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A STUDY ON A GENERALIZED
SWEEPING METHOD FOR CSG
MODELING

Yoshiyuki SHIROMA

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1. INTRODUCTION

1.1 Historical Summary of Geometric Modeling

The advent of computers and peripheral devices such as plotters and CRT displays has stimulated the development of computer aided geometric modeling schemes. The resulting geometric modeling systems have been adopted as the primary medium of geometric specifications for industrial designs and productions. Today, it is well recognized that three kinds of computer aided geometric modeling schemes have been developed through historical steps, namely, the **wireframe, surface, and solid modeling schemes**[1][2]. First we wish to summarize both the advantages and disadvantages of these schemes. In doing so, we shall attempt to show why solid modeling is now emerging as the best choice.

Though 3-D wireframe systems are common today, the first computer aided geometric modeling systems were 2-D wireframe systems. However, these were essentially used as computer aided drafting systems. Even now the 2-D wireframe drawing is still used as a common medium between designers to confirm the process planing. In general, wireframe data consist of straight lines, circular arcs, and parametrically defined curves such as Bezier curves[3] and B-spline curves[3], but these curves are segmented into straight lines in order that they make a simple data structure. This simple data structure provides the quick way of displaying object shapes, and such characteristic is particularly useful when

selecting projections and designing interactively. It is very difficult, however, to eliminate hidden lines only by using wireframe data, because wireframe's simple data structure cannot contain geometric/topological entities such as surfaces/faces which are required for the hidden line elimination. Moreover, since there is no way to ensure the completeness of object models in wireframe systems, nonsense object models may be designed. Thus it is recognized that the wireframe modeling is ambiguous. A typical example of the wireframe's ambiguity is illustrated in Fig.1-1. Nevertheless, the wireframe modeling is useful to show three-view drawings, and to show a sequence of dynamic motions where different projections must quickly be applied. To overcome the drawback and to enhance the advantage of the wireframe modeling, it is desirable that object models should first be represented more completely in the computer, and then wireframe data should be derived from the complete internal models.

Surface modeling schemes are excellent tools for the representation of complex shapes. Ships' hulls, aircraft, car bodies and turbine blades can be represented by these schemes. Mathematically, the surface modeling is based on interpolations and approximations by surface patches such as those known as Coons patches[4], Bezier surfaces[4], and B-spline surfaces[4]. Though each surface modeling scheme has particular interpolation function, all these schemes have one thing in common: they adopt the parametric representation of surfaces. The parametric representation provides designers with local operations to modify

the shape by means of changing the parameters. Thus interactive designs can easily be done in surface modeling systems. The greatest advantage of the surface modeling is the capability to represent complex shapes, as mentioned above. Nevertheless the surface modeling has disadvantages, too. The individual surfaces in surface modeling systems are completely described mathematically, and may represent a part of boundaries of a solid object. The resulting surface model, however, does not ensure a solid object because there is no geometric/topological information in the data of the individual surfaces to connect each other for completion of a solid model. Thus surface models are often incomplete like wireframe models. An incomplete model may be fatal if it is passed over to application programs for further analyses. A piecewise modeling process in the surface modeling system EUCLID[5] is illustrated in Fig.1-2.

In surface modeling systems, since the surface is defined by parametric equations, coordinates of any point in the parametric space can easily be returned to those of the corresponding point in Cartesian space. It is very difficult, however, to transform coordinates of any point reversely from Cartesian space to the parametric space, because it is very difficult to transform parametric equations into either implicit or explicit forms in Cartesian space. Nevertheless, the space point evaluation (whether the point is inside, outside, or on the boundary of an object) is performed by giving the coordinates of any point in Cartesian space. Hence, it is impossible to evaluate a space point by using the surface model. As the space point evaluation is often required in

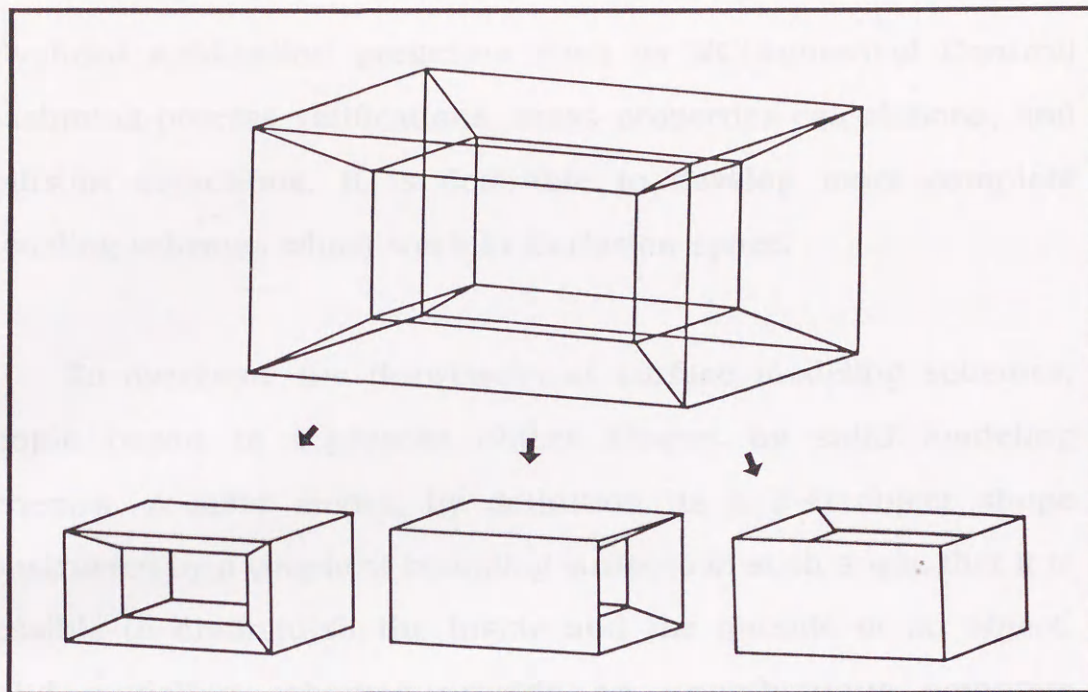


Fig.1-1 An example of wireframe's ambiguity

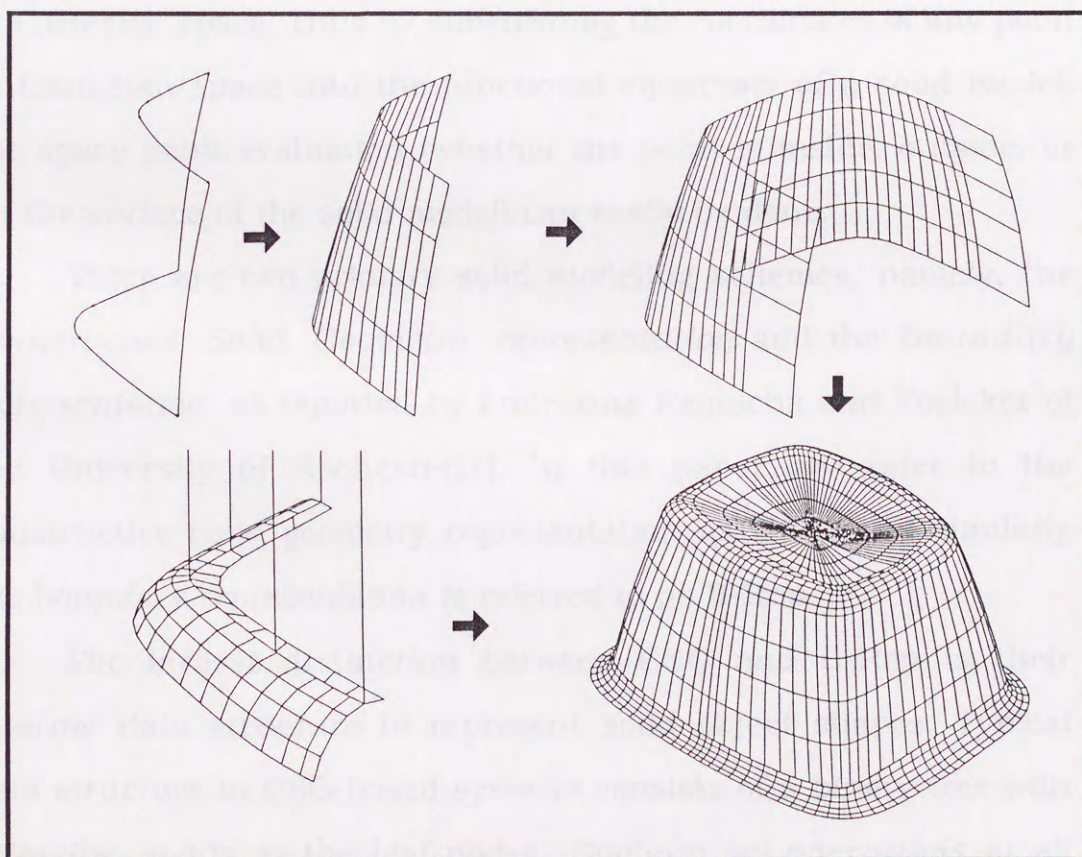


Fig.1-2 A piecewise modeling process in the surface modeling system EUCLID (Courtesy of Matra Datavision)

advanced application programs such as NC(Numerical Control) machining-process verifications, mass properties calculations, and collision detections, it is desirable to develop more complete modeling schemes which work in Cartesian space.

To overcome the drawbacks of surface modeling schemes, people began to represent object shapes by solid modeling schemes. A solid model, by definition, is a 3-D object shape constructed by a couple of bounding surfaces in such a way that it is possible to distinguish the inside and the outside of an object. Solid modeling schemes provide an unambiguous complete representation of real objects. In general, a solid model is defined in Cartesian space. Thus by substituting the coordinates of any point in Cartesian space into the functional equations of a solid model, the space point evaluation (whether the point is inside, outside, or on the surface of the solid model) can easily be done.

There are two primary solid modeling schemes, namely, the *Constructive Solid Geometry representation* and the *Boundary representation*, as reported by Professors Requicha and Voelcker of the University of Rochester[1]. In this paper, we refer to the constructive solid geometry representation as **CSG**, and similarly the boundary representation is referred to as **B-Rep**.

The largest distinction between CSG and B-Rep is their internal data structure to represent solid object shapes. Typical data structure in CSG-based systems consists of a binary tree with *primitive solids* at the leaf nodes, *Boolean set operations* at all internal nodes, and the root node which represents the complete

solid object shape as shown in Fig.1-3-(a). Typical primitive solids provided in CSG-based systems are *blocks* (i.e., *rectangular blocks*), *cylinders*, *spheres*, *cones*, *tori*, and *wedges*. Boolean set operations are *union*, *difference*, and *intersection*. Fig.1-4 shows a simple example of Boolean set operations applied to a block and a cylinder.

A solid object shape in B-Rep systems is represented by its bounding surfaces as illustrated in Fig.1-3-(b). The data structure of B-Rep consists of explicit geometric/topological entities such as *surfaces/faces*, *curves/edges*, and *points/vertices*. Three kinds of topological entities are connected each other by pointers to ensure topological completeness. The data structure of B-Rep also contains *face loops* of each face in order to distinguish the inside from the outside of the solid.

Some distinctive features of CSG and B-Rep are caused by their data structure. For example, CSG is concise and always valid in representing solid object shapes, but it is expensive to generate the wireframe drawing directly from CSG. On the other hand, B-Rep requires a large amount of data and great care to ensure valid correspondence to solid objects, but it is very easy to generate the wireframe drawing from B-Rep because B-Rep has bounding edges and vertices in its geometric/topological entities. Thus, it can be said that neither CSG nor B-Rep is uniformly better than the other. Further discussions on CSG and B-Rep are taken up in a later section of this paper.

The biggest disadvantage of solid modeling schemes is the lack of the capability to represent complex solid shapes. In this

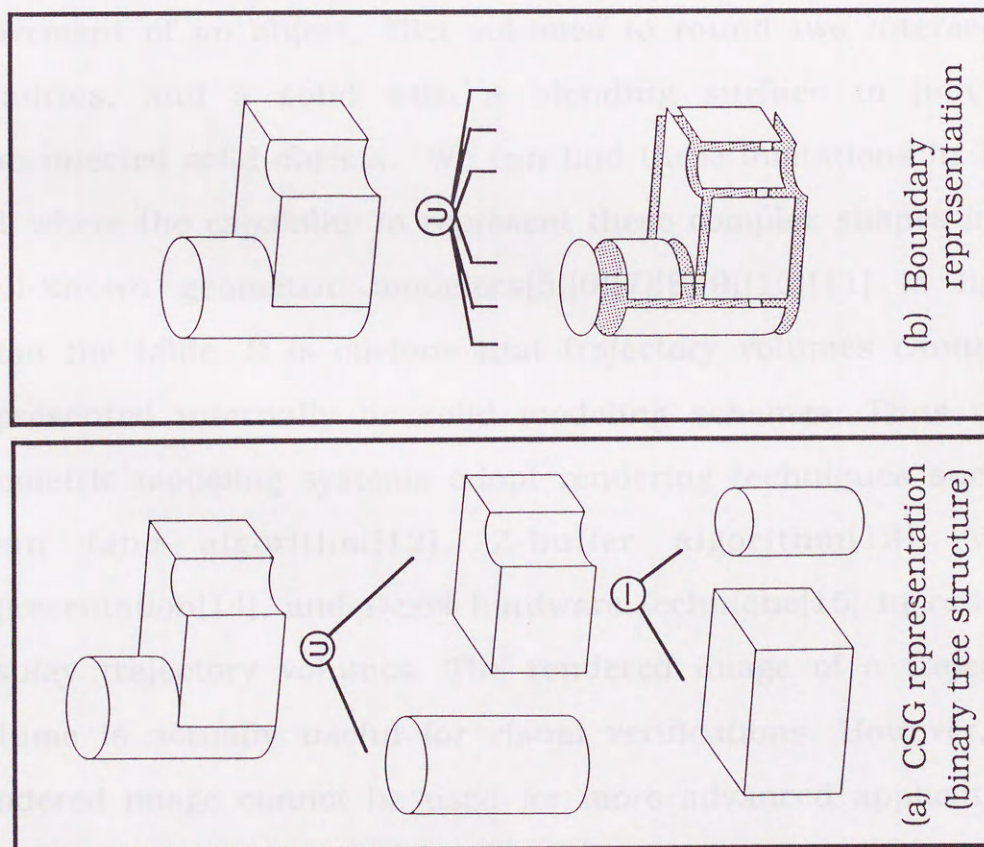


Fig.1-3 Solid modeling schemes

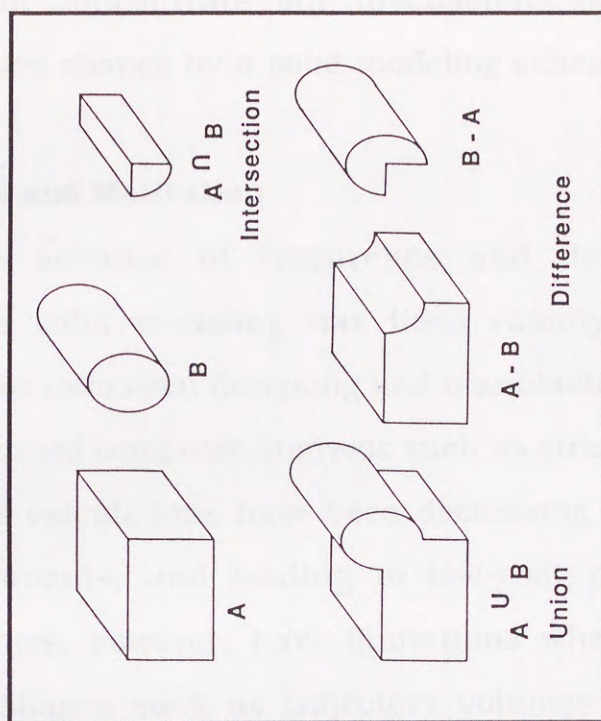


Fig.1-4 Boolean set operations

paper, we shall concentrate our discussions on problems to represent complex shapes by a solid modeling scheme.

1.2 Background and Motivation

With the advance of researches and developments in CAD/CAM, the solid modeling has been rapidly becoming an important tool for industrial designing and manufacturing. Actually, solid modeling based computer analyses such as stress analyses and mass properties calculations have been decreasing time-consumed physical experiments, and leading to low-cost products. Solid modeling schemes, however, have limitations when representing complex solid shapes such as trajectory volumes formed by the movement of an object, fillet volumes to round two intersecting quadrics, and a solid with a blending surface to join two disconnected solid objects. We can find these limitations in Table 1-1 where the capability to represent these complex shapes in the well-known geometric modelers[5][6][7][8][9][10][11] is listed. From the table, it is obvious that trajectory volumes cannot be represented internally by solid modeling schemes. Thus many geometric modeling systems adopt rendering techniques such as Scan Line algorithm[12], Z-buffer algorithm[13], Voxel representation[14], and Dixel hardware technique[15] in order to display trajectory volumes. The rendered image of a trajectory volume is actually useful for visual verifications. However, the rendered image cannot be used for more advanced applications, because mathematical definitions of the rendered image cannot be consistent with those of the moving object or its environments

	Surface based systems			Solid based systems		
	DUCT	EUCLID	CATIA	GEOMOD	ROMULUS	TIPS-1
Curved Surfaces	Bezier surfaces	Bezier surfaces	Surfaces defined by polynomial functions	Surfaces defined by NURBS	Cylindrical, spherical, conical, and toroidal surfaces	Cylinder, sphere, cone, and torus
Solid Definition Method					B-Rep	CSG
Sweeping Method	Generalized sweeps	Generalized sweeps	Generalized sweeps	Generalized sweeps (skinning operators)	Simple sweeps (lift and spin operators)	Simple sweeps (revolve operator)
Blending Surfaces	YES	YES	YES	YES	NO	NO
Filletts	YES	YES	YES	YES	NO	NO
Trajectory volumes	NO	NO	NO	NO	NO	NO

Table 1-1 Internal representations of surfaces and solids in the well-known geometric modelers, and their capability to represent the complex shapes

which are defined by a solid modeling scheme. Thus it is impossible to execute set operations between the rendered image of the trajectory volume and the internal solid model of its environments even though such operations are often required in NC machining-process verifications.

In addition, the blending surface and the fillet have been handled by surface modeling schemes as listed in Table 1-1. Thus surface modeling schemes are currently being incorporated into solid modeling systems. Surface modeling schemes, however, are not consistent with those of solid modeling. Actually, in such systems, the solid and the surface modeling schemes are used properly by estimating geometric complexity of an object shape.

From the viewpoint of the modeling consistency, these complex shapes should be represented by solid modeling schemes in solid modeling systems. This is the motivation to start this study, aiming at representing such complex shapes by a 3-D solid modeling scheme.

1.3 Objectives

1.3.1 The primary problems

This paper deals with the solid representation problems of complex shapes such as trajectory volumes formed by the movement of an object, fillet volumes to round two intersecting quadrics, and a solid with a blending surface to join two disconnected solid objects. These complex shapes are illustrated in Fig.1-5. Primary problems to be solved are as follows:

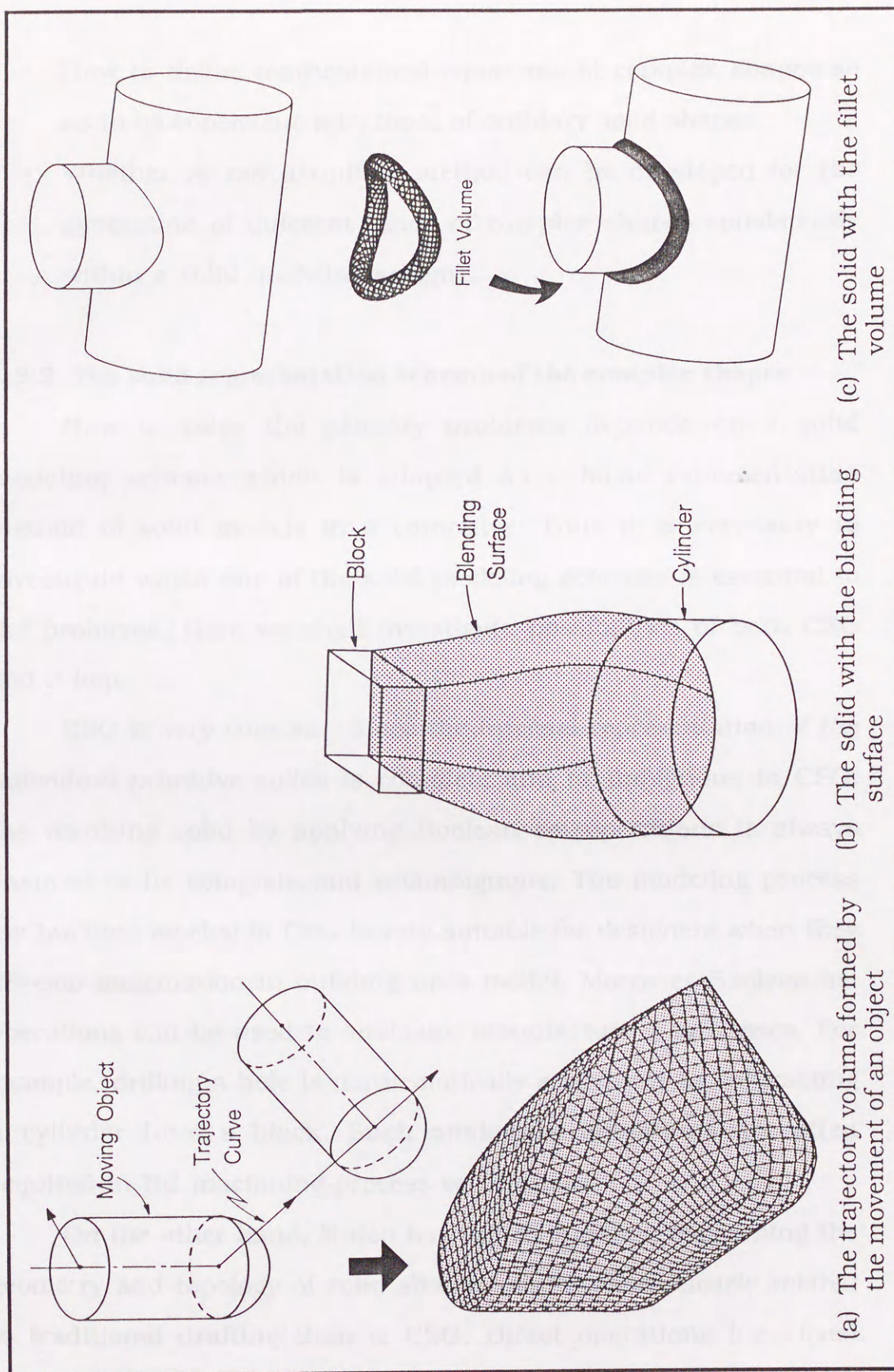


Fig.1-5 Examples of complex solid shapes

- How to define mathematical equations of complex shapes so as to be consistent with those of ordinary solid shapes.
- Whether or not a unified method can be developed for the generation of different kinds of complex shapes consistently within a solid modeling scheme.

1.3.2 The solid representation scheme of the complex shapes

How to solve the primary problems depends on a solid modeling scheme which is adopted as a basic representation method of solid models in a computer. Thus it is necessary to investigate which one of the solid modeling schemes is essential to the problems. Here we shall investigate possibilities of both CSG and B-Rep.

CSG is very concise. Since the internal representation of the individual primitive solids is complete and unambiguous in CSG, the resulting solid by applying Boolean set operations is always ensured to be complete and unambiguous. The modeling process (by building blocks) in CSG is very suitable for designers when they develop imagination in building up a model. Moreover Boolean set operations can be used to analogize manufacturing processes. For example, drilling a hole is mathematically equivalent to subtracting a cylinder from a block. Such analogous operations are often required in NC machining-process verifications.

On the other hand, B-Rep has full information concerning the geometry and topology of solid shapes, and is more closely related to traditional drafting than is CSG. Direct operations (i.e., local operations) on boundaries of solids are available in B-Rep. Thus it is

very easy to chamfer edges or to round corners by using the local operations, but the local operations may produce incomplete solids. Furthermore, the modeling process in B-Rep is often tedious like the surface modeling, and requires great care to make complete solids. Though practical conversion algorithms from CSG to B-Rep have been addressed[10][16][17][18][19], there is no reliable algorithm to convert reversely from B-Rep to CSG. From above considerations, we decided to adopt CSG as a unified internal representation method of solid object shapes when solving the primary problems.

1.3.3 Proposal of a generalized sweeping method for the CSG modeling

In order to solve the primary problems consistently, the different kinds of complex shapes should be handled uniformly. In this paper, these complex shapes are considered uniformly to be solid models of loci formed by the movement of an object. So first, we shall clarify the effect of moving of an object.

We are familiar with the simple motions such as *translations* (i.e., linear-translations) and *rotations*. Let us investigate the translational and the rotational motions. In generating solid shapes, *the simple sweeping method*[20][21] is well known, which is available both in CSG and B-Rep. The method allows both *extrusion* (i.e., lift) and *rotation* (i.e., revolution or spin) as illustrated in Fig.1-6. The extrusive sweep uses a straight line for its sweeping path while the rotational sweep uses a circular arc. For example, a solid of revolution is generated by revolving a plane

curve about a straight line (i.e., axis of revolution) in its plane. Let us define now a circular arc on a plane perpendicular to the axis of revolution. Then we can find that revolving a plane curve about a straight line is equivalent to sweeping the plane curve along the circular arc. From the viewpoint of geometric modeling of a locus formed by the movement of an object, the extrusive/rotational sweep is equivalent to the translational/rotational motion. Therefore, *motion is considered to be sweep* in this paper.

Only the simple sweeping method is used restrictively in the current CSG-based systems as listed in Table 1-1. Moreover, in many CSG-based systems, the simple sweeping method is only used as an input medium, and the input data are translated to the canonical CSG data[21], because its capability is considered to be too small to use as primary representation. It is necessary to extend and generalize the simple sweeping method so that it can be used for more advanced applications. Therefore, a generalized sweeping method for the CSG modeling is proposed in this paper, and is applied to generate the solid model of the locus formed by the movement of an object. The basic idea of the method is as follows: a 2-D contour (i.e., a closed curve) is swept in 3-D space along a path(i.e., a 3-D curve) to generate a solid shape as illustrated in Fig. 1-7.

Objectives of this paper are to propose a generalized sweeping method for generation of the complex shapes within CSG scheme, and to clarify mathematical fundamentals with respect to the objective problems, and to show basic examples performed

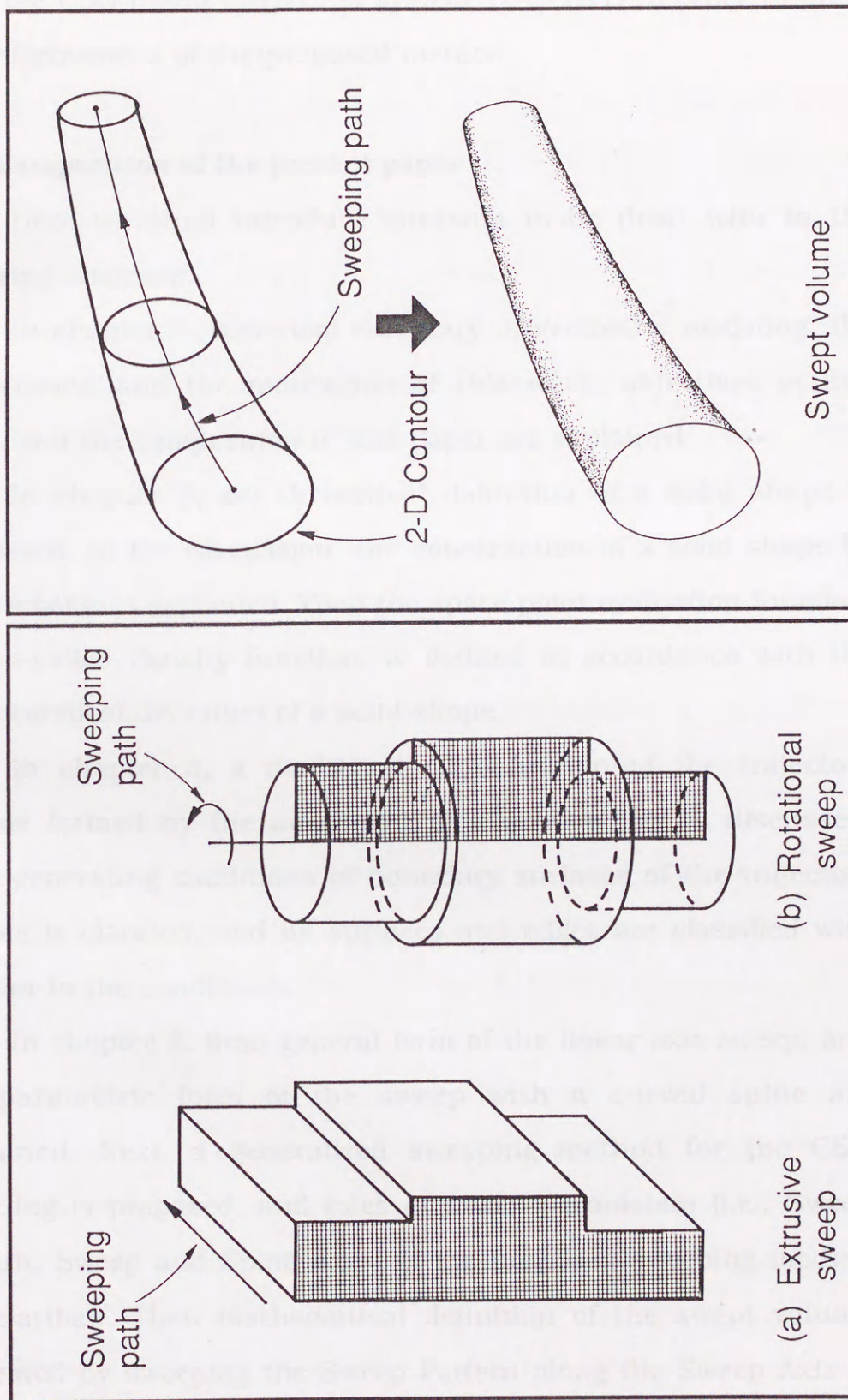


Fig.1.1-7 Concept of the generalized sweeping method

Fig.1.1-6 The simple sweeping method

with the CSG-based CAD/CAM system TIPS-1[11] in order to prove the effectiveness of the proposed method.

1.4 Composition of the present paper

Here we shall introduce contents to be dealt with in the following chapters.

In chapter 1, historical summary of geometric modeling, the background and the motivation of this work, objectives of this work, and the composition of this paper are explained.

In chapter 2, set theoretical definition of a solid shape is discussed. In the discussion, the construction of a solid shape by CSG scheme is explained. Then the space point evaluation function, the so-called Penalty function, is defined in accordance with the set theoretical definition of a solid shape.

In chapter 3, a mathematical definition of the trajectory volume formed by the movement of a solid object is discussed. Then generating conditions of boundary surfaces of the trajectory volume is clarified, and its surfaces and edges are classified with relation to the conditions.

In chapter 4, first, general form of the linear axis sweep, and the parametric form of the sweep with a curved spine are explained. Next, a generalized sweeping method for the CSG modeling is proposed, and roles of sweep parameters (i.e., Sweep Pattern, Sweep and Spine Axes) of the proposed sweeping method are clarified. Then mathematical definition of the swept volume generated by sweeping the Sweep Pattern along the Sweep Axis is

clarified. Finally, the problems to be solved by applying the proposed method are explained.

In chapter 5, generation of a swept volume formed by the movement of the NC cutter is discussed. In the discussion, an enveloped pattern of the swept volume is introduced as the Sweep Pattern in order to apply the proposed sweeping method. The enveloped pattern is considered to be the cross-sectional shape obtained by cutting the swept volume with the plane cross-section. Hence, if the enveloped pattern can be derived, an entire swept volume can be generated directly by sweeping the enveloped pattern along the tool path (i.e., the Sweep Axis). Therefore, how to derive an enveloped pattern is clarified in detail. Finally, some results of experiments are shown.

In chapter 6, generation of a fillet volume to round two intersecting quadrics is discussed. In the discussion, how to derive equations of intersection lines between two quadrics is clarified. It is also clarified that the fillet volume to round two intersecting quadrics can be generated by sweeping a fillet pattern along intersecting lines. Finally, some results of experiments are shown.

In chapter 7, generation of a fillet volume to round corners is discussed. In the discussion, equations of a fillet volume to round an edge, and of a fillet volume to round a vertex are derived. Then a method to smoothly connect the vertex and the related edges simultaneously is clarified. Finally, some results of experiments are shown.

In chapter 8, generation of a fillet volume to join two disconnected solids is discussed. In the discussion, a method to

generate fillet volume by blending two "cross-sections" of two solids is clarified. In the method, the current Sweep Pattern in the middle of the sweep is expressed by blending two static Sweep Patterns (i.e., the start and end Sweep Patterns). Thus how to derive both blending functions and implicit functions of the current Sweep Pattern is explained in detail. Finally, some results of experiments are shown.

In chapter 9, the program structure, data tables, and shape description statements of the developed prototype system for the swept volume are explained. The system is incorporated into the CSG-based CAD/CAM system TIPS-1, and performed on the HITAC M-680 computer at Hokkaido University. Finally, some results of experiments are shown in order to prove the effectiveness of the prototype system.

In chapter 10, concluding remarks of this paper are stated.

1.5 Conclusion

In this chapter, the advancement of geometric modeling is summarized first in relation to well known geometric modeling schemes. Next, background and motivation of this work are described. Then primary problems to be solved in this paper are focused on. After considering which solid modeling scheme (CSG or B-Rep) is essential to the primary problems, CSG is adopted as a unified internal representation method of solid object shapes when solving the primary problems.

In order to solve the primary problems consistently, we decided that the different kinds of complex shapes to be dealt with

in this paper are handled uniformly as solid models of loci formed by the movement of an object.

Since motion of an object is analogous to sweep of an object, motion is considered to be sweep in this paper. Therefore, a generalized sweeping method for the CSG modeling is proposed in order to generate a solid model of a locus formed by the movement of an object.

Then objectives of this paper are described. Finally, the composition of the following chapters are explained.

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2. SET THEORETICAL DEFINITION OF A SOLID SHAPE

2.1 Introduction

CSG (an acronym for constructive solid geometry representation) is adopted as a unified internal representation method of solid object shapes in this study, as mentioned previously. The basic idea of CSG is that a complicated solid shape is represented as "additions" and "subtractions" of simple primitive solids by using Boolean set operations. Before dealing with the solid representation problems of complex shapes by CSG scheme, we wish to clarify the principle of CSG mathematically. For this purpose, we shall define a solid shape set-theoretically in this chapter.

2.2 Set operations

In order to clarify the basis of set theory, we shall first define Boolean set operations such as *unions*, *intersections*, and *differences*[1]. Let us consider now two sets A and B. Let a letter "x" be an arbitrary element of the set. The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or B, i.e.,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\} . \quad (2-1)$$

Here "or" is used in the sense of "and/or" of logical operations. The intersection of two sets A and B, denoted by $A \cap B$, is the set of elements which belong to A and B, i.e.,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\} . \quad (2-2)$$

The difference of two sets A and B, denoted by $A-B$, is the set of elements which belong to A but which do not belong to B, i.e.,

$$A - B = \{x \mid x \in A, x \notin B\} . \quad (2-3)$$

The complement of a set B, denoted by $\bar{B}$, is the set of elements which do not belong to B, i.e.,

$$\bar{B} = \{x \mid x \in U, x \notin B\} . \quad (2-4)$$

In other words, $\bar{B}$ is the difference of the universal set U and B. By using Equations(2-2) and (2-4), we can replace Equation(2-3) as follows:

$$A - B = A \cap \bar{B} . \quad (2-5)$$

This means that we can use the intersection operation instead of the difference one. This transformation is essential in order to evaluate the space point (whether the point is inside, outside, or on the surface of the solid shape) for the CSG modeling, as taking up in section 2.4. The concept of the above Boolean set operations are illustrated by using Venn diagrams in Fig.2-1. Here sets are represented by simple plane areas and U, the universal set, by the area in the entire rectangle. The shaded areas show the results of the corresponding operations respectively.

2.3 Set theoretical expression of a solid shape

Let us consider now a constructing process of a solid shape by simple primitive solids. Each primitive solid can be used for both

addition and subtraction. So let a letter "P" symbolize additional primitive solids, and similarly a letter "Q" symbolize subtractive ones, for convenience. Then a solid shape, denoted by S, can be defined as follows[2]:

$$\left. \begin{aligned} S &= P_1 \cup P_2 \cup \dots \cup P_k - (Q_1 \cup Q_2 \cup \dots \cup Q_m) \\ &= \bigcup_{i=1}^k P_i - \bigcup_{i=1}^m Q_i . \end{aligned} \right\} (2-6)$$

Here the symbol "U" represents an addition while the symbol "-" represents a subtraction. It is considered that the additional and subtractive operations for constructing the solid shape are equivalent to the union and difference operations of set theory respectively. In Equation(2-6), the letter "k" is a positive integer which means the number of additional primitive solids. Similarly the letter "m" means the number of subtractive primitive solids. The constraint condition of the above definition is that we can never define the additional primitive solid within 3-D space that is once occupied by the subtractive one. In other words, the subtractive primitive solid has priority in the space allocation over the additional one. From now on, we refer to an additional primitive solid as a *P-mode Primitive* and to a subtractive primitive solid as a *Q-mode Primitive*. From Equation(2-6), it can be said that we may store all the data of Q-mode Primitives separately from that of P-mode Primitives. This means that the order of additions and subtractions of the Primitives in the middle of modeling stage does not affect the result of the modeling. Therefore it is not necessary for designers to pay attention to the order of additions and subtractions when constructing the solid shape. The concept of

Equation(2-6) is illustrated in Fig.2-2. The scheme, according to Equation(2-6), is referred to as *the scheme of double-layer structure*. Under the constraint condition, the shape description with the binary tree structure of CSG, as mentioned previously, is equivalent to that of the double-layer structure. The Primitive is composed of a couple of bounding surfaces as shown in Fig.2-3 and Fig.2-4. We assume that each bounding surface is given a direction toward the inside of the Primitive in order to define a closed space which corresponds to the Primitive. Then the Primitive can be defined by intersections of the bounding surfaces as follows:

$$\left. \begin{aligned} P_i &= P_{i1} \cap P_{i2} \cap \cdots \cap P_{il} = \bigcap_{j=1}^l P_{ij}, \\ \text{similarly,} \\ Q_i &= Q_{i1} \cap Q_{i2} \cap \cdots \cap Q_{in} = \bigcap_{j=1}^n Q_{ij}. \end{aligned} \right\} (2-7)$$

Here the positive integers "l" and "n" mean the numbers of bounding surfaces of which the Primitive is composed. By substituting Equation(2-7) into Equation(2-6), the solid shape can be expressed as follows:

$$S = \bigcup_{i=1}^k \bigcap_{j=1}^l P_{ij} - \bigcup_{i=1}^m \bigcap_{j=1}^n Q_{ij}. \quad \left. \right\} (2-8)$$

Let a letter " $\overline{Q}$ " be the complement of the union set of all the Q-mode Primitives, and a letter "x" be an arbitrary element of the set. Namely:

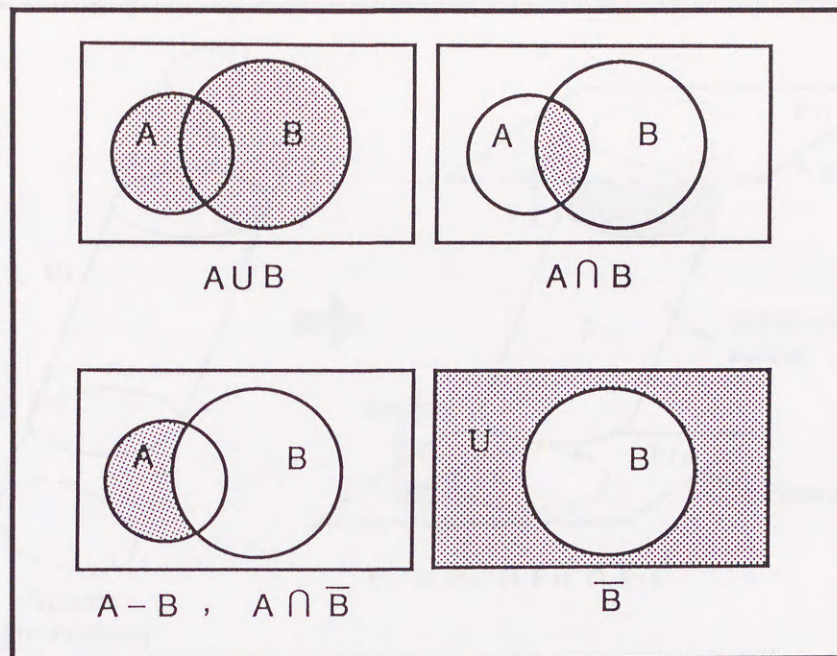


Fig.2-1 Venn diagrams of Boolean Set Operations

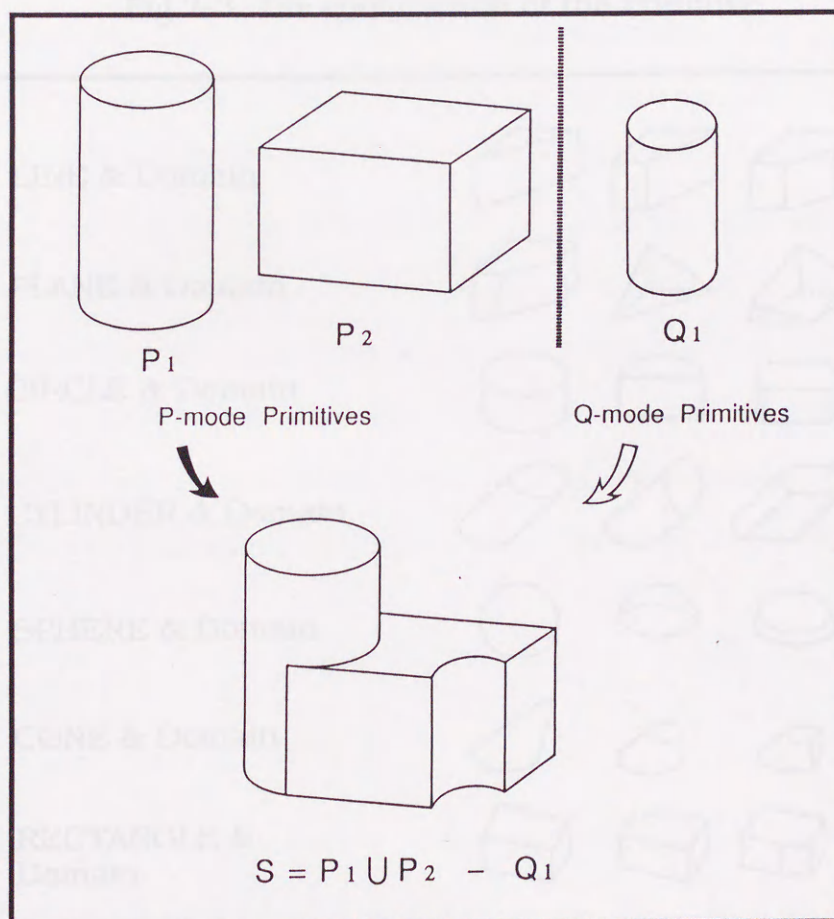


Fig.2-2 Concept of CSG representation by the scheme of double-layer structure

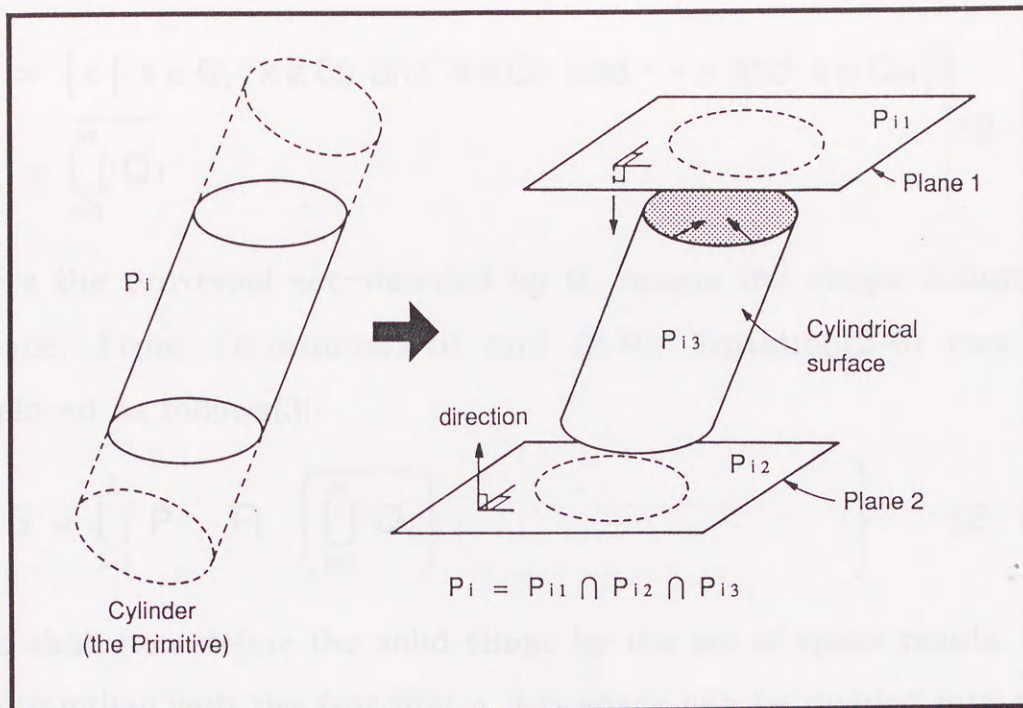


Fig.2-3 The composition of the Primitive

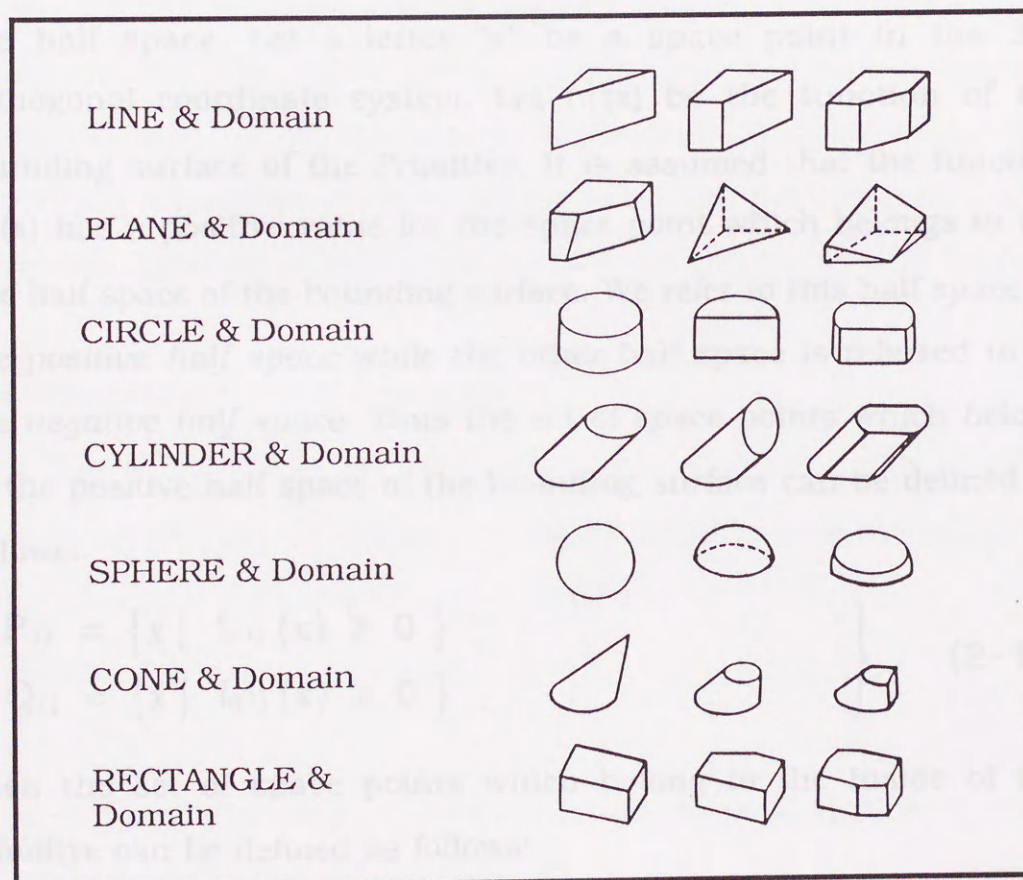


Fig.2-4 Examples of the Primitives (in case of the TIPS-1 system)

$$\begin{aligned}\overline{Q} &= \left\{ x \mid x \in U, x \notin Q_1 \text{ and } x \notin Q_2 \text{ and } \dots \text{ and } x \notin Q_m \right\} \\ &= \overline{\bigcup_{i=1}^m Q_i} .\end{aligned}\quad (2-9)$$

Here the universal set, denoted by U , means the shape definition space. From Equations(2-5) and (2-9), Equation(2-6) can be replaced as follows[3]:

$$S = \left\{ \bigcup_{i=1}^k P_i \cap \left(\overline{\bigcup_{i=1}^m Q_i} \right) \right\} \quad (2-10)$$

We shall now define the solid shape by the set of space points. We are familiar with the fact that a 3-D space can be divided into *two half spaces* by a surface. Thus we shall introduce the principle of the half space. Let a letter "x" be a space point in the 3-D orthogonal coordinate system. Let $f_{ij}(x)$ be the function of the bounding surface of the Primitive. It is assumed that the function $f_{ij}(x)$ has a positive value for the space point which belongs to the one half space of the bounding surface. We refer to this half space as the *positive half space* while the other half space is referred to as the *negative half space*. Thus the set of space points which belong to the positive half space of the bounding surface can be defined as follows:

$$\begin{aligned}P_{ij} &= \{ x \mid f_{p_{ij}}(x) \geq 0 \} , \\ Q_{ij} &= \{ x \mid f_{q_{ij}}(x) \geq 0 \} .\end{aligned}\quad (2-11)$$

Then the set of space points which belong to the inside of the Primitive can be defined as follows:

$$\begin{aligned}
P_i &= \{ \mathbf{x} \mid f_{p i 1}(\mathbf{x}) \geq 0 \text{ and } f_{p i 2}(\mathbf{x}) \geq 0 \text{ and } \cdots \text{ and } f_{p i l}(\mathbf{x}) \geq 0 \} \\
&= \bigcap_{j=1}^l \{ \mathbf{x} \mid f_{p i j}(\mathbf{x}) \geq 0 \} , \\
\text{similarly,} & \\
Q_i &= \bigcap_{j=1}^n \{ \mathbf{x} \mid f_{q i j}(\mathbf{x}) \geq 0 \} .
\end{aligned}
\quad \left. \vphantom{\begin{aligned} P_i &= \{ \mathbf{x} \mid f_{p i 1}(\mathbf{x}) \geq 0 \text{ and } f_{p i 2}(\mathbf{x}) \geq 0 \text{ and } \cdots \text{ and } f_{p i l}(\mathbf{x}) \geq 0 \} \\ &= \bigcap_{j=1}^l \{ \mathbf{x} \mid f_{p i j}(\mathbf{x}) \geq 0 \} , \\ \text{similarly,} & \\ Q_i &= \bigcap_{j=1}^n \{ \mathbf{x} \mid f_{q i j}(\mathbf{x}) \geq 0 \} . \end{aligned}} \right\} \quad (2-12)$$

The concept of Equation(2-12) is illustrated in Fig.2-5. By substituting Equation(2-12) into Equation(2-6), we can define the solid shape by the set of space points as follows:

$$S = \bigcup_{i=1}^k \bigcap_{j=1}^l \{ \mathbf{x} \mid f_{p i j}(\mathbf{x}) \geq 0 \} - \bigcup_{i=1}^m \bigcap_{j=1}^n \{ \mathbf{x} \mid f_{q i j}(\mathbf{x}) \geq 0 \} . \quad (2-13)$$

As a consequence, the solid shape can be generated by the set of space points. The space point evaluation (whether the point is inside, outside, or on the surface of the solid shape) is, however, practically inevitable in order to generate the solid shape. So we shall define the space point evaluation functions in the next section.

2.4 Penalty function of a solid shape

Let $f_{ij}(\mathbf{x}) = 0$ be an implicit function of the bounding surface of a Primitive. Let us define the following functions by using the terms on the left-hand side of the function $f_{ij}(\mathbf{x}) = 0$, i.e.,

$$\begin{aligned}
F_{p i j} &= f_{p i j} , \\
F_{q i j} &= f_{q i j} .
\end{aligned}
\quad \left. \vphantom{\begin{aligned} F_{p i j} &= f_{p i j} , \\ F_{q i j} &= f_{q i j} . \end{aligned}} \right\} \quad (2-14)$$

If these functions have positive values for an arbitrary space point which belongs to the respective positive half space of the bounding surfaces, then the following functions can be defined as

correspondence of the Primitive as shown in Equations(2-7) and (2-12):

$$\left. \begin{aligned}
 F_{pi} &= \text{MIN} \{ F_{pi1}, F_{pi2}, \dots, F_{pil} \} \\
 &= \text{MIN}_{j=1}^l \{ F_{pij} \}, \\
 \text{similarly,} \\
 F_{qi} &= \text{MIN}_{j=1}^n \{ F_{qij} \}.
 \end{aligned} \right\} \quad (2-15)$$

Equation(2-15) means that the minimum value in all the functional values of bounding surfaces is opted as the representative value of the Primitive. It is obvious that these functions have positive values for a space point which belongs to the inside of the Primitive. Similarly we can define the functions for the union set of the Primitives as follows:

$$\left. \begin{aligned}
 F_p &= \text{MAX} \{ F_{p1}, F_{p2}, \dots, F_{pk} \} \\
 &= \text{MAX}_{i=1}^k \{ F_{pi} \}, \\
 \text{similarly,} \\
 F_q &= \text{MAX}_{i=1}^m \{ F_{qi} \}.
 \end{aligned} \right\} \quad (2-16)$$

Equation(2-16) means that the maximum value in all the functional values of the Primitives is opted as the representative value of the union set of the Primitives. It is clear that these functions have positive values for a space point which belongs to the union set.

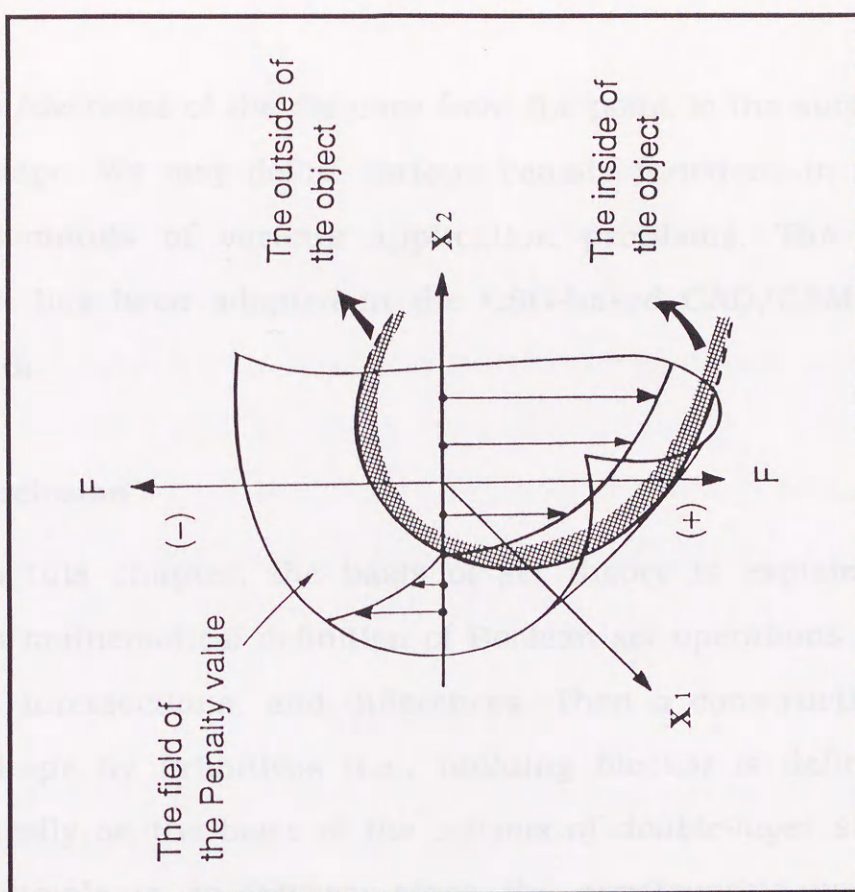
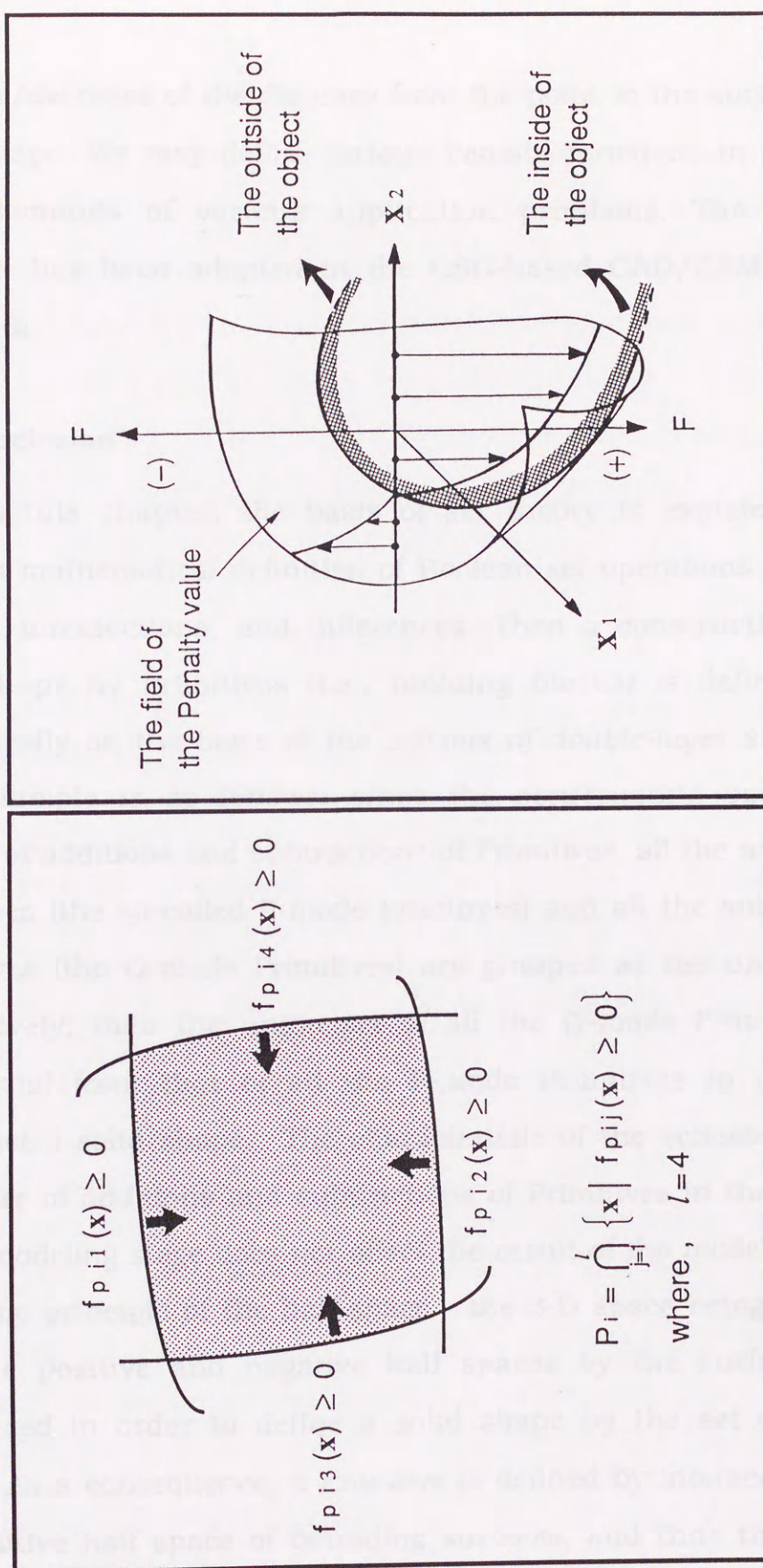
From Equation(2-16), the function $\bar{F}_q$, which corresponds to the complement of the union set of all the Q-mode Primitives as defined in Equation(2-9), can be defined as follows:

$$\bar{F}_q = - \left. \begin{matrix} \text{MAX}_{i=1}^m \{ F_{qi} \} \end{matrix} \right\} \quad (2-17)$$

Equation(2-17) means that the maximum value in all the functional values of the Q-mode Primitives is opted first, and that the opted value is multiplied by -1 in order to make the representative value of the complement of the union set of all the Q-mode Primitives. By using Equations(2-16) and (2-17), the function of a solid shape, denoted by F, can be defined as follows:

$$\begin{aligned} F &= \text{MIN} \left\{ \begin{matrix} \text{MAX}_{i=1}^k \{ F_{pi} \} & , & - \text{MAX}_{i=1}^m \{ F_{qi} \} \end{matrix} \right\} \\ &= \text{MIN} \{ F_p , \bar{F}_q \} . \end{aligned} \quad (2-18)$$

The function F corresponds to Equation(2-10). The function F has a positive/negative value for a space point which belongs to the inside/outside of a solid shape, having zero for a space point on the bounding surface of a solid shape. Therefore we can apply the function F to the space point evaluation. We refer to the function F as *Penalty function*[4][5] in this study. The aspect of the Penalty function is illustrated in Fig.2-6. The characteristic of the Penalty function is that the absolute value of the function for a space point is getting large/small gradually in accordance with the



increase/decrease of the distance from the point to the surface of a solid shape. We may define various Penalty functions in order to meet demands of various application problems. The Penalty function has been adopted in the CSG-based CAD/CAM system TIPS-1[6].

2.5 Conclusion

In this chapter, the basis of set theory is explained first through mathematical definition of Boolean set operations such as unions, intersections, and differences. Then a construction of a solid shape by Primitives (i.e., building blocks) is defined set-theoretically on the basis of *the scheme of double-layer structure*. The principle is as follows: since the constructing operations consist of additions and subtractions of Primitives, all the additional Primitives (the so-called P-mode Primitives) and all the subtractive Primitives (the Q-mode Primitives) are grouped as the union sets respectively; then the union set of all the Q-mode Primitives is subtracted from that of all the P-mode Primitives in order to construct a solid shape. The characteristic of the scheme is that the order of additions and subtractions of Primitives in the middle of the modeling stage does not affect the result of the modeling.

The principle of the half space - the 3-D space being divided into the positive and negative half spaces by the surface - is introduced in order to define a solid shape by the set of space points. As a consequence, a Primitive is defined by intersections of the positive half space of bounding surfaces, and thus the set of space points which belong to the inside of a solid shape is defined.

According to the equation of a solid shape defined by the set of space points, the space point evaluation is practically inevitable in order to generate a solid shape. Therefore the space point evaluation function, the so-called Penalty function, is defined in accordance with the set theoretical definition of a solid shape. The Penalty function has a positive/negative value for a space point which belongs to the inside/outside of a solid shape, having zero for a space point on the bounding surface of a solid shape.

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3. DEFINITION OF THE TRAJECTORY VOLUME FORMED BY THE MOVEMENT OF A SOLID OBJECT

3.1 Introduction

The geometric model of the locus formed by the movement of an object is called a *trajectory volume* or a *swept volume*. Here we refer to it as trajectory volume.

The interference checking between a moving object and its environments is essential especially in simulating dynamic motions of robots and in NC machining-process verifications. In order to calculate the interference, the trajectory volume must be represented in the computer. It has, however, been considered that the trajectory volume is too complicated to represent consistently with the ordinary solid shape. This is due to the difficulty of defining the solid model of the trajectory volume so as to be consistent with the ordinary one. Thus the trajectory volume has been handled and displayed by means of rendering techniques such as Scan Line algorithm[1], Z-buffer algorithm[2], Voxel representation[3], and Dixel hardware technique[4]. Though the rendered image of the trajectory volume is actually useful for visual verifications, the rendered image cannot be used for more advanced applications. For example, it is impossible to execute set operations between the rendered image and the internal solid models of its environments even though such operations are often required in NC machining-process verifications and collision detections. From the viewpoint of modeling consistency, the trajectory volume should be

represented internally in accordance with the ordinary solid shape. Therefore we wish to define the solid model of the trajectory volume mathematically so as to be handled within CSG scheme. In this chapter, first, we shall define the envelope of a family of surfaces and the trajectory volume formed by the movement of a solid object. Then we clarify generating conditions of the surface of the trajectory volume, and classify its boundary surface and edge with relation to the condition.

3.2 The envelope of a family of surfaces

3.2.1 The envelope and the characteristic curve

We shall clarify the mathematical basis concerning motion of a geometry. Let us consider now a one-parameter family of surfaces as illustrated in Fig.3-1. Let the family of surfaces be given by

$$f(x, y, z, t) = 0 \quad , \quad (3-1)$$

where t is the family parameter which is constant for any given surface. Let two neighboring surfaces be given by

$$\left. \begin{aligned} f(x, y, z, t) &= 0 \quad , \\ f(x, y, z, t + \Delta t) &= 0 \quad . \end{aligned} \right\} \quad (3-2)$$

These two surfaces will, in general, intersect in a curve Γ . Now any linear combination as

$$a f(x, y, z, t) + b f(x, y, z, t + \Delta t) = 0 \quad (3-3)$$

represents a surface through the intersection of the two given surfaces. Here coefficients a and b are constants and they are not both zero. Hence the surfaces

$$\left. \begin{aligned} f(x, y, z, t) &= 0, \\ \frac{f(x, y, z, t + \Delta t) - f(x, y, z, t)}{\Delta t} &= 0, \end{aligned} \right\} \quad (3-4)$$

intersect in the curve Γ . Here $\Delta t \neq 0$. As $\Delta t \rightarrow 0$, the curve Γ approaches a limiting position, and is given by

$$\left. \begin{aligned} f(x, y, z, t) &= 0, \\ \frac{\partial}{\partial t} f(x, y, z, t) &= 0. \end{aligned} \right\} \quad (3-5)$$

This curve is called *the characteristic curve of the surface for the parameter t* . Each t determines a characteristic curve, and totality of all the characteristic curves is a surface called *the envelope (or the enveloping surface) of the family $f(x, y, z, t) = 0$* . The equation of the envelope can be obtained by eliminating t from Equation(3-5). The detailed discussion of the envelope is found in the literature[5][6][7].

