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**Study on flow transition of dilute bubbly fluids
-circular pipe flow and bubble convection-**

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Part I

Introduction

Chapter 1

Background

Over the 135 years have passed since the epoch making study by O. Reynolds [4], flow transition scenario of single-phase fluids has been identified to predict and control flow behaviors in laminar-to-turbulent transition from the view point of hydrodynamic stability theorem [24]. Fundamental concepts of the theorem is to investigate the stability of flow fields to perturbation, i.e. velocity, pressure, and temperature fields called base flows. The stability theorem shows whether base flows are observed or not due to perturbation. When flow parameters exceed critical values, base flows become unstable to infinitesimal perturbation (linear stability), resulting in different base flows. As perturbations grow to finite amplitudes due to the nonlinearity, the perturbations saturate to base flows and induce secondary flows. Flow transition provoked by the perturbations shows a variety of flow patterns.

Instability of boundary layer arises in a bounded shear flow. The instability mechanism has been investigated intensively for a laminar boundary layer along a flat plate [Fig. 1.1 [1]. When the destabilizing effect of inertial force overcomes the stabilization of viscous force, the base flow, i.e., laminar boundary layer along a flat plate, becomes unstable to infinitesimal perturbation

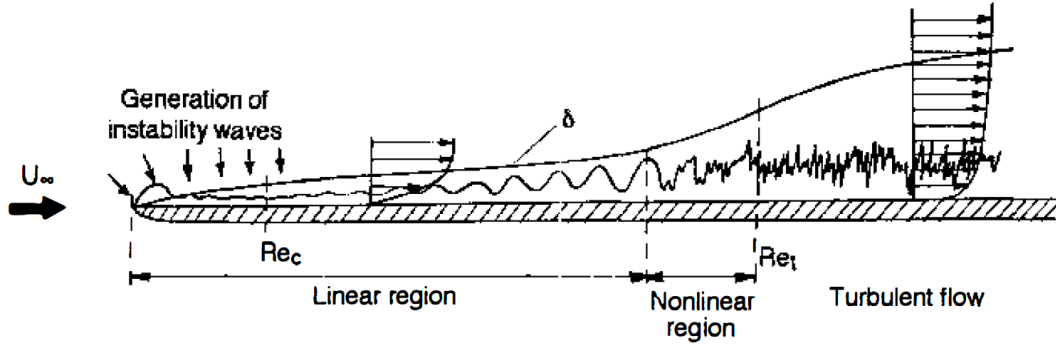


Figure 1.1: Qualitative sketch of the process of turbulence onset in a laminar boundary layer [1]

consisting of traveling waves called Tollmien-Schlichting (TS) waves. Lambda vortices emerge as secondary flows and produce peak-valley structures on TS waves when the perturbation grows to a finite amplitude.

Centrifugal instability appears in a system where a fluid is enclosed between two rotating coaxial cylinders. Taylor-Couette (TC) flow is cited as a classical example of the instability where a swirling flow with angular velocity of the inner cylinder is imposed on the base state. Increase of the angular velocity leads to the linear instability of the base state. Centrifugal force tends to destabilize the base state against the stabilization of viscous force. Steady toroidal vortices, i.e., Taylor-vortex, are observed when the nonlinearity is significant in the flow behaviors. Flow transition in TC flow shows a variety of vortical structures embedded in the dynamics [2] [Fig. 1.2].

Thermal instability often arises when a fluid is heated from below and cooled above [25]. In a classical problem of the instability, i.e. Rayleigh-Benard instability, a quiescent fluid layer with the bottom hotter than the top is imposed on the base state. The base state becomes unstable to infinitesimal perturbation when a temperature difference across the fluid layer exceeds a critical value. The destabilization of thermal buoyancy overcomes the stabilization of viscous force since more buoyant fluid (hotter) in the lower overturns

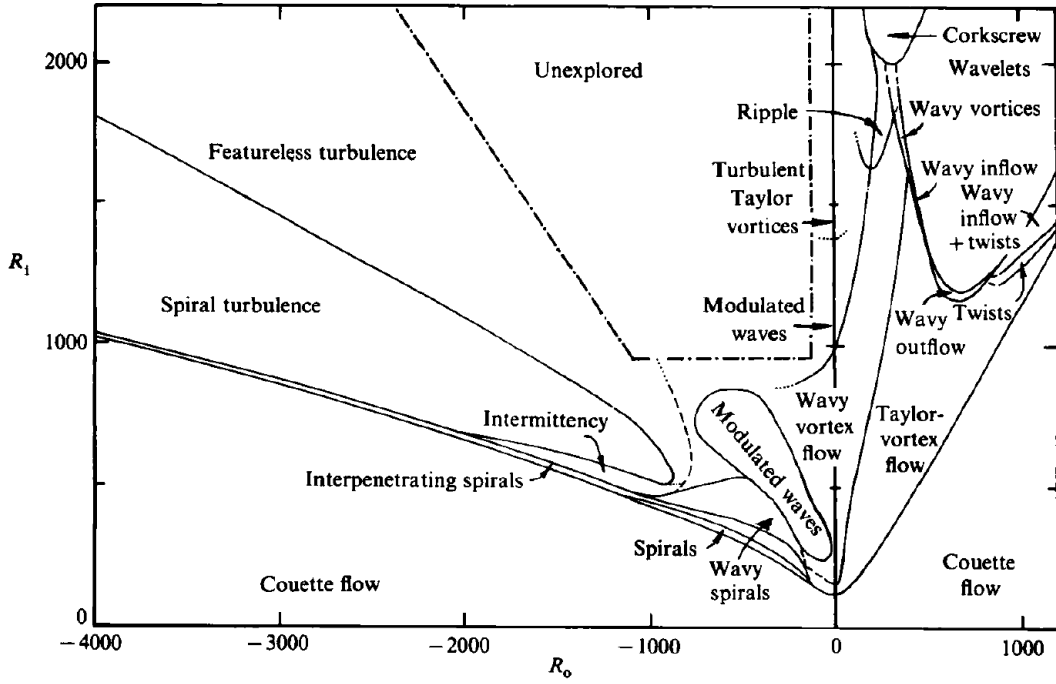


Figure 1.2: A variety of flow patterns is observed due to the nonlinear development in TC flow [2]

less buoyant fluid (cooler) in the upper. Secondary flows developing from the linear instability show steady counter-rotating convection rolls, i.e., Rayleigh-Bénard (RB) convection. Busse [3] showed the nonlinear development of the rolls due to various secondary mechanism [Fig. 1.3].

In the fields of engineering, processing technologies involve multi-phase flows, i.e. fluid systems where different fluid phases or fluid and solid phases coexist simultaneously (e.g. [26]). Gas-liquid two phase flows are particular of interest because of a large variety of applications for energy saving, e.g., frictional drag reduction of naval vessels by bubble injection [13] and bubble column reactors as an efficient gas-liquid contactor for mass and heat transfer [27]. These previous works have focused on modulation of turbulence and vortical structures resulting from mutual interactions between the carrier and dispersed phases. In our laboratory, experimental results showed

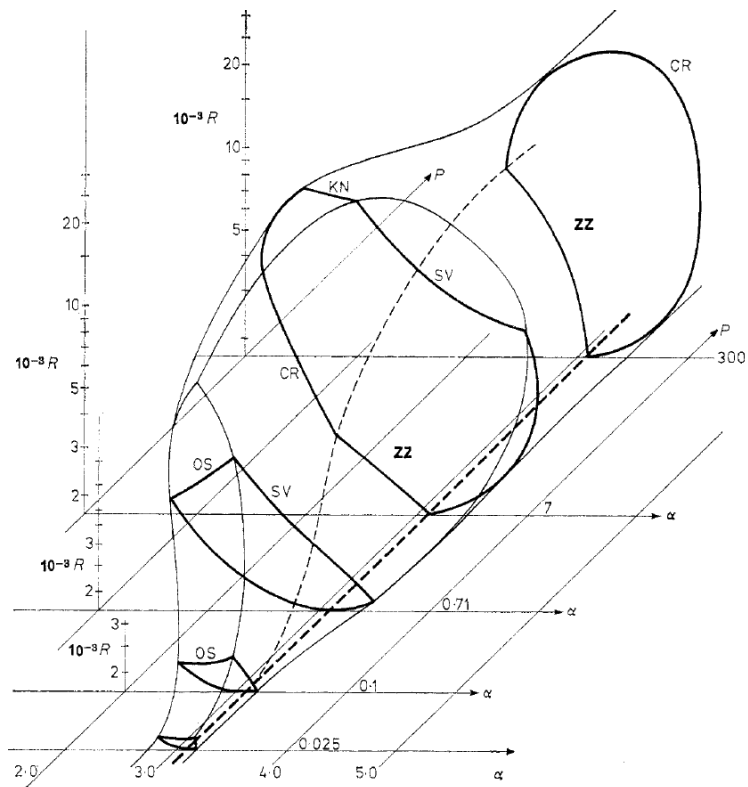


Figure 1.3: Regime for stable convection rolls in the three-dimensional parameter space: the oscillatory (OS), the skewed varicose (SV), the cross-roll (CR), the knot (KN), and the zig-zag (ZZ) instabilities are observed as the secondary flows [3].

uniformly dispersed bubbles have sensitive interactions with vortical structures in Taylor-Couette flow, i.e., bubbles are preferentially accumulated in Taylor vortices [14]. The interactions lead to enhancement of vortices compared to those in single-phase flow. In a horizontal channel flow, transition from stream-wise vortical structures to turbulent bursting phenomena was impeded due to accumulated bubbles in the vortical structures [13]. Typical length scale of the bubbles was sufficiently small to keep spherical shape, i.e. diameter of the bubbles was less than $O(100 \mu\text{m})$ so called microbubble. Einstein [28] and Taylor [29] reported that a uniform suspension of dispersed phase in a flow at low density affects the fluid only through effective viscosity. Bulk volume fraction of the dispersed microbubbles was less than $O(0.1\%)$, i.e. effective viscosity was approximated as that of the single-phase fluid. These results imply that flow transition scenario of dilute microbubbly fluid is different due to the mutual interactions between vortical structures and accumulated microbubbles in vortices. NS equation for pure liquid flow does not hold in the dilute dispersed bubbly flow.

A two-fluid formulation was introduced to model two-phase flows as continuous fields by applying an average operation over volume, time, or ensemble to the kinematic and dynamic laws for the phases, in which the liquid and gas phases are regarded as two dynamical continua exchanging momentum and energy with each phases [30, 31, 32, 33]. While two-fluid models become mathematically ill-posed as an initial-value problem [34], well-posedness is compensated for the models when the dilute suspension of the dispersed phase and small relative velocity between the phases [31, 35, 36, 37]. This formulation has provided rich insight into bubble-laden flows in dilute limit since the bubble dynamics is coupled with translational motion equation of spherical bubbles considering force balance on the bubble. Hydrodynamic forces on

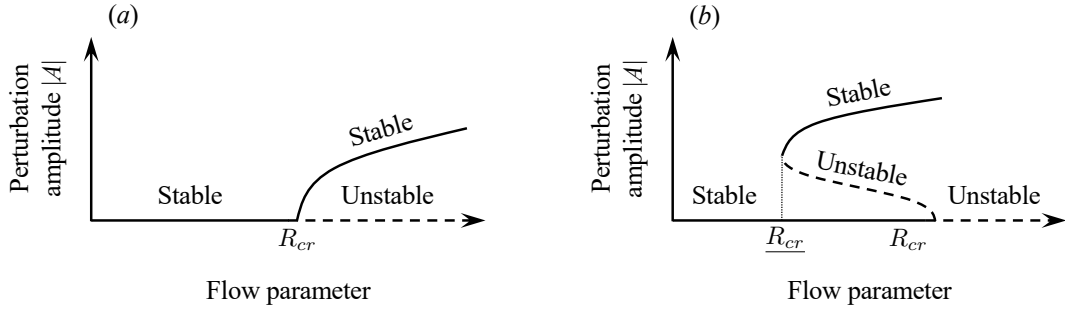


Figure 1.4: Bifurcation diagrams of (a) supercritical and (b) subcritical instabilities: perturbation amplitude is shown as a function of a flow parameter. R_{cr} is a critical parameter above which base flow becomes unstable to finite amplitude perturbation and bifurcations occur from the base flow solution. $\underline{R}_{cr}$ indicate a lower critical parameter below which the bifurcated solution does not exist so that the curve of subcritical bifurcation returns with decrease .

a spherical bubble consist of the added-mass, the viscous drag, and the shear-induced lift, and the buoyancy (e.g. [38]). These forces affect the dynamics of liquid phase through mass and momentum exchange between the phases. Flow transition scenario of dilute bubbly fluids, however, remain unsolved from the view point of the stability theorem. Hydrodynamic stability theorem for single-phase fluids can be applied to describe the transition scenario of dilute bubbly fluids.

The linear stability theory of a steady base flow gives a spectrum of independent modes with velocity perturbation. Velocity fields $\mathbf{u}(\mathbf{x}, t)$ are decomposed into a base flow $\bar{\mathbf{U}}(\mathbf{x})$ and a perturbation $\mathbf{u}'(\mathbf{x}, t)$:

$$\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{U}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t) = \hat{A}(t)e^{ik\mathbf{x}+st}. \quad (1.1)$$

where k is the wavenumber and $\hat{A}(t)$ is the complex amplitude of the perturbation with the complex growth rate $s = \sigma + i\omega$ consisting of a growth rate σ and a frequency ω . When a given flow parameter R is lower than a critical value R_{cr} , all the s have a negative real part, i.e., the base flow is stable to the

perturbation. At $R = R_{cr}$ marginally stable conditions, there exists a s having a positive real part with $s = s_1 = \sigma_1 + \omega_1$, i.e., the base flow becomes unstable to the perturbation. When R is slightly above R_{cr} , the growth rate σ_1 of the first mode of the perturbation is positive while σ for all the other modes are negative. The other modes, however, also grow with sufficiently large values of R resulting in the nonlinear interaction between the modes with generating harmonics and distorting the base flow.

The developments of the perturbation amplitude as a function of a flow parameter are shown Fig. 1.4. The basic flow $|A| = 0$ becomes linearly unstable to the perturbation at $R = R_{cr}$ and bifurcates to other flow at a larger flow parameter, called *supercritical bifurcation* [Fig. 1.4(a)]. The onset of thermal convection was found supercritical. Heat conduction becomes unstable to infinitesimal perturbation when Rayleigh number exceeds a critical value. The formation of convection rolls is observed in the nonlinear development of the flow behaviors at larger Rayleigh number.

Finite amplitude perturbation, however, triggers flow transition at $R < R_{cr}$ while all infinitesimal perturbation are stable, called *subcritical bifurcation* [Fig. 1.4(b)]. The bifurcated solution at $|A| = 0$ return with increase of the perturbation amplitude. Flow transition in a circular pipe shows subcritical bifurcation. Finite amplitude perturbation leads to the abrupt onset of turbulence at Reynolds number lower than a critical value while Hagen-Poiseuille flow is linear stable to infinitesimal perturbation for all Reynolds number.

Motivation of this study is to give a basic concept of dilute gas-liquid two-phase flow transition based on experiments, stability theorem, and numerical simulations. The epoch-making study of laminar-to-turbulent flow transition in a circular pipe by O. Reynolds [4] and the onset of bubble-induced convection are revisited to study flow transitions of dilute bubbly fluids.

Chapter 2

Flow transition in the pipe

Flow transition in a circular pipe has attracted much attention in various fields of scientific research because of their importance in industrial application. The experiments by O. Reynolds [4] are cited as one of the most famous studies on laminar-to-turbulent flow transition in a pipe. O. Reynolds performed dye visualization experiments for flows in the pipe [Fig. 2.1]. The results showed that regular or eddy motions depend on the characteristic scales of length and velocity. At a low flow velocity, the flow remained regular, i.e. laminar flow. At a larger flow velocity, the laminar flow broke down and becomes turbulent with a chaotic motion. The threshold between laminar and turbulent flow is

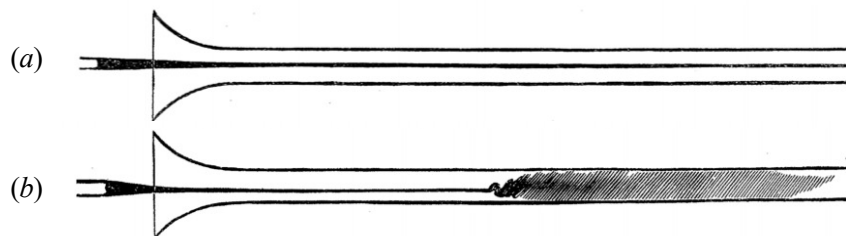


Figure 2.1: Flow regimes in a circular pipe flow illustrated by O. Reynolds [4]: at a low flow velocity, the fluid motions remain regular, namely (a) laminar flow. At a larger flow velocity, the laminar flow becomes turbulent with a chaotic motion, namely (b) turbulent flow.

often given as a dimensionless number, now called Reynolds number,

$$Re = \frac{UD}{\nu} \quad (2.1)$$

where U is the bulk mean velocity, D is the pipe diameter, and ν is the kinematic viscosity.

Flow transition scenario in a circular pipe has been identified, over the 135 years have passed since his experiments. Hagen-Poiseuille flow, a non-trivial solution of Navier-Stokes equation, exists for all range of Reynolds number. The fact implies that the flow would be laminar up to infinite Reynolds number. This contradicts the afore-mentioned experimental results by O. Reynolds. This discrepancy is explained from sensitivity of the flow to perturbation. O. Reynolds reported that perturbation decays as low as $Re \sim 2,000$ while the laminar flow is observed up to $Re \sim 13,000$ when carefully designed experimental facilities minimizing inlet flow and background perturbation.

His investigation implies the existence of two critical Reynolds numbers and a minimal amplitude of perturbation to trigger the flow transition. One is an upper critical Reynolds number $\overline{Re_c}$ above which Hagen-Poiseuille flow becomes unstable to infinitesimal perturbations (linear stability) in the main flow, i.e. the flow does not remain laminar. The other is a lower critical Reynolds number $\underline{Re_c}$ below which the steady Hagen-Poiseuille flow remains stable even for finite amplitude perturbation, i.e. the laminar flow does not become turbulent.

Numerical computation by a linearized NS equation [39, 40] showed that the steady Hagen-Poiseuille flow remains stable to infinitesimal perturbations up to $\overline{Re_c} = 10^7$. This implies that all sufficiently small perturbations decay in the flow. Later careful experiments showed $\underline{Re_c} = 50,000$ [41] and larger

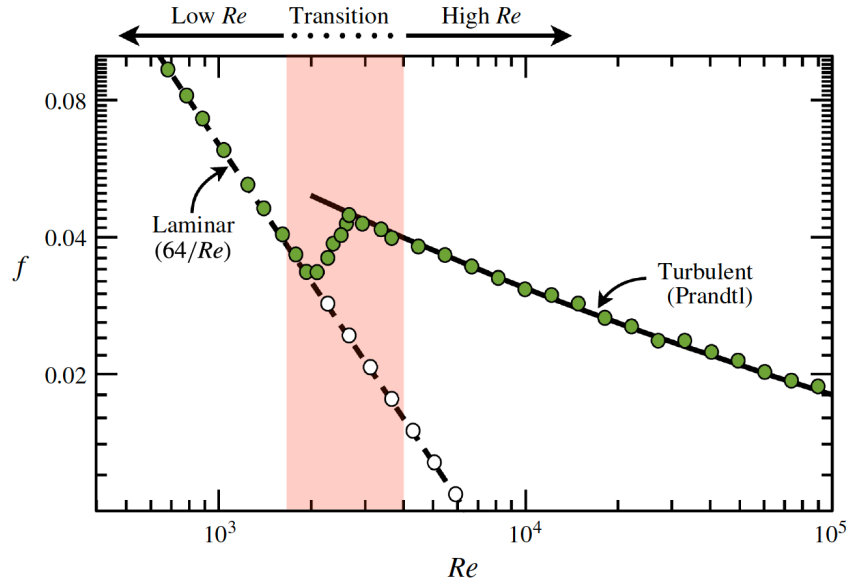


Figure 2.2: Diagram for the dimensionless friction factor f defined as Eq. 2.2; the friction factor for the laminar and the turbulent flow are shown in the dashed and solid lines, respectively. Laminar-to-turbulent flow transition occurs in the range of Reynolds number shown in the shading where the friction factor jumps between the laminar and the turbulent flow [5].

critical values as $Re_c = 100,000$ [42]. Turbulent transition in the pipe, thus, occurs below the upper critical Reynolds number due to finite amplitude perturbations, i.e., subcritical transition.

The abrupt onset is observed in variations of dimensionless friction factor defined as a function of Reynolds number [Fig. 2.2] [5], i.e.

$$f = \frac{\Delta p D}{\frac{1}{2} \rho L U^2} \quad (2.2)$$

where Δp is the pressure drop between two points in streamwise direction over distance L . The friction factor shows the drag law $f = 64/Re$ and the Blasius formula at low and high Re flows, respectively. In the laminar-to-turbulent transition regime, the friction resistance jumps from the former to the latter. In the fluid engineering, prediction and control of the flow transition have

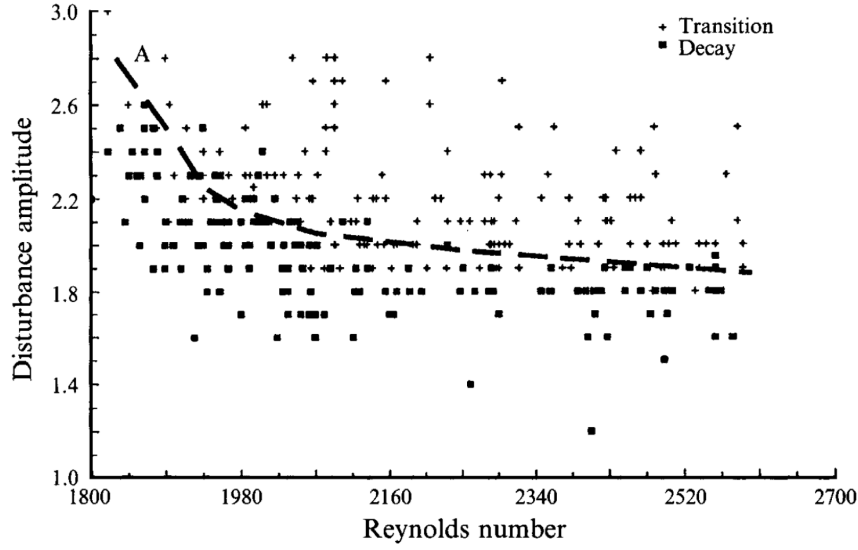


Figure 2.3: Perturbation amplitude required to trigger the flow transition as a function of Reynolds number. The amplitude is the proportional to the volumetric flux of the injected fluid [6].

been required since the friction resistance in the pipe is varied rapidly when the flows become turbulent.

In experimental studies, perturbation injection systems have been adopted to introduce finite amplitude perturbation into the laminar base flow to trigger the flow transition. Perturbations are continuously generated by obstacles [43, 44], a periodically moving wall [45], and a porous wall [46] set in the pipe. In contrast to the above injection system, the base flow is disturbed by injecting or withdrawing liquid as a single jet normal to the main flow [6, 47, 48, 49, 50, 51, 9]. The amplitude is defined as the ratio of the volumetric flux of injected or withdrawn fluid from that of the main flow. Experiments by Darbyshire & Mullin [6] indicated a stability curve above which the laminar flow becomes turbulent and below which the disturbed flow return to the laminar flow, respectively [Fig. 2.3], i.e. the threshold is not determined uniquely at a fixed Reynolds number. The results also showed monotonic decrease of the amplitude with increase of Reynolds number. The critical

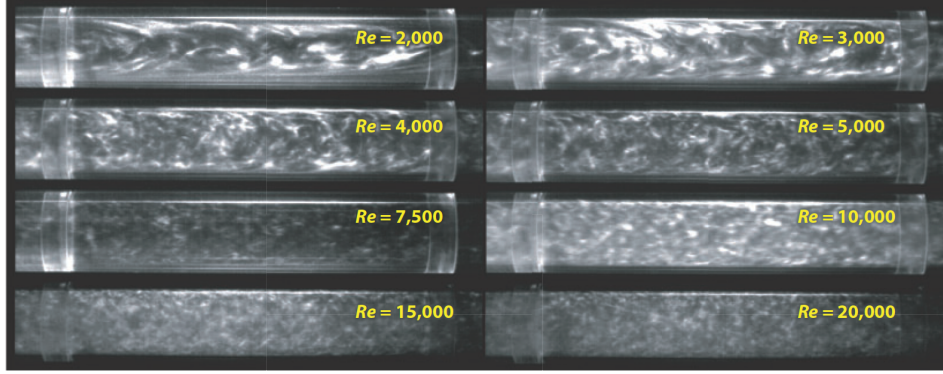


Figure 2.4: Various pictures of disturbed flow patterns with varying Reynolds number between 2,000 and 20,000: flow direction is from left to right. 6.5 diameters of the pipe along the pipe axis are visualized in the pictures [7].

amplitude of perturbation was found to scale as Re^{-1} with consideration of the balance between viscous energy dissipation and nonlinear development of flows in NS equation [47]. The stability curves were obtained thanks to velocity measurements and flow visualizations of the transitional flow since these observations reflect on coherent structures embedded in the dynamics. Results of flow visualization [7] showed a variety of flow patterns in the pipe flows using flakes with varying Reynolds number [Fig. 2.4].

In the pipe flows of $Re \sim 2,000$, the onset of the transition is found to start with an appearance of a localized turbulence between laminar flows, named a turbulent puff [43] (or a flash [4]). Puffs travel at a speed around $0.9U$ with realized size about $20\text{--}30D$ along the pipe axis [52]. Streamwise velocity profile on the pipe axis reflects on the localized structure of puff [Fig. 2.5] [6]. The boundary between the laminar and the puff at the leading edge (the upstream side) is distinct while that at the trailing edge (the downstream side) is unclear. Doorne & Westerweel [8] discussed the dynamics of a turbulent puff as follows: (i) laminar flow relatively faster than a puff, thus, enters the trailing edge of the puff; (ii) the laminar flow produces the kinetic energy to sustain the turbulence

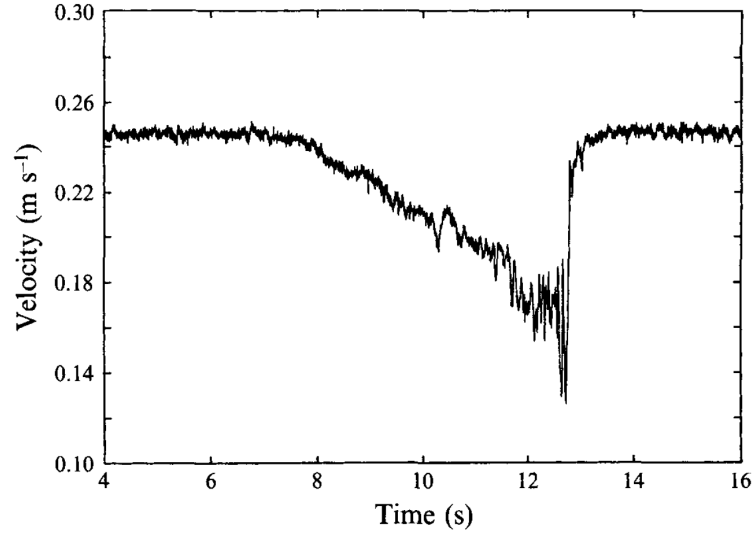


Figure 2.5: Time trace of streamwise velocity profile of a turbulent puff measured by LDV on the pipe axis at $Re = 2,300$: flow direction is from right to left [6].

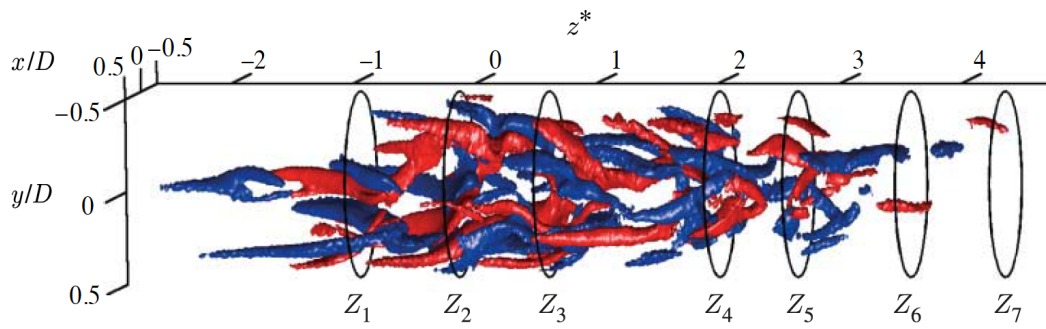


Figure 2.6: Isosurface of counter rotating streamwise vortices ($\pm 3.5s^{-1}$) of a turbulent puff at $Re = 2,000$ measured by Stereoscopic PIV [8]

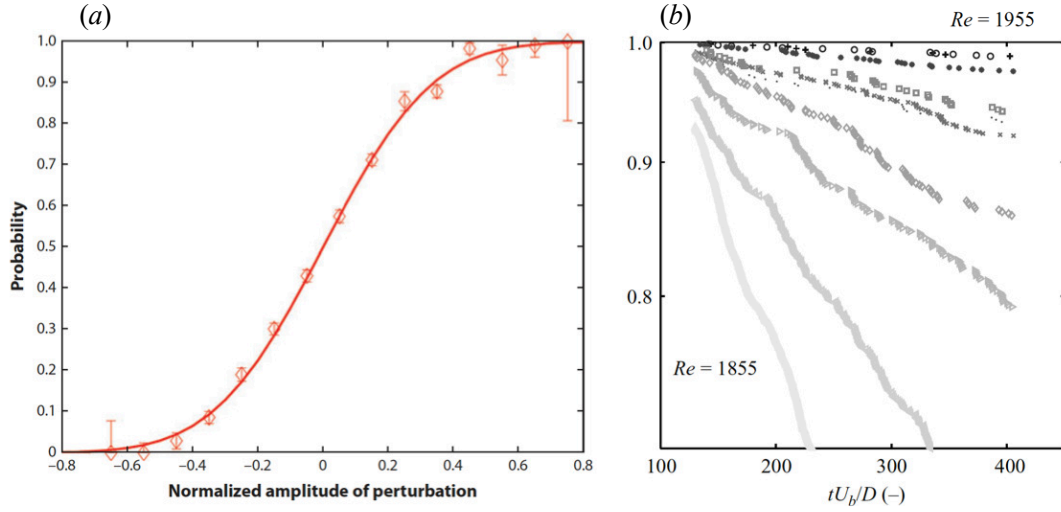


Figure 2.7: Probability of observation of a turbulent puff as a function of (a) the perturbation amplitude at different Reynolds numbers $Re = 2, 170, 3,000, \text{ and } 4,000$ [7] and (b) the time tU/D at different Reynolds numbers in increments of 10 [9].

in the puff; (iii) the turbulence decays due to viscous energy dissipation at the leading edge and returns to laminar flow. Barkley [5] modeled the behavior of this energy evolution as an advection-reaction-diffusion equation, resulting in good qualitative agreement with experimental observations. Experiments by Hof *et al.* [10] showed that streamwise velocity profiles of puffs have inflection points at the trailing edge where production of turbulent kinetic energy in puffs are found at the points thanks to DNS [Fig. 2.8]. A direct numerical simulation (DNS) based on a spectral method was performed by Shimizu & Kida [53] for pipe flows developing from Hagen-Poiseuille flow. The results showed a self-sustenance process of a turbulent puff as follows: (i) turbulence in a puff creates low-speed streaks (relatively slower than laminar flow) and two counter rotating streamwise vortices similar to wall-bounded turbulence; (ii) at the trailing edge, shear layers are created due to velocity differences between the streaks and the laminar flow penetrating the puff; (iii) the shear layers are rolled up by Kelvin-Helmholtz instability, resulting in production of turbulence

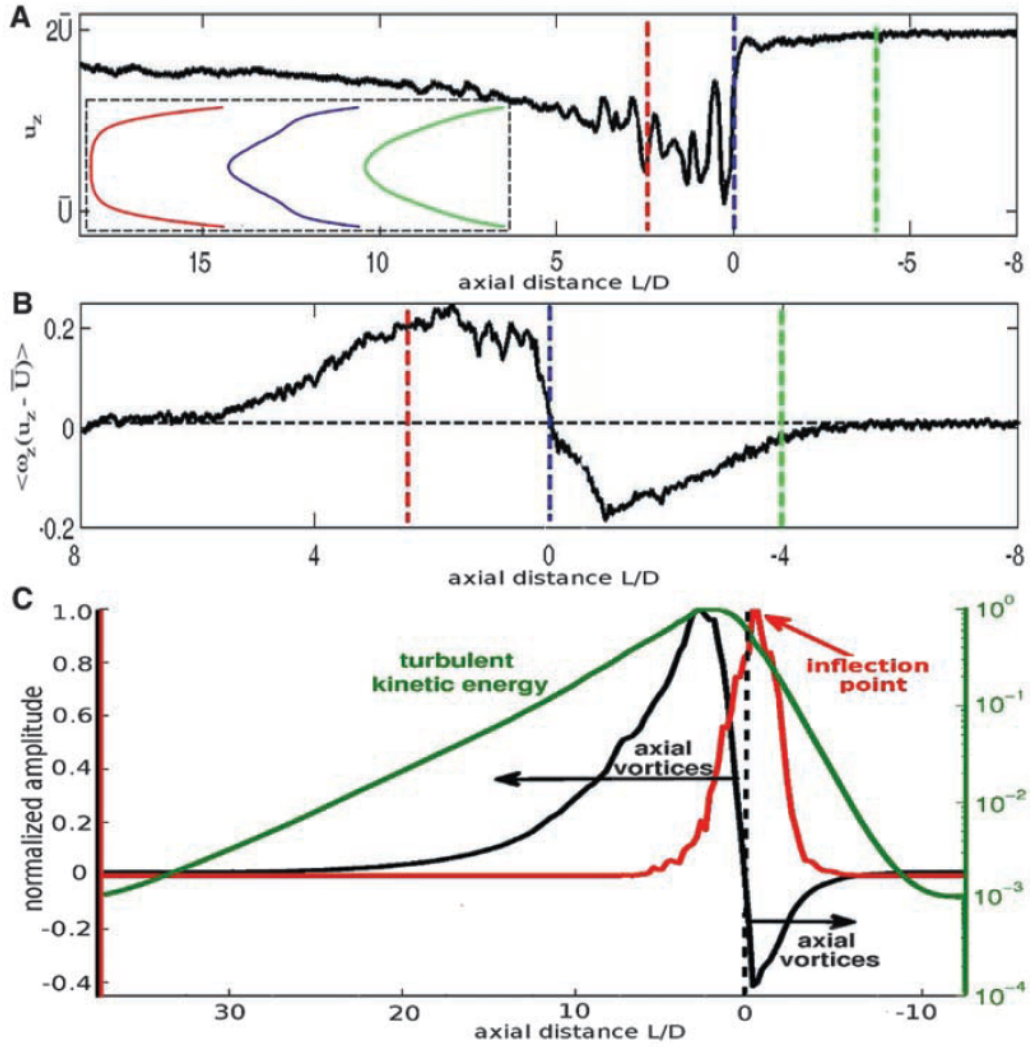


Figure 2.8: (A) A streamwise velocity profile of a turbulent puff has inflection points at the trailing edge as shown azimuthally averaged velocity profiles in the downstream positions. (B) Transport of streamwise vorticity shows that the vorticity moves downstream and upstream for positive and negative axial distance, respectively. The creation of the vorticity is found at the trailing edge of the puff. (C) Production of turbulent kinetic energy is found at the inflection points [10].

