Supplemental Material for
"Experimental Evidence of Cubic Rashba Effect in an Inversion-Symmetric Oxide"

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In this Supplemental Material, we present details of the localization/antilocalization analysis using the Iordanskii, Lyanda-Geller, Pikus (ILP) theory. The general formula used for a theoretical fit is as follows [S1,S2].

$$\begin{aligned}\Delta\sigma(B) = -\frac{e^2}{4\pi^2\hbar}\Bigg[&\frac{1}{a_0} + \frac{2a_0 + 1 + \dfrac{B_{\mathrm{SO1}} + B_{\mathrm{SO3}}}{B}}{a_1\left(a_0 + \dfrac{B_{\mathrm{SO1}} + B_{\mathrm{SO3}}}{B}\right) - 2\dfrac{B_{\mathrm{SO1}}}{B}} \\ &- \sum_{n=0}^{\infty}\left\{\frac{3}{n} - \frac{3a_n^2 + 2a_n\dfrac{B_{\mathrm{SO1}} + B_{\mathrm{SO3}}}{B} - 1 - 2(2n+1)\dfrac{B_{\mathrm{SO1}}}{B}}{\left(a_n + \dfrac{B_{\mathrm{SO1}} + B_{\mathrm{SO3}}}{B}\right)a_{n-1}a_{n+1} - 2\dfrac{B_{\mathrm{SO1}}}{B}[(2n+1)a_n - 1]}\right\} \\ &+ \Psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right)\Bigg],\end{aligned} \tag{S1}$$

where

$$\begin{aligned}&a_n = n + \frac{1}{2} + \frac{B_\phi}{B} + \frac{B_{\mathrm{SO1}} + B_{\mathrm{SO3}}}{B}, \quad B_\phi = \frac{\hbar}{4eD\tau_\phi}, \\ &B_{\mathrm{SO1}} = \frac{\hbar}{4eD}\cdot|\mathbf{\Omega}_1|^2\tau_1, \quad B_{\mathrm{SO3}} = \frac{\hbar}{4eD}\cdot|\mathbf{\Omega}_3|^2\tau_3, \quad \frac{1}{\tau_{\mathrm{m}}} = \int(1-\cos\mathrm{m}\theta)W(\theta)\mathrm{d}\theta,\end{aligned} \tag{S2}$$

Ψ is the digamma-function, D is the diffusion constant, τ_ϕ is the phase relaxation time, the index $\mathrm{m} = 1$ or 3, and $W(\theta)$ is the probability of scattering by an angle θ. Note that the terms in the original ILP theory which merely shift the overall curve in the y-direction is omitted. To calculate Eq. S1, the infinite sum was added up to $n = 2\times10^5$.

When $B_{\mathrm{SO1}} = 0$ ($|\mathbf{\Omega}_1| = 0$), Eq. S1 has the following closed form [S2]:

$$\begin{aligned}\Delta\sigma(B) = \frac{e^2}{2\pi^2\hbar}\Bigg\{&\Psi\left(\frac{1}{2} + \frac{B_\phi}{B} + \frac{B_{\mathrm{SO3}}}{B}\right) + \frac{1}{2}\Psi\left(\frac{1}{2} + \frac{B_\phi}{B} + 2\frac{B_{\mathrm{SO3}}}{B}\right) \\ &- \frac{1}{2}\Psi\left(\frac{1}{2} + \frac{B_\phi}{B}\right) - \ln\frac{B_\phi + B_{\mathrm{SO3}}}{B} - \frac{1}{2}\ln\frac{B_\phi + 2B_{\mathrm{SO3}}}{B} + \frac{1}{2}\ln\frac{B_\phi}{B}\Bigg\}.\end{aligned} \tag{S3}$$

In the main text, fitting was performed for the condition $|\mathbf{\Omega}_1| = 0$ or $|\mathbf{\Omega}_3| = 0$, leaving two free parameters B_ϕ and B_{SO} (B_{SO1} or B_{SO3}). Here, we analyze our data retaining all three fitting parameters (B_ϕ, B_{SO1}, and B_{SO3}). For this purpose, we first define the ratio $r = B_{\mathrm{SO1}}/B_{\mathrm{SO3}}$. Second, for several values of r which is set between 0 (*i.e.*, $|\mathbf{\Omega}_1| = 0$) and infinity ($|\mathbf{\Omega}_3| = 0$), parameters are adjusted to fit the data around $B = 0$. The result as well as the parameters used for the fitting are shown in Fig. S1 and Table SI, respectively.