3.2.2 An example of the envelope of a moving sphere

Let us consider a sphere of radius ρ that is moving along a circular path with radius R ($\rho < R$) on the xy -plane in the 3-D orthogonal coordinate system as illustrated in Fig.3-2. Let the center of circular path be on the origin of the 3-D orthogonal coordinate system. Then the x , y , and z components of the equation of the moving sphere are given as follows:

$$\left. \begin{aligned} x &= (R + \rho \cos \theta) \cos t, & \dots (a) \\ y &= (R + \rho \cos \theta) \sin t, & \dots (b) \\ z &= \rho \sin \theta, & \dots (c) \\ \text{where } 0 \leq t \leq 2\pi, \quad 0 \leq \theta \leq 2\pi. \end{aligned} \right\} \quad (3-6)$$

Here t is the family parameter which designates the instance of the sphere at time t . By eliminating t from the terms on the right-hand side of (a) and (b), we get

$$(x - R \cos t)^2 + (y - R \sin t)^2 - \rho^2 \cos^2 \theta = 0. \quad (d)$$

By squaring the terms on both sides of (c), we get

$$z^2 - \rho^2 \sin^2 \theta = 0. \quad (e)$$

Adding the terms on both sides of (d) and (e) respectively, the equation of the family of the moving sphere is obtained as follows:

$$f(x, y, z, t) = (x - R \cos t)^2 + (y - R \sin t)^2 + z^2 - \rho^2 = 0. \quad (3-7)$$

Then partial derivative of f with respect to t is

$$\frac{\partial f}{\partial t} = -x \sin t + y \cos t = 0. \quad (3-8)$$

From Equation(3-8), the parameter t is obtained as

$$\left. \begin{aligned} t &= \tan^{-1} \left(\frac{y}{x} \right), \\ \text{where } x &\neq 0, \quad 0 \leq t < \frac{\pi}{2}, \quad \frac{\pi}{2} < t < \frac{3\pi}{2}, \quad \frac{3\pi}{2} < t \leq 2\pi. \end{aligned} \right\} \quad (3-9)$$

By substituting Equation(3-9) into (3-7), we can obtain the envelope of the moving sphere. As a consequence, the envelope is a torus for this example.

3.3 Set theoretical definition of the trajectory volume

Let us consider now motion of a solid object as illustrated in Fig.3-3. Let t ($t_0 \leq t \leq t_1$) be a motion parameter which designates time instances of the motion, and the initial and final instances of

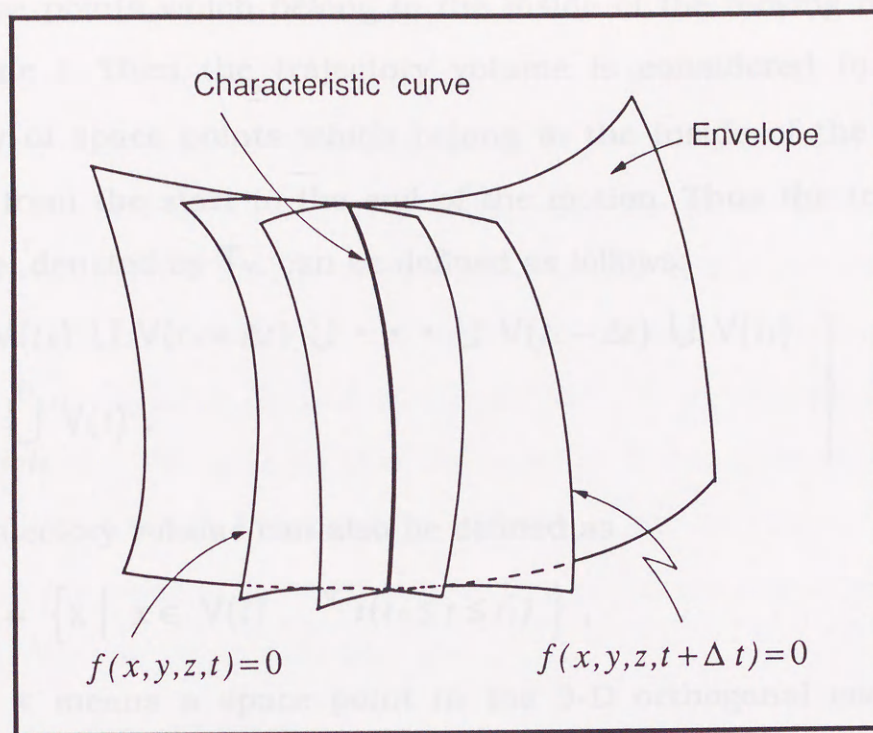


Fig.3-1 The envelope and the characteristic curve

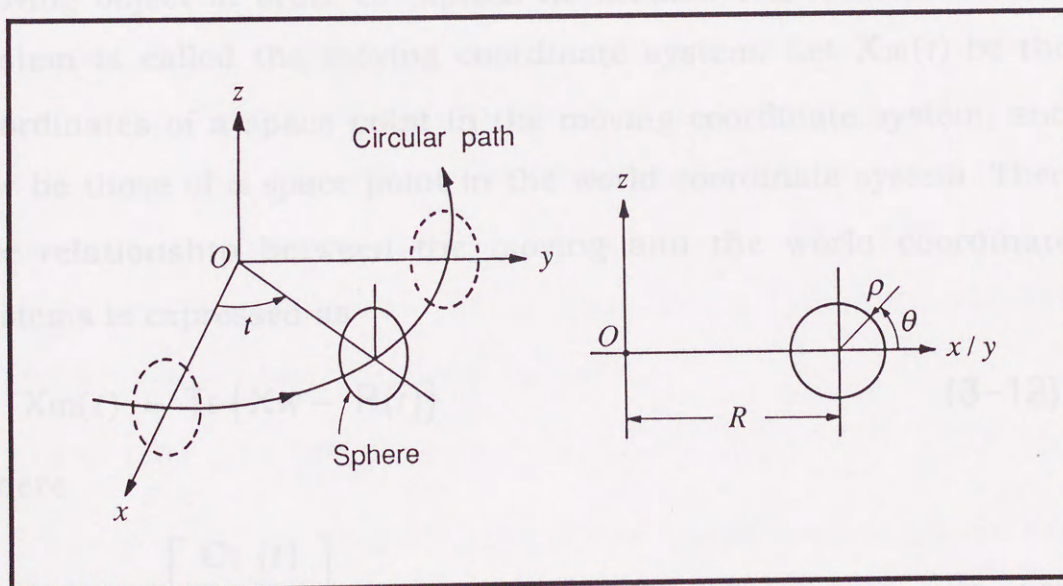


Fig.3-2 A sphere moving along a circular path

the motion be designated by t_0 and t_1 respectively. Let $V(t)$ be a set of space points which belong to the inside of the moving object at the time t . Then the trajectory volume is considered to be the totality of space points which belong to the inside of the moving object from the start to the end of the motion. Thus the trajectory volume, denoted by T_v , can be defined as follows:

$$\left. \begin{aligned} T_v &= V(t_0) \cup V(t_0 + \Delta t) \cup \cdots \cup V(t_1 - \Delta t) \cup V(t_1) \\ &= \bigcup_{t=t_0}^{t_1} V(t) . \end{aligned} \right\} \quad (3-10)$$

The trajectory volume can also be defined as

$$T_v = \left\{ x \mid x \in V(t) , \quad \forall t(t_0 \leq t \leq t_1) \right\} , \quad (3-11)$$

where x means a space point in the 3-D orthogonal coordinate system. It is convenient to attach a local coordinate system to the moving object in order to express its motion. The local coordinate system is called the moving coordinate system. Let $X_m(t)$ be the coordinates of a space point in the moving coordinate system, and X_w be those of a space point in the world coordinate system. Then the relationship between the moving and the world coordinate systems is expressed as

$$X_m(t) = Tr \{ X_w - R(t) \} , \quad (3-12)$$

where

$$Tr = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \end{bmatrix} . \quad (3-13)$$

Here $R(t)$ is an origin of the moving coordinate system, Tr is the coordinate transformation matrix, and C_i ($i=1,2,3$) are axis vectors of the moving coordinate system. Spatial relationship between these two coordinate systems is illustrated in Fig.3-4. We assume that the moving object has an ordinary shape and its solid model can be addressed by CSG scheme. Let a solid model of the moving object be represented by CSG scheme, then the mathematical function of the moving object is defined as Equation(2-18) in chapter 2. Let F be its function. We assume that the function F has positive value for a space point which belongs to the inside of the moving object, and has zero for a space point on the surface of the moving object. Then the set $V(t)$ can be defined as

$$V(t) = \{X_w \mid F(X_m(t)) \geq 0\}, \quad (3-14)$$

where $t_0 \leq t \leq t_1$. Equation(3-14) suggests that the space point evaluation (whether the point is inside, outside, or on the surface of the moving object) should be done in the moving coordinate system, and that the moving coordinates of a space point which belongs to the inside of the moving object are to be transformed into the world coordinates. From Equations(3-11) and(3-14), the trajectory volume can be defined as

$$Tv = \{X_w \mid F(X_m(t)) \geq 0, \forall t(t_0 \leq t \leq t_1)\}. \quad (3-15)$$

According to Equation(3-15), the space point evaluation in the moving coordinate system is practically inevitable in order to generate the trajectory volume. There is no exact way, however, to evaluate the space point in accordance with the dynamic motion of

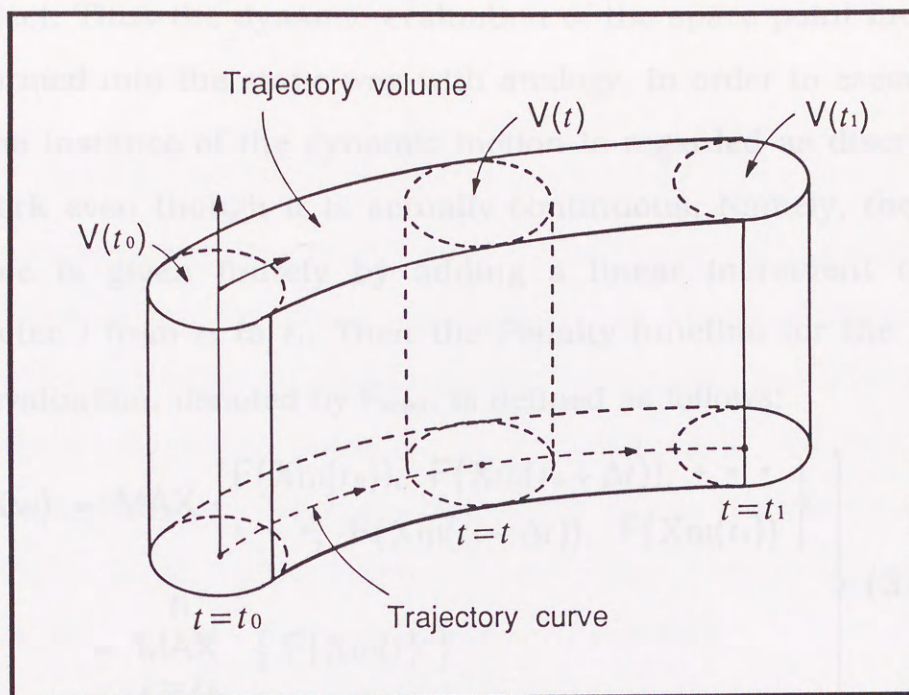


Fig.3-3 The trajectory volume formed by the movement of a solid object

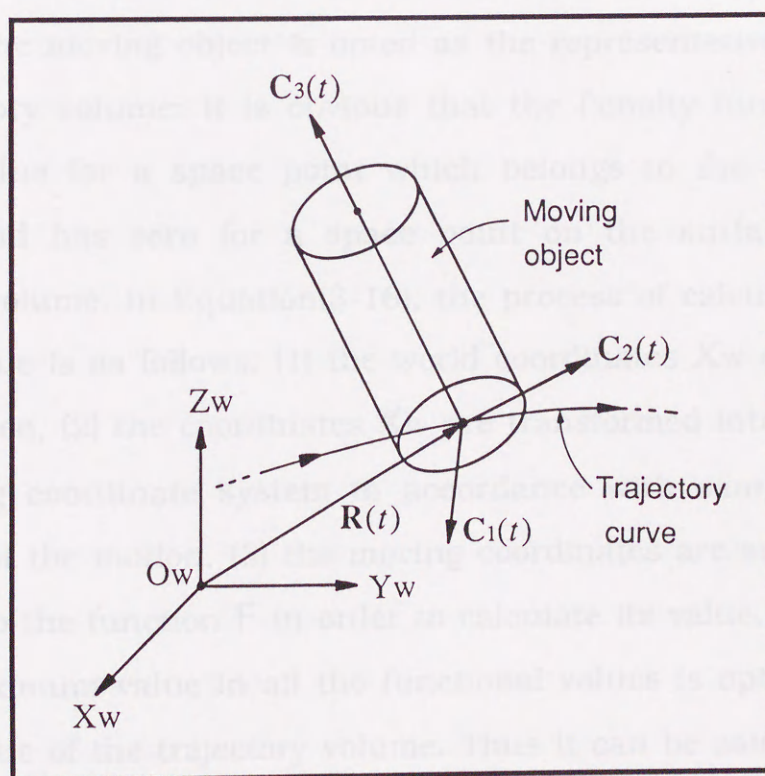


Fig.3-4 Relationship between the world and the moving coordinate systems

the object. Thus the dynamic evaluation of the space point must be transformed into the static way with analogy. In order to execute it, the time instance of the dynamic motion is regarded as discrete in this work even though it is actually continuous. Namely, the time instance is given finitely by adding a linear increment of the parameter t from t_0 to t_1 . Then the Penalty function for the space point evaluation, denoted by $F_{\max}$, is defined as follows:

$$F_{\max}(X_w) = \left. \begin{aligned} & \text{MAX} \left\{ F(X_m(t_0)), F(X_m(t_0 + \Delta t)), \dots \right\} \\ & \left\{ \dots, F(X_m(t_1 - \Delta t)), F(X_m(t_1)) \right\} \end{aligned} \right\} \quad (3-16)$$

$$= \text{MAX}_{t=t_0}^{t_1} \{ F(X_m(t)) \} .$$

Equation(3-16) means that the maximum value in all the functional values of the moving object is opted as the representative value of the trajectory volume. It is obvious that the Penalty function has positive value for a space point which belongs to the trajectory volume, and has zero for a space point on the surface of the trajectory volume. In Equation(3-16), the process of calculating the Penalty value is as follows: (1) the world coordinates X_w of a space point is given, (2) the coordinates X_w are transformed into those in the moving coordinate system in accordance with sampled time instances of the motion, (3) the moving coordinates are substituted in turn into the function F in order to calculate its value, and then (4) the maximum value in all the functional values is opted as the Penalty value of the trajectory volume. Thus it can be said that the computing time of the Penalty value depends on the number of sampled time instances of the motion. By using Equation(3-16), the

trajectory volume formed by the movement of a solid object is finally defined as follows:

$$Tv = \{Xw \mid F_{\max}(Xw) \geq 0\}. \quad (3-17)$$

As a consequence, the trajectory volume can be generated in accordance with the above definition. The above definition is effective for not only CSG-based motion simulations but also Voxel-based dynamic motion simulations. The definition, however, does not express boundary surfaces and edges of the trajectory volume explicitly. Thus we wish to discuss the boundary surface and edge of the trajectory volume in the following two sections.

3.4 Generating conditions for the surface of the trajectory volume

The boundary surface of the trajectory volume is often required in order to develop advanced algorithms/applications for motion simulations. For example, if geometric/topological data of the boundary surface of the trajectory volume can be used, the shaded image of the trajectory volume can be displayed more quickly. In this section, we wish to clarify generating conditions for the boundary surface of the trajectory volume[8][9], and to classify the surface into four categories with relation to the conditions.

Let us consider now motion of a solid object as illustrated in Fig.3-5. We assume that a normal of the surface of the moving object and a trajectory curve of the motion are given, and a tangent of the motion at an arbitrary point on the surface of the moving object can be determined. Let $\mathbf{n}$ be a normal vector of the surface of the moving object, and its direction from the inside to the outside of

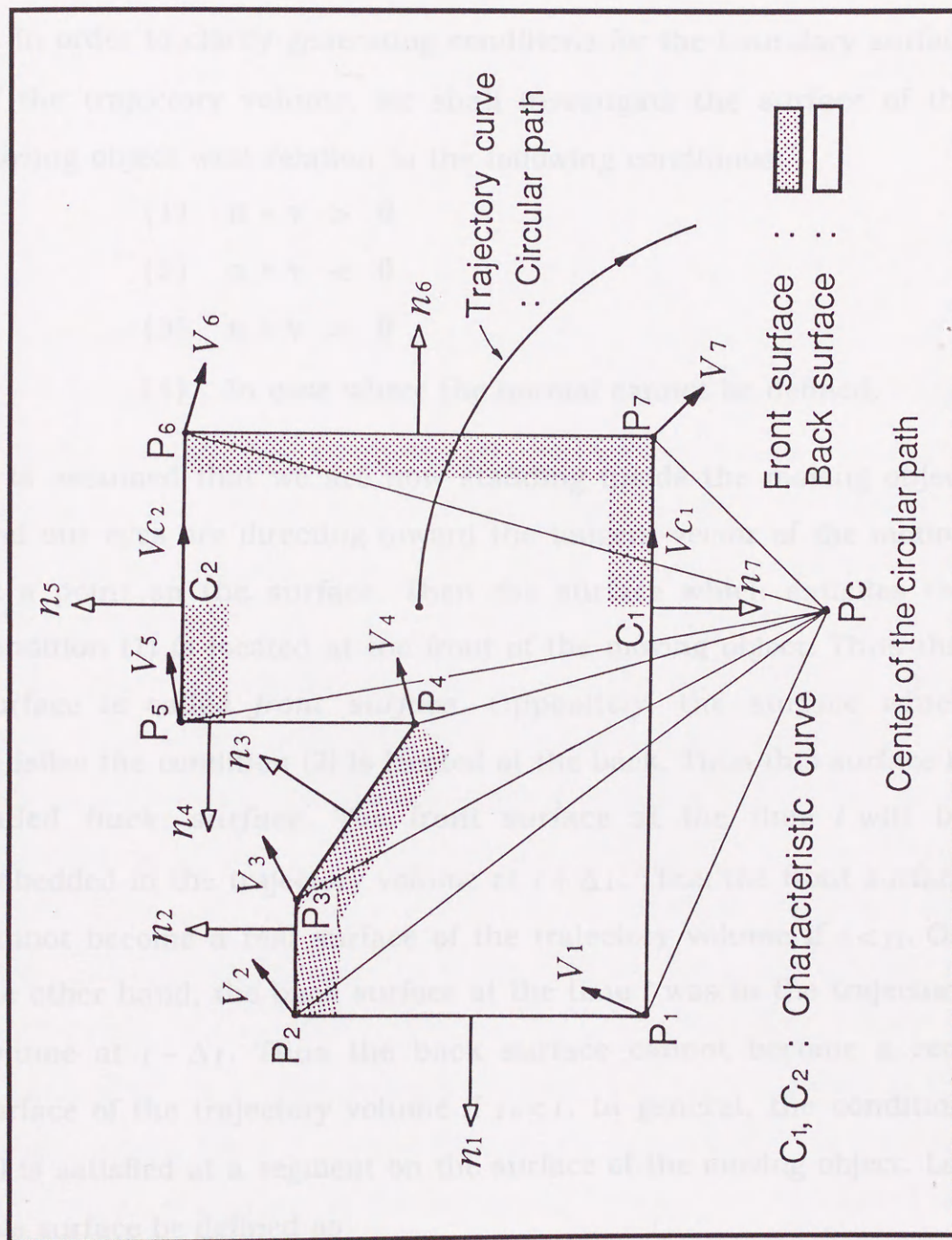


Fig.3-5 The front and back surfaces of a moving object

the moving object be considered as a positive direction. Let $\mathbf{v}$ be a tangent vector of the motion at the point on the surface of the moving object, and $\mathbf{n} \cdot \mathbf{v}$ be a dot product of the two vectors, $\mathbf{n}$ and $\mathbf{v}$. In order to clarify generating conditions for the boundary surface of the trajectory volume, we shall investigate the surface of the moving object with relation to the following conditions:

$$(1) \quad \mathbf{n} \cdot \mathbf{v} > 0$$

$$(2) \quad \mathbf{n} \cdot \mathbf{v} < 0$$

$$(3) \quad \mathbf{n} \cdot \mathbf{v} = 0$$

(4) In case where the normal cannot be defined.

It is assumed that we are now standing inside the moving object and our eyes are directing toward the tangent vector of the motion at a point on the surface. Then the surface which satisfies the condition (1) is located at the front of the moving object. Thus this surface is called *front surface*. Oppositely, the surface which satisfies the condition (2) is located at the back. Thus this surface is called *back surface*. The front surface at the time t will be embedded in the trajectory volume at $t + \Delta t$. Thus the front surface cannot become a real surface of the trajectory volume if $t < t_1$. On the other hand, the back surface at the time t was in the trajectory volume at $t - \Delta t$. Thus the back surface cannot become a real surface of the trajectory volume if $t_0 < t$. In general, the condition (3) is satisfied at a segment on the surface of the moving object. Let this surface be defined as

$$f_{ij}(Xm(t)) = 0. \quad (3-18)$$

Here the function $f_{ij}(X_m(t))$ is considered to be a continuously differentiable. At the segment on the surface, the condition (3) is expressed as

$$\left. \frac{df_{ij}}{dX_m} \cdot \frac{dX_m}{dt} = 0. \right\} \quad (3-19)$$

Namely,

$$\frac{d}{dt} f_{ij}(X_m(t)) = 0. \quad (3-20)$$

Equations(3-18) and (3-20) are equivalent to the condition of a characteristic curve as defined in Equation(3-5). Thus it can be said that the condition (3) is formed on the characteristic curve of the surface for the parameter t . A surface which satisfies the condition (3) is divided into sub-surfaces (i.e., the front and back surfaces) by the characteristic curve as illustrated in Fig.3-6. The geometric entity which satisfy the condition (4) is an edge of the moving object. This edge is often located at the boundary between the front and back surfaces. The front and back surfaces cannot become a real surface of the trajectory volume if $t_0 < t < t_1$, as mentioned above. When $t_0 < t < t_1$, only the boundary between the front and back surfaces can become the surface of the trajectory volume. In other words, when $t_0 < t < t_1$, the surface of the trajectory volume is a locus of the boundary between the front and back surfaces of the moving object. From the above discussion, generating conditions for the boundary surface of the trajectory volume are expressed as follows:

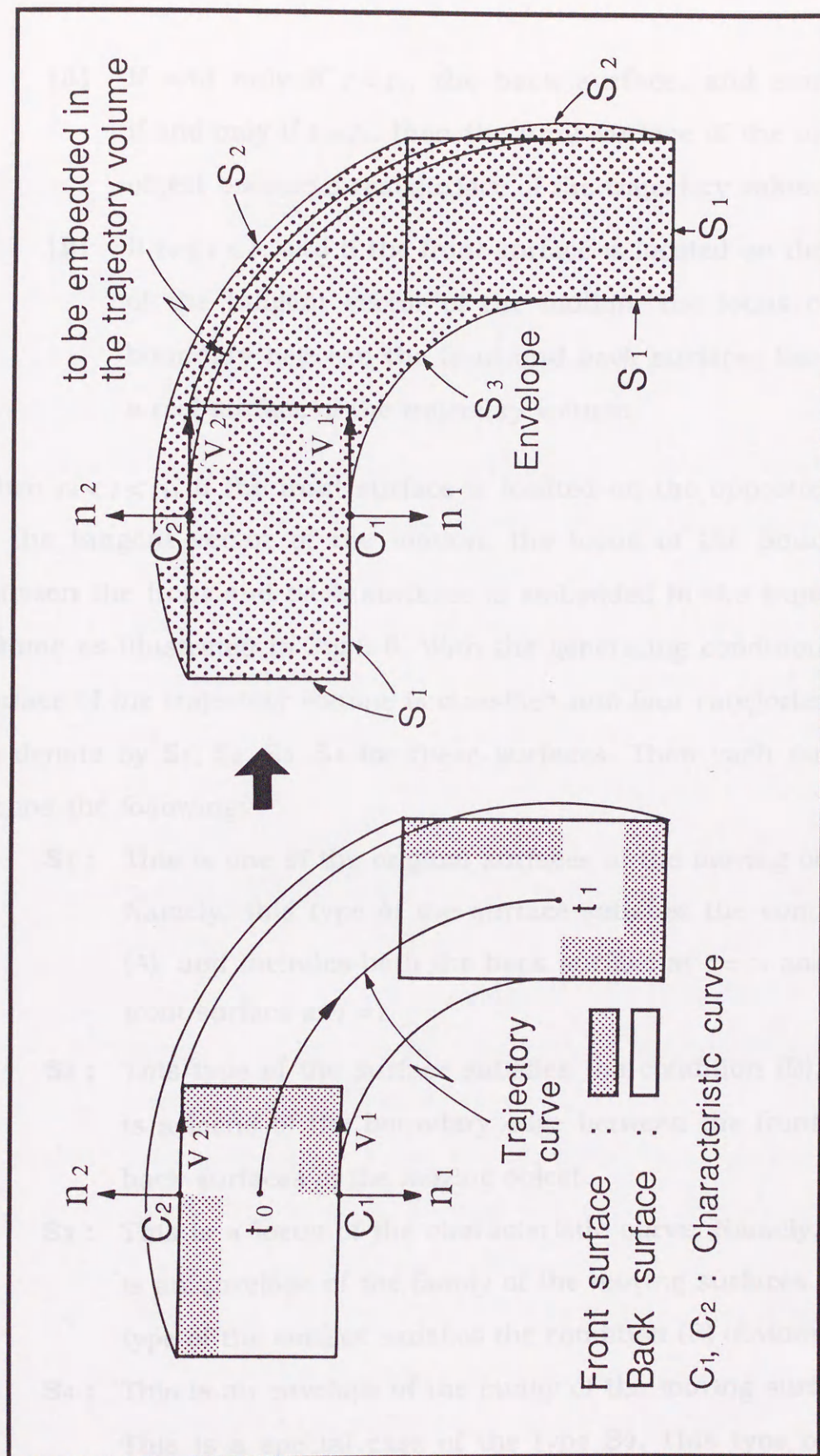


Fig.3-6 An example of the trajectory volume and its classified surfaces

- (A) If and only if $t = t_0$, the back surface, and similarly if and only if $t = t_1$, then the front surface of the moving object become a real surface of the trajectory volume.
- (B) If $t_0 < t < t_1$ and if the front surface is located on the side of the tangent vector of the motion, the locus of the boundary between the front and back surfaces becomes a real surface of the trajectory volume.

When $t_0 < t < t_1$, if the front surface is located on the opposite side of the tangent vector of the motion, the locus of the boundary between the front and back surfaces is embedded in the trajectory volume as illustrated in Fig.3-6. With the generating condition, the surface of the trajectory volume is classified into four categories. Let us denote by S_1, S_2, S_3, S_4 for these surfaces. Then each surface means the following:

- S₁ :** This is one of the original surfaces of the moving object. Namely, this type of the surface satisfies the condition (A), and includes both the back surface at $t = t_0$ and the front surface at $t = t_1$.
- S₂ :** This type of the surface satisfies the condition (B), and is a locus of the boundary edge between the front and back surfaces of the moving object.
- S₃ :** This is a locus of the characteristic curve. Namely, this is an envelope of the family of the moving surfaces. This type of the surface satisfies the condition (B) obviously .
- S₄ :** This is an envelope of the family of the moving surfaces. This is a special case of the type S_3 . This type of the

surface is generated just like that a plane moves in a direction perpendicular to its normal.

A simple example of the classified surfaces is shown in Fig.3-6.

3.5 Classification of edges of the trajectory volume

The edge of the trajectory volume can be classified into the following five categories:

- E1 : This is one of the original edges of the moving object. This type of the edge appears at the initial and final positions of the moving object.
- E2 : This is a special case of the type E1. Namely, this is a boundary edge between the front and back surfaces of the moving object at $t = t_0$ and $t = t_1$. In other words, this type of the edge appears as an intersection line between the S_1 (i.e., the original surface of the moving object) and S_2 (i.e., the locus of the boundary edge) surfaces.
- E3 : This is a locus of the original vertex of the moving object. This type of the edge appears as an intersection line between both the S_2 surfaces.
- E4 : This is a locus of the end point of the characteristic curve. This type of the edge appears as an intersection line between the S_2 and S_3 surfaces.
- E5 : This type of the edge occurs by intersecting real surfaces of the trajectory volume.

A simple example of the classified edges is shown in Fig.3-7.

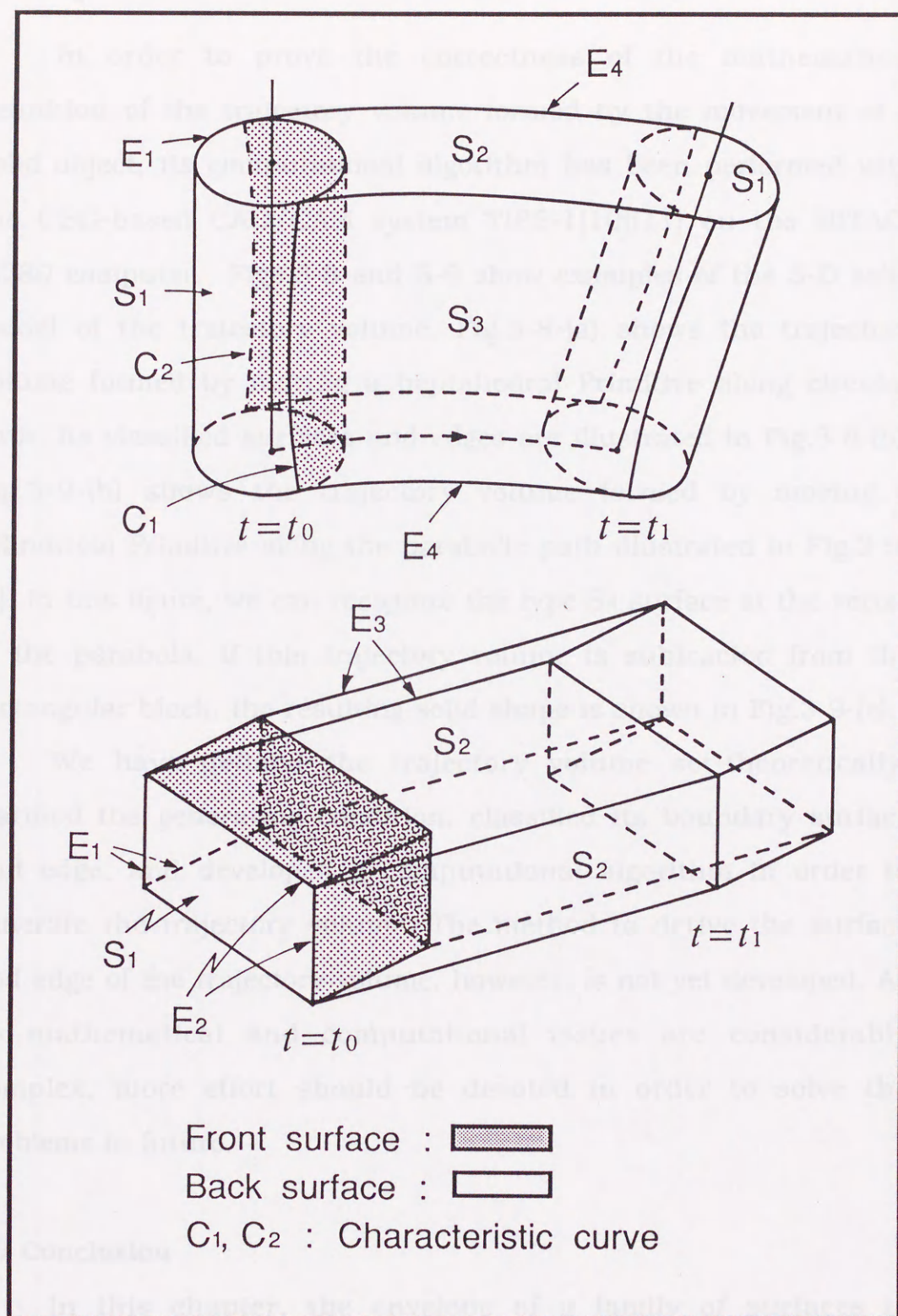


Fig.3-7 The classified surfaces and edges of the trajectory volume

3.6 Experiments and the consideration

In order to prove the correctness of the mathematical definition of the trajectory volume formed by the movement of a solid object, its computational algorithm has been performed with the CSG-based CAD/CAM system TIPS-1[10][11] on the HITAC-M280 computer. Figs.3-8 and 3-9 show examples of the 3-D solid model of the trajectory volume. Fig.3-8-(a) shows the trajectory volume formed by moving a heptahedral Primitive along circular path. Its classified surfaces and edges are illustrated in Fig.3-8-(b). Fig.3-9-(b) shows the trajectory volume formed by moving a cylindrical Primitive along the parabolic path illustrated in Fig.3-9-(a). In this figure, we can recognize the type S_4 surface at the vertex of the parabola. If this trajectory volume is subtracted from the rectangular block, the resulting solid shape is shown in Fig.3-9-(c).

We have defined the trajectory volume set-theoretically, clarified the generating condition, classified its boundary surface and edge, and developed a computational algorithm in order to generate the trajectory volume. The method to derive the surface and edge of the trajectory volume, however, is not yet developed. As its mathematical and computational issues are considerably complex, more effort should be devoted in order to solve the problems in future.

3.7 Conclusion

In this chapter, the envelope of a family of surfaces is explained first in order to understand the mathematical basis concerning motion of a solid object. Then the equation of the

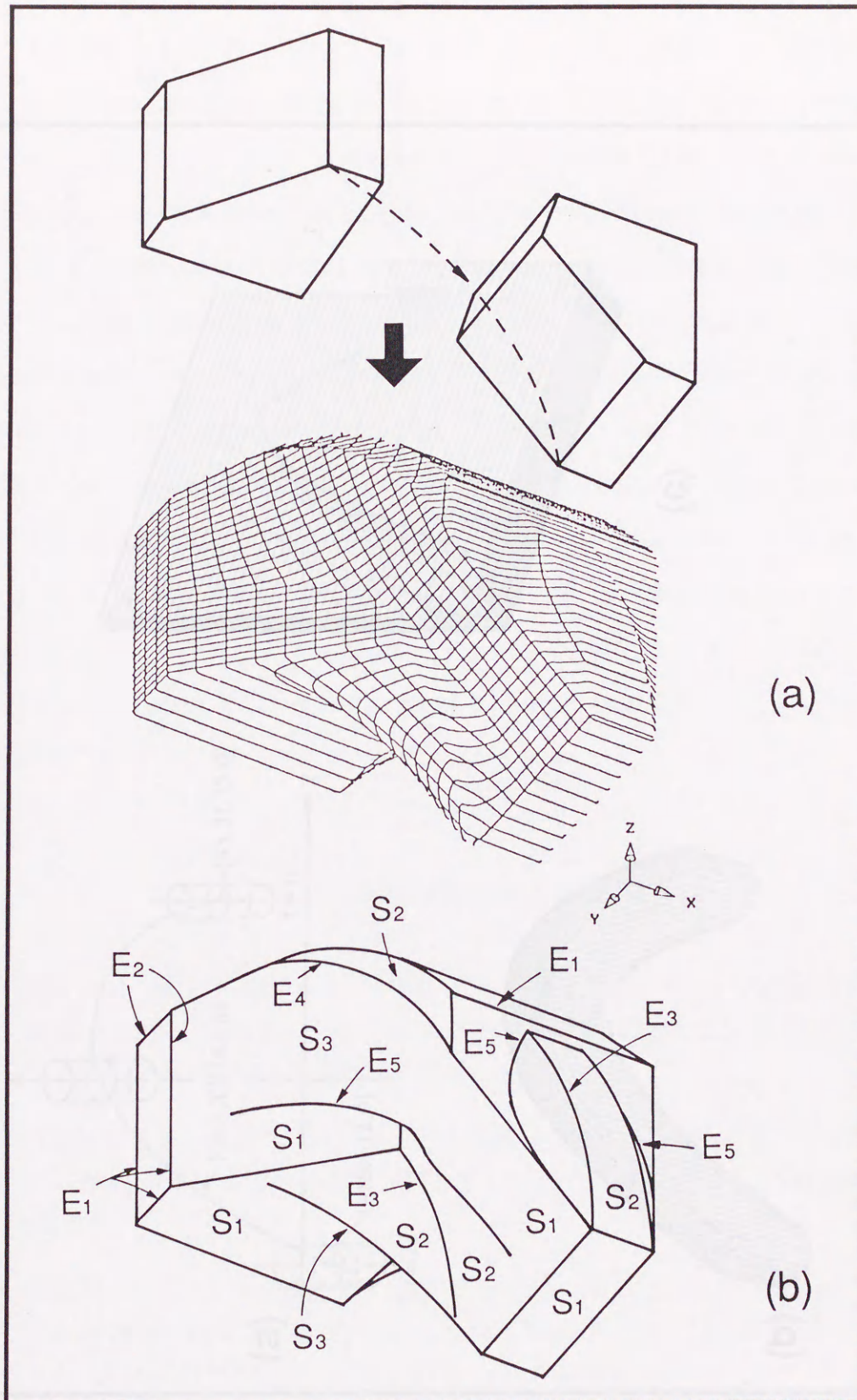


Fig.3-8 The trajectory volume formed by the movement of the heptahedral Primitive, and its classified surfaces and edges

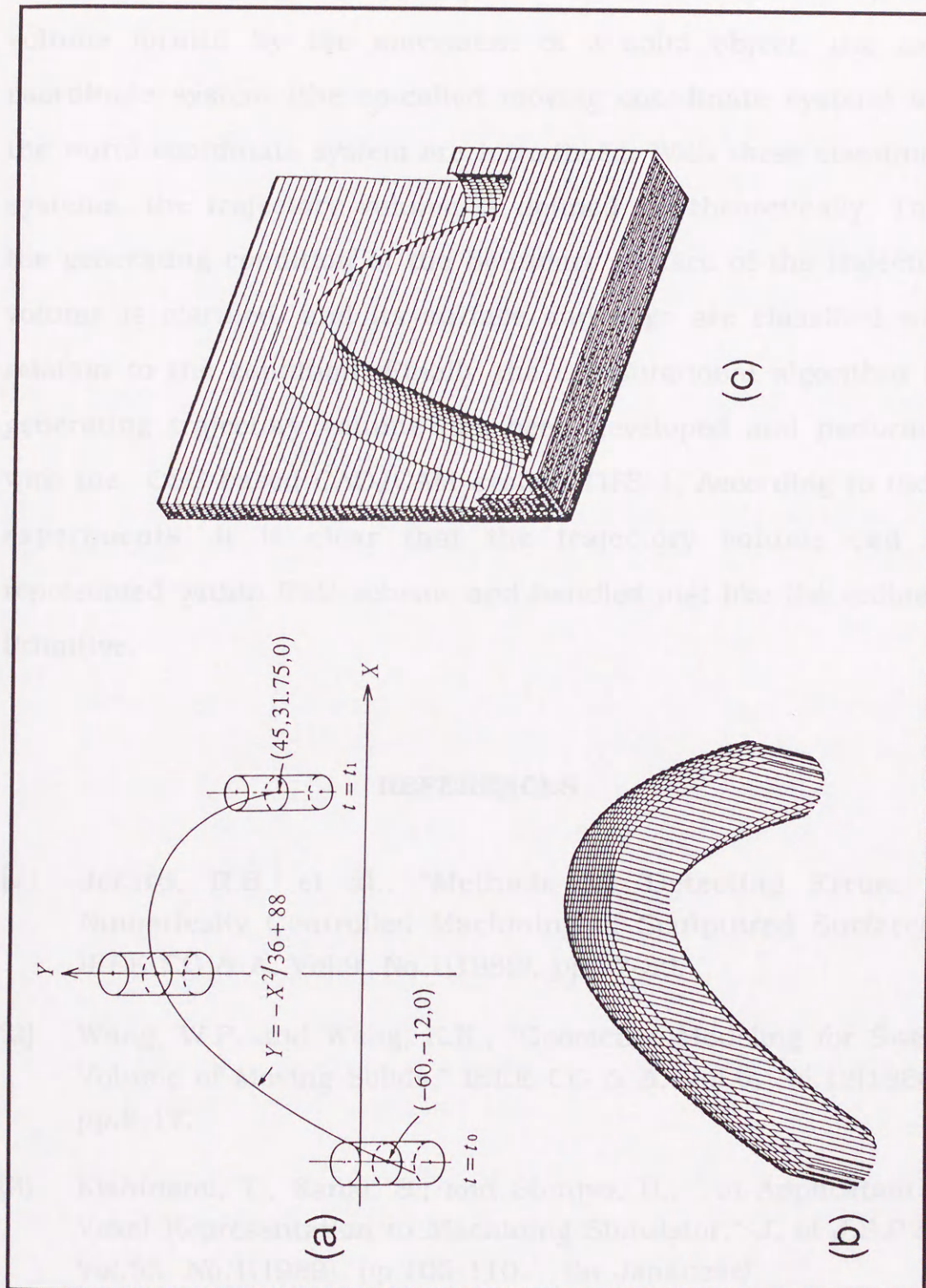


Fig.3-9 The trajectory volume formed by the movement of a cylindrical Primitive

envelope is actually derived for the simple example in which a sphere moves along a circular path. When defining the trajectory volume formed by the movement of a solid object, the local coordinate system (the so-called moving coordinate system) and the world coordinate system are introduced. With these coordinate systems, the trajectory volume is defined set-theoretically. Then the generating condition of the boundary surface of the trajectory volume is clarified, and its surface and edge are classified with relation to the condition. Finally the computational algorithm for generating trajectory volume has been developed and performed with the CSG-based CAD/CAM system TIPS-1. According to these experiments, it is clear that the trajectory volume can be represented within CSG scheme and handled just like the ordinary Primitive.

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4. A GENERALIZED SWEEPING METHOD

4.1 Introduction

There are many products which possess a natural axis, about which they generally have some degree of symmetry. These products are usually designed by defining a number of plane cross-sections normal to the axis, and interpolating them, or by defining a plane cross-section, and sweeping it along the path (i.e., the axis). These techniques are called the *cross-sectioning* or the *sweep*. In this paper, we refer to the technique as the sweeping method. The sweeping method is one of the earliest developed geometric modeling techniques[1][2]. In the early days, only a straight line and a circular arc were adopted for the sweeping path. Such sweeping method is called *the simple sweeping method*. In other words, the simple sweeping method consists of two sweeping techniques called the *extrusive* and *rotational sweeps*, as mentioned in chapter 1. With the advent of the sculptured surface modeling, the sweeping method has been more generalized, namely, more complicated 3-D curves such as parametrically defined curves have been used for the sweeping path. Such sweeping method is called *the generalized sweeping method*. Recently, the generalized sweeping method is available in almost all the surface modeling systems[3][4][5][6]. Only the simple sweeping method, however, is used restrictively in the current CSG-based systems. Moreover, in many CSG-based systems, the simple sweeping method is only used as an input medium and the input data are translated to the

canonical CSG data[7], because its capability is considered to be too small to use as primary representation. It is necessary to extend and generalize the simple sweeping method so that it can be used for more advanced applications. Therefore, a generalized sweeping method for the CSG modeling is proposed in this paper, and is applied to generate the solid model of the locus formed by sweeping an object along the path. In this chapter, we shall clarify the mathematical basis of the conventional sweeping method, and then explain the proposed sweeping method.

4.2 Linear axis sweep

4.2.1 General form of the linear axis sweep

Let us consider now the linear axis sweep. We assume that a plane cross-section to be swept is perpendicular to the linear axis. Hereafter, unless specially noted, the cross-section refers to a plane cross-section. We shall choose the z-axis as the axis of the sweep, so that the individual cross-sections will be members of the family of curves $x=x(u,v)$, $y=y(u,v)$. Here the parameter v distinguishes the individual cross-sections, and will be called *the profile parameter*. The cross-section parameter u describes the individual points on a given cross-sectional curve.

The variation of the cross-section along the z-axis can be described by defining z as a function of the parameter v . Thus the surface $\mathbf{r}$ to be generated by the linear axis sweep can be expressed as

$$\mathbf{r} = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(v)\mathbf{k}, \quad (4-1)$$

where $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ are unit vectors of the x, y , and z -axes respectively. Let an intersection point between the cross-section and the z -axis be an origin of the cross-section, for convenience. If the distance between the origin and a point on the cross-sectional curve is given by $\ell(u, v)$ as illustrated in Fig.4-1-(a), the x and y components of the point on the cross-sectional curve can be given as

$$\left. \begin{aligned} x(u, v) &= \ell(u, v) \cos u, \\ y(u, v) &= \ell(u, v) \sin u. \end{aligned} \right\} \quad (4-2)$$

Substituting Equation(4-2) into (4-1), the surface can be expressed as follows:

$$\mathbf{r} = \ell(u, v) \cos u \mathbf{i} + \ell(u, v) \sin u \mathbf{j} + z(v) \mathbf{k}. \quad (4-3)$$

This is the general form of the surface to be generated by the linear axis sweep.

4.2.2 The surface generated by the extrusive sweep

When the cross-section is extruded along the z -axis, the profile of the resulting surface is not varied even if the profile parameter v is varied, namely, $\ell = \ell(u)$ for all the v . (See Fig.4-1-(b).) Thus the surface to be generated by the extrusive sweep can be expressed as follows:

$$\mathbf{r} = \ell(u) \cos u \mathbf{i} + \ell(u) \sin u \mathbf{j} + z(v) \mathbf{k}. \quad (4-4)$$

4.2.3 The surface generated by the rotational sweep

Let us consider a plane which is parallel to the z -axis. If a plane curve on the plane is rotated about the z -axis, the resulting

surface is a surface of revolution. In this case, the sweeping path is considered to be a circular arc, and the cross-sectional curve on the cross-section which is perpendicular to the z -axis is a circle with radius $\ell(v)$, as illustrated in Fig.4-1-(c). Thus the surface to be generated by the rotational sweep can be expressed as follows:

$$\mathbf{r} = \ell(v) \cos u \mathbf{i} + \ell(v) \sin u \mathbf{j} + z(v) \mathbf{k} . \quad (4-5)$$

The surface of revolution can also be generated by sweeping a circular cross-section along the z -axis. In this case, the profile can be varied by defining z as a function of the parameter v . If the profile curve is equal to the plane curve for the rotational sweep, the resulting surface is equivalent to the surface of revolution generated by the rotational sweep.

4.3 General form of the sweep with a curved spine

Let us consider the design of the piping and ducting shapes. If the pipes and ducts are curved, their surfaces are usually designed in terms of a number of cross-sections normal to some *mean flow line*. This *mean flow line* is often known as *the spine*. If the spine curve is represented by the parametric form $\mathbf{r} = \mathbf{r}_s(v)$, then we may use two normal vectors (i.e., the principal normal vector $\mathbf{N}(v)$ and the binormal vector $\mathbf{B}(v)$) as unit vectors in a local coordinate system on the normal plane at each point of the spine. (See Fig.4-2.) If the cross-sectional curve on this normal plane is described parametrically by

$$\mathbf{s}(u, v) = f(u, v) \mathbf{N}(v) + g(u, v) \mathbf{B}(v) , \quad (4-6)$$

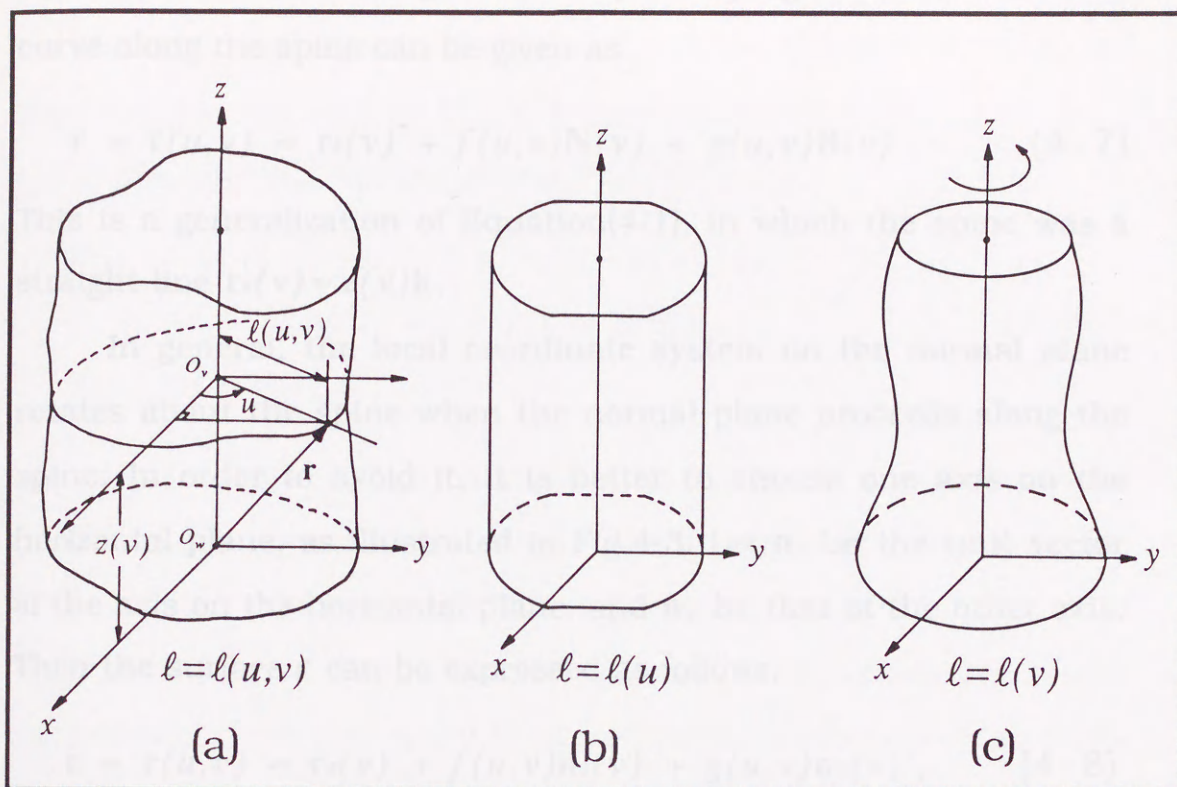


Fig.4-1 The linear axis sweep

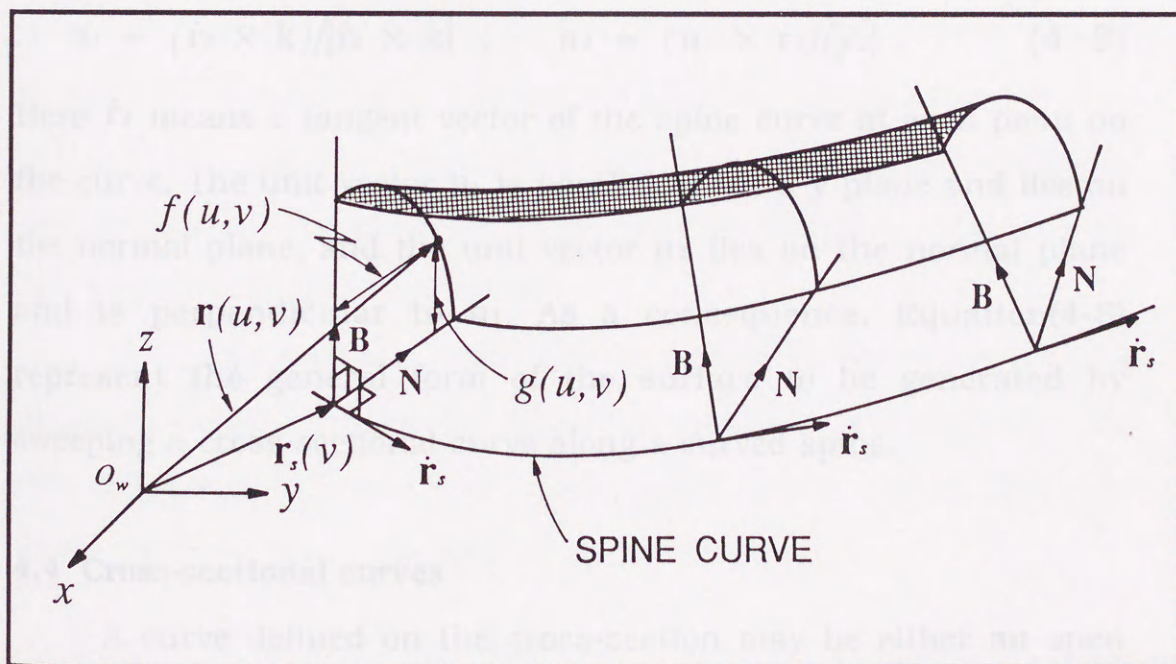


Fig.4-2 The sweep with a curved spine

then the surface to be generated by sweeping the cross-sectional curve along the spine can be given as

$$\mathbf{r} = \mathbf{r}(u, v) = \mathbf{r}_s(v) + f(u, v)\mathbf{N}(v) + g(u, v)\mathbf{B}(v) . \quad (4-7)$$

This is a generalization of Equation(4-1), in which the spine was a straight line $\mathbf{r}_s(v) = z(v)\mathbf{k}$.

In general, the local coordinate system on the normal plane rotates about the spine when the normal plane proceeds along the spine. In order to avoid it, it is better to choose one axis on the horizontal plane, as illustrated in Fig.4-3. Let $\mathbf{n}_1$ be the unit vector of the axis on the horizontal plane, and $\mathbf{n}_2$ be that of the other axis. Then the surface $\mathbf{r}$ can be expressed as follows:

$$\mathbf{r} = \mathbf{r}(u, v) = \mathbf{r}_s(v) + f(u, v)\mathbf{n}_1(v) + g(u, v)\mathbf{n}_2(v) , \quad (4-8)$$

where

$$\mathbf{n}_1 = (\dot{\mathbf{r}}_s \times \mathbf{k})/|\dot{\mathbf{r}}_s \times \mathbf{k}| , \quad \mathbf{n}_2 = (\mathbf{n}_1 \times \dot{\mathbf{r}}_s)/|\dot{\mathbf{r}}_s| . \quad (4-9)$$

Here $\dot{\mathbf{r}}_s$ means a tangent vector of the spine curve at each point on the curve. The unit vector $\mathbf{n}_1$ is parallel to the x-y plane and lies on the normal plane, and the unit vector $\mathbf{n}_2$ lies on the normal plane and is perpendicular to $\mathbf{n}_1$. As a consequence, Equation(4-8) represent the general form of the surface to be generated by sweeping a cross-sectional curve along a curved spine.

4.4 Cross-sectional curves

A curve defined on the cross-section may be either an open loop or a closed loop, and its equation may be defined by explicit, implicit or parametric forms. We wish to define the cross-sectional

curve in this section. We shall take the x - y plane as the cross-section, and the z -axis as the spine, for convenience. If we apply the Bezier cubic curve[8] to define a cross-sectional curve for a given value of v , its equation is expressed as follows:

$$\left. \begin{aligned} s(u, v) &= (1-u)^3 \mathbf{p}_0(v) + 3u(1-u)^2 \mathbf{p}_1(v) \\ &\quad + 3u^2(1-u) \mathbf{p}_2(v) + u^3 \mathbf{p}_3(v), \\ \text{and } z &= z(v), \end{aligned} \right\} \quad (4-10)$$

where $\mathbf{p}_0(v)$, $\mathbf{p}_1(v)$, $\mathbf{p}_2(v)$ and $\mathbf{p}_3(v)$ indicate the position vectors of the control points of the polygon for the cross-sectional curve at $z=z(v)$. If the cross-sectional curve possesses some symmetry, it is only necessary to design part of the curve, the remainder of the curve being generated using the coordinate transformation method. For example, the Bezier curve, as shown in Fig.4-4, has a 4-fold rotational symmetry. Thus we need only to define the segment in the first quadrant by writing $\mathbf{p}_0 = a\mathbf{i}$, $\mathbf{p}_1 = a\mathbf{i} + b\mathbf{j}$, $\mathbf{p}_2 = b\mathbf{i} + a\mathbf{j}$ and $\mathbf{p}_3 = a\mathbf{j}$. The remaining quadrants are obtained by reflections in the O - x and O - y axes. The longitudinal variation of the cross-section in Equation(4-10) is controlled by the polygons of the curves $\mathbf{r}_0(v) = \mathbf{p}_0(v) + z(v)\mathbf{k}$ and so on.

The cross-sectional curve may be defined by the union set of a number of closed curves. Let us define the cross-sectional curve by this way. Let S_i ($i=1, 2, \dots, m$) be elemental closed curves. Then the cross-sectional curve, denoted by S , is defined as

$$\left. \begin{aligned} S &= S_1 \cup S_2 \cup \dots \cup S_m \\ &= \bigcup_{i=1}^m S_i. \end{aligned} \right\} \quad (4-11)$$

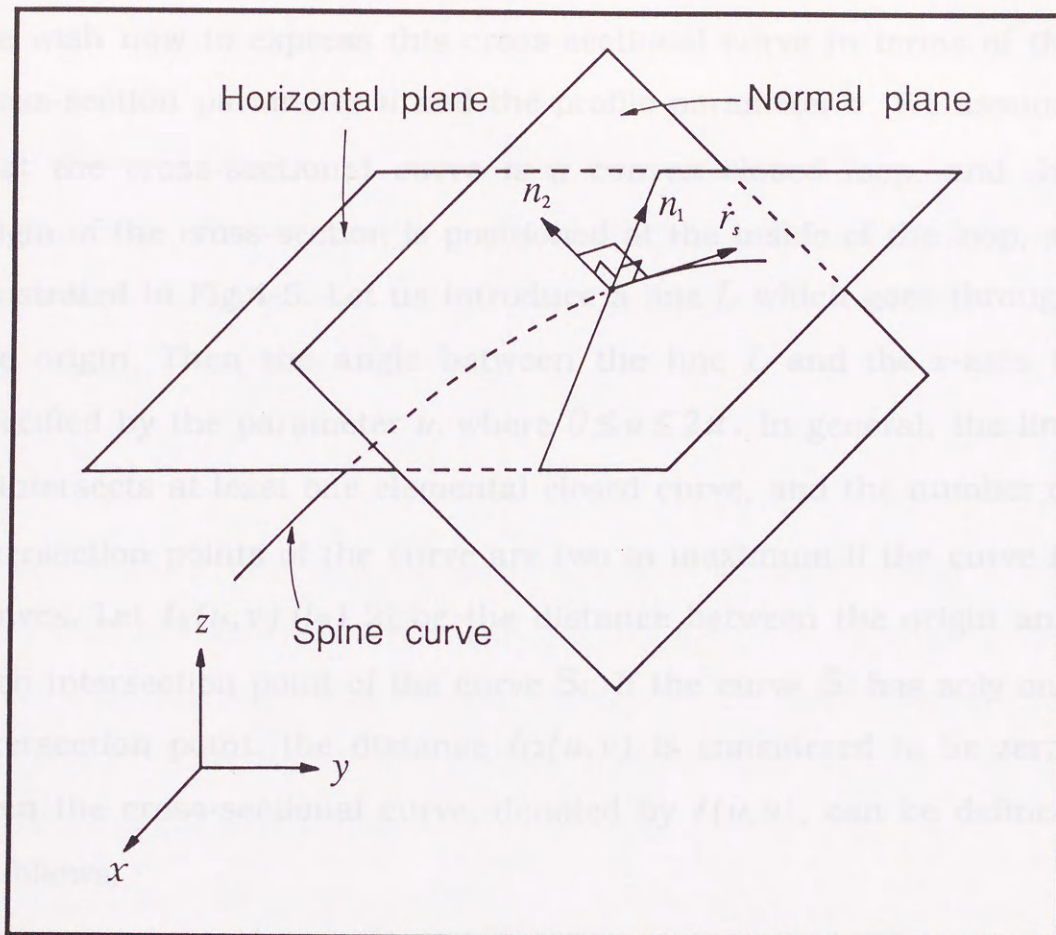


Fig.4-3 Spatial relationship among the vectors $\mathbf{n}_1$, $\mathbf{n}_2$ and $\dot{\mathbf{r}}_s$

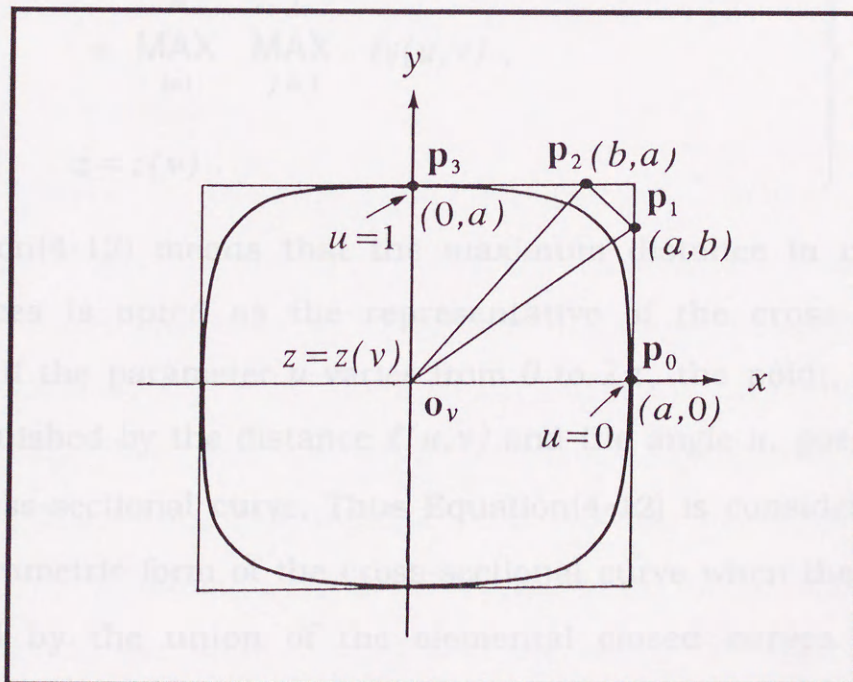


Fig.4-4 The cross-sectional curve defined by the Bezier curve

We wish now to express this cross-sectional curve in terms of the cross-section parameter u and the profile parameter v . We assume that the cross-sectional curve is a convex closed loop, and the origin of the cross-section is positioned at the inside of the loop, as illustrated in Fig.4-5. Let us introduce a line L which goes through the origin. Then the angle between the line L and the x -axis is specified by the parameter u , where $0 \leq u \leq 2\pi$. In general, the line L intersects at least one elemental closed curve, and the number of intersection points of the curve are two in maximum if the curve is convex. Let $\ell_{ij}(u, v)$ ($j=1,2$) be the distance between the origin and each intersection point of the curve S_i . If the curve S_i has only one intersection point, the distance $\ell_{i2}(u, v)$ is considered to be zero. Then the cross-sectional curve, denoted by $\ell(u, v)$, can be defined as follows:

$$\left. \begin{aligned} \ell(u, v) &= \text{MAX} (\ell_{11}(u, v), \ell_{12}(u, v), \dots, \ell_{i2}(u, v)) \\ &= \text{MAX}_{i=1}^m \text{MAX}_{j=1}^2 \ell_{ij}(u, v), \\ \text{and } z &= z(v). \end{aligned} \right\} \quad (4-12)$$

Equation(4-12) means that the maximum distance in candidate distances is opted as the representative of the cross-sectional curve. If the parameter u varies from 0 to 2π , the point, which is distinguished by the distance $\ell(u, v)$ and the angle u , goes around the cross-sectional curve. Thus Equation(4-12) is considered to be the parametric form of the cross-sectional curve when the curve is defined by the union of the elemental closed curves. If some elemental closed curves do not intersect with the line L , these

curves are precluded from the calculation with respect to Equation(4-12).

The cross-sectional curve may be defined by the intersection of a number of open curves. If the cross-sectional curve, denoted by S , is defined by this way, the equation is expressed as follows:

$$\left. \begin{aligned} S &= S_1 \cap S_2 \cap \cdots \cap S_m \\ &= \bigcap_{i=1}^m S_i, \end{aligned} \right\} \quad (4-13)$$

where S_i ($i=1,2,\cdots,m$) are elemental curves. We assume that all the elemental curves are open ones. (See Fig.4-6.) If we shall define the cross-sectional curve parametrically in the same way for the definition by the union set, the parametric equation of the cross-sectional curve is as follows:

$$\left. \begin{aligned} \ell(u,v) &= \text{MIN} (\ell_1(u,v), \ell_2(u,v), \cdots, \ell_m(u,v)) \\ &= \text{MIN}_{i=1}^m \ell_i(u,v), \\ \text{and } z &= z(v). \end{aligned} \right\} \quad (4-14)$$

Here $\ell_i(u,v)$ is the distance between the origin and the intersection point of the open curve S_i . Equation(4-14) means that the minimum distance in candidate distances is opted as the representative of the cross-sectional curve. If the parameter u varies from 0 to 2π , the point, which is distinguished by the distance $\ell(u,v)$ and the angle u , goes around the cross-sectional curve. Thus Equation(4-14) is considered to be the parametric form of the cross-sectional curve when the curve is defined by the intersection of the elemental open curves. If some elemental open

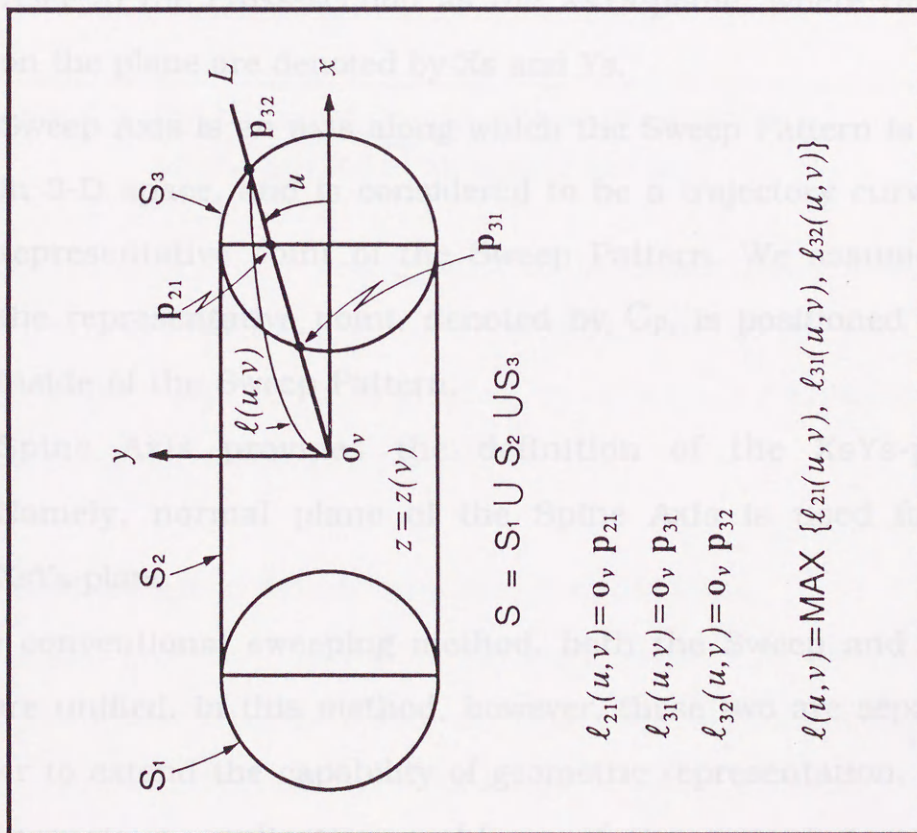


Fig.4-5 The cross-sectional curve defined by the union of closed curves

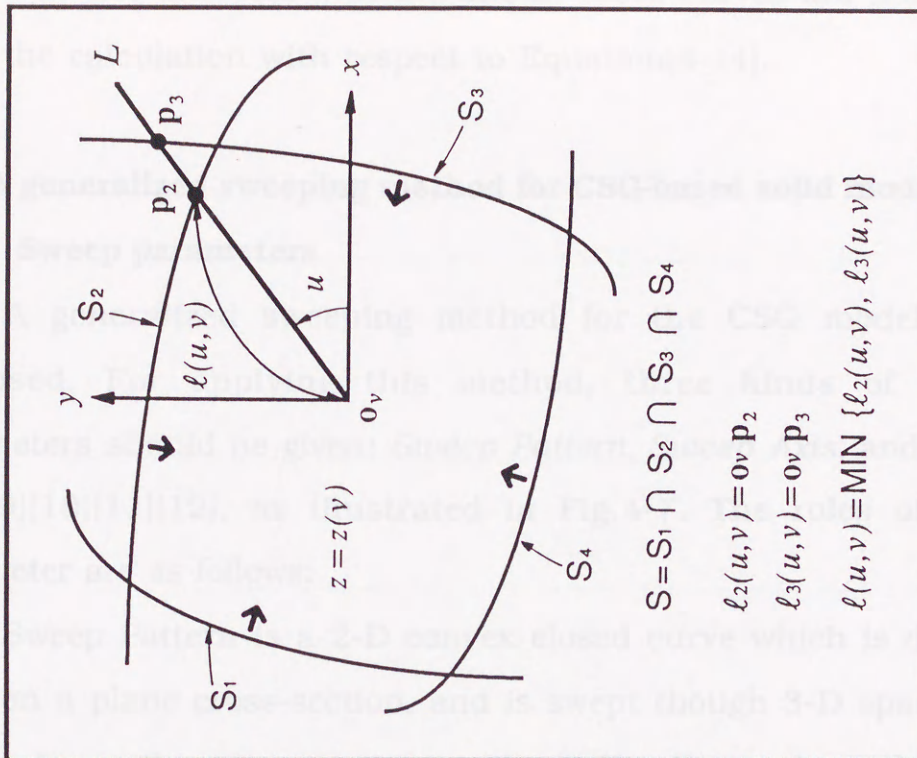


Fig.4-6 The cross-sectional curve defined by the intersection of open curves

curves do not intersect with the line L , these curves are precluded from the calculation with respect to Equation(4-14).

