



HOKKAIDO UNIVERSITY

Title	Self-Triggered Pinning Consensus Control for Multi-Agent Systems
Author(s)	Andoh, Shun; Kobayashi, Koichi; Yamashita, Yuh
Citation	IEICE transactions on fundamentals of electronics communications and computer sciences, E103.A(2), 443-450 https://doi.org/10.1587/transfun.2019MAP0009
Issue Date	2020-02
Doc URL	https://hdl.handle.net/2115/76920
Rights	copyright©2020 IEICE
Type	journal article
File Information	Self-Triggered Pinning Consensus Control for Multi-Agent Systems.pdf



Self-Triggered Pinning Consensus Control for Multi-Agent Systems

Shun ANDOH[†], *Nonmember*, Koichi KOBAYASHI^{†a)}, and Yuh YAMASHITA[†], *Members*

SUMMARY Pinning control of multi-agent systems is a method that the external control input is added to some agents (pinning nodes), e.g., leaders. By the external control input, consensus to a certain target value and faster consensus are achieved. In this paper, we propose a new method of self-triggered predictive pinning control for the consensus problem. Self-triggered control is a method that both the control input and the next update time are calculated. Using self-triggered control, it is expected that the communication cost can be reduced. First, a new finite-time optimal control problem used in self-triggered control is formulated, and its solution method is derived. Next, as an on-line algorithm, two methods, i.e., the multi-hop communication-based method and the observer-based method are proposed. Finally, numerical examples are presented.

key words: consensus, multi-agent systems, pinning control, self-triggered control

1. Introduction

The consensus problem of multi-agent systems is to find a control input such that the agents reach a particular ordered state by using only information on neighborhood agents. This problem has been widely studied (see, e.g., [5], [6], [13]–[15], [18]). In the conventional consensus problems, the states of the agents converge to the average of initial states, and the convergence speed depends on a given graph expressing neighborhood agents. To achieve consensus to the other target value and faster consensus, it is important to consider the external control inputs. From this viewpoint, pinning control has been proposed (see, e.g., [16], [17]).

Pinning control is a method that the external control input is added to some agents (pinning nodes), e.g., leaders. In many existing methods of pinning control, a simple controller using only information on neighborhood agents is utilized. Hence, it is important to develop optimization-based methods such as model predictive control (MPC). MPC is a control method that the control input is generated by solving the finite-time optimal control problem at each discrete time (see, e.g., [4], [12]). In [18], predictive pinning control for the consensus problem has been proposed. In this method, a controller can be obtained based on the policy of MPC, and the convergence rate is analyzed theoretically. We must assume that the pinning nodes are connected to all nodes. One of the authors has proposed in [11] a new method of

predictive pinning control using MPC. In this method, we introduced the notion of a controller node, which is connected to only pinning nodes. The controller node solves the optimization problem in MPC, and send the external control inputs to the pinning nodes. In [11], at each discrete time, the conventional finite-time optimal control problem is solved, and the obtained control input is sent to pinning nodes. However, from the viewpoint of the communication load, reduction of the number of communications between the controller node and pinning nodes is important while considering the control performance.

In this paper, we propose a method of self-triggered predictive pinning control for the consensus problem of multi-agent systems. Self-triggered control is a method that both the control input and the next update time are calculated (see, e.g., [6]–[10]). Using the concept of self-triggered control, the communication load can be reduced. In a similar way to [11], we introduce the controller node.

First, the finite-time optimal control problem, which can be used in self-triggered control, is newly formulated based on [8], and its solution method is derived. In [8], the infinite-time optimal control problem is considered, and the algebraic Riccati equation must be solved. However, the conventional solution method for the algebraic Riccati equation cannot be applied to the system in the consensus problem (see also Sect. 3). Hence, we consider a new type of the finite-time optimal control problem.

Next, we consider a method of self-triggered predictive control using this finite-time optimal control problem. Here, two methods of the multi-hop communication-based method and the observer-based method are proposed. In the multi-hop communication-based method, the controller node collects the states of all agents using multi-hop communication. This method is an extension of the method in [11]. In the observer-based method, the controller node estimates the states of all agents using only the states of the pinning nodes. Since there exists an estimation error, the control performance in the former is better than that in the latter.

Finally, the proposed two methods are demonstrated by numerical examples.

Notation: Let $\mathcal{R}$ denote the set of real numbers. Let I_n and $0_{m \times n}$ denote the $n \times n$ identity matrix and the $m \times n$ zero matrix, respectively. For simplicity of notation, we sometimes use the symbol 0 instead of $0_{m \times n}$, and the symbol I instead of I_n . Let 1_n denote the n -dimensional column vector whose elements are all one.

Manuscript received April 14, 2019.

Manuscript revised August 30, 2019.

[†]The authors are with the Graduate School of Information Science and Technology, Hokkaido University, Sapporo-shi, 060-0814 Japan.

a) E-mail: k-kobaya@ssi.ist.hokudai.ac.jp

DOI: 10.1587/transfun.2019MAP0009

2. Preliminaries

First, the consensus problem of multi-agent systems is explained. Next, the outline of pinning control is explained.

