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A Study on State Estimation for Cyber Physical Systems

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A Study on State Estimation for Cyber Physical Systems*

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Abstract

This thesis discusses estimation theories for cyber physical systems. Some social concepts for the next generation based on IoT technology have been discussed. In a context of control theory, cyber physical systems are regard as a new system for the next generation. In this thesis, estimation of quadrotor UAV and distributed estimation over delayed sensor networks are studied as a main problem of cyber physical systems.

In Chapter 3, we propose a maximum-likelihood-estimation method for a quadrotor UAV given the existence of sensor delays. The state equation of the UAV is nonlinear, and thus, we propose an approximated method that consists of two steps. The first step estimates the past state based on the delayed output through an extended Kalman filter. The second step involves calculating an estimate of the present state by simulating the original system from the past to the present. It is proven that the proposed method provides an approximated maximum-likelihood-estimation. The effectiveness of the estimator is verified by performing experiments.

In Chapter 4, a distributed delay-compensated observer for wireless sensor network with delay is proposed. Each node of the sensor network aggregates data from the other nodes and sends the aggregated data to the neighbor nodes. In this communication, each node also compensates communication delays among the neighbor nodes. Therefore, all node can synchronize sensor measurements by using the scalable and local communication in real-time. All nodes estimate the state variables of a system simultaneously. The observer in each node is similar to the exiting delay-compensated observer with multi-sensor delays. Convergence rates of the proposed observer can be arbitrarily designed regardless of communication delays. The effectiveness of the proposed method is verified by numerical simulation.

In Chapter 5, we consider the design problem of an unknown-input observer for distributed network systems under existence of communication delays. In the proposed method, each node estimates all states and calculates inputs from own estimate. It is assumed that the controller of each node is given by an observer-based controller. In the calculation of each node, input values of the other nodes cannot be utilized. Therefore, each node calculates alternative inputs instead of the unknown input of the other nodes. The alternative inputs are generated by own estimate based on the feedback controller of the other nodes given by the assumption. Each node utilizes these values in the calculation of the estimation and the delay compensation instead of the unknown inputs. The stability of the estimation error of the proposed observer is proven by

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a Lyapunov-Krasovskii functional. The stability condition is given by a linear matrix inequality (LMI). Finally, a result of a numerical simulation is shown to verify the effectiveness of the proposed method.

Keywords: Cyber Physical Systems, Quadrotor UAV, Distributed Estimation, Delay Compensation, Unknown Input Observer

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Chapter 1. Introduction

1.1 New Social Concept for Next Generation

Society 5.0 is an IoT based social concept for a next generation which has been proposed in some countries including Japan [1, 2, 3]. Paper [4] indicates that similar concepts are proposed in the other countries, for example, Industrial Internet of Things in U.S.A [5, 6] or Industry 4.0 in Germany [7, 8, 9, 10, 11]. Internet of Things (IoT) is a concept which means that any things, e.g. machines, sensors, actuators and storage, are connected by the internet. IoT based Integration of many research fields, for example measurement, communication and computer technology, create the additional value which causes a paradigm shift. After this paradigm shift, optimal and sustainable societies can be realized in next generation.

The IoT based concept like Society 5.0 expects innovations of automation which realize higher level of operational efficiency and productivity. Transportation systems in smart cities are introduced IoT and automatic driving technology to optimize fuels and amount of traffic [12, 13, 14, 15, 16, 17, 18]. Based on the measurements which are collected from each car, the IoT technology can realize relaxation of traffic congestion, optimization of charge planing for electric cars and improvement of efficiency of car sharing. In concepts of smart factories, classical factory are updated to flexible and intelligent systems by the IoT technology [19, 20, 21]. In the smart factories, a collaboration of the IT and computer technologies realizes the high-level self-optimization, for examples a self-control of production amounts and detection of faults or cyber attacks. A smart grid is a new concept of energy management systems for next generation whose purpose is a stable and sustainable energy supply [22, 23, 24, 25, 26]. In the smart grid, smart meters are utilized for the demand response which controls a power consumption of customers and promotes the introduction of the natural energies. In smart agriculture, many sensors observe growing situation of agricultural corp and managements of temperature, fertilizer and water are automated. Since the automation is introduced in the agriculture, improvement of labor shortage and optimization of the amount of water and fertilizer are realized.

For the innovations of the IoT based automation, new systems control theory are required. Paper [27] indicates importance of theory for efficient and reliable integration of each systems in a case of vehicle systems for next generation, and recites frameworks which is required in new system theories as follows:

- The frameworks which optimize, design and analyze abstracted compartments of various subsystems, for example network system, computer systems ad physical systems,
- The frameworks which analyze and understand the interaction among subsystems,

- The frameworks which ensure safety, stability, and performance of the systems.

These framework are also required for the other IoT based concept, for example smart grids and smart factories. In addition, the author considers that the frameworks for automated decision making based on the measurement of sensor are also required in the IoT based concept for the next generation. Therefore, this thesis reports theoretical works about the control of cyber physical systems.

1.2 Contribution of This Thesis

Cyber physical systems are constructed by real spaces, i.e. physical system, and virtual spaces, i.e. cyber systems. The automation based on IoT technologies can be considered as automatic controls of the cyber physical systems. Therefore, many researches study the analysis, optimize and control of the cyber physical systems. In particular, we consider studies about a quadrotor UAV and distributed estimation over sensor networks as a cyber physical systems in this thesis.

The studies of the cyber physical systems include controls and analysis of multi-agent systems because the agents communicate each others over wireless networks to execute tasks. For example, formation controls [28, 29, 30, 31], consensus controls [32, 33, 34], coverage control[35, 36, 37, 38, 39] and dynamical monitoring problems [40, 41, 42, 43] are included in the control problem of the multi-agent systems. We can utilize quadrotor UAVs as a flexible and versatile agents because they can move and hover in 3D region unlike two wheel vehicle robots or fixed-wing aircrafts[44, 45, 46, 47]. This thesis indicates that indoor measurements of the 3D position of the quadrotor UAV has some technical issues. Therefore, one result of this thesis include the state estimation problem for the indoor quadrotor UAVs. Contributions of this result are:

- to propose a maximum-likelihood-estimation (MLE) for nonlinear systems with a sensor delay,
- to show an effectiveness of the proposed method from experimental results by using A.R Drone 2.0 and Microsoft Kinect.

Wireless sensor networks (WSN), which are systems where sensors are connected by a wireless network, are considered as the observation systems for the cyber physical systems. Since distirbuted controler design are fundamental problems for the control of cyber physical systems, distributed estimation problems over sensor network are also fundamental ones. In the distributed estimation problems over sensor network, we need to satisfy both specification of network and control systems. In particular, the network systems require a reduction of a communication amount, and the control systems need a real-time communication of measurements over sensor networks. Therefore, the other result of this thesis include a data-aggregation based observer for delayed sensor network. Contributions of this result are:

- to introduce a system representation of the data aggregation for the state estimations,

- to propose a sacalable data-aggregation communication with the delay compensation,
- to propose distributed observer over sensor networks for both known and unknown input cases.

1.3 Outline

This thesis is organized as follows. Following this chapter, Chapter 2 introduce the mathematical preliminaries, including notation and the fundamental result of previous works.

Chapter 3 shows the result of a MLE method for quadrotor UAV with delayed measurements. It is proven that the proposed method can provide MLE from the delayed measurements in the sense of linear approximation. Experimental results shows that the proposed method are effective against the quadrotor UAVs.

Chapter 4 discusses distributed estimation problems over delayed sensor networks. For the reduction of the communication amount, a data-aggregation is utilize in a proposed method. In addition, by using model based delay compensation, a design of distributed estimation problems become finite pole assignments.

In Chapter 5, the result of Chapter 4 are extended in a case of unknown input observer problems. Based on Lyapunov-Krasovskii functional analysis, the observer design problems becomes numerical calculation of a linear matrix inequality, which can be solved as a semidefinite programming.

Chapter 6 gives the conclusion of this thesis.

Chapter 2. Notation

The following notation is used in this thesis.

$\mathbb{R}^n$	n dimensional Euclidean space for some integer n
$\mathbb{R}^{n\times m}$	$n \times m$ dimensional matrices
$\mathbb{S}^n_{++}$	a set of $n \times n$ dimensional positive definite matrices
0_n	n dimensional zero vector
$0_{n\times m}$ (or simple 0)	$n \times m$ zero matrix
A^T	the transpose of matrix A
$\emptyset$	empty set
$\mathrm{He}(\cdot)$	the function which means $\mathrm{He}(X) := X + X^T$
$\mathrm{diag}(A_i)_{i\in S}$	the matrix arranged A_i diagonally with the index set S
$\mathrm{row}(A_i)_{i\in S}$	the matrix arranged A_i horizontally with the index set S
$\mathrm{col}(A_i)_{i\in S}$	a matrix arranged A_i vertically with the index set S

For example, if $S = \{1, 2, 3\}$, then $\mathrm{diag}(A_i)_{i\in S}$, $\mathrm{row}(A_i)_{i\in S}$ and $\mathrm{col}(A_i)_{i\in S}$ are

$$\begin{aligned}\mathrm{diag}(A_i)_{i\in S} &= \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix},\\ \mathrm{row}(A_i)_{i\in S} &= \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix},\\ \mathrm{col}(A_i)_{i\in S} &= \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}.\end{aligned}$$

The following notation is also defined.

- For the square matrix X, let $X < (>) 0$ express that X is negative (positive) definite.

Chapter 3. Delay-Compensated Maximum-Likelihood-Estimation

3.1 Introduction

Recently owing to the presence of cheap sensor devices in the market, several studies have reported on the applications of low-cost sensors. In this chapter, we propose a new application of low-cost sensors in unmanned aerial vehicle (UAV) systems flying in an indoor environment. Expensive motion-tracking sensors are often used with respect to position controls of UAVs in an indoor environment while UAVs in outdoor space use a global positioning system (GPS). The results of the present study demonstrate that low cost sensors provide a sufficient performance as opposed to expensive sensors. However, there are a few problems with respect to the accuracy and speed of low-cost sensors. For example, Microsoft Kinect is a low-cost depth image sensor that involves large noises and time delays. It is necessary to investigate a sensor-delay compensation method for real time controls of UAVs[48, 49, 50, 51]. Additionally, the low image resolution of Kinect reduces the accuracy of measured values. The sensor noise on the 3D position caused by the image discretization depends on the distance between the measured object and the sensor. The observer gain should be adjusted based on the distance.

Observer designs for UAV systems were proposed by several studies. Difficulties exist with respect to observer design and include nonlinearity of UAV dynamics and the existence of aerodynamics and parametric uncertainty of the model. Benallegue [52] proposed a high order sliding mode (HOSM) observer with an exact linearization controller based on Levant's exact differentiator [53]. The HOSM observer estimates the state of the system under the existence of external disturbances, although it severely suffers from the effect of sensor noises. Therefore, the HOSM observer is not suitable for low-cost sensors. A few estimation methods reduce the effect of observation noises. An example of this is the maximum-likelihood-estimation. Specifically, the estimate of the extended Kalman filter constitutes an approximate maximum likelihood estimation for a nonlinear dynamical system.

With respect to the sensor time-lag problem, extant studies of delay compensated methods proposed delay-compensated observers [54, 55]. Watanabe [54] relied on an appropriate coordinate transformation to design a delay compensated observer. The determination of the feedback gain of Watanabe's observer constitutes a finite pole assignment problem. Kristic [55] considered delay dynamics as a transport partial differential equation (PDE). He proposed a delay compensated observer for the ODE-PDE system. However, a delay-compensated observer that provides a maximum likelihood estimation does not exist.

In this study, we propose an extended-Kalman-filter-based delay-compensated observer for a quadrotor UAV. The structure of the proposed observer is the same as that of Watanabe's delay compensated observer, and an extended Kalman filter is a portion of the observer. The estimation value of the proposed observer corresponds to the maximum likelihood estimation in a linear approximation sense. The effectiveness of the proposed method is confirmed by performing experiments.

This chapter is organized as follows. The state equation and output equation of UAV system are defined in Section 3.2, and the delay-compensated maximum-likelihood-estimation observer is proposed in Section 3.4. The results of practical experiments using the proposed method are shown in Section 3.5. Finally, Section 3.6 discusses the conclusions of this chapter.

3.2 Modeling and Problem Formulation

3.2.1 Description of State Equation

The purpose of this study involves proposing a delay-compensated maximum-likelihood-estimation method for quadrotor UAVs. In this section, we describe the dynamics and an output equation of the proposed system. Parrot AR.Drone 2.0 is utilized as the quadrotor UAV in the experiment in Section 3.5, and thus, we consider the motion equation of this type of UAV.

Fig.3.1 illustrates the global coordinate of the UAV system. We define notations x_g, y_g, and z_g as the 3D Cartesian coordinates of the UAV position; and $\psi \in [-\pi, \pi)$, $\theta \in (-\pi/2, \pi/2)$, and $\phi \in (-\pi/2, \pi/2)$ as the yaw, pitch, and roll angles, respectively. For the purposes of notational simplicity, we use S_* and C_* as opposed to sin and cos functions, respectively. The horizontal dynamics for x_g and y_g of the quadrotor UAV are dominated by the following:

$$\begin{aligned} \ddot{x}_g &= \frac{F}{m}(C_\theta S_\phi C_\psi - S_\theta S_\psi) - k_x \dot{x}_g |\dot{x}_g| + A_x \\ \ddot{y}_g &= \frac{F}{m}(C_\theta S_\phi S_\psi + S_\theta C_\psi) - k_y \dot{y}_g |\dot{y}_g| + A_y, \end{aligned} \tag{3.1}$$

where m denotes the mass of the UAV, F is the total thrust force, k_x and k_y denote air friction coefficients, and A_x and A_y denote disturbance terms caused by the quasi-stationary flow of air. It is assumed that A_x and A_y follow random walk models as follows:

$$\dot{A}_x = \xi_x, \quad \dot{A}_y = \xi_y, \tag{3.2}$$

where ξ_x and ξ_y denote signals generated by stochastic processes. Generally, several external disturbances are caused by environmental air flow and ground effects in a UAV system, and these disturbances affect the whole UAV dynamics. However, AR.Drone 2.0 controls lifting velocity by employing a minor control loop based on information acquired by an altitude sensor. Thus, we assume that the disturbances of the environmental air flow exist in the dynamics of only x_g and y_g while vertical aerodynamics disturbances are ignored. It should be noted that the disturbances A_x and A_y are estimated online by the proposed extended Kalman filter. In accordance with the control command of AR.Drone 2.0, we choose the control inputs as the desired values of the roll angle ϕ, pitch angle θ, lifting speed $\dot{z}_g$, and yaw rate $\dot{\psi}$. It is assumed

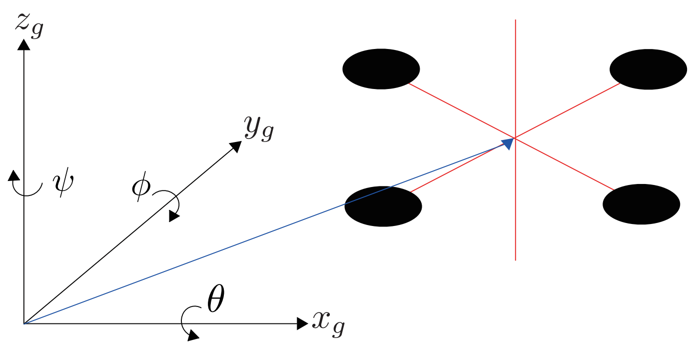

Figure 3.1: Global Coordinate

that u_1, u_2, u_3, and u_4 denote the input signals for the roll angle, pitch angle, rise velocity, and yaw rate, respectively. The inputs only correspond to desired values, and the actual values of ϕ, θ, $\dot{z}_g$, and $\dot{\psi}$ are regulated to the input values by built-in inner control loops in the UAV. The actual values include system errors as follows:

$$\begin{aligned} \phi &= u_1 + \xi_1 \\ \theta &= u_2 + \xi_2 \\ \dot{z}_g &= u_3 + \xi_3 \\ \dot{\psi}_g &= u_4 + \xi_4 \end{aligned} \tag{3.3}$$

where ξ_1, ξ_2, ξ_3, and ξ_4 denote system noises added in u_1, u_2, u_3, and u_4, respectively. We define the input vector u as follows:

$$u(t) = (u_1, u_2, u_3, u_4)^T. \tag{3.4}$$

We also define the system noise vector ξ and the state vector x as follows:

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_x, \xi_y)^T, \tag{3.5}$$

$$x = (x_g, y_g, v_g, w_g, z_g, \psi, A_x, A_y)^T. \tag{3.6}$$

From (5.1) and (3.2), we obtain the state equation as follows:

$$\begin{aligned} \dot{x} &= f(x, u, \xi) \\ &= \begin{pmatrix} v_g \\ w_g \\ \frac{F}{m}(C_{u_2+\xi_2} S_{u_1+\xi_2} C_\psi - S_{u_2+\xi_2} S_\psi) \\ \qquad - k_x v_g |v_g| + A_x \\ \frac{F}{m}(C_{u_2+\xi_2} S_{u_1+\xi_1} S_\psi + S_{u_2+\xi_2} C_\psi) \\ \qquad - k_y w_g |w_g| + A_y \\ u_3 + \xi_3 \\ u_4 + \xi_4 \\ \xi_x \\ \xi_y \end{pmatrix}. \end{aligned} \tag{3.7}$$

It is assumed that each component of the noise vector ξ follows a white Gaussian process with known covariances and are not correlated with each other. Strictly speaking, (3.7) should be represented as a stochastic differential equation. However, we use the ordinal differential equation form for purposes of simplicity.

Remark 3.1 (Dynamics of AR.Drone 2.0)
As other works (e.g.[52]) point out, the dynamics of quadrotor UAVs are more complex than (3.7). However, AR.Drone 2.0 has built-in inner control loops which regulate the roll angle ϕ, pitch angle θ, lifting speed $\dot{z}_g$, and yaw rate $\dot{\psi}$ from measurements of equipped sensors. Users can utilize these values as the input commands. From these reason, this paper ignores the inner-loop dynamics regulated by AR.Drone 2.0, and adopts the state equation (3.7).

3.2.2 Description of Output Equation

In this study, Microsoft Kinect [56], [57] is utilized as a position sensor, and we assume that the yaw angle is measured by a magnetic field sensor quipped on the UAV. We define the output equation based on these sensors. Microsoft Kinect measures the 3D-position of a UAV (x_g, y_g, z_g) with a delay due to transmission delay and calculation time. In contrast, the magnetic field sensor on the UAV provides the yaw angle ψ in almost real-time. In order to simplify the problem, we insert an additional delay with respect to the measurement of yaw angle and make all sensor delays uniform. We define a vector $y_{\text{true}}(t) = (x_g(t-D), y_g(t-D), z_g(t-D), \psi(t-D))^T$.

We define (u_p, v_p) as the pixel coordinate value of the UAV and d as its corresponding depth data as measured by a Kinect sensor. The result of [56] indicates that the transformation from the 3-D position to the depth coordinate as shown in Fig. 3.2 is expressed as follows:

$$\begin{aligned} u_p &= \frac{x_g(t-D)}{y_g(t-D)} f_u + u_0 \\ v_p &= -\frac{z_g(t-D)}{y_g(t-D)} f_v + v_o \\ d &= \frac{b}{y_g(t-D)} - a \end{aligned} \tag{3.8}$$

where (u_0, v_0) denotes the pixel coordinate value of the image center; and a, b, f_v, and f_u denote conversion parameters. It is assumed that a map $\Lambda(y_{\text{true}})$ denotes $y_{\text{true}} \mapsto (u_p, v_p, d, \psi(t-D))^T$. The observation noises affect the measurements in the actual observation of Microsoft Kinect and the magnetic field sensor. $y_{\text{raw}} = (\bar{u}_p, \bar{v}_p, \bar{d}, \bar{\psi})^T$ as

$$\begin{aligned} \bar{u}_p &= u_p + \sigma_u \delta_u \\ \bar{v}_p &= v_p + \sigma_v \delta_v \\ \bar{d} &= d + \sigma_d \delta_d \\ \bar{\psi}(t-D) &= \psi(t-D) + \sigma_\psi \delta_\psi, \end{aligned} \tag{3.9}$$

where δ_u, δ_v, δ_d, and δ_ψ denote Gaussian white noises with zero mean value and variances 1; and σ_u^2, σ_v^2, σ_d^2, and σ_ψ^2 denote the variances of observation values. The observed values are

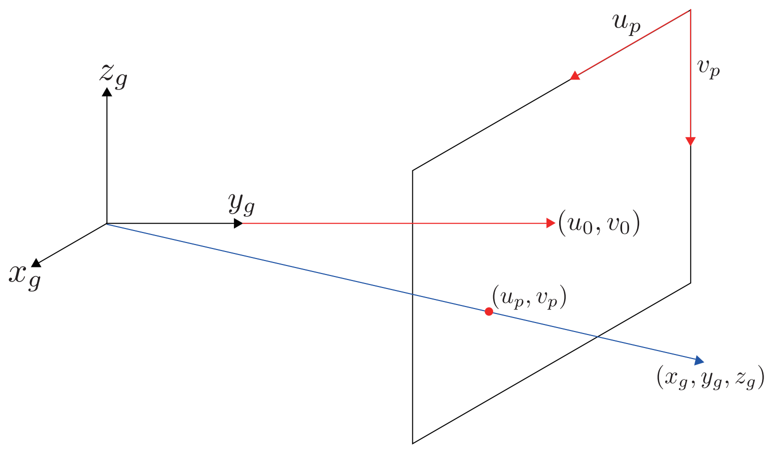

Figure 3.2: Relationship Between Pixel Coordinate and Global Coordinate

calculated from the raw data from sensors by using the inverse map of $\Lambda(\cdot)$ as follows:

$$y(t) = \begin{pmatrix} \bar{x}_g(t-D) \\ \bar{y}_g(t-D) \\ \bar{z}_g(t-D) \\ \bar{\psi}_g(t-D) \end{pmatrix} = \Lambda^{-1}(y_{\mathrm{raw}}) = \begin{pmatrix} (\bar{u}_p - u_0)\frac{b}{f_u(a+\bar{d})} \\ \frac{b}{a+\bar{d}} \\ -(\bar{v}_p - v_0)\frac{b}{f_v(a+\bar{d})} \\ \bar{\psi}(t-D) \end{pmatrix}. \tag{3.10}$$

Finally, the output equation of the system is expressed as follows:

$$\begin{aligned} y(t) &= h(x(t-D), \Delta) = \Lambda^{-1}(\Lambda(y_{\mathrm{true}}) + \mathrm{diag}(\sigma)\Delta) \\ &= \begin{pmatrix} \frac{b(f_u x_g(t-D) + \sigma_u y_g(t-D)\delta_u)}{f_u(b+\sigma_d y_g(t-D)\delta_d)} \\ \frac{b y_g(t-D)}{b + y_g(t-D)\sigma_d \delta_d} \\ \frac{b(f_v z_g(t-D) - \sigma_v y_g(t-D)\delta_v}{f_v(b+\sigma_d y_g(t-D)\delta_d)} \\ \psi(t-D) + \sigma_\psi \delta_\psi \end{pmatrix}, \end{aligned} \tag{3.11}$$

where $\sigma = (\sigma_u, \sigma_v, \sigma_d, \sigma_\psi)^T$ and $\Delta = (\delta_u, \delta_v, \delta_d, \delta_\psi)^T$.