4.5 A generalized sweeping method for CSG-based solid modeling

4.5.1 Sweep parameters

A generalized sweeping method for the CSG modeling is proposed. For applying this method, three kinds of sweep parameters should be given; *Sweep Pattern*, *Sweep Axis*, and *Spine Axis*[9][10][11][12], as illustrated in Fig.4-7. The roles of each parameter are as follows:

- Sweep Pattern is a 2-D convex closed curve which is defined on a plane cross-section, and is swept through 3-D space. We refer to the cross-section as the X_sY_s -plane, where the axes on the plane are denoted by X_s and Y_s .
- Sweep Axis is an axis along which the Sweep Pattern is swept in 3-D space, and is considered to be a trajectory curve of a representative point of the Sweep Pattern. We assume that the representative point, denoted by C_p , is positioned at the inside of the Sweep Pattern.
- Spine Axis provides the definition of the X_sY_s -plane. Namely, normal plane of the Spine Axis is used for the X_sY_s -plane.

In the conventional sweeping method, both the Sweep and Spine Axes are unified. In this method, however, these two are separated in order to extend the capability of geometric representation.

In various application problems of the sweeping method, there are cases where the Spine Axis cannot be given explicitly.

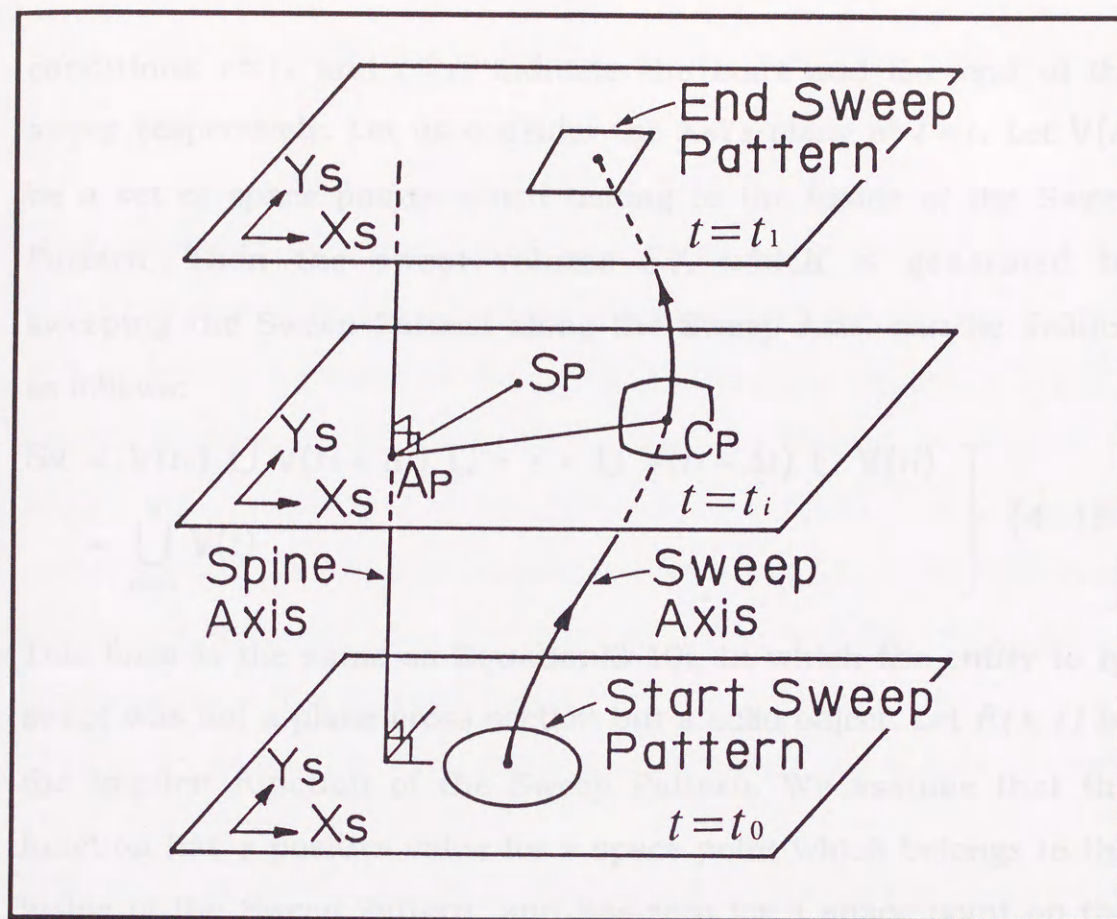


Fig.4-7 Spatial relationship among parameters of the proposed sweeping method

In order to apply the sweeping method to those cases, however, a rule of defining the X_sY_s -plane is required instead of the Spine Axis. The rule can be defined so as to adapt to each case. Examples of the rule are explained in chapters 5 and 6.

4.5.2 Set theoretical definition of the swept volume

As the geometric model of the locus formed by sweeping an object along the path is called a *trajectory volume* or a *swept volume*, hereafter we refer to it as swept volume. We shall define a swept volume set-theoretically in this section. Let t be a motion parameter defined on the Sweep Axis, where $t_0 \leq t \leq t_1$. The

conditions $t = t_0$ and $t = t_1$ indicate the start and the end of the sweep respectively. Let us consider the XsYs-plane at $t = t$. Let $V(t)$ be a set of space points which belong to the inside of the Sweep Pattern. Then the swept volume S_v , which is generated by sweeping the Sweep Pattern along the Sweep Axis, can be defined as follows:

$$\left. \begin{aligned} S_v &= V(t_0) \cup V(t_0 + \Delta t) \cup \dots \cup V(t_1 - \Delta t) \cup V(t_1) \\ &= \bigcup_{t=t_0}^{t_1} V(t) . \end{aligned} \right\} \quad (4-15)$$

This form is the same as Equation(3-10), in which the entity to be swept was not a plane cross-section but a solid object. Let $F(\mathbf{x}, t)$ be the implicit function of the Sweep Pattern. We assume that the function has a positive value for a space point which belongs to the inside of the Sweep Pattern, and has zero for a space point on the curve of the Sweep Pattern. Then the set $V(t)$ can be defined as

$$\left. \begin{aligned} V(t) &= \{ \mathbf{x} \mid F(\mathbf{x}, t) \geq 0 \} , \\ \text{where } t_0 \leq t \leq t_1 . \end{aligned} \right\} \quad (4-16)$$

Here $\mathbf{x}$ means the orthogonal coordinates of a space point. Thus a set of space points which belong to an entire swept volume can be expressed as follows:

$$S_v = \bigcup_{t=t_0}^{t_1} V(t) = \{ \mathbf{x} \mid F(\mathbf{x}, t) \geq 0, \forall t(t_0 \leq t \leq t_1) \} . \quad (4-17)$$

Equation(4-17) suggests that the space point evaluation (whether the point is inside, outside, or on the curve of the Sweep Pattern) is inevitable in order to generate the swept volume.

4.5.3 Problems to be solved for the application of the sweeping method

In evaluating a space point, if a XsYs-plane can be determined with respect to the space point, and if this XsYs-plane includes the space point, we can use the space point evaluation only by using this XsYs-plane as a substitute for using all the XsYs-planes during the sweep. Namely, in this case, we can use Equation(4-16) instead of (4-17). This means that the problem of the 3-D solid modeling can be reduced to that of the 2-D point evaluation for the plane curve. There are, however, two problems to be solved for evaluating a space point by this method. Namely,

- (1) How to lead a XsYs-plane in 3-D space from only a given space point.
- (2) How to derive the implicit function of the Sweep Pattern on the XsYs-plane.

How to solve these problems depends on the object to apply this sweeping method. Therefore we shall apply the sweeping method practically to the solid modeling problems in the following chapters.

4.6 Conclusion

In this chapter, first, the general form of the linear axis sweep is defined parametrically, and equations of the extrusive and rotational sweeps are derived from it respectively. Then the parametric form of the sweep with a curved spine is defined. This parametric form is considered to be the equation of the generalized sweeping method. The cross-sectional curve to be swept is defined

both parametrically and set-theoretically. Then a generalized sweeping method for the CSG modeling is proposed. Concerning the proposed method, the roles of the sweep parameters (i.e., Sweep Pattern, Sweep and Spine Axes) are described. Then the swept volume generated by sweeping the Sweep Pattern along the Sweep Axis is defined set-theoretically. Finally, problems to be solved for the application of the proposed method are clarified. The characteristic of the proposed method is that the 3-D solid modeling problem can be reduced to that of the 2-D cross-section.

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5. SOLID MODELING OF NC CUTTER MOVEMENT TRAJECTORY

5.1 Introduction

With the advance of NC (Numerical Control) machine tools, the NC machining has been widely used in manufacturing stage of industrial goods. The NC machining provides simpler, repeatable, and quick ways to cut the desired part out of a block of material. The correctness of the generated NC code, however, is often uncertain. Thus one of the most important problems to be solved for the NC machining is how to verify the correctness of the NC code. In order to solve it, two kinds of the verification methods have been applied conventionally as illustrated in Figs.5-1-(a) and (b); namely, *the point model* and *physical model methods*. In the point model method, the tool path (i.e., the trajectory curve formed by the movement of the cutter control point) is displayed and verified by the know-how and sense of imagination that NC programmers must use when operating the NC machine tool. In the physical model method, the NC machining tool is actually run in order to cut the miniature model of the desired shape out of a well-cut material such as wood, clay and wax, and then the resulting physical model is verified practically. The point model method is very simple, inexpensive and actually useful for brief verification of the simple program with three-axis motions of the NC machining. It is very difficult, however, to verify the complicated program with five-axis motions of the NC machining, because this method cannot take the cutter orientation and the cutting surface of the NC cutter

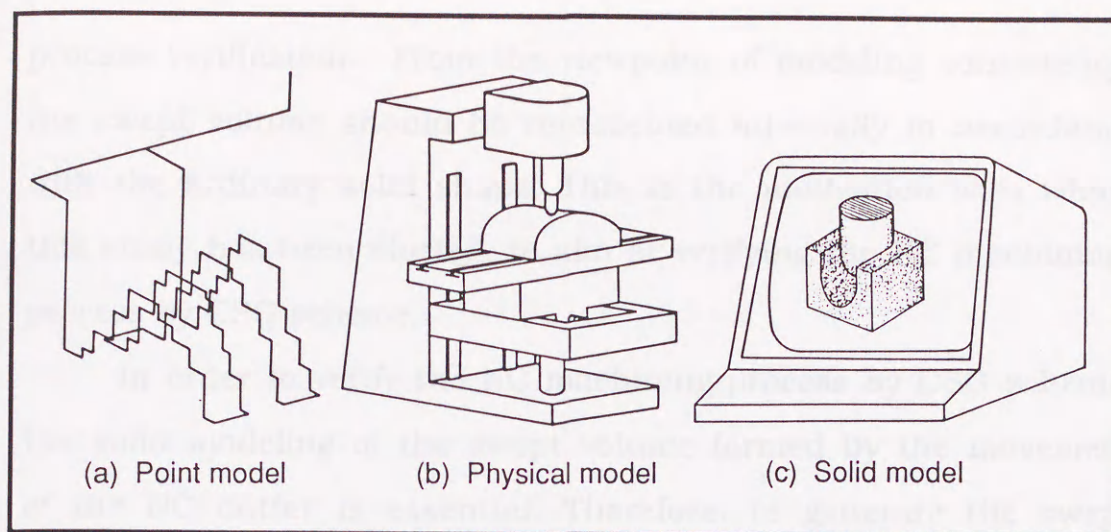


Fig.5-1 The NC code verification by the point, the physical, and the solid models

into consideration. This is due to the adoption of the point model instead of the solid one of the NC cutter. On the other hand, the physical model method is considerably strict, but is very time-consuming and expensive. In order to overcome the disadvantage of the above two methods, a solid modeling based NC machining-process verification is required. (See Fig.5-1-(c).)

Some interesting attempts have been reported[1][2][3][4] recently for the NC verification. In these attempts, the swept volume formed by the movement of the NC cutter has been handled and displayed by means of rendering techniques such as Scan Line algorithm[1], Z-buffer algorithm[2], Voxel representation[3], and Dixel hardware technique[4]. Though the rendered image of the swept volume is actually useful for visual verifications, the rendered image cannot be used for more advanced applications. For example, it is impossible to execute set operations between the rendered image and the internal solid models of its environments even though such operations are often required in the NC machining-

process verification. From the viewpoint of modeling consistency, the swept volume should be represented internally in accordance with the ordinary solid shape. This is the motivation with which this study has been started, to aim at verifying the NC machining-process by CSG scheme.

In order to verify the NC machining-process by CSG scheme, the solid modeling of the swept volume formed by the movement of the NC cutter is essential. Therefore, to generate the swept volume, we shall apply a new method[5] based on the proposed sweeping method. In this chapter, we shall describe the applied method.

5.2 Related works

Early efforts to achieve the graphic verification of the machining-process was devoted by Gossard and Tsuchiya[6], who developed a microcomputer-based system which verifies 2-D turned parts. Voelcker and Hunt[7] developed an experimental system by using the PADL modeling system[8]. That system verified several three-axis NC programs. Fridshal[9], at General Dynamics, developed the TIPS-1[10][11] based NC simulator. In that system, several special Primitives for the swept volume of the cutter movement was introduced. Wang[2][12] developed a system for real-time graphic verification of the NC tool path by using an extended Z-buffer algorithm and the scan-line algorithm. In this system, the enveloping surface of the swept volume was computed by sweeping a set of the critical curve and the edge on the bounding surface of the cutter shape. Hook[4] developed a hardware system for real-

time shaded display of the NC milling. Jerard[1] developed a sculptured surface based system for the NC machining verification. Kishinami[3] developed a voxel based NC machining simulator. Minami[13], at Toshiba Machine Co., developed the CSG-based commercial system GP/SIM for the NC machining-process verification. Kawashima[14], at Hitachi Ltd., developed a NC machining verification system. In this system, a combined data structure with Oct-tree and CSG was introduced so as to evaluate the correctness of the NC code quantitatively.

5.3 The principle of the NC machining-process simulation

The geometric simulation for the NC machining-process verification can be performed basically by calculating the interference between the swept volume formed by the movement of the NC cutter and its environments (i.e., the workpiece and the fixture to hold the workpiece). Mathematically, the interference can be defined by set theory. Let P_{rd} be the product model, W the workpiece, E the fixture, and S_v the swept volume. Then the machined part, denoted by P_m , can be expressed as

$$P_m = W - S_v . \quad (5-1)$$

We can get the delta volume by calculating the difference of the product model and the machined part. The delta volume can be expressed as

$$\Delta_u = P_m - P_{rd} , \quad (5-2)$$

$$\Delta_o = P_{rd} - P_m . \quad (5-3)$$

If $\Delta_u \neq \emptyset$, the under-cut area remains in the machined part, and reversely, if $\Delta_o \neq \emptyset$, there is the over-cut area in the machined part. Obviously, if $\Delta_u = 0$ or $\Delta_o = 0$, the machined part is equivalent to the product model. The collision G_c between the NC cutter and the fixture is expressed as

$$G_c = E \cap S_v . \quad (5-4)$$

If $G_c = \emptyset$, there is no collision. Fig.5-2 illustrates the principle of the geometric simulation for the NC machining-process verification. The product model, the workpiece and the fixture can be represented by using the ordinary Primitive. Thus if the swept volume can be represented by CSG scheme, these interferences can be calculated consistently within the unified method as shown in Fig.5-3. Therefore we shall try to represent the swept volume by CSG scheme in the following sections.

5.4 Description of the problem

We have already defined the swept volume and generated it in chapter 3. As a matter of course there are, however, different approaches to generate the swept volume. Here, we shall apply the proposed sweeping method to generate it. We shall limit the discussion to the swept volume formed by the movement of the NC cutter.

Let us consider now a moving solid object as illustrated in Fig.5-4. Let P be the moving object, and t ($t_0 \leq t \leq t_1$) be a motion parameter which designates time instances of the motion, and the initial and final instances of the motion be designated by t_0 and t_1

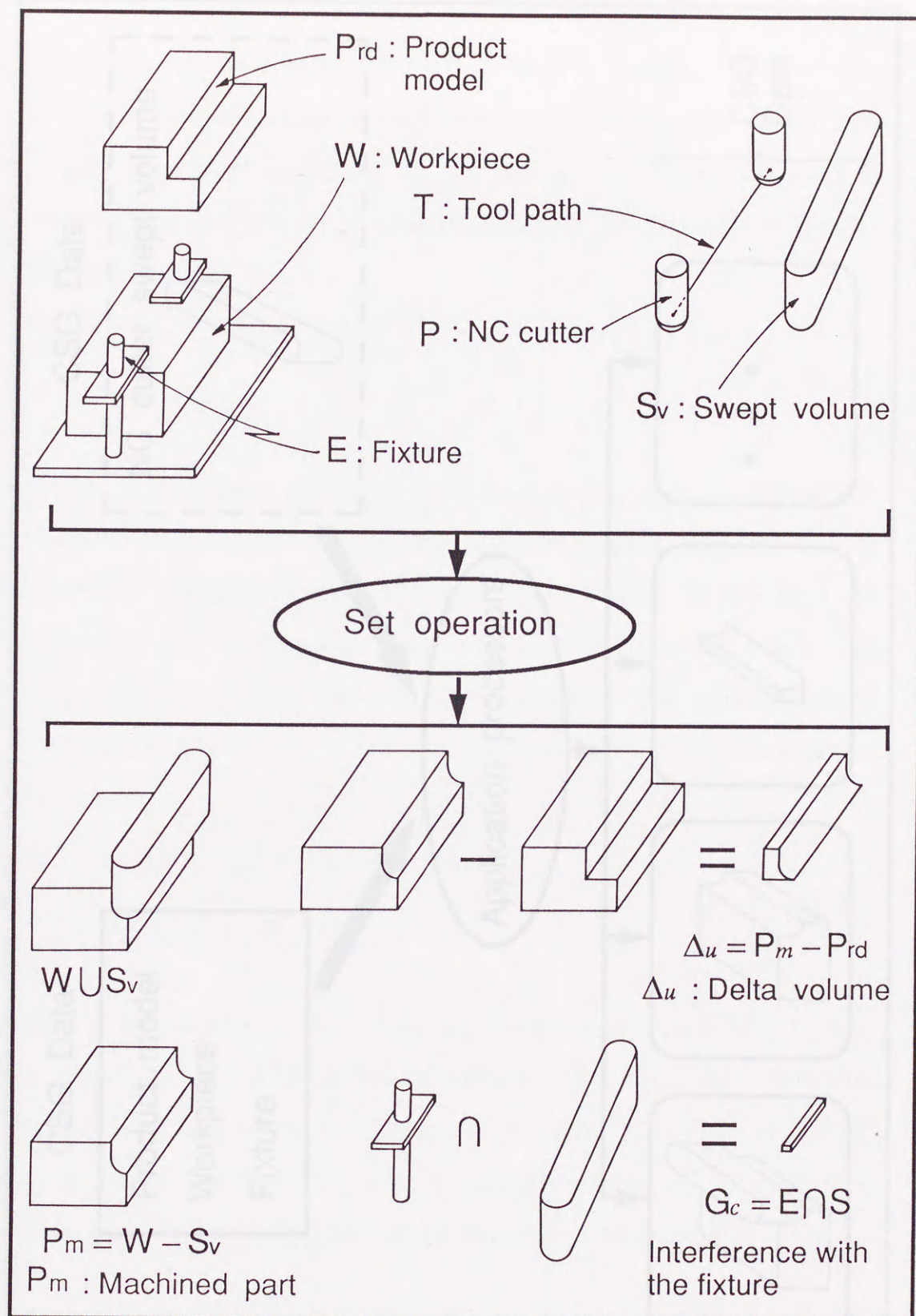


Fig.5-2 The principle of the NC simulation

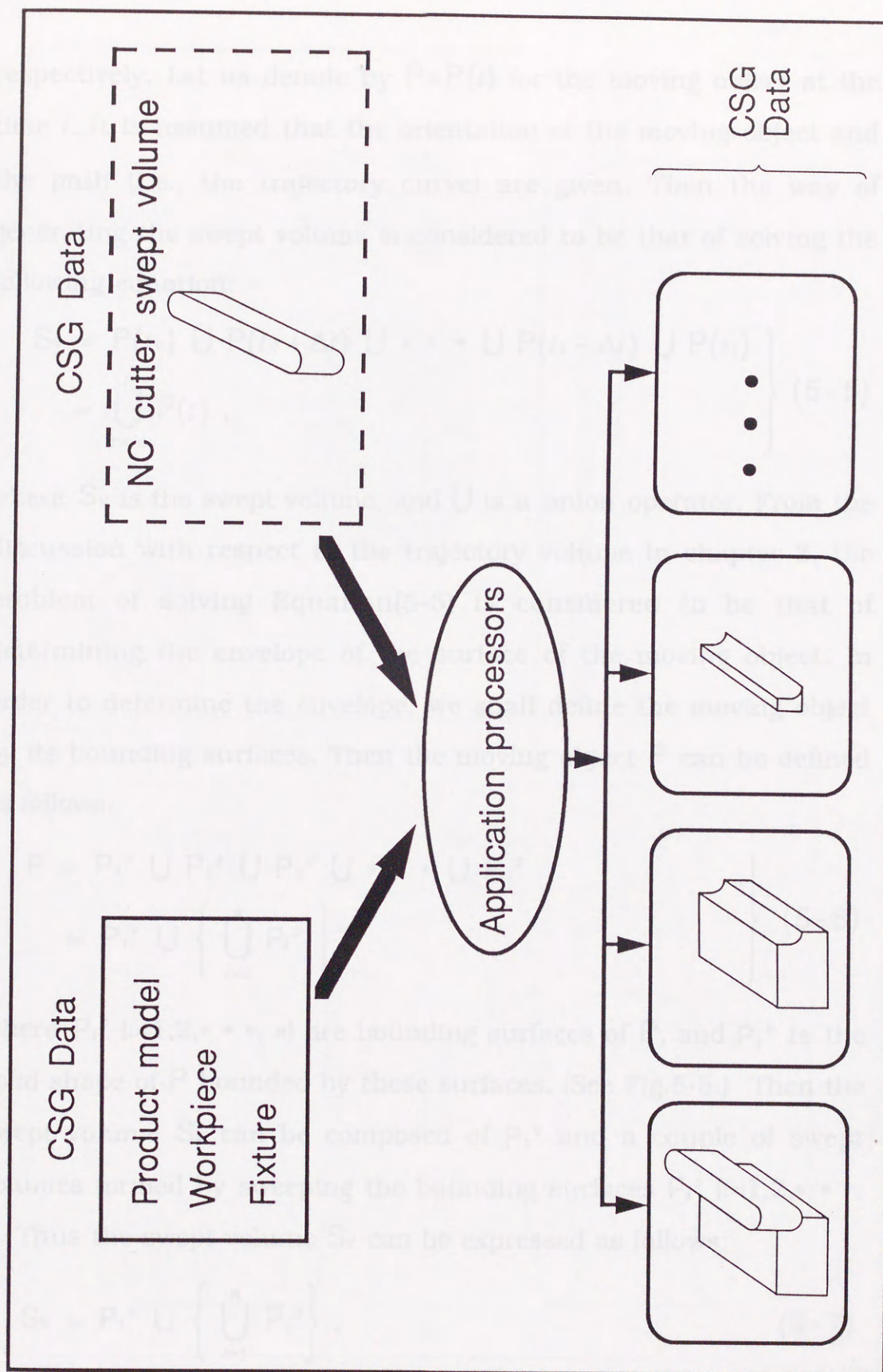


Fig.5-3 General flow for the NC simulation within CSG scheme

respectively. Let us denote by $P=P(t)$ for the moving object at the time t . It is assumed that the orientation of the moving object and the path (i.e., the trajectory curve) are given. Then the way of generating the swept volume is considered to be that of solving the following equation:

$$\left. \begin{aligned} S_v &= P(t_0) \cup P(t_0 + \Delta t) \cup \cdots \cup P(t_1 - \Delta t) \cup P(t_1) \\ &= \bigcup_{t=t_0}^{t_1} P(t), \end{aligned} \right\} \quad (5-5)$$

where S_v is the swept volume, and $\cup$ is a union operator. From the discussion with respect to the trajectory volume in chapter 3, the problem of solving Equation(5-5) is considered to be that of determining the envelope of the surface of the moving object. In order to determine the envelope, we shall define the moving object by its bounding surfaces. Then the moving object P can be defined as follows:

$$\left. \begin{aligned} P &= P_1^v \cup P_1^s \cup P_2^s \cup \cdots \cup P_n^s \\ &= P_1^v \cup \left\{ \bigcup_{i=1}^n P_i^s \right\}, \end{aligned} \right\} \quad (5-6)$$

where P_i^s ($i=1,2,\cdots,n$) are bounding surfaces of P , and P_1^v is the solid shape of P bounded by these surfaces. (See Fig.5-5.) Then the swept volume S_v can be composed of P_1^v and a couple of swept volumes formed by sweeping the bounding surfaces P_i^s ($i=1,2,\cdots,n$). Thus the swept volume S_v can be expressed as follows:

$$S_v = P_1^v \cup \left\{ \bigcup_{i=1}^n \bar{P}_i^s \right\}, \quad (5-7)$$

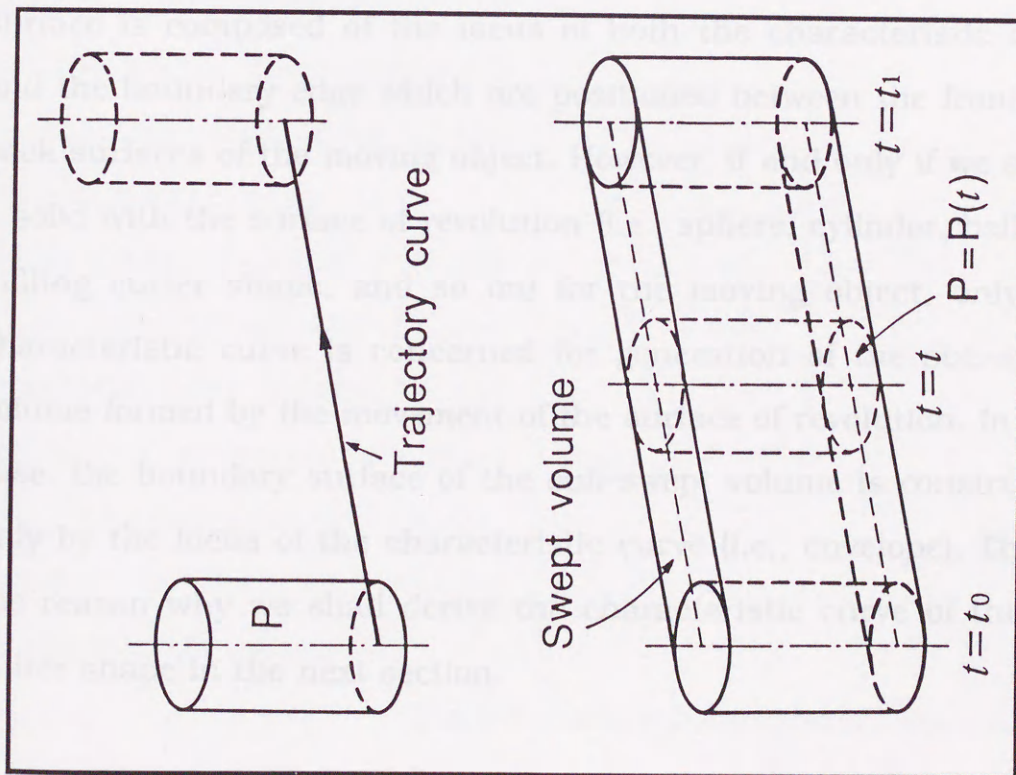


Fig.5-4 The moving solid object and its swept volume

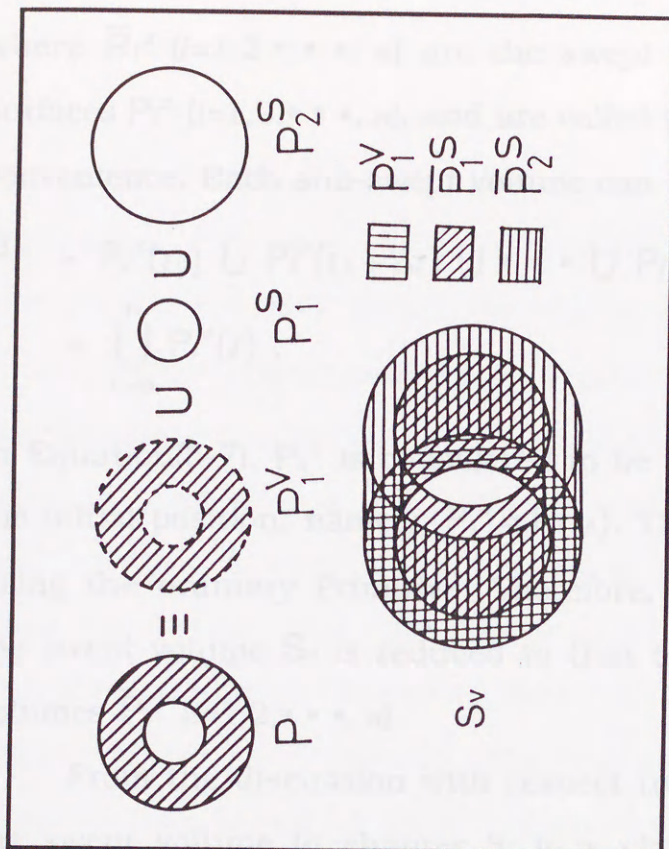


Fig.5-5 Decomposition of the moving solid object and the composition of the swept volume

where $\bar{P}_i^s$ ($i=1,2,\dots,n$) are the swept volumes of the bounding surfaces P_i^s ($i=1,2,\dots,n$), and are called the sub-swept volumes, for convenience. Each sub-swept volume can be expressed as

$$\left. \begin{aligned} \bar{P}_i^s &= P_i^s(t_0) \cup P_i^s(t_0 + \Delta t) \cup \dots \cup P_i^s(t_1 - \Delta t) \cup P_i^s(t_1) \\ &= \bigcup_{t=t_0}^{t_1} P_i^s(t) . \end{aligned} \right\} (5-8)$$

In Equation(5-7), P_1^v is considered to be the moving object itself at the initial position; namely, $P_1^v = P(t_0)$. This can be represented by using the ordinary Primitive. Therefore, the problem to generate the swept volume S_v is reduced to that to generate the sub-swept volumes $\bar{P}_i^s$ ($i=1,2,\dots,n$).

From the discussion with respect to the boundary surface of the swept volume in chapter 3, it is clarified that the boundary surface is composed of the locus of both the characteristic curve and the boundary edge which are positioned between the front and back surfaces of the moving object. However, if and only if we adopt a solid with the surface of revolution (i.e., sphere, cylinder, ball-end milling cutter shape, and so on) for the moving object, only the characteristic curve is concerned for generation of the sub-swept volume formed by the movement of the surface of revolution. In that case, the boundary surface of the sub-swept volume is constructed only by the locus of the characteristic curve (i.e., envelope). This is the reason why we shall derive the characteristic curve of the NC cutter shape in the next section.

5.5 Parametric representation of the characteristic curve

In general, the characteristic curve is defined mathematically by the simultaneous equations shown in Equation(3-5). In order to determine the envelope, the equations must be solved by eliminating a family parameter t from them. If the shape of the moving object and its motion are restricted to the simple case, the simultaneous equations can be solved analytically as explained the example in the section 3.2.2. That example, however, is a rare case. In general, it is very difficult to solve the equations analytically. Thus we shall use the parametric representation of the characteristic curve instead of Equation(3-5) for generation of the swept volume. We assume that the solid with the surface of revolution is adopted for the NC cutter shape. It is assumed that the position and the orientation of the NC cutter are represented by the trajectory curve of the representative point (i.e., the cutter control point on the axis of the revolution) and by the orientation of the axis of the revolution respectively. Then the characteristic curve, denoted by $C(t)$, can be represented as follows:

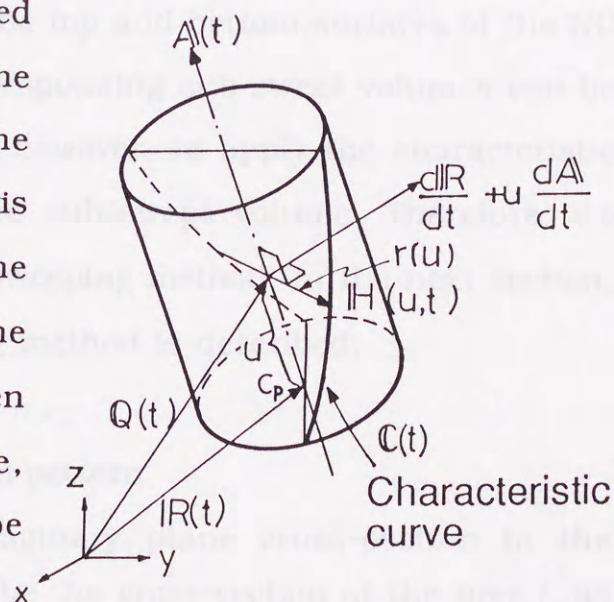


Fig.5-6 Parametric representation of the characteristic curve of the NC cutter

$$C(t) = R(t) + u \cdot A(t) + r(u) \cdot H(u, t), \quad (5-9)$$

where

$$\left. \begin{aligned} Q(u,t) &= R(t) + u \cdot A(t) , \\ H(u,t) &= \frac{\partial Q}{\partial t} \times A / \left| \frac{\partial Q}{\partial t} \times A \right| , \\ A &\neq k(\partial Q / \partial t) , \quad (k = \text{const.}) . \end{aligned} \right\} \quad (5-10)$$

Here $R(t)$ is a position vector of the cutter control point C_p , and $A(t)$ is a vector of the axis of revolution. $H(u,t)$ is a vector which is perpendicular to both the vectors $A(t)$ and $\partial Q / \partial t$ as shown in Fig.5-6. The parameter u designates the distance from the cutter control point C_p to an objective point on the axis $A(t)$, and $r(u)$ is the radius of the NC cutter on the cross-section at $u=u$. Theoretically, if the characteristic curve $C(t)$ is calculated from $t=t_0$ to $t=t_1$, the sub-swept volume formed by the envelope of the surface of the NC cutter can be generated. Similarly, if the characteristic curves of both the top and bottom surfaces of the NC cutter are calculated, the corresponding sub-swept volumes can be generated. It is very difficult, however, to apply the characteristic curve directly to generate the sub-swept volume. Therefore, we shall introduce the proposed sweeping method. In the next section, how to introduce the sweeping method is described.

5.6 Definition of the enveloped pattern

Let us consider an imaginary plane cross-section in the middle of the tool path. Let L_t be the cross-section at the time t , as illustrated in Fig.5-7. When the NC cutter goes through the cross-section, it leaves a trace on the cross-section. In general, this trace turns out to be a closed profile curve, and the shape and the

dimension of the profile curve are varied in accordance with the position of the NC cutter. Let us define the profile curve. We assume that the NC cutter leaves its trace on the cross-section L_t from $t = t_i$ to $t = t_j$, where $(t_0 \leq t_i \leq t \leq t_j \leq t_1)$. Let $E(t_i)$ be the profile curve left by the NC cutter at $t = t_i$. Then the union set of all the profile curves on the cross-section L_t are

$$\left. \begin{aligned} E_p &= E(t_i) \cup E(t_i + \Delta t) \cup \dots \cup E(t_j - \Delta t) \cup E(t_j) \\ &= \bigcup_{t=t_i}^{t_j} E(t) . \end{aligned} \right\} (5-11)$$

The union set E_p constructs the 2-D pattern with the maximum profile. This pattern is called *the enveloped pattern*. The enveloped pattern is considered to be the cross-sectional one obtained by cutting the swept volume S_v with the cross-section L_t . Thus the enveloped pattern can also be defined as

$$E_p = S_v \cap L_t . \quad (5-12)$$

As the swept volume S_v is composed of the sub-swept volumes, so is the enveloped pattern composed of sub-enveloped patterns left by the individual bounding surfaces of the NC cutter. Let E_i^s ($i=1, 2, \dots, n$) be the sub-enveloped patterns corresponding to the bounding surfaces of the NC cutter. Then the enveloped pattern can also be defined as

$$E_p = E_1^s \cup E_2^s \cup \dots \cup E_n^s = \bigcup_{i=1}^n E_i^s . \quad (5-13)$$

Actually, the sub-enveloped pattern is the trace left by the characteristic curve of the surface of the NC cutter. Thus the

characteristic curve represented in the previous section can be used for derivation of the sub-enveloped pattern. If the enveloped pattern E_p can be derived consequently, the swept volume S_v can be generated reversely by sweeping the enveloped pattern along the tool path. This is the principle of the application of the sweeping method to generate the swept volume.

5.7 Classification of the shape and the motion of the NC cutter

In general, the enveloped pattern depends on the shape and the motion of the NC cutter. Before deriving the enveloped pattern, we shall classify the shape and the motion of the NC cutter.

5.7.1 The NC cutter shapes

The prepared NC cutter shapes are illustrated in Fig.5-8; namely, a sphere, a cylinder and the ball-end milling cutter. A cylinder can be used as the flat-end milling cutter. Since the shape of the ball-end milling cutter is represented by the union set of the sphere and the cylinder, the enveloped pattern of the ball-end milling cutter can be represented by that of the sphere and the cylinder.

5.7.2 The NC cutter motions

The motion of the NC cutter is classified on the following assumptions:

- The tool path is given as the trajectory curve of the cutter control point C_p .

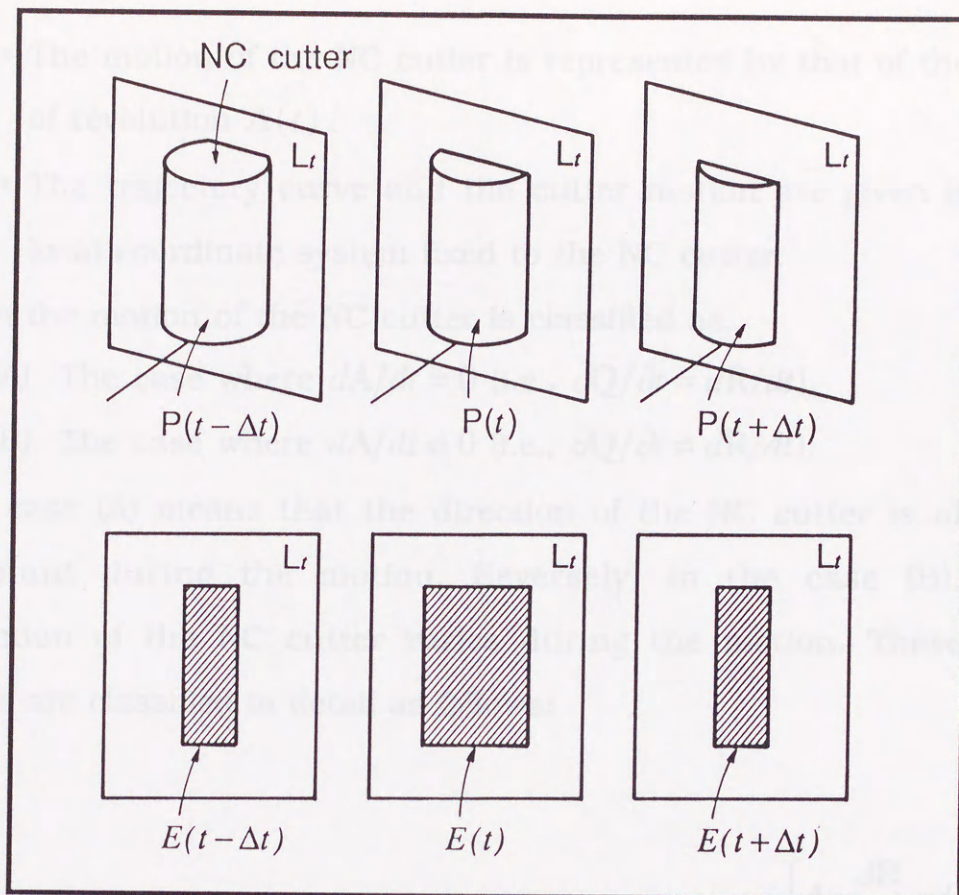


Fig.5-7 Concept of the enveloped pattern

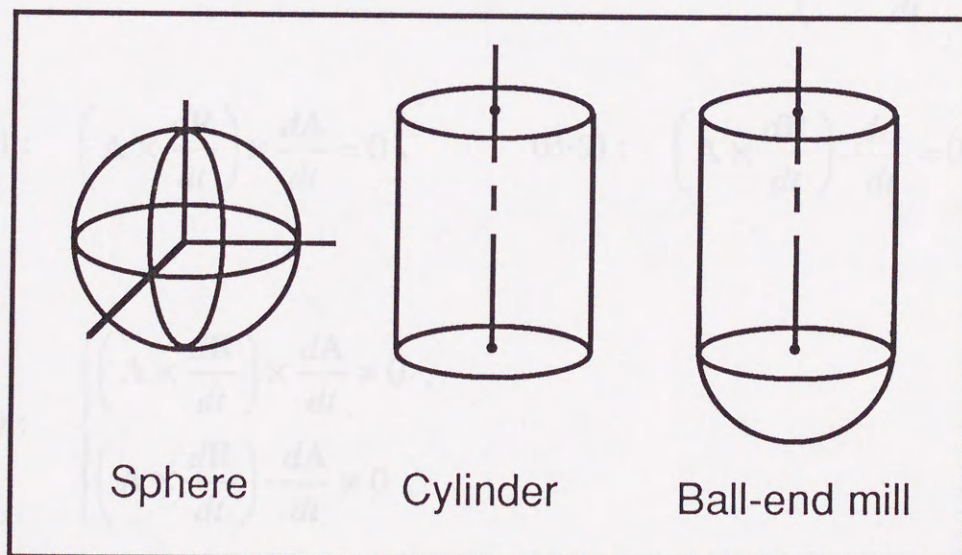


Fig.5-8 The prepared NC cutter shapes

- The motion of the NC cutter is represented by that of the axis of revolution $\mathbf{A}(t)$.
- The trajectory curve and the cutter motion are given in the local coordinate system fixed to the NC cutter.

Then the motion of the NC cutter is classified as,

(A) The case where $d\mathbf{A}/dt = 0$ (i.e., $\partial\mathbf{Q}/\partial t = d\mathbf{R}/dt$).

(B) The case where $d\mathbf{A}/dt \neq 0$ (i.e., $\partial\mathbf{Q}/\partial t \neq d\mathbf{R}/dt$).

The case (A) means that the direction of the NC cutter is always constant during the motion. Reversely, in the case (B), the direction of the NC cutter varies during the motion. These two cases are classified in detail as follows:

$$(A-1) : \mathbf{A} \cdot \frac{d\mathbf{R}}{dt} = 0, \quad (A-2) : \mathbf{A} \times \frac{d\mathbf{R}}{dt} = 0, \quad (A-3) : \begin{cases} \mathbf{A} \cdot \frac{d\mathbf{R}}{dt} \neq 0, \\ \mathbf{A} \times \frac{d\mathbf{R}}{dt} \neq 0. \end{cases}$$

$$(B-1) : \left(\mathbf{A} \times \frac{d\mathbf{R}}{dt} \right) \times \frac{d\mathbf{A}}{dt} = 0, \quad (B-2) : \left(\mathbf{A} \times \frac{d\mathbf{R}}{dt} \right) \cdot \frac{d\mathbf{A}}{dt} = 0,$$

$$(B-3) : \begin{cases} \left(\mathbf{A} \times \frac{d\mathbf{R}}{dt} \right) \times \frac{d\mathbf{A}}{dt} \neq 0, \\ \left(\mathbf{A} \times \frac{d\mathbf{R}}{dt} \right) \cdot \frac{d\mathbf{A}}{dt} \neq 0. \end{cases}$$

These classified motions are illustrated in Fig.5-9 respectively. Hereafter, we refer to the case (A-1) as the (A-1) motion, and the other cases are referred to in the same form. The (A-1) motion

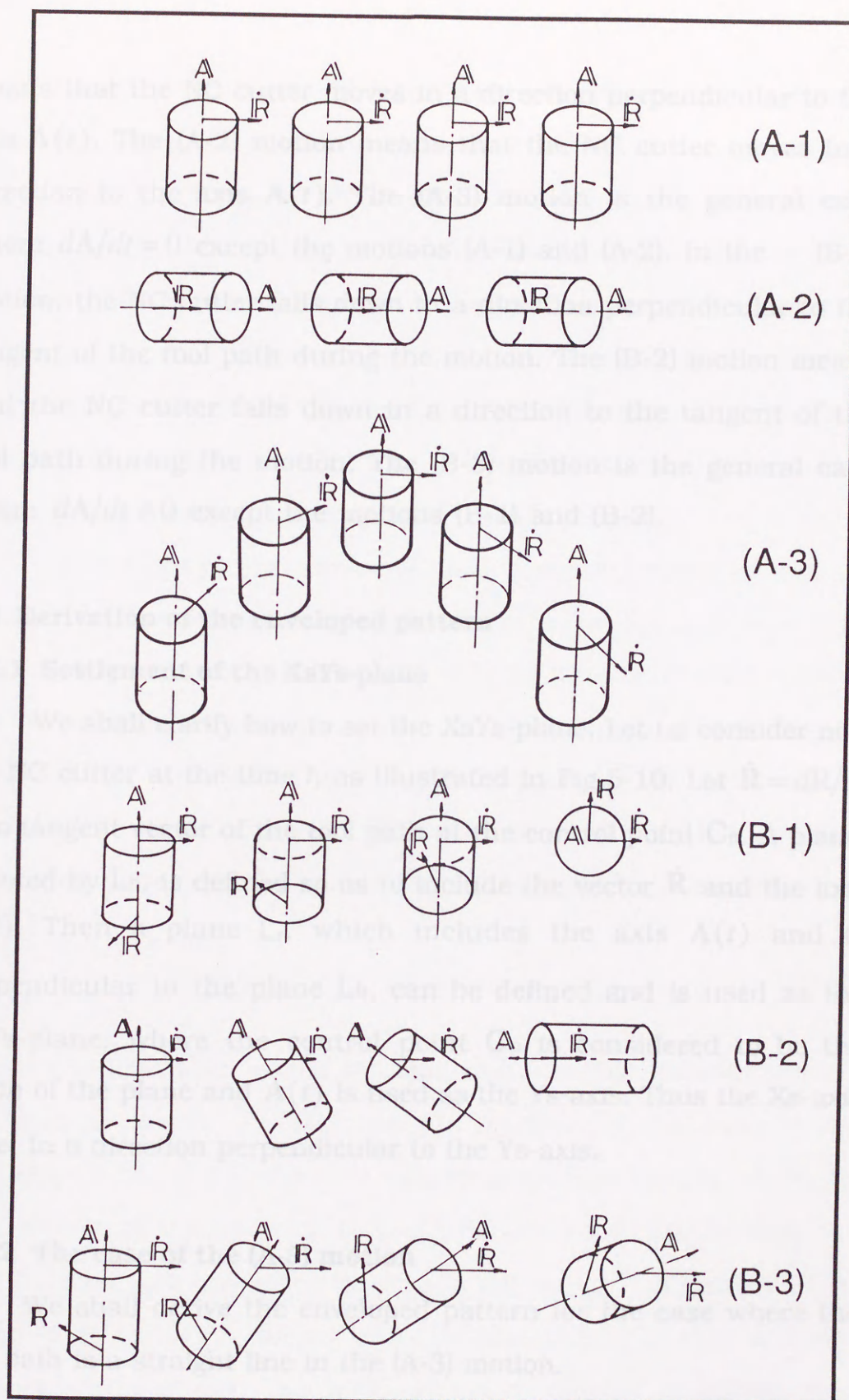


Fig.5-9 The classified motions of the NC cutter

means that the NC cutter moves in a direction perpendicular to the axis $A(t)$. The (A-2) motion means that the NC cutter moves in a direction to the axis $A(t)$. The (A-3) motion is the general case where $dA/dt = 0$ except the motions (A-1) and (A-2). In the (B-1) motion, the NC cutter falls down in a direction perpendicular to the tangent of the tool path during the motion. The (B-2) motion means that the NC cutter falls down in a direction to the tangent of the tool path during the motion. The (B-3) motion is the general case where $dA/dt \neq 0$ except the motions (B-1) and (B-2).

5.8 Derivation of the enveloped pattern

5.8.1 Settlement of the XsYs-plane

We shall clarify how to set the XsYs-plane. Let us consider now the NC cutter at the time t , as illustrated in Fig.5-10. Let $\dot{\mathbf{R}} = d\mathbf{R}/dt$ be a tangent vector of the tool path at the control point C_p . A plane, denoted by L_b , is defined so as to include the vector $\dot{\mathbf{R}}$ and the axis $A(t)$. Then a plane L_t , which includes the axis $A(t)$ and is perpendicular to the plane L_b , can be defined and is used as the XsYs-plane, where the control point C_p is considered to be the origin of the plane and $A(t)$ is used as the Ys-axis. Thus the Xs-axis is set in a direction perpendicular to the Ys-axis.

5.8.2 The case of the (A-3) motion

We shall derive the enveloped pattern for the case where the tool path is a straight line in the (A-3) motion.

Let a cylinder be the flat-end milling cutter, and L_t be the XsYs-plane at the time t , as illustrated in Fig.5-11. Let θ be the

angle between the vector $\dot{\mathbf{R}}$ and the top surface of the cylinder. When the vector $\dot{\mathbf{R}}$ directs to the upper side of the top surface of the cylinder, the sign of θ is positive. Reversely, if the vector $\dot{\mathbf{R}}$ directs to the lower side of the top surface, the sign of θ is negative. In the case mentioned above, the characteristic curves of the cylindrical surface are straight lines, and appear as the intersection lines of the cylinder and the XsYs-plane. Thus the sub-enveloped pattern left by the cylindrical surface becomes a rectangle, as shown in Fig.5-11. (See the shaded area.)

Let us derive the sub-enveloped pattern formed by the top surface of the cylinder. In order to express the sub-enveloped pattern, a parameter s is introduced. This parameter distinguishes the proceeded distance of the cutter control point C_p during the time Δt . The top surface of the cylinder at $t = t - \Delta t$ leaves a trace on the XsYs-plane at $t = t$, as illustrated in Fig.5-12. Then the sub-enveloped pattern of the top surface of the cylinder is expressed as

$$\left. \begin{aligned} Y_s &= h + s \cdot \sin \theta \\ X_s &= \pm \sqrt{r^2 - (s \cdot \cos \theta)^2} \end{aligned} \right\} \quad (5-14)$$

where r is the radius of the cylinder, and h is the distance between the control point C_p and the top surface of the cylinder. In Equation(5-14), if the objective sub-enveloped pattern is left by the top surface of the cylinder at $t = t - \Delta t$, then $s = -|s|$. Reversely, if the objective sub-enveloped pattern is left by the top surface of the cylinder at $t = t + \Delta t$, then $s = |s|$. By eliminating the parameter s from Equation(5-14), the sub-enveloped pattern is obtained as follows:

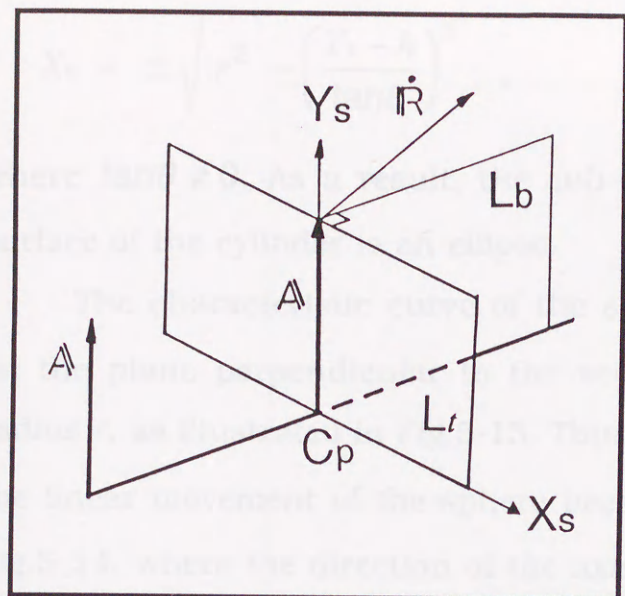


Fig.5-10 The settlement of the X_sY_s -plane

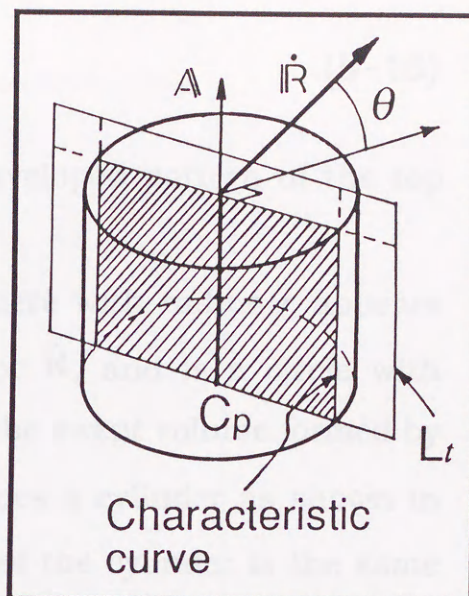


Fig.5-11 The sub-enveloped pattern left by the cylindrical surface
(The case of the (A-3) motion)

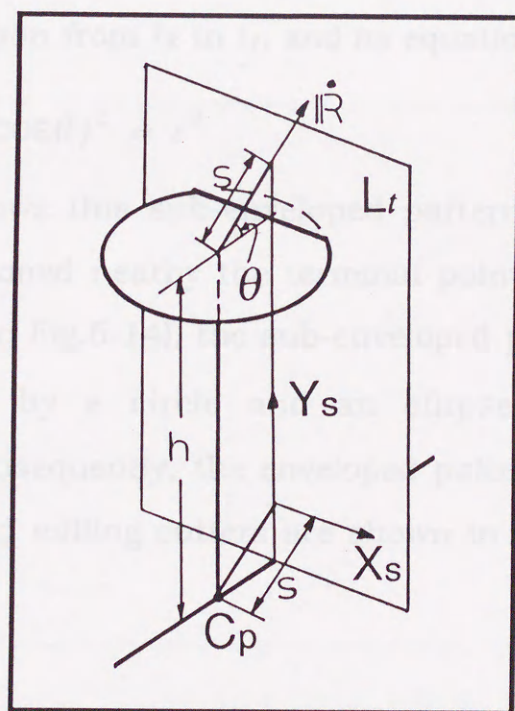


Fig.5-12 Derivation of the sub-enveloped pattern left by the top surface of the cylinder
(The case of the (A-3) motion)

$$X_s = \pm \sqrt{r^2 - \left(\frac{Y_s - h}{\tan \theta} \right)^2}, \quad (5-15)$$

where $\tan \theta \neq 0$. As a result, the sub-enveloped pattern of the top surface of the cylinder is an ellipse.

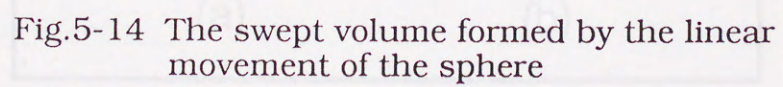
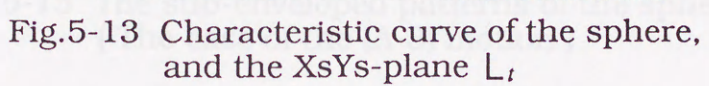
The characteristic curve of the sphere with radius r appears on the plane perpendicular to the vector $\dot{\mathbf{R}}$, and is a circle with radius r , as illustrated in Fig.5-13. Thus the swept volume formed by the linear movement of the sphere becomes a cylinder as shown in Fig.5-14, where the direction of the axis of the cylinder is the same as the vector $\dot{\mathbf{R}}$. But this cylinder does not include the remained volume at the initial and final positions of the sphere. In Fig.5-14, the sub-enveloped pattern of the sphere becomes an ellipse within the moving section from t_k to t_l , and its equation is as follows:

$$X_s^2 + (Y_s \cdot \cos \theta)^2 = r^2. \quad (5-16)$$

Fig.5-15-(a) shows this sub-enveloped pattern. When the moving sphere is positioned nearby the terminal point of the motion (for instance, at t_m in Fig.5-14), the sub-enveloped pattern of the sphere is constructed by a circle and an ellipse, as illustrated in Fig.5-15-(b). Consequently, the enveloped patterns of both the flat-end and ball-end milling cutters are shown in Figs.5-16-(a) and (b) respectively.

5.8.3 The case of the (B-1) motion

We shall derive the enveloped pattern for the case where the tool path is a straight line in the (B-1) motion. In this case, the NC



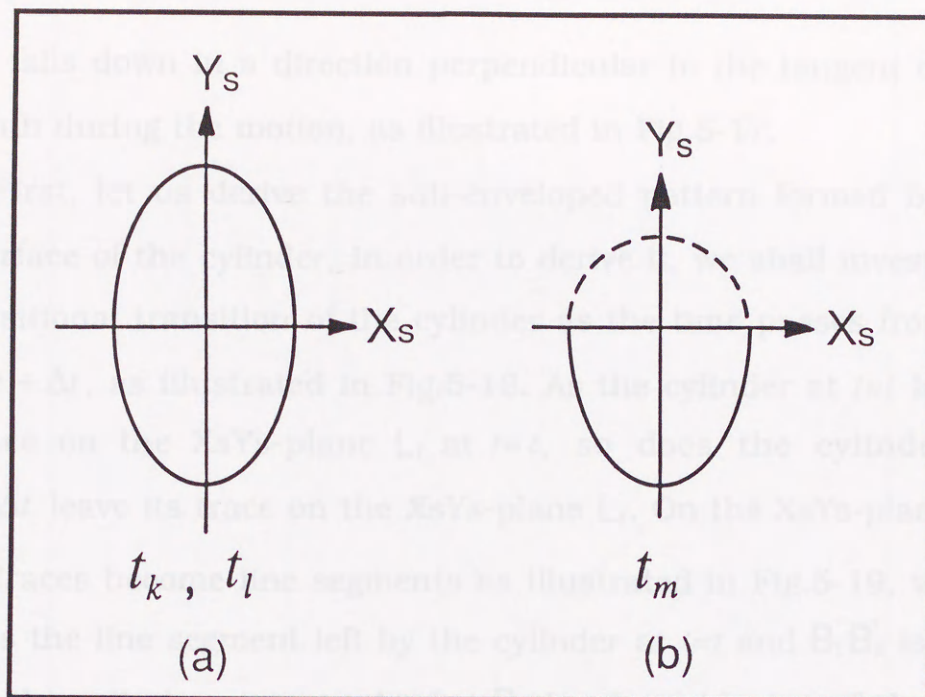


Fig.5-15 The sub-enveloped patterns of the sphere
(The case of the (A-3) motion)

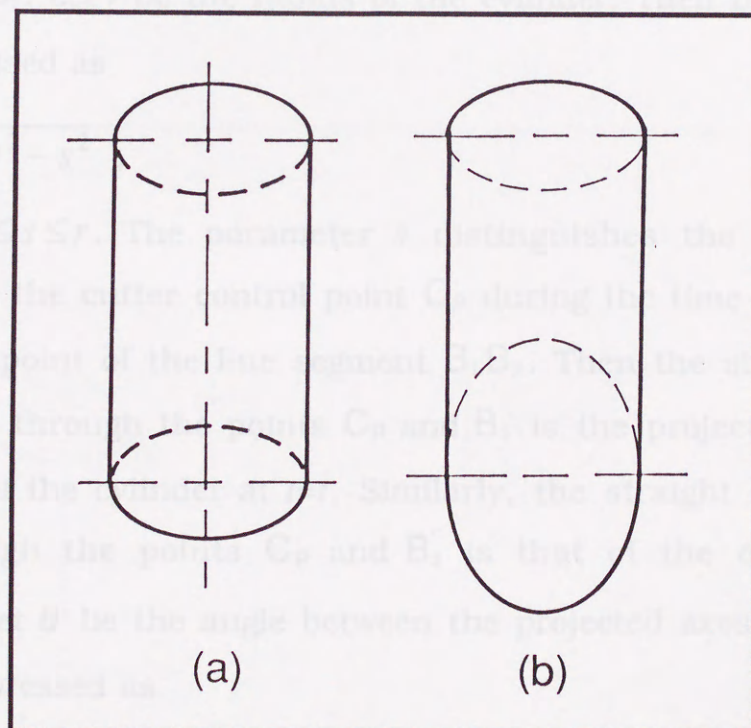


Fig.5-16 The derived enveloped patterns of the
flat-end and ball-end milling cutters
(The case of the (A-3) motion)

cutter falls down in a direction perpendicular to the tangent of the tool path during the motion, as illustrated in Fig.5-17.

First, let us derive the sub-enveloped pattern formed by the top surface of the cylinder. In order to derive it, we shall investigate the positional transition of the cylinder as the time passes from $t=t$ to $t=t+\Delta t$, as illustrated in Fig.5-18. As the cylinder at $t=t$ leaves its trace on the XsYs-plane L_t at $t=t$, so does the cylinder at $t=t+\Delta t$ leave its trace on the XsYs-plane L_t . On the XsYs-plane L_t , these traces become line segments as illustrated in Fig.5-19, where B_1B_2 is the line segment left by the cylinder at $t=t$ and $B'_1B'_2$ is that left by the cylinder at $t=t+\Delta t$. Let B_3 be the midpoint of the line segment B_1B_2 , and r_s be the distance between the midpoint B_3 and the point B'_1 . Let r be the radius of the cylinder. Then the distance r_s is expressed as

$$r_s = \sqrt{r^2 - s^2}, \quad (5-17)$$

where $-r \leq s \leq r$. The parameter s distinguishes the proceeded distance of the cutter control point C_p during the time Δt . Let B_3 be the midpoint of the line segment B_1B_2 . Then the straight line which goes through the points C_p and B_3 is the projection of the axis $A(t)$ of the cylinder at $t=t$. Similarly, the straight line which goes through the points C_p and B'_3 is that of the cylinder at $t=t+\Delta t$. Let θ be the angle between the projected axes. Then the angle is expressed as

$$\theta = s \cdot |\dot{A}|, \quad (5-18)$$

where $\dot{A} = dA/dt$. Thus the coordinates of the point B'_1 are

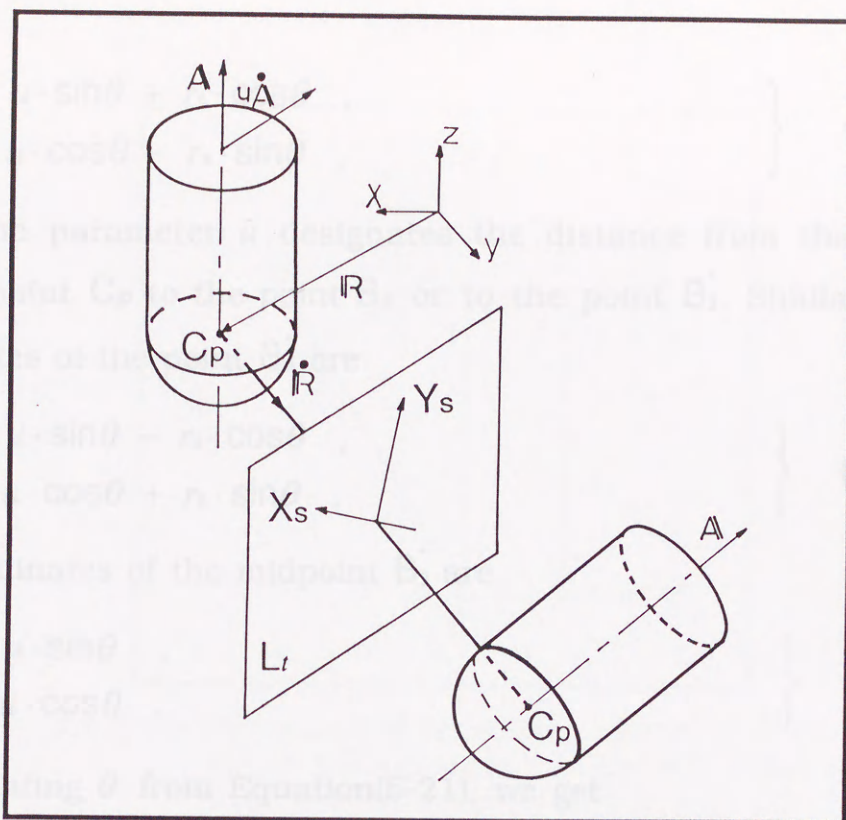


Fig.5-17 Spatial relationship between the NC cutter and the X_sY_s -plane
(The case of the (B-1) motion)

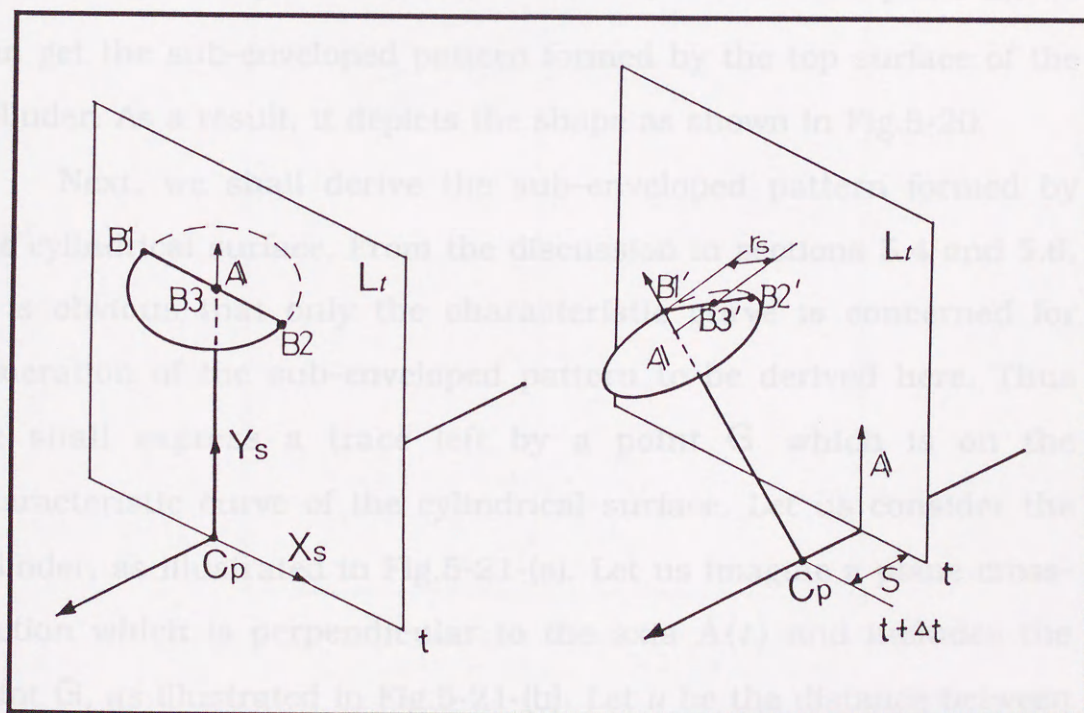


Fig.5-18 The position and the orientation of the top surface of the NC cutter at $t=t$ and $t = t + \Delta t$

$$\left. \begin{aligned} X_s &= u \cdot \sin\theta + r_s \cdot \cos\theta , \\ Y_s &= u \cdot \cos\theta - r_s \cdot \sin\theta , \end{aligned} \right\} \quad (5-19)$$

where the parameter u designates the distance from the cutter control point C_p to the point B_3 or to the point B'_3 . Similarly, the coordinates of the point B'_2 are

$$\left. \begin{aligned} X_s &= u \cdot \sin\theta - r_s \cdot \cos\theta , \\ Y_s &= u \cdot \cos\theta + r_s \cdot \sin\theta . \end{aligned} \right\} \quad (5-20)$$

The coordinates of the midpoint B'_3 are

$$\left. \begin{aligned} X_s &= u \cdot \sin\theta , \\ Y_s &= u \cdot \cos\theta . \end{aligned} \right\} \quad (5-21)$$

By eliminating θ from Equation(5-21), we get

$$X_s^2 + Y_s^2 = u^2 . \quad (5-22)$$

By plotting the Equations(5-19) and (5-20) on the X_sY_s -plane L_t , we can get the sub-enveloped pattern formed by the top surface of the cylinder. As a result, it depicts the shape as shown in Fig.5-20.

