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A New Type of Self-Organization Associated with Chaotic Dynamics in Neural Networks

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Abstract

A new type of self-organized dynamics is presented, in relation with chaos in neural networks. One is chaotic itinerancy and the other is chaos-driven contraction dynamics. The former is addressed as a universal behavior in high-dimensional dynamical systems. In particular, it can be viewed as one possible form of memory dynamics in brain. The latter gives rise to singular-continuous nowhere-differentiable attractors. These dynamics can be related to each other in the context of dimentionality and of chaotic information processings. Possible roles of these complex dynamics in brain are also discussed.

1 Introduction

A mechanism of a temporal representation of input information has been highlighted in relation with chaotic neurodynamics as well as a so called temporal code. Irrespective of the coding scheme at a single neuron's level, the dynamics of neural networks can afford the temporal representation of external or internal inputs. In this respect, we proposed the notion of Hermeneutics in brain [1, 14], based on the chaotic dynamics and chaotic itinerancy.

Freeman's findings of chaotic behaviors in entire olfactory bulb's activity showed the significance of chaos and chaotic itinerancy in animal's motivated behaviors [2, 3, 4, 5, 6]. Possible bifurcations and complex behaviors in the olfactory bulb have been studied by Érdi *et al* [7, 8] with semi-biological neural networks. Based on the Freeman's observations, the feedback pathway from olfactory cortex to the bulb is prerequisite for the emergence of chaos in the bulb. The interactions at least between the bulb and the cortex is thus necessary to simulate the observed dynamics. Liljenström proposed an appropiate model for this aim [9]. Although we will not refer in the present paper to these issues in detail, owing to limited space, the olfactory systems are apparently an appropriate system to investigate roles of chaotic dynamics as well as the thalamo-cortical complex [10, 11, 12].

In this paper, we present two kinds of self-organized dynamics in neural systems. One is chaotic itinerancy, and the other a chaos-driven contraction dynamics giving rise to singular-continuous nowhere-differentiable attractors.

2 Chaotic Itinerancy

2.1 Chaotic Itinerancy in Neural Systems

We proposed with unanimous cooperation [13, 14, 15, 16, 17] the notion of chaotic itinerancy as a new universality class in high-dimensional dynamical systems. Here, "high-dimension" means that the number of interacting variables is at least more than three. In far-from-equilibrium systems, there exist several (macroscopic) modes excited, which could be frozen as a dissipative structure [18] or a slaving mode [19]. Such modes could, however, be inactive when chaotic components begin to interact weakly with other modes, thereby some other modes are excited. If such chaotic components are not so strong as to permit all other modes to cooperate with such chaotic modes, excited modes can change successively, depending on the system's past states. Thus, chaotic itinerancy can be addressed as an appropriate gadget to understand the mechanism of complex behaviors in high-dimensional dynamical systems such as hydrodynamic turbulence, chemical turbulence, many-body problems, immune systems, ecosystems, social systems, *etc* as well as brain. For more detailed discussions on chaotic itinerancy in the context of "complex systems research", Kaneko and Tsuda [20] is recommended.

We discuss here chaotic itinerancy in relation with dynamic behaviors in neural systems which can be more complex than usual low-dimensional chaos. Our main concern here is a dynamic process of memory. We proposed [21] a model neural network to investigate the dynamic retrieval process of memory, and found the chaotic retrieval. Each memory is represented by fixed point in phase space due to Hebbian learning algorithm, but the retrieval process becomes dynamic if one introduces local inhibition, skew-product transformations, and a variable (a state-dependent) field. By further studies [22, 23, 24], the decrease of the number of connections per neuron, or the presence of refractoriness can also assure the chaotic transition among memory representations. Recently, a similar behavior has been reported in various systems (see for example, [25, 26]).

Let us describe the skew product transformations used in our model neural networks (see also [28]).

$$y_i(t+1) = f^p(x_i(t)) = \begin{cases} \tanh(\gamma x_i(t)) & \text{with probability } p, \\ y_i(t) & \text{with probability } 1-p. \end{cases} \tag{1}$$

Here, y_i and x_i denote the activity and the membrane potential of i-th neuron, respectively, and t a discrete time. If one adopts some chaotic model for the above stochastic renewal process, the dynamics can be transformed by the skew product defined on the space $T = \Gamma \times \Sigma$, where $\Gamma = [-1,1]^n$ denotes the state space of the neurodynamics and $\Sigma = [0,1)$ the space which the probability measure is defined on. Let $y \in \Gamma$ and $\sigma \in \Sigma$. If one adopts, for instance, a Bernoulli model for the stochastic renewal process, the following skew product transformations hold.

$$T(y(t), \sigma(t)) = \begin{cases} (\tanh(\gamma x_i(t)), \sigma(t)/p) & \text{if } 0 \le \sigma(t) < p, \\ (y(t), (\sigma(t)-p)/(1-p)) & \text{if } p \le \sigma(t) < 1. \end{cases} \tag{2}$$

Since a stochastic process is, in general, provided by some chaotic dynamics, the stochastic renewal of neurodynamics can be viewed as a "chaos-driven contraction dynamics".

In usual static neural networks, only a single association of memory is permitted, thereby the learning and the retrieval phases must be separated. On the contrary, a concurrent process of the (Hebbian) learning and the chaotic retrieval can be performed with the above skew product transformations, and consequently it was found that chaotic itinerancy can subserve the enhancement of memory capacity of the neural networks, and dynamic novelty filter, too [27, 28].