3.2.3 Problem Formulation

In this study, we consider a maximum-likelihood-estimation problem for the quadrotor UAV system (3.7) and (3.11). The definition of the maximum likelihood estimation is as follows:

Definition 3.1 (Maximum Likelihood Estimation)
Given the output $y(t')$ and input $u(t')$ $(t_0 \leq t' \leq t)$ with an estimation of the initial state $x(t_0)$, the maximizer $x(t) = \hat{x}(t)$ of the conditional probability density function $p(x(t) \mid y(\cdot), u(\cdot), \mathrm{E}(x(t_0)))$ under (3.7) and (3.11) is termed as a maximum likelihood estimation of the current state.

However, it is difficult to solve the maximum-likelihood-estimation problem for a nonlinear system such as the quadrotor UAV. Therefore, we make the following assumption:

Assumption 3.1
It is assumed that the noises ξ and Δ have a normality and are sufficiently small. Furthermore, it is assumed that the initial estimation error $x(t_0) - \mathrm{E}(x(t_0))$ is also sufficiently small.

Given Assumption 3.1, the high-order terms $O(\xi^2)$ and $O(\Delta^2)$ are ignored, and we consider the problem of obtaining $\mathrm{E}(x(t) \mid y(\cdot), u(\cdot), \mathrm{E}(x(t_0)))$ as opposed to $\mathrm{argmin}_{x(t)}\, p(x(t) \mid y(\cdot), u(\cdot), \mathrm{E}(x(t_0)))$. The problem considered is summarized as follows:

Problem 3.1
With respect to the system (3.7) and (3.11), it is assumed that the covariance matrices of ξ and Δ are given. Then, design the delay-compensated observer of the system (3.7) and (3.11) which approximately provides the maximum likelihood estimation under Assumption 3.1.

3.3 Linear Cases

With respect to linear dynamical systems, the Kalman filter [58] provides the maximum likelihood estimation. Additionally, the concept of a prediction method for linear systems is also proposed in [59]. However, an actual delay-compensated method is not shown in [58, 59]. Conversely, a few types of delay-compensated observers were proposed in [54, 55]. A previous study [54] considers a linear system as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t-D) \end{aligned} \tag{3.12}$$

with a time delay D. An appropriate coordinate transformation for (3.12) is used, and the feedback-gain design of an observer which is proposed in [54] corresponds to a finite pole assignment problem.

Lemma 3.1 (Watanabe's Observer [54])
We consider the following observer:

$$\begin{aligned} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + e^{AD}L\bigg(&Ce^{-AD}\hat{x}(t) \\ -y(t) - C\int_{t-D}^{t} &e^{A(t-D-\theta)}Bu(\theta)d\theta\bigg), \end{aligned} \tag{3.13}$$

where (A, C) is observable. The estimation errors between (3.12) and (3.13) converge to 0 if and only if $A - LC$ is stable.

An extant study [55] reveals that the observer (3.13) is decomposed into two equations as follows:

Lemma 3.2
The coordinate transformation is as follows:

$$\Xi(t) = e^{-AD}\hat{x}(t) - \int_{t-D}^{t} e^{A(t-D-\theta)}Bu(\theta)d\theta, \tag{3.14}$$

Thus, observer (3.13) is expressed as follows:

$$\dot{\Xi}(t) = A\Xi(t) + Bu(t-D) + L(y(t) - C\Xi(t)) \tag{3.15}$$

$$\hat{x}(t) = e^{AD}\Xi(t) + \int_{t-D}^{t} e^{A(t-\theta)}Bu(\theta)d\theta. \tag{3.16}$$

This lemma is confirmed by a direct calculation. Lemma 3.2 indicates that the delay-compensated observer (3.13) includes the estimation (3.15) and the prediction (3.16). The structure of equation (3.15) is the same structure as that of typical full state observers, and the state $\Xi(t)$ converges to $x(t-D)$. The equation (3.16) is the solution of the system (3.12) which has the initial value $\Xi(t)$. We utilize memories of the past inputs $u(\theta)$ ($\theta \in [t-D, t]$), which are stored in the controller module including the observer to calculate $\hat{x}(t)$ in (3.16).

In this study, we consider the case where the output feedback gain L in (3.15) is selected as the Kalman gain. In this case, $\Xi(t)$ provides the maximum-likelihood-estimation value of the state at time $t-D$. The estimate $\hat{x}(t)$ potentially corresponds to the delay-compensated maximum likelihood estimation although previous studies do not offer an exact proof for the same. Furthermore, the quadrotor UAV exhibits nonlinear dynamics, and the linear observer (3.13) cannot be directly applied to the fore-mentioned problem. Therefore, we propose a new estimation method for nonlinear systems with sensor delays.

3.4 Main Result

3.4.1 Proposed Observer

Our method is based on Lemma 3.2 and includes an extended Kalman filter. In keeping with Lemma 3.2, our proposed method is decomposed into a filtering step and a prediction step. The extended Kalman filter provides an approximated maximum-likelihood-estimation value at time $t-D$ in the filtering step. The prediction step provides a current estimation value as a solution of the original system. The next subsection discusses a proof to demonstrate that the estimated value of this method corresponds to a maximum-likelihood-estimation value in a linear approximation sense along an estimated trajectory.

We define the state of the filtering step as $\hat{z}(t)$. This state denotes an estimate of $x(t-D)$. We estimate $x(t-D)$ by the extended Kalman filter as follows:

$$\begin{aligned}
\dot{\hat{z}}(t) &= f(\hat{z}(t), u(t-D), 0) + K(t)(h(\hat{z}(t)) - y(t)) \\
K(t) &= -P(t)\bar{C}(t)R^{-1}(t) \\
\dot{P}(t) &= \bar{A}(t)P(t) + P(t)\bar{A}^T(t) + \bar{B}(t)Q(t-D)\bar{B}^T(t) \\
&\quad - P(t)\bar{C}^T(t)R^{-1}(t)\bar{C}(t)P(t),
\end{aligned} \tag{3.17}$$

where $\bar{A}(t)$, $\bar{B}(t)$, and $\bar{C}(t)$ denote the linear approximation coefficients of the equation (3.7) along $\hat{z}(t)$ and are given as follows:

$$\begin{aligned}\bar{A}(t) &= \frac{\partial f}{\partial x}(\hat{z}(t), u(t-D), 0)\\ \bar{B}(t) &= \frac{\partial f}{\partial \xi}(\hat{z}(t), u(t-D), 0)\\ \bar{C}(t) &= \frac{\partial h}{\partial x}(\hat{z}(t), 0).\end{aligned} \tag{3.18}$$

In equation (3.17), $P(t)$, $Q(t)$, and $R(t)$ denote covariance matrices of $\hat{x}(t)$, $\xi(t)$, and $y(t) - y_{\mathrm{true}}(t)$, respectively, near the origin. The covariance matrices for the system noises are calculated as $Q(t) = \mathrm{E}(\xi(t)\xi(t)^T)$, and this corresponds to a positive-definite diagonal matrix. In the calculation of $R(t)$, we utilize the power expansion of (3.10) with respect to Δ as follows:

$$\begin{aligned}y(t) &= h(x(t-D), 0) + J(t)\Delta(t) + O(\Delta^2)\\ J(t) &= \frac{\partial \Lambda^{-1}}{\partial \Delta}(y_{\mathrm{raw}}(t))\\ &= \begin{bmatrix} \sigma_u \frac{\bar{y}_g}{f_u} & (\bar{u}_p(t) - u_0)\sigma_y \frac{\bar{y}_g^2}{f_u} & 0 & 0\\ 0 & \sigma_y \bar{y}_g^2 & 0 & 0\\ 0 & -(\bar{v}_p(t) - v_0)\sigma_y \frac{\bar{y}_g^2}{f_v} & \sigma_v \frac{\bar{y}_g}{f_v} & 0\\ 0 & 0 & 0 & \sigma_\psi \end{bmatrix},\end{aligned} \tag{3.19}$$

where $\sigma_y = -\sigma_d / b$. It should be noted that $J(t)$ is not equivalent to $(\partial h/\partial \Delta)(y_{\mathrm{true}}, 0)$ although the difference is absorbed by the high-order term $O(\Delta^2)$. From the equation (3.19), we can calculate $R(t)$ as follows:

$$\begin{aligned}R(t) &= \mathrm{E}(J\Delta(t)(J\Delta(t))^T)\\ &= J(t)\mathrm{E}(\Delta(t)\Delta(t)^T)J(t)^T = J(t)J(t)^T.\end{aligned} \tag{3.20}$$

The covariance matrix $R(t)$ depends on $\bar{y}_g$. The value of $\bar{y}_g$ increases, and this increases the eigenvalues of $R(t)$ that correspond to the Kinect's output.

The prediction step provides the current estimate value $\hat{x}(t)$. We consider virtual time $\tau \in [0, D]$ and a value $\hat{\mu}(\tau, t)$. The fore-mentioned values are used to define the following nonlinear system:

$$\begin{aligned}\frac{\partial \hat{\mu}(\tau, t)}{\partial \tau} &= f(\hat{\mu}(\tau, t), u(t - \tau + D), 0) \ (0 \le \tau \le D)\\ \hat{\mu}(0, t) &= \hat{z}(t),\end{aligned} \tag{3.21}$$

In the equation (3.21), the initial value of $\hat{\mu}$ with respect to τ corresponds to the past estimate $\hat{z}(t)$, which is the estimated value of the filtering step. The prediction step advances its state $\hat{\mu}(\tau, t)$ along the original system based on the memory of the input. The estimate $\hat{\mu}(\tau, t)$ denotes an estimate value of $x(t - D + \tau)$, and thus the current estimate $\hat{x}(t)$ is given by $\hat{\mu}(D, t)$. Consequently, the approximated delay-compensated maximum-likelihood-estimation observer is obtained from equation (3.17) and (3.21).

3.4.2 Proof of Approximate Maximum-Likelihood-Estimation

The observers (3.17) and (3.21) are proposed in the previous subsection. This subsection demonstrates that the proposed observer corresponds to the approximated delay-compensated maximum-likelihood-estimation method. We consider a stochastic differential equation as follows:

$$\begin{aligned} d\mu(\tau,t) &= A(\tau,t)\mu(\tau,t)d\tau + H(\tau,t)u(\tau-D+t)d\tau \\ &\quad + B(\tau,t)d\xi(\tau-D+t) + F(\tau,t)d\tau \\ E(\mu(0,t)) &= \hat{z}(t), \quad \mathrm{cov}(\mu(0,t),\mu(0,t)) = P(t), \end{aligned} \tag{3.22}$$

where $A(\tau,t)$, $B(\tau,t)$, $H(\tau,t)$, and $F(\tau,t)$ denote the linear approximation coefficients of the equation (3.7) along $\hat{\mu}(\tau,t)$, which are given by the following expression:

$$\begin{aligned} A(\tau,t) &= \frac{\partial f}{\partial x}(\hat{\mu}(\tau,t),u(t-D+\tau),0) \\ B(\tau,t) &= \frac{\partial f}{\partial \xi}(\hat{\mu}(\tau,t),u(t-D+\tau),0) \\ H(\tau,t) &= \frac{\partial f}{\partial u}(\hat{\mu}(\tau,t),u(t-D+\tau),0) \\ F(\tau,t) &= f(\hat{\mu}(\tau,t),u(t-D+\tau),0). \end{aligned} \tag{3.23}$$

The equation (3.22) is a linear approximation stochastic system of the equation (3.21). For the system (3.22), the following lemma holds:

Lemma 3.3
With respect to the system (3.22), assumption 3.1 holds and let us define $e_\mu(\tau,t)$ as $e_\mu(\tau,t) = \mu(\tau,t) - \hat{\mu}(\tau,t)$. Subsequently, $\mathrm{E}(e_\mu(\tau,t)) = 0$ is satisfied.

Proof
The proof is demonstrated by considering a time evolution of a probability density $e_\mu(\tau,t)$. We consider a stochastic differential equation as follows:

$$\begin{aligned} de_\mu(\tau,t) &= A(\tau,t)e_\mu(\tau,t)dt + B(\tau,t)d\xi(\tau-D+t) \\ E(e_\mu(0,t)) &= 0, \quad \mathrm{cov}(e_\mu(0,t),e_\mu(0,t)) = 0 \\ \mathrm{cov}(\xi(t-D+\tau),\xi(t-D+\tau)) &= Q(t-D+\tau). \end{aligned} \tag{3.24}$$

Equation (3.24) suggests that a time evolution of the probability density $p(e_\mu,\tau)$ follows Kolmogorov's forward equation as follows:

$$\begin{aligned} \frac{\partial p(e_\mu,\tau)}{\partial \tau} &= -\sum_{i=1}^{n}\sum_{j=1}^{n}[A(\tau,t)]_{ij}\frac{\partial}{\partial e_{\mu i}}(e_{\mu j}p(e_\mu,\tau)) \\ &+ \frac{1}{2}\sum_{i,j=1}^{n}[B(\tau,t)Q(t-D+\tau)B(\tau,t)^T]_{ij}\frac{\partial^2 p(e_\mu,\tau)}{\partial e_{\mu i}\partial e_{\mu j}}. \end{aligned} \tag{3.25}$$

A solution of the diffusion equation (3.25) is given as follows:

$$p(e_\mu,\tau) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Gamma(\tau)|^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}e_\mu^T\Gamma^{-1}(\tau)e_\mu\right\}, \tag{3.26}$$

Table 3.1: Parameter of Microsoft Kinect

a	b	u_0	v_0	f_u	f_v
-1097	-365	320	240	599.56	599.01

Table 3.2: Parameters of Observation Noises

σ_u	σ_y	σ_v	σ_ψ
10.0	0.01	10.0	0.001

where

$$\begin{aligned}
&\Gamma(\tau) = \Phi(\tau, 0)\gamma(\tau)\Phi(\tau, 0)^T \\
&\gamma(\tau) = \\
&\quad \int_0^\tau \Phi(\theta, 0)^{-1} B(\tau, t) Q(t - D + \tau) B(\tau, t)^T (\Phi(\theta, 0)^{-1})^T d\theta \\
&\frac{d}{d\tau}\Phi(\tau, \tau') = A(\tau, t)\Phi(\tau, \tau') \\
&\Phi(\tau, \tau) = I.
\end{aligned}$$

From (3.26), $\mathrm{E}(e_\mu(\tau, t)) = 0$ is obtained.

From Lemma 3.3, the following theorem holds for the proposed observer.

Theorem 3.1
The estimate $\hat{x}$ generated by the observer (3.17) with (3.21) provides the maximum likelihood estimation of the state $x(t)$ of the system (3.7) approximately under Assumption 3.1.

Proof
The extended Kalman filter (3.17) guarantees that the estimated value of the filtering step $\hat{z}(t)$ corresponds to the approximated maximum likelihood estimation at time $t - D$. Thus, the proof demonstrates that $\hat{x}(t)$ of the prediction step constitutes a maximum likelihood estimation at time t in a linear approximation sense. The normality of the noise $\xi(t-D+\tau)$ in (3.22) guarantees that the maximum likelihood estimation of $\hat{\mu}(\tau, t)$ is $E(\mu(\tau, t))$ and that it is equivalent to $\mathrm{E}(e_\mu(\tau, t)) = 0$.

It should be noted that Assumption 3.1 is necessary for the validity of the linear approximation and the normality of $p(\cdots)$.

3.5 Experimental Result

In this section, the results of the experiments are discussed to demonstrate the validity of the proposed method. As previously mentioned, AR.Drone 2.0 and Microsoft Kinect are used in the

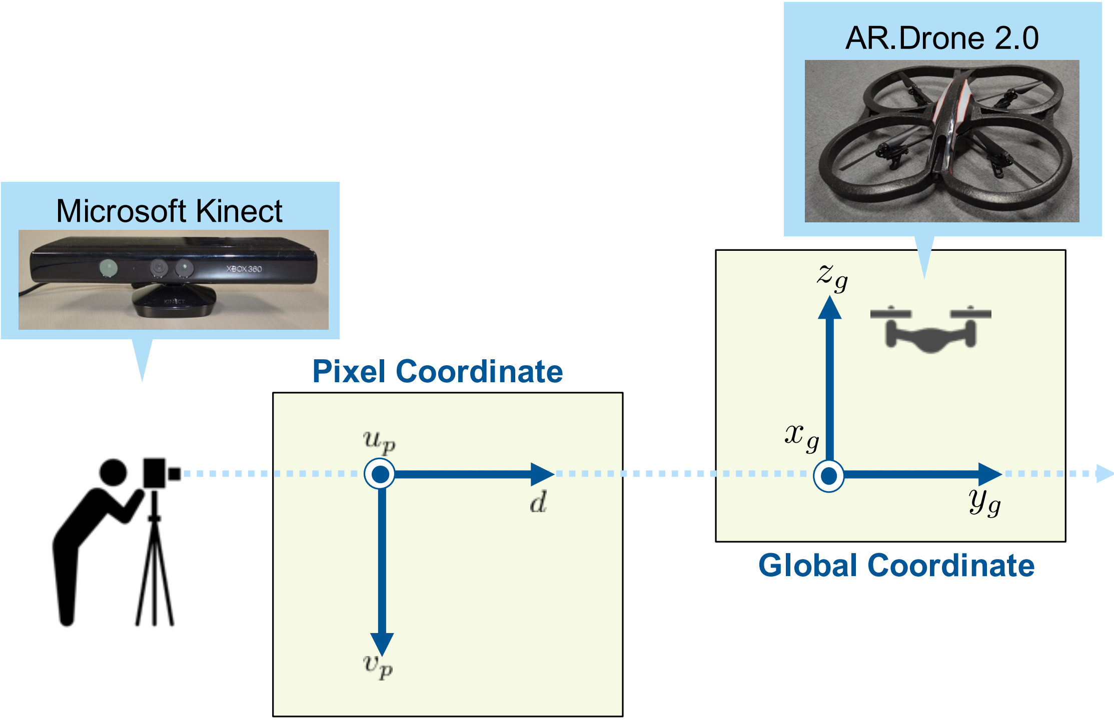

Figure 3.3: Setting of Experiment

experiment like Fig. 3.3. The parameters of Microsoft Kinect and variances of the observation noises are listed in Tables 3.1 and 3.2. As a result of a series of experiments, other parameters are chosen as follows: $Q(t) = \mathrm{diag}(3.0{\times}10^{-3}, 3.0{\times}10^{-3}, 5.0{\times}10^{-4}, 1.0{\times}10^{-4}, 1.0{\times}10^{-4}, 1.0{\times}10^{-4})$ and $D = 0.25$.

Two results of the experiments are discussed. In the first experiment, the UAV flies on a x_g–y_g plane by using u_1, u_2, and u_4. The result of this experiment is indicated in Figs. 3.4–3.8. In order to confirm the accuracy of the proposed method, we compare the results of the proposed method with future outputs $y(t + D)$ and their time-differentiated values $\dot{y}(t + D)$. Figures 3.4, 3.5, 3.8 and 3.9 show the estimations calculated by proposed method and the future output $y(t + D)$. In Figs. 3.6 and 3.7, the estimations of $\dot{x}_g$ and $\dot{y}_g$ are plotted, respectively, with the differential values of the future sensor measurement which are calculated by exact differentiators. The exact differentiator, which is proposed in [53], is based on sliding mode techniques and provides exact differential values, if the second order differential of the original signal is bounded. It should be noted that it is not possible to use future measurements in an actual real-time control system. It is confirmed that the accuracy of the proposed method is acceptable. Generally, with respect to the differentiation-based method, high-frequency components of the observation noises affect the accuracy of the estimate. Conversely, the proposed method reduces impulsive noise effects. Therefore, we confirm that the proposed estimation method satisfies the delay compensation and the reduction of noise effects.

In the first experiment, the UAV does not move in the z_g direction. In this second experiment,

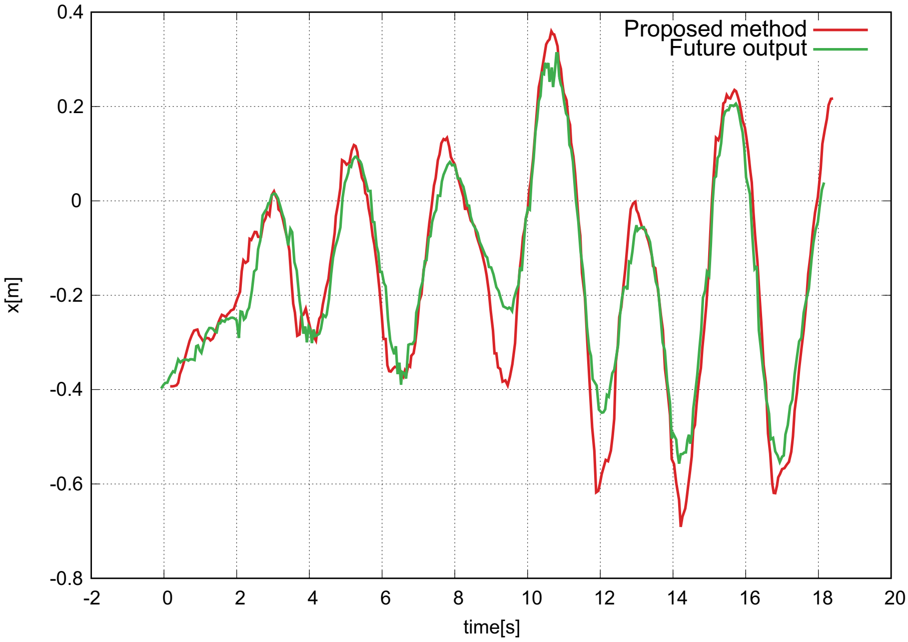

Figure 3.4: Estimation Result of $x_g(t)$

we use u_3 to move the UAV to the z_g direction. The result is shown in Fig. 3.9. This figure implies that a large error exists between the estimation value of the proposed method and feature measurement based value with respect to a decrease in the UAV. It is assumed that the reason for this phenomenon corresponds to a mismatch between the actual falling speed of AR.Drone 2.0 and its command input u_3. Thus, it is expected that a modification of the UAV model improves the accuracy of the z_g-axis.

3.6 Conclusion

In this chapter, we propose a delay-compensated maximum-likelihood-estimation observer for a quadrotor UAV to use low-cost sensors with time delay and observation noises. The proposed observer is designed by combining an extended Kalman filter and Watanabe's delay-compensated method. The observer can be decomposed into a filtering step and a prediction step. The filtering step is based on the extended Kalman filter. The prediction step provides a final estimate by employing a simulation along the original system. Both the fore-mentioned steps accomplish delay compensation and reduction in noise effects. Additionally, we show that the estimation value of the method constitutes a maximum-likelihood-estimation value in the sense of a linear approximation. The effectiveness of the proposed estimation method is verified by performing practical experiments. A future study will involve considering a particle filter based observer with delay compensation for cases with large-amplitude noises.