Next, we shall derive the sub-enveloped pattern formed by the cylindrical surface. From the discussion in sections 5.4 and 5.6, it is obvious that only the characteristic curve is concerned for generation of the sub-enveloped pattern to be derived here. Thus we shall express a trace left by a point G which is on the characteristic curve of the cylindrical surface. Let us consider the cylinder, as illustrated in Fig.5-21-(a). Let us imagine a plane cross-section which is perpendicular to the axis $A(t)$ and includes the point G , as illustrated in Fig.5-21-(b). Let u be the distance between

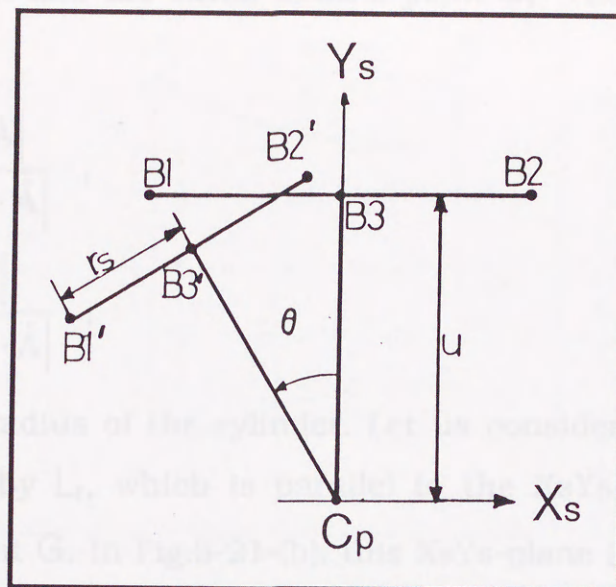


Fig.5-19 Derivation of the sub-enveloped pattern left by the top surface of the NC cutter (The case of the (B-1) motion)

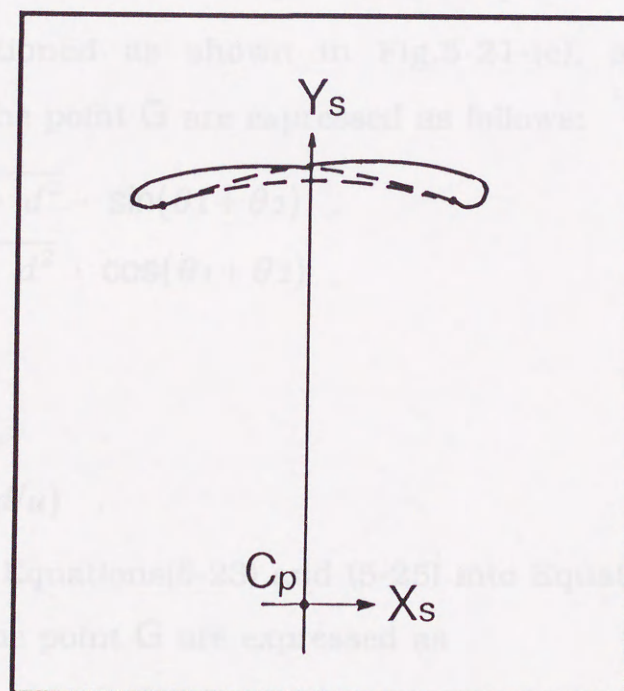


Fig.5-20 The sub-enveloped pattern left by the top surface of the NC cutter (The case of the (B-1) motion)

the cross-section and the cutter control point C_p . Then the point G is expressed as

$$\left. \begin{aligned} s &= \frac{r \cdot u \cdot |\dot{A}|}{|\dot{R} + u \cdot \dot{A}|} , \\ d &= \frac{r \cdot |\dot{R}|}{|\dot{R} + u \cdot \dot{A}|} , \end{aligned} \right\} \quad (5-23)$$

where r is the radius of the cylinder. Let us consider now a X_sY_s -plane, denoted by L'_t , which is parallel to the X_sY_s -plane L_t and includes the point G. In Fig.5-21-(b), this X_sY_s -plane L'_t is shown as the straight line which goes through the point G and is perpendicular to the vector $\dot{R}$. The L'_t is considered to be the X_sY_s -plane at $t = t - \Delta t$. On the X_sY_s -plane L'_t , the point G and the axis $A(t)$ are positioned as shown in Fig.5-21-(c), and thus the coordinates of the point G are expressed as follows:

$$\left. \begin{aligned} X_s &= \sqrt{u^2 + d^2} \cdot \sin(\theta_1 + \theta_2) , \\ Y_s &= \sqrt{u^2 + d^2} \cdot \cos(\theta_1 + \theta_2) , \end{aligned} \right\} \quad (5-24)$$

where

$$\left. \begin{aligned} \theta_1 &= |\dot{A}| \cdot s , \\ \theta_2 &= \tan^{-1}(d/u) . \end{aligned} \right\} \quad (5-25)$$

By substituting Equations(5-23) and (5-25) into Equation(5-24), the coordinates of the point G are expressed as

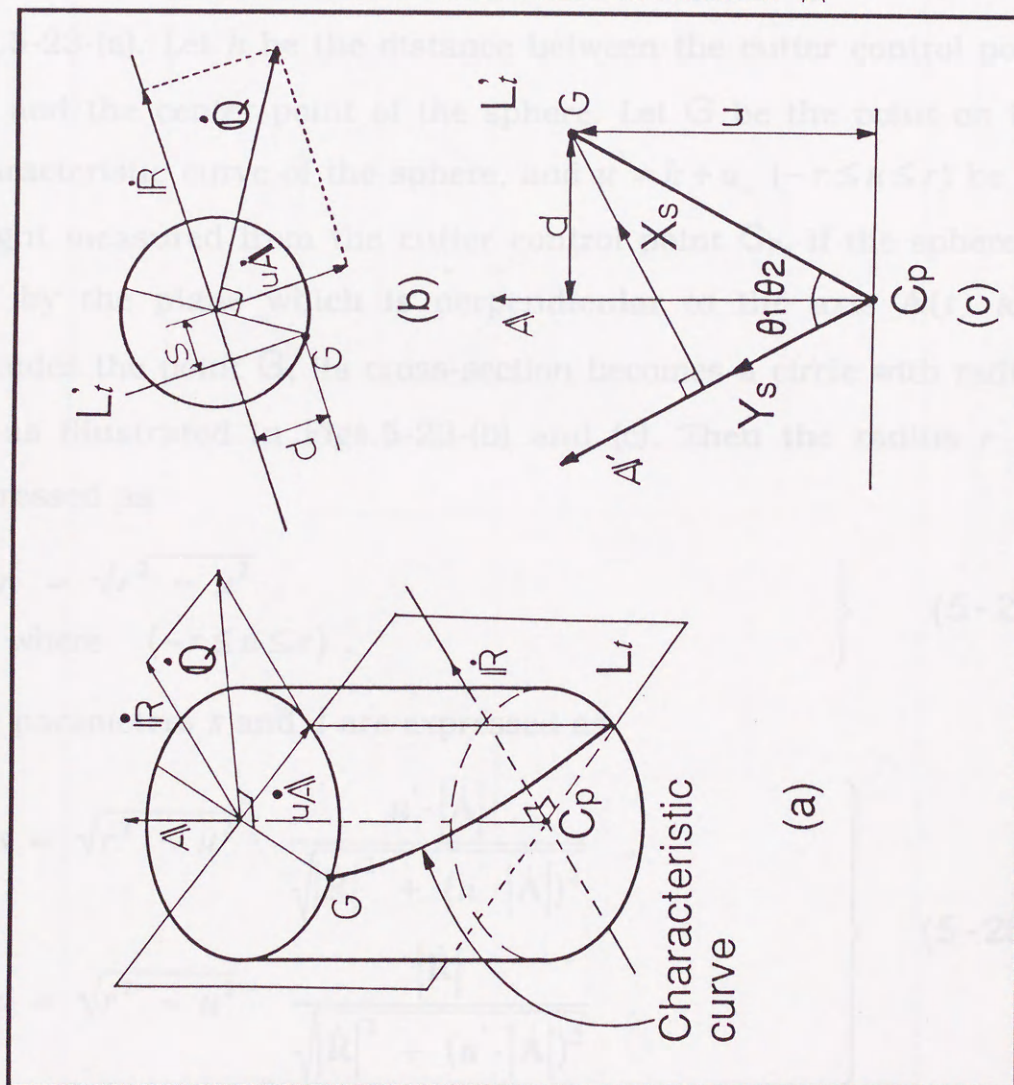


Fig.5-21 Characteristic curve of the cylindrical surface,
and the X_sY_s -plane L_i
(The case of the (B-1) motion)

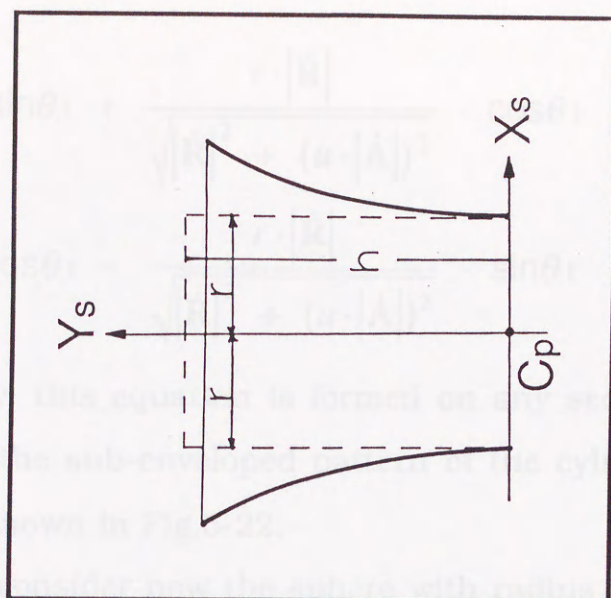


Fig.5-22 The sub-enveloped pattern
of the cylindrical surface
(The case of the (B-1) motion)

$$\left. \begin{aligned} X_s &= u \cdot \sin\theta_1 + \frac{r \cdot |\dot{\mathbf{R}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + (u \cdot |\dot{\mathbf{A}}|)^2}} \cdot \cos\theta_1 , \\ Y_s &= u \cdot \cos\theta_1 - \frac{r \cdot |\dot{\mathbf{R}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + (u \cdot |\dot{\mathbf{A}}|)^2}} \cdot \sin\theta_1 . \end{aligned} \right\} \quad (5-26)$$

Consequently, this equation is formed on any section of the X_sY_s -plane. Thus the sub-enveloped pattern of the cylindrical surface is depicted as shown in Fig.5-22.

Let us consider now the sphere with radius r as illustrated in Fig.5-23-(a). Let h be the distance between the cutter control point C_p and the center point of the sphere. Let G be the point on the characteristic curve of the sphere, and $u' = h + u$, $(-r \leq u \leq r)$ be its height measured from the cutter control point C_p . If the sphere is cut by the plane which is perpendicular to the axis $A(t)$ and includes the point G , its cross-section becomes a circle with radius r' , as illustrated in Figs.5-23-(b) and (c). Then the radius r' is expressed as

$$\left. \begin{aligned} r' &= \sqrt{r^2 - u^2} , \\ \text{where } (-r \leq u \leq r) . \end{aligned} \right\} \quad (5-27)$$

The parameters s and d are expressed as

$$\left. \begin{aligned} s &= \sqrt{r^2 - u^2} \cdot \frac{u' \cdot |\dot{\mathbf{A}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + (u' \cdot |\dot{\mathbf{A}}|)^2}} , \\ d &= \sqrt{r^2 - u^2} \cdot \frac{|\dot{\mathbf{R}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + (u' \cdot |\dot{\mathbf{A}}|)^2}} . \end{aligned} \right\} \quad (5-28)$$

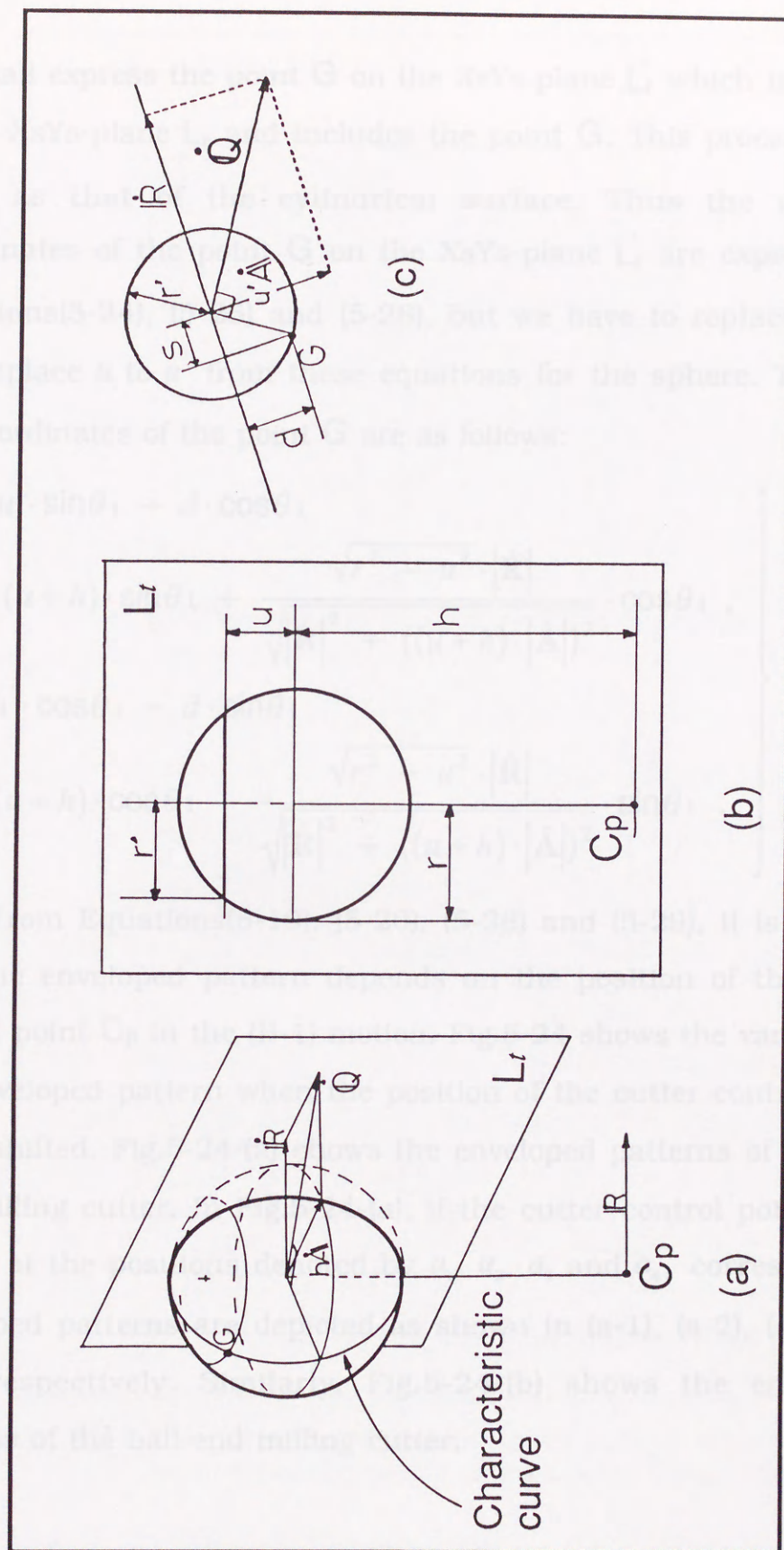


Fig.5-23 Characteristic curve of the sphere, and the XsYs-plane L_1
(The case of the (B-1) motion)

We shall express the point G on the X_sY_s -plane L'_t which is parallel to the X_sY_s -plane L_t and includes the point G . This process is the same as that of the cylindrical surface. Thus the resulting coordinates of the point G on the X_sY_s -plane L'_t are expressed as Equations(5-24), (5-25) and (5-26), but we have to replace r to r' , and replace u to u' from these equations for the sphere. Therefore the coordinates of the point G are as follows:

$$\left. \begin{aligned} X_s &= u' \cdot \sin\theta_1 + d \cdot \cos\theta_1 \\ &= (u+h) \cdot \sin\theta_1 + \frac{\sqrt{r'^2 - u'^2} \cdot |\dot{\mathbf{R}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + ((u+h) \cdot |\dot{\mathbf{A}}|)^2}} \cdot \cos\theta_1, \\ Y_s &= u' \cdot \cos\theta_1 - d \cdot \sin\theta_1 \\ &= (u+h) \cdot \cos\theta_1 - \frac{\sqrt{r'^2 - u'^2} \cdot |\dot{\mathbf{R}}|}{\sqrt{|\dot{\mathbf{R}}|^2 + ((u+h) \cdot |\dot{\mathbf{A}}|)^2}} \cdot \sin\theta_1. \end{aligned} \right\} \quad (5-29)$$

From Equations(5-19), (5-20), (5-26) and (5-29), it is obvious that the enveloped pattern depends on the position of the cutter control point C_p in the (B-1) motion. Fig.5-24 shows the variation of the enveloped pattern when the position of the cutter control point C_p is shifted. Fig.5-24-(a) shows the enveloped patterns of the flat-end milling cutter. In Fig.5-24-(a), if the cutter control point C_p is placed at the positions denoted by a_1 , a_2 , a_3 and a_4 , corresponding enveloped patterns are depicted as shown in (a-1), (a-2), (a-3) and (a-4) respectively. Similarly, Fig.5-24-(b) shows the enveloped patterns of the ball-end milling cutter.

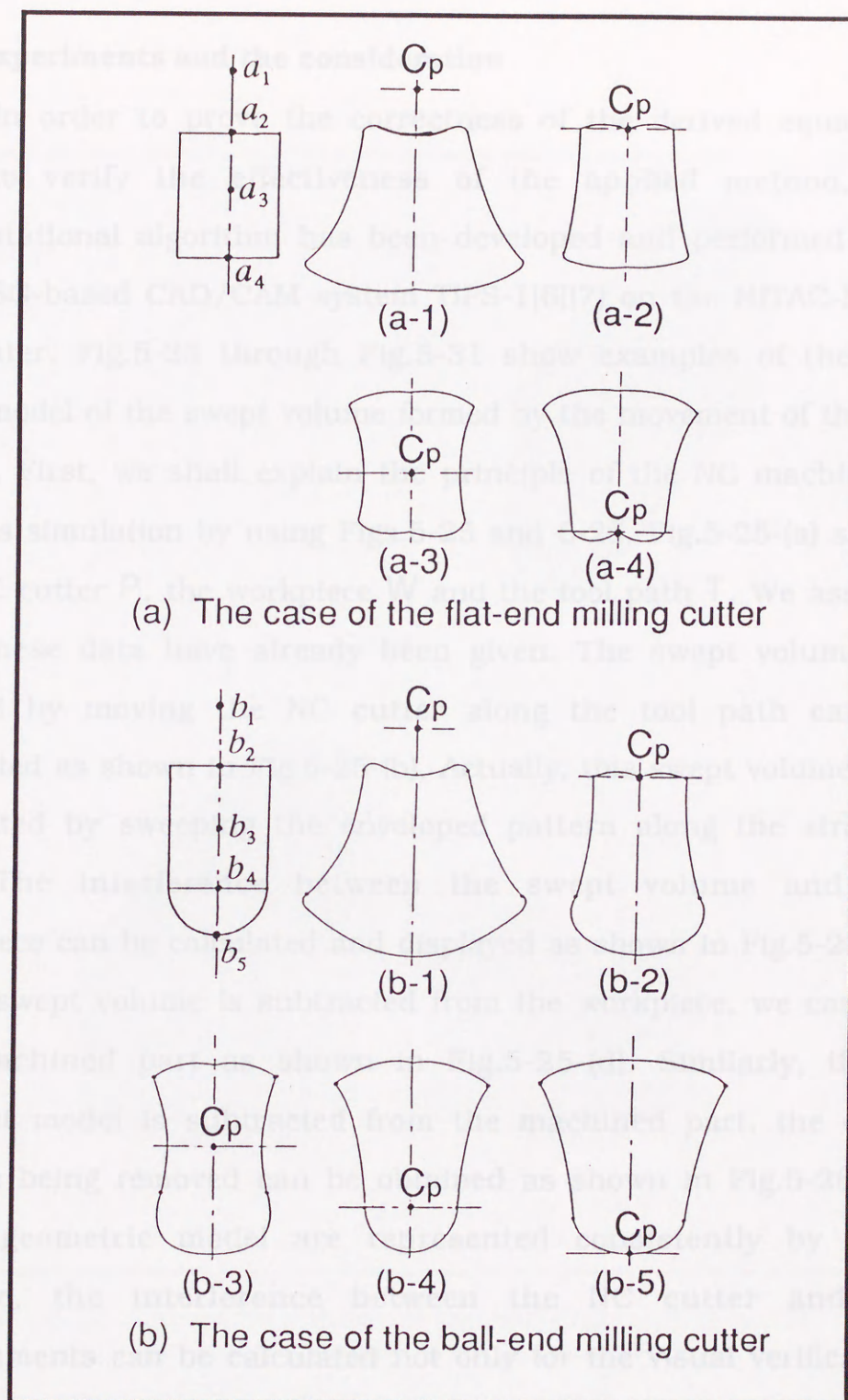


Fig.5-24 The variation of the enveloped pattern when the position of the cutter control point C_p is shifted

5.9 Experiments and the consideration

In order to prove the correctness of the derived equations and to verify the effectiveness of the applied method, the computational algorithm has been developed and performed with the CSG-based CAD/CAM system TIPS-1[6][7] on the HITAC-M280 computer. Fig.5-25 through Fig.5-31 show examples of the 3-D solid model of the swept volume formed by the movement of the NC cutter. First, we shall explain the principle of the NC machining-process simulation by using Figs.5-25 and 5-26. Fig.5-25-(a) shows the NC cutter P , the workpiece W and the tool path T . We assume that these data have already been given. The swept volume S_v formed by moving the NC cutter along the tool path can be generated as shown in Fig.5-25-(b). Actually, this swept volume was generated by sweeping the enveloped pattern along the straight line. The interference between the swept volume and the workpiece can be calculated and displayed as shown in Fig.5-25-(c). If the swept volume is subtracted from the workpiece, we can get the machined part as shown in Fig.5-25-(d). Similarly, if the product model is subtracted from the machined part, the delta volume being removed can be obtained as shown in Fig.5-26. As these geometric model are represented consistently by CSG scheme, the interference between the NC cutter and its environments can be calculated not only for the visual verification but also for the quantitative one. The NC cutter motion used in Figs.5-25 and 5-26 is the case (A-3). Figs.5-27-(a) and (b) show the NC machining-process when the ball-end milling cutter moves along the circular path and cuts the workpiece. In Fig.5-27-(a), the

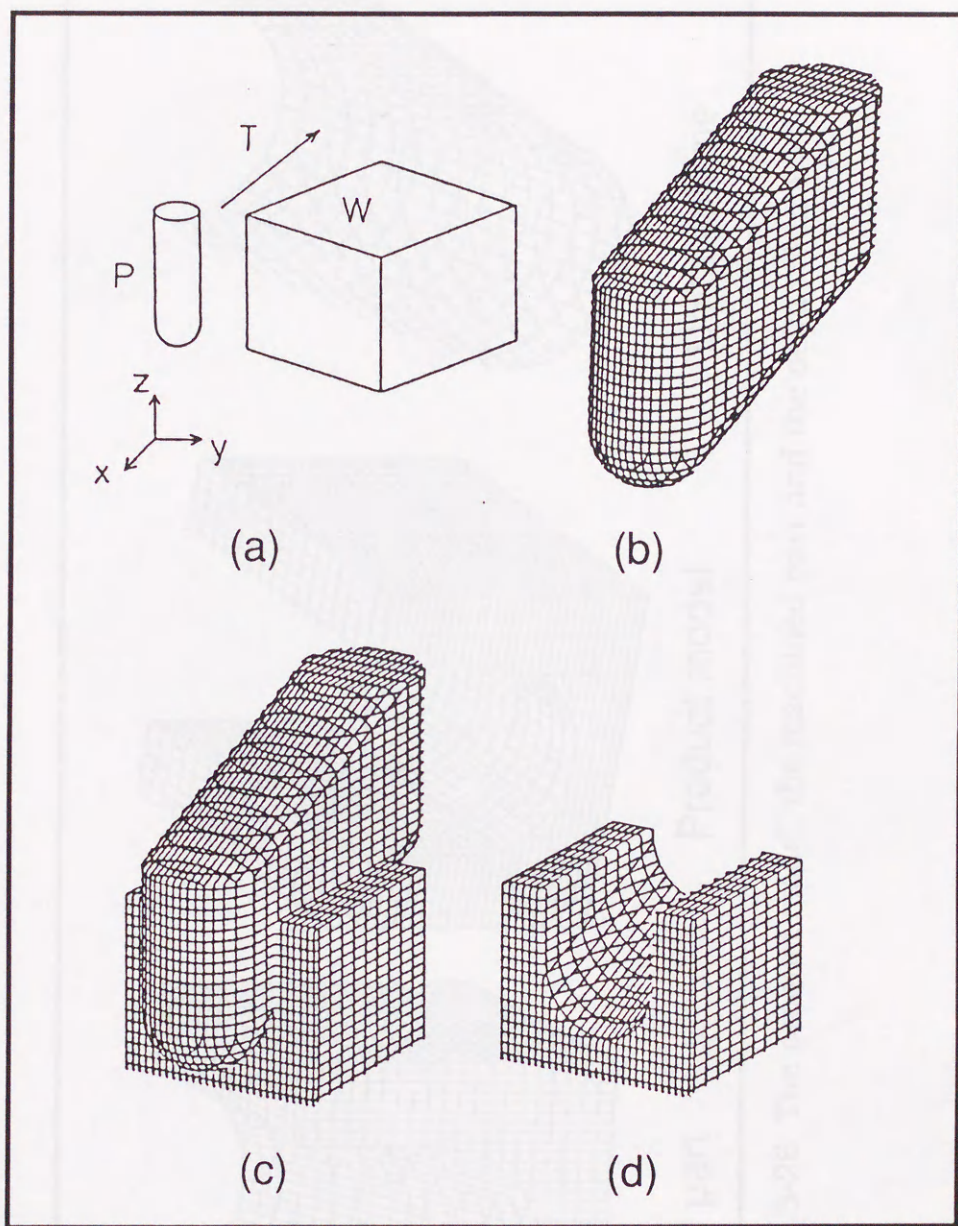


Fig.5-25 The swept volume of the ball-end milling cutter, and its interference with the workpiece

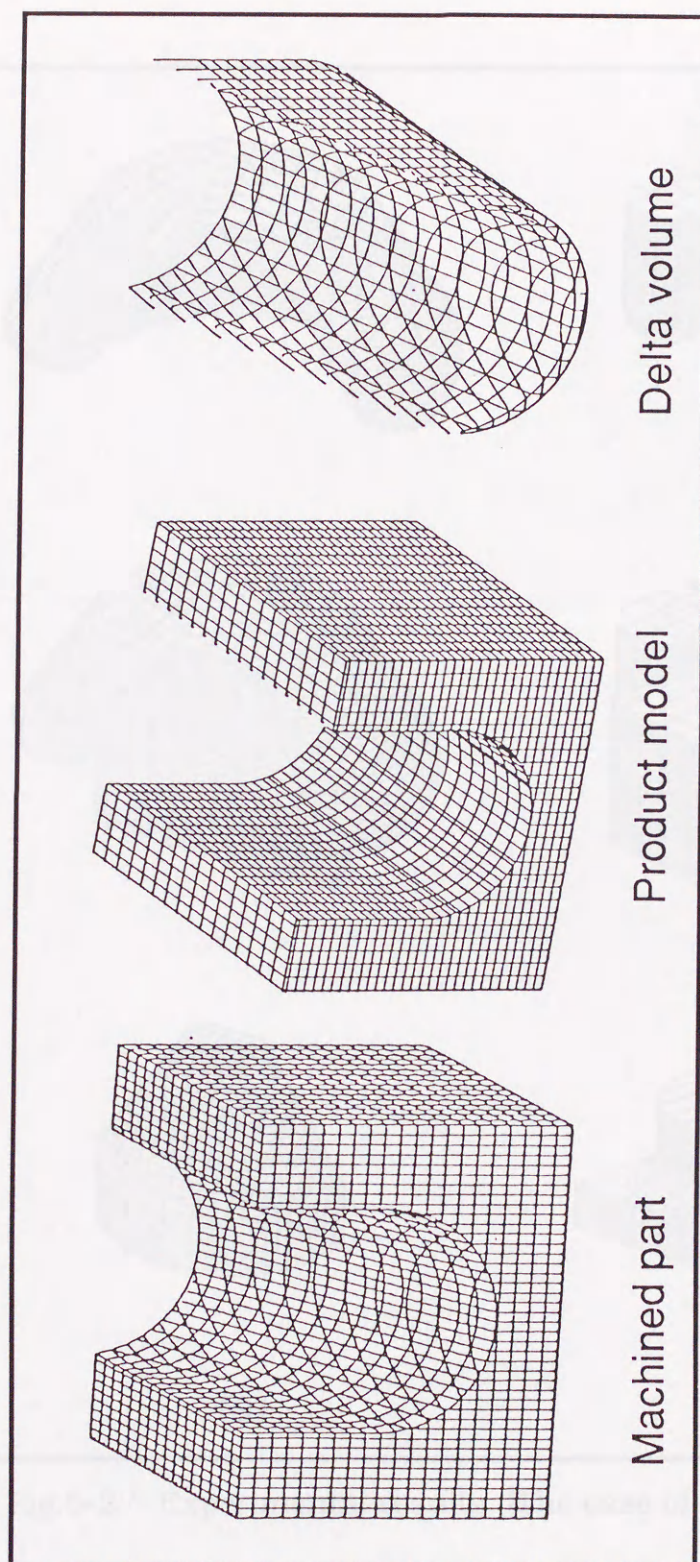


Fig.5-26 The product model, the machined part and the delta volume

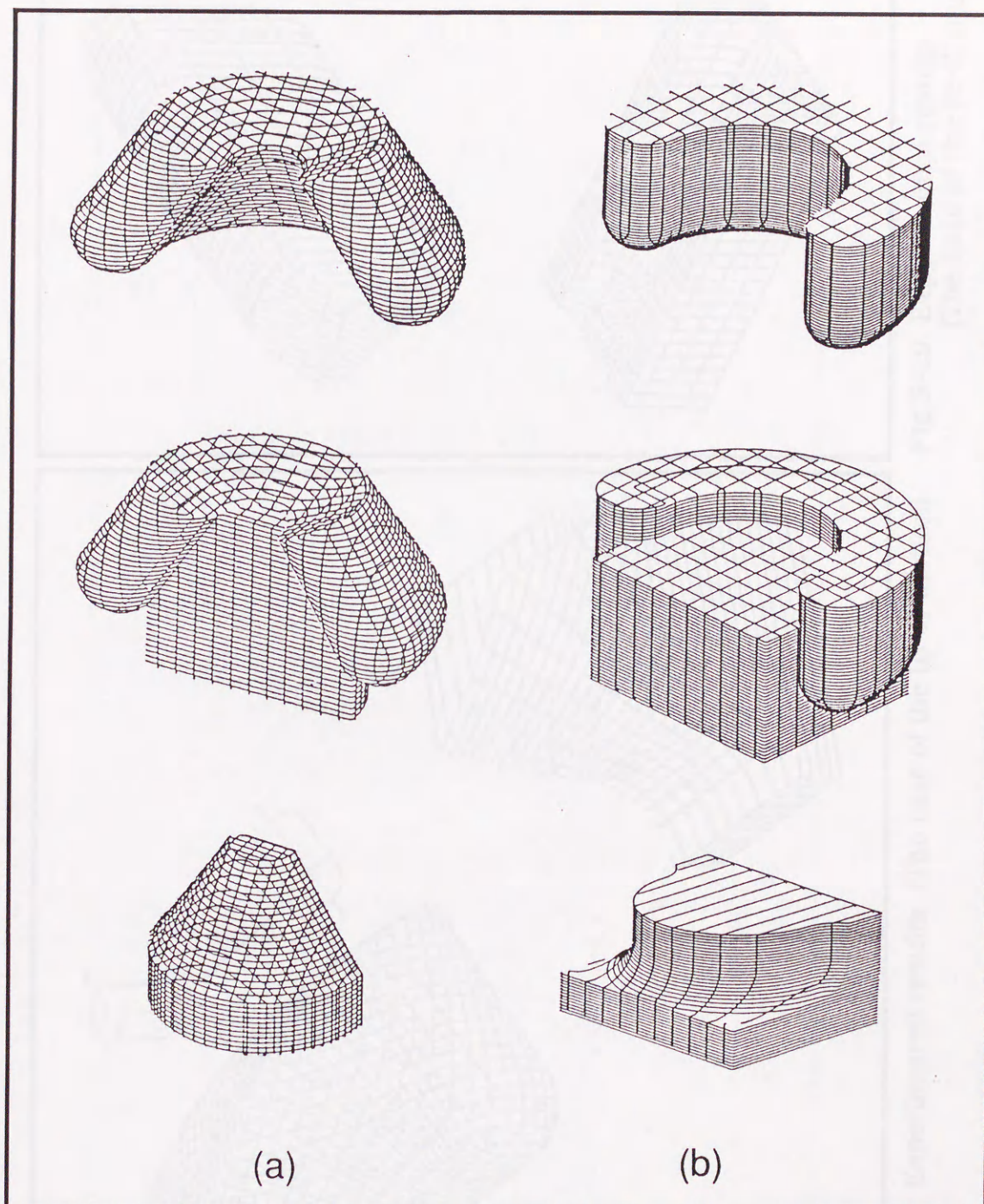


Fig.5-27 Experimental results (The case of the (A-1) motion)

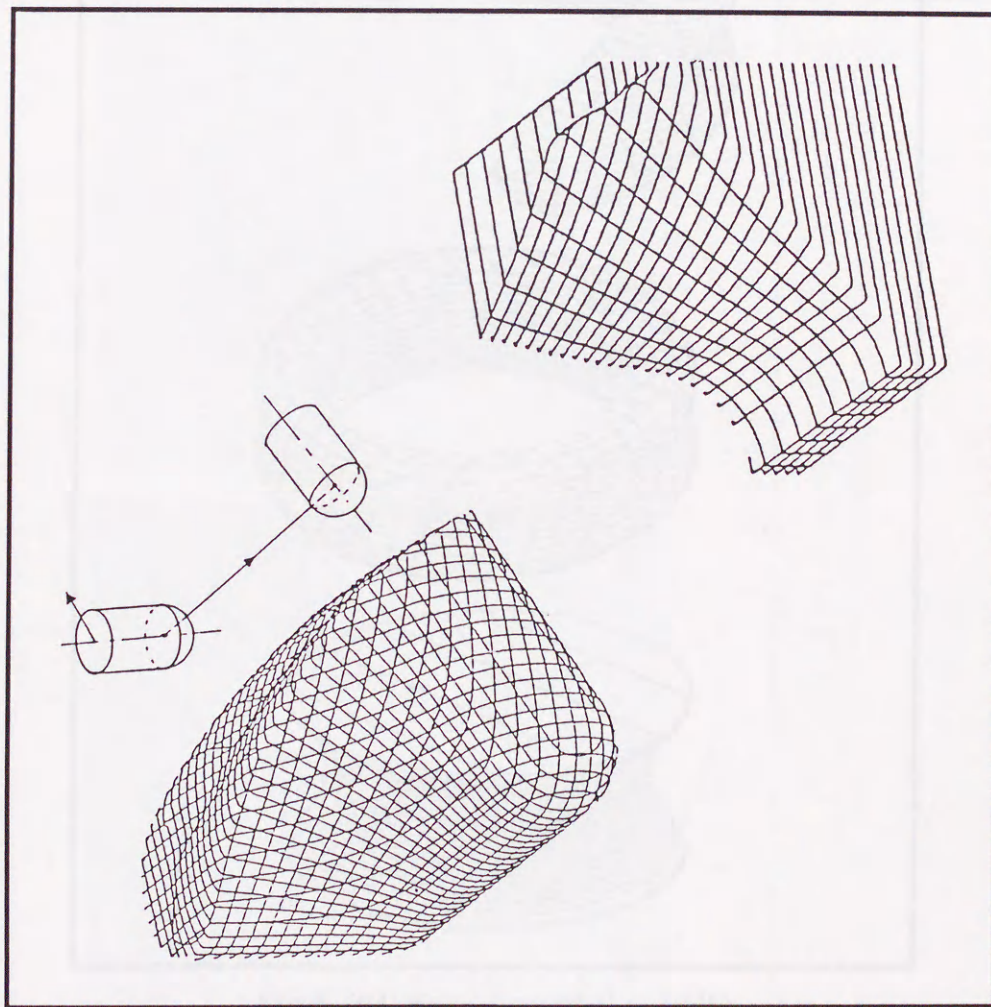


Fig.5-28 Experimental results (The case of the (B-1) motion)

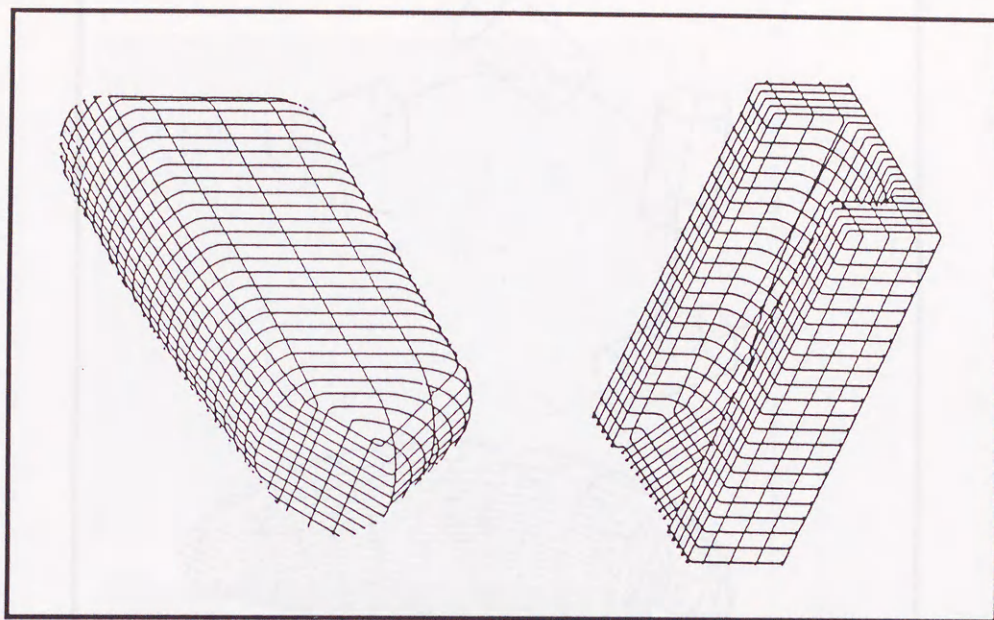


Fig.5-29 Experimental results
(The case of the (B-2) motion)

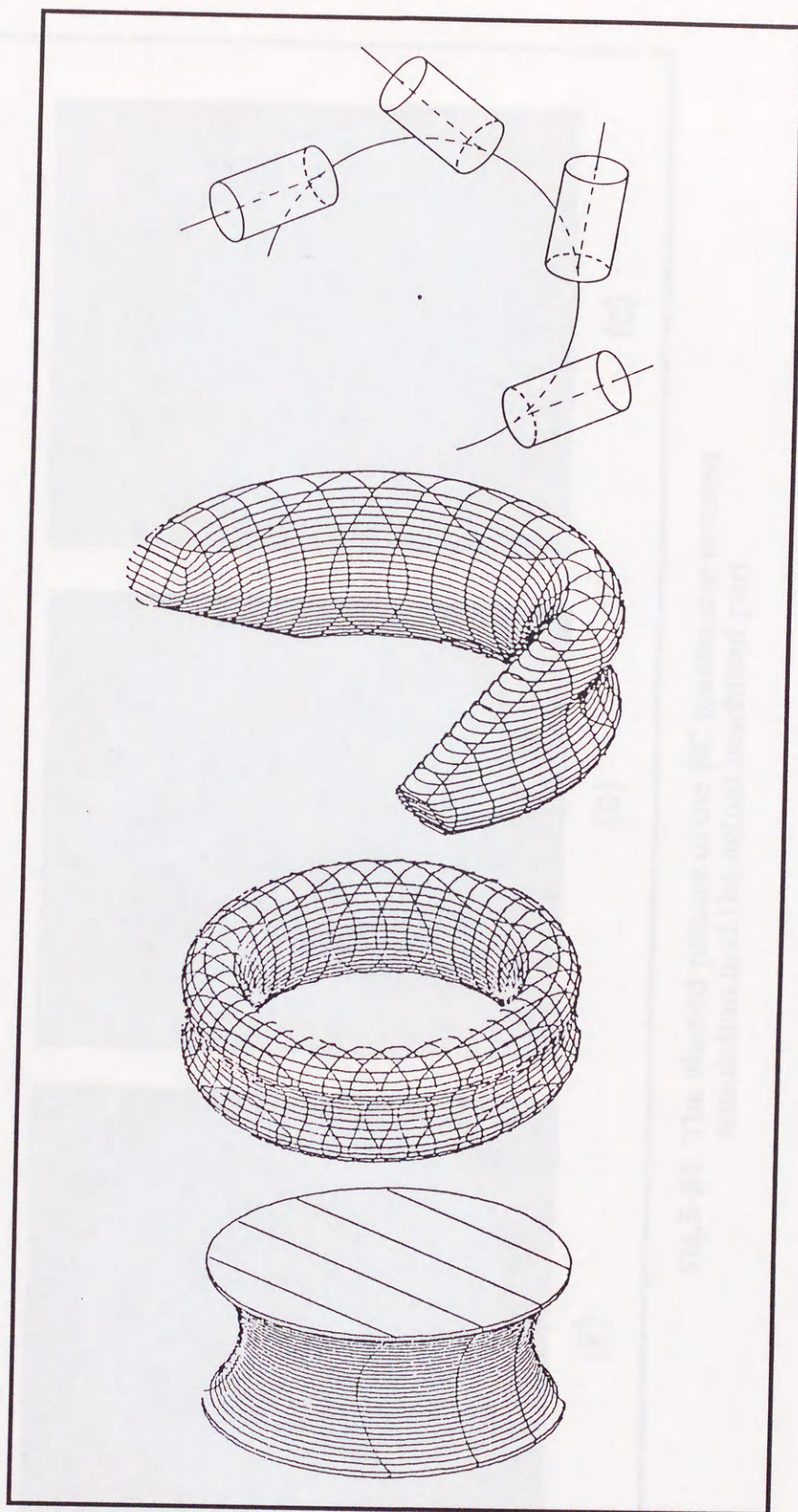


Fig.5-30 Experimental results
(The case of the (B-2) motion)

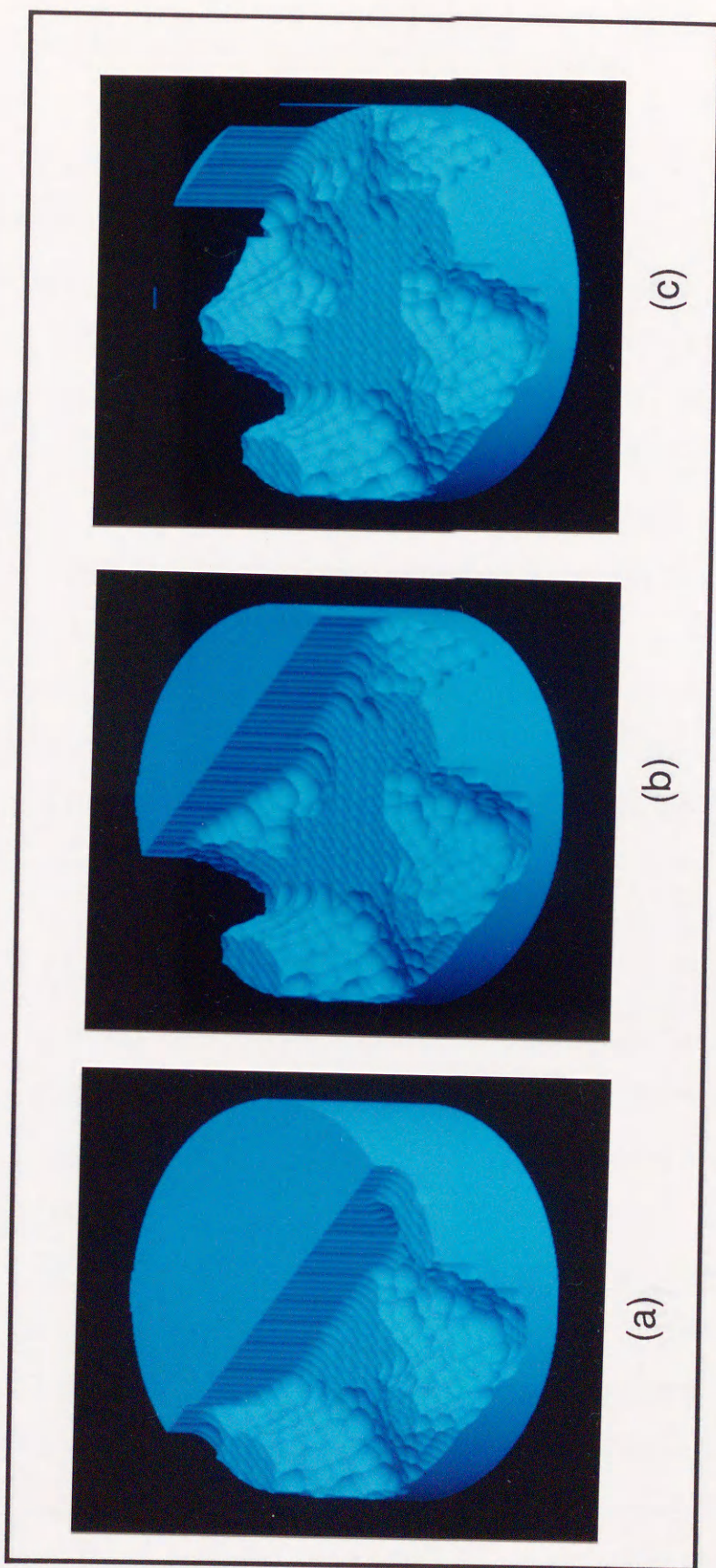


Fig.5-31 The shaded picture of the NC machining-process simulation and the actual machined part

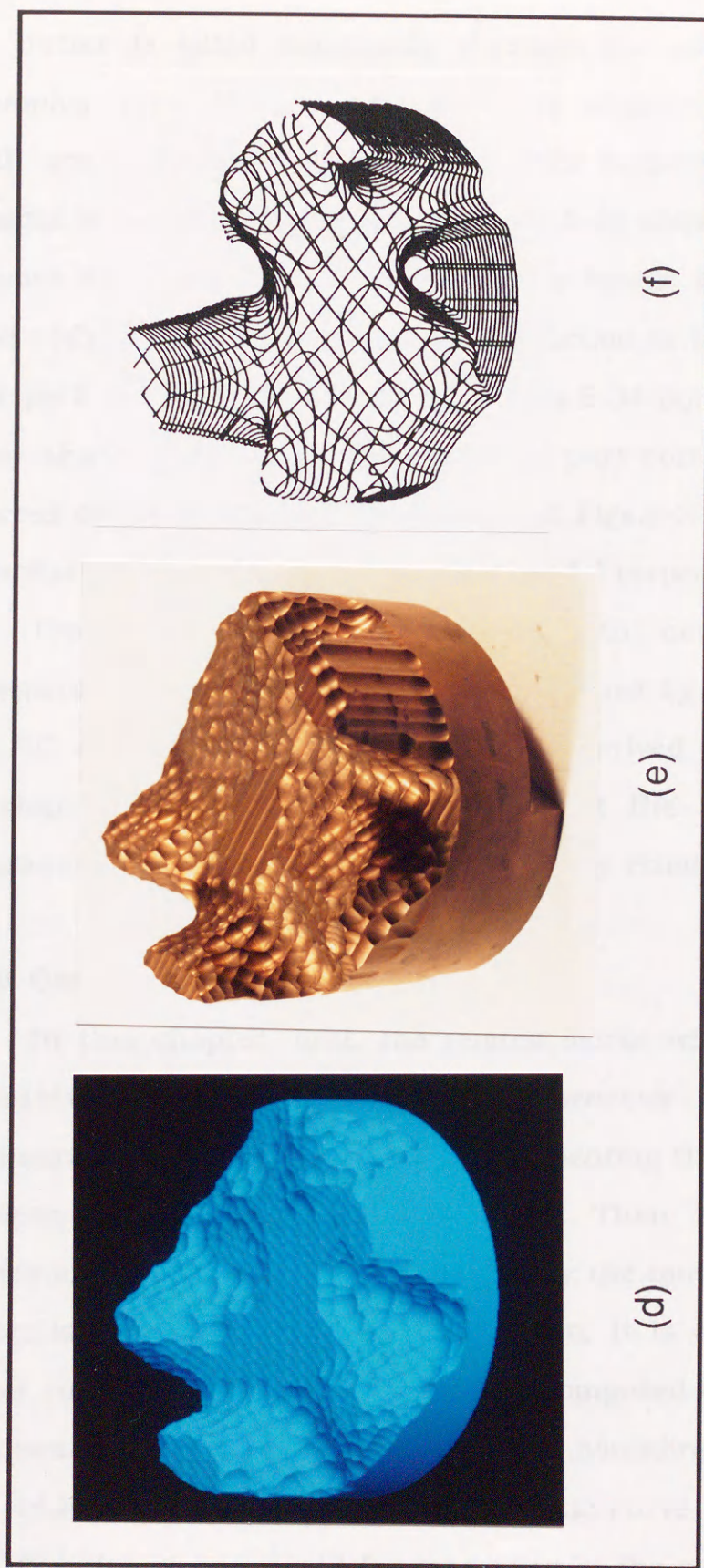


Fig.5-31 The shaded picture of the NC machining-process simulation and the actual machined part

NC cutter is tilted constantly through the motion. These are examples of the (A-1) motion. Examples of the motions (B-1) and (B-2) are shown in Figs.5-28 and 5-29 respectively, where the straight line is used as the tool path. Fig.5-30 shows the machining-process when the flat-end milling cutter moves along the circular path with the fall-down motion in a direction to the tangent of the tool path (i.e., the (B-2) motion). Figs.5-31-(a), (b), (c), and (d) show shaded pictures of the machined part corresponding to the process of the machining operation, and Figs.5-31-(e) and (f) show its actual machined part and product model respectively.

From these examples, it is clear that the new method applied to generate the swept volume which is formed by the movement of the NC cutter is effective enough, and derived equations of the enveloped pattern are correct, and that the generated swept volume can be handled just like the ordinary Primitive.

5.10 Conclusion

In this chapter, first, the related works with the geometric simulation for the NC machining-process verification are summarized. Next, the principle of representing the NC machining-process is expressed set-theoretically. Then the problem for generation of the swept volume formed by the movement of the NC cutter is discussed. From the discussion, it is clarified that the swept volume of the NC cutter can be composed of the sub-swept volumes formed by the movement of the bounding surfaces of the NC cutter, and that only the characteristic curve of the surface of the NC cutter is concerned for generation of the sub-swept volume.

Thus, how to represent the characteristic curve is discussed, and as a consequence, the characteristic curve is represented parametrically.

As the direct application of the characteristic curve to generate the sub-swept volume is very difficult, the sweeping method proposed in chapter 4 is introduced. It is necessary, however, to give the 2-D Sweep Pattern in order to apply the sweeping method. Thus the enveloped pattern (i.e., the Sweep Pattern) of the swept volume is defined. The enveloped pattern is considered to be the cross-sectional one obtained by cutting the swept volume with the plane cross-section. Actually, the enveloped pattern is composed of traces left by the individual characteristic curves of the individual surfaces of the NC cutter when the NC cutter is supposed to go through the cross-section. Therefore, if the enveloped pattern can be derived, the entire swept volume can be generated directly by sweeping the enveloped pattern along the tool path (i.e., the Sweep Axis). As the shape of the enveloped pattern depends on the shape and the motion of the NC cutter, the shape and the motion of the NC cutter are classified respectively, and then the enveloped pattern is derived mathematically in accordance with the classification.

In order to prove the correctness of the derived equations of the enveloped pattern and to verify the effectiveness of the applied method, the computational algorithm has been developed and performed with the CSG-based CAD/CAM system TIPS-1. According to the experiments, it is clear that the new method applied to generate the swept volume is effective enough, and derived

equations of the enveloped pattern are correct, and that the generated swept volume can be handled just like the ordinary Primitive.

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6. SOLID MODELING OF THE FILLET VOLUME

- ON THE FILLET FOR ROUNDING THE INTERSECTION LINE BETWEEN TWO QUADRICS -

6.1 Introduction

With the advance of CAD/CAM, requirements for solid modeling schemes have been increased. Most of the requirements are concerned with the capability to represent *blending surfaces*. The term *blending surface* usually means either a surface which forms a smooth transition between different, but intersecting surfaces, or a surface which smoothly joins two or more disconnected surfaces. However, the blending surface is one of the most difficult ones to be represented by solid modeling schemes. Thus the blending surface has been represented by surface modeling schemes[1][2][3]. The surface modeling schemes, however, are not consistent with those of solid modeling. From the viewpoint of the modeling consistency, the blending surface should be represented by the solid modeling schemes in 3-D solid modeling systems. This is the motivation to start this study, aiming at representing the blending surface by CSG scheme.

When building up a desired shape by the Primitives in CSG-based systems, a large number of intersections occur between surfaces. In these intersections, the most complicated case appears between quadrics. Thus the blending surface to connect two intersecting quadrics smoothly is more complicated than any other case. If the blending surface of the most complicated case can be

represented by CSG scheme, the other cases can easily be addressed. This is the reason why the case of two intersecting quadrics is chosen as the object of the blending problem to be solved in this work. Here we shall clarify the CSG representation method for the solid with this blending surface.

Since the blending surface used for the smooth connection between intersecting surfaces is called *the fillet*, we refer to the solid with the blending surface as *the fillet volume* in this paper. It has been considered that the fillet volume to round the intersecting quadrics is very difficult to be represented by CSG scheme. This is caused by the following reasons.

- As the intersection line between quadrics becomes a 3-D curve, and moreover as the form of the curve is varied spatially by positions of quadrics, it is very hard to derive the intersection line analytically.
- Even if the intersection line can be derived, there is no established way to generate the fillet volume from it.

Although some attempts have been reported[4][5][6] recently for generation of the fillet volume, these attempts must solve higher degree (at least higher than 4 degree) algebraic equations. In general, it is very difficult to solve equations analytically for any case of the intersecting quadrics. Thus we shall apply a new method[7][8][9] based on the proposed sweeping method to represent the fillet volume by CSG scheme. In the following sections, we shall describe the applied method.

6.2 Description of the problem

Let us consider now two intersecting quadrics as shown in Fig.6-1. Let C_1 and C_2 be the quadrics. We assume that the quadrics can be expressed as follows:

$$\left. \begin{array}{l} C_1 : g(X) = 0 , \\ C_2 : h(X) = 0 , \end{array} \right\} \quad (6-1)$$

where X designates coordinates of a space point defined in the 3-D orthogonal coordinate system. The quadrics can be offset in the normal direction. We assume that the offset quadrics can be expressed as follows:

$$\left. \begin{array}{l} C_{1off} : G(X, a) = 0 , \\ C_{2off} : H(X, b) = 0 , \end{array} \right\} \quad (6-2)$$

where variables " a " and " b " designate offset values of the quadrics C_1 and C_2 respectively. If the values are zero, the offset quadrics are obviously equivalent to the original ones. Thus the following equation can be formed:

$$\left. \begin{array}{l} G(X, 0) = g(X) , \\ H(X, 0) = h(X) . \end{array} \right\} \quad (6-3)$$

The intersection line between the quadrics C_1 and C_2 can be obtained by solving the following simultaneous equations:

$$\left. \begin{array}{l} g(X) = 0 , \\ h(X) = 0 . \end{array} \right\} \quad (6-4)$$

If the offset quadrics are used in order to specify connecting lines between the quadrics and the fillet volume, these lines can be obtained by solving the following two simultaneous equations:

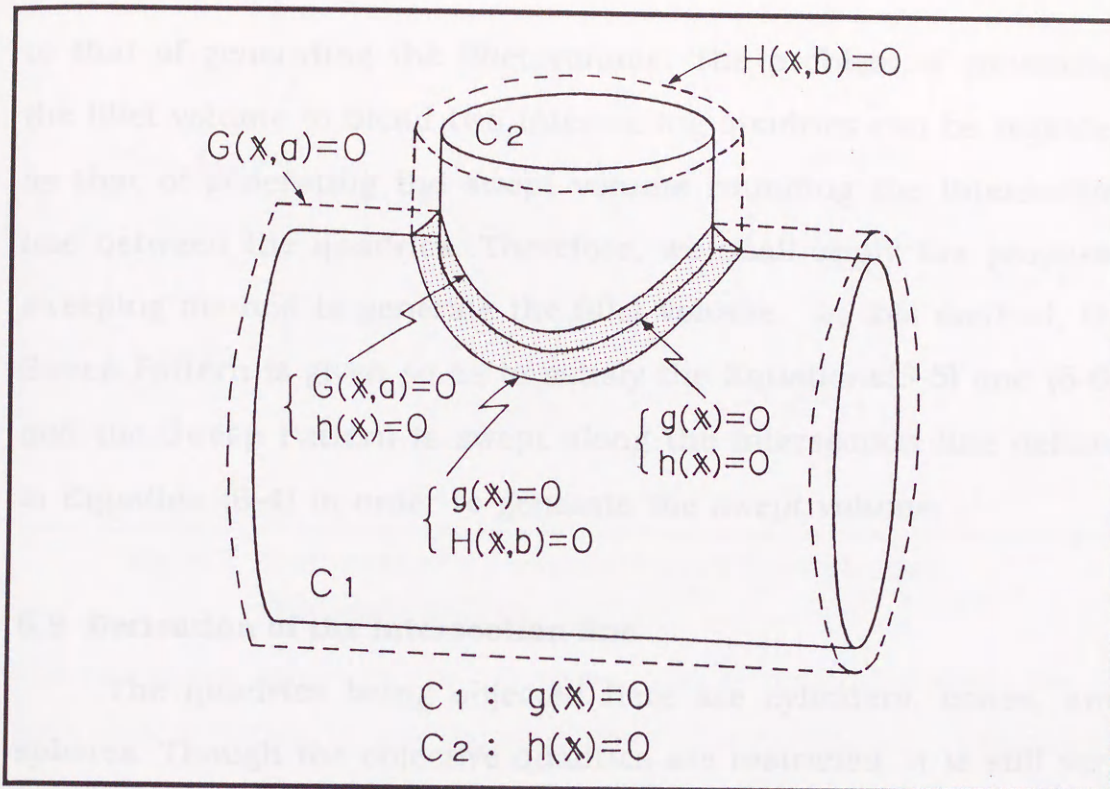


Fig.6-1 Definition of the connecting lines between the quadrics and the fillet volume

$$\left. \begin{array}{l} G(X,a)=0, \\ h(X)=0 \end{array} \right\} \quad (6-5)$$

$$\left. \begin{array}{l} g(X)=0, \\ H(X,b)=0 \end{array} \right\} \quad (6-6)$$

The solution of Equation(6-5) represents the connecting line on C_2 , and that of Equation(6-6) means the connecting line on C_1 . These connecting lines are actually a part of the fillet volume. It is very difficult, however, to derive a mathematical function of an entire fillet volume even if we can solve Equations(6-5) and (6-6) analytically. As the purpose of this work is to represent the fillet volume by CSG scheme, we shall change the viewpoint from the problem of deriving the mathematical function of the fillet volume

to that of generating the fillet volume. The problem of generating the fillet volume to blend two intersecting quadrics can be regarded as that of generating the swept volume rounding the intersection line between the quadrics. Therefore, we shall apply the proposed sweeping method to generate the fillet volume. In this method, the Sweep Pattern is given so as to satisfy the Equations(6-5) and (6-6), and the Sweep Pattern is swept along the intersection line defined in Equation (6-4) in order to generate the swept volume.

6.3 Derivation of the intersection line

The quadrics being objected here are cylinders, cones, and spheres. Though the objective quadrics are restricted, it is still very difficult to derive the intersection line between them by solving the simultaneous equations shown in Equation(6-4) directly. Thus we shall represent the quadrics parametrically, and then derive the intersection line. The cylinder, the cone and the sphere, which are illustrated in Fig.6-2, are expressed[10] parametrically as Equations (6-7), (6-8), and (6-9) respectively:

$$P_{cy} = P_0 + a \cdot W + r(U \cdot \cos\theta + V \cdot \sin\theta) \quad , \quad (6-7)$$

$$P_{co} = P_0 + a \cdot W + a \cdot \tan\alpha(U \cdot \cos\theta + V \cdot \sin\theta) \quad , \quad (6-8)$$

$$P_{sp} = P_0 + r \cdot \cos\phi \cdot W + r \cdot \sin\phi(U \cdot \cos\theta + V \cdot \sin\theta) \quad , \quad (6-9)$$

where W is the unit vector of the axis of the revolution, U and V are unit vectors of the axes of the UV-plane which is perpendicular to the axis W , and P_0 is the position vector of the representative point of the quadric. In these equations, the parameter θ means

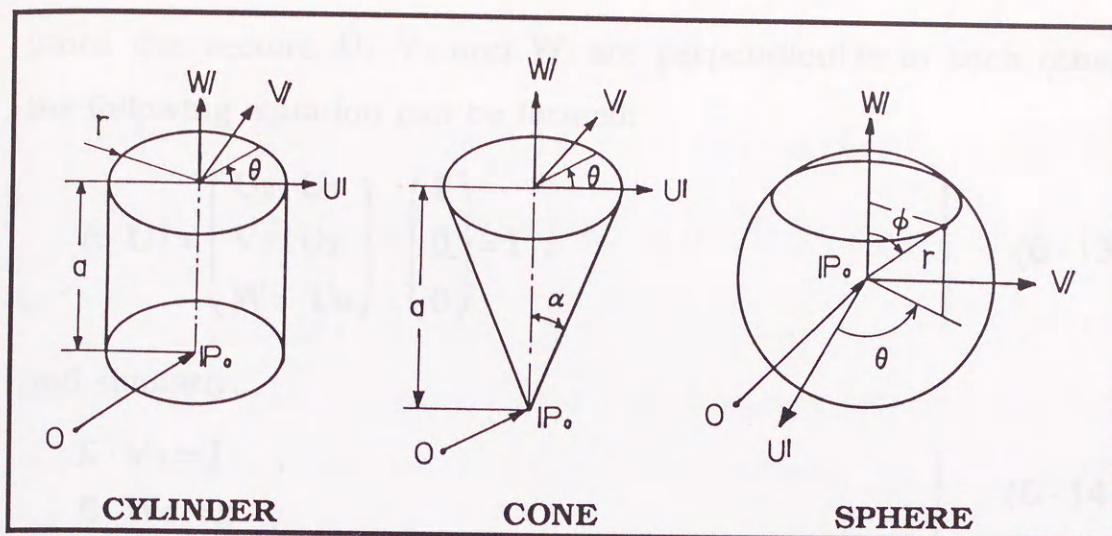


Fig.6-2 Parametric representation of the cylinder, the cone, and the sphere

the rotational angle on the UV-plane and is measured from the U-axis in a direction to counter-clockwise, the parameter a designates the distance between the point P_0 and the UV-plane, r is the radius of the sphere and the cylinder, and $2 \cdot \alpha$ is the vertex angle of the cone. The coordinate system used for representing the sphere is similar to that used in locating points on the earth's surface by means of longitude and latitude. The angle θ corresponds to the longitude and ϕ to the colatitude.