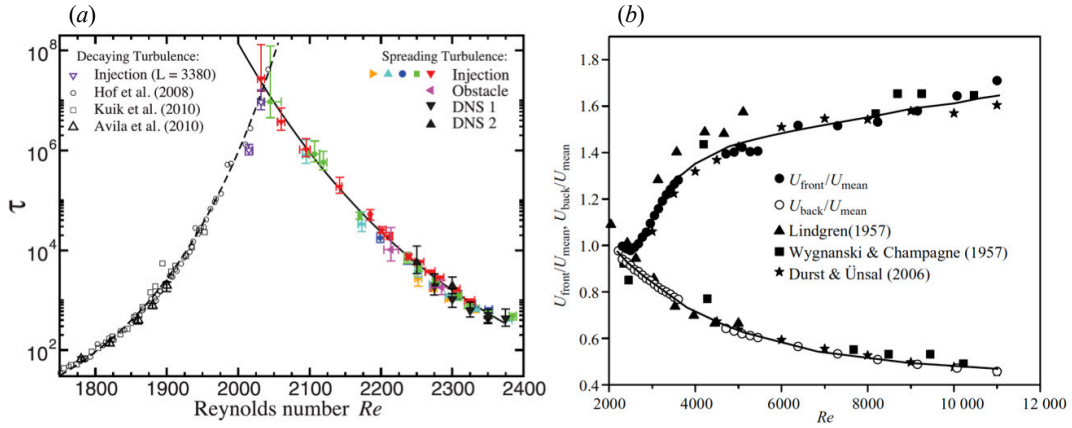


Figure 2.9: (a) Time-scale competition of a turbulent puff for returning to laminar flow or for splitting with varying Reynolds numbers: the intersection corresponds to $Re_c = 2,040 \pm 10$ [11]. (b) Propagation velocities at the leading and the trailing edge of puffs and slugs [12].

in the puff. For the pipe flows at $Re = 2,000$, counter rotating streamwise vortex pairs of a turbulent puff were observed by Doorne & Westerweel [8] with stereoscopic PIV (SPIV) [Fig. 2.6]. Experimental studies by Mullin [7] showed that puff creation is determined by probability and the probability increases with the perturbation amplitude thanks to 5,797 experiments for different Reynolds number [Fig. 2.7(a)]. Quantitative measurement of the pressure and the velocity in the pipe flow by Kuik *et al.* [9] determined the finite lifetime of turbulent puffs depending on Reynolds number, i.e. puffs travel in a downstream distance and finally returns to laminar state [Fig. 2.7(b)].

Transition from a puff to fully-developed turbulent flow starts for a larger Reynolds number. In the pipe flows for a larger Reynolds number, an increase of turbulence in a puff leads to puff splitting, i.e. new puffs are created downstream of the puff [43, 12]. For determining the lower critical Reynolds number, Avila *et al.* [11] showed time-scale competition of a puff for returning to laminar flow or for splitting with varying Reynolds number [Fig. 2.9(a)]. The critical value of Re was found to be $2,040 \pm 10$ due to their experiments and

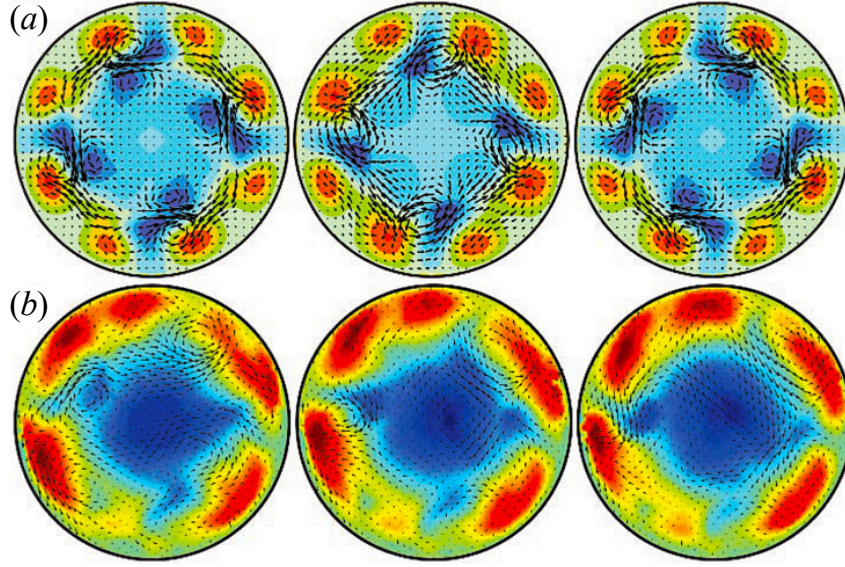


Figure 2.10: Streak patterns of turbulent puffs in the pipe flows observed by (a) experiments and (b) numerical simulations: velocity components in the plane and in downstream direction are indicated by arrows and color coding where red and blue represent velocities faster and slower than the laminar flow.

computer simulations. At a larger Reynolds number, the multiple puffs merge and change into turbulent slug with fluid motions similar to fully-developed turbulence. The leading and the trailing edges of turbulent slug were found to travel, respectively, one-third faster and slower than the mean velocity, implying that turbulent slug expands as it propagates along the pipe and extend the entire pipe, i.e. leading to sustained turbulence [Fig. 2.9(b)] [12].

For determining the exact solutions of the pipe flow, a non-trivial solution of the NS equation for a Reynolds number was considered numerically and theoretically. The existence of stationary exact solutions named three-dimensional traveling waves was found in the pipe for $Re > 1250$ [Fig. 2.10(a)] [54]. The waves travel at the phase velocity and envelop streamwise vortices accompanied with streaks. The exact solutions reflects on the dynamics of transitional pipe flow while the solutions are unstable to perturbation. Experiments by Hof *et al.* [48] showed that these flow structures are coupled with streaks and

streamwise vortices of the realized size about $0.75D$ embedded in puffs.

Chapter 3

Modulation on flow transition

Modulation on flow transition by the presence of dispersed bubbles has been investigated to control flow properties for industrial application industrial applications, e.g. frictional drag reduction of naval vessels since the first report by McCormick & Bhattacharyya [55]. Gas bubbles injected to flows have an impact on the transition and developed turbulence. Murai [13] showed a regime diagram of the mechanism of drag reduction due to bubble injection as a function of flow speed and bubble size in the review article [Fig. 3.1]. At low flow speed and small bubble size in the figure, i.e. bubbles with $O(100 \mu m)$ in the diameter called *microubbble* are injected into flows, the rate of the drag reduction G shows ~ -1 at the maximum value. Laboratory experiments, in contrast to, indicated that the rate is $G = O(10^{-1})$ in dilute microbubbly fluids with the volume fraction $\alpha = O(10^{-4})$ [56, 57, 58, 59, 60, 61]. This discrepancy has been focused from the view points of modulation on flow transition due to mutual interactions between dilute dispersed bubble and coherent structures embedded in transitional and turbulent flows. Mutual interactions between vortical structures and microbubbles with the volume fraction $O(10^{-4})$ were observed in Taylor-Couette flows [14]. The experimental results showed that

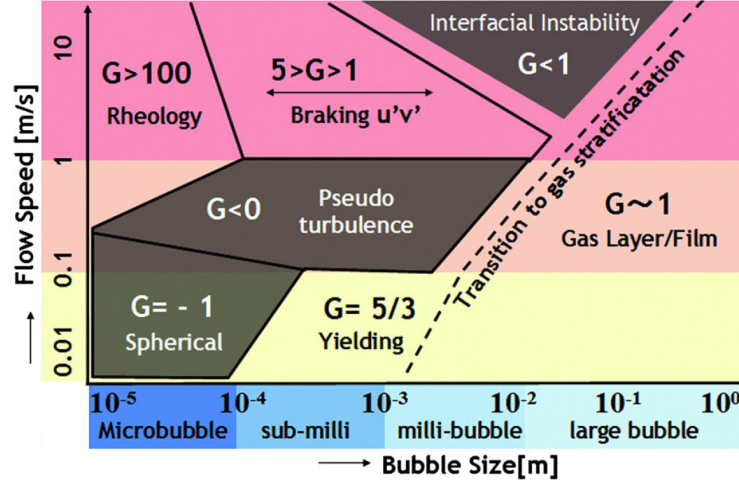


Figure 3.1: Regime diagram of the mechanism of drag reduction due to bubble injection as a function of flow speed and bubble size: G is the rate of the drag reduction defined as $G = \alpha \Delta D_R / D_R$ where D_R and ΔD_R are drag without bubbles and reduced drag due to bubble injection with the bulk volume fraction α [13].

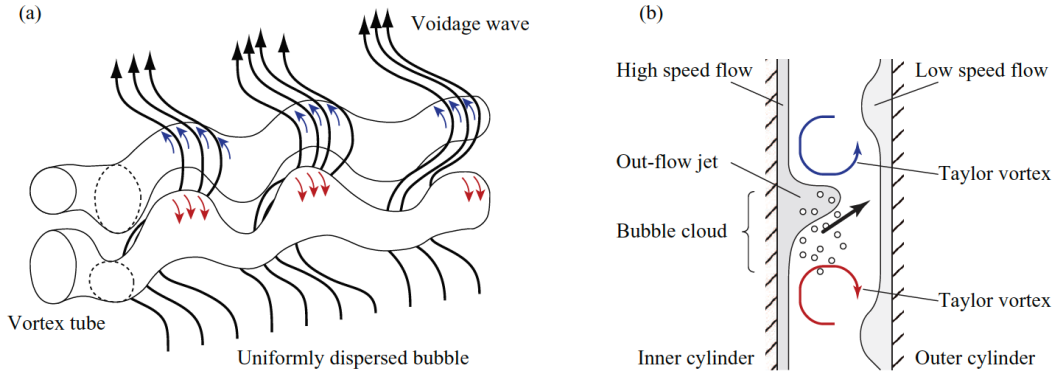


Figure 3.2: Schematic illustration of (a) migration of uniformly dispersed microbubbles in Taylor vortex tubes and (b) ascending motion of a microbubble cluster in the cross section of Taylor vortices [14].

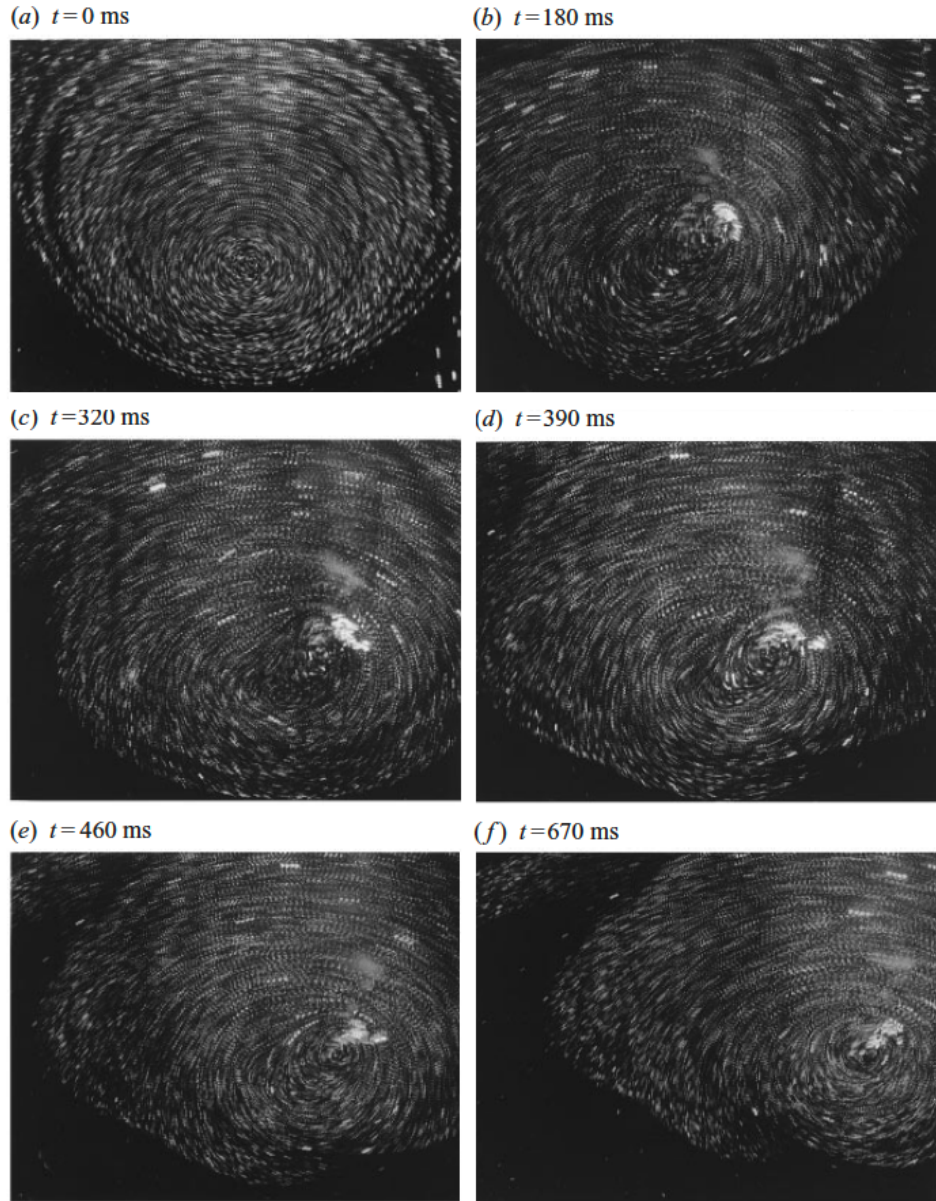


Figure 3.3: Vortex rings with entrained microbubbles for different times: the flow is from right to left [15].

powers of the primary wavy motion on Taylor vortices in the dilute bubbly fluids are enhanced compared to that in the single-phase flows due to preferential concentration of microbubbles. Homogeneously dispersed microbubbles are preferentially concentrated in the vortices and migrate between vortices [Fig. 3.2]. In horizontal channel flows, bursting phenomena of streamwise vortices are impeded by the concentration [13]. Experimental studies by Sridar & Katz [15] to investigate effects of entrained bubbles on the structures of the vortex rings showed migration of the vortex core and increase of the vortices compared to that in single-phase flows due to the preferential concentration of dilute microbubbles in the vortex. A point bubble model has been introduced to predict motions of a small spherical bubble in flow transition and turbulence. The model consists of force balances between self-inertia, buoyancy, added-mass, viscous drag, and lift (e.g. [38]). Climent & Magnaude [62] performed DNS for flows of two-dimensional Lamb-Ossen vortex with a single small rising bubbles based on a point bubble model considering two-way coupling effects between the phases and neglecting bubble-bubble interactions due to the dilute limits. The results confirmed that local vorticity variation due to two-way coupling is around 20% of the local undisturbed flow similar to the previous works.

In pipe flows, modulation on laminar-to-turbulent transition was observed due the presence of dispersed microbubbles. Experimental studies were conducted to investigate effects of the bubbles on frictional drag reduction in circular pipe flows. Experiments on flow transition of bubbly fluids in a horizontal pipe [18] and a vertical pipe [16, 17] showed the increase of $\overline{Re}_c$ with the gas volume fractions where the volume fractions were varied between 0.2% and 0.5% [Fig. ??, i.e. relaminarization of the flows. The volume fraction was large enough to realize *milky bubbly flows*. These observations, however, do

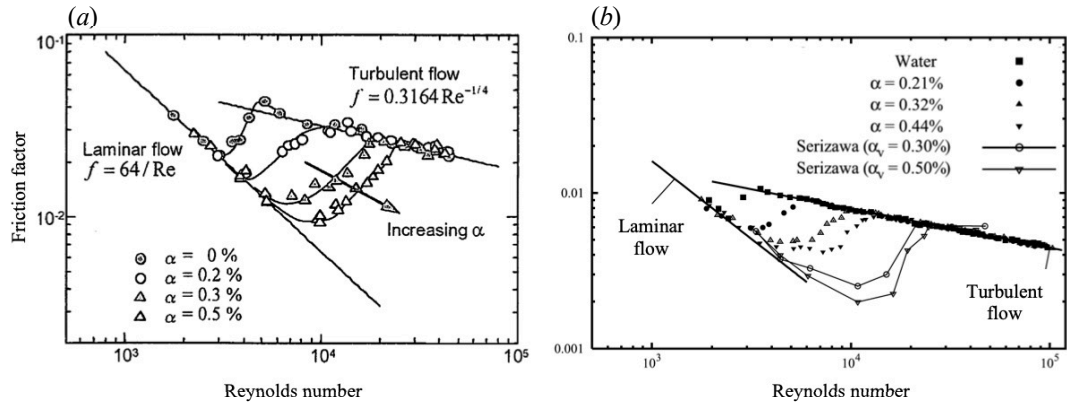


Figure 3.4: Two-phase friction factor in (a) a vertical pipe [16, 17] and (b) a horizontal pipe [18] flow, respectively: α in the figures represents the gas volume fraction in the flows.

not report any results for the transition of dilute bubbly fluids.

Chapter 4

Bubble convection

Inhomogeneous bubble distributions are also observed while bubbles are injected into stationary liquids, in contrast to the preferential concentration presented in the previous chapter. This inhomogeneous distributions induce liquid convective flows, called buoyancy-driven bubbly flows. These flows occur in a large variety of applications, e.g., bubble column reactors. Bubble column reactors are widely used in chemical and biological engineering processes as an efficient gas-liquid contactor for mass and heat transfer [27, 63]. Bubbles injected to liquids ascend due to Archimedian buoyancy forces, exchanging mass and heat at liquid-gas interfaces. In a bubble column reactor, size distributions of rising and falling bubbles in space and time were observed depending on the volume fraction and the ascending and descending velocities of bubbles [64]. Bubble column with phase change showed a much wider size distribution of bubbles [65]. Depending on the bubble size, the flow rate of gas injection, and the geometrical shape of the reactor [66, 67, 68], the system exhibits a variety of flow patterns resulting from multiscale phenomena embedded in the dynamics of dispersed two-phase systems. When the gas flux is small, bubbles can rise in liquid with a uniform spatial distribution [66, 69]. The ascending

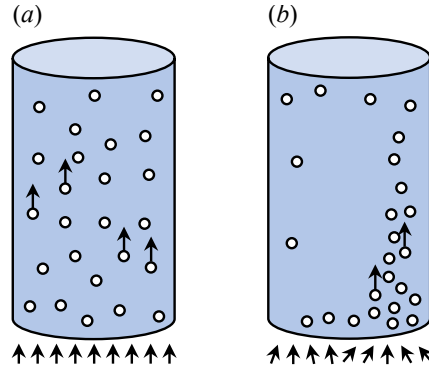


Figure 4.1: Flow transition from (a) homogeneous to (b) heterogeneous regime occurs in bubble column depending on the bubble, size, the gas flow rate, and the geometrical shape of the reactor.

bubble motion does not induce mean flow of the liquid phase [63]. Flows in this homogeneous regime are destabilized at large gas flux and become heterogeneous, where the bubble distribution is no longer uniform. The transition from homogeneous to heterogeneous regimes occurs with convective motion in the liquid phase of a length scale comparable with the system size [Fig. 4.1]. This large-scale convection is crucial for the design of reactors [70]. Theoretical and numerical investigations on the generation of large-scale convections have been performed for semibatch and continuous operation conditions of reactors [70, 71, 72]. Different instabilities associated with the pressure drop in gas injector arrays [70] and with lift forces on rising bubbles close to the lateral wall [71] are reported.

Convection in bubbly flows has also been investigated from the viewpoint of stability of fluid systems with inverse density profiles [19, 73, 20]. When a lower zone of a two-phase layer is richer in bubbles than in the upper zone, the layer is overturned by gravity-driven instabilities similar to the Rayleigh-Taylor (RT) instability. The threshold of the gravity-driven instabilities of bubble layers is often given as a Rayleigh number Ra based on the Archimedean buoyancy of the gas-dispersed liquid phase. Experiments on a RT -like instability of

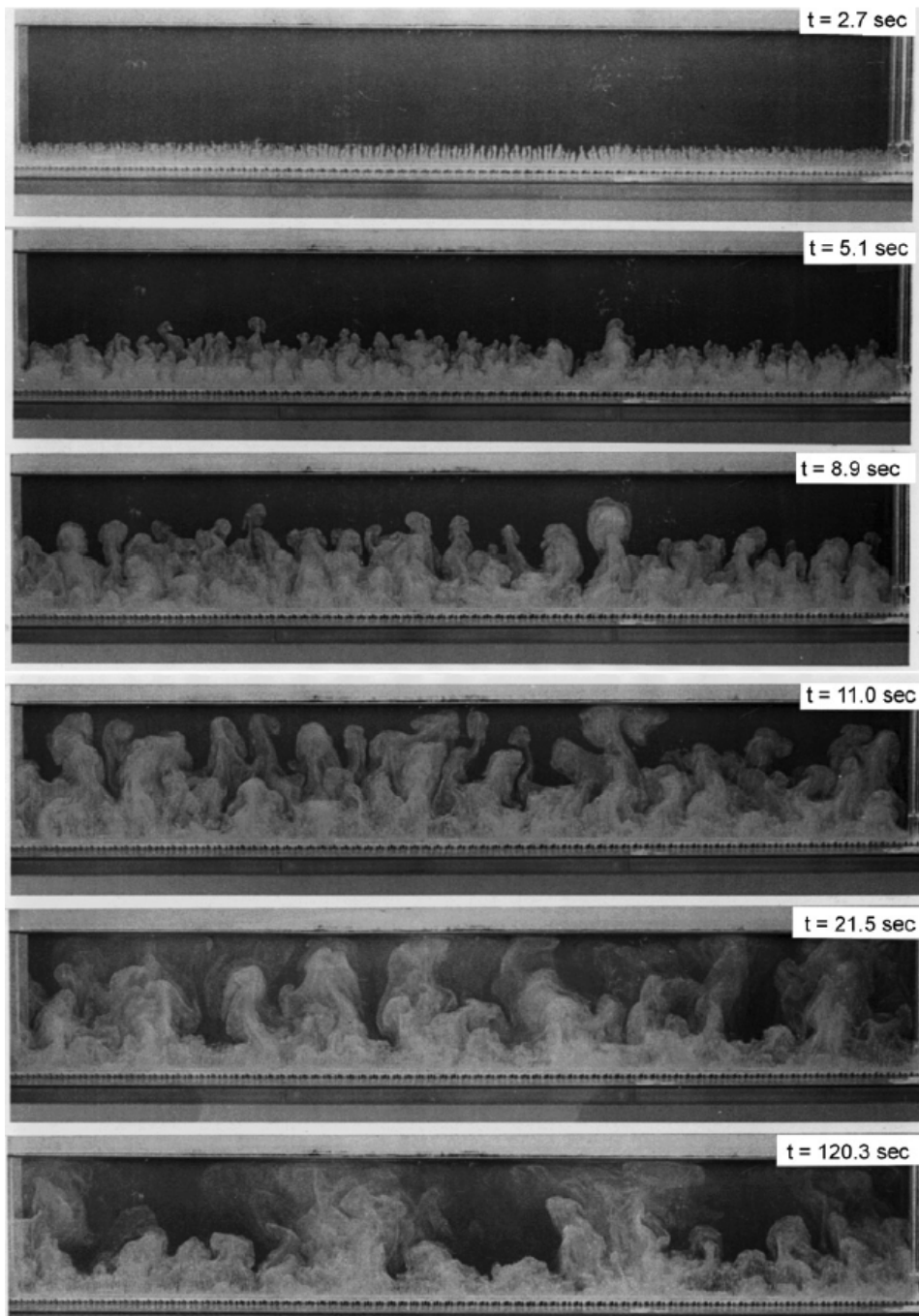


Figure 4.2: The formation of bubble plumes after the development of RT-like instability of bubble layers [19]

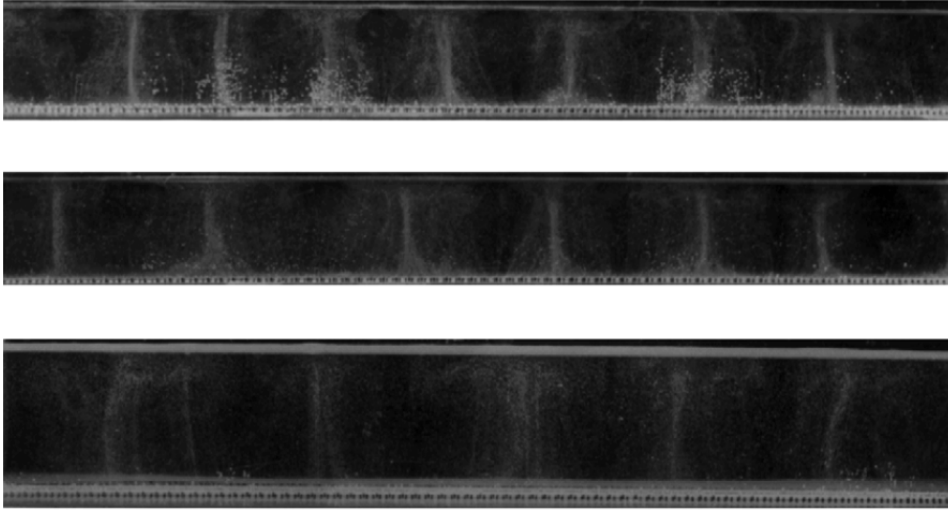


Figure 4.3: Steady counter-rotating convection rolls for different gas injection fluxes [19]

bubble layers showed the formation of bubble plumes [Fig. 4.2 [19]. For $Ra \gtrsim 2.5 \times 10^5$, steady counter-rotating convection rolls are observed by Kimura [73] and Iga & Kimura [19] [Fig. 4.3]. A direct numerical simulation based on a point bubble model was performed by Climent & Magnaudet [20] for flows developing from a particular two-layer state, in which a pure liquid layer is superposed on a bubble layer [Fig. 4.5]. The results confirm the formation of convection rolls for $Ra \sim 2.0 \times 10^5$ [Fig. 4.5]. The point bubble model is based on an Euler-Lagrange approach. Bubbles are regarded as point sources of buoyancy subjected to drag and lift forces. A spatially filtered Navier-Stokes equation is coupled with the equations of motion for individual point bubbles. This model does not take into account diffusion-like processes (*hydrodynamic diffusion*) in the bubble dynamics, which prevents the formation of highly buoyant bubble clusters [74] and results from direct hydrodynamic bubble-bubble interactions [75]. The results of Climent & Magnaudet [20] suggest that no wavenumber selection exists for convection roll formation [Fig. 4.5].