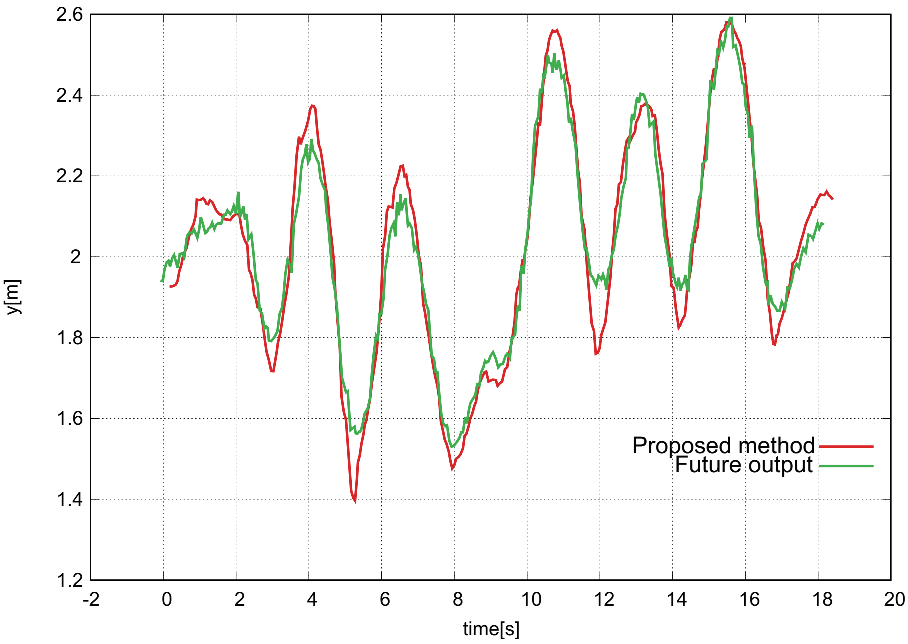

Figure 3.5: Estimation Result of $y_g(t)$

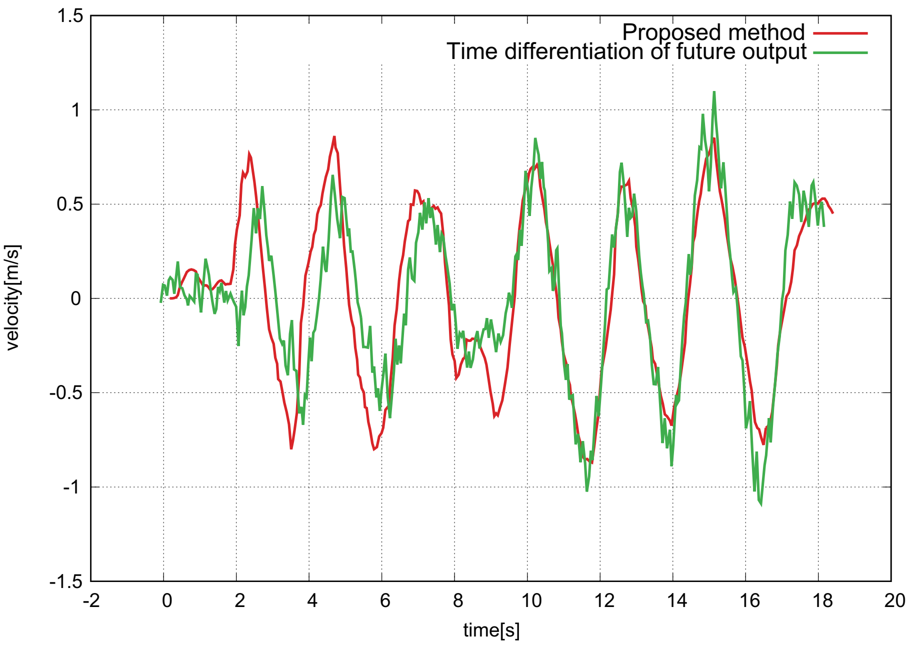

Figure 3.6: Estimation Result of $\dot{x}_g(t)$

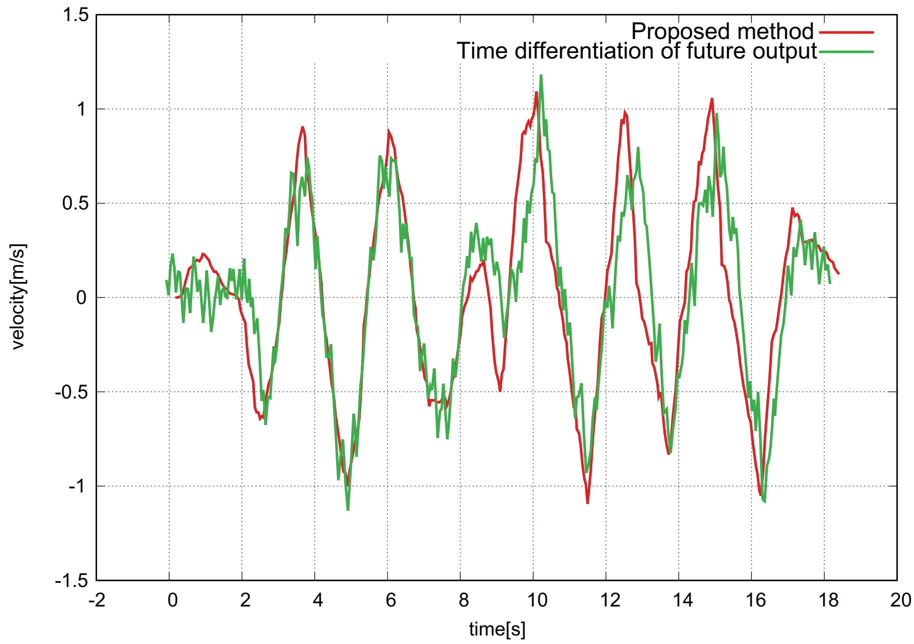

Figure 3.7: Estimation Result of $\dot{y}_g(t)$

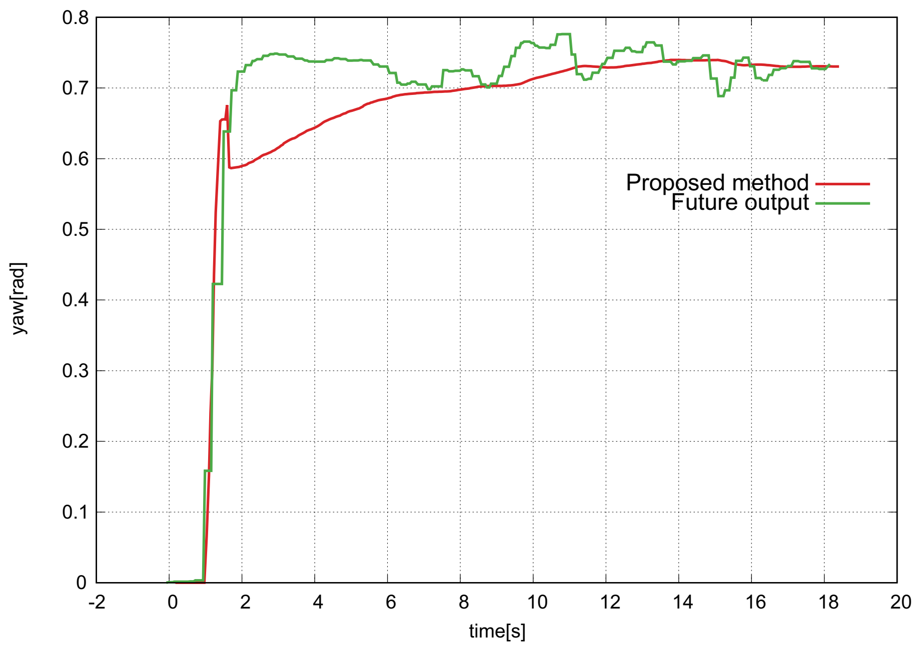

Figure 3.8: Estimation Result of $\psi(t)$

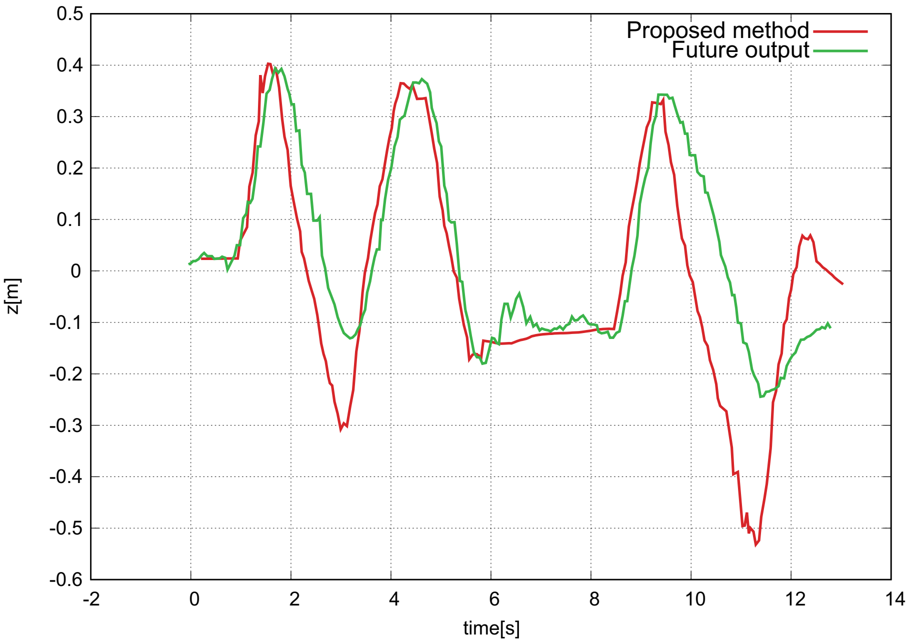

Figure 3.9: Estimation Result of $z(t)$

Chapter 4. Distributed Estimation over Delayed Wireless Sensor Network

4.1 Introduction

Sensor networks with numerous sensors have attracted much attention because the development of micro-electro-mechanical systems (MEMS) has improved the performance of compact sensors and communication elements [60]. Many sensors can realize a wide-range of complex observations in large-scale systems. Redundant sensors improve the accuracy and robustness of an observation, and enable a fault-tolerant observation. A flexible sensing system can be realized by connecting sensors via a wireless network. Such a sensor network can be used in various applications, including area surveillance or the active monitoring of forests and agricultural lands.

There have been many studies on the applications of sensor networks. Distributed estimation methods have also been proposed to reduce wasteful communication paths. Olfati-Saber et al. [61, 62, 63, 64] proposed a distributed Kalman filter based on a consensus filter. The consensus filter is an application of consensus controls, and provides the average consensus of all the sensors included in a network. The consensus filter can calculate the consensus value through communications between adjacent nodes. An estimate of the distributed Kalman filter is obtained from the original Kalman filter using the consensus value [58]. Olfati-Saber et al. also proposed a Kalman-consensus filter, which executes the Kalman filtering and consensus calculation simultaneously. A gossip algorithm is also a distributed consensus algorithm for sensor networks [65]. In the gossip algorithm, each node selects the data sent from the other nodes at random. This reduces the communication traffic of the sensor network, and allows this algorithm to obtain a consensus value.

These studies focus on the communication efficiency. The communication delay is another problem of a sensor network with delay. In particular, the communication delays in a large wireless network cannot be ignored. However, the distributed estimation methods discussed in previous studies do not consider the communication delay. Delay compensated observers, which do not assume a network structure, have been proposed in [54, 55, 66]. In particular, Watanabe et al. [54] and Tsubakino et al. [66] considered the case where the output vector includes multiple delays. The delays included in the network are non-uniform, because they depend on the communication paths. The design of Watanabe's observer resolves itself into a finite pole assignment problem.

In this chapter, we bring a network structure to Watanabe's delay-compensated observer. A distributed estimation method with delay compensation is proposed here. Watanabe defined an output vector, where each element of the vector is a sum of multiple measurements of a physical quantity with different delays. The output form of Watanabe's observer is useful in aggregating the observed values using distributed data aggregation methods. Data aggregation methods for a sensor network are proposed in [67]. In this paper, we introduce the delay compensation proposed in [54] for tree-based data aggregation. All of the observed values of the sensor network are aggregated through communications between the neighbor-node pairs in a tree network. The observed values of the sensor network are aggregated by the tree-based communication at the root node. The communication delays are compensated by the memory of the input stored by each node. An observer of the root node can estimate the state from the aggregated data at its own node. We also propose an intercommunication protocol to aggregate the observed values in all the nodes, which is based on the fact that each node of a tree network can be a root node. Finally, the distributed observer can estimate the states at all the nodes. A dimension of the communication data among the nodes corresponds to a dimension of the output vector. Because the dimension of the output vector is independent of the number of nodes, the proposed communication law in this chapter is scalable.

4.2 Problem Formulation

Figure 4.1 shows a system with distributed controllers. The dynamics of the plant in Fig. 4.1 can be expressed by

$$\dot{x} = Ax + Bu, \tag{4.1}$$

where $x \in \mathbb{R}^n$ is the system state, and $u \in \mathbb{R}^m$ represents the input values of the distributed controllers. Some states of system (4.1) can be measured by sensors as

$$y = Cx, \tag{4.2}$$

where $y \in \mathbb{R}^p$ is an output vector that consists of all of the redundant raw measurements, and therefore $\operatorname{rank} C$ may be less than p, and p may be larger than n. For the scalability of the network communication with respect to the number of sensors, this chapter considers aggregations of sensor measurements. The raw output y is aggregated into a q-dimensional vector y_{agr} as

$$y_{\mathrm{agr}} = Fy = C_{\mathrm{agr}}x, \tag{4.3}$$

where $C_{\mathrm{agr}} = FC$. The designs of F and q are important because these affect the performances of the state estimation or control, as indicated in [68]. However, this chapter focuses on other problems, and we assume that F is given.

There are two networks in Fig. 4.1, i.e., a sensor network and an actuator network that share the same node set with N elements. Each node may have the sensors and a controller. Each controller collects the observed information from the other nodes via the sensor network to estimate the state x, and calculates a part of the input elements from the estimated value. By renumbering the elements of y, we can decompose matrix F as

$$F = \left[\begin{array}{c|c|c} F_1 & \cdots & F_N \end{array}\right],$$

where F_i corresponds to the output of the i-th node. Therefore, the output of node i, which is mapped to the aggregated output space, is defined as

$$\begin{aligned} y_1 &= [\ F_1 \mid 0 \mid \cdots \mid 0\ ]y = C_1 x \\ &\vdots \\ y_N &= [\ 0 \mid \cdots \mid 0 \mid F_N\ ]y = C_N x, \end{aligned} \tag{4.4}$$

where $C_i = [\ 0 \mid \cdots \mid 0 \mid F_i \mid 0 \mid \cdots \mid 0\ ]C$. If the current outputs y_i can be obtained with no transmission delay, the aggregated output coincides with y_{agr}, i.e.

$$y_{\mathrm{agr}} = \sum_{i=1}^{N} y_i. \tag{4.5}$$

Each node needs all of the input values to estimate the state. In this chapter, it is assumed that the dimension of the input is smaller than that of the output. Each node sends the input values via a high-speed actuator network with a limited capacity. On the other hand, the observed values of each of the nodes are sent via a sensor network with sufficient bandwidth but low communication speed. Therefore, the communication delay in the transmission of the input values is sufficiently smaller than that for the observed values. We have ignored the communication delay in the broadcast of the input values, and it is assumed that all the nodes can obtain the input instantly.

We represent the sensor network by an undirected graph. The set of nodes is denoted by $V := \{1, 2, ..., N\}$, and the set of edges is denoted by $E \subseteq V \times V$. By using V and E, the undirected graph is expressed by $G := (V, E)$. In this chapter, it is assumed that the graph G is a connected graph. For the connected graph G, there always exists at least one tree that is a subgraph of G and includes all the nodes of V. This tree is denoted by $T := (V, \hat{E})$, where $\hat{E}$ satisfies $\hat{E} \subseteq E$. Node i can mutually communicate with the neighbor nodes. The set of neighbors of node i connected by T is denoted by $J_i := \{j; (i, j) \in \hat{E}\}$. Once a root node of the undirected tree is chosen, the parent node and child nodes of each node are automatically determined. A set of the child nodes of node i is denoted by H_{ir}, where the r-th node is chosen as the root node. For example, the set of child nodes of node 4 in Fig. 4.2 is $H_{2r} = \{1, 5\}$ if $r = 2, 4, 8,$ or 9. A communication delay from node j to i is denoted by D_{ij}. If $j \notin J_i$, D_{ij} becomes the total delay in the path from node j to i. For example, D_{81} of the tree in Fig. 4.2 is $D_{81} = D_{84} + D_{42} + D_{21}$. Each node stores the history of input $u(\tau)$, $\tau \in [t, t - D_{i,\max}]$, where $D_{i,\max}$ is the maximum delay expressed by $D_{i,\max} = \max_{j \in J_i} D_{ij}$.

Remark 4.1
In many cases, it is assumed that $D_{ij} = D_{ji}$, which is a natural assumption. However, the method proposed in this paper does not need this assumption.

Remark 4.2
The sensor network may have relay nodes. A relay node does not have a sensor but can communicate with the other nodes. The output matrix of the relay node is $C_i = 0$. Several systems can be realized using relay nodes. For example, a system with a single centralized controller can be expressed by a network that includes a controller and has a relay node as the root node.

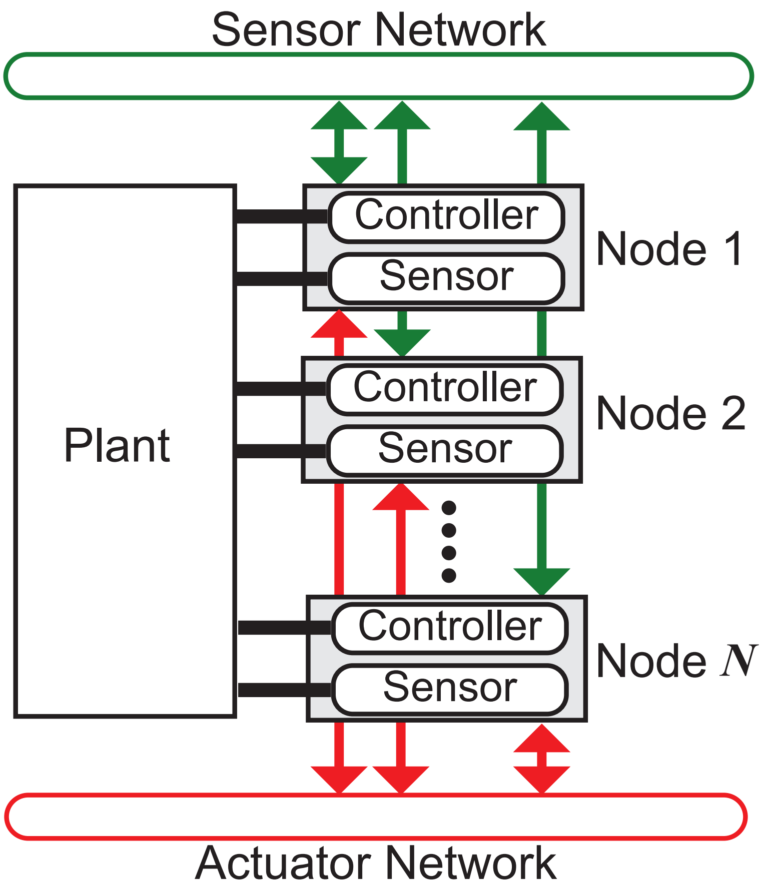

Figure 4.1: System with Actuator and Sensor Networks

To simplify the problem, we divide the problem into the following two parts:

Problem 4.1
Node r is fixed as the root node of T. Design the full state observer at the root node using the information from its own child nodes.

Problem 4.2
Design the full state observers at all the nodes, which use the information obtained via the networks.

We will obtain a result for Problem 4.1 in subsection 4.3.2, and then extend it to Problem 4.2 in subsection 4.3.3.

4.3 Proposed Method

4.3.1 Multi-Delay Compensated Observer

The data received at each node includes multiple delays because the communication delays in the sensor network depend on the selection of communication paths. An observer with

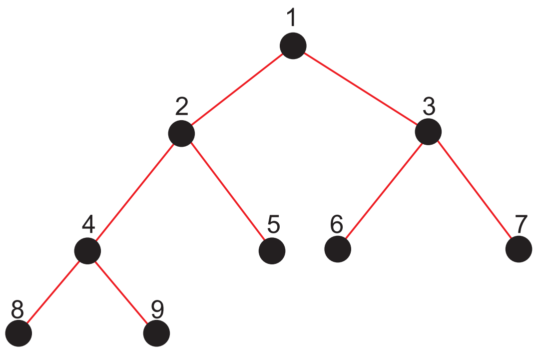

Figure 4.2: Example of a Tree

multi-sensor delays was proposed in [54]. Watanabe et al. [54] defines the outputs $C_i x$ for each corresponding delay D_i. Thus the output from all the measurements is expressed by

$$y_{\text{cen}}(t) = \sum_{i=1}^{N} C_i x(t - D_i). \tag{4.6}$$

We can compensate the delays in (4.6) by predicting the system behavior.

Lemma 4.1 (Delay-Compensation Based on Prediction)
Consider system (4.1) with output (4.6). For this system, the following equation holds:

$$y_{\text{cen}}(t) + \sum_{i=1}^{N} C_i e^{-AD_i} \int_{t-D_i}^{t} e^{A(t-\tau)} Bu(\tau) d\tau = \hat{C} x(t), \tag{4.7}$$

where $\hat{C} = \sum_{i=1}^{N} C_i e^{-AD_i}$.

Proof
The solution for system (4.1) is expressed by

$$x(t) = e^{AD_i} x(t - D_i) + \int_{t-D_i}^{t} e^{A(t-\tau)} Bu(\tau) d\tau. \tag{4.8}$$

Lemma 4.1 can be proven by solving (4.8) with respect to $x(t - D_i)$ and inserting it into (4.6).

From Lemma 4.1, the state estimation for the system (4.1) with (4.6) becomes a finite pole assignment problem as follows.

Lemma 4.2 (Watanabe's Delay-Compensated Observer [54])
Consider system (4.1), the output (4.6), and the observer

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) \\ &+L\left(y_{\text{cen}}(t) + \sum_{i=1}^{N} C_i e^{-AD_i} \int_{t-D_i}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \right. \\ &\left. - \sum_{i=1}^{N} C_i e^{-AD_i} \hat{x}(t) \right), \end{aligned} \tag{4.9}$$

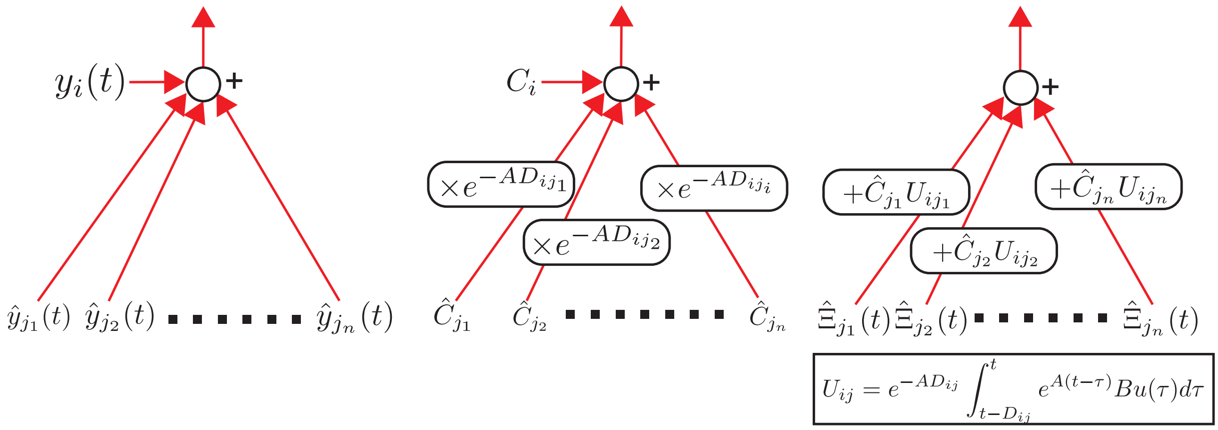

Figure 4.3: Aggregation Method for Sensor Network

and suppose that $(A, \hat{C})$ is an observable pair. Then, the estimation error $\tilde{x} = x - \hat{x}$ converges to zero, if and only if $A - L\hat{C}$ is Hurwitz.