Let us explain the derivation of the intersection line by using the case of two cylinders. Then the cylinders are given as follows:

$$C_1 : P = P_0 + r_1(U_1 \cdot \cos\theta_1 + V_1 \cdot \sin\theta_1) + a_1 \cdot W_1 \quad , \quad (6-10)$$

$$C_2 : Q = Q_0 + r_2(U_2 \cdot \cos\theta_2 + V_2 \cdot \sin\theta_2) + a_2 \cdot W_2 \quad . \quad (6-11)$$

By using the vectors U , V , and W , we can define a column matrix as follows:

$$E = \begin{pmatrix} U_2 \\ V_2 \\ W_2 \end{pmatrix} \quad . \quad \left. \vphantom{\begin{pmatrix} U_2 \\ V_2 \\ W_2 \end{pmatrix}} \right\} \quad (6-12)$$

Since the vectors U , V , and W are perpendicular to each other, the following equation can be formed:

$$E \cdot U_2 = \begin{pmatrix} U_2 \cdot U_2 \\ V_2 \cdot U_2 \\ W_2 \cdot U_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{i} , \quad \left. \vphantom{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} \right\} \quad (6-13)$$

and similarly,

$$\left. \begin{aligned} E \cdot V_2 &= \mathbf{j} , \\ E \cdot W_2 &= \mathbf{k} , \end{aligned} \right\} \quad (6-14)$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors which have the directions of the positive x , y , and z -axes of the 3-D orthogonal coordinate system. By transforming the Equations(6-10) and (6-11) with the matrix E , we can get the following equations:

$$P' = P_0' + r_1(U \cdot \cos\theta_1 + V \cdot \sin\theta_1) + a_1 \cdot W , \quad (6-15)$$

$$Q' = Q_0' + r_2(\mathbf{i} \cdot \cos\theta_2 + \mathbf{j} \cdot \sin\theta_2) + a_2 \cdot \mathbf{k} , \quad (6-16)$$

where,

$$\left. \begin{aligned} P_0' &= E \cdot P_0 = (p_x \ p_y \ p_z) , \\ Q_0' &= E \cdot Q_0 = (q_x \ q_y \ q_z) , \\ U &= E \cdot U_1 = (U_x \ U_y \ U_z) , \\ V &= E \cdot V_1 = (V_x \ V_y \ V_z) , \\ W &= E \cdot W_1 = (W_x \ W_y \ W_z) . \end{aligned} \right\} \quad (6-17)$$

As Equations(6-15) and (6-16) represent the cylinders C_1 and C_2 respectively, the following equation is formed on the intersection line of the C_1 and C_2 :

$$P' = Q' . \quad (6-18)$$

The x, y, and z components of Equation(6-18) can be expressed as:

$$p_x + r_1(U_x \cdot \cos\theta_1 + V_x \cdot \sin\theta_1) + W_x \cdot a_1 = q_x + r_2 \cdot \cos\theta_2 , \quad (6-19)$$

$$p_y + r_1(U_y \cdot \cos\theta_1 + V_y \cdot \sin\theta_1) + W_y \cdot a_1 = q_y + r_2 \cdot \sin\theta_2 , \quad (6-20)$$

$$p_z + r_1(U_z \cdot \cos\theta_1 + V_z \cdot \sin\theta_1) + W_z \cdot a_1 = q_z + a_2 . \quad (6-21)$$

Let us solve these equations for the parameter a_1 . By squaring both sides of Equations(6-19) and (6-20) respectively, we can get the following equations:

$$\left. \begin{aligned} \{p_x - q_x + r_1(U_x \cdot \cos\theta_1 + V_x \cdot \sin\theta_1) + W_x \cdot a_1\}^2 \\ = r_2^2 \cdot \cos^2 \theta_2 , \end{aligned} \right\} \quad (6-22)$$

$$\left. \begin{aligned} \{p_y - q_y + r_1(U_y \cdot \cos\theta_1 + V_y \cdot \sin\theta_1) + W_y \cdot a_1\}^2 \\ = r_2^2 \cdot \sin^2 \theta_2 . \end{aligned} \right\} \quad (6-23)$$

By adding both sides of Equations(6-22) and (6-23) respectively, the following equation is obtained:

$$\{f_x(\theta_1) + W_x \cdot a_1\}^2 + \{f_y(\theta_1) + W_y \cdot a_1\}^2 = r_2^2 , \quad (6-24)$$

where,

$$\left. \begin{aligned} f_x(\theta_1) &= p_x - q_x + r_1(U_x \cdot \cos\theta_1 + V_x \cdot \sin\theta_1) , \\ f_y(\theta_1) &= p_y - q_y + r_1(U_y \cdot \cos\theta_1 + V_y \cdot \sin\theta_1) . \end{aligned} \right\} \quad (6-25)$$

Grouping the a_1 terms, we obtain

$$A \cdot a_1^2 + 2 \cdot B \cdot a_1 + C = 0 , \quad (6-26)$$

where coefficients A, B, and C are,

$$\left. \begin{aligned} A &= W_x^2 + W_y^2 , \\ B &= W_x \cdot f_x(\theta_1) + W_y \cdot f_y(\theta_1) , \\ C &= f_x^2(\theta_1) + f_y^2(\theta_1) - r_2^2 . \end{aligned} \right\} \quad (6-27)$$

Let us solve Equation(6-26). In order to express a solution of Equation(6-26), set the following equation:

$$D = B^2 - A \cdot C . \quad (6-28)$$

(A) The case where $A \neq 0$

i) If $D \geq 0$, then

$$a_1 = \frac{(-B \pm \sqrt{D})}{A} . \quad (6-29)$$

In this case, as the coefficients B and C are the function of θ_1 , a_1 obviously becomes the function of θ_1 . (See Equation(6-27).)

ii) If $D < 0$, then there is no solution. Namely, there is no value of a_1 corresponding to any value of θ_1 .

(B) The case where $A = 0$

This case corresponds to that of $W_x = 0$ and $W_y = 0$ in Equation(6-27). Hence the coefficient B is zero. Thus Equation(6-26) becomes as follows:

$$f_x^2(\theta_1) + f_y^2(\theta_1) - r_2^2 = 0 . \quad (6-30)$$

In this case, a_1 cannot be determined. But the intersection line of the two cylinders becomes a straight line which satisfies Equation(6-30). This means that the two cylinders intersect such that their axes of revolution W_1 and W_2 are parallel to each other. An example of this case is illustrated in Fig.6-3-(a). Fig.6-3-(b) shows the case where two cylinders intersect so as to

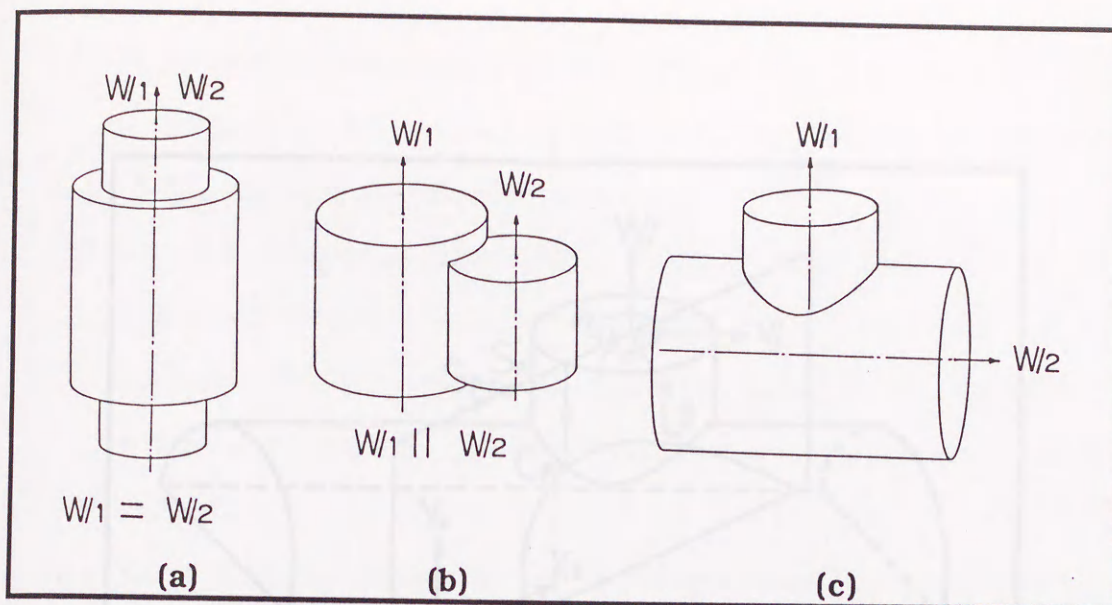


Fig.6-3 The form of the intersection between two cylinders

unify their axes of revolution W_1 and W_2 . As the cases shown in Figs.6-3-(a) and (b) can be handled by the conventional CSG scheme, here we preclude the cases in the discussion for generation of the fillet volume.

Substituting a_1 shown in Equation(6-29) into Equation(6-10), the resulting Equation(6-10) becomes a function of only θ_1 . As a consequence, the resulting Equation(6-10) represents the intersection line between the two cylinders. The intersection line is used as the Sweep Axis in this work. This derivation method is applicable to any combination between the cylinder, the cone, and the sphere[10].

6.4 Sweep parameters

6.4.1 Settlement of the XsYs-plane

We shall clarify how to set the XsYs-plane in this section. Let us consider two intersecting quadrics as shown in Fig.6-4. Then the XsYs-plane is set from a given space point as follows:

- (1) If an arbitrary space point, denoted by S_p , is given, then draw a perpendicular from the point S_p to the quadric C_1 . We refer to the quadric as the objective one.
- (2) As the perpendicular goes through the axis W of the objective quadric, an intersection point, denoted by A_p , between the perpendicular and the axis W is determined, where the point S_p is considered not to be positioned on the axis W .
- (3) Then set the UV-plane so as to contain the point A_p and perpendicular to the axis W .
- (4) Project the space point S_p to the UV-plane in order to determine the angle θ_1 . Let S_p' be the projected point of S_p . Then θ_1 can easily be determined by the positional relation among the projected point S_p' and the axes U and V .
- (5) Substituting the determined θ_1 into Equations(6-25), (6-27), and (6-29), the parameter a_1 is obtained.
- (6) Substituting a_1 into Equation(6-10), an intersection point, which corresponds to the space point S_p and is on the intersection line between the two cylinders, is determined.
When determining the intersection point, there are cases where two intersection points are obtained as illustrated in Fig.6-5. These cases correspond to the condition $D \geq 0$ of Equation(6-28). When $D \geq 0$, there are two solutions of a_1 which correspond to one value of θ_1 . This is the reason why the two intersection points are obtained.
- (7) Let C_p be the intersection point. Then set the plane containing the point C_p and the axis W .

Even if two intersection points are obtained, this plane contains both of them. We use this plane as the XsYs-plane in this work. The XsYs-plane obviously includes the space point S_p and its projected point S_p' .

6.4.2 The case where the XsYs-plane cannot be set normally

As mentioned above, the method for the settlement of the XsYs-plane cannot be applied to the case where the space point S_p is positioned on the axis W of the objective quadric C_1 . In this case, the plane containing the two axes W and U is used for the XsYs-plane.

There are cases where the intersection point C_p cannot be determined as illustrated in Fig.6-6. Hence we cannot set the XsYs-plane normally in these cases. This is caused by the following reason:

The angle θ_1 has an effective region (i.e., $\theta_{min} \leq \theta_1 \leq \theta_{max}$ in Fig.6-6.) which corresponds to the intersection line of the two quadrics, and the effective region is decided automatically by connecting positions of the quadrics. Nevertheless, the angle θ_1 can take any value corresponding to any space point. As a consequence, if the angle θ_1 takes its value from the ineffective region, then the intersection point cannot be determined. Without

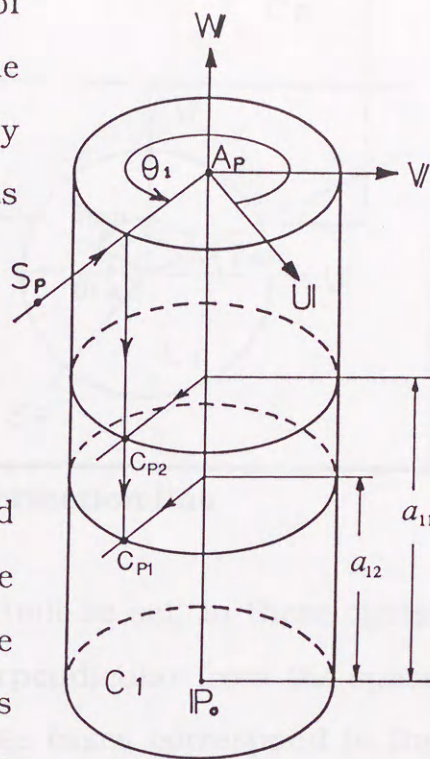
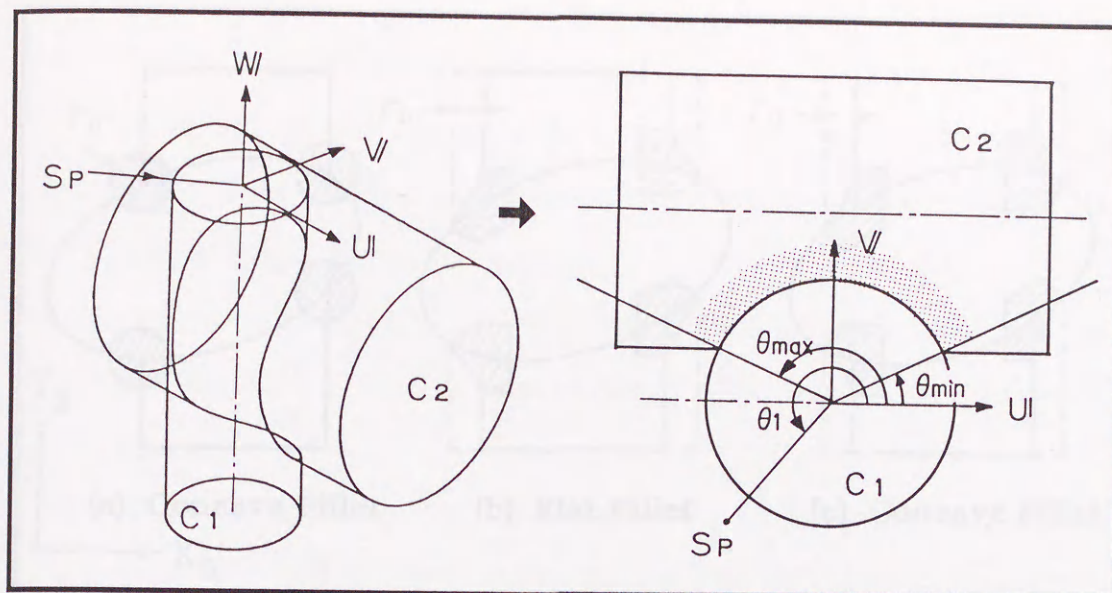


Fig.6-5 The case where two intersection points are led from one space point



**Fig.6-6 An existing region of the intersection line
between two quadrics**

the intersection point, the XsYs-plane cannot be set. In these cases, the objective quadric to be drawn the perpendicular from the space point S_p is changed from C_1 to C_2 . These cases correspond to the condition $D < 0$ of Equation(6-28). When using C_2 as the objective quadric, it is necessary to solve Equations(6-10) and (6-11) for the parameter a_2 instead of a_1 . The derivation method is applicable to these cases in the same form as described in section 6.3.

6.4.3 The Sweep Pattern

Prepared Sweep Patterns are illustrated in Fig.6-7. Namely, *concave fillet*, *convex fillet*, and *flat fillet patterns*. (See shaded areas in the figure.) These come from throat area shapes of the fillet welding. A circle is used as the convex fillet pattern. Thus only the radius r_0 of the circle should be given when modeling the convex fillet volume. In general, since the leg length of the concave fillet and flat fillet patterns can be varied when the pattern is swept

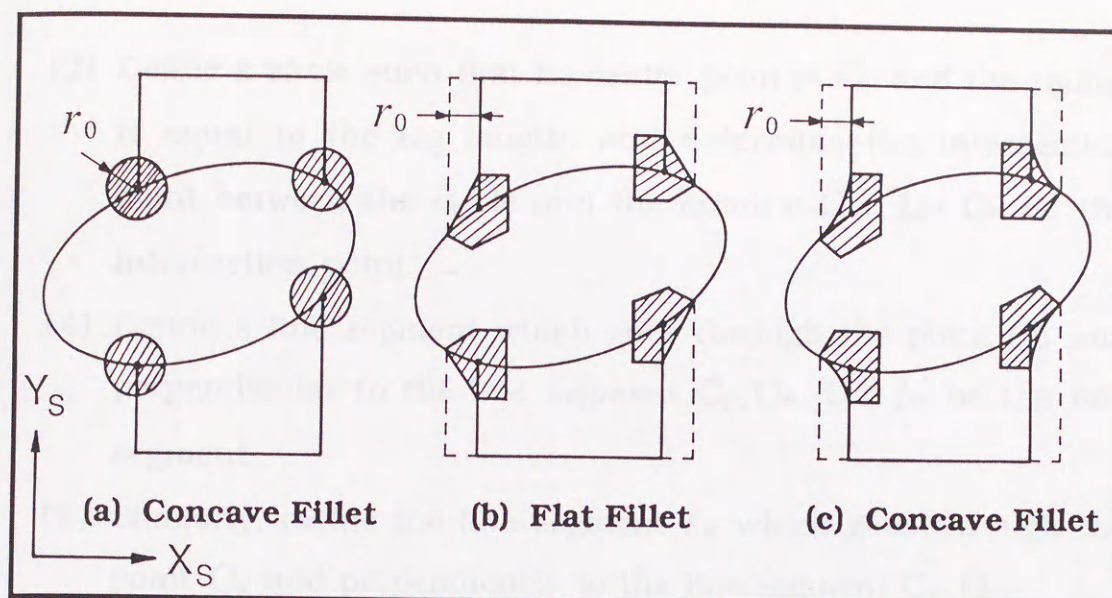


Fig.6-7 Prepared Sweep Patterns

along the intersection line of the quadrics, it is very difficult to give the leg length for specifying a dimension of the pattern. Thus the offset value r_0 of the quadric is given in order to specify the dimension, and then the leg length is calculated when it is necessary. Let us explain how to determine the leg length of the concave fillet pattern, the radius, and the center point of the circle for the concave fillet pattern. Let us consider cross-sections of the quadrics on the X_sY_s -plane as illustrated in Fig.6-8. Let C_1 be the objective quadric, and C_2 be the other. We assume that the intersection point C_p has already been determined. Then the parameters for the concave fillet pattern are determined as follows:

- (1) If the offset value r_0 of the quadric C_1 is given, then determine the intersection point, denoted by O_s , between the offset quadric of C_1 and C_2 .
- (2) Calculate the distance of the line segment C_p, O_s . We take the distance as the leg length of the concave fillet pattern.

- (3) Define a circle such that its center point is C_p and the radius is equal to the leg length, and determine the intersection point between the circle and the quadric C_1 . Let O_R be the intersection point.
- (4) Define a line segment which goes through the point O_R and perpendicular to the line segment C_p, O_R . Let ℓ_R be the line segment.
- (5) Similarly, define the line segment ℓ_s which goes through the point O_s and perpendicular to the line segment C_p, O_s .
- (6) Finally, determine the intersection point C_f between the two line segments ℓ_R and ℓ_s . The resulting point C_f is used as the center point of the circle for the concave fillet pattern. It is obvious that the radius of the circle is equal to the distance between the two points C_f and O_R .

If we define the line segment which goes through the two points O_s and O_R , the flat fillet pattern can be made.

6.5 Definition of the Penalty function

When generating the fillet volume by CSG scheme, the space point evaluation (whether the point is inside, outside, or on the surface of the fillet volume) is inevitable. If the Penalty function[11][12] of the fillet volume can be defined, the space point evaluation can easily be done by means of the function. As mentioned in chapter 2, we may define various Penalty functions in order to meet demands of various application programs. Therefore we shall define the Penalty function of the fillet volume in this section. Let us consider now the XsYs-plane as shown in Fig.6-9.

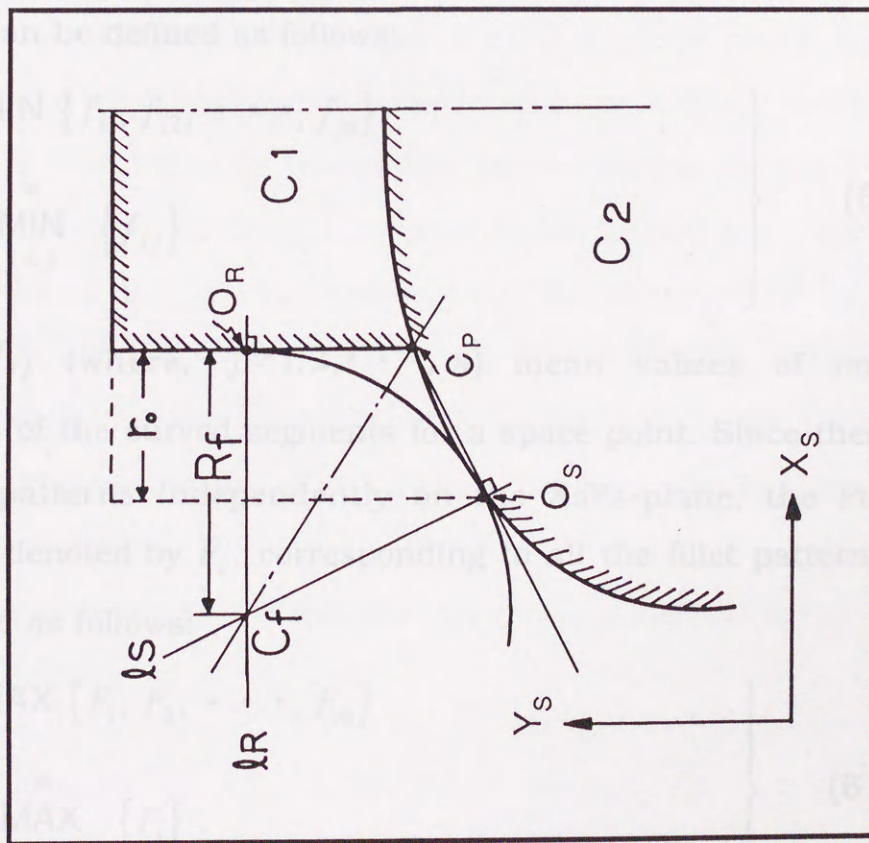


Fig.6-8 The settlement of the leg length of the concave fillet, and of the center point of the circular arc for the concave fillet

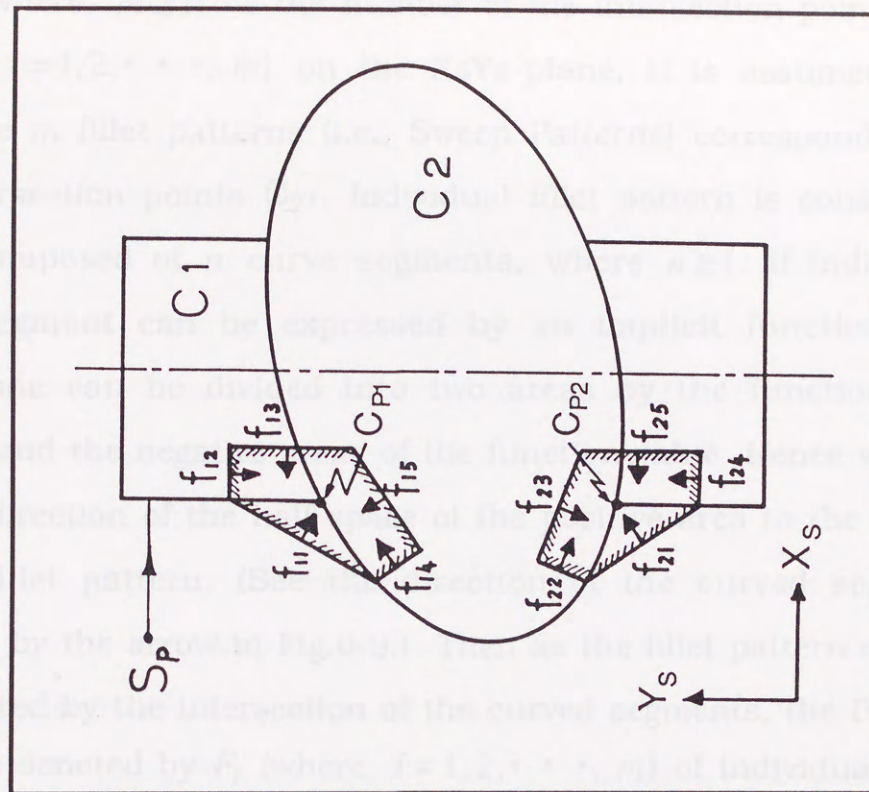


Fig.6-9 The Sweep Pattern represented by directed segments

Let m (where, $m \geq 1$) be the number of the intersection points C_{pi} (where, $i=1,2,\dots,m$) on the XsYs-plane. It is assumed that there are m fillet patterns (i.e., Sweep Patterns) corresponding to the intersection points C_{pi} . Individual fillet pattern is considered to be composed of n curve segments, where $n \geq 1$. If individual curve segment can be expressed by an implicit function, the XsYs-plane can be divided into two areas by the function: the positive and the negative areas of the function value. Hence we can set the direction of the half space of the positive area to the inside of the fillet pattern. (See the direction of the curved segment specified by the arrow in Fig.6-9.) Then as the fillet pattern can be constructed by the intersection of the curved segments, the Penalty function, denoted by F_i (where, $i=1,2,\dots,m$) of individual fillet pattern can be defined as follows:

$$\left. \begin{aligned} F_i &= \text{MIN} \{f_{i1}, f_{i2}, \dots, f_{in}\} \\ &= \text{MIN}_{j=1}^n \{f_{ij}\}, \end{aligned} \right\} \quad (6-31)$$

where f_{ij} (where, $j=1,2,\dots,n$) mean values of implicit functions of the curved segments for a space point. Since there are m fillet patterns independently on the XsYs-plane, the Penalty function, denoted by F_c , corresponding to all the fillet patterns can be defined as follows:

$$\left. \begin{aligned} F_c &= \text{MAX} \{F_1, F_2, \dots, F_m\} \\ &= \text{MAX}_{i=1}^m \{F_i\}. \end{aligned} \right\} \quad (6-32)$$

By substituting Equation(6-32) into (6-31), the Penalty function F_c can be expressed as follows:

$$F_c = \max_{i=1}^m \min_{j=1}^n \{f_{ij}\} . \quad (6-33)$$

The function F_c has a positive/negative value for a space point which belongs to the inside/outside of the fillet pattern, and has zero for a space point on the curved segment which constructs the fillet pattern.

There are cases where two XsYs-planes can be set simultaneously from one space point as shown in Fig.6-10. In these cases, as the Penalty function is defined on the two XsYs-planes independently, two Penalty values can be calculated for one space point. However, the correspondence of one space point and one Penalty value must be kept in order to evaluate a space point. Thus we shall define the rule to opt the representative of the Penalty values. Let "C1-plane" and "C2-plane" be the XsYs-planes which are set by drawing the perpendiculars from the space point S_p to the quadrics C1 and C2, respectively. Let F_{c1} be the Penalty value for the space point S_p on the C1-plane, and F_{c2} be that on the C2-plane. Then the representative of the Penalty values, denoted by F , is defined as follows:

If both the C1-plane and the C2-plane can be set from the space point S_p , then

$$F = \max \{F_{c1}, F_{c2}\} . \quad (6-34)$$

If the C1-plane can only be set from the space point S_p , then

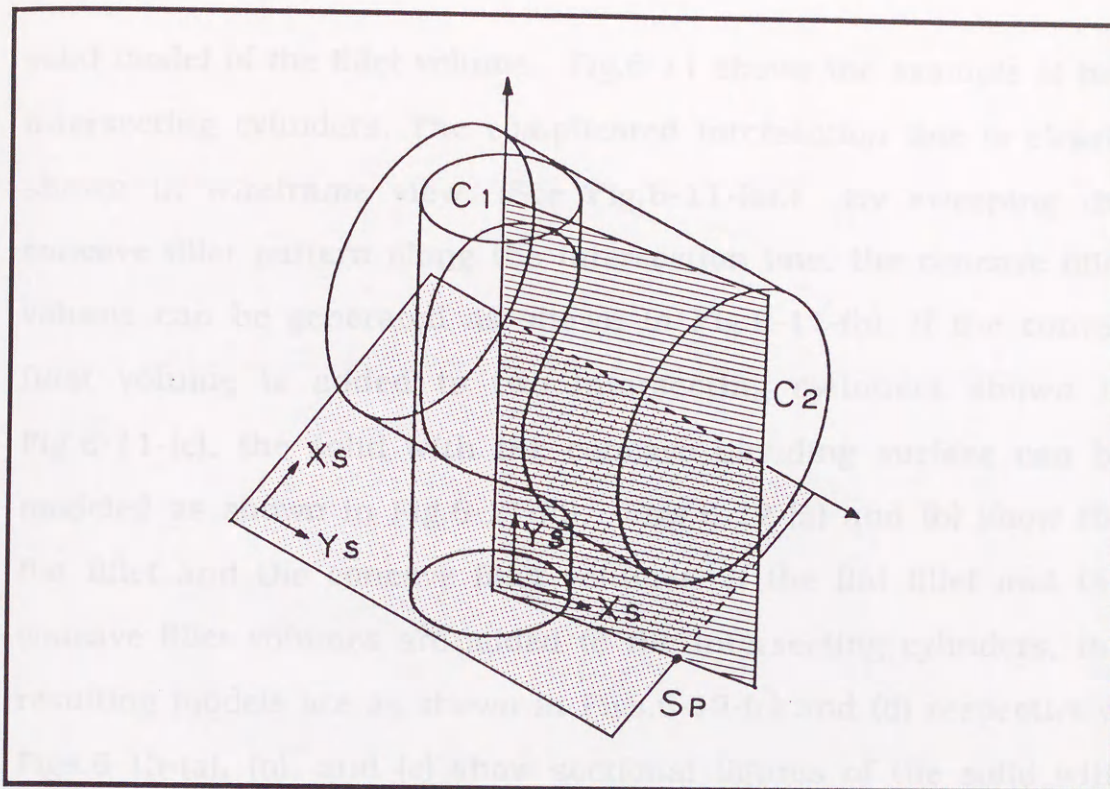


Fig.6-10 Two XsYs-planes led from one space point

$$F = F_{c1} . \quad (6-35)$$

Reversely, if the C2-plane can only be set from the space point S_p , then

$$F = F_{c2} . \quad (6-36)$$

By judging the representative value F for a space point during the sweep, the fillet volume can be generated as a set of space points.

6.6 Experiments and the consideration

In order to prove the correctness of the derived equations and to verify the effectiveness of the applied method, the computational algorithm has been developed and performed with the CSG-based CAD/CAM system TIPS-1[11] on the HITAC-M280 computer. Figures 6-11 through 6-17 show examples of the 3-D

solid model of the fillet volume. Fig.6-11 shows the example of two intersecting cylinders. The complicated intersection line is clearly shown in wireframe view. (See Fig.6-11-(a).) By sweeping the concave fillet pattern along the intersection line, the concave fillet volume can be generated as shown in Fig.6-11-(b). If the convex fillet volume is added to two intersecting cylinders shown in Fig.6-11-(c), the solid with the concave blending surface can be modeled as shown in Fig.6-11-(d). Figs.6-12-(a) and (b) show the flat fillet and the concave fillet volumes. If the flat fillet and the concave fillet volumes are added to the intersecting cylinders, the resulting models are as shown in Figs.6-12-(c) and (d) respectively. Figs.6-13-(a), (b), and (c) show sectional figures of the solid with the concave fillet, flat fillet, and convex fillet volumes respectively. In Fig.6-13-(c), we can find that the two cylinders are blended smoothly by the concave fillet volume. The two cylinders used in Figures 6-11 through 6-13 intersect with a right angle. Fig.6-14 shows the example of two cylinders intersecting with an acute angle. From Fig.6-14, we can find that the radius of the circle for the concave fillet is varied during the sweep. Another example of the automatic variation of the dimension of the Sweep Pattern is shown in Fig.6-15. Fig.6-16 shows the example that the intersection line is composed of two separate closed curves. Thus two separate fillet volumes are generated automatically in this case. Fig.6-17-(a) shows the example of two intersecting spheres, and Fig.6-17-(b) shows that of two cones.

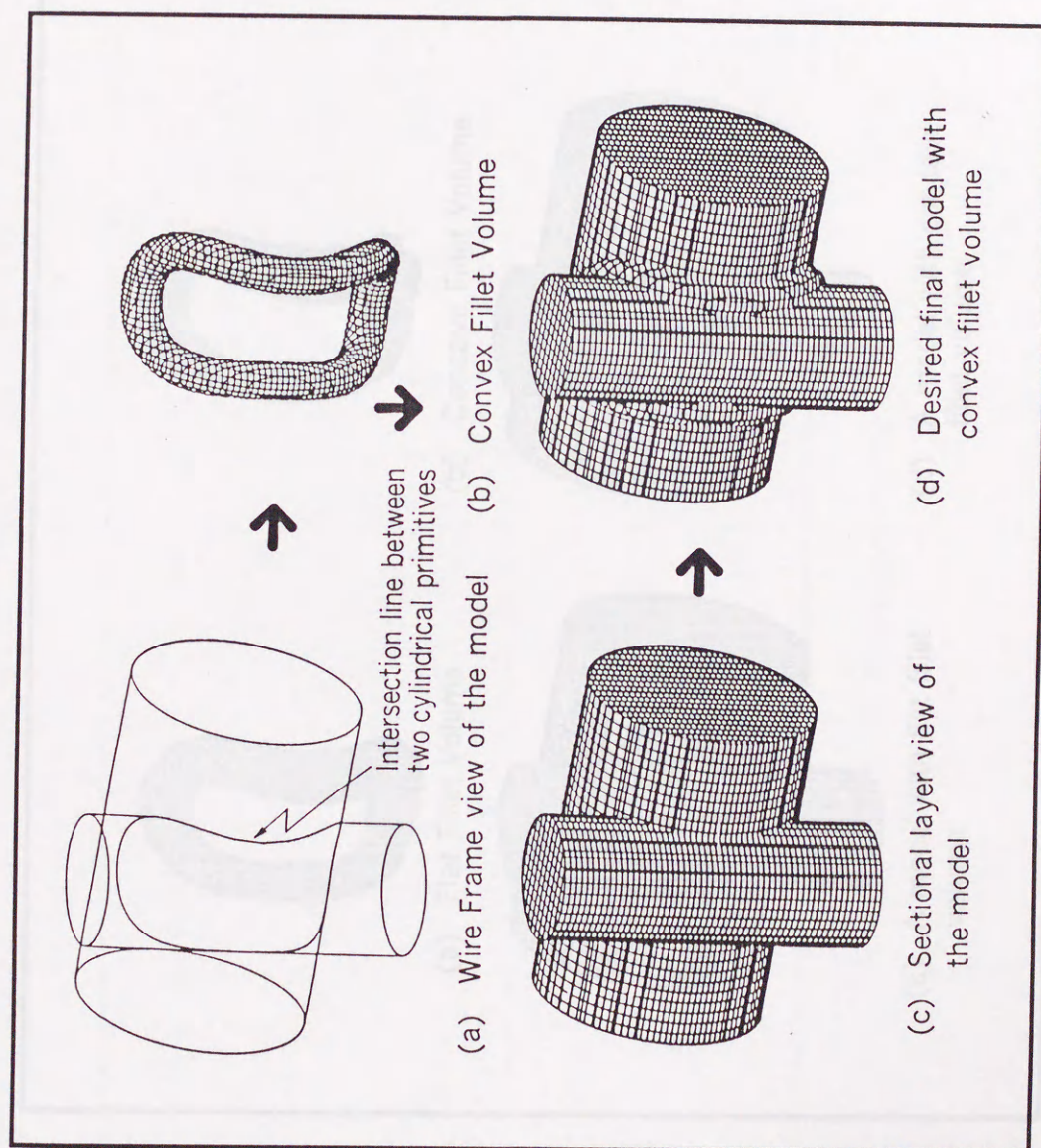


Fig.6-11 Experimental results

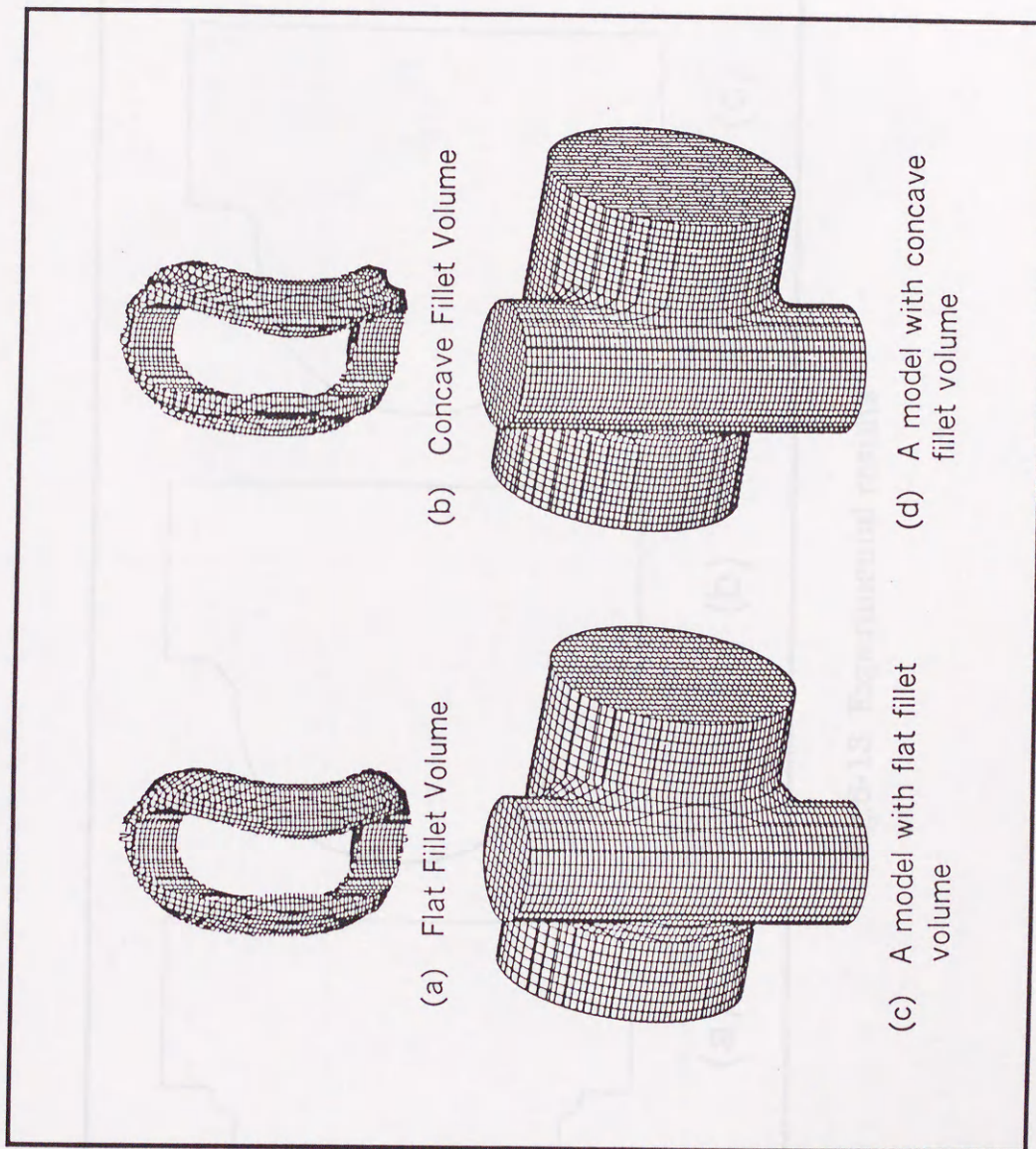


Fig.6-12 Experimental results

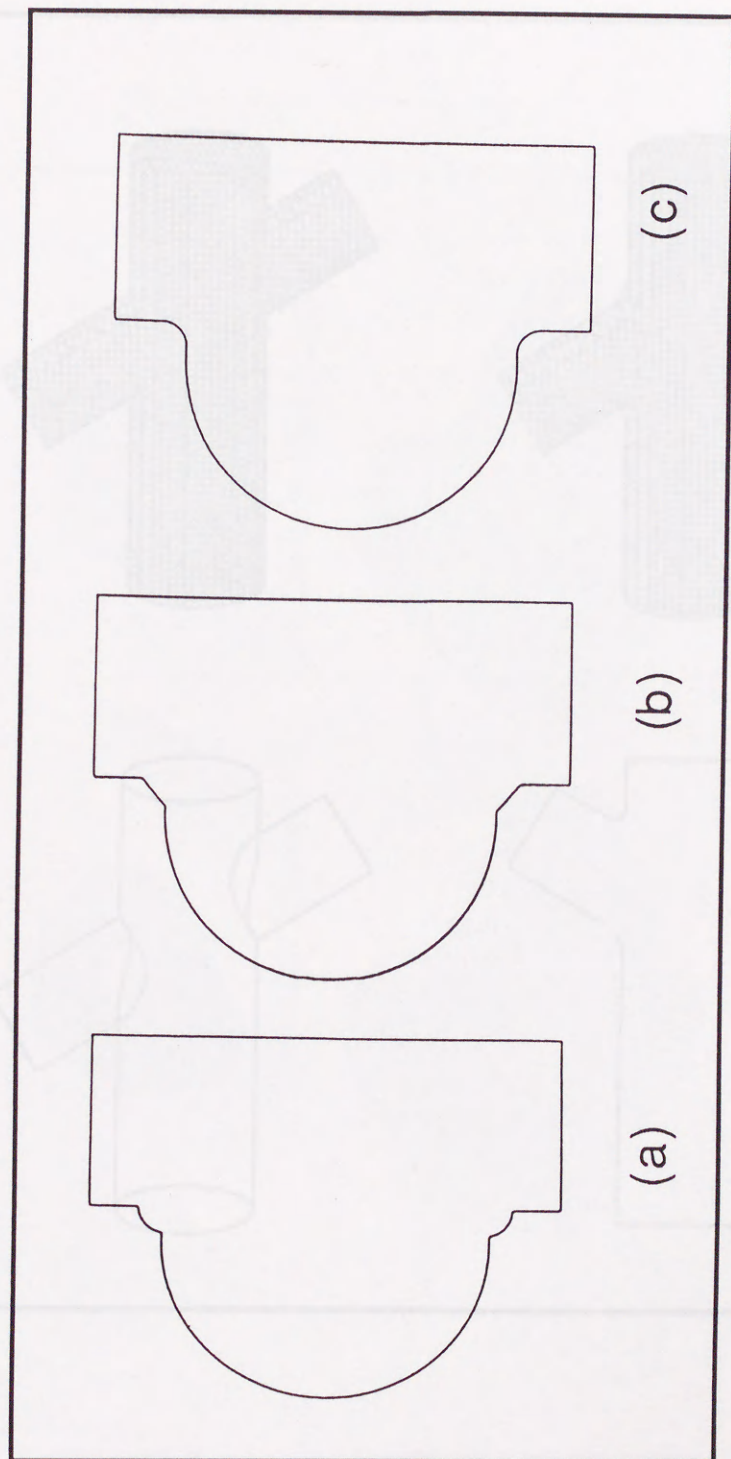


Fig.6-13 Experimental results

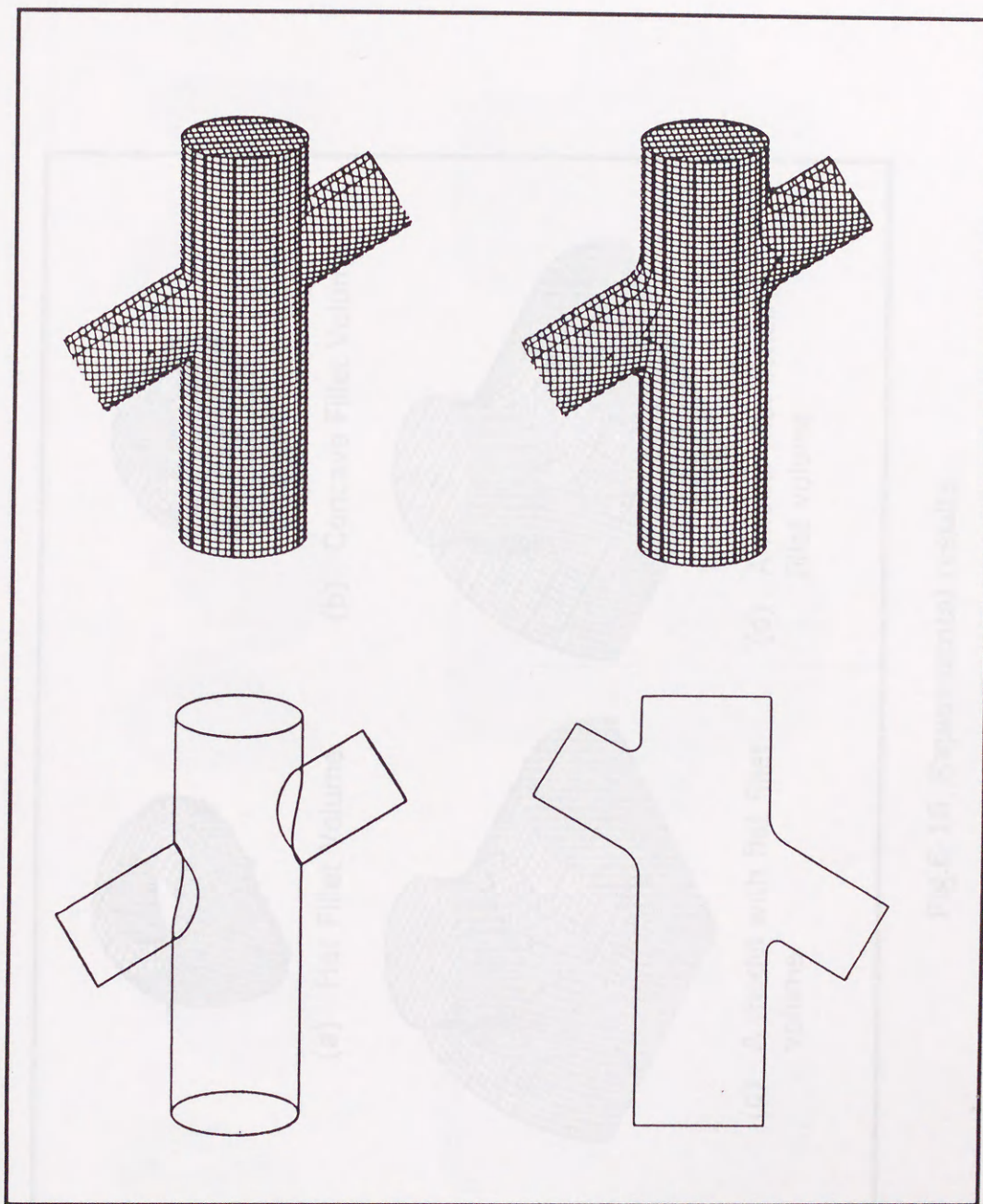


Fig.6-14 Experimental results

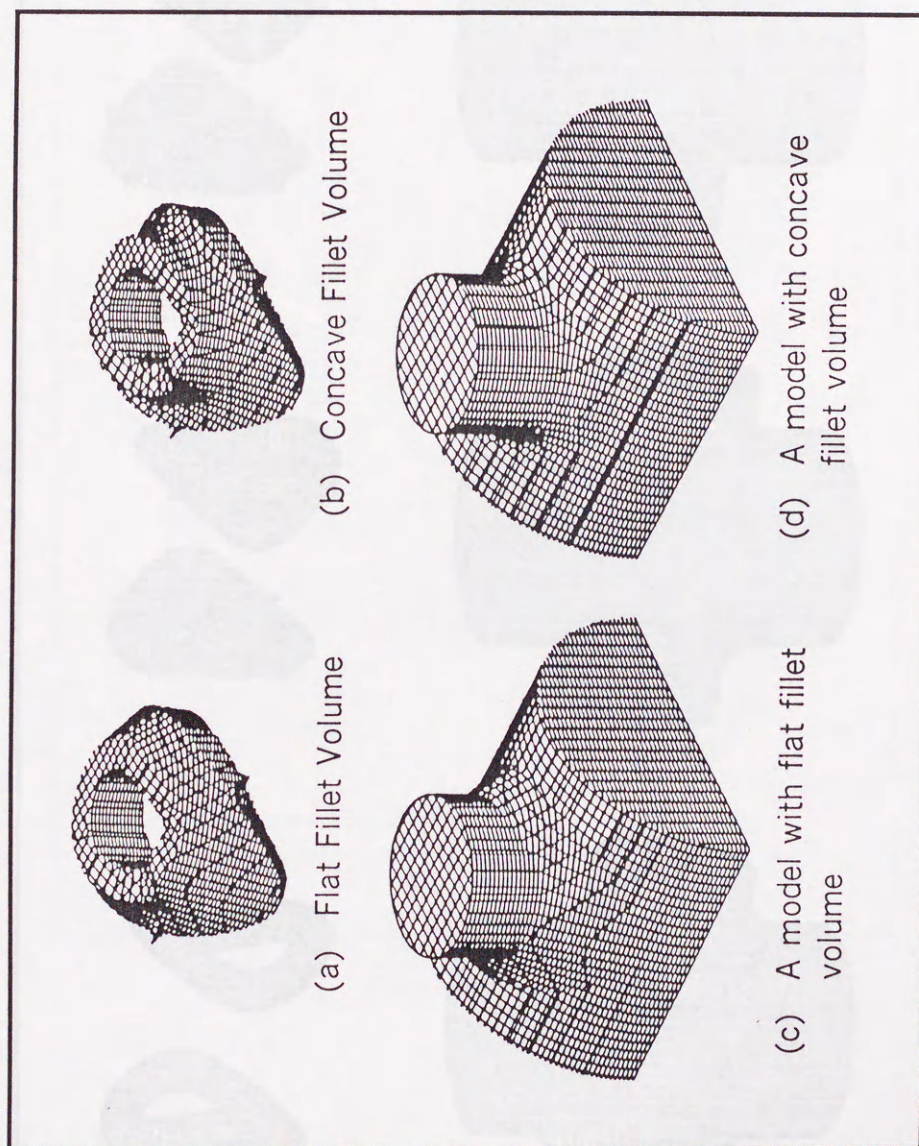


Fig.6-15 Experimental results

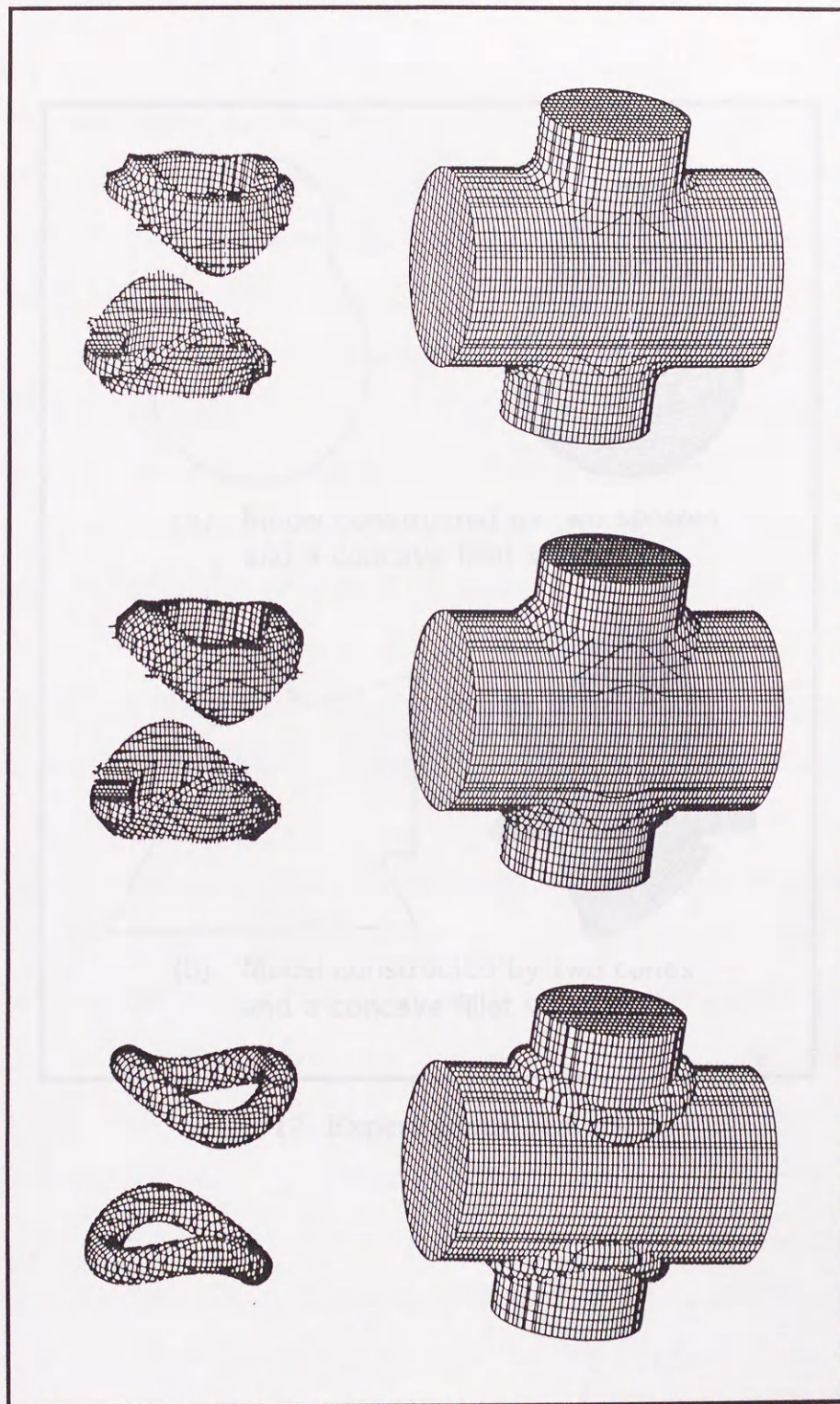


Fig.6-16 Experimental results

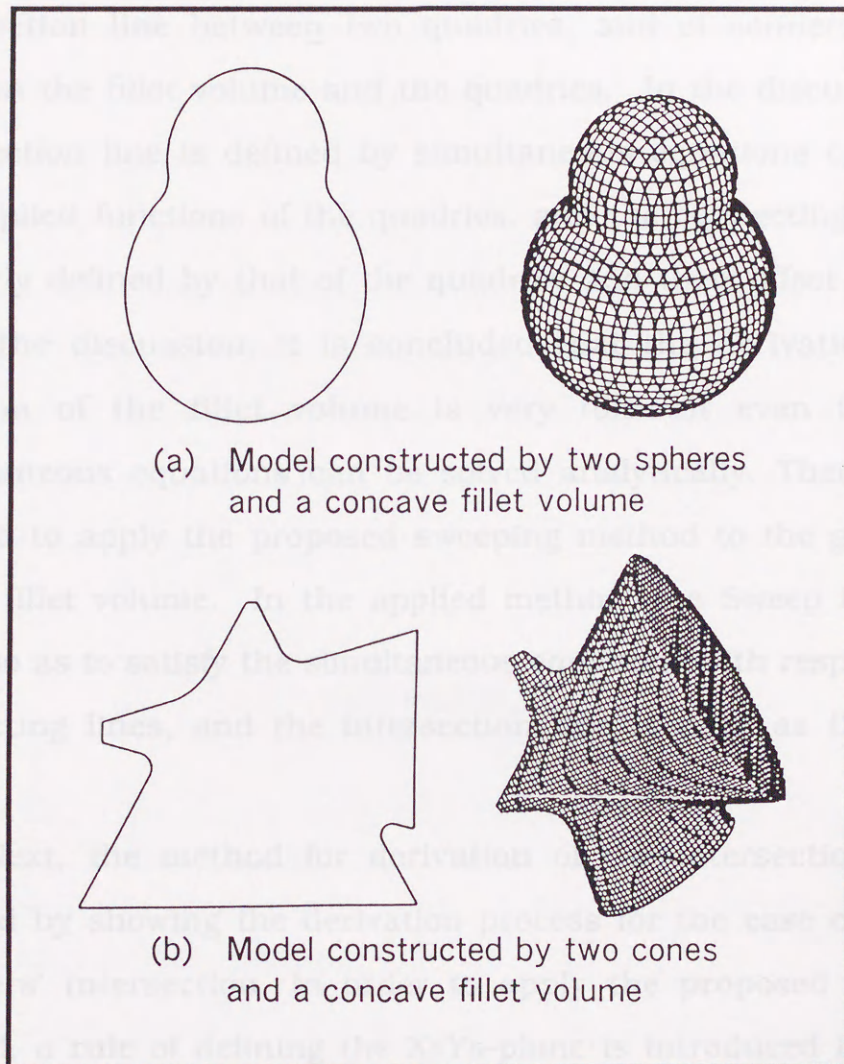


Fig.6-17 Experimental results

6.7 conclusion

In this chapter, first, the problem for generation of the fillet volume is discussed through the mathematical definitions of the intersection line between two quadrics, and of connecting lines between the fillet volume and the quadrics. In the discussion, the intersection line is defined by simultaneous equations concerning the implicit functions of the quadrics, and the connecting lines are similarly defined by that of the quadrics and their offset quadrics. From the discussion, it is concluded that the derivation of the function of the fillet volume is very difficult even if all the simultaneous equations can be solved analytically. Therefore, we decided to apply the proposed sweeping method to the generation of the fillet volume. In the applied method, the Sweep Pattern is given so as to satisfy the simultaneous equations with respect to the connecting lines, and the intersection line is used as the Sweep Axis.

Next, the method for derivation of the intersection line is clarified by showing the derivation process for the case of the two cylinders' intersection. In order to apply the proposed sweeping method, a rule of defining the XsYs-plane is introduced instead of the Spine Axis, and the rule is explained in detail through the settlement of the XsYs-plane from a space point.

The convex fillet, concave fillet, and flat fillet patterns are prepared for the Sweep Pattern. The convex fillet pattern is given by a circle. It is necessary to give the leg length of the concave fillet and flat fillet patterns in order to specify their dimensions. The leg length, however, can be varied during the sweep. Thus the

offset value of the quadric is given instead of the leg length in this work, and the way of calculating the leg length from the offset value is clarified.

The Penalty function of the fillet pattern is defined for the space point evaluation when generating the fillet volume. Finally, the computational algorithm of the applied method has been developed and performed with the CSG-based CAD/CAM system TIPS-1. According to the experiments, it is clear that the method applied to generate the fillet volume is effective enough, derived equations of the intersection line between the quadrics are correct, and that the generated fillet volume can be handled just like the ordinary Primitive.

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7. SOLID MODELING OF THE FILLET VOLUME

- ON THE FILLET FOR CORNER ROUNDING -

7.1 Introduction

Most mechanical parts have corners such as edges and vertices, but most of these corners are rounded. This is resulted from design decisions based on the aesthetic, structural, and functional criteria, and on the consideration for the ease of manufacturing and assembling. Thus the rounding operation with the blending surface is essential when the shape of the mechanical part is practically designed. Though the simple case of the blending surface to round the corner can be substituted by the spherical and cylindrical surfaces in general, it has been considered to be difficult to represent the blending surface by the solid modeling schemes. Thus the blending surface has been represented by the surface modeling schemes[1][2][3]. Surface modeling schemes, however, are not consistent with those of solid modeling. From the viewpoint of the modeling consistency, the blending surface to round the corner should be represented by the solid modeling schemes in 3-D solid modeling systems. Thus this work started aiming at representing the solid with the blending surface (i.e., the fillet volume) by CSG scheme.

There are many kinds of corners. In these corners, a straight line edge and a vertex constructed only by the straight line edges are found very often in the shape of mechanical parts. Thus the

above cases of edges and vertices are opted as the object of the corner rounding problem to be solved in this work.

Although some attempts have been reported[4][5][6][7][8] recently for generation of the fillet volume to round the corner, these attempts must solve the higher degree algebraic equations. For example, suppose the case where three edges meet at a vertex. If the fillet volume to round each edge has degree 4, then the fillet volume to round the vertex has at least degree 8. In general, it is very difficult to solve such higher degree equations analytically. Moreover these equations are difficult to be controlled intuitively. More simple approaches are required. Thus we shall apply a new method[9][10][11] based on the proposed sweeping method to represent the fillet volume by CSG scheme. In the following sections, we shall describe the new method.

7.2 The corner rounding problem

It has been considered that the fillet volume to round the corner is very difficult to be represented by CSG scheme. This is caused by the following reasons:

- As the kinds of the Primitives are limited to the analytical solids such as spheres and cylinders, the simple case of the fillet volume can only be represented by the Primitives.
- There is no established way of connecting two or more fillet volumes smoothly. Hence it is difficult to round a vertex and the related edges simultaneously.
- As the CSG data do not contain the geometric/topological entities such as surfaces/faces, curves/edges and

points/vertices explicitly, it is difficult to operate the entities in order to connect the fillet volume and the related Primitives smoothly.

We shall solve the above problems as follows:

- The capability to represent the fillet volume is extended by applying the proposed sweeping method. This advantage is resulted from the uniqueness that the Sweep and Spine Axes are separated in the proposed method though these axes are unified in the conventional method.
- In order to round a vertex and the related edges simultaneously, parameters for the fillet volume to round the vertex and that to round the edge are related.
- In order to connect the fillet volume and the related Primitives smoothly, the offset parameter of the Primitive is introduced and used instead of the geometric/topological entities of the Primitive. Actually, the offset parameter is added to the data of each Primitive, and it works automatically so as to offset the related surface of the Primitive in accordance with the fillet volume.

7.3 The corner rounding method

7.3.1 Classification of the edge and the vertex

As the vertex is composed of more than two edges, the fillet volume to round the vertex is more complicated than that to round each edge. Moreover the shape of the fillet volume to round the vertex is varied in accordance with the type of the vertex. Thus it is

better to classify the edge and the vertex, and then to generate the fillet volume on the basis of the classification.

The edge can be classified into two types, namely, the *concave* and *convex edges*. (See Fig.7-1.) Though there are cases where more than three edges meet at the vertex, we shall classify the vertex by the types of the three edges which meet at the vertex. Then the vertex can be classified as follows.

- *The 3 convex vertex* : This vertex is composed of three convex edges.
- *The 3 concave vertex* : This vertex is composed of three concave edges.
- *The 2 convex 1 concave vertex* : This vertex is composed of two convex edges and one concave edge.
- *The 2 concave 1 convex vertex* : This vertex is composed of two concave edges and one convex edge.

These classified vertices are illustrated in Fig.7-2. When the vertex is composed of more than three edges, it is regarded as the composite vertex of the above classified vertices.

7.3.2 The corner rounding by the additional fillet volume

In general, there are two ways to round the corner: namely, the additional and subtractive methods, as illustrated in Fig.7-3-(a). If only one edge is considered as the object of the rounding and if the edge is convex (or concave), then the subtractive (or the additional) method is conventionally applied. However, if the vertex requires both the additional and subtractive methods simultaneously to round more than two edges which meet at the

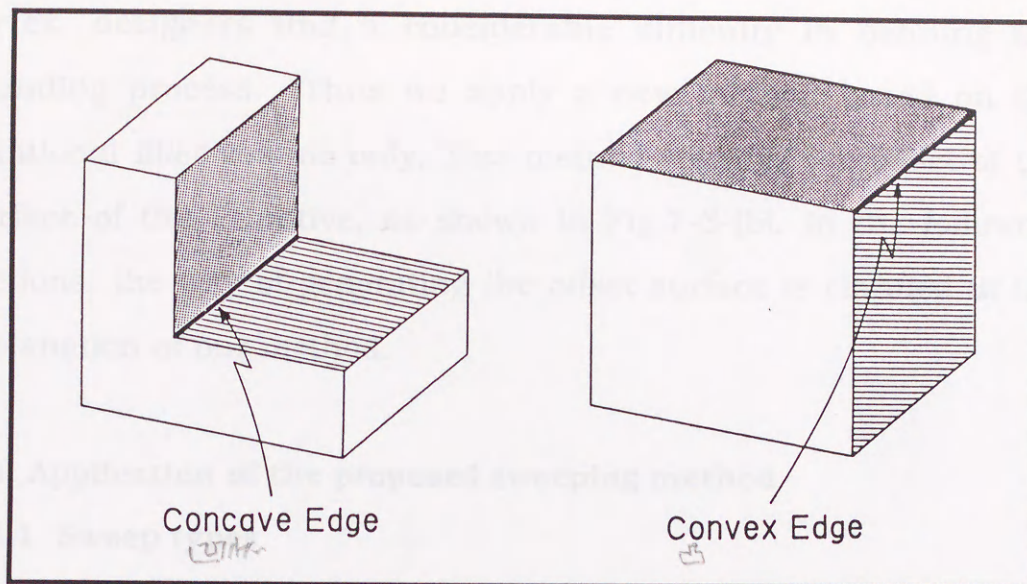


Fig.7-1 The concave and convex edges

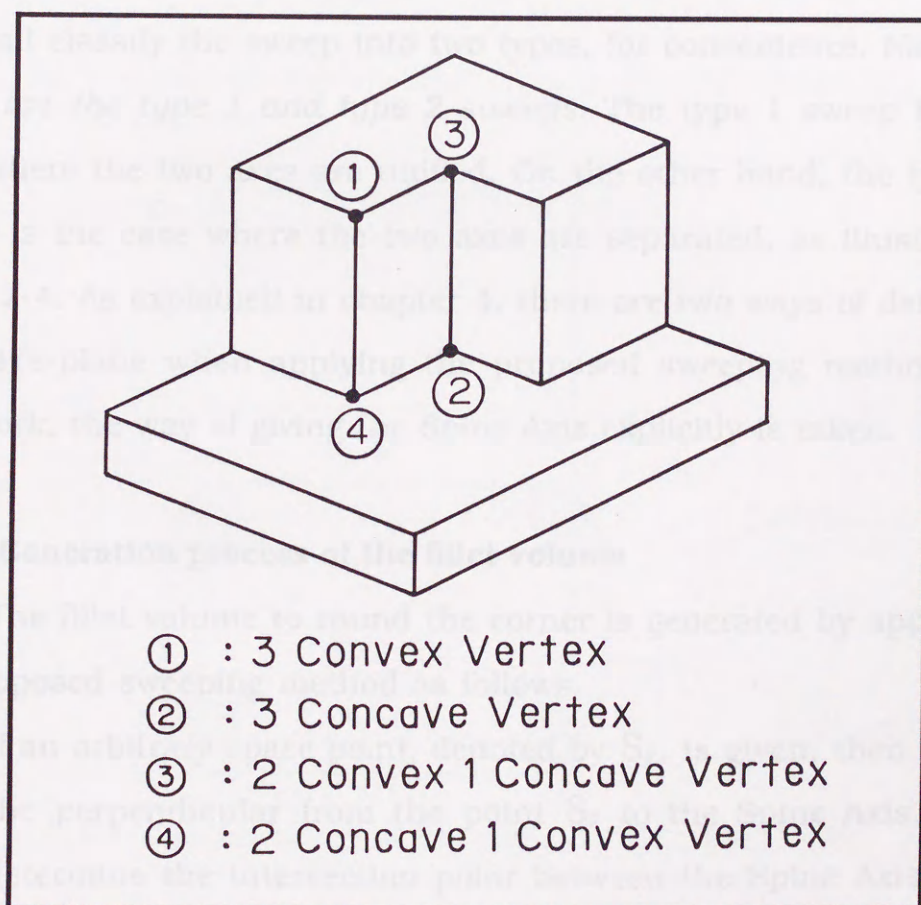


Fig.7-2 The classified vertices

vertex, designers find a considerable difficulty in defining the rounding process. Thus we apply a new method based on the additional fillet volume only. This method requires the offset of the surface of the Primitive, as shown in Fig.7-3-(b). In the following sections, the way of generating the offset surface is clarified in the explanation of our method.