2.1 Consensus Problem

Let $G = (\mathcal{V}, \mathcal{E})$ denote an undirected connected graph, where the $\mathcal{V} = \{1, 2, \dots, n\}$ is the set of nodes (vertices) and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. For simplicity of discussion, we consider undirected graphs, but we can easily extend the following discussion to that in the case of directed graphs. Each node corresponds to each agent, and each edge corresponds to a communication link between agents. If there is the edge from the node i to j (i.e., the edge from j to i), then the information about the state in the node i (j) can be transmitted to the node j (i). Let $A \in \{0, 1\}^{n \times n}$ denote the adjacency matrix of G . We assume that there is no self-loop, that is, the (i, i) -th element of A is zero. Let $\mathcal{N}_i \subset \mathcal{V}$ denote the set of nodes that are adjacent to the node i . Then, the degree matrix D is defined by $D := \text{diag}(|\mathcal{N}_1|, |\mathcal{N}_2|, \dots, |\mathcal{N}_n|)$. In addition, the graph Laplacian matrix L is defined by $L := D - A$.

Next, the dynamics of the agent i are defined by the following discrete-time single integrator:

$$x_i(k+1) = x_i(k) + u_i(k), \quad (1)$$

where $x_i \in \mathcal{R}$ and $u_i \in \mathcal{R}$ are the state and the control input of the agent i , respectively. For the system (1), the consensus problem is formulated as follows.

Problem 1: It is said that the agents have reached consensus if $\lim_{k \rightarrow \infty} (x_i(k) - x_j(k)) = 0$ holds for all $i, j \in \mathcal{V}$. Then, find a control input such that the agents have reached consensus, where $u_i(k)$ must be given by a function with respect to only $x_i(k)$ and $x_j(k)$, $j \in \mathcal{N}_i$.

The solution for this problem is given by the following lemma (see [13] for further details).

Lemma 1: Consider the controller $u_i(k) = -\varepsilon \sum_{j \in \mathcal{N}_i} (x_i(k) - x_j(k))$, where $\varepsilon \in (0, 1/d_{\max})$, $d_{\max} = \max(|\mathcal{N}_1|, |\mathcal{N}_2|, \dots, |\mathcal{N}_n|)$. Then, the agents have reached consensus, that is, $\lim_{k \rightarrow \infty} x_i(k) = \sum_{i=1}^n x_i(0)/n$ holds.

From this lemma, we see that all states converge to the average of the initial states. The closed-loop system can be obtained by $x(k+1) = Px(k)$, where $x = [x_1 \ x_2 \ \dots \ x_n]^T$ and $P = I_n - \varepsilon L$.

2.2 Pinning Control

In pinning control, we introduce pinning nodes. Pinning nodes may be regarded as “leaders”. Pinning control is a method that only agents corresponding to pinning nodes are controlled by the external signal from the upper layer. Using pinning control, consensus to the different target value (not the average of the initial states) is achieved. We can also

consider faster consensus.

We define the set of pinning nodes as follows: $\mathcal{V}_p = \{1, 2, \dots, m\} \subset \mathcal{V}$, $m \ll n$. In pinning nodes, the external control input $v_i(k) \in \mathcal{R}$ is added as follows:

$$u_i(k) = -\varepsilon \sum_{j \in \mathcal{N}_i} (x_i(k) - x_j(k)) + v_i(k), \quad i \in \mathcal{V}_p, \quad (2)$$

$$u_i(k) = -\varepsilon \sum_{j \in \mathcal{N}_i} (x_i(k) - x_j(k)), \quad i \in \mathcal{V} \setminus \mathcal{V}_p. \quad (3)$$

The closed-loop system consisting of (1), (2), and (3) can be obtained by

$$x(k+1) = Px(k) + Bv(k), \quad (4)$$

where $v = [v_1 \ v_2 \ \dots \ v_m]^T$ and $B = [I_m \ 0_{m \times (n-m)}]^T$.

In this paper, we propose a method to find $v(k)$ based on the policy of MPC.

3. Finite-Time Optimal Control Problem for Self-Triggered Control

In this section, based on [8], we newly formulate the finite-time optimal control problem for the system (4). This problem is utilized in self-triggered predictive pinning control proposed in the next section. Here, we define $\tilde{x}(k) := x(k) - x_d$, where $x_d \in \mathcal{R}^n$ is the target state given in advance. The problem is given as follows.

Problem 2:

$$\begin{aligned} \text{given} \quad & x(t) = x_t \text{ (} t \text{ is the current time)} \\ \text{find} \quad & T \in \{1, 2, \dots, \bar{T}\}, \\ & \bar{v} = [v^T(t) \ v^T(t+1) \ \dots \ v^T(t+N-1)]^T \\ & \in \mathcal{R}^{mN}, \\ \min \quad & J = \frac{\alpha}{T} + J_s, \\ & J_s = \sum_{k=t}^{t+N-1} \left\{ \tilde{x}^T(k) Q \tilde{x}(k) + v^T(k) R v(k) \right\} \\ & + \tilde{x}^T(t+N) Q_f \tilde{x}(t+N) \\ \text{s.t.} \quad & \text{System (4),} \\ & v(t) = v(t+1) = \dots = v(t+T-1), \quad (5) \\ & v(t+T+r\bar{T}) = v(t+T+r\bar{T}+1) = \dots \\ & = v(t+T+r\bar{T}+\bar{T}-1), \quad r \in \{0, 1, \dots, f-1\} \quad (6) \\ & v(t+T+f\bar{T}) = \dots = v(t+N-1). \quad (7) \end{aligned}$$

In this problem, $\alpha \geq 0$, $Q \geq 0$, $R > 0$, and $Q_f \geq 0$ are a given weight, and N is a given prediction horizon. For a given N , the following relation:

$$N = T + f\bar{T} + s, \quad (8)$$

i.e., $N - T \equiv s \pmod{\bar{T}}$ is satisfied ($\bar{T}$ is a given upper bound of T), where $f \geq 0$ and $s \geq 0$ can be determined by fixing T .