Proof
From Lemma 4.1, the dynamics of $\tilde{x}$ can be expressed by

$$\dot{\tilde{x}} = (A - L\hat{C})\tilde{x}. \tag{4.10}$$

Therefore, $\tilde{x}$ tends to zero as $t \to \infty$ if and only if $A - L\hat{C}$ is Hurwitz.

We notice that Watanabe's delay-compensated observer does not need to handle y, as defined by (4.2). The single output vector y_{cen} defined by (4.6), which includes all the delayed sensor signals, and the input signal $u(\tau)$ $(t - \max_i(D_i) \le \tau \le t)$ are only required for the external signals of the observer (4.9). This property is effective for reducing the network traffic, because y includes redundant information. In addition, it is not assumed that the number of delay values N is smaller than dimension of the output m. Thus, outputs that have the same elements but include different delays can be aggregated into one value. Based on these results, this paper solves Problems 4.1 and 4.2 in the following subsections.

4.3.2 Tree-Data-Aggregation-Based Observer

In this subsection, we consider Problem 4.1. Let r be a root node of T. To collect information on the sensor network, each node executes the following communication. Let $\hat{y}_i(t)$ be an aggregated output value at node i. Each node aggregates its own measurements and the data received from child nodes, and sends these data to the parent node. The data sent from node i are expressed by

$$\hat{y}_i(t) = y_i(t) + \sum_{j \in H_{ir}} \hat{y}_j(t - D_{ij}). \tag{4.11}$$

Leaf node l, which has no child node, i.e., $H_{lr} = \emptyset$, does not receive any data from the other nodes. Therefore, $\hat{y}_l(t) = y_l(t)$ for each leaf node l.

The aggregated measurements in each node include communication delays, which depend on the communication paths. To compensate these delays, each node calculates delay-compensation terms using the memory of the input, and sends the correction terms to the parent node. Let $\hat{\Xi}_i(t)$ be a variable that includes delay-compensation terms at node i and $\bar{C}_i$ be a coefficient matrix that is recursively defined by

$$\begin{cases} \bar{C}_i = C_i + \sum_{j\in H_{ir}} \bar{C}_j e^{-AD_{ij}} & (H_{ir} \neq \emptyset), \\ \bar{C}_i = C_i & (H_{ir} = \emptyset). \end{cases} \tag{4.12}$$

Node i receives $\hat{\Xi}_j(t - D_{ij})$ from node $j \in H_{ir}$ and calculates $\hat{\Xi}_i(t)$ to compensate D_{ij} ($j \in H_{ir}$) as

$$\begin{aligned} \hat{\Xi}_i(t) = \sum_{j\in H_{ir}} \Bigg(& \hat{\Xi}_j(t - D_{ij}) \\ & + \bar{C}_j e^{-AD_{ij}} \int_{t-D_{ij}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \Bigg). \end{aligned} \tag{4.13}$$

Each leaf node l does not need to calculate $\hat{\Xi}_l(t)$ because there are no data sent from the other nodes, which means that $\hat{\Xi}_l(t) = 0$ ($H_{lr} = \emptyset$).

Then, the following lemma holds for the communication law (4.11).

Lemma 4.3 (Data Aggregation on Tree Networks)
The aggregated value at the root node can be expressed by

$$\hat{y}_r(t) = \sum_{i=1}^{N} C_i x(t - D_{ri}), \tag{4.14}$$

which includes all the measurements on the network with delays.

Proof
Let $\hat{H}_r^h$ be a set of nodes that can be reached from r via a simple path with length h. It is expressed by

$$\hat{H}_r^h = \begin{cases} \{r\} & (h = 0) \\ H_{rr} & (h = 1) \\ \left\{ i \in H_{jr} | j \in \hat{H}_r^{h-1} \right\} & (h > 1). \end{cases}$$

Moreover, we define

$$\tilde{H}_r^h = \bigcup_{i=0}^{h} \hat{H}_r^i.$$

Using (4.11) twice, a relation

$$\hat{y}_r(t) = \sum_{i\in\tilde{H}_r^1} y_i(t - D_{ri}) + \sum_{j\in\hat{H}_r^2} \hat{y}_j(t - D_{rj}) \tag{4.15}$$

can be obtained. Because $\hat{y}_i(t) = y_i(t)$ when node i satisfies $H_{ir} = \emptyset$, $\hat{y}_r(t)$ is recursively given by (4.14).

The delay compensated terms (4.13) satisfy the following lemma.

Lemma 4.4 (Delay Compensation on Tree Networks)
Under the communication laws (4.11) and (4.13),

$$\hat{y}_r(t) + \hat{\Xi}_r(t) = \bar{C}_r x(t)$$

holds, i.e., the sum of the aggregated output and compensating term at the root node can be expressed by a linear map of the current state.

Proof
By applying (4.12) twice,

$$\bar{C}_r = \sum_{i \in \tilde{H}_r^1} C_i e^{-AD_{ri}} + \sum_{j \in \hat{H}_r^2} \bar{C}_j e^{-AD_{rj}} \tag{4.16}$$

is obtained. Note that $\bar{C}_i = C_i$ if $H_{ir} = \emptyset$. Therefore, matrix $\bar{C}_r$ is recursively given by

$$\bar{C}_r = \sum_{i=1}^{N} C_i e^{-AD_{ri}}. \tag{4.17}$$

The aggregated value $\hat{\Xi}_r(t)$ is given by (4.13). In addition, the data received from the child nodes are expressed as

$$\begin{aligned}\hat{\Xi}_j(t) = \sum_{k \in H_{jr}} \Bigg(& \hat{\Xi}_k(t - D_{jk}) \\ & + \bar{C}_k e^{-AD_{jk}} \int_{t-D_{jk}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \Bigg).\end{aligned} \tag{4.18}$$

By substituting (4.18) in (4.13), we get

$$\begin{aligned}\hat{\Xi}_i(t) = & \sum_{i \in \tilde{H}_r^1} C_i e^{-AD_{ri}} \int_{t-D_{ri}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \\ & + \sum_{j \in H_r^2} \Bigg(\hat{\Xi}_k(t - D_{rj}) \\ & + \bar{C}_j e^{-AD_{rj}} \int_{t-D_{rj}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \Bigg).\end{aligned} \tag{4.19}$$

If node i is the leaf node, $\hat{\Xi}_i(t) = 0$. Thus, $\hat{\Xi}_r(t)$ is recursively given by

$$\hat{\Xi}_r(t) = \sum_{i=1}^{N} C_i e^{-AD_{ri}} \int_{t-D_{ri}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau. \tag{4.20}$$

From (4.17), (4.20), and Lemmas 4.1 and 4.3, we can prove Lemma 4.4.

Lemmas 4.3 and 4.4 indicate that the root can collect all of the measurements on the networks with delay-compensation. Therefore, the root node can estimate the state from $\hat{y}_r(t)$ and $\Xi_r(t)$.

Theorem 4.1
Assume that $(A, \bar{C}_r)$ is an observable pair. Then, the observer of the root node

$$\begin{aligned}\dot{\hat{x}}_r(t) &= A\hat{x}_r(t) + Bu(t) \\ &\quad + L_r\left(\hat{y}_r(t) + \hat{\bar{\Xi}}_r(t) - \bar{C}_r\hat{x}_r(t)\right)\end{aligned} \tag{4.21}$$

can estimate the state, i.e., $\hat{x}_r(t) \to x(t)$ as $t \to \infty$, if and only if $A - \bar{C}_r L_r$ is Hurwitz.

Proof
Let $\tilde{x}_r(t) = x(t) - \hat{x}_r(t)$. From Lemma 4.4, the estimation error $\tilde{x}_r(t)$ satisfies the following equation:

$$\dot{\tilde{x}}_r(t) = (A - \bar{C}_r L_r)\tilde{x}_r(t). \tag{4.22}$$

Thus, Theorem 4.1 is proven.

4.3.3 Delay-Compensated Observer for Sensor Network

In the previous subsection, the observed information are aggregated in the communication paths, and finally the root node can obtain an aggregated value for all the nodes' information. However, with the exception of the root, all of the nodes only have part of the information observed by all the sensors. Each node needs the observed information of all the other nodes to estimate the state at the node. Because every node of a tree can be a root, each node can collect the observed values of all the other nodes in the same way as the method discussed in subsection 4.3.2. However, wasteful communications will occur if we individually design the communication laws to allow the different roots to collect data. Therefore, in this subsection, we propose an efficient intercommunication-based data aggregation method to estimate the state at all the nodes.

Let $\bar{y}_{ij}$ and $\bar{\Xi}_{ij}$ denote the data sent from node i to j, which will be defined later. The data sent from node i to j are the aggregated information from the neighbor nodes of node i, except for j, i.e., $J_i \setminus \{j\}$. Therefore, $\bar{y}_{ij}$ and $\bar{\Xi}_{ij}$ can be defined by

$$\bar{y}_{ij}(t) = y_i(t) + \sum_{k \in J_i \setminus \{j\}} \bar{y}_{ki}(t - D_{ik}), \tag{4.23}$$

$$\begin{aligned}\bar{\Xi}_{ij}(t) = \sum_{k \in J_i \setminus \{j\}} \Bigg(&\bar{\Xi}_{ki}(t - D_{ik}) \\ &+ \bar{C}_{ki} e^{-AD_{ik}} \int_{t-D_{ik}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \Bigg),\end{aligned} \tag{4.24}$$

where $\bar{C}_{ij}$ is the matrix expressed by

$$\bar{C}_{ij} = C_i + \sum_{k \in J_i \setminus j} \bar{C}_{ki} e^{-AD_{ik}}. \tag{4.25}$$

Note that matrix $\bar{C}_{ij}$ can be obtained by an offline calculation. The aggregated values of each node, $\hat{y}_i(t)$ and $\hat{\Xi}_i(t)$, are given by

$$\hat{y}_i(t) = y_i(t) + \sum_{j \in J_i} \bar{y}_{ji}(t - D_{ij}), \tag{4.26}$$

$$\begin{aligned} \hat{\Xi}_i(t) = \sum_{j \in J_i} \Bigg(& \bar{\Xi}_{ji}(t) \\ & + \bar{C}_{ji} e^{-AD_{ij}} \int_{t-D_{ij}}^{t} e^{A(t-\tau)} Bu(\tau) d\tau \Bigg), \end{aligned} \tag{4.27}$$

and the output matrix after the delay-compensation is recursively defined by

$$\hat{C}_i = C_i + \sum_{j \in J_i} \bar{C}_{ji} e^{-AD_{ij}}. \tag{4.28}$$

Using the intercommunication law in (4.23) and (4.24), the following theorem holds.

Theorem 4.2
The aggregated value of each node $\hat{y}_i(t)$ includes the outputs of all the sensors. The delays included in $\hat{y}_i(t)$ can be compensated by $\hat{\Xi}_i(t)$. Let $\hat{x}_i$ be an estimate of the state calculated at node i. The observer in node i is defined by

$$\begin{aligned} \dot{\hat{x}}_i(t) = & A\hat{x}_i(t) + Bu(t) \\ & + L_i \left(\hat{y}_i(t) + \hat{\Xi}_i(t) - \hat{C}_i \hat{x}_i(t) \right), \end{aligned} \tag{4.29}$$

where $(A, \hat{C}_i)$ is an observable pair. Then, the error dynamics of (4.29) for node i are asymptotically stable if $A - L_i\hat{C}_i$ is Hurwitz.

Proof
The intercommunication law of (4.23) and (4.24) implies that $\bar{y}_{ij}(t)$ and $\bar{\Xi}_{ij}(t)$ are equal to the aggregated data expressed by (4.11) and (4.13), respectively, when node j is the parent node of node i. Similarly, $\bar{C}_{ij}$ in (4.25) coincides with $\bar{C}_i$ of (4.12), when the parent node is j. Thus, the aggregated values of node i, which are defined in (4.26) and (4.27), become the tree-based aggregated values at the root node. Matrix (4.28) also becomes the aggregated matrix whose root node is its own node. Therefore, the error dynamics of the observer of each node (4.29) are asymptotically stable if $A - \hat{C}_i L_i$ is Hurwitz.

Remark 4.3 (Observability of Sensor Network)
In Theorem 2, it is assumed that all the pairs $(A, \hat{C}_i)$ are observable. Thus, the observability of the sensor network depends on the network topology because $\hat{C}_i$ includes the delay values. In general, the condition that pair A and non-delay output matrix C_{agr} are observable does not guarantee that $(A, \hat{C}_i)$ is observable. However, we can expect that the sensor network becomes observable if (A, C_{agr}) is observable and the communication delays are sufficiently small.

4.3.4 Adaptation Algorithm for Modification of Network Topology

In this subsection, we show an algorithm for the recalculation of the parameters in each node when the topology of the network is modified. The parameters that depend on the network topology in each node are $\hat{C}_i$, $\bar{C}_{ij}$, and L_i. The proposed algorithm calculates these parameters through local calculations and mutually communications between nodes.

We define logical variables $\delta_i(t)$ as $\delta_i(t) \in \{\mathsf{T}, \mathsf{F}\}$, which indicates whether J_i is modified. Node i sets $\delta_i(t)$ to "T" (true) if J_i has been modified at t, and otherwise $\delta_i(t) = \mathsf{F}$ (false). The signals $\bar{\delta}_{ij}(t)$ represent the propagation of the modification from node i. If node i needs to tell the present of the modification to node j, $\bar{\delta}_{ij}(t)$ becomes "T," which means

$$\bar{\delta}_{ij}(t) = \delta_i(t) \vee \left(\bigvee_{k \in J_i \setminus \{j\}} \bar{\delta}_{ki}(t - D_{ik}) \right).$$

According to $\delta_i(t)$ and $\bar{\delta}_{ij}(t)$, each node recalculates or updates each parameter. If $\bar{\delta}_{ij}(t)$ is "T," node i executes an event to update $\bar{C}_{ij}$ based on (4.25). Let $\hat{\delta}_i(t)$ be

$$\hat{\delta}_i(t) = \delta_i(t) \vee \left(\bigvee_{j \in J_i} \bar{\delta}_{ji}(t - D_{ij}) \right).$$

Node i needs to recalculate L_i and $\hat{C}_i$ when $\hat{\delta}_i(t)$ is "T." The triggered node updates $\hat{C}_i$ as (4.28), and chooses L_i such that $A - L_i\hat{C}_i$ becomes stable. Algorithm 1 have summarized the above procedure.

To execute Algorithm 1, each node needs to prepare C_i and $e^{-AD_{ij}}$ for all j which are candidates for the neighborhood nodes. Each node can utilize unsteady Kalman filter algorithms to recalculate L_i. The unsteady Kalman filter algorithms need to calculate Riccati differential equation, but do not need the real-time calculation of eigenvalues or an inverse matrix. Therefore, the assumption that each node has the ability to execute Algorithm 1 is reasonable.

Algorithm 1 Recalculation of $\hat{C}_i$, $\bar{C}_{ij}$ and L_i in Each Time Sequence

if J_i is modified at t **then**
    $\delta_i(t) \leftarrow \mathsf{true}$
else
    $\delta_i(t) \leftarrow \mathsf{false}$
end if
for all j such that $j \in J_i$ **do**
    if $\bar{\delta}_{ji}(t - D_{ij}) = \mathsf{true}$ **then**
        $\bar{C}_{ji} \leftarrow \bar{C}_{ji}^{\mathrm{new}}$
    end if
end for
for all j such that $j \in J_i$ **do**
    $\bar{\delta}_{ij}(t) \leftarrow \delta_i(t) \vee \left(\bigvee_{k \in J_i \setminus \{j\}} \bar{\delta}_{ki}(t - D_{ik})\right)$
    if $\bar{\delta}_{ij}(t) = \mathsf{true}$ **then**
        $\bar{C}_{ij}^{\mathrm{new}} \leftarrow C_i + \sum_{k \in J_i \setminus j} \bar{C}_{ki} e^{-AD_{ik}}$
    end if
end for
$\hat{\delta}_i(t) \leftarrow \delta_i(t) \vee \left(\bigvee_{k \in J_i} \bar{\delta}_{ki}(t - D_{jk})\right)$
if $\hat{\delta}_i(t) = \mathsf{true}$ **then**
    $\hat{C}_i \leftarrow C_i + \sum_{j \in J_i} \bar{C}_{ji} e^{-AD_{ij}}$
    Calculate L_i such that $A - L_i\hat{C}_i$ becomes stable.
end if

4.4 Contribution of This Chapter

4.4.1 Comparison between Consensus Based Observer and Proposed Method

As related works of the proposed method, many consensus based observers are proposed [69, 70, 71, 72, 73, 74, 75, 76, 77]. In both the consensus based observers and the proposed method, all nodes over sensor network G estimate the full state. Although each node communicates the raw measurement over delayed sensor networks in the proposed method, the consensus based observers communicates the estimates. As a basic example, Kalman Consensus Filter (KCF) [62] are explain in this subsection. Let $d_{(i,j)} = 0$ for all $(i, j) \in E$ and consider the system (4.1) with system noises as

$$\dot{x} = Ax + Bu + Dw, \tag{4.30}$$

where w is a while noise with corvariance matrix $Q \in \mathbb{S}^m_{++}$. An output vector with a observation noise in each node is given by

$$y_i = H_i x + v_i \in \mathbb{R}^{q_i} \tag{4.31}$$

where v_i is a while noise with corvariance matrix $R_i \in \mathbb{R}^{q_i}_{++}$. KCF is given by

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i + Bu + K_i(y_i(t) - C_i\hat{x}_i(t)) + \gamma \sum_{j \in J_i} P_i(\hat{x}_j - \hat{x}_i) \\ K_i &= P_i C_i^T R_i^{-1} \\ \dot{P}_i &= AP_i + P_i A^T + DQD^T - P_i H_i^T R_i^{-1} H_i P_i. \end{aligned} \tag{4.32}$$

where $\gamma > 0$. Then, the stability condition of KCF is given by the following theorem.

Theorem 4.3 (Olfati-Saber [62])
Let the pair (A, H_i) for all $i \in V$ be detectable, $w = 0_m$ and $v_i = 0_{q_i}$. Then, the estimation error $\tilde{x}_i = x - \hat{x}_i$ of (4.32) converges to 0.

As theorem 4.3 indicates, KCF can estimate only a part of the state which is observable from local sensors. Therefore, communication between adjacent nodes in KCF can improve an accuracy of the estimates and convergence rates, but can not expand the observable space in each node. Some works improve this technical issue, but these requires all-to-all communications over the sensor networks. On the other hand, our proposed method can expand the observable space from the data-aggregation based communication, i.e even if (A, C_i) is unobservable pair, our proposed method permits that $(A, \hat{C}_i)$ is the observable pair. This is verified by numerical simulations in subsection 4.5.2.

4.4.2 Comparison between No Delay-Compensation Observer and Proposed Method

The past works consider a finite pole assignment method for a stabilization of linear delay systems which have infinite pole [54, 78, 79, 80, 81, 82]. In this chapter, the model based prediction proposed in [54] is utilized to realize the finite pole assignment method in an observer design. Therefore, we can select any convergence rates of each observer without system constraints.

On the other hand, a linear matrix inequality (LMI) based controller design methods for delay systems have already proposed [83, 84, 85, 86, 87, 88, 89, 90, 91, 92]. Based on these works, distributed observers without the delay-compensation can be designed. Let us consider (4.1) and $y_1(t-D_{1i}), \cdots, y_N(t-D_{Ni})$. The observer without the delay-compensation is expressed by

$$\begin{aligned}\dot{x}_i(t) = A\hat{x}_i(t) + Bu(t) + L_i\left(y_i(t) - C_i\hat{x}_i(t)\right)\\ + \sum_{j=1, j\neq i}^{N} L_{ji}\left(y_j(t-D_{ji}) - C_j\hat{x}_i(t-D_{ji})\right).\end{aligned} \tag{4.33}$$

Then, the stability condition of (4.33) is given by the following theorem.

Theorem 4.4
If there exists $P_i \in \mathbb{S}^n_{++}$, $Q_j \in \mathbb{S}^n_{++}$ $(j \in V/\{i\})$ and $M_{ji} = P_i L_{ji}$ $(j \in V)$ which satisfy

$$\Psi_i = \begin{pmatrix} \mathrm{He}\left(PA + C_i M_{ii}\right) + \sum_{j\in V/\{i\}} Q_j & \mathrm{row}\left(M_{ji}^T C_j^T\right)_{j\in V/\{i\}} \\ \mathrm{col}\left(C_j M_{ji}\right)_{j\in V/\{i\}} & \mathrm{diag}\left(-Q_j\right)_{j\in V/\{i\}} \end{pmatrix} < 0, \tag{4.34}$$

the estimation error $\xi_i = x - \hat{x}_i$ of (4.33) converges 0.

Proof
Let us consider Lyapunov Krasovskii functional candidate V as

$$V = \xi_i(t)^T P_i \xi(t) + \sum_{j\in V/\{i\}} \int_{t-D_{ji}}^{t} \xi_i(\tau)^T Q_j \xi(\tau) d\tau.$$

The time differentiation of V are given by

$$\dot{V} = e_i(t)^T \Psi_i e_i(t),$$

where

$$e_i = \left[\xi_i(t)^T, \mathrm{row}\left(\xi_i(t-D_{ji})^T\right)_{j\in V/\{i\}}\right]^T.$$

The time differentiation of V satisfies $\dot{V} < 0$ except for $e_i = 0$ if theorem 4.4 holds.

To calculate the estimate by using (4.33), each node needs to collect the non-aggregated measurement in the other nodes. Therefore, unlike in the case of the proposed method, LMI based observer requires unscalable communications. The stability condition shown in theorem 4.4 is conservative because Lyapunov Krasovskii analysis is utilized in the proof. In addition, to obtain the solution of LMI, we need to solve semidifinite programmings. Therefore, it is difficult for us to obtain the reasonable solution, which is shown in the numerical simulation in subsection 4.5.3.

4.5 Numerical Simulation

4.5.1 Switching of Tree Network

Let us consider the linear system which is expressed by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3.645 & -10.935 & -11.97 & -5.7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3 \end{pmatrix} \cos(\pi t). \tag{4.35}$$

A total of 50 sensors in the networks measure x_1, x_2, or x_3. We set the observation matrix of each node as

$$C_i = \begin{cases} \begin{pmatrix} 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \text{if} \quad i \equiv 0 \mod 3 \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \text{if} \quad i \equiv 1 \mod 3 \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 \end{pmatrix}, & \text{if} \quad i \equiv 2 \mod 3. \end{cases} \tag{4.36}$$

The output matrix after the aggregation is

$$C_{\mathrm{agr}} = \begin{pmatrix} 0.32 & 0 & 0 & 0 \\ 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0.34 & 0 \end{pmatrix}. \tag{4.37}$$

Note that A and C_{agr} are observable. The communication delays between two neighbor nodes are uniformly 0.25 seconds. In the numerical simulation, the three topologies in Fig. 4.4 are switched every 4 seconds. The topologies are generated by a BA model which is proposed in [93]. The BA model is a randomized algorithm which generates a scale free network. We can find the scale free networks in the real wold, for example social network or world wide web.