7.4 Application of the proposed sweeping method

7.4.1 Sweep types

Though the Sweep and Spine Axes are separated in the proposed sweeping method, these axes can be unified. In this work, we shall classify the sweep into two types, for convenience. Namely, these are *the type 1 and type 2 sweeps*. The type 1 sweep is the case where the two axes are unified. On the other hand, the type 2 sweep is the case where the two axes are separated, as illustrated in Fig.7-4. As explained in chapter 4, there are two ways of defining the XsYs-plane when applying the proposed sweeping method. In this work, the way of giving the Spine Axis explicitly is taken.

7.4.2 Generation process of the fillet volume

The fillet volume to round the corner is generated by applying the proposed sweeping method as follows.

- (1) If an arbitrary space point, denoted by S_p , is given, then draw the perpendicular from the point S_p to the Spine Axis, and determine the intersection point between the Spine Axis and the perpendicular.

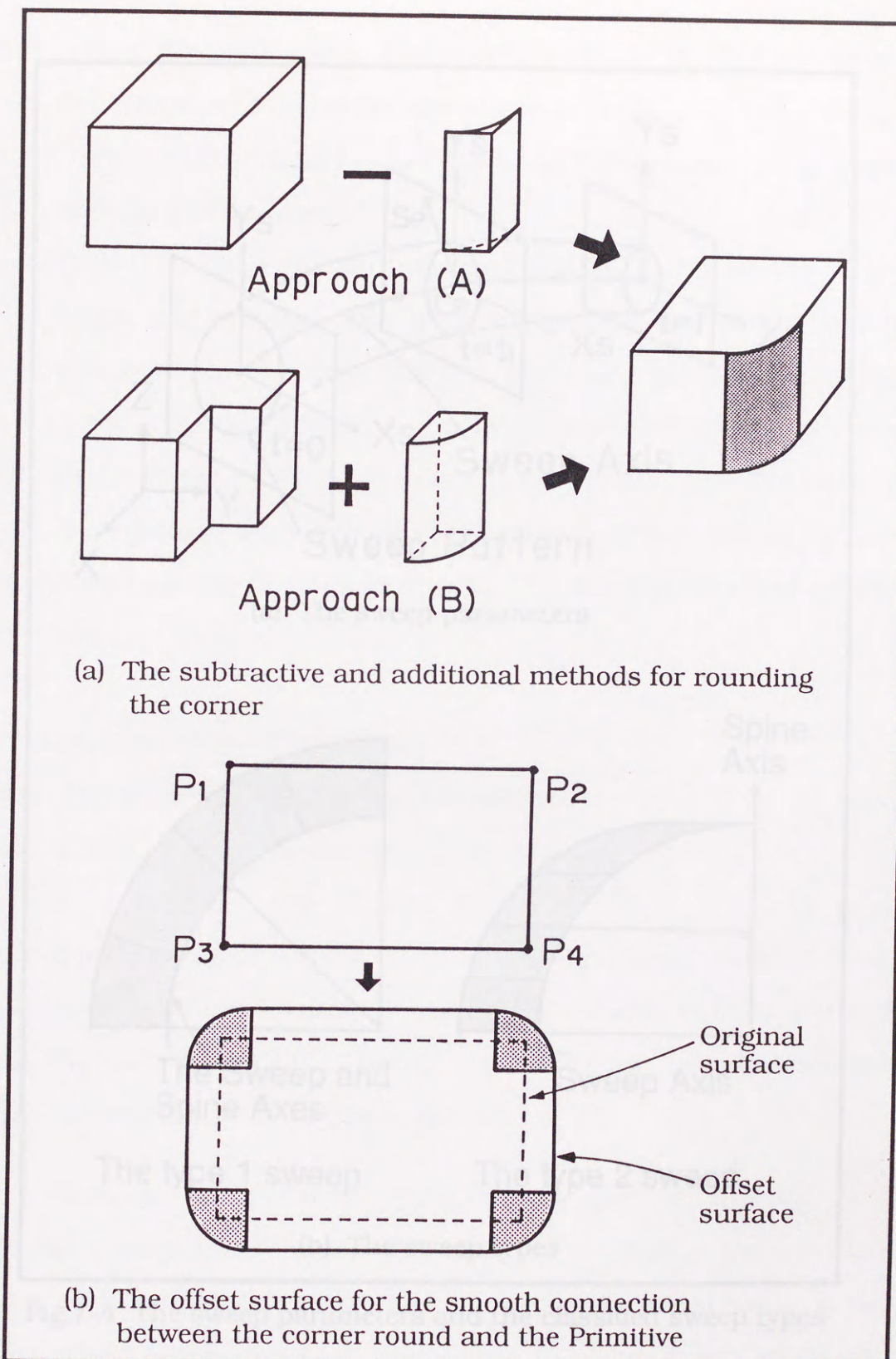


Fig.7-3 The corner rounding

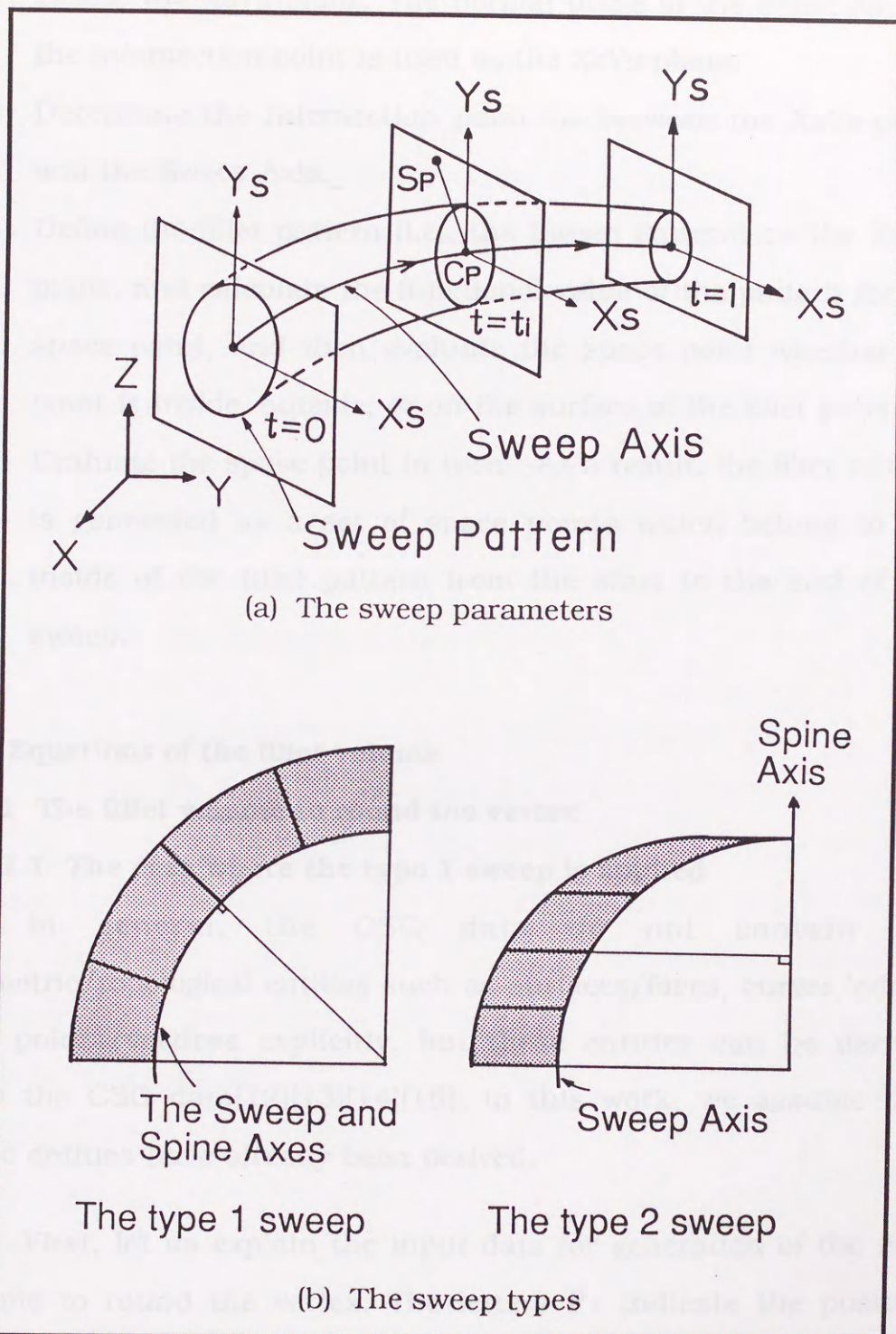


Fig.7-4 The sweep parameters and the classified sweep types

- (2) Define the XsYs-plane. The normal plane of the Spine Axis at the intersection point is used as the XsYs-plane.
- (3) Determine the intersection point C_p between the XsYs-plane and the Sweep Axis.
- (4) Define the fillet pattern (i.e., the Sweep Pattern) on the XsYs-plane, and calculate the functional value of the pattern for the space point, and then evaluate the space point whether the point is inside, outside, or on the surface of the fillet pattern.
- (5) Evaluate the space point in turn. As a result, the fillet volume is generated as a set of space points which belong to the inside of the fillet pattern from the start to the end of the sweep.

7.5 Equations of the fillet volume

7.5.1 The fillet volume to round the vertex

7.5.1.1 The case where the type 1 sweep is applied

In general, the CSG data do not contain the geometric/topological entities such as surfaces/faces, curves/edges, and points/vertices explicitly, but these entities can be derived from the CSG data[12][13][14][15]. In this work, we assume that these entities have already been derived.

First, let us explain the input data for generation of the fillet volume to round the vertex. The letters P_0 indicate the position vector of the vertex, and V_1 , V_2 , W represent the vectors of the three edges which meet at the vertex P_0 . The letter " a " is the Sweep Axis control parameter, and " R " is the radius of the circle

used as the fillet pattern. A circular arc is used as the Sweep Axis, and is provided to be defined on the V_1V_2 -plane or on the plane which is parallel to the V_1V_2 -plane. We refer to the plane as *the Sweep Axis defining plane*. The control parameter a means the distance from the vertex P_0 and is measured along the edge V_1 (or V_2). The control parameter a works so as to define the center point P_c and the radius ℓ of the circular arc. The relationship among the input data is illustrated in Fig.7-5. When the type 1 sweep is applied, a circular arc is used as the Spine and Sweep Axes. On the other hand, when the type 2 sweep is applied, the straight line which is perpendicular to the edge V_1 (or V_2) and is defined on the V_1V_2 -plane is used as the Spine Axis.

Let us derive the equation of the fillet volume to round the 3 convex vertex P_0 as illustrated in Fig.7-6. Let the type 1 sweep be applied. It is assumed that the Sweep Axis defining plane is projected to the xy -plane of the 3-D orthogonal coordinate system. If an arbitrary space point $S_p(x, y, z)$ is given, then the representative point $C_p(x_{cp}, y_{cp}, z_{cp})$ of the Sweep Pattern is determined as follows:

$$\left. \begin{aligned} x_{cp} &= x_c + (x - x_c) \cdot \ell / d , \\ y_{cp} &= y_c + (y - y_c) \cdot \ell / d , \\ z_{cp} &= z_c , \\ \text{where ,} \\ d &= \sqrt{(x - x_c)^2 + (y - y_c)^2} , \\ \ell &= a \cdot \tan(\theta/2) , \quad \ell \geq 0 , \quad d > 0 , \end{aligned} \right\} \quad (7-1)$$

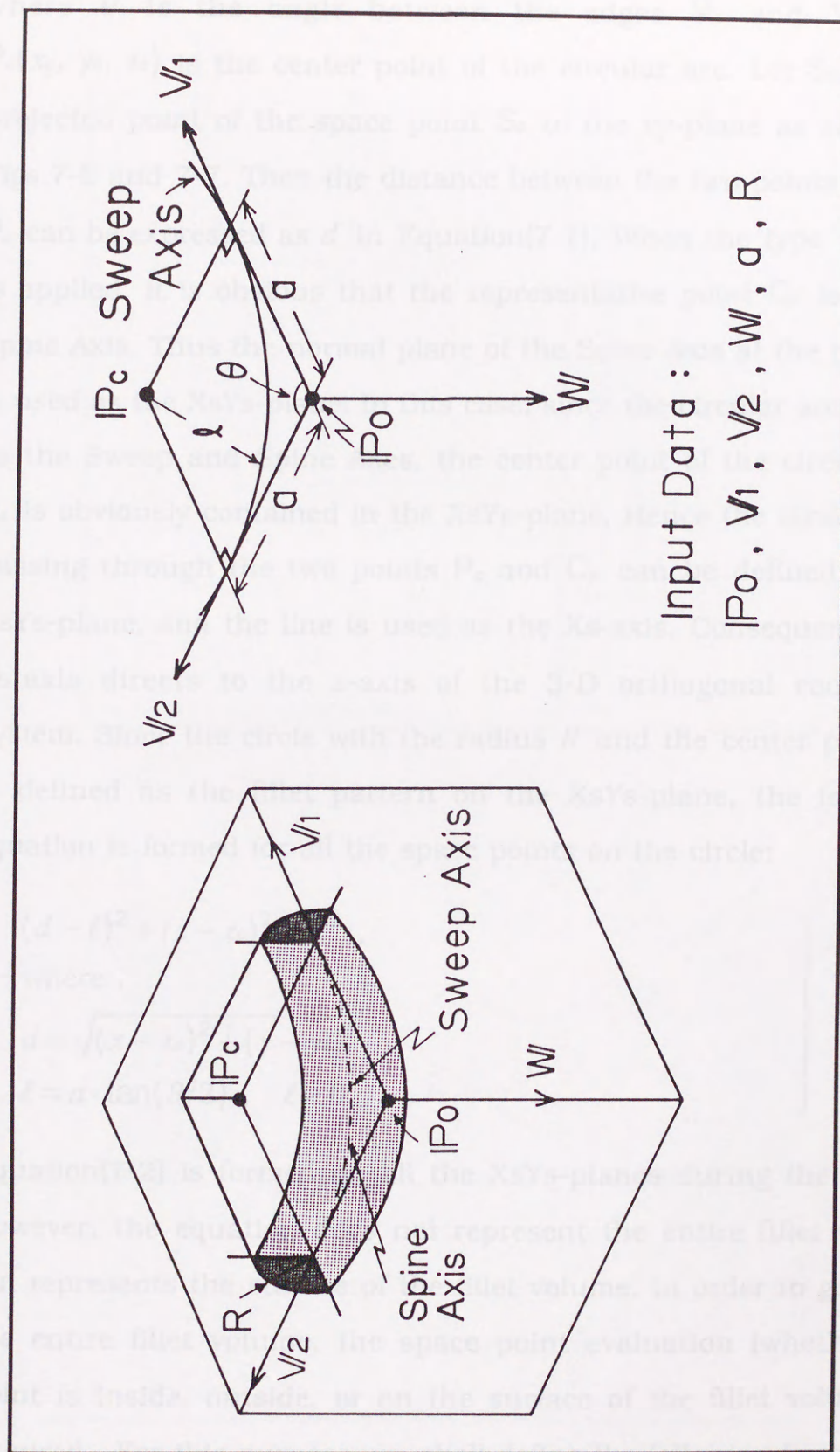


Fig.7-5 The input data for the fillet volume to round the vertex

where θ is the angle between the edges V_1 and V_2 , and $P_c(x_c, y_c, z_c)$ is the center point of the circular arc. Let S_p' be the projected point of the space point S_p to the xy -plane as shown in Figs.7-6 and 7-7. Then the distance between the two points S_p' and P_c can be expressed as d in Equation(7-1). When the type 1 sweep is applied, it is obvious that the representative point C_p is on the Spine Axis. Thus the normal plane of the Spine Axis at the point C_p is used as the $XsYs$ -plane. In this case, since the circular arc is used as the Sweep and Spine Axes, the center point of the circular arc P_c is obviously contained in the $XsYs$ -plane. Hence the straight line passing through the two points P_c and C_p can be defined on the $XsYs$ -plane, and the line is used as the Xs -axis. Consequently, the Ys -axis directs to the z -axis of the 3-D orthogonal coordinate system. Since the circle with the radius R and the center point C_p is defined as the fillet pattern on the $XsYs$ -plane, the following equation is formed for all the space points on the circle:

$$\left. \begin{aligned} (d - \ell)^2 + (z - z_c)^2 &= R^2, \\ \text{where,} \\ d &= \sqrt{(x - x_c)^2 + (y - y_c)^2}, \\ \ell &= a \cdot \tan(\theta/2), \quad \ell \geq 0. \end{aligned} \right\} \quad (7-2)$$

Equation(7-2) is formed on all the $XsYs$ -planes during the sweep. However, the equation does not represent the entire fillet volume but represents the surface of the fillet volume. In order to generate the entire fillet volume, the space point evaluation (whether the point is inside, outside, or on the surface of the fillet volume) is required. For this purpose, we shall define the following function:

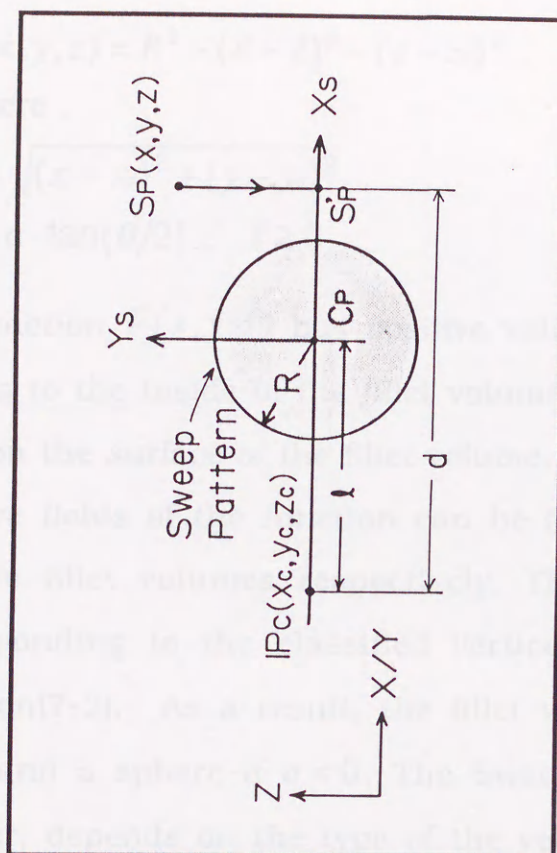
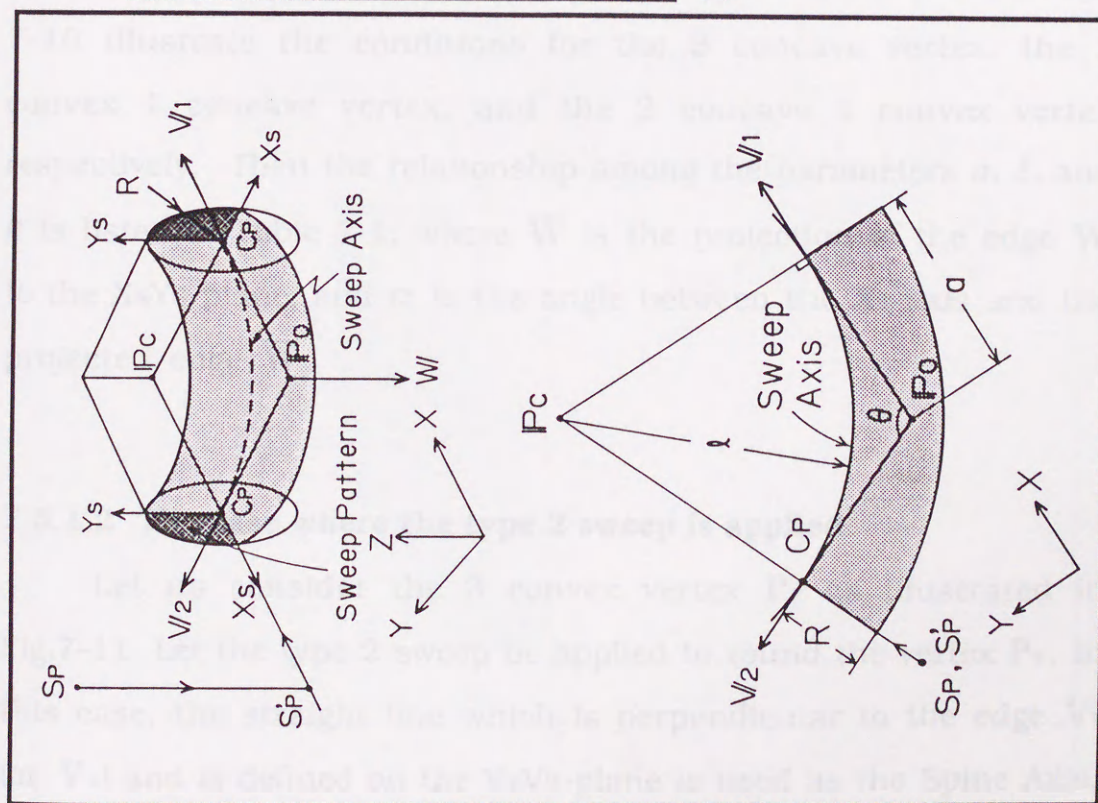


Fig. 7-7 Relationship among the points S_p , C_p , and P_c on the XsYs-plane

Fig. 7-6 Spatial relationship among the parameters for generation of the fillet volume to round the 3 convex vertex (The case where the type 1 sweep is applied)

$$\left. \begin{aligned}
 F(x, y, z) &= R^2 - (d - \ell)^2 - (z - z_c)^2, \\
 \text{where,} \\
 d &= \sqrt{(x - x_c)^2 + (y - y_c)^2}, \\
 \ell &= a \cdot \tan(\theta/2), \quad \ell \geq 0.
 \end{aligned} \right\} (7-3)$$

The function $F(x, y, z)$ has positive value for a space point which belongs to the inside of the fillet volume, and has zero for a space point on the surface of the fillet volume. Moreover the positive and negative fields of the function can be adapted to the convex and concave fillet volumes respectively. Thus all the fillet volumes corresponding to the classified vertices can be represented by Equation(7-3). As a result, the fillet volume becomes a torus if $a > 0$, and a sphere if $a = 0$. The Sweep Axis setting conditions, however, depends on the type of the vertex. Figures 7-8 through 7-10 illustrate the conditions for the 3 concave vertex, the 2 convex 1 concave vertex, and the 2 concave 1 convex vertex respectively. Then the relationship among the parameters a , ℓ , and R is listed in Table 7-1, where $\bar{W}$ is the projection of the edge W to the XsYs-plane, and α is the angle between the Xs-axis and the projected edge $\bar{W}$.

7.5.1.2 The case where the type 2 sweep is applied

Let us consider the 3 convex vertex P_0 as illustrated in Fig.7-11. Let the type 2 sweep be applied to round the vertex P_0 . In this case, the straight line which is perpendicular to the edge V_1 (or V_2) and is defined on the V_1V_2 -plane is used as the Spine Axis, and the circular arc defined on the V_1V_2 -plane is used as the Sweep

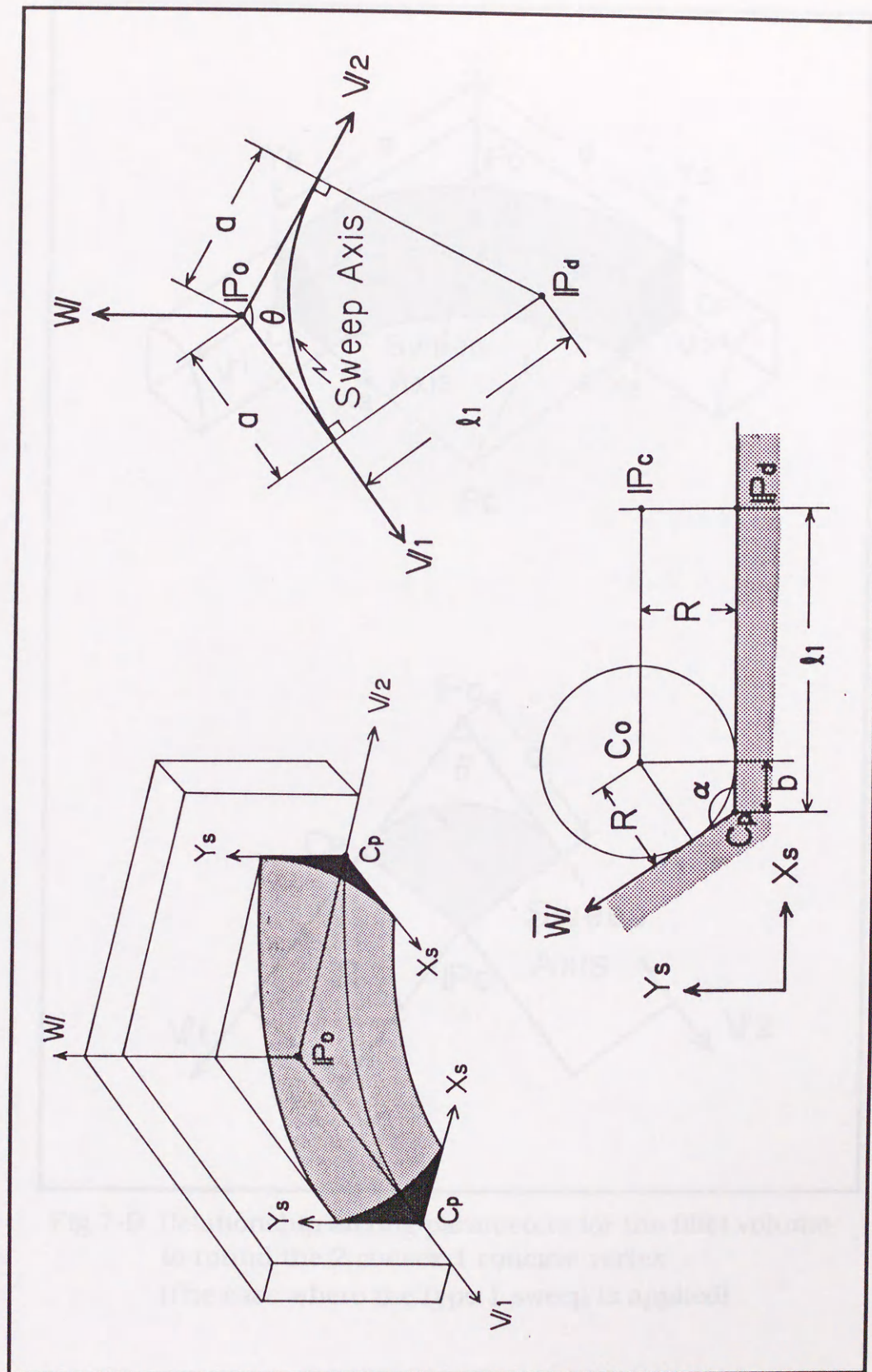


Fig.7-8 Relationship among parameters for the fillet volume to round the 3 concave vertex (The case where the type 1 sweep is applied)

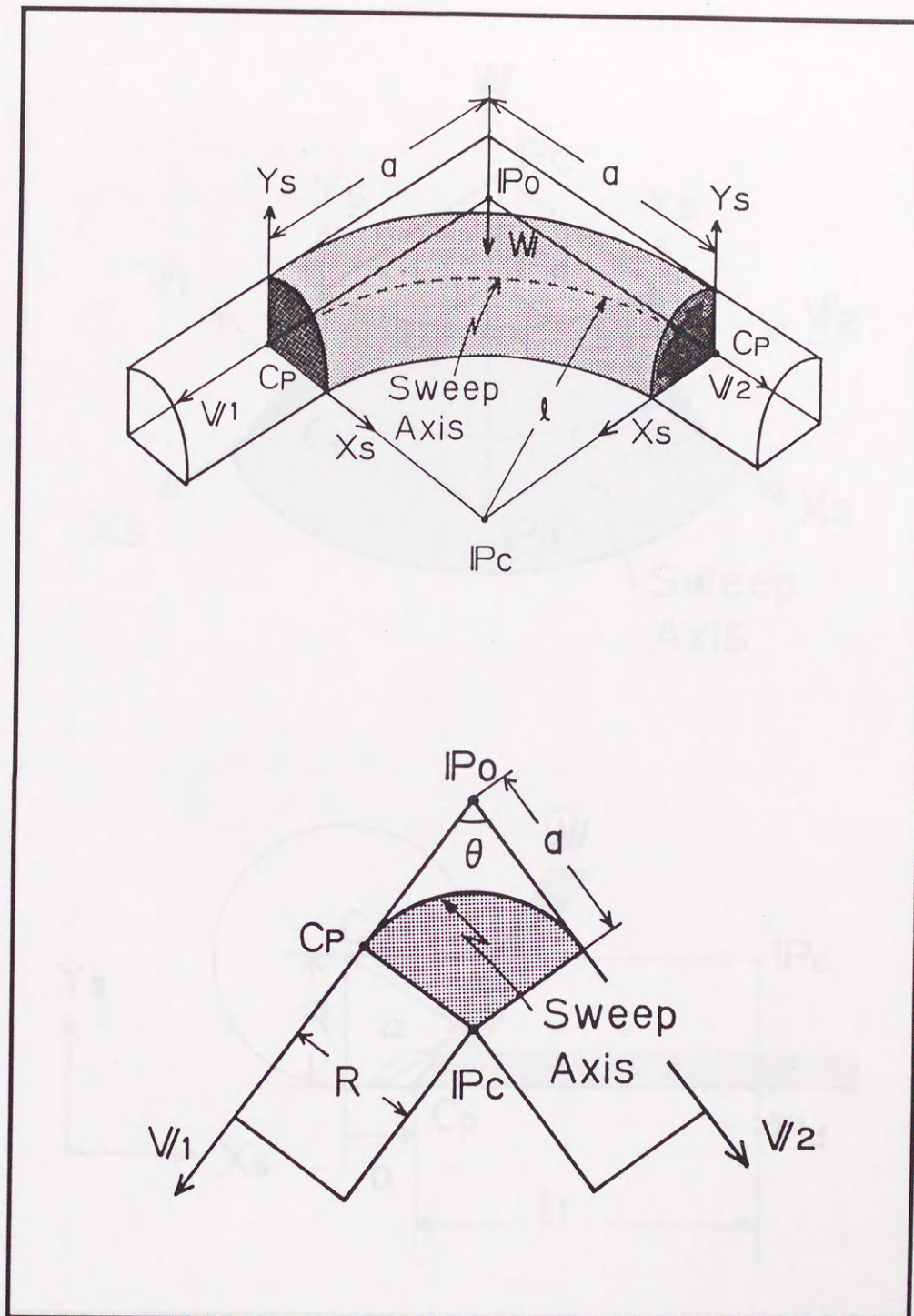


Fig.7-9 Relationship among parameters for the fillet volume to round the 2 convex 1 concave vertex
(The case where the type 1 sweep is applied)

Vertex type	Radius of the Sweep Axis : ℓ	Condition of ℓ for the type 1 sweep	Condition of ℓ for the type 2 sweep
3 Convex Vertex	$\ell = a \cdot \tan(\theta/2)$	$\ell \geq 0$	$\ell > 0$
3 Concave Vertex	$\ell = \ell_1 - b$ where $\ell_1 = a \cdot \tan(\theta/2)$ $b = R \cdot \cot(\alpha/2)$	$\ell \geq 0$	$\ell > 0$
		$\ell_1 \geq b > 0$	$\ell_1 > b > 0$
2 Convex 1 Concave Vertex	$\ell = a \cdot \tan(\theta/2)$ $h = R \cdot \cot(\theta/2)$	$\ell \geq R > 0$	$\ell > h > 0$
2 Concave 1 Convex Vertex	$\ell = \ell_1 + b$ where $\ell_1 = a \cdot \tan(\theta/2)$ $b = R \cdot \cot(\alpha/2)$	$\ell \geq b > 0$	$\ell > b > 0$

Table 7-1 Relationship among the sweep axis control parameter (a), the radius of the Sweep Axis (ℓ), and the radius of the fillet pattern (R)

Axis. It is provided that the Spine Axis becomes the y-axis when the V_1V_2 -plane is projected to the xy-plane of the 3-D orthogonal coordinate system. If an arbitrary space point $S_p(x, y, z)$ is given, then the representative point $C_p(x_{cp}, y_{cp}, z_{cp})$ of the Sweep Pattern is determined as follows:

$$\left. \begin{aligned} x_{cp} &= x_c \pm \sqrt{\ell^2 - (y - y_c)^2} , \\ y_{cp} &= y , \\ z_{cp} &= z_c , \\ \text{where ,} \\ \ell &= a \cdot \tan(\theta/2) , \quad \ell > 0 . \end{aligned} \right\} \quad (7-4)$$

As shown in Fig.7-11, the point $C_p(x_{cp}, y_{cp}, z_{cp})$ expressed in Equation(7-4) is not on the Spine Axis. In order to define the fillet pattern on the X_sY_s -plane, it is necessary to determine a point which is on the Spine Axis and has the one-to-one correspondence with the space point $S_p(x, y, z)$. Thus we shall determine the point as follows:

- (1) If an arbitrary space point S_p is given, then the point S_p is projected to the xy-plane. Let S_p' be the projected point.
- (2) Draw the perpendicular from the projected point S_p' to the Spine Axis, and determine the intersection point. As both the Spine Axis and the perpendicular are straight lines and are defined on the same plane, there is only one intersection point if they are not parallel to each other. Thus the intersection point has the one-to-one correspondence with the space point S_p . Let Q_p be the intersection point.

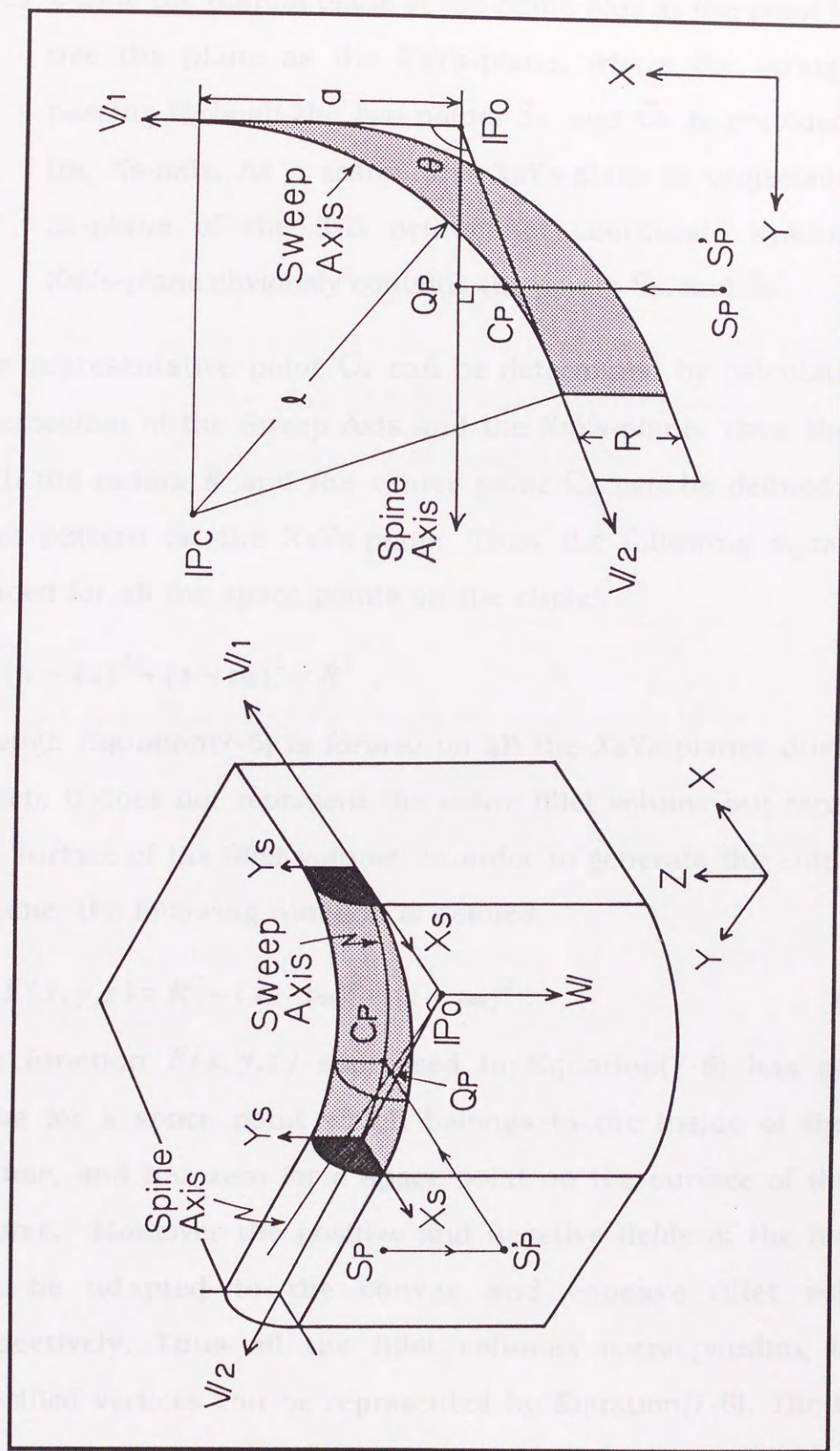


Fig.7-11 Relationship among parameters for the fillet volume to round the 3 convex vertex (The case where the type 2 sweep is applied)

(3) Define the normal plane of the Spine Axis at the point Q_p , and use the plane as the XsYs-plane, where the straight line passing through the two points S_p and Q_p is provided to be the Xs-axis. As a result, the XsYs-plane is projected to the zx-plane of the 3-D orthogonal coordinate system. The XsYs-plane obviously contains the points S_p , and S_p' .

The representative point C_p can be determined by calculating the intersection of the Sweep Axis and the XsYs-plane. Thus the circle with the radius R and the center point C_p can be defined as the fillet pattern on the XsYs-plane. Then the following equation is formed for all the space points on the circle:

$$(x - x_{cp})^2 + (z - z_{cp})^2 = R^2 \quad (7-5)$$

Though Equation(7-5) is formed on all the XsYs-planes during the sweep, it does not represent the entire fillet volume but represents the surface of the fillet volume. In order to generate the entire fillet volume, the following function is defined.

$$F(x, y, z) = R^2 - (x - x_{cp})^2 - (z - z_{cp})^2 \quad (7-6)$$

The function $F(x, y, z)$ expressed in Equation(7-6) has positive value for a space point which belongs to the inside of the fillet volume, and has zero for a space point on the surface of the fillet volume. Moreover the positive and negative fields of the function can be adapted to the convex and concave fillet volumes respectively. Thus all the fillet volumes corresponding to the classified vertices can be represented by Equation(7-6). The type 2 sweep can be applied if and only if the edge W is perpendicular to

the V_1V_2 -plane. The relationship among the parameters a , ℓ , and R is listed in Table 7-1.

7.5.2 The fillet volume to round the edge

Let us discuss now the generation of the fillet volume to round the edge W as illustrated in Fig.7-12. As the straight line edge is opted to be rounded, two straight lines are prepared as the Sweep and Spine Axes, and these are expressed as follows:

the Sweep Axis :

$$\frac{x - x_t}{l} = \frac{y - y_t}{m} = \frac{z - z_t}{n} ,$$

the Spine Axis :

$$\frac{x - x_n}{\lambda} = \frac{y - y_n}{\mu} = \frac{z - z_n}{\nu} ,$$

(7-7)

where l, m, n are the direction cosines of the Sweep Axis, and $P_t(x_t, y_t, z_t)$ is the point on the Sweep Axis. Similarly, λ, μ, ν are the direction cosines of the Spine Axis and $P_n(x_n, y_n, z_n)$ is the point on the Spine Axis. Let us derive the equation of the fillet volume. If an arbitrary space point S_p is given, then draw the perpendicular from the point S_p to the Spine Axis, and determine the intersection point. Let $Q(x_q, y_q, z_q)$ be the intersection point between the perpendicular and the Spine Axis. Then the normal plane of the Spine Axis at the point Q is defined and used as the X_sY_s -plane. Thus the X_sY_s -plane can be expressed as follows:

$$(x - x_q) \cdot \lambda + (y - y_q) \cdot \mu + (z - z_q) \cdot \nu = 0 . \quad (7-8)$$

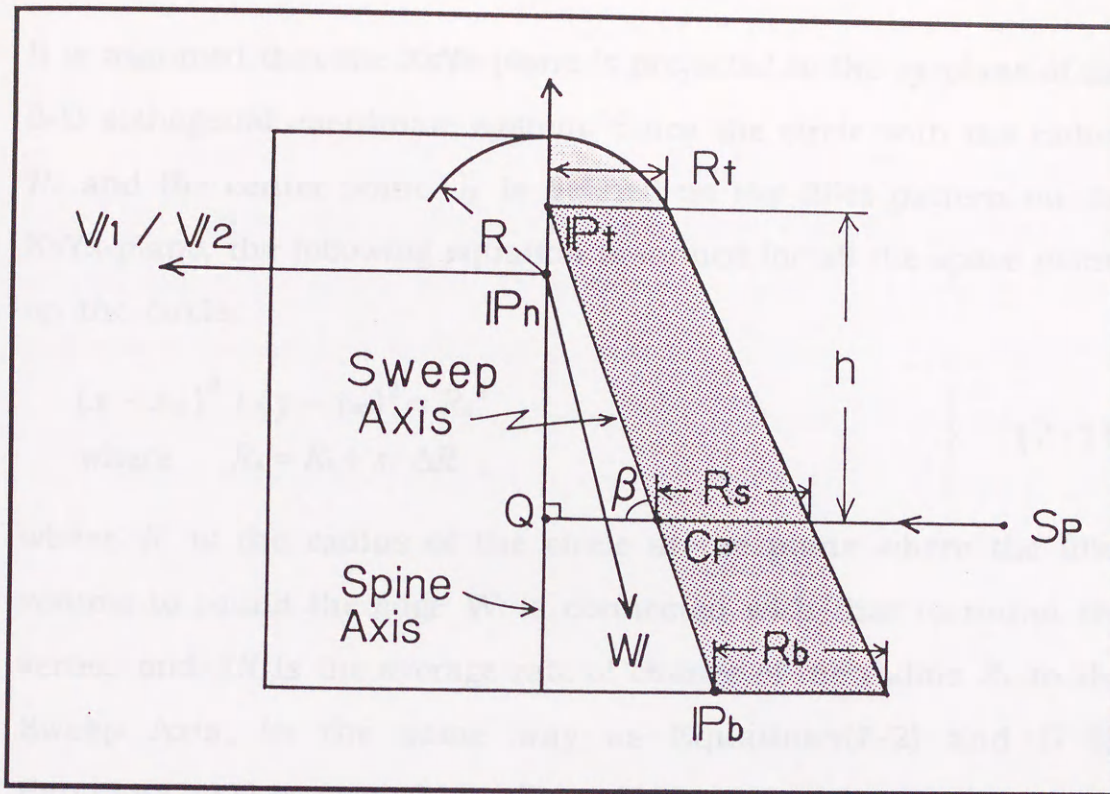


Fig.7-12 Relationship among parameters for the fillet volume to round the edge W

The angle between the Sweep Axis and the $XsYs$ -plane can be expressed as

$$\sin(\beta) = |l \cdot \lambda + m \cdot \mu + n \cdot v| \quad (7-9)$$

If the type 1 sweep is applied, $\sin(\beta) = 1$, and if the type 2 sweep is applied, then $0 < \sin(\beta) < 1$. Let h be the distance between the point P_t and the $XsYs$ -plane. Then the intersection point $C_p(x_{cp}, y_{cp}, z_{cp})$ between the Sweep Axis and the $XsYs$ -plane is expressed as follows:

$$\left. \begin{aligned} x_{cp} &= x_t + s \cdot l, \\ y_{cp} &= y_t + s \cdot m, \\ z_{cp} &= z_t + s \cdot n, \end{aligned} \right\} \quad (7-10)$$

where : $s = h / \sin(\beta)$.

It is assumed that the XsYs-plane is projected to the xy-plane of the 3-D orthogonal coordinate system. Since the circle with the radius R_s and the center point C_p is defined as the fillet pattern on the XsYs-plane, the following equation is formed for all the space points on the circle:

$$\left. \begin{aligned} (x - x_{cp})^2 + (y - y_{cp})^2 &= R_s^2, \\ \text{where : } R_s &= R_t + s \cdot \Delta R, \end{aligned} \right\} \quad (7-11)$$

where R_t is the radius of the circle at the plane where the fillet volume to round the edge W is connected with that to round the vertex, and ΔR is the average rate of change of the radius R_s to the Sweep Axis. In the same way as Equations(7-2) and (7-5), Equation(7-11) is formed on all the XsYs-planes during the sweep. However, Equation(7-11) does not represent the entire fillet volume but represents the surface of the fillet volume. In order to generate the entire fillet volume to round the edge, the following function is defined.

$$\left. \begin{aligned} F(x, y, z) &= R_s^2 - (x - x_{cp})^2 - (y - y_{cp})^2 = 0, \\ \text{where : } R_s &= R_t + s \cdot \Delta R, \end{aligned} \right\} \quad (7-12)$$

The function $F(x, y, z)$ expressed in Equation(7-12) has positive value for a space point which belongs to the inside of the fillet volume, and has zero for a space point on the surface of the fillet volume. Moreover the positive and negative fields of the function can be adapted to the convex and concave fillet volumes respectively.

7.6 The connection of fillet volumes for rounding the vertex and the edges simultaneously

An entire fillet volume to round the vertex and the three edges simultaneously can be generated by connecting the fillet volume to round the vertex and that to round each edge. Here we shall discuss the connection of the fillet volumes. Since the Sweep and Spine Axes for the fillet volume to round the vertex P_0 are defined on the V_1V_2 -plane or on the plane which is parallel to the V_1V_2 -plane, and moreover the $XsYs$ -planes at the start and the end of the sweep are perpendicular to the edges V_1 and V_2 , the fillet volume to round the vertex P_0 can easily be connected with the fillet volumes to round the edges V_1 and V_2 . However, the connection between the fillet volume to round the vertex P_0 and that to round the remaining third edge W is very complicated. In our method, therefore, the parameters of the fillet volume to round the edge W are set so as to connect smoothly with the fillet volume to round the vertex P_0 . Let us explain the connecting method by using the 3 convex vertex P_0 , as illustrated in Figs.7-13-(a) and (b). It is assumed that the fillet volume to round the vertex P_0 is generated by applying the type 1 sweep, for convenience. Then the parameters are set as follows:

(1) The settlement of the Spine Axis l_{sp}

- The straight line which passes through the center point P_c of the circular arc and is perpendicular to the V_1V_2 -plane is

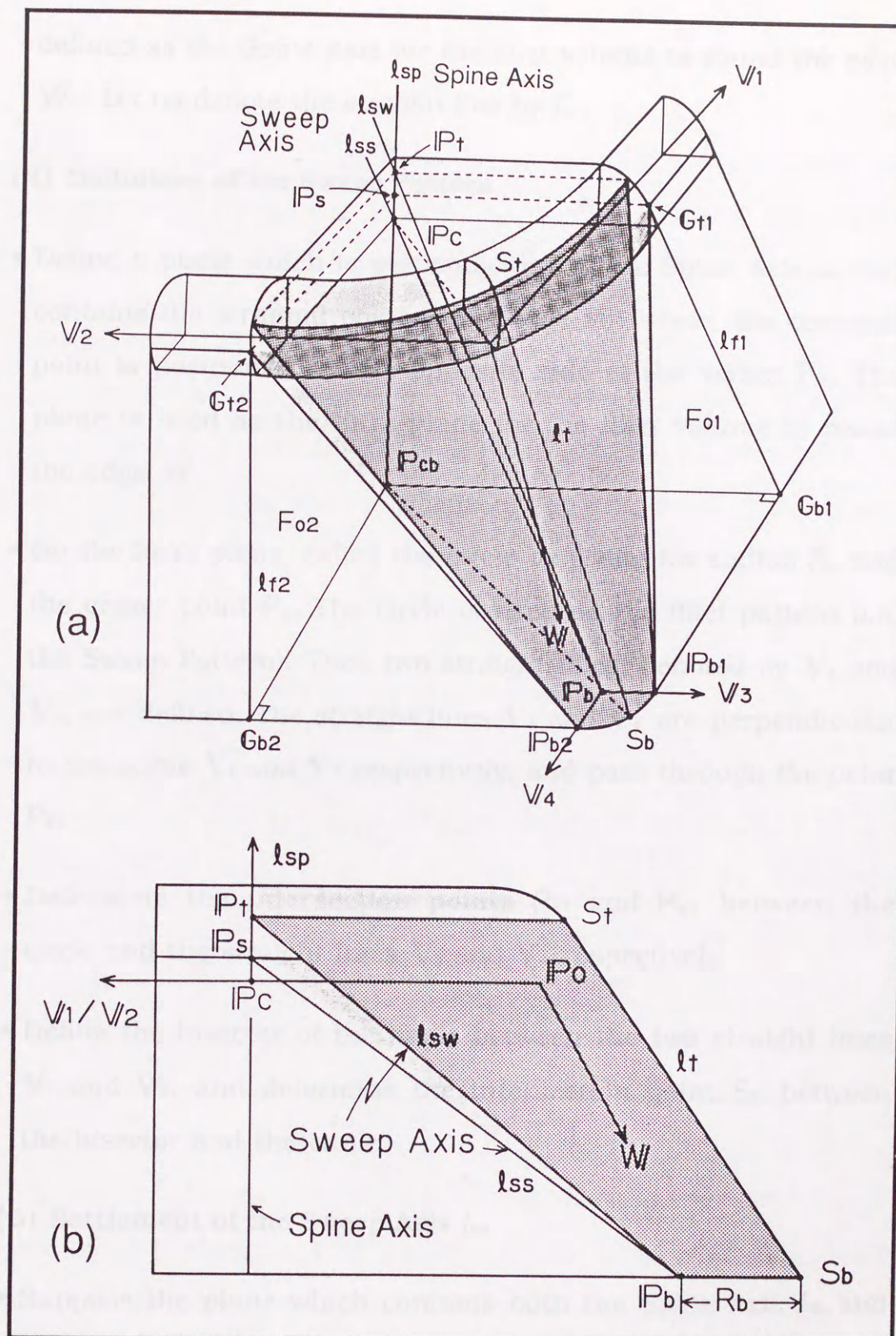


Fig.7-13 Smooth connection of two fillet volumes for rounding both the vertex P_0 and the edge W

defined as the Spine Axis for the fillet volume to round the edge W . Let us denote the straight line by l_{sp} .

(2) Definition of the Sweep Pattern

- Define a plane which is perpendicular to the Spine Axis l_{sp} and contains the terminal point of the edge W , where the terminal point is positioned on the opposite side of the vertex P_0 . The plane is used as the XsYs-plane for the fillet volume to round the edge W .
- On the XsYs-plane, define the circle by giving the radius R_b and the center point P_b . The circle is used as the fillet pattern (i.e., the Sweep Pattern). Then two straight lines, denoted by V_3 and V_4 , are defined. The straight lines V_3 and V_4 are perpendicular to the edges V_1 and V_2 respectively, and pass through the point P_b .
- Determine the intersection points P_{b1} and P_{b2} between the circle and the straight lines V_3 and V_4 respectively.
- Define the bisector of the angle between the two straight lines V_3 and V_4 , and determine the intersection point S_b between the bisector and the circle.

(3) Settlement of the Sweep Axis l_{sw}

- Suppose the plane which contains both the Spine Axis l_{sp} and the bisector of the angle between the edges V_1 and V_2 . If the fillet volume to round the vertex P_0 is cut by the supposed

plane, the sectional figure of the fillet volume is obtained. It is obvious that the sectional figure is equivalent to the fillet pattern when the type 1 sweep is applied. Then draw the tangent line l_t from the point S_b to the fillet pattern in order to determine the tangent point S_t .

- Define the XsYs-plane containing the tangent point S_t , and determine the intersection point P_t between the XsYs-plane and the Spine Axis l_{sp} . Then the straight line passing through the two points P_b and P_t is defined as the Sweep Axis for the fillet volume to round the edge W . Let us denote the Sweep Axis by l_{sw} .

(4) Determination of the radius of the circle

- Though the circle is used as the fillet pattern for the fillet volume to round the edge W , the radius of the circle is varied when the circle is swept along the Sweep Axis l_{sw} . Thus the radius must be determined on each XsYs-plane during the sweep. As shown in Fig.7-13-(b), the tangent line l_t becomes the profile line of the fillet volume to round the edge W . Moreover the Sweep Axis l_{sw} is considered to be the locus formed by sweeping the center point of the circle. Thus the distance between the Sweep Axis l_{sw} and the tangent line l_t becomes the radius of the circle. Actually, as both the Sweep Axis l_{sw} and the tangent line l_t intersect to the XsYs-plane, two intersection points are obtained on the XsYs-plane. Thus the

distance between the two intersection points becomes the radius of the circle.

(5) Generation of the offset surface of the Primitive

In our method, the additional fillet volume is used only for rounding the corners, as mentioned previously. The method requires the offset of the surface of the Primitive, and the offset surface is generated as follows:

- Calculate the intersection point P_{cb} between the Spine Axis l_{sp} and the XsYs-plane containing the point P_b .
- Define the straight line which passes through the point P_{b1} and is parallel to the edge V_1 . Then draw the perpendicular from the point P_{cb} to the straight line, and determine the intersection point G_{b1} between the perpendicular and the straight line.
- Suppose the plane containing the Spine Axis l_{sp} and the two points P_{cb} and G_{b1} . If the fillet volume to round the vertex P_0 is cut by the supposed plane, the sectional figure of the fillet volume is obtained. Then draw the tangent line l_{t1} from the point G_{b1} to the sectional figure in order to determine the tangent point G_{t1} .
- Define the plane containing the three points P_{b1} , G_{b1} , and G_{t1} . This plane is used as the offset of the original plane which is defined by the two edges V_1 and W . Let us denote the offset

plane by F_{01} . Similarly, determine the offset plane F_{02} of the original plane which is defined by the two edges V_2 and W .

(6) The introduction of the joint volume

When the type 2 sweep is applied, the offset plane cannot be connected smoothly with the fillet volume to round the edge W . In this case, the third volume is necessary in order to smoothly connect the offset plane to the fillet volume. We refer to the third volume as *the joint volume*. The sweep parameters for the joint volume is set as follows:

- The Spine Axis l_{sp} for the fillet volume to round the edge W is used for the joint volume too. Thus the fillet patterns for these volumes are defined on the same XsYs-plane. The sweep Axis l_{ss} for the joint volume is defined in the same way as l_{sw} . Then the terminal point P_s of the Sweep Axis l_{ss} is determined by using the tangent point G_{t1} (or G_{t2}).

(7) Smooth connection among the offset plane, the fillet volume, and the joint volume

Suppose an XsYs-plane for both the joint volume and the fillet volume to round the edge W . On the XsYs-plane, both the joint volume and the fillet volume to round the edge W obviously become the two circles C_1 and C_2 respectively, as shown in Fig.7-14. These circles can be connected smoothly as follows:

- Determine the intersection point between the Sweep Axis l_{ss} and the XsYs-plane. Let us denote the intersection point by P_{c1} .

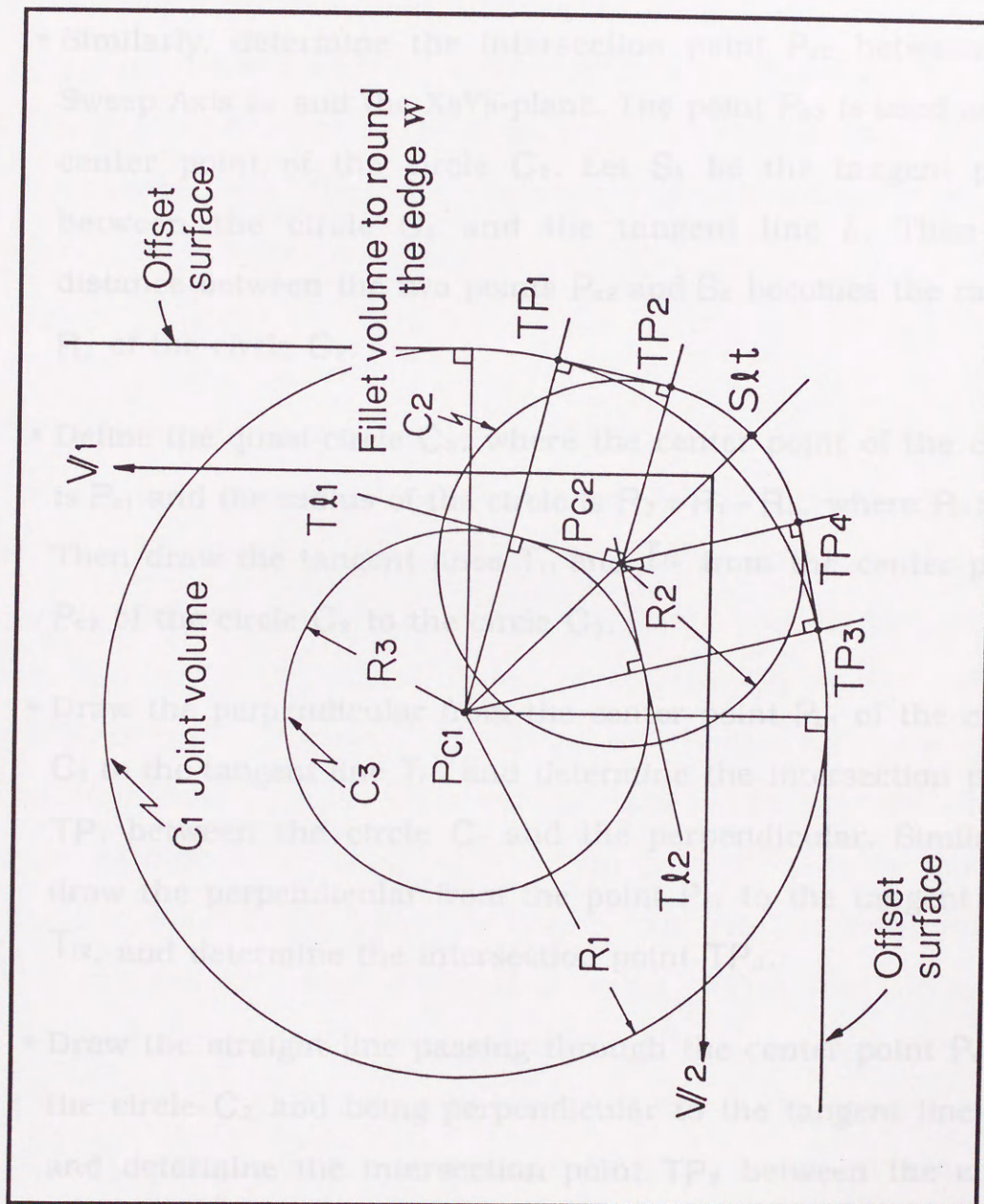


Fig.7-14 Smooth connection between the joint volume and the fillet volume to round the edge W

The point P_{c1} is used as the center point of the circle C_1 . As the joint volume is defined so as to tangent to the offset plane, the distance between the center point P_{c1} and the offset plane becomes the radius R_1 of the circle C_1 .

- Similarly, determine the intersection point P_{c2} between the Sweep Axis l_{sp} and the XsYs-plane. The point P_{c2} is used as the center point of the circle C_2 . Let S_n be the tangent point between the circle C_2 and the tangent line l_i . Then the distance between the two points P_{c2} and S_n becomes the radius R_2 of the circle C_2 .
- Define the quasi-circle C_3 , where the center point of the circle is P_{c1} and the radius of the circle is $R_3 = R_1 - R_2$, where $R_1 > R_2$. Then draw the tangent lines T_{i1} and T_{i2} from the center point P_{c2} of the circle C_2 to the circle C_3 .
- Draw the perpendicular from the center point P_{c1} of the circle C_1 to the tangent line T_{i1} , and determine the intersection point TP_1 between the circle C_1 and the perpendicular. Similarly, draw the perpendicular from the point P_{c1} to the tangent line T_{i2} , and determine the intersection point TP_3 .
- Draw the straight line passing through the center point P_{c2} of the circle C_2 and being perpendicular to the tangent line T_{i1} , and determine the intersection point TP_2 between the circle C_2 and the straight line. Similarly, draw the straight line passing through the point P_{c2} and being perpendicular to the tangent line T_{i2} , and determine the intersection point TP_4 .

- Define the straight line passing through the two points TP_1 and TP_2 . Similarly, define the straight line passing through the two points TP_3 and TP_4 . Then these straight lines are tangent to the circles C_1 and C_2 respectively. This means that the joint volume and the fillet volume to round the edge W are connected smoothly by the straight lines. Moreover the joint volume is tangent to the offset plane, as mentioned previously. Therefore, the offset plane is connected smoothly with the fillet volume and the joint volume.

7.7 Experiments and the consideration

In order to prove the correctness of the derived equations and to verify the effectiveness of the applied method, the computational algorithm has been developed and performed with the CSG-based CAD/CAM system TIPS-1[16][17] on the HITAC-M280 computer. Figures 7-15 through 7-19 show examples of the 3-D solid model of the fillet volume. Let us explain the corner rounding process by using Fig.7-15. Suppose the planner solid, as shown in Fig.7-15-(a). First, the Sweep Axis defining plane for the fillet volume to round the vertex is set. Then the two edges contained in the plane can be rounded by the cylindrical Primitives as shown in Fig.7-15-(b). Next, if we give the sweep parameters of the fillet volumes for the vertex and the remained third edge, the offset planes are generated automatically as shown in Fig.7-15-(c) and then the fillet volume to round both the vertex and the remained third edge simultaneously is added automatically so as to

complete the desired shape as shown in Fig.7-15-(d). In this example, the type 1 sweep is applied to generate fillet volumes. On the other hand, fillet volumes shown in Fig.7-16 are generated by applying the type 2 sweep. Figs.7-16-(a) and (b) are examples of the fillet volume to round the vertex, and Figs.7-16-(c) and (d) are those to round edges. From Fig.7-16, we can find that three fillet volumes having different radii are connected smoothly with the fillet volume to round the vertex. Other examples of smooth connection among the fillet volumes are shown in Fig.7-17. In the figure, we can find various shapes of the corner rounding. Fig.7-18-(a) shows one quarter model of bathtub, Fig.7-18-(b) shows the cavity of the solid model with fillet volumes, and Figs.7-18-(c) and (d) show examples of a bowl and a plate.

There are cases where more than three edges meet at a vertex. Such vertex is regarded as the composite vertex of the classified vertices. Thus the fillet volume to round such vertex can be generated by connecting the corresponding fillet volumes of the classified vertices. Figs.7-19-(a) and (b) show cases where four edges meet at the vertex, and similarly Figs.7-19-(c) and (d) show cases where six edges meet at the vertex. In the cases shown in Figs.7-19- (a) and (c), the convex fillet volume to round the vertex is generated while both the concave and convex fillet volumes are generated for the cases shown in Figs.7-19-(b) and (d).

