) and inclination angle (β) can be represented in the displacement differentiation form as

$$h_{I(II)xt}(x_{t_i}, y_{t_i}, a, \beta) = \frac{H}{2K_{I(II)}} \frac{\partial U_{I(II)xt}(x_{t_i}, y_{t_i}, a, \beta)}{\partial a}, \quad (4.1)$$

$$h_{I(II)yt}(xt_i, yt_i, a, \beta) = \frac{H}{2K_{I(II)}} \frac{\partial U_{I(II)yt}(xt_i, yt_i, a, \beta)}{\partial a}, \quad (4.2)$$

$h_{I(II)xt}$ = weight function components along xt axis for Mode I and Mode II

$h_{I(II)yt}$ = weight function components along yt axis for Mode I and Mode II

$K_{I(II)}$ = stress intensity factors for Mode I and Mode II

H = effective modulus which is E for plane stress and $E/(1-\nu^2)$ for plane strain

E = Young's modulus

ν = Possion's ratio

$U_{I(II)xt}$ = displacement components along xt axis for Mode I and Mode II deformation

$U_{I(II)yt}$ = displacement components along yt axis for Mode I and Mode II deformation

The decoupled displacement components, $U_{I(II)xt}$ and $U_{I(II)yt}$, for Mode I and Mode II within the symmetric region in the crack-tip neighborhood can be determined according to [46].

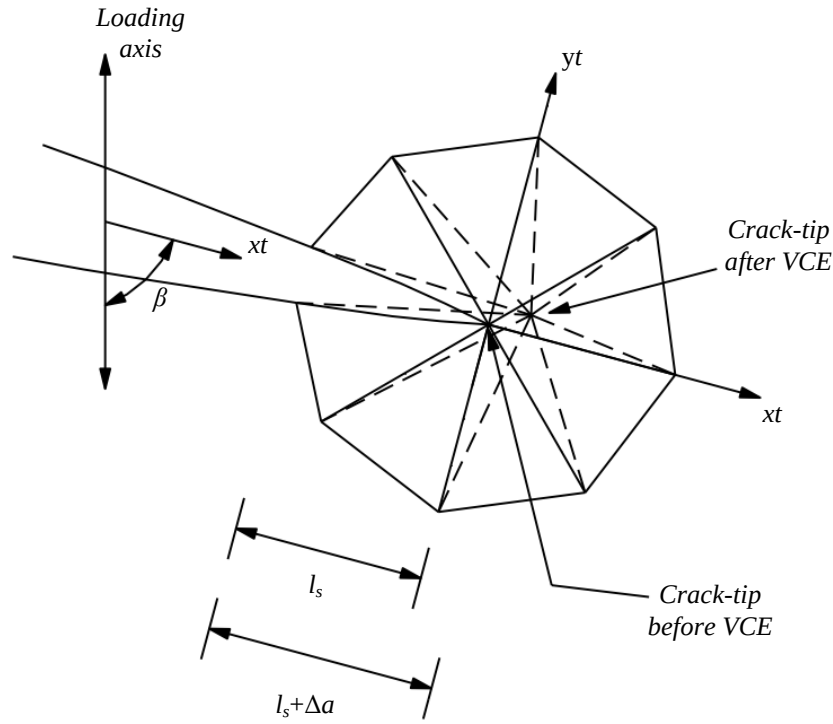


Figure 4.1 Symmetric mesh in crack-tip neighborhood with respect to the global xt axis

Based on the definition of strain energy release rate, in VCE technique the strain energy release rate is the change in potential energy in a given loading system produced by a virtual crack extension. By applying the VCE technique with symmetric mesh in the crack-tip neighborhood to the mixed mode fracture problems, the decoupled strain energy release rate G_I for Mode I and G_{II} for Mode II can be obtained from the decomposed displacement components $\{U_I\}$ and $\{U_{II}\}$, the changes in global stiffness $\Delta[K]$ and decomposed nodal force components Δf_I and Δf_{II} as follows:

$$G_I = -\frac{1}{2}\{U_I\}^T \frac{\partial[K]}{\partial a} \{U_I\} + \{U_I\}^T \frac{\partial\{f_I\}}{\partial a}, \quad (4.3a)$$

and

$$G_{II} = -\frac{1}{2}\{U_{II}\}^T \frac{\partial[K]}{\partial a} \{U_{II}\} + \{U_{II}\}^T \frac{\partial\{f_{II}\}}{\partial a}. \quad (4.3b)$$

Then, K_I and K_{II} can be obtained from the energy-based strain energy release rate according to the following relations

$$G_I H = K_I^2, \quad (4.4a)$$

and

$$G_{II} H = K_{II}^2. \quad (4.4b)$$

The decouple displacement derivatives, $\partial U_{I(II)xt}(xt_i, yt_i, a, \beta)$ and $\partial U_{I(II)yt}(xt_i, yt_i, a, \beta)$ for the entire structure, can be obtained through following process. To simplify the problem, the inclined angle β is considered as constant for a given crack. For a given β , the decoupled Mode I and Mode II displacement components can be expressed functionally as

$$\{U_{I(II)}\} = \{U_{I(II)}(xt, yt, a)\}. \quad (4.5)$$

Applying the chain rule of differentiation with respect to the crack length (a) produces the following equation after rearrangement:

$$\frac{\partial\{U_{I(II)}\}}{\partial a} = \frac{d\{U_{I(II)}\}}{da} - \frac{\partial\{U_{I(II)}\}}{\partial xt} \cdot \frac{dxt}{da} - \frac{\partial\{U_{I(II)}\}}{\partial yt} \cdot \frac{dyt}{da}. \quad (4.6)$$

Since the stresses, strains and displacements of Mode I and Mode II are independent with each other and should satisfy the equilibrium equation and compatibility condition, we can obtain that

$$[K]\{U_{I(II)}\} - \{f_{I(II)}\} = 0, \quad (4.7)$$

where $[K]$ is the matrix of global stiffness of original crack geometry.

Taking total differentiation of **Eq.(4.7)** with respect to crack length and after rearranging, we have

$$\frac{d\{U_{I(II)}\}}{da} = [K]^{-1} \left[\frac{d\{f_{I(II)}\}}{da} - \frac{d[K]}{da} \{U_{I(II)}\} \right]. \quad (4.8)$$

As the changes of the elemental stiffness for the entire structure and the decoupled nodal forces occur only in a few elements around the crack-tip as a result of VCE, $d[K]/da$ and $d\{f_{I(II)}\}/da$ of **Eq.(4.8)** can be expressed as:

$$\frac{d[K]}{da} = \sum_{i=1}^{N_c} \frac{[k_i]_{a+\Delta a} - [k_i]_a}{\Delta a}, \quad (4.9)$$

and

$$\frac{d\{f_{I(II)}\}}{da} = \sum_{i=1}^{N_f} \frac{\{f_{I(II)}\}_{a+\Delta a} - \{f_{I(II)}\}_a}{\Delta a}. \quad (4.10)$$

where

Δa = VCE in direction collinear with the oblique crack

$[k_i]$ = elemental stiffness matrix

$\{f_{I(II)}\}$ = elemental matrix after VCE

N_c = number of elements around the crack-tip

N_f = number of crack-face elements with nodal perturbation of $f_{I(II)}$ as a result of

VCE

The last two terms of **Eq.(4.6)** serve as the correction factors of changing the total displacement derivatives to partial displacement derivatives for the oblique cracks, which are null for nodes without geometric changes as a result of VCE. For the VCE, which is collinear with an oblique crack, we have $dyt/da = 0.0$.

By substituting **Eq.(4.6)** into **Eq.(4.1)** and **Eq.(4.2)**, the nodal weight functions for Mode I and Mode II, with crack length (a) and inclination angle (β) at (xt_i, yt_i) locations, can be expressed as

$$h_{I(II)xt}(xt_i, yt_i, a, \beta) = \frac{H}{2K_{I(II)}} \left\{ \frac{d\{U_{I(II)xt}\}}{da} - \frac{\partial\{U_{I(II)xt}\}}{\partial xt} \frac{dxt}{da} \right\}, \quad (4.11a)$$

$$h_{I(II)yt}(xt_i, yt_i, a, \beta) = \frac{H}{2K_{I(II)}} \left\{ \frac{d\{U_{I(II)yt}\}}{da} - \frac{\partial\{U_{I(II)yt}\}}{\partial xt} \frac{dxt}{da} \right\}. \quad (4.11b)$$

4.3. Modeling in finite element analysis

4.3.1. Finite element model

This study focuses on 2-D elastic plane problems and the finite element model is assumed under plane strain conditions where two material properties, Elastic modulus E and Poisson's ratio ν , need to be defined. As illustrated in the formulation section, the elements at the crack-tip neighborhood have a significant influence on the accuracy of the VCE technique.

For the finite element modeling of fracture problems, firstly, the crack-tip singularity, $\sqrt{r}$ and $1/\sqrt{r}$ displacement and stress variations, should be represented adequately in finite element method. Accordingly, the crack-tip vicinity is modeled with the degenerated quarter-point quadratic elements where the singularity is implicitly included. The degenerated triangular quarter-point element can be formed from standard 8-noded quadratic elements by defining duplicate node numbers for nodes K , L and O and shift node P and N to the quarter-point as shown in **Figure 4.2**.

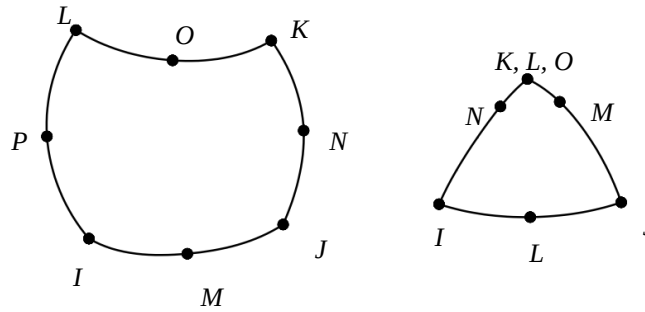


Figure 4.2 Element geometry

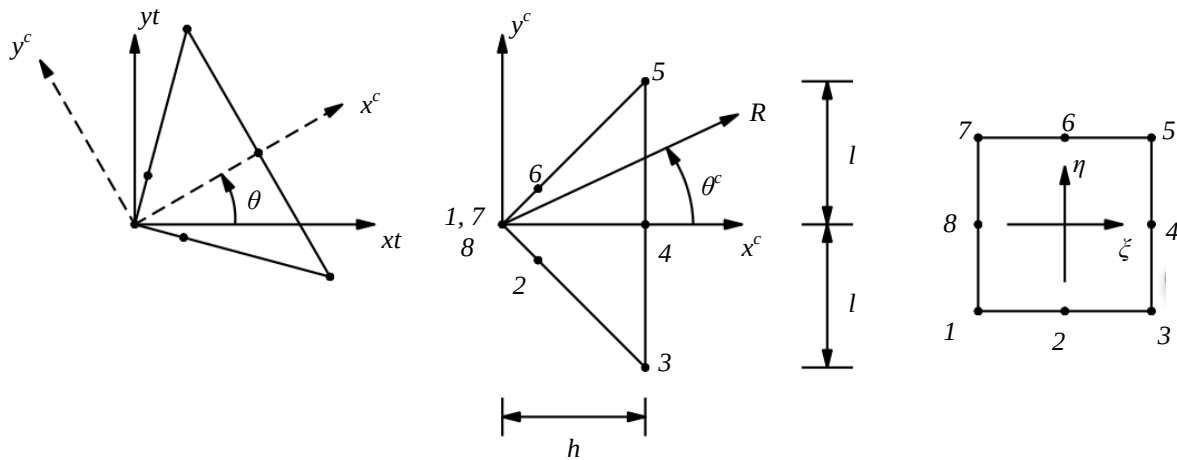


Figure 4.3 2-D triangular quarter point elements and the parent element

For the standard 8-noded plane isoparametric element, the geometry is mapped into the normalized square space shape (ξ, η) , $(-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)$ through the transformations as shown in **Figure 4.3**.

$$x^c = \sum_{i=1}^8 N_i(\xi, \eta) x_i^c, \quad (4.12a)$$

$$y^c = \sum_{i=1}^8 N_i(\xi, \eta) y_i^c, \quad (4.12b)$$

$$N_i(\xi, \eta) = \left[(1 + \xi \xi_i)(1 + \eta \eta_i) - (1 - \xi^2)(1 + \eta \eta_i) - (1 - \eta^2)(1 + \xi \xi_i) \right] \xi_i^2 \eta_i^2 / 4 \\ + (1 - \xi^2)(1 + \eta \eta_i)(1 - \xi_i^2) \eta_i^2 / 2 + (1 - \eta^2)(1 + \xi \xi_i)(1 - \eta_i^2) \xi_i^2 / 2, \quad (4.13)$$

where $\xi_i, \eta_i = \pm 1$ for the corner nodes and zero for the mid-side nodes, x_i^c, y_i^c are the nodal coordinates of the element. For the element shown in **Figure 4.3** we have

$$x_1^c = x_7^c = x_8^c = 0 \quad x_2^c = x_6^c = \frac{h}{4} \quad x_3^c = x_4^c = x_5^c = h, \quad (4.14a)$$

and

$$y_1^c = y_7^c = y_8^c = y_4^c = 0 \quad y_2^c = -y_6^c = -\frac{l}{4} \quad y_3^c = -y_5^c = -l. \quad (4.14b)$$

Substituting **Eq.(4.14)** into **Eq.(4.12)** and after collecting terms

$$x^c = \frac{h}{4}(1 + \xi)^2, \quad (4.15a)$$

and

$$y^c = \frac{l}{4}\eta(1 + \xi)^2. \quad (4.15b)$$

Therefore, the Jacobian of transformation $[J_2]$ between the normalized local coordinates (ξ, η) and coordinates (x^c, y^c) is given by

$$[J_2] = \begin{bmatrix} \frac{\partial x^c}{\partial \xi} & \frac{\partial y^c}{\partial \xi} \\ \frac{\partial x^c}{\partial \eta} & \frac{\partial y^c}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{h}{2}(1 + \xi) & \frac{l}{2}\eta(1 + \xi) \\ 0 & \frac{l}{4}(1 + \xi)^2 \end{bmatrix}, \quad (4.16)$$

and its determinant

$$\det[J_2] = \frac{hl}{8}(1+\xi)^3, \quad (4.17)$$

inverting

$$[J_2]^{-1} = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} = \begin{bmatrix} \frac{2}{h(1+\xi)} & \frac{-4\eta}{h(1+\xi)^2} \\ 0 & \frac{4}{l(1+\xi)^2} \end{bmatrix}. \quad (4.18)$$

To facilitate calculation, the origin of the global coordinate system is located at the crack-tip. As shown in **Figure 4.1**, the global coordinate system can be obtained through conducting a counterclockwise rotation by the angle, θ . Thus, the formulae for coordinates (xt, yt) can be expressed as

$$xt = x^c \cos \theta - y^c \sin \theta, \quad (4.19a)$$

and

$$yt = x^c \sin \theta + y^c \cos \theta. \quad (4.19b)$$

Correspondingly, a Jacobian of transformation $[J_1]$ between the local coordinates (x_c, y_c) and global coordinates (xt, yt) is given by

$$[J_1] = \begin{bmatrix} \frac{\partial xt}{\partial x^c} & \frac{\partial yt}{\partial x^c} \\ \frac{\partial xt}{\partial y^c} & \frac{\partial yt}{\partial y^c} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}, \quad (4.20)$$

inverting

$$[J_1]^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}. \quad (4.21)$$

Consequently, the Jacobian transformation of $[J]$ between the normalized square shape local coordinates (ξ, η) and global coordinates (xt, yt) is given by

$$[J] = [J_2] \cdot [J_1]. \quad (4.22)$$

Taking the advantage of Jacobian matrix $[J]$, differentials of displacement with respect to the global coordinates (xt, yt) for isoparametric elements are given as

$$\left\{ \begin{array}{c} \frac{\partial \{U_{I(II)i}\}}{\partial \xi} \\ \frac{\partial \{U_{I(II)i}\}}{\partial \eta} \end{array} \right\} = [J]^{-1} \cdot \left\{ \begin{array}{c} \frac{\partial [N_i U_{I(II)i}]}{\partial \xi} \\ \frac{\partial [N_i U_{I(II)i}]}{\partial \eta} \end{array} \right\}, \quad (4.23)$$

where $\{U_{I(II)i}\}$ is the decoupled nodal displacement of Mode I or Mode II at i 's nodal location. N_i is the shape function at i 's nodal location.

Similarly, displacements (u , v) within the element are interpolated by the same functions $N_i(\xi, \eta)$ of **Eq.(4.13)** from nodal displacements u_i , v_i . Taking into account of constraint conditions, i.e. nodal displacements of nodes 1, 7 and 8 are exactly the same, the differential of displacements in the local coordinates for any point at a distance r from the crack-tip on the radial line R are given as

$$\frac{\partial u}{\partial \xi} = a_0 + a_1(1 + \xi), \quad (4.24a)$$

$$\frac{\partial u}{\partial \eta} = b_1(1 + \xi) + b_2(1 + \xi)^2, \quad (4.24b)$$

$$\frac{\partial v}{\partial \xi} = c_0 + c_1(1 + \xi), \quad (4.24c)$$

$$\frac{\partial v}{\partial \eta} = d_1(1 + \xi) + d_2(1 + \xi)^2, \quad (4.24d)$$

where a_0 , a_1 , b_1 , b_2 , c_0 , c_1 , d_1 , d_2 are constants for any given set of nodal displacements along the radial line R . And it is obviously that

$$r = \sqrt{\left((x^c)^2 + (y^c)^2\right)}. \quad (4.25)$$

Substituting **Eq.(4.15)** into **Eq.(4.26)** and after rearrangement, we get

$$(1 + \xi) = \sqrt{r} / \sqrt{\frac{l}{4} \sqrt{\left[\left(\frac{h}{l}\right)^2 + \eta^2\right]}}. \quad (4.26)$$

And the derivatives u , and v with respect to x_c and y_c reduce to

$$\frac{\partial u}{\partial x^c} = \frac{A_0}{\sqrt{r}} + A_1, \quad (4.27a)$$

$$\frac{\partial u}{\partial y^c} = \frac{B_0}{\sqrt{r}} + B_1, \quad (4.27b)$$

$$\frac{\partial v}{\partial x^c} = \frac{C_0}{\sqrt{r}} + C_1, \quad (4.27c)$$

$$\frac{\partial v}{\partial y^c} = \frac{D_0}{\sqrt{r}} + D_1, \quad (4.27d)$$

where $A_0, A_1, B_0, B_1, C_0, C_1, D_0, D_1$ are also constants for any given set of nodal displacements and independent of r .

Therefore, the $\sqrt{r}$ displacement and correspondingly the $1/\sqrt{r}$ stress characteristic of elasticity at the crack-tip can be represented by the degenerated quarter-point quadratic elements, if nodes 1, 7 and 8 at the crack-tip are constrained to have the same displacement.

Secondly, the result sensitivity for the element size is an eternal concern in finite element analysis especially for crack-tip elements in this study where the crack-tip elements determine the accuracy. The effect of the singular element size was previously studied by [47]. In both studies, triangular quarter-point elements surrounding the crack were employed with an outer mesh of quadrilateral elements. The results on stress intensity factors calculation showed that a local refinement of mesh in the crack-tip zone improves the accuracy while an acceptable accuracy can be achieved even for relatively coarse mesh if the transition elements (regular eight-noded rectangular elements surrounding the triangular quarter-point elements) are employed [48, 49]. [48] pointed out that few percents error can be generally achieved for $0.1 \leq l_s/a \leq 0.3$, which satisfies almost all engineering purposes. The 1% level errors were achieved for $l_s/a = 0.03$ in [47, 48]. Although from a practical point of view it does not seem desirable to use smaller elements, considering that error of stress intensity factors may be transformed to the nodal weight functions and then to the further fitting weigh functions, smaller element size with $l_s/a = 0.04$ is employed in this study. Except for the crack-tip field, the remaining elastic body can be meshed with standard 8-noded quadratic elements.

4.3.2. Result output

According to the previous formulation on stress intensity factors and nodal weight functions for the combined mode I and mode II fracture problems, the global stiffness matrices and the nodal displacement vectors under a reference load should be output from finite element analysis to conduct analysis of a combined mode I and mode II fracture problem based on the VCE technique. After conducting finite element analysis, we can get the nodal displacement vectors under the global coordinate system (x, y) . With these nodal displacement vectors, the Mode I and Mode II components for field displacements within the symmetric region in the crack-tip neighborhood can be obtained following [47], where a subtraction of corresponding nodal displacements between symmetrical nodes with respect to the x axis is conducted. Since the corresponding nodal displacements of the symmetrical nodes are generally with very similar values, the subtraction may eliminate all effective numbers if the digit length of the values is not enough. A subtraction is conducted between the global stiffness before and after a VCE to get the change of global stiffness due to the VCE as well. Due to the microscale VCE, the change of global stiffness matrices may be cut-off for matrix elements with insufficient digit length. Therefore, sufficient digit length for the element of nodal displacement vectors and global stiffness matrices should be ensured.

4.4. Case study

This study is dedicated to providing some references for the fracture analysis of a shear crack in an RC beam. The geometry and the finite element mesh of an oblique edge crack are shown in **Figure 4.4**. Any constraint conditions can be employed for calculation and results discussion because as reported in [45], the dependence of weight function on constrain conditions for a given crack geometry can be circumvented through combining all self-equilibrium forces, which include applied surface tractions and reaction forces included from the selected constraint conditions, with nodal weight functions of different constraint conditions for a same crack geometry. Constraint conditions shown in **Figure 4.5** are employed in all calculations in this study because stress states under these constraint conditions is clear and convenient for analyzing. In this section, the introduced method is applied for analyzing an oblique edge crack geometry with oblique angle $\beta=45^\circ$, $h_2/h_1=1.5$, $h/w=5$ and $0.1 \leq a/w \leq 0.8$.

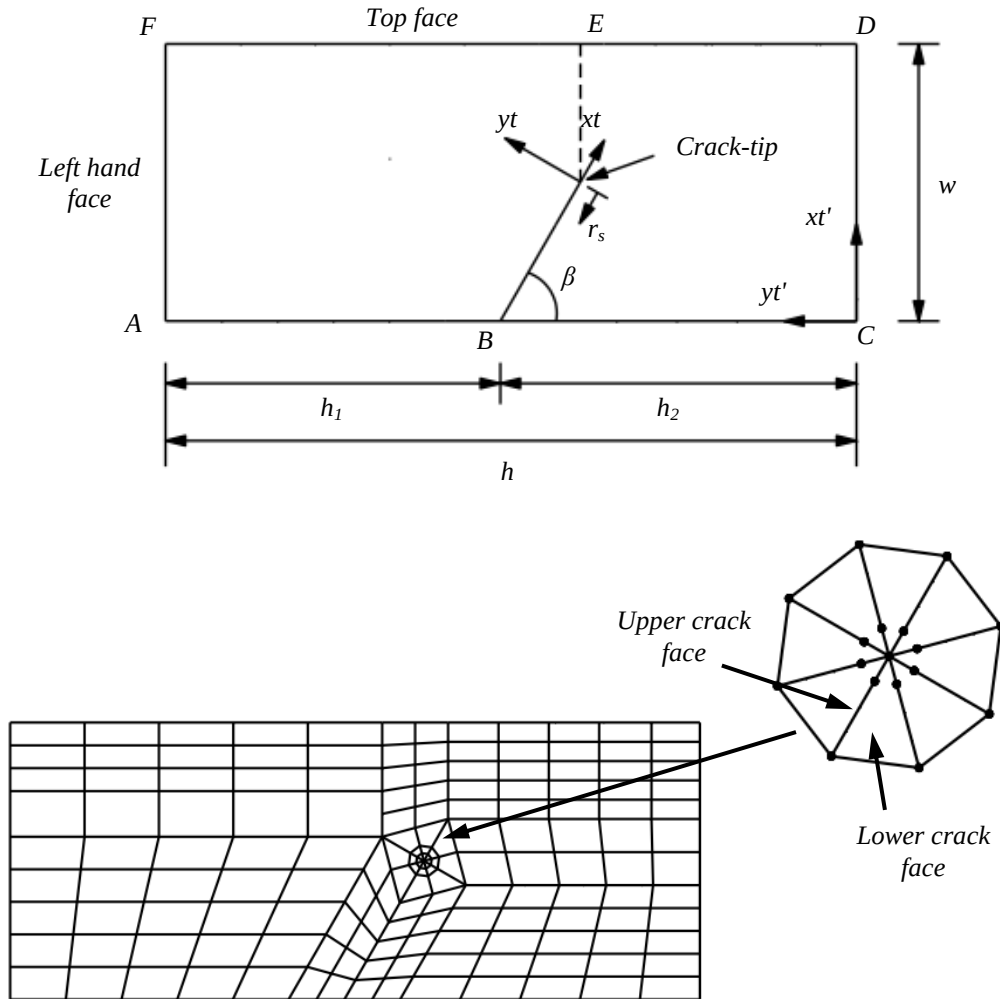


Figure 4.4 Geometry and Finite Element Model for an oblique edge crack

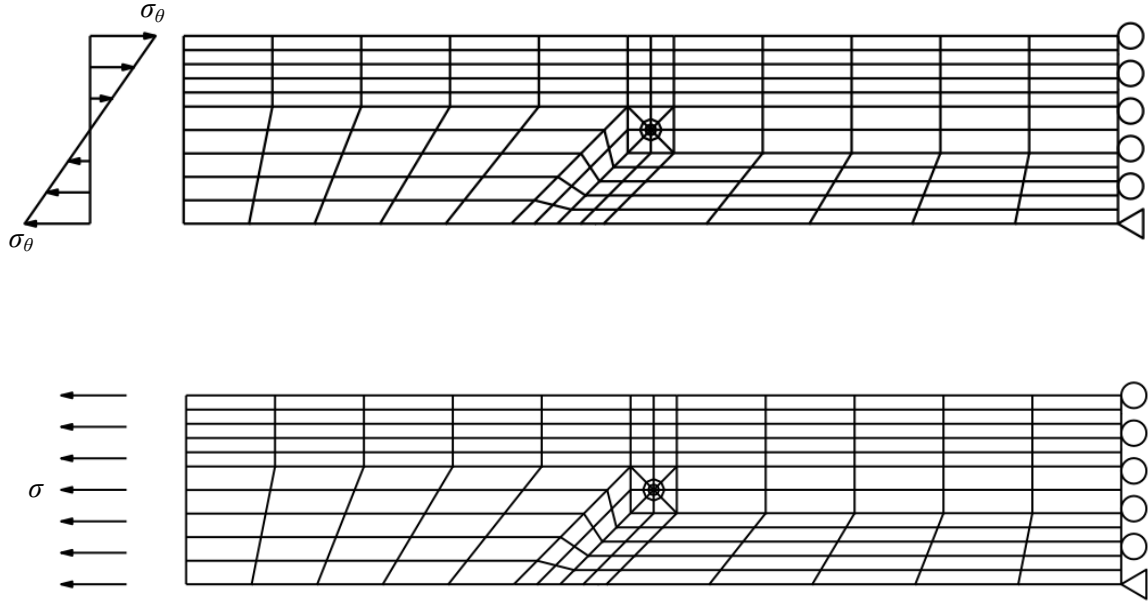


Figure 4.5 Finite element model under different loads

4.4.1. Strain energy release rate

Applying the VCE technique to decoupling characteristics of the field parameters in a crack-tip neighborhood produces an efficient finite element evaluation of strain energy release rates for mixed mode fracture problems. The accuracy assessment of the numerical strain energy release rates through a combination of the VCE technique and symmetrical mesh at the crack-tip vicinity is conducted through comparing with results from a J-Integral method [50], which is a generally accepted approach of obtaining strain energy release rates for the oblique edge crack geometries under pure bending load condition. The finite element mesh of a crack geometry and constraint conditions are shown in **Figure 4.5**. The J-Integral is originally formulated as a closed line-integral of strain energy density and work done by tractions around the crack-tip and then extended to the finite element application based on a domain integral expression of J-Integral derived in [51]. The line integral method and the domain integral method were compared in [52], more extensive applicability and better accuracy have been achieved by the domain integral method. Therefore, the domain integral method is employed in obtaining J-Integral parameters (and hence strain energy release rates) for the elastic bodies in this study.

Before conducting the VCE technique for strain energy release rates, the amount of collinear crack extension Δa in **Figure 4.1**, which is one of key operational parameters for the VCE technique, should be determined. [44] calculated strain energy release rates corresponding to a wide range of $10^{-14} \leq \Delta a/l_s \leq 10^0$ for Mode I problems and pointed out that there are two causes for the solution degradations. On one side, due mainly to the round-off error of the output global stiffness matrices and nodal displacement vectors, $\Delta a/l_s$ should not be small than 10^{-13} , on the other side, when $\Delta a/l_s > 10^{-3}$ the solution deterioration was observed due to the excessive shape distortion of the degenerated crack-tip elements as a result of virtual crack extension. For practical purposes, $\Delta a/l_s$ ratio value of 10^{-5} was suggested for numerical computations on the Mode I problems. As the round-off error is one of the mainly causes for solution deterioration

suggested from the study on Mode I problems, the solution stable range $\Delta a/l_s$ should be determined again for mixed mode problems even using the same precise level of stiffness and displacements. The reasons are: (1) The process of obtaining Mode I and Mode II field nodal displacements within the symmetric region in the crack-tip neighborhood involves a subtraction of corresponding nodal displacements between symmetrical nodes with respect to the xt axis. This subtraction may cut off certain effective digits; (2) The Mode I and Mode II field nodal displacements may be in different orders of magnitude, as a result, the computation precisely sufficient for the calculation of Mode I strain energy release rates may be insufficient for the Mode II strain energy release rates. The normalized GI and GII values corresponding to different $\Delta a/l_s$ are shown in **Figure 4.6**. It appears that normalized GI and GII solutions approach the converged GI and GII values as $\Delta a/l_s$ ratio decreases before showing accuracy degradation at $\Delta a/l_s$ ratio equaling 10^{-10} and 10^{-7} , respectively, due to the round-off error mentioned in the upward sections. As the $\Delta a/l_s$ ratio increasing, both GI and GII solution degradations are observed when $\Delta a/l_s > 10^{-3}$. Therefore, for practical purposes, the midpoint, $\Delta a/l_s$ ratio value of 10^{-5} is used to generate the numerical results in this study, which is sufficient to ensure the solution stability.