The control input is constrained by (5), (6), and (7). In the first time interval $[t, t + T - 1]$, the control input is a constant, and both T and $v(t)$ in (5) are decision variables. The scalar T is chosen from the finite set $\{1, 2, \dots, \bar{T}\}$. In the time interval $[t + T + r\bar{T}, t + T + r\bar{T} + \bar{T} - 1]$, the control input is a constant. Also in the time interval $[t + T + f\bar{T}, t + N - 1]$, the control input is a constant. Hence, in the time interval $[t + T, t + N - 1]$, only $v(t + T + r\bar{T})$ in (6) and $v(t + T + f\bar{T})$ in (7) are decision variables.

When this problem is applied to self-triggered predictive pinning control, $v(t)$, $t \in [t, t + T - 1]$ is applied to the system (4). Hence, T corresponds to the communication interval. The communication load is reduced by choosing a longer T , but the control performance decreases. By setting α appropriately, we can consider the trade-off between the communication load and the control performance.

In [8], the infinite-time optimal control problem is considered. In this paper, the finite-time optimal control problem is considered. This is because the matrix P in the system (4) has an eigenvalue 1, and the discrete-time algebraic Riccati equation cannot be solved by the conventional method (e.g., dare in MATLAB).

Now, we consider deriving a solution of Problem 2. The dimension of decision variables may be changed depending on T . In other words, f and s in (8) may be changed depending on T . Hence, it is difficult to directly reduce Problem 2 to a some optimization problem. First, for a fixed T , we consider finding a control input sequence $\bar{v}$ minimizing J_s . Then, f and s is uniquely determined. The solution of the system (4) can be obtained by $x(t + k) = P^k x_t + \sum_{i=1}^k P^{i-1} B v(t + k - i)$. From this expression, (5) and (6), we can obtain

$$\bar{x} = \bar{P}x_t + \bar{B}\bar{w}, \quad (9)$$

where

$$\begin{aligned} \bar{x} &= [x_t^\top(t) \ x_t^\top(t+1) \ \cdots \ x_t^\top(t+N)]^\top, \\ \bar{w} &= [v^\top(t) \ v^\top(t+T) \ v^\top(t+T+\bar{T}) \\ &\quad \cdots \ v^\top(t+T+f\bar{T})]^\top, \end{aligned}$$

$$\bar{P} = \begin{bmatrix} I & P^\top & (P^2)^\top & \cdots & (P^N)^\top \end{bmatrix}^\top,$$

$$\bar{B} = \bar{B}_1 \bar{B}_2,$$

$$\bar{B}_1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ PB & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ P^{N-1}B & \cdots & PB & B \end{bmatrix},$$

$$\begin{aligned} \bar{B}_2 &= \text{block-diag}(\bar{B}_{21}(T), \bar{B}_{22}, \dots, \bar{B}_{22}, \bar{B}_{23}) \\ &\in \mathcal{R}^{(T+f\bar{T}+s)m \times (f+2)m}, \end{aligned}$$

$$\bar{B}_{21}(T) = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} \in \mathcal{R}^{Tm \times m}, \quad \bar{B}_{22(23)} = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} \in \mathcal{R}^{\bar{T}m(sm) \times m}.$$

In the case of $s = 0$, the vector $\bar{w}$ is given by

$$\begin{aligned} \bar{w} &= [v^\top(t) \ v^\top(t+T) \ v^\top(t+T+\bar{T}) \\ &\quad \cdots \ v^\top(t+T+(f-1)\bar{T})]^\top, \end{aligned}$$

and the size of $\bar{B}_2$ is $(T + f\bar{T})m \times (f+1)m$. The cost function J_s can be rewritten as

$$J_s = (\bar{x} - \bar{x}_d)^\top \bar{Q}(\bar{x} - \bar{x}_d) + \bar{w}^\top \bar{R}\bar{w}, \quad (10)$$

where $\bar{x}_d = [x_d^\top \ x_d^\top \ \cdots \ x_d^\top]^\top$, $\bar{Q} = \text{block-diag}(Q, Q, \dots, Q, Q_f)$, and $\bar{R} = \text{block-diag}(R, R, \dots, R)$. By substituting (9) into (10), we can obtain

$$\begin{aligned} J_s &= \bar{w}^\top (\bar{R} + \bar{B}^\top \bar{Q} \bar{B}) \bar{w} + 2\bar{w}^\top \bar{B}^\top \bar{Q}(\bar{P}x_t - \bar{x}_d) \\ &\quad + (\bar{P}x_t - \bar{x}_d)^\top \bar{Q}(\bar{P}x_t - \bar{x}_d). \end{aligned} \quad (11)$$

The optimal $\bar{w}$ satisfies $\partial J_s / \partial \bar{w} = 0$. Let $\bar{v}^*$ denote the optimal control input sequence. Noting that (5) and (6) must be satisfied, $\bar{v}^*$ can be obtained by

$$\bar{v}^* = -\bar{B}_2(\bar{R} + \bar{B}^\top \bar{Q} \bar{B})^{-1} \bar{B}^\top \bar{Q}(\bar{P}x_t - \bar{x}_d).$$

Let J_s^* denote the optimal value of J_s . By substituting the optimal $\bar{w}$ ($= -(\bar{R} + \bar{B}^\top \bar{Q} \bar{B})^{-1} \bar{B}^\top \bar{Q}(\bar{P}x_t - \bar{x}_d)$) into (11), J_s^* can be obtained by $J_s^* = (\bar{P}x_t - \bar{x}_d)^\top (\bar{Q} - \bar{Q} \bar{B}(\bar{R} + \bar{B}^\top \bar{Q} \bar{B})^{-1} \bar{B}^\top \bar{Q})(\bar{P}x_t - \bar{x}_d)$. More precisely, $\bar{v}^*$ and J_s^* are a function with respect to x_t and T . Hereafter, $\bar{v}^*$ and J_s^* are denoted by $\bar{v}^*(x_t, T)$ and $J_s^*(x_t, T)$, respectively.