Ruzicka & Thomas [74] discuss the analogy between the homogeneous-

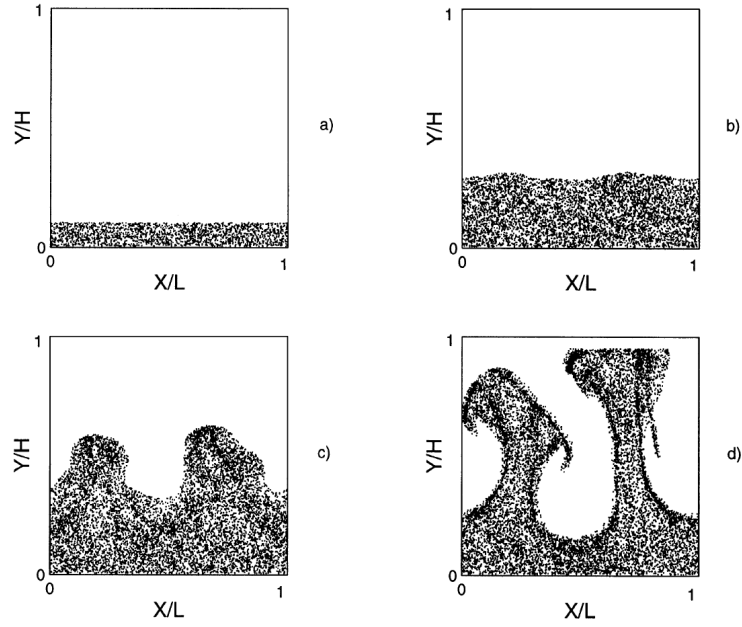


Figure 4.4: Time development of RT-like instability of bubbly layers resulting in the formation of bubble plumes [20]

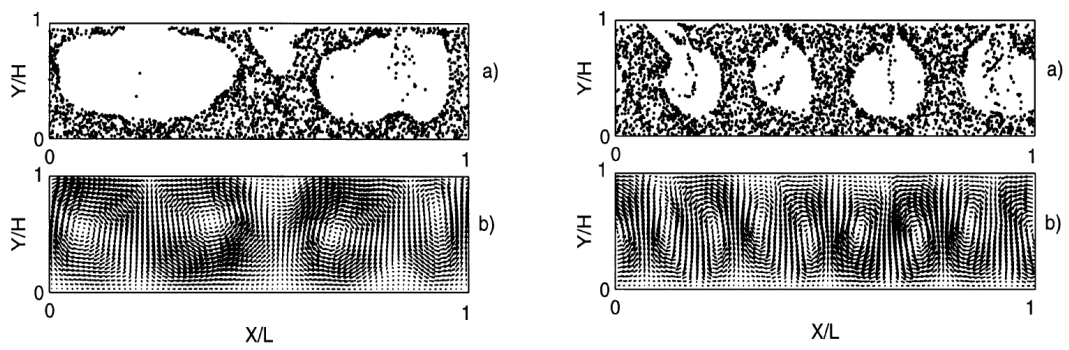


Figure 4.5: Steady counter-rotating convection rolls with different wavenumbers while the same parameter values: (a) bubble distributions and (b) liquid velocity fields [20]

heterogeneous regime transition of bubbly flows and the Rayleigh-Bénard (RB) instability. The authors consider rising bubbles distributed uniformly in a quiescent liquid layer, assuming hydrodynamic diffusion in the behavior of bubble swarms. They examine the analogy in detail and deduce Ra , Prandtl number Pr , and Nusselt number Nu characterizing bubble convections. Ruzicka & Thomas [74] estimate the critical value of Ra to be about 10^5 thanks to their experiments [76]. Though the consideration of the buoyancy driven instability by Ruzicka & Thomas [74] remains qualitative, the analogy indicates the existence of a wavenumber selection mechanism. This contradicts the results of the afore-mentioned simulation of bubbly flows [20]. This discrepancy might be explained *either* from the difference in the base states considered in these studies *or* from the effects of hydrodynamic diffusion, which are absent in the simulation by Climent & Magnaudet [20].

Chapter 5

Objective

The epoch making study by O. Reynolds [4] is revisited to investigate an impact of dilute dispersed microbubbles on turbulent puffs [43] observed at the onset of flow transition in a circular pipe using well-designed experimental facilities reducing perturbation, e.g., external vibrations and pipe distortion. Impulsive jets are injected into the laminar flows at moderate Reynolds number, $Re = 1,900$, to create localized turbulent puffs following the previous studies [6, 9, 47, 48, 49, 50, 51]. Laminar pipe flow of dilute microbubbly fluids would be stable without injecting perturbation at this Reynolds number, i.e., dilute dispersed microbubbles do not modify the base flow. At the onset of the transition, in contrast, the bubbles may have sensitive interactions with vortical structures of turbulent puffs as shown by the literatures [13, 14, 15, 62].

Revisiting the problem of homogeneous-heterogeneous regime transition, the wavenumber selection mechanism in the linear stage of transition is examined by performing a linear stability analysis for ascending bubbles in a quiescent liquid layer. In contrast to the existing works on bubble column reactors [70, 71, 72], the transition provoked by the inherent dynamics of bubbly flows is focused in this study, i.e., without consideration any effect of gas

injection systems and of lateral walls. The analysis is based on an Euler-Euler approach by a spatially filtered Navier-Stokes equation for liquid phase coupled with an Eulerian momentum equation for bubbles. In contrast to the Euler-Euler approach of Ruzicka & Thomas [74], no hydrodynamic diffusion is taken into account. The present work would thus determine whether the hydrodynamic diffusion is essential in the wavenumber selection of bubble-induced convection rolls. For simplicity, only the added-mass, drag, lift and buoyancy forces are considered in the momentum exchanges between gas and liquid phases as in the simulation [20].

Part II

Flow transition of bubbly fluids in a circular pipe

Chapter 6

Experimental setup

6.1 Pipe flow facility

Hagen-Poiseuille flow in the base is formed by constant-mass-flux pipe flow facility [Fig. 6.1]. The working fluid is water. A pipe is made of acryl resin with $D = 32$ mm inner diameter and 3 mm thick, and 8,000 mm ($= 250D$) length. The pipe is connected to a head tank through a bellmouse with tranpet shape to prevent flow separation in the inlet. The pipe is supported by aluminum pillars to prevent the deflection. Stationary water in the pipe is driven from left to right by a piston system. The piston system is mounted at $250D$ downstream from the inlet. The system consists of a metal cylinder with 250 mm inner

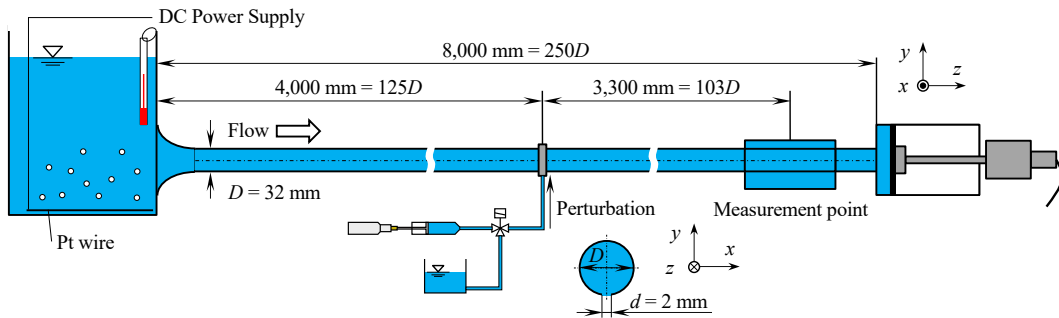


Figure 6.1: Schematic of the constant-mass-flux pipe flow facility

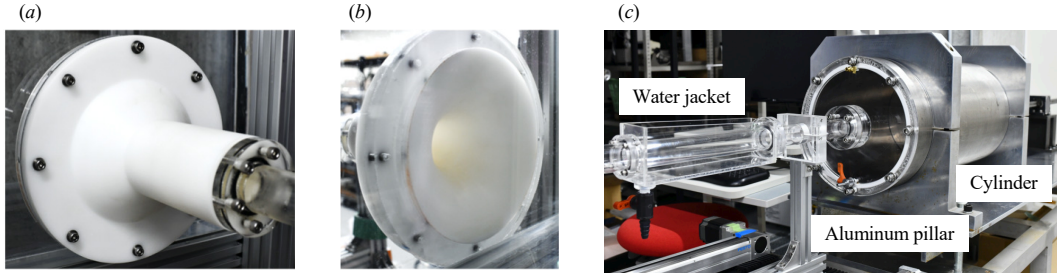


Figure 6.2: Schematic of the constant-mass-flux pipe flow facility

Table 6.1: Specification of the cylinder system

Apparatus	Manufacture	Product name
Piston	Yamamoto Engineering	Custom-made
Motor	Oriental Motor Co., Ltd.	LSD20A-W
Motion controller	Oriental Motor Co., Ltd.	EMP400-1

diameter and a piston inside the cylinder. The piston is connected to and is controlled by a stepping motor. Signals are sent from a motion controller to the stepping motor under computer control. Piston speed is kept constant thanks to this control system. Flows in the pipe, therefore, remain constant mass flux regardless of flow transition. In this thesis, Reynolds number is defined by

$$Re = \frac{UD}{\nu} \quad (6.1)$$

where U , D , and ν are mean velocity in the pipe, the pipe diameter, and the kinematic viscosity of the working fluid. A measurement point for laser Doppler velocimetry and stereoscopic particle image velocimetry is located at $106D$ downstream from the inlet. A rectangular water jacket filled with water is mounted at the point to reduce laser reflection on the pipe wall.

Results of flow visualization using flake particles show that the flow remains laminar up to $Re = 25,000$ thanks to well-designed facility, precise alignment of the pipe, and isolation of external vibrations. Sufficiently long settling time around 30 minutes between piston runs is required for development of laminar

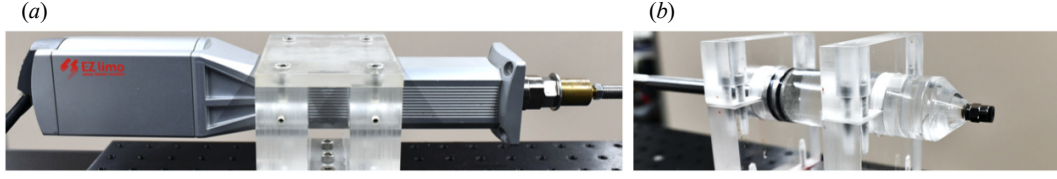


Figure 6.3: Schematic of the constant-mass-flux pipe flow facility

Table 6.2: Specification of the perturbation system

Apparatus	Manufacture	Product name
Actuator	Oriental Motor Co., Ltd.	EZHC4A-10
Solenoid valve	CKD	USG2-M5-1-DC12V
Pulse generator	Stanford research system, inc.	DG645
Polyethylene tube	Sansyo Co., Ltd.	Hibiki size: 9

pipe flow at this high Reynolds number. In contrast to, at the onset of flow transition in the pipe, i.e., at Reynolds number $Re \sim 2000$, the settling time is found at least 10 minutes. These settling times are related to decay of perturbation in the head tank and imply the sensitivity of the flow transition to inlet perturbation.

6.2 Perturbation system

Finite amplitude perturbation is required to cause the flow transition since Hagen-Poiseuille flow is linearly stable. In this study, the transition is triggered by injecting an impulsive jet through a hole with 2 mm diameter on the pipe wall located at $127D$ downstream from the inlet. The jet is injected orthogonally to the main flow with a volumetric flux controlled by a perturbation system. The perturbation system consists of an actuator, a syringe, a pulse generator, and solenoid valve. The syringe, the reservoir tank, and the pipe are connected through the solenoid valve. Pulse signals are sent from the pulse generator to a stepping motor connected with the actuator under com-

Table 6.3: Specification of the microbubble generator

Apparatus	Manufacture	Product name
DC power supply	Kikusui Electronics Corp.	Custom made
Platinum wire	The Nilaco Corp.	$\phi = 0.01$ mm, 99.98%

puter control. The actuator pushes the syringe with a constant speed owing to this control. The solenoid valve permits to switch the flow channel from the syringe to the pipe or reservoir tank. When the actuator starts to move, a pressure spike is generated in the injected impulsive jet. The jet is, therefore, injected to the main flow with switching the solenoid valve after the jet is discharged to the reservoir tank in a time t_{pre} . When the position is 0, the jet is discharged into the reservoir tank. When the position is changed to 1, the jet is injected into the main flow at a given volumetric flux. Perturbation amplitude A is defined as the ratio of volumetric fluxes

$$A = \frac{Q_{inj}}{Q_{pipe}} \quad (6.2)$$

where Q_{inj} and Q_{pipe} are injection amount of the jet and flow rate in the pipe. After the injection of jet to the main flow, the jet is also discharged to the reservoir in a time t_{post} . In this experiments, t_{pre} and t_{post} are fixed at 10 s and 1.5 s, respectively. This system ensures the reproducibility of the perturbations.

6.3 Apparatus for microbubble generation

In this experimental study, water electrolysis is utilized to generate microbubbles. A platinum wire with $100\mu\text{m}$ in diameter is used as an anode and a cathode. These electrode are mounted on the bottom wall in the head tank and connected with DC electrical power source.

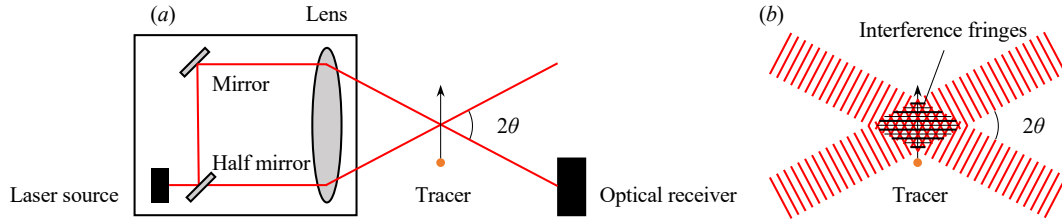


Figure 6.4: (a) A schematic diagram of a LDV system and (b) the measurement region with the interference fringes

6.4 Laser Doppler velocimetry (LDV)

6.4.1 Principles

This system measures local and instantaneous velocity of particles following fluid flows. JIS test powders 1, No. 11 with $2\mu\text{m}$ in diameter with the volume fraction less than 0.01% are dispersed as tracer particles for the measurement. The transition in the pipe shows supercritical behaviors when particles are dispersed in the flow while depending on the volume fraction [77, 78]. The ratio of the particle and pipe diameters determines formation of preferential concentration of the dispersed phase, known as Segré-Silberberg effects [79, 80, 81]. The effects dominate for the ratio larger than 0.07 [79, 80, 81]. The values of the particle volume fraction and the ratio in our experimental conditions do not trigger these modulations. The stability of the laminar flow with the particles is observed, i.e. the flows remains laminar up to $\overline{Re}_c = 25,000$.

LDV is a velocimetry based on Doppler effect. A laser light from the source is divided into two lasers. One goes straight while the other is reflected by a half mirror. These two lasers enter a lens horizontally and cross at a section, where the interface fringes are formed due to the optical path difference. This section corresponds to a measurement volume of the LDV system. Intensity of scattered light is varied when the particles pass through the fringes. The

Table 6.4: Specification of the LDV system

Apparatus	Manufacture	Product name
Laser source	Univ Tech Co., Ltd.	Custom made
Optical receiver	Univ Tech Co., Ltd.	Custom made
Frequency counter	System Kobo	LDV signal processor
Low pass filter	NF corporation	Filter for LDV
Oscilloscope	Iwatsu Test Instrument	Analog Oscilloscope SS-7804
AD board	Contec	AD12-16U(PCI)E
Tracer particle	Hosokawa Micron Corp.	JIS test powder 1, No. 11

variation is received by an optical device and is converted into Doppler frequency by a frequency counter. Doppler frequency and the velocity have a relationship,

$$f_D = \frac{2m \sin \theta}{\lambda} u_D \quad (6.3)$$

where f_D , m , u_D , θ , and λ are Doppler frequency, refraction rate, velocity in the parallel direction of the lasers, the half angle between the two lasers at the measurement volume, and a wavelength of the laser in vacuum condition, respectively. In a gas-liquid two phase flow, signal dropout occurs when gas phase enters the measurement volume due to the lack of tracer particles. The threshold of this dropout was given based on the gas volume fraction, i.e. the LDV system works correctly for the gas volume fraction less than 2%. The experimental condition does not exceed this threshold since the maximum volume fraction of microbubbles is less than 0.018%.

6.4.2 Setup of LDV

Turbulent puffs show the characteristic velocity fluctuation in the streamwise direction at the pipe center. LDV is, thus, utilized to measure the fluctuation to observe puff creation. The measurement point is determined based

on development length of turbulent pipe flows. Prandtl & Tietjens reported development length of turbulent flow in a pipe as,

$$l_T = (25 - 40)D. \quad (6.4)$$

The measurement point is, thus, performed $106D$ downstream from the perturbation injection point.

6.5 Stereoscopic particle image velocimetry

6.5.1 Principles

Particle image velocimetry (PIV) [82] is an imaging-based technique to measure two components of velocity in flow fields based on migrations of tiny tracer particles in the fields illuminated by a laser light sheet. The tracer particles follows the fluid motions when the particles are neutral buoyant and the size is smaller than characteristic length scales of fluid motions. Positions of the tracer particles at two different times separated by a set time interval taken by photographic film or single digital camera determine the displacements between the two images. PIV analysis to determine particle velocity is, in general, based on the cross correlation. The images are divided into small areas called interrogation windows. The cross correlation is applied between the windows to find the mean displacement of the particles. Subpixel interpolations are required for interpolation of the displacements between pixels [83, 84]. A Gaussian fitting to the first peak of the correlation plane show the accuracy of about 0.1 pixel in the sub-pixel interpolations [83]. Velocity of the tracer particles is calculated by the displacements over the time interval.

PIV techniques have been introduced to obtain three components of fluid

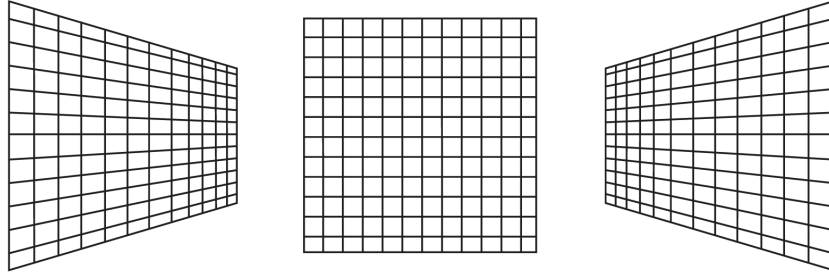


Figure 6.5: The perspective distortions of the images recorded by two cameras due to the angle displacement arrangement [21]

velocity in a plane or in a volume, e.g., holographic PIV [85, 86], three-dimensional particle tracking velocimetry [87], tomographic PIV [88], and stereoscopic PIV (SPIV) [89]. In this thesis, SPIV system is utilized to measure three components of velocity in two-dimensional plane, i.e. in entire pipe sections for visualizing creation and development of vortical structures embedded in a turbulent puff as the same approach with the previous studies (e.g., [8, 48]). In the system, positions of the tracer particles in a two-dimensional field illuminated by a laser sheet are recorded by two digital cameras at different view axes or with an offset distance. The two particle images are matched to reconstruct three components of velocity vectors in the image planes based on methods of the lens translation, the angle displacement, and the displacement with Scheimpflug condition, providing a large measurement window for SPIV as the advantage [21].

In these methods, recorded image is distorted since the image sensors of the cameras are tilted with respect to the laser sheet [Fig. 6.5]. Mapping function between the coordinates of the physical space and image sensors is determined to reduce the disadvantage [90]. Soloff *et al.* [91] proposed a mapping function based on general polynomial function. The coordinates of the physical space

(x, y, z) are associated with that of the image sensors (X, Y) :

$$X = F_X(x, y, z) \quad (6.5)$$

$$Y = F_Y(x, y, z) \quad (6.6)$$

where F_X and F_Y are the mapping function depending on the system. Differentiating the above equations with respect to time, velocity in the image planes are given as

$$\begin{aligned} U &= \left(\frac{\partial X}{\partial t} \right) \\ &= \frac{\partial F_X}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial F_X}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial F_X}{\partial z} \frac{\partial z}{\partial t} \\ &= \frac{\partial F_X}{\partial x} u + \frac{\partial F_X}{\partial y} v + \frac{\partial F_X}{\partial z} w, \end{aligned} \quad (6.7)$$

$$\begin{aligned} V &= \left(\frac{\partial Y}{\partial t} \right) \\ &= \frac{\partial F_Y}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial F_Y}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial F_Y}{\partial z} \frac{\partial z}{\partial t} \\ &= \frac{\partial F_Y}{\partial x} u + \frac{\partial F_Y}{\partial y} v + \frac{\partial F_Y}{\partial z} w, \end{aligned} \quad (6.8)$$

where (u, v, w) are velocity of the particles in physical space. In PIV measurement, the velocity is approximated as the displacement over a time interval between the two particle images, i.e., $(u, v, w) = (\Delta x / \Delta t, \Delta y / \Delta t, \Delta z / \Delta t)$ and $(U, V) = (\Delta X / \Delta t, \Delta Y / \Delta t)$. The velocity in the physical space (u, v, w) is, therefore, associated with that of particles in image (U, V) following matrix:

$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \frac{\partial F_X}{\partial x} & \frac{\partial F_X}{\partial y} & \frac{\partial F_X}{\partial z} \\ \frac{\partial F_Y}{\partial x} & \frac{\partial F_Y}{\partial y} & \frac{\partial F_Y}{\partial z} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (6.9)$$

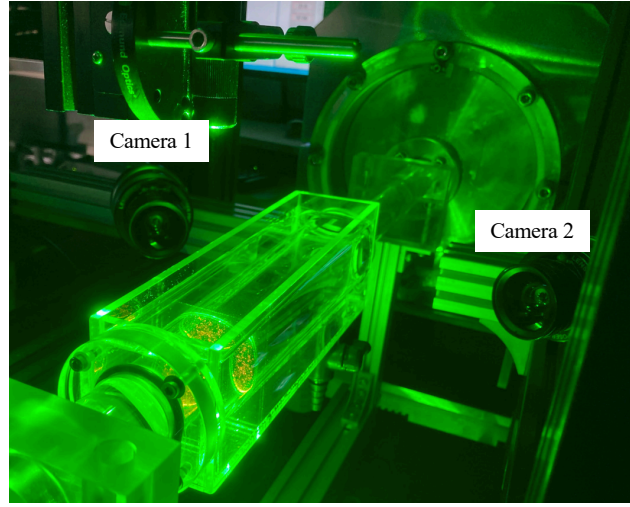


Figure 6.6: Setup of SPIV system: fluorescent microparticles are dispersed in the flow. The flow is from left to right.

In SPIV system with two cameras, the above matrix is rewritten as:

$$\begin{bmatrix} U^1 \\ V^1 \\ U^2 \\ V^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial F_X^1}{\partial x} & \frac{\partial F_X^1}{\partial y} & \frac{\partial F_X^1}{\partial z} \\ \frac{\partial F_Y^1}{\partial x} & \frac{\partial F_Y^1}{\partial y} & \frac{\partial F_Y^1}{\partial z} \\ \frac{\partial F_X^2}{\partial x} & \frac{\partial F_X^2}{\partial y} & \frac{\partial F_X^2}{\partial z} \\ \frac{\partial F_Y^2}{\partial x} & \frac{\partial F_Y^2}{\partial y} & \frac{\partial F_Y^2}{\partial z} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (6.10)$$

where (U^1, V^1, U^2, V^2) is the particle velocity in two images recorded by two cameras and $(F_X^1, F_Y^1, F_X^2, F_Y^2)$ is the mapping function for the two cameras [Fig. 6.6]. In vector representations, the matrix is given as

$$\mathbf{U} = \nabla F \mathbf{u}. \quad (6.11)$$

The mapping function was given as a polynomial function [91], e.g., a fourth order polynomial function in the plane parallel to the laser sheet and a second

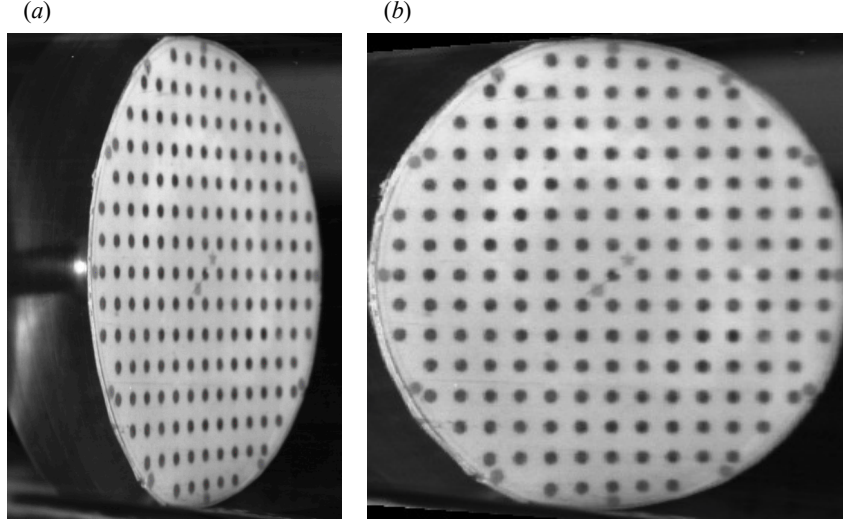


Figure 6.7: (a) The image of the calibration plate from camera 2 presented in Fig. 6.6 and (b) the dewarped image based on the mapping function

order along the direction normal to the laser sheet [21]:

$$\begin{aligned}
 F(x, y, z) = & a_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5xy + a_6y^2 \\
 & + a_7xz + a_8yz + a_9z^2 + a_{10}x^3 + a_{11}x^2y + a_{12}xy^2 \\
 & + a_{13}y^3 + a_{14}x^2z + a_{15}xyz + a_{16}y^2z + a_{17}xz^2 \\
 & + a_{18}yz^2 + a_{19}x^4 + a_{20}x^3y + a_{21}x^2z^2 + a_{22}xy^3 \\
 & + a_{23}y^4 + a_{24}x^3z + a_{25}x^2yz + a_{26}xy^2z + a_{27}y^3z \\
 & + a_{28}x^2z^2 + a_{29}xyz^2 + a_{30}y^2z^2.
 \end{aligned} \tag{6.12}$$

The x and y directions, in this study, corresponds in pipe sections and the z direction to that of the pipe axis.

For determining the coefficients $a_i (i = 0, \dots, 30)$ based on a least square method, a calibration plate with dots [Fig. 6.7(a)] is traversed at different positions across the depth of the laser sheet, i.e., the axial direction of the pipe. The image of the calibration plate in Fig. 6.7(a) is dewarped based on

Table 6.5: Specification of the tracer particles

Product name	Fluostar Rhodamine B-encapsulating microspheres
Manufacture	BMG Labtech
Substrate material	Carboxy-modified acrylate resin
Refractive index	1.560 (polymer)
Temperature	Resistant up to 100°C
Fluorescence dye	Rhodamine B (Excitation 550 nm / Emission 580 nm)
Density	1.1 g/cm ³
Diameter	15 μ m
Size uniformaty	Less than 20% coefficient of variation

the mapping function as shown in Fig. 6.7 (b). From Eq. 6.11, the velocity $\mathbf{u}$ in the physical space is obtained:

$$\mathbf{u} = [\nabla F^T \nabla F]^{-1} [\nabla F^T \mathbf{U}] \quad (6.13)$$

where ∇F^T is the tranpose of the matrix ∇F .

6.5.2 Tracer particles

Polymer particles encapsulating the fluorescent dye, Rhodamine B, are dispersed in the main flow. The particles emit the orange-colored fluorescence when they are illuminated by a green laser. The emission enables to distinguish the particles from the dispersed microbubbles in RGB color space since the bubbles shine by the reflected light of the green laser. The details of the particles as shown in the following Table.

6.5.3 Setup of SPIV

The measurement point of the SPIV is located at the same point with LDV. Positions of the fluorescent particles and microbubbles in the pipe are recorded with the two cameras to reconstruct 2D-3C velocity fields based on the afore-

mentioned principles of SPIV and to visualize microbubble distributions. The particles and microbubbles are, respectively, detected in RGB color space.

Chapter 7

Experimental conditions

7.1 Gas phase

7.1.1 Gas volume fraction

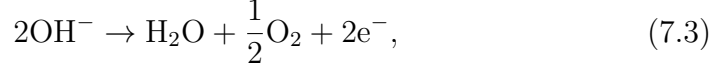
Volume fraction of microbubbles is defined as

$$\alpha = \frac{Q_b}{Q_{pipe}} \quad (7.1)$$

where Q_b and Q_{pipe} are the volumetric flux ratio of the generation and the main flow in the pipe. The order of α is approximated from Faraday's law coupled with the gas state equation and ionic equation for water electrolysis with the assumptions as follows: (i) the generated bubbles are ideal gas; (ii) atmospheric pressure $P = 101.3$ kPa; (iii) gas constant $R = 8.3143$ J/(mol · K); (iv) water in room temperature $T = 293$ K. The gas state equation for Q_b in per unit time is described by

$$Q_b = \frac{nRT}{P} \quad (7.2)$$

where n is amount of substance. Ionic equations of water electrolysis for anode and cathode, respectively, are described as



where electron is indicated by e in the above equations. Assuming ideal faradaic efficiency, the amount of bubble generation is proportional to the total electrical charge. Amount of substance for electron n_e per unit time is given as

$$n_e = \frac{I}{F} \quad (7.5)$$

where I and F are an electrical current and the faradaic constant, i.e. $F = 96,485$ C/mol. Eqs. [7.3](#) show that the generations of hydrogen and oxygen bubbles are 1/2 and 1/4 per unit 1 mol electron. The total amount of substance for hydrogen n_{H_2} and oxygen n_{O_2} is, therefore,

$$n_{\text{all}} = n_{\text{H}_2} + n_{\text{O}_2} = \frac{3}{4}n_e = \frac{3I}{4F}. \quad (7.6)$$

Substituting the above equation into Eq. [7.2](#), the resulting equation shows that the amount of bubble generations is coupled with electrical current:

$$Q_b = \frac{3IRT}{4FP}. \quad (7.7)$$

Eq. [7.1](#) is, thus, rewritten as

$$\alpha = \frac{3IRT}{4FPQ_{\text{pipe}}}. \quad (7.8)$$

This equation estimates the maximum values of the gas volume fraction in experimental conditions as less than 0.018%.

7.1.2 Bubble size

Size of bubbles generated by water electrolysis depends on the diameter of electrode [92]. The generated bubbles has a size distribution [93]. The mean diameter based on ensemble average is $30 \pm 20 \mu m$ in this experimental condition.

7.1.3 Dimensionless parameters for the dispersed phase

Bubbles migrating in flows are subject to forces from ambient fluid. Bubble shape, thus, depends on the size, slip velocity between the phases, and the viscous stress from the liquid phase. Modulation of flows was observed due to bubble deformation in space and time. The threshold is given as Capillary number Ca and Weber number We . Ca is the ratio of viscous shear to surface tension defined as,

$$Ca = \frac{2R_b\mu\tau}{\gamma} \quad (7.9)$$

where μ is viscosity coefficient of liquid phase, τ is viscous stress, and γ is surface tension. In the pipe flow transition, the maximum value of viscous stress would be less than that at the pipe wall τ_{wall} :

$$\tau \leq \tau_{wall} = \frac{\rho f U^2}{8}. \quad (7.10)$$

For approximating the order of the viscous stress, the friction factor f is assumed as Blasius resistance formula, i.e., $f = 0.3164 Re^{-0.25}$ in the turbu-

lent flow since the dispersed bubbles migrate in the transitional flow where Reynolds number between $1,700 \lesssim Re \lesssim 2,400$. The order of τ is, thus, estimated by the equation

$$\tau \leq \tau_{wall} = \frac{0.03955\rho U^2}{Re^{0.25}}. \quad (7.11)$$

Substituting the above equation into Eq. [7.9](#), the Capillary number is given as

$$Ca = \frac{0.791R_b\mu\rho U^2}{\Gamma Re^{0.25}}. \quad (7.12)$$

Weber number We measures the relative importance of the inertia compared to the surface tension:

$$We = \frac{2R_b\rho U^2}{\gamma}, \quad (7.13)$$

assuming the bubble velocity as mean velocity in the pipe. Bubble deformations due to the shear and inertial force are assumed negligibly small when Ca and We are sufficiently small than unity. These conditions are satisfied for the experimental conditions. The orders of these two parameters, indeed, are $Ca = O(10^{-6})$ and $We = O(10^{-3})$ for bubbles of $2R_b \sim O(100\mu\text{m})$ in water at room temperature.