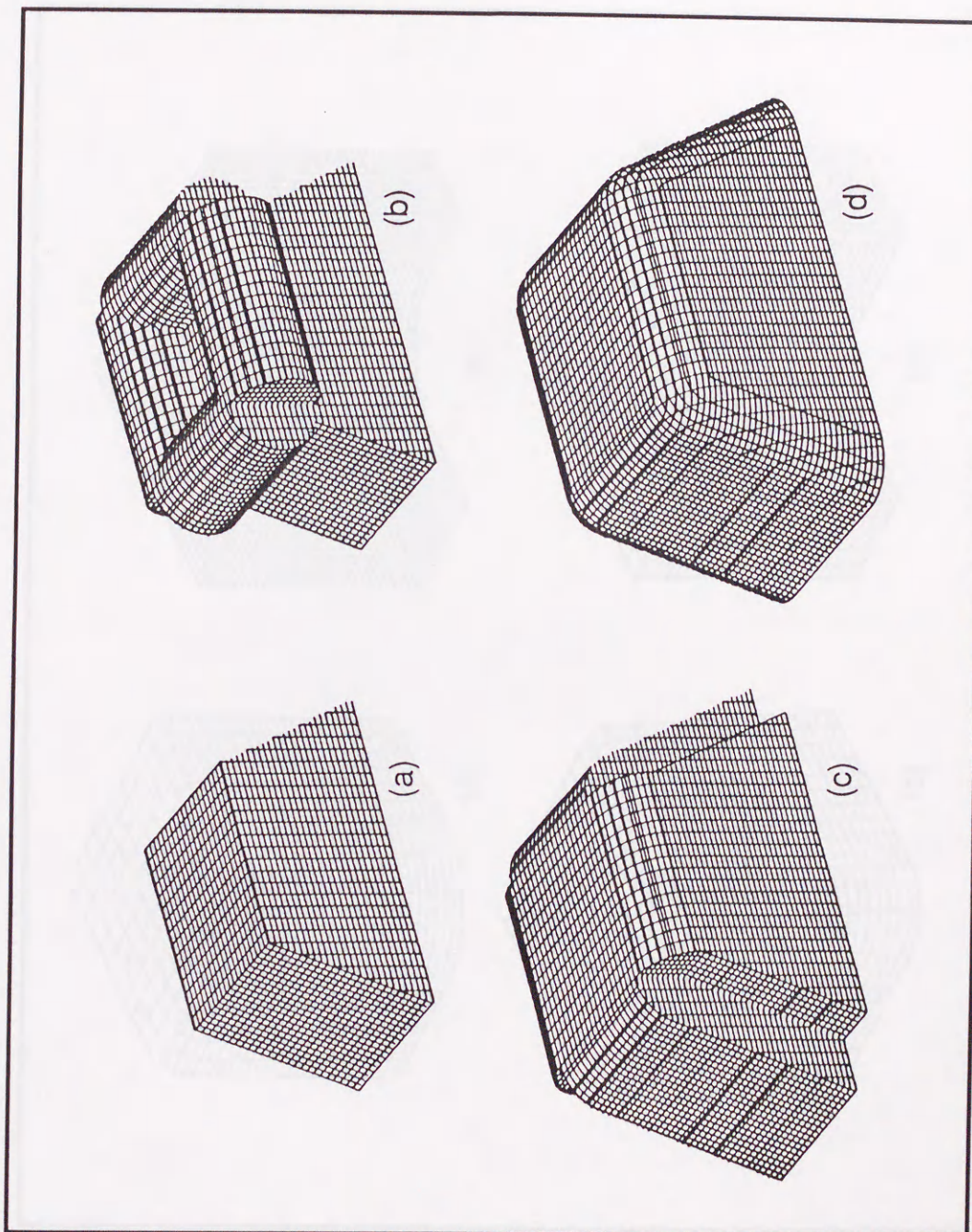


Fig.7-15 Modeling process of a solid object with fillet volumes
by CSG scheme

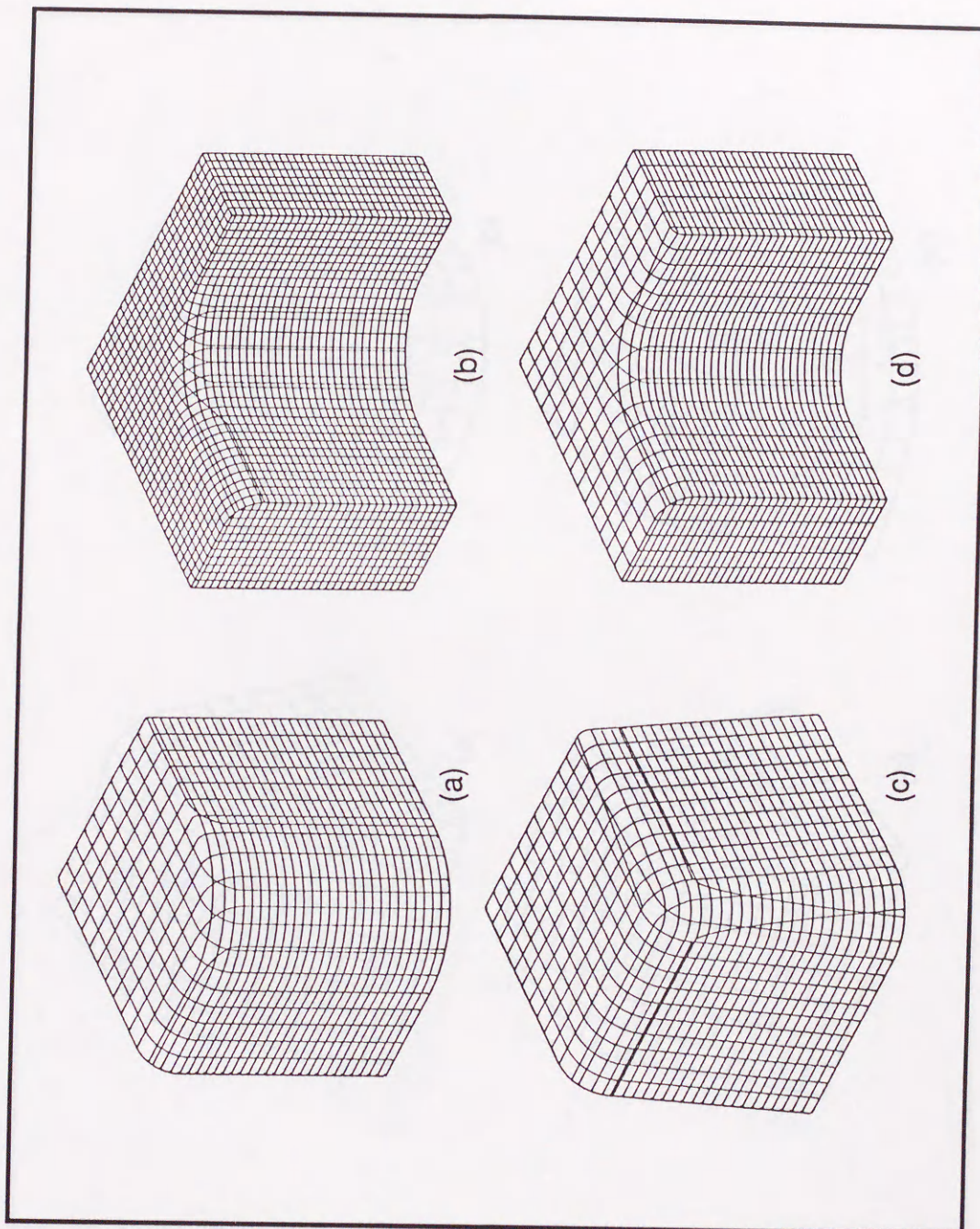


Fig.7-16 Experimental results

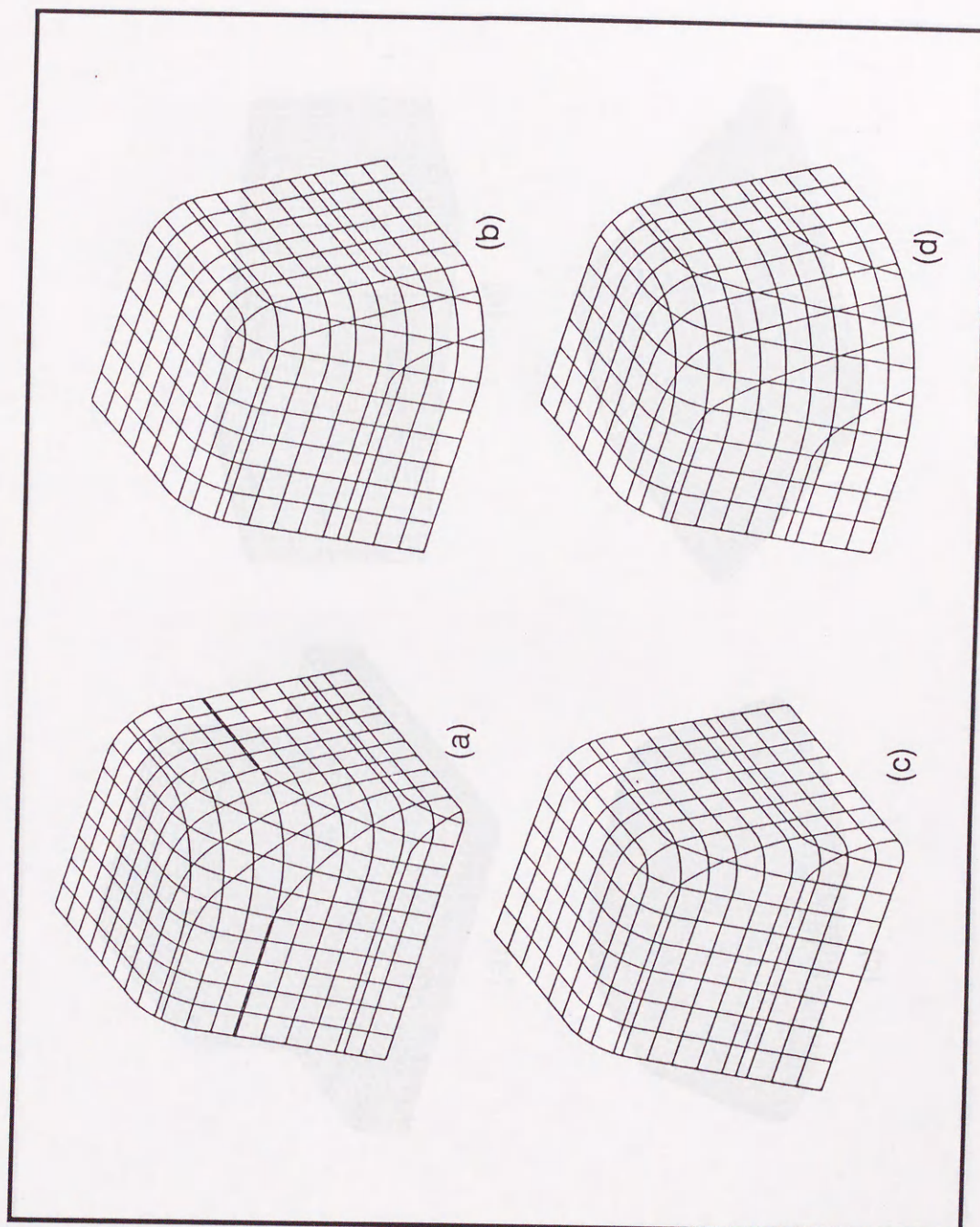


Fig.7-17 Experimental results

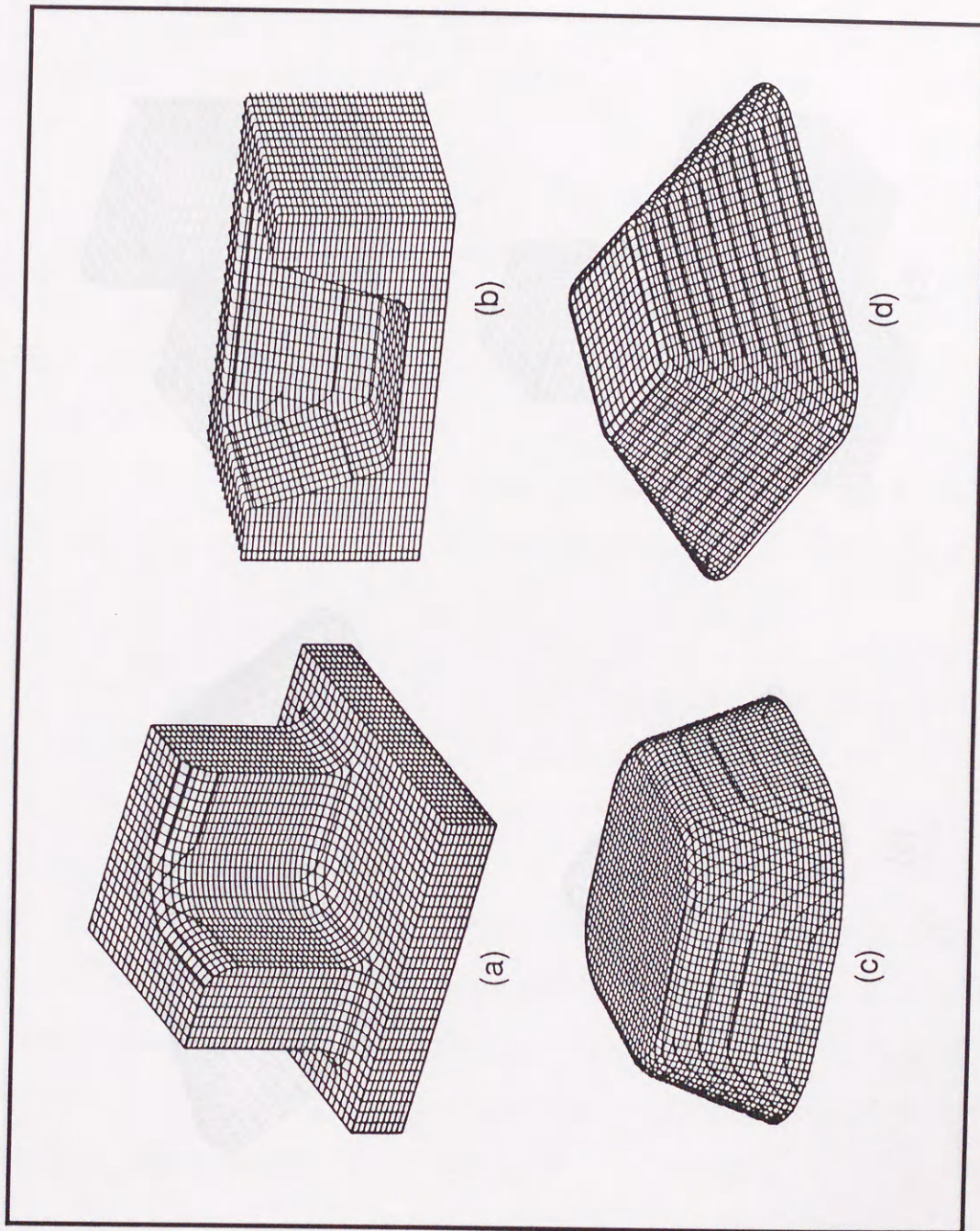


Fig.7-18 Experimental results

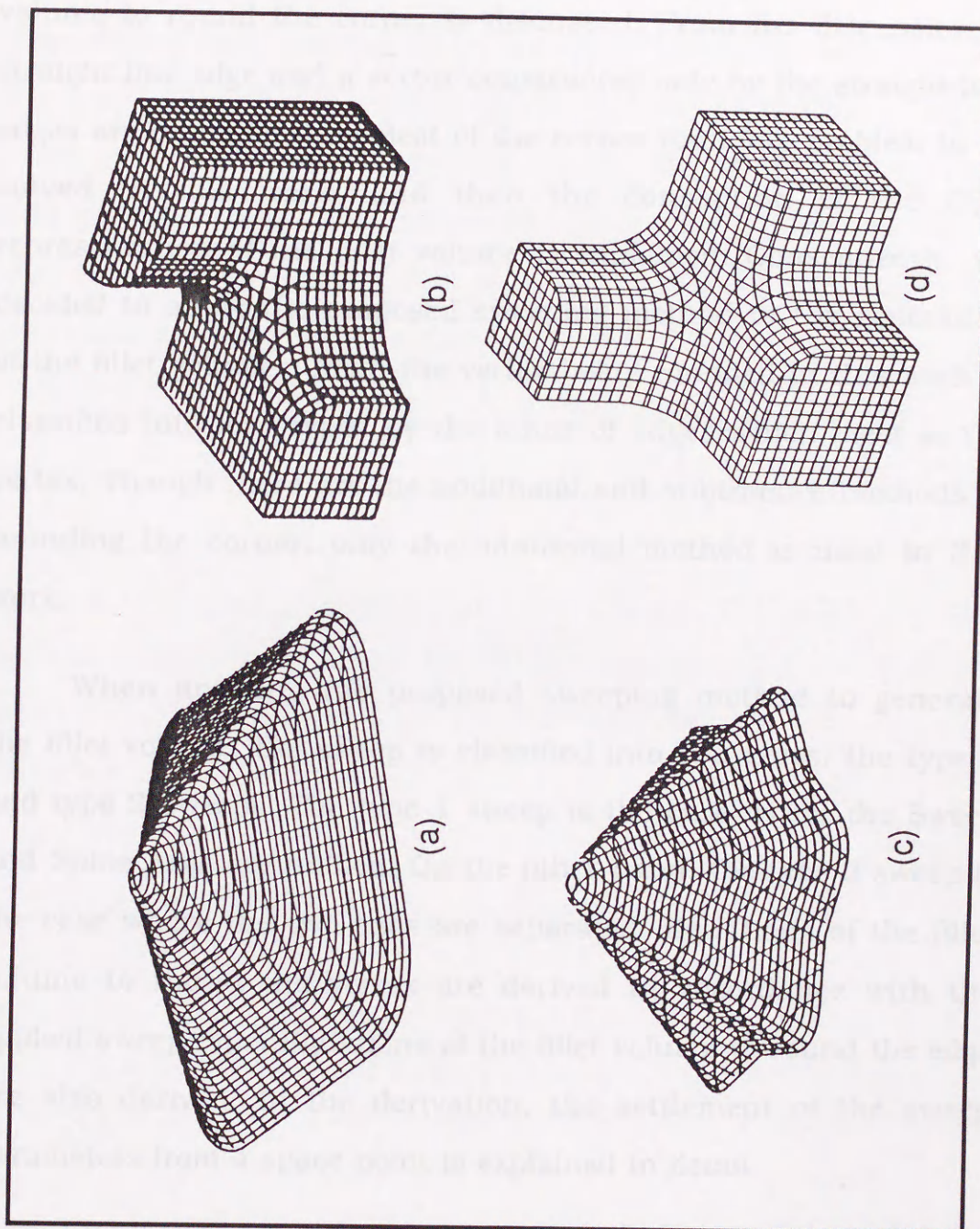


Fig.7-19 Experimental results

7.8 conclusion

In this chapter, first, the problem for generation of the fillet volume to round the corner is discussed. From the discussion, a straight line edge and a vertex constructed only by the straight line edges are opted as the object of the corner rounding problem to be solved in this work, and then the conception of the CSG representation of the fillet volume is explained. Consequently, we decided to apply the proposed sweeping method to the generation of the fillet volume. Next, the vertex being rounded in this work is classified into four types by the kinds of edges which meet at the vertex. Though there are the additional and subtractive methods in rounding the corner, only the additional method is used in this work.

When applying the proposed sweeping method to generate the fillet volume, the sweep is classified into two types: the type 1 and type 2 sweeps. The type 1 sweep is the case where the Sweep and Spine Axes are unified. On the other hand, the type 2 sweep is the case where the two axes are separated. Equations of the fillet volume to round the vertex are derived in accordance with the applied sweep type. Equations of the fillet volume to round the edge are also derived. In the derivation, the settlement of the sweep parameters from a space point is explained in detail.

In order to round the vertex and the related three edges simultaneously, the smooth connection among the related fillet volumes is required. Thus the connecting method is clarified in

detail. As this method requires the offset of the surface, the way of generating offset surface of the Primitive is clarified. When the type 2 sweep is applied to generate the fillet volume to round the third edge, the offset plane cannot be connected smoothly with the fillet volume. In this case, an intermediate solid, called the joint volume, is introduced in order to smoothly connect the offset plane and the fillet volume. Thus the settlement of the sweep parameters for the joint volume is explained, and then the connecting method by the joint volume is clarified in detail.

Finally, the computational algorithm of the applied method has been developed and performed with the CSG-based CAD/CAM system TIPS-1. According to the experiments, it is clear that the method applied to generate the fillet volume is effective enough, derived equations of the fillet volume are correct, and that the generated fillet volume can be handled just like the ordinary Primitive.

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8. SOLID MODELING OF THE FILLET VOLUME

- ON THE FILLET FOR JOINING TWO DISCONNECTED SOLIDS -

8.1 Introduction

Geometric modeling of the blending surface to join two disconnected objects smoothly is an old subject in CAD/CAM fields, and many efforts have been devoted to the subject. Today, several practical methods for the representation of the blending surface have been reported[1][2][3], and used in industrial designs. Its characteristic is that these methods are based on surface modeling schemes. On the other hand, solid modeling schemes have limitation when representing the blending surface. Thus surface modeling schemes are currently being incorporated into solid modeling systems. Surface modeling schemes, however, are not consistent with those of solid modeling. Actually, in these systems, the solid and the surface modeling schemes are used properly by estimating geometric complexity of an object shape. From the viewpoint of the modeling consistency, the blending surface should be represented by the solid modeling schemes in solid modeling systems. This is the motivation to start this study, aiming at representing the blending surface by CSG scheme.

It has been considered that the blending surface to join two disconnected objects smoothly is very difficult to be represented by CSG scheme. Mathematically, this is due to the difficulty to express the blending surface by implicit functions in the 3-D orthogonal coordinate system. In fact, it is very hard to derive implicit

functions which express an entire blending surface. It is possible, however, to derive implicit functions of a 2-D closed curve which represents a sectional figure of an entire blending surface. Therefore, we shall apply a new method[4][5] based on the proposed sweeping method to represent the blending surface by CSG scheme. The basic idea of this method is as follows. A 2-D closed curve is swept in 3-D space along a 3-D curve to generate a swept volume. Since the 2-D closed curve which is in the middle of the sweep is represented by blending two static 2-D closed curves at the start and end positions of the sweep, the enveloping surface of the 2-D closed curve during the sweep becomes an entire blending surface. The characteristic of the new method is that implicit functions of the 2-D closed curve can be substituted for those of an entire blending surface. Consequently, the solid with the blending surface (i.e., the fillet volume) can be generated by applying the new method.

8.2 Description of the problem

Let us imagine two disconnected solids, as shown in Fig.8-1. The fillet volume to join such disconnected solids smoothly is an object of this work. In order to represent the fillet volume by CSG scheme, the mathematical function of the fillet volume should be derived. Let us consider now the problem of deriving the function. Let the solids be a cube and a cylinder, for the convenience of explanation. Let S_1 be a plane cross-section of the cylinder, and S_2 be that of the cube. In this work, it is restricted that these two solids are joined by the fillet volume between the cross-sections S_1

and S_2 . We assume that the closed curves appeared on the cross-sections S_1 and S_2 can be expressed as follows:

$$\left. \begin{array}{l} S_1 : g(\mathbf{X}) = 0 , \\ S_2 : h(\mathbf{X}) = 0 , \end{array} \right\} \quad (8-1)$$

where $\mathbf{X}$ designates coordinates of a space point defined in the 3-D orthogonal coordinate system. Though we have a tendency to consider that the fillet volume is generated by blending two "solids", we are rather concerned with two "cross-sections" to blend to generate the fillet volume. Thus the function of the fillet volume is considered to be defined in the following form:

$$Q(g, h) = 0 . \quad (8-2)$$

It is very difficult to derive such a function of an entire fillet volume. It is possible, however, to derive the function of a 2-D closed curve made by cutting the entire fillet volume with a plane cross-section. Thus we shall change the object for the derivation problem from the entire fillet volume to the 2-D closed curve. Let us imagine a plane cross-section between the sections S_1 and S_2 . (See Fig.8-1.) We refer to the plane cross-section as *the current cross-section*, and similarly the sections S_1 and S_2 are referred to as *the start* and *end cross-sections* respectively. Let t ($0 \leq t \leq 1$) be a parameter which designates the position of the current cross-section, and the start and end cross-sections be designated by $t=0$ and $t=1$ respectively. As the closed curve on the current cross-section can be represented by blending those on the start and end cross-sections, the function of the closed curve can be defined as follows:

$$f(g, h) = 0 \quad , \quad (8-3)$$

$$\left. \begin{aligned} f(g, h) &\equiv \{1 - Bf(t)\} \cdot g(\mathbf{X}) + Bf(t) \cdot h(\mathbf{X}) \quad , \\ \text{where } 0 &\leq t \leq 1 \quad . \end{aligned} \right\} \quad (8-4)$$

Here $Bf(t)$ is the blending function. As $\mathbf{X}$ and t are variables of the function $f(g, h)$, Equations(8-3) and (8-4) can also be defined as

$$F(\mathbf{X}, t) = 0 \quad , \quad (8-5)$$

$$\left. \begin{aligned} F(\mathbf{X}, t) &\equiv \{1 - Bf(t)\} \cdot g(\mathbf{X}) + Bf(t) \cdot h(\mathbf{X}) \quad , \\ \text{where } 0 &\leq t \leq 1 \quad . \end{aligned} \right\} \quad (8-6)$$

Still it is difficult to derive a function of an entire fillet volume even if we use Equations(8-5) and (8-6). As the purpose of this work is to represent the fillet volume by CSG scheme, we shall change the viewpoint from the problem of deriving the mathematical function of the fillet volume to that of generating the fillet volume. We can find that the parameters of the proposed sweeping method shown in Fig.8-2 are analogous to those of the fillet volume shown in Fig.8-1. Thus the problem of generating the fillet volume to join two disconnected solids can be regarded as that of generating the swept volume between two cross-sections of the disconnected solids. Therefore, we shall apply the proposed sweeping method to generate the fillet volume. In the following sections, we shall describe our method.

8.3 The Sweep and Spine Axes curves

A straight line and a circular arc are opted as the Spine Axis curves in this work. In these cases, the XsYs-plane is led from a space point as follows:

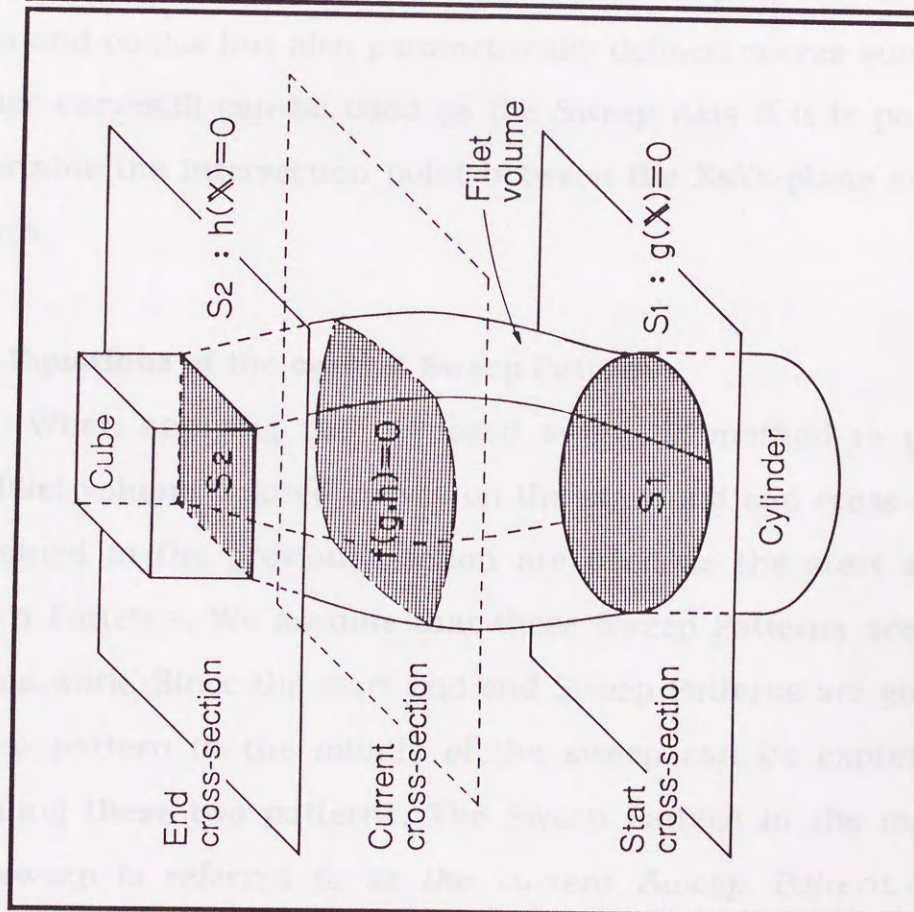


Fig.8-1 The fillet volume to join disconnected two solids

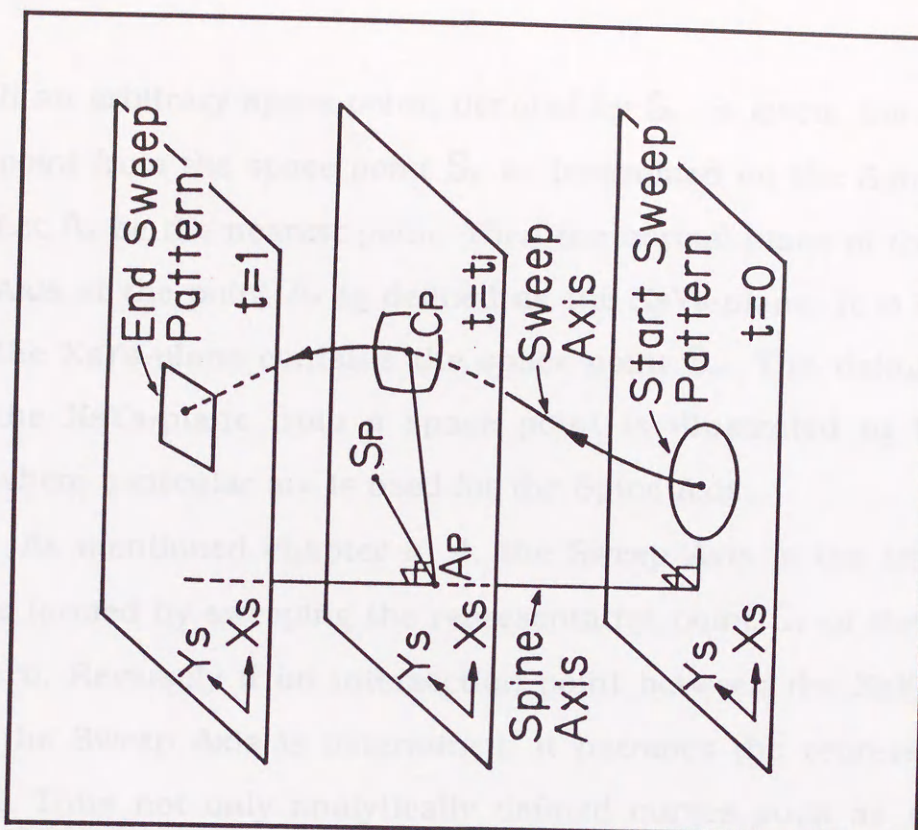


Fig.8-2 Sweep parameters of the proposed sweeping method

- If an arbitrary space point, denoted by S_p , is given, the nearest point from the space point S_p is determined on the Spine Axis. Let A_p be the nearest point. Then the normal plane of the Spine Axis at the point A_p is defined as the XsYs-plane. It is obvious the XsYs-plane contains the space point S_p . The definition of the XsYs-plane from a space point is illustrated in Fig.8-3, where a circular arc is used for the Spine Axis.

As mentioned chapter in 4, the Sweep Axis is the trajectory curve formed by sweeping the representative point C_p of the Sweep Pattern. Reversely if an intersection point between the XsYs-plane and the Sweep Axis is determined, it becomes the representative point. Thus not only analytically defined curves such as straight lines and conics but also parametrically defined curves such as the Bezier curves[6] can be used as the Sweep Axis if it is possible to determine the intersection point between the XsYs-plane and these curves.

8.4 Equations of the current Sweep Pattern

When applying the proposed sweeping method to generate the fillet volume, closed curves on the start and end cross-sections explained in the previous section are used as the start and end Sweep Patterns. We assume that these Sweep Patterns are convex in this work. Since the start and end Sweep Patterns are given, the Sweep pattern in the middle of the sweep can be expressed by blending these two patterns. The Sweep Pattern in the middle of the sweep is referred to as *the current Sweep Pattern*. In this section, we shall derive mathematical functions of the current

Sweep Pattern. We assume that the parameter t which designates the position of the current Sweep Pattern is set on the Sweep Axis, and the start and end Sweep Patterns are designated by $t=0$ and $t=1$ respectively, as shown in Fig.8-2. Let us consider now the XsYs-plane where the position parameter is $t=t$ ($0 \leq t \leq 1$), as shown in Fig.8-4. We assume that the start and end Sweep Patterns have been projected to the XsYs-plane, and similarly the representative point C_p of the Sweep Pattern has already been determined. The representative point C_p is positioned at the inside of the Sweep Pattern, and used as the origin of the XsYs-plane. The derivation process of the functions of the current Sweep Pattern is as follows:

- If an arbitrary space point, denoted by S_p , is given, the XsYs-plane is set, as explained in section 8-3. Next, on the XsYs-plane, the straight line passing through the origin C_p and the space point S_p is set. Let L_{sp} be the straight line. Then the angle between the straight line L_{sp} and the Xs-axis is determined. Let θ be the angle, where $0 \leq \theta \leq 2\pi$. It is obvious that the angle θ relates to the position of the space point S_p .
- An intersection point between the straight line L_{sp} and the start Sweep Pattern is determined. Let Q_s be the intersection point. Then the distance from the origin C_p to the point Q_s is calculated. The distance can be varied in accordance with the angle θ . Thus we denote the distance by $\ell_s(\theta)$.
- In the same way, an intersection point between the straight line L_{sp} and the end Sweep Pattern, denoted by Q_e , is determined. Then the distance from the origin C_p to the point Q_e is calculated. Let $\ell_e(\theta)$ be the distance.

• As the current Sweep Pattern is not yet determined, we assume a quasi-point which is considered to be an intersection point between the straight line L_{sp} and the current Sweep Pattern. Let Q_x be the quasi-point. The quasi-point Q_x obviously relates to the angle θ . As the shape and the dimension of the current Sweep Pattern is varied in accordance with the position of the XsYs-plane during the sweep, the quasi-point Q_x on the current Sweep Pattern also relates to the position parameter t . Thus the distance from the origin C_p to the quasi-point Q_x becomes the function of both the angle θ and the position parameter t . Let $\ell(t, \theta)$ be the distance.

Then the distance $\ell(t, \theta)$ can be expressed by blending the two distances $\ell_s(\theta)$ and $\ell_e(\theta)$ as follows:

$$\left. \begin{aligned} \ell(t, \theta) &= \{1 - Bf(t)\} \cdot \ell_s(\theta) + Bf(t) \cdot \ell_e(\theta) , \\ \text{where } 0 \leq t \leq 1, \quad 0 \leq \theta \leq 2\pi . \end{aligned} \right\} \quad (8-7)$$

Here $Bf(t)$ is the blending function. The position of the quasi-point Q_x has the one-to-one relation with the angle θ . If the angle θ is varied from $\theta = 0$ to $\theta = 2\pi$, the quasi-point Q_x goes around the bounding curve of the current Sweep Pattern. Therefore, Equation(8-7) is considered to express the current Sweep Pattern. The blending function $Bf(t)$, however, is unknown. Thus we shall attempt to derive the blending function.

On the XsYs-plane where the position parameter is $t = 0$, the shape and the dimension of the current Sweep Pattern must be equivalent to those of the start Sweep Pattern. Thus the start condition of the current Sweep Pattern can be expressed as follows:

$$\left. \begin{array}{l} \ell(0, \theta) = \ell_s(\theta) , \\ \text{where } 0 \leq \theta \leq 2\pi . \end{array} \right\} \quad (8-8)$$

By substituting Equation(8-8) into Equation(8-7), the start condition of the blending function is obtained as follows:

$$Bf(0) = 0 . \quad (8-9)$$

Similarly, the end conditions of both the current Sweep Pattern and the blending function are as follows:

$$\left. \begin{array}{l} \ell(1, \theta) = \ell_e(\theta) , \\ \text{where } 0 \leq \theta \leq 2\pi , \end{array} \right\} \quad (8-10)$$

$$Bf(1) = 1 . \quad (8-11)$$

There are indefinite number of functions which satisfy Equations(8-9) and (8-11) simultaneously. Thus we shall show some examples which satisfy the following additional conditions:

- Derive the n degree functions for the parameter t , where $(n=1,2,3,\dots)$, and all the coefficients of the function are integers:

Then derived functions are,

$$Bf_1(t) = t^n , \quad (8-12)$$

$$Bf_2(t) = 2t - t^n , \quad (8-13)$$

where $n = 1, 2, 3, \dots$.

There are cases where the shapes and the dimensions of both the start and end Sweep Patterns are exactly the same, namely, $\ell_s(\theta) = \ell_e(\theta)$ for $\theta = 0 \sim 2\pi$. In these cases, if Equation(8-7) is used for the expression of the current Sweep Pattern, the dimension of the current Sweep Pattern cannot be varied during the sweep. Thus we introduce the following equation for the current Sweep Pattern:

$$\left. \begin{aligned} \ell(t, \theta) &= \ell_s(\theta) \cdot Bf(t) , \\ \text{where } 0 \leq t \leq 1 , \quad 0 \leq \theta \leq 2\pi , \\ \ell_s(\theta) &= \ell_e(\theta) , \quad \text{for } \theta = 0 \sim 2\pi . \end{aligned} \right\} \quad (8-14)$$

Both the start and end conditions of the current Sweep Patterns are,

$$\ell(0, \theta) = \ell_s(\theta) , \quad (8-15)$$

$$\ell(1, \theta) = \ell_e(\theta) , \quad (8-16)$$

$$\text{where } 0 \leq \theta \leq 2\pi .$$

By substituting Equations(8-15) and (8-16) into Equation(8-14), both the start and end conditions of the blending function are obtained as follows:

$$Bf(0) = 1 , \quad (8-17)$$

$$Bf(1) = 1 , \quad (8-18)$$

Like the case of Equations(8-9) and (8-11), there are indefinite number of functions which satisfy Equations(8-17) and (8-18) simultaneously. Thus we shall show some examples of the blending function satisfying the conditions mentioned above. Then these are expressed as follows:

$$Bf_3(t) = (1-t)^n + t^n , \quad (8-19)$$

$$Bf_4(t) = 2 - (1-t)^n - t^n , \quad (8-20)$$

$$\text{where } n = 1, 2, 3, \dots$$

The blending function plays significant roles not only for defining the current Sweep Pattern but also for controlling the profile of the fillet volume to be generated. However, the blending functions expressed in Equations(8-12), (8-13), (8-19), and (8-20) are led automatically by the constraint conditions at the start and the end of the sweep. Thus we cannot control the profile of the fillet volume

intuitively if these functions are applied. In order to introduce the controllable blending function, intermediate conditions of the blending function during the sweep should be designated by ourselves. Actually these conditions can easily be given as shown in Fig.8-5, where P_1, P_2 , and P_3 are the intermediate conditions, and P_0 and P_4 are the start and end conditions respectively. The intermediate conditions play a role of control points when interpolating the start and end conditions. There are well known interpolation methods[6][7][8] based on polynomial functions. These polynomial functions can be used as the controllable blending function of the current Sweep Pattern. For instance, if the Bezier interpolation function[6] is applied, the blending function is expressed as follows:

$$\left. \begin{aligned} Bf_s &= \sum_{i=0}^m P_i J_{m,i}(t) , \\ J_{m,i}(t) &= \binom{m}{i} t^i (1-t)^{m-i} , \\ \text{where } 0 \leq t \leq 1 . \end{aligned} \right\} \quad (8-21)$$

Here P_i ($i=1, 2, \dots, m$) are control points of the blending function Bf_s , and $J_{m,i}$ is the Bernstein basis function[6]. It is obvious that other polynomial functions can be used as the blending function of the current Sweep Pattern.

8.5 An implicit representation of the Sweep Pattern

Although Equations(8-7) and (8-14) represent the bounding curve of the current Sweep Pattern, both the inside and the outside

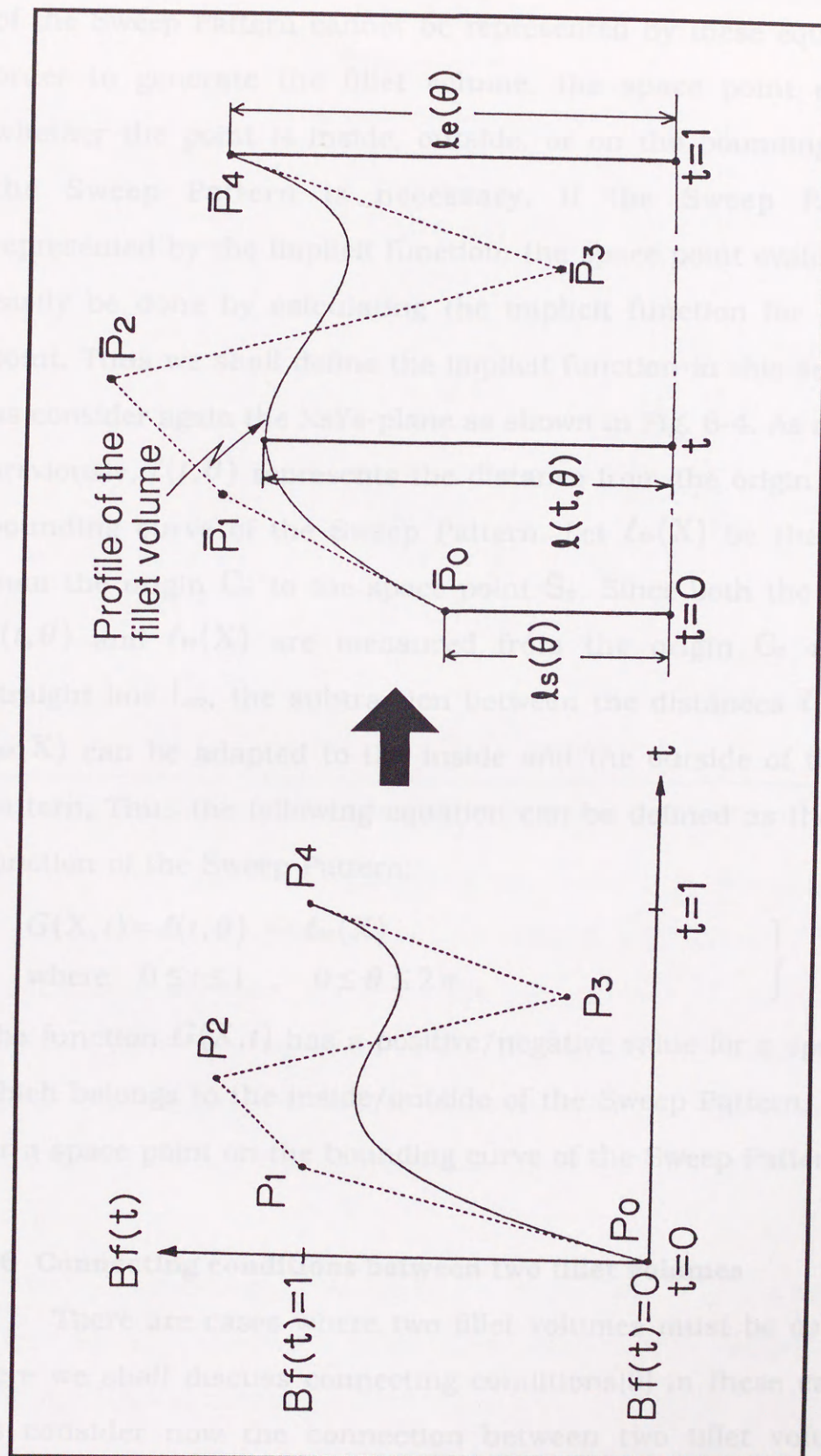


Fig.8-5 Application of the Bezier interpolation function to the blending function of the Sweep Pattern, and variation aspect of the profile of the fillet volume

of the Sweep Pattern cannot be represented by these equations. In order to generate the fillet volume, the space point evaluation whether the point is inside, outside, or on the bounding curve of the Sweep Pattern is necessary. If the Sweep Pattern is represented by the implicit function, the space point evaluation can easily be done by calculating the implicit function for the space point. Thus we shall define the implicit function in this section. Let us consider again the XsYs-plane as shown in Fig. 8-4. As mentioned previously, $\ell(t, \theta)$ represents the distance from the origin C_p to the bounding curve of the Sweep Pattern. Let $\ell_{sp}(X)$ be the distance from the origin C_p to the space point S_p . Since both the distances $\ell(t, \theta)$ and $\ell_{sp}(X)$ are measured from the origin C_p along the straight line L_{sp} , the subtraction between the distances $\ell(t, \theta)$ and $\ell_{sp}(X)$ can be adapted to the inside and the outside of the Sweep Pattern. Thus the following equation can be defined as the implicit function of the Sweep Pattern:

$$\left. \begin{aligned} G(X, t) &= \ell(t, \theta) - \ell_{sp}(X) . \\ \text{where } 0 \leq t \leq 1, \quad 0 \leq \theta \leq 2\pi . \end{aligned} \right\} \quad (8-22)$$

The function $G(X, t)$ has a positive/negative value for a space point which belongs to the inside/outside of the Sweep Pattern, and zero for a space point on the bounding curve of the Sweep Pattern.

8.6 Connecting conditions between two fillet volumes

There are cases where two fillet volumes must be connected. Here we shall discuss connecting conditions[9] in these cases. Let us consider now the connection between two fillet volumes as illustrated in Fig.8-6. It is assumed that these fillet volumes are

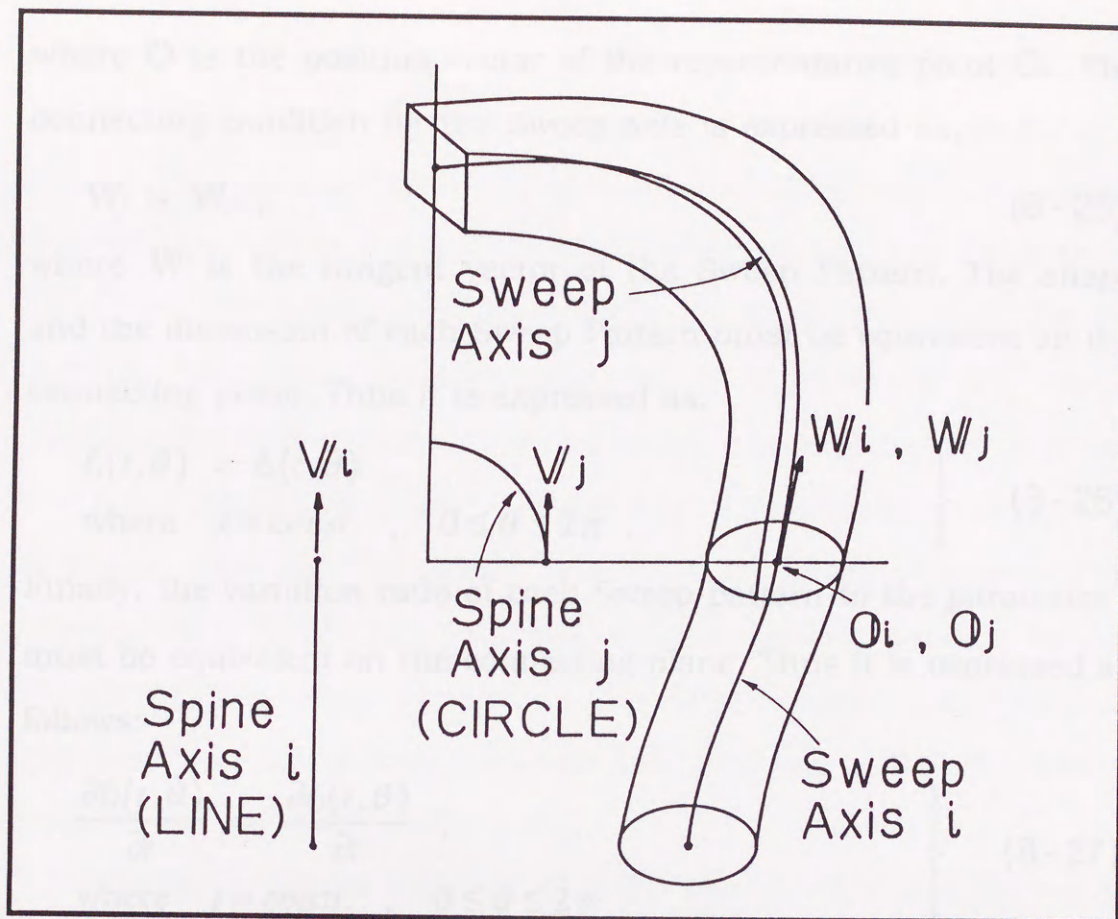


Fig.8-6 Connecting conditions between two fillet volumes

connected at the common plane cross-section. We refer to the cross-section as *the connecting plane*. Let letters "i" and "j" symbolize these two solids, for the convenience of explanation. In order to make the common connecting plane, the tangent vectors of the Spine Axes of two fillet volumes must be parallel each other on the plane. Thus it is expressed as,

$$V_i = V_j , \quad (8-23)$$

where V is the tangent vector of the Spine Axis. On the connecting plane, the representative point C_p of each Sweep Pattern must be unified. Thus it is expressed as

$$O_i = O_j , \quad (8-24)$$

where $\mathbf{O}$ is the position vector of the representative point C_p . The connecting condition for the Sweep Axis is expressed as,

$$\mathbf{W}_i = \mathbf{W}_j, \quad (8-25)$$

where $\mathbf{W}$ is the tangent vector of the Sweep Pattern. The shape and the dimension of each Sweep Pattern must be equivalent on the connecting plane. Thus it is expressed as,

$$\left. \begin{aligned} \ell_i(t, \theta) &= \ell_j(t, \theta), \\ \text{where } t &= \text{const.}, \quad 0 \leq \theta \leq 2\pi. \end{aligned} \right\} \quad (8-26)$$

Finally, the variation ratio of each Sweep pattern to the parameter t must be equivalent on the connecting plane. Thus it is expressed as follows:

$$\left. \begin{aligned} \frac{\partial \ell_i(t, \theta)}{\partial t} &= \frac{\partial \ell_j(t, \theta)}{\partial t}, \\ \text{where } t &= \text{const.}, \quad 0 \leq \theta \leq 2\pi. \end{aligned} \right\} \quad (8-27)$$

If the above connecting conditions are not satisfied, discontinuity occurs between the two connected fillet volumes.

8.7 Experiments and the consideration

In order to prove the correctness of the derived equations and to verify the effectiveness of the applied method, the computational algorithm has been developed and performed with the CSG-based CAD/CAM system TIPS-1[10][11] on the HITAC-M680 computer. Figures 8-7 through 8-16 show examples of the 3-D solid model of the fillet volume. Fig.8-7 shows fillet volumes to join the disconnected circular and rectangular cross-sections. Fillet volumes shown in Figs.8-7-(a), (b), and (c) are generated by using Equation(8-12) as the blending function of the Sweep Pattern, and

similarly the blending function expressed in Equation(8-13) is used for the fillet volumes shown in Figs.8-7-(d) and (e). From Fig.8-7, we can find that the blending surface of the fillet volume becomes convex or concave if the degree of the blending function is more than two. For instance, if Equation(8-12) is applied, the blending surface becomes convex, and reversely it becomes concave if Equation(8-13) is applied. As a result, the blending surface becomes the ruled surface as shown in Fig.8-7-(a) if the blending function is a linear function. The Bezier interpolation function expressed in Equation(8-21) is used as the controllable blending function of the Sweep Pattern when generating the fillet volumes as shown in Fig.8-8. From this figure, it can be said that the profile of the fillet volume can be varied aesthetically.

In the examples shown in Figs.8-7 and 8-8, the straight line is used as both the Sweep and Spine Axes of the fillet volumes. However, in the examples shown in Fig.8-9, the Bezier curve and the straight line are used as the Sweep and Spine Axes respectively. As all the examples shown in Figures 8-7 through 8-9 are the cases where the shapes or the dimensions of the start and end Sweep Patterns are different, the bounding curve of the current Sweep Pattern is expressed by Equation(8-7). However, there are cases where the shapes and the dimensions of the start and end Sweep Patterns are equivalent. In these cases, the bounding curve of the current Sweep Pattern is expressed by Equation(8-14) in order to vary the profile of the fillet volume. Fig.8-10 shows examples of these cases. From Fig.8-10, we can find that the profile of the fillet volume is varied. Figs.8-11-(a) and (b) illustrate the Sweep and

Spine Axes curves for the fillet volumes shown in Figs.8-11-(c) and (d) respectively. In these examples, the Bezier curve is used as the Sweep Axis. If these two fillet volumes are connected, the resulting fillet volume is shown in Fig.8-12-(b). Fig.8-12-(a) shows the smooth connection of the two Sweep Axes. Examples shown in Figs.8-12-(b) and (c) satisfy all the connecting conditions except Equation(8-27). By applying our method, the solid of revolution can easily be generated as shown in Fig.8-13. In this method, the straight line is used as both the Sweep and Spine Axes, and two circles are defined such that the center point of each circle is positioned on the Sweep Axis. These circles are used as the start and end Sweep Patterns. The fillet volume to round the vertex can also be generated as shown in Fig.8-14. From Fig.8-14, we can find that three fillet volumes having different radii are connected smoothly by the fillet volume to round the vertex. If the Xs- and Ys-axes are designated so as to rotate between the start and end Sweep Patterns, the twisted solid can be generated as shown in Fig.8-15. All the fillet volumes generated by our method can be handled just like the Primitive in CSG-based systems. Thus the CSG-based engineering analysis program can easily be applied as shown in Fig.8-16, where mass property is calculated.

8.8 conclusion

In this chapter, first, the problem for generation of the fillet volume to join two disconnected solids is discussed through the definition of the mathematical function of the fillet volume. In the discussion, the following are clarified: it is very difficult to define

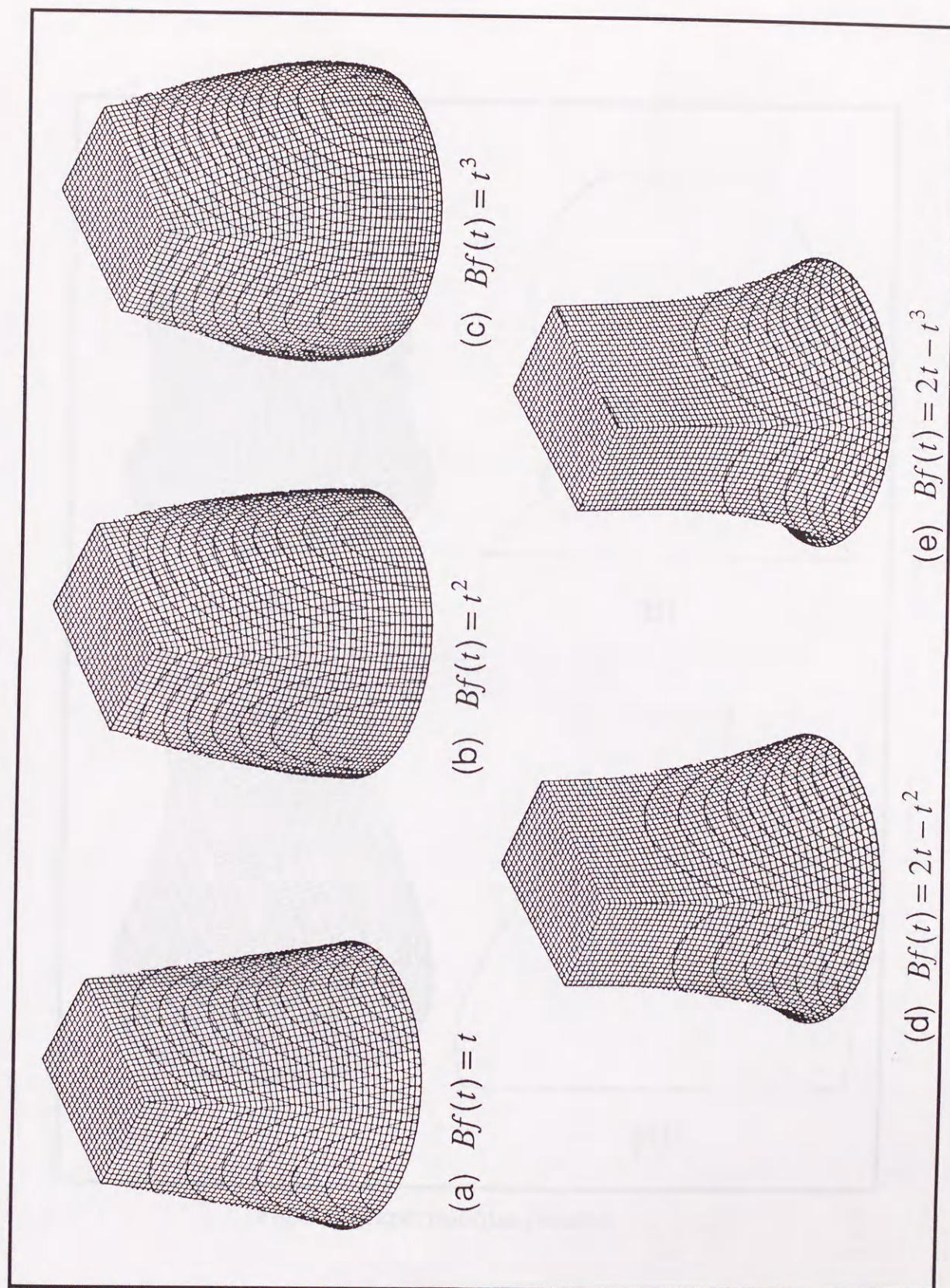


Fig.8-7 Experimental results

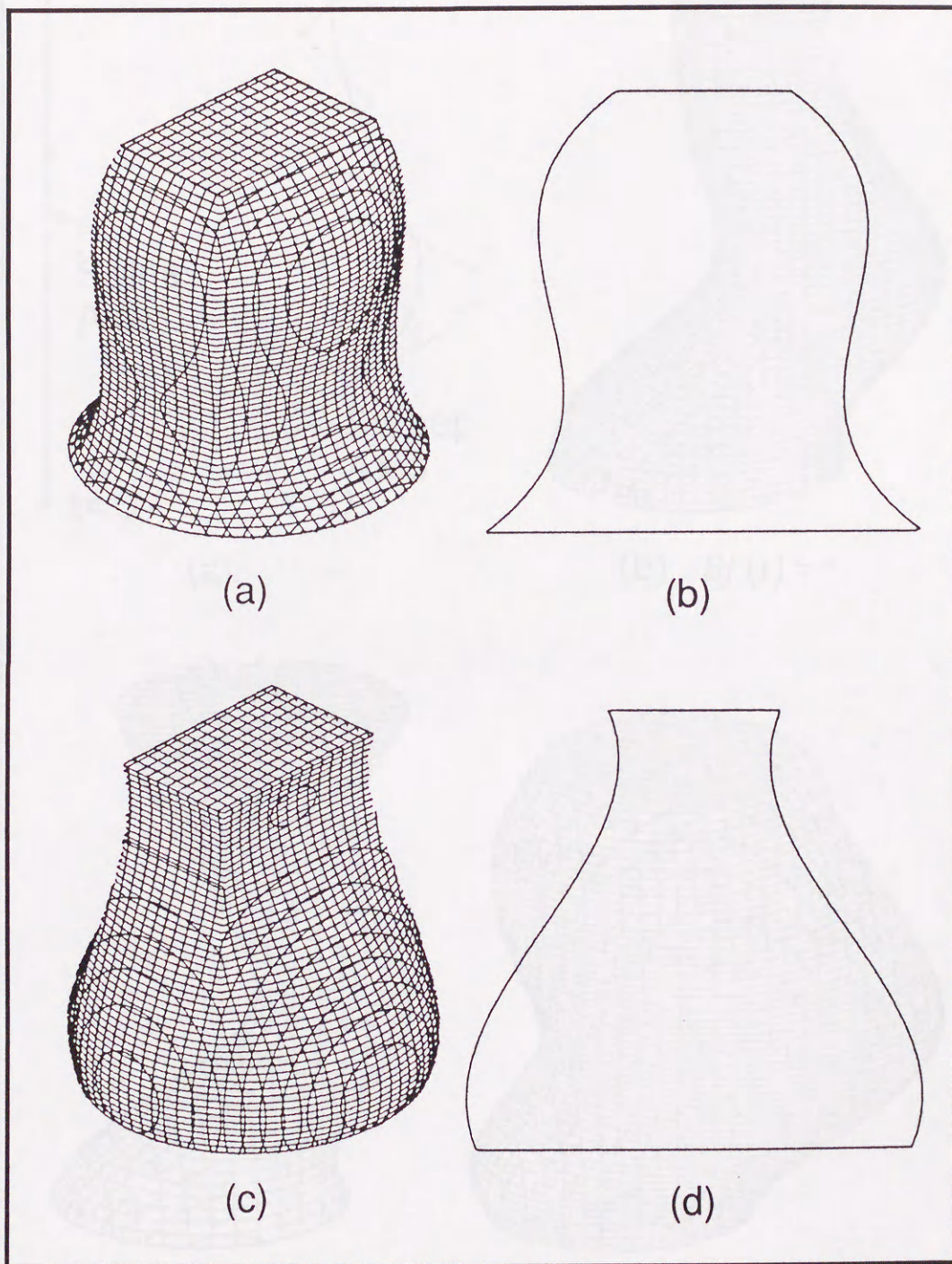


Fig.8-8 Experimental results

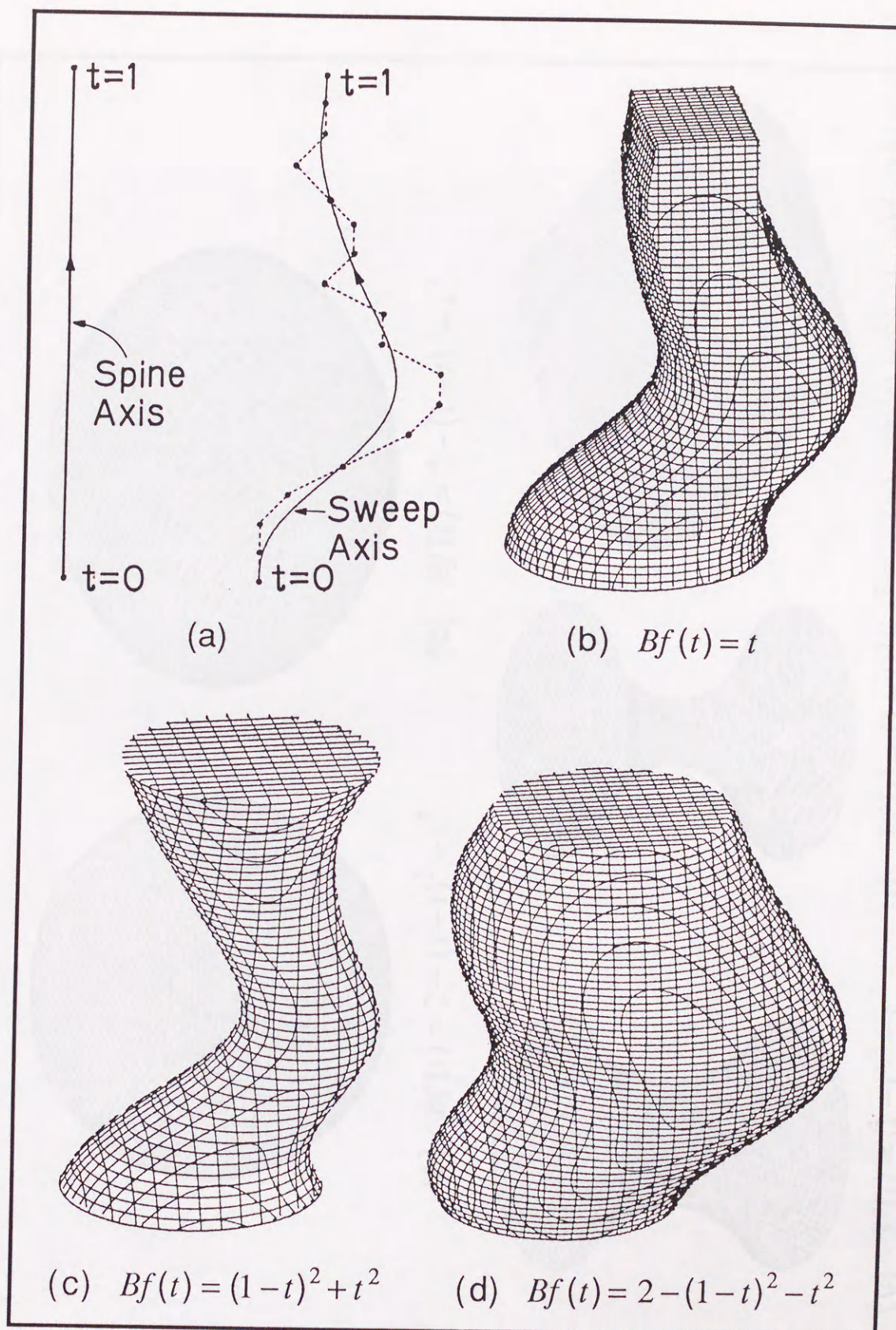
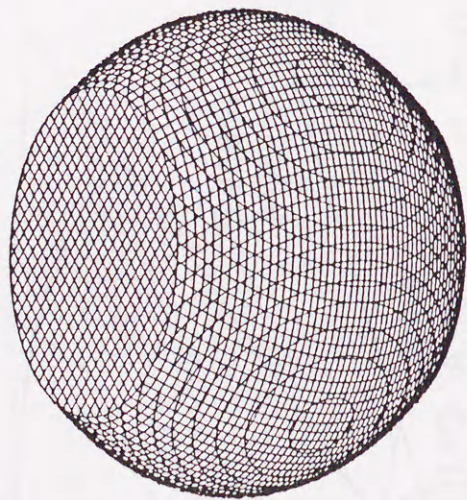
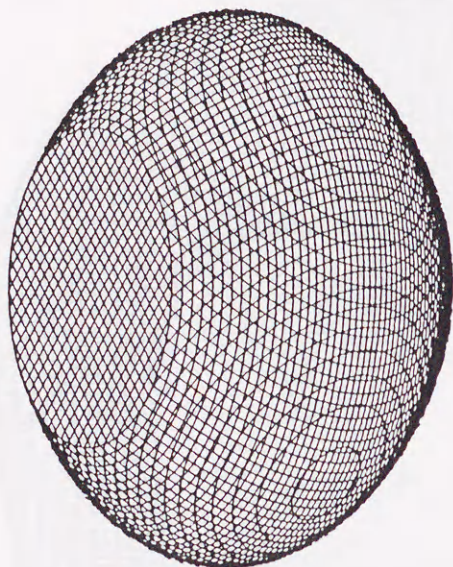