Information theory developed in a series of papers [29, 30, 31], where chaos was viewed as an information channel provides a potential performance of chaotic itinerancy. Suppose the network exhibiting chaotic itinerancy. If the network is further connected to other similar ones to form a larger scale of networks, then the large network accomplishes the correct transmission of information along the "channel" created in phase space by chaotic itinerancy. By these studies, a hypothesis was posed that chaotic itinerancy may work even in the interpretation process of conscious mind (chaotic Hermeneutics [14]), since the dynamic memory process must be the basis of mind.

2.2 Basic Features of Chaotic Itinerancy

Let us discuss basic features of chaotic itinerancy. Suppose the coexistence of attractors which can be fixed points, limit cycles, tori, or strange attractors. Furthermore, a current state is assumed to be some one attractor. By some instability, such an attractor becomes unstable, but other attractors could keep their stability since the instability can be remained local. Hence, the orbits move along the unstable manifold, and approach to another attractor. If the instability occurs only for the current attractor, a motion from one to another attractor can be expected. This instability can be realized by saddle-node bifurcations. This is, however, not an actual case, since the transition itself must be chaotic in chaotic itinerancy. A slight modification of the instability is necessary to realize chaotic transitions. A possible modification is made such that homoclinic orbits emerge between the destabilized attractor and one of saddles intervening the initial attractors.

Our observations concerning dimensionality of chaotic itinerancy may permit to relate this neuro-chaotic itinerancy to singular-continuous nowhere-differentiable attractors. In chaotic itinerancy, almost Lyapunov exponents condense into the neighborhood of the zero value. The characteristics of the spectrum $(\lambda_1, \lambda_2, \cdots, \lambda_n)$ is as follows:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_j \geq 0 \geq \lambda_{j+1} \geq \cdots \geq \lambda_n, \tag{3}$$

$$\lambda_1, \lambda_2, \cdots, \lambda_i = O(1), \tag{4}$$

$$\lambda_{i+1}, \lambda_{i+2}, \cdots, \lambda_j = o(1), \tag{5}$$

$$|\lambda_{j+1}|, |\lambda_{j+2}|, \cdots, |\lambda_{n-1}| = o(1), \tag{6}$$

$$|\lambda_n| \gg |\lambda_k| \ \ (k = 1, \cdots, n-1). \tag{7}$$

These relations can give a large Lyapunov dimension; even the case that the space dimension only exceeds the Lyapunov dimension by less than unity. On the other hand, if the number of negative Lyapunov exponents is large enough, the topological dimension remains low. Thus, the attractor's dimension may exceed the topological dimension by more than unity, which is unexpected in low-dimensional chaos.

3 Singular-Continuous Nowhere-Differentiable Attractors

3.1 Continuous but Nowhere-Differentiable Functions

Continuous but nowhere-differentiable functions have been widely investigated [32, 33, 34]. In particular, Yamaguti [35], and Yamaguti and Hata [36] elucidated a beautiful relation between fractals [37] and chaos. Recently, perhaps the simplest example of this class of functions has been proposed by Katsuura [38]. Katsuura's function (named by Rössler) is obtained from the graph of asymptotic form of unit square transformed by a specific contraction mapping. The following Katsuura map defines a contraction of any subset of the unit squre $X = [0,1] \times [0,1]$ when an appropriate metric is introduced.

Let us define contraction mappings $w_i{:}X \to X$ $(i = 1, 2, 3)$.

$$w_1(x,y) = (\frac{x}{3}, \frac{2y}{3}), \tag{8}$$

$$w_2(x,y) = (\frac{2-x}{3}, \frac{1+y}{3}), \tag{9}$$

$$w_3(x,y) = (\frac{2+x}{3}, \frac{1+2y}{3}) \tag{10}$$

Each of these mappings has a fixed point, (0,0), $(\frac{1}{2},\frac{1}{2})$, and (1,1), respectively. Here, let $F(X)$ be a collection of all nonempty closed subsets of X. For every $A \in F(X)$,

$$w(A) := w_1(A) \cup w_2(A) \cup w_3(A). \tag{11}$$

For every A, $B \in F(X)$, the Hausdorff metric is defined.

$$d_H(A, B) := \inf\{\epsilon > 0 | N_\epsilon(A) \supset B \cap N_\epsilon(B) \supset A\}, \tag{12}$$

where $N_\epsilon(\bullet)$ is an ϵ-neighborhood of $\bullet$. Then, w is a contraction mapping on $F(X)$ under this metric.

Let $D_0 = (x,x) \in X$ and $D_n = w(D_{n-1})$, then D_n is a graph of continuous function f_n : $[0,1] \to [0,1]$. In particular, a continuous function f_∞ : $[0,1] \to [0,1]$ exists. The mapping w has a unique fixed point D^* in $F(X)$, since w is a contraction mapping on $F(X)$. Therefore, for any $A \in F(X)$, a series $\{w^n(A)\}$ converges to D^* with respect to the metric d_H. Then, D^* is a graph of f_∞, namely D_n converges to D^*. Katsuura proved [38] that the function f_∞ is nowhere-differentiable on [0,1].