The presence of the dispersed phase modulates turbulence and vortical structures depending the size, the volume fraction, and the density, e.g., induced flow separation is observed past a large particle in high Reynolds number flow. For understanding flows around the dispersed phase, particle Reynolds number is introduced:

$$Re_p = \frac{2R_p U_p}{\nu}, \quad (7.14)$$

where Re_p and U_p are particle radius and mean particle velocity. Assuming Stokes' regime as migration of the dispersed phase, Stokes rising velocity is given as,

$$u_{St} = \frac{gR_p^2}{3\nu}. \quad (7.15)$$

Eq. [7.14](#) is, thus, rewritten as,

$$Re_p = \frac{2gR_p^3}{3\nu^2}. \quad (7.16)$$

For the microbubbles and the tracer particles, the orders of particle Reynolds numbers are, respectively, $O(10^{-2})$ and $O(10^{-6})$ sufficiently smaller than unity neglecting induced flow separation behind the dispersed phase.

Dispersed bubbles and particles do not follow fluid flows depending on Stokes number based on the ratio of the characteristic time:

$$St = \frac{\tau_b}{\tau_f} \quad (7.17)$$

where τ_b and τ_f are the time of dispersed phase response to flows and that of the flows, respectively. For a dispersed phase with small particle Reynolds number, Stokes number is given as

$$St = \frac{2\rho_p R_p^2}{9\mu\tau_f} \quad (7.18)$$

where ρ_p is the particle density. Assuming the time of the flows as the ratio of the pipe diameter to mean velocity in the pipe, i.e. $\tau_f = D/U$, the Stokes

number is given as,

$$St = \frac{2\rho_p R_p^2 U}{9\mu D} \quad (7.19)$$

The orders of Stokes number for the tracer particles and the microbubbles are, respectively, $O(10^{-6})$ and $O(10^{-8})$. These values imply that the dispersed phase follows the main flows.

$$\begin{aligned} &= \frac{\sqrt{g R_b} R_b}{\nu}, \\ Ca &= \frac{\rho g R_b^2}{\gamma}, \end{aligned} \quad (7.20)$$

where γ is surface tension and g is gravitational acceleration. Bubble shape keeps spherical and non-deformable when these two dimensionless parameters are sufficiently small: $Ga \lesssim 10^2$ and $Eu \lesssim 10^{-1}$.

7.2 Liquid phase

7.2.1 Effective viscosity

Bulk density of gas-liquid two-phase flow is given by

$$\rho^* = \varepsilon_G \rho_G + (1 - \varepsilon_G) \rho. \quad (7.21)$$

Einstein and Taylor showed the effective viscosity coefficient of liquids with gas phase,

$$\mu^* = \left(1 + \frac{\mu + \frac{5}{2}\mu_G}{\mu + \mu_G} \varepsilon_G \right) \mu. \quad (7.22)$$

Assuming the mass of the bubbles as negligibly smaller than that of the surrounding fluid, i.e., $\rho_G \ll \rho$, Eqs. (7.21)-(7.22) are, thus, written as

$$\begin{aligned}\rho^* &= (1 - \varepsilon_G)\rho, \\ \mu^* &= (1 + \varepsilon_G)\mu.\end{aligned}\tag{7.23}$$

From the above equations, the effective viscosity is written as,

$$\nu^* = \frac{\mu^*}{\rho^*} = \frac{(1 + \varepsilon_G)\mu}{(1 - \varepsilon_G)\rho} \sim \frac{(1 + 2\varepsilon_G)\mu}{\rho}.\tag{7.24}$$

Assuming the distribution and the volume fraction as uniform and dilute in the bulk fluid, i.e., $\varepsilon_G \sim 0$ the effective viscosity is given as,

$$\nu^* = \frac{\mu}{\rho} = \mu.\tag{7.25}$$

Dilute dispersed microbubbles, thus, does not modify the viscosity of the fluid when the distribution is uniform in flows. The fluid properties, however, would be modified when the microbubbles are preferentially concentrated in the flows since the local volume fractions breaks the assumption of $\varepsilon_G \sim 0$.

7.2.2 Base flow

As we mentioned in section 5.1, flow in the pipe is driven by the piston withdrawing. It enables that the mass flux in the pipe is fixed, therefore Re is kept constant regardless of the flow transition. In order to investigate influences of microbubbles on generation of a localized turbulent puff, probability fluctuation of puff creation is required to increase gently with increasing perturbation amplitude. For satisfying with this requirement, Re is fixed at 1,900 in this study.

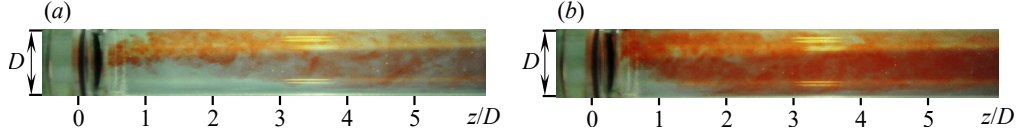


Figure 7.1: Image of perturbation formed at $Re = 1,900$ when Hagen-Poiseuille flow is disturbed by a jet. The scale indicate in units distance D from the perturbation injection point. Dye visualization when (a) $A = 0.03$ and (b) $A = 0.05$

7.2.3 Perturbation

Experimental investigation into the threshold boundary between laminar and disordered pipe flows showed a probability curve of puff creation depending on perturbation amplitudes [94]. Some peaks of the curve were observed at small perturbation amplitude. Results of dye visualizations showed that hairpin-like vortices are created at small perturbation amplitudes. The authors suggested that hairpin-like vortices enhance to create a turbulent puff. In this study, for preventing puff creation through hairpin vortices, noisy jets are injected to the main flow. The perturbation amplitudes are ranged from 0.0 %, 3.0 %, 4.0 %, and 5.0 %. Visualized image of noisy jets at the perturbation injection point is shown in Fig. 7.1. Hagen-Poiseuille flow is disturbed by noisy jets without creation of the hairpin vortices. Velocity fields in the pipe flow are measured by SPIV to observe flow structures embedded in turbulent puffs.

Chapter 8

Results

8.1 Velocity fields measured by SPIV

8.1.1 Single-phase flows

Time trace of streamwise velocity on the pipe axis measured by SPIV in the single-phase flow shows creation of a puff [Fig. 8.1]. The flow direction is from right to left, i.e., downstream and upstream sides of the puff are left and right of the trace. The centerline velocity gradually decreases during the passage of the puff between $t = 0$ s and 2.8 s. Large spikes are observed between 1 s and 3 s similar to the results of LDV measurement [43, 6] and of SPIV measurement [8]. Experiments on the flow transition by Han *et al.* [95] and Doorne & Westerweel [8] showed that a spike at the trailing edge reflects on creations of hairpin-like vortices embedded in a puff. The centerline velocity of the laminar flow is observed after the passage of the puff at time $t = 3.5$ s. Velocity fluctuations in the laminar state at $t = 3.5$ are assumed to be SPIV measurement noise with the order of (2 mm/s).

Velocity and streamwise vorticity fields in the pipe sections at different times $T_i (i = 0, \dots, 6)$ appended in the upper of Fig. 8.1 are visualized to

show the flow structures in Fig. 8.2 and Fig. 8.3. Low-speed fluids close to the pipe wall are lifted up toward to the center of the pipe section [Fig. 8.2(i)(1)] due to a pair of streamwise vorticities [Fig. 8.3(i)(1)]. Strong ejections of low-speed fluids from the vicinity of the pipe walls into the central region of the pipe sections are observed at the different times [Fig. 8.2(ii)-(iv)(2)] where the streamwise velocity profile shows the large spikes shown in Fig. 8.1. The ejection in Fig. 8.2(iii)(iv)(2) is created due to a pair of elongated vortical structures, i.e. hair-pin like vortices [Fig. 8.3(iv)(3)]. Velocity fields show a sweep of high-speed fluids to the pipe wall from the central region of the pipe section [Fig. 8.2(iii)(2) and (iv)(4)]. A pair of the vortices close to the wall is found to create the sweep [Fig. 8.3(iv)(4)].

Streamwise vortical structure of a puff is a component of the self-sustenance process of the puff presented in Sec. 2. Isosurface of the vortical structure are reconstructed in Fig. 8.4. Positive and negative vortices entangle complicatedly each other.

8.1.2 Two-phase flows

Time trace of centerline velocity measured by SPIV in the two-phase flows shows creation of a turbulent puff [Fig. 8.5]. Large spikes are also observed during the passage of the puff as the same with the result of single-phase flows. In contrast, velocity defect in the downstream side is larger than that of the single-phase flow (see the time between $t = 0$ s and $t = 1$ s in Figs. 8.1 and 8.5).

Flow structures embedded in the puff are visualized in velocity and streamwise vorticity fields at the different times appended in the upper of Fig. 8.5. Lift up of low-speed fluids toward to the center of the pipe section is observed close to the wall [Fig. 8.6]. The lift up is coupled with a pair of streamwise

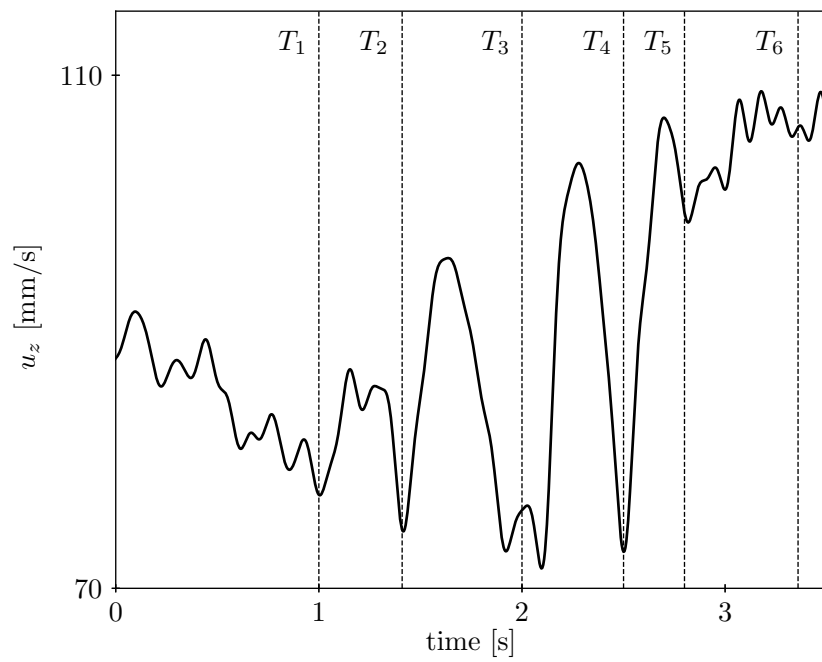


Figure 8.1: A streamwise velocity profile of a turbulent puff on the pipe axis in the single-phase flow measured by SPIV: downstream and upstream sides of the puff are left and right, respectively.

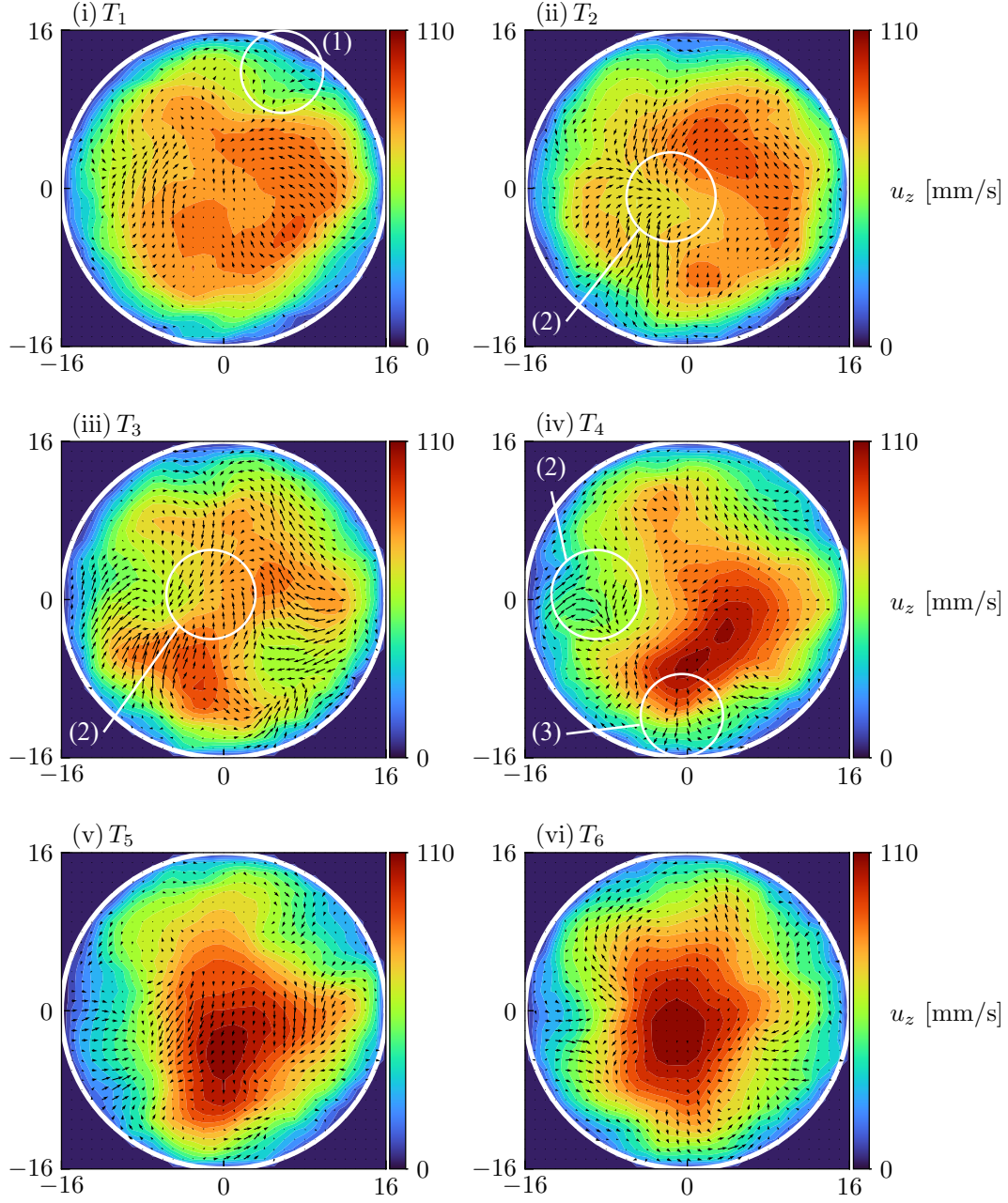


Figure 8.2: Velocity fields of the turbulent puff in the pipe sections in the single-phase flow measured by SPIV for different times appended in Fig. 8.1; the liquid velocity (u_x, u_y) and u_z are shown by vectors and colors, respectively.

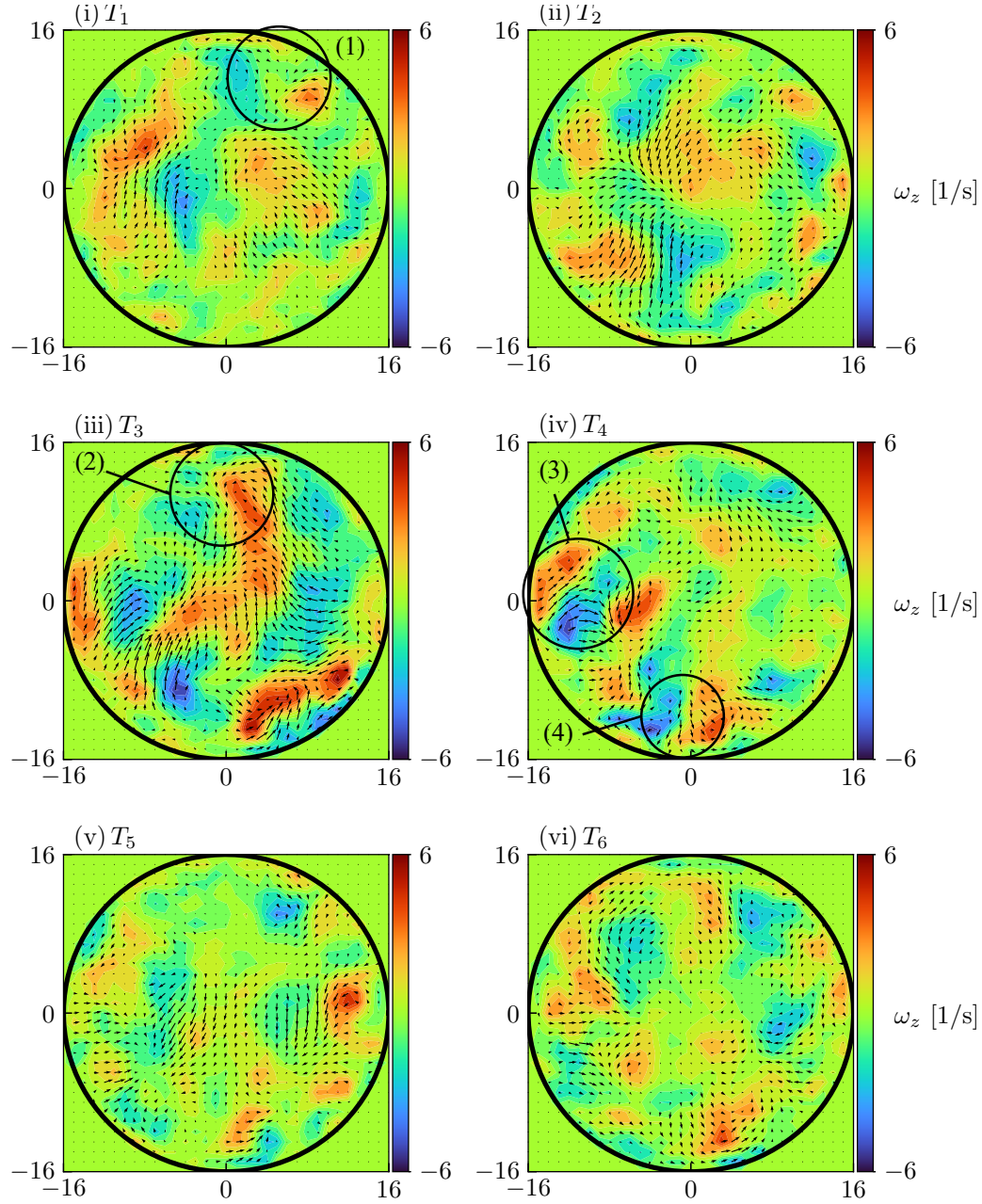


Figure 8.3: Streamwise vorticity distributions of the puff in the pipe sections in the single-phase flow measured by SPIV for different times appended in Fig. 8.1; the liquid velocity (u_x, u_y) and ζ_z are shown by vectors and colors, respectively.

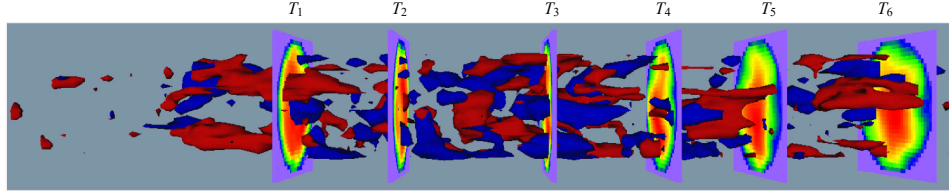


Figure 8.4: Three-dimensional visualization for isosurface of counter-rotating vortex pairs of a puff measured by SPIV in the single-phase flow: red and blue represents positive 3.5 s^{-1} and negative -3.5 s^{-1} vortices

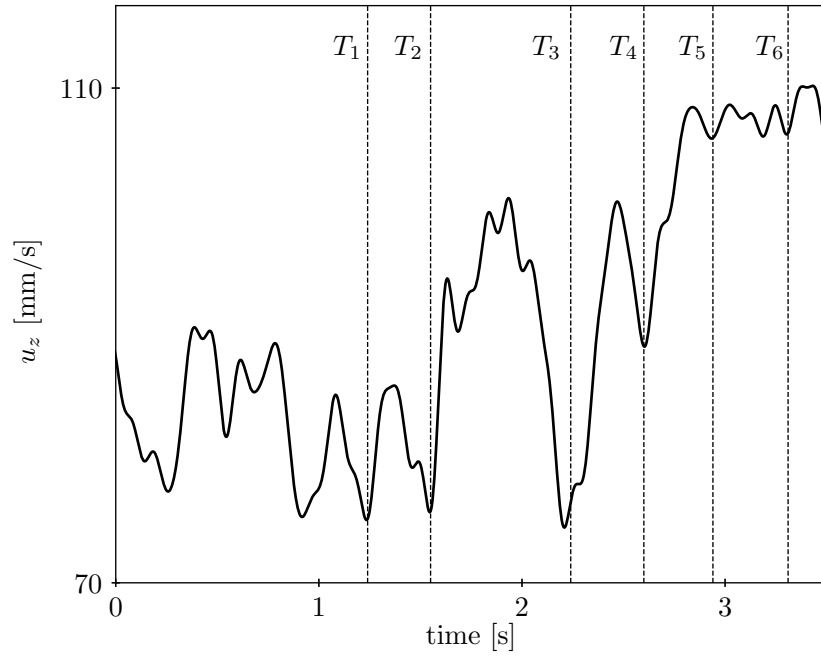


Figure 8.5: Streamwise velocity profile of the turbulent puff on the pipe axis in the two-phase flow measured by SPIV

vortices [Fig. 8.6]. The velocity fields show strong ejections of low-speed fluids into the pipe center [Fig. 8.6(ii)-(iv)(2)]. Velocity fields show a sweep of high-speed fluids toward to the pipe wall from the center of the pipe section [Fig. 8.2(iii)(2) and (iv)(4)] where a pair of streamwise vortices is observed [Fig. 8.3(iii)(3)]. Strong ejections of low-speed fluids close to the pipe walls into the center of the pipe sections are observed at the times where the streamwise velocity profile shows the spikes, i.e., at $T_2 - T_4$ in Fig. 8.5. The ejections are coupled with harppin-like vortical structures [Fig. 8.7].

Isosurface of the vortical structure are reconstructed in Fig. 8.4. The vorticity distributions shrink in radial direction compared to that in the single-phase flows while positive and negative vortices entangle complicatedly each other as the same with the results of single-phase flows. The velocity defect of the streamwise velocity profiles, in fact, reflects on the distributions.

8.2 Bubble distribution

Projected bubble positions in a laminar flow and during the passage of the puff are shown in Figs. 8.9 and 8.10. The number of the detected bubbles is $N_b = 5231$ and $N_b = 4614$, respectively. The distributions in the laminar flow and the puff tend to be inhomogenized in a certain region while bubbles are located in the entire pipe section. Some bubbles are observed close to the upper wall of the pipe section.

Monchaux *et al.* [96] showed that voronoi diagram can be efficiently used to identify clusters from data. The diagram for the projected bubble positions are shown in Figs. 8.11 and 8.12. Dark purple cells (clusters) are surrounded by cells of various sizes and shapes. The cells close to the upper wall of the pipe section reflects on the concentration due to the buoyancy force. Comparing

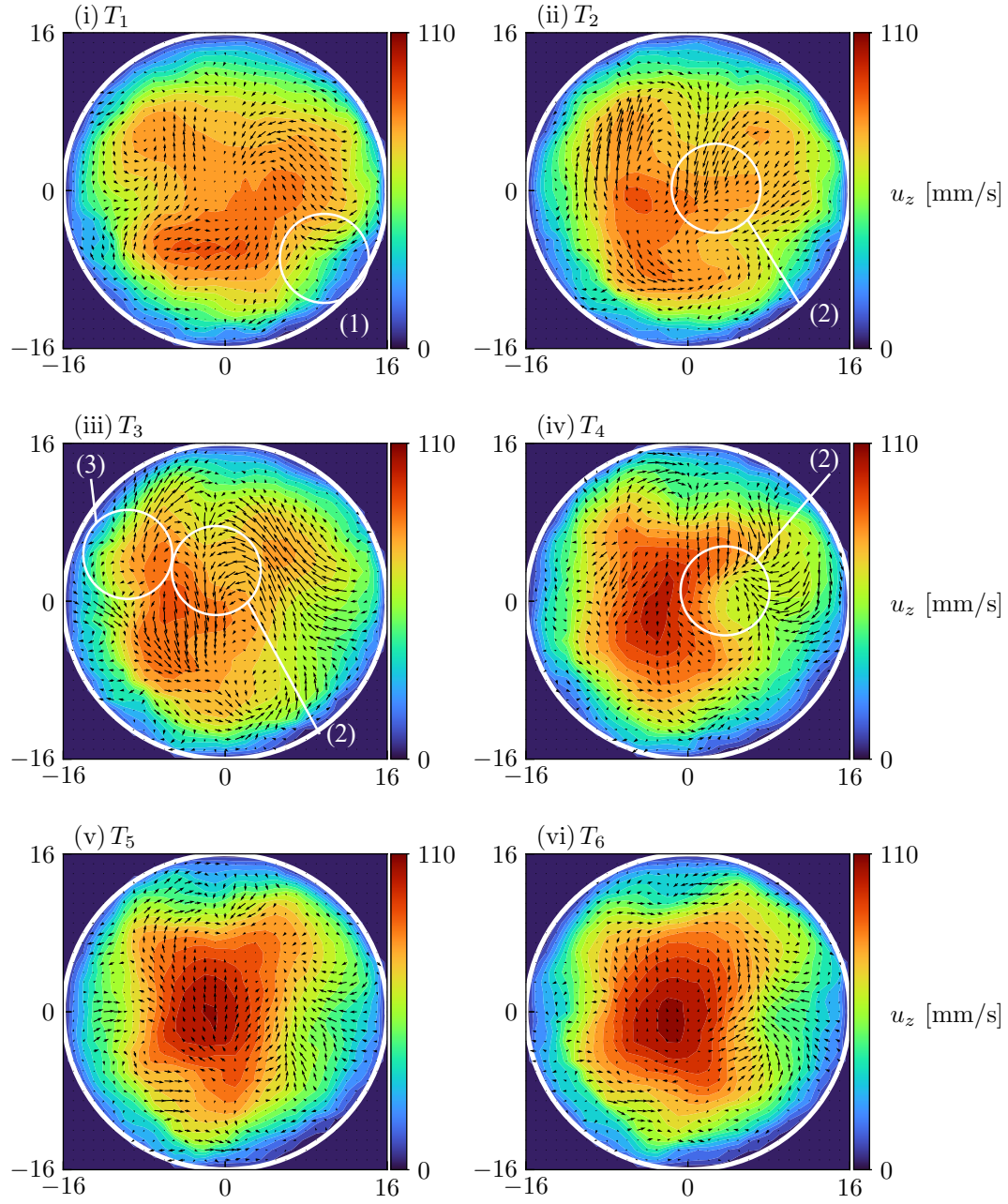


Figure 8.6: Streamwise velocity fields of the turbulent puff in the pipe sections in the two-phase flow measured by SPIV flow for the different times appended in Fig. [8.5](#)

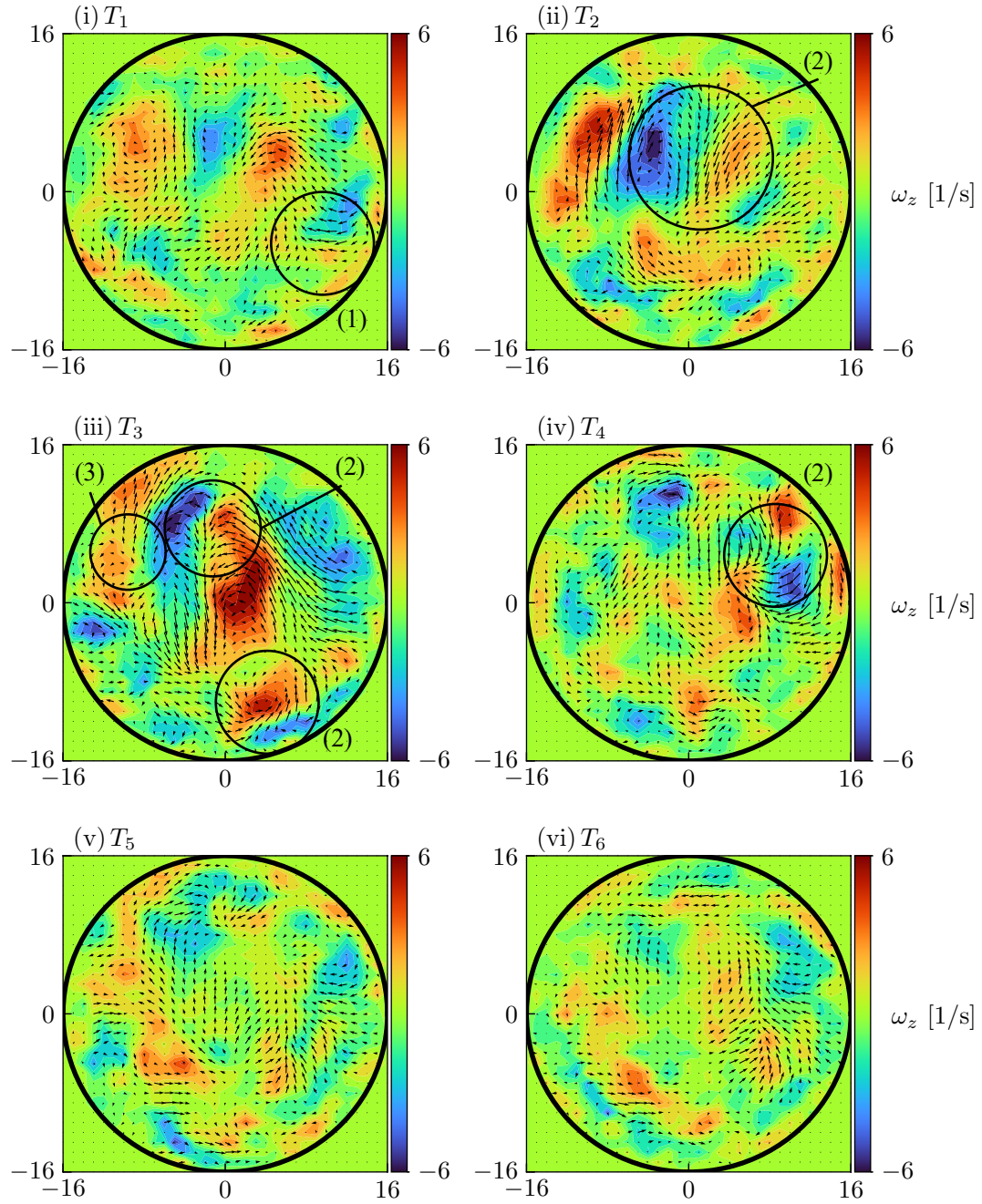


Figure 8.7: Streamwise vorticity distributions of the turbulent puff in the pipe sections in the two-phase flow measured by SPIV for the different times appended in Fig. [8.5](#)

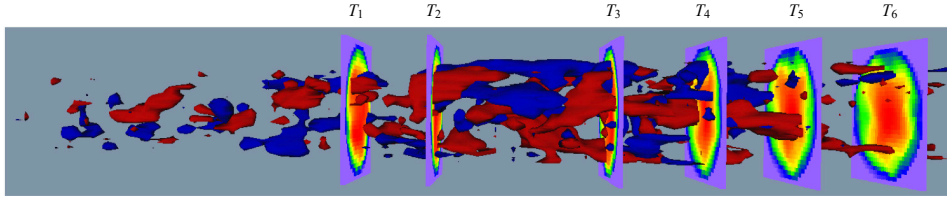


Figure 8.8: Three-dimensional visualization for isosurface of counter-rotating vortex pairs of a puff measured by SPIV in the two-phase flow: red and blue represents positive 3.5 s^{-1} and negative -3.5 s^{-1} vortices

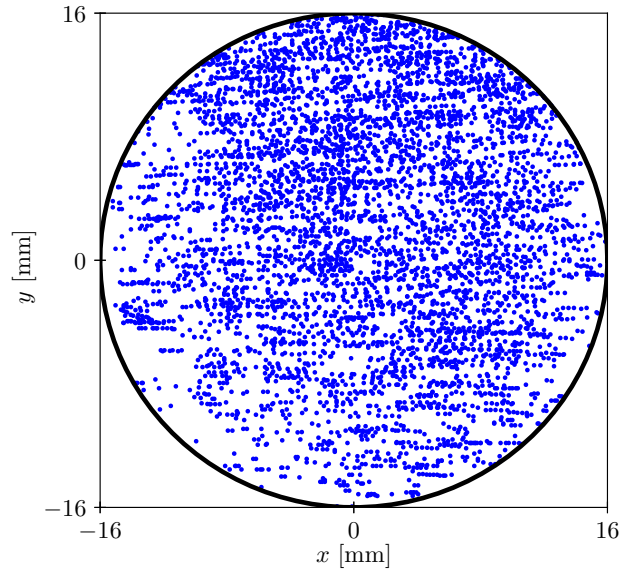


Figure 8.9: Projected bubble distribution in a laminar flow

between the diagrams in the laminar flow and the puff, the results does not show preferential concentration of microbubbles in the puff.

