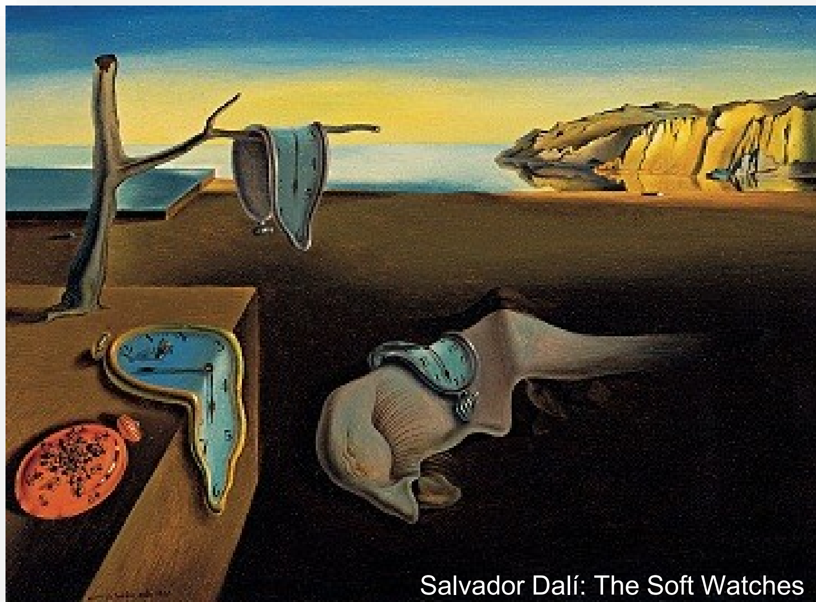
Units in the changing times

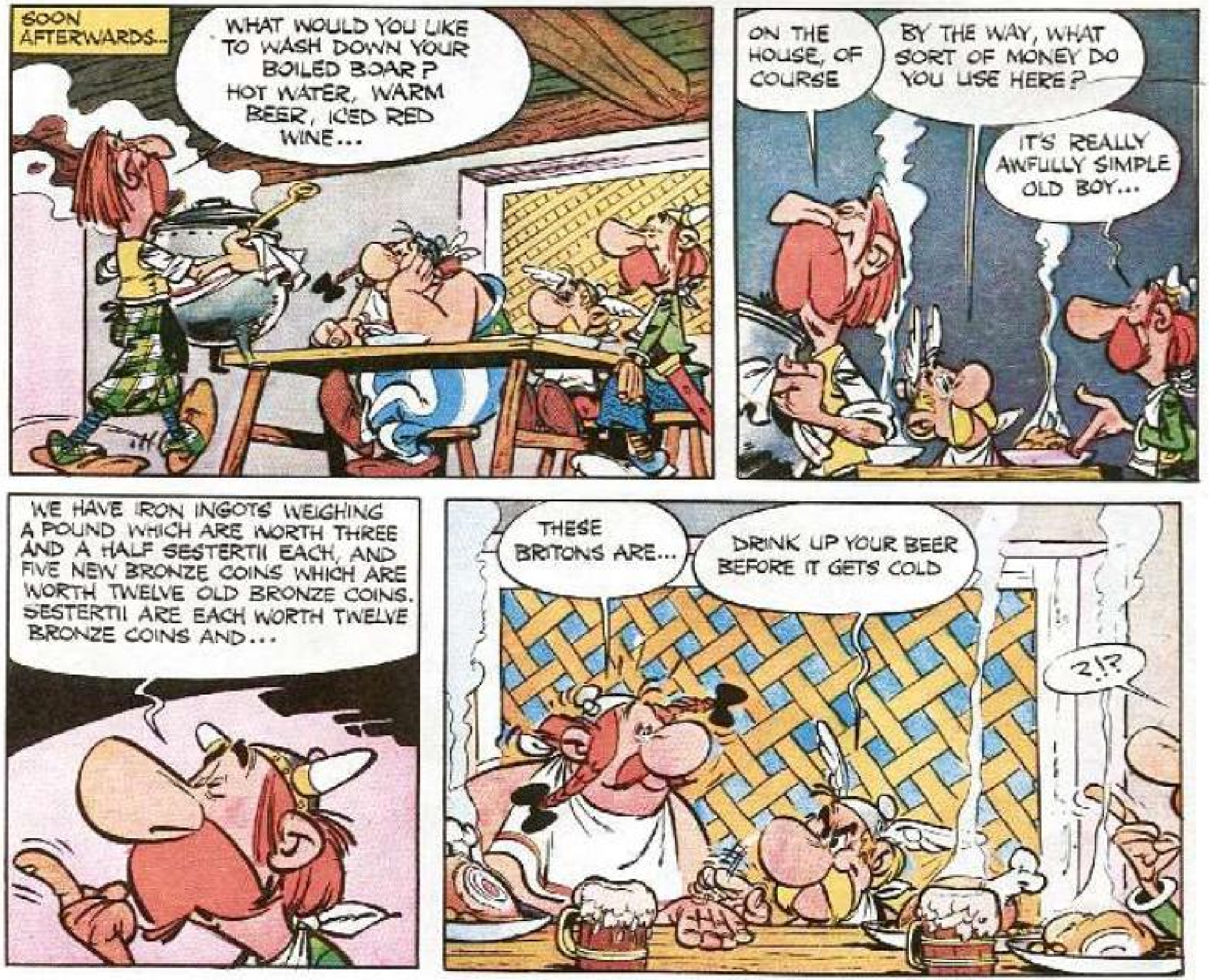
On the occasion of the 2019 redefinition of the SI base units