Finally, we can derive an optimal solution of Problem 2. Let T^* denote the optimal T . Then, we can obtain the following theorem.

Theorem 1: The optimal T and the optimal control input sequence of Problem 2 are given by

$$T^* = \arg \min_{T \in \{1, 2, \dots, \bar{T}\}} \left\{ \frac{\alpha}{T} + J_s^*(x_t, T) \right\}$$

and $\bar{v}^*(x_t, T^*)$, respectively. In addition, the optimal value of the cost function is given by

$$J^* = \frac{\alpha}{T^*} + J_s^*(x_t, T^*).$$

Proof : From $\min_{T, \bar{v}} J = \min_T \{\alpha/T + J_s^*(x_t, T)\}$, we can obtain this theorem.

From this theorem, we see that the optimal solution can be obtained by calculating $\alpha/T + J_s^*(x_t, T)$ for each T .

4. Self-Triggered Predictive Pinning Control

In the proposed self-triggered predictive pinning control methods, both the time interval and the control input are generated by solving Problem 2. In this section, we propose two methods of self-triggered predictive pinning control, i.e., the multi-hop communication-based method and the observer-based method. The former is an extension of the method in [11]. Self-triggered control has not been studied in [11]. The latter is an extension of the conventional observer method. In both methods, we add a *controller node* [11] to the graph. The controller node is adjacent to only

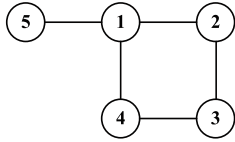


Fig. 1 Example of undirected graphs. The pinning node is 1, and the controller node is 5.

pinning nodes, and solves Problem 2 based on information from pinning nodes.

First, we present simple examples of the proposed methods. Next, we explain general procedures of the proposed methods. We define the notation. Let $\hat{x}_i(k_1|k_2)$ denote the estimated value of $\hat{x}_i(k_1)$ at time k_2 . We also define $\hat{x}(k_1|k_2) := [\hat{x}_1(k_1|k_2) \ \hat{x}_2(k_1|k_2) \ \cdots \ \hat{x}_n(k_1|k_2)]^T$.

4.1 Example on the Multi-Hop Communication-Based Method

Consider the undirected graph in Fig. 1. We assume that $m = 1$, that is, only the node 1 is the pinning node. The multi-agent system consists of the nodes 1, 2, 3, 4. The node 5 is the controller node.

We suppose that the initial states $x_i(0)$, $i = 1, 2, 3, 4$ are unknown in the controller node, and the information about $x_i(0)$ is sent from each node to the controller node through a given graph (i.e., there are communication delays). After the controller node receives the information about $x_i(0)$, we can use $x_i(0)$ in the controller node. Hence, we must use the estimated initial state until the controller node receives all initial states.

The procedure of the multi-hop communication-based method is summarized.

At time $t = 0$, Problem 2 is solved using $\hat{x}(0|0)$ given in advance. We suppose that $T^* = 2$ and $\bar{v}^*(\hat{x}_0, 2) =: [v^*(0) \ v^*(1) \ \cdots \ v^*(N-1)]^T$ are obtained. Then, only $v^*(0)$ is sent and applied to the node 1.

At time $t = 1$, from $T^* = 2$, the controller node performs no calculation. The pinning node stores $x_1(0)$, $x_2(0)$, $x_4(0)$, and $x_1(1)$.

At time $t = 2$, we must solve Problem 2. The controller node receives the message about $x_1(0)$, $x_2(0)$, $x_4(0)$, and $x_1(1)$. Then, the estimated states $\hat{x}(1|2)$ and $\hat{x}(2|2)$ are calculated by

$$\begin{aligned} \hat{x}(1|2) &= P \begin{bmatrix} x_1(0) \\ x_2(0) \\ \hat{x}_3(0|0) \\ x_4(0) \end{bmatrix} + Bv^*(0), \\ \hat{x}(2|2) &= P \begin{bmatrix} x_1(1) \\ \hat{x}_2(1|2) \\ \hat{x}_3(1|2) \\ \hat{x}_4(1|2) \end{bmatrix} + Bv^*(0), \end{aligned}$$

respectively. Using $\hat{x}(2|2)$, Problem 2 is solved. We suppose that $T^* = 1$ and $\bar{v}^*(\hat{x}_2, 1) =: [v^{**}(2) \ v^{**}(3) \ \cdots \ v^{**}(N+1)]^T$ are obtained. Then, only $v^{**}(2)$ is sent to the node 1.

At time $t = 3$, the controller node receives the message about $x_3(0)$, $x_2(1)$, $x_4(1)$, and $x_1(2)$. Then, the estimated states $\hat{x}(1|3)$, $\hat{x}(2|3)$, and $\hat{x}(3|3)$ are calculated by

$$\begin{aligned} \hat{x}(1|3) &= P \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} + Bv^*(0), \\ \hat{x}(2|3) &= P \begin{bmatrix} x_1(1) \\ x_2(1) \\ \hat{x}_3(1|3) \\ x_4(1) \end{bmatrix} + Bv^*(0), \\ \hat{x}(3|3) &= P \begin{bmatrix} x_1(2) \\ \hat{x}_2(2|3) \\ \hat{x}_3(2|3) \\ \hat{x}_4(2|3) \end{bmatrix} + Bv^{**}(2), \end{aligned}$$

respectively. Since $T^* = 1$ is obtained at time $k = 2$, we must solve Problem 2.