8.3 Lagrangian tracking of bubbles

8.3.1 Flow fields

DNS based on a spectral method by Shimizu & Kida [53] reproduced turbulent puffs as mentioned in Chap. 2. By using the DNS results kindly provided to us by Dr. Shimizu, migrations of dispersed microbubbles in a turbulent puff

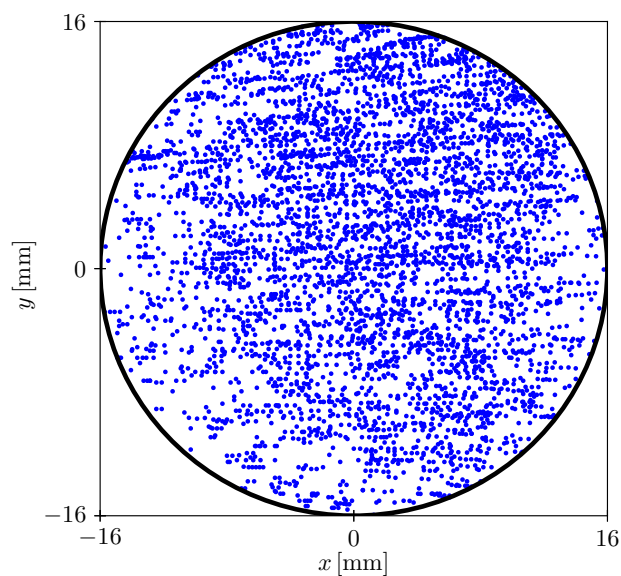


Figure 8.10: Projected bubble distribution during the passage of the puff, i.e., the time between $t = 0$ s and $t = 3$ s in Fig. 8.5

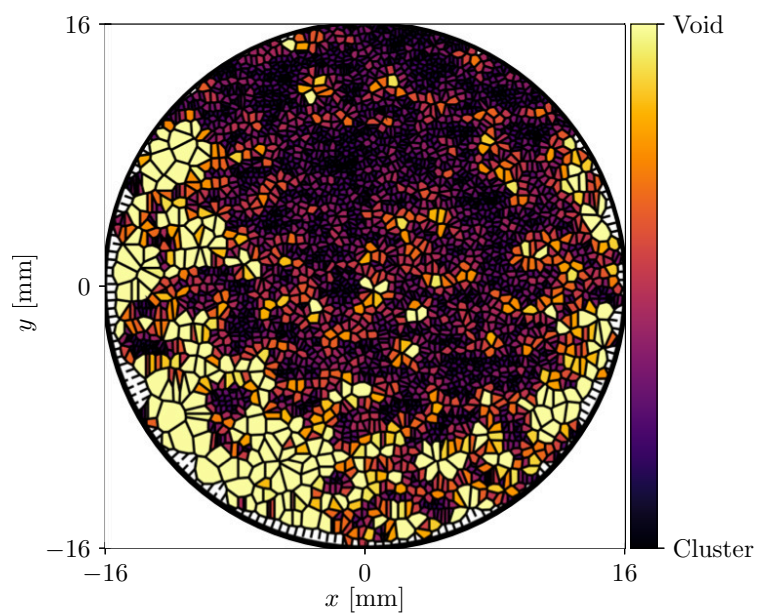


Figure 8.11: Voronoi diagram for the projected bubble positions in Fig. 8.9 with color coding for clusters (dark purple) and voids (white).

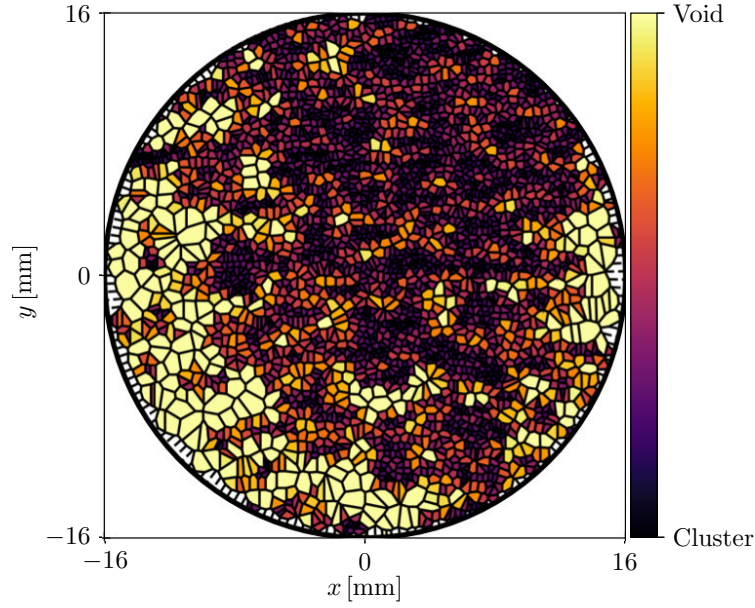


Figure 8.12: Voronoi diagram for the projected bubble positions in Fig. 8.10 with color coding for clusters (dark purple) and voids (white).

are tracked based on a one-way Euler-Lagrangian simulation.

A turbulent puff at $Re = 1,900$ reproduced by the DNS [53] shows a streamwise velocity profile [Fig. 8.13], streamwise velocities in pipe sections [Fig. 8.14], and streamwise vortices [Fig. 8.15] in good qualitative agreement with the experimental results presented in Sec. 8.1.

8.3.2 Translational motion equation of a bubble

The dispersed phase consists of N_b clean spherical microbubbles with radius R_b . Dispersed bubbles experience the Archimedian buoyancy force and forces from surrounding fluids. A translational motion equation for a spherical bubble was proposed by Magnaudet & Eames [38], consisting of the buoyancy, viscous

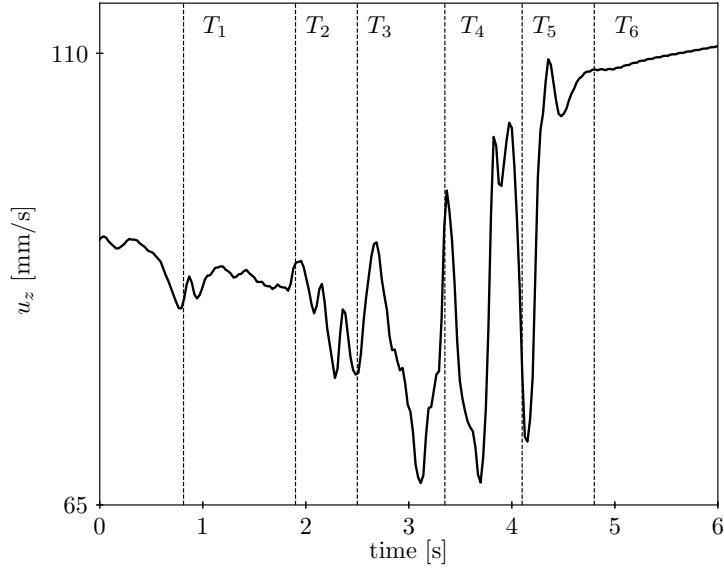


Figure 8.13: Streamwise velocity profile of a turbulent puff on the pipe axis

drag, added-mass, pressure gradient, and shear-induced lift:

$$\frac{d\mathbf{x}_b}{dt} = \mathbf{u}_b \quad (8.1)$$

$$\rho_b V_b \frac{d\mathbf{u}_b}{dt} = (\rho_b - \rho) V_b \mathbf{g} \quad (8.2)$$

$$- \rho V_b \frac{3}{8 R_b} C_D \|\mathbf{u}_b - \mathbf{u}\| (\mathbf{u}_b - \mathbf{u}) \quad (8.3)$$

$$+ \rho V_b C_A \left(\frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{u}_b}{dt} \right) \quad (8.4)$$

$$+ \rho V_b \frac{D\mathbf{u}}{Dt} \quad (8.5)$$

$$- \rho V_b C_L (\mathbf{u}_b - \mathbf{u}) \times (\nabla \times \mathbf{u}), \quad (8.6)$$

where $\mathbf{x}_b$ and $\mathbf{u}_b$ are the bubble position and the bubble velocity, ρ_b and $V_b = 4\pi R_b^3/3$ are the density and the volume of a bubble, ρ and $\mathbf{u}$ are the density and the liquid velocity at the bubble position $\mathbf{x}_b$. The differential operator D/Dt stands for the material derivative, i.e., $D\mathbf{u}/Dt = \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}$ at the bubble position $\mathbf{x}_b$. DNS by Mei *et al.* for flows around the bubbles

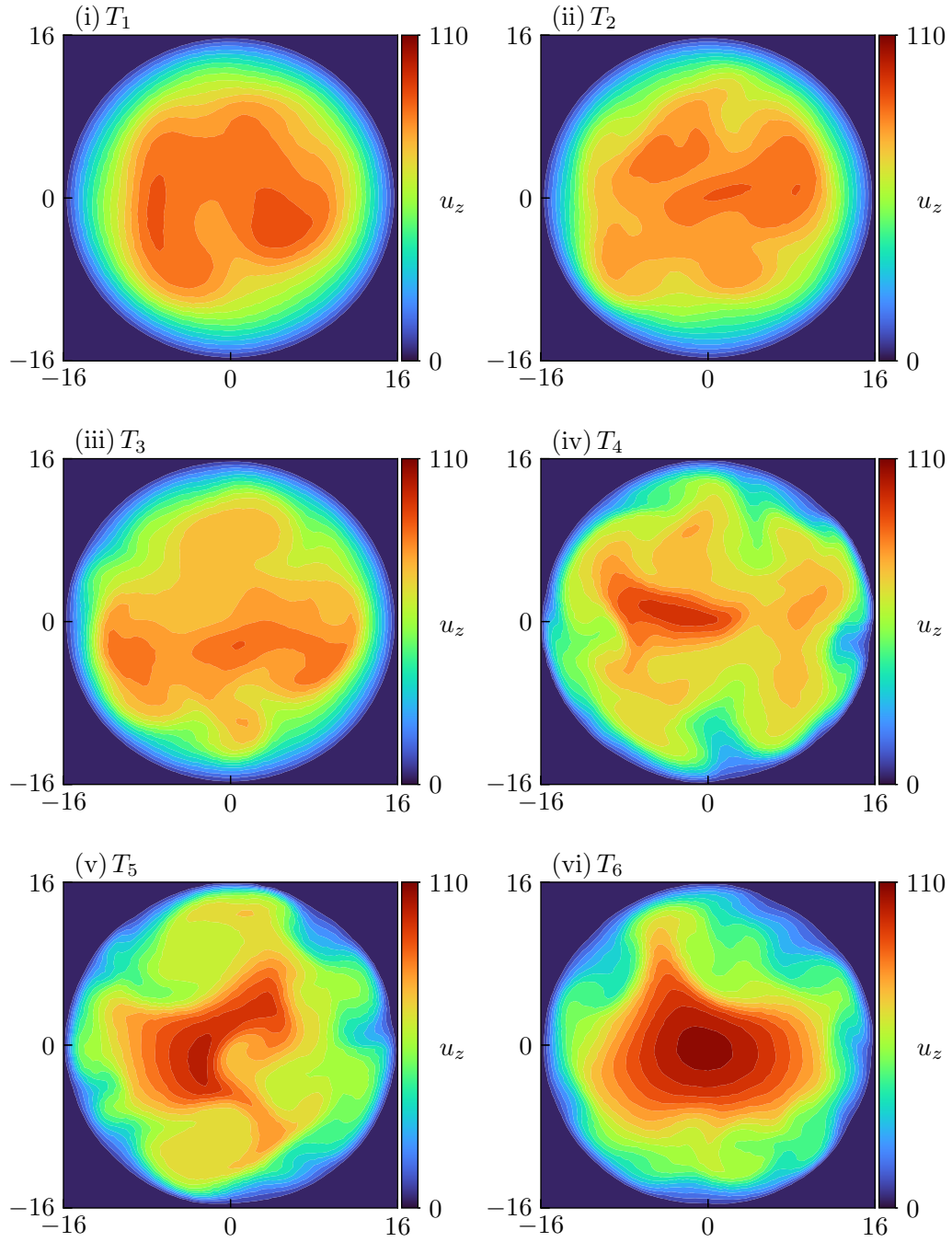


Figure 8.14: Streamwise velocity distribution of a turbulent puff in the pipe sections for different times appended in Fig. [8.13](#)

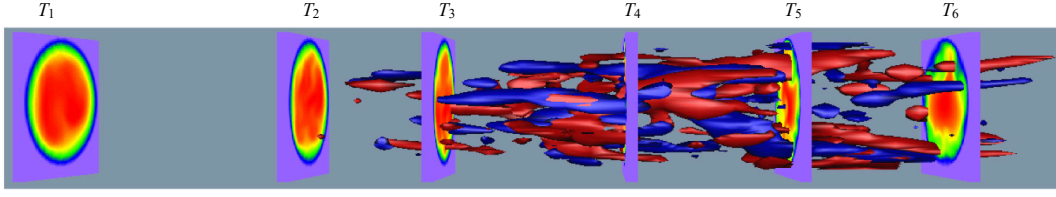


Figure 8.15: Isosurface of three-dimensional vortical structures of a turbulent puff where red and blue represents positive 3.5 s^{-1} and negative -3.5 s^{-1} vortices

showed the drag coefficient C_D in any bubble Reynolds number Re_b :

$$C_D = \frac{16}{Re_b} \left\{ 1 + \left[\frac{8}{Re_b} + \frac{1}{2} \left(1 + \frac{3.315}{\sqrt{Re_b}} \right) \right]^{-1} \right\}. \quad (8.7)$$

The bubble Reynolds number is defined as

$$Re_b = \frac{2R_b \|\mathbf{u}_b - \mathbf{u}\|}{\nu} \quad (8.8)$$

where ν is the kinematic viscosity of liquid. The added-mass and lift coefficients, C_A and C_L , are assumed constant and given by $C_A = 1/2$ [38, 97] and $C_L = 1/2$ [98, 99], supposing spherical bubbles in shear flows.

Migrations of microbubbles are computed by solving the time intergration of the above equations with a second-order Runge-Kutta scheme. Time steps for the integration are set $dt = \tau_b/5$ where $\tau_b = R_b^2/6\nu$ is the bubble relaxation time. The time integration is solved between $t = 0$ and $t = \tau_{puff}$ where $\tau_{puff} = 2.5D/U$ is the time of the production of turbulence kinetic energy at the trailing edge of a puff [53]. The liquid velocity $\mathbf{u}$ and the Lagrangian acceleration $D\mathbf{u}/Dt$ at bubble positions are interpolated with a bicubic spline interpolation. The Lagrangian tracking has been used for, e.g., bubble motions in turbulent Taylor-Couette flow [100, 101] and upflowing mixing layer dispersed with bubbles [62].

8.3.3 Initial and boundary conditions

$N_b = 10,000$ bubbles with $R_b = 20 \mu m$ are dispersed in the turbulent puff, i.e., the time between 0 s and 6 s in Fig. 8.13. The initial positions are determined by a random number generator based on an algorithm called Mersenne twister [102]. The initial velocities are set equal to liquid velocities at the initial positions. An elastic collision is imposed for boundary conditions on the pipe wall.

8.3.4 Bubble distribution

Projected bubble positions at the terminal state show the inhomogeneous distributions [Fig. 8.16]. Some bubbles attach on the upper wall due to the buoyancy and are uniformly distributed in the plane. The voronoi diagram for the projected bubble positions is shown in Fig. 8.17. The results indicate that microbubbles are preferentially concentrated in the puff by the time $\tau_{puff} = 2.5D/U$ of the production of kinetic energy.

8.4 Streamwise velocity profile on the pipe axis

Probability of observation of a puff are determined in the flows with and without microbubbles to examine influences of the bubbles on puff creations based on streamwise velocity profiles on the pipe axis measured by LDV. Streamwise velocity profiles of the laminar flow are observed while finite amplitude perturbation is injected into the main flows [Fig. 8.18(a)]. This observations imply that the injected perturbations decay and return to laminar flows up to the measurement point from the perturbation injection point. The velocity is, indeed, equal to $2U$ of the centerline velocity in the laminar flows. This results, thus, indicate the correct operation of the LDV system. The other

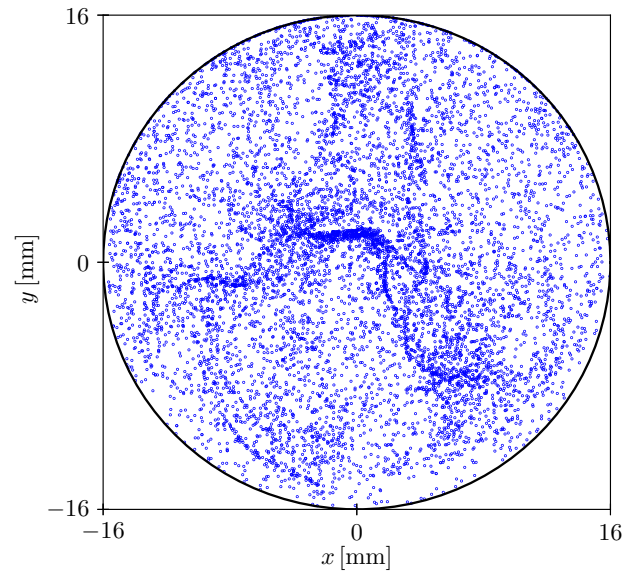


Figure 8.16: Projected bubble positions in the pipe at terminal state

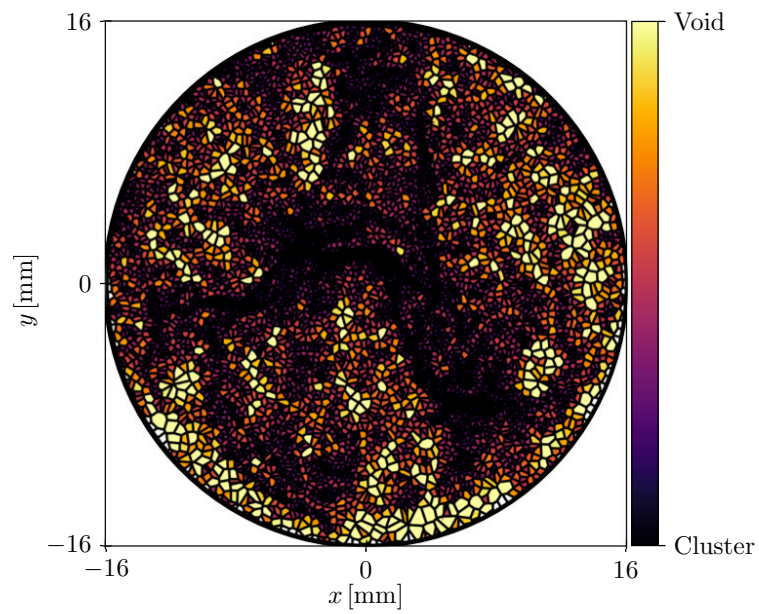


Figure 8.17: Voronoi diagram for the projected bubble positions in Fig. 8.16 with color coding for clusters (dark purple) and voids (white).

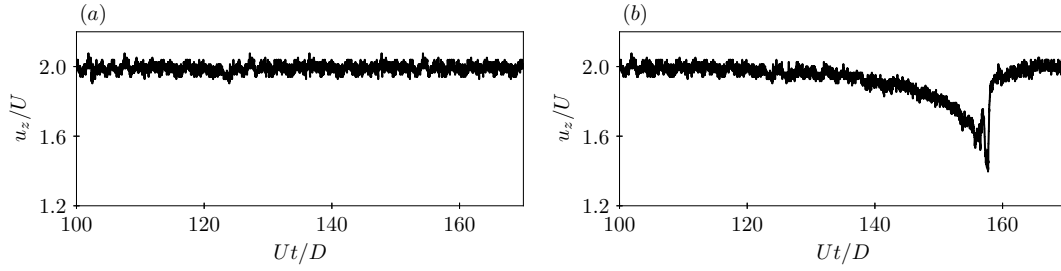


Figure 8.18: Streamwise velocities on the pipe axis for (a) decayed perturbation (laminar flow) and (b) turbulent puff measured by LDV: the abscissa is measurement time normalized by the pipe diameter D and mean velocity in the pipe U . The ordinate is the measured velocity normalized by the mean velocity. Flow direction is from right to left.

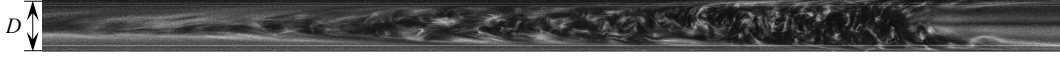


Figure 8.19: A turbulent puff at $Re = 1,900$ visualized using flakes: flow direction is from left to right.

[Fig. 8.18(b)] shows a streamwise velocity profile of a turbulent puff on the pipe axis. The velocity of laminar flow gradually decreases in the time direction and is the minimum at the trailing edge. The minimum velocity recovers to the centerline velocity $2U$ since laminar flow relatively higher than the puff enters the downstream side of the puff. This trend is good agreement with the measured velocity profile by previous studies presented in Chap. ?? and reflects on flow structures of turbulent puffs. Localized structure of a puff between laminar flows are observed in a space-time diagram of a turbulent puff visualized using flakes [Fig. 8.19]. Turbulence in the puff becomes more complicated with approaching the leading edge. No turbulent motion is observed in the downstream side of the puff since the flow recovers to laminar state.

8.5 Probability of observation of turbulent puffs

Experiments conducted at least 40 trials under conditions of single-phase flows and fluid containing dilute microbubbles for the perturbation amplitudes.

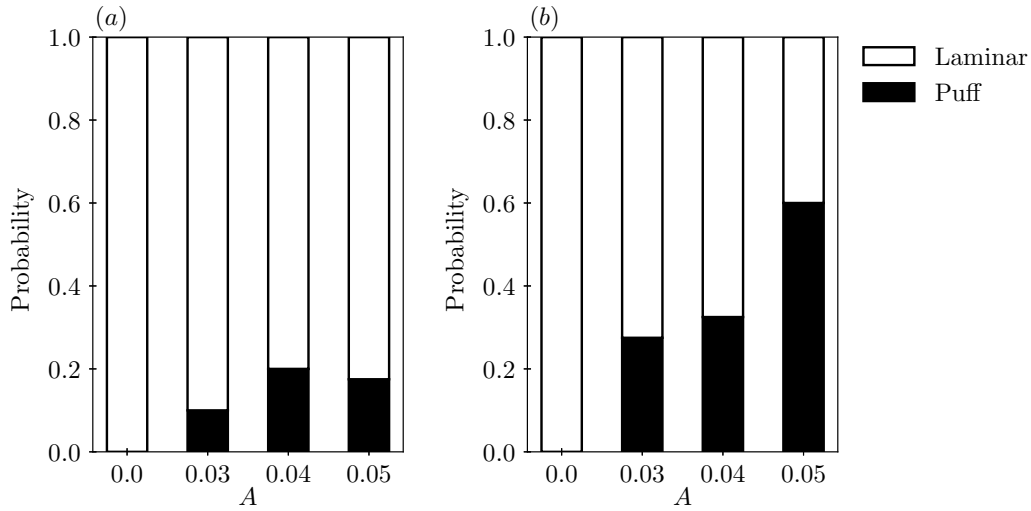


Figure 8.20: Probability of observations of decayed perturbation (laminar flow) and a puff shown in the white and the black areas, respectively, in flows (a) without microbubbles and (b) with microbubbles: the abscissa is the perturbation amplitude and the ordinate is the creation probability.

Probability of observation of decayed perturbations (laminar flows) or puffs are measured in the flows without and with microbubbles at $Re = 1,900$ as shown in Fig. 8.20 (a) and (b), respectively. Experiments were conducted at least 40 perturbation injections for these amplitudes. If 5 velocity profiles of puffs [Fig. 8.18(b)] are observed at the measurement point in 40 experiments for an amplitude, the probability of observation of puffs is estimated to be $5/40$. The estimated probability, thus, contains an error value about 0.158. This implies that the probability larger than the value are reliable and assured statistically.

In the flows without microbubbles [Fig. 8.20(a)], puff creation is not observed at $A = 0$, i.e. without perturbation injection. This results show that the flows are stable to intrinsic perturbation due to, e.g. thermal insulation, external vibrations and alignment of the pipe sections. As a larger amplitude of the perturbation, turbulent puffs are observed, where the maximum probability is found at most 20%.

The probability of observation of a puff at $A = 0$ in the flows with microbubbles remains zero [Fig. 8.20(a)] as the same with that in the flows without microbubbles. This results imply that the main flow remains laminar while microbubbles are dispersed in the flow. In this two-phase flows, laminar state is, indeed, observed using flakes and dye visualizations up to $Re = 25,000$ as the same value of $\overline{Re_c}$ with single-phase flows. In contrast to the results of single-phase flows, the probability in the two-phase flows shows an effect of microbubbles. The probability of observation of puffs is increased with the perturbation amplitudes. The effect is found significant when the largest amplitude within the examined range, i.e. the probability shows the maximum value 70% at $A = 0.05$.

Chapter 9

Discussion

9.1 Decaying perturbations

Some streamwise velocity profiles on the pipe axis measured by LDV show intermediate states between laminar flows and turbulent puffs [Fig. 9.1]. While the velocity decreases gradually in the downstream direction as the same with that of turbulent puffs as shown in Fig. 8.1(b), sharp increases of the velocity at the leading edge are not observed in intermediate states. The minimum values of the velocity are larger than that of turbulent puffs, implying that turbulence of the states are much weaker than that of turbulent puffs. Darbyshire & Mullin [6] also observed the intermediate states at the measurement point in $22D$ downstream distance from the perturbation injection point due to LDV measurement for the centerline velocity. Decay of the intermediate states was also found in the distance [6]. Experiments for determining the finite lifetime of puffs by Kuik *et al.* [9] showed the probability of the observations of puffs as a function of downstream distance from a perturbation injection point and Reynolds number [Fig. 2.7(b)]. In my experimental conditions, i.e. the measurement point in $106D$ downstream distance from the perturbation injection

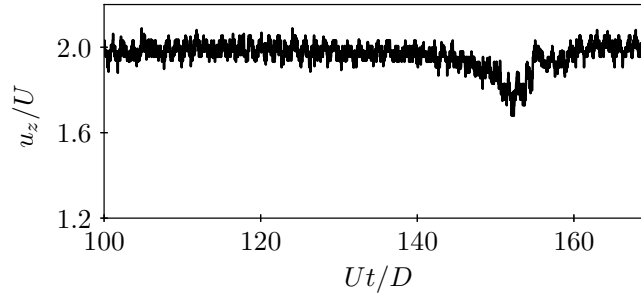


Figure 9.1: Streamwise velocity profiles of a decaying perturbation on the pipe axis measured by LDV: the abscissa is the measurement time normalized by the pipe diameter D and the mean velocity U in the pipe and the ordinate is streamwise velocity normalized by the mean velocity.

point and $Re = 1,900$, when injected perturbations create puffs, the puffs would not decay and not return to laminar states. The intermediate states, thus, represent decaying perturbations, i.e., injected perturbations do not create turbulent puffs and are decaying toward to laminar flows. In Fig. 8.20, the decaying perturbations are identified with decayed perturbations (the white areas).

9.2 Quantitative identification of puff observation

The probability are constructed based on the trends of the streamwise velocity profiles on the pipe axis. Quantitative methods to identify the observation of decayed perturbations (laminar flows), decaying perturbations, and turbulent puffs are proposed in this section based on a velocity ratio and a correlation coefficient.

The streamwise velocity profiles of puffs show the gradual decrease from

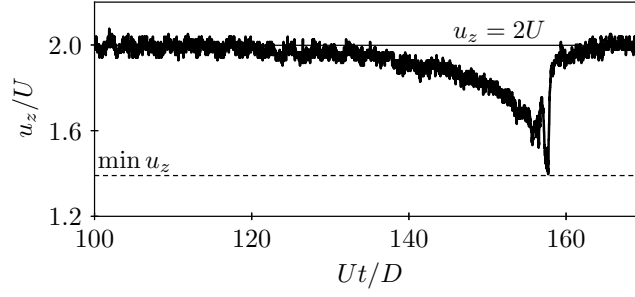


Figure 9.2: Definition sketch of the VR for a streamwise velocity profile: the minimum value and the velocity profile of laminar pipe flow on the pipe axis are indicated by the solid and dashed lines, respectively. The abscissa is the measurement time normalized by the pipe diameter D and mean velocity in the pipe and the ordinate is streamwise velocity normalized by the mean velocity.

the centerline velocity of laminar flow $2U$ and the minimum velocity at the trailing edge mentioned in Sec. 8.4. The ratio of the centerline velocity and the minimum velocity for puffs would determine the threshold of observation of puffs:

$$\Delta u_z = \frac{u_{\min}}{2U} \quad (9.1)$$

where $u_{\min}$ is the minimum velocity of puffs ($\min u_z$) (see the dashed line in Fig. 9.2). For the velocity profiles of the laminar flow [Fig. 8.18(a)], the puff [Fig. 8.18(b)], and the decaying perturbation [Fig. 9.1], the values of Δu_z are, e.g., estimated as 0.95, 0.70, and 0.84, respectively. The values of Δu_z are estimated for streamwise velocity profiles to construct the probability density distributions.