Salvador Dalí: The Soft Watches

Ralf Greve

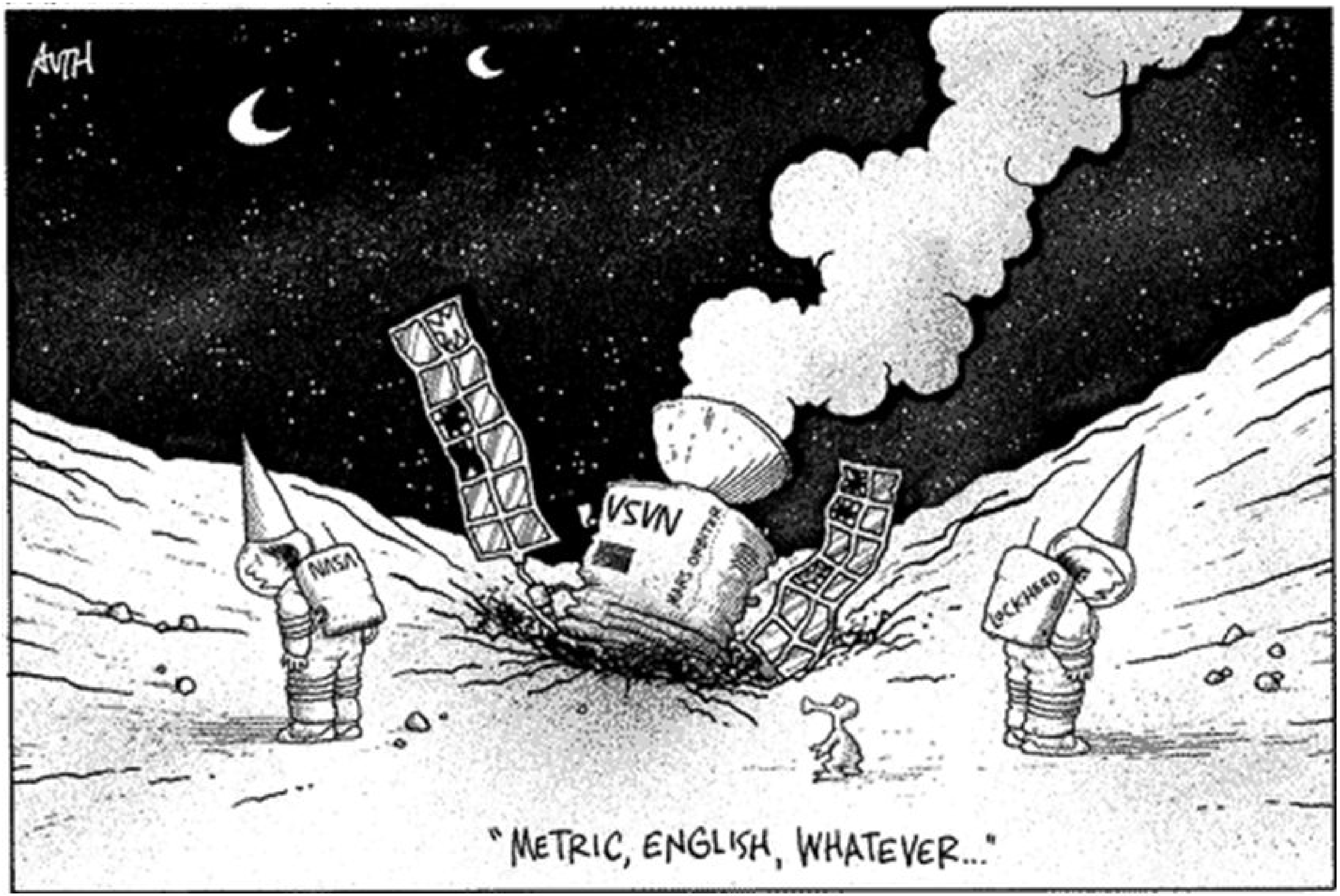
Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan

“These Britons are crazy” (~ 50 BCE)



Goscinny/Uderzo (1966): Asterix in Britain

Mars Climate Orbiter lost in translation (1999)



Newspaper cartoon. Source: Slideplayer.com

Metric system introduced in Revolutionary France (1790s)

Six new decimal units:

- Metre, for length

  1/10,000,000 of the distance between the North Pole and the Equator through Paris

- Are (100 m^2), for area of land
- Stère (1 m^3), for volume of firewood
- Litre (1 dm^3), for volumes of liquid
- Gram, for mass

  Mass of one cubic centimetre of water

- Franc, for currency

Decimal multiples by Greek prefixes (kilo-, centi-, milli-, etc.)

SI base units

SI (Système International d'unités):
Modern form of the metric system, established in 1960

Second	(s)	for time
Metre	(m)	for length
Kilogram	(kg)	for mass
Ampere	(A)	for electric current
Kelvin	(K)	for temperature
Mole	(mol)	for amount of substance (since 1971)
Candela	(cd)	for luminous intensity

Second

- Historically defined as 1/86400 of a mean solar day.

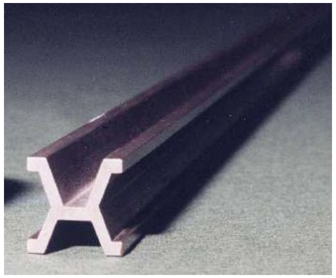
- Ephemeris second (1956):
  Fraction 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hours ephemeris time.

- Atomic second (1967 – today):
  Duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom
  (at a temperature of 0 K).

Can be interpreted as fixing the value of a physical constant:
Caesium frequency $\Delta\nu_{\mathrm{Cs}} = 9{,}192{,}631{,}770\ \mathrm{s}^{-1}$.

Metre

- Historically: 1/10,000,000 of the distance ...
- International prototype metre (1889, 1927): Distance, at 0°C, between the two lines on the standard bar of platinum–iridium kept at the International Bureau of Weights and Measures (BIPM) in Paris.



Credit: BIPM

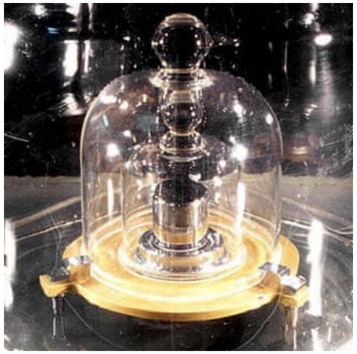
- Wavelength definition (1960): 1,650,763.73 wavelengths of the orange-red emission line in the electromagnetic spectrum of the krypton-86 atom in a vacuum.
- Fixed value of the speed of light (1983–today):

$$c = 299{,}792{,}458 \text{ m s}^{-1}.$$

Kilogram

- Historically: Mass of one litre of distilled water at 0°C.

- International prototype kilogram (1889): Mass of the platinum–iridium cylinder kept at the International Bureau of Weights and Measures in Paris.



Credit: AFP/Getty Images

- Fixed value of the Planck constant (2019):

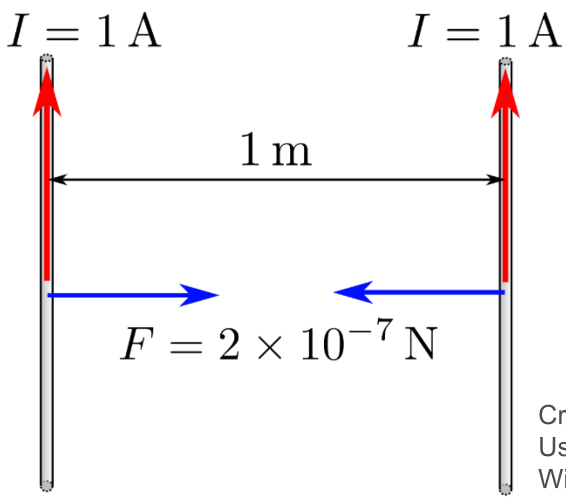
$$h = 6.62607015 \times 10^{-34}\ \mathrm{J\ s}\ \ [\mathrm{kg\ m^2\ s^{-1}}].$$

Energy and equivalent mass of a photon:

$$E = h\nu \quad \Rightarrow \quad m = \frac{h\nu}{c^2}$$

Ampere

- Force between two parallel, current-carrying wires of infinite length 1 metre apart in a vacuum (1946):
  The constant current that will produce an attractive force of 2×10^{-7} N per metre of length.