Figures 4.5, 4.6, 4.7, and 4.8 show the time responses of the estimation errors on x_1, x_2, x_3, and x_4, respectively. From Figs. 4.5, 4.6, 4.7, and 4.8, we can confirm that the observer proposed in this chapter can estimate the state. The effects of the parameter modifications appear at approximately $t = 4$ sec. However, the estimation errors converge to zero regardless of the parameter modifications. After the estimation errors converge to zero, the parameter modifications do not affect the estimates. Therefore, these numerical simulation results verify the effectiveness of the proposed observer and data aggregation method.

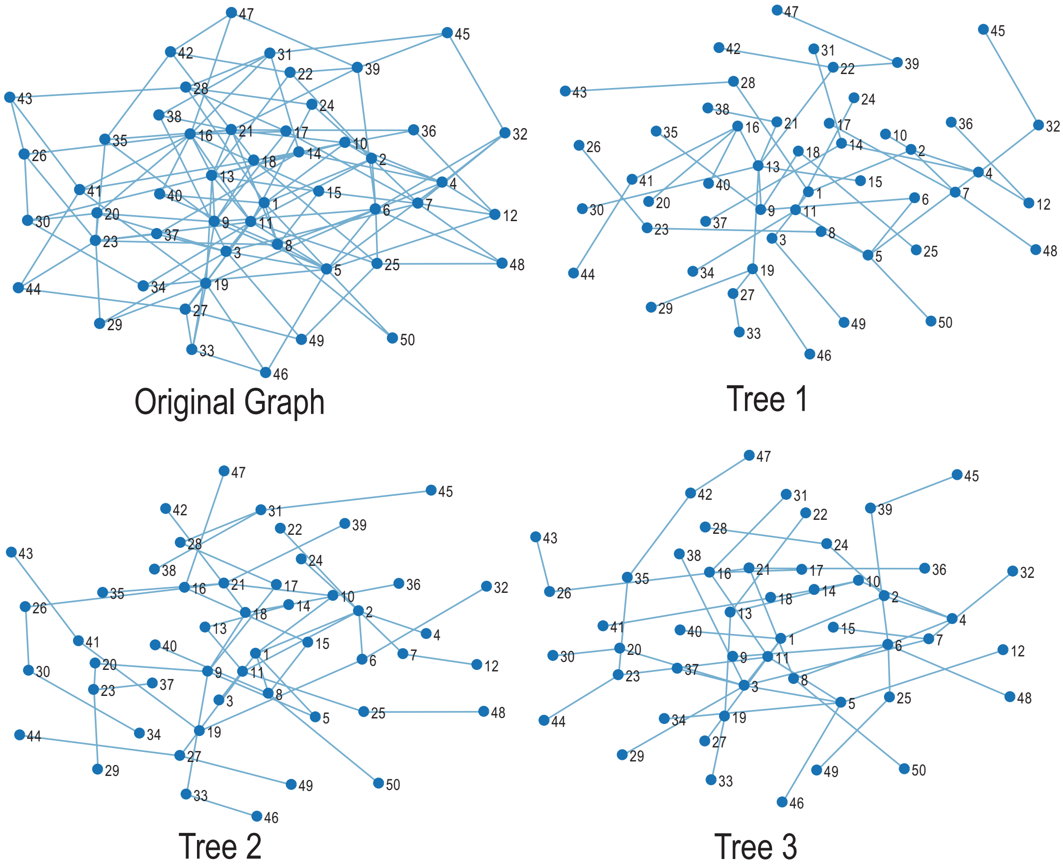

Figure 4.4: Topologies of Graphs in Simulation

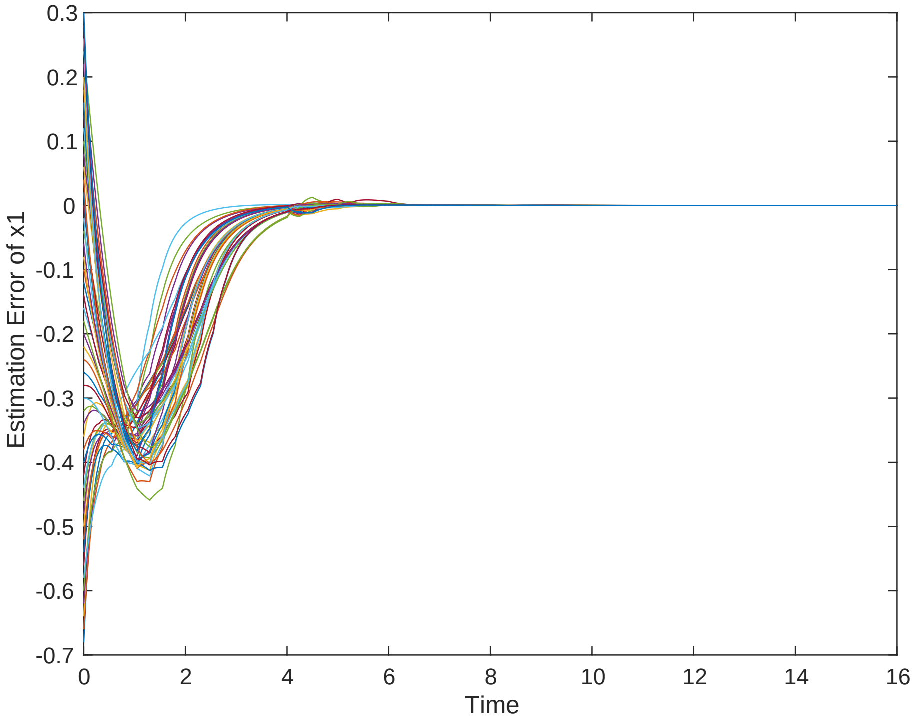

Figure 4.5: Time Responses of Errors for x_1

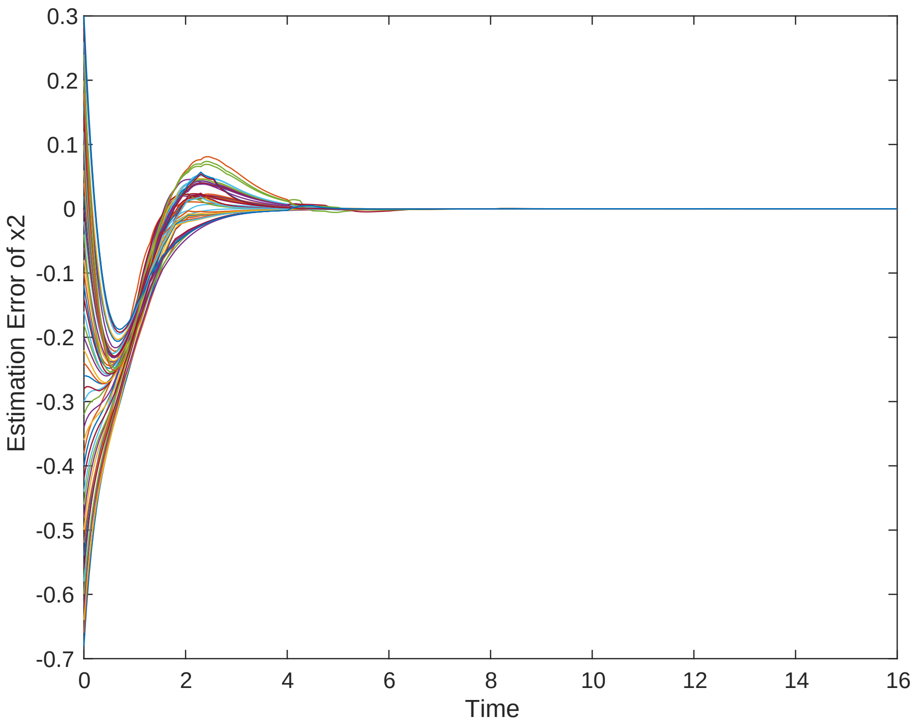

Figure 4.6: Time Responses of Errors for x_2

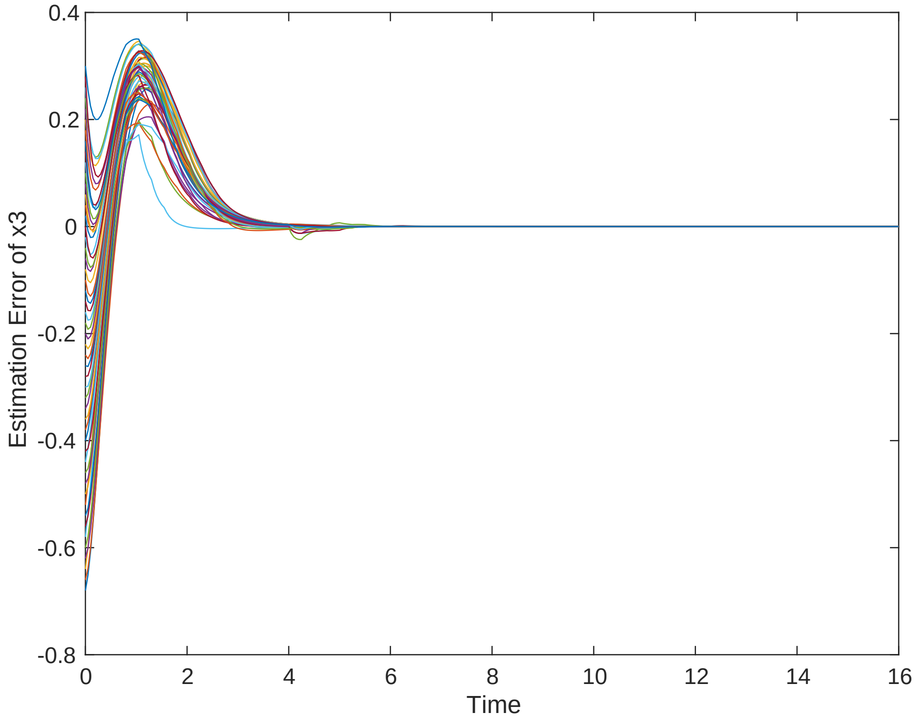

Figure 4.7: Time Responses of Errors for x_3

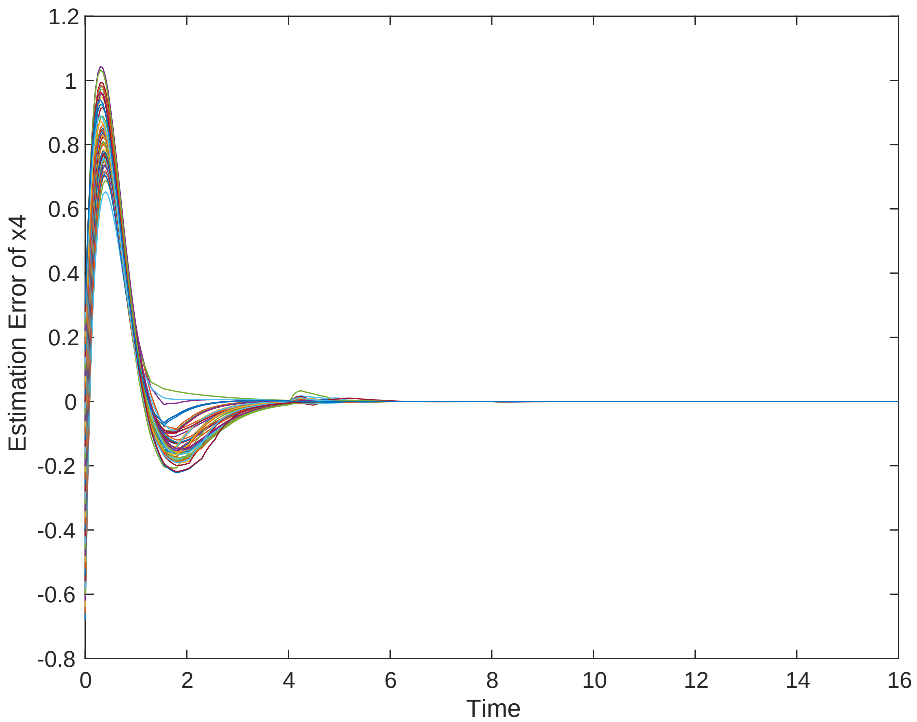

Figure 4.8: Time Responses of Errors for x_4

4.5.2 Verification of Improvement on Observability

In this subsection, we confirm that our proposed method can improve observability of each node unlike the consensus based filters. Let us consider the plant which is illustrated by Fig. 4.9, where subplant are expressed by

$$\Sigma_k := \begin{cases} \begin{pmatrix} \dot{x}_{k,1} \\ \dot{x}_{k,2} \end{pmatrix} = A_k \begin{pmatrix} x_{k,1} \\ x_{k,2} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) \\ z_k = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_{k,1} \\ x_{k,2} \end{pmatrix} \end{cases},$$

and the eigen values of A_K are give by

$$\lambda(A_k) = \begin{cases} (-1, -2) & \text{if} \quad k = 1 \\ (-0.1 + i, -0.1 - i) & \text{if} \quad k = 2, 3 \end{cases}.$$

In the simulation of this subsection, we utilize Fig. 4.11 and Fig. 4.10 as sensor networks for KFC and the proposed method, respectively. Note that Fig. 4.11 is one of spanning trees of Fig. 4.10. Local outputs y_i $(i \in V)$ are given by

$$y_i = \begin{cases} (0, z_2, 0)^T & \text{if} \quad i \equiv 0 \mod 3 \\ (0, 0, z_3)^T & \text{if} \quad i \equiv 1 \mod 3 \\ (z_1, 0, 0)^T & \text{if} \quad i \equiv 2 \mod 3. \end{cases}$$

Because each nodes can not estimates whole states of Fig. 4.9 from only y_i, a collection of the other outputs are required. The observable states from each output are summarized in Table 4.1. For the proposed method, we select observer gain L_i such that poles of $A - L_i\hat{C}_i$ becomes $(-2, -2.2, -3.9, -4.5, -3, -3)$. On the other hand, the parameters for KCF are given by $Q = 0.1$ and $R_i = 1.0 \times 10^{-4}$.

Time responses of the estimation errors in this subsection are shown by Figs. 4.12, 4.13, 4.14, 4.15, 4.16 and 4.17. From Figs. 4.12 and 4.13, we can confirm that both the proposed method and KCF can estimate $(x_{1,1}, x_{1,2})$ with sufficient convergence rates because these states are observable from all local outputs y_i $(i \in V)$. On the other hand the other states, which are unobservable from some local sensors, can not be estimated by KCF. However, we can confirm that our proposed method can estimate full states in all nodes from Figs. 4.14, 4.15, 4.16 and 4.17. From these results, we conclude that our proposed method can expand observable space in each node from the scalable communication over the sensor networks.

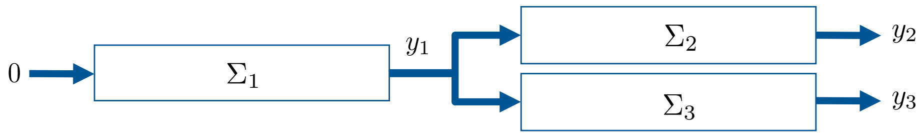

Figure 4.9: Plant for the Simulation

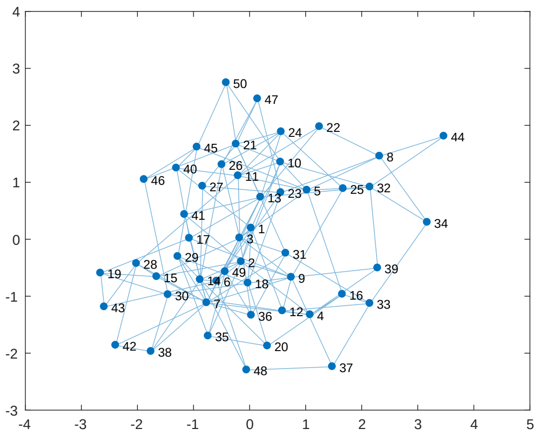

Figure 4.10: Original Graph

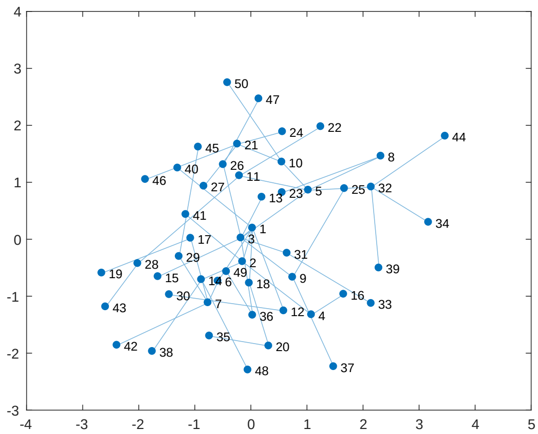

Figure 4.11: Spanning Tree

Table 4.1: Observalble State

	$(x_{1,1}, x_{1,2})$	$(x_{2,1}, x_{2,2})$	$(x_{3,1}, x_{3,2})$
y_1	Observable		
y_2	Observable	Observable	
y_3	Observable		Observable