3.2 Singular-Continuous Nowhere-Differentiable Functions

Rössler *et al* [39] constructed a singular-continuous but nowhere-differentiable function, based on Katsuura's function. A singular continuity is generally defined on Cantor set. A typical example of singular-continuous nowhere-differentiable function has been introduced by Rössler *et al*, which is given by the following equations:

$$c_1(x,y) = (\frac{x}{5}, \frac{2y}{3}), \tag{13}$$

$$c_2(x,y) = (\frac{3-x}{5}, \frac{1+y}{3}), \tag{14}$$

$$c_3(x,y) = (\frac{4+x}{5}, \frac{1+2y}{3}). \tag{15}$$

For any $A \in F(X)$, define

$$c(A) := c_1(A) \cup c_2(A) \cup c_3(A). \tag{16}$$

This gives a contraction mapping under the Hausdorff metric (12), hence S_n recursively defined by $S_n = c(S_{n-1})$ for given $S_0 = (x,x)$ is the graph of the function g_n on [0,1]. In particular, the graph S_∞ of g_∞ consists of isolated points, and g_∞ is singular-continuous nowhere-differentiable.

3.3 Dimensionality

Let us estimate, according to Rössler [39], the dimensions of S_∞ derived from the above Rössler's model . For the first time, it is obvious that the topological dimension $dim_t(S_\infty)$ $= 0$. On the other hand, the Hausdorff dimension $dim_h(S_\infty)$ is simply estimated by using the self-similarity that $dim_h(S_\infty) = \frac{log\frac{25}{3}}{log5} = 1.317\cdots$. Namely,

$$dim_h(S_\infty) - dim_t(S_\infty) > 1.0. \tag{17}$$

On the other hand, the dimensions of the graph of Katsuura's function f_∞ which is 'simply' nowhere-differentiable are $dim_t(D_\infty) = 1$ and $dim_h(D_\infty) = \frac{log5}{log3} = 1.4649\cdots$, hence

$$dim_h(D_\infty) - dim_t(D_\infty) < 1.0. \tag{18}$$

The above two models, (8) $\sim$ (10) and (13) $\sim$ (15), are not dynamical systems but simply contraction mappings. Our interest here is an attractor of differentiable dynamical systems, which can be represented by nowhere-differentiable or singular-continuous nowhere-differentiable functions. An attractor represented by the Weierstrass function was discussed by Moser [40] in the context of structural stability. Recently, Kaneko has found a similar model exhibiting a fractal torus [41]. A singular-continuous nowhere-differentiable attractor was first found by Yorke et al [42, 43], which was named strange nonchaotic attractors. Actually, Rössler *et al* [39] have proposed the singular-continuous nowhere-differentiable function disscussed above as a mechanism of strange nonchaotic attractors, and proposed [44] another models of this class, where the name "superfat attractors" was given. According to Rössler, the eqn.(17) should generally hold for singular-continuous nowhere-differentiable attractors. This means that the Rössler model, (13) $\sim$ (15), provides a ground for the Kaplan-Yorke conjecture [45] that a dimension of chaotic attractors can exceed its topological dimension by more than unity. This can happen by the presence of negative Lyapunov exponents whose absolute values are smaller than positive exponents, as seen in neuro-chaotic itinerancy.

3.4 Singular-Continuous Nowhere-Differentiable Attractors in Axiom A

It is valuable to see the structure and characteristics of Axiom A systems showing singular-continuous nowhere-differentiable attractors. Studying these issues will provide a hint to construct a neural system whose dynamics can be performed on Cantor sets, though neural systems with "Cantor code" are, in general, not an Axiom A system. Such an Axiom A system has been given by Rössler *et al* [46].

$$x_{n+1} = 9x_n \pmod{2\pi}, \tag{19}$$
$$y_{n+1} = 0.3y_n - 0.7\cos(9x_n), \tag{20}$$
$$z_{n+1} = 0.3z_n + 0.7\sin(9x_n), \tag{21}$$
$$w_{n+1} = 0.3w_n - 0.7\sin(2\times 9x_n). \tag{22}$$

This model was proposed as a Poincaré map of a flow of supposed 5-dimensional vector field. This model is actually Axiom A, for periodic orbits are dense on the nonwandering set and the nonwandering set is hyperbolic. This is one of the typical chaos-driven contraction mappings.

Lyapunov spectrum of this system is (2.197, -1.204, -1.202, -1.204), thus the Lyapunov dimension, which is equivalent to the Hausdorff dimension in this case, is $dim_\lambda = 2.825$. On the other hand, the topological dimension is $dim_t = 1.0$, since all of three diections than x-coordinate are responsible for contraction, where only a set of isolated points is eventually formed. Hence the same inequality as in (17) holds. This relation of dimensions means that the attractor is sparsely distributed in phase space. This may provide a merit for information processings with Cantor coding[47].

Let us look the attractor on the cross-section which was chosen not to see the first variable creating chaos. Then, we have a three-dimensional Cantor set shown in Fig.1. There seemingly exists the two-dimensional singular-continuous nowhere-differentiable function, though it is not proved here. In the projection to $y-z$ plane the graph becomes nowhere-differentiable, as shown in Fig.2 [46]

–Fig.1–

–Fig.2–

4 Nowhere-Differentiable Attractors in Neural Systems

4.1 A Basic Model Neural Network

In this section, we discuss the possibility of nowhere-differentiable attractors in brain dynamics. For this aim, we propose a model neural network of small scale. Our model consists of only three neurons, two of which are called here a "static" neuron and the other called a "dynamic" neuron. A static neuron's activity eventually relaxes to stationary firings which is represented by a fixed point in phase space, whereas a dynamic neuron fires chaotically, thus it is represented by chaotic attractor. It would be convenient to use a chaotic neuron model introduced by Aihara [48] in order to model both static and dynamic neurons in terms of a family of chaotic map.