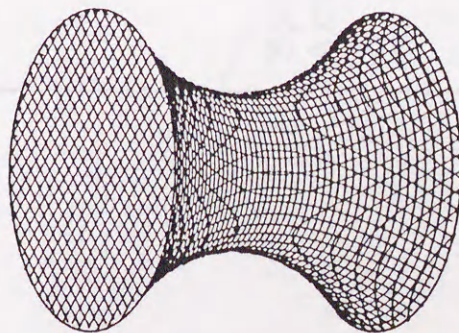
Fig.8-9 Experimental results



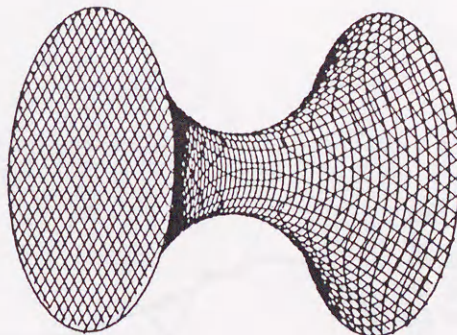
(a) $Bf(t) = 2 - (1-t)^2 - t^2$



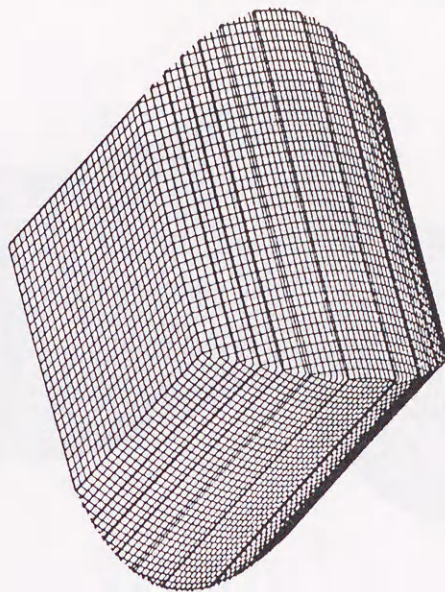
(b) $Bf(t) = 2 - (1-t)^3 - t^3$



(c) $Bf(t) = (1-t)^2 + t^2$



(d) $Bf(t) = (1-t)^3 + t^3$



(e) $Bf(t) = 2 - (1-t)^2 - t^2$

Fig.8-10 Experimental results

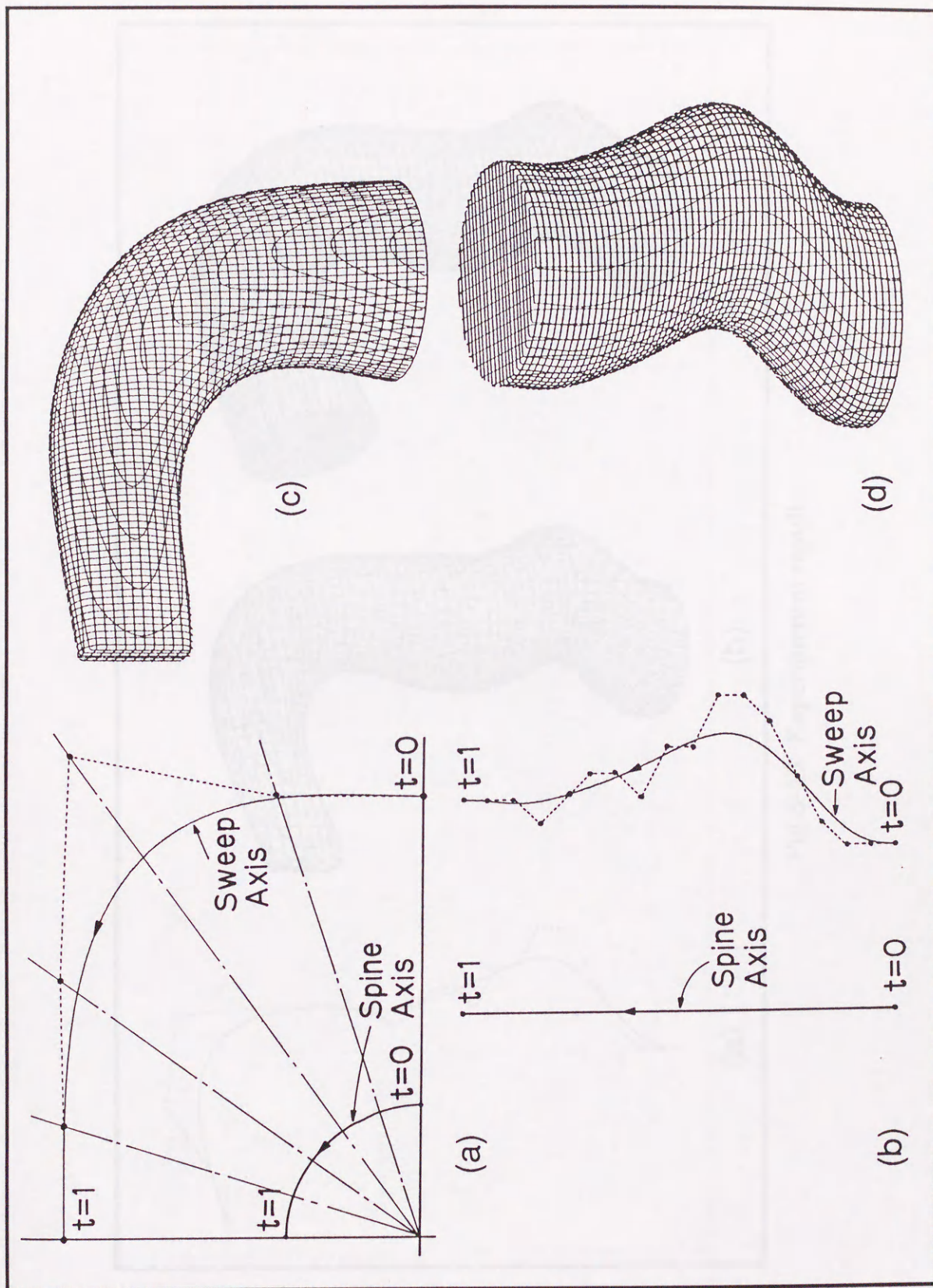


Fig.8-11 Experimental results

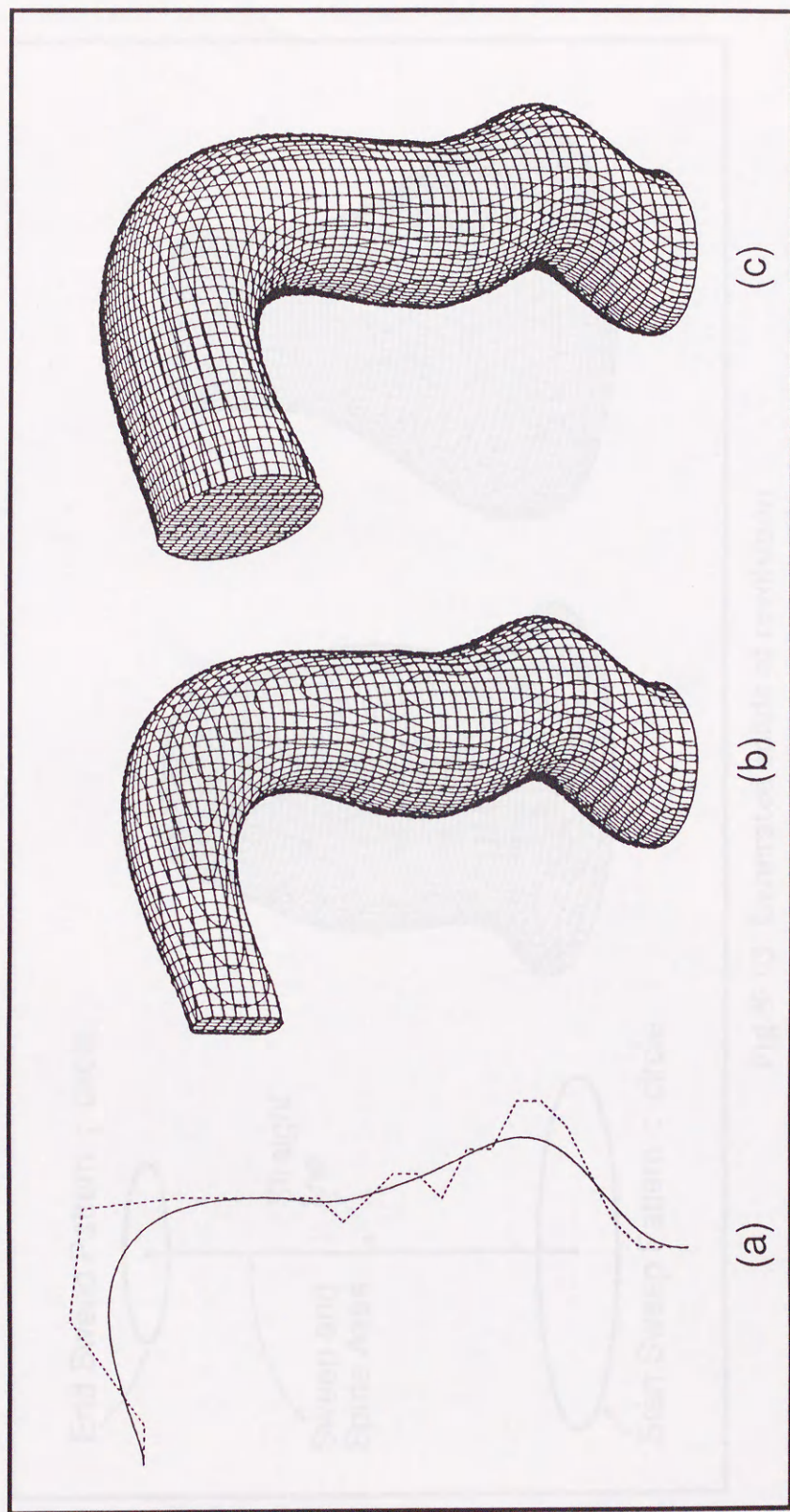


Fig.8-12 Experimental results

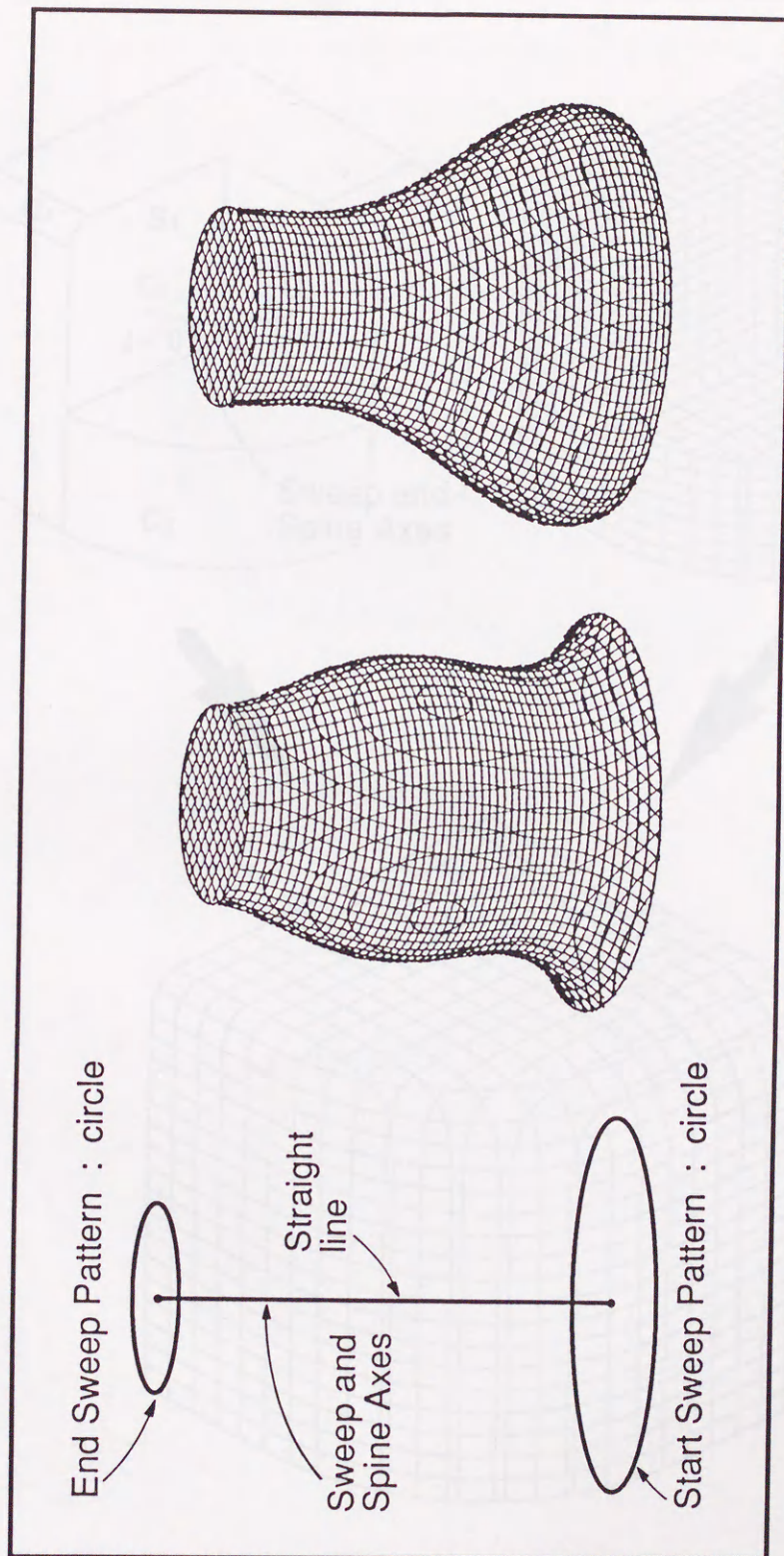


Fig.8-13 Generated solids of revolution

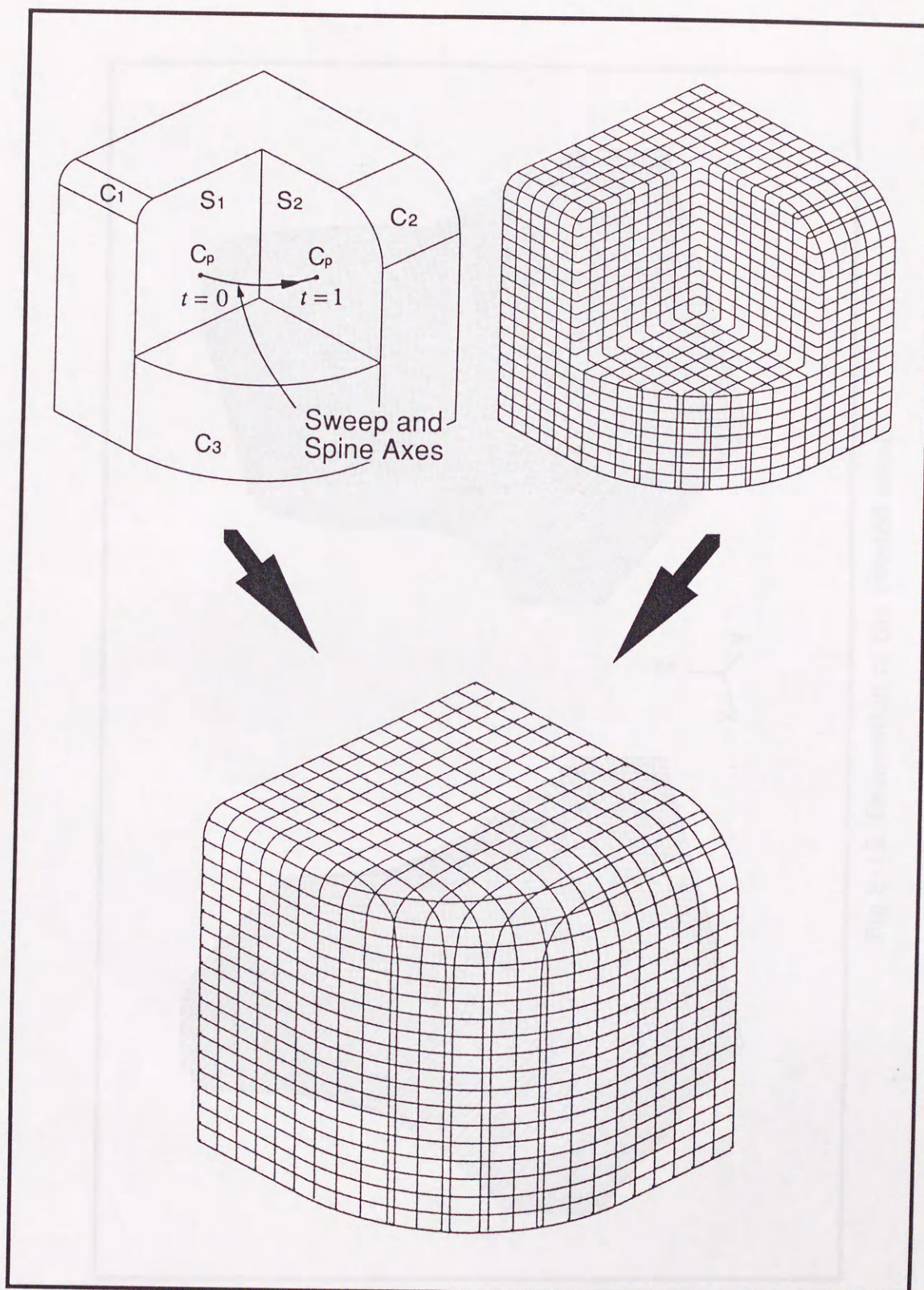


Fig.8-14 The fillet volume to round the vertex

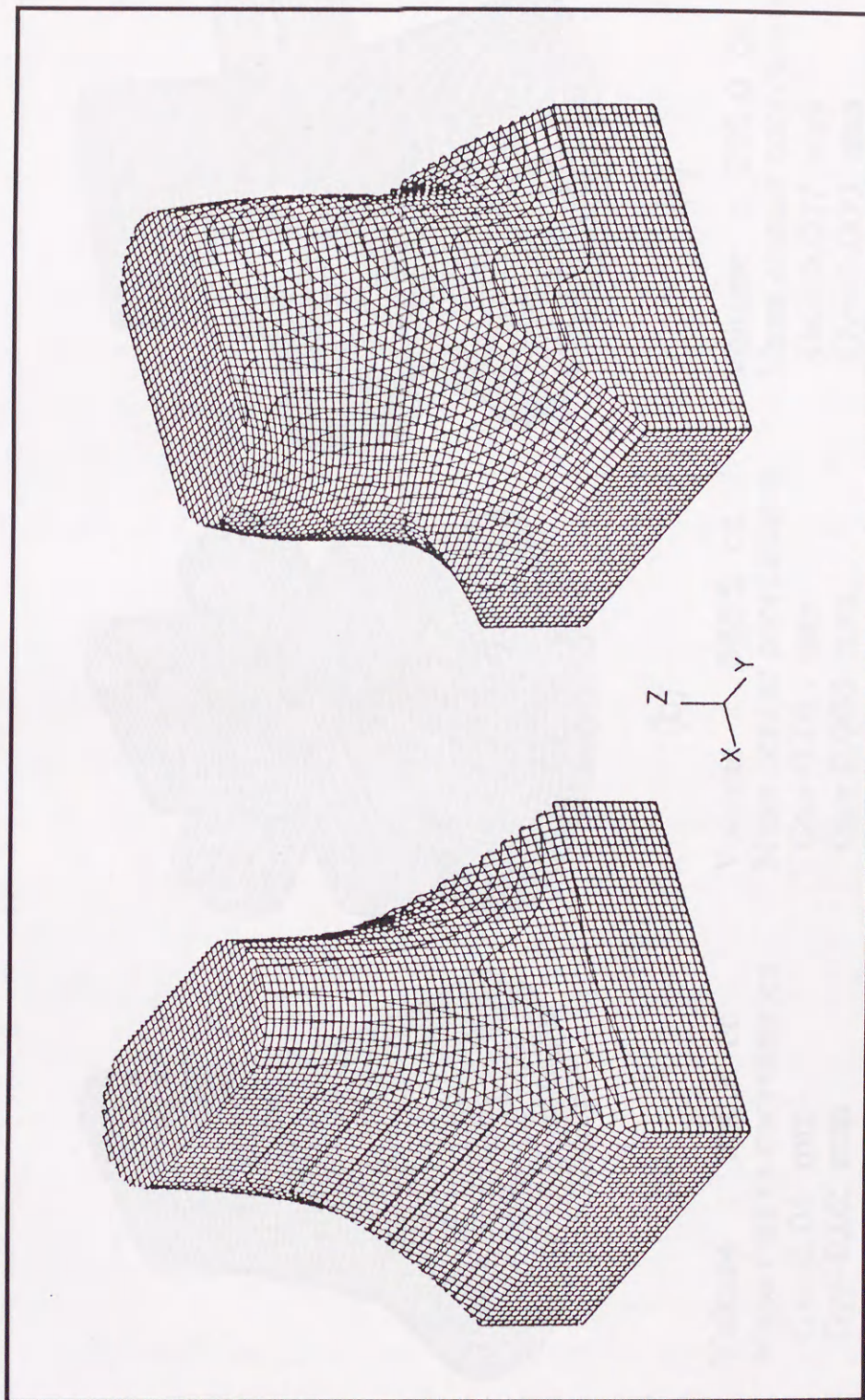


Fig.8-15 Generation of the twisted solid

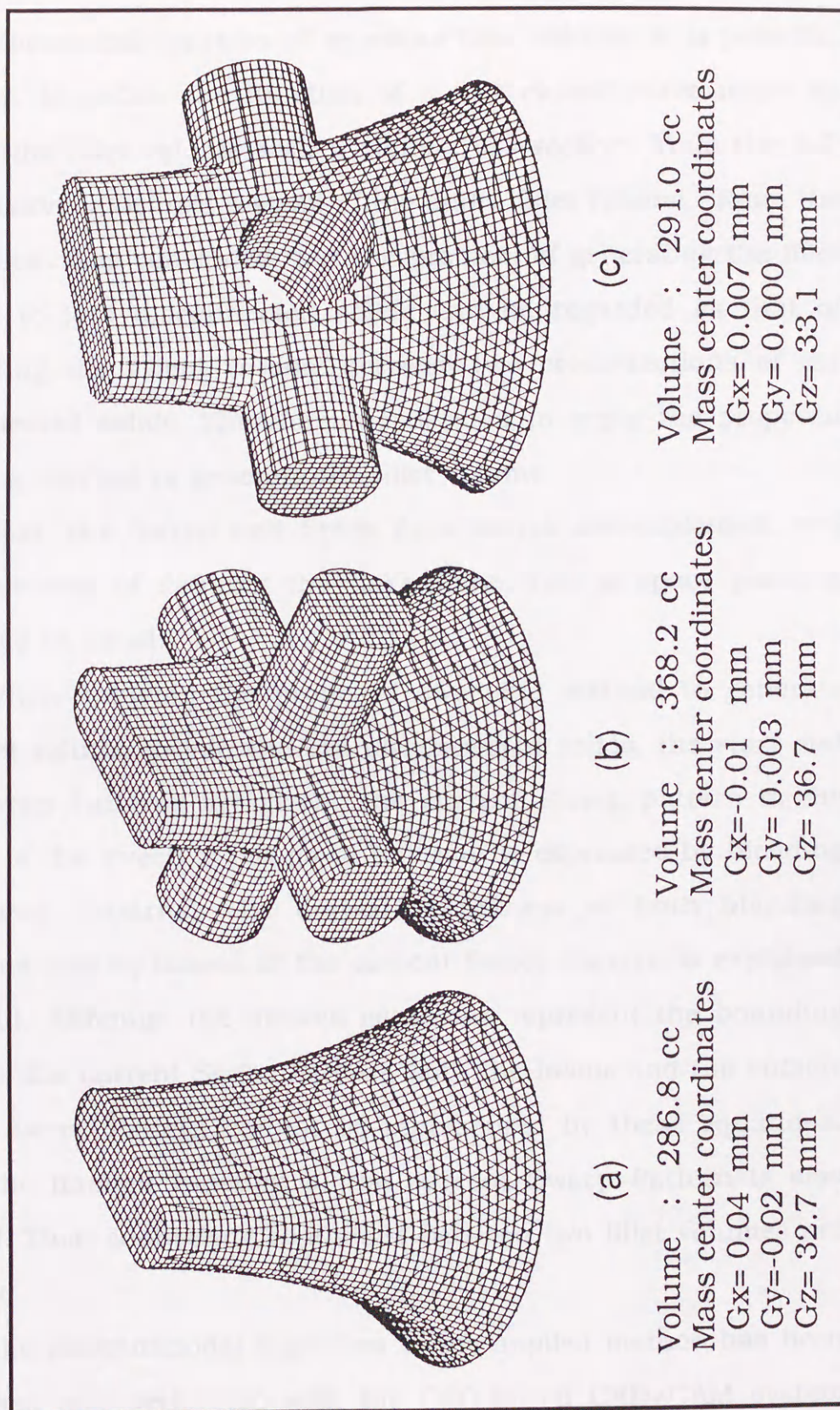


Fig.8-16 Calculation of mass properties

the mathematical function of an entire fillet volume; it is possible, however, to define the function of a 2-D closed curve made by cutting the fillet volume with a plane cross-section. Thus the 2-D closed curve is defined instead of the entire fillet volume. From the discussion, it is concluded that the problem of generating the fillet volume to join disconnected solids can be regarded as that of generating the swept volume between two cross-sections of the disconnected solids. Therefore, we decided to apply the proposed sweeping method to generate the fillet volume.

Next, the Sweep and Spine Axis curves are explained, and then the way of defining the XsYs-plane from a space point is explained in detail.

When applying the proposed sweeping method to generate the fillet volume to join the two disconnected solids, the start and end Sweep Patterns are given. The current Sweep pattern in the middle of the sweep is not given but can be expressed by blending these two patterns. The derivation process of both blending functions and equations of the current Sweep Pattern is explained in detail. Although the derived equations represent the bounding curve of the current Sweep Pattern, both the inside and the outside of the Sweep Pattern cannot be represented by these equations. Thus the implicit function of the current Sweep Pattern is also derived. Then connecting conditions between two fillet volumes are clarified.

The computational algorithm of the applied method has been developed and performed with the CSG-based CAD/CAM system TIPS-1. According to the experiments, it is clear that the method

applied to generate the fillet volume is effective enough, derived equations are correct, and that the generated fillet volume can be handled just like the ordinary Primitive.

Finally, the applied method has the following advantages:

- The method allows the Sweep Axis to use not only analytically defined curves such as conic curves but also parametrically defined curves such as the Bezier curves. Thus the complex sweep trajectory curve is representable.
- Since polynomial functions such as the Bezier interpolation functions can be used as the blending function of the current Sweep Pattern, we can control the blending function intuitively. This characteristic contributes to extend the modeling capability of CSG scheme.

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9. A PROTOTYPE SYSTEM OF THE SPECIAL PRIMITIVE

FOR THE SWEEPED VOLUME

9.1 Introduction

A generalized sweeping method for the CSG modeling is proposed and explained in chapter 4, and the generation of the swept volume by the proposed sweeping method is clarified through discussions and experiments in chapters from 5 to 8. However, it is not yet clarified how to describe parameters of the swept volume, and how to incorporate the swept volume into CSG systems. We also need to clarify what the canonical data of the swept volume are. These questions are clarified in this chapter through explanation of the developed prototype system[1] for the swept volume.

9.2 The system structure

An outline of the program structure of the prototype system is illustrated in Fig.9-1. The system is incorporated into the CSG-based CAD/CAM system TIPS-1[2], implemented on the HITAC M-680 computer at Hokkaido University, and can be run either in batch or interactively via Tektronix 4014 graphic terminal under the control of the VOS-3 operating system.

The input data to the system are prepared in APT-like description statements. The shape description statements are processed by two pre-processors, T1PRPR and SWPRPR. The T1PRPR interprets the input statements for the TIPS-1 system,

and generates a data table T1LIST. The T1LIST contains all geometric information of an object in a form of CSG data structure. The SWPRPR interprets the input statements for the swept volume, and generates data tables (i.e., SWPVLM, SWPAXS, SPNAXS, SSWPTN, and ESWPTN) concerning with the Sweep and Spine Axes, and with the Sweep Pattern. The T1LIST and the data tables of the swept volume are further processed by T1MHPR which results in two data tables T1MESH and M3LIST. These data tables contain geometric information in a form of 3-D mesh cells, and constitute a common geometry data-base from which application routines (or post-processors) can draw information for further processing. More detailed discussion of the tables T1MESH and M3LIST is found in the literature[2].

9.3 The CONTOUR statement and the subroutine CONTOR

In the TIPS-1 system, the shape description statement CONTOUR is prepared for a special, non-standard, and open-ended Primitive which has to be defined by the user. With the CONTOUR statement, the swept volume can be incorporated into the CSG-based CAD/CAM system TIPS-1. Though the pattern function of the ordinary Primitive has already been prepared in the subroutine FPATEN, the special Primitive defined by the CONTOUR statement is handled differently in the system. Namely, if a CONTOUR statement appears in the input data stream, the FPATEN calls the subroutine CONTOR. Since the system allows the user to program the subroutine CONTOR, any non-standard Primitive can be added to the system if its mathematical function can be programmed in

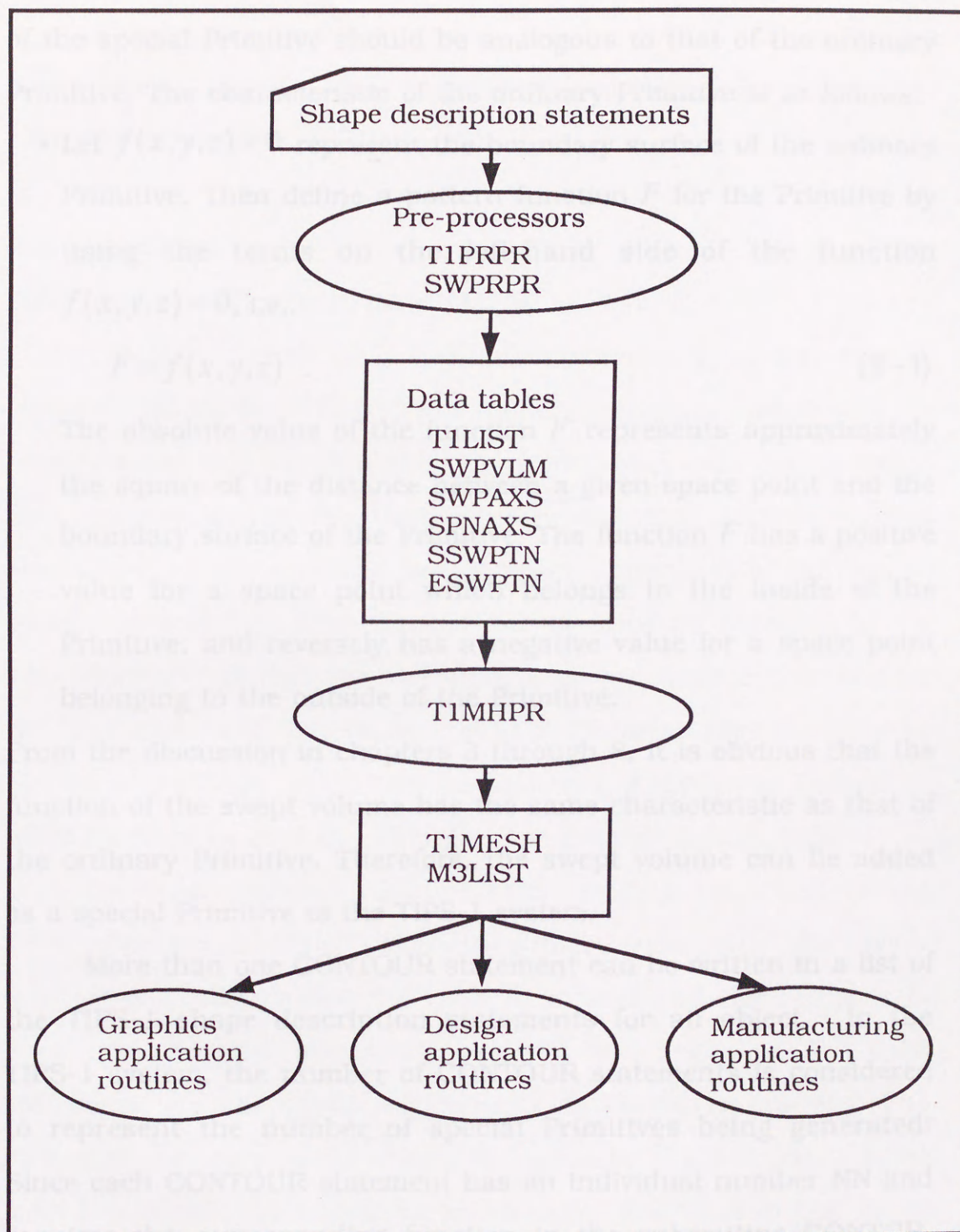


Fig.9-1 The program structure of the prototype system

the CONTOR. Then the characteristic of the mathematical function of the special Primitive should be analogous to that of the ordinary Primitive. The characteristic of the ordinary Primitive is as follows:

- Let $f(x, y, z) = 0$ represent the boundary surface of the ordinary Primitive. Then define a pattern function F for the Primitive by using the terms on the left-hand side of the function $f(x, y, z) = 0$, i.e.,

$$F = f(x, y, z) \quad (9-1)$$

The absolute value of the function F represents approximately the square of the distance between a given space point and the boundary surface of the Primitive. The function F has a positive value for a space point which belongs to the inside of the Primitive, and reversely has a negative value for a space point belonging to the outside of the Primitive.

From the discussion in chapters 3 through 8, it is obvious that the function of the swept volume has the same characteristic as that of the ordinary Primitive. Therefore, the swept volume can be added as a special Primitive to the TIPS-1 system.

More than one CONTOUR statement can be written in a list of the TIPS-1 shape description statements for an object. In the TIPS-1 system, the number of CONTOUR statements is considered to represent the number of special Primitives being generated. Since each CONTOUR statement has an individual number NN and requires the corresponding function in the subroutine CONTOR, the algorithm of the subroutine is subdivided by the pointer NN so as to define the corresponding function. Moreover each subdivided

part can be programmed independently. Therefore, more than one special Primitive can be added to the system through the subroutine CONTOR. More detailed discussion of the subroutine CONTOR is found in the literature[2].

9.4 Data tables

9.4.1 CSG canonical data table T1LIST

The table T1LIST is a two-dimensional array of size $N \times M$, where $N=20$. In the T1LIST, the shape description statements are column-wise; that is, with the initial statement (i.e., the so-called *Part statement*) in the first column, the first statement of a Primitive (i.e., the so-called the *Segment statement*) in the second column and so on. Thus the number of columns occupied in the T1LIST equals to the number of Segment statements plus one for the Part statement. The dimension M of the array may therefore be chosen accordingly, while the dimension N of the array is fixed in $N=20$ by the format of the TIPS-1 shape description language.

To show contents of the T1LIST during execution of the TIPS-1 program, the T1LIST is printed out with the array transposed, that is, with the rows N vertical and the columns M horizontal. In other words, the contents of a Segment (or a Part) statement are displayed horizontally to facilitate reading. The format of the transported T1LIST is shown in Fig.9-2. The pointer from the CONTOUR statement to the table SWPVLM of the swept volume is stored in the twentieth row of the T1LIST. More detailed discussion of the T1LIST is found in the literature[2].

TILIST(I,J): Data table for constructive representation of objects

J	I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
1	1	Part	Part Data			Comment		Rotation Angle			Coordinate			Domain of Part						System				
		Name (A4)	MJ (I2)(I2)	CC (I2)(I2)	N (I2)(I2)	M (I2)(I2)	MP (I2)(I2)	MQ (I2)(I2)			PHI (F4)	PSI (F4)	THETA (F4)	X (F4)	Y (F4)	Z (F4)	LX (F4)	UX (F4)	LY (F4)	UY (F4)	LZ (F4)	UZ (F4)	ISN (I2)(I2)	
		Seg.	Pattern Data			Constants			Coordinate			Domain of Segment						System						
MP	2	Name (A4)	MD (I2)(I2)	CC (I2)(I2)	PN (I2)(I2)	DM (I2)(I2)	FC (I2)(I2)	SD (I2)(I2)	ARG1 (F4)	ARG2 (F4)	ARG3 (F4)	ARG4 (F4)	ARG5 (F4)	ARG6 (F4)	X (F4)	Y (F4)	Z (F4)	LX (F4)	UX (F4)	LY (F4)	UY (F4)	LZ (F4)	UZ (F4)	ISN (I2)(I2)
		P-Mode Segments																						
		Q-Mode Segments																						
MQ	MJ	D-Mode Segments & Error Segments																						
		Blank																						

Fig.9-2 The transported TILIST

9.4.2 The table SWPVLM

SWPVLM is a pointer table between the T1LIST and tables for the Sweep and Spine Axes, and for the Sweep Pattern. This table is defined as a two-dimensional array of size NV x MV, where NV and MV are positive integers. The dimension MV of the array is equal to that of the T1LIST, and the dimension NV is fixed in NV=10. The format of the SWPVLM is illustrated in Fig.9-3, and its contents are as follows:

(1) The first row of SWPVLM

Row #	Symbol	Definition
1	NSW	The number of the swept volumes to be generated.

(2) The second to NV-th row of SWPVLM

Row #	Symbol	Definition
1	SGN	A pointer to the T1LIST. The pointer designates the column number of T1LIST in which the data of the corresponding CONTOUR statement are stored.
2	SWTP	SWPT designates the sweep type.
	SWTP=1	The type 1 sweep is applied.
	SWTP=2	The type 2 sweep is applied.
3	NSWP	The number of curved segments with which a Sweep Axis is constructed.
4	NSPN	The number of curved segments with which a Spine Axis is constructed.

NV MV	1	2	3	4	5	6	7	8	9	10
1	NSW									
2	SGN	SWTP	NSWP	NSPN	SNCC	SSWP	ENCC	ESWP		
3	"	"	"	"	"	"	"	"		
•	"	"	"	"	"	"	"	"		
•	"	"	"	"	"	"	"	"		
MV	"	"	"	"	"	"	"	"		

Fig.9-3 The SWPVLM table

Row #	Symbol	Definition
5	SNCC	The number of closed curves for the start Sweep Pattern.
6	SSWP	SSWP designates the type of the start Sweep Pattern.
	SSWP=1	The Sweep Pattern is composed of single closed curve.
	SSWP=2	The Sweep Pattern is composed of a product set of closed curves.
	SSWP=3	The Sweep Pattern is composed of a union set of closed curves.
7	ENCC	The number of closed curves for the end Sweep Pattern.
8	ESWP	ESWP designates the type of the end Sweep Pattern, and its value has the same meaning as that of SSWP.
9		A working slot for the system.
10		A working slot for the system.

9.4.3 The table SWPAXS

The table SWPAXS is prepared for storing the data of the Sweep Axis. This table is defined as a three-dimensional array of size $IW \times JW \times KW$, where IW , JW , and KW are positive integers. The dimension KW of the array is equal to the column M of the T1LIST, and the dimensions IW and JW are fixed in $IW=40$ and $JW=20$ respectively. There are KW storages in the column of the

SWPAXS. (i.e., SWPAXS(\$, \$, JJ), where JJ=1~KW.) We refer to the storages as *the slots* KW. The slots KW have the one-to-one relation with the slots MV of the table SWPVLM. The slots JW are prepared for the curved segments with which a Sweep Axis is constructed. Thus if the Sweep Axis is composed of a single curve, only one slot of JW is used. The slots IW are used for storing the input data of a curved segment. The format of the shape description statement of curved segments is illustrated in Fig.9-4, and these data are stored in the slots IW as follows:

<u>Row #</u>	<u>Definition</u>
SWPAXS(1,\$,\$)	Name of the statement. (A4)
SWPAXS(2,\$,\$)	Pattern code of the curved segment.
SWPAXS(2,\$,\$)=1	LINE.
SWPAXS(2,\$,\$)=2	CIRCULAR ARC.
SWPAXS(2,\$,\$)=3	ELLIPSE.
SWPAXS(2,\$,\$)=4	PARABOLA.
SWPAXS(2,\$,\$)=5	SPIRAL of Archimedes.
SWPAXS(3,\$,\$)	Dimension code. This code designates the defining plane of the curved segment.
SWPAXS(3,\$,\$)=XY	The xy-plane.
SWPAXS(3,\$,\$)=YZ	The yz-plane.
SWPAXS(3,\$,\$)=ZX	The zx-plane.
SWPAXS(3,\$,\$)=W	An arbitrary plane in the 3-D space.
SWPAXS(3,\$,\$)=N	The defining plane cannot be determined. This code is effective only for the case where SWPAXS(2,\$,\$)=1.

(1) The case where SWPAXS(2,\$,\$)=1.

SWPAXS(I,\$,\$), where I=4~6	Position vector of the start point.
SWPAXS(I,\$,\$), where I=7~9	Position vector of the end point.
SWPAXS(I,\$,\$), where I=10~12	Direction vector from the start point to the end point.
SWPAXS(I,\$,\$), where I=13	Distance from the start point to the end point.

(2) The case where SWPAXS(2,\$,\$)=2.

SWPAXS(I,\$,\$), where I=4~6	Position vector of the center point.
SWPAXS(I,\$,\$), where I=7~9	Position vector of the start point.
SWPAXS(I,\$,\$), where I=10~12	Position vector of the end point.
SWPAXS(I,\$,\$), where I=13~15	Normal vector of the defining plane.
SWPAXS(I,\$,\$), where I=16	Radius of the circular arc.
SWPAXS(I,\$,\$), where I=17~19	Position vector of the third point on the arc. This point is used for defining a course from the start point to the end point.

(3) The case where SWPAXS(2,\$,\$)=3.

SWPAXS(I,\$,\$), where I=4~6	Position vector of the center point.
SWPAXS(I,\$,\$), where I=7~9	Position vector of the start point.
SWPAXS(I,\$,\$), where I=10~12	Position vector of the end point.
SWPAXS(I,\$,\$), where I=13~15	Normal vector of the defining plane.
SWPAXS(I,\$,\$), where I=16	Rotational angle of the major axis.

SWPAXS(I,\$,\$), where I=17	The major axis of an ellipse.
SWPAXS(I,\$,\$), where I=18	The minor axis of an ellipse.
SWPAXS(I,\$,\$), where I=19~21	Position vector of the third point on the ellipse. This point is used for defining a course from the start point to the end point.

(4) The case where SWPAXS(2,\$,\$)=4.

SWPAXS(I,\$,\$), where I=4~6	Position vector of the vertex of a parabola.
SWPAXS(I,\$,\$), where I=7~9	Position vector of the start point.
SWPAXS(I,\$,\$), where I=10~12	Position vector of the end point.
SWPAXS(I,\$,\$), where I=13~15	Normal vector of the defining plane.
SWPAXS(I,\$,\$), where I=16~18	Direction vector of the axis of the parabola.
SWPAXS(I,\$,\$), where I=19	Rotational angle α of the axis of the parabola.
SWPAXS(I,\$,\$), where I=20	The coefficient "a" of the equation of the parabola.

(5) The case where SWPAXS(2,\$,\$)=5.

SWPAXS(I,\$,\$), where I=4~6	Position vector of the origin of a spiral of Archimedes.
SWPAXS(I,\$,\$), where I=7~9	Position vector of the start point.
SWPAXS(I,\$,\$), where I=10~12	Position vector of the end point.
SWPAXS(I,\$,\$), where I=13~15	Normal vector of the defining plane.
SWPAXS(I,\$,\$), where I=16	The start angle.

SWPAXS(I,\$,\$), where I=17	The end angle.
SWPAXS(I,\$,\$), where I=18	The coefficient k of the spiral curve.

9.4.4 The table SPNAXS

The table SPNAXS is prepared for storing the data of the Spine Axis. This table is defined as a three-dimensional array of size $IW \times JW \times KW$, where IW , JW , and KW are positive integers. A line and a circular arc can be used as the Spine axis curves, and these data are stored in the same format as the SWPAXS table. The format of the shape description statement of curved segments for the Spine Axis is the same as that of the Sweep Axis.

9.4.5 The table SSWPTN

The table SSWPTN is prepared for storing the data of the start Sweep Pattern. This table is defined as a three-dimensional array of size $IP \times JP \times KP$, where IP , JP , and KP are positive integers. The dimension KP of the array is equal to the column M of the T1LIST, and the dimensions JP and IP are fixed in $JP=10$ and $IP=20$ respectively. The slots KP have the one-to-one relation with the slots MV of the table SWPVLM. The slots JP are prepared for the closed curves with which a Sweep Pattern is composed. More than one slot of JP are necessary when a Sweep Pattern is composed of a union (or product) set of closed curves while only one slot is used if the Sweep Pattern is composed of single closed curve. The slots IP are used for storing the element of a Sweep Pattern. The format of the shape description statement of the

Sweep Pattern is illustrated in Fig.9-5, and these data are stored in the slots IP as follows:

Row #	Definition
SSWPTN(1,\$,\$)	Name of the statement for the start Sweep Pattern. (A4)
SSWPTN(2,\$,\$)	Element code of the Sweep Pattern.
SSWPTN(2,\$,\$)=10	CIRCLE.
SSWPTN(2,\$,\$)=20	RECTANGLE.
SSWPTN(2,\$,\$)=30	ELLIPSE.
SSWPTN(3,\$,\$)	Side code of the element.
SSWPTN(3,\$,\$)=1	Inside of the element is defined.
SSWPTN(3,\$,\$)=2	Outside of the element is defined.

The case where SSWPTN(2,\$,\$)=10.

SSWPTN(I,\$,\$), where I=4~5	The coordinates of the center point.
SSWPTN(I,\$,\$), where I=6	Radius of the circle.
SSWPTN(I,\$,\$), where I=7	Square of the radius.

The case where SSWPTN(2,\$,\$)=20.

SSWPTN(I,\$,\$), where I=4~5	The coordinates of the starting point.
SSWPTN(I,\$,\$), where I=6	The size of the abscissa "a". (See Fig.9-5.)
SSWPTN(I,\$,\$), where I=7	The size of the ordinate "b". (See Fig.9-5.)
SSWPTN(I,\$,\$), where I=8	Rotational angle α .
SSWPTN(I,\$,\$), where I=9	$\cos(\alpha)$.

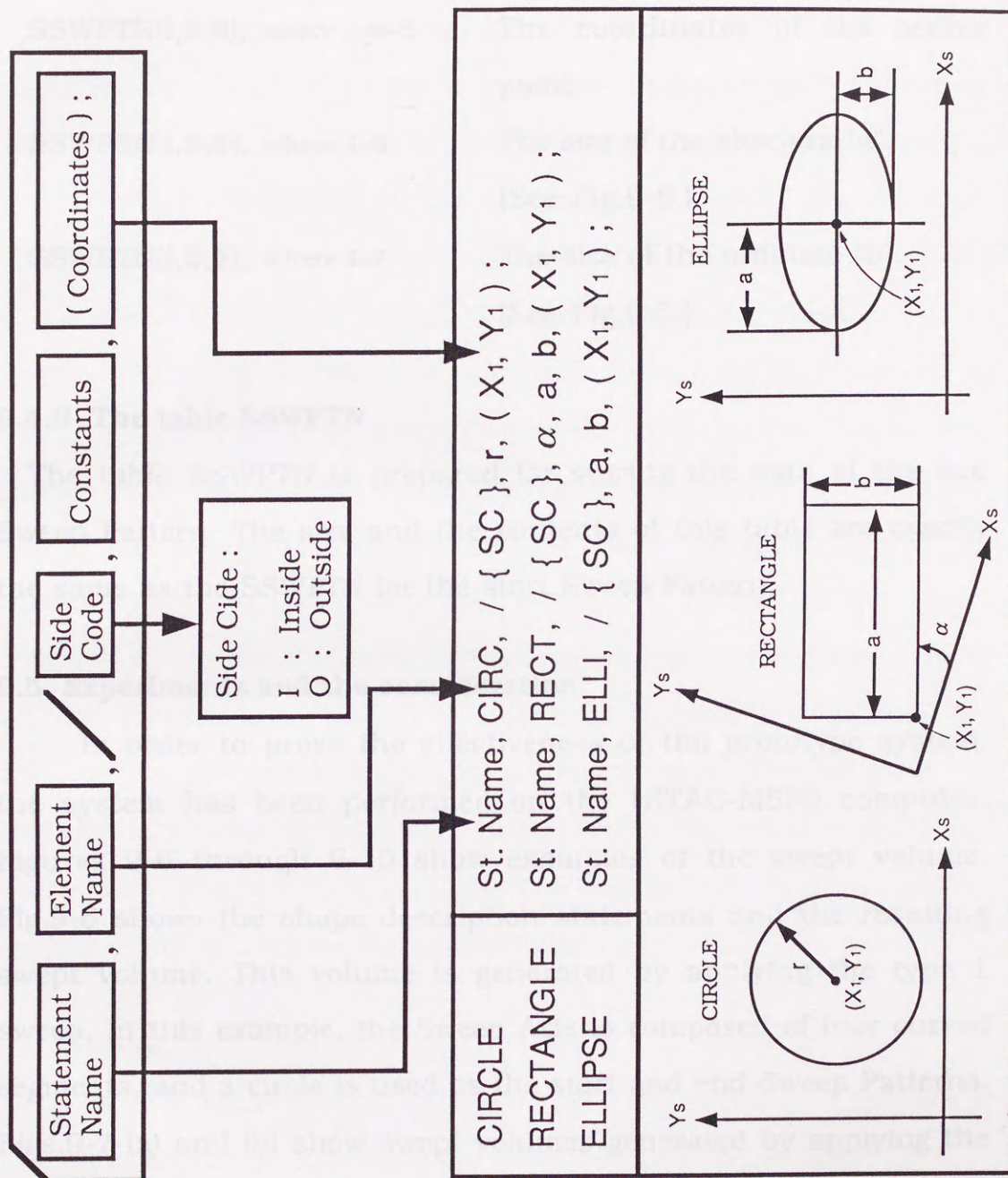


Fig.9-5 The shape description statement of the Sweep Pattern

SSWPTN(I,\$,\$), where I=10 $\sin(\alpha)$.

The case where SSWPTN(2,\$,\$)=30.

SSWPTN(I,\$,\$), where I=4~5 The coordinates of the center point.

SSWPTN(I,\$,\$), where I=6 The size of the abscissa "a".
(See Fig.9-5.)

SSWPTN(I,\$,\$), where I=7 The size of the ordinate "b".
(See Fig.9-5.)

9.4.6 The table ESWPTN

The table ESWPTN is prepared for storing the data of the end Sweep Pattern. The size and the contents of this table are exactly the same as the SSWPTN for the start Sweep Pattern.

9.5 Experiments and the consideration

In order to prove the effectiveness of the prototype system, the system has been performed on the HITAC-M680 computer. Figures 9-6 through 9-10 show examples of the swept volume. Fig.9-6 shows the shape description statements and the resulting swept volume. This volume is generated by applying the type 1 sweep. In this example, the Sweep Axis is composed of four curved segments, and a circle is used as the start and end Sweep Patterns. Figs.9-7-(a) and (b) show swept volumes generated by applying the type 2 sweep. Namely, an ellipse defined on the xy-plane is used as the Sweep Axis, and a circumscribed circle of the ellipse is used as the Spine Axis. Since the dimension of the start and end Sweep Patterns is changed in Fig.9-7-(a), the resulting swept volume has

surface discontinuity on the plane where the start and end Sweep Patterns meet. In Fig.9-7-(b), the start and end Sweep Patterns are exactly the same, and these pattern are composed of a union set of three circles. Fig.9-8 shows a simplified exhaust manifold of an automobile and its shape description statements. Other special Primitives can be added to the system. Figs.9-9-(a) and (b) show the helical and the conical springs respectively. If the helical spring is subtracted from the cylindrical Primitive, the resulting solid is shown in Fig.9-9-(c). In these examples, the spring is generated by using a helix and the base circle of the helix as the Sweep and Spine Axes respectively. The special Primitive for the involute helical (or spur) gear is shown in Fig.9-10. In this example, the 2-D involute curve (i.e., the tooth profile curve) is defined as the Sweep Pattern, a straight line perpendicular to the Sweep Pattern is used as the Sweep Axis, and then the helical gear is generated by rotating the Sweep Pattern along the Sweep Axis. It is obvious that the spur gear can be generated if the Sweep Pattern is not rotated during the sweep. The cycloid gear can also be generated if the 2-D cycloid curve is used as the Sweep Pattern.

9.6 Conclusion

In this chapter, first, the program structure of a prototype system is explained. The explanation is summarized as follows:

- The prototype system is developed for the special Primitive (i.e., the swept volume), and incorporated into the CSG-based CAD/CAM system TIPS-1.


```

*P CR16, CR16 (-60,250,-35,220,-120,90;
CON1/P,CONTOUR,90/1, (); ← The CONTOUR statement
*E
*S CR16,CR16;
1 ← NSW
1 ← SWTP
4 ← NSW
SEG1,LINE,N,(200,0,0,67.05,38.89,0);
SEG2,ARC,ZX,(67.05,38.89,0,0,0,60,0,25,-15);
SEG3,ARC,YZ,(0,25,-15,0,47.14,-81.41,-7.5,85.3,-30.5);
SEG4,LINE,N,(-7.5,85.3,-30.5,-20,160,40);
1 ← SNCC
1 ← SSWP
SCR1,CIRC,/I,15,(0,0); ← The start Sweep Pattern
1 ← ENCC
1 ← ESWP
ECR1,CIRC,/I,15,(0,0); ← The end Sweep Pattern
*E

```

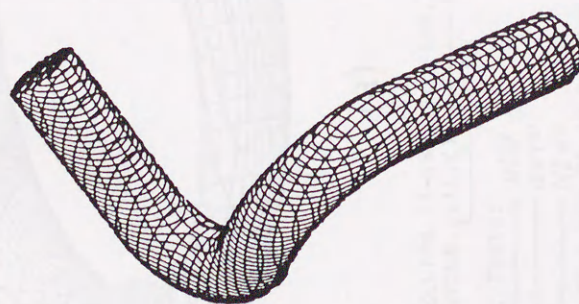
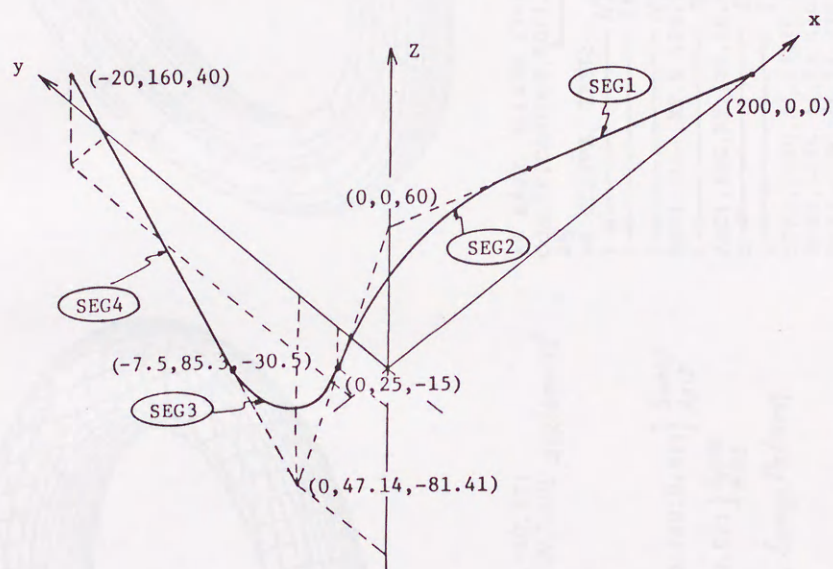


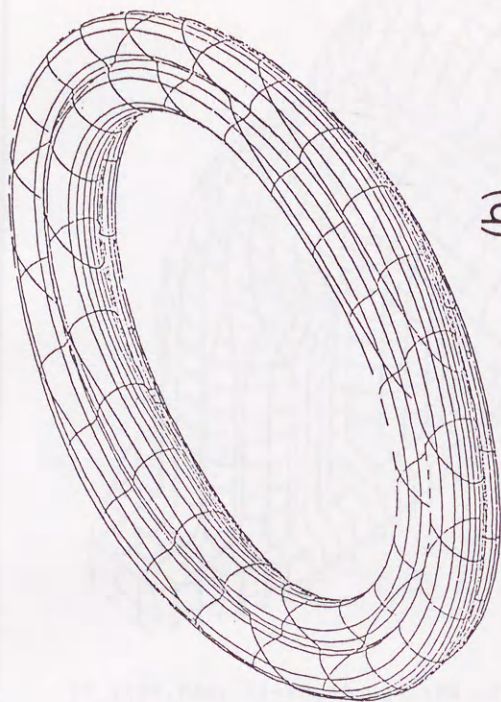
Fig.9-6 An example of the shape description statement of the curved segment, and the resulting swept volume

(a)

```

*P EX04, ELLIPS (,-150,150,-100,100,-30,30;
CON2/P, CONTOUR, 51/I, (,); ← NN The CONTOUR statement
*E
*S ELLIPS, TEST2;
1 → NSW
2 → SWTP
1 → NSWP
SEG1, ELLI, XY, 0, 130, 70, (0,0,0,130,0,0,0,0); } Sweep
1 → NSPN } Axis
ARC1, ARC, XY, 130, (0,0,0,130,0,0,130,0,0); } Spine
1 → SNCC } Axis
1 → SSWP
SCRI, CIRC, /I, 10, (0,0); } The start Sweep Pattern
1 → ENCC
1 → ESWP
ECRI, CIRC, /I, 20, (0,0); } The end Sweep Pattern
*E

```



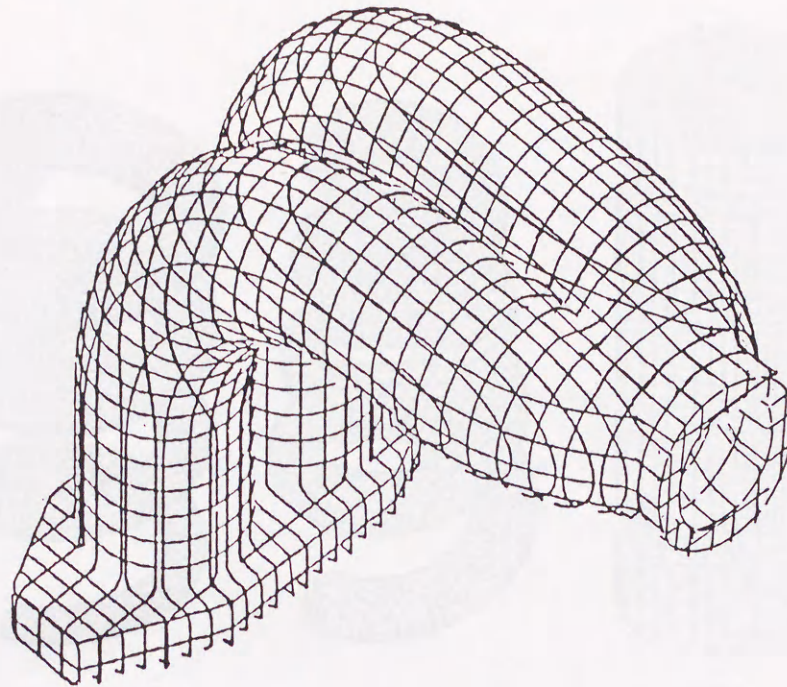
(b)

* P EX03, ELLIPS (-150,150,-100,100,-40,40;
CON1/P, CONTOUR, 50/1,()); ← NN The CONTOUR statement

** E ELLIPS, TEST;

1 → NSW
2 → SWTP
3 → NSWP
SEG1, ELLI, XY, 0, 130, 70, (0, 0, 0, 130, 0, 0, 0); Sweep Axis
1 → NSPN
ARC1, ARC, XY, 130, (0, 0, 0, 130, 0, 0, 130, 0, 0); Spine Axis
3 → SNCC
3 → SSWP
SCR1, CIRC, /1, 11, (0, 0);
SCR2, CIRC, /1, 11, (10, -17, 32); } The start Sweep Pattern
SCR3, CIRC, /1, 11, (10, 17, 32);
3 → ENCC
3 → ESWP
ECR1, CIRC, /1, 11, (0, 0);
ECR2, CIRC, /1, 11, (0, -17, 32); } The end Sweep Pattern
ECR3, CIRC, /1, 11, (0, 17, 32);
** E

Fig.9-7 Experimental results



```

*P TEST,EX01 (-50,50,-20,155,-20,170;
REC1/P,RECTANGLE,/1,0,46,46(-23,-23),,,,0,11;
CR10/P,CIRCLE,/1,23,(17.5,0),,,,0,11;
CR20/P,CIRCLE,/1,11,(44,0),,,,0,11;
LN10/P,LINE,/L,(30,19.5,,50,9.22)29,50.5,-5,,0,11;
LN20/P,LINE,/L,(30,-19.5,,50,-9.22)29,50.5,,5,0,11;
MR10/P,MIRROR/X,/1,CR10,CR20,LN10,LN20,(0),,,,0,11;
CON1/P,CONTOUR,50/1, ();
CON2/P,CONTOUR,60/1, ();
*E
*S
2 ←———— NSW
1 ←———— SWTP
5 ←———— NSWP
SG11,LINE,ZX,(-17.5,0,0,-17.5,0,60);
SG12,ARC,YZ,50,(-17.5,50,60,-17.5,50,110,-17.5,0,60);
SG13,LINE,XY,(-17.5,50,110,-17.5,81,110);
SG14,ARC,XY,55,(37,79.5,110,-9,120,110,-17.5,80,110);
SG15,LINE,XY,(0,120,110,0,170,110);
1 ←———— SNCC
1 ←———— SSWP
SC11,CIRC,/1,17.5,(0,0); ————— The start Sweep Pattern
1 ←———— ENCC
1 ←———— ESWP
EC11,CIRC,/1,17.5,(0,0); ————— The end Sweep Pattern
1 ←———— SWTP
5 ←———— NSWP
SG21,LINE,ZX,(17.5,0,0,17.5,0,45);
SG22,ARC,YZ,65,(17.5,65,45,17.5,65,110,17.5,0,45);
SG23,LINE,XY,(17.5,65,110,17.5,81,110);
SG24,ARC,XY,55,(-37,79.5,110,17.5,80,110,9,120,110);
SG25,LINE,XY,(0,120,110,0,170,110);
1 ←———— SNCC
1 ←———— SSWP
SC21,CIRC,/1,17.5,(0,0); ————— The start Sweep Pattern
1 ←———— ENCC
1 ←———— ESWP
EC21,CIRC,/1,17.5,(0,0); ————— The end Sweep Pattern
*E

```

Fig.9-8 An example of the exhaust manifold

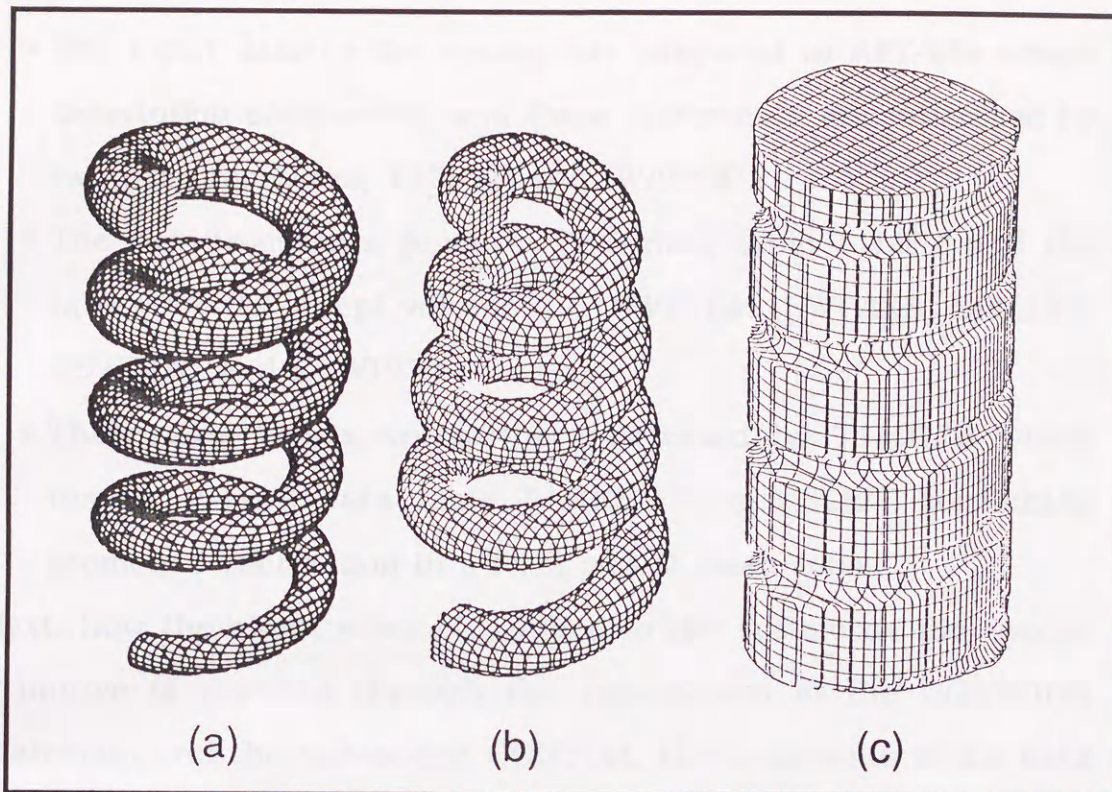


Fig.9-9 Examples of the swept volume for the spring

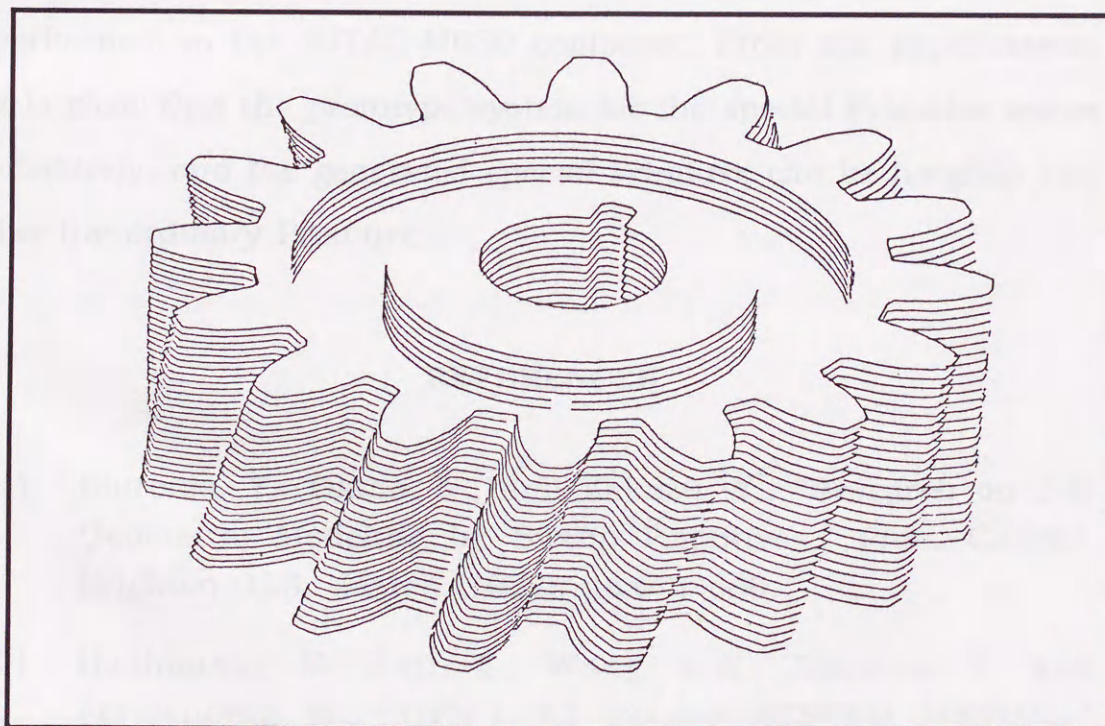


Fig.9-10 An example of the involute helical gear

- The input data to the system are prepared in APT-like shape description statements, and these statements are processed by two pre-processors, T1PRPR and SWPRPR.
- The two processors generate CSG data table T1LIST and the tables for the swept volume (i.e, SWPVLM, SWPAXS, SPNAXS, SSWPTN, and ESWPTN).
- These data tables are further processed by T1MHPR which results in two data table T1MESH and M3LIST containing geometric information in a form of 3-D mesh cells.

Next, how the swept volume is added to the system as the special Primitive is clarified through the explanation of the CONTOUR statement and the subroutine CONTOR. Then, contents of the data tables for the Sweep and Spine Axes, and for the Sweep Pattern are explained in detail. Finally, the prototype system has been performed on the HITAC-M680 computer. From the experiments, it is clear that the prototype system for the special Primitive works effectively, and the generated special Primitive can be handled just like the ordinary Primitive.

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10. CONCLUSION

This paper is concluded as follows:

- (1) A generalized sweeping method for the CSG modeling is proposed. In this method, sweep parameters (i.e., the Sweep Pattern, the Sweep and Spine Axes) should be given, and then a swept volume is generated by sweeping the Sweep Pattern (i.e., a 2-D closed curve) along the Sweep Axis (i.e. a curve in 3-D space). The Spine Axis provides the definition of the Sweep Pattern defining plane (i.e., the so-called XsYs-plane). Namely, normal plane of the Spine Axis is used as the XsYs-plane. In the conventional sweeping method, both the Sweep and Spine Axes are unified. In this method, however, these two are separated in order to extend the capability of geometric representation. The characteristic of the proposed sweeping method is that the solid modeling problem in 3-D space can be reduced to that on the 2-D plane.
- (2) The method for generation of the swept volume formed by the movement of the NC cutter is clarified. In this method, the proposed sweeping method is applied as follows: consider an imaginary plane cross-section in the middle of the tool path; if the NC cutter goes through the cross-section, it leaves a trace (i.e., a closed profile curve) on the cross-section; the profile curve is used as the enveloped pattern (i.e., the Sweep

Pattern); consequently the swept volume is generated by sweeping the enveloped pattern along tool path.

- (3) The method for generation of the fillet volume to round two intersecting quadrics is clarified. In this method, the concave, the convex, and the flat fillet patterns are prepared for the Sweep Pattern, the intersection line between two quadrics is used as the Sweep Axis, and a rule of defining the XsYs-plane is introduced instead of the Spine Axis.
- (4) The method for generation of the fillet volume to round corners such as edges and vertices is clarified. In order to round the vertex and the related three edges simultaneously, the method for smooth connection of fillet volumes is also clarified.
- (5) The method for generation of the fillet volume to join two disconnected solids is clarified. In this method, the start and end Sweep Patterns are given. The current Sweep pattern in the middle of the sweep is expressed by blending these two patterns. Thus the derivation process of both blending functions and implicit functions of the current Sweep Pattern is explained in detail.
- (6) Generating conditions of the boundary surface of the trajectory volume formed by the movement of a solid object are clarified, and surfaces and edges of the trajectory volume are classified with relation to the condition.

- (7) A prototype system for the swept volume has been developed, incorporated into the CSG-based CAD/CAM system TIPS-1, and performed on the HITAC M-680 computer at Hokkaido University. According to the experiments, it is clear that the proposed sweeping method has high capability to represent complicated solid shapes such as swept volumes formed by the movement of the NC cutter, fillet volumes to round two intersecting quadrics, and fillet volumes to join two disconnected solids. In addition, it is also clear that set operations between two swept volumes or between a swept volume and a Primitive can be executed just like between two Primitives.
- (8) It is clarified that not only the simple swept volumes (i.e., the extruded and the rotational solids) but also ordinary Primitives can be generated by applying the proposed sweeping method. Furthermore, the proposed sweeping method is applicable within any CSG-based solid modeling system. Therefore it is expected that the proposed sweeping method will contribute to enhance modeling capability of CSG-based solid modeling systems.

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