



HOKKAIDO UNIVERSITY

Title	The 28th Sapporo Symposium on Partial Differential Equations
Author(s)	Jimbo, Shuichi
Description	The 28th Sapporo Symposium on Partial Differential Equations Organizers: T. Ozawa, Y. Giga, S. Jimbo, K. Tsutaya, Y. Tonegawa, G. Nakamura ² . Venues: Department of Mathematics, Faculty of Science, Hokkaido University July 23, 2003 (Wednesday) 9:30-10:30 Gregory SEREGIN (Steklov Institute/Keio Univ.) Interior regularity of L_3 -solutions to the Navier-Stokes equations 11:00-12:00 Takaaki NISHIDA (Kyoto Univ.) Heat convection problems and computer assisted proof 14:30-15:00 Akihiro SHIMOMURA (Gakushuin Univ.) Modified wave operators for Maxwell-Schrodinger equations 15:15-15:45 Hideaki SUNAGAWA (Osaka Univ.) Remarks on the large time asymptotics for nonlinear Klein-Gordon systems 16:00-16:30 Hirokazu NINOMIYA (Ryukoku Univ.) Curved traveling front of Allen-Cahn equations July 24, 2003 (Thursday) 9:30-10:30 Masahiro YAMAMOTO (Univ. Tokyo) Uniqueness in inverse scattering problems with a single incident wave 11:00-12:00 Shinya NISHIBATA (Tokyo Inst. Tech.) Asymptotic behavior of spherically symmetric solutions to the compressible Navier Stokes equation with external forces 14:30-15:00 Dening LI (West Virginia Univ.) Conical shock waves in supersonic flow 15:15-15:45 Yasushi TANIUCHI (Shinshu Univ.) Remarks on global solvability of 2-D Boussinesq equations with nondecaying initial data July 25, 2003 (Friday) 9:30-10:30 Ryuichi SUZUKI (Kokushikan Univ.) Blow-up of solutions of quasilinear parabolic equations with localized reactions 11:00-12:00 SAKAGUCHI (Ehime Univ.) Initial behavior of solutions of diffusion equations and symmetries of domains"
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The curved traveling front of the Allen-Cahn equation

Hirokazu Ninomiya (Ryukoku University)
and

Masaharu Taniguchi (Tokyo Institute of Technology)

In this talk we consider the Allen-Cahn equation:

$$u_t = \Delta u + f(u) \quad (x, y, t) \in \mathbf{R}^2 \times \mathbf{R}_+ \quad (1)$$

where f is of “bistable type”. The typical example of the nonlinear term f is

$$f(u) = u(1 - u)(u - a), \quad 0 < a < \frac{1}{2}. \quad (2)$$

The constant states 0 and 1 are stable under the diffusion-free system. By the assumption of a , the region of the state 1 is getting larger and larger and finally it covers the whole space. When the state 1 propagates, we can observe the characteristic profiles. In the one-dimensional space, one of the typical solutions is a *traveling wave* solution which never changes its shape without translation. Substituting $u(x, t) = \Phi(x - ct)$, we have

$$\Phi_{\xi\xi} - c\Phi_{\xi} + f(\Phi) = 0.$$

Actually for the nonlinearity (2), we have

$$\Phi(\xi) = \frac{1}{2} \left(1 - \tanh \frac{\xi}{2\sqrt{2}} \right), \quad c = \sqrt{2} \left(\frac{1}{2} - a \right).$$

As the appropriate singular limit, the interface between two states 1 and 0 becomes sharp and we can get the interface equation (see e.g. [4]):

$$V = H + k \quad (3)$$

where V is a normal velocity, H is the curvature, and k is a given constant. This equation is also observed in the filamentary vortex of the Ginzburg–Landau equation confined in a plane [3] and the BZ reaction [6].

The typical solutions of (3) are the circles and lines. In the case $k \neq 0$, unfortunately, some interfaces may possess some self-intersection points eventually, even if the initial interface has none. If the interface is represented by the graph $y = v(x, t)$, the equation (3) is reduced to

$$v_t = \frac{v_{xx}}{1 + v_x^2} + k\sqrt{1 + v_x^2} \quad x \in \mathbf{R}, t > 0, \quad (4)$$

Deckelnick et al in [3] proved the existence of the traveling curved front and studied the stability of the front under some restricted assumptions for u_0 . The authors relaxed the assumption for the initial data and classified all the traveling fronts in [7, 8]. They proved the following (see [7, Proposition 1.1, Theorem 1.2]).

Theorem 1 *Any traveling front of (3) with velocity $v(0, c)$ is one of the three, after appropriate translations,*

- (i) *lines $y = m_*x$, and $y = -m_*x$*
- (ii) *a traveling curved front $\Gamma_c(t)$ which possesses two asymptotes $y = \pm m_*x$,*
- (iii) *stationary circles with radius $1/|k|$ only in the case $c = 0$,*

where $m_* := \sqrt{c^2 - k^2}/k$. Moreover the explicit form of the traveling curved front $\Gamma_c(t) = \{y = \varphi(x) + ct\}$ with speed $c(\geq k)$ is given in

$$x(\theta; c) := \frac{\theta}{c} + \frac{k}{c\sqrt{c^2 - k^2}} \log \left| \frac{1 + \sqrt{\frac{c+k}{c-k}} \tan \frac{\theta}{2}}{1 - \sqrt{\frac{c+k}{c-k}} \tan \frac{\theta}{2}} \right|,$$

$$y(\theta; c) := -\frac{1}{c} \log \left(\frac{c \cos \theta - k}{c - k} \right),$$

for $\theta \in (-\arctan m_*, \arctan m_*)$.

The traveling curved front $\Gamma_c(t)$ is “V-shaped”, which connects two asymptotes. The existence of this traveling front is also reported in [2, 3] and in a liquid BZ reaction [6].

The asymptotic stability of the curved traveling front in (4) is discussed in [3, 8]. It is proved that the traveling curved front is asymptotically stable, if the initial perturbation is restricted to

$$BC_0^1 := \{v \in C^1(\mathbf{R}) \mid \sup_{-\infty < x < \infty} (|v(x)| + |v_x(x)|) < \infty, \lim_{|x| \rightarrow \infty} v(x) = 0\}.$$

and that if you take the perturbation space

$$BC^1 := \{v \in C^1(\mathbf{R}) \mid \sup_{-\infty < x < \infty} (|v(x)| + |v_x(x)|) < \infty\},$$

instead of BC_0^1 , the traveling curved front is not asymptotically stable (see [8, Theorem 1.1 and Theorem 4.1]).

By the above observation, we can expect that a “V-shaped” traveling wave solution of (1) exists. Actually we have the following theorem.

Theorem 2 *There exists a traveling wave solution $u(x, y, t) = U(x, y - ct)$ of (1) such that*

$$\lim_{R \rightarrow \infty} \sup_{(x,y) \in D_R} \left| U(x, y) - \Phi \left(\frac{k}{c}(y - m_*|x|) \right) \right| = 0$$

where

$$D_R := \{(x, y) \mid x^2 + y^2 \geq R^2\}.$$

Bonnet and Hamel [1] showed the existence of the “V-shaped” traveling wave solutions, if f is of the “ignition temperature” type (mono-stable type) instead of (2). Hamel and Monneau [5] shows the uniqueness of the traveling front of the corresponding singular limit problem.

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