We adopt here an mutually interacting static neurons: one is excitatory and the other inhibitory. A dynamic neuron which is assumed to be excitatory drives these two static neurons. Interacting static neurons form a contraction mapping. Hence the present system belongs to the class of chaos-driven contraction mapping. The model equations are as follows.

$$
\begin{aligned}
x_{n+1} &= f_1(-\sum_{r=0}^{n} b_1^r x_{n-r} + I), && (23)\\
y_{n+1} &= f_2(-\sum_{r=0}^{n} b_2^r y_{n-r} + c_{zy} z_n + c_{xy} x_n), && (24)\\
z_{n+1} &= f_2(-\sum_{r=0}^{n} b_3^r z_{n-r} + c_{yz} y_n + c_{xz} x_n), && (25)
\end{aligned}
$$

where $0 < b_1, b_2, b_3 < 1$, c_{uv} are a synaptic strength from neuron u to v and $c_{zy} < 0$, $c_{yz}, c_{xy}, c_{xz} > 0$. The functions f_i $(i = 1, 2)$ is a sigmoid function described as follows.

$$
f_i(x) = \frac{1}{1 + e^{-\gamma_i x}}, \quad (i = 1, 2). \qquad (26)
$$

In the simulation, $\gamma_1 = 70$ and $\gamma_2 = 1.5$ were given in order to assure the chaos-driven contraction mapping.

This system looks dependent on a whole history of states, but it can be reduced to the dynamical system of six-dimension when the terms of history-dependent internal states are replaced by each one variable. Let $X_n \equiv -\sum_{r=0}^{n} b_1^r x_{n-r} + I$, $Y_n \equiv -\sum_{r=0}^{n} b_2^r y_{n-r} + c_{zy} z_n + c_{xy} x_n$, and $Z_n \equiv -\sum_{r=0}^{n} b_3^r z_{n-r} + c_{yz} y_n + c_{xz} x_n$. Reduced model equations are as follows.

$$
\begin{aligned}
X_{n+1} &= b_1 X_n - f_1(X_n) + \alpha, && (27)\\
Y_{n+1} &= b_2 Y_n - f_2(Y_n) + c_{zy}(f_2(Z_n) - b_3 f_2(Z'_n)) + c_{xy}(f_1(X_n) - b_1 f_1(X'_n)), && (28)\\
Z_{n+1} &= b_3 Z_n - f_2(Z_n) + c_{yz}(f_2(Y_n) - b_2 f_2(Y'_n)) + c_{xz}(f_1(X_n) - b_1 f_1(X'_n)), && (29)\\
X'_{n+1} &= X_n, && (30)\\
Y'_{n+1} &= Y_n, && (31)\\
Z'_{n+1} &= Z_n. && (32)
\end{aligned}
$$

The last three variables, X', Y', and Z' simply give a linear effect, in particular, $X'_n - X_n$ plane simply defines just one-dimensional map $X_{n+1} = F(X_n)$, where $F(X_n) = b_1 X_n - f_1(X_n) + \alpha$. Figure 3 shows a map of static neuron ((a)) and dynamic one ((b)) when each is isolated.

–Fig.3 (a), (b)–

The parameters b_1, b_2, and b_3 provide a measure of volume contraction. In the simulations, we assume that $b \equiv b_1 = b_2 = b_3$. We will see, in the following subsection, the effect of this parameter values to the dimensionality.

4.2 Neurodynamics on Cantor Set

Let us see the dynamics on the cross-section, for instance, $X = 0.05$ (± 0.0005), which provides two values for variable X', whereby two branches of Cantor set are obtained (Fig.4). In order to characterize the attractor seen on the cross-section, we calculated various quantities: invariant measure, Lyapunov spectrum, transition probability distribution, mutual information, approximated K-S entropy, and dimension. These quantities except for the Lyapunov spectrum and dimension were calculated with some partition, thus coarse-grained ones. We changed partition in several systematic ways, and concluded the presence of invariant characters.

–Fig.4–

The numerical fact that the invariant distribution and the transition probability distribution are almost equivalent derives the almost time-invariance of mutual information with a small value depending on the cross-section and its precision for determination. There are some probabilities that a few digits become different between the invariant distribution and the transition probability distribution. These digits contribute to a non-zero value of the mutual information.

There are two kinds of information processings on Cantor set. One is the localization of information on Cantor set, and the other the concurrent process of write-in ("learning") and read-out ("retrieval"). The information can be distributed on subsets of Cantor set, but information can never flow among any subsets. This is, however, different from zero-information flow seen in stationary distribution. Only the shared information in the present case is the information that "missing bits" possess, whose bits provide the difference between invariant measure and transition probability distribution.

On the other hand, the concurrent process emerges as a microscopic mechanism of the stationarity. In general, an expanding map provides a read-out process of information, whereas a contraction map provides a write-in process. In usual chaotic dynamics, an expanding and a contracting phase appears successively, associated with its dynamics. In the present case, these phases progress concurrently, because of chaos-driven contraction mapping. A simpler alternative to conceive this process would be a dissipative baker's transformation. The details of the computational results will appear elsewhere [49].

We also calculated the noise effect of such an attractor. At the small noise level, entropy, conditional entropy, and mutual information slightly decrease when increasing the noise level. Under the assumption of finite observation, entropy is producted only on the attractor restricted on the cross-section, namely the zero-entropy production in the place other than the attractor. Since the orbits can move out of the attractor due to noise, entropy can decrease by noise as long as the noise does not increase the admissible orbits on the attractor. The numerical calculations about this issue should, in particular, be done very much carefully. The details will be discussed elsewhere [49].