With the $\Delta a/l_s$ ratio value of 10^{-5} , the normalized strain energy release rate values evaluated from Virtual Crack Extension method for the oblique crack with different a/w ratios are listed and compared with solutions from the domain integral method in

Table 4.1. It is found that the maximum and average absolute discrepancy is 1.057% and 0.425% respectively between the solutions from these two methods. This confirms that strain energy release rates of a combined mode I and mode II fracture problem can be evaluated accurately by applying VCE technique with symmetric mesh in the crack-tip neighborhood.

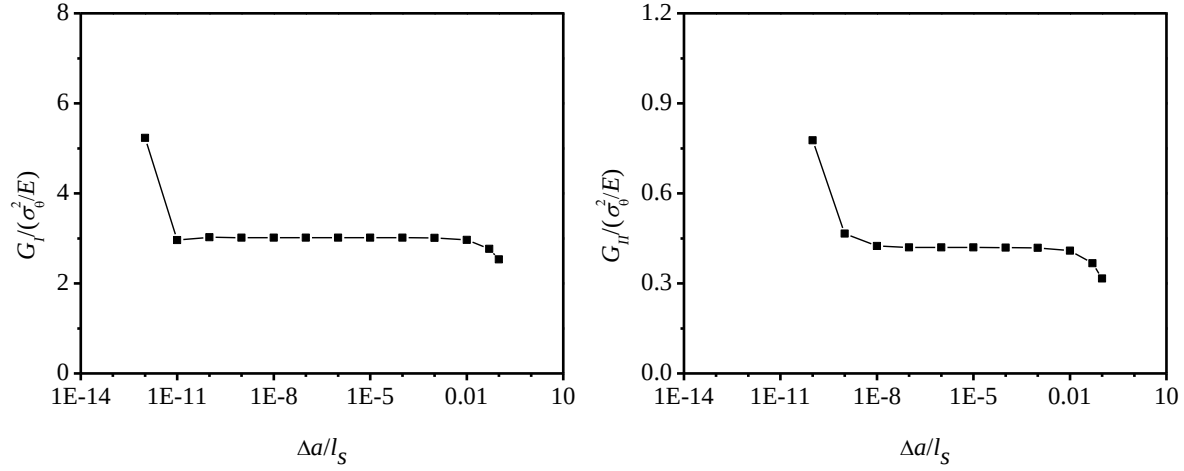


Figure 4.6 Effect of incremental crack size of virtual crack extension on solution stability

Table 4.1 Strain energy release rates calculated by different methods

a/w	Virtual Crack Extension			$J/(\sigma_\theta^2 a/E)$	Discrepancy (%)
	$G_I/(\sigma_\theta^2 a/E)$	$G_{II}/(\sigma_\theta^2 a/E)$	$G/(\sigma_\theta^2 a/E)$		
0.1	1.264	0.318	1.582	1.579	-0.204
0.2	1.339	0.295	1.635	1.617	-1.057
0.3	1.571	0.298	1.869	1.861	-0.432
0.4	2.056	0.332	2.388	2.381	-0.307
0.5	3.014	0.420	3.434	3.420	-0.406
0.6	5.090	0.625	5.714	5.687	-0.482
0.7	10.558	1.178	11.736	11.680	-0.483
0.8	31.312	3.229	34.541	34.551	0.029

Note: (1) Virtual Crack Extension: Strain energy release rates determined from applying the VCE technique into finite element;

(2) J: Strain energy release rates determined based on the J-Integral method.

4.4.2. Weight function

Even though strain energy release rates and corresponding stress intensity factors for mixed mode problems can be determined through applying the VCE technique into finite element analysis, it is still inconvenient to conduct calculations repeatedly for different load conditions. Therefore, a further step of this study is obtaining the weight function through applying the VCE technique into finite element analysis because once weight functions for a particular cracked body are determined, the stress intensity factor for any loading system applied to the body can be calculated by a simple integration.

4.4.2.1. Nodal weight functions along different boundaries

With respect to the (x', y') coordinates, **Figure 4.7** shows the weight function components along the left-hand face for oblique edge crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=5.0$ in relation to the distance away from the point A in **Figure 4.4**.

For a given group of a/w and h/w ratios, $h_{Ixt'}$ and $h_{IIxt'}$ keep almost constant while $h_{Iyt'}$ and $h_{IIyt'}$ decrease with an increase of the distance away from the point A. Specially, in all h_{Ixt} and $h_{IIxt'}$, there is a consistent increase of absolute value with an increase of both a/w ratio.

Considering that loads are generally applied on the top-face of an RC beam, weight functions on the top-face are presented as well in this study. Regarding the point E which is the closest point away from the crack-tip on the top-face as the origin and defining the direction from F to D as the positive direction, plots of weight function components for the top-face of crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=5.0$ are shown in **Figure 4.8**. In h_{Ixt} and $h_{IIxt'}$, a consistent decrease from maximum to almost zero is observed from point F to E due to the decreasing force lever, while $h_{Iyt'}$ and $h_{IIyt'}$ stay at a certain plateau in most of F-E region and drop dramatically to almost zero just adjacent E. When the load applied location passes E, the force lever turns into zero and consequently both h_{Ixt} , $h_{IIxt'}$ and $h_{Iyt'}$, $h_{IIyt'}$ remain almost zero in most of E-D region. Due to the increasing content of local disturbing in the stresses around the crack-tip as the applied load approaching the crack-tip, a slight fluctuation is observed in all curves within a small region adjacent the origin point E. The characteristics shown in the weight function curves for the top-face can be interpreted similarly as for the left-hand face.

In general, an inclined shear crack in an RC beam is bridged by a series of forces acting on both crack faces owing to concrete aggregates or reinforcement. The evaluation of crack-face weight functions are of great significance. For the mixed-mode stress intensity factors, which are evaluated with the weight function concept with the crack-face loading, the explicit crack-face weight functions with respect to (xt, yt) coordinates h_{Ixt} , h_{IIxt} , h_{Iyt} and h_{IIyt} are more convenient to use than that of (xt', yt') coordinates $h_{Ixt'}$, $h_{IIxt'}$, $h_{Iyt'}$ and $h_{IIyt'}$. The crack-face weight functions h_{Ixt} , h_{IIxt} , h_{Iyt} and h_{IIyt} along the lower and upper crack faces of the oblique edge crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=5.0$ are shown respectively in **Figure 4.9** and **Figure 4.10**, where r_s is the distance away from the crack-tip. It is noted that the $1/\sqrt{r}$ singular behaviors of the crack-face weight functions in crack-tip neighborhood are limited to the primary weight function components (h_{IIxt} and h_{Iyt}) while the secondary crack-face weight functions (h_{Ixt} and h_{IIyt}) are nonsingular in the crack-tip neighborhood.

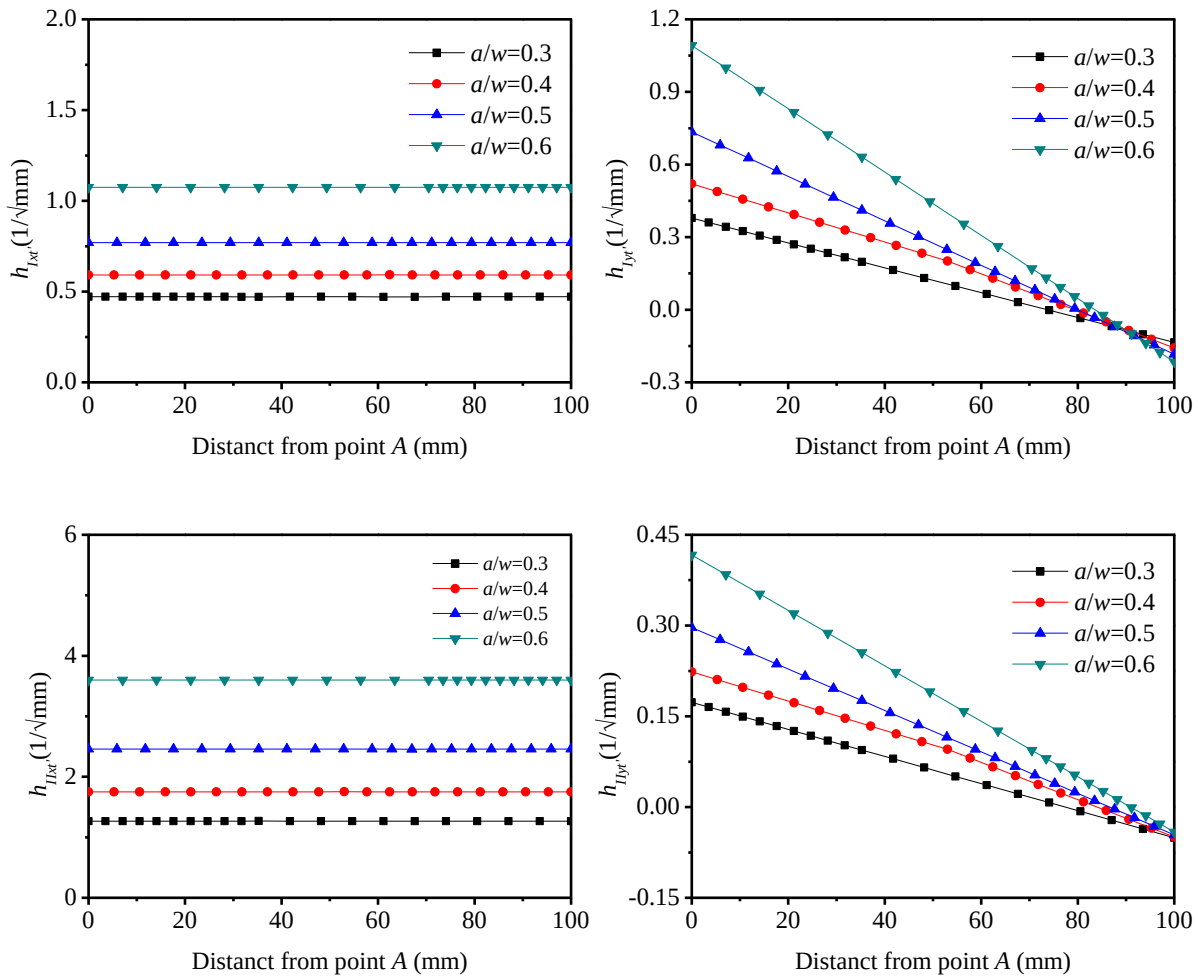


Figure 4.7 Weights function along the left-hand face

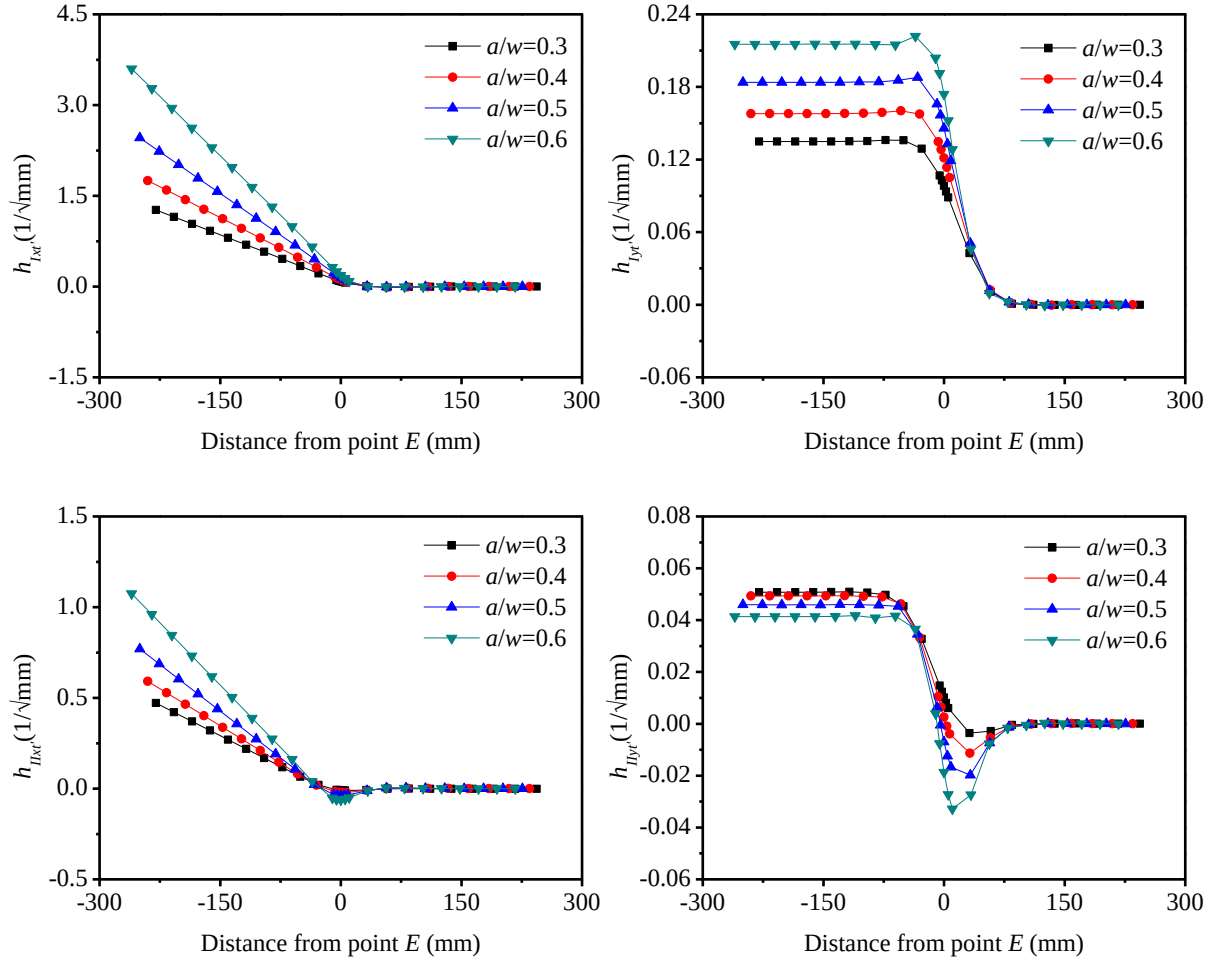
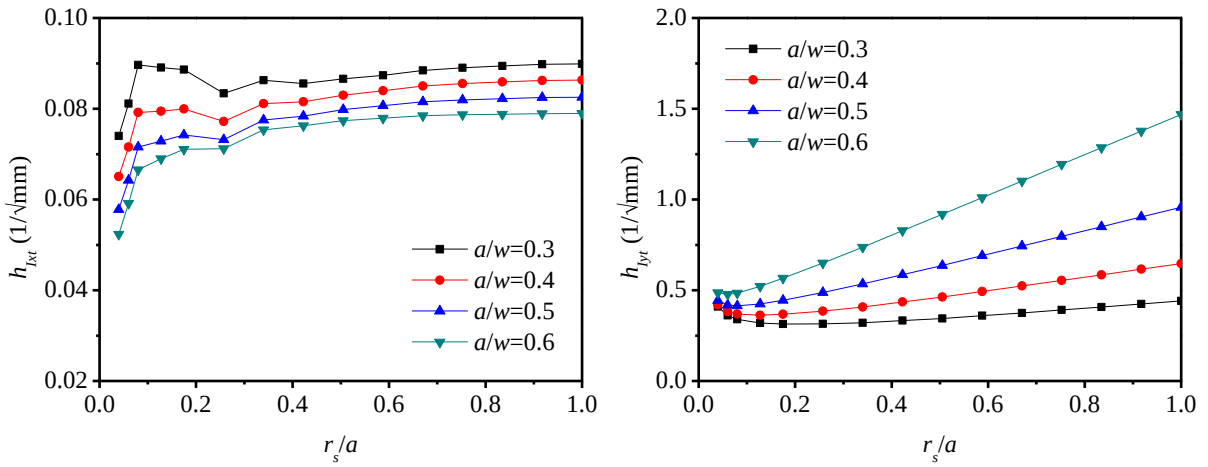


Figure 4.8 Weight functions along the top-face



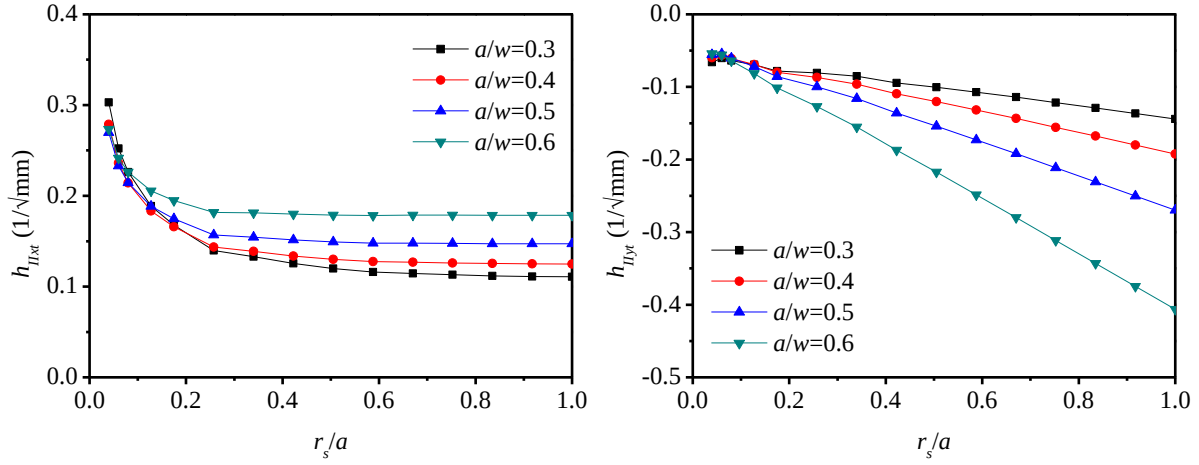


Figure 4.9 Weights function along the upper crack-face

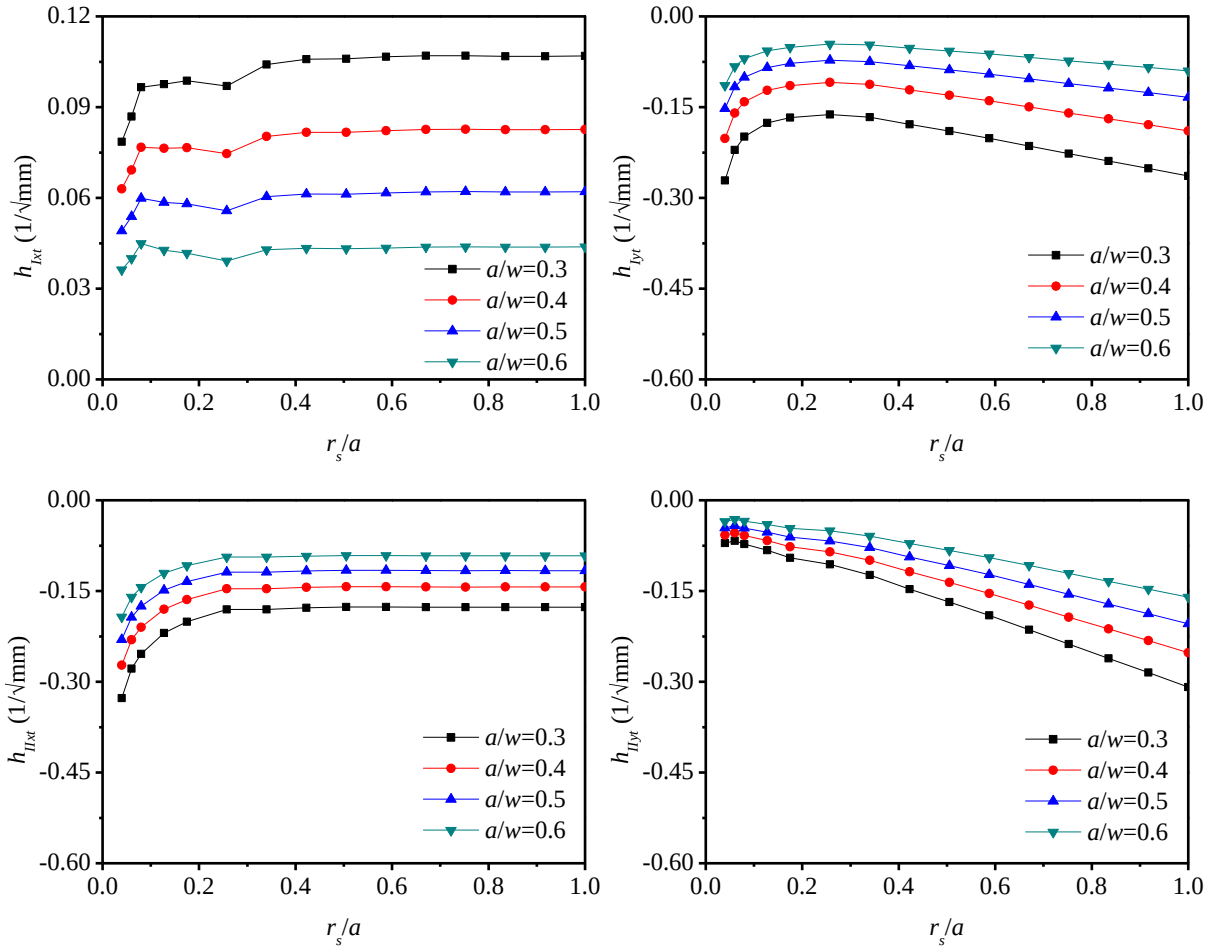


Figure 4.10 Weight functions along the lower crack-face

4.4.2.2. Weight function verification

Considering the load independent characteristic of weight function, the nodal weight functions of the oblique edge crack geometries determined under the pure bending loads can be applied to the evaluation of stress intensity factors and the corresponding strain energy release rates for the mixed fracture mode under remote tension loads. The weight functions on the left-hand face of the oblique edge crack geometries with $h/w=5.0$ and $0.2 \leq a/w \leq 0.7$ are calculated using the pure bending load and shown in **Figure 4.11**. With the weight functions from pure bending loads, the strain energy release rates evaluated following weight function method and J-Integral method for the oblique edge crack elastic geometries under pure tension load conditions are listed in **Table 4.2**. The less than 1 percent discrepancies for all a/w ratios further confirm the applicability and reliability of the weight functions determination for mixed mode fracture problems through applying VCE technique with symmetric mesh around crack-tip.

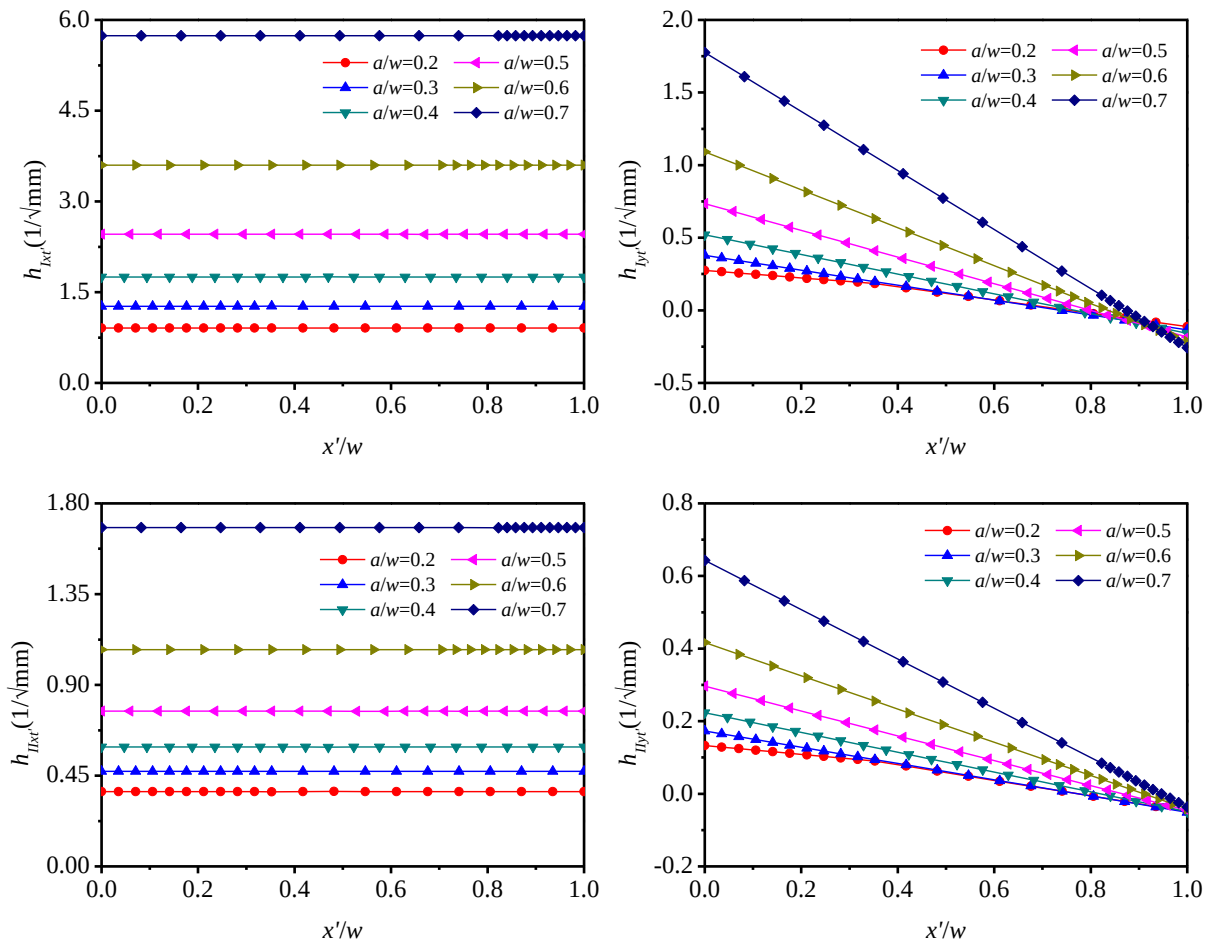


Figure 4.11 Weight functions along the left-hand face

Table 4.2 Strain energy release rates calculated by different methods

a/w	Weight Function method			$J/(\sigma^2 a/E)$	Discrepancy (%)
	$G_I/(\sigma^2 a/E)$	$G_{II}/(\sigma^2 a/E)$	$G/(\sigma^2 a/E)$		
0.2	2.136	0.568	2.704	2.686	-0.679
0.3	3.187	0.802	3.990	3.976	-0.352
0.4	5.283	1.219	6.502	6.487	-0.238
0.5	9.754	2.022	11.776	11.746	-0.262
0.6	20.597	3.775	24.372	24.295	-0.319
0.7	53.019	8.457	61.476	61.297	-0.293

Note: (1) Weight Function method: Strain energy release rates determined based on the weight function method employing the weight function from the pure bending load;

(2) J: Strain energy release rates determined based on the J-Integral method.

4.4.2.3. Fitting and interpolation of crack face weight functions

Taking advantage of the well-known superposition principle, any load can be equivalent to a certain crack face load. As a result, the stress intensity factors due to the load may be determined with the crack-face weight functions which are obtainable through fitting the crack-face nodal weight functions with base functions. Generally, the polynomial bases are sufficient for the accurate fitting of the secondary crack-face nodal weight function components while the primary crack face weight function components should be fitted with special bases because the singular behavior at the crack-tip neighborhood is observed in the primary crack-face nodal weight function components h_{Iyt} and h_{IIxt} as illustrated in previous sections. It is obviously that the primary crack-face weight function components with a given crack length (a) for the oblique edge crack geometry should be a function of the ratio (r_s/a) between distance (r_s) from the crack-tip and the crack length (a). Therefore, these primary components of a given oblique angle β in (xt , yt) coordinates can be fitting by the following equations where the superscripts U and L referring to the upper and lower crack faces respectively.

$$h_{Iyt}^{U(L)}(r_s/a, a, \beta) = \sum_{n=1}^{N_I} A_n(a) (r_s/a)^{(n/2-1)}, \quad (4.28a)$$

$$h_{IIxt}^{U(L)}(r_s/a, a, \beta) = \sum_{n=1}^{N_{II}} B_n(a) (r_s/a)^{(n/2-1)}, \quad (4.28b)$$

where

$A_n(a)$ = coefficients that are a function of crack length for a given oblique angle β .

$N_{I(II)}$ = maximum number of terms required to achieve accurate fitting of the nodal weight functions.

$N_I=6$ and $N_{II}=6$ are found to provide accurate fitting results of for oblique edge crack geometry. Both the primary nodal weight functions and the fitting weight function curves along the lower and upper crack faces for the oblique edge crack geometries with $\beta=45^\circ$ and $0.1 \leq a/w \leq 0.8$ are shown in **Figure 4.12**.

For the crack-tip node, the nodal weight function is unobtainable because the determinant of Jacobian matrix $[J]$ as illustrated in **Eq.(4.23)** approaches to infinite for this node. The coefficients, $A_n(a)$ and $B_n(a)$ corresponding to different crack lengths are listed in **Table 4.3**.