Probability of observation of laminar flows, decaying perturbations, and turbulent puffs are measured with varying threshold values for the velocity ratio Δu_z [Fig. 9.3]. Values of Δu_z for laminar flows would show unity when the streamwise velocity profiles does not include any noise. Threshold values for laminar flows are, therefore, set between 0.9 and 1.0 considering fluctua-

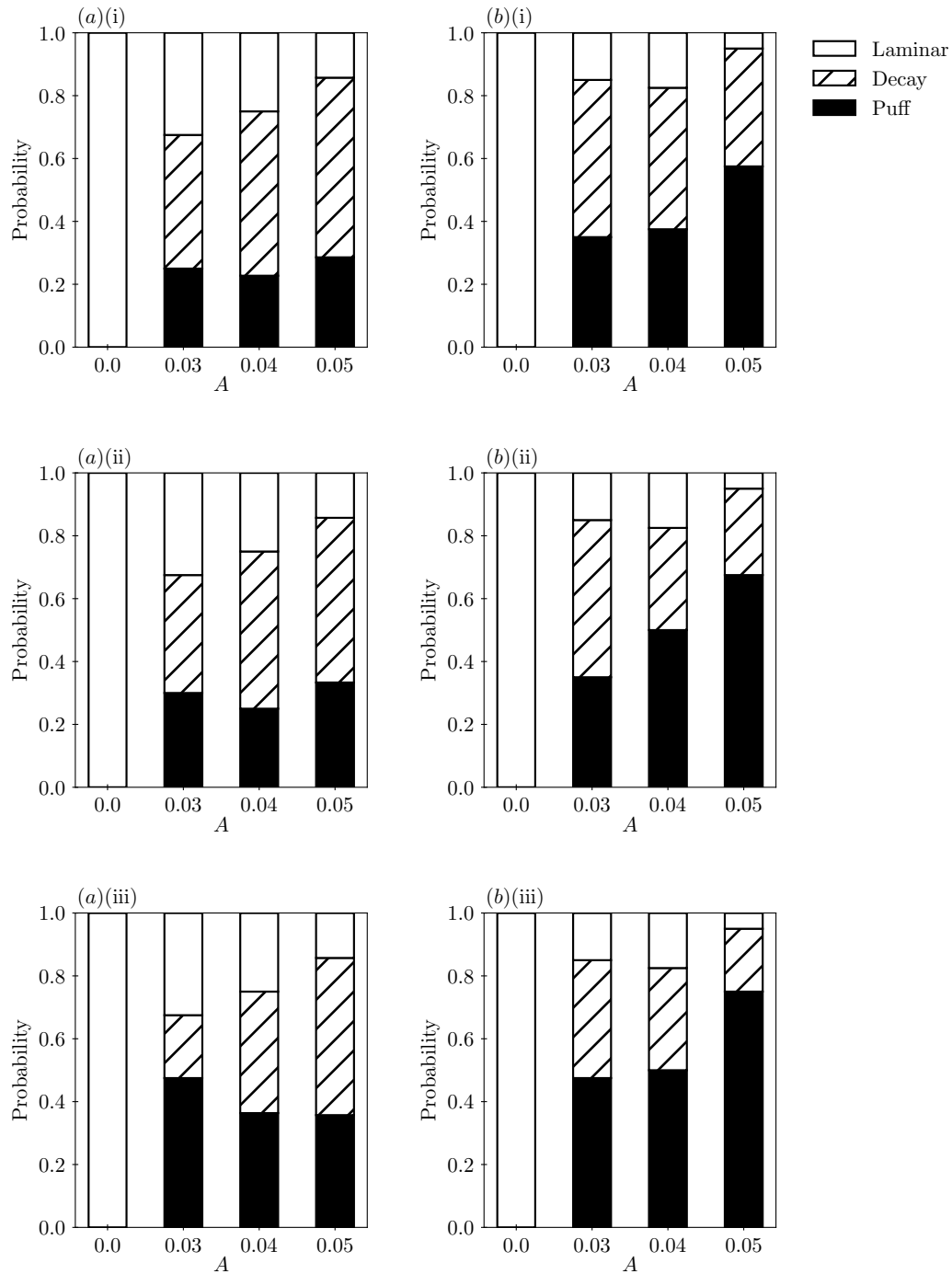


Figure 9.3: Probability of observation of laminar flows, decaying perturbations, and tubulent puffs in flows without microbubbles (a) and with microbubbles (b) are shown in the white, hacthed, and black areas, respectively. The roman numbers (i), (ii), and (iii) corresponds to the cases for the different threshold values of Δu_z shown in Table 9.1. The abscissa is the perturbation amplitude and the ordinate is the probability.

Table 9.1: Different threshold values for the velocity ratio Δu_z to classify flow states into laminar flows, decaying perturbations, and turbulent puffs: probability of observation of laminar flows, decaying perturbations, and turbulent puffs are measured based on the threshold values [Fig. 9.3]

Flow state	Case i	Case ii	Case iii
Laminar flow	0.9-1.0	0.9-1.0	0.9-1.0
Decaying perturbation	0.6-0.9	0.7-0.9	0.8-0.9
Turbulent puff	0.0-0.6	0.0-0.7	0.0-0.8

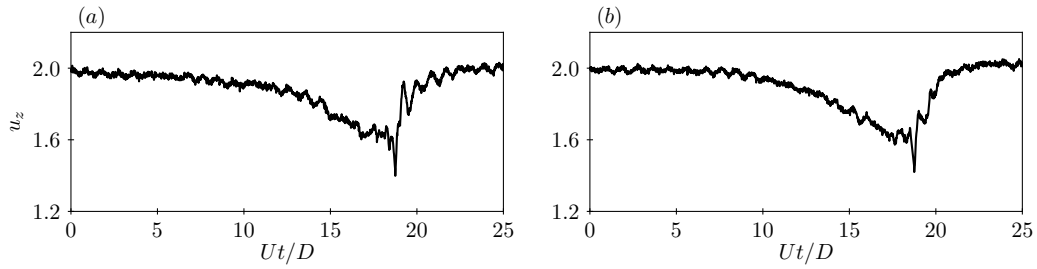


Figure 9.4: Averaged streamwise velocity profiles of puffs on the pipe axis in flows (a) without microbubbles and (b) with microbubbles: the abscissa is measurement time normalized by the pipe diameter D and the mean velocity in the pipe U . The ordinate is the normalized velocity by the mean velocity.

tions due to measurement noise. Probability of observation of puffs in flows without microbubbles changes with varying the threshold values, e.g. the probability changes between 0.3 and 0.5 at $A = 0.03$ since the minimum value of turbulent puffs fluctuates between 0.6 and 0.7 at this Reynolds number. The results, however, show the increases of observation probability of turbulent puffs in flows with microbubbles as the same with the results in Fig. 8.20. The threshold given as the velocity ratio, thus, would determine flow states.

Table 9.2: Different threshold values for the correlation coefficient to classify flow states into laminar flows, decaying perturbations, and turbulent puffs: probability of observation of laminar flows, decaying perturbations, and turbulent puffs are measured based on the threshold values [Fig. 9.5]

	Laminar flow	Decaying perturbation	Turbulent puff
r	0.0-0.4	0.4-0.7	0.7-1.0

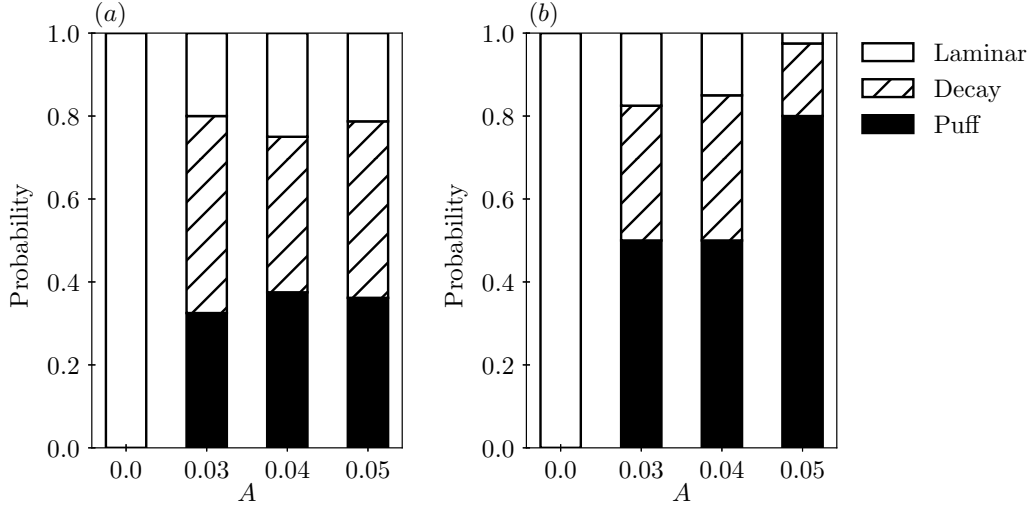


Figure 9.5: Probability of observation of laminar flows, decaying perturbations, and turbulent puffs in flows without microbubbles (a) and with microbubbles (b) are shown in the white, hatched, and black areas, respectively. The flow patterns are identified based on the threshold values of the correlation coefficient in Table 9.2. The abscissa is the perturbation amplitude and the ordinate is the probability.

The streamwise velocity profiles of turbulent puffs show the gradual decrease toward the trailing edge and the sharp increase at the edge mentioned in Sec. 8.4. Estimating a correlation coefficient for the velocity of puffs, flow states are identified to measure observation probability. The correlation coefficient r is defined as

$$r = \frac{\int_t [u_{z,puff}(t) - \bar{u}_{z,puff}(t)][u_z(t) - \bar{u}_z(t)] dt}{\int_t [u_{z,puff}(t) - \bar{u}_{z,puff}(t)]^2 dt [u_z(t) - \bar{u}_z(t)]^2 dt} \quad (9.2)$$

where $u_{z,puff}(t)$ and $u_z(t)$ are the velocity of puffs and sample date. $\bar{u}_{z,puff}(t)$ and $\bar{u}_z(t)$ are the arithmetic mean of $u_{z,puff}(t)$ and $u_z(t)$. Darbyshire and Mullin [6] measured centerline velocity by LDV and reported that realized size and distinctiveness of a region within the puff varies between repetitions of experiments even if Reynolds number and perturbation parameters are kept

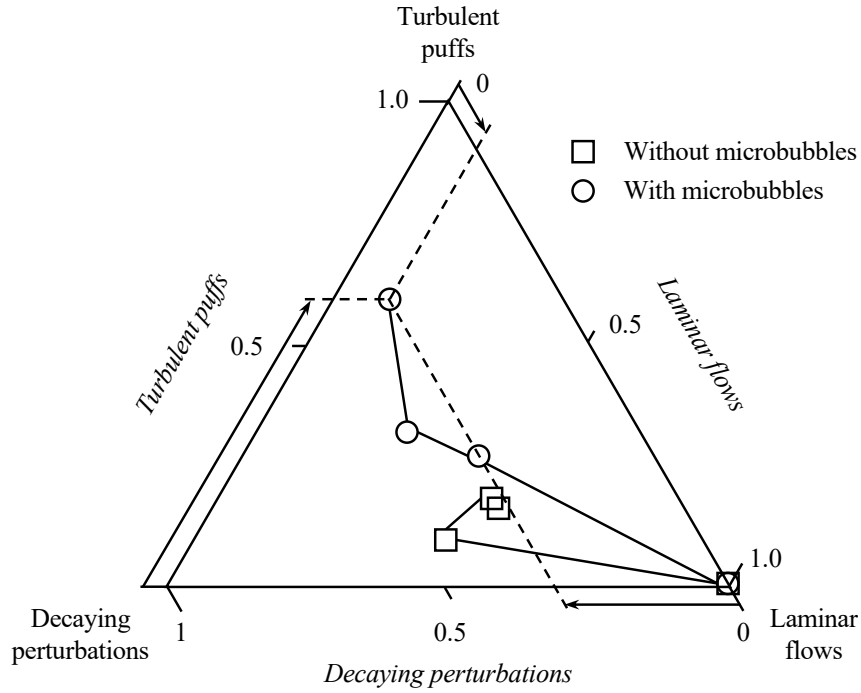


Figure 9.6: Triangular coordinate system for the results in Fig. 8.20

constant. Shifting streamwise velocity profiles of puffs at $A = 0.05$ in Fig. 8.20 in time direction to align at the minimum velocity, the ensemble average are estimated for the single-phase and two-phase flows with realized size of puff $25D$ between $20D$ and $30D$ referred in Chap. 2 [Fig. 9.4]. In general, the relationship between two variables is considered as strong, medium, and weak when the coefficient is between 0.7 and 1.0, 0.4 and 0.7, and 0.0 and 0.4, respectively. Flow patterns are classified into laminar flows, decaying perturbations, and turbulent puffs based on the threshold value [Table 9.2] to measure probability of observation of puffs [Fig. 9.5]. The results show the increases of observation probability of turbulent puffs by adding microbubbles as the same with the results in Fig. 8.20. These two methods are useful to identify flow states based on the velocity profiles.

Triangular coordinate system is introduced to understand the results in Fig. 8.20 visually [Fig. 9.6]. Let probability of observation of laminar flows,

decaying perturbations, and the turbulent puffs be $P_{laminar}$, P_{decay} , and P_{puff} , respectively, the results of three components ($P_{laminar}, P_{decay}, P_{laminar}$), where $P_{laminar} + P_{decay} + P_{puff} = 1$, are converted to two components (X, Y). Equations for the conversion are given as

$$X = \frac{1}{2} \frac{P_{laminar} + 2P_{puff}}{P_{laminar} + P_{decay} + P_{puff}} \quad (9.3)$$

$$Y = \frac{\sqrt{3}}{2} \frac{P_{laminar}}{P_{laminar} + P_{decay} + P_{puff}} \quad (9.4)$$

The results of, e.g., $A = 0$ in Fig. 8.20(a) are presented as the square and the circle at the lower right vertex (laminar flows) in the figure. The results of, e.g., $A = 0.05$ in Fig. 8.20(b) are presented at the intersection of the three dashed lines in the figure. In flows without microbubbles, the positions of circles remain close to decaying perturbations with increase of perturbation amplitudes. In contrast to, in flows with microbubbles, the position of the square moves toward turbulent puffs with increase of perturbation amplitudes. Dilutely dispersed microbubbles, therefore, would increase the creation of turbulent puffs.

9.3 Flow transition of milky bubbly fluids

The results of the increase of puff observations in the dispersed bubbly flows imply an enhancement of flow transition. This is marked with the increase of the critical Reynolds number in milky bubbly flows reported by Serizawa *et al.* [16, 17] and Shatat *et al.* [18] as mentioned in Chap. 3. In the milky bubbly flows, microbubbles with the high volume fraction are dispersed uniformly in the main flows. The distributions would modify the liquid viscosity based on the effective viscosity presented in Sec. 7.2, i.e., the liquid viscosity is increased

with the bubble volume fraction in the milky bubbly flows.

The results in a vertical pipe flow presented in Fig. 3.4(a) [16, 17] show that critical Reynolds numbers are $\sim 2,000$ in single-phase flows denoted by the square plots and $\sim 3,500$ in two-phase flows when the volume fraction is 0.2% denoted by the circle plots. Substituting the value of the volume fraction into ε_G in Eq. 7.24, the viscosity and Reynolds number of the bubbly fluids are, respectively, increased and decreased by 0.4% compared to that of single-phase fluids at fixed mean velocity and diameter. The order of the decrease is $O(10)$ when Reynolds number in single-phase flows is $\sim 2,000$. In the results for a horizontal pipe flow shown in Fig. 3.4(b) [18], the same order is found when the microbubbles are dispersed in the main flows. The order is not large enough to show the increase of the critical Reynolds numbers. While microbubbles are dispersed uniformly in the milky bubbly flow, the bubbles would have interactions with flow structures of turbulent puffs.

9.4 Inflection points in streamwise velocity profiles of puffs

Rayleigh [103] showed that if velocity profiles have an inflection point, flows become unstable to perturbations. Streamwise velocity profiles of turbulent puffs have inflection points at the trailing edge where the production of the turbulent kinetic energy is found [10] as mentioned in Chap. 2. The authors [10] also observed immediate collapse of localized turbulence due to modification of the inflection points by adding body forces to the flows, resulting in relaminarization of the flows.

In upward two-phase flows through a vertical pipe, radial gas distributions have the local maximum points close to the wall while the order of bubble

size and gas volume fraction are $O(1 \text{ mm})$ and $O(1 \text{ \%})$, respectively [104]. Modification of the inflection points would occur since local buoyancy is created close to the wall by the heterogeneous distributions of bubbles, resulting in relaminarization observed by Serizawa [16, 17]. In horizontal pipe flows, the modification would be observed at the inflection point in the upper half of the pipe due to heterogeneous bubble distributions created by the buoyancy as bubbles rise upward direction with migrating in horizontal direction.

9.5 Subcritical and supercritical bifurcations

Experiments on particle-laden flows in a horizontal pipe showed subcritical and supercritical transition depending on the volume fraction [77, 78]. To trigger the flow transition in the pipe, finite amplitude perturbations were injected into main flows where neutral buoyant particles with the diameter in $O(100 \text{ }\mu\text{m})$ were dispersed with varying the volume fraction between 0% (single-phase flow) and 24% [78]. The results [78] showed that the transition was found supercritical at high, i.e., 24% otherwise subcritical, i.e., at low 2.8% and intermediate 12.5% particle concentrations. The dynamics of particle-laden pipe flows arises from Newtonian-type turbulence at the low, a particle-induced turbulence at the high, and a coexistence at the intermediate fractions. In the transition of milky bubbly fluids [16, 17, 18] shown in Fig. 3.4 in Chap. 3, the jumps from the laminar to turbulent flows are not observed in contrast to the single-phase transition, i.e., the transition would become supercritical with increase of bubble volume fractions as the same scenario with the particle-laden pipe flows [77, 78]. The dynamics of the dispersed bubbly flows would be different with the single-phase flows depending on the concentrations.

Experiments on gravitaional instabilities of particle layers by Harada *et*

al. [105] showed particle- and fluid-like behaviors for flows developing from a particular two-layer state, where a particle layer is superposed on a pure liquid layer. Depending on the competition of length scales between particle-particle distance and wavelength of perturbations on the interface of the two-layers, the two-phase flows become immisible or misible, i.e. particle layers are assumed continuous or dispersed in the surrounding fluids. In the mikly bubbly flows, bubble-bubble distance is smaller than length scales of turbulent puffs, e.g. the wavelength of streamwise vortices [48] and the diameter of the vortices in the pipe section [8] as mentioned in Chap. 2. Experimental studies with varying the volume fraction between dilute and milky bubbly fluids, thus, would indicate the differences of the impacts on flow transition.

Part III

Bubble-induced convection

Chapter 10

Linear stability analysis

10.1 Theoretical model

10.1.1 Governing equation

We consider two-dimensional bubbly flows in a horizontal liquid layer generated by uniform injection of monodisperse spherical bubbles from the bottom wall [Fig. 10.1]. Bubbles rise in the layer due to Archimedean buoyancy forces and are ejected from the top free surface. The velocity $W_{G,0}$ and flux $J_{G,0}$ of gas injection are maintained at constant values. We have supposed spherical and non-deformable bubbles, assuming the Galilei number $Ga = \sqrt{gR_b}R_b/\nu$ and Eötvös number $Eo = \rho g R_b^2/\gamma$ are sufficiently small: $Ga \lesssim 10^2$, $Eo \lesssim 10^{-1}$ [106, 107], where R_b and γ are the bubble radius and the surface tension. These hypothesis are satisfied, e.g., for bubbles of $R_b \sim 0.5$ mm in water at room temperature.

A point particle model for a spherical and non-deformable bubble was proposed by Magnaudet & Eames [38]. It consists of a translational motion equation considering force balance on the bubble as a function of the liquid velocity

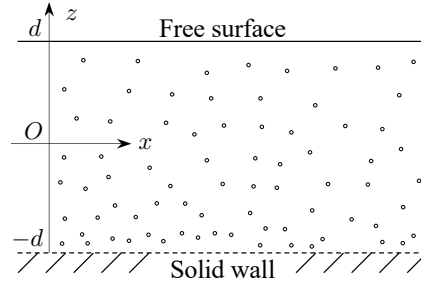


Figure 10.1: Schematic illustration of the problem: small spherical bubbles are injected into the quiescent liquid layer from the bottom wall. The injection is assumed uniform in space and constant in time. Injected bubbles ascend in the liquid layer due to the Archimedean buoyancy force and are ejected from the top free surface of the layer.

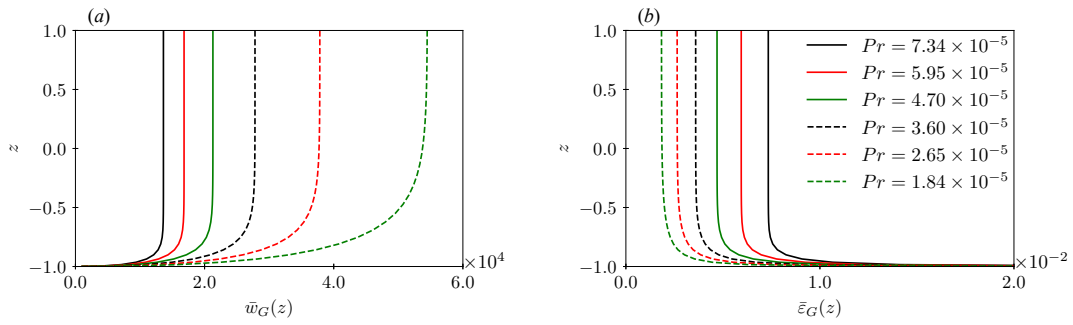


Figure 10.2: Profiles in the base state for (a) bubble ascending velocity $\bar{w}_G(z)$ and (b) gas volume fraction $\bar{\varepsilon}_G(z)$ with different Pr defined as the equation (10.30): the gas injection velocities and gas injection volume fractions at $z = -1$ of the profiles equal to $w_{G,0} = 1000$ and $\varepsilon_{G,0} = 0.02$, respectively.

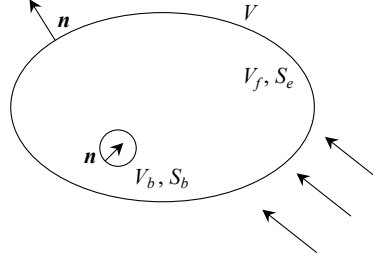


Figure 10.3: A spherical bubble of volume V_b is dispersed in a liquid of volume V_f with external surface S_e . The vector $\mathbf{n}$ is the outward normal.

field at the bubble location,

$$\frac{d\mathbf{x}_b}{dt} = \mathbf{u}_b \quad (10.1)$$

$$\rho_b V_b \frac{d\mathbf{u}_b}{dt} = \mathbf{F}_A + \mathbf{F}_P + \mathbf{F}_D + \mathbf{F}_L + \mathbf{F}_B, \quad (10.2)$$

where added-mass $\mathbf{F}_A$, pressure gradient $\mathbf{F}_P$, viscous drag $\mathbf{F}_D$, shear-induced lift $\mathbf{F}_L$, and Archimedean buoyancy $\mathbf{F}_B$ on the bubble are defined as,

$$\mathbf{F}_A = \rho V_b C_A \left(\frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{u}_b}{dt} \right), \quad (10.3)$$

$$\mathbf{F}_P = \rho V_b \frac{D\mathbf{u}}{Dt}, \quad (10.4)$$

$$\mathbf{F}_D = -\rho V_b \frac{3}{8R_b} C_D \|\mathbf{u}_b - \mathbf{u}\| (\mathbf{u}_b - \mathbf{u}), \quad (10.5)$$

$$\mathbf{F}_L = -\rho V_b C_L (\mathbf{u}_b - \mathbf{u}) \times (\nabla \times \mathbf{u}), \quad (10.6)$$

$$\mathbf{F}_B = (\rho_b - \rho) V_b \mathbf{g}. \quad (10.7)$$

C_A , C_D , and C_L are the added-mass, the drag, and the lift coefficients. C_A and C_L are assumed constant and given by 1/2, supposing small spherical bubbles [38].

Point bubbles affect the dynamics of the liquid phase only through a reaction force. A cumulative reaction force between gas and liquid phases is modeled following derivations of the previous works [15, 108, 109, 110, 111, 112].

We consider a control volume V filled with a fluid whose volume V_f containing a single bubble whose volume V_b , i.e., $V = V_f + V_b$. Let us denote external surface of the fluid and the surface of the bubble as S_e and S_b , respectively. The definition sketch for the control volume is shown in Fig. 10.3. The momentum conservation for the fluid in volume V_f is written as

$$\int_{V_f} \rho_f \frac{D\mathbf{u}}{Dt} dV = \int_{S_e} \boldsymbol{\sigma} \cdot \mathbf{n} dS + \int_{S_b} \boldsymbol{\sigma} \cdot \mathbf{n} dS + \int_{V_f} \rho_f \mathbf{g} dV, \quad (10.8)$$

where $\boldsymbol{\sigma}$ and $\mathbf{n}$ are the stress tensor and outward unit normal vector. The force balance equation for a spherical bubble is

$$\rho_b V_b \frac{d\mathbf{u}_b}{dt} = - \int_{S_b} \boldsymbol{\sigma} \cdot \mathbf{n} dS + \rho_b V_b \mathbf{g}, \quad (10.9)$$

where ρ_b is density of the bubble. From Eqs. (10.8)-(10.9) and $V = V_f + V_b$ and assuming the flow gradients as constant over the bubble volume,

$$\begin{aligned} \int_V \rho_f \frac{D\mathbf{u}}{Dt} dV &= \int_V (\nabla \cdot \boldsymbol{\sigma} + \rho_f \mathbf{g}) dV \\ &+ \int_V \left[\rho_f \left(\frac{D\mathbf{u}}{Dt} - \mathbf{g} \right) + \rho_b \left(\mathbf{g} - \frac{d\mathbf{u}_b}{dt} \right) \right] \frac{V_b}{V} \delta(\mathbf{x} - \mathbf{x}_b(t)) dV \end{aligned} \quad (10.10)$$

where $\mathbf{x}$ and $\mathbf{x}_b(t)$ are position vector for the fluid and the bubble in the control volume. The force from the bubble to the liquid is represented as δ -forcing in the second term on the right hand side of the above equation, i.e.,

$$\rho_f \mathbf{f}_b(\mathbf{x}, t) = \left[\rho_f \left(\frac{D\mathbf{u}}{Dt} - \mathbf{g} \right) + \rho_b \left(\mathbf{g} - \frac{d\mathbf{u}_b}{dt} \right) \right] \frac{V_b}{V} \delta(\mathbf{x} - \mathbf{x}_b(t)). \quad (10.11)$$

Since the density of gas bubble ρ_b is sufficiently smaller than that of the liquid,

the above equation is written as

$$\mathbf{f}_b(\mathbf{x}, t) = \left(\frac{D\mathbf{u}}{Dt} - \mathbf{g} \right) \frac{V_b}{V} \delta(\mathbf{x} - \mathbf{x}_b(t)). \quad (\because \rho_b \ll \rho_f) \quad (10.12)$$

Climent & Magnaudet [20], Mazzitelli *et al.* [112], and Oresta *et al.* [113] modeled, respectively, large-scale structures of bubble-induced convection in a liquid layer, bubble-laden isotropic turbulent flows and RB convection with vapor bubbles as a momentum equation with the δ -forcing, where the reaction from bubbles and buoyancy forces are coupled with the motions of point bubbles determined in a Lagrangian approach.

The dynamics of this immiscible two-phase flow system are modeled using the Euler-Euler approach, in which the liquid and gas phases are regarded as two dynamical continua exchanging momentum and energy with each other. Soo [114] proposed a volume averaging method for dispersed two-phase flows with introducing a mesoscale L_{meso} for the flows similar to the procedure by Drew [30]. Averaging over a mesoscale L_{meso} , motion equation for dispersed phase are considered as a continuous phase. The scale L_{meso} is larger than the characteristic length scales of dispersed phase (e.g., bubble radius) and of a bubble-bubble distance since the averaging over the mesoscale are required to be smooth with respect to space and time. The characteristic length scale of dispersed phase, i.e., bubble radius R_b is assumed sufficiently smaller than the characteristic length scale of flows L_{flow} , i.e., wavelength of convection rolls. The radius is smaller than a typical bubble-bubble distance in dilute dispersed bubbly flows. The scale L_{meso} is required to be smaller than the scale L_{flow} to resolve flow development. Rightley & Lasheras [111] proposed a requirement to these scales,

$$R_b \ll \frac{R_b}{\varepsilon_G^{1/3}} \ll L_{meso} \ll L_{flow}. \quad (10.13)$$

where ε_G is volume fraction of the dispersed phase. Averaging over the mesoscale L_{meso} , the mass and momentum conservations for gas phase are modeled as [30, 31, 32, 110, 115, 116, 117, 118, 119],

$$\frac{\partial \varepsilon_G}{\partial t} + \nabla \cdot (\varepsilon_G \mathbf{u}_G) = 0, \quad (10.14)$$

$$C_A \frac{D_G \mathbf{u}_G}{Dt} = (1 + C_A) \frac{D\mathbf{u}}{Dt} - \frac{3}{8R_b} C_D \|\mathbf{u}_G - \mathbf{u}\| (\mathbf{u}_G - \mathbf{u}) - C_L (\mathbf{u}_G - \mathbf{u}) \times (\nabla \times \mathbf{u}) - \mathbf{g}, \quad (10.15)$$

where $\mathbf{u}_G = (u_G, w_G)$ and $\mathbf{u} = (u, w)$ are the velocity fields of gas and liquid, respectively. The bubble radius and the gravitational acceleration are denoted by R_b and $\mathbf{g}$. The differential operators D_G/Dt and D/Dt stand for the material derivative, i.e., $D_G \mathbf{u}_G/Dt = \partial_t \mathbf{u}_G + \mathbf{u}_G \cdot \nabla \mathbf{u}_G$ and $D\mathbf{u}/Dt = \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}$. In the gas momentum equation [Eq. (10.14)], the effects of bubble-bubble interactions inducing bubble clustering and hydrodynamic diffusion have been omitted, assuming dilute bubbly flows. The first to third terms on the right-hand side of Eq. (10.15) arise from the momentum exchange with liquid phase. These terms represent the effects of liquid inertia, drag and shear-induced lift on bubble motion, respectively. The added-mass and lift coefficients, C_A and C_L , are assumed constant and given by 1/2, supposing small bubbles [38]. For simplicity, the drag coefficient is modeled by

$$C_D = \frac{48}{Re_b}, \quad \text{with } Re_b = \frac{2R_b \|\mathbf{u}_G - \mathbf{u}\|}{\nu}, \quad (10.16)$$

where Re_b is the bubble Reynolds number. This drag model has been obtained by computing the viscous energy dissipation around a spherical bubble in a steady potential flow [120]. The bubble Reynolds number is of $O(10^2)$ for bubbles of $R_b \sim 0.5$ mm ascending in water at room temperature with the terminal

velocity. Drunzhinin & Elghobashi [110] and Ferrante & Elghobashi [119] modeled, respectively, bubble-laden turbulent flows, spatially developing turbulent boundary with bubbles and bubbly Taylor-Green vortex flows as the same mass and momentum conservations.

Averaging over the same length scale, the mass and momentum conservations for liquid phase are modeled as [30, 31, 32, 110, 115, 116, 117, 118, 119],

$$\frac{\partial}{\partial t}(1 - \varepsilon_G) + \nabla \cdot (1 - \varepsilon_G)\mathbf{u} = 0, \quad (10.17)$$

$$\begin{aligned} \frac{\partial}{\partial t}(1 - \varepsilon_G)\mathbf{u} + \nabla \cdot (1 - \varepsilon_G)\mathbf{u}\mathbf{u} = & -\frac{1}{\rho}\nabla(1 - \varepsilon_G)p \\ & + \nu\nabla[(1 - \varepsilon_G)(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)] + (1 - \varepsilon_G)\mathbf{g} + \mathbf{f}. \end{aligned} \quad (10.18)$$

where the density, the kinematic viscosity, and pressure field of liquid are denoted by ρ , ν , and p . $\mathbf{f}$ is a coupling term between the gas and liquid phases. In the liquid mass conservation Eq. (10.17) and the liquid momentum Eq. (10.18), we have assumed dilute bubbly flows and considered liquid flows effectively incompressible. We have also restricted our attention to flows at scales larger than the mean distance between bubbles. The mass and momentum conservations are modeled as [109, 110, 111],

$$\nabla \cdot \mathbf{u} = 0, \quad (10.19)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{u} + \mathbf{g} + \varepsilon_G\left(\frac{D\mathbf{u}}{Dt} - \mathbf{g}\right). \quad (10.20)$$

Bubbles affect the dynamics of the liquid phase only through the mesoscale cumulative reaction force $\mathbf{F} = \varepsilon_G(D\mathbf{u}/Dt - \mathbf{g})$ from the bubbles to the liquid. Considering N -th spherical bubbles dispersed in the control volume V , the cumulative reaction force is modeled as the sum of the δ -forcing in Eq. (10.10) [110],

[111],

$$\mathbf{F} = \sum_{i=1}^N \mathbf{f}_b(\mathbf{x}, t) = \varepsilon_G \left(\frac{D\mathbf{u}}{Dt} - \mathbf{g} \right). \quad (10.21)$$

In the liquid mass conservation Eq. (10.19) and the liquid momentum Eq. (10.20), we have assumed dilute bubbly flows and considered liquid flows effectively incompressible. The same equation has been used in a Lagrangian analysis of the influence of bubbles on a carrier phase [109], in an investigation on the liquid-gas interfacial coupling in a free-shear flow of a dilute bubble suspension [111], and bubble-laden isotropic decaying turbulence using the Euler-Euler approach in direct numerical simulations [110].