After $t = 3$, the estimated state $\hat{x}(k|k)$ can be calculated in a similar way. The discrete time at which Problem 2 is solved is determined using T^* .

Using the above procedure, we can achieve self-triggered control. However, in implementation, we must consider multi-hop communications (see, e.g., [3]).

4.2 Example on the Observer-Based Method

To avoid to use multi-hop communications, we introduce an observer that estimates all states from only the states in the pinning nodes.

From the definition of pinning nodes, the measured output is given by $y(k) = Cx(k)$, $C = [I_m \ 0_{m \times (n-m)}]$. Here, we consider the following full-order observer:

$$\hat{x}(k+1|k) = P\hat{x}(k|k) + Bv(k) + L(y(k) - C\hat{x}(k|k)), \quad (12)$$

where L is the observer gain.

Consider the undirected graph in Fig. 1 again. The multi-agent system consists of the nodes 1, 2, 3, 4. Only the node 1 is the pinning node, and the node 5 is the controller node. In this case, $y(k) = x_1(k)$ holds. We suppose that the initial states $x_i(0)$, $i = 1, 2, 3, 4$ are unknown in the controller node.

The procedure of the observer-based method is summarized.

At time $t = 0$, Problem 2 is solved using $\hat{x}(0|0)$ given in advance. We suppose that $T^* = 2$ and $\bar{v}^*(\hat{x}_0, 2) =: [v^*(0) \ v^*(1) \ \cdots \ v^*(N-1)]^T$ are obtained. Then, only $v^*(0)$ is sent and applied to the node 1. The pinning node stores the measured output $y(0)$.

At time $t = 1$, from $T^* = 2$, the controller node performs no calculation. The pinning node stores $y(1)$.

At time $t = 2$, the controller node collects the measured outputs $y(0)$ and $y(1)$ from the pinning node. Then, the estimated state $\hat{x}(2|2)$ can be calculated as follows:

$$\hat{x}(1|2) = P\hat{x}(0|0) + Bv^*(0) + L(y(0) - C\hat{x}(0|0)),$$

$$\hat{x}(2|2) = P\hat{x}(1|2) + Bv^*(0) + L(y(1) - C\hat{x}(1|2)).$$

Using $\hat{x}(2|2)$, Problem 2 is solved. We suppose that $T^* = 1$ and $\bar{v}^*(\hat{x}_2, 1) = [v^{**}(2) \ v^{**}(3) \ \dots \ v^{**}(N+1)]^\top$ are obtained. Then, only $v^{**}(2)$ is sent to the node 1.

At time $t = 3$, the controller node receives the message about $y(2)$. Then, the estimated state $\hat{x}(3|3)$ can be calculated as follows:

$$\hat{x}(3|3) = P\hat{x}(2|2) + Bv^{**}(2) + L(y(2) - C\hat{x}(2|2)).$$

Since $T^* = 1$ is obtained at time $k = 2$, we must solve Problem 2 using $\hat{x}(3|3)$.

After $t = 3$, the estimated state $\hat{x}(k|k)$ can be calculated in a similar way.

Although the pinning nodes must store the measured outputs, implementation of the observer-based method is easier than that of the multi-hop communication-based method.

4.3 Proposed Procedures

We explain the proposed procedures of two proposed methods. Here, we assume that the number of the controller nodes is 1.

First, the procedure of the multi-hop communication-based method is given as follows.

Procedure of the multi-hop communication-based method:

Step 1: Set $t = 0$ and $\hat{x}(0|0)$.

Step 2: Find optimal T and $v(t)$ by solving Problem 2 with the estimated state.

Step 3: Apply $v(t)$ obtained to the pinning nodes.

Step 4: Update $t := t + 1$ and $T := T - 1$.

Step 5: If $T = 0$, then the controller node collects the information about the states through pinning nodes, and calculates the estimated state $\hat{x}(t|t)$. Return to Step 2. Otherwise return to Step 4.

Next, the procedure of the observer-based method is given as follows.

Procedure of the observer-based method:

Step 1: Set $t = 0$ and $\hat{x}(0|0)$.

Step 2: Find optimal T and $v(t)$ by solving Problem 2 with the estimated state.

Step 3: Apply $v(t)$ obtained to the pinning nodes.

Step 4: Update $t := t + 1$ and $T := T - 1$.

Step 5: If $T = 0$, then the controller node collects the measured outputs $y(t-1), y(t-2), \dots, y(t-T)$ (i.e., the states of the pinning nodes) from the pinning nodes, and estimates the current state $\hat{x}(t|t)$ using an observer. Return to Step 2. Otherwise return to Step 4.

The coefficient matrices in $J_s^*(x_t, T)$ and $\bar{v}^*(x_t, T)$ can

be calculated off-line. Hence, in both methods, the on-line computation time for solving Problem 2 is short.

We have some remarks.

First, the total communication cost is not reduced by the multi-hop communication-based method. The momentary communication cost of each node can be reduced by adding pinning nodes. Because the communication cost to the controller node is distributed. If the communication delay is allowed, the communication from pinning nodes to the controller node may be executed in order. The communication delay can be regarded as an input delay, which can be considered (see also Section 5.1).