In terms of the application of weight functions, in some cases, such as the calculation of crack opening displacements based on the integral equation in [25], the weight function should be formulated as a function of both crack length (a) and distance ratio away from the crack-tip (r_s/a). Thus, in this study, the coefficients, $A_n(a)$ and $B_n(a)$, are expressed as a function of crack length (a) for a given oblique angle through interpolating the corresponding coefficients for different crack length shown in **Table 4.3**. The applicability and reliability of the interpolation is confirmed through the less than 1% discrepancy between the weight functions obtained from applying the interpolated weight functions and the direct VCE technique for $a/w=0.35, 0.45, 0.55$ as shown in **Figure 4.13**.

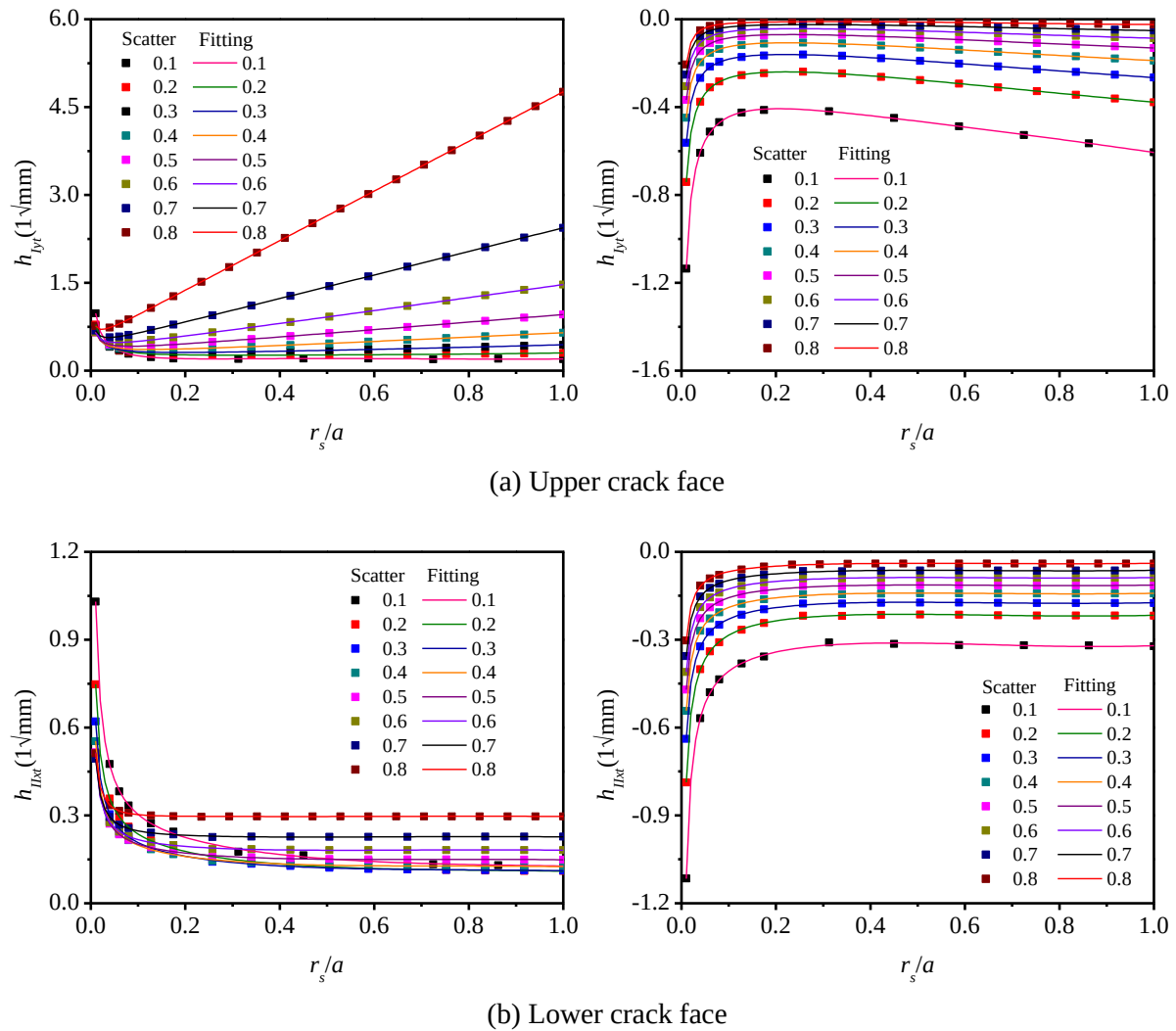
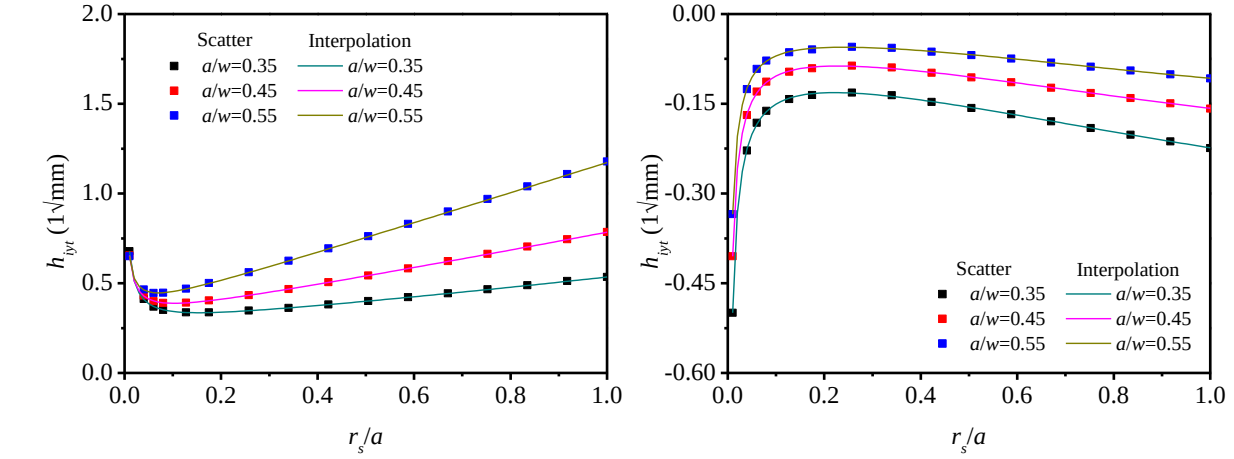
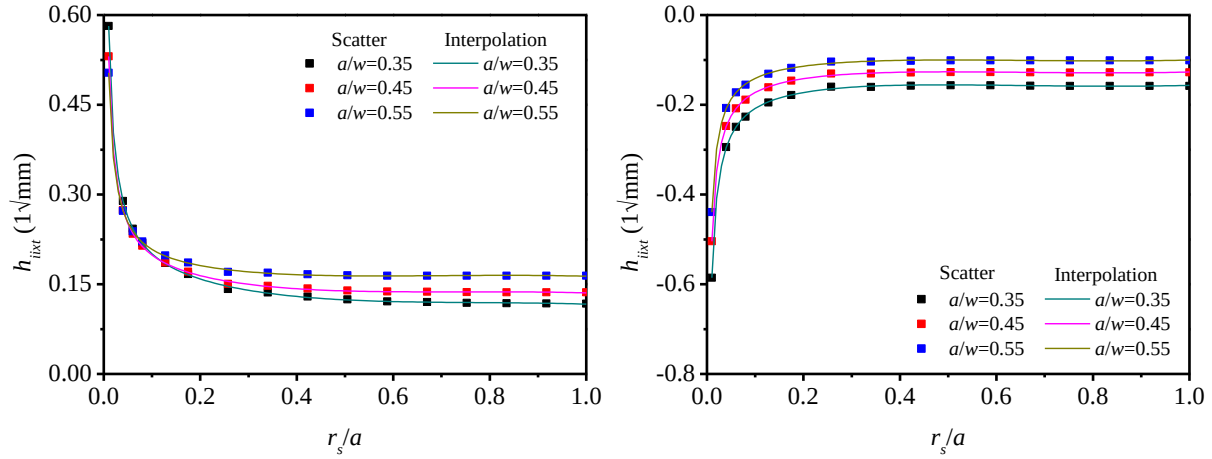


Figure 4.12 Primary weight function along upper and lower crack faces



(a) Upper crack face



(b) Lower crack face

Figure 4.13 Primary weight functions from interpolated results and the VCE technique

Table 4.3 Coefficients of **Eq.(4.28)** for crack-face weight function

(a) h_{ijt} on upper crack face						
a/w	A_1	A_2	A_3	A_4	A_5	A_6
0.1	-0.092734	-0.229009	2.383365	-6.921558	7.379385	-2.718690
0.2	-0.080541	0.038203	-0.131954	-0.766838	1.203308	-0.564631
0.3	-0.067423	0.013743	-0.420176	0.037796	0.141774	-0.147851
0.4	-0.059254	-0.018566	-0.559689	0.303690	-0.377589	0.063093
0.5	-0.053520	-0.052535	-0.646945	0.270632	-0.652596	0.176646
0.6	-0.049182	-0.090396	-0.716896	-0.051040	-0.796518	0.234919
0.7	-0.045670	-0.139725	-0.783600	-0.878386	-0.835279	0.244057
0.8	-0.043304	-0.204715	-0.958865	-2.807243	-1.055604	0.308214

(b) $h_{I_{yt}}$ on lower crack face

a/w	A_1	A_2	A_3	A_4	A_5	A_6
0.1	0.111336	0.021621	-0.12338	1.517708	-1.50778	0.587017
0.2	0.084595	-0.17307	0.777969	-1.06263	1.184471	-0.43361
0.3	0.069391	-0.1949	0.726072	-1.02747	1.086364	-0.39387
0.4	0.060614	-0.22225	0.749106	-1.12875	1.130974	-0.40135
0.5	0.054837	-0.25015	0.810916	-1.2877	1.228063	-0.42502
0.6	0.050751	-0.27922	0.904473	-1.49009	1.359472	-0.45891
0.7	0.04775	-0.3123	1.039534	-1.76236	1.552487	-0.51361
0.8	0.044793	-0.34309	1.199906	-2.10641	1.836285	-0.60747

(c) $h_{I_{xt}}$ on upper crack face

a/w	B_1	B_2	B_3	B_4	B_5	B_6
0.1	0.131802	-0.44081	1.833506	-3.32233	2.800416	-0.87519
0.2	0.091909	-0.28242	1.354536	-2.71649	2.529538	-0.86776
0.3	0.076298	-0.24426	1.255316	-2.54506	2.411011	-0.84208
0.4	0.066731	-0.20921	1.163286	-2.34709	2.250624	-0.79935
0.5	0.060091	-0.17488	1.083916	-2.13085	2.043538	-0.7334
0.6	0.055304	-0.1429	1.034728	-1.92274	1.798936	-0.64223
0.7	0.052347	-0.12152	1.086784	-1.86518	1.633669	-0.55949
0.8	0.051027	-0.11057	1.317747	-2.13715	1.715409	-0.5401

(d) $h_{I_{xt}}$ on lower crack face

a/w	B_1	B_2	B_3	B_4	B_5	B_6
0.1	-0.13498	0.427064	-2.35207	4.672481	-4.70868	1.775827
0.2	-0.09009	0.212744	-1.2018	2.357066	-2.45043	0.956048
0.3	-0.07384	0.182489	-0.99382	1.893965	-1.92035	0.737893
0.4	-0.06439	0.176023	-0.90237	1.703452	-1.69844	0.644279
0.5	-0.0581	0.182118	-0.86356	1.629122	-1.60587	0.602862
0.6	-0.05369	0.198288	-0.86797	1.644464	-1.60506	0.596248
0.7	-0.05062	0.22618	-0.92195	1.753932	-1.68949	0.618835
0.8	-0.04719	0.244479	-0.90486	1.673915	-1.56931	0.563969

4.5. Parametric study on shear span vs. beam depth ratios

For an RC beam, the failure mode is strongly dependent on the shear span to beam depth ratio (h/w). [53] summarized the failure crack pattern of RC beams with h/w varying from 1.5 to 8.0 under four-point bending load. It was found that the inclined shear crack leading to failure developed only in RC beams with h/w ratios smaller than 8.0. For very large h/w , only a series of almost parallel fracture cracks were observed on the failure RC beams. Therefore, the oblique crack geometries as shown in **Figure 4.4** with oblique angle $\beta=45^\circ$ and $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=2.5, 5.0, 7.5$ are employed for the detailed parametric study.

With respect to the (xt', yt') coordinates, **Figure 4.14** shows the weight function components along the left-hand face for the oblique edge crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=2.5, 5.0, 7.5$ in relation to the distance away from the point A in **Figure 4.4**. For a given group of a/w and h/w ratios, $h_{Ixt'}$ and $h_{IIxt'}$ keep almost constant while $h_{Iyt'}$ and $h_{IIyt'}$ decrease with an increase of the distance away from the point A. In all $h_{Ixt'}$ and $h_{IIxt'}$, there is a consistent increase of absolute value with an increase of h/w ratio. For different a/w ratios, the relation between the constant value of $h_{Ixt'}$ and $h_{IIxt'}$ and h/w ratios are shown in **Figure 4.15**. It is found that for each a/w ratio $h_{Ixt'}$ and $h_{IIxt'}$ increase linearly with the increasing of h/w , which means the $h_{Ixt'}$ and $h_{IIxt'}$ for different a/w and h/w ratios can be obtained through linear interpolation of the presented results. In terms of $h_{Iyt'}$ and $h_{IIyt'}$, they increase with respect to the increase of a/w ratio, nevertheless stay nearly unchanged for a fixed a/w ratio and changing h/w ratio.

All the orderly trends represented in the curves of the weight function components can be interpreted theoretically as following. As shown in **Figure 4.16**, for a 2-D crack geometry subjected to any arbitrary combined Mode I and Mode II load condition, the linear elastic stress field around a crack-tip can be expressed with as simple analytical form as

$$\begin{Bmatrix} \sigma_{xtxt} \\ \sigma_{ytyt} \\ \sigma_{xtyt} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{Bmatrix} \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)] \\ \cos(\theta/2)[1 + \sin(\theta/2)\sin(3\theta/2)] \\ \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2) \end{Bmatrix} + \frac{K_{II}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin(\theta/2)[2 + \cos(\theta/2)\cos(3\theta/2)] \\ \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2) \\ \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)] \end{Bmatrix} \quad (4.29)$$

thus, for any point around the crack-tip, θ and r are constants. Then,

$$\begin{Bmatrix} \sigma_{xtxt} \\ \sigma_{ytyt} \\ \sigma_{xtyt} \end{Bmatrix} = [C] \cdot \begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix}, \quad (4.30)$$

where $[C]$ is a matrix of constants. According to the weight function concept, SIFs due to applied load is an integration of the applied load and the weight function at their points of application. Thus, the weight functions at a point on a crack geometry is equivalent to the SIFs if a unit concentrated load is applied on the point. Hence the $h_{Ixt'}$, $h_{IIxt'}$ and $h_{Iyt'}$, $h_{IIyt'}$ at a point on the left-hand face can be obtained if a unit load is applied at the point along the xt' and yt' axes, respectively. In the elastic mechanics scale, it is easily imaginable that the stresses of any point around the crack-tip stay the same if a unit load applied along the xt' axis moving from point A to F in **Figure 4.4** owing to the unchanged force lever from the crack-tip and vary linearly if the unit load applied along the yt' axis. Similarly, following **Eq. (4.30)**, the SIFs should experience the same trends manifested in the stresses. Therefore, $h_{Ixt'}$, $h_{IIxt'}$ and $h_{Iyt'}$, $h_{IIyt'}$ should be as shown in **Figure 4.14**.

With the same origin point and positive direction definitions in Section 4.4.2, plots of the weight function components for the top-face of a series of typical crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=2.5, 5.0, 7.5$ are shown in **Figure 4.17**. The characteristics exhibited in the series of geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=5.0$ are observed as well. The weight function components for $h/w=2.5$ and $h/w=7.5$ are almost the cutting and extension of the

corresponding weight function components for $h/w=5.0$. Therefore, if a group of h_{Ixt} , h_{IIxt} and h_{Iyt} , h_{IIyt} weight function components for the top-face of a crack geometry with a span wider than the disturbed region are given, the weight function components for a crack geometry with a small span or a larger span can be obtained through cutting from or extending the given weight function components, respectively.

As for the crack face weight function components, h_{Ixt} , h_{IIxt} , h_{Iyt} and h_{IIyt} along the lower and upper crack faces of the oblique edge crack geometries with $a/w=0.3, 0.4, 0.5, 0.6$ and $h/w=2.5, 5.0, 7.5$ are shown respectively in **Figure 4.18** and **Figure 4.19**. It is found that all series of weight functions for a fixed a/w ratio stay almost the same for changing h/w ratios, which means the repeated computation of crack-face weight functions for different h/w ratios can be obviated for practical engineering purposes of determination of stress intensity factors with the weight function concept.

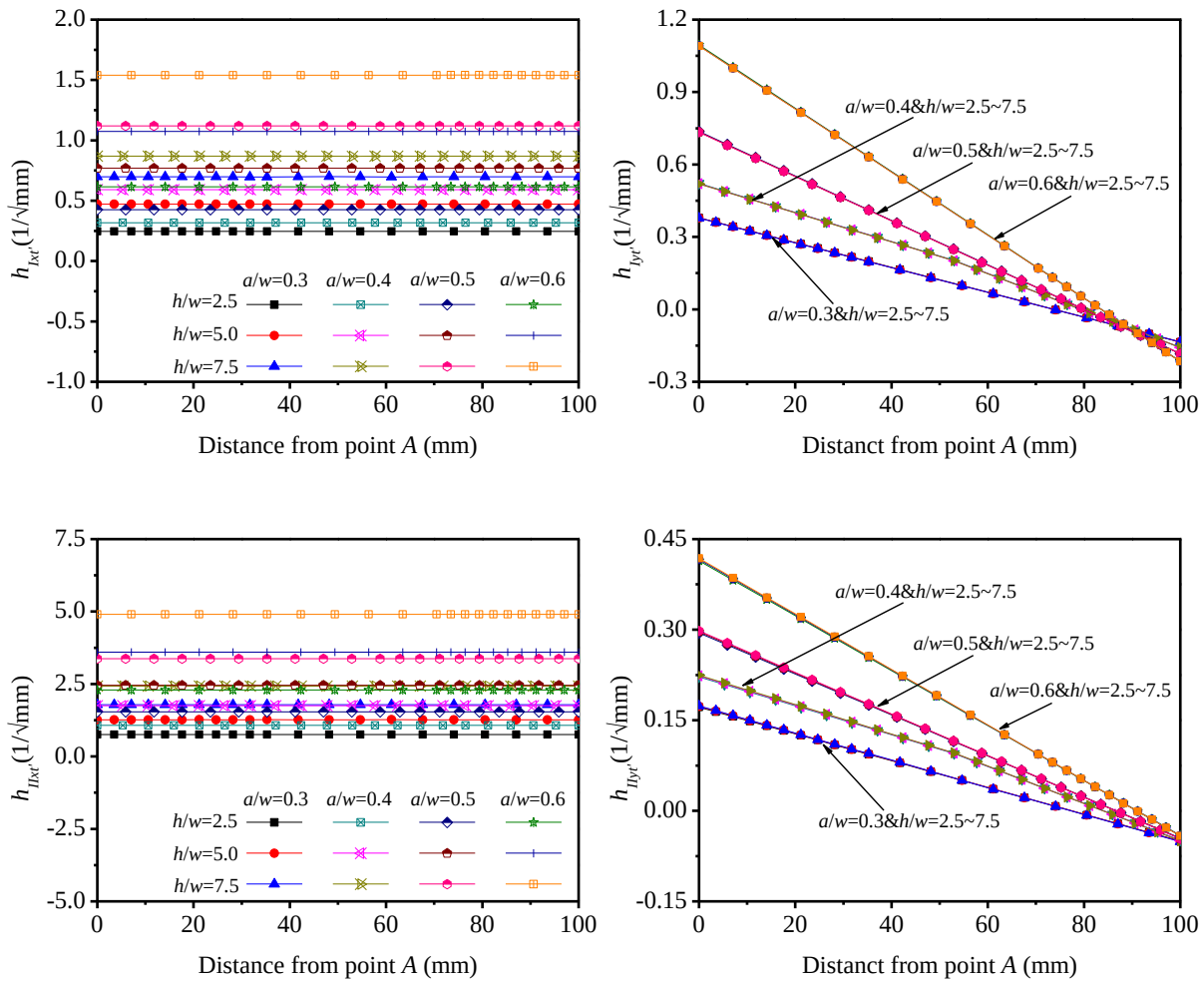


Figure 4.14 Nodal weight functions left-hand face

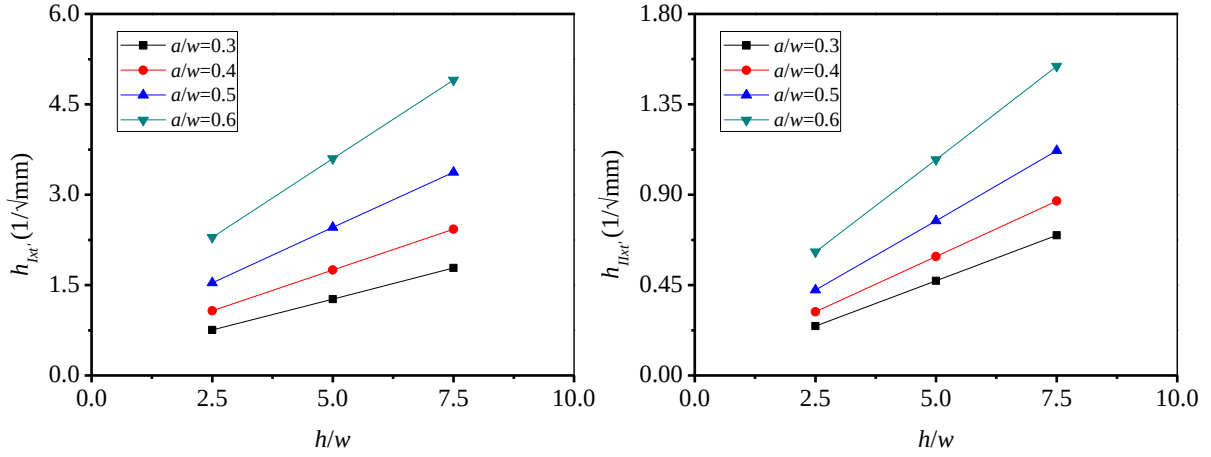


Figure 4.15 Nodal weight functions left-hand face for different a/w ratios

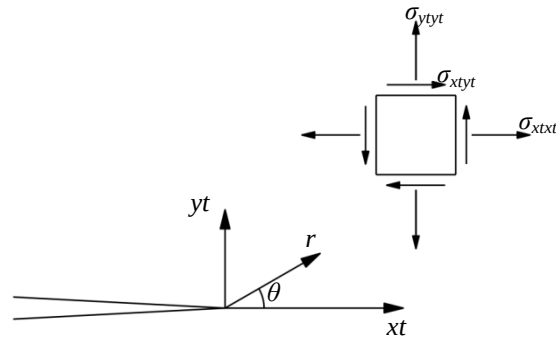
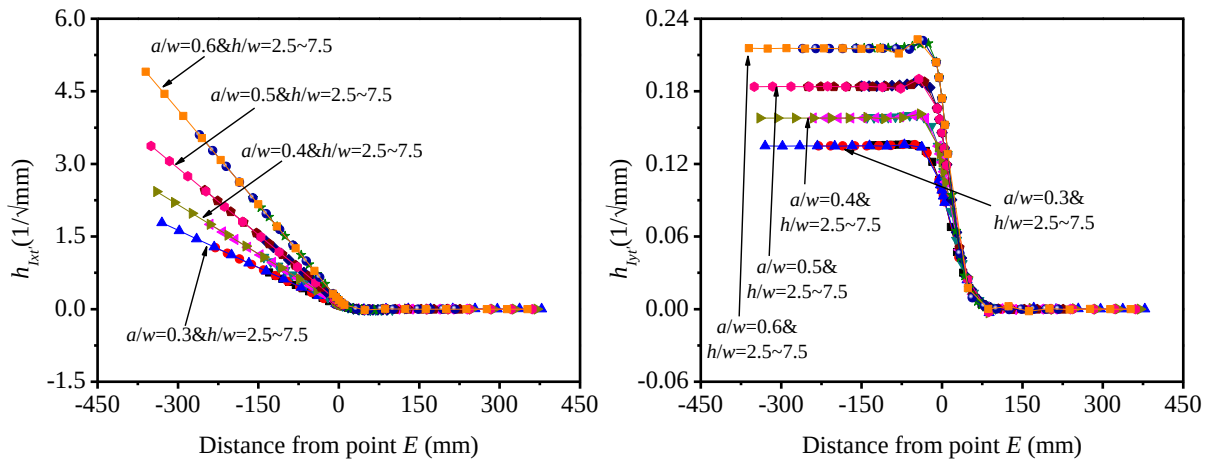