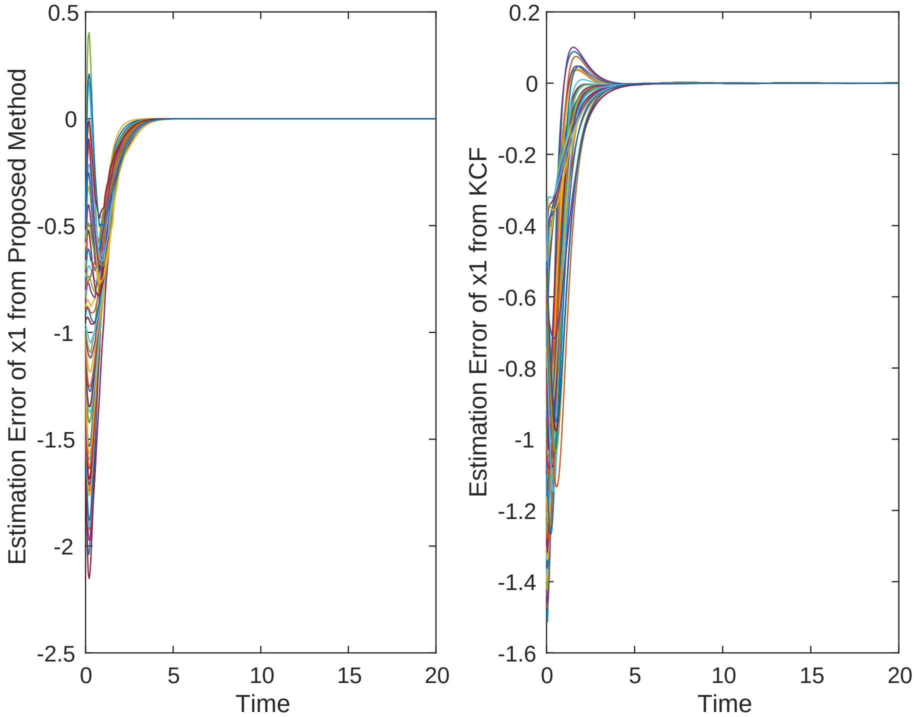

Figure 4.12: Time Responses of Errors for $x_{1,1}$ from Proposed Method and KCF

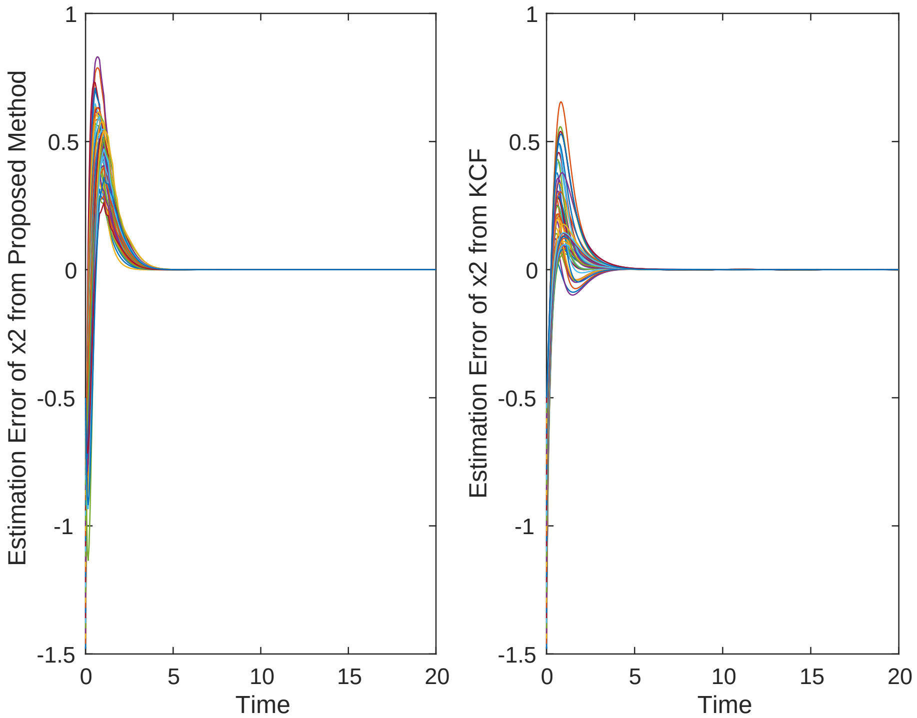

Figure 4.13: Time Responses of Errors for $x_{1,2}$ from Proposed Method and KCF

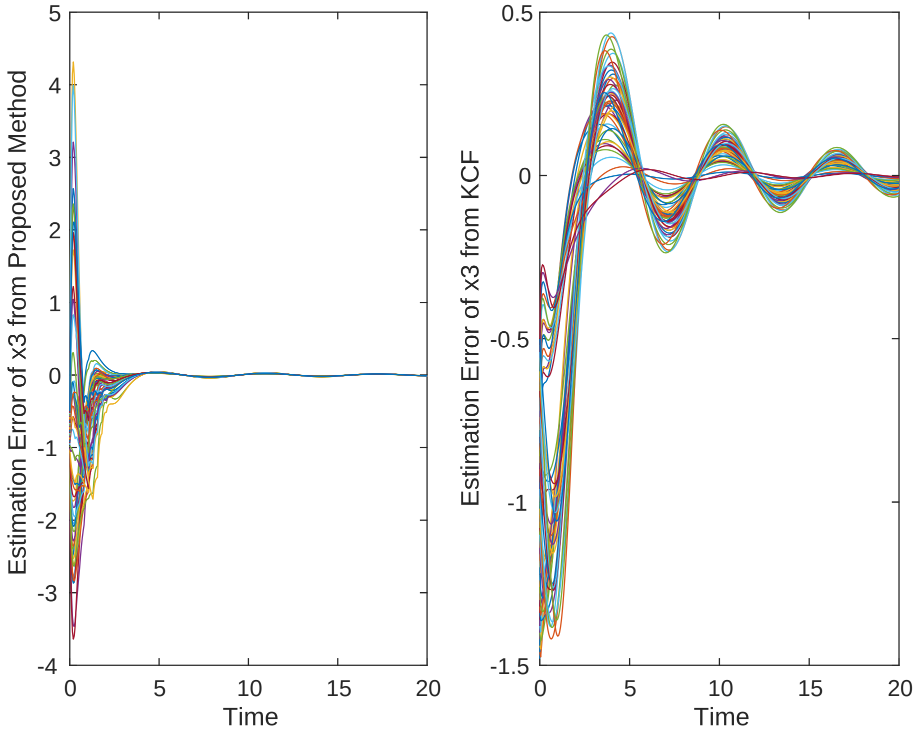

Figure 4.14: Time Responses of Errors for $x_{2,1}$ from Proposed Method and KCF

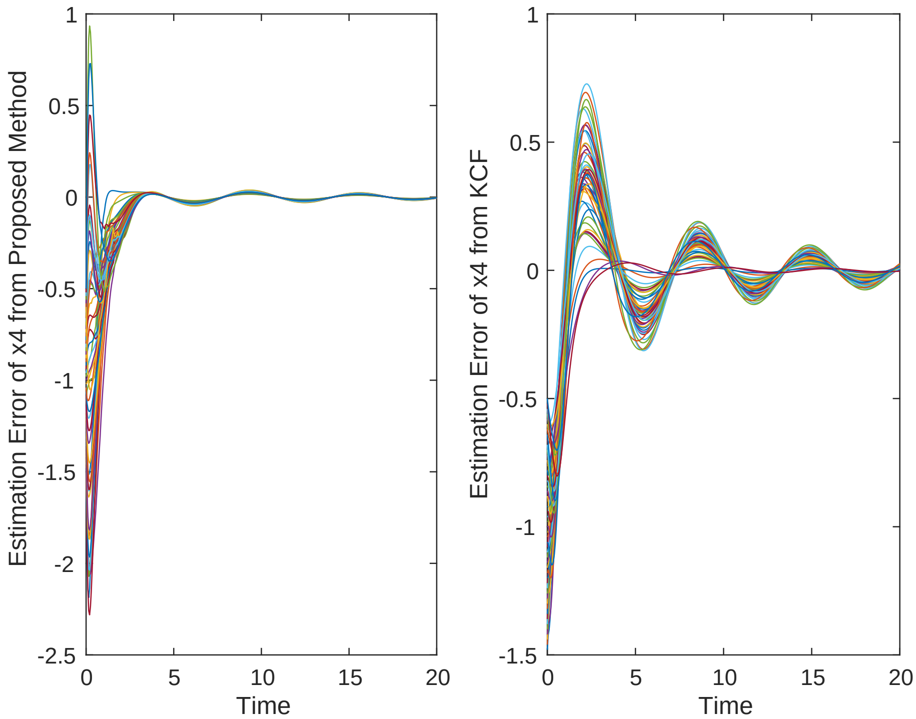

Figure 4.15: Time Responses of Errors for $x_{2,2}$ from Proposed Method and KCF

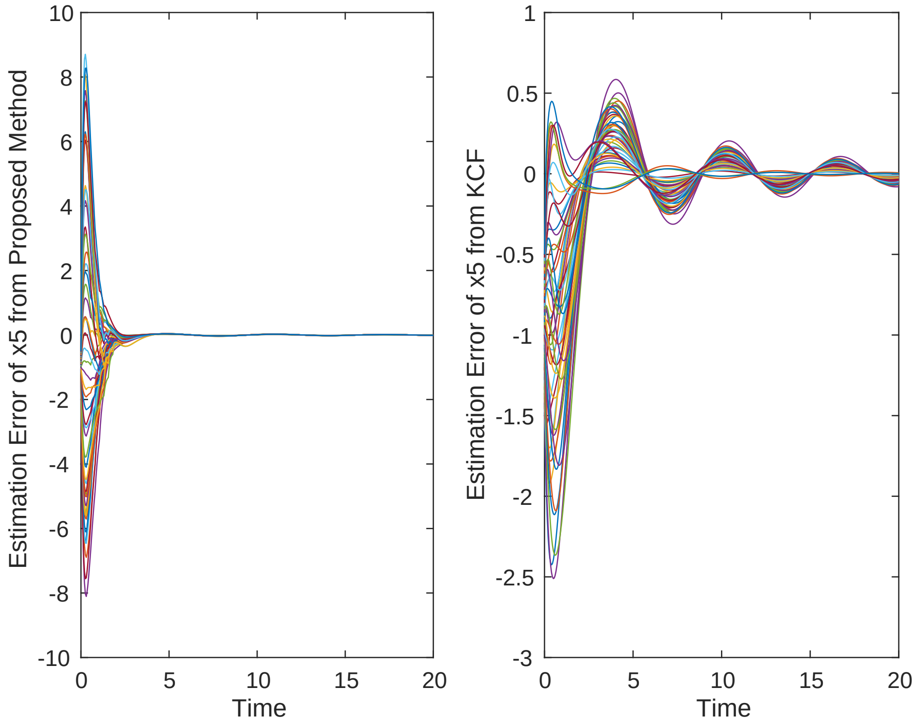

Figure 4.16: Time Responses of Errors for $x_{3,1}$ from Proposed Method and KCF

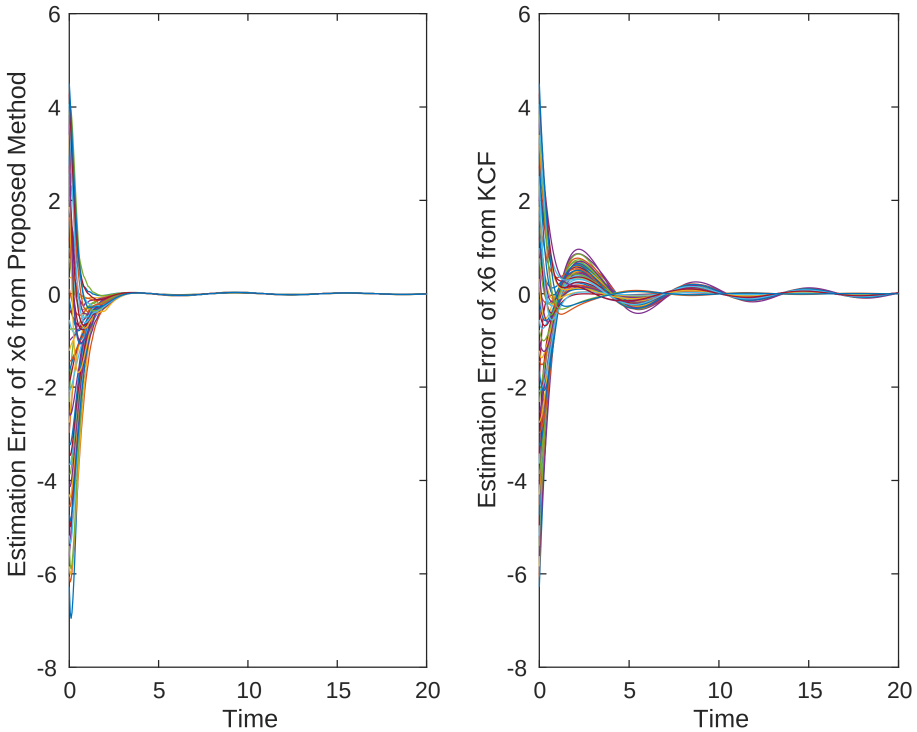

Figure 4.17: Time Responses of Errors for $x_{3,2}$ from Proposed Method and KCF

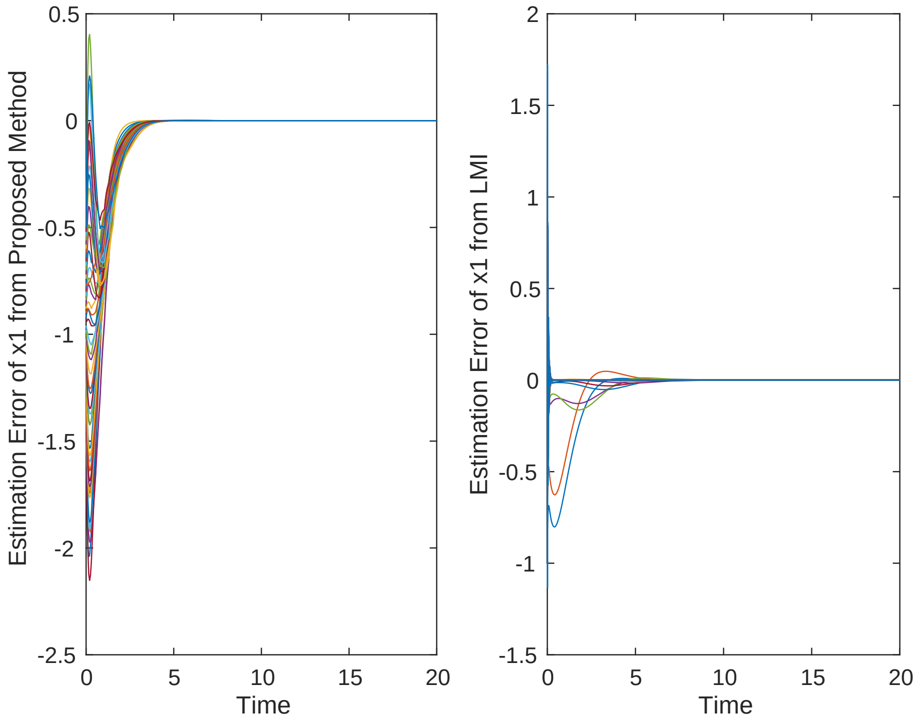

Figure 4.18: Time Responses of Errors for $x_{1,1}$ from Proposed Method and LMI Based Observer

4.5.3 Verification of Delay Compensation

In this subsection, we confirm that effect of the delay compensation in the proposed method by comparing to LMI observer given by theorem 4.4. As simulation conditions in this subsection, we utilize the same system and networks in subsection 4.5.2. Time responses of the estimation errors in this subsection are shown by Figs. 4.18, 4.19, 4.20, 4.21, 4.22 and 4.23.

From these results, it is confirmed that observer gains for the no-delay output tend to be very high. In particular, from Figs. 4.18 and 4.19 whose estimates are observable from only local output, we can confirm high convergence rates. In generally, high-gain observers, for example the LMI based observer, are undesirable because they are very sensitive against observation noises. On the other hand, we can select reasonable gains of the observer in each node because the proposed method can assign the poles of the observer to any values.

For the delayed output, observer gains calculated from LMI tend to be very low. Therefore, some nodes can not estimates the state with the sufficient convergence rates in Figs. 4.20, 4.21, 4.22 and 4.23. Because existences of Lyapunov-Krasovskii functional is a conservative condition, the semidefinite programming based on the LMI has a limited space of solution. Therefore, it is difficult to improve the convergence rates of the LMI based observer.

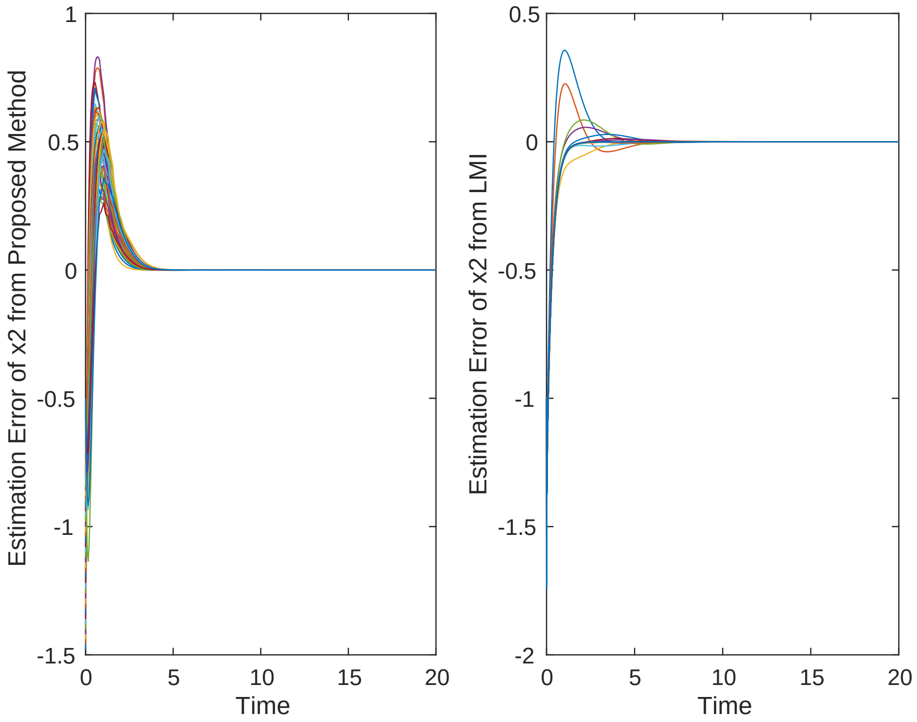

Figure 4.19: Time Responses of Errors for $x_{1,2}$ from Proposed Method and LMI Based Observer

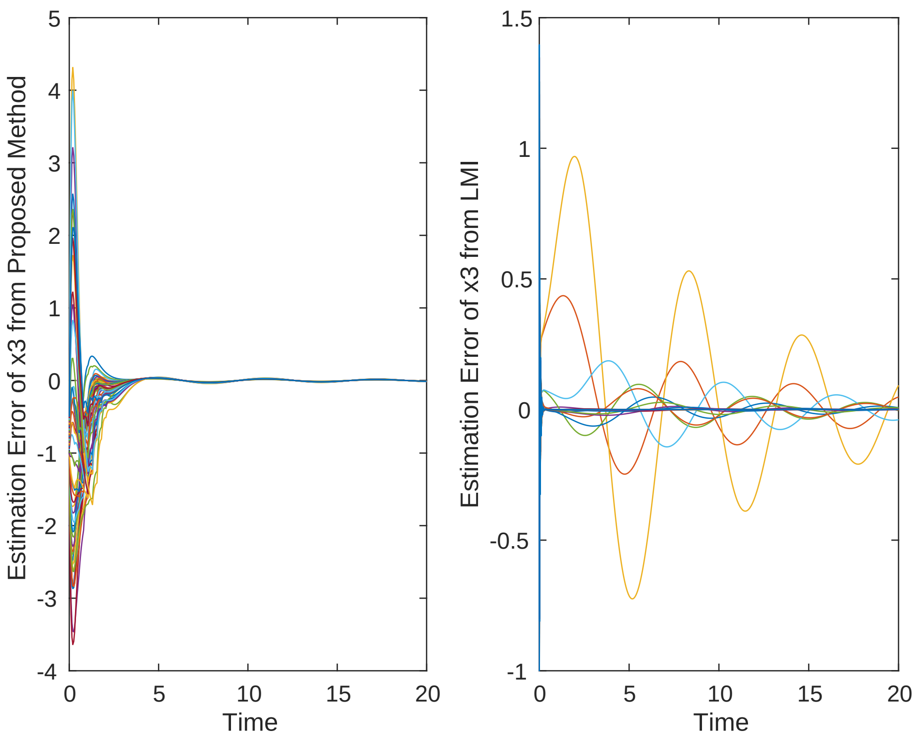

Figure 4.20: Time Responses of Errors for $x_{2,1}$ from Proposed Method and LMI Based Observer

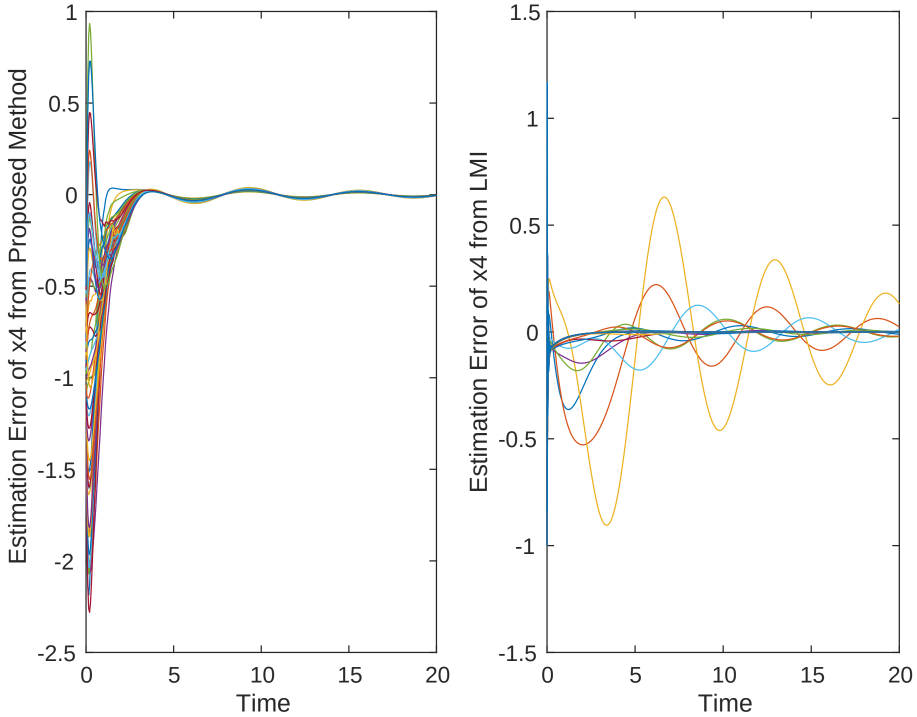

Figure 4.21: Time Responses of Errors for $x_{2,2}$ from Proposed Method and LMI Based Observer

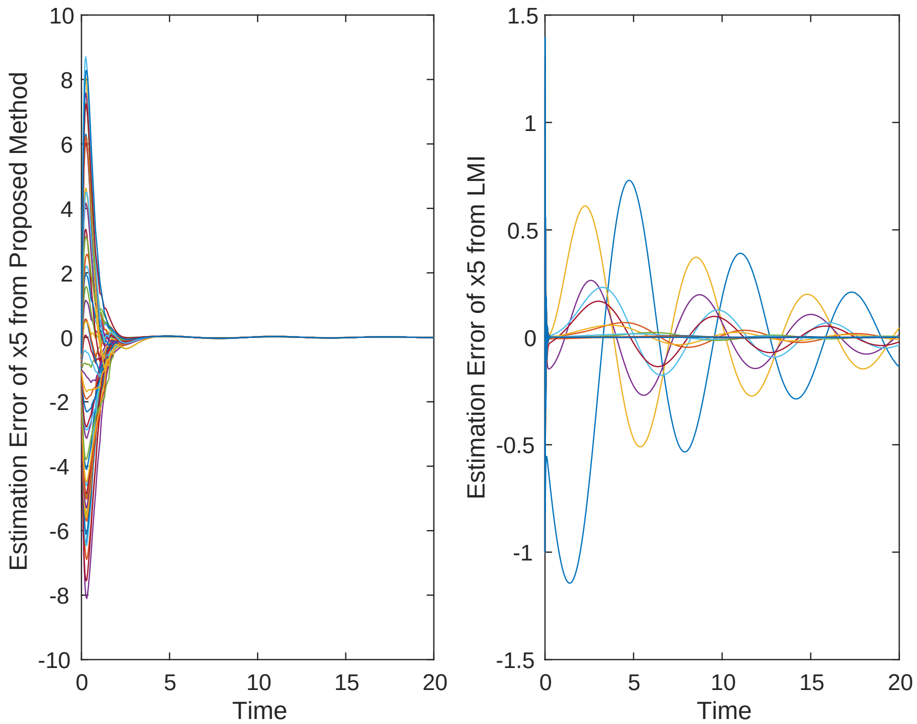

Figure 4.22: Time Responses of Errors for $x_{3,1}$ from Proposed Method and LMI Based Observer

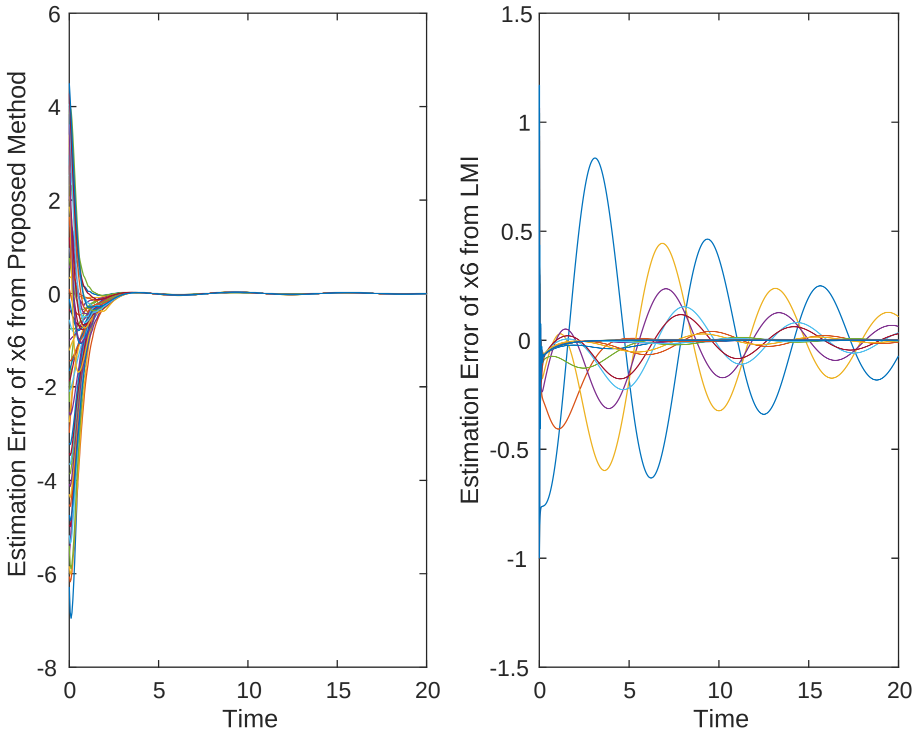

Figure 4.23: Time Responses of Errors for $x_{3,2}$ from Proposed Method and LMI Based Observer

4.6 Conclusion

This chapter propose a data-aggregation-based delay compensated observer for a wireless sensor network. The proposed method aggregates the values measured by all the sensors to each node. The communication delay between the neighbor nodes is instantly compensated by each node. Therefore, all of the nodes of a sensor network can estimate the state of the system. The dimensions of the signals on all the communication paths is $2q$, which is independent of node number N. This implies that the proposed communication laws are scalable with respect to the network size. A numerical simulation verifies the effectiveness of the proposed method.

In the proposed method, it is assumed that all the nodes can obtain inputs in real time via a fast network. To remove this assumption, we will consider an observer-based distributed controller in the next chapter. Moreover, the estimated state of the proposed method and output of the sensor networks have redundancy. We believe that this redundancy could enable us to realize a fault tolerant design for the distributed observers or controllers in wireless networks. This will also be the focus of our future study.

Chapter 5. Observer Design for Distributed Control System

5.1 Introduction

In recent years, sensor networks which include a lot of sensors have attracted much attention. Many sensors improve an accuracy of a sensing on the sensor network. Redundancies of sensors are able to realize a fault tolerant sensing. The past studies of the applications to sensor networks have the most interest about communication efficiencies. In order to reduce a communication traffic, distributed consensus algorithms are proposed. In [62, 61, 64, 63], a distributed observer which are combined a consensus and Kalman filter are proposed. The consensus filter can calculate an average consensus value from the communication between neighbor nodes on the network. An estimate of the observer proposed by Olfati-Sabe is calculated from the Kalman filter based on the consensus value. A gossip algorithm [65] is also a distributed consensus algorithm for the sensor network. In this algorithm, each node selects received data from the neighbor nodes on the network at random. Finally, each node obtains the consensus value with less communication. In the case that there exist communication delays in the network, a data aggregation protocol is useful to collect the information of the other nodes. We can calculate the estimate from aggregated data by using a delay compensated observer. In addition, it is indicated in [67] that the data aggregation protocol can reduce a power consumption of each node.

Distributed estimation in sensor networks can be applied to distributed control systems. In the observer based distributed control, each estimator which is collocated with the actuator has to estimate the state without inputs of the other actuator except for own node. From these reasons, some unknown input observers have been proposed in the past studies. in [94], a delayed unknown input observer for a discrete-time system was proposed. The delayed unknown input observer provides only a delayed estimate. However, this observer can reconstruct the unknown inputs with delays. On the other hand, in [95, 96], the unknown input observers for a continuous-time system were proposed. These observer can calculate the estimate from the no delay output without some inputs. However, the unknown input observer proposed in [95, 96] cannot calculate the state in real time under the existence of the communication delay.

We proposed a data aggregation based estimation for the sensor network with the communication delay in [97]. In this work, it is assumed that each node is connected by the actuator network without the communication delay. Therefore, all nodes can obtain all input values of the other nodes and estimate the state from the delay compensated observer proposed in [54]. However, in many cases, the assumption of the actuator network without the communication delay is not satisfied. Thus, in this chapter, we consider unknown input observer problems in the sensor network with the communication delays. The communication delays depend on communication paths in the network. Therefore, we need a multi-delay compensated observer for the estimation on the network. The multi-delay compensated observer are also proposed in [66] except for [54]. However, both observers utilize a memory of the inputs to compensate the delay included in the outputs. Thus, instead of the unknown input, the alternative input is needed in the calculation of unknown input observers with the delay compensation. In this chapter, we propose an unknown input observer by using alternative inputs. In our method, all nodes calculate full states from the information obtained from the sensor network. It is assumed that an input of each node generated by an observer-based controller. Thus, each node calculates the input values from own estimate. Instead of the inputs generated by the other nodes, each node calculates the alternative inputs from own estimate based on the feedback controller of the other node given by the assumption. We can expect that the alternative inputs converge to a real input of each node if converge rates of each observer are sufficiently high. The estimate and the delay compensated term of each node is calculated from the alternative inputs instead of the unknown inputs. A stability of an estimation error of the proposed observer is proven by a Lyapunov-Krasovskii functional proposed in [98]. Finally, the stability condition of the proposed observer is given by LMIs.

This chapter is organized as follows. A problem formulation is explained in section 5.2. We explain the detail of the alternative input observer and show a main result in section 5.3. Results of a numerical simulation are shown in section 5.4. Section 5.5 gives a conclusion.

5.2 Problem Formulation

We consider a distributed network system showed by Fig. 5.1. The network has N nodes which include sensors and actuators. Each node collects the observed value of the other nodes via a network and estimates a state. An input of each node is calculated from the estimate of own node like Fig. 5.2. The dynamics of the plant in Fig. 5.1 can be represented by

$$\begin{aligned} \dot{x} &= Ax + \sum_{i=1}^{N} B_i u_i \\ y &= Cx \end{aligned} \tag{5.1}$$

where $x \in \mathbb{R}^n$ is a state and $u_i \in \mathbb{R}^{m_i}$ is a distributed input of node i, and $y \in \mathbb{R}^p$ is the output of the system. The output y consists of all redundant raw measurements. Therefore, $\operatorname{rank} C$ may be less than p, and p may be larger than n.

In order to represent the network, we utilize an undirected graph. The set of nodes is defined by $V := \{1, 2, ..., N\}$, and the set of edges is defined by $E \subseteq V \times V$. From V and E, the network is expressed by $G := (V, E)$. In this chapter, it is assumed that G is connected. If G is connected, there exists at least one tree $T := (V, \hat{E})$ which is a subgraph of G, where $\hat{E}$ is a set of the edges which satisfies $\hat{E} \subseteq E$.

The nodes which have own sensors provides its measurements to the sensor network. Let us define the scalable communication as follows.

Definition 5.1 (Scalable Communication on T)
When the dimension of communication data on all edges in $\hat{E}$ is independent of $N = |V|$, we call the communication scalable.

This paper considers aggregations of the sensor measurements by scalable communications. The redundant raw output y is aggregated into a q-dimensional vector

$$y_{\mathrm{agr}} = Fy = C_{\mathrm{agr}}x, \tag{5.2}$$

where $F \in \mathbb{R}^{q\times p}$ represents the relationship between y and y_{agr}, and $C_{\mathrm{agr}} = FC$. By renumbering the elements of y, we can decompose the matrix F as

$$F = [\ F_1 \,|\, \cdots \,|\, F_N\ ],$$

where F_i corresponds to the output of i-th node. Therefore, the outputs of the nodes, which are mapped to the aggregated output space, are defined as

$$\begin{aligned} y_1 &= [\ F_1 \,|\, 0 \,|\, \cdots \,|\, 0\ ]y = C_1 x \\ &\vdots \\ y_N &= [\ 0 \,|\, \cdots \,|\, 0 \,|\, F_N\ ]y = C_N x, \end{aligned} \tag{5.3}$$

where $C_i = [\ 0 \,|\, \cdots \,|\, 0 \,|\, F_i \,|\, 0 \,|\, \cdots \,|\, 0\ ]C$. If one can obtain the current outputs y_i with no transmission delay, the aggregated output of y_i coincides with y_{agr}, i.e.

$$y_{\mathrm{agr}} = \sum_{i=1}^{N} y_i. \tag{5.4}$$

We suppose that the physical meaning of measurements should be preserved in the aggregation. For example, the aggregations of positions and velocities are prohibited by this assumption. In addition, we also prohibit the aggregations which cancel the measurements each other.

The nodes which have actuators calculates input values from an estimate of the state. The estimate of node i is denoted by $\hat{x}_i$. Based on $\hat{x}_i$, the input of node i is given by

$$u_i = K_i\hat{x}_i. \tag{5.5}$$

The nodes which have no sensor provide no information to the sensor network and the corresponding F_i has no column. Therefore, we model them as the nodes whose output matrices are zero matrices, i.e. $C_i = 0$.

In the sensor network, each node communicates the information of the measurements to the other nodes via the paths in E. We define a set of the neighbor of node i connected by T like $J_i := \{j; (i,j) \in \hat{E}\}$. Node i can mutually communicate to the nodes included by J_i. Once a root of the undirected tree T is fixed, a parent node and child nodes of each node are also determined. The set of the child nodes of node j is represented by P_{ij}, where i is a fixed root node. In Fig. 4.2, if $i \neq 4, 8, 9$, the set of the child nodes in node 4 is $P_{i4} = \{8, 9\}$.

A communication delay from node i to j is denoted by $d_{(i,j)}$. In the case that $j \notin J_i$, $d_{(i,j)}$ represents a total delay on the simple path from node i to j. For example, $d_{(1,8)}$ in Fig. 4.2 is $d_{(1,8)} = d_{(1,2)} + d_{(2,4)} + d_{(4,8)}$.

Each node collects the observed data of the other node by a data aggregation protocol as

$$\bar{y}_{ij}(t) = y_i(t) + \sum_{k \in J_i \setminus \{j\}} \bar{y}_{ki}(t - d_{(k,i)}), \tag{5.6a}$$

$$\hat{y}_i(t) = y_i(t) + \sum_{j \in J_i} \bar{y}_{ji}(t - d_{(j,i)}), \tag{5.6b}$$

where $\bar{y}_{ij}$ is the send data from node i to j, and $\hat{y}_i$ is an aggregated output in node i. We can represent $\hat{y}_i$ as

$$\hat{y}_i(t) = \sum_{j=1}^{N} C_i x(t - d_{(i,j)}). \tag{5.7}$$

When the communication delay is sufficiently small to be ignored, $\hat{y}_i$ for all i becomes y_{agr}.

In addition, it is also assumed that node i cannot obtain the input of the other nodes. In this chapter, we solve Problem5.1 as a main problem.

Problem 5.1
Design the full state observer at all nodes by using only the information from own neighbor nodes.

Remark 5.1
In many cases, it is assumed that $d_{(i,j)} = d_{(j,i)}$. This is a natural assumption. However, our proposed method does not need this one.

5.3 Main Result

5.3.1 Alternative Input Observer with No Delays

Consider the following problem.

Problem 5.2
Solve Problem 5.1 under the assumption that the communication delay $d_{(i,j)}$ is 0.

In [96, 95], Problem 5.2 is considered as an unknown input observer problem and a linear system with unknown inputs is expressed as

$$\begin{aligned} \dot{x} &= Ax + Bu + Dv, \\ y &= Cx, \end{aligned} \tag{5.8}$$

where u is a known input and v is an unknown input. In the problem formulation of this chapter,

$$Bu = B_i u_i, D = \sum_{i \neq j} B_j, C = C_{\text{agr}}.$$

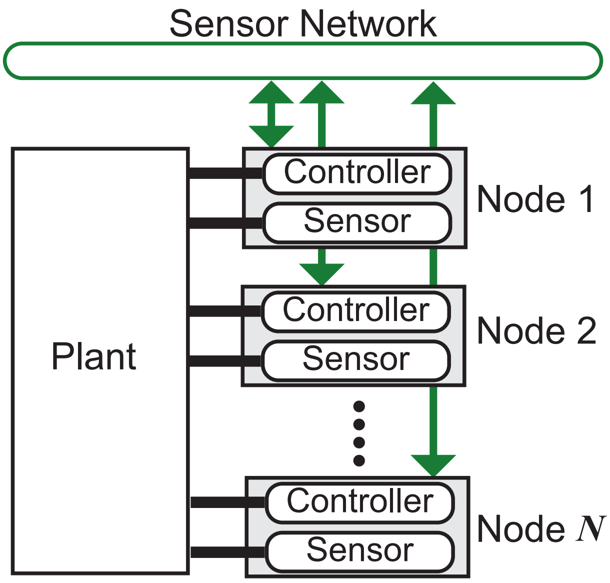

Figure 5.1: Distributed Network System

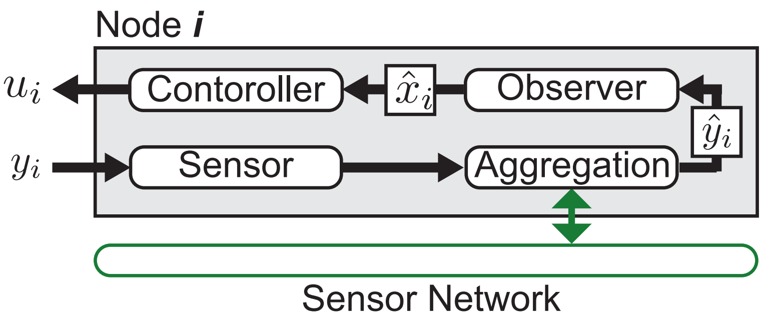

Figure 5.2: Detail of Each Node

The unknown input observer of system (5.8) can be expressed by

$$\begin{aligned} \dot{z} &= Nz + Ly + Gu, \\ \hat{x} &= z - Ey. \end{aligned} \tag{5.9}$$

In [96], a stability condition of the observer (5.9) is indicated as algebraic equations of N, L, G and E. On the other hand, in [95], the stability condition of the observer (5.9) is shown as the LMI. A difference between a general unknown input observer and our problem formulation is an assumption that the input of each node is given by (5.5). Note that the estimate of each node converges to $\hat{x}_1 = \cdots = \hat{x}_N = x$ if the estimation errors of each node are stable. If convergence rates of the estimation errors are sufficiently high, we can expect that each node can estimate the unknown input of the other node from own estimate. Therefore, we utilize an alternative input $\hat{u}_{ij}$ instead of u_j in the calculation of the observer on node i like Fig. 5.3. By using $\hat{x}_i$, the alternative input $\hat{u}_{ij}$ is calculated by

$$\hat{u}_{ij} = K_j \hat{x}_i.$$

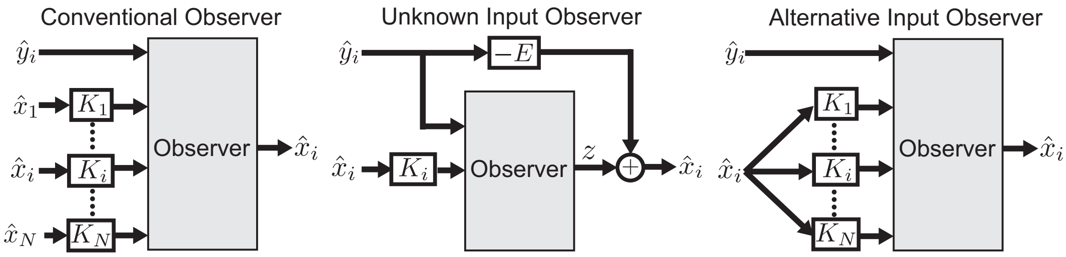

Figure 5.3: Conventional, unknown, and alternative input observers in node i

Based on the alternative inputs, the observer in node i can be expressed by

$$\dot{\hat{x}}_i = A\hat{x}_i + B_i u_i + \sum_{i \neq j} B_i \hat{u}_{ij} + L_i(\hat{y}_i - C_{\mathrm{agr}}\hat{x}_i). \tag{5.10}$$

The stability condition of the estimation error on (5.10) is given by Theorem 5.1.

Let $\bar{B}_i$ be a matrix in which $B_j K_j$ are horizontally arranged except for ith block element. In the $\bar{B}_i$, ith block element is the negative sum of $B_j K_j$ except for $B_i K_i$. Thus, $\bar{B}_i$ can be expressed by

$$\bar{B}_i = \left[B_1 K_1, \cdots, B_{i-1} K_{i-1}, -\sum_{i \neq j} B_j K_j, \cdots, B_N K_N \right].$$

For example, if $N = 3$, $\bar{B}_2$ can be expressed by

$$\bar{B}_2 = [B_1 K_1, -(B_1 K_1 + B_3 K_3), B_3 K_3].$$

By using $\bar{B}_i$, $\bar{A}$ is defined by

$$\bar{A} = \mathrm{diag}\,(A)_{i \in V} - \begin{pmatrix} \bar{B}_1 \\ \vdots \\ \bar{B}_N \end{pmatrix}.$$

Theorem 5.1

If there exist $P_i \in \mathbb{S}^n_{++}$ and the observer gains L_i which satisfy

$$\mathrm{He}\left(\mathrm{diag}\,(P_i)_{i \in V}\,(\bar{A} - \mathrm{diag}\,(L_i C_{\mathrm{ide}})_{i \in V})\right) < 0, \tag{5.11}$$

the estimation error $\xi_i = x - \hat{x}_i$ of the observer (5.10) converges zero.

Proof

Let $\xi \in \mathbb{R}^{nN}$ be a vector which is expressed by

$$\xi = [\xi_1^T, \cdots, \xi_N^T]^T.$$

By using ξ, the error dynamics of (5.10) can be expressed by

$$\dot{\xi} = (\bar{A} - \mathrm{diag}\left(L_i C_{\mathrm{agr}}\right)_{i \in V})\xi.$$

We consider a Lyapunov function candidate V as

$$V = \xi^T \operatorname{diag}(P_i)_{i \in V}\, \xi.$$

By time differentiating V, we can get

$$\dot{V} = \xi^T \operatorname{He}\left(\operatorname{diag}(P_i)_{i \in V}\, (\bar{A} - \operatorname{diag}(L_i C_{\mathrm{ide}})_{i \in V})\right) \xi.$$

Therefore, we can get Theorem 5.1 as a stability condition of the estimation error of the observer (5.10).

Both the alternative input observer (5.10) and the unknown input observer (5.9) are also solutions of Problem 5.2. However, the unknown input observer cannot provide the estimate in real time for the network with the communication delay. Therefore, we propose a distributed observer with the delay compensation by using the alternative inputs in next subsection.

5.3.2 Alternative Input Observer with Delay Compensation

An observer with sensor delays proposed by Watanabe [54]. They indicate that a delayed output $C_i x(t - d_{(i,j)})$ can be compensated by adding delay compensation term $\Xi(t)$, which can be expressed by

$$C_i x(t - d_{(i,j)}) + \Xi(t) = C_i e^{-Ad_{(i,j)}} x(t), \tag{5.12a}$$

$$\Xi(t) = C_i e^{-Ad_{(i,j)}} \int_{t-d_{(i,j)}}^{t} e^{A(t-\tau)} \sum_{i=1}^{N} B_i u_i(\tau) d\tau. \tag{5.12b}$$

Each node obtains the all observed data of the other node via the aggregated data (5.6a) with multiple delays which depend on communication paths. Therefore, a distributed calculation of the delay compensation terms is necessary. In addition, the unknown input is needed to execute the calculation of the delay compensation in each node. In this paper, we utilize the alternative input in the calculation of the delay compensation. The delay compensation term sent from node i to j can be expressed by

$$\begin{aligned}\bar{\Xi}_{ij}(t) = \sum_{k \in J_i \setminus \{j\}} \Bigg(& \bar{\Xi}_{ki}(t - d_{(k,i)}) \\ & + \bar{C}_{(k,i)} e^{-Ad_{(k,i)}} \int_{t-d_{(k,i)}}^{t} e^{A(t-\tau)} \sum_{l=1}^{N} B_l \hat{u}_{il}(\tau) d\tau \Bigg),\end{aligned}$$

where $\bar{C}_{(i,j)}$ is a matrix defined by

$$\bar{C}_{(i,j)} = C_i + \sum_{k \in J_i \setminus \{j\}} \bar{C}_{(k,i)} e^{-Ad_{(k,i)}}. \tag{5.13}$$

Note that $\hat{u}_{ii} = u_i$ and the calculation of (5.13) can be executed in off line. Each node calculates $\hat{\Xi}_i$ as

$$\begin{aligned}\hat{\Xi}_i(t) = \sum_{j \in J_i} \Bigg(& \bar{\Xi}_{ji}(t - d_{(j,i)}) \\ & + \bar{C}_{(j,i)} e^{-Ad_{(j,i)}} \int_{t-d_{(j,i)}}^{t} e^{A(t-\tau)} \sum_{k=1}^{N} B_i \hat{u}_{ik}(\tau) d\tau \Bigg).\end{aligned} \tag{5.14}$$

By using $\hat{y}_i$, $\hat{\Xi}_i$ and the alternative inputs, the observer of node i can be expressed by

$$\begin{aligned}\dot{\hat{x}}_i(t) = & A\hat{x}_i(t) + \sum_{j=1}^{N} B_j \hat{u}_{ij}(t) \\ & + L_i \left(\hat{y}_i(t) + \hat{\Xi}_i(t) - \hat{C}_i \hat{x}_i(t) \right),\end{aligned} \tag{5.15}$$

where $\hat{C}_i$ is a matrix given by

$$\hat{C}_i = C_i + \sum_{j \in J_i} \bar{C}_{(j,i)} e^{-Ad_{(j,i)}}.$$

In the case that each node can obtain all inputs in real time, the estimation error of the observer (5.15) is asymptotically stable if and only if $A - L_i \hat{C}_i$ is Hurwitz. On the other hands, the stability condition of the estimation error for (5.15) under existence of the alternative inputs is given by Theorem 5.2.

Let $\bar{E}(i)$ be set of edges whose elements are a part of simple paths from node j to i for all $j \neq i$. For example, $\bar{E}(1)$ in Fig. 4.2 is given by

$$\begin{aligned}\bar{E}(1) = \{ & (2,1), (3,1), (4,2), (5,2), (6,3), \\ & (7,3), (8,4), (9,4)\}.\end{aligned}$$

The density of $\bar{E}(i)$ is $N-1$ regardless of i. For each i, we define $P_i \in \mathbb{S}^n_{++}$ and $M_i \in \mathbb{R}^{n \times p} = P_i L_i$. In addition, for each i and $e \in \bar{E}(i)$, the matrices ${}^iQ_e \in \mathbb{S}^{nN}_{++}$, ${}^iW_e \in \mathbb{S}^{nN}_{++}$ and ${}^iR_e \in \mathbb{S}^n_{++}$ are defined. By using ${}^iR_e$, ${}^i\Omega_e$ is given by

$${}^i\Omega_{e=(j,k)} = \int_0^{d_e} e^{A\theta} \bar{B}_j {}^iR_e^{-1} \bar{B}_j^T e^{A^T\theta} d\theta. \tag{5.16}$$

From $\bar{A}$, P_i, M_i, $\hat{C}_i$ and ${}^iQ_e$, the matrix Φ is calculated by

$$\begin{aligned}\Phi = & \operatorname{He}\left(\operatorname{diag}(P_i)_{i \in V} \bar{A} - \operatorname{diag}\left(M_i \hat{C}_i \right)_{i \in V} \right) \\ & + \sum_{i=1}^{N} \sum_{e \in \bar{E}(i)} {}^iQ_e.\end{aligned}$$

We set $\tilde{C}_i$, ${}^i\Omega$ as

$$\tilde{C}_i = \operatorname{row}\left(\bar{C}_e \right)_{e \in \bar{E}(i)}, {}^i\Omega = \operatorname{diag}\left({}^i\Omega_e \right)_{e \in \bar{E}(i)}.$$

Let us consider the function

$$V_1 = \xi^T(t)\mathrm{diag}\,(P_i)_{i\in V}\,\xi(t).$$

Then the following lemma helds.

Lemma 5.1
The time differentiation of V_1 satisfies the following inequality:

$$\begin{aligned}
\dot{V}_1 &\le \xi^T(t)\Lambda\xi(t) \\
&+ \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \xi^T(t)P_iL_i\bar{C}_e{}^i\Omega_e\bar{C}_e^T L_i^T P_i^T \xi(t) \\
&+ \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \int_{t-d_{(k,i)}}^{t-d_{(j,i)}} \xi^T(\tau)\bar{B}_j^{T\,i}R_e\bar{B}_j\xi(\tau)d\tau, \\
\Lambda &= \mathrm{He}\left(\mathrm{diag}\,(P_i)_{i\in V}\left(\bar{A} - \mathrm{diag}\left(L_i\hat{C}_i\right)_{i\in V}\right)\right).
\end{aligned}$$

Proof
Let ${}^i\hat{\Xi}_{jk}$ be a delay compensated term sent from node j to k when node i is root node.The term ${}^i\hat{\Xi}_{jk}$ can be expressed by

$$\begin{aligned}
{}^i\hat{\Xi}_{jk}(t) = \sum_{l\in P_{ij}} \Bigg(& {}^i\hat{\Xi}_{lj}(t - d_{(l,j)}) \\
&+ \bar{C}_{(l,j)}e^{-Ad_{(l,j)}} \int_{t-d_{(l,j)}}^{t} e^{A(t-\tau)} \sum_{p=1}^{N} B_p\hat{u}_{jp}(\tau)d\tau \Bigg).
\end{aligned} \tag{5.17}$$

In the protocol (5.14), each node does not need to distinguish where node is a root. For example, ${}^i\hat{\Xi}_{13}, i = 3, 6, 7$ are equal to $\bar{\Xi}_{13}$ in Fig. 4.2. But the notation of (5.14) is not convenient for the proof, we use equation (5.17) in this subsection. Note that ${}^i\hat{\Xi}_{ii} = \hat{\Xi}_i$. Let an error between actual inputs and the alternative input be $\tilde{u}_{ij} = u_j - \hat{u}_{ij}$. By substituting $\hat{u}_{ij} = u_j - \tilde{u}_{ij}$ to the equation (5.17), we can get

$$\begin{aligned}
{}^i\hat{\Xi}_{jk}(t) &= {}^i\Xi_{jk}(t) + {}^i\tilde{\Xi}_{jk}(t), \\
{}^i\Xi_{jk}(t) &= \sum_{l\in P_{ij}} \Bigg({}^i\Xi_{lj}(t - d_{(l,j)}) \\
&\quad + \bar{C}_{(l,j)}e^{-Ad_{l,j}} \int_{t-d_{(l,j)}}^{t} e^{A(t-\tau)} \sum_{p=1}^{N} B_pu_p(\tau)d\tau \Bigg), \\
{}^i\tilde{\Xi}_{jk}(t) &= \sum_{l\in P_{ij}} \Bigg({}^i\tilde{\Xi}_{lj}(t - d_{(l,j)}) \\
&\quad - \bar{C}_{(l,j)}e^{-Ad_{(l,j)}} \int_{t-d_{(l,j)}}^{t} e^{A(t-\tau)} \sum_{p=1}^{N} B_p\tilde{u}_{jp}(\tau)d\tau \Bigg).
\end{aligned}$$

The term ${}^{i}\Xi_{jk}$ means the actual delay compensation term from node j to k. On the other hands, ${}^{i}\tilde{\Xi}_{jk}$ means the error between the actual delay compensation term and the one based on the alternative inputs. Therefore, ${}^{i}\Xi_{ii}$ can be expressed by

$$ {}^{i}\Xi_{ii}(t) = \sum_{j=1}^{N} C_i e^{-Ad_{(j,i)}} \int_{t-d_{(j,i)}}^{t} e^{A(t-\tau)} \sum_{k=1}^{N} B_k u_k(\tau) d\tau. $$

By adding ${}^{i}\Xi_{ii}$ to $\hat{y}_i$, we can compensate the communication delays as

$$ \hat{y}_i(t) + {}^{i}\Xi_{ii}(t) = \hat{C}_i x(t). $$

Let $\tilde{\Xi}$ be $\tilde{\Xi} = [{}^{1}\tilde{\Xi}_{11}^{T}, \cdots, {}^{N}\tilde{\Xi}_{NN}^{T}]^{T}$. Note that the following equation is held:

$$ \sum_{j=1}^{N} B_j \tilde{u}_{ij} = -\bar{B}_i \xi. $$

Thus, the dynamics of estimation error ξ for observer (5.15) can be expressed by

$$ \dot{\xi}(t) = \left(\bar{A} - \mathrm{diag}\left(L_i \hat{C}_i\right)_{i\in V}\right) \xi(t) - \mathrm{diag}\left(L_i\right)_{i\in V} \tilde{\Xi}(t), \tag{5.18a} $$

$$ \begin{aligned} {}^{i}\tilde{\Xi}_{jk}(t) = \sum_{l\in P_{ij}} \Bigg(& {}^{i}\tilde{\Xi}_{lj}(t - d_{(l,j)}) \\ & + \bar{C}_{(l,j)} e^{-Ad_{(l,j)}} \int_{t-d_{(l,j)}}^{t} e^{A(t-\tau)} \bar{B}_j \xi(\tau) d\tau \Bigg). \end{aligned} \tag{5.18b} $$

The time differentiation of V_1 can be expressed by

$$ \dot{V}_1 = \xi(t)^T \Lambda \xi(t) - 2 \sum_{i=1}^{N} \xi(t)^T P_i L_i {}^{i}\tilde{\Xi}_{ii}(t). \tag{5.19} $$

By substituting (5.18b) to the second term of the right side on equation (5.19), we can get

$$ \begin{aligned} & - 2\xi^T(t) P_i L_i {}^{i}\tilde{\Xi}_{ii}(t) \\ & = - 2 \sum_{j\in P_{ii}} \xi^T(t) P_i L_i {}^{i}\tilde{\Xi}_{ji}(t - d_{(j,i)}) \\ & \quad - 2 \sum_{j\in P_{ii}} \int_{t-d_{j,i}}^{t} \xi^T(t) P_i L_i \bar{C}_{(j,i)} e^{A(t-\tau-d_{ij})} \bar{B}_i \xi(\tau) d\tau. \end{aligned} \tag{5.20} $$

Let a and b be $a, b \in \mathbb{R}^n$ and R be $R \in \mathbb{S}_{++}^{n}$. For a, b and R, the following inequality holds:

$$ 2a^T b \le a^T R^{-1} a + b^T R b. $$

Therefore, the integral term in (5.20) can be expressed by

$$\begin{aligned}
&-2\int_{t-d_{(j,i)}}^{t} \xi^T(t)P_iL_i\bar{C}_{(j,i)}e^{A(t-\tau-d_{(j,i)})}\bar{B}_i\xi(\tau)d\tau \\
&\leq 2\int_{t-d_{(j,i)}}^{t} |\xi^T(t)P_iL_i\bar{C}_{(j,i)}e^{A(t-\tau-d_{(j,i)})}\bar{B}_i\xi(\tau)|d\tau \\
&\leq \xi^T(t)P_iL_i\bar{C}_{(j,i)}{}^i\Omega_{(j,i)}\bar{C}_{(j,i)}^TL_i^TP_i^T\xi(t) \\
&+\int_{t-d_{(j,i)}}^{t} \xi^T(\tau)\bar{B}_i^T{}^iR_{(j,i)}\bar{B}_i\xi(\tau)d\tau.
\end{aligned}$$

The first term in (5.20) can be expressed by

$$\begin{aligned}
&-2\xi^T(t)P_iL_i{}^i\tilde{\Xi}_{ji}(t-d_{(j,i)}) \\
&=-2\sum_{k\in P_{ij}}\xi^T(t)P_iL_i{}^i\tilde{\Xi}_{kj}(t-d_{(k,i)}) \\
&-2\sum_{k\in P_{ij}}\int_{t-d_{(k,i)}}^{t-d_{(j,i)}}\xi^T(t)P_iL_i\bar{C}_{(k,j)}e^{A(t-\tau-d_{(k,i)})}\bar{B}_j\xi(\tau)d\tau.
\end{aligned} \tag{5.21}$$

The integral term in (5.21) is also expressed by

$$\begin{aligned}
&-2\int_{t-d_{(k,i)}}^{t-d_{(j,i)}} \xi^T(t)P_iL_i\bar{C}_{(k,j)}e^{A(t-\tau-d_{(k,i)})}\bar{B}_j\xi(\tau)d\tau \\
&\leq \xi^T(t)P_iL_i\bar{C}_{(k,j)}{}^i\Omega_{(k,j)}\bar{C}_{(k,j)}^TL_i^TP_i^T\xi(t) \\
&+\int_{t-d_{(k,i)}}^{t-d_{(j,i)}} \xi^T(\tau)\bar{B}_j^T{}^iR_{(k,j)}\bar{B}_j\xi(\tau)d\tau.
\end{aligned}$$

If node j is the leaf node, ${}^i\tilde{\Xi}_{kj}(t)$ satisfies that ${}^i\tilde{\Xi}_{kj}(t)=0$. Thus, $\dot{V}_1$ is recursively given by

$$\begin{aligned}
\dot{V}_1 \leq&\xi^T(t)\Lambda\xi(t) \\
&+\sum_{i=1}^{N}\sum_{e=(k,j)\in\bar{E}(i)}\xi^T(t)P_iL_i\bar{C}_e{}^i\Omega_e\bar{C}_e^TL_i^TP_i^T\xi(t) \\
&+\sum_{i=1}^{N}\sum_{e=(k,j)\in\bar{E}(i)}\int_{t-d_{(k,i)}}^{t-d_{(j,i)}}\xi^T(\tau)\bar{B}_j^T{}^iR_e\bar{B}_j\xi(\tau)d\tau.
\end{aligned} \tag{5.22}$$

For the stability analysis of the estimation error $\xi_i = x-\hat{x}_i$ of (5.15), let us consider $V = V_1+V_2+V_3$ as a Lyapunov Krasovskii functional candidate, where V_2 and V_3 are expressed by

$$V_2 = \sum_{i=1}^{N}\sum_{e=(k,j)\in\bar{E}(i)}\int_{t-d_{(k,i)}+d_e}^{t}\xi^T(\tau){}^iQ_e\xi(\tau)d\tau, \tag{5.23a}$$

$$\begin{aligned}
V_3 &= \sum_{i=1}^{N}\sum_{e=(k,j)\in\bar{E}(i)}\int_{-d_e}^{0}\int_{t-d_{(k,i)}+d_e+\theta}^{t-d_{(k,i)}+d_e}{}^iV_e(\tau)d\tau d\theta \\
{}^iV_e(\tau) &= \xi^T(\tau){}^iW_e\xi(\tau).
\end{aligned} \tag{5.23b}$$

Then, the stability condition of (5.15) is obtained by the following theorem.

Theorem 5.2

If there exit the matrices P_i, M_i, ${}^iQ_e$, ${}^iW_e$ and ${}^iR_e$ which satisfy

$$\begin{pmatrix} \mathrm{diag}\left({}^i\Omega^{-1}\right)_{i\in V} & \mathrm{diag}\left(\tilde{C}_i^T M_i^T\right)_{i\in V} \\ \mathrm{diag}\left(M_i\tilde{C}_i\right)_{i\in V} & -\Psi \end{pmatrix} > 0 \tag{5.24a}$$

$$d_e{}^iW_e - {}^iQ_e < 0 \tag{5.24b}$$

$$\bar{B}_j^T{}^iR_e\bar{B}_j - {}^iW_e < 0, \tag{5.24c}$$

the estimation error $\xi_i = x - \hat{x}_i$ of (5.15) is stable.

Proof

The time differentiation of V_2 and V_3 are given by

$$\begin{aligned}\dot{V}_2 = \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \Big(&\xi^T(t){}^iQ_e\xi(t) \\ &- \xi^T(t-d_{(j,i)}){}^iQ_e\xi(t-d_{(j,i)})\Big)\end{aligned} \tag{5.25a}$$

$$\begin{aligned}\dot{V}_3 = \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \Bigg(&d_e\xi^T(t-d_{(j,i)}){}^iW_e\xi(t-d_{(j,i)}) \\ &- \int_{t-d_{(k,i)}}^{t-d_{(j,i)}} \xi^T(\tau){}^iW_e\xi(\tau)d\tau\Bigg).\end{aligned} \tag{5.25b}$$

From Lemma 5.1 and the equation (5.25), we can get the inequality about $\dot{V}$ as

$$\begin{aligned}\dot{V} &\leq \xi^T(t)\Psi\xi(t) \\ &+ \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \xi^T(t-d_{(j,i)})(d_e{}^iW_e - {}^iQ_e)\xi(t-d_{(j,i)}) \\ &+ \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} \int_{t-d_{(k,i)}}^{t-d_{(j,i)}} \xi^T(\tau)(\bar{B}_j^T{}^iR_e\bar{B}_j - {}^iW_e)\xi(\tau)d\tau,\end{aligned}$$

where Ψ is given by

$$\Psi = \Phi + \sum_{i=1}^{N} \sum_{e=(k,j)\in\bar{E}(i)} P_iL_i\bar{C}_e{}^i\Omega_e\bar{C}_e^TL_i^TP_i^T.$$

The time differentiation of V satisfies $\dot{V} < 0$ except for $\xi = 0$ if (5.24b), (5.24c) and the following matrix inequalities holds:

$$\Psi < 0. \tag{5.26}$$

By applying Schur complements to (5.26), (5.24a) is obtained.

The equation (5.24) is not the LMI for P_i, M_i, ${}^iQ_e$, ${}^iW_e$ and ${}^iR_e$. However the equation (5.16) can be transformed into

$$\mathrm{He}\left(A{}^i\Omega_e\right) = e^{Ad_e}{}^iR_e^{-1}e^{A^Td_e} - {}^iR_e^{-1}.$$

Therefore, the equation (5.24) becomes the LMI for P_i, M_i, ${}^iQ_e$, ${}^iW_e$, ${}^iR_e$ and ${}^i\Omega_e$ with equality constraints. However, in the numerical calculation of a semi-definite programming problem, the equality constraint is undesirable. Thus, it is convenient that ${}^iR_e$ is given in the actual calculation.

From the equation (5.11), we can confirm that if we can select the observer gains of each node sufficiently high, the estimation error of each node converges to 0 in the no delay case. This result is not contradictory to the motivation of the introduction to the alternative inputs. In the existence of the communication delay, the stability condition (5.24a) can be transformed into the equation (5.26). From the equation (5.26), it is also necessary that the observer gains of each node are high to estimate the state. On the other hand, it is also confirm that when we raise the observer gain too high, the equation (5.26) is not satisfied. Thus, there exist some systems which has no solution of (5.24). Qualitative characteristics of the infeasible condition of (5.24) are described in section 5.4 from results of numerical simulations.

5.4 Numerical Simulation

In this section, we consider the liner system with 10 nodes which can be expressed by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0.050 & -0.105 \\ 0.015 & 0.085 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \sum_{i=1}^{10} B_i u_i. \tag{5.27}$$

The network which observes and controls the system (5.27) has a topology shown by Fig. 5.4. The odd-numbered nodes can measure x_1 and inject the input to x_2. On the other hand, the even-numbered nodes can measure x_2 and inject the input to x_1. Therefore, input and output matrices of each node are given by

$$B_i = \begin{cases} \begin{pmatrix} 1 & 0 \end{pmatrix}^T & \text{if} \quad i \equiv 1 \mod 2 \\ \begin{pmatrix} 0 & 1 \end{pmatrix}^T & \text{if} \quad i \equiv 0 \mod 2 \end{cases}$$

$$C_i = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} & \text{if} \quad i \equiv 1 \mod 2 \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \text{if} \quad i \equiv 0 \mod 2. \end{cases}$$

The communication delays on the network is uniformly 0.25. The feedback gains of each node K_i are determined such that the eigenvalues of following matrix (5.28) becomes $(-0.05, -0.1)$.

$$A + \sum_{i=1}^{10} B_i K_i \tag{5.28}$$

We give the parameter matrices ${}^iR_e$ as uniformly $I_2 \times 0.001$. Under the above conditions, we solve the LMI (5.24) to find the observer gains of each node. We can confirm that some observer gain become height compared to system matrix of (5.27). For example, we can get the gain for

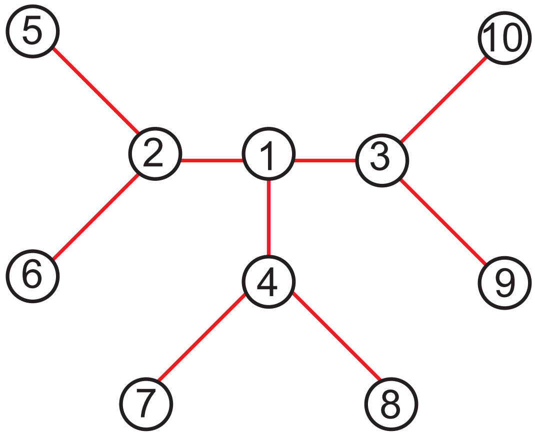

Figure 5.4: Topology of Network

the 2-th node as

$$\begin{pmatrix} 36.9160 & -10.9542 \\ -10.0549 & 2.9931 \end{pmatrix}.$$

Time responses of the estimation errors on x_1 and x_2 are shown in Fig. 5.5 and 5.6. It is also confirmed that the estimation errors converge to 0.

From a series of the other numerical simulations, we conclude that the condition (5.24) is very severe for the system condition. Of course, there exist the cases that the LMI (5.24) becomes feasible like above example. However, in many case, for example the case that a feedback gain is high, the system matrix A has a large unstable eigenvalue or the communication delay is large, the LMI (5.24) becomes infeasible. On the other hand, if the communication delay is enough small to be ignored, we can estimate the state from the alternative input observer with high gains. In addition, if each node can obtain all input of the other node, the observer (5.15) can estimate state if and only if $(A, \hat{C}_i)$ is a observable pair. Therefore, it is important for us to collect either the input or the output in real time to estimate the state without delays.

5.5 Conclusion

In this chapter, we proposed the alternative input observer for the sensor network with the communication delays. Each node estimates the state and calculate the input from own estimate. In the calculation of the observer and the delay compensation, each node utilizes the alternative input which is calculated from own estimate instead of input of the other nodes. The stability condition of the alternative input observer is given by the LMI.

Through the numerical experiments, it is confirmed that the stability condition that we proposed is sensitive against the system condition. We introduce the stability condition from the Lyapunov-Krasovskii functional analysis, which means the result of this paper is conservative. We may be able to relax the stability condition of alternative input observer under existence of

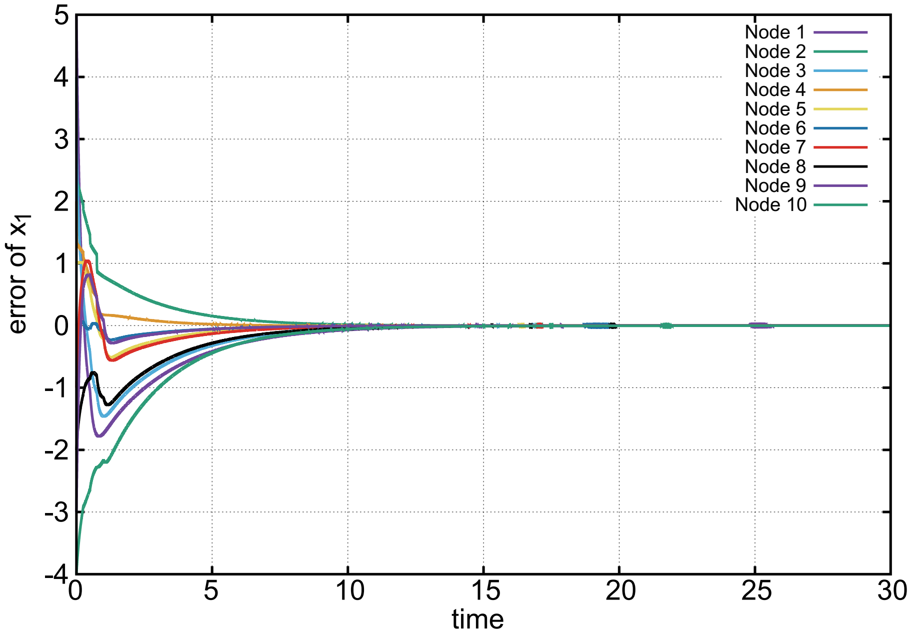

Figure 5.5: Time Response of Estimation Error on x_1

communication delay. However, we conclude that it is important for us to collect either the input or the output in real time to estimate the state without the delays.

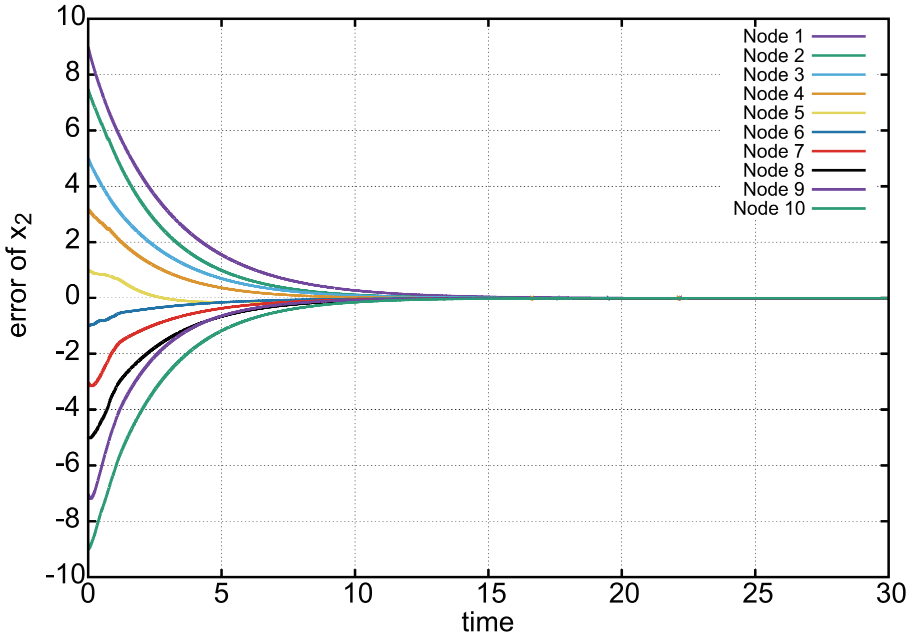

Figure 5.6: Time Response of Estimation Error on x_2

Chapter 6. Conclusion

This thesis discusses state estimation problem of cyber physical systems for the next generation. In particular, an estimation of a quadrotor UAV and a distributed estimation over delayed sensor networks are selected as main problmes of the cyber physical systems.

Chapter 3 proposes a maximum-likelihood-estimation method for a quadrotor UAV given the existence of sensor delays. Since state equation of the UAV is given by nonlinear, an approximated extimation method that has filter and predictor are required. The filter of the proposed mehtod estimates the past state based on the delayed output through an extended Kalman filter. The predictor involves calculating an estimate of the present state by simulating the original system from the past to the present. It is proven that the proposed method provides an approximated maximum-likelihood-estimation. The effectiveness of the estimator is verified by performing experiments. A future work will involve considering a particle filter based observer with delay compensation for cases with large-amplitude noises.

Chapter 4 propose a distributed delay-compensated observer over wireless sensor network. To reduce the communication amount, data aggregation of measurements are utilized. In the data aggregation of each node, communication delays are compensated by using given system dynamics and stored inputs. Therefore, all node can synchronize sensor measurements by using the scalable and local communication in real-time. All nodes estimate the state variables of a system simultaneously. Convergence rates of the proposed observer can be arbitrarily designed regardless of communication delays. The effectiveness of the proposed method is verified by numerical simulation. Future works involve analyzing the effect of data-aggregation against reductions of energy consumption and noise affects.

In Chapter 5, we extend the result in Chapter 4 into an unknown-input observer for distributed network systems under existence of communication delays. The proposed method provides the distributed observer over the delayed sensor networks, which requires only the input generated by own node. Other input values are replaced by surmised values using own state estimate. State-estimation error of a node affects the delay compensation, which is used by the observers in the other nodes, and therefore the stability analysis of the proposed method is not straightforward. In this chapter, a stability condition is obtained via Lyapunov-Krasovski functional, and the parameters of the distributed observer over the delayed sensor network, which assure the stability, are obtained as a solution of a semidefinite programming. Finally, a result of a numerical simulation is shown to verify the effectiveness of the proposed method.

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List of Publication of the Author

Journal Papers

1. Ryosuke Adachi and Yuh Yamashita,"Delay-Compensated Maximum-Likelihood-Estimation Method and its Application for Quadrotor UAVs", IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, Vol.101-A, N0.4, pp.678-684, 2018.

Reviewed Conference Papers

1. Ryosuke Adachi and Yuh Yamashita: Delay-Compensated Maximum Likelihood Estimation Method for Quadrotor UAV, 2015 IEEE Multi-Conference on Systems and Control, pp. 601-606, 2015, Sydney, Australia.

2. Ryosuke Adachi, Yuh Yamashita and Koichi Kobayashi: Data Aggregation based Estimation for Sensor Network with Communication Delay, Proc. of the 20th IFAC World Congress, pp. 2543-2548, Toulouse, 2017.

3. Ryosuke Adachi, Yuh Yamashita and Koichi Kobayashi: Observer Design for Distributed Network Systems with Communication Delays, Proc. of the 5th IFAC Conference on Analysis and Control of Chaotic Systems, pp. 191-196, Eindhoven, 2018.