Increasing the value of b, the Lyapunov dimension almost monotonously increases except for the bifurcations to periodic orbits. Topological dimension is always unity which chaos as a "driver" is responsible for, if a unique value is asymptotically determined on each point of Cantor set, such as an asymptotic form of the Rössler's function, *i.e.*, a singular- continuous nowhere-differentiable function. This is true if a bounded function with arbitrarily high periodic components forces a linear function [44, 50]. In the present neural networks, a static neuron is modeled by almost linear map as shown in Fig4(a). A driving neuron is chaotic, whereby the presence of arbitrarily high periodicity is assured. Hence, it is highly plausible that a unique value is actually defined on each element of Cantor set. Thus, the neural attractor presented here can be represented by a singular-continuous nowhere-differentiable function of Rössler's type. According to the increase of the paramer value b, this function becomes what defined on higher-dimensional space. By this fact, the dimension gap becomes increasing. Together with the above noise effect, this implies that the Cantor coding can be robust for small noise.

5 Conclusion and Discussions

A new type of self-organized dynamics was addressed as a new universality class of high-dimensional dynamical systems. Several characteristics of chaotic itinerancy and singular-continuous nowhere-differentiable attractos were elucidated. In particular, the dimensionality implies that a side view of the former in some direction or on some cross-section may manifest the latter.

We investigated these dynamics by means of the model neural networks. Thus, it would be interesting to ask if they can occur in brain. The complex dynamics like chaotic itinerancy has actually been observed in animal olfactory systems, and interpreted as mechanisms of memory search, dynamic link of memory, input-driven pattern recognition and a self-control of motivation levels [4, 5, 6, 14, 25, 28]. Singular-continuous nowhere-differentiable attractors have not yet been found in brain. It would, however, be natural to expect it in brain's dynamic process of information, since it is conceivable also in brain that chaotic neural activities drive stationary ones in order to assure a concurrent process of write-in and read-out of input information.

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Figure Captions

Fig. 1 Attractor seen on the cross-section $x = 0.1 \pm 0.0001$ in the model (19) $\sim$ (22).

Fig. 2 Projection of the attractor shown in Fig. 1 to $y - z$ plane.

Fig. 3 The graph of chaotic neuron map adopted. (a) for a static neuron, and (b) for a dynamic one.

Fig. 4 Attractor seen on the cross-section $x = 0.05 \pm 0.0005$ in the model (27) $\sim$ (32).