Figure 4.16 Elastic stress field at crack-tip



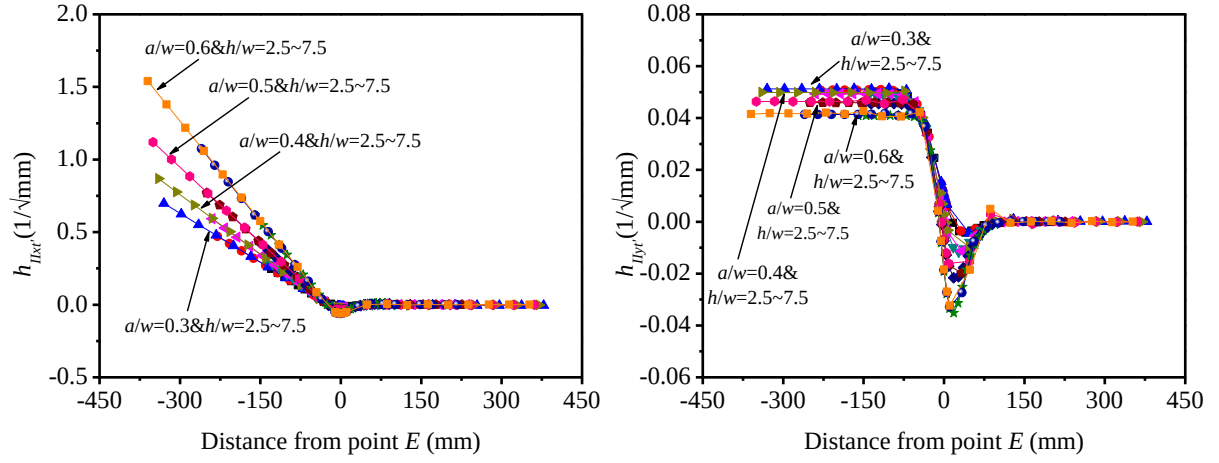


Figure 4.17 Nodal weight functions on top-face

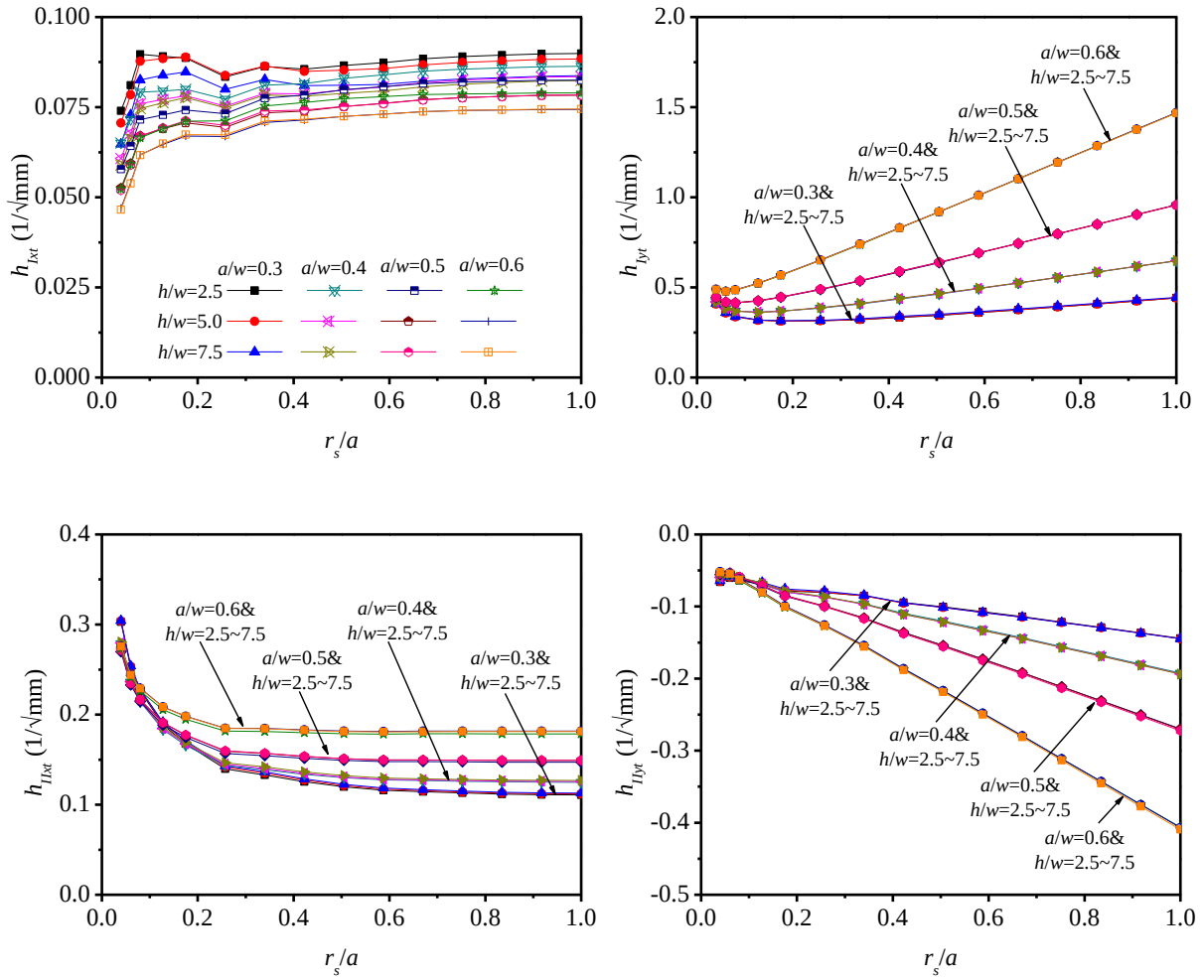


Figure 4.18 Nodal weight functions on upper crack-face

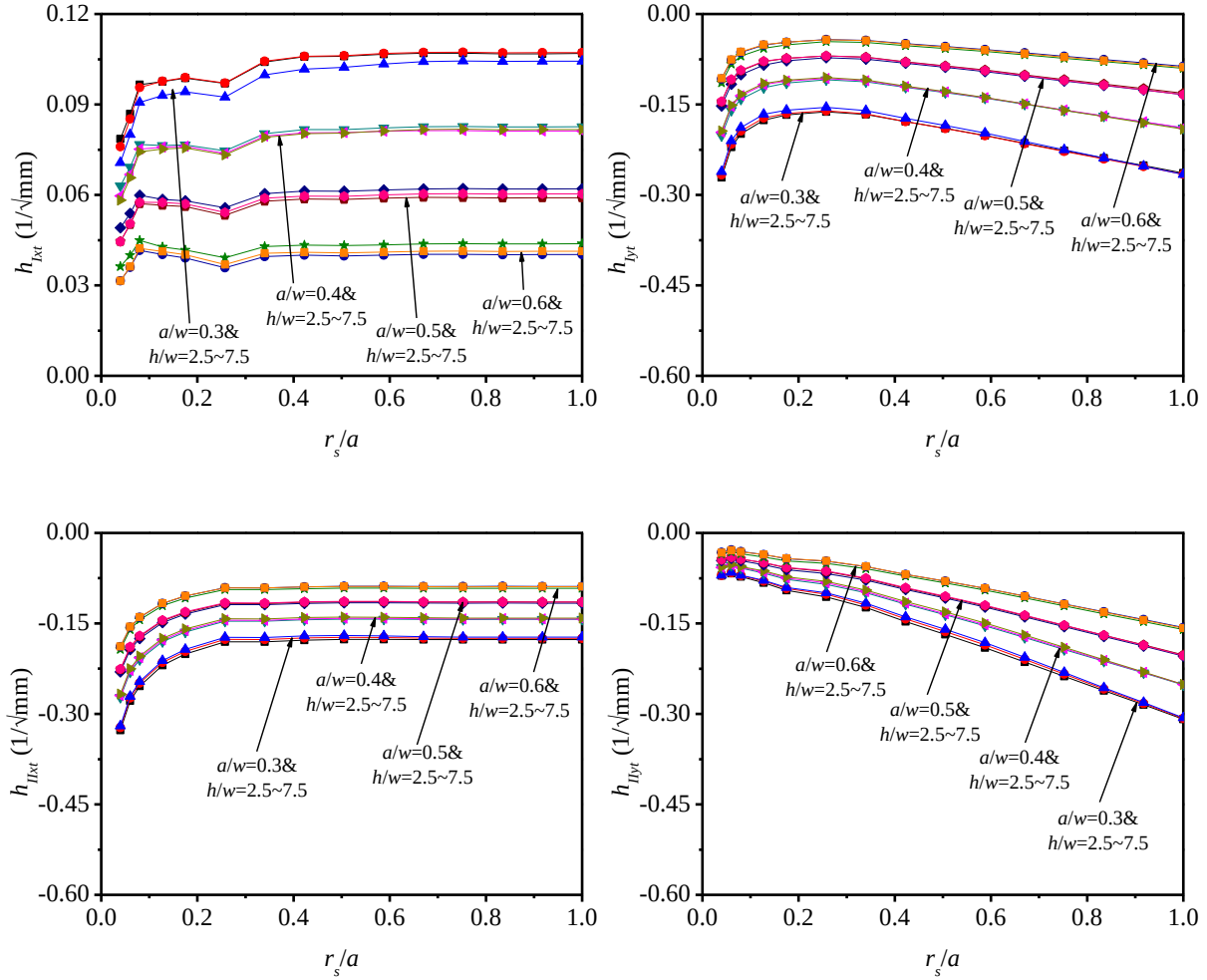


Figure 4.19 Nodal weight functions on lower crack-face

4.6. Summary and conclusions

Fracture mechanics provides a possible theoretical approach of analyzing a shear crack in an RC beam, which propagates under mixed model loading conditions. In this chapter, an efficient finite element method, where a VCE technique is coupled with symmetry mesh in the crack-tip neighborhood, was used in evaluating both strain energy release rates and weight functions for the shear crack in an RC beam.

For VCE technique, the crack extension ratio ($\Delta a/l_s$) is one of key operational parameters. It was found that result stable ranges of this parameter for Mode I and Mode II components were different even using same precise stiffness matrices and nodal displacement vectors. Therefore, the stable ranges should be determined beforehand for a mixed mode fracture problem.

Accuracy assessment of strain energy release rate evaluation based on the VCE technique method was conducted through comparing results from VCE technique and a generally accepted J-Integral method, where less than 1% discrepancies were achieved in almost all cases.

Weight functions on all generally load applied faces for an RC beam, such as the left-hand face, top-face, upper and lower crack faces, were evaluated. A singular behavior at the crack-tip neighborhood was observed in primary crack-face weight functions. To facilitate application, these crack-face weight functions were formulated successfully as a function of crack length and distance away from the crack-tip through fitting and interpolating the primary crack-face nodal weight functions. The fitted and interpolated weight functions were employed in calculating crack-face weight functions for other crack lengths and then compared with the corresponding nodal weight functions obtained directly from VCE technique. A less than 1% discrepancy between the primary crack-face weight functions from these two approaches confirmed the applicability and reliability of the fitting and interpolating process.

Considering the shear failure of an RC beam is heavily influenced by the shear span to beam depth ratio, parametric study was conducted on this ratio. It was found that the weight function along different boundaries varied orderly with respect to shear span to beam depth ratio. This orderly trend can then be followed in determining weight functions along all boundaries of an RC beam with varying shear span/beam depth ratio using the corresponding weight functions of two RC beams with different shear span/beam depth ratios.

CHAPTER 5 FATIGUE LIFE PREDICTION OF AN RC BRIDGE SLAB BASED ON FRACTURE MECHANICS

5.1. Introduction

As introduced in Chapter 2, since the previous design theories of RC bridge slabs were based on the flexural capacity, and the RC bridge slabs were designed to be lack of reinforcement and thickness. As a result, a punching shear failure mode was often observed in RC bridge slabs under service.

To figure out the fatigue behavior of RC slabs under service, a huge amount of experiments were conducted employing a cyclic moving load, where the punching shear failure mode was reproduced. According to the fatigue characteristics observed in experiments, it was concluded that the fatigue life of RC bridge slabs is determined by the punching shear capacity of a critical beam [8, 9]. Treating the punch shear strength of the critical beam as a normalizing parameter, a series of S-N equations for life prediction of RC bridge slabs under moving load were formulated through fitting the huge set of experimental data statistically.

Numerically analysis based on finite element method was conducted in [16, 17], where the fatigue crack pattern, failure mode and fatigue life were successfully modeled.

However, the existing researches have some defects. Since the empirical S-N equations were derived merely through statistically fitting experimental data, the inner degradation and failure mechanisms cannot be reflected and captured, which makes them resemble "black box". As for the numerical analysis in finite element method, even though the smeared crack model which is relatively time-saving and easy for convergence was employed, the calculation was still very time-consuming and a convergence problem was usually encountered due to the widely distributed crack element before failure.

In this study, a theoretical method for fatigue life prediction of RC bridge slabs subjected to a cyclic moving load is proposed based on the nonlinear fracture mechanics. On the basis of previous researches, the fatigue life prediction is conducted focusing on the propagation of a couple of punching shear cracks in a critical RC beam because these cracks determine the final failure of the RC slabs. As mentioned in Chapter 1, two possible dominant degradation mechanisms for RC structure under service level fatigue moving loads, i.e. the concrete bridging stress degradation and the bond slip degradation between concrete and rebar are taken into consideration. In this method, the cracking state parameters of the punching shear crack can be determined after every loading cycle. With these parameters, the failure moment, i.e. fatigue life, is identified following some failure criterion.

5.2. A reinforced concrete slab and the corresponding critical reinforced concrete beam

In this study, the fatigue analysis is conducted on a RC slab in [13], where a RC slab specimen was casted following the Specification for Steel Highway Birdges (1956) considering that most of the target RC slabs failed due to punching shear were constructed from the second half of 1950s and the beginning of 1960s. The test specimens were made of Portland cement. A

coarse aggregate with maximum size of 20mm was used. The rebar diameters along the major and minor reinforcement directions are 16mm and 13mm, respectively. The properties of concrete and steel rebars are listed in **Table 5.1**. The tested specimens were 3300mm along the moving load direction, 2650mm along the direct vertical to the moving load and 160mm depth. The cover depth is 32mm. The detail information for specimen dimensions and reinforcement arrangement are shown in **Figure 5.1**.

Figure 1.1 shows the moving load experimental set-up. The schematic diagram for the loading plate is shown as **Figure 5.2**.

According to the fatigue behavior of RC slabs under moving load, [8] proposed an empirical equation for fatigue life prediction employing the punching shear capacity of a critical RC beam. The schematic diagram for determining the punching shear capacity in [8] is shown in **Figure 5.3**. The meanings of all parameters in the **Figure 5.3** are listed as following:

- x_m : compression depth from sectional analysis ignoring the tensile effect of concrete (mm);
- h : depth of slab (mm);
- c : cover depth for tensile rebars (mm);
- f_{cv} : shear strength of concrete (MPa);
- f_t : tensile strength of concrete (MPa);
- d_e : effective depth for tensile rebar (mm);
- b : dimension of the load plate along moving load direction (mm);
- l_w : dimension of the load plate along direction vertical to the moving load (mm);
- B : width of the critical beam.

Both the shear strength (f_{cv}) and tensile strength (f_t) of concrete are determined using concrete compression strength (f_c) as:

$$f_{cv} = 0.656 \times f_c^{0.606}, \quad (5.1)$$

$$f_t = 0.269 \times f_c^{2/3}. \quad (5.2)$$

It is found that the punching shear failure is assumed along lines, 45 ° with respect to the bottom surface and the width of the critical RC beam is determined by adding two time d_e and b . This relation is employed in determining the critical beam in this study as well.

Employing the values of parameters for the RC slab shown in **Figure 5.1**, the critical RC beam corresponding to the RC slab are obtained and shown in **Figure 5.4**. In this study, a fatigue life prediction of the RC slab is conducted on this RC beam and focused on a couple of punching shear cracks symmetrical with respect to the moving load.

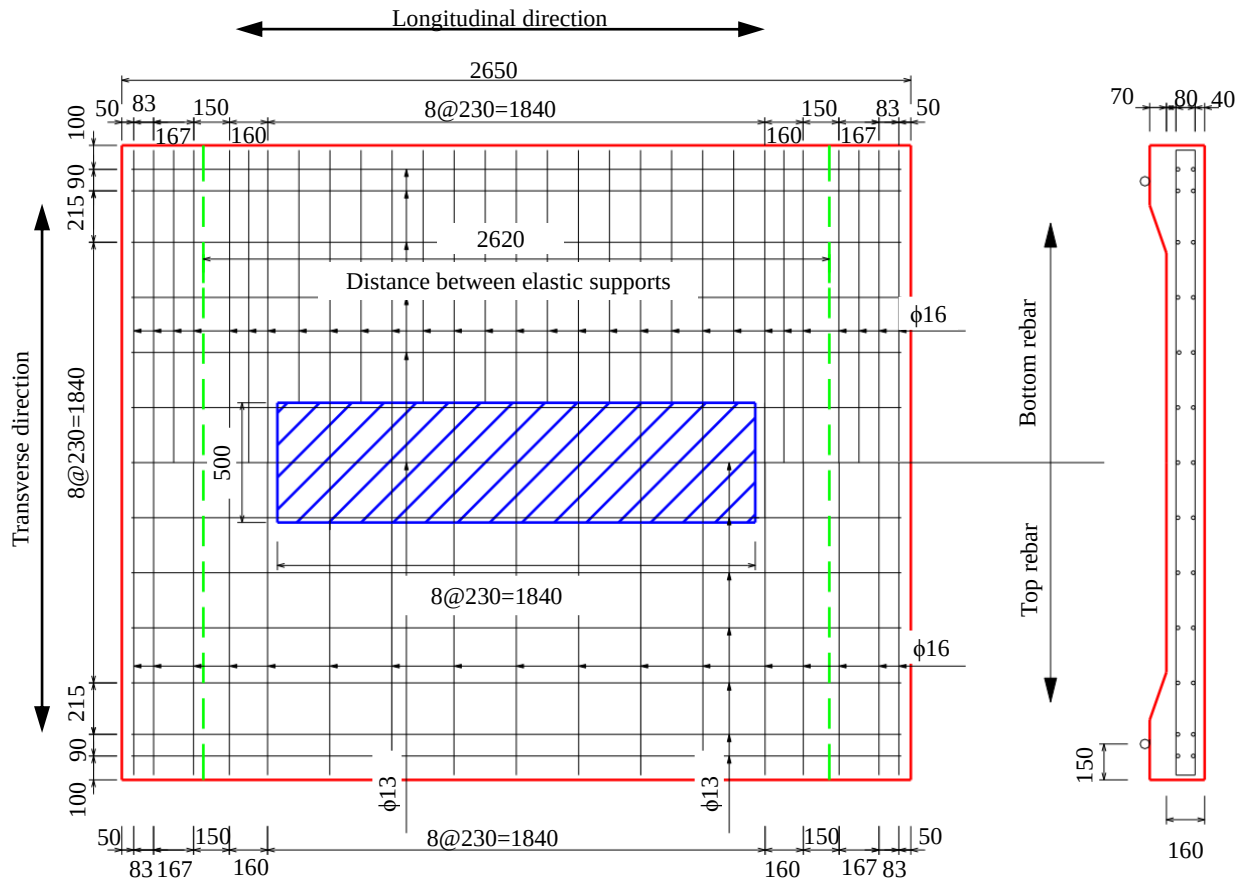


Figure 5.1 RC slab geometry and reinforcement arrangement

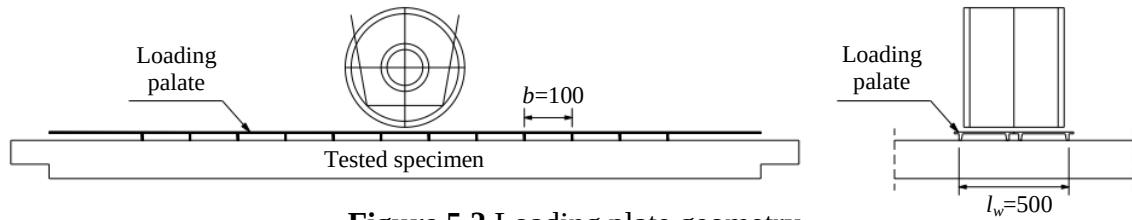


Figure 5.2 Loading plate geometry

Table 5.1 Material properties

Concrete	Compression strength (MPa)	43.2
	Elastic modulus (MPa)	25442
Steel rebar (SR235)	Yielding strength (MPa)	341
	Tensile strength (MPa)	462
	Elastic modulus (MPa)	200,000

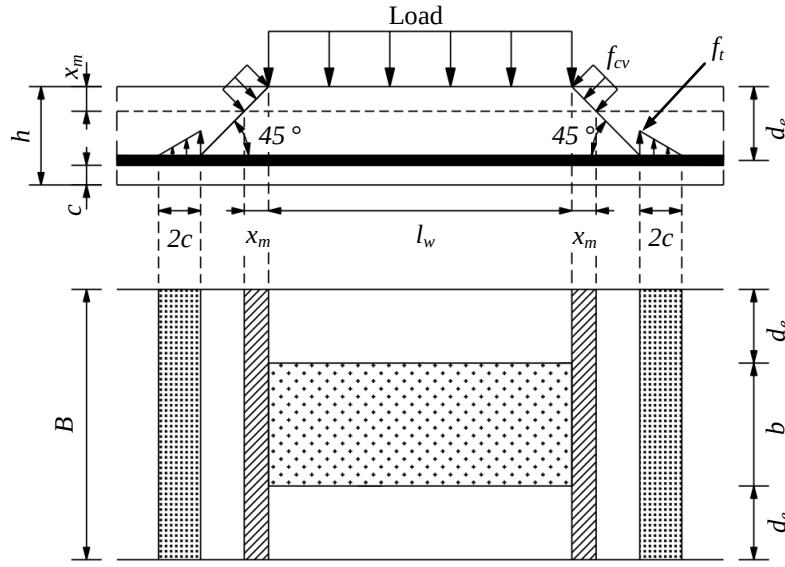


Figure 5.3 Punching shear failure model for the critical beam proposed by [8]

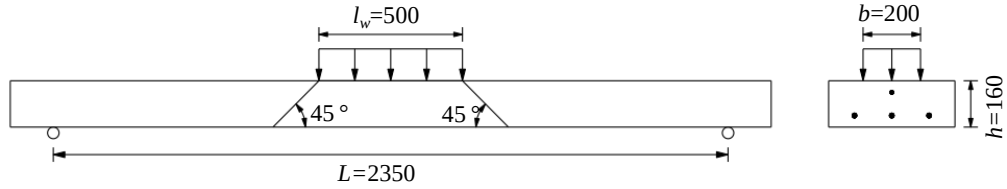


Figure 5.4 Geometry of the simplified RC beam and the punching shear cracks

5.3. Problem formulation

For the critical RC beam subjected to some general loads, the stresses acting on a section obtained through cutting the beam vertical from the tip of the punching shear cracks are shown in **Figure 5.5**.

Comparing with the concrete cracking analysis of an RC beam in Chapter 3, the post-cracking resistance of concrete should be included for a rational and accurate analysis of an RC slab due to following reasons: (1) reinforcement ratio of an RC slab is generally smaller than that of an RC beam. As a result, crack bridging effect of concrete may be not negligible compared with that from rebars; (2) In the experiment on the RC beam, a notch was set in the middle span to ensure that a major crack initiates and propagates in the pure bending range under a model I loading condition. Thus, this major crack should be wider than the punching shear crack in an RC slab subjected to moving load, whereas the bridging stress of concrete decreases with the increasing of crack width; (3) The depth for the model I crack in Chapter 3 was determined following the criterion for crack advance in bridged crack model under monotonic loading, which is the crack-tip stress intensity factor due to applied load and bridging elements equaling to the fracture toughness of reinforced concrete from experiment. This means the energy dissipation due to bridging effect of concrete was included implicitly.

Since cracking of reinforced concrete element obeys the general law of fracture mechanics, it is accepted that cracks appear in a direction perpendicular to that of principal tensile stresses in the concrete. As a result, no shear stress transferred in the cracked concrete is assumed, which is also exploited in theories of Matsui.

Theoretically, for every loading cycle, if the effects from actions (applied load) and reactions (concrete and rebars stresses) are available, the crack states, such as crack depth and width, can be calculated and whether the RC beam fails or not can be determined as well. Therefore, the life prediction method in this study is establish following: (1) Determine all the actions and reactions; (2) Calculate the crack opening state parameters; (3) Evaluate the relation between apply loads and load capacity after every load cycle using the information obtained in (1) and (2).

5.3.1. Basic assumptions

(1) Plane cross-section assumption: The normal strain along the height is a linear function of a distance away from a neutral axis.

(2) The cracked cross-section is assumed to be around the neutral axis. This assumption is employed by [54]. As a result of this assumption, the opening of the inclined crack is almost perpendicular to its direction and no shear stress is transferred by aggregates

(3) The crack has a linear crack opening profile which was employed by [55] and is coincident with the plane cross-section assumption.

From the plane cross-section assumption, the strain distribution (ε_{II}) at the uncracked concrete and the normal strain of rebar on both compression (ε_{ru}) and tension zone (ε_{rb}) can be related to α and β as

$$\varepsilon_{II}(x) = \varepsilon_t \left(1 - \frac{x - \alpha h}{\beta h - \alpha h} \right), \quad (5.3)$$

$$\varepsilon_{ru} = -\varepsilon_t \frac{h - (c + d_b/2) - \alpha h}{(\beta - \alpha)h}, \quad (5.4)$$

$$\varepsilon_{rb} = \varepsilon_t \frac{h - (c + d_b/2)}{(\beta - \alpha)h}, \quad (5.5)$$

where c and d_b are the cover depth and rebar diameter, respectively. With these strains, the stress distribution of uncracked concrete (σ_{II}) and normal rebar forces (T_{rb} and T_{ub}) can be determined if the stress-strain relations for uncracked concrete and rebars are available.

From the cross-section rotating assumption, the longitudinal rebars undergo both normal and shear strains. From the tensorial consideration of strains at the crack location, the normal strain (ε_{rb}) and shear strain (γ_{rb}) for the longitudinal rebars of the beam can be related as:

$$\gamma_{rb} = \varepsilon_{rb} \cdot \tan \varphi, \quad (5.6)$$

where φ is the angle between crack and vertical direction as shown in **Figure 5.6**. This relation is valid as long as stresses in rebars are in the elastic phase which is generally ensured for beams or slabs failure due to shear.

Taking into account that the relation between shear elastic modulus and normal elastic modulus is

$$G_s = E_s / 2(1 + \nu), \quad (5.7)$$

where ν is Poisson's ratio which is about 0.3 for steel. Thus

$$G_s \approx 0.4E_s, \quad (5.8)$$

and then, one can obtain the relation between shear stress and normal stress as

$$\tau_{rb} = 0.4\sigma_{rb} \tan \varphi, \quad (5.9)$$

correspondingly, relation between shear force and normal force is

$$V_{rb} = 0.4T_{rb} \tan \varphi. \quad (5.10)$$

This shear force is the well-known dowel effect of rebars. According to this analysis, it is implied that the dowel effect of rebars at a crack location is caused by a pure shear deformation of the rebars.

From the linear crack opening profile assumption, the crack width at any location (x) along the crack can be expressed by the CMOD (δ) and crack depth (αh) as

$$w(x) = \delta \left(1 - \frac{x}{\alpha h} \right), \quad (5.11)$$

where $w(x)$ is the crack width at location x . Generally, the concrete bridging stress is a function of crack width. Therefore, the concrete bridging distribution σ_l can be expressed by δ following an appropriate concrete bridging model.

Until now, except for the shear stress of uncracked concrete, all the reaction stresses of concrete and rebars can be expressed with three unknown variables, i.e. α , β and δ , if the concrete bridging model and stress to strain relations for uncracked concrete and rebars are available. To determine these three unknown variables after every loading cycle, at least three independent equations are needed.

5.3.2. Establishment of three independent equations

Referencing to the **Figure 5.5**, there are three equilibrium equations, i.e. force equilibrium equations along x and y directions and moment equilibrium equation, can be established. However, as the shear stress distribution for cracked cross section is not fully understood until now, force equilibrium equation along y axis cannot be established. Thus, two available equilibrium equations are given as:

$$\int_0^{\alpha h} \sigma_I(x) dx + \int_{\alpha h}^h \sigma_{II}(x) dx + T_{rb} + T_{ru} = 0, \quad (5.12)$$

$$\int_0^{\alpha h} \sigma_I(x) [(h-x) + (\alpha h - x)] dx + \int_{\alpha h}^h \sigma_{II}(x) (h-x) dx + T_{rb} \cdot (h - a_s) + V_{rb} \cdot (\alpha h - a_s) + T_{ru} \cdot a_s = M_A. \quad (5.13)$$

In order to obtain a complete solution, another relationship is necessary. According to the cracking analysis of reinforced concrete structures in Chapter 3, the total CMOD (δ) can be decomposed as CMODs due to applied load and bridging effects and bond slip. Thus

$$\delta = \delta_A(\alpha) + \delta_I(\sigma_I(x), \alpha) + \delta_{rb}(T_{rb}, V_{rb}, \alpha) + \delta_s(T_{rb}), \quad (5.14)$$

where δ_A is CMOD due to applied load. δ_I and δ_{rb} are CMODs due to concrete bridging stress and rebar bridging forces, respectively. As introduced in Chapter 3, all of these three CMODs can be calculated employing an integral equation which is based on fracture mechanics if weight functions for a crack geometry are available. δ_s is CMOD due to bond slip which can be determined following a certain bond slip model.

Obviously, these three developed equations are independent with each other. Similarly, the same three equations for any number of loading cycles can be established as well if the corresponding degraded models are used. Through solving these three equations, the three unknown variables (α , β and δ) can be determined.

Now, the tasks are (1) Express all the stresses and forces appeared in **Eq.(5.12)** to **Eq.(5.14)** by α , β and δ based on the introduced assumptions, some available constitutive models for all materials and a certain bond slip model; (2) Determine the weight functions for the punching shear crack geometry.

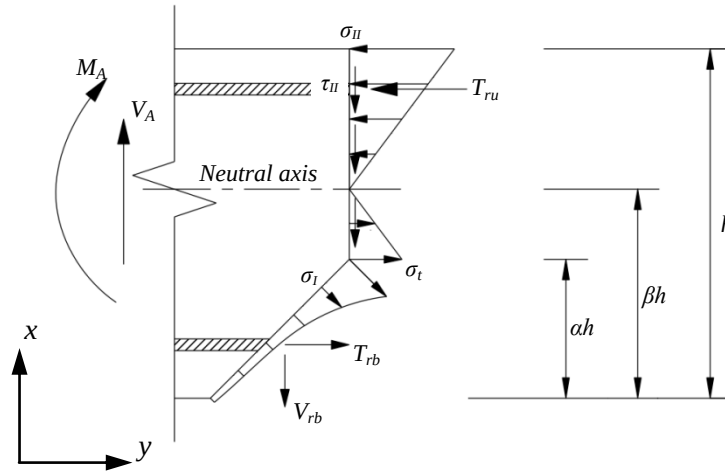


Figure 5.5 Sectional stresses under applied loads

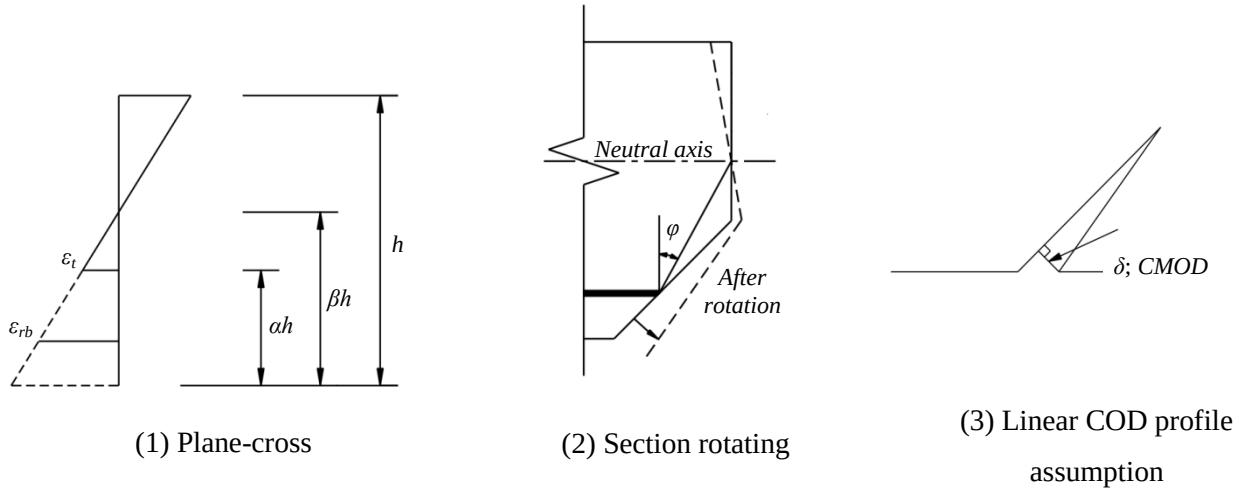


Figure 5.6 Schematic diagrams for basic assumptions

5.4. Sectional moment

For a cracked reinforced concrete structure under bending load, the sectional stiffness is mainly from concrete in the compression zone and rebars in the tension zone, both of which can be regarded as staying in elastic range under service load conditions. Therefore, for the critical RC beam, the sectional moment due to applied load can be calculated though conducting finite element analysis of a RC beam as **Figure 5.7**, where the stiffness along both the longitudinal and transverse directions are determined according to the fully cracked cross sections. The obtained nodal sectional moment and the corresponding fitted sectional moment for a unit width of the critical RC beam are shown in **Figure 5.8** with isolated points and continuous curve, respectively. The punching shear cracks are assumed propagating in the range enclosed by a couple of green lines in **Figure 5.8**.

5.5. Stresses along crack faces due to applied loads

It is found from **Eq.(5.12)** to **Eq.(5.14)** that stresses due to applied loads are only used for calculating the CMOD due to applied loads. As illustrated in Chapter 3, the CMOD due to applied loads is calculated in terms of stresses that would exist on the crack planes in the absence of the crack. Therefore, only stresses along the assumed crack plane are needed and these stresses can be calculated through conducting sectional elastic analysis without considering the crack.

5.5.1. Bending stress

Under the obtained sectional moment, a linear bending stress would have existed along any cross section in the absence of the crack. The linearly varying bending stress is expressed as

$$\sigma_A(x) = \sigma_{\max} \left(1 - \frac{2x}{h} \right), \quad (5.15)$$

where σ_{max} is the maximum stress of extreme fibers based on sectional analysis of uncracked cross-section. As rebars are considered as crack bridging elements separately, the neutral axis should be at the middle of the cross section.