Next, the communication cost in each node may become large by the multi-hop communication-based method. In order to overcome this issue, it is important to consider addition and assignment of pinning nodes. It is also important to give an appropriate route in advance.

The multi-hop communication-based method should be used for the case where each node is implemented by a device that is appropriate for multi-hop communications. If each node is a low performance device, the observer-based method should be used. From the viewpoints of both the control performance and the communication interval, the multi-hop communication-based method is better than the observer-based method (see also the next section). Users may choose one of them from several viewpoints such as the control performance, the communication cost, and the economic cost.

5. Numerical Example

To show the effectiveness of the proposed method, we present numerical examples. First, we consider a multi-agent system with fifty agents. Next, we consider one hundred multi-agent systems.

5.1 Example 1

Consider a multi-agent system with fifty agents. The graph expressing communication links is given by Fig. 2. This graph was generated by using Barabási-Albert model [1], [2]. In this graph, the number of nodes is 51, and one controller node (the node 51) is included. The number of the pinning nodes is 20 (i.e., the nodes 1, 2, ..., 20 are the pinning node). The parameter ε in the matrix P is given by $\varepsilon = 0.95/d_{\max} = 0.95/13 = 0.0731$.

In Problem 2, the weighting matrices, Q , R , and Q_f are given by $Q = 100I$, $R = 1$, and $Q_f = Q$, respectively. The weight α is given by $\alpha = 100$. The prediction horizon N is given by $N = 10$. The target state x_d is given by the origin. The upper bound $\bar{T}$ of T is given by $\bar{T} = 8$. The initial state and the initial estimated state are given by $x(0) = [1 \ 2 \ \dots \ 50]^\top$ and $\hat{x}(0|0) = 25.5 \times 1_{50 \times 1}$, respectively.

In this example, to consider the communication delay, (4) is changed to $x(k+1) = Px(k) + Bv(k-1)$. Defining a new state as $[x(k) \ v(k-1)]$, we can apply the proposed method straightforwardly. The target state is also enlarged, and is

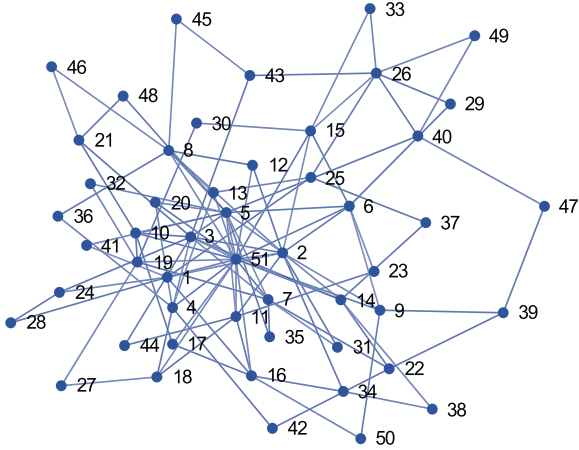


Fig. 2 Undirected connected graph with 50 nodes and the controller node (the node 51).

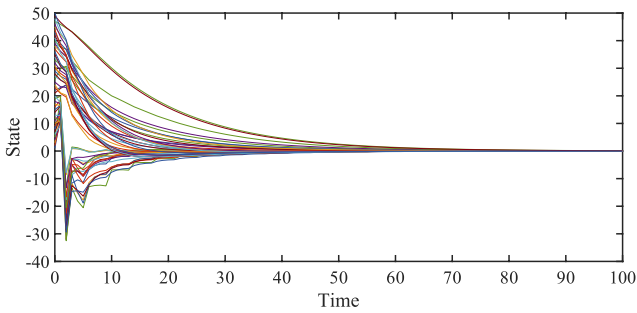


Fig. 3 Time response of the state $x(k)$ in the multi-hop communication-based method.

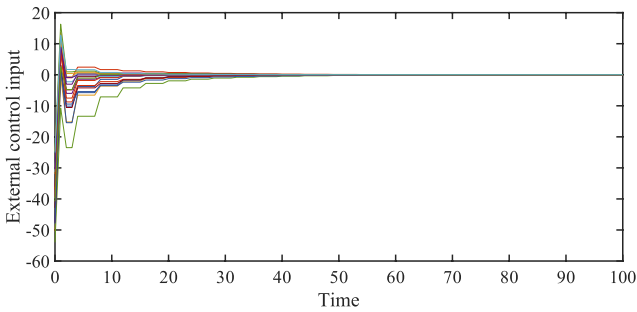


Fig. 4 External control input $v(k)$ in the multi-hop communication-based method.

given by the origin (i.e., the past external control input is given by $u(-1) = 0_{20 \times 1}$).

First, we present the computation result in the multi-hop communication-based method. Figure 3 shows the time response of the state $x(k)$. From this figure, we see that the state converges to the origin. Figure 4 shows the external control input. From this figure, we see that self-triggered control is achieved. The estimation error converges to zero at time 4, because there are no disturbances.

Next, we present the computation result in the observer-based method. Here, we use the full-order observer (12), where the observer gain L is designed such

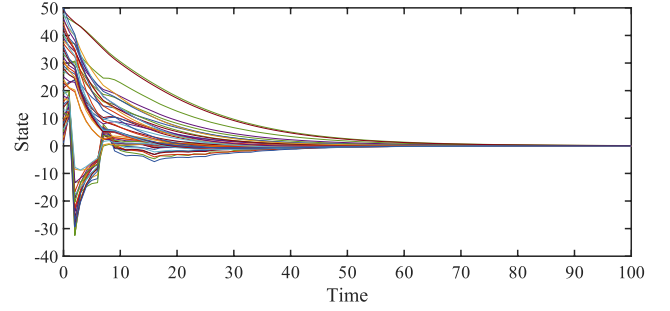


Fig. 5 Time response of the state $x(k)$ in the observer-based method.