Credit:
User:Danmichaelo/
Wikimedia Commons

- Fixed value of the elementary charge (2019):

$$e = 1.602176634 \times 10^{-19}\ \mathrm{C}\ \ [\mathrm{A\ s}].$$

Kelvin

- Triple point of water (1954):
  Absolute zero = 0 K, triple point of water = 273.16 K.

  Derived unit Celsius: θ [°C] = T [K] – 273.15.

- Fixed value of the Boltzmann constant (2019):
  $$k_\mathrm{B} = 1.380649 \times 10^{-23}\ \mathrm{J\,K^{-1}}\ \ [\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}].$$

  Ideal gas law: $pV = Nk_\mathrm{B}T$

  Thermal energy per microscopic DOF: $E = \frac{1}{2}k_\mathrm{B}T$

Mole

- Historic background (early 1800s):
  Could measure relative atomic masses
  (m_{H} = 1 u, m_{He} ≈ 4 u, m_{O} ≈ 16 u, …), but 1 g = xxx u?

  → 1 mol H = 1 g, 1 mol He ≈ 4 g, 1 mol O ≈ 16 g, etc.

- Carbon reference (1967):
  Amount of substance of a system which contains as many particles as there are atoms in 12 g of ^{12}C.

- Fixed value of the Avogadro constant (2019):
  $$N_{\mathrm{A}} = 6.02214076 \times 10^{23}\ \mathrm{mol}^{-1}$$
  (one mole contains exactly $6.02214076 \times 10^{23}$ particles).

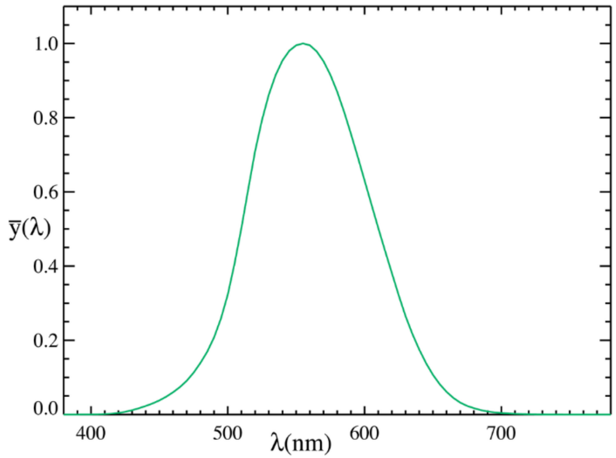
Candela

- Fixed value of the luminous efficacy (1979–today):

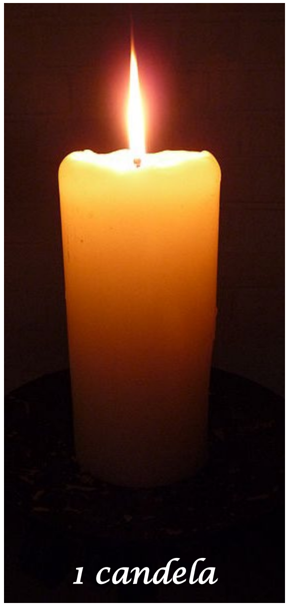
$$K_{\mathrm{cd}} = 683\ \mathrm{lm\ W^{-1}}$$

(for monochromatic radiation of frequency 540 THz ≙ 555 nm).

- For other frequencies/wavelengths to be weighted by the luminosity function for the human eye:



Credit: User:PAR~commonswiki/ Wikimedia Commons



Credit: User:4028mdk09/ Wikimedia Commons

- Luminous intensity = luminous flux (power) per solid angle:

$$1\ \mathrm{cd} = 1\ \mathrm{lm\ sr^{-1}}.$$

A little side step: Wall-plug efficacy
(aka luminous efficacy of the source)