As for any point along the crack plane as shown in **Figure 5.5**, σ_{max} should be a function of x as well because the sectional moment for the cross section along the concerned point is a function of x . Thus, the bending stress that would exist along the crack is expressed as

$$\sigma_A(x) = \sigma_{max}(x) \left(1 - \frac{2x}{h} \right). \quad (5.16)$$

5.5.2. Shear stress

To get applied load related shear stress, sectional shear forces should be determined beforehand. Due to the separating effect of the parallel cracks along the direction vertical to the moving load, the shearing force cannot be transferred effectively to the neighbor beams when the load is moved on the critical RC beam as pointed out by Matsui. Generally, this shear force is too strong for a single RC beam because the applied load is designed to be sustained by the whole slab. This is also the reason why punching shear cracks appear symmetrically with respect to the moving load in experiments. Therefore, the sectional shear force due to applied load can be calculated though beam analysis. Considering the symmetry of the load, the shear forces for every section in the punching shear range (see **Figure 5.8**) is equivalent to half of the applied load (P).

With the sectional shear force, the shear stress for any point along the crack plane in the absence of the crack is

$$\tau_A(x) = \tau_{max} \left(1 - \frac{4x^2}{h^2} \right), \quad (5.17)$$

where τ_{max} is maximum shear stress of the middle fibers based on sectional analysis. For a rectangular cross section, $\tau_{max} = \frac{3Q}{2A}$, where Q and A are sectional shear force and area, respectively. Thus, Q equals to $P/2$.

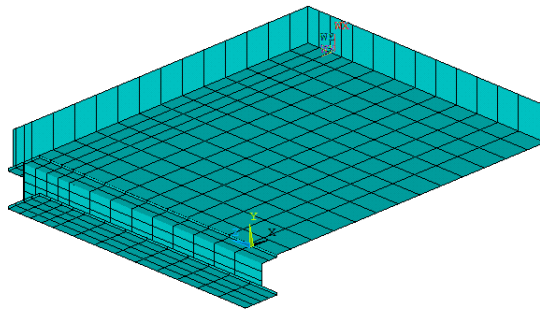


Figure 5.7 FEM of a quarter of RC slab

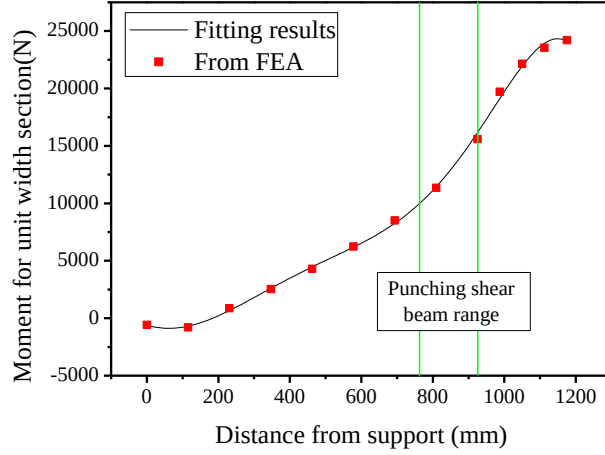


Figure 5.8 Mid-span sectional moment along transverse direction for unit width

5.6. Stresses for uncracked concrete

Under service load condition, the uncracked concrete under both tension and compression is considered as staying in a linear elastic stage, which is reasonable especially for RC beams failure due to shear because the shear capacity is generally much lower than the flexural capacity. From a assumed strain distribution as shown in **Figure 5.6**, the stress for uncracked concrete, $\sigma_{II}(x)$, can be related to α and β by:

$$\sigma_{II}(x) = \sigma_t \left(1 - \frac{x - \alpha h}{\beta h - \alpha h} \right), \quad (5.18)$$

where σ_t equals to the tensile strength of concrete (f_t). βh and αh are the depth of tensile zone and crack depth.

No degradation is assumed for uncracked concrete under both tension and compression in this study.

5.7. Stresses for cracked concrete

Crack propagation and fracture of concrete are primarily bound up with the softening behavior of concrete in tension, which is a primarily material property of concrete. In recent years, researches on the tensile behavior of concrete have increased enormously, and the concept of fracture mechanics has entered the field of concrete structures. Especially the nonlinear fracture mechanics, in which a softening zone ahead of a visible crack-tip was assumed, has shed new light on the behavior of concrete structures. In nonlinear fracture mechanics on concrete, the crack is assumed to propagate when the stress at the crack-tip reaches the tensile strength (see **Figure 5.9**). When the crack opens, the stress is not assumed to fall to zero at once, but to decrease with increasing crack width. This cracked range is named as a fictitious crack, referred to as the crack process zone where microcracks appear [56]. The microcracks, after the size exceeds a critical value, coalesce into a macrocrack, this is, a visible discrete crack. As the concrete in the fracture process zone is albeit damaged, it is still able to transfer stresses which

are the so called “Concrete bridging stress”. The fracture of concrete is significantly influenced by the fracture process zone and concrete bridging stress [57, 58].

Therefore, in this study, the concrete bridging stress in the fracture process zone is determined based on a concrete crack bridging model, which can be reflected by a relationship between concrete bridging stress and crack opening width.

5.7.1. Concrete bridging model

A concrete bridging model represented by the relation between stress and crack opening in the softening zone can be determined in deformation-controlled uniaxial tensile tests. Unfortunately, a bridging effect of concrete after cracking is mainly from the randomly oriented aggregates bridging, which is generally governed by the characteristics of aggregates, such as the grading, maximum particle size, surface texture, and so on. The detailed physics of aggregate bridging in concrete have not been fully understood. In the present study, an empirical model proposed by [59], which fits a wide range of experimental data extremely well, is adopted. In this model, the concrete bridging $\sigma_{a,1}$ is expressed as a function of crack width W :

$$\sigma_{a,1} = \frac{\sigma_{ma}}{1 + \left(\frac{W}{W_0}\right)^p}, \quad (5.19)$$

where σ_{ma} is the maximum bridging stress due to aggregate action at $W=0$, which is equal to the tensile strength of the material f_t . W_0 is the crack with corresponding to a reduction of the stress carrying capacity of 50% of the tensile strength. p is a shape factor. The concrete bridging model is simply illustrated as **Figure 5.10**.

It has been reported that an aggregate bridging model with $p=1.2$ and $W_0=0.015\text{mm}$ fits a wide range of experimental data extremely well, including normal and high strength concrete. Thus, these values are used in this study as well.

5.7.2. Concrete bridging stress degradation

It has been claimed that a fatigue of concrete mainly occurs in the fracture process zone which is represented by the degradation of concrete bridging stress. Under repetitive load condition, the interface of aggregates and mortar and aggregates themselves in the fracture process zone experience damage due to the crack opening and closing process. As the bridging stress from concrete is mainly from the aggregates, the stress transferring ability of concrete degrades, which means, with the same crack opening as the previous loading cycle, less stress can be transferred through cracked concrete.

Obviously, the speed and amount of degradation are closely related to the crack opening process, i.e. maximum crack opening, minimum crack opening and number of cycles [58]. [60] analyzed a large number of experimental data and related the degradation of crack bridging stress under cyclic load with the number of cycles, the maximum and minimum crack openings. For the target case of this study, a RC slab under cyclic moving load, the degradation model for concrete under single cyclic tension is employed because no reversal process occurs. And then the complicated model in [60] turns into a simple model, where the crack bridging law is

modeled with the assumption that the bridging degradation with load cycle obeys a linear function of $\log(N)$ as

$$\frac{\sigma_{a,N}}{\sigma_{a,1}} = 1 - d \log(N), \quad (5.20)$$

where $\sigma_{a,N}$ and $\sigma_{a,1}$ denote bridging stress at the N th cycle and 1st cycle respectively, d is the stress degradation factor which reflects the rate of aggregate bridging degradation which can be determined from experiments. As the maximum crack width corresponding to the zero load, d can be approximately related with the maximum crack width W_{max} by

$$d = d_0 + \gamma W_{max}, \quad (5.21)$$

where d_0 is the stress degradation factor at $W=0$ and γ is the slope of the linear relation of bridging degradation factor d and maximum crack width W_{max} . Through analysis of experimental results of experimental results of plain concrete under cyclic uniaxial tensile tests, it is found that when $d_0=0.08$ and $\gamma=4\text{mm}^{-1}$, the model predictions fit test results reasonably well.

For a concrete structure under cyclic load, the bridging law without degradation will be used to get the crack opening state after the first load cycle. In the second cycle, since bridging degradation occurs in the fracture zone, the load capacity from concrete bridging stress cannot reach the same level as in the 1st cycle with the already formed cracked state as illustrated by **Eq. 5.19**. The crack will propagate for an additional length to reach the load level as shown as **Figure 5.11**.

5.7.3. Stresses of cracked concrete

Until now, the concrete bridging model for the first loading cycle and the concrete degradation model are available as **Eq.(5.19)** and **Eq.(5.20)**. Substituting **Eq.(5.19)** into **Eq.(5.20)**, the concrete bridging model for every loading cycle is formulated as a function a crack opening width (W) and number of cycles (N) as

$$\sigma_{a,N} = \frac{f_t}{1 + \left(\frac{W}{W_0}\right)^p} \cdot [1 - d \log(N)]. \quad (5.22)$$

From the assumed linear crack opening profile, the stress distribution at the cracked concrete (σ_I) can be related to α and δ by:

$$\sigma_I(x) = \frac{f_t}{1 + \left(\frac{\delta(1-x/\alpha h)}{W_0}\right)^p} \cdot [1 - d \log(N)]. \quad (5.23)$$

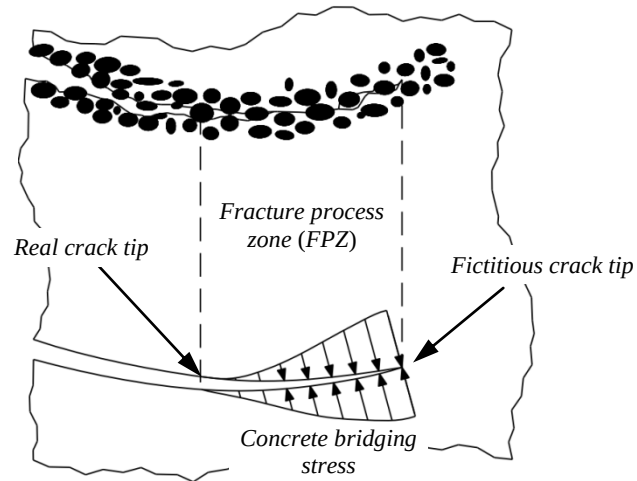


Figure 5.9 Fracture process zone of concrete

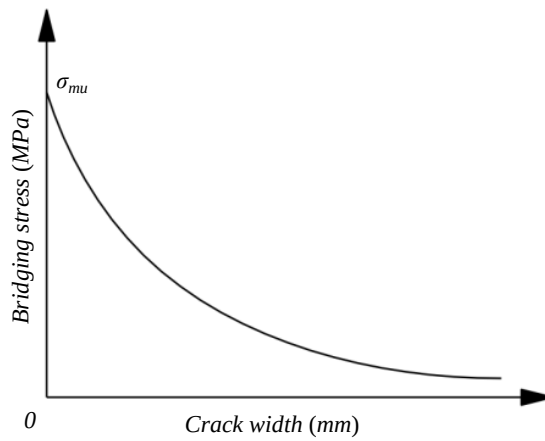


Figure 5.10 Concrete bridging model

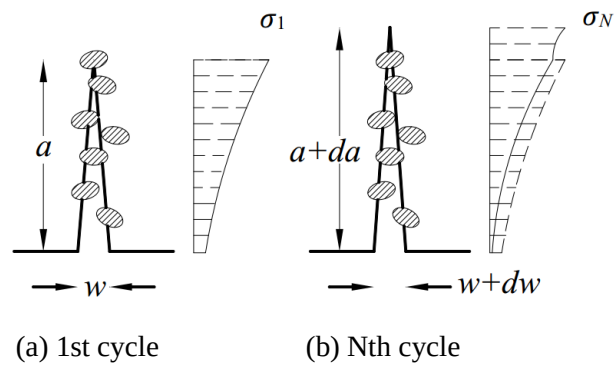


Figure 5.11 Crack propagation due to bridging stress degradation

5.8. Rebar stresses

In cracked reinforced concrete structures, concrete between cracks can develop local tensile stresses after cracks occur because of a bond stress transferred at the concrete/rebars interface, which is known as tension stiffening effect. Due to this effect, the rebar stress should vary along the rebar between cracks and peak at cracks. Therefore, generally there are two kinds of rebar stresses, i.e. average rebar stress and peak rebar stress at a crack, that are concerned from structure analysis according to different purposes. In this study, the peak rebar stress at a crack is needed.

Unfortunately, even though the average rebar stress can be obtained according to various approaches, such as smeared element model, an effective method of determining rebar stress at a crack is still unavailable as far as I know, whereas the rebar stress peaks at cracks. To get the peak rebar stress at a crack, a new method is proposed though integrating strengths of both the smeared crack model and a bond slip model.

The smeared crack model is primarily developed for finite element modeling a composite behavior of rebars surrounded by concrete, where the locality, i.e. the local physically uniform stress and strain distribution within the cracked elements, need not to be explicitly considered. In the smeared crack model, a space-averaged stress-strain relation of rebars is formulated. As the smeared element model possesses many advantages, such as easily applicability and rapid convergence, this model is commonly employed in conducting structural analysis aiming at some overall behaviors, such as structural deformation, crack orientation, and the average rebar stress as well. The reliability of determine the average stress of rebars in cracked RC members have been verified in many researches through comparing with experiment results and application.

Obviously, the smeared crack model cannot be employed directly for determining the peak stress of rebars at a crack which are necessary for this study. However, considering that the source of rebar stress variation is the bond in the rebar/concrete interface, theoretically the rebar stress distribution between cracks can be determined following a bond slip model. With the rebar stress distribution along a rebar, a relation between peak rebar stress and average rebar stress can be determined following certain principle as well. Finally, the peak stress at a crack can be identified using the obtained peak stress to average stress relation and the average rebar stress from smeared crack model.

Therefore, in this study and for the first loading cycle, the peak rebar stress at a crack is obtainable following as: (1) Determine the average rebar stress based on the smeared crack model; (2) Determine the relations between peak rebar stresses and average rebar stresses for different rebar stress levels; (3) Obtain peak rebar stresses using the average rebar stresses and the relations determined in (1) and (2), respectively.

For the 2nd to Nth load cycle, some degradations that influence the rebar stresses occur. Among them the most influential and commonly considered degradations are rebar stiffness and rebar/concrete bond slip degradations. As introduced in Chapter 1, the rebar stiffness degradation can be neglected because almost no degradation is found for rebar stress lower than the yielding strength, which can be ensured under service load conditions. However, the bond slip degradation should be accounted for because this degradation appears even under relatively low

load level. To obtain the rebar stress variation due to bond slip degradation, the procedure of determining peak rebar stress in the first cycle can be followed except that a degraded bond slip model should be employed.

5.8.1. Determine average rebar stress based on smeared crack model

5.8.1.1. Modified rebar constitutive model considering bond slip effect

The essential concept of smeared crack model is a space-averaged constitutive model, according to which the averaged stress of a material is derived from the averaged strain developed. Therefore, a space-averaged constitutive model for rebars surrounded by concrete should be obtained beforehand.

Obviously, the space-averaged stress-strain relation for rebars differs from that of bare bars with no interaction with the concrete. Thus, modifying rebar constitutive model considering bond-slip effect is of great significance to obtain a reliable simulation. Firstly, suppose that a RC member is subjected to certain loads, at the moment when the rebar starts yielding close to cracks other volumes of rebars remain elastic due to the tensile stiffness effect. Therefore, the yield strength of the modified rebar constitutive model must be less than the yield strength of bare bars. Secondly, considering two same rebars, i.e. one bare bar and one rebar surrounded by concrete, subjected to the same pure uniaxial tension load, the loading end displacement for the concrete surrounded rebar must be larger than that of the bare bar due to slip between rebar and concrete as shown in **Figure 5.12**. Thus, the elastic modulus of rebar should be modified to account for the extra elongation due to slip.

According to the introduction, there are three things should be determined before reaching the space-averaged stress-strain relation of rebars in concrete which are the average crack space, reduced yielding strength and the stress-strain relationship (**Figure 5.13**).

Firstly, for a RC structure subjected to overall flexural load, such as beams under three or four-point bending load, slabs under fixed pulsating and moving load, the average spacing of flexural cracks is given by [61] as

$$S_{rm} = \frac{2}{3} \times \frac{d_b}{3.6\rho_{eff}}, \quad (5.24)$$

where ρ_{eff} is effective reinforcement ratio and can be obtained from

$$\rho_{eff} = \frac{A_s}{A_{c,eff}}, \quad (5.25)$$

where $A_{c,eff}$ is the effective concrete area in tension which usually obtained by some simplified approaches, such as those in Eurocode 2 or [61]. For rectangular cross sections:

$$A_{c,eff} = 2.5(h - d_e)B, \quad (5.26)$$

where h is total depth of cross section, d_e is effective depth of cross section and B is width of cross section. An extended formula for $A_{c,eff}$ can be developed by taking into consideration the size effect, which was proposed by [62].

$$A_{c,eff} = m(h - d_e)B, \quad (5.27)$$

in which:

$$m = \begin{cases} h/(h - d_e) & 0 \leq h/(h - d_e) \leq 5 \\ 3.33 + 0.33h/(h - d_e) & 5 < h/(h - d_e) \leq 35 \\ 15 & 35 < h/(h - d_e) \end{cases} \quad (5.28)$$

Substituting the values of parameters from the critical beam, the average crack spacing is determined as 259.36mm.

Secondly, the real yielding strength of rebars embedded in concrete is determined following a method proposed on the basis of experiments and mechanisms in [63], where the modified yielding strength (f_y^*) is obtained from:

$$\frac{f_y^*}{f_y} = (0.93 - 2Y), \quad (5.29)$$

where the effective yield stress is a function of $Y = (f_t/f_y)^{1.5}/\rho$. In this equation, ρ is the reinforcement ratio and f_t represents the tensile strength of concrete.

Thirdly, the stress-strain relationship is modified according to the method in [63] as well. As shown in **Figure 5.12**, the total displacement of rebar is the sum of its own mechanical elongation and the relative slip between rebar and concrete. To consider bond slip in rebar stress-strain relationship, the equivalent strain (relative slip divides transmission length) of bond slip effect is added to the strain of rebar. In addition, as reported by many literatures based on experimental and analytical method, the stress-strain relation of rebar embedded in concrete remains linearly before reaching the effective yield stress. The modified elastic modulus of rebar can be simply written as

$$E_s^* = \frac{f_y^*}{\varepsilon_s + (s_y/l_{tran})}, \quad (5.30)$$

where f_y^* is the effective yield stress of rebar which can be obtained from **Eq.(5.29)**, ε_s is the strain of rebar corresponding to the stress of f_y^* in bare rebar model. s_y is the bond slip at the effective yield stress. l_{tran} is the transmission length.

In terms of the transmission length, as shown in **Figure 5.14**, the transmission lengths for adjacent cracks can be considered as almost the same. Thus, no slip occurs at the middle point between two adjacent cracks and the transmission length l_{tran} should be equivalent to half of the minimum crack spacing as

$$l_{tran} = \frac{S_{r(min)}}{2}, \quad (5.31)$$

where $S_{r(min)}$ is the length between two adjacent crack, which is equivalent to the minimum crack spacing. [64] have reported that the ratio between the minimum and the average spacing is between 0.67 and 0.77. Therefore, the transmission length can be given as

$$l_{tran} = \frac{0.67}{2} S_{rm}. \quad (5.32)$$

Thus, the transmission length for the beam in this study is 86.9mm.

As for the relative slip at effective yielding stress, s_y , it can be determined based on a bond slip model which will be introduced in the upcoming section.

Following the procedure described, the modified stress-strain relationship for rebars staying in elastic state is finally established by considering the bond-slip effect. After rebar stress reaching the effective yielding strength, the hardening stiffness of rebar model is simply calculated though multiplying the modified elastic modulus of rebars with a coefficient. Even though the value of the coefficient depends on many factor, such as yielding strength and elastic modulus, it has been suggested by [64] that 0.03 is an acceptable value for it. Thus, the hardening modulus of rebars is given as

$$E_{sp}^* = 0.03E_s^*, \quad (5.33)$$

where E_{sp}^* is the hardening stiffness of rebar.

5.8.1.2. Bond slip model

In this section, the local bond slip model employed in Chapter 3 is unavailable because the long enough embedment boundary condition cannot be ensured. Thus a unique bond-slip-strain relation proposed in [65] for non-deteriorated conditions will be employed. The applicability of this model is confirmed though conducting analysis on bond tests under various boundary conditions including varying embedment lengths in the literature. The bond-slip-strain model is simply expressed as

$$\frac{\tau}{f_c} = \frac{0.73(\ln(1+5s'))^3}{1+\varepsilon \times 10^5}, \quad (5.34)$$

where $\ln$ is natural logarithm, ε is the rebar strain, s' is the normalized slip equaling $1000S/d_b$, τ and S are bond stress and slip at any point along rebar, respectively, f_c is concrete strength in MPa. The unit of slip (S) and rebar diameter (d_b) should be the same.

As illustrate in **Figure 5.14**, the local slip between concrete and rebar can be regarded as zero at the middle point between cracks. This point is defined as the origin of z coordinate. Under this coordinate setup, the general relations among bond stress, slip and rebar strain expressed by **Eq.(3.24)** and **Eq.(3.25)** are applicable for this section as well.

Substituting **Eq.(3.24)** and **Eq.(3.25)** into **Eq.(5.34)** and after rearrangement, a second order of differential equation of actual local slip (S) with respect to the location along the rebars (z) is obtained as

$$\frac{E_s \cdot d_b}{4} \cdot \frac{dS}{dz} \cdot \frac{d^2S}{dz^2} \times 10^{-5} + \frac{E_s \cdot d_b}{4} \cdot \frac{d^2S}{dz^2} - 0.73 \cdot f_c \cdot \left(\ln \left(1 + \frac{5000S}{d_b} \right) \right)^3 = 0. \quad (5.35)$$

Obviously, this equation is even more complicate than the equation for the bond slip model employed in Chapter 3. It has no theoretical solution and should be solved numerically as well.

To solve the differential equation, firstly, the available boundary conditions should be figured out. With the transmission length determined in section, the boundary conditions that this differential equation should satisfy are:

$$z = 0 \Rightarrow \begin{cases} S = 0 \\ \tau = 0 \end{cases}, \quad (5.36)$$

where boundary condition $\tau=0$ at any point is implicitly contained in **Eq.(5.34)** when the $S=0$ is satisfied. Thus, the number of boundary conditions illustrated in **Eq.(5.36)** is actually one, which is not enough for solving a second order of differential equation. The loading end strain to loading end slip relation cannot be obtained directly following the numerical procedure described in Chapter 3. However, as the first step, a point on the loading end strain versus loading end slip relation can be determined.

For example, if the rebar strain at location $z=l_{tran}$ is assumed as 0.001, a series of bond stress, rebar strain and slip distribution curves in the transmission length can be obtained following the numerical procedure described in section for any assumed slip at $z=l_{tran}$, $S(l_{tran})$. Among all of the assumed $S(l_{tran})$, there is one $S(l_{tran})$ that can ensure the boundary conditions at $z=0$ satisfied. This $S(l_{tran})$ is the loading end slip of the point with loading end strain equaling 0.001 in the strain verse slip curve.

Similarly, by assuming different loading end rebar strains, the corresponding loading end slips can be determined. One set of strain and slip is the coordinate of a point on the strain verse slip curve. Connecting these points, the strain verse slip curve is reached as shown in **Figure 5.15**.

And then the loading end slip (s_y) for loading end rebar stress equaling the effect yielding strength (f_y^*) can be determined following the obtained strain verse slip curve. Substituting s_y into **Eq.(5.30)**, the effective elastic modulus E_s^* is determined.

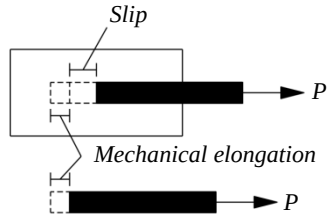


Figure 5.12 Rebar embedded in concrete and bare rebar

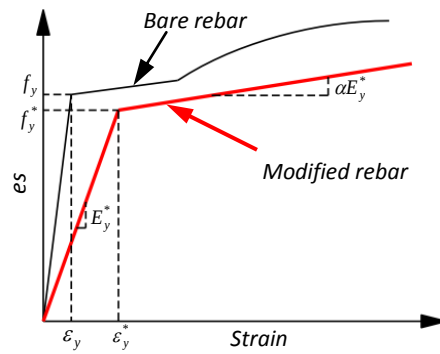


Figure 5.13 Modified stress-strain relationship for rebar embedded in concrete

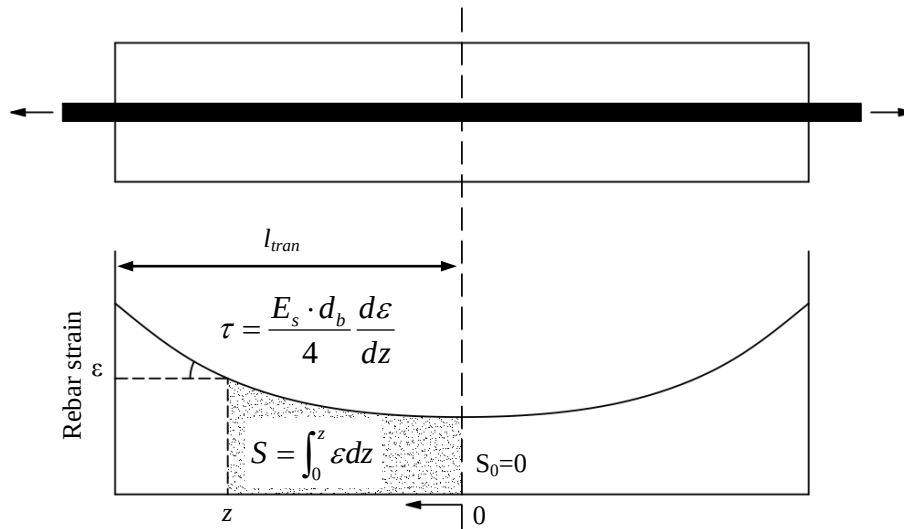


Figure 5.14 Definition of local bond stress and slip

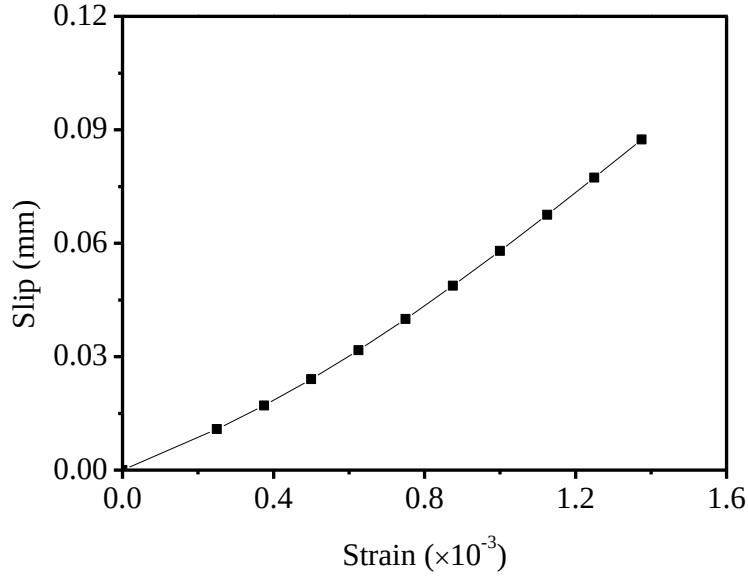


Figure 5.15 Loading end slip vs. loading end rebar strain

5.8.2. Determine the relation between average rebar stress and peak rebar stress

As introduced above, the bond-slip-strain model can be used for computing the bond stress, rebar strain and bond slip distributions for different boundary conditions. The boundary condition for this study is that non-zero strain and zero slip for the end point of transmission length (the middle point between two adjacent cracks). Under different assumed rebar strains at the crack, the bond stress, rebar strain and bond slip distributions in a transmission length (86.9mm) are shown as **Figure 5.16**.

The average rebar is calculated following the principle that the area under the average strain line is equivalent the area under the strain distribution curve as shown in **Figure 5.17**. Thus, the average rebar strain is defined following [66] as

$$\varepsilon_{sa} = \frac{1}{l_{tran}} \int_0^{l_{tran}} \varepsilon_s(z) dz. \quad (5.37)$$

This definition is coincident with the average strain concept for materials in smeared crack model.

Under this definition, the relation between the peak rebar strain at crack and the average rebar strain is obtained, which is illustrated as a ratio as

$$e_s = \frac{\varepsilon_{s \max}}{\varepsilon_{sa}}, \quad (5.38)$$

where $\varepsilon_{s \max}$ is the peak rebar strain at a crack. The relation between e_s and peak rebar strain is shown in **Figure 5.18**.