The upper surface of the liquid layer is assumed as flat and shear free for simplicity. At the bottom wall, no-slip conditions on the liquid velocity, constant gas velocity, and constant gas flux are imposed,

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad w = 0, \quad \text{at } z = d, \quad (10.22)$$

$$\mathbf{u} = \mathbf{0}, \quad w_G = W_{G,0}, \quad \varepsilon_G W_{G,0} = J_{G,0}, \quad \text{at } z = -d. \quad (10.23)$$

10.1.2 Dimensionless governing equation

Adopting scales d of length, ν/d of velocity, d^2/ν of time, and $J_{G,0}/W_{G,0}$ of bubble volume fraction, we nondimensionalize the governing equations (10.14)–(10.20) to obtain

$$\frac{\partial \varepsilon_G}{\partial t} + \nabla \cdot (\varepsilon_G \mathbf{u}_G) = 0, \quad (10.24)$$

$$\begin{aligned} \frac{D\mathbf{u}_G}{Dt} = 3 \frac{D\mathbf{u}}{Dt} - \frac{18}{r_b^2} (\mathbf{u}_G - \mathbf{u}) \\ - (\mathbf{u}_G - \mathbf{u}) \times (\nabla \times \mathbf{u}) + \frac{18}{Pr \cdot r_b^2} \mathbf{e}_z, \end{aligned} \quad (10.25)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (10.26)$$

$$\begin{aligned} \frac{D\mathbf{u}}{Dt} = & -\nabla p + \Delta \mathbf{u} - \frac{9}{Pr \cdot r_b^2} \mathbf{e}_z \\ & + \varepsilon_G \frac{Ra \cdot r_b^2}{9Pr} \left(\frac{D\mathbf{u}}{Dt} + \frac{9}{Pr \cdot r_b^2} \mathbf{e}_z \right). \end{aligned} \quad (10.27)$$

The boundary condition (10.22) & (10.23) read in dimensionless form,

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad w = 0, \quad \text{at } z = 1, \quad (10.28)$$

$$\mathbf{u} = \mathbf{0}, \quad w_G = w_{G,0}, \quad \varepsilon_G = 1, \quad \text{at } z = -1. \quad (10.29)$$

We have introduced the following four dimensionless parameters,

$$\begin{aligned} Pr &= \frac{\nu}{V_\infty d} = \frac{9\nu^2}{gR_b^2 d}, \quad Ra = \frac{gd^2 J_{G,0}}{V_\infty^2 \nu} = \frac{81\nu d^2 J_{G,0}}{gR_b^4}, \\ w_{G,0} &= \frac{W_{G,0} d}{\nu}, \quad r_b = \frac{R_b}{d}. \end{aligned} \quad (10.30)$$

The first two parameters are Prandtl and Rayleigh numbers. They compare characteristic time scales: $Pr = T_R/T_\nu$ and $Ra = T_\nu T_R/T_B^2$, where T_R is the bubble residence time $T_R = d/V_\infty$, T_ν is the momentum diffusion time $T_\nu = d^2/\nu$, and T_B is the buoyancy time $T_B = \sqrt{d/g_r}$ with a reduced gravity $g_r = (J_{G,0}/V_\infty)g$. Iga & Kimura [19] and Climent & Magnaudet [20] employ Pr and Ra similar to Eqs. (10.30) in their numerical investigations on bubbly convection flows. Throughout the present work, the injection velocity $w_{G,0}$ and bubble radius r_b are assumed to be $w_{G,0} = 1000$ and $r_b = 0.01$.

The laterally uniform steady state would be formed, if the residence time T_R is short compared with the overturning time scale T_{OV} : $T_R/T_{OV} \lesssim 1$. The time scales T_{OV} is estimated by $T_{OV} = [\nu/(Ag)^2]^{1/3} = (4\nu/\Delta\varepsilon_G^2 g^2)^{1/3}$ where A is the Atwood number and given by $A = \Delta\varepsilon_G/2$ for dilute bubbly flows and $\Delta\varepsilon_G$ is the total variation of gas volume fraction $\Delta\varepsilon_G = J_{G,0}/W_{G,0} - J_{G,0}/V_\infty$.

The condition $T_R/T_{OV} \lesssim 1$ for the formation of homogeneous state is satisfied in dilute bubbly flows, as $T_R/T_{OV} \approx \varepsilon_{G,0}^{2/3} \cdot (\nu^2/g)^{1/3} \cdot d/R_b^2$, where $\varepsilon_{G,0}$ is the gas volume fraction at the wall, $\varepsilon_{G,0} = J_{G,0}/W_{G,0}$, e.g., for bubbles of $R_b = 0.5$ mm with $\varepsilon_{G,0} = 0.03$ in water at room temperature $\nu = 1.0$ mm²/s on the earth $g = 9810$ mm/s². The density decrease between the bottom and top of the sublayer is given by $\rho\Delta\varepsilon_G$,

10.2 Base state and perturbation equation

Two-dimensional flows would respect the translational symmetry of the system along the x direction when the flux of gas injection is small. We thus assume a steady bubbly flow in the homogeneous regime, where the flow fields are laterally uniform:

$$\mathbf{u} = \mathbf{0}, \quad \mathbf{u}_G = \bar{w}_G(z) \mathbf{e}_z, \quad \varepsilon_G = \bar{\varepsilon}_G(z). \quad (10.31)$$

The unit vector in the z direction is denoted as $\mathbf{e}_z$. We consider perturbations around the base state,

$$\mathbf{u} = \delta\mathbf{u}', \quad p = \bar{p} + p', \quad \mathbf{u}_G = \bar{w}_G\mathbf{e}_z + \delta\mathbf{u}'_G, \quad \varepsilon_G = \bar{\varepsilon}_G + \delta\varepsilon'_G, \quad (10.32)$$

where perturbation components are indicated by primes and $\mathbf{u}' = u'\mathbf{e}_x + w'\mathbf{e}_z$, $\mathbf{u}'_G = u'_G\mathbf{e}_x + w'_G\mathbf{e}_z$ ($\mathbf{e}_x$: the unit vector in the x direction). δ is a 'small' (strictly infinitesimal) parameter, i.e., assuming $\delta \ll 1$. Substituting Eqs. (10.32) into Eqs. (10.24)-(10.27), we have equations for powers of δ .

$O(\delta^0)$

$$\frac{d}{dz}(\bar{\varepsilon}_G \bar{w}_G) = 0, \quad (10.33)$$

$$\bar{w}_G \frac{d\bar{w}_G}{dz} = -\frac{18\bar{w}_G}{r_b^2} + \frac{18}{Pr \cdot r_b^2}, \quad (10.34)$$

$$0 = -\nabla \bar{p} - \frac{9}{Pr \cdot r_b^2} \left(1 - \frac{Ra \cdot r_b^2}{9Pr} \bar{\varepsilon}_G \right) \mathbf{e}_z \quad (10.35)$$

We have invoked Eq. (10.16) for the drag force. The boundary conditions (10.29) read,

$$\bar{w}_G = w_{G,0}, \quad \bar{\varepsilon}_G = 1, \quad \text{at } z = -1. \quad (10.36)$$

The momentum equation (10.34) indicates that the gas phase is accelerated toward the terminal velocity $V_\infty = gR_b^2/9\nu$. Some profiles of velocity $\bar{w}_G$ calculated from Eq. (10.34) are shown in Fig. 10.2 (a). The gas velocity increases in a sublayer attached to the wall. Outside this sublayer the gas flows at a constant velocity V_∞ .

According to Eqs. (10.33) and (10.36), the gas volumetric flux $\bar{\varepsilon}_G \bar{w}_G$ is constant and given by $w_{G,0}$. The volume fraction $\bar{\varepsilon}_G$ is thus calculated from $\bar{w}_G$ as

$$\bar{\varepsilon}_G = \frac{w_{G,0}}{\bar{w}_G}. \quad (10.37)$$

Some profiles of $\bar{\varepsilon}_G$ are shown in Fig. 10.2 (b). As expected from Eq. (10.37) and from the $\bar{w}_G$ profiles in Fig. 10.2(a), a sublayer rich in bubbles is formed on the bottom wall. As $\bar{w}_G$ increases from WG_0 to V_∞ , the volume fraction decreases from $J_{G,0}/WG_0$ to a smaller value $J_{G,0}/V_\infty$ in the sublayer. The stratification in the mean density of liquid-gas mixture is potentially unstable

to the gravity inside the sublayer. For $O(\delta^1)$, we have linearized perturbation equations:

$O(\delta^1)$

$$\begin{aligned} \frac{\partial \varepsilon'_G}{\partial t} = & - \left(\bar{w}_G \frac{\partial}{\partial z} + \frac{d\bar{w}_G}{dz} \right) \varepsilon'_G \\ & - \bar{\varepsilon}_G \frac{\partial u'_G}{\partial x} - \left(\bar{\varepsilon}_G \frac{\partial}{\partial z} + \frac{d\bar{\varepsilon}_G}{dz} \right) w'_G, \end{aligned} \quad (10.38)$$

$$\begin{aligned} \frac{\partial \mathbf{u}'_G}{\partial t} + \bar{w}_G \frac{\partial \mathbf{u}'_G}{\partial z} + \frac{d\bar{w}_G}{dz} w'_G \mathbf{e}_z = & 3 \frac{\partial \mathbf{u}'}{\partial t} \\ & - \frac{18}{r_b^2} (\mathbf{u}'_G - \mathbf{u}') + \bar{w}_G \zeta' \mathbf{e}_x, \end{aligned} \quad (10.39)$$

$$\nabla \cdot \mathbf{u}' = 0, \quad (10.40)$$

$$\frac{\partial \mathbf{u}'}{\partial t} = -\nabla p' + \Delta \mathbf{u}' + \bar{\varepsilon}_G \frac{Ra \cdot r_b^2}{9Pr} \frac{\partial \mathbf{u}'}{\partial t} + \varepsilon'_G \frac{Ra}{Pr^2} \mathbf{e}_z, \quad (10.41)$$

where ζ' is the liquid vorticity perturbation: $\zeta' = \partial_z u' - \partial_x w'$ and π' is the perturbation of a reduced pressure. The boundary conditions on the perturbation fields are obtained from Eqs. (10.28) & (10.29). They are

$$\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} = w' = 0, \quad \text{at } z = 1, \quad (10.42)$$

$$u' = w' = w'_G = \varepsilon'_G = 0, \quad \text{at } z = -1. \quad (10.43)$$

We perform a modal analysis to determine the stability of the base state, expanding the perturbation fields $\mathbf{u}'_G(x, z, t)$, $\mathbf{u}'(x, z, t)$, and $\varepsilon'_G(x, z, t)$ into normal modes:

$$(\mathbf{u}'_G, \mathbf{u}', \varepsilon'_G) = \int_{-\infty}^{\infty} (\hat{\mathbf{u}}_G(z), \hat{\mathbf{u}}(z), \hat{\varepsilon}_G(z)) e^{st+ikx} dk, \quad (10.44)$$

where k is the wavenumber and $s = \sigma + i\omega$ is the complex growth rate consisting of a growth rate σ and a frequency ω . Hatted quantities are the complex amplitudes of perturbation fields. The complex amplitudes $(\hat{\mathbf{u}}_G(z), \hat{\mathbf{u}}(z), \hat{\varepsilon}_G(z))$

are expanded as Chebyshev series:

$$(\hat{u}_G(z), \hat{w}_G(z), \hat{u}(z), \hat{w}(z), \hat{\varepsilon}_G(z)) = \sum_{l=0}^L (u_{G,l}, w_{G,l}, u_l, w_l, \varepsilon_{G,l}) T_l(z) \quad (10.45)$$

where $T_l(z) = \cos(lq)$, ($z = \cos q$) is Chebyshev polynomials of the first kind and L is truncation levels of Chebyshev series.

For a mode of wavenumber k , Eqs. (10.38)–(10.43) can be cast into the following form,

$$\mathcal{A}(k, Pr, Ra, w_{G,0}, r_b) \hat{\mathbf{X}} = s \mathcal{B} \hat{\mathbf{X}}, \quad (10.46)$$

where $\hat{\mathbf{X}} = (u_{G,l}, w_{G,l}, u_l, w_l, \varepsilon_{G,l})^{tr}$ are unknown variables. A linear differential operator $\mathcal{A}$ with respect to z depends on the wavenumber k and the parameters $(Pr, Ra, w_{G,0}, r_b)$. The coefficient matrix $\mathcal{B}$ is constant. We discretize the eigenvalue problem (10.46) with the Lobatto-Gauss-collocation points

$$z_k = \cos(k\pi/L), \quad k = 0, \dots, L. \quad (10.47)$$

The eigenvalues s and the unknown variables $\hat{\mathbf{X}}$ are determined by the QZ decomposition.

10.3 Results

10.3.1 Stability diagram

The growth rate σ of the least stable mode determined from the eigenvalue problem (10.46) changes its sign from negative to positive with increasing Rayleigh number Ra from zero at a given wavenumber k . Determining the marginal stability condition where the sign of σ changes for different wavenumbers while keeping r_b and $w_{G,0}$ constant, we obtain the marginal stability

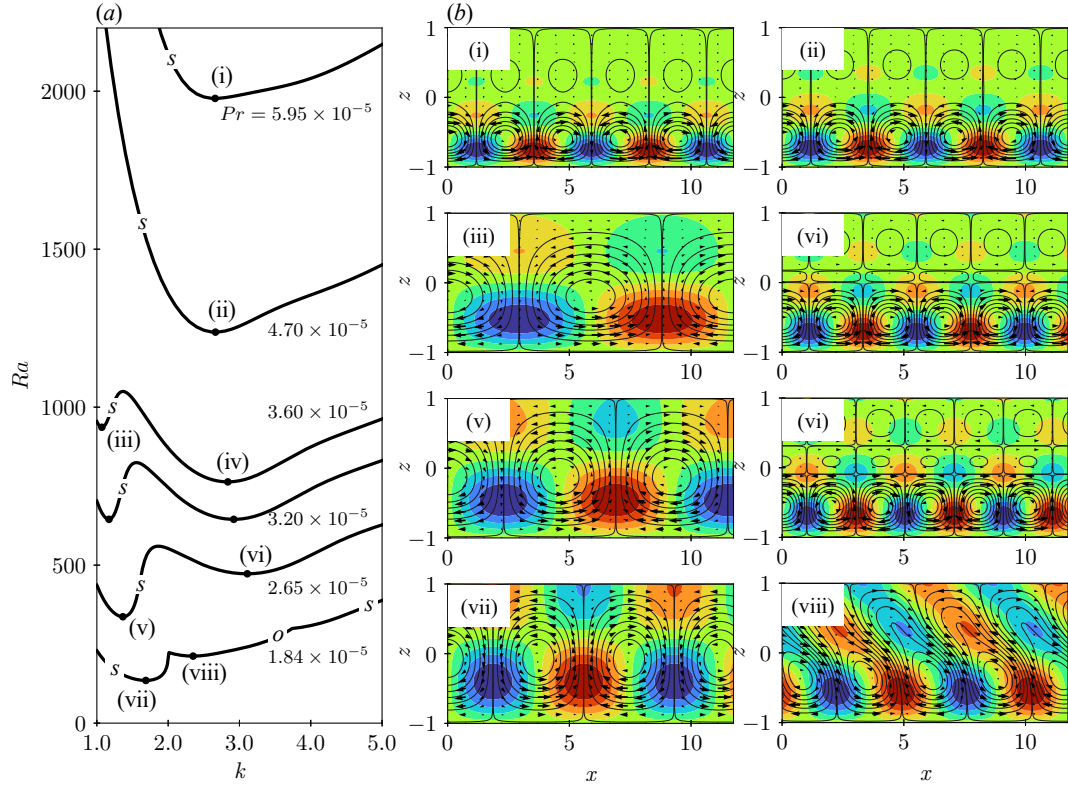


Figure 10.4: (a) Marginal stability curves, above which the quiescent state is unstable, for different values of the Prandtl number Pr in the $k - Ra$ space. The gas injection velocity and the bubble radius are fixed at $w_{G,0} = 1000$ and $r_b = 0.01$. The temporal natures of marginally stable mode are indicated on curves by s (stationary) and o (oscillatory). (b) Eigenvectors at the local minimum points (i)-(ix). The liquid velocity $\mathbf{u}' = (u', w')$ and the gas fraction ε'_G are shown by vectors and colors, respectively. Bubble-rich and bubble-poor cells correspond to red and white zones. Some streamlines are also shown in (b) to visualize the structures of liquid flows.

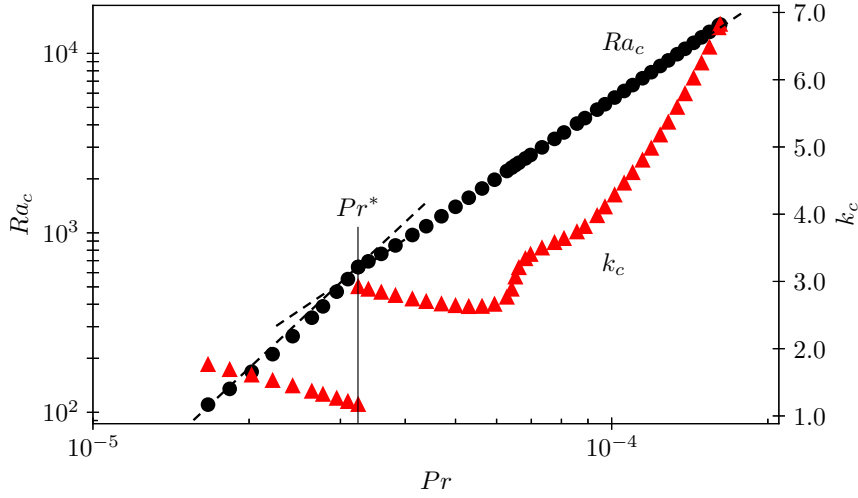


Figure 10.5: Variation of critical parameters (k_c , Ra_c) as function of Pr . The whole- and multi-layer modes are critical on the left and right of a codimension-two point $Pr = Pr^*$, respectively. The best fitting power laws $Ra_c \propto Pr^n$ are shown in dashed lines. The exponent n is 2.68 and 1.93 for the left and right lines.

curve $\sigma(k, Ra) = 0$ [Fig. 10.4(a)]. The curve shifts downward with decreasing Prandtl number Pr , changing its shape. It has only a single minimum point for $Pr > 4.13 \times 10^{-5}$, while there exist two local minima otherwise. At $Pr = 3.25 \times 10^{-5}$ ($=: Pr^*$) the two minima are found at the same value of $Ra = 645$ ($=: Ra^*$). Marginally stable modes are stationary except for a range of k found at small Pr (see the lowest curve in the figure).

Eigenvectors of some marginal modes are shown in Fig. 10.4(b) for the gas volume fraction ε'_G and liquid velocity $\mathbf{u}'$. The perturbation gas volume fraction ε'_G exhibits different structures consisting of horizontal arrays of bubble-rich and bubble-poor cells [Fig. 10.4(b)]. For stationary modes [Fig. 10.4(b) i–viii], the horizontal flow of the liquid phase converges and diverges in cells of large and small ε'_G , respectively. In the lowest layer attached to the bottom wall, where the unstable density stratification is the most significant, the liquid moves upward inside bubble-rich cells and downward inside bubble-poor cells.

Convection rolls on the bottom do not extend to the top free surface of fluid layer for large k [Fig. 10.4(b) i–iii, v, vii]: multi-layer convection rolls are observed, though the flow in convection rolls away from the bottom wall is much weaker than that in the rolls on the wall. For small k , in contrast, convection rolls on the wall occupy the entire fluid layer [Fig. 10.4(b) iv, vi, viii]. We call them whole-layer convection rolls. The gas velocity field $\mathbf{u}'_G$ exhibits the same flow patterns as the liquid velocity field $\mathbf{u}'$. The slip velocity $\mathbf{u}'_G - \mathbf{u}'$ is only of the order of a few percent of $\mathbf{u}'$. Pr determines the types of convection rolls, i.e., either multi- or whole-layer mode, at critical conditions. Oscillatory eigenmodes [Fig. 10.4(b) ix] are also composed of multi-layer cells of gas volume fraction. The perturbation flow rises and falls in bubble-rich and bubble-poor cells attached to the bottom, respectively. In contrast to stationary modes, the structures of cells and flows of oscillatory modes lack $x \rightarrow -x$ reflection symmetry.

Variations of the critical parameters (k_c, Ra_c) determined from the marginal curves are shown in Fig. 10.5. The critical Rayleigh number Ra_c increases with Pr , while the critical wavenumber k_c varies nonmonotonically. The behavior of (k_c, Ra_c) for $Pr < Pr^*$ is distinct from the behavior for $Pr > Pr^*$, since whole-layer and multi-layer modes are critical in the former and latter cases, respectively. At the codimension-two point $Pr = Pr^*$, both types of modes are critical. In the present work, the ratio T_R/T_{OV} at critical conditions is $O(1)$. This order estimation would imply the two time scale competition at the onset of bubble-induced convection.

Whole-layer convection rolls were observed in the numerical simulation by Climent & Magnaudet [20]. These authors reported convection rolls of different geometrical characteristics for the same set of parameter values $(Pr, Ra, r_b) = (2.5 \times 10^{-4}, 2.07 \times 10^5, 2 \times 10^{-3})$. The aspect ratio Γ of the wavelength of

observed convection pattern to the fluid layer thickness is either 1 or 0.5. For this Prandtl number and bubble radius, the present linear stability analysis does not predict any instability.

Experiments on a RT-like instability of bubble layers by Iga & Kimura [19] showed the formation of whole-layer convection rolls at $Pr = O(10^{-2})$, $Ra = O(10^5)$ and $r_b = O(10^{-3})$. The numerical simulations performed by these authors showed also the formation of whole-layer convection for experimental values of Pr and Ra . The ratio Γ was found to increase with Ra and to take a value 2.7 close to the onset of convection formation. For the critical modes reported in Fig. 10.5, the ratio $\Gamma (= \pi/k_c)$ is smaller than 1.3 for multi-layer mode and between 1.5 and 3.1 for whole-layer mode. Bubbly convection observed by Iga & Kimura [19] might correspond to whole-layer modes in the present work. The present linear stability theory predicts, however, no instability at $Ra \sim 10^5$ for the values of (Pr, r_b) considered by Iga & Kimura [19]. In fact, for tiny bubbles in a viscous liquid such as considered in Iga & Kimura [19], the base state [Fig. 10.2(b)] has a uniform bubble distribution throughout the fluid layer, $\bar{\varepsilon}_G = \text{const.}$, so that the density stratification is stable to gravity. The instability investigated by Iga & Kimura [19] would be related to a particular initial configuration, i.e., an unstable superposition of a pure liquid layer over a bubble layer, and would be distinct from the instability considered in the present work.

The model of drag coefficient affects the stability of the bubbly flow system, as the inverse density profile in the base state is different between models. In addition to the stability analysis obtained with $C_D = 48/\text{Re}_b$, we have performed similar analyses with different models of drag coefficient: $C_D = 16/\text{Re}_b$ [22, 23] for bubbles in a pure liquid in the low Reynolds number limit, $C_D = 24/\text{Re}_b$ [121] for bubbles in a liquid contaminated by surfactants also in the

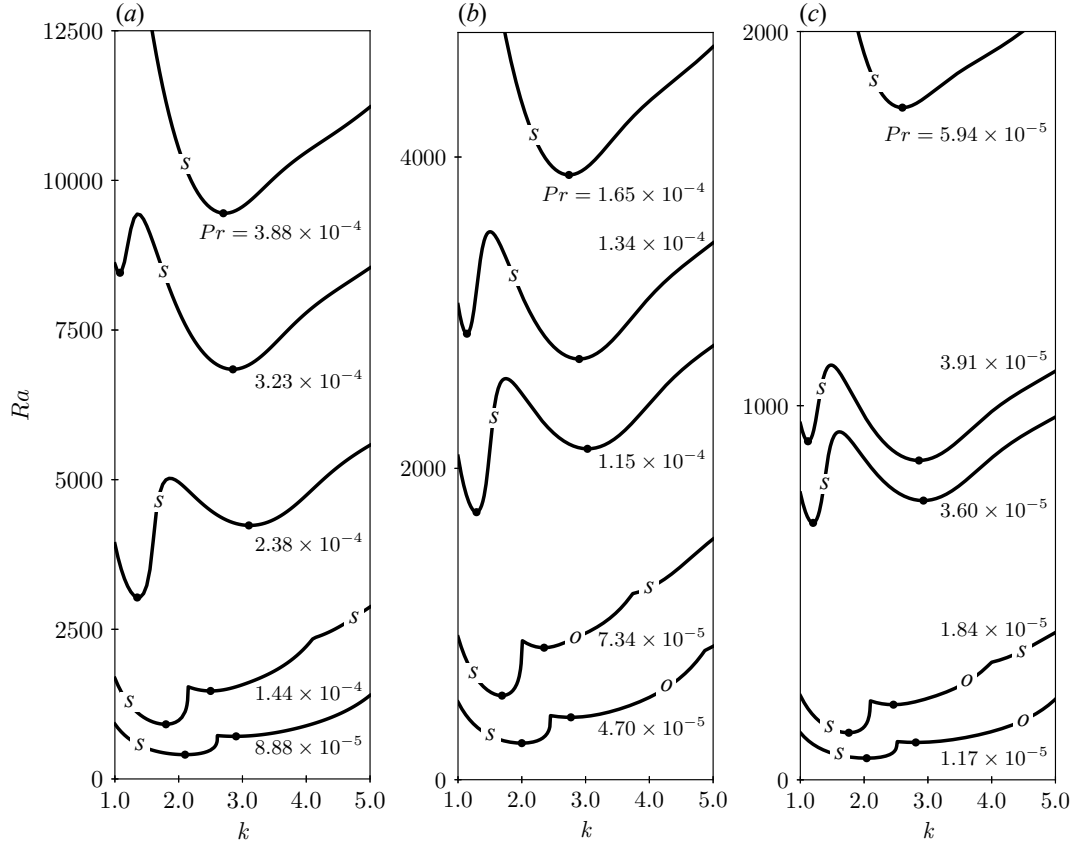


Figure 10.6: Marginal stability curves, above which the quiescent state is unstable, for different values of Prandtl number Pr in the $k - Ra$ space. The bubble radius and the bubble injection velocity are fixed at $r_b = 0.01$ and $w_{G,0} = 1000$. The drag coefficient is modeled by $C_D = 16/\text{Re}_b$ [22, 23].

low Reynolds number limit, and C_D in Eq. 8.7 by Mei [122] which interpolates smoothly C_D for bubbles in a pure liquid from the low Reynolds number limit ($C_D = 16/\text{Re}_b$) to the large Reynolds number limit ($C_D = 48/\text{Re}_b$). Marginal stability conditions are quantitatively different between models. For models giving larger drags (i.e., for models with a larger value of $C_D \text{Re}_b$), the inverse density profile is confined in a thinner sublayer of fluid on the bottom wall and the critical value of Ra becomes larger. The analyses predict, however, the qualitatively same behavior of the bubbly flow system as that reported in the main text. In all the analyses, multi- and whole-layer critical modes are observed and the transition between these two modes is predicted with varying Pr [Fig. 10.6].

The value of the r_b affects the stability of the bubbly flow layer significantly, as the inverse density profile in the base state is sensitive to r_b . For example, the critical value of Ra is $Ra_c = 73.96$ for $r_b = 0.014$ and $Ra_c = 2997$ for $r_b = 0.006$ [Fig. 10.7]. These values are, respectively, 0.22 times smaller and 8.91 times larger than the critical Ra for $r_b = 0.01$. However, the predicted behavior of the bubbly flow system is qualitatively the same as that reported in the main text. Multi- and whole-layer critical modes are observed and the transition between these two flow patterns is predicted with varying Pr .

The inverse density profile of the base state changes only slightly with varying the injection velocity $w_{G,0}$. As a consequence, the variation of $w_{G,0}$ does not affect the stability, unless $w_{G,0}$ is very close to the dimensionless terminal rising velocity $v_\infty = V_\infty/\nu/d$. For example, no qualitative and quantitative difference is produced by a variation of $w_{G,0}$ in a range $0 < w_{G,0}/v_\infty < 0.15$ for $Pr = 2.65 \times 10^{-5}$.

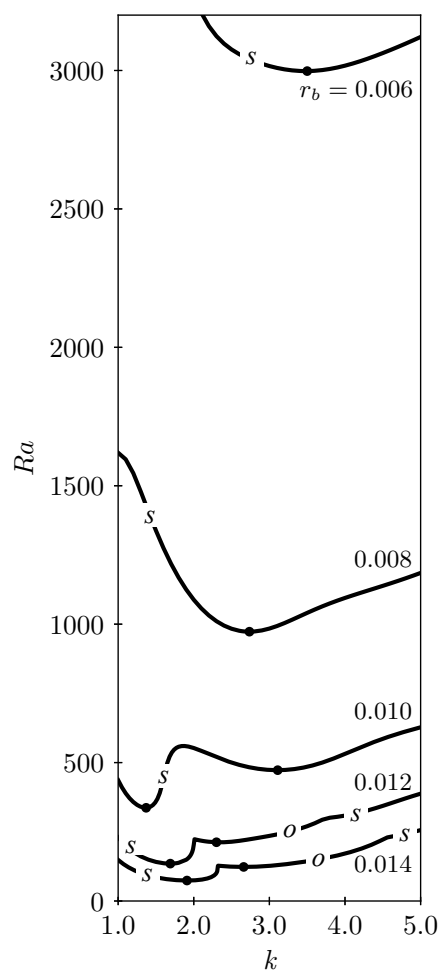


Figure 10.7: Marginal stability curves for different values of bubble radius r_b in the $k - Ra$ space. Prandtl number Pr and the injection velocity are fixed at $Pr = 2.65 \times 10^{-5}$ and $w_{G,0} = 1000$. The temporal natures of marginally stable mode are indicated on curves by s (stationary) and o (oscillatory).

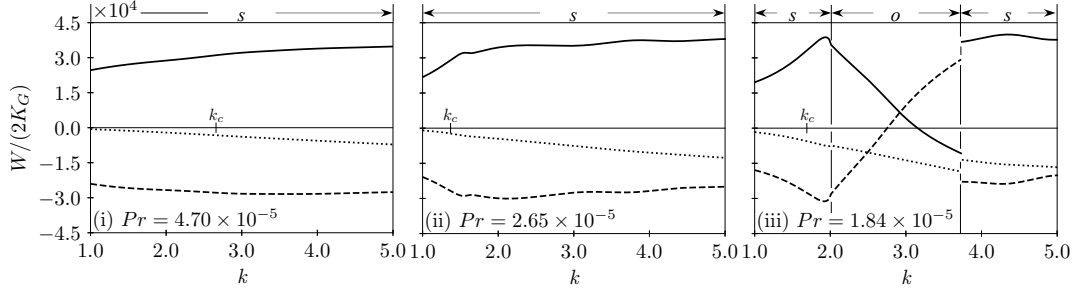


Figure 10.8: Powers providing energy to perturbation gas flows at marginally stable conditions: W_I (solid line), W_D (dotted line), and W_L (dashed line) represent the powers of liquid inertia, drag and lift forces, respectively. The powers have been normalized by twice the kinetic energy K_G of perturbation flows. Different panels show results for different values of the Prandtl number Pr . The values of critical wavenumber k_c are marked on the k axis. The temporal natures of marginally stable mode are indicated by s (stationary) and o (oscillatory) at the top of each panel.