or: **A short shopping guide for light bulbs**

Wall-plug efficacy = emitted luminous flux / electrical input power

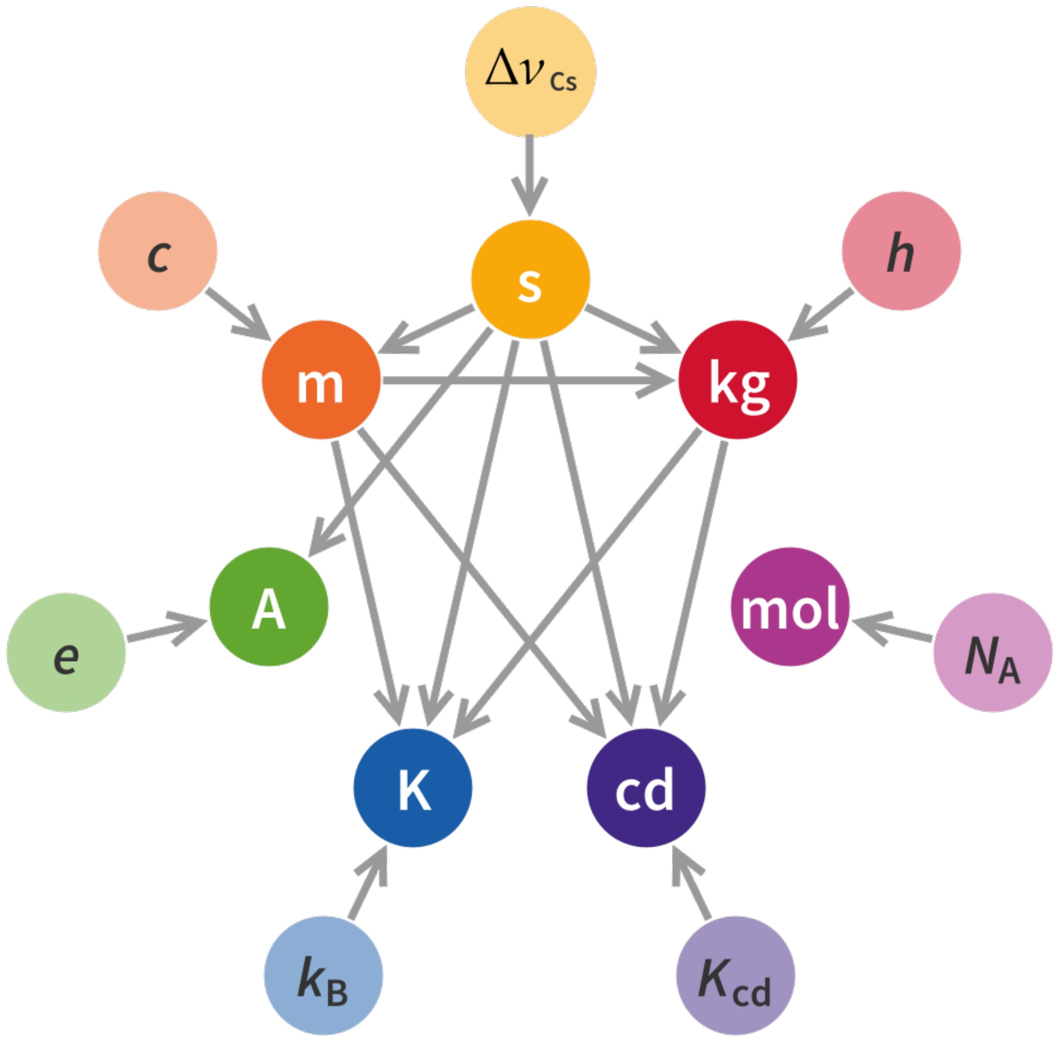
Type of bulb	Wall-plug efficacy	Input power for 700 lm
Tungsten light bulb	10 – 30 lm/W	60 W (~12 lm/W)
Compact fluorescent ("energy-saving") lamp	50 – 80 lm/W	12 W (~60 lm/W)
LED light bulb	60 – 160 lm/W	8 W (~90 lm/W)

Source: https://de.wikipedia.org/wiki/Lichtausbeute

factor ~5 more efficient

factor ~8 more efficient

SI system after the 2019 redefinition



Credit: Emilio Pisanty/Wikimedia Commons

Some thoughts

- Using physical constants for defining the base units 

- Mole – who needs that (as a base unit)?
  Historical significance, but it's just a number…

- Candela – who on Earth needs that (as a base unit)???
  It's just a radiation power (per unit solid angle) weighted by the standard luminosity function (physiology, not physics)…

Some more thoughts

Using physical constants for defining the base units

could set them all to 1 !

1 time unit = 1/(caesium frequency $\Delta\nu_{\mathrm{Cs}}$) ≈ 1.088×10^{-10} s

1 length unit = c × 1 time unit ≈ 0.0326 m

1 mass unit = $(h\Delta\nu_{\mathrm{Cs}})/c^2$ ≈ 6.777×10^{-41} kg

etc…

Very common in theoretical physics!

THANK YOU