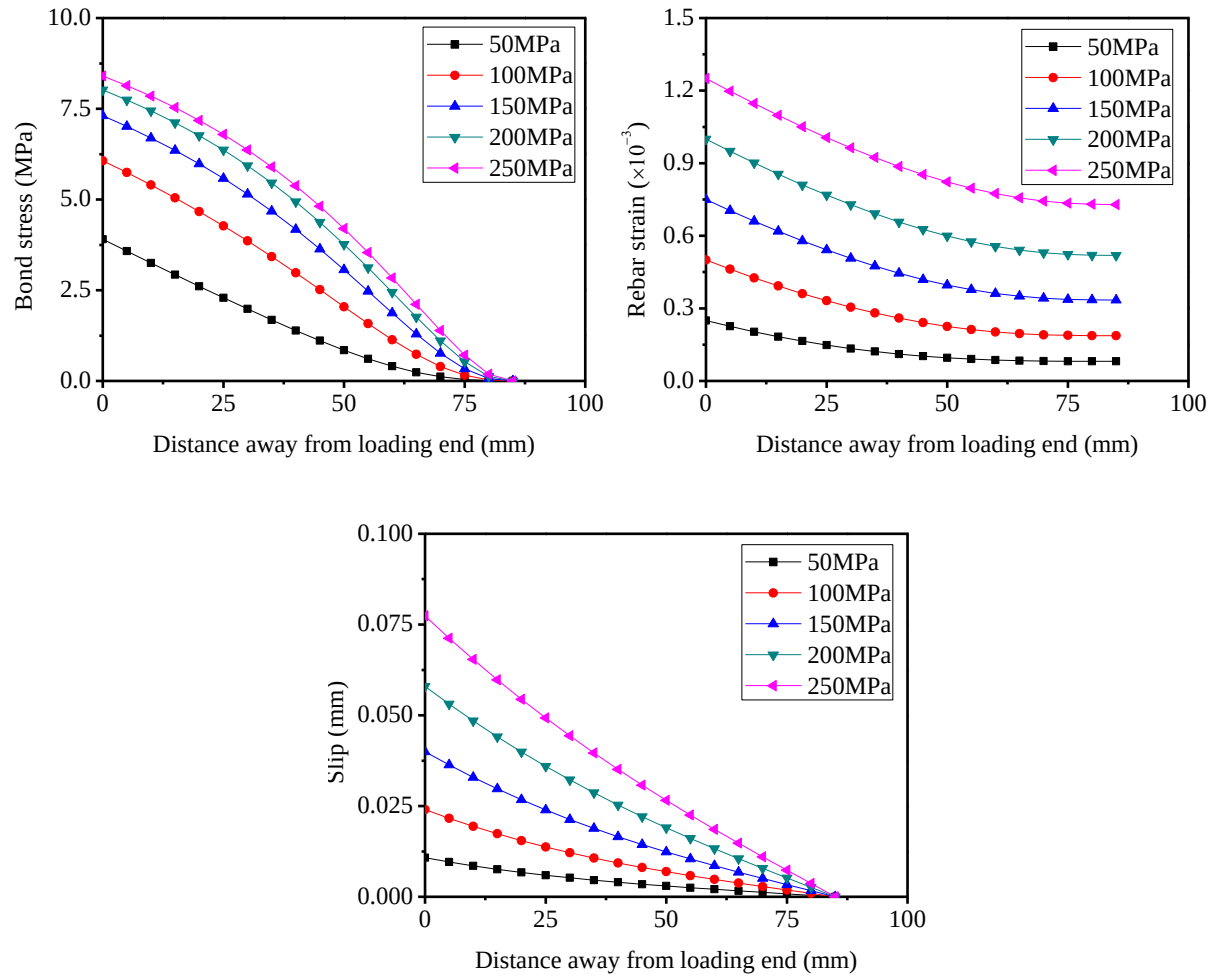


Figure 5.16 Bond stress, rebar strain and bond slip distributions

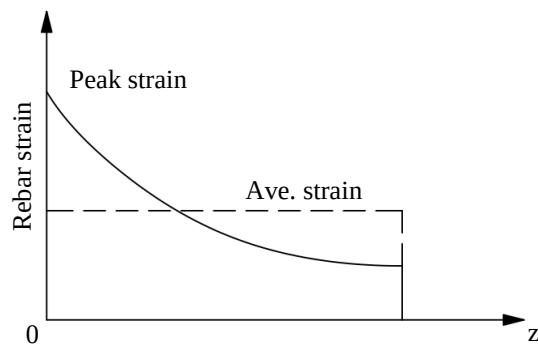


Figure 5.17 Relation between peak rebar strain and average rebar strain

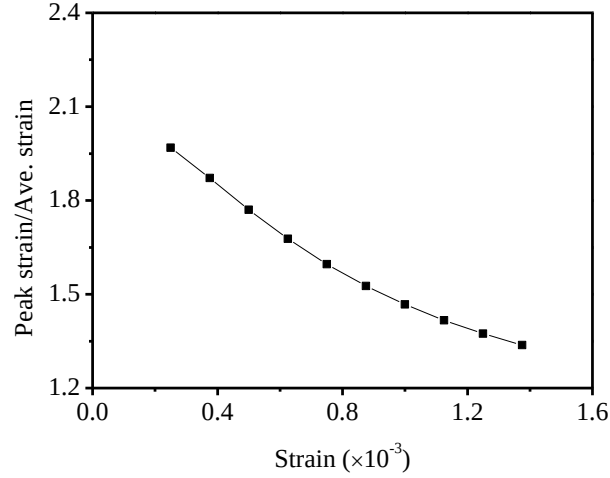


Figure 5.18 Peak strain/ave. strain vs. loading end strain

5.8.3. Rebar normal stress and shear stress at crack

Following the plane cross-section assumption, the normal strain of rebar in the tension zone has been expressed by α and β as **Eq.(5.5)**. This normal strain can be regarded as the averaged rebar normal strain. Substituting this normal strain into the modified rebar constitutive model, the average rebar normal stress can be determined. Under service load conditions, it is generally accepted that the rebars stay in the elastic range. Thus, the average normal stress of rebar in the tension zone can be simply expressed as

$$\sigma_{sa} = E_s^* \cdot \varepsilon_t \frac{h - (c + d_b/2)}{(\beta - \alpha)h} . \quad (5.39)$$

Since the average rebar strain and the peak rebar strain is related by the ratio from **Eq.(5.38)**, the rebar normal stress at crack is given as

$$\sigma_{rb} = e_s \cdot \sigma_{sa} , \quad (5.40)$$

and then, the shear stress of rebar in the tension zone can be determined following the relation illustrated in **Eq.(5.8)** and is given as

$$\tau_{rb} = 0.4 \cdot E_s^* \cdot \varepsilon_t \frac{h - (c + d_b/2)}{(\beta - \alpha)h} \cdot \tan \varphi . \quad (5.41)$$

Until now, for the first loading cycle, the rebar stress at a crack is expressed by α and β based on a famous modified rebar constitutive model and a bond slip model.

5.8.4. Rebar stress variation due to bond slip degradation

For the cracked RC slab under cyclic moving load, the rebars crossing cracks can be regarded as under repeated pullout loading. Inevitably, the resistance of relative movement between concrete and rebars reduces due to the deteriorated interface bond properties. Generally, this mechanism is manifested by a degradation of the local bond stress-slip relationship. As the bond

affects the cracking behavior directly and further environment deterioration of reinforce concrete members indirectly, a huge amount of researches on identifying the properties of bond between rebar and concrete under repeated loading have been conducted. Based on the pullout test results of 308 specimens, [67] concluded that if fatigue failure does not occur under repeated loading, then the previous repeated load does not negatively influence the bond strength, compared with that of monotonic loading. An even about 12% increasing of residual bond strength was found in [68] where 202 specimens were tested. Even though bond strength is not apparently affected by the repeated loading, the degradation of bond stiffness is observed in all studies because the loading end slip increased during repeated loading. However, it is found by [69] that the degradation speed is not influenced by the bond stress level. At least up to the stress level, Experimental bond stress/Bond strength=0.5, the degradation speed stay in almost constant. This phenomenon facilitates the developing of an accurate and easily applicable bond slip degradation model.

In this study, a comprehensive bond slip degradation model proposed by [70] is employed. Since the bond slip model is derived based on pullout test results of specimens with a short embedment length of $2d_b$ (two times of rebar diameters), the bond stress can be regarded as constantly distributed in the embedment length. Therefore, the degraded bond stress-slip relations are the local bond stress-slip relation. In the experiment, similar phenomenon reported by [67, 68] were found as well. As the bond slip degradation speed is almost the same for different stress level, the bond slip degradation is simply formulated with a cycle dependent slip (S_N), which can be expressed as a function of initial slip (S_1) by first cyclic loading and the number of cycles (N) as shown in

$$S_N = S_1 N^m. \quad (5.42)$$

The power m of **Eq.(5.42)** was found to be 0.098 which is the average value of the powers for different load levels in the tests. Similar values for this power were reported according to test results, such as a 0.107 is used by [67].

Since the employed bond slip model and bond slip degradation model were formulated with local parameters, such as local bond stress and local bond slip, it is reasonable combining them to get the bond stress-slip-strain degradation model as

$$\frac{\tau_N}{f_c} = \frac{0.73 \cdot \left(\ln \left(1 + 5s_N / N^m \right) \right)^3}{1 + \varepsilon \times 10^5}. \quad (5.43)$$

In addition, general relations among bond stress, slip and rebar strain as expressed by **Eq.(3.24)** and **Eq.(3.25)** are available as well. Substituting these relations into **Eq. (5.43)**, the second order differential equation of degraded local slip (S_N) with respect to the location along the rebars (z) can be obtained as

$$\frac{E_s \cdot d_b}{4} \cdot \frac{dS_N}{dz} \cdot \frac{d^2 S_N}{dz^2} \times 10^{-5} + \frac{E_s \cdot d_b}{4} \cdot \frac{d^2 S_N}{dz^2} - 0.73 \cdot f_c \cdot \left(\ln \left(1 + \frac{5000S_N / N^b}{d_b} \right) \right)^3 = 0. \quad (5.44)$$

This equation can be solved numerically following definitely the same procedure described in section 5.8.1.2.

For example, assuming rebar stress at a crack is 150MPa, the obtained bond stress, rebar strain and slip distributions for different numbers of loading cycles are shown as **Figure 5.19**. With these strain distributions, the peak strain versus average strain relation for different number of loading cycles are obtained. Assuming different loading end rebar stresses at the crack, the relation between bond slips at the crack vs. number of cycles and peak stress/average stress ratios vs. number of cycles are obtained as shown in **Figure 5.20** and **Figure 5.21**, respectively.

Therefore, all the results required from the bond slip model to determine rebar stress at a crack, i.e. bond slip at crack and strain distribution, can be provided by solving the degraded bond slip model.

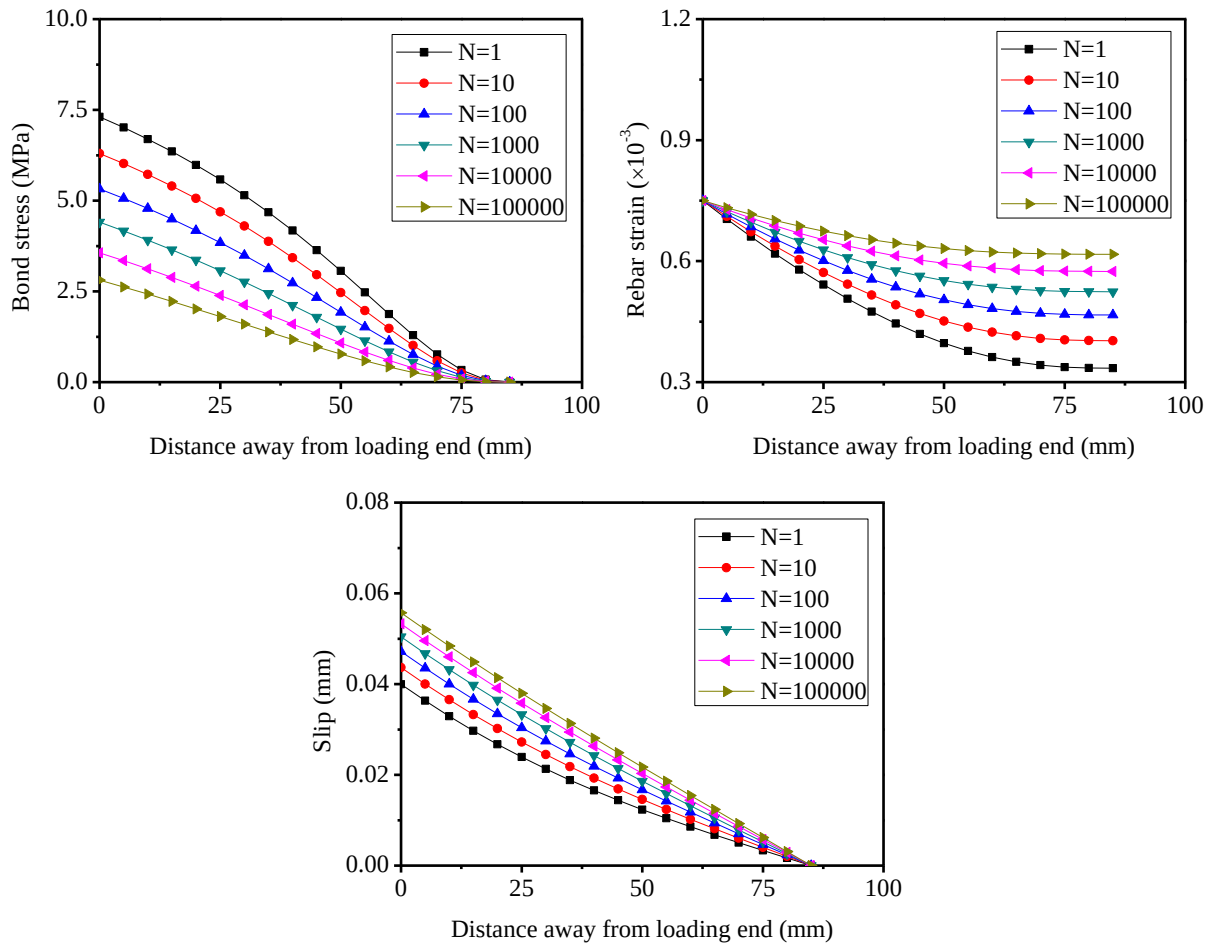


Figure 5.19 Bond stress, rebar strain and bond slip distributions for different loading cycles

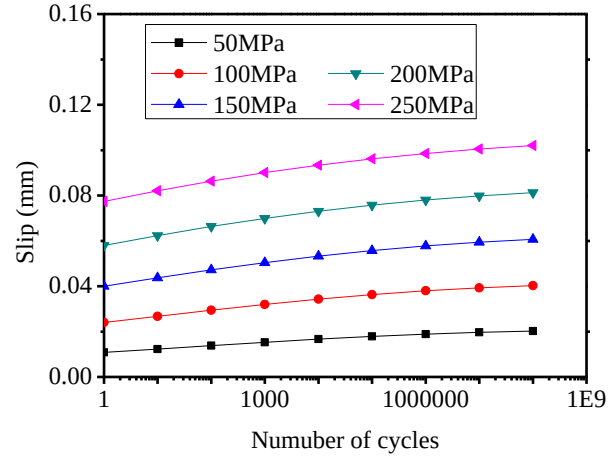


Figure 5.20 Loading end slip vs. number of cycles

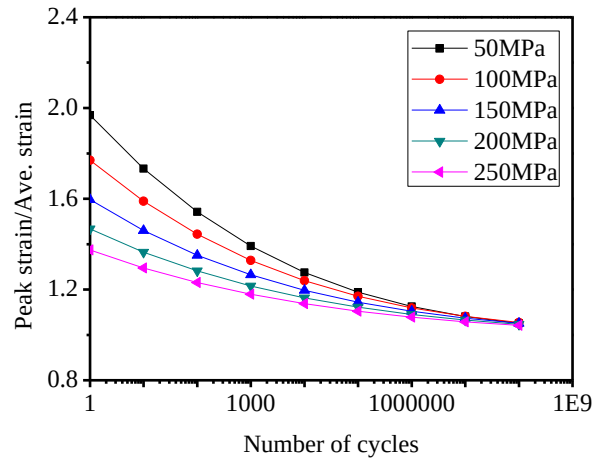


Figure 5.21 Peak strain/ave. strain ratios vs. number of cycles

5.9. Weight functions for punching shear cracks

Obviously, the punching shear cracks propagate under mixed mode loading condition. There is no closed form analytical solution of weight function for the crack geometries. Therefore, the weight function determination method based on finite element method as introduced in Chapter 4 is employed for the punching shear cracks. Taking advantage of symmetrical characteristics, only half of the cracked beam is modeled in finite element analysis. The finite element model is shown as **Figure 5.22**. From finite element analysis, both Mode I and Mode II nodal weight function components for all nodes can be determined.

However, not every weight function components are necessary. Firstly, for a crack under mixed mode loading, the crack mouth displacement (CMD) is the vector sum of CMOD and crack mouth sliding displacement (CMSD) (see **Figure 5.23**). For a crack, the CMOD and CMSD are independent with each other under the (x_t, y_t) coordinates. Thus, following the integral equation in Chapter 3, the CMOD and CMSD can be determined employing the Mode I weight function components and Mode II weight function components along both crack faces,

separately. In this study, the compatibility equation employed is the CMOD decomposition equation, which means only Mode II crack-face weight function components are not necessary.

In addition, under the (x_t, y_t) coordinates where x_t axis is along the crack faces, the secondary Mode I crack-face weight function components are almost the same for both crack faces, whereas the stresses acting on both crack faces, i.e. concrete bridging stress, rebar forces and stresses due to applied load in the absence of the crack, are along the opposite directions with each other. Therefore, the secondary Mode I crack-face weight function components are not necessary.

In summary, only the primary Mode I crack-face weight function components are needed to determine the CMOD following the integral equation in Chapter 3. The primary Mode I crack-face nodal weight functions for varying crack lengths from $0.1h$ to $0.8h$ are shown with isolated points in **Figure 5.24** and **Figure 5.25**.

For the primary crack-face weight function components which exhibit the singular behavior at the crack-tip neighborhood, before applying into the integral equation, the weight function should be formulated as a function of crack length (a) and locations along the crack face (x_t), which can be achieved through conducting fitting and interpolation methods on the obtained nodal weight functions as Chapter 4. The fitted and interpolated results are shown with lines from **Figure 5.24** and **Figure 5.25**, which are very close to the corresponding nodal weight functions. The coefficients of **Eq.(4.28)**, $A_n(a)$, for different crack lengths are listed in **Table 5.2**.

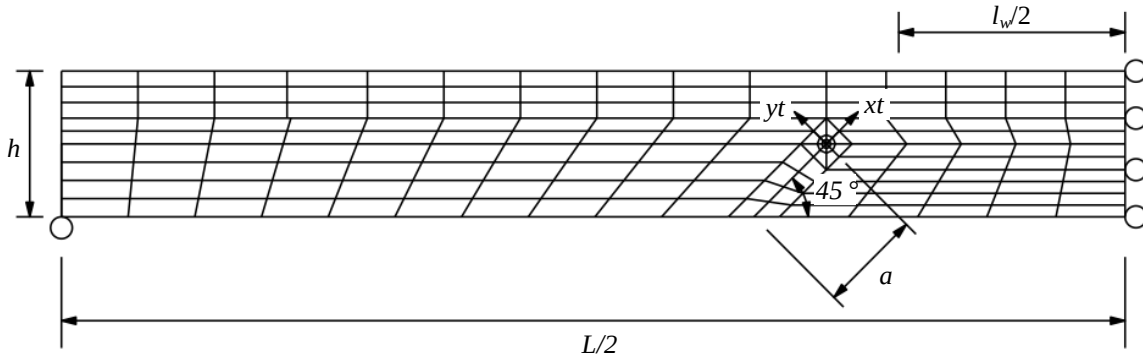


Figure 5.22 Finite element model of beam with punching shear crack

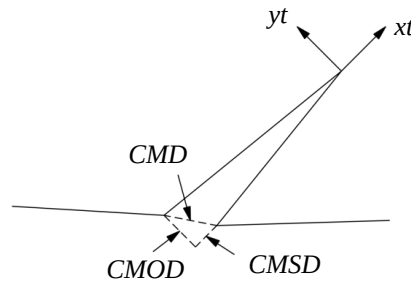


Figure 5.23 Composition of crack mouth displacement

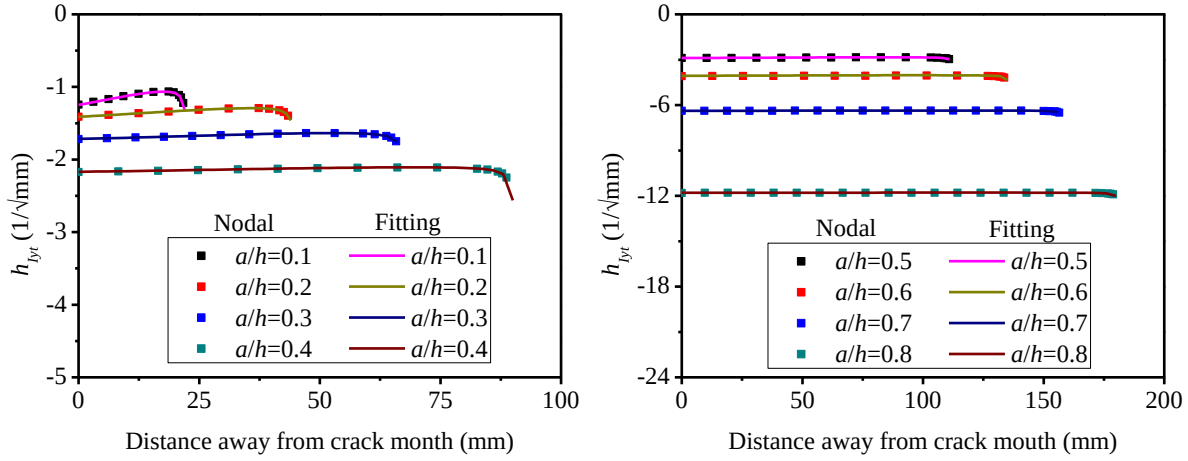


Figure 5.24 Lower crack face weight functions

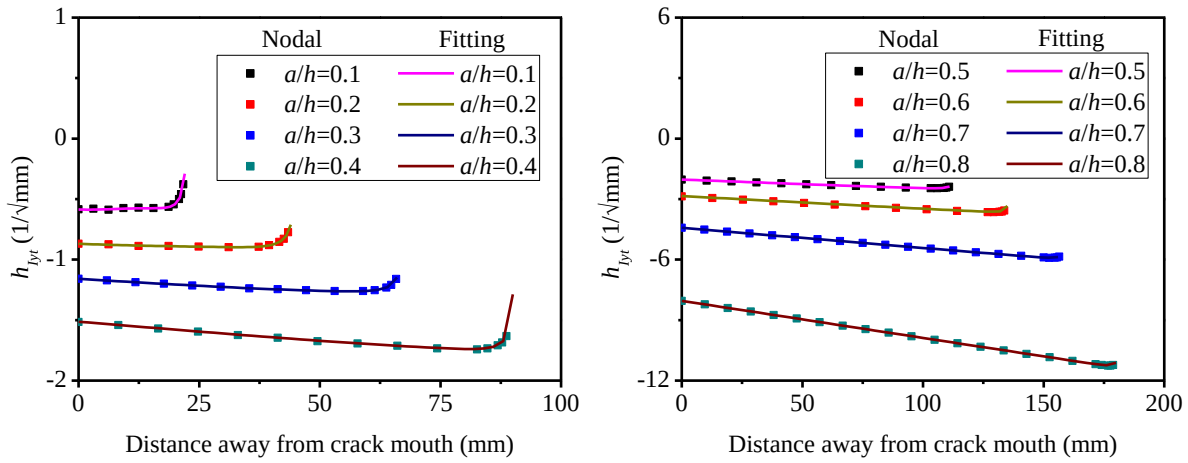


Figure 5.25 Upper crack face weight functions

Table 5.2 Coefficient of Eq.(4.28) for crack-face weight function

(a) h_{yt} on upper crack face						
a/h	A_1	A_2	A_3	A_4	A_5	A_6
0.1	0.095325	-0.81518	-0.6728	3.01417	-3.61359	1.40792
0.2	0.076475	-1.229	0.300353	0.471683	-0.93546	0.447291
0.3	0.082329	-1.88376	1.8781	-2.953	2.43775	-0.72058
0.4	0.067261	-2.31535	1.83629	-2.96565	2.70014	-0.8379
0.5	0.057521	-2.97793	1.59234	-2.39204	2.48576	-0.80108
0.6	0.048301	-4.11985	1.44075	-1.82717	2.3872	-0.79636
0.7	0.045151	-6.43961	1.79617	-2.00927	3.34654	-1.16432
0.8	0.038268	-11.7722	1.62454	0.136764	2.93857	-1.01693

(b) h_{lyt} on lower crack face

a/h	A_1	A_2	A_3	A_4	A_5	A_6
0.1	-0.15312	0.017506	-3.31022	5.72595	-5.30235	1.77782
0.2	-0.10427	-0.56176	-2.15668	3.55006	-3.18703	1.04828
0.3	-0.07583	-1.0855	-1.77154	3.21366	-3.07314	1.07549
0.4	-0.06189	-1.62864	-1.6679	3.16305	-3.05024	1.07416
0.5	-0.05341	-2.42612	-1.4791	2.83921	-2.73695	0.965452
0.6	-0.04513	-3.6793	-1.43193	2.8929	-2.84349	1.02034
0.7	-0.03771	-6.05252	-1.15938	2.29735	-2.2108	0.790112
0.8	-0.03272	-11.5145	-1.06392	2.06675	-1.93843	0.679847

5.10. Fatigue life prediction

Until now, for every loading cycle, all the cross-section stresses and forces are expressed with α , β and δ and the weight functions for the punching shear crack are available. Thus, through solving the three independent equations (**Eq.(5.12)** to **Eq.(5.13)**), the crack depth ratio (α), tensile depth ratio (β) and CMOD (δ) are obtained. These three variables representing crack opening state can then be employed in evaluating the failure moment based on some failure criterion.

5.10.1. Cracking states after cycles of load

The crack depth/beam depth ratio at several fatigue loading levels are plotted with the number of loading cycles on the semi-logarithmic scale as shown in **Figure 5.26**. Generally, the fatigue growth of a bridged crack in a concrete structure involves of three distinct stages: initial decelerated growth, steady state growth and final accelerated growth. Most of the fatigue life was observed to be spent in the second stage. It is found that the theoretical simulation successfully captured the first two stages, whereas the third stage cannot be observed in the curves.

This can be explained as follows. To facilitate understanding, the crack-tip stress intensity factor (SIF) which reflects the crack-tip stress state is employed for explanation. For the punching shear crack, the net crack-tip SIF is a resultant of SIF from external applied load and all bridging effects. The SIF amplitude due to external applied load increases with the length of the crack emanating from the tension face. At the same time, the magnitude of the negative crack-tip SIF due to crack bridging (rebar forces and concrete bridging stresses) increases with the propagation of the crack as well.

The initial decelerated growth is caused by the effective reduction of the net crack-tip stress intensity factor amplitude due to the rapidly increased crack bridging related SIF with the development of the crack length. The steady state growth is caused by the constant value of net crack-tip stress intensity factor when the increase of SIF due to applied load is balanced with the increase of negative SIF due to bridging stress. The reason why the final accelerated crack growth stage cannot be observed is that the RC beam is assumed to fail in a brittle shear failure mode according to certain failure criterion.

However, the initial decelerated stage in the crack depth/beam depth ratio to number of loading cycles curve for higher load level is not as apparent as that for lower load levels. This is because the punching shear crack is more fully developed in the first loading cycle under the higher load level, manifested with a higher crack depth and a wider crack opening. Due to the wide crack opening, the bridging effect from nonlinear concrete bridging stress maybe small or even negligible compared with that from rebars. Thus, the structure degradation can be regarded as mainly from the degradation of bond slip, which follows a steady state growth as shown in **Figure 5.20**. As a result, the crack propagation curve seems to start from the steady state growth for higher load level as observed in **Figure 5.26**.

Similar to the characteristics exhibited in the crack depth/beam depth ratio to number of loading cycles relations, the two stages are also observed in **Figure 5.27**, where the relationships between tensile depth/beam depth ratio and the number of loading cycles for several fatigue loading levels are plotted on the semi-logarithmic scale.

The evolution of CMODs with respect to the increasing number of loading cycles for several fatigue loading levels are drawn on a semi-logarithmic scale **Figure 5.28**. It is noticed that the CMOD evolution of the punching shear crack depends on the load level. However, the two stages observed in the **Figure 5.26** and **Figure 5.27** are not apparently exhibited. Especially for the second stage where a continuous increasing rather than a steady stable of CMOD is observed for all load levels. As the crack depth reaches a stable state in the second stage, the CMOD due to applied load and crack bridging stress should stay almost unchanged. Thus, the CMOD increasing in the second stage is expected to be mainly from the degradation of bond slip. This conjecture is confirmed by the similar trends between CMOD evolution exhibited in **Figure 5.28** and the growing loading end slip with respect to the increasing number of loading cycles as shown in **Figure 5.20**.

In summary, fatigue loading produces generally continuous increase of crack depth, tensile depth and CMOD.