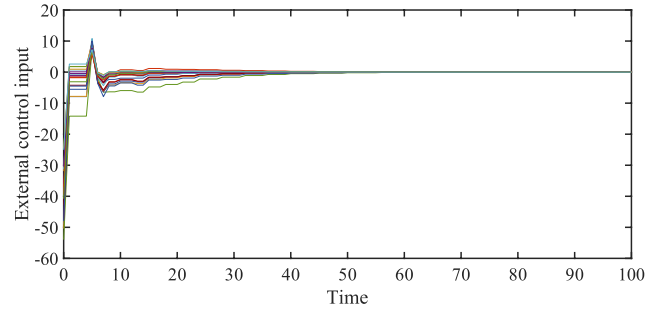


Fig. 6 External control input $v(k)$ in the observer-based method.

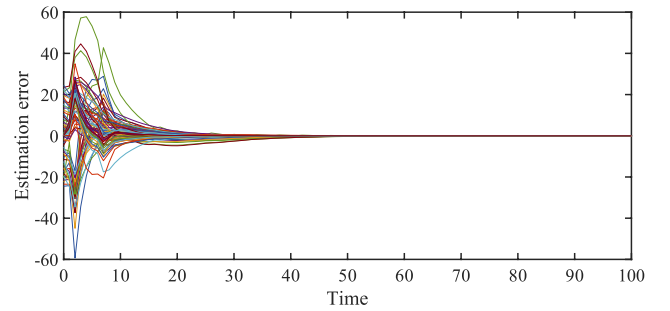


Fig. 7 Estimation error in the observer-based method.

that the eigenvalues of $P - LC$ are given by the row vector $[0.7 \times 1_{20}^T \ 0.71 \times 1_{20}^T \ 0.72 \times 1_{10}^T \ 0.75 \times 1_{10}^T \ 0.82 \times 1_{10}^T]$.

Figure 5 shows the time response of the state $x(k)$. From this figure, we see that the state converges to the origin. Figure 6 shows the external control input. From this figure, we see that also in this method, self-triggered control is achieved. Figure 7 shows the estimation error. From this figure, we see the estimation error converges to the origin.

Finally, we compare the control performances of these two methods. In the time interval $[0, 100]$, the mean value of T^* obtained was 4.86 (the multi-hop communication-based method), 4.29 (the observer-based method). To evaluate the control performance from the state and the external control input, we define $\bar{J} := \sum_{k=0}^{100} \{ \sum_{i=1}^{50} 100x_i(k) + \sum_{i=1}^{20} v_i(k) \}$. In the multi-hop communication-based method, $\bar{J}$ was $\bar{J} = 2.42 \times 10^7$. In the observer-based method, $\bar{J}$ was $\bar{J} = 2.54 \times 10^7$. From these values, we see that owing to the effect of the estimation error, the performance in the multi-hop communication-based method is better than that in the

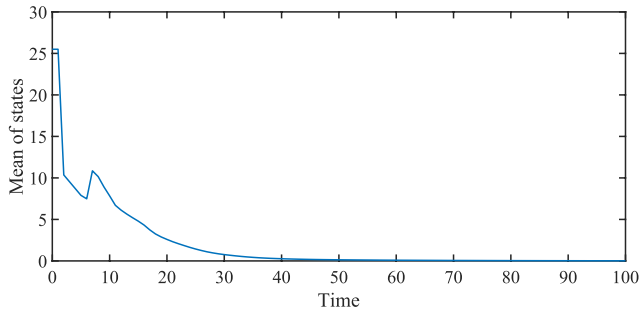


Fig. 8 Mean of all state at each time.

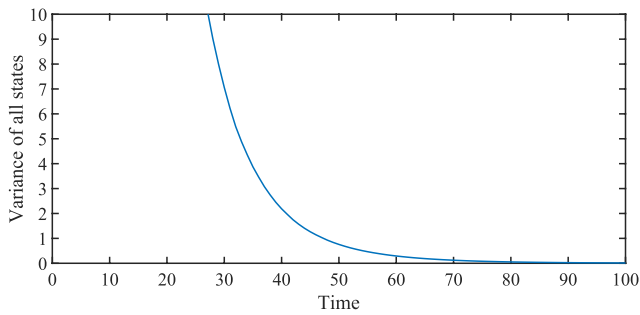


Fig. 9 Variance of all state at each time.

observer-based method from both control and communication viewpoints.

5.2 Example 2

Consider one hundred multi-agent systems with fifty agents. One hundred graphs expressing communication links are randomly generated by using Barabási-Albert model. Problem setting is the same as that in the previous subsection. Here, we consider only the observer-based method.

Figures 8 and 9 show the mean and the variance of all states at each time. From these figures, we see that all states converge in a neighborhood of the origin. Furthermore, in the time interval $[0, 100]$, the mean value of T^* obtained was 4.30.

Thus, the effectiveness of the proposed method is presented by a lot of examples.

6. Conclusion

In this paper, we studied self-triggered pinning consensus control for multi-agent systems. First, a solution method for the finite-time optimal control problem used in self-triggered control was derived. Next, two methods, i.e., the multi-hop communication-based method and the observer-based method were proposed. Depending on computer environment and so on, we can choose one of these methods. Finally, we demonstrated two methods by numerical examples.