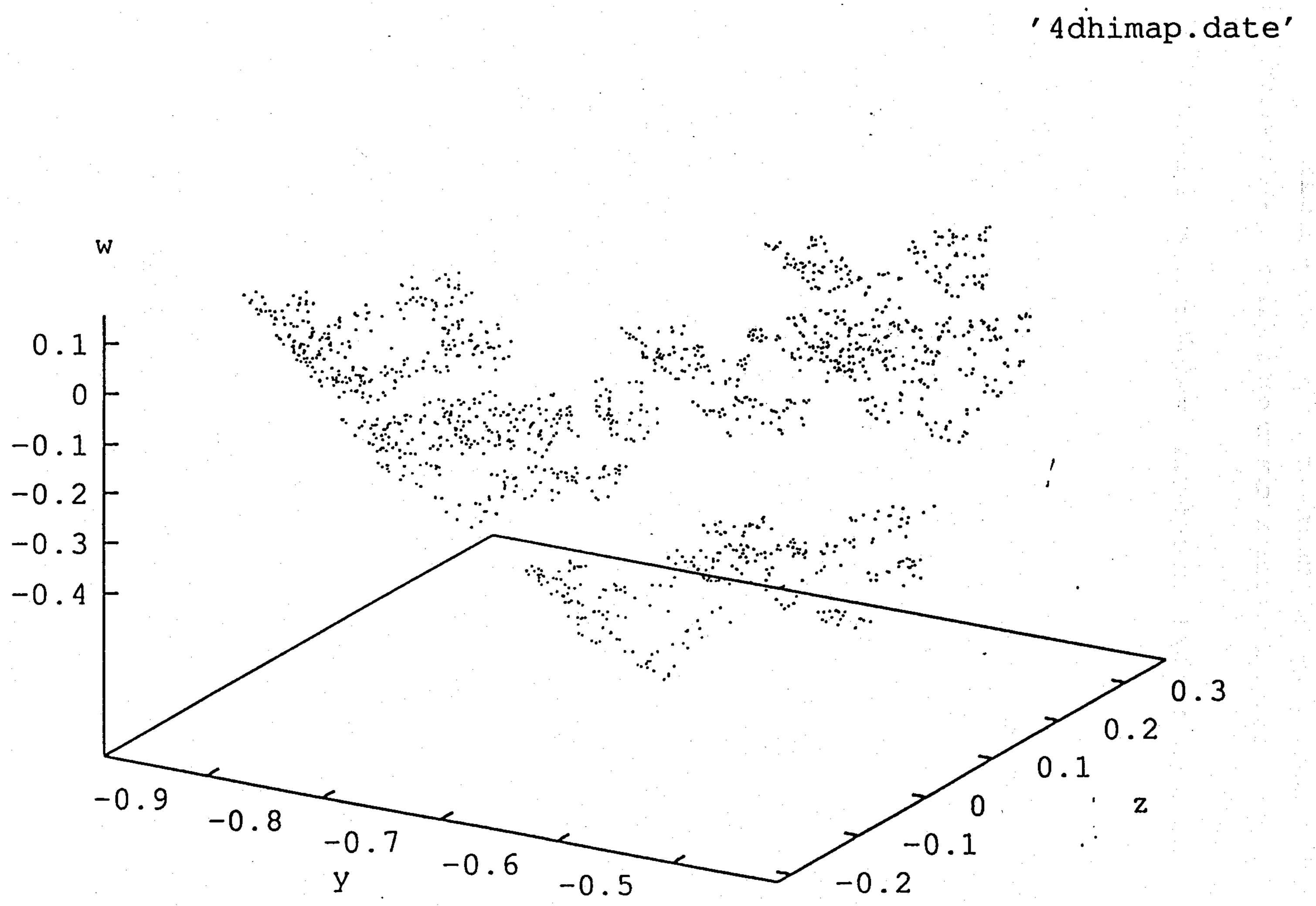

Fig. 1.
J. Tsuda

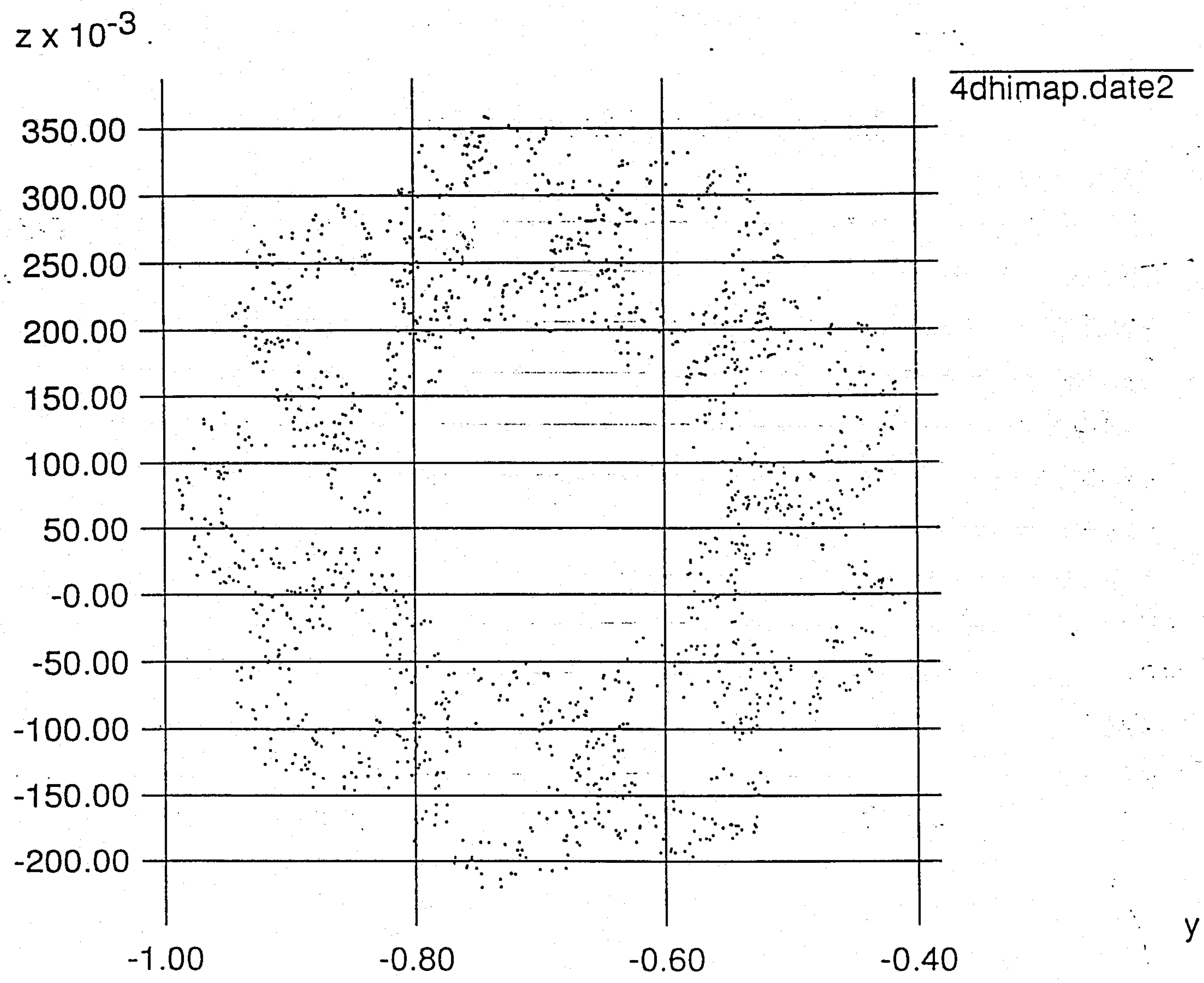
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Fig. 2.