TABLE SI: Parameters used for the fitting shown in Fig. S1

$r = B_{SO1}/B_{SO3}$	B_ϕ (mT)	B_{SO1} (mT)	B_{SO3} (mT)
0	13	0	34
0.2	13	6.6	33
0.5	13	15.5	31
1	13	27.5	27.5
2	13	43	21.5
∞	13	78	0

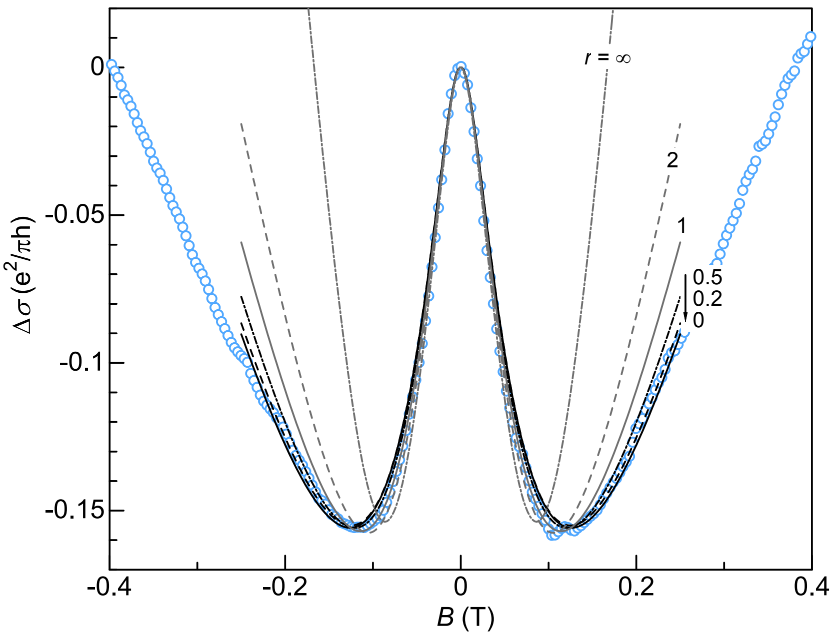

FIG. S1: Theoretical fits (curves) to experimental data $\Delta\sigma$ (open circles) of STO-B at $E_G = 4.38$ MV/cm given by the ILP theory. Several values of $r = \dfrac{B_{SO1}}{B_{SO3}}$ are considered. The theoretical curves approach the experimental data as $r \to 0$, *i.e.*, as $|\mathbf{\Omega}_1| \to 0$.

Next, we show the results of changing B_ϕ for each value of r with fixed B_{SO1} and B_{SO3} (Figs. S2-S6). In these fittings, parameters are adjusted so that the magnetic field at the minimum of theoretical $\Delta\sigma$ curves coincides with that of experimental data.

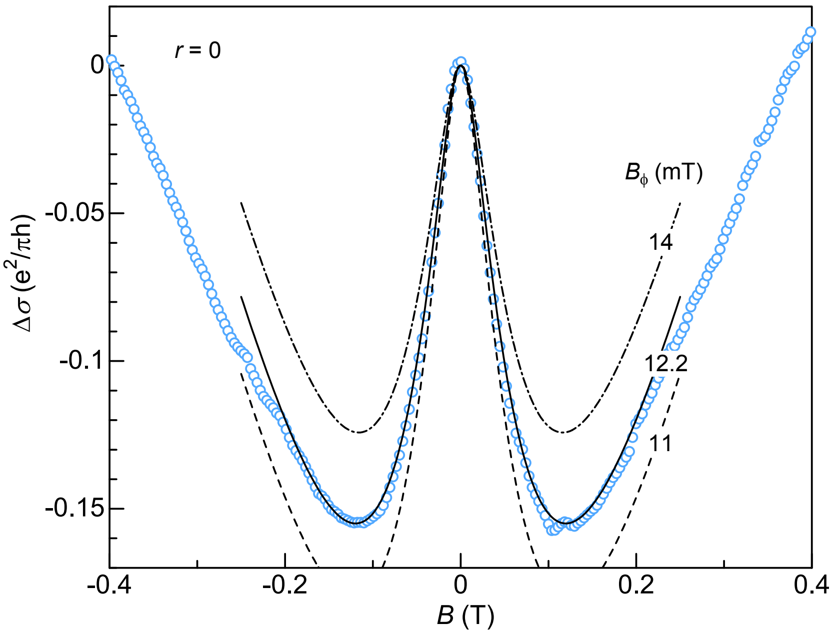

FIG. S2: The result of changing B_ϕ for $r = 0$ with the following fixed parameters: B_{SO1}=0 mT, B_{SO3}=32.1 mT.

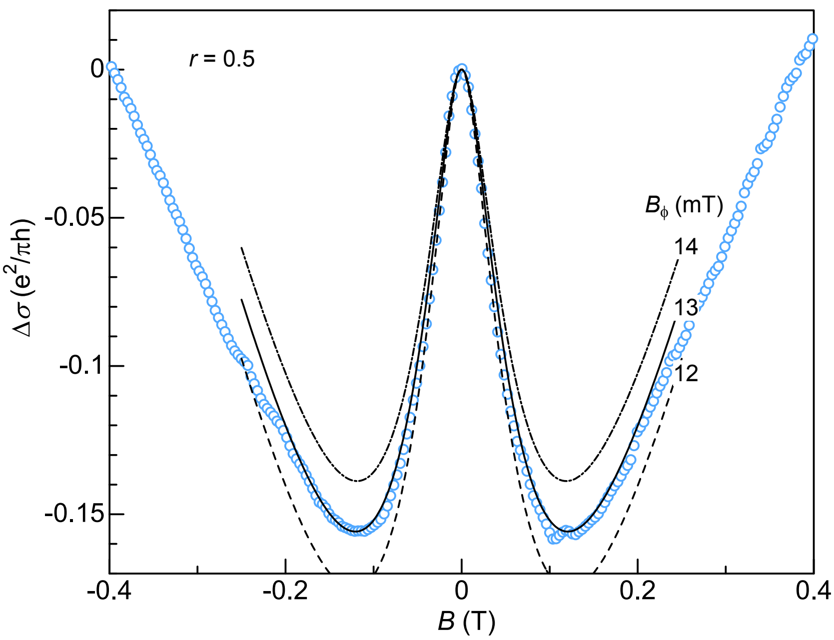

FIG. S3: The result of changing B_ϕ for $r = 0.5$ with the following fixed parameters: B_{SO1}=15.5 mT, B_{SO3}=31 mT.

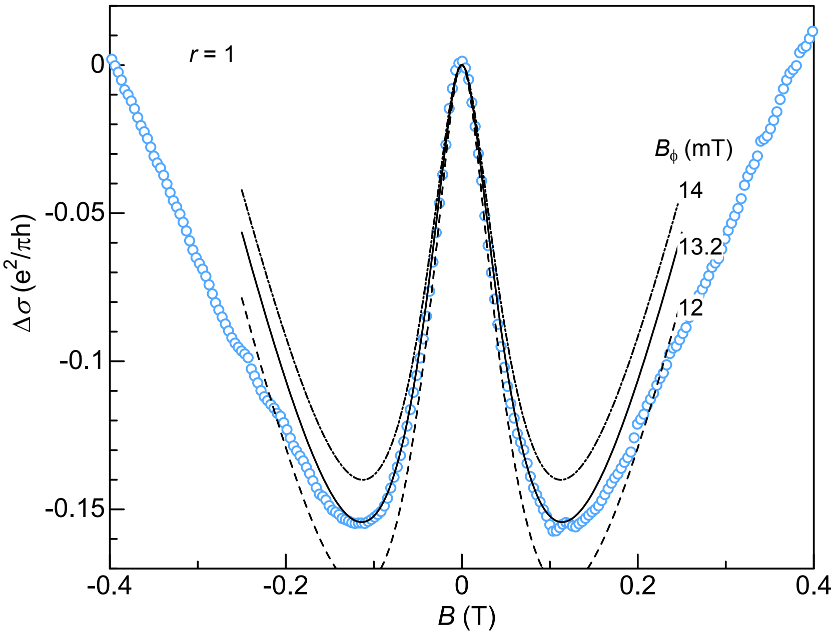

FIG. S4: The result of changing B_ϕ for $r = 1$ with the following fixed parameters: $B_{\rm SO1}$=27.5 mT, $B_{\rm SO3}$=27.5 mT.

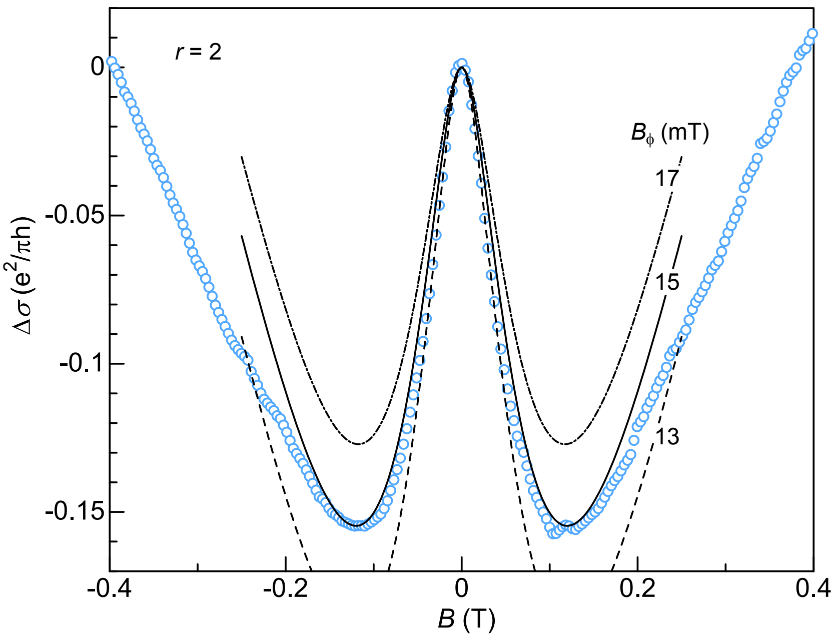

FIG. S5: The result of changing B_ϕ for $r = 2$ with the following fixed parameters: $B_{\rm SO1}$=49 mT, $B_{\rm SO3}$=24.5 mT.

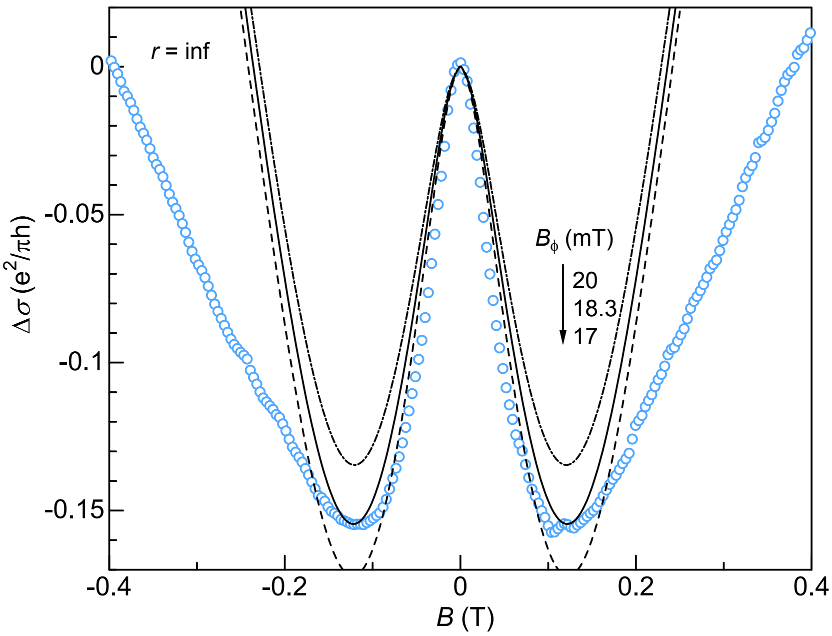

FIG. S6: The result of changing B_ϕ for $r = \infty$ with the following fixed parameters: B_{SO1}=110 mT, B_{SO3}=0 mT.

[S1] S. V. Iordanskii, Y. B. Lyanda-Geller, and G. E. Pikus, JETP Lett. **60**, 206 (1994).
[S2] W. Knap, *et al.*, Phys. Rev. B **53**, 3912 (1996).