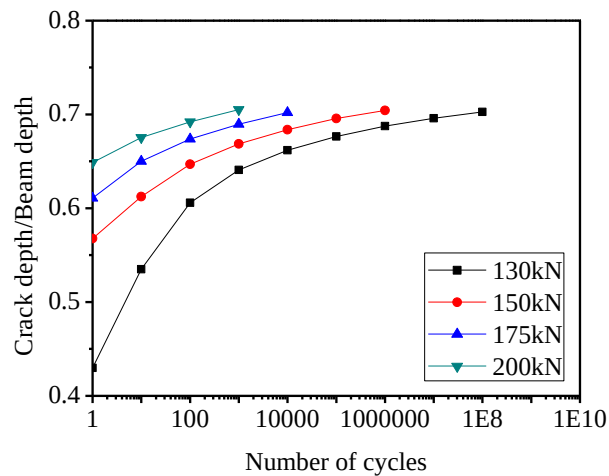


Figure 5.26 Crack depth/beam depth ratio vs. number of loading cycles

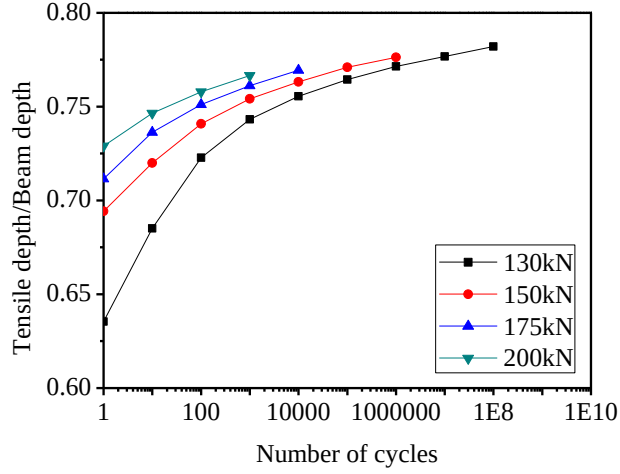


Figure 5.27 Tensile depth/beam depth ratio vs. number of loading cycles

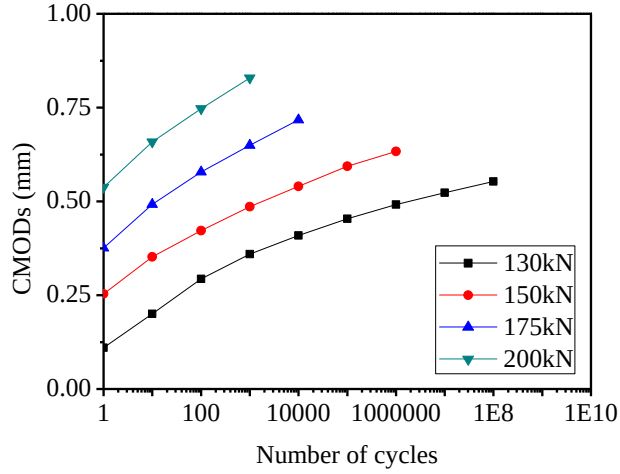


Figure 5.28 CMODs vs. number of loading cycles

5.10.2. Fatigue life from the proposed method

In this method, the final fatigue failure of RC slabs under cyclic moving load is determined following certain brittle failure criterion.

Firstly, the punching shear crack observed in experiments propagates almost along the line 45 °with respect to the bottom surface of the RC beam. Thus, load capacity along this line should be checked after every loading cycle, which can be determined based on the method proposed by [8]. In this method, the punching capacity is assumed from the concrete in the compression zone and the dowel effect of rebars. The schematic diagram for this method is shown as **Figure 5.3** and the load capacity (P_{sx}) is given as

$$P_{sx} = 2 \cdot f_{cv} \cdot x_m \cdot B + 2 \cdot f_t \cdot c \cdot B, \quad (5.45)$$

where the compression depth (x_m) can be related to β as

$$x_m = (1 - \beta) \cdot h. \quad (5.46)$$

Thus,

$$P_{sx} = 2 \cdot f_{cv} \cdot (1 - \beta) \cdot h \cdot B + 2 \cdot f_t \cdot c \cdot B. \quad (5.47)$$

With the obtain β shown in **Figure 5.27**, the load capacity of the cracked beam can be evaluated and compared with the applied load.

Secondly, concrete on the top of the crack-tip are under almost pure compression [53]. This pure compression stress distribution implies a failure due to splitting of the compressed concrete. As shown in **Figure 5.29**, before the crack-tip reaching the upper rebars, this brittle splitting failure mode cannot occur because the crack needs to pass the ductile upper rebars. After the crack-tip reaching the upper rebars, the stress state of compressed concrete and the possible cracking path for concrete splitting as shown in **Figure 5.29** are the same as reported in [54]. Therefore, a load capacity determination method introduced in [54] can be employed. Substituting values of parameters from the critical RC beam in this chapter, For the time when the crack reaches the upper rebars, a load capacity from [54] is determined as 103.8kN for the time when the crack-tip reaches the upper rebars. This value is smaller than the employed load levels. Thus, a splitting failure of compressed concrete occurs as long as the crack-tip reaches the upper rebars.

In addition, the existence of upper rebars weakens the concrete section and the splitting capacity after the critical crack-tip reaching the upper rebars. As a result, the splitting crack propagates along the upper rebar were observed in experiments (see **Figure 5.30**).

In summary, for the selected load levels, the fatigue failure moment, i.e. fatigue life, is determined according to two criterion: (1) The applied load is larger than a punching shear load capacity calculated by **Eq. (47)** using the tensile depth ratio after every loading cycle as shown in **Figure 5.27**. (2) A brittle splitting failure of compressed concrete is assumed to occur if the critical punching shear crack-tip reaches the upper rebars. The fatigue life of RC slabs under different loading levels is plotted on a double-logarithmic scale as shown in **Figure 5.31**.

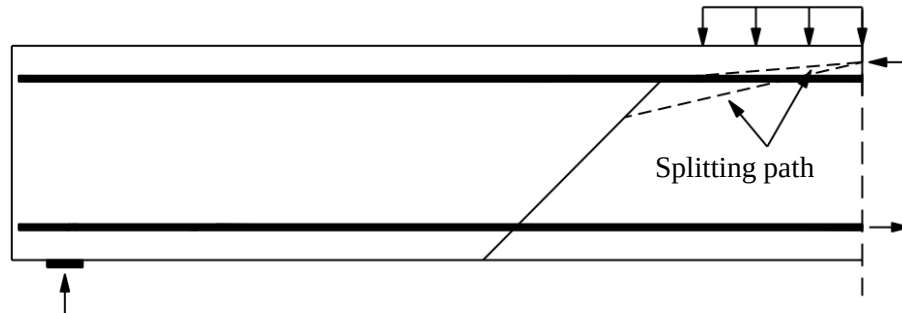


Figure 5.29 Splitting paths in concrete under compression

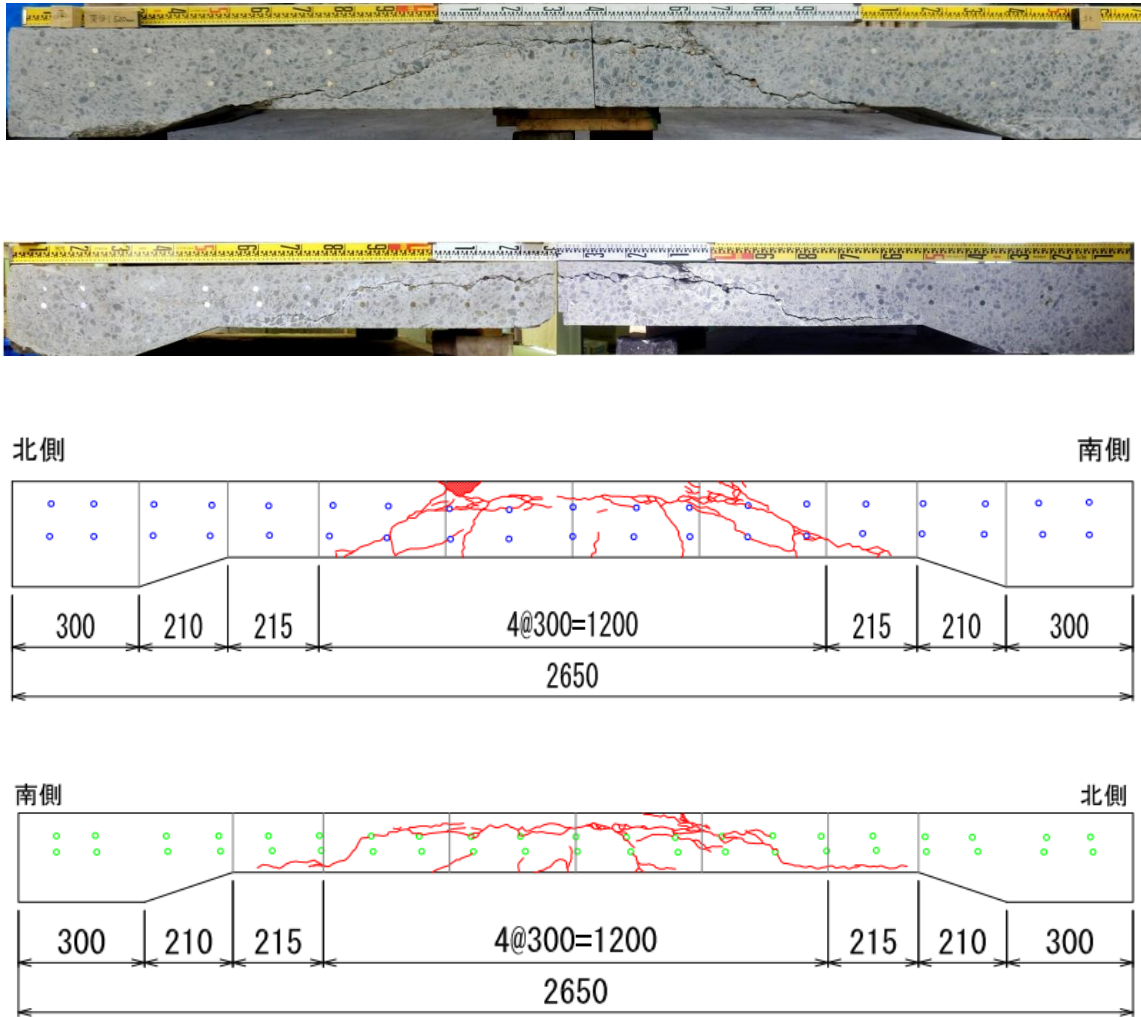


Figure 5.30 Mid-span crack pattern of several RC slabs

5.10.3. Method verification through comparing with other methods

Firstly, based on the proposed in this study, the fatigue life for RC slabs under different loading levels can be determined according to the failure criterion introduced in section 5.10.2.

Moreover, the fatigue life can be calculated following the empirical life prediction equations derived by different research groups, such as Matsui and Abe research teams, and institutions, such as Japan Society of Civil Engineers (JSCE) and Public Work Research Institute (PWRI). The reported equations are listed from **Eq. (2.1)** to **Eq. (2.4)**. All these equations are obtained through statistically fitting a huge amount of data from experiment on RC slabs under cyclic moving loads. Therefore, these equations reflect the internal mechanisms and can be employed in life prediction of similar problems as the employed case in this study.

To illustrate the relationship between fatigue life and load condition, an S-N diagram is commonly employed. This S-N diagram which shows how many load repetitions that a structure can sustain under a given load level is widely used to investigate and to understand the fatigue life of a structure.

All the S-N diagrams from the proposed theoretical method and the empirical equations and experimental fatigue life are plotted on a double-logarithmic scale as shown in **Figure 5.31** where the vertical and horizontal axes represent a load parameter normalized with the punching shear capacity determined from **Eq.(5.47)** and number of loading cycles, respectively. It is found that the theoretical S-N relation agrees well with the fatigue life of the RC slab from experiment and is almost the average results of the four reported equations derived statistically. These good agreements verified the reliability of the proposed theoretical method.

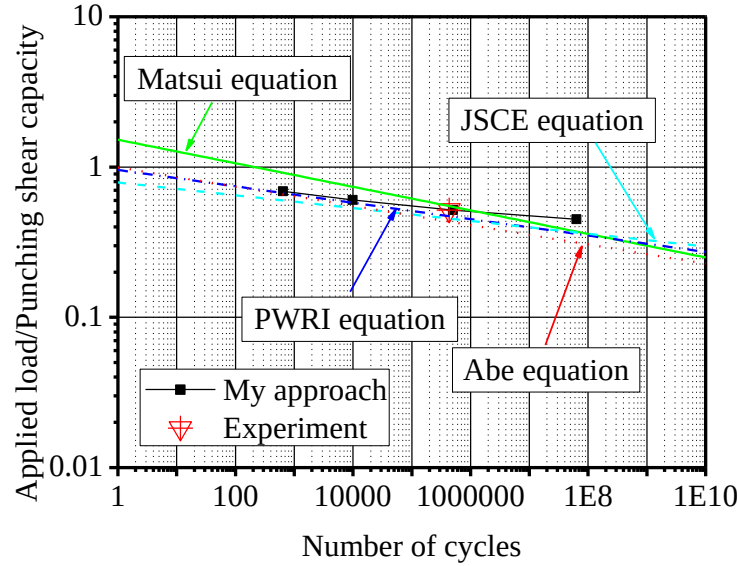


Figure 5.31 Comparison between fatigue life from different approaches

5.11. Summary and conclusions

This chapter proposed a theoretical fatigue life prediction method for RC bridge slabs subjected to cyclic loads based on the nonlinear fracture mechanics. The reliability of this method is then verified through comparing with experimental results and some empirical life predicting equations.

According to the cracking process and failure crack pattern of RC bridge slabs under traveling wheel loads and referencing to the theory basis for previous research on the same field, the failure life prediction problem of RC slabs was simplified into the fatigue life prediction of critical RC beams focusing on the propagation of a couple of punching shear cracks symmetrical with respect to the moving load.

For the critical punching shear crack, the cracking states after the first loading cycle were determined following the idea that the effects from sectional actions (applied loads) are equivalent to those from all sectional reactions (rebar stresses, stresses of cracked concrete and uncracked concrete).

To determine the reactions, firstly, all the material strains and the crack opening profile were expressed by three cracking state parameters, i.e. crack depth ratio, tensile depth ratio, and

CMOD based on three widely employed assumptions. Then, the stresses of cracked and uncracked concrete were simply calculated following a concrete bridging model and a constitutive model of concrete, separately.

However, due to the bond slip effect between concrete and rebars, the rebar stresses should vary along the rebars between cracks and peak at the cracks. To obtain the peak rebar stresses at cracks, a method combining strengths of a smeared crack model and a bond slip model was proposed, where the average rebar stress was determined following a smeared crack model firstly, and then the peak rebar stress was calculated following its relation with the average rebar stress from the bond slip model.

As a result, for the first cycle, all stresses and forces acting on the section along the punching shear cracks were expressed by the three cracking state parameters. These three parameters were calculated through solving three independent equations, i.e. two force equilibrium equations and one CMOD decomposition equation, in this study.

In the CMOD decomposition equation, the total CMOD was regarded as a joint result from applied load, concrete bridging stress, rebar stress and rebar/concrete slip. Except for the CMOD due to bond slip which was determined employing a bond slip model, the CMODs due to other effects were calculated following a fracture mechanics based integral equation. As the closed form analytical weight functions, which are necessary for the integral equation, for the shear cracks under mixed mode loading conditions were unavailable, a finite element analysis based virtual crack extension technique was employed in determine the weight functions.

For materials under repetitive loads, performance degradations are inevitable. Thus, for 2nd to Nth, the concrete bridging stress and the bond slip resistance degradations were accounted for through introducing a concrete bridging degradation model and a bond slip degradation model, respectively, because these two degradations were claimed to be the governing factors for RC structure degradation under service level fatigue load. Due to these degradations, the critical punching shear cracks propagated to a new state. After every loading cycle, the three cracking state parameters for the new crack were determined through solving the updated three equations, where the degraded material models were employed, as well.

With the three cracking state parameters after every loading cycle, the failure moment, i.e. fatigue life, was identified following some failure criterion. Under different load levels, the fatigue lifes from method in this study, experiment and some empirical equations obtained through fitting a huge set of experimental data were draw together into S-N diagrams. The good agreements verified the reliability of this method.

In addition, since this method was established based on the nonlinear fracture mechanics and the degradation mechanisms of RC structures under repetitive loads, it processes many advantages compared with the existing empirical equations, such as figuring out crack propagation, enclosing inner mechanisms, indentifying dominant factors for fatigue failure through conducting parametric study and providing advice for structure design based on parametric study results.

CHAPTER 6 DETERMINATION OF THE DOMINANT DEGRADATION

MECHANISMS OF RC SLABS UNDER MOVING LOAD

6.1. Introduction

Fatigue is a process of progressive permanent internal structural change in a material subjected to repeated loading. Generally, an RC slab subjected to fatigue loads consists of 2 materials (concrete and steel rebar) under 3 stress states (compression, tension and shear) which collaborate with each other using 1 interface (concrete/rebar interface). Therefore, the degradation of the RC structures is a result of 7 possible degradations, i.e. the interface and materials under different stress states. However, a fatigue analysis considering all these degradations should be very complicated and the contributions from different degradations are different. Therefore, to conduct fatigue analysis of an RC structure under repeated load efficiently and accurately, the dominant degradation mechanisms should be identified beforehand.

[71] analyzed the fatigue strength of steel-concrete sandwich beams using the finite element method. Tailored constitutive models were employed according to fatigue effects by reducing stiffness and strength of material models. This reduction is mainly based on the empirical formulation from previous experimental analysis. This investigation indicated that the effect of reduction in compressive stiffness was found to be negligible. [72] conducted fatigue analysis of plain and reinforced concrete beams based on the concrete bridging degradation concept, where it was found that the participation of concrete bridging stress is very small in resisting bending moment compared with that from reinforcing rebars. [17] presented a numerical fatigue simulation of RC slabs under a moving load. A direct path-integral scheme with fatigue constitutive models for concrete tension, compression and rough crack shear is used to predict the life-cycle of RC slabs, where the important role of shear transfer decay was reported based on a parametric study. Regarding the bridging stress degradation of concrete as a primary mechanism for the propagations of cracks inducing failure, fatigue failure characteristics of RC slabs subjected to cyclic moving load were successfully predicted in [16, 73].

In most existing researches on fatigue analysis of RC structures, the dominant mechanisms were identified, employed and considered based on experience from experimental analysis or numerical sensitivity studies. However, the employed dominant degradation mechanisms for accurately analyzing one case may be not applicable for another case because it can be found that the dominant degradation mechanisms vary for different structures and failure modes.

As the degradation of a material subjected to repeated loading is directly related to the experienced stress history, i.e. stress levels, stress amplitudes, and number of loading cycles. Therefore, whether a degradation of a material under a stress state should be considered in fatigue analysis can be determined if the stress history of that material is obtained.

In this study, as introduced in Chapter 5, the stresses of concrete and reinforcing rebars acting on the cross-section along the punching shear cracks of the critical RC beam were expressed by the three cracking state parameters, i.e. crack depth/beam depth ratio, tensile depth/beam depth ratio and CMOD, which are obtainable through solving the three equations.

With the three parameters corresponding to different number of cycles, the stresses and the sectional resistance from concrete and rebars can be calculated. This information can be further utilized in identifying the dominant degradation mechanisms. As this approach is on the basis of the source of material degradation, it is theoretically more straight-forward and reliable.

In this chapter, focusing on the critical RC beam cutting from a RC slab under moving load, the stress states of concrete and rebars over the entire life range are calculated utilizing the obtained three cracking state parameters after every loading cycle. And then, to figure out sectional resistances from different components, i.e. cracked concrete, uncracked concrete, bottom and upper rebars, forces, moments and CMODs due to each component are calculated and compared as well. Furthermore, judgments on which degradation mechanisms should be included in fatigue analysis of RC slabs under moving loads are made based on the obtained stress state of each material and sectional resistant contribution of each component.

6.2. Stress states of different components

Figure 6.1 shows the relationship between of rebar normal and shear stresses at cracks and the number of loading cycles. It is noticed that the rebar stress increases with the increasing number of cycles and approaches an asymptote-resembled line. The value of this line depends on the loading level and is much lower than the yielding strength of rebars even under a relatively high load level, 200kN. This means the Baushinger effect of tensioned rebar which consists of reduction of yield stress after a reverse and decrease of curvature in the transition zone between the elastic and the plastic branches can be neglected. In addition, it is found that the rebar stress experienced a relative high speed of rising in the initial several loading cycles, especially for lower load levels. This can be explained as: under the initial narrow crack opening, the concrete bridging stress accounts for a large proportion of the total bridging stress. The stress transferring ability of cracked concrete decreases due to the damage caused by the crack opening and closing process, which is considered through introducing the concrete bridging degradation model. As a result, the forces which were supposed to be sustained by the cracked concrete are transferred to the rebars. With increasing number of cycles, the opening and closing process keep smoothing both faces of the crack and finally the concrete bridging stress reduces to almost zero. Correspondingly, the rebar stresses reach a plateau state. The final asymptote-resembled rebar stress level can be regarded as the rebar stresses computed without accounting for the tensile effect of concrete. As shown in **Figure 5.26** and **Figure 5.27**, the crack grows with a very slow speed in the steady state, which mean the internal force lever between the stress resultant point of components in the compression zone and the bottom rebars stays almost unchanged. Thus, the rebar stress should remain almost constant under the same sectional moment from applied load as indicated in **Figure 6.1**. The notable rebar stress increasing due to the concrete bridging stress degradation confirms the necessity of including this degradation in the fatigue analysis of a RC slab subjecting to cyclic moving load indirectly.

The variations of the maximum compression stress of concrete at several fatigue loading levels are plotted with the number of loading cycles on a semi-logarithmic scale as shown in **Figure 6.2**. Even though the absolute value of maximum compression stress keeps growing, the final values under all the selected loading levels are in a relatively low level, less than 30% of compression strength. Under this stress level, the fatigue life of concrete can be up to 10^{12} loading cycles following the S-N relationship presented in [74]. This fatigue life is much longer

than the ACI provisions employed threshold value for fatigue life, ten million cycles, which mean the degradation of concrete in the compression zone is negligible. The degradation of concrete was also reported as negligible if the maximum stress is lower than 30% of compression in [75], where a versatile model on analyzing the fatigue degradation of compressed concrete was proposed through introducing a low-cycle concept. All these results indicate that the degradation of concrete in the compression zone is negligible for an accurate fatigue analysis of a RC slab under cyclic moving load.

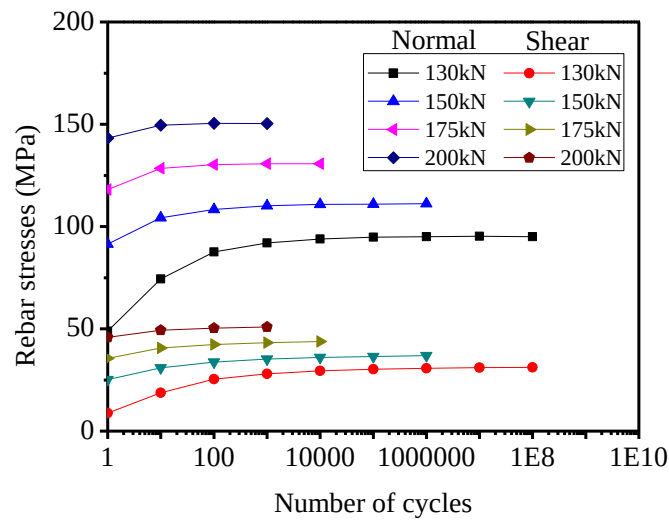


Figure 6.1 Rebar stresses vs. number of loading cycles

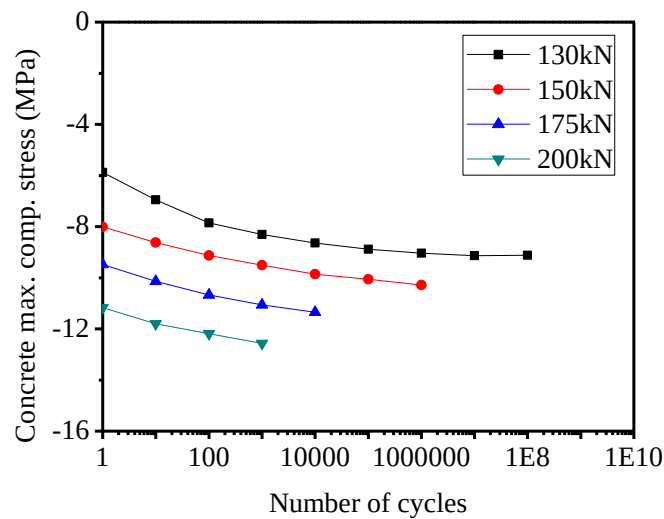


Figure 6.2 Maximum compression stress of concrete vs. number of loading cycles

6.3. Contributions from different components

For the punching shear crack in the critical RC beam under applied loads, the sectional rotation and crack opening are resisted by the stresses from all components, i.e. bottom rebars, cracked concrete, uncracked concrete and upper rebars. There should be some components playing dominant roles in balancing the applied load, which can be identified if the contributions from all components are computed. On the basis of the contributions from all components in the whole life time, a more straight-forward and confident image on the fatigue behavior of RC slabs under cyclic moving load can be figured out. Furthermore, the determined dominant components provide advice and references to researchers who are purposed at developing simulation methods as efficient as possible.

In this section, for several loading levels, the forces, moments and CMODs due to different components in the cross-section along the punching shear crack are computed after certain number of cycles using the obtained three cracking state parameters. Since a two-dimensional crack model is assumed for the cracked RC beam, all the forces and moments given in this section are the forces and moments acting on unit width of the beam. Based on the obtained results, some discussions on the dominant sectional rotation and crack opening resistant components are given, respectively.

6.3.1. Forces from different components

For a selected cross-section, the forces from all components make up a self-equilibrium system. According to the force acting direction, the components can be roughly divided into two groups, i.e. bottom rebars and cracked concrete, uncracked concrete and upper rebars.

The relationships between forces from bottom rebars and number of loading cycles for several loading levels are shown in **Figure 6.3(a)**, where the curve trends are exactly the same as the trends observed in the bottom rebar stress curves (**Figure 6.1**). This is because the forces of the bottom rebar can be related to the corresponding stresses with a constant rebar area.

Figure 6.3(b) shows the relationship between forces from cracked concrete and number of loading cycles for several loading levels. For each curve, the forces decline rapidly to a small amplitude and then reach a steady state, which means the concrete bridging stress degradation occurs quickly after the initiation of a crack. That is why the fatigue life of a plain concrete structure is much shorter than a reinforced concrete structure as reported in [55]. From the four curves, it is found that, due to the wider crack opening under a higher load level, the forces from cracked concrete decrease with the increasing applied load level. If **Figure 6.3(b)** is investigated jointly with **Figure 6.3(a)**, the relative relationships between forces from bottom rebars and cracked concrete are obtainable. **Figure 6.3(a)** and **Figure 6.3(b)** indicate that the forces of bottom rebars are generally larger compared to the forces for cracked concrete, and that the difference between them becomes larger as the increasing number of loading cycles. Under all load levels, the forces from cracked concrete decline to almost negligible compared with the bottom rebar forces before final failure.

Figure 6.3(c) and **Figure 6.3(d)** show the variations the forces from uncracked concrete and upper rebars, which are mostly in the compression zone, after each number of loading cycles in semi-logarithmic scales, respectively. It is seen from **Figure 6.3(c)** that the forces from

uncracked concrete depend on the load level and, except for curve for 130kN, the forces from uncracked concrete remain almost unchanged in other cases. This is because, as shown in **Figure 5.26**, only the punching shear crack under 130kN loading condition experienced a remarkable uplifting of tensile depth/beam depth ratio, which reflects the internal force lever between compressed and tensioned components. In terms of the forces from upper rebars as shown in **Figure 6.3(d)**, the absolute values of these forces are much lower than those from the other components. In addition, due to the continuous growing tensile depth, the sectional neutral axis may surpass the center point of upper rebars and correspondingly the stress state of upper rebars is transformed from compression into tension as indicated in **Figure 6.3(d)**.

Figure 6.4 compares the forces from different components versus number of loading cycles for applied load ranging from 130kN to 200kN. The variations of relative magnitude relationships among all the components over load levels and number of cycles can be more easily visible from **Figure 6.4**. It is found that the magnitude of force from upper rebars is always much smaller than that from other components, which means the effect of upper rebars on resisting sectional rotation can be simply neglected conducting calculation. For the contribution from the cracked concrete which is generally regarded as zero in sectional analysis of RC beams, even though the magnitude decrease with the increasing load level, it cannot be neglected for structural analysis of RC slabs under service loading levels compared with the contribution from bottom rebars. However, to determine no matter static or fatigue ultimate load capacities, the bridging effect of concrete can be just ignored as assumed by design codes.

6.3.2. Moments from different components

Considering the variation of height of the sectional neutral axis for different load levels and number of cycles, the moments from all components are computed with respect to the upper surface of the beam and given in this section. For the moments from both bottom and upper rebars as shown in **Figure 6.5(a)** and **Figure 6.5(d)**, the curves exhibit the same trends as observed in the corresponding forces owing to the constant force levers for all rebars.

In the semi-logarithmic scale, **Figure 6.5(b)** shows the variations of moments from cracked concrete with the increasing number of loading cycles for several load levels. As the reduction of moment from cracked concrete is a result of the combined action of reductions of both cracked concrete forces and forces levers, the moments from cracked concrete drop at a higher speed compared with the trends of cracked concrete forces as indicated in **Figure 6.3(b)**.

The relationships between moment from uncracked concrete and number of loading cycles are plotted in a semi-logarithmic scale for several loading levels as shown in **Figure 6.5(c)**. It is found that the curve for 130kN can be divided into two stages: decelerated decrease stage, stable stage. The two stages can be explained as follows. Generally, the stress distribution of uncracked concrete is shown as **Figure 6.7**. With respect to top point of the cross-section, moments from uncracked concrete above and below the neutral axis are in opposite signs. As illustrated in **Figure 6.3(c)**, the forces from uncracked concrete are with a negative sign in all situations, which means the absolute value of the resultant force from uncracked concrete above the neutral axis should be larger than that from uncracked concrete below the neutral axis. However, the force lever for the uncracked concrete below the neutral axis is larger than that of above the neutral axis. If the distance from the neutral axis to the top point is smaller than 2 times of the distance from the crack-tip to the neutral axis, the moment from uncracked concrete should be in a

positive sign as observed in the beginning several cycles of the curve for 130kN. Owing to crack growing, the crack-tip approaches to the neutral axis, and the moment from uncracked concrete below the neutral axis decreases rapidly. As a result, the decelerated decrease stage is observed in the curve for 130kN. This decrease stops when the crack growth reaches the steady stable stage, and the moment from uncracked concrete enters the stable stage as well. According to the interpretation of the curve for 130kN, the range and presence of decelerated decrease stage depend on the extent of crack propagation in the initial number of cycles. Since the crack becomes more developed after several initial loading cycles for higher loading levels, the range of the decelerated decrease stage shortens and then even disappears with the increasing loading levels as shown in **Figure 6.5(c)**.