Z. Tsuda

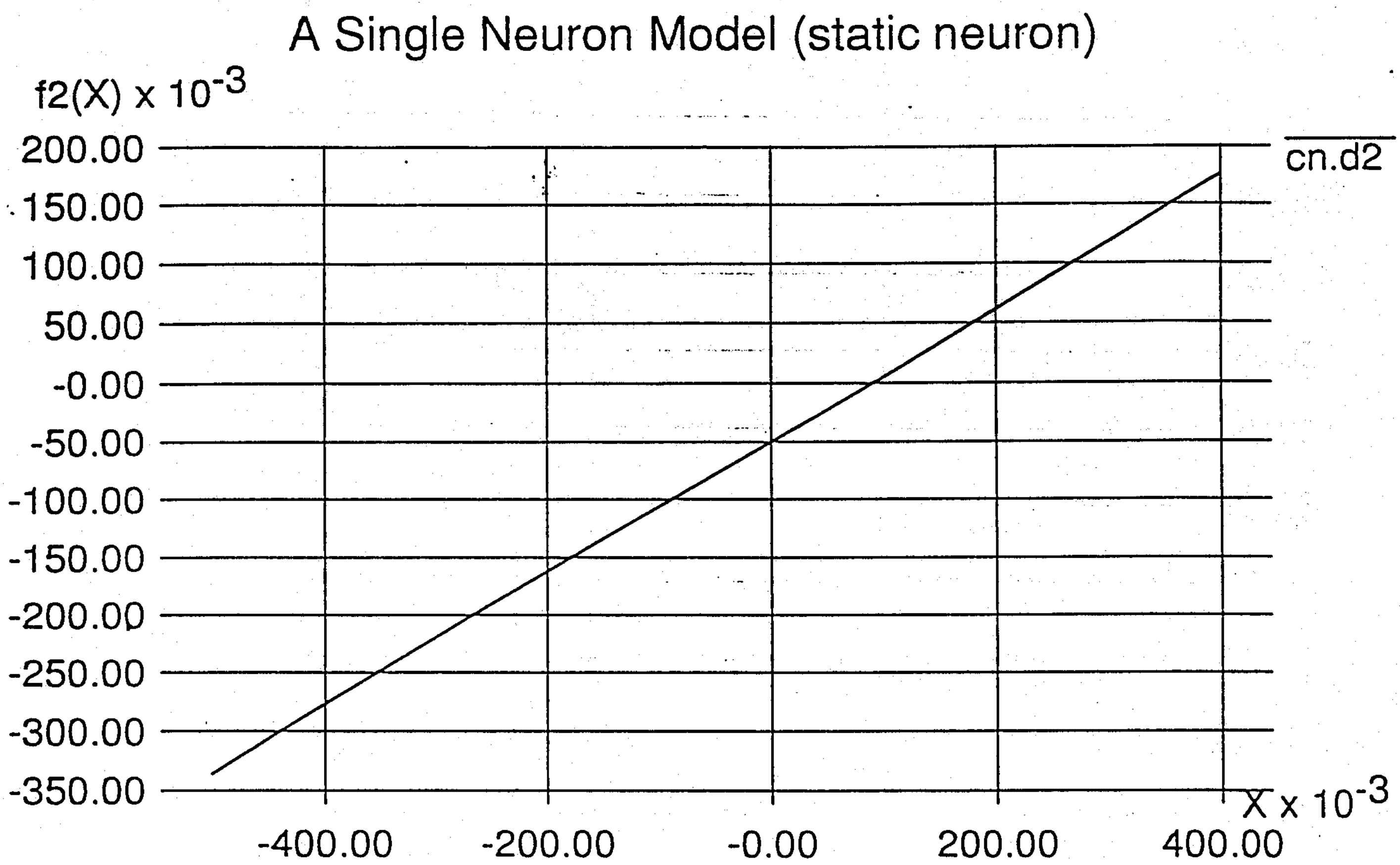

Fig. 3 (a)