In future work, we will study an extension of the proposed method to distributed pinning control. That is, we

will consider the case of multiple controller nodes. The effectiveness of the proposed method was presented by a lot of examples, but stability of the closed-loop system was not shown theoretically. As one of the future efforts, we will analyze stability of the closed-loop system. In addition, we assume here that complete information of the graph structure is obtained. It is important to consider pinning control under imperfect information of graphs.

This work was partly supported by JSPS KAKENHI Grant Numbers JP17K06486, JP19H02157, JP19H02158.

References

- [1] R. Albert and A.-L. Barabási, “Statistical mechanics of complex networks,” *Rev. Mod. Phys.*, vol.74, no.1, pp.47–97, 2002.
- [2] A.-L. Barabási and R. Albert, “Emergence of scaling in random networks,” *Science*, vol.286, no.5439, pp.509–512, 1999.
- [3] T. Braun, A. Kasser, M. Kihl, V. Rakocevic, V. Siris, and G. Heijnen, “Multihop wireless networks,” *Traffic and QoS Management in Wireless Multimedia Networks*, Y. Koucheryavy et al. eds., *Lecture Notes in Electrical Engineering*, vol.31, pp.201–265, 2009.
- [4] E.F. Camacho and C.B. Alba, *Model Predictive Control*, 2nd ed., Springer, 2007.
- [5] G. Ferrari-Trecate, L. Galbusera, M.P.E. Marciandi, and R. Scattolini, “Model predictive control schemes for consensus in multi-agent systems with single- and double-integrator dynamics,” *IEEE Trans. Autom. Control*, vol.54, no.11, pp.2560–2572, 2009.
- [6] K. Hamada, N. Hayashi, and S. Takai, “Event-triggered and self-triggered control for discrete-time average consensus problems,” *SICE J. Control, Measurement, and System Integration*, vol.7, no.5, pp.297–303, 2014.
- [7] W.P.M.H. Heemels, K.H. Johansson, and P. Tabuada, “An introduction to event-triggered and self-triggered control,” *Proc. 51st IEEE Conf. on Decision and Control*, pp.3270–3285, 2012.
- [8] E. Henriksson, D.E. Quevedo, E.G.W. Peters, H. Sandberg, and K.H. Johansson, “Multiple-loop self-triggered model predictive control for network scheduling and control,” *IEEE Trans. Control Syst. Technol.*, vol.23, no.6, pp.2167–2180, 2015.
- [9] S. Iwase, N. Hayashi, and S. Takai, “A gradient-based approach for discrete-time average consensus with self-triggered control,” *SICE J. Control, Measurement, and System Integration*, vol.9, no.3, pp.122–127, 2016.
- [10] K. Kobayashi and K. Hiraishi, “Self-triggered model predictive control with delay compensation for networked control systems,” *IEICE Trans. Fundamentals*, vol.E96-A, no.5, pp.861–868, May 2013.
- [11] K. Kobayashi, “Predictive pinning control with communication delays for consensus of multi-agent systems,” *IEICE Trans. Fundamentals*, vol.E102-A, no.2, pp.359–364, Feb. 2019.
- [12] D.Q. Mayne, J.B. Rawlings, C.V. Rao, and P.O.M. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol.36, no.6, pp.789–814, 2000.
- [13] R. Olfati-Saber and R.M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Trans. Autom. Control*, vol.49, no.9, pp.1520–1533, 2004.
- [14] W. Ren and R.W. Beard, “Consensus seeking in multiagents systems under dynamically changing interaction topologies,” *IEEE Trans. Autom. Control*, vol.50, no.5, pp.655–661, 2005.
- [15] W. Ren, R.W. Beard, and E.M. Atkins, “Information consensus in multivehicle cooperative control,” *IEEE Control Syst. Mag.*, vol.71, no.2, pp.71–82, 2007.
- [16] H. Su and X. Wang, *Pinning Control of Complex Networked Systems: Synchronization, Consensus and Flocking of Networked Systems via Pinning*, Springer, 2013.
- [17] X. Wang and H. Su, “Pinning control of complex networked systems: A decade after and beyond,” *Annual Reviews in Control*,

vol.38, no.1, pp.103–111, 2014.

- [18] H.-T. Zhang, M.Z.Q. Chen, and G.-B. Stan, “Fast consensus via predictive pinning control,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol.58, no.9, pp.2247–2258, 2011.



Shun Andoh received the B.E. degree in 2018 from Hokkaido University. Since 2018, he has been pursuing the M.E. degree with the Graduate School of Information Science and Technology, Hokkaido University. His research interests include multi-agent systems.



Koichi Kobayashi received the B.E. degree in 1998 and the M.E. degree in 2000 from Hosei University, and the D.E. degree in 2007 from Tokyo Institute of Technology. From 2000 to 2004, he worked at Nippon Steel Corporation. From 2007 to 2015, he was an Assistant Professor at Japan Advanced Institute of Science and Technology. Since 2015, he has been an Associate Professor at the Graduate School of Information Science and Technology, Hokkaido University. His research interests include discrete event and hybrid systems. He is a member of IEEE, IEEJ, IEICE, ISCIE, and SICE.



Yuh Yamashita received his B.E., M.E., and Ph.D. degrees from Hokkaido University, Japan, in 1984, 1986, and 1993, respectively. In 1988, he joined the faculty of Hokkaido University. From 1996 to 2004, he was an Associate Professor at the Nara Institute of Science and Technology, Japan. Since 2004, he has been a Professor of the Graduate School of Information Science and Technology, at Hokkaido University. His research interests include nonlinear control and nonlinear dynamical systems. He is a member of SICE, ISCIE, IEICE, RSJ, and IEEE.