10.3.2 Energy budget

We consider the energy transfer from the base to perturbation flows in order to obtain insights into the instability mechanism. The transfer to liquid perturbation flows will be similar to that in RB convection because of the similarity of Eqs. (10.38) & (10.41) with the corresponding equations for thermal convection [24]. The perturbation flows gain and lose energy due to buoyancy and viscous energy dissipation, respectively. To confirm this scenario we can derive an evolution equation for the kinetic energy of liquid flows by taking the inner product of Eq. (10.41) with $\mathbf{u}'$. After integrating the resulting equation over the whole liquid layer and averaging it with respect to the horizontal direction, we obtain

$$\frac{dK}{dt} = W_B - D_V, \quad (10.48)$$

where the kinetic energy K , the power of the buoyancy force W_B and the rate of viscous energy dissipation D_V are defined by

$$K = \left\langle \left(1 - \bar{\varepsilon}_G \frac{Ra \cdot r_b^2}{9Pr} \right) \frac{u'^2 + w'^2}{2} \right\rangle, \quad (10.49)$$

$$W_B = \frac{Ra}{Pr^2} \langle \varepsilon'_G w' \rangle, \quad (10.50)$$

$$D_V = \langle \Phi \rangle, \quad (10.51)$$

where the dissipation function Φ is given as $\Phi = 2(\partial_x u')^2 + (\partial_z u' + \partial_x w')^2 + 2(\partial_z w')^2$ and the angle brackets denote the following integral operation,

$$\langle \bullet \rangle = \frac{k}{2\pi} \int_{-1}^1 \int_0^{2\pi/k} \bullet \, dx dz. \quad (10.52)$$

Equation (10.48) confirms that convection develops when the buoyancy overcomes the viscous energy dissipation.

We should now turn to the energy transfer in gas phase, as the driving buoyancy effect W_B is created by a heterogeneous distribution of the gas phase due to perturbation gas flows. By similar procedures used to obtain Eq. (10.48), we derive the following evolution equation from Eq. (10.39).

$$\frac{dK_G}{dt} = W_I + W_D + W_L, \quad (10.53)$$

where the kinetic energy K_G , the powers of the liquid inertia W_I , of the drag W_D and of the lift W_L are defined by,

$$K_G = \left\langle \frac{u_G'^2 + w_G'^2}{2} \right\rangle, \quad (10.54)$$

$$W_I = - \left\langle \bar{w}_G \frac{\partial}{\partial z} \frac{\|\mathbf{u}'_G\|^2}{2} + \frac{d\bar{w}_G}{dz} w_G'^2 \right\rangle + 3 \left\langle \frac{\partial \mathbf{u}'}{\partial t} \cdot \mathbf{u}'_G \right\rangle, \quad (10.55)$$

$$W_D = -\frac{18}{r_b^2} \langle (\mathbf{u}'_G - \mathbf{u}') \cdot \mathbf{u}'_G \rangle, \quad (10.56)$$

$$W_L = \langle \bar{w}_G \zeta' u'_G \rangle. \quad (10.57)$$

Energy budget of the gas flow are computed for marginal modes for different values of Pr [Fig. 10.8]. For stationary modes, the liquid inertia provides energy to perturbation flows (i.e., $W_I > 0$) regardless of the type of convection roll, while the drag and lift have stabilizing effects (i.e., $W_D, W_L < 0$). The liquid inertia and the lift become stabilizing and destabilizing for some oscillatory modes, e.g., when $Pr = 1.84 \times 10^{-5}$ for wavenumbers k between 2.9 and 3.8.

Chapter 11

Bifurcation analysis

11.1 Perturbation equation

For $O(\delta^2)$, we have nonlinear perturbation equations:

$O(\delta^2)$

$$\begin{aligned} \frac{\partial \varepsilon'_G}{\partial t} + \left(\bar{w}_G \frac{\partial}{\partial z} + \frac{d\bar{w}_G}{dz} \right) \varepsilon'_G + \bar{\varepsilon}_G \frac{\partial u'_G}{\partial x} \\ + \left(\bar{\varepsilon}_G \frac{\partial}{\partial z} + \frac{d\bar{\varepsilon}_G}{dz} \right) w'_G + \nabla \cdot (\varepsilon'_G \mathbf{u}'_G) = 0, \end{aligned} \quad (11.1)$$

$$\begin{aligned} \frac{\partial \mathbf{u}'_G}{\partial t} + \bar{w}_G \frac{\partial \mathbf{u}'_G}{\partial z} + \frac{d\bar{w}_G}{dz} w'_G \mathbf{e}_z + \mathbf{u}'_G \cdot \nabla \mathbf{u}'_G = 3 \left\{ \frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{u}' \cdot \nabla \mathbf{u}') \right\} \\ - \frac{18}{r_b^2} (\mathbf{u}'_G + \mathbf{u}') + (\bar{w}_G + w'_G - w') \zeta' \mathbf{e}_x - (u'_G - u') \zeta' \mathbf{e}_z, \end{aligned} \quad (11.2)$$

$$\nabla \cdot \mathbf{u}' = 0, \quad (11.3)$$

$$\begin{aligned} \frac{\partial \mathbf{u}'}{\partial t} + \mathbf{u}' \cdot \nabla \mathbf{u}' = -\nabla \pi' + \Delta \mathbf{u}' \\ + \bar{\varepsilon}_G \frac{Ra \cdot r_b^2}{9Pr} \frac{\partial \mathbf{u}'}{\partial t} + \varepsilon'_G \frac{Ra}{Pr^2} \mathbf{e}_z + \varepsilon'_G \frac{Ra \cdot r_b^2}{9Pr} \frac{\partial \mathbf{u}'}{\partial t}. \end{aligned} \quad (11.4)$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad w = 0, \quad \text{at } z = 1, \quad (11.5)$$

$$\mathbf{u} = \mathbf{0}, \quad w_G = w_{G,0}, \quad \varepsilon_G w_{G,0} = 1, \quad \text{at } z = -1. \quad (11.6)$$

We perform a modal analysis to determine the nonlinear development of the flows from the linear instability, expanding the perturbations fields $\mathbf{u}'_G(x, z, t)$, $\mathbf{u}'(x, z, t)$, and $\varepsilon'_G(x, z, t)$ into Fourier-Chebyshev series:

$$(\mathbf{u}'_G, \mathbf{u}', \varepsilon'_G) = \sum_{m=-M}^M (\hat{\mathbf{u}}_{G,m}(z), \hat{\mathbf{u}}_m(z), \hat{\varepsilon}_{G,m}(z)) e^{im\alpha(x-ct)}. \quad (11.7)$$

$\hat{\mathbf{u}}_{G,m}(z)$, $\hat{\mathbf{u}}_m(z)$, $\hat{\varepsilon}_{G,m}(z)$ are the complex amplitudes of Fourier spectra of mode m , α is the wavenumber, c is the complex phase velocity. The complex amplitudes are expanded into Fourier-Chebyshev spectra:

$$\begin{aligned} &(\hat{u}_{G,m}(z), \hat{w}_{G,m}(z), \hat{u}_m(z), \hat{w}_m(z), \hat{\varepsilon}_{G,m}(z)) \\ &= \sum_{m=-M}^M \sum_{l=0}^L (u_{G,lm}, w_{G,lm}, u_{lm}, w_{lm}, \varepsilon_{G,lm}) T_l(z), \end{aligned} \quad (11.8)$$

where $u_{G,lm}$, $w_{G,lm}$, u_{lm} , w_{lm} , and $\varepsilon_{G,lm}$ are Fourier-Chebyshev spectra. Equations (10.14)–(10.20) are discretized at the Lobatto-Gauss-collocation points in Eq. 10.47. Substituting Eq. 11.8 into Equations (11.1)–(10.43), the problem can be cast into the following form,

$$\mathbf{F}(\mathbf{X}) = 0, \quad (11.9)$$

where the unknown variable $\mathbf{X}$ consists of $(u_{G,lm}, w_{G,lm}, u_{lm}, w_{lm}, \varepsilon_{G,lm})^{tr}$. We solve Eq. 11.9 to obtain nonlinear flows at different values of Ra by the Newton-Rhapson iterative scheme with the tolerance

$$|\mathbf{X}^{It} - \mathbf{X}^{It-1}| < 10^{-8}, \quad (11.10)$$

where It is the iteration number. Linear solutions obtained by the linear stability analysis presented in Sec. 10.3 are assumed to be the first initial

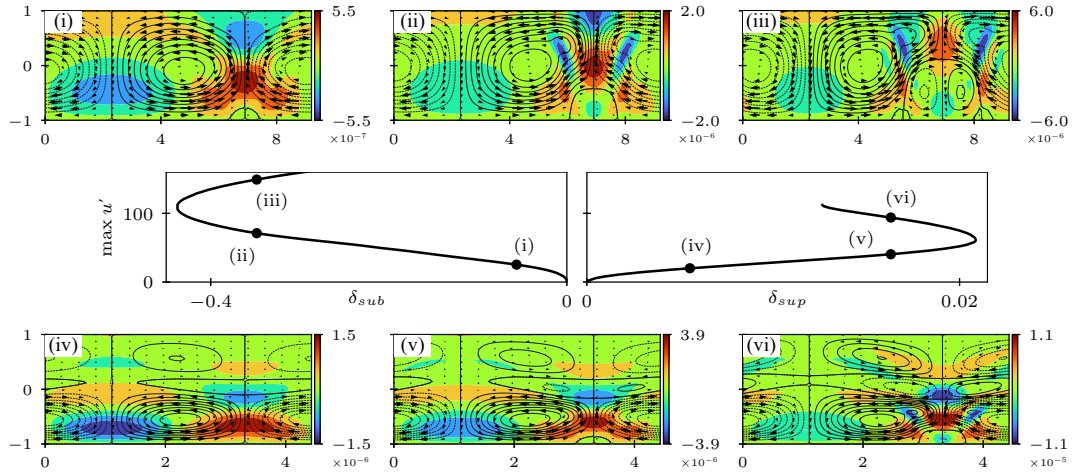


Figure 11.1: Bifurcation diagrams and eigenvectors on the branches for different values of bifurcation parameters (a) $\delta_{sub} = (Ra - Ra_{wl})/Ra$ and (b) $\delta_{sup} = (Ra - Ra_{ml})/Ra$. Ra_{wl} and Ra_{ml} are the critical Rayleigh numbers on the marginally stable conditions in Fig. ??(a)(iv) and (v), respectively. The ordinate is the maximum value of horizontal perturbation velocity in liquid phase. The gas injection velocity and the bubble radius are fixed at $w_{G,0} = 1000$ $r_b = 0.01$. In insets, the liquid velocity perturbation $\mathbf{u} = (u', w')$ and the gas volume fraction perturbation ε'_G are indicated by arrows and colours, respectively. Bubble-rich and bubble-poor cells correspond to red and white zones. Some streamline are also shown in the fields to visualize the structures of liquid flows.

solution of the scheme.

11.2 Results

11.2.1 Bifurcation diagram

The bifurcation diagrams from the critical conditions for whole- and multi-layer modes are shown in Fig. ?. Some eigenvectors for the gas volume fraction ε'_G and liquid velocity $\mathbf{u}'$ on the branches are shown in the different panels. For the whole-layer critical mode in the left side of Fig. 11.1, the nonlinear development shows subcritical bifurcation. $\max u'$ increases with the decrease of bifurcation parameter δ_{sub} up to the values of $\delta_{sub} \sim -0.45$. At this value,

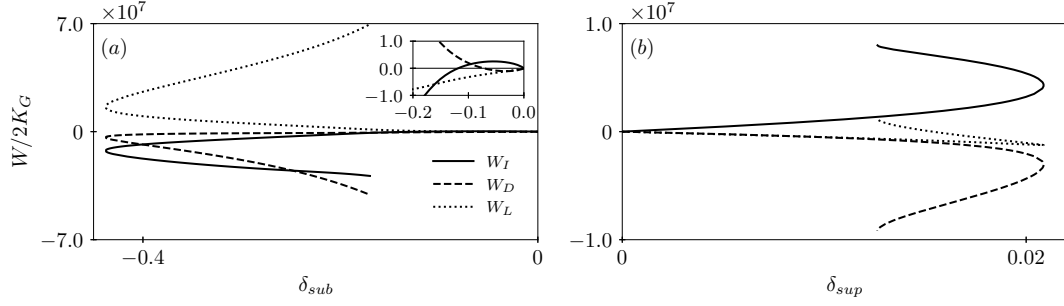


Figure 11.2: Energy transfer from the base to perturbation gas flows of (a) the subcritical and (b) the supercritical bifurcation: W_I (solid line), W_D (dotted line), and W_L (dashed line) represent the powers of the liquid inertia, the drag, and the lift, respectively. The powers are normalized by twice the kinematic energy K_G of gas flow perturbation.

the lower branch also bifurcates to the upper branch. In contrast to the lower branch, $\max u'$ increases with the value of δ_{sub} on the upper branch. For the multi-layer critical mode in the right side of Fig. 11.1, the bifurcation shows supercritical behavior. $\max u'$ increases with the bifurcation parameter δ_{sup} up to the values of $\delta_{sup} \sim 0.02$ where the lower branch bifurcates to the upper branch. On the upper branch, in contrast to, the decrease of δ_{sup} leads to the increase of $\max u'$.

The eigenvectors show that equally sized convection rolls develop at low flow nonlinearity, i.e., at small $\max u'$. Liquid flow in the convection rolls develops when the nonlinearity becomes significant. Sharp ascending plumes and associated high gas concentration regions are observed at large flow nonlinearity, i.e., at large $\max u'$. For the supercritical bifurcation, while convection rolls do not extend to the entire fluid layer as the same with the critical multi-layer mode, liquid flow in the convection rolls in the highest liquid layer becomes stronger than that in the rolls of the critical mode.

11.2.2 Energy budget

Adopting the same procedures presented in Sec. 10.3.2, the energy transfer from the base to liquid and gas perturbation flows is considered in order to understand the nonlinear development of the flows. The evolution equation for the kinetic energy of liquid perturbation flows is given as:

$$\frac{d\mathcal{K}}{dt} = W_B - D_V, \quad (11.11)$$

where the kinetic energy $\mathcal{K}$ are defined by

$$\mathcal{K} = \left\langle \left[1 - (\bar{\varepsilon}_G + \varepsilon'_G) \frac{Ra \cdot r_b^2}{9Pr} \right] \frac{u'^2 + w'^2}{2} \right\rangle. \quad (11.12)$$

Equation (11.11) confirms that convection develops when the buoyancy overcomes the viscous energy dissipation as the same with the results presented in Sec. 10.3.2.

The evolution equation for the gas perturbations flows is also given as:

$$\frac{d\mathcal{K}_G}{dt} = \mathcal{W}_I + \mathcal{W}_D + \mathcal{W}_L, \quad (11.13)$$

where the kinetic energy $\mathcal{K}_G$, the powers of the liquid inertia $\mathcal{W}_I$, of the drag $\mathcal{W}_D$ and of the lift $\mathcal{W}_L$ are defined by,

$$\mathcal{K}_G = \left\langle \frac{u_G'^2 + w_G'^2}{2} \right\rangle, \quad (11.14)$$

$$\begin{aligned} \mathcal{W}_I = & - \left\langle \bar{w}_G \frac{\partial}{\partial z} \frac{\|\mathbf{u}'_G\|^2}{2} + \frac{d\bar{w}_G}{dz} w_G'^2 + (\mathbf{u}'_G \cdot \nabla \mathbf{u}'_G) \cdot \mathbf{u}'_G \right\rangle \\ & + 3 \left\langle \frac{\partial \mathbf{u}'}{\partial t} \cdot \mathbf{u}'_G + (\mathbf{u}' \cdot \nabla \mathbf{u}') \cdot \mathbf{u}'_G \right\rangle, \end{aligned} \quad (11.15)$$

$$\mathcal{W}_D = -\frac{18}{r_b^2} \langle (\mathbf{u}'_G - \mathbf{u}') \cdot \mathbf{u}'_G \rangle, \quad (11.16)$$

$$\mathcal{W}_L = \langle \bar{w}_G \zeta' u'_G + (u' w'_G - w' u'_G) \zeta' \rangle. \quad (11.17)$$

Energy budgets of the gas perturbation flows are computed for the non-linear solutions on the bifurcation diagrams. For the subcritical bifurcation [Fig. 11.2 (a)], the power from the added-mass has destabilizing effects (i.e., $\mathcal{W}_I > 0$), while the drag and the lift have stabilizing effects (i.e., $\mathcal{W}_D < 0, \mathcal{W}_L < 0$) for bifurcation parameters δ_{sub} between -0.1 and 0 . The powers from the added-mass and the lift, in contrast to, become negative and positive with decrease of δ_{sub} while those from the drag remains negative as shown in the inset. For the supercritical bifurcation [Fig. 11.2(b)], the power from the added-mass has destabilizing effects (i.e., $\mathcal{W}_I > 0$), while those from the drag and the lift have stabilizing effects (i.e., $\mathcal{W}_D < 0, \mathcal{W}_L < 0$) at small δ_{sup} . The power of the lift decreases monotonically with increase of δ_{sup} up to $\delta_{sup} \sim 0.02$ where the lower branch bifurcates to the upper branch. $\mathcal{W}_L$ on the bifurcated branch increases with decrease of δ_{sup} . $\mathcal{W}_I$ and $\mathcal{W}_D$ increase and decrease monotonically, regardless of the variations of δ_{sup} .

Chapter 12

Discussion

12.1 Wavenumber selection mechanism

The results on the critical parameters presented in Sec. [10.3](#) are in marked contrast with the observations on the RT-like instability by Climent & Magnaudet [\[20\]](#). These authors observed the formation of whole-layer convection of different aspect ratios ($\Gamma = 1$ and 0.5) for the same set of parameter values $(Pr, Ra) = (2.5 \times 10^{-4}, 2.07 \times 10^5)$ and suggested the absence of wavenumber selection. The present results (Fig. [10.4](#)) show, in contrast, that a unique mode, *either* a multi-layer mode ($\Gamma \leq 1.3$) *or* a whole-layer mode ($\Gamma = 1.5 - 3.1$), is selected depending on Pr . Like the present work, Climent & Magnaudet [\[20\]](#) assumes no hydrodynamic diffusion and is based on the same governing equations as Eqs. [\(10.14\)](#)–[\(10.20\)](#) except for the Lagrangian approach to the bubble dynamics. The absence of wavenumber selection would hence not be related to the absence of hydrodynamic diffusion. Our analysis showed that the selection mechanism exists even in the absence of bubble-bubble interactions. Different convection patterns observed in Climent & Magnaudet [\[20\]](#) would thus indicate the multistability of the bubbly flow system rather than the absence of

wavenumber selection. Different attractors would be present under a given flow condition. A bifurcation analysis and a stability analysis of secondary flows could elucidate this point.

12.2 Instability mechanism at the linear stage

As the energy budget equation (10.48) shows, the instability of bubbly flows found in Sec. 10.3 is provoked by the buoyancy. More buoyant lower fluid overturns less buoyant upper fluid, once the buoyancy overcomes the stabilization by the viscous force. This principle mechanism is similar to that of thermal instabilities. There exists, however, essential differences between these instabilities in the transport of buoyancy sources. In thermal convection, the heat diffusion tends to homogenize temperature fields to produce stabilizing effects. In bubble convection, in contrast, stabilization arises from the transport of bubbles by the lift forces as shown by the energy budget analysis in Sec. ??.

In fact, the lift force $\bar{w}_G \zeta' \mathbf{e}_x$ in the gas momentum equation (10.39) is directed from bubble-rich to bubble-poor cells on the wall [see Fig. 10.4(b), noting that $\zeta' > 0$ and < 0 inside clockwise and counter-clockwise convection rolls, respectively]. Thus, the lift force tends to homogenize the bubble distribution in the bottom layer of fluid where the buoyancy drives convection. Stabilizing effects of lift force are also reported in the literature on bubble columns [123].

12.3 Analogy with thermal convection

The transport of buoyancy sources in bubble convection can give rise to multi-layer convection rolls, even though the mean density increases monotonically in the base state [Fig. 10.2(b)]. Multi-layer convection rolls are observed in double

diffusive systems and have been investigated in geophysical and astrophysical contexts. For example, a variety of convection patterns can be produced in the Earth's mantle depending on the competition between destabilizing thermal density anomaly and stabilizing chemical density anomaly [124]. When the former effect predominates the latter one, whole-layer convection rolls develop. Otherwise, multi-layer convection rolls emerge [125, 126]. In ordinary thermal convection, superposed convection rolls can be observed in particular configurations, e.g., above a localized heat source in a stably stratified fluid layer [127], where the thermal stratification is unstable to gravity only slightly above the heat source. Otherwise, multi-layer structures are rarely formed even for nonlinear temperature profiles due to internal heating [128]. In the present two-phase system, critical multi-layer modes are observed for small Pr . The formation of multi-layer convection could be explained from the perturbation in bubble distribution.

In contrast to the ordinary thermal convection, the cells of buoyancy sources, i.e., the cells of positive ε'_G [see Fig. 10.4(b)] attached on the bottom wall extend only up to half the height of convection rolls. Above each of these cells, there is a cell of negative buoyancy sources, i.e., cells of negative ε'_G , in which the upward motion of convective flow is impeded by a net downward force resulting from a small number of bubbles. This implies a stabilizing effect at the upper part of convection rolls as in double diffusive systems exhibiting multi-layer convection rolls.

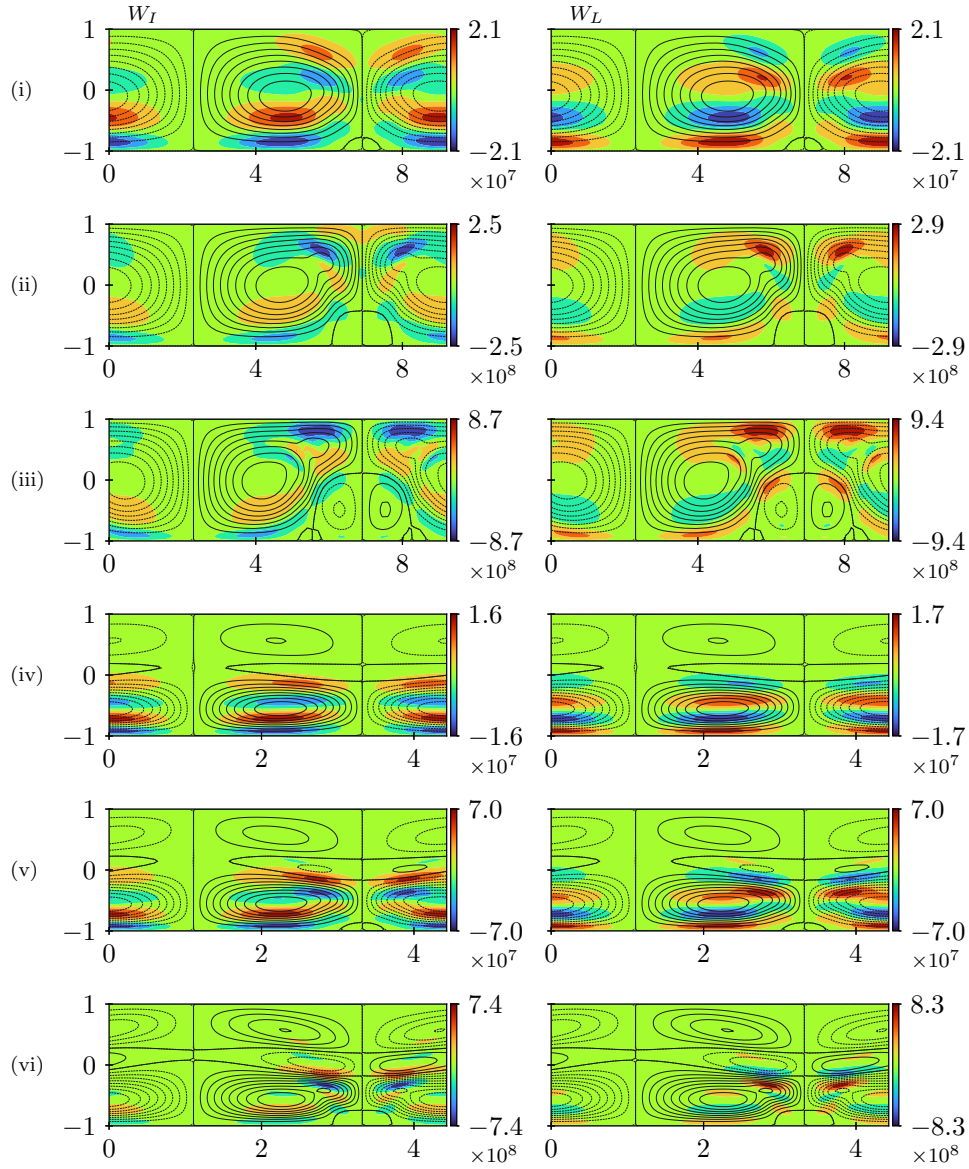


Figure 12.1: Local powers providing energy from the base to perturbation gas flows at the nonlinear stage: different panels show results for the nonlinear flows [Fig. 11.1(i)-(vi)]. Left and right columns represent the local powers of the inertial and the lift, respectively. Some streamlines are indicated to visualize the structures of liquid flows.

12.4 Instability mechanism at the nonlinear stage

Energy budget analysis for the linear stage presented in Sec. 10.3.2 shows that the instability of bubbly flows is provoked by the buoyancy and the stabilization arises from the transport of bubbles by the lift forces. Destabilizing effects of the lift are also observed when the nonlinearity is significant. This variations of the powers of the lift are in marked contrast with the stabilizing effects in thermal convection, i.e., the heat diffusion homogenizing temperature fields has stabilizing effects regardless of flow nonlinearities. Local powers providing the energy are computed for the eigenvectors [Fig. 11.1(i)-(vi)] to understand the contributions of the lift in the nonlinear stage. The powers of the $\mathcal{W}_I$ and $\mathcal{W}_L$ exhibits different distributions coupled with liquid flows [Fig. 12.1]. At low flow nonlinearity in the subcritical [Fig. 12.1(i)] and supercritical bifurcation [Fig. 12.1(iv)(v)], the local power of the lift produces destabilizing and stabilizing effects in convection rolls. The maximum and the minimum values of the power are observed close to the bottom wall where the buoyancy drives the convection. In these convection rolls, however, the lift globally has stabilizing effects as shown by the energy budget analysis in Sec. 11.2.2. As the flow develops due to nonlinearity in subcritical [Fig. 12.1(ii)(iii)] and supercritical bifurcations [Fig. 12.1(vi)], the local power of the lift produces strong destabilizing effect in the highest and the middle liquid layers, respectively. The lift transports bubbles in bubble-rich cells to the same direction of liquid convective flows, resulting in the enhancement of the flows. In fact, the formation of gas plume with high gas volume fraction is found when the nonlinearity becomes significant in the flow [Fig. 11.1(ii),(iii),(vi)].

Part IV

Conclusion

Revisiting the experimental study of flow transition in a circular pipe by O. Reynolds [4], impact of dilute dispersed microbubbles on puff creation has been examined by experiments and numerical simulations. Localized turbulent puffs were created at fixed $Re = 1,900$ by injecting a small amount of fluids into the laminar base flow as the same perturbation system of the previous works for flow transition of single-phase fluids in the pipe (e.g., [6]). Streamwise velocity profiles on the pipe axis measured by LDV were utilized to observe puff creation. The profiles of puffs are in good qualitative agreement with results of previous studies.

Probability of observation of puffs were measured based on a trend of the profiles in the pipe flow of single-phase and dilute bubbly fluids with the volume fraction smaller than 0.018%. For measuring the probability quantitatively, methods to identify flow regimes in the transitional pipe flows were proposed based on a velocity ratio and a correlation coefficient. These results indicate that the probability increases in the flows with dilute microbubble dispersion and would imply modulation on puff creation due to the preferential concentration.

The increase implies the discrepancy with respect to relaminarization in horizontal and vertical pipe flows of milky bubbly fluids [16, 17, 18]. This discrepancy would be explained from the effective viscosity, modification of inflection points, or the competition of length scales between bubble-bubble distance and wavelength of perturbations.

In dilute limit, modulation on flow transition has been observed due to the preferential concentration of bubbles in vortical structures. Liquid velocity fields were measured by SPIV system to examine the modulation in the two-phase system. Fluorescent particles were dispersed in the flows to visualize both phases. Bubble distributions in the puff were visualized based on RGB

color values. Streamwise velocity profiles on the pipe axis measured by SPIV show that the velocity defect coupled with the flow elements in a puff increases with the bubble dispersions. Streamwise vorticity distributions, in fact, shrink in radial direction. Voronoi diagram obtained from projected bubble positions in experiments and numerical simulations shows preferential concentration of bubbles in a turbulent puff.

Revisiting the problem of homogeneous-heterogeneous regime transition, the wavenumber selection mechanism in the linear stage of transition is examined by performing a linear stability analysis for ascending bubbles in a quiescent liquid layer. Marginal stability curves $Ra = Ra(k)$ were determined for different values of Pr . While the curves have single minima for $Pr > 4.13 \times 10^{-5}$, there exist multiple minima otherwise. The marginal modes are stationary except for limited ranges of wavenumber at small Pr . Critical conditions (k_c, Ra_c) for $Pr < Pr^* = 3.25 \times 10^{-5}$ are distinct from those for $Pr > Pr^*$. In the former and latter cases, the critical modes consist of whole-layer and multilayer convection rolls, respectively. At a condimension two point $(Pr, Ra) = (Pr^*, Ra^*)$, both multi- and whole-layer modes are critical.

The energy transfer from the base to perturbation flows has been examined in order to understand the instability mechanism. Liquid perturbation flows gain and lose energy due to buoyancy and viscous dissipation, respectively. The driving buoyancy is created by a heterogeneous distribution of bubbles. For critical modes, gas perturbation flows, which generate this non-equilibrated bubble distribution, gain energy from the inertial forces on bubbles. The lift forces on bubbles tend, in contrast, to damp the gas flows through homogenization of the bubble distribution close to the wall. The drag forces are also found to have stabilizing effects.

These results show that the mechanism of wavenumber selection exists

even in the absence of bubble-bubble interactions. Different convection patterns observed in CM99 [20] would indicate the multistability of the bubbly flow system. A nonlinear bifurcation analysis could elucidate the presence of different attractors and their stability.

Nonlinear development from the linear instability has been investigated by performing a bifurcation analysis. The development are found subcritical and supercritical depending on the critical modes, whole- or multi-layer convection rolls. Liquid flow in the convection rolls develops when the nonlinearity becomes significant. Sharp ascending plumes and associated high gas concentration regions are observed at large flow nonlinearity. Examining energy budgets of the gas perturbation flows in the nonlinear stages, the results show that the power of the lift changes when the nonlinearity is significant. Preferential concentration of bubbles in a puff creates inhomogeneous buoyancy distributions in the flow.

The buoyancy distributions induce convective motion in the flows, resulting in the competition of length scale between vortical structure and bubble-induced convection. As presented in Sec.??, the formation of vortical structures embedded in transitional flows are observed in subcritical and supercritical bifurcations, e.g., TS waves, lambda vortices, Bénard cell, Taylor vortex, and turbulent puffs. The flow transition scenarios, thus, would be modulated due to mutual interactions between the vortical structures and dilute small bubbles. The alternation in dilute bubbly flows would be elucidated based on the presented approaches in this thesis.

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