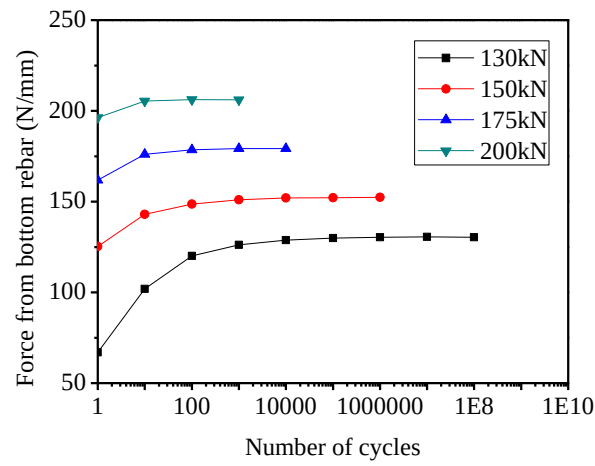
6.3.3. CMODs from different components

As illustrated in Chapter 5, the crack opening is a result of combined action of applied load, rebar forces, bond slip and concrete bridging stresses, whereas the crack opening due to bond slip is included in the crack opening due to rebar forces through modifying the rebar constitutive model. Therefore, only the evolutions of three CMODs, i.e. CMOD due to applied load, CMOD due to rebar forces and CMOD due to concrete bridging stresses, are presented in this section.

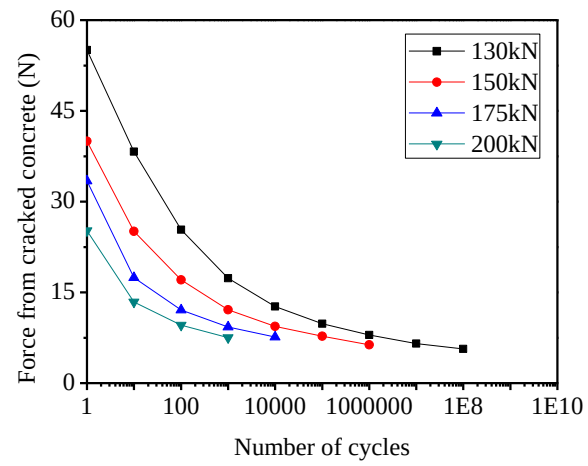
The evolutions of CMODs due to rebar forces with the increasing number of loading cycles are drawn in **Figure 6.8(a)** for several loading cycles. The trend of these curves maybe difficult to be understood theoretically because the CMODs due to rebar forces are calculated following the complicated energy based integral equation as introduced in Chapter 3. However, these curves are easily understandable from the physical meaning of crack opening. For an existing crack, the crack faces can be regarded as a cantilever beams rotating around the crack-tip. Thus, the rebar force lever increases with the propagating of the crack. At the same time, the magnitude of rebar forces increases as well. Therefore, the growing speed of the magnitude of CMOD due to rebar forces is the addition of the speeds for crack propagation shown in **Figure 5.26** and rebar force increasing shown in **Figure 6.3**.

The relationships between CMODs due to cracked concrete and number of loading cycles for several loading levels are shown in **Figure 6.8(b)**, where the characteristics of all curves can be explained similarly as the explanation for the CMODs due to rebar forces. The resultant forces of cracked concrete decrease with the increasing number of cycles; meanwhile, the resultant force points approach the crack-tip because the concrete bridging stress degrades more dramatically for locations close to the crack mouth. Consequently, the CMOD due to cracked concrete drops to a low magnitude rapidly as shown in **Figure 6.8(b)**.

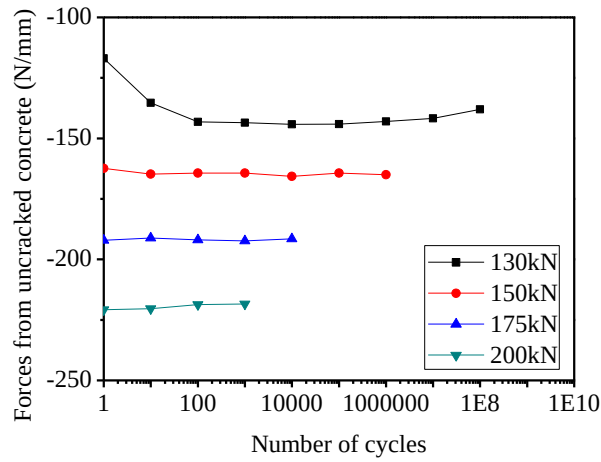
Figure 6.8 present the variations CMODs from different components versus number of loading cycles in semi-logarithmic scales for several loading levels. It can be found that, except for the initial certain number of loading cycles under 130kN, the crack bridging effect is mainly from the rebars. Therefore, to calculate the width a crack in degraded RC structures or RC structures under relatively high loading levels, the crack bridging effect of concrete can be simply regarded as negligible. This is why many researches on determination of crack widths of RC structures can achieve an acceptable accuracy without considering the effect of concrete bridge stress.



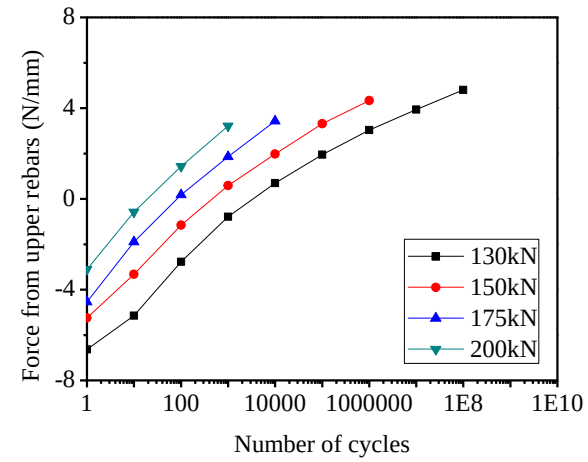
(a) Forces from bottom rebar



(b) Forces from cracked concrete

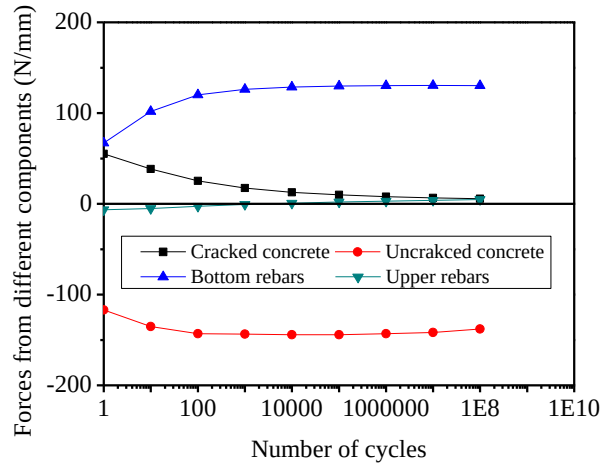


(c) Forces from uncracked concrete

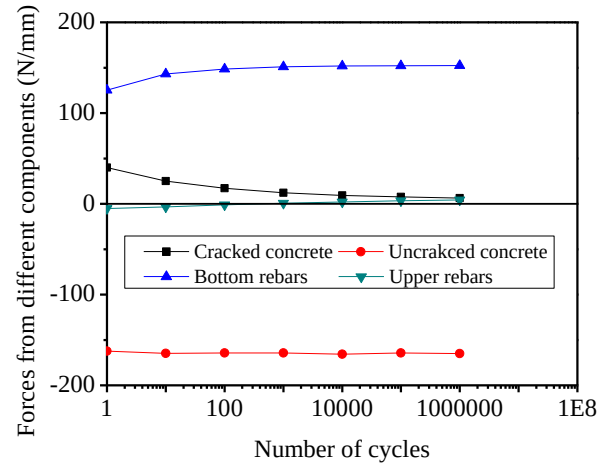


(d) Force from upper rebars

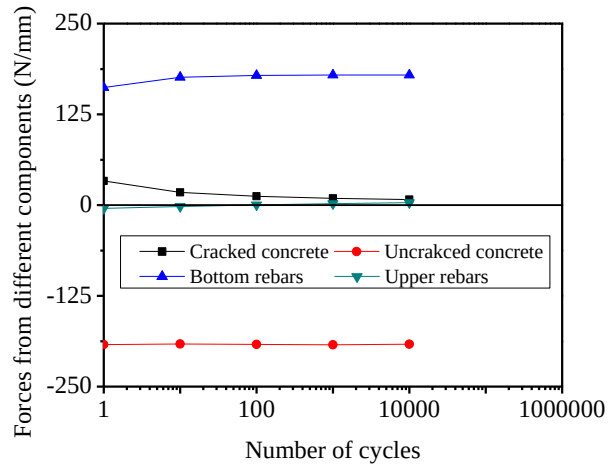
Figure 6.3 Forces from different components vs. number of loading cycles for several loading levels



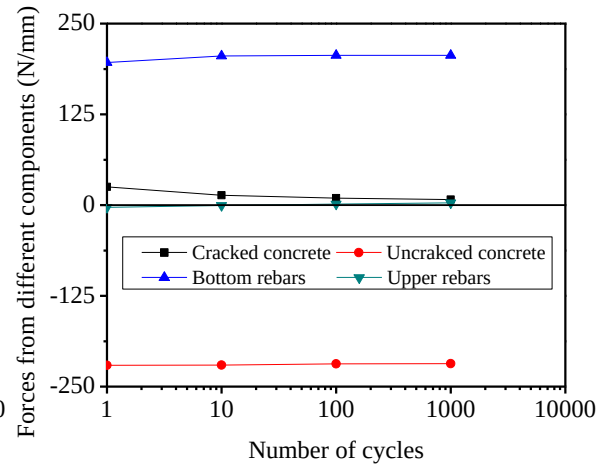
(a) Load=130kN



(b) Load=150kN



(a) Load=175kN



(b) Load=200kN

Figure 6.4 Comparison of forces from different components vs. number of loading cycles for several loading levels

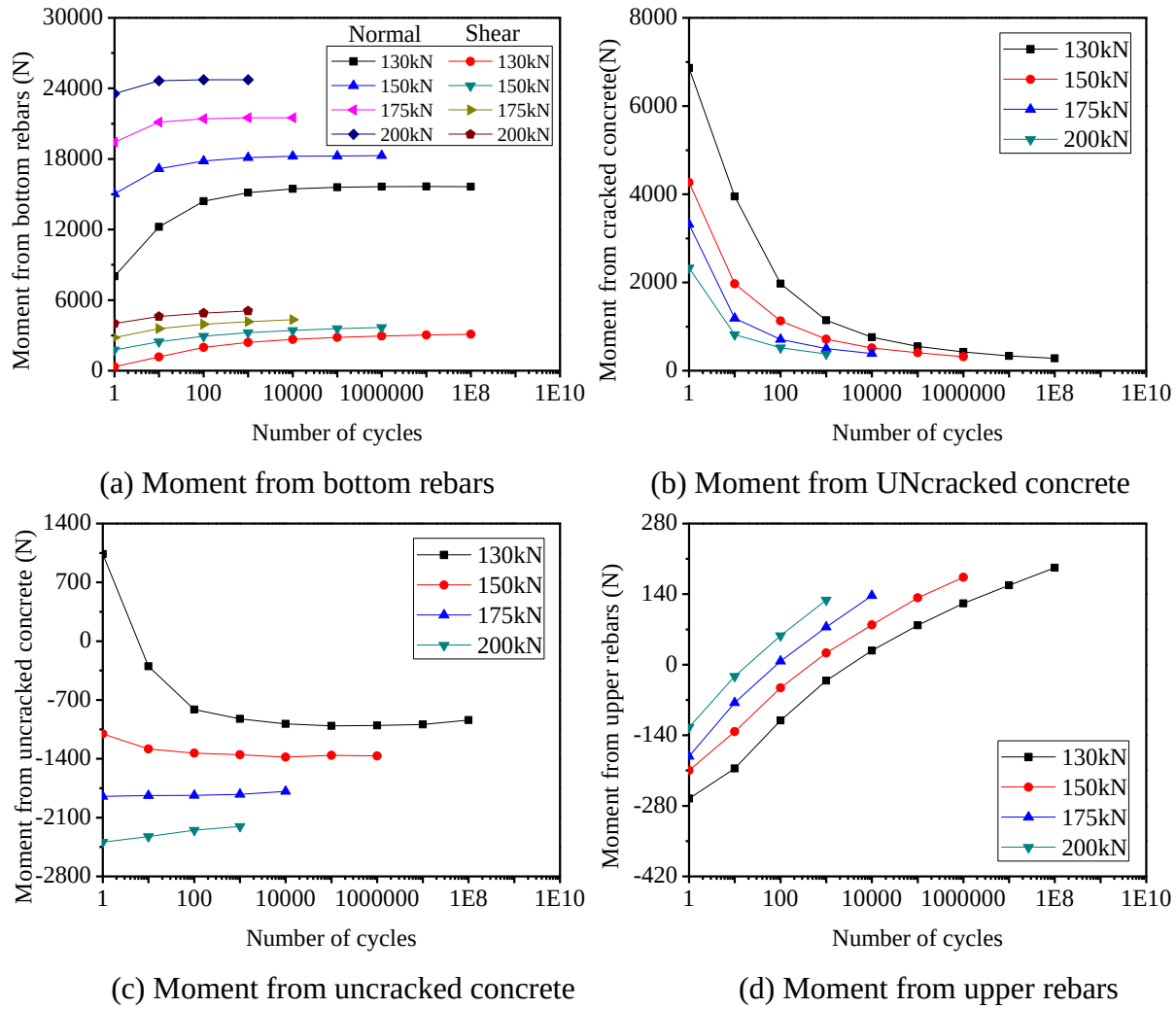


Figure 6.5 Moments from different components vs. number of loading cycles for several loading levels

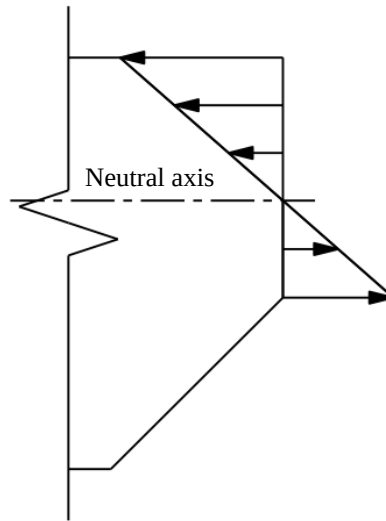


Figure 6.6 Stress distribution in the uncracked concrete

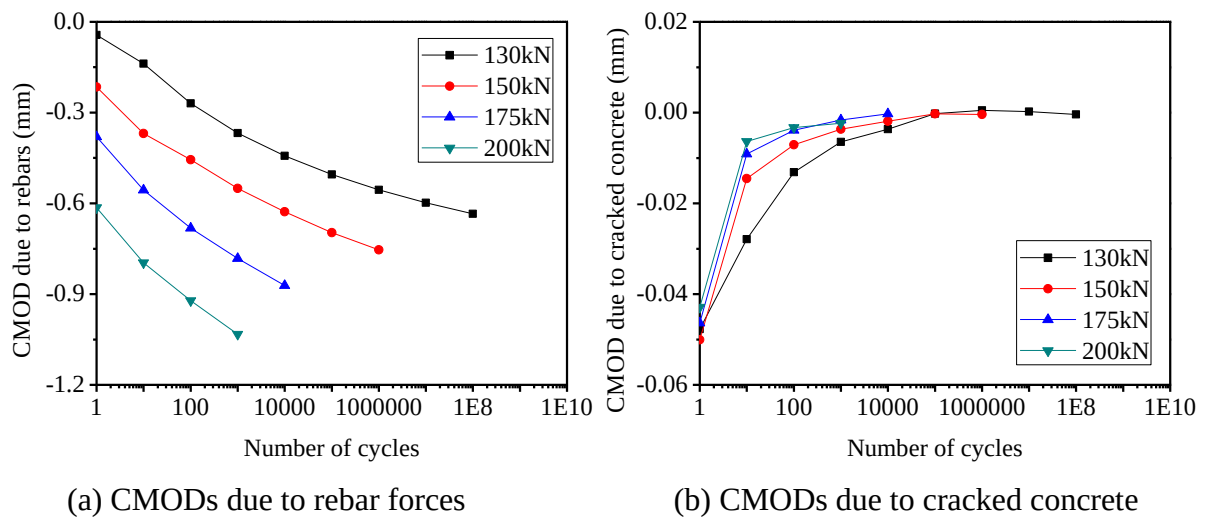
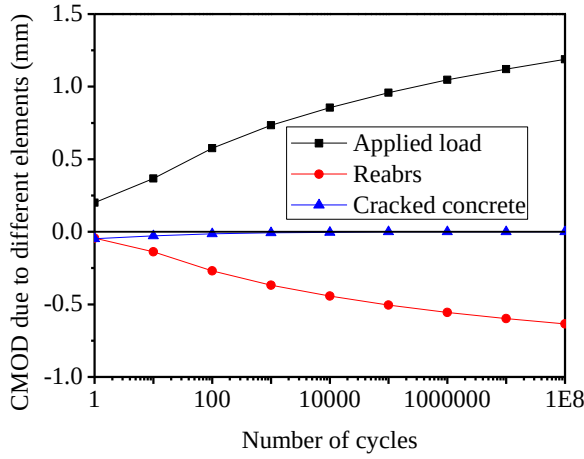
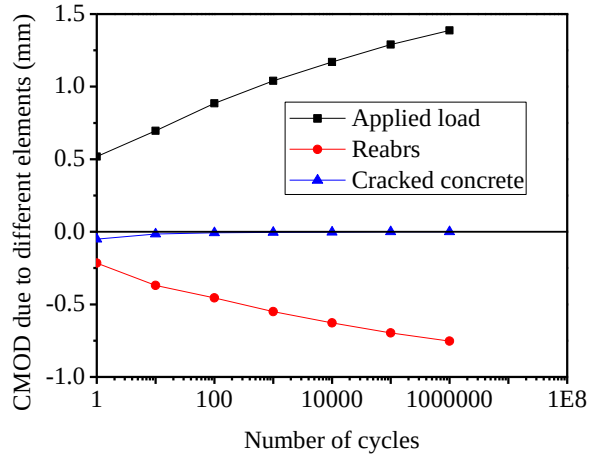


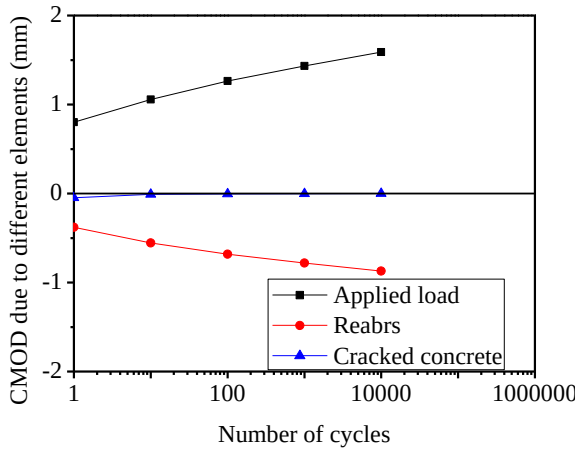
Figure 6.7 CMODs due to different components vs. number of loading cycles for several loading levels



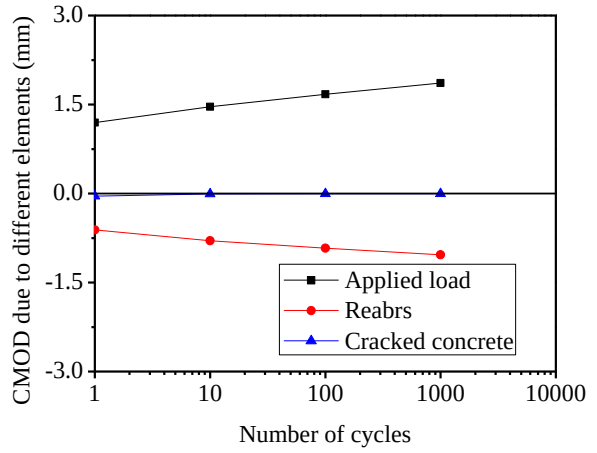
(a) Load=130kN



(b) Load=150kN



(a) Load=130kN



(b) Load=150kN

Figure 6.8 Comparison of CMODs from different components vs. number of loading cycles for several loading levels

6.4. Summary and conclusions

In the chapter, the stresses and forces from all the components in the cross-section along the punching shear crack in a RC beam simplified from a RC slab subjected to moving loads were computed using the three cracking state parameters after certain number of loading cycles. These stresses and forces provided a reliable, confident and straight-forward reference for determining the primary degradation mechanisms and dominant sectional rotation and crack bridging components.

Using the three obtained cracking state parameters after every loading cycle, the stresses of bottom rebars and concrete in the compression zone were calculated. Since the rebar stresses and the maximum compression stresses of concrete were found to be lower than the yielding strength

of rebar and 30% of compression strength of concrete, respectively, under all the selected service-level loading conditions in the whole life range, it can be concluded that the degradations of tensioned rebars and compressed concrete are negligible for an accurate fatigue analysis of an RC slab under cyclic moving loads.

To identify the dominant components resisting sectional rotation and crack opening, forces, moments and CMODs from all the components, i.e. bottom rebars, cracked concrete, uncracked concrete and upper rebars, were computed as well. The obtained results were coincident with the conceptions widely possessed on the structure analysis of RC beams that the bottom rebars and uncracked concrete play dominant roles in resisting sectional rotation and crack opening and the contribution of upper rebars is negligible. However, the contribution from the cracked concrete which is generally regarded as zero in sectional analysis of RC beams should be carefully considered, especially for fatigue analysis of RC beams under service loading conditions.

Conclusively, since the results in this chapter covered a wide range of loading level and number of loading cycles, they can be referenced by researches according to their research purposes.

CHAPTER 7 CONCLUSIONS AND FUTURE WORKS

7.1. Major conclusions

In this study, a theoretical life prediction method for RC slabs subjected to cyclic moving loads was developed based on a nonlinear fracture mechanics for concrete structures. In this method, the concrete bridging stress degradation and the bond slip degradation in the rebar/concrete interface were introduced as two primary mechanisms for structural degradation and fatigue crack propagation. Conclusions are summarized as follows.

1. A fatigue life prediction of RC slabs under cyclic moving loads was simplified into a life prediction of a critical RC beam focusing on a couple of punching shear crack based on both the observed crack process and failure crack pattern of the RC slabs and the theoretical basis for existing researches.

2. As the crack width related concrete bridging stress degradation is considered as a dominant degradation mechanism, a crack width determination method for RC beams under applied loads was proposed based on fracture mechanics with a bond slip model. In this method, the total crack width was decomposed into crack widths due to applied loads and crack bridging stresses and crack width due to bond slip, which were calculated based in a fracture mechanics based integral equation and an appropriate bond slip model. The developed method was then employed in analyzing a tested RC beam under several static loading levels. A good agreement between crack widths from analysis and experiment confirmed the reliability of this method.

3. In the integral equation, weight functions were needed. In this study, nodal weight functions on all generally load applied faces for an RC beam with oblique edge shear cracks were evaluated exploiting a finite element method based virtual crack extension technique. A singular behavior at the crack-tip neighborhood was observed in primary crack-face weight functions. To facilitate application, these crack-face weight functions were formulated successfully as a function of crack length and distance away from the crack-tip through fitting and interpolating the primary crack-face nodal weight functions.

4. Considering the shear failure of an RC beam is heavily influenced by the shear span to beam depth ratio, parametric study was conducted on this ratio. It was found that the weight function along different boundaries varied orderly with respect to shear span to beam depth ratio. This orderly trend can then be followed in determining weight functions along all boundaries of an RC beam with varying shear span/beam depth ratio using the corresponding weight functions of two RC beams with different shear span/beam depth ratios.

5. Using the developed crack width and weight function determination methods, a theoretical fatigue life prediction of the critical RC beam was established focusing on the punching shear cracks. In this method, a method for determining rebar stresses at cracks was proposed combining a smeared crack model and a bond slip model. Considering concrete bridging degradation and bond slip degradation, cracking state parameters, i.e. crack depth, tensile depth and crack width, of the punching shear cracks were obtained after every loading cycle. These cracking state information were then employed in determining the fatigue life according to some widely-employed brittle failure criterion. Through comparing with fatigue life

from experiment and some empirical equations, the accuracy and reliability of this method were verified by good agreements.

6. Using the three obtained cracking state parameters after every loading cycle, stresses and forces from all components in the cross-section along the punching shear crack were calculated. These stresses and forces provided a reliable, confident and straight-forward reference for determining the primary degradation mechanisms and dominant sectional rotation and crack bridging components. It was concluded that the degradations of tensioned rebars and compressed concrete are negligible for an accurate fatigue analysis of an RC slab under cyclic moving loads. In addition, even though the bottom rebars and uncracked concrete played dominant roles in resisting sectional rotation and crack opening, the contribution from the cracked concrete should be carefully considered as well, especially for fatigue analysis of RC slabs under service loading conditions.

7.2. Future works

Some suggestions regarding possible further research topics are proposed below.

1. Parametric study on design parameter

Compared with existing empirical and numerical approaches, the proposed method can not only account for degradation mechanisms of RC slabs under repetitive loads, but also save a lot of computing time. Therefore, this method is suitable for parametric study on design parameters including slab thickness, concrete strength, reinforcement ratio, rebar arrangement, etc. These parametric studies can be referenced in improving and modifying design codes on RC bridge slabs.

2. Study on influence of boundary conditions

Obviously, for RC slabs under repetitive loads, boundary conditions should have some influences on the fatigue life. However, as the introduced theories for empirical equations, the influence of boundary conditions cannot be captured by these equations.

According to the methods of computing sectional shear force and moment due to applied loads, the influence of boundary conditions is mainly on the sectional moment which is obtained through finite element analysis of cracked RC slabs in the proposed method. Therefore, studies on influences of boundary conditions can be conducted. And then, optimized schemes on boundary condition set-up can be proposed based on the results.

3. Environment related deteriorations

RC slabs in reality are subjected both repetitive moving loads and certain environmental actions. As a result, the fatigue life is shortened under this coupling action. It is meaningful for studies on effects of environment related deteriorations.

For example, as introduced in Chapter 5, concrete in the compression zone plays an important role for RC slabs failed in a punching shear failure mode. For the concrete, it is inherently full of flaws, such as pores, air voids, lenses of bleed water under coarse aggregates.

These flaws make the concrete sensitive to environmental actions like water and freezing-thaw cycles. The acceleration of punching shear failure of RC slabs is frequently observed in cold regions as Hokkaido.

Besides, the influence of environment related deterioration on the concrete/rebar interface bond effect should be studied as well, because this bond effect determines the cooperation between concrete and rebar and the interface is very vulnerable to environmental actions.

4. Life extension due to structural strengthening

To increase the punching shear capacity and extend fatigue life of RC bridge slabs, strengthening techniques have been employed, such as externally bonded FRC sheets. The effectiveness of these techniques should be checked through predicting the related life extension.

APPENDIX PUBLICATION LIST

Part 1: Journal papers:

- 1) Deng, P. R. and Matsumoto, T. "Estimation of the Rebar Force in RC Members from the Analysis of the Crack Mouth Opening Displacement Based on Fracture Mechanics", Journal of Advanced Concrete Technology, Vol. 15, No. 2, pp. 81-93, 2017a.
- 2) Deng, P. R. and Matsumoto, T. "Weight Function Determination for Shear Cracks in Reinforced Concrete Beams Based on Finite Element Method", Engineering Fracture Mechanics, Vol. 177, No. 2, pp. 61-78, 2017b.
- 3) Deng, P. R. and Matsumoto, T. "Determination of the Relation between the Inclined Angle and Weight Functions for Shear Cracks for Reinforced Concrete Beams using Virtual Crack Extension Technique". (In preparing)
- 4) Deng, P. R. and Matsumoto, T. "Estimation of Rebar Force at Crack Location Based on Modified Rebar Model and Bond-slip Model". (In preparing)
- 5) Deng, P. R. and Matsumoto, T. "Life Prediction of Reinforce Concrete Slabs Based on Fracture Mechanics". (In preparing)

Part 2: Conference papers

- 6) Deng, P. R., Matsumoto, T., Hayashikawa, H. and He, X. W. "Noise Sensitive Study on Inverse Analysis of Rebar Force in RC Beams", JSCE Hokkaido branch No. 71, Paper report collection No. A-30, January 2015.
- 7) Deng, P. R. and Matsumoto, T. "Rebar Force Estimation from Crack Mouth Opening Displacement", 3rd International Conference on Computational Design in Engineering, pp 32, December, 2015.
- 8) Deng, P. R. and Matsumoto, T. "Rebar Force Estimation using Crack Mouth Opening Displacement Based on Fracture Mechanics with Bond Slip Model", JSCE Hokkaido branch No. 72, Paper report collection No. A-20, January 2016.
- 9) Deng, P. R. and Matsumoto, T. "Rebar Force Estimation through Inverse Analysis of CMOD Based on Fracture Mechanics", 14th East Asia-Pacific Conference on Structural Engineering and Construction, pp. 849-857, January 2016.
- 10) Deng, P. R. and Matsumoto, T. "Weight Function Determinations for Shear Cracks in Reinforce Concrete Beams with Virtual Crack Extension Technique", JSCE Hokkaido branch No. 73, Paper report collection No. A-14, February 2017.

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