T. Tsuda

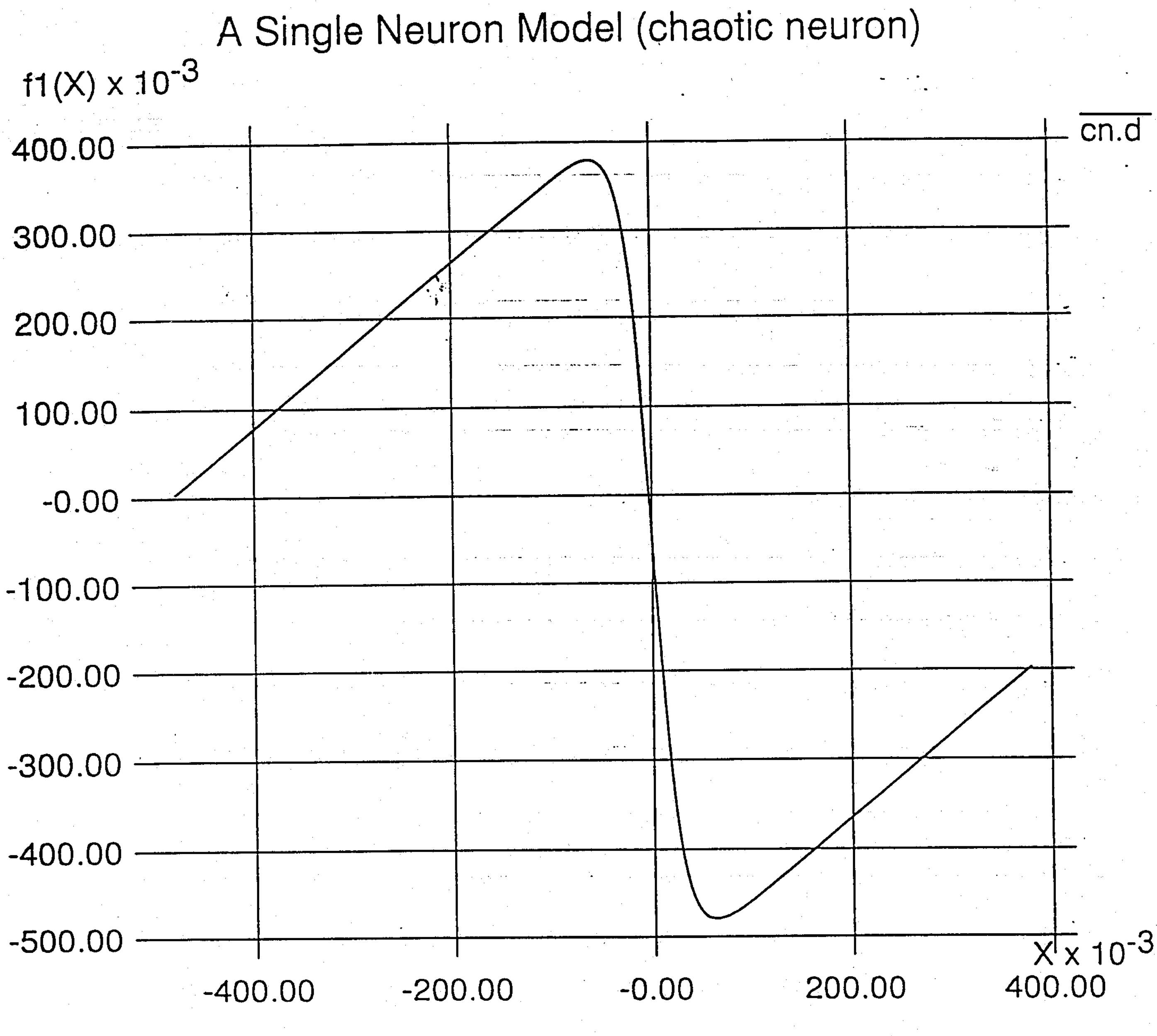

Fig 3. (b)

Z. Tsuda

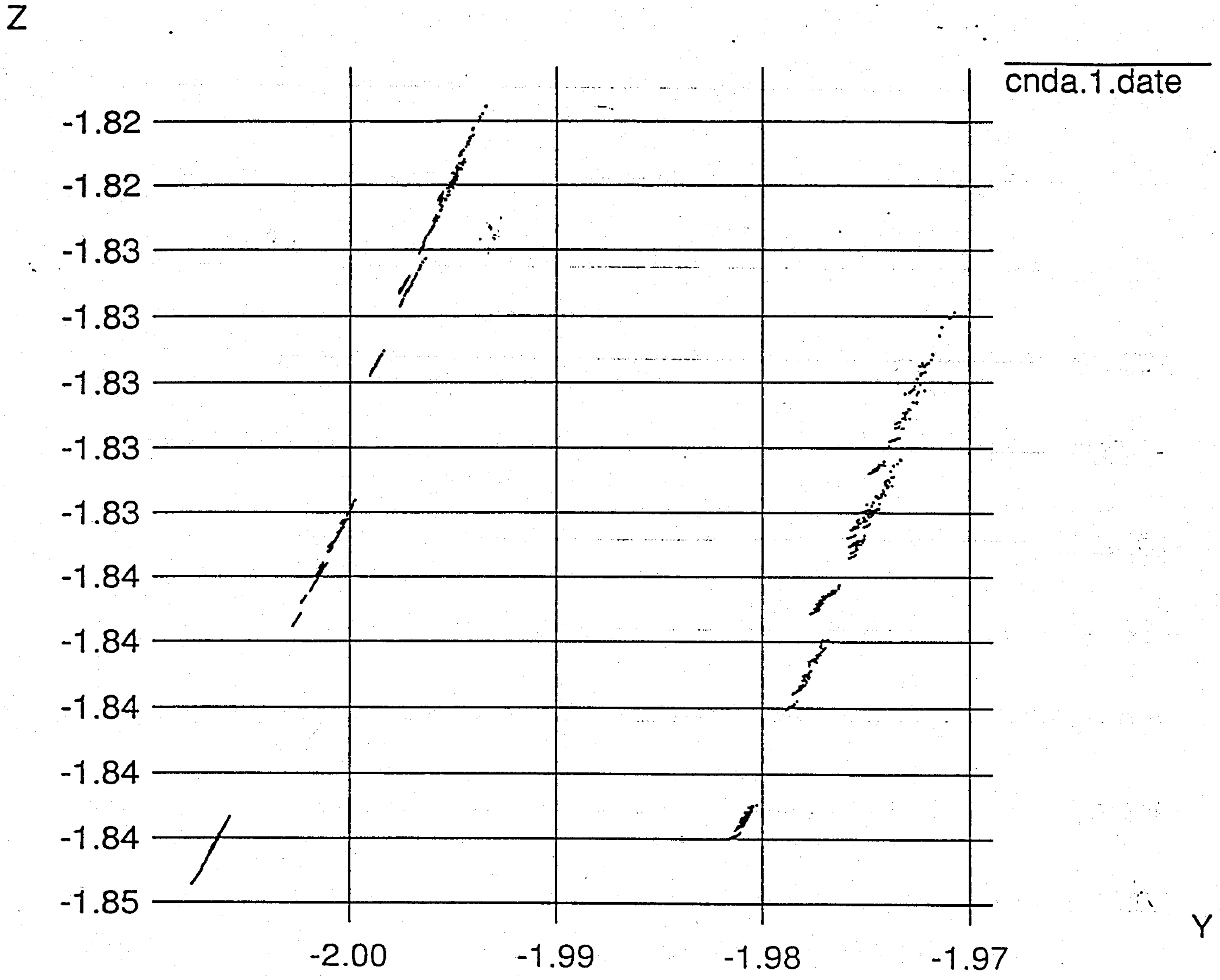
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Fig. 4.