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Study on thermal convection by advancement of color-based optical measurements

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ABSTRACT

Study on thermal convection has been continued more than a century since Henri Bénard's systematic experiments. Thermal convection has attracted worldwide interests not only as a key phenomenon in the field of thermo-fluid physics, but also as a large-scale fluid motion in the fields of geo- and astro-physics. Further, thermal convection is a significant platform when considering a way of prediction and control of a heat transport system from the perspective of engineering. Since the driving force of fluid flows originates in thermal buoyancy for relaxing density inhomogeneity of the fluid, temperature field measurements are sometimes required in addition to velocity field measurements for comprehensive understandings, even though temperature field measurements require much effort than velocity field measurements. Flow field measurements, such as particle image velocimetry (PIV) and particle tracking velocimetry (PTV), have been developed as monochrome digital image processing techniques, as velocity fields can be easily measured by tracking particle displacements across different image frames. Meanwhile, rich color information which can be acquired by widespread color cameras has not been fully utilized in flow measurements to date. Addition of color information has the potential to improve the measurement ability of optical measurement techniques by keeping the optical system simple. Thus, this thesis aims to develop color-based optical measurements which append a function to measure physical quantities, such as temperature, in addition to velocity fields for the purpose of comprehensive understandings of various thermal convection phenomena. This thesis comprises four parts involving the introduction (Part I) and the summary (Part IV).

In Part II, four different color-based optical measurement techniques are established. Chapter 1, 2, and 3 demonstrate ways of utilization of color information as out-of-plane physical coordinates. Color-based optical measurements are systematized as two different problem settings; classification problem and regression problem. The classification problem treats color information as thresholds for separating distinct differences identifiable on color images; the use of two differently colored laser sheets with different illuminating planes enables to separate a color image into two different planes only from the color information. In Chapter 1, this color image separation technique is utilized to simultaneously measure convective motions in two immiscible fluid layers. With the simultaneous velocity field measurements, coupling structures of convection cells formed in each layer can be identified as two-dimensional interfacial interactions. Chapter 2 demonstrates a possibility to measure out-of-plane velocity

component only by a single camera and two different laser sheets. The method named “dual-plane ensemble correlation” and color image separation enable pixelwise out-of-plane velocity field measurements with such a simple optical system. The regression problem considers color information as raw data representing certain physical quantities. Color PTV method established in Chapter 3 utilizes continuous rainbow color changes as continuous out-of-plane coordinate changes. With a combination of artificial neural network (ANN), originally recorded ambiguous color information can be calibrated to out-of-plane coordinates as a multi-regression problem without unreasonable assumptions. Chapter 4 proposes a novel technique that can identify fluid deformation by coloration originated in birefringence. The results suggest a new insight leading to direct and pixelwise fluid deformation measurements only by utilizing color information.

In Part III, multi-timescale physics of various thermal convection phenomena are studied utilizing thermochromic liquid crystals (TLC). Efficacy of the TLC particles is carefully studied and subsequent special image processing schemes are provided in Chapter 1. The TLC particles can be utilized not only for temperature field visualization but also for velocity field measurements in thermal convection studies thanks to their high traceability and high dispersibility. In Chapter 2, developing stages of horizontal convection (HC) are examined through long-time laboratory experiments. It is found that the degree of background stable temperature stratification plays an important role in determining roll structures formed during transient evolution from conduction to thermally-equilibrated states. A novel nondimensional parameter representing balance of braking effect by the stable temperature stratification and driving effect by the baroclinic torque describes well the temporal evolution of the developing HC. Transient states and equilibrated states of rotating Rayleigh–Bénard convection (RBC) are discussed deeply in Chapter 3 and 4, respectively. On the transient states during spin-up of rotating RBC studied in Chapter 3, it is revealed that break down of thermally coherent structures, toroidal roll formations, azimuthally aligned vortex formations, and plume emissions forming columnar vortices appear in order. Mechanisms of these stepwise transitions are studied in detail. Such beautiful stepwise transitions are found to be realized only when multiple timescales, e.g., Ekman spin-up and viscous timescales, are balanced in a certain manner. Dynamics of columnar vortices formed in equilibrated rotating RBC are studied in Chapter 4. In order to measure local and instantaneous heat transport carried by a single columnar vortex, simultaneous three-dimensional velocity and temperature measurement systems are constructed. Stereoscopic PIV and coloration of the TLC particles enable such complicated measurements by employing two color cameras, a light source, and a scanning system. Horizontal motions of the columnar vortices are quantified as diffusive motions with inelastic collisions, and its motion seems to be motivated by thermal forcing. Relationship between the heat transport characteristics and the dynamics of a single columnar vortex is elucidated by statistical analysis. The results suggest the source of random-walk-like horizontal diffusive motions of the columnar vortices, which has been an open issue in past studies.

DECLARATION

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where states otherwise by reference or acknowledgment, the work presented is entirely my own.

Daisuke Noto
August 24, 2021

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- Daisuke Noto and Yuji Tasaka, “Dual-plane ensemble correlation for pixelwise 2D-3C velocity field measurements using a single camera”, *Experiments in Fluids*, Vol. 62, No. 111 (2021), DOI: <https://doi.org/10.1007/s00348-021-03208-5>.
- Daisuke Noto, Yuji Tasaka, and Yuichi Murai, “Coupling structures of two-layer natural convection in a cylindrical vessel investigated by simultaneous two-layer visualization”, *Experimental Thermal and Fluid Science*, Vol. 127, 110394 (2021), DOI: <https://doi.org/10.1016/j.expthermflusci.2021.110394>.
- Daisuke Noto, Yuji Tasaka, Takatoshi Yanagisawa, and Takehiro Miyagoshi, “Stepwise transitions in spin-up of rotating Rayleigh–Bénard convection”, *Journal of Fluid Mechanics*, Vol. 911, A43 (2021), DOI: <https://doi.org/10.1017/jfm.2020.1052>.
- Daisuke Noto, Yuji Tasaka, Jumpei Hitomi, and Yuichi Murai, “Applicability evaluation of the stress-optic law in Newtonian fluids toward stress field measurements”, *Physical Review Research*, Vol. 2, p043111 (2020), DOI: <https://doi.org/10.1103/PhysRevResearch.2.043111>.
- Daisuke Noto, Yuji Tasaka, Takatoshi Yanagisawa, Takehiro Miyagoshi, and Yuichi Murai, “Vortical structure formed during spin-up of thermal convection”, *Nagare (Japan Society of Fluid Mechanics, in Japanese)*, Vol. 38, pp. 427–430 (2019).
- Daisuke Noto, Yuji Tasaka, Takatoshi Yanagisawa, and Yuichi Murai, “Horizontal diffusive motion of columnar vortices in rotating Rayleigh–Bénard convection”, *Journal of Fluid Mechanics*, Vol. 871, pp. 401–426 (2019), DOI: <https://doi.org/10.1017/jfm.2019.313>.

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- Sten Anders, Daisuke Noto, Yuji Tasaka, and Sven Eckert, “Simultaneous optical measurement of temperature and velocity fields in solidifying liquids”, *Experiments in Fluids*, Vol. 61, Np. 113 (2020), DOI: <https://doi.org/10.1007/s00348-020-2939-3>.
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- Daisuke Noto, Yuji Tasaka, Takatoshi Yanagisawa, Hyun Jin Park, and Yuichi Murai, “Vortex tracking on visualized temperature fields in a rotating Rayleigh–Bénard convection”, *Journal of Visualization*, Vol. 21, pp. 987–998 (2018), DOI: <https://doi.org/10.1007/s12650-018-0510-6>.

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Part I

Introduction

PREFACE

Great efforts have been dedicated all over the world to quantitatively measure motions of various fluid flows via optical visualization. That is, people have tried to quantify the fluid flows as they observe with the naked eyes. This may be the most straightforward approach to measure and understand fluid dynamics. Optical approach for flow visualization can be found earlier in some artistic works, e.g. turbulent vortex structures drawn precisely by Leonard da Vinci [Fig. 1.1(a)] and oceanic waves illustrated by a Japanese woodcut artist Katsushika Hokusai [Fig. 1.1(b)]. These are not only fantabulous as artistic works, but also excellent as flow visualizations from a perspective of fluid dynamics. Needless to say, these works were drawn based on the observations by the naked eyes of the artists. Detailed vortical structures drawn in Leonard da Vinci's work were proved to resemble well with recent experimental work introduced by Kemp [87]. Also, the multi-fractal structures of waves were compared with recent observations by Dudley et al. [44]. From these facts, it is possible to deduce that the two artists possessed outstanding observation skills (or eyes) yielding to capture high-speed fluid motions. It is, however, worth respecting their careful observation skills, and such attitudes facing seriously to fluid phenomena are essential even in modern times with high specification optics.

Film cameras made fluid flow visualization easier, irrespective of the observation skills through the naked eyes of experimentalists. The use of camera had yielded to freeze fluid motions and various flow quantities, such as velocity, density, and temperature, became possible to be qualitatively (or sometimes quantitatively) captured. Early efforts to capture beautiful and informative visualization can be found in a famous album assembled by Van Dyke [195]. For instance, dye visualization and seeding particles with long exposure photographing clearly identified streamlines of various liquid flows around bluff bodies. Numerous flow features were revealed through experimental works assembled in Van Dyke [195], and these are still regarded as textbook-like photographs in flow visualization even though most of these only provided qualitative information. Such qualitative works can leave a lot of space to be explored by the imaginations of experimentalists. These impressive works might require lots of trials and errors

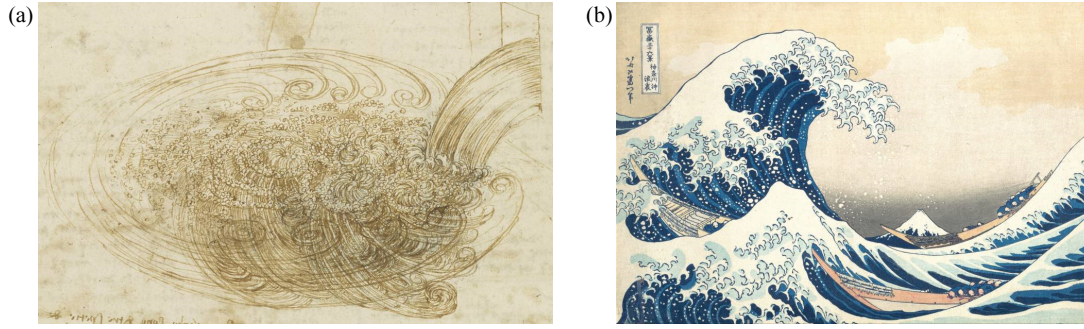


Figure 1.1: Works drawing fine flow structures: (a) “*Studies of Turbulent Water*” from Leonardo da Vinci’s Codex Leicester (Royal Collection Trust / © Her Majesty Queen Elizabeth II 2021) and (b) “*Great Wave Off Kanagawa*” depicted as a woodcut art by Katsushika Hokusai (a public domain from the Metropolitan Museum of Art).

in preparation and photographing stages, and in this sense, they can be called artists as well.

Ever since the commercialization of digital cameras, the optical measurements of fluid flows have been advanced a lot. With digitally recorded images, experimentalists became possible to measure various fluid quantities through data processing. The visualization (photographing) of fluid flows became more systematic compared to those using film cameras, because transient flow structures can be captured as temporally consecutive images. In other words, everybody can visualize fluid flows without special cares in experiments as arbitrary post-processing can be performed after image acquisitions. One of the techniques which have shown the biggest advancement is particle image velocimetry (PIV). PIV measures displacements of particles seeded in the fluids between consecutive image frames. A number of algorithms to measure particle velocity have been introduced since the principle of PIV was provided in 1980s. Increase of the number of cameras has extended measurable velocity components and dimension. Stereoscopic PIV allow measuring two-dimensional (2D) three-component (3C) velocity fields adopting two cameras, and this method was earlier reviewed by Prasad [149] and references therein. Tomographic PIV, which was firstly introduced by Elsinga et al. [46], enabled to measure three-dimensional (3D) 3C velocity fields adopting four or more cameras. Even now the development of PIV is active throughout the world toward precise measurements of velocity fields and additional quantities measurable from the velocity information, i.e. recent efforts for PIV have been dedicated for improving accuracy and dynamic range of velocity measurements. Nowadays, it is possible to consider the development of PIV has entered the period of maturity.

One of the main objectives of this study is to reveal fluid dynamics of various thermal convection systems via conventional and advanced optical visualization methods. Thermal convection is one of the most important fluid phenomena which can be seen in various applications in the fields of fluid engineering, geophysics, astrophysics, and so on. Another objective is to develop various approaches for fluid flow measurements, which had not been proposed so far, through optical visualizations in experiments. By utilizing additional information, e.g. color information, recorded on visualized digital images apart from particle displacements required for quantifying velocity fields, more sophisticated flow measurement techniques will be developed.

THERMAL CONVECTION

Natural convection is one of the most important fluid motions, which is induced spontaneously to ease density contrast by buoyant force in addition to diffusion. In particular, thermal convection is a kind of convective motions driven by density inhomogeneity produced by temperature contrast within the fluid, as most fluids change their density with temperature variations, i.e. hot fluid is lighter than cold fluid and vice versa. History of systematic investigations on thermal convection can be retroactive to 1900, Henri Bénard found regularities of flow patterns inside thin paraffin or whale oil layers heated from below [16]. John William Strutt (the third Baron Rayleigh) provided theoretical explanation to this phenomenon in 1916 [159], and presently ongoing systematic study on thermal convection begun since this. After more than a century of worldwide research history, even now thermal convection has attracted a variety of research interests from fluid dynamics, geo- and astrophysics, and even from engineering applications.

Fluid dynamics — Thermal convection has been studied from a perspective of fundamental physics since the earliest works [16, 159]. This is because thermal convection is a common issue in various thermo-fluid dynamics far from equilibrium entailing instabilities, spatio-temporal evolution, chaos, and turbulence. An extensive review book [117] provided recent insights of thermal convection associating with the works by Bénard.

Flow patterns observed in various thermal convection systems are shown in Fig. 2.1. In Bénard–Marangoni convection imposing uniform heating from below under a free surface condition, balance of surface tension and buoyancy induces hexagonal cell patterns, the so-called Bénard cells, in a shallow fluid layer as shown in Fig. 2.1(a) [94]. By introducing horizontal plate at the top of the fluid layer, the surface tension is no longer dominant and the system is termed Rayleigh–Bénard convection. In such vertically symmetric system, circular convective rolls can form as shown in Fig. 2.1 [94], and other various regular patterns like rectangular and hexagonal cells are suggested in other works [27, 102, 171]. With increasing vertical temperature difference, more complicated flow structures form. Thermal boundary layers are separated to

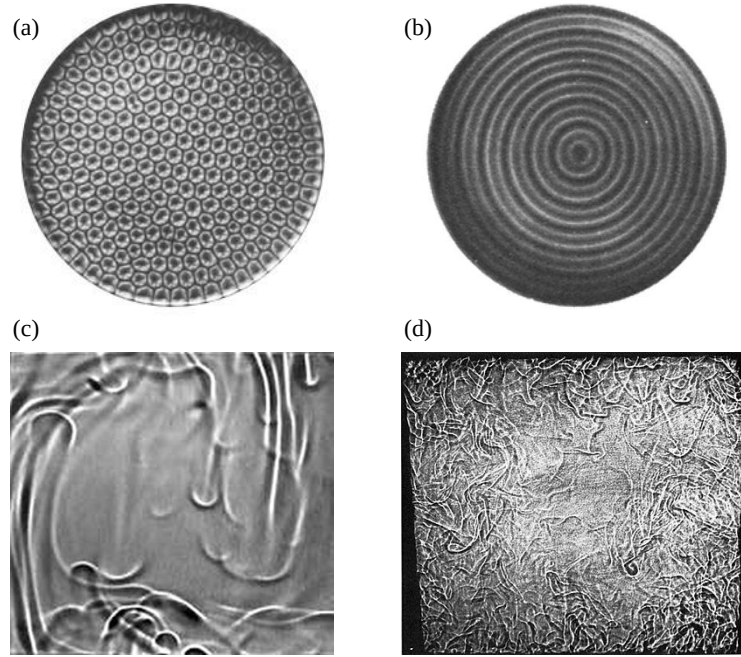


Figure 2.1: Various flow patterns in thermal convection: (a) Bénard cells in Bénard–Marangoni convection [94], (b) convective rolls in Rayleigh–Bénard convection [94], (c) plume emissions forming a large-scale circulation [212], and (d) thermal turbulence [222]. (a) and (b) are top views and (c) and (d) are side views.

form thermal plumes and global flows clustered by the plumes are formed in a certain preferred direction as shown in Fig. 2.1(c). This global trend is termed large-scale circulation. Further increase in vertical temperature difference yields fine plume emissions as shown in Fig. 2.1(d), and the flows are regarded as highly turbulent and called thermal turbulence.

As above, pattern formations alone are complicated and fascinating, thermal convection has been a research target in the fundamental physics of fluids.

Geo- and astrophysics — One of the largest motivations to investigate the physics of thermal convection is applications to the large-scale fluid motions in geophysical and astrophysical fluid dynamics. Various fluid motions based on thermal convection can be found even on the Earth, for instance, oceanic and atmospheric flows, dynamics of mantle convection, and convection in the outer core are often referred to as the applications. In considering thermal convection on the Earth, additional complexities are essential to take into account in addition to thermal forcing. The most representative complexity is rotation, since the large-scale fluid motions can be influenced by the Earth’s rotation due to their long timescale physics. The intuitive example of thermal convection influenced by rotation is typhoon in the atmosphere [Fig. 2.2(a)]. Rotation provides the Coriolis force and the centrifugal force, and these basically suppress thermal convection due to its two dimensionality. In addition, roughness and non-uniformity of heating and cooling sources greatly increase the complexities of thermal convection. As for the convection in the outer core, magnetic force (Lorentz force) affects motions of the liquid metals, and the research field is shifted to the magnetohydrodynamics (MHD). This MHD convection

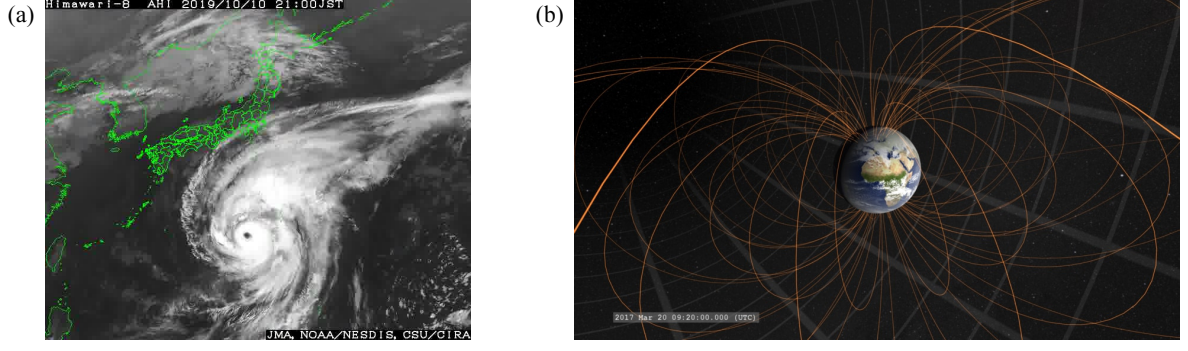


Figure 2.2: Convection-related phenomena in geophysics: (a) The 19th typhoon in 2019 approaching to Japan observed by the artificial satellite Himawari (by courtesy of JMA, NOAA/NESDIS, and CSU/CIRA) and (b) Earth's magnetosphere sustained by the dynamo effect of the outer core (by courtesy of NASA's Scientific Visualization Studio).

is a key factor to understand the dynamo effect of the Earth as well as the effect of rotation [Fig. 2.2(b)]. Planetary dynamo is an active research topic as summarized in an extensive review [80, 92]. A recent review paper also provides a summary of convective motions observed in the sun [169].

Fluid engineering — Thermal convection has also been investigated as one of the heat transport systems, not as the fluid dynamics itself. In many cooling and heating systems in industrial environments, e.g. heat exchanger and heat sink, the heat transport is facilitated by external forces like fans and pumps, and these are called forced convection. Apart from such forced convection, natural convection can be found in various industrial environments. For instance, convection in storage tanks of liquefied natural gas is frequently argued topic from the perspective of safety energy management (this will be detailed in § 1 of Part. II). Convective motions inside melting liquid metals are inevitable phenomena in considering continuous metal/alloy casting procedures. Needless to say, proper control of thermal convection is required to maintain the quality of such products.

As above, thermal convection is a universal topic in various research fields, and deep understandings for each specified condition are essential to control and predict fluid motions induced by thermal convection. Governing equations for thermal convection are, in general, formulated considering the Boussinesq fluids. That is, the density variation of the fluid only by the thermal expansion is considered. Under this condition, the continuity equation, the momentum (Navier–Stokes) equation, and the energy equation are given respectively as

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1)$$

$$\rho_0 \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \rho_0 \nu \nabla^2 \mathbf{u} - \rho(T) g \hat{\mathbf{z}}, \quad (2.2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T. \quad (2.3)$$

Here, ρ_0 is the density at a reference temperature T_0 . ν , κ , and g are the kinematic viscosity and the thermal diffusivity of the fluid, and the gravity acceleration, respectively. The temperature dependence of the density is approximated as

$$\rho(T) \simeq \rho_0 [1 - \beta (T - T_0)], \quad (2.4)$$

where β is the thermal expansion coefficient of the fluid. Without flow, the fluid will be in the hydrostatic equilibrium state

$$\nabla p_H = \frac{dp}{dz} = -\rho_0 g. \quad (2.5)$$

By introducing a modified pressure as the deviation from the hydrostatic pressure as $p^* = p - p_H$, one obtains

$$\rho_0 \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p^* + \rho_0 \nu \nabla^2 \mathbf{u} - \rho_0 \beta (T - T_0) g \hat{\mathbf{z}}. \quad (2.6)$$

Dividing by the reference density ρ_0 , the momentum equation can be rewritten as

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_0} \nabla p^* + \nu \nabla^2 \mathbf{u} - \beta (T - T_0) g \hat{\mathbf{z}}. \quad (2.7)$$

For nondimensionalization of the governing equations by following the method by Drazin and Reid [42], let θ be the temperature deviation (perturbation temperature) from the thermal conduction profile, one gets

$$T = T_0 - \frac{\Delta T}{L} + \theta, \quad (2.8)$$

where ΔT is the vertical temperature difference across the full depth of the fluid layer L . Adopting L , L^2/κ and ΔT as scales for length, time, and temperature, one obtains nondimensional equations as

$$\nabla \cdot \mathbf{u} = 0 \quad (2.9)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + Pr \nabla^2 \mathbf{u} + Pr Ra \theta \hat{\mathbf{z}}, \quad (2.10)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta = w + \nabla^2 \theta. \quad (2.11)$$

Here all the variables are nondimensionalized. The Rayleigh number Ra and Prandtl number Pr are the dimensionless parameters, which can be written in terms of the fluid properties as

$$Ra = \frac{g \beta \Delta T L^3}{\kappa \nu}, \quad (2.12)$$

and

$$Pr = \frac{\nu}{\kappa}. \quad (2.13)$$

Thermal convection can be thus characterized by the two dimensionless parameters and specific boundary conditions.

Even under the Boussinesq approximation, of course, temperature always appears as one of the unknown variables due to the addition of the energy equation. Accordingly, temperature fields should be quantified to fully understand the fluid motions of thermal convection as well as velocity fields. Temperature field measurements in laboratory, however, are a kind of bottleneck tasks. This is because a number of temperature sensors, e.g. thermistor, thermocouple, and so on, need to be embedded in the measurement domains. Such invasive approaches are straightforward, however, will affect flow fields as external disturbances and thus not desirable. In addition, a number of temperature sensors embedded in the fluid layer will prevent optical visualization to quantify velocity fields. Another approach is employment of the infrared camera, which enables non-invasive temperature measurements. The infrared cameras can acquire temperature as two-dimensional fields, however, only sense the surface temperatures. Accordingly, temperatures inside the fluids are not possible to measure through the casing of test fluids. For these reasons, investigation of thermal convection is much complicated even under single phase flows.

OPTICAL MEASUREMENTS OF FLUID FLOWS

As explained in § 2, at least velocity and temperature fields should be measured to fully understand the physics of thermal convection. Ever since the commercialization of digital cameras, such flow field measurements have been conducted through optical measurements techniques. That is, physical quantities of fluid flows have been quantified via image processing for the digitally recorded images.

Particle image velocimetry — One of the most important physical quantities of flows is velocity $\mathbf{u}$. If there is no contribution of heat, the governing equations can be reduced to only the continuity equation and the momentum equation. Since the velocity is only the primary unknown variables, measurements of velocity fields are essential to quantify the flow fields.

The most famous approach to optically measure the velocity fields is the so-called particle image velocimetry (PIV). An extensive review paper for PIV [5] and a practical guide book [154] can be found elsewhere. Worldwide efforts have been dedicated continuously to advance and disseminate PIV as apparent from the international PIV challenge [84, 184–186]. To conduct PIV measurements, first tracer particles are seeded into the fluid flows, and a certain cross-section is illuminated by a thin light sheet for the purpose of scattering intense lights from the seeded particles. PIV algorithm then seeks the most plausible displacements of particle clouds within a two-dimensional interrogation window captured in the two consecutive image frames by searching the most similar brightness pattern. Since the thin light sheet is utilized for illumination, the displacement vectors possess only the in-plane components inside the illuminated plane, i.e. two-dimensional and two-component (2D-2C) velocity fields are obtained. An example of PIV is shown in Fig. 3.1, which is produced using the PIV standard images distributed by Okamoto et al. [128]. A number of algorithms have been developed to improve the qualities of the displacement vector fields. In principle, PIV tracks average motions of particles within

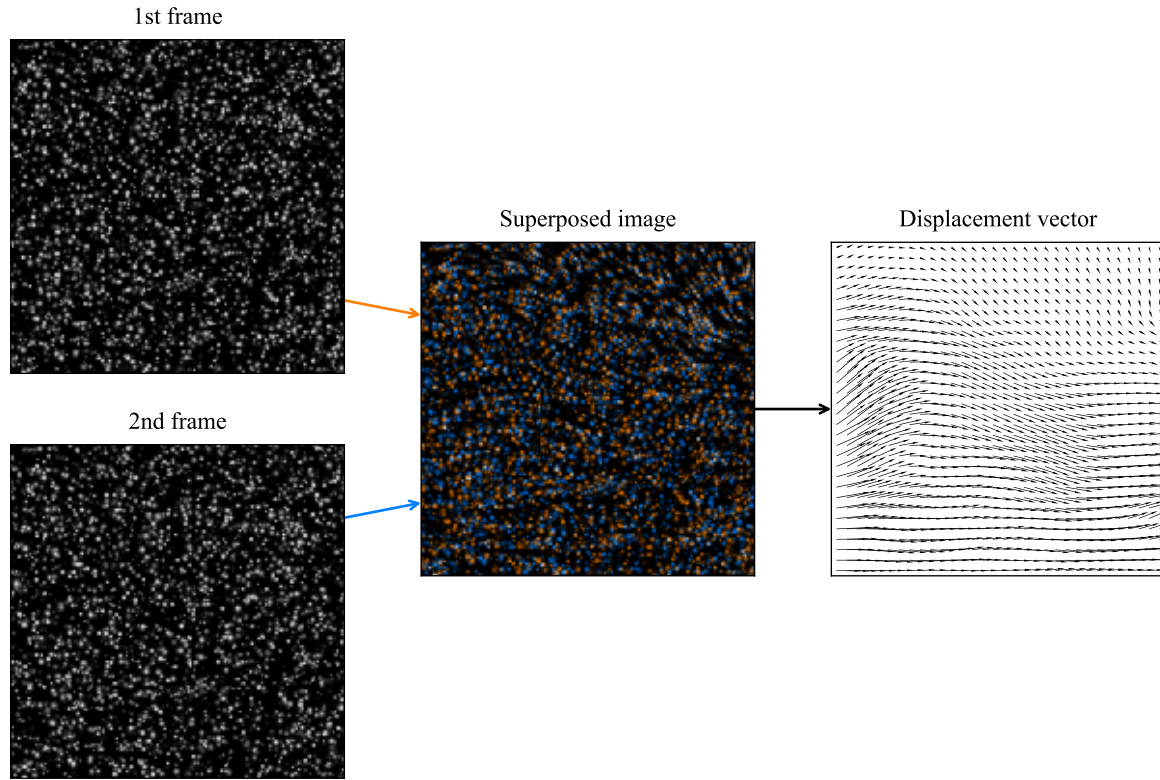


Figure 3.1: Particle image velocimetry (PIV). The superposed image of the first and second frames are shown in the middle with artificially colored particles to qualitatively see the particle displacements. These images are available from the PIV standard image directed by the Visualization Society of Japan [128] (available from [here](#)).

the interrogation window between the two frames, the spatial and temporal information are utilized from the digitally recorded images. A similar method, particle tracking velocimetry (PTV), tracks motions of individual particles directly, and various tracking algorithms have been proposed together with the developments of PIV. In any cases, binary or grayscale digital images are sufficient to acquire displacement vector fields via PIV and PTV due to their algorithms explained above.

Recent incredible developments of PIV are supported by the increase in the complexities of optical configurations. These advanced PIV methods basically increase the available dimension and component of velocity fields, which is limited to 2D-2C in the standard PIV. Three typical optical setups for different PIV methods are illustrated in Fig. 3.2, and the standard PIV is shown in Fig. 3.2(a). The first notable advancement is stereoscopic PIV, which determines the out-of-plane velocity component on the basis of geometrical configurations of two cameras as illustrated in Fig. 3.2(b) [e.g. 149, 150, 181]. That is, the stereoscopic PIV surpasses the limitation of in-plane (2C) velocity field measurements of the standard PIV only by the addition of one more camera, i.e. two-dimensional three-component (2D-3C) velocity field measurements become possible. The second great advancement is tomographic PIV introduced by Elsinga et al. [46], which utilizes four or more cameras and a volumetric illumination to determine three-dimensional three-component (3D-3C) velocity fields. These advanced methods have

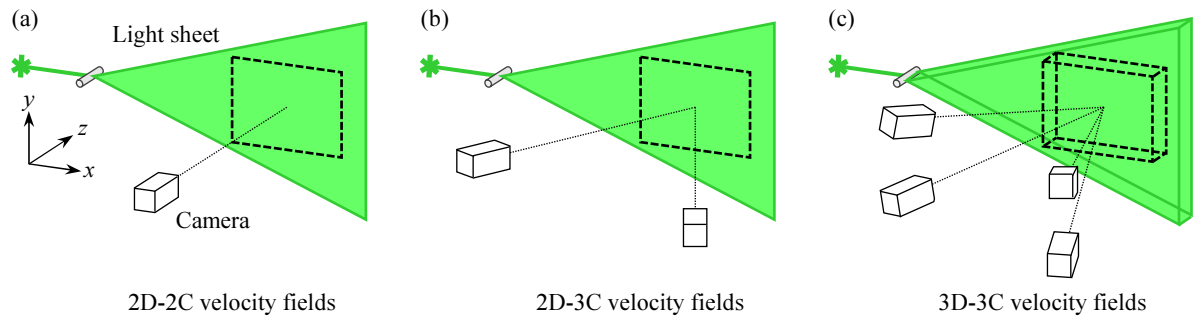


Figure 3.2: Advancement of PIV technique: (a) standard PIV, (b) stereoscopic PIV, and (c) tomographic PIV.

been also implemented and extended by a number of researchers throughout the world like the standard PIV, and many commercial softwares can be available. For these advanced PIV methods, the requirement is only grayscale images as such methods are based on identification of particle displacements.

Color information in flow measurements — While various PIV methods addressed above only require grayscale images, it stands to reason in terms of the quality of particle images. This is because of the difference in the image sensors of monochrome camera and color camera. In Fig. 3.3, conceptual illustrations of the different image sensors are shown. Monochrome camera directly receives light intensity by the array of light receiving elements consisting of a single pixel [Fig. 3.3(a)]. In contrast, color camera requires color filters in front of each light receiving element so as to record a single color information in each pixel. In general, the color filter a complementary metal-oxide-semiconductor (CMOS) camera or a charge-coupled device (CCD) camera is in the form of a checkerboard pattern, the so-called Bayer pattern. Quarter numbers of all the pixels are assigned to red and blue pixels, and the rest half pixels are assigned to green pixels, and these pixels are aligned alternately. Apart from cameras possessing special light receiving elements, color reconstruction procedures (debayering or demosaicing) are required after image acquisitions to obtain full color images due to the Bayer pattern; i.e. missing color brightness values are interpolated using those of surrounding pixels. Otherwise, the obtained images also show checkerboard-like patterns. Finally, the grayscale images can be acquired through a certain color-to-grayscale conversion scheme. That is, the grayscale images obtained by a monochrome camera and a color camera are not equivalent, and the actual spatial resolutions of the color images are approximately one-fourth of those captured by a monochrome camera as the demosaicing process is a kind of spatial smoothing methods [48]. Considering that PIV methods only require grayscale images in principle, monochrome cameras are preferable rather than color cameras, since particle images can be obtained in a high spatial resolution yielding to identify particle outlines finely.

Such a low bit-depth approach of PIV, however, cannot utilize full capacity of specifications of recent digital cameras; it may be a waste of acquired information. Thanks to expanding use of color cameras and the advancement of production techniques, color cameras with high spatial

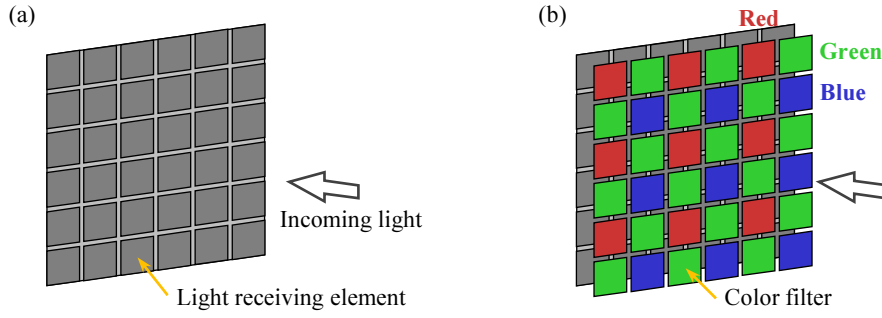


Figure 3.3: Image sensors of (a) monochrome camera and (b) color camera.

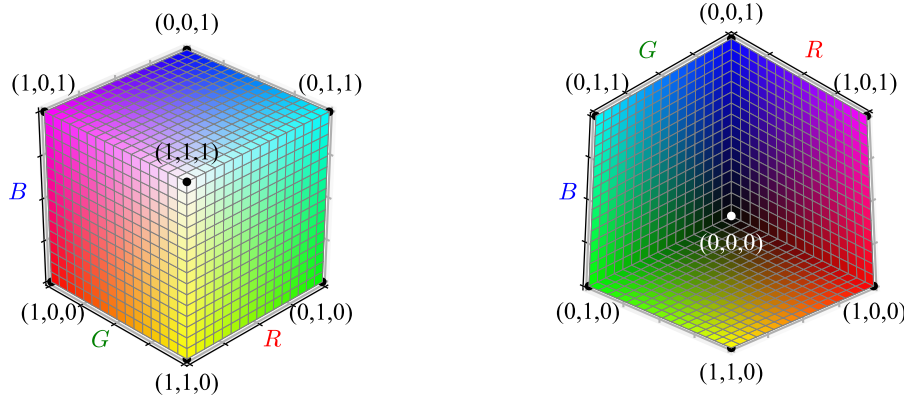


Figure 3.4: Available colors expressed in the cubical RGB -space.

resolutions are affordable unless high-speed image acquisitions or other irregular specifications are not necessary. A few to tens of megapixel cameras are now regarded as typical consumer-grade cameras. Such color cameras are considered to be sufficiently high resolution in practical use of PIV, even if the demosaicing process reduces the resolution. After the demosaicing process, three brightness values, red R , green G , and blue B , are recorded to express a single color of the pixel. The available colors are shown in Fig. 3.4. The three brightness values basically range from 0 (minimum) to 1 (maximum) as analog inputs, and discretized to digital values as explained above. Suppose that each pixel can originally record light intensity in 8-bit format, brightness value ranges from 0 to $2^8 - 1 = 255$. In this condition, the interpolated color images possess 24-bit format brightness values ($2^{24} = 16,777,216$ levels) in each pixel, meaning 8-bit format for each R , G , and B channel. As a result, the available colors of a single pixel can be expressed as a cubical RGB space like Fig. 3.4. Considering that grayscale images obtained from color images are sufficiently informative for the PIV or PTV analysis, such abundant color information recorded in a larger bit-depth format will be acquired as simply supplemental information to the velocity fields.

Utilization of highly informative color values of visualized images will increase the available physical quantities without increasing the complexities of the optical hardwares like stereoscopic or tomographic PIV. Of course, such additional physical quantities are required to be represented by color information through experimental contrivance. Some ideas for utilizing color

information have been already introduced so far, to extract time t , spatial position $\mathbf{x}$, velocity $\mathbf{u}$, temperature T , and so on. The first idea is extraction of temporal information t by colors. Goss et al. [54] and Gogineni et al. [53] utilized two different colored laser sheets pulsed at different timing within a very short duration. A slight long exposure time of a color camera yields to capture two differently colored particles on a single frame. On the basis of double pulsed PIV, the brightness values recorded in two color channels are correlated and in-plane velocity vector fields can be acquired. The second idea is determination of out-of-plane spatial positions $\mathbf{x}$. In standard PIV configuration, only a plane illuminated by a thin light sheet is utilized as the measurement plane. The distance from the camera or other reference position like walls to the illuminated plane is mechanically fixed, and thus particle images within the predetermined plane can be acquired. If two or more planes are illuminated simultaneously by different colored light sheets, a single particle image can be separated to multiple images corresponding to the illuminated planes using color information. As the colored illuminations are mechanically fixed at predetermined spatial position, the color information represents spatial positions. Toward discrimination of the out-of-plane spatial position, a number of efforts has been dedicated. For instance, Pick and Lehmann [142] performed simultaneous multi-layer stereoscopic PIV by adopting layered color filters and a white light as the light source, and achieved six-layer 2D-3C velocity fields measurements. Such a color filter approach or difference colored light sheet approach first separate a color image into multiple images corresponding to the illuminated planes. Thus, the available post-processing will be just the standard PIV acquiring 2D-2C velocity fields, and these velocity fields at different planes are stacked, resulting in multiple 2D-2C velocity fields. A more advanced approach is volumetric color gradation illumination, i.e. rainbow illumination, to determine the out-of-plane positions steplessly. Watamura et al. [200] showed a promising result in utilization of rainbow illumination by a liquid crystal projector. A different approach of color-aided flow measurements is liquid crystal thermometry (LCT), which utilizes thermochromic liquid crystals (TLC). Since the TLC scatters colors corresponding to temperatures, color information on digital images can be regarded to represent temperature T . Extensive review papers for this technique can be found elsewhere [4, 85, 86, 133, 210].

Incorporating these former efforts, color-aided optical measurements of fluid flows are worth developing to achieve advanced flow measurements. The requirement for flow measurements utilizing color information is to handle color information as a supplemental information to the 2D-2C velocity fields obtained by the standard PIV. This is necessarily satisfied to establish color-aided measurement methods as practically useful ones toward comprehensive quantification of physical quantities appeared in the governing equations.

COLOR-BASED OPTICAL MEASUREMENTS IN THERMAL CONVECTION

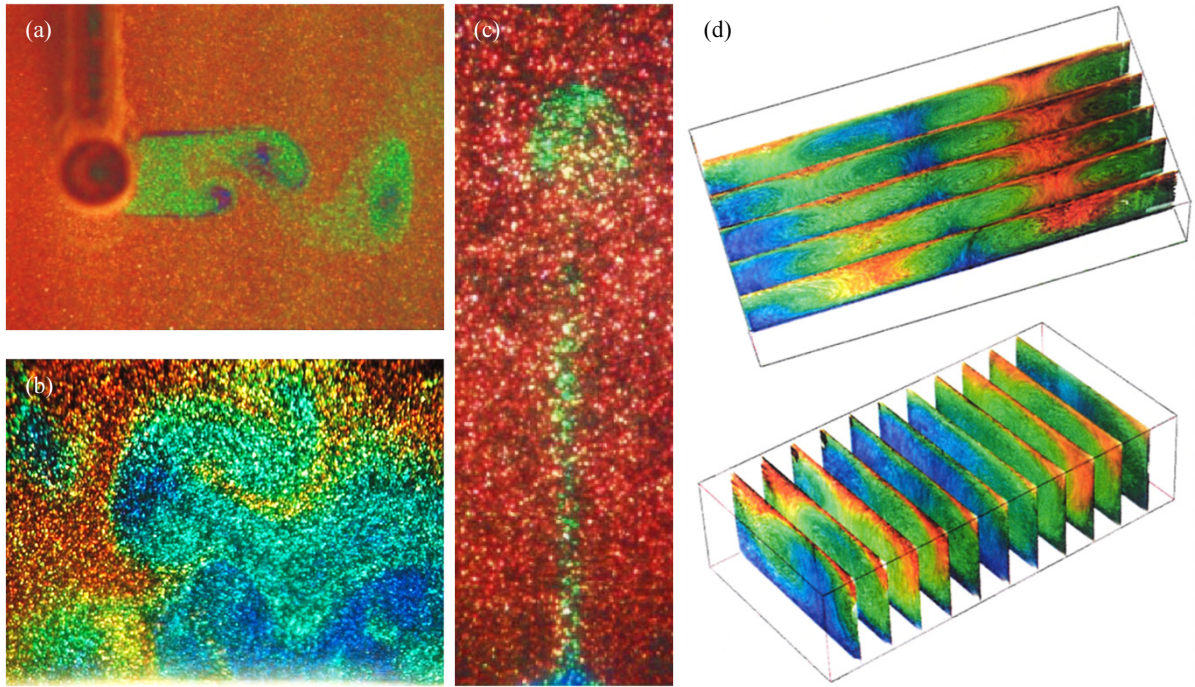


Figure 4.1: Examples of thermo-fluid flows visualized by utilizing TLC particles: (a) down-stream of a heated cylinder in a cross flow [133], (b) flow over heated waves [65], (c) emission of a thermal plume from a heated disk [144], and (d) three-dimensional assembly of visualized temperature fields in Rayleigh–Bénard convection [34].

As discussed in Chapter 2, temperature field measurements are essential for comprehensive understanding of thermal convection phenomena, in addition to velocity field measurements. As briefly introduced in Chapter 3, thermochromic liquid crystals (TLC) has a potential to display temperature as coloration. In particular, micro-particles encapsulating the TLC, TLC particles, and illumination by a white light sheet are possible to display temperature fields at the illuminated planes in the fluids as colors. In this sense, the TLC particles are the key factor for

understanding thermal convection via color-based optical measurements.

A number of experimental works utilizing the TLC particles have been reported earlier. Examples of the visualization of various thermo-fluid flows utilizing the TLC particles are shown in Fig. 4.1. In Fig. 4.1(a), a wake flow observed at down-stream of a heated cylinder in a cross flow visualized by Park et al. [133] is shown. Since the TLC particles scatter colored lights corresponding to the fluid temperature as red, green, and blue with the increase in temperature, it is possible to identify heated wakes after the cylinder in Fig. 4.1(a). Similarly, Günther and von Rohr [65] tried to visualize the flow passing over heated waves as shown in Fig. 4.1(b). Application to thermal convection can be also referred from the work by Pottebaum and Gharib [144]; they visualized an emission of a hot thermal plume from a heated disk as shown in Fig. 4.1(c). A more advanced application can be found in the work by Ciofalo et al. [34]; they visualized three-dimensional temperature fields of Rayleigh–Bénard convection by illuminating multiple measurement planes with a flush light source, and the assemblies of the visualized images are shown in Fig. 4.1(d).

The visualization results shown in Fig. 4.1 suggest that the coloration can be quantified as the temperature of the fluids. In addition, uneven brightness distributions originating from the spherical particles can be identified. That is, the standard methods for measuring velocity fields, PIV and PTV, can be applied to the particle images obtained from the TLC particles as long as the particles can be extracted adequately from the color images. Accordingly, the use of TLC particles yields simultaneous quantification of both the velocity fields and the temperature fields without additional tracer particles and temperature sensors.

All of the earlier works shown in Fig. 4.1 tried to quantify the coloration of the TLC particles as temperature fields. However, no deterministic method has been proposed, and color-to-depth calibration was carried in an ad hoc manner for each condition. This is because the coloration of the TLC particles strongly depends on the optical environment in the laboratories. Details of the coloration mechanism of the TLC particles will be provided in Appendix A, however, it is apparent that the use of the TLC particles provide lots of information which cannot be access via the standard methods such as PIV/PTV and temperature sensors. For these reasons, the systematization of the color-to-depth calibration of the TLC particles is essential to boost experimental study on thermal convection.

OBJECTIVE AND OUTLINE

In order to satisfy the requirements discussed so far, this thesis consists of four parts containing two main objectives. Outline of this thesis is explained as follows.

Part I (this part) describes the background and the objectives of this study.

The two main objectives are in Part II and III as follows.

One of the objectives is “*development of color-based flow measurement methods*”, and detailed in Part II. As discussed in Chapter 3, addition of color information to the standard optical measurement techniques possessed possibilities to achieve advanced flow measurements. Such functional techniques are required when one faces challenging measurement situations or requirements in fluid flow at which the standard (commercial) methods are not applicable. Therefore, practical measurement methods, which can be easily implemented and effective under various challenging experimental conditions, have been developed.

First, to study 3D coupling structures of immiscible thermal convection, color information is utilized to determine the out-of-plane positions. In Chapter 1, color-aided image separation technique is developed to identify visualized cross-sections at different depth positions illuminated by two differently colored laser sheets. Adopting the image separation technique, simultaneous stereoscopic PIV measurements at the two horizontal cross-sections of immiscible two-layer thermal convection are performed. Throughout the experiments imitating the liquefied natural gas storage tanks, three-dimensional coupling structures of the convection cells are investigated in detail using a cylindrical fluid container.

Second, the color separation technique is further developed for the purpose of advancement of a single-view approach, which maximizes the available color information as much as possible. In Chapter 2, a novel 3C velocity field measurement method is established by adopting the image separation technique developed in Chapter 1. The method termed “dual-plane ensemble correlation” is developed as an extension of the so-called single-pixel PIV and dual-plane PIV

introduced in former works. The method can measure 2D-3C velocity fields with single-pixel resolution only by a single camera, and its performance is validated by comparing the standard PIV technique.

As the third topic, the color information is utilized to determined stepless out-of-plane positions. In situ color-to-depth calibration scheme is developed for generalization of the color PTV method. In Chapter 3, a more advanced approach for utilizing color information, 3D color PTV, is proposed. A volumetric rainbow-gradient light is used for illumination and the depth positions are obtained by solving a multi-regression problem associating color and depth. Employment of artificial neural network to solve the multi-regression and proposal of special experimental/analytic procedures yield in situ application of the color PTV. The developed method is validated by comparing the standard PIV technique, and ensured its performance.

In Chapter 4, applicability of the stress-optic law in Newtonian fluids is evaluated through color image processing. Birefringence induces interference colors corresponding to fluid deformations. Addition of tiny flake particles provides birefringence to Newtonian fluids, and a unique relationship between colors and shear stress is proved. This implies the possibility of direct stress fields (fluid deformation fields) measurements through color image processing without obtaining velocity fields.

Another objective of this thesis is “*study on thermal convection utilizing thermochromic liquid crystals*”, and the details are discussed in Part III. As mentioned in Chapter 2, thermal convection is one of the most essential fluid motions on the Earth. Since the fluid motions are driven by thermal buoyancy, the energy equation needs to be considered to fully understand the physics of thermal convection. In experiments, this means the requirement of temperature field measurements. Thus dynamics of thermal convection are studied utilizing thermochromic liquid crystal (TLC) particles by accumulating all the knowledge attained in the developments of color-aided measurement methods. In particular, horizontal convection and rotating convection are employed as measurement targets, as these involve multi-timescale phenomena to be revealed.

In Chapter 1, details of TLC particles and its practical use are presented. Thermal convection is a measurement target entailing a very long timescale physics of $O(10^0 - 10^1 \text{ hour})$, and thus standard techniques are difficult to apply due to measurement limitations in the laboratory. Usefulness of the TLC particles from the perspective of measurements in thermal convection is discussed. Exemplary experiments and color image processing schemes are then provided for the later applications.

In Chapter 2, transient behaviors of developing horizontal convection are investigated through hours of optical measurements. Conventionally, horizontal convection has been studied in its thermally-equilibrated states; the thermal buoyancy directly drives flow vertically, the baroclinic torque due to the horizontal temperature gradients, and the restoring force by the stable temperature stratification are all balanced. In developing stages, the physics totally becomes different due to the absence of the thermal buoyancy produced at the destabilizing source. Through-

out parametric studies, a developing nondimensional parameter to characterize the transient developments is found.

Rotating thermal convection consists of multi-timescale phenomena lying from $O(1\text{ s})$ to $O(1\text{ hour})$, as this system is significant from the perspective of the fluid dynamics and fluid engineering. In Chapter 3, spin-up states from non-rotating to rotating thermal convection are examined. Multi-timescale physics embedded in the spin-up process is revealed by the PIV analysis and qualitative observation of temperature fields utilizing the TLC particles.

In Chapter 4, dynamics and heat transport characteristics of columnar vortices formed in rotating thermal convection are studied. As an integration of the color-aided measurement methods, simultaneous velocity and temperature measurement methods are developed to reveal the physics of rotating thermal convection. To quantify heat transport characteristics optically, 3D-3C velocity and temperature measurement techniques are developed; scanning stereoscopic PIV and scanning liquid crystal thermometry are established. Relationship between random-walk-like diffusive motions of columnar vortices and heat transport carried by individual vortices is discussed.

Throughout this thesis, practically useful color-aided flow measurement methods are developed. Further, the implementation of the established methods is demonstrated in study on thermal convection. Summary and perspectives of this study will be provided in Part IV.

Part II

Advanced flow measurements utilizing color information

TWO-LAYER NATURAL CONVECTION INVESTIGATED BY TWO-COLORED LASER VISUALIZATION

Preface

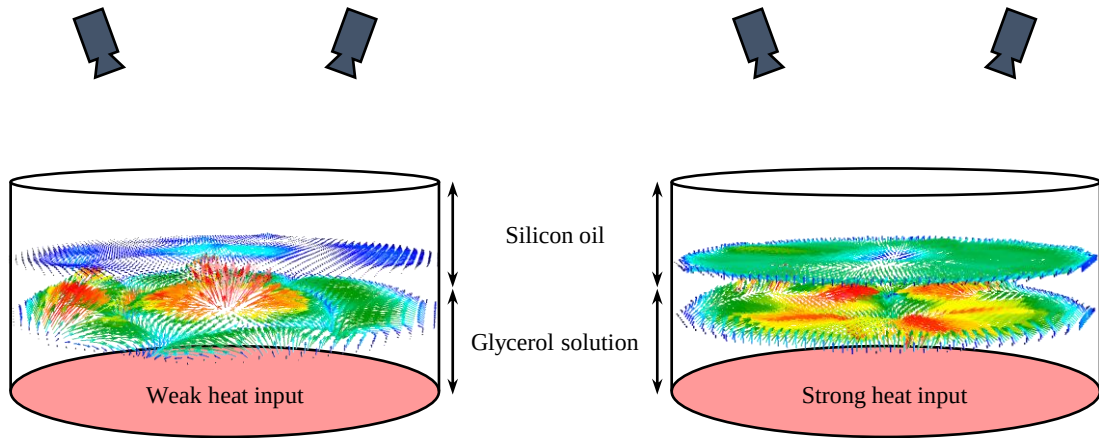


Figure 1.1: Graphical abstract of this chapter.

As a first attempt for utilizing color information in flow measurements, a color image separation technique is established. This may be the simplest implementation in the use of color information. In this chapter, the color information is utilized to distinguish different cross-sections inside test fluids illuminated by different colored laser sheets. That is, the color information represents the spatial positions x , which are out from the in-plane positions within the image sensors of the camera. This kind of techniques was reported earlier by some works [e.g. 142], and promising results in the use of color information to distinguish the seeded particles existing in the different illuminated planes were shown. The advantage is only the addition of

one more light source with a different color yields simultaneous measurements of two or more planes, which cannot be available in a standard optical configuration for PIV.

In this chapter, coupling structures of buoyancy induced thermal convection in two immiscible layers with and without weak background rotation were investigated in laboratory experiments using a glycerol solution and a silicon oil as the test fluids. A cylindrical vessel was employed to yield three-dimensional (3D) development of the two-layer convection, and the influence of background rotation on convective motion was investigated to imitate the rotation of the Earth. Using color image processing, velocity fields at two horizontal cross-sections, one in each layer, of the vessel were measured simultaneously by stereoscopic particle image velocimetry to identify the interactions on interface between the layers. The temporal evolution of the velocity fields showed that thermal convection, first developed in the lower layer subsequently progresses ahead of the upper layer both with and without rotation. The rotation suppressed developments of convection in both of the two layers in cases of small thermal forces. The coupling structures of the 3D convection cells were evaluated from the perspectives of cross-correlations and the dominance ratio, representing the accumulation of the local couplings, by using simultaneously measured velocity information. The coupling structures were found to be well characterized by the dominance ratio, as various local couplings were coexisting in the 3D domain. The global coupling structures were in the thermal coupling, and those were weakened by the rotation.

The results shown in Chapter 1 were published as Noto et al. [125] and the details are below.

Author Daisuke Noto, Yuji Tasaka, and Yuichi Murai

Title Coupling structures of two-layer natural convection in a cylindrical vessel investigated by simultaneous two-layer visualization

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1.1 Introduction

Research in the fields of geophysics and geoscience has investigated topics relating to natural convection phenomena coexisting with stratification, multi-layer, background rotation, and others, for instance atmospheric flows, oceanic flows, and flows in the interior of the Earth [157, 183, 216, 217]. The influences of these complexities on convection are key factors in understanding details of fluid motions also inside large-scale artificial structures, for instance, a liquefied natural gas (LNG) tank, other than natural phenomena. From the perspective of the fluid engineering, the safety management of the LNG storage tanks is one of the important issues, since the risk of rollover accidents has been pointed out [14, 15, 116, 174]. This matter is now revisited by engineers due to the expansion of the world LNG industry since 2010, upon updated energy policy for low CO₂ emission rate. The rollover is a phenomenon accompanying a sudden generation of full-scale penetrative convection of a density stratified LNG with continuous heat input (Fig. 1.2). The rollover causes explosive pressure-rise due to boil off gas (BOG) emission and damages the tank with its huge kinetic energy of convection [218]. Operators of LNG storage tanks need continuous monitoring of the flow field inside tanks to be able to identify conditions warning of potential rollovers. Convection structures and the accompanying sign of the rollover in stratified liquids differs depending on the initial pattern of the density stratification in fluid layers. In linear stratification, double diffusivity of the density and the temperature influences initial growth of the convection [37, 221]. For a stepwise stratification, the case focused on in this study, vertical coupling of convection cells across density gaps produced by immiscible fluids or large density differences become a key phenomenon.

In the early studies, the coupling pattern was experimentally investigated using two immiscible liquids and visualized qualitatively by particle pathlines [147] and temperature fields [148]. Interfacial interactions of convection in multiple fluid layers have been characterized as the coupling structures, mechanical and thermal couplings [145, 146], and these are schematically illustrated in Fig. 1.3. The mechanical coupling, shown in Fig. 1.3(a), emerges when the mo-

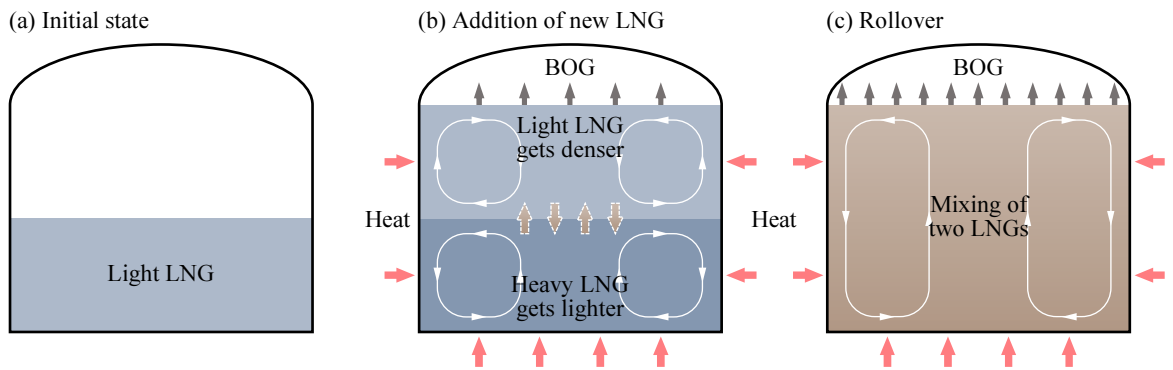


Figure 1.2: Schematic illustration of process leading to the rollover phenomenon: (a) initial state, (b) addition of new LNG with a different density to the existing one, and (c) rollover leading to an explosive pressure-rise by BOG emission..

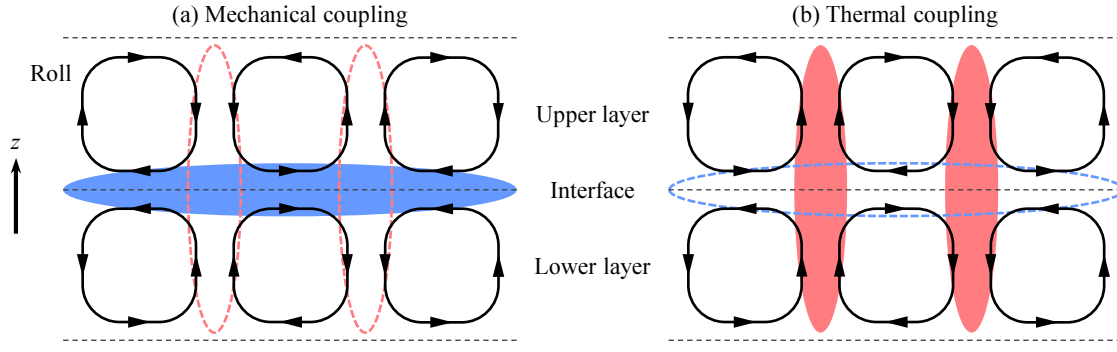


Figure 1.3: Schematic diagrams of two types of coupling structures of convection rolls in a 2D system: (a) mechanical coupling and (b) thermal coupling.

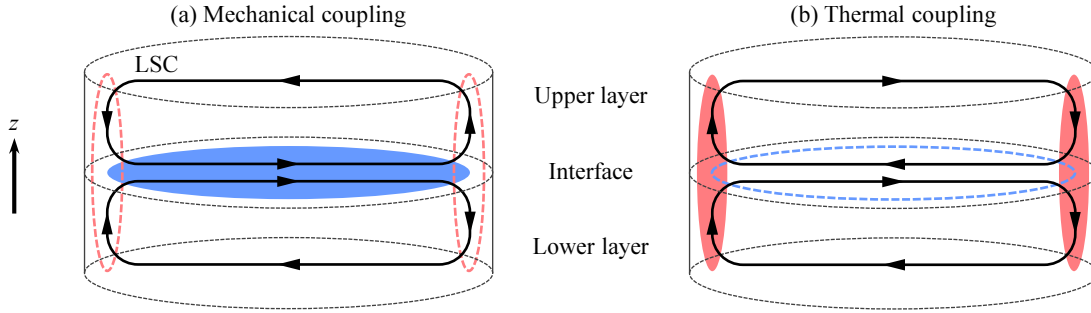


Figure 1.4: Schematic diagrams of two types of coupling structures of large scale circulations (LSCs) in a 3D system: (a) mechanical coupling and (b) thermal coupling.

mentum transfer across the interfaces of the fluid layers becomes prominent. The momentum generated in one layer is transferred to the adjoining layers to fulfill both the requirements for continuity of the tangential velocity at the interfaces and the balance in the shear stress. As a result, the circulating motion of the convection cells engages with each other in a gearwheel-like fashion as illustrated in Fig. 1.3(a). The thermal coupling transfers heat across the interfaces of the fluids with a relatively weak influence from the momentum transfer, and the positions of vertical flows line up as illustrated in Fig. 1.3(b). The two types of coupling structures and their conditions of appearance in two-dimensional (2D) containers are now well understood [11, 45, 107], and are easily characterized by identifying the presence of identical horizontal or vertical flow directions as illustrated in Fig. 1.3.

These past findings mostly dealt with the interaction of convection cells in idealized 2D domains, i.e. coupling and decoupling of 2D roll structures of Rayleigh–Bénard convection (RBC). Convection in nature and LNG tanks, however, possesses a three-dimensional (3D) structure. In addition, the rotation of the Earth may influence the convection structures depending on the length scale of the convection, as in general, rotation stabilizes thermal convection and enhances two-dimensionality [27]. Thus both the effects of 3D domain and rotation need to be clarified to deeply understand the fluid mechanics/physics of multi-layer convection from the perspective of the coupling structures. The coupling mechanism in a 3D domain has been only little investigated to date, except for a recent extensive experimental work [213]. In the study,

coupling structures of the large-scale circulations (LSCs) of turbulent two-layer RBC in a cylinder were investigated in detail, in terms of temperature measurements. In turbulent convection, the LSCs, clustering of thermal plumes, dominate the whole system, and it is possible to discuss the coupling structures in a global fashion as shown in Fig. 1.4. In contrast, laminar thermal convection possesses multiple cell structures unlike turbulent thermal convection dominated by numerous thermal plumes forming a large scale circulation. In actual LNG tanks, such laminar thermal convection is more crucial as the rollover problem happens at soon after the onset of thermal convection. The coupling structures of laminar convection structures like rolls and cells, however, may not be easily characterized in a 3D system. This is because convection cells develop locally, and each cell can form coupling structures with that in the adjoining fluid layer. Accordingly, characterization of the coupling structures in a 3D system will be complicated, even in laminar convection, as it requires considering local mutual interactions on the interface between the multiple layers.

The present study aims to clarify the coupling structures emerging in immiscible two-layer convection via laboratory experiments. In particular, the coupling structures of laminar convection cells in a 3D system are investigated as extended research of the past 2D multi-layer laminar convection [e.g. 40, 145–148, 157]. The experiments are conducted in a cylindrical fluid container for yielding 3D flow developments, and the influence of background rotation is also investigated as detailed in Sec. 1.2. To elucidate the interactions of convection on the interface between the two layers, simultaneous velocity field measurements at two horizontal cross-sections are established utilizing two different-colored lasers in Sec. 1.3. The 3D developments of flow fields with and without background rotation are evaluated through variations of the flow patterns and Rayleigh numbers in Sec. 1.4. Summarizing the results quantified from the experimental investigations, this study attempts to explain the coupling structures in the two-layer convection using all the accessible velocity information revealed by the simultaneous visualization method in Sec. 1.5, and a summary will be provided in Sec. 1.6.

1.2 Experiment with two-layer convection in a cylindrical container

1.2.1 Experimental setup

Schematic diagrams of the experimental fluid container and the setup are shown in Fig. 1.5 and 1.6, respectively. For enabling 3D developments of multi-layer convection, the present experiments employed a cylindrical fluid layer as shown in Fig. 1.5. The fluid layer was formed by a transparent acrylic cylindrical container with an inner diameter of $D = 80$ mm, a height of 40 mm, and a thickness of 5 mm. A 10 mm-thick copper plate embedded with a silicone rubber heater was positioned at the bottom of the fluid layer. The cylindrical container was placed at

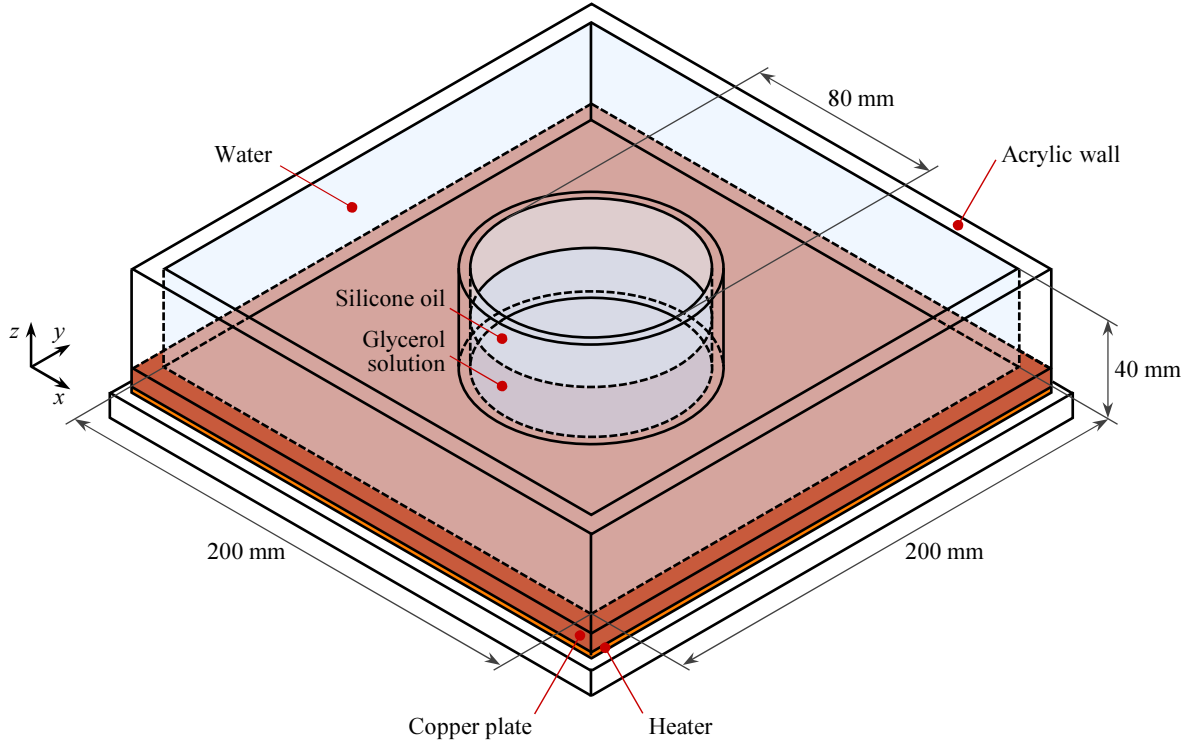


Figure 1.5: Schematic illustrations of the fluid container.

the center of a rectangular vessel consisting of 15 mm-thick acrylic walls of $200 \times 200 \text{ mm}^2$ horizontally and 40 mm vertically. Temperature measurements were performed with five small thermistor sensors, one at the bottom (inside the copper plate immediately beneath the fluid layer), three for the fluid-fluid interface, and one in the air immediately above the fluid layer indicated with small circles filled with yellow in Fig. 1.6(a) and (b). The fluid container was mounted on a turntable as shown in Fig. 1.6(c). The turntable was composed of two stages each with a 1 m-diameter stainless plate, and was able to rotate around the z -axis coinciding with the gravitational direction at a fixed angular velocity Ω in the range of $0.0 - 5.0 \text{ rad/s}$. Further detailed descriptions of the fluid vessel and the turntable can be found in our earlier reports [122, 123].

Two immiscible fluids, a 80-wt% glycerol solution (GS) and a 100-cSt silicon oil (SO) (KF-96-100cSt, Shin-Etsu Chemical Co., Ltd.) were employed as the test fluids. Physical properties of the fluids at room temperature ($T = 20^\circ\text{C}$) are listed in Table 1.1. Equal volumes of these two fluids were poured into the cylindrical container, giving a thickness of each fluid layer of $d = 20 \text{ mm}$, with the interface between the fluids located at $z = 20 \text{ mm}$. Due to the higher density of the GS than that of the SO, the fluids settled into clearly separated strata with the lower layer GS and upper layer SO as shown in Fig. 1.6(b). Here, the interface between the two layers is close to flat at the inner surface of cylindrical container, and the deformation of the interface is considered negligibly small even in the rotating conditions. From this, the thickness of the two layers was each a constant $d = 20 \text{ mm}$, and the aspect ratio of the layers becomes $\Gamma = D/d = 4$.

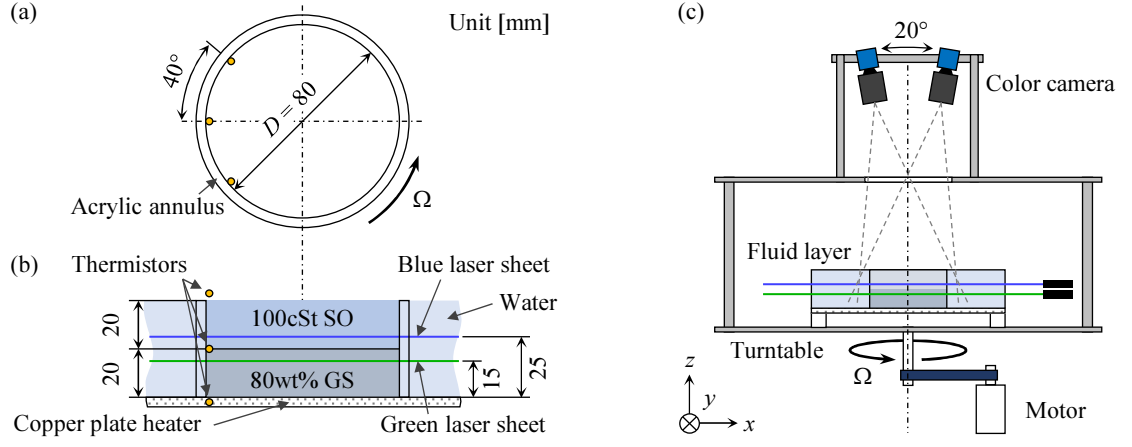


Figure 1.6: Schematic illustrations of the experimental setup: (a) top view and (b) side view of the cylindrical fluid container made of the acrylic side wall and the copper plate heater. (c) Overview of the turntable illustrated together with the container and the optical equipment. The size of the turntable is exaggerated for visibility.

Table 1.1: Physical properties of the glycerol solution [2] and the silicon oil (after manufacturer's data sheet) at room temperature ($T = 20^\circ\text{C}$).

	Glycerol solution (80 wt%)	Silicon oil (100 cSt)
Density ρ [kg/m^3]	1.21×10^3	9.62×10^2
Viscosity μ [$\text{kg}/(\text{m} \cdot \text{s})$]	6.01×10^{-2}	1.07×10^{-1}
Kinematic viscosity ν [m^2/s]	4.97×10^{-5}	1.11×10^{-4}
Thermal expansion β [$1/\text{K}$]	6.15×10^{-4}	9.50×10^{-4}
Thermal diffusivity κ [m^2/s]	1.17×10^{-7}	1.11×10^{-7}
Thermal conductivity λ [$\text{W}/(\text{m} \cdot \text{K})$]	3.25×10^{-1}	1.60×10^{-1}
Prandtl number Pr [—]	424	999

These two fluids were selected to allow a small viscosity difference, $\mu_{\text{GS}}/\mu_{\text{SO}} = 0.56$, in order to realize varieties of coupling structures. This is because the mechanical coupling is preferred to emerge for small differences of the viscosity [216, 217], while the thermal coupling is originally predominant in two-layer convection for a better heat transport [213].

To evaluate the coupling structures of the two-layer convection, flow fields of the two layers need to be measured simultaneously. For this, two CMOS color cameras (DFK33UP5000, The Imaging Source Co.) were mounted on the turntable to record the particle images at the cross-sections of the fluid layer from the top of the fluid container for the velocity measurements by stereoscopic particle image velocimetry (SPIV). Fluorescent microsphere particles (mean diameter $15 \mu\text{m}$ and mean density $1.1 \text{ g}/\text{cm}^3$, FLUOSTAR, EBM Co.) and polyethylene spheres (mean diameter $50 \mu\text{m}$ and mean density $0.92 \text{ g}/\text{cm}^3$, Flowbeads, Sumitomo Seika Chemicals Co., Ltd.) were seeded in the GS and SO layers, respectively. Two different-colored laser sheets, a 500-mW green laser sheet and a 500-mW blue laser sheet, illuminated at $z = 15 \text{ mm}$ of the GS and $z = 25 \text{ mm}$ of the SO layer respectively as indicated in Fig. 1.6(b) and (c). The green

laser excited the fluorescent dye encapsulated in the particles in the GS layer, and red-colored fluorescence was observed, and the particles in the SO layer directly scattered blue light of the blue laser. The same lens with a focal length of 35 mm and an aperture of F5.6, resulting in a focal depth of ~ 33 mm were mounted to the cameras, to capture sufficiently focused particle images of the whole cross-sections. The details of the subsequent SPIV analysis and accompanying image processing to decompose flow fields in the two layers will be described in Sec. 1.3. The gap between the cylindrical container and the rectangular vessel was filled with water to reduce unnecessary optical refractions of the incident laser sheets, and homogeneous illumination was provided to the cross-sections of the fluid layer.

1.2.2 Experimental conditions

Convection driven by thermal buoyancy is commonly described by two dimensionless parameters as follows: The Prandtl number defined as the ratio of the kinematic viscosity ν and the thermal diffusivity κ as

$$Pr = \frac{\nu}{\kappa} \quad (1.1)$$

reflects the relation of the thickness of the viscous and thermal boundary layers. The Rayleigh number is defined as the strength of the thermal buoyancy against the viscous and thermal diffusion, and is expressed in the form

$$Ra = \frac{g\beta\Delta T d^3}{\kappa\nu}, \quad (1.2)$$

where g , β , ΔT , and d denote the gravitational acceleration, the thermal expansion coefficient, the temperature difference imposed over the depth of the fluid layer, and the depth of the fluid layer, respectively. Under the conditions of constant heat fluxes, the flux Rayleigh number Ra_f is frequently used to describe the flow structures, and it is expressed as

$$Ra_f = \frac{g\beta q d^4}{\lambda\kappa\nu}, \quad (1.3)$$

where λ represents the thermal conductivity of the fluid. The influence of the rotation is dictated by the convective Rossby number

$$Ro = \frac{1}{2\Omega} \sqrt{\frac{g\beta\Delta T}{d}}, \quad (1.4)$$

to characterize flow states in rotating convection.

In the present study, the temporal evolution of the two-layer convection was investigated by imposing two different constant heat fluxes, a weak heat input of $q = 375 \text{ W/m}^2$ and a

strong heat input of $q = 1573 \text{ W/m}^2$, to the stable stratification states at $t = 0 \text{ min}$. The initial Ra_f values in the weak heat input cases were $Ra_f = 1.9 \times 10^5$ and 2.8×10^5 in the lower and upper layers, i.e. $Ra_f = O(10^5)$. Those for the strong heat input cases were $Ra_f = 8.1 \times 10^5$ and 1.2×10^6 , i.e. $Ra_f = O(10^6)$. Each heat input condition was examined with and without background rotation. A set angular velocity of $\Omega = 0.025 \text{ rad/s}$ was imposed in the rotation case. To ensure the robustness of the results, the initial conditions are carefully fixed. For the no rotation cases, the heat input was imposed after leaving the fluids for more than 30 min so as not to allow initial perturbations. For the rotation cases, the heat input was imposed after more than 30 min from the start of rotation to ensure a rigid-body rotation of the fluids. In addition, new fluids at room temperature were used in every run. With these careful preparations, the initial velocities in the fluids could be minimized as much as possible. The observed flow developments are expected to be statistically identical in each condition, in terms of the length- and time-scales. The Ra values realized at the present settings varied in the range $O(10^3 - 10^5)$ during the heating, and the corresponding Ro values were $O(10^0 - 10^1)$. The water in the gap between the cylindrical and the rectangular vessels was also heated together with the test fluids to maintain the ambient temperature of the fluid layer close to that of the test fluids.

The top of the fluid container is open to the air at room temperature. Water filled surrounding the cylindrical container was also heated by the heater at the bottom. Since the Pr of water, $Pr \sim 7$ at 20°C , is much smaller than those of the two immiscible fluids, the continuous heating from the bottom yielded thermal turbulence in the water jacket. That is, the bulk temperature of water increasing with time was almost constant across the volume due to the formation of the isothermal core. Thus the heat transport through the sidewall of the fluid layer was considered negligible and the side wall of the container was considered thermally insulated compared to the top of the fluid container. In this open-top system, the vertically asymmetric thermal condition is considered to play a key role for determining flow structures. The lower GS layer is expected to evolve first, and subsequently, the upper SO layer start convective motions, unlike a vertically symmetric system of RBC, where convection develops from both the top and bottom [213]. In addition, the fluid temperatures will increase owing to the constant heat input without cooling. This leads to changes in the physical properties of the fluids during the measurements.

1.3 Two-layer SPIV measurement

To simultaneously quantify the velocity fields containing the out-of-plane velocity in each layer, the particle images recorded by the two cameras were analyzed together by the SPIV analysis. The present setup limited the angle of one camera axis to the other to approximately 20° , and the resulting accuracy of the out-of-plane velocity w was around 5 to 6 times poorer than the in-plane-velocity (u and v). In addition, the camera configuration was a simple angular offset using a normal lens. The quality of the measurement, however, would be sufficient to characterize

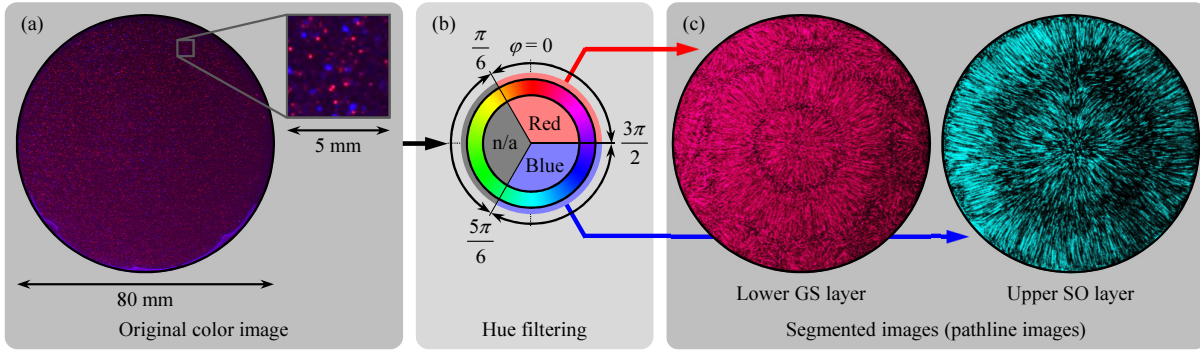


Figure 1.7: Examples of the hue filtering method decomposing a single image with two-colored particles to two images: (a) original color image obtained by the left camera with an enlarged section shown at the upper right, (b) schematic illustration of the hue filtering method, and (c) pathline images compiled using the segmented particle images from the left camera. The pathlines are colored with pseudo colors to enhance visibility and drawn for 20 s for the GS layer and 80 s for the SO layer. The displayed images in (a) and (c) are dewarped by the perspective transform.

the coupling structures of the two layers. With the present setup, the image resolution was 1000×1000 pixels for an $80 \times 80 \text{ mm}^2$ (0.08 mm/pixel) image. The long-time measurements for 90 min generally cause the floatation or the sedimentation of the tracer particles. However, the present study employed very viscous fluids, and these issues were negligibly small.

To analyze the particle images in the two horizontal cross-sections separately, an image decomposition process was required prior to the subsequent SPIV analysis. This is because the obtained images from a single camera contained two differently colored particles as shown in Fig. 1.7(a), and the particles need to be assigned to their respective layer as red particles for the GS layer and blue particles for the SO layer. A similar effort distinguishing particles in images containing six differently colored particles into six layers was reported in [142]. Here, a hue value φ is employed for extraction of the red and blue particles on the images, and it is calculated pixelwise as

$$\varphi_{i,j} = \tan^{-1} \left[\frac{\sqrt{3}(G_{i,j} - B_{i,j})}{2R_{i,j} - G_{i,j} - B_{i,j}} \right], \quad (1.5)$$

where R , G , and B are the 8-bit brightness values recorded for each pixel (i, j) in the format of 24-bit color images. When φ is defined in a range of $[0, 2\pi]$, red corresponds to 0 or 2π , green $2\pi/3$, and blue $4\pi/3$, as suggested in Fig. 1.7(b). To properly distinguish pixels, threshold values for φ were empirically determined. In the present study, pixels with $0 \leq \varphi < \pi/6$ and $3\pi/2 < \varphi < 2\pi$ were attributed as red (lower GS layer), those with $5\pi/6 < \varphi < 3\pi/2$ as blue (upper SO layer). The pixels with $\pi/6 \leq \varphi \leq 5\pi/6$ were considered to be absent of the tracer particles, and thus those were attributed as not available (n/a) pixels. These threshold values may differ depending on the experimental environment, including the light scattering characteristics of the particles, the white balance of the cameras, and so on. The brightness values of the pixels regarded as red or blue were then recorded separately in red and blue particle images. For the

red particle image, the pixels assigned as blue and n/a were recorded as 0 (black), while the pixels attributed as red and n/a were recorded as 0 for the blue particle image. The segmented images are then blurred by the median filter with a kernel size of 5×5 pixels to fill the black pixels defected in the hue filtering procedure. Examples of the image segmentation using the hue filtering to form pathline images are shown in Fig. 1.7(c); the left is the 20 s-pathline image of the lower GS layer and the right is the 80 s-pathline of the upper SO layer. The two pathline images show two different flow structures as a smaller length scale for the lower and larger length scale for the upper layers.

The SPIV analysis was applied to the particle images segmented in the hue filtering method. The in-plane velocities $(u, v)_L$ of the left camera and $(u, v)_R$ of the right camera were individually measured, and subsequently merged by using the simple pinhole camera model. These procedures were performed separately for both the red and blue particle images. Here the perspective transform matrix for the mapping function of the SPIV was utilized, as the lens aberrations may be negligibly small in the present condition. As a result, spatial distributions of three-component velocity vector (u, v, w) were obtained at two horizontal slices at $z = 15$ and 25 mm. The grids for the SPIV were made as Cartesian coordinates (regular grid) every 1.6 mm ($= 20$ pixels) both in x and y directions, and the resultant number of grids were 51×51 in the x - and y -directions. The frame rate for the image acquisition was set sufficiently high so as to conduct PIV analysis for the two different fluid layers with different velocity ranges by reducing the frame rate after the image acquisition.

1.4 Quantitative observations of two-layer convection

1.4.1 Time development of flow structures

The temporal evolution of the horizontal velocity fields for the weak heat input without rotation is shown in Fig. 1.8. The two cross-sections of the GS and the SO layers are individually shown in Fig. 1.8(a) for the upper SO layer and Fig. 1.8(b) for the lower GS layer. The in-plane velocity vectors (u, v) indicated by the black arrows are superposed on the color contours representing the out-of-plane velocity w , and the vectors are plotted one of every two both in the x - and y -directions for better visibility. The time elapsed from the start of the imposing heating is noted above each panel with a time interval of 20 min from $t = 10$ to 90 min. At $t = 10$ min, the lower layer formed two convection cells, a small one at the center and a large toroidal one surrounding the central one, while the upper layer remained largely unchanged. The convection cells in the lower layer developed with time as organized structures with smaller length scales at $t = 30$ min; the large convection cell at the center and the six small convection cells aligned in the azimuthal direction while surrounding the central one are shown in Fig. 1.8(b). During $t = 10$ – 30 min in Fig. 1.8(b), the flow direction in the lower layer reversed as apparent from

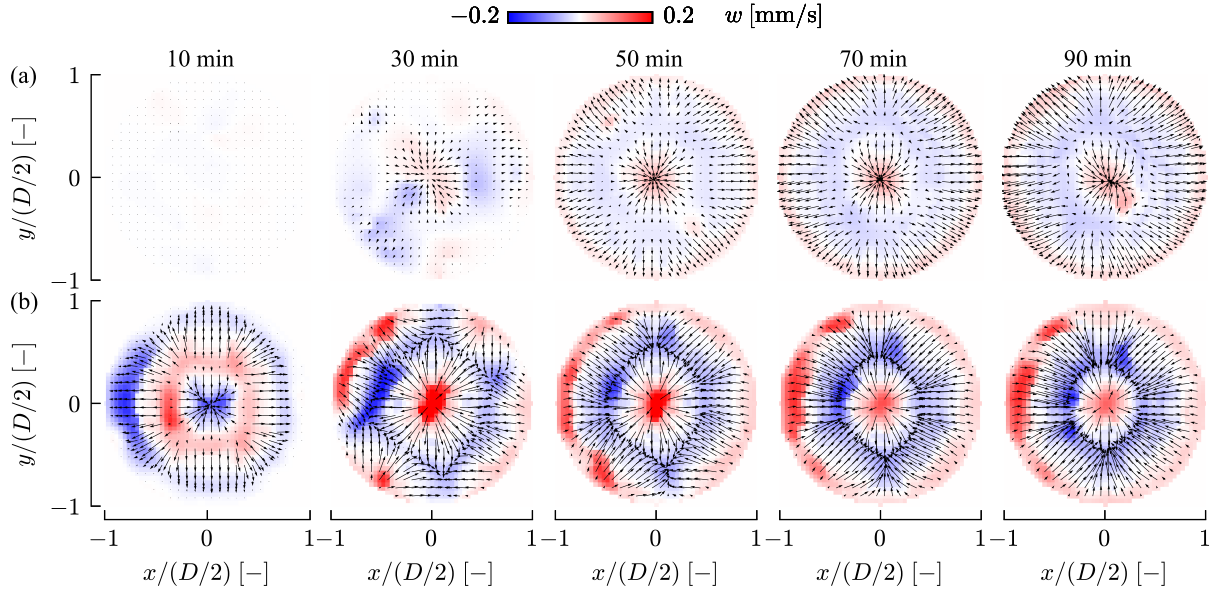


Figure 1.8: Instantaneous velocity fields measured by the SPIV for the weak heat input $Ra_f = O(10^5)$ without rotation $Ro = \infty$: (a) the upper SO layer at $z = 25$ mm and (b) the lower GS layer at $z = 15$ mm. The in-plane velocity vectors (u, v) represented by black arrows are superposed on the color contours showing the out-of-plane velocity w . For the sake of clarity, one of two vectors is displayed both in the x - and y -directions. The time elapsed from the start of the experiment is noted above the images. The scale for the vectors are adjusted in each snapshot for visibility.

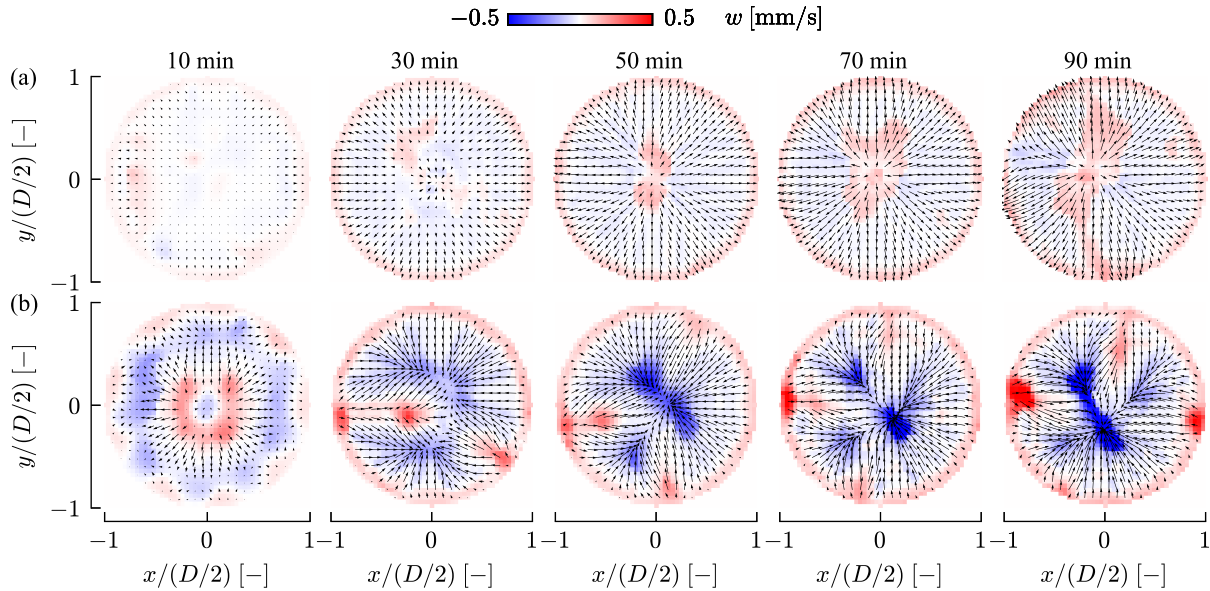


Figure 1.9: Instantaneous velocity fields measured by the SPIV for the strong heat input $Ra_f = O(10^6)$ without rotation $Ro = \infty$: (a) the upper SO layer at $z = 25$ mm and (b) the lower GS layer at $z = 15$ mm. The in-plane velocity vectors (u, v) represented by black arrows are superposed on the color contours showing the out-of-plane velocity w . For the sake of clarity, one of two vectors is displayed both in the x - and y -directions. The time elapsed from the start of the experiment is noted above the images. The scale for the vectors are adjusted in each snapshot for visibility.

the directions of the black arrows and the signs of w . At $t = 30$ min, the upper layer formed convection structures, which are much weaker than those in the lower layer [Fig. 1.8(a)], and the weak convection cells organized into two large structures, central and toroidal surrounding the central cell, at $t = 50$ min. Whereas, the convection structures in the lower layer were sustained after $t = 30$ min [Fig. 1.8(b)].

The results for the strong heat input without rotation are shown in Fig. 1.9. At $t = 10$ min, convection cells emerged in the lower layer [Fig. 1.9(b)] like those in the weak heat input case at the same moment shown in Fig. 1.9(b), while the upper layer remained stationary at $t = 10$ min [Fig. 1.9(a)]. The convection cells in the lower layer had evolved greatly at $t = 30$ min, and the fluid layer was almost separated into two regions horizontally at $y \sim 0$ by two linearly aligned down-welling flows [Fig. 1.9(b)]. At the same moment, the upper layer formed convection structures, a central small one and a surrounding larger one [Fig. 1.9(a)]. The two large convection cells formed in the lower layer [Fig. 1.9(b)] evolved into a unified down-welling structure at $t = 50$ min, and the upper layer also unified the convection structures to a single large convection cell [Fig. 1.9(a)]. The principal convection structures in the two layers formed at $t = 50$ min did not change greatly after this.

1.4.2 Flow structures influenced by rotation

As above, rotation imposed on thermal convection influences the development of thermal convection negatively as rotation promotes the two-dimensionality of the convection structures. The results for the weak heat input case with rotation are shown in Fig. 1.10. At $t = 10$ min in Fig. 1.10(b), the lower layer did not develop any convection structures unlike the case without rotation [Fig. 1.8(b)], and the upper layer also remained stationary [Fig. 1.10(a)]. The lower layer formed convection cells, a central large one with a surrounding toroidal one, at $t = 30$ min [Fig. 1.10(b)], showing a delay in the onset of convection from that without rotation [Fig. 1.8(b)]. With time, the central cell in the lower layer decreased in size till $t = 90$ min, and correspondingly the toroidal roll widened [Fig. 1.10(b)]. The upper layer generated weak convection structures at $t = 50$ min [Fig. 1.10(a)] and developed to a large single structure eventually at $t = 90$ min.

The results for the strong heat input case with rotation shown in Fig. 1.11 suggest similar developments of convection structures to those without rotation shown in Fig. 1.9, unlike the different developments in the weak heat input cases discussed above. From the results shown above, the influence of rotation was noticeable as different developments of flow fields from those of the no rotation cases, while the convection cells subject to the weak rotation had 3D structures as well as those of the cases without rotation. The influence of the weak rotation was apparent especially in the weak heat input (large Ro value) condition. The uncomplicated spatial patterns of convection cells shown in Fig. 1.11 relative to those without rotation shown in Fig. 1.9 indicate the suppression of convection that was as would be predicted from the above discussion. The strong heat input case was less affected by the rotation and the developed flow

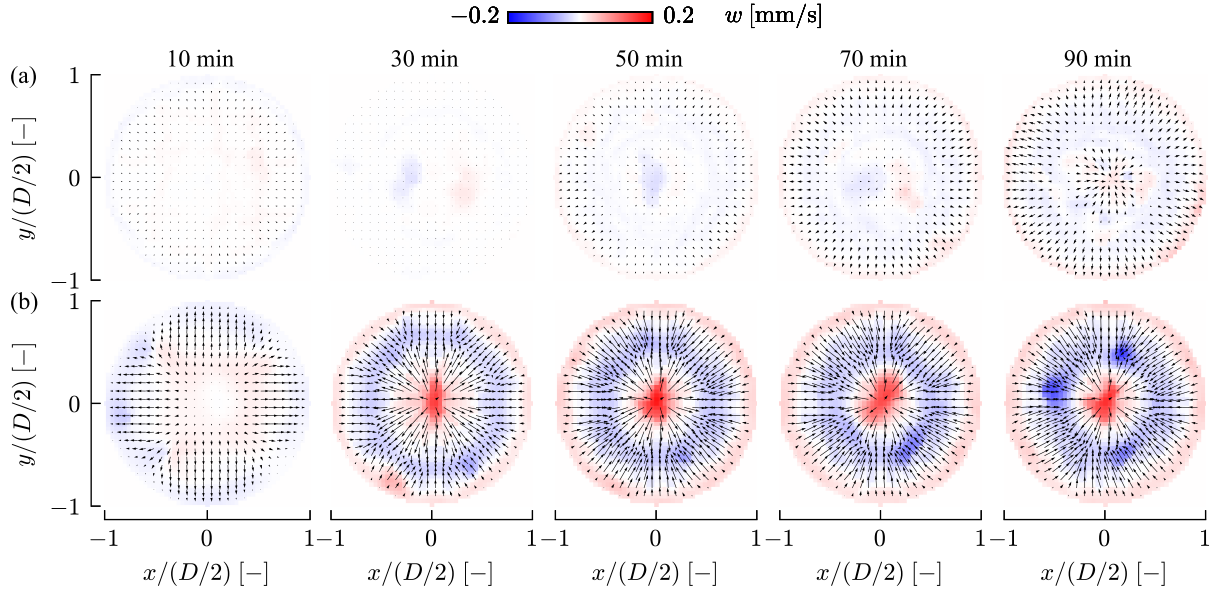


Figure 1.10: Instantaneous velocity fields measured by the SPIV for the weak heat input $Ra_f = O(10^5)$ influenced by weak rotation $Ro = O(10^0 - 10^1)$: (a) the upper SO layer at $z = 25$ mm and (b) the lower GS layer at $z = 15$ mm. The in-plane velocity vectors (u, v) represented by black arrows are superposed on the color contours showing the out-of-plane velocity w . For the sake of clarity, one of two vectors is displayed both in the x - and y -directions. The time elapsed from the start of the experiment is noted above the images. The scale for the vectors are adjusted in each snapshot for visibility.

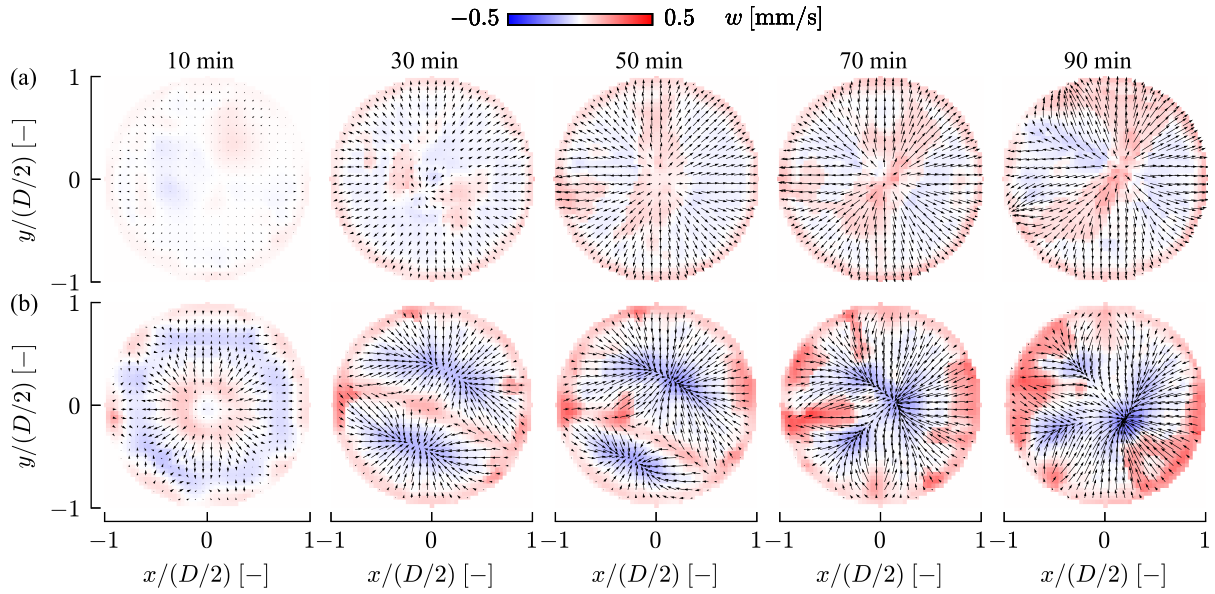


Figure 1.11: Instantaneous velocity fields measured by the SPIV for the strong heat input $Ra_f = O(10^7)$ influenced by weak rotation $Ro = O(10^0 - 10^1)$: (a) the upper SO layer at $z = 25$ mm and (b) the lower GS layer at $z = 15$ mm. The in-plane velocity vectors (u, v) represented by black arrows are superposed on the color contours showing the out-of-plane velocity w . For the sake of clarity, one of two vectors is displayed both in the x - and y -directions. The time elapsed from the start of the experiment is noted above the images. The scale for the vectors are adjusted in each snapshot for visibility.

structures displayed in Fig. 1.9 and Fig. 1.11 were not principally different.

1.4.3 Two-layer convection observed in Ra values

The temperature differences and the corresponding Ra values represent the amount of heat transported by convection motion, and indicate the degree of development of convection as bulk values. In the present study, heat was supplied from the bottom and the flow structures are spontaneously established by the convection only. The convection motion in the lower layer leads to an increase in temperature at the interface and the Ra value in the lower layer, which is anticipated to increase until the onset of convection in the upper layer. However, the development of Ra values after the onset of convection in both of the fluid layers are difficult to deduce because the mutual interactions between the two layers need to be considered.

To investigate the development of the Ra values, the present study directly measured the temperature profiles with the five thermistors shown in Fig. 1.6(a) and (b). The Ra value in the lower layer Ra_1 is calculated using the temperature difference $\Delta T_1 = T_{\text{bot}} - T_{\text{int}}$, where T_{bot} and T_{int} are the temperature at the bottom and interface of the fluid layers, and T_{int} is the averaged value of the temperature values of the three thermistors set on the side wall of the fluid container aligned along the interface level ($z = 20$ mm). The Ra value in the upper layer Ra_2 is calculated using the temperature difference $\Delta T_2 = T_{\text{int}} - T_{\text{air}}$, where T_{air} is the temperature measured in the air immediately above the fluid. Please note that the temperature dependence of the physical properties of the test fluids is considered by using mean temperatures, $(T_{\text{bot}} + T_{\text{int}})/2$ and $(T_{\text{int}} + T_{\text{air}})/2$, as the mean temperature is more than 20 K above the initial room temperature after 90 min of the strong heat input.

The Ra values plotted in the Ra_1 - Ra_2 domain are shown in Fig. 1.12, where the solid and dotted lines represent the cases with and without rotation. The temporal evolution of the Ra values is noted by the diamonds plotted for every 20 min from imposing the heating. The error margins for each Ra plot are $\sim 2\%$ at the worst, which originated in the relative deviations of the three thermistor sensors located at the interface. For all cases, the Ra_1 values initially assume large Ra_1 values after the heating. For the weak heat input case without rotation, the Ra_1 is an almost constant $Ra_1 \sim 1.2 \times 10^4$ after the onset of convection in the lower layer, while the Ra_2 increases with time and exceeds Ra_1 at $t \sim 60$ min. The development of the Ra values corresponds to those of the flow fields shown in Fig. 1.8; initially the lower layer started the convection and subsequently the upper layer reached onset of convection, and the flow structures in the lower layer were sustained after $t \sim 30$ min. The result of the weak heat input case with rotation shows constant profiles of $Ra_1 \sim 8.5 \times 10^3$ while Ra_2 increases with time, and the changes in the Ra values follow that without rotation. The Ra_1 values in the case with rotation are, however, lower than those without rotation, and the Ra_2 values with rotation record similar magnitudes to those with rotation after a 20 to 30 min delay. This time lag can be seen in the flow fields shown in Fig. 1.8(a) and 1.10(a) as the delay in the onset of convection in the upper

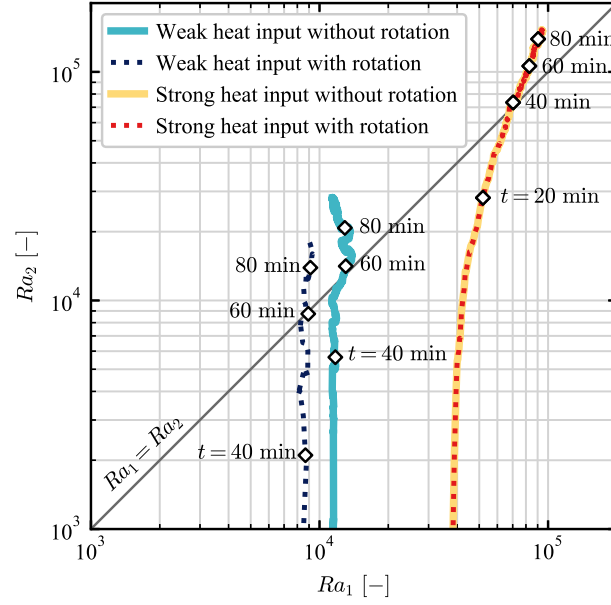


Figure 1.12: Plots of the Ra values in the lower (Ra_1) and upper layers (Ra_2). The solid lines show the cases without rotation as light blue for the weak heat input and yellow for the strong heat input, and the dotted lines show the rotation as dark blue for the weak heat input and red for the strong heat input. The diamonds represent the duration of the heating. The $t = 20$ min data for the weak heat input cases are not presented in the plot, because the Ra_2 values stayed below 10^3 at the time.

layer. The smaller Ra_1 values with rotation are the result of suppression of convection due to the background rotation. The two Ra values in the strong heat input cases with and without rotation are also shown in Fig. 1.12, and the yellow solid and red dotted curves match well. The Ra values of $O(10^4 - 10^5)$ in the strong heat input cases are larger than those of $O(10^3 - 10^4)$ in the weak heat input cases. For the strong heat input cases, the Ra_2 exceeds the Ra_1 at $t \sim 40$ min, and both the Ra_1 and the Ra_2 continue to increase throughout the experiments. This is because of the increase in the mean temperature decreasing the Pr values, and the convection structures in the lower layer develop even after the onset of convection in the upper layer. These developments in the Ra values express the flow fields which do not reach any specific terminal state shown in Fig. 1.9 and 1.11.

In principle, rotation suppresses the development of convection as it promotes two-dimensionality in horizontal directions. This is consistent with the difference in the length scale of the convection cells formed in the lower layer shown in Fig. 1.8(b) and Fig. 1.10(b); the case without rotation shows separated convection cells surrounding the central cell while with rotation it shows a large toroidal roll. The flow patterns in the upper layer shown in Fig. 1.8(a) and Fig. 1.10(a) are similar with the time lag in the development of the convection structures. Considering that the development of the convection in the strong heat input cases are very similar, the influence of rotation is prominent especially with the strong rotation relative to the thermal forces, with the Ro values $Ro \sim 10 - 20$, while those in the strong heat input case are $Ro \sim 20 - 60$. The initial states for each experimental condition were carefully set up as described in Sec. 1.2.2. The global trends discussed above thus are repeatable, in terms of length- and time-scale, while the

positions of local convection structures especially in the lower layer change in each experiment due to the cylindrical fluid layer. The local structures in the lower layer, however, do not have to be repeatable, as the convection in the upper layer develops following the convection in the lower layer. Accordingly, it is possible to consider the global trends of the development of two-layer convection and accompanying coupling structures in the present study as robust results.

1.5 Evaluation of coupling structures

To evaluate the mutual interactions between the two fluid layers in a 3D domain, methods to identify the coupling structures must be defined. In the realized flow fields, the magnitudes of all the three velocity components are in $O(10^{-1} - 10^0 \text{ mm/s})$, regardless of the directions. Accordingly, all the velocity components can be dealt with as equally significant quantities in evaluating the coupling structures. The present study suggests two different methods for evaluating the coupling structures by using the detailed velocity information and discusses the effect of rotation on the coupling structures.

1.5.1 Cross-correlation

One of the methods is cross-correlation, which quantifies similarities between multiple velocity values, and may be useful in characterizing coupling structures. For two datasets of a single velocity component, ξ_1 and ξ_2 , the cross-correlation C_ξ is defined as

$$C_\xi = \frac{\sum (\xi_1 - \bar{\xi}_1) (\xi_2 - \bar{\xi}_2)}{\sqrt{\sum (\xi_1 - \bar{\xi}_1)^2} \sqrt{\sum (\xi_2 - \bar{\xi}_2)^2}}, \quad (1.6)$$

where $\bar{\xi}_1$ and $\bar{\xi}_2$ are the mean values of each dataset. Here, the cross-correlations C_u , C_v , and C_w for the various velocity components u , v , and w are calculated to characterize the coupling structures. Please note that, the same velocity component from the two fluid layers, u_1 and u_2 , v_1 and v_2 , w_1 and w_2 , are utilized for the computation of the cross-correlations. The larger (smaller) values of C_u and C_v and the smaller (larger) values of C_w represent the flow structures globally dominated by the stronger mechanical (thermal) coupling. The cross-correlations are calculated from both the 1D and 2D velocity profiles, assuming limited (1D) and ideal (2D) measurement environments in the application stages. For the 1D cross-correlations, the C_ξ values are calculated using only 1D velocity profiles along a diameter at four θ positions, $\theta = 0, \pi/4, \pi/2$, and $3\pi/4$.

The temporal evolution of the cross-correlations in the cases without rotation are shown in Fig. 1.13, here the red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers. The onset of convective motions were identified by computing temporal evolution of the kinematic energy in each layer, and the sudden increase in the kinematic energy

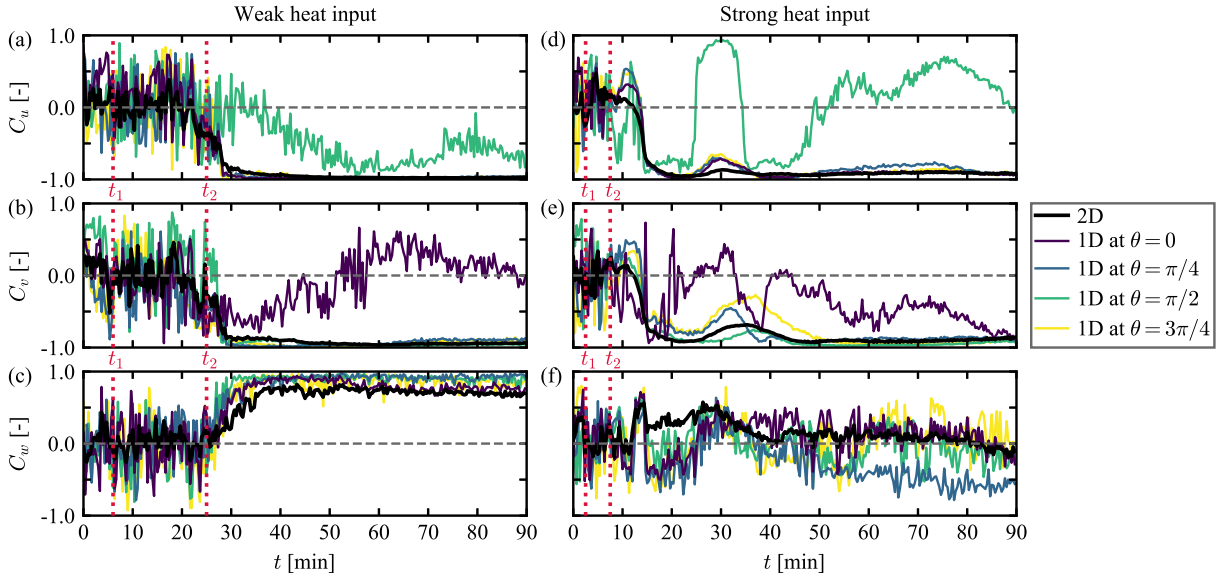


Figure 1.13: Cross-correlations without rotation, $Ro = \infty$. The cross-correlations C_u , C_v , and C_w are plotted in (a), (b), and (c) for the weak heat input case $Ra_f = O(10^5)$, and in (d), (e), and (f) for the strong heat input case $Ra_f = O(10^6)$. The 2D black lines in each panel represent the cross-correlations for the 2D velocity profiles, and the gray lines show the cross-correlations for the 1D velocity profiles in the different θ positions, $\theta = 0, \pi/4, \pi/2$, and $3\pi/4$. The red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers.

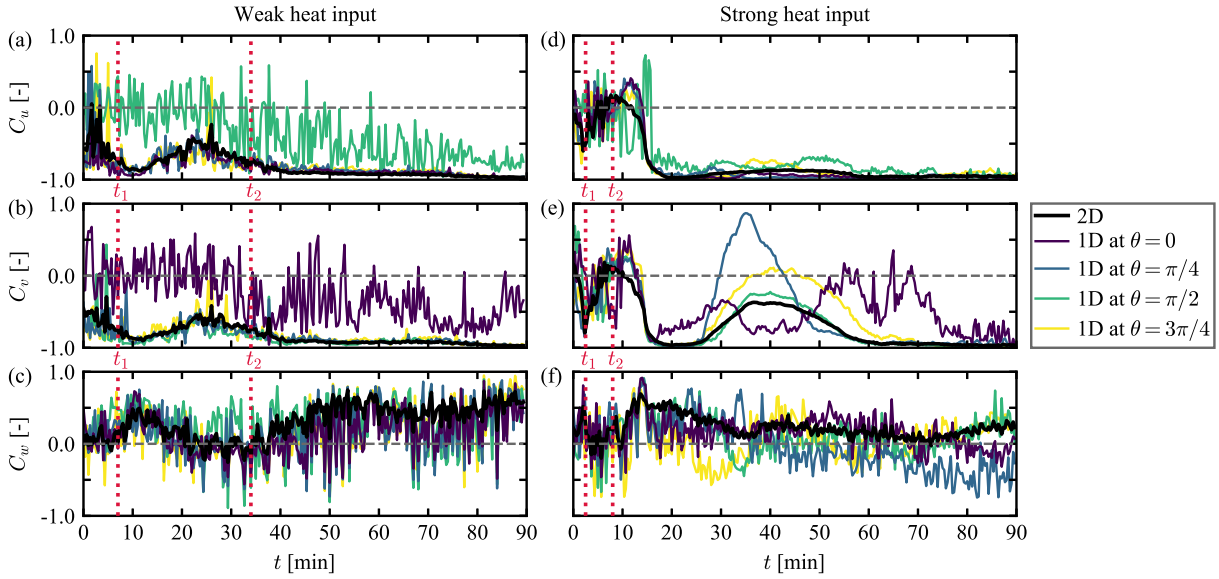


Figure 1.14: Cross-correlations with rotation, $Ro = O(10^0 - 10^1)$. The cross-correlations C_u , C_v , and C_w are plotted in (a), (b), and (c) for the weak heat input case $Ra_f = O(10^5)$, and in (d), (e), and (f) for the strong heat input case $Ra_f = O(10^6)$. The 2D black lines in each panel represent the cross-correlations for the 2D velocity profiles, and the gray lines show the cross-correlations for the 1D velocity profiles in the different θ positions, $\theta = 0, \pi/4, \pi/2$, and $3\pi/4$. The red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers.

was determined as the onset of the convection in the layer. For the case with weak heat input [Fig. 1.13(a-c)], all the cross-correlations for the first 20 min during which no convection occurred in the upper layer fluctuate around zero with large deviations. At $t \sim 25$ min, the 2D C_u and C_v drop to ~ -1 , and the 2D C_w increases steeply to ~ 0.8 suggesting a global thermal coupling structure, and may indicate the onset of convection in the upper layer. The changes in the 2D C_u and C_v also appear in three of the four 1D curves not in the 1D C_u at $\theta = \pi/2$ and the 1D C_v at $\theta = 0$, while all of the 1D C_w values show trends similar to the 2D curves. The results with strong heat input [Fig. 1.13(d-f)] also display fluctuations around zero for the first 10 min at all the cross-correlations. This duration of the unstable period is shorter than the 20 min-fluctuations in the case with the weak heat input [Fig. 1.13(a-b)], suggesting a more rapid onset of convection in the upper layer with the strong heat input. After the first 10 min of fluctuations in the case with the strong heat input, the 2D C_u and C_v profiles are negative till $t = 90$ min [Fig. 1.13(d) and (e)], while both the 1D and 2D C_w profiles stay around zero throughout the measurements [Fig. 1.13(f)].

The cross-correlations with rotation are detailed in Fig. 1.14. The 2D C_u and C_v profiles with the weak heat input subject to the rotation [Fig. 1.14(a) and (b)] are negative from the beginning of the heating till $t = 90$ min, while the 2D C_w profiles fluctuate around zero and become stable around 0.5 from $t \sim 40$ min. Three of the four 1D C_u and C_v curves not in the 1D C_u at $\theta = \pi/2$ and the 1D C_v at $\theta = 0$, are similar to the corresponding 2D profiles, while both the 1D and 2D C_w profiles show a similar temporal evolution. The results with the strong heat input with rotation [Fig. 1.14(d-f)] suggest that the profiles of the cross-correlations are largely similar to those in the case without rotation [Fig. 1.13(d-f)]. This reflects the negligible influence of the rotation on convection as discussed with the evolution of the flow fields (Fig. 1.9 and 1.11).

The 1D cross-correlations cannot describe the global fashions, as these are sensitive to the velocity signals localized along the diameter. For instance, the 1D C_u at $\theta = \pi/2$ in Fig. 1.13(d) shows an abrupt increase at $t = 25 - 35$ min. Most of the u values along a diameter at $\theta = \pi/2$ was recorded as zero and significant cross-correlation could not be computed at the duration. An occasional coincidence of u values in the two layers due to an abrupt flow evolution leads to such a sudden increase in the C_u . Similarly, the 1D C_v at $\theta = 0$ in Fig. 1.13(b) and (d) show irregular signals, while the 2D C_u and C_v do not show abrupt changes. From these results, the 1D observations may not be sufficient to identify coupling structures of the 3D system, and the coupling structures need to be considered as the interfacial interactions of the two layers. For this reason, an observation of a vertical cross-section may not provide the global trend of the 3D system.

As above, the 2D cross-correlations seem to describe the coupling structures. The cross-correlation is, however, a representative value for describing a global relation between two datasets, and local relations at each grid point are disregarded or underestimated. Besides, the cross-correlation cannot represent the concordances of the flow directions, because it compares

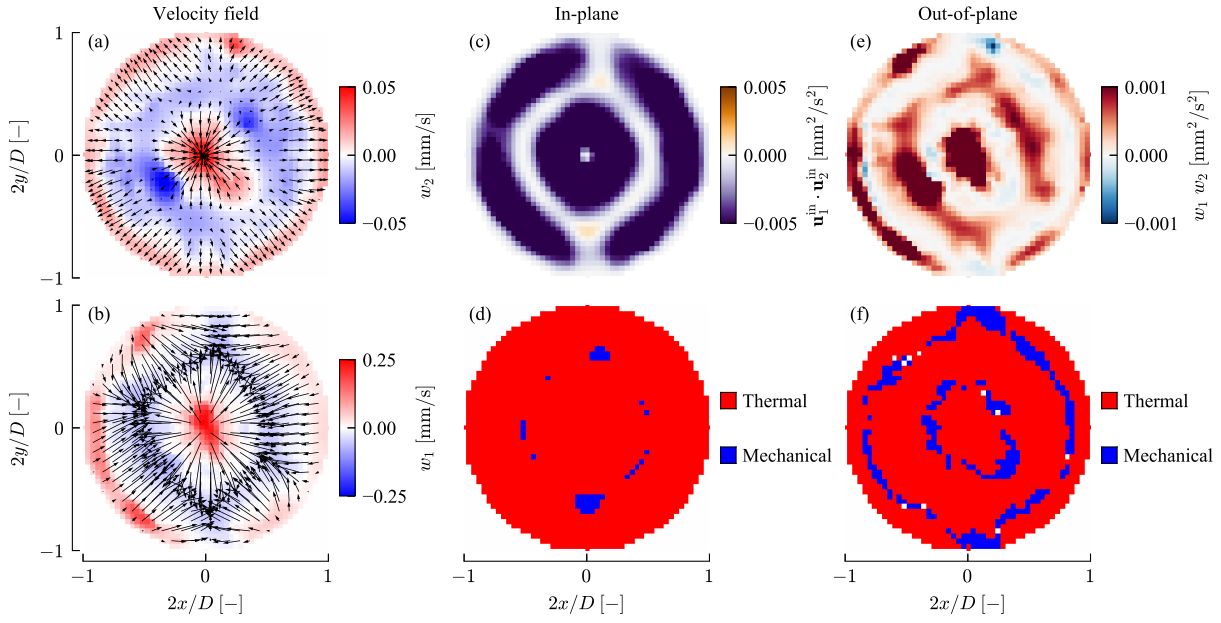


Figure 1.15: An example of the local coupling states for the weak heat input without rotation at $t = 42$ min: (a) and (b) velocity fields at the upper and lower layer, respectively showing the in-plane velocity vectors as black arrows and the out-of-plane velocity as color contours. (c) map of the inner products of the in-plane velocity vectors $\mathbf{u}_1^{\text{in}} \cdot \mathbf{u}_2^{\text{in}}$, and (d) map of the local coupling states defined using the in-plane criterion. (e) map of the multiplied out-of-plane velocity $w_1 w_2$, and (f) map of the local coupling states defined using the out-of-plane criterion.

the deviations of each dataset regardless of the absolute values of the datasets.

1.5.2 Dominance ratio

To evaluate the coupling structures of the whole system, an integration of local coupling structures must be considered, not just the cross-correlations. The local coupling structures can be defined gridwise by the relative directions of two velocity vectors in the two layers; a pair of velocity vectors in identical horizontal directions represent the mechanical coupling [Fig. 1.3(a)], and that with identical vertical directions indicate the thermal coupling [Fig. 1.3(b)]. A relative angle $\psi_{i,j}$ between two horizontal (in-plane) velocity vectors of different cross-sections at a grid point (i, j) , $\mathbf{u}_{1,i,j}^{\text{in}} = [u_1, v_1]_{i,j}^T$ and $\mathbf{u}_{2,i,j}^{\text{in}} = [u_2, v_2]_{i,j}^T$, appears as the inner product of the two vectors

$$\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} = |\mathbf{u}_{1,i,j}^{\text{in}}| |\mathbf{u}_{2,i,j}^{\text{in}}| \cos \psi_{i,j}. \quad (1.7)$$

The sign of $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$ represents the local coupling structure as $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} > 0$ for the mechanical and $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} < 0$ for the thermal coupling. For the vertical (out-of-plane) velocities, $w_{1,i,j}$ and $w_{2,i,j}$, the sign of $w_{1,i,j} w_{2,i,j}$ also suggests the local coupling structure as $w_{1,i,j} w_{2,i,j} > 0$ for the thermal and $w_{1,i,j} w_{2,i,j} < 0$ for the mechanical coupling. Here, grid points where $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} = 0$ or $w_{1,i,j} w_{2,i,j} = 0$ cannot be characterized for any specified coupling structure, and all the grid points can be ternarized to the mechanical, thermal, or unspecified coupling structures. In the present study, all the measured velocity data were utilized

without setting any thresholds for the velocity values.

Examples of the in-plane and out-of-plane criteria, $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$ and $w_{1,i,j} w_{2,i,j}$, calculated from the flow fields measured in the experiments are shown in Fig. 1.15. Instantaneous velocity fields in the upper and lower layer for the case with the weak heat input without rotation at $t = 42$ min are shown in Fig. 1.15(a) and (b). The in-plane velocity vectors $\mathbf{u}^{\text{in}}$ represented by the black arrows are superposed on the color contours showing the out-of-plane velocities w . At the displayed moment, the cross-correlations C_u , C_v , and C_w suggest a global thermal coupling [Fig. 1.13(a-c)]. The corresponding map of the inner products of the in-plane velocity vectors $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$ is shown in Fig. 1.15(c). There are three large coherent regions with negative values, one at the center and two at the periphery. The boundaries of these three regions coincide with those of the convection cells shown in Fig. 1.15(b). A map of the local coupling structures after the ternarization of $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$ is shown in Fig. 1.15(d), here most regions of the fluid layer are occupied by thermal coupling colored red, and blue regions indicating mechanical coupling appear at the borders of the convection cells. A map of the multiplied out-of-plane velocities $w_{1,i,j} w_{2,i,j}$ is shown in Fig. 1.15(e). The patterns in Fig. 1.15(e) are more complicated than those in Fig. 1.15(c), and most of the grids display positive values. In particular, there are large positive values at the center in Fig. 1.15(e), where there are strong up-welling vertical flows in the flow fields [Fig. 1.15(a) and (b)]. A map of local coupling structures after the ternarization is shown in Fig. 1.15(f), here most regions of the fluid layer show the local thermal coupling (red), and the regions assigned as the local mechanical coupling (blue) appear along the border of the convection cells. The results displayed in Fig. 1.15(d) and (f) show the thermal coupling at most of the grid points, here Fig. 1.15(f) shows larger regions assigned as the mechanical coupling along the borders of the convection cells than those in Fig. 1.15(d).

To describe the global coupling structures from the integration of the local coupling structures, the number of grid points of specific local coupling structures were counted using the Heaviside step function H ,

$$H(\zeta) = \begin{cases} 1 & : \zeta > 0 \\ 0 & : \zeta < 0 \end{cases}, \quad (1.8)$$

for a variable ζ representing $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$ or $w_{1,i,j} w_{2,i,j}$. For the in-plane criterion $\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}$, the numbers of grid points of local mechanical and thermal coupling structures are calculated as

$$N_{\text{m}}^{\text{in}} = - \sum H \left(\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} \right), \quad (1.9)$$

and

$$N_{\text{t}}^{\text{in}} = \sum H \left(-\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}} \right), \quad (1.10)$$

respectively. Here, for convenience, these two values are defined as $N_m^{\text{in}} \leq 0$ and $N_t^{\text{in}} \geq 0$. The total number of the grid points assigned to either coupling structure is calculated as

$$N^{\text{in}} = \sum H(|\mathbf{u}_{1,i,j}^{\text{in}} \cdot \mathbf{u}_{2,i,j}^{\text{in}}|). \quad (1.11)$$

Similarly, for the out-of-plane criterion $w_{1,i,j} w_{2,i,j}$, the numbers of grid points of local mechanical and thermal coupling structures are calculated as

$$N_m^{\text{out}} = - \sum H(-w_{1,i,j} w_{2,i,j}), \quad (1.12)$$

and

$$N_t^{\text{out}} = \sum H(w_{1,i,j} w_{2,i,j}), \quad (1.13)$$

respectively. These two values are defined as $N_m^{\text{out}} \leq 0$ and $N_t^{\text{out}} \geq 0$ for ease of the subsequent calculations, and the total number of grid points of either coupling structure is calculated as

$$N^{\text{out}} = \sum H(|w_{1,i,j} w_{2,i,j}|). \quad (1.14)$$

The degree of global coupling structure is defined as the dominance ratio ϕ for both the in-plane and the out-of-plane criteria as

$$\phi^{\text{in}} = \frac{N_m^{\text{in}} + N_t^{\text{in}}}{N^{\text{in}}}, \quad (1.15)$$

and

$$\phi^{\text{out}} = \frac{N_m^{\text{out}} + N_t^{\text{out}}}{N^{\text{out}}}. \quad (1.16)$$

Here ϕ is defined in $\phi \in [-1, 1]$, and $\phi \sim -1$ indicates global mechanical coupling and $\phi \sim 1$ represents global thermal coupling structures. For example, the dominance ratios calculated for the case shown in Fig. 1.15 are $\phi^{\text{in}} = 0.957$ and $\phi^{\text{out}} = 0.707$, and the whole may be regarded as a global thermal coupling structure at this moment, as could be expected from the cross-correlations shown in Fig. 1.13(a-c).

Temporal evolution of the dominance ratios are shown in Fig. 1.16 for cases without rotation and Fig. 1.17 for cases with rotation. The black solid lines represent ϕ values, and the positive (pink-filled) and negative (blue-filled) regions represent the evolution of the ratios of N_t and N_m to N . The red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers. The results for the weak heat input case without rotation are shown in Fig. 1.16(a) and (b), and both the ϕ^{in} and ϕ^{out} profiles fluctuate around zero for the first 20 min and increase toward a thermal coupling. Similar to the cross-correlations shown in Fig. 1.13(a-c). For the

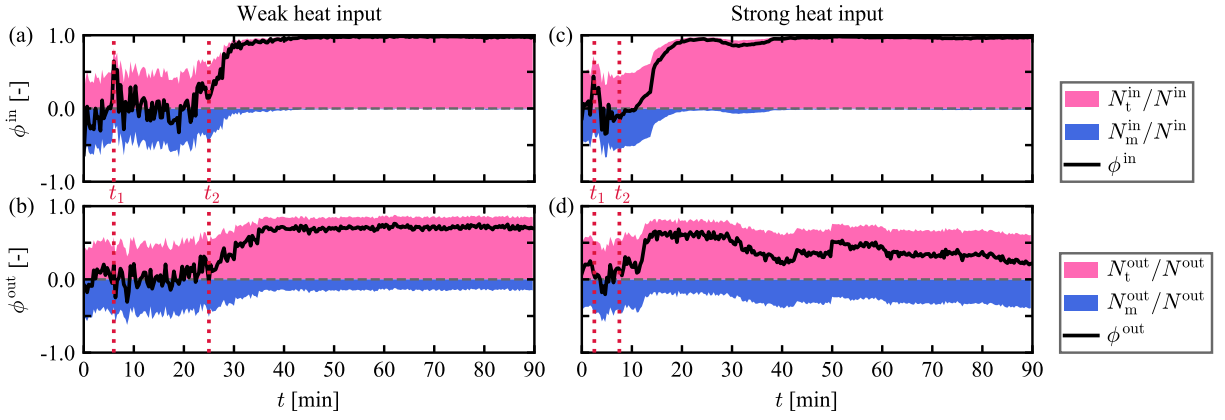


Figure 1.16: Temporal evolution of the dominance ratios ϕ without rotation, $Ro = \infty$: (a) ϕ^{in} and (b) ϕ^{out} for the weak heat input case $Ra_f = O(10^5)$, and (c) ϕ^{in} and (d) ϕ^{out} for the strong heat input case $Ra_f = O(10^6)$. Black solid lines represent ϕ , and positive and negative filled regions represent the ratios of N_t and N_m to N , respectively. The red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers.

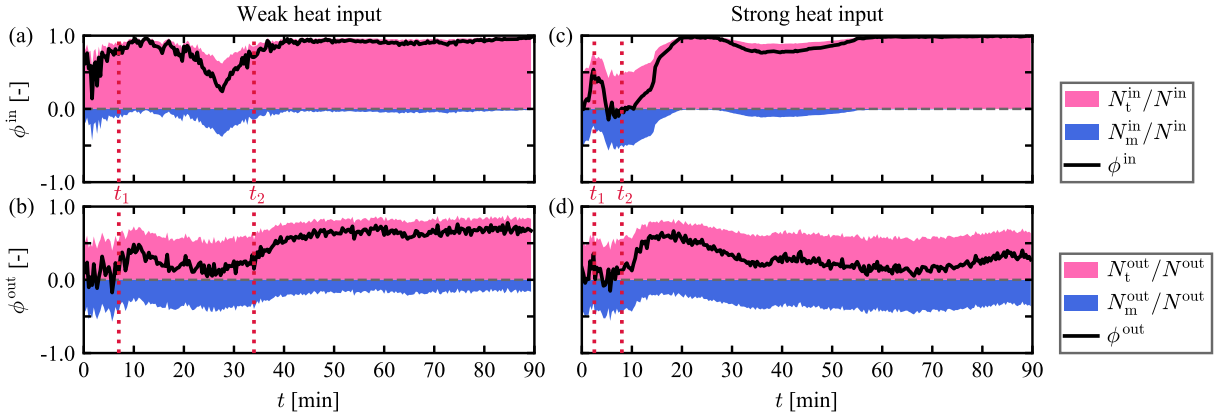


Figure 1.17: Temporal evolution of the dominance ratios ϕ with the weak rotation, $Ro = O(10^0 - 10^1)$: (a) ϕ^{in} and (b) ϕ^{out} for the weak heat input case $Ra_f = O(10^5)$, and (c) ϕ^{in} and (d) ϕ^{out} for the strong heat input case $Ra_f = O(10^6)$. Black solid lines represent ϕ , and positive and negative filled regions represent the ratios of N_t and N_m to N , respectively. The red dotted lines indicate the time of the onset of convection in the lower (t_1) and upper (t_2) layers.

strong heat input case without rotation shown in Fig. 1.16(c), the ϕ^{in} profiles start to increase at $t \sim 10$ min and stay at large values suggesting global thermal coupling till $t = 90$ min. The ϕ^{out} values [Fig. 1.16(d)] are smaller than those of ϕ^{in} [Fig. 1.16(c)]. The evolution of the dominance ratios and the cross-correlations are very similar, however, the ϕ^{in} profiles shown in Fig. 1.16(c) are more stable than the C_u and C_v profiles in Fig. 1.13(d) and (e).

The results for the weak heat input case with rotation are shown in Fig. 1.17(a) and (b), and both the ϕ^{in} and ϕ^{out} are positive throughout with a sudden decrease of the ϕ^{in} at $t \sim 27$ min. Compared to the cross-correlations in the same condition [Fig. 1.14(a-c)], the dominance ratios suggest a stronger global thermal coupling structure here. The results for the strong heat input case with the rotation [Fig. 1.17(c) and (d)] are largely similar to those without rotation [Fig. 1.16(c) and (d)], while the ϕ^{out} values in Fig. 1.17(d) are slightly smaller than those in Fig. 1.16(d).

The principal trends of the global coupling structures described by the dominance ratios (Fig. 1.16 and 1.17) are similar to the 2D cross-correlations (Fig. 1.13 and 1.14). The in-plane criteria showed $C_u \sim -1$, $C_v \sim -1$, and $\phi^{\text{in}} \sim 1$ providing the global thermal coupling for all conditions, and cannot express unique flow states. However, the dominance ratios are positive or negative while the 2D cross-correlations sometimes showed random-like fluctuations around zero for long periods, and the dominance ratios are stable after the onset of convection while the 2D cross-correlations show large deviations. The out-of-plane criteria C_w [Fig. 1.13(c, f) and Fig. 1.14(c, f)] and ϕ^{out} [Fig. 1.16(b, d) and Fig. 1.17(b, d)] showed unique values in each condition. Based on these results, the in-plane criteria cannot be regarded as sufficiently sensitive to represent the different flow structures, however the coupling structures can be determined using the out-of-plane velocities. As the dominance ratio for the out-of-plane velocity ϕ^{out} only requires the sign of the vertical velocities, directions of the vertical flows, it is possible to define the local coupling structure at the measurement point. Incorporating all of this discussion, the coupling structures of multi-layer convection in a 3D domain can be evaluated from the aspect of the dominance ratio ϕ^{out} , an integration of the local coupling structures.

1.5.3 Effect of rotation on coupling structures

The effect of rotation on the coupling structures is quantified as the probability density function (P.D.F.) of the dominance ratios as shown in Fig. 1.18. The dominance ratios after $t = t_2$, the onset of convection in the upper layer, are taken for the P.D.F. of Fig. 1.18. The circles and triangles are for the ϕ^{in} and ϕ^{out} , and temporal mean values of these, $\langle \phi^{\text{in}} \rangle$ and $\langle \phi^{\text{out}} \rangle$, are shown as the blue dashed and red dotted lines. For all the conditions, the peaks of the P.D.F. are on the positive side of the ϕ , corresponding to a global thermal coupling structure. Both the P.D.F. and mean values of ϕ^{in} do not show significant differences among the conditions with and without rotation. However, it is noteworthy that the peak of P.D.F. of ϕ^{out} under the condition of strong heat input with rotation [Fig. 1.18(d)] shifts to left compared to that without rotation [Fig. 1.18(b)]. This suggests that the global thermal coupling is weakened due to the presence of rotation. From the weak heat input conditions, shown in Fig. 1.18(a) and (c), a similar trend can be identified, while the peak shift is smaller than that in Fig. 1.18(d).

As discussed in Sec. 1.4.3, rotation suppresses the developments of convection, especially in the weak heat input condition as shown in Fig. 1.12. In addition, rotation also affects the coupling structures as above. The global thermal coupling structures are weakened by rotation, in the developing stages of the two-layer convection investigated in the present study. The effect of rotation on the coupling structures, however, may not be universal and vary depending on the conditions like Ro values, and this requires further investigation in the future.

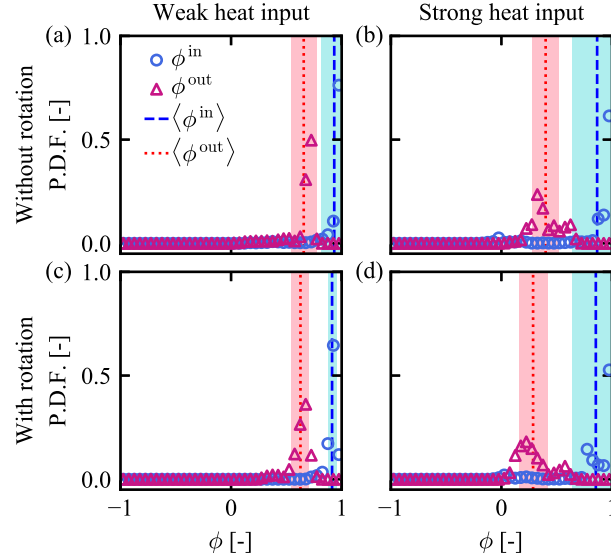


Figure 1.18: Probability density function (P.D.F.) of the dominance ratios after the onset of convection in the two fluid layers: (a) weak heat input without rotation, (b) strong heat input with rotation, (c) weak heat input with rotation, and (d) strong heat input with rotation. Circles and triangles are for ϕ^{in} and ϕ^{out} , respectively. Vertical lines represent temporal mean values of the dominance ratios, $\langle \phi^{\text{in}} \rangle$ (blue dashed) and $\langle \phi^{\text{out}} \rangle$ (red dotted). Colored areas represent the error bars (standard deviations) corresponding to the temporal means.

1.6 Summary

To investigate coupling structures of buoyancy induced convection in two immiscible layers, laboratory experiments imposing constant heat fluxes at the bottom were performed using a glycerol solution and a silicon oil as the test fluids. A cylindrical vessel that allowed to form 3D convection structures was employed, and the influence of background rotation on the convection motion imitating the rotation of the Earth was investigated. To evaluate coupling structures in the 3D domain, an experimental and an analytic scheme to simultaneously measure the flow fields of the two layers via the stereoscopic particle image velocimetry (SPIV) was established. Two horizontal cross-sections, one for each layer, were illuminated simultaneously by two differently colored lasers to decompose a single particle image recorded in a camera into two images via the image decomposition method utilizing hue information. The SPIV measurement was applied to the decomposed particle images, and the horizontal distributions of velocity vectors with three components at the two different fluid layers were simultaneously quantified.

The developments of the velocity fields indicate that the lower layer was the first reach onset of convection and subsequently the upper layer initiated convection. As expected, the influence of the background rotation stood out especially in the weak heat input case, and convection motion in both of the two layers was suppressed. The different developments of the convection motion by the contrasting conditions could be also observed in the Rayleigh numbers as well as the flow fields.

The coupling structures in the 3D domain were evaluated quantitatively to consider the mutual interactions of the two layers, by using the simultaneously measured velocity information

of each layer. The cross-correlations of the multiple velocity components tend to blur the degree of global coupling structures because these do not openly display the local trends. Meanwhile, the dominance ratio, considering the combinations of the flow directions of adjoining layers, indicated better descriptions of the degree of global coupling structures in a 3D domain as the ratio of the local coupling structures. The coupling structures evaluated in terms of the dominance ratio were largely in the thermal coupling for all the explored conditions. However, the global thermal coupling structures were weakened by the addition of rotation.

DUAL-PLANE ENSEMBLE CORRELATION FOR PIXELWISE 2D-3C VELOCITY FIELD MEASUREMENTS

Preface

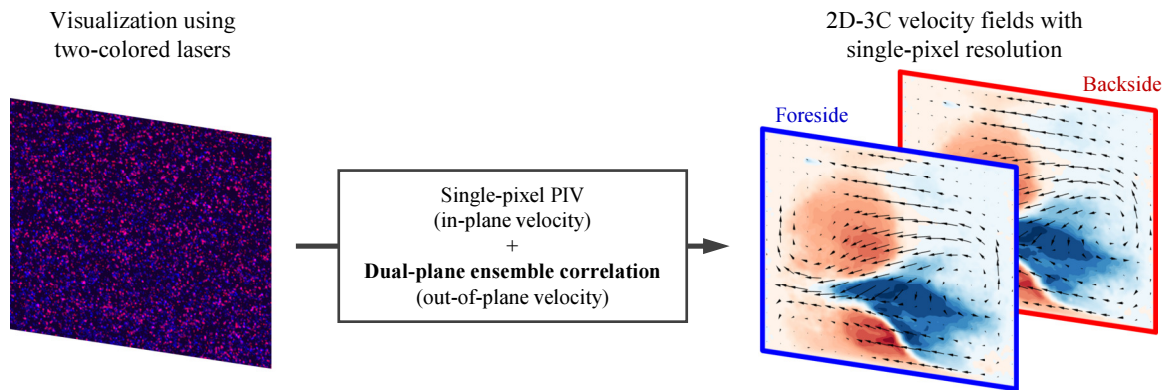


Figure 2.1: Graphical abstract of this chapter.

In Chapter 1, two planes illuminated by different colored laser sheets were successfully separated based on color information of the visualized images. The simple color-based separation technique distinguished motions of particles seeded in two immiscible test fluids. Since there was no mixing of the fluids, the separation of the differently-colored particles directly means the separation of the two liquid phases. In addition to the established method, employment of one more camera for stereoscopic PIV configuration allowed measuring 2D-3C velocity fields at two different planes. Employment of another camera, however, does not ease the complexity of the optical configurations in stereoscopic PIV and tomographic PIV.

The concept of the color-based plane separation is, of course, applicable to the simultaneous visualization of the two different planes in a continuous single phase. If only a single camera is employed, 2D-2C velocity fields at two different planes will be obtained by proceeding the color-based image separation, and the standard PIV, while it is possible to measure the out-of-plane velocity components by employing another camera. In the single-phase flows, however, the seeded particles can appear both the two illuminated planes unlike the immiscible two-phase flows. Utilizing this fact, it may be possible to measure the out-of-plane velocity components without adopting another camera. A similar concept was proposed earlier by Cenedese and Paglialunga [26], Gharib et al. [52] as dual-plane PIV, while it could not expand its applications due to the measurement limitations.

In Chapter 2, as an extension of single-pixel particle image velocimetry (SP-PIV), a novel method to determine both the out-of-plane and in-plane velocity fields with single-pixel resolution is presented. With this approach, called dual-plane SP-PIV, mean velocity vector fields with three components (3C) can be measured with single-pixel resolution using a single-color camera and two differently colored laser sheets. The method computes the ensemble correlation across two planes illuminated simultaneously by the two laser sheets. The method is demonstrated via an experiment in which a rotating flow is induced by an impeller. The method is validated by comparing its results with those of conventional PIV. The results show good accordance in terms of the magnitudes and spatial distributions of the out-of-plane velocities and provide a new approach to 3C velocity field measurements with high spatial resolution.

The results shown in Chapter 2 were published as Noto and Tasaka [121] and the details are below.

Author Daisuke Noto and Yuji Tasaka

Title Dual-plane ensemble correlation for pixelwise 2D-3C velocity field measurements using a single camera

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2.1 Introduction

Single-pixel particle image velocimetry (SP-PIV) was first introduced by Westerweel et al. [208] to measure mean velocity fields with single-pixel resolution, and it has since been extended to enable the measurement of unsteady and turbulent flows [83, 120, 130, 168].

Comparison of the different correlation methods employed in the conventional PIV and the SP-PIV is schematically illustrated in Fig. 2.2. The conventional PIV is based on the spatial correlation; an interrogation window is set to include at least two or three particles within it, and search the most plausible displacement of the interrogation window between the two consecutive frames. While the PIV can measure instantaneous velocity fields using particle images acquired only in two frames, the resultant velocity vectors can be regarded as a local spatial average due to the requirement of the interrogation window with a certain measurement volume of $N_x \times N_y$ pixels. For instance, the direct cross-correlation method of the PIV calculates the correlation maps as

$$R_{\text{PIV}}(x, y, \xi, \eta) = \frac{\sigma_{01}(x, y, \xi, \eta)}{\sigma_0(x, y)\sigma_1(x, y, \xi, \eta)}, \quad (2.1)$$

where $\sigma_{01}(x, y, \xi, \eta)$ is the covariance of the image pair, $I_0 = I(t = t_0)$ and $I_1 = I(t = t_0 + \Delta t)$,

$$\sigma_{01}(x, y, \xi, \eta) = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} [I_0(x+i, y+j) - \bar{I}_0] [I_1(x+i+\xi, y+j+\eta) - \bar{I}_1]. \quad (2.2)$$

$\sigma_0(x, y)$ and $\sigma_1(x, y, \xi, \eta)$ are the standard deviations of each image computed as,

$$\sigma_0(x, y) = \sqrt{\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} [I_0(x+i, y+j) - \bar{I}_0]^2}, \quad (2.3)$$

and

$$\sigma_1(x, y, \xi, \eta) = \sqrt{\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} [I_1(x+i+\xi, y+j+\eta) - \bar{I}_1]^2}, \quad (2.4)$$

where $\bar{I}_0$ and $\bar{I}_1$ are the mean brightness values within the interrogation window. The displacement vectors can be acquired as the maximal correlation position of the correlation map space (ξ, η) .

In contrast, the SP-PIV is based on single-pixel ensemble correlation, which computes pixelwise cross-correlation maps using temporal brightness signals extracted from sequential particle images, unlike the conventional PIV using image pairs. This can be written as

$$R_{\text{SPPIV}}(x, y, \xi, \eta) = \frac{\sigma_{0\Delta t}(x, y, \xi, \eta)}{\sigma_0(x, y)\sigma_{\Delta t}(x, y, \xi, \eta)}, \quad (2.5)$$

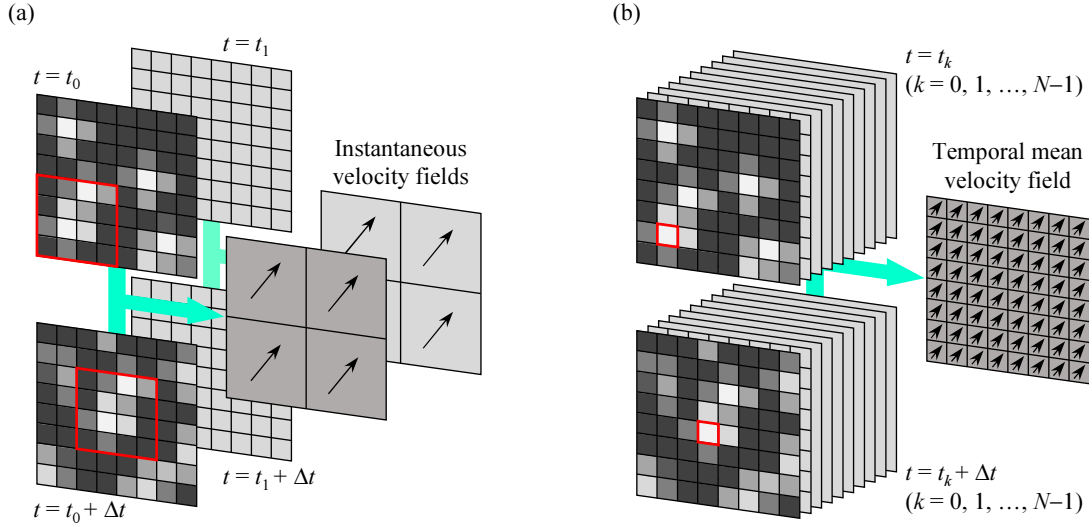


Figure 2.2: Conceptual images of (a) the standard PIV and (b) the SP-PIV.

where $\sigma_{0\Delta t}(x, y, \xi, \eta)$ is the covariance of the pair of brightness signals from the continuous image series, $I = I(t = t_k)$ and $I_{\Delta t} = I(t = t_k + \Delta t)$ with $k \in [0, N - 1]$,

$$\sigma_{0\Delta t}(x, y, \xi, \eta) = \sum_{k=0}^{N-1} [I(x, y, t_k) - \bar{I}] [I_{\Delta t}(x + \xi, y + \eta, t_k + \Delta t) - \bar{I}_{\Delta t}]. \quad (2.6)$$

$\sigma_0(x, y)$ and $\sigma_{\Delta t}(x, y, \xi, \eta)$ are the standard deviations of each brightness signal computed as,

$$\sigma_0(x, y) = \sqrt{\sum_{k=0}^{N-1} [I_0(x, y, t_k) - \bar{I}_0]^2}, \quad (2.7)$$

and

$$\sigma_{\Delta t}(x, y, \xi, \eta) = \sqrt{\sum_{k=0}^{N-1} [I_{\Delta t}(x + \xi, y + \eta, t_k + \Delta t) - \bar{I}_{\Delta t}]^2}, \quad (2.8)$$

where $\bar{I}_0$ and $\bar{I}_{\Delta t}$ are the mean brightness values of each brightness signal. Since the SP-PIV does not require spatial correlation unlike the conventional PIV, the resultant velocity fields allow pixelwise spatial resolution, even though the temporal resolution is discarded.

In early stage of the PIV developments, the out-of-plane velocity component was attempted to determined by adopting the idea of dual-plane PIV, which was earlier introduced by Cenedese and Paglialunga [26] and Gharib et al. [52]. Dual-plane PIV determines the out-of-plane velocity component using only a single camera and multiple sheet illuminations at different depths. This method, however, computes spatio-temporal correlations across multiple illuminated planes, and thus does not ensure sufficiently high accuracy in determining the out-of-plane velocity components when it is adopted in highly 3D flow fields.

The difference in these correlation calculations suggests that the SP-PIV is more robust in the

flow fields with large fluid deformations. This takes an advantage especially in the determination of the out-of-plane velocity fields. In this study, the idea of the dual-plane PIV is incorporated into the SP-PIV by implementing two parallel laser sheets with different colors to determine both the in-plane and out-of-plane velocity fields with single-pixel resolution. This incorporation will take the advantages of the two differently developed methods, high spatial resolution and third velocity component determination. The principle is addressed in Sec. 2.2, and the algorithm is implemented to highly 3D flows shown in Sec. 2.3. The results are validated by comparing the conventional PIV and a summary is provided in Sec. 2.4.

2.2 Principle

A conceptual image of the proposed method, called dual-plane ensemble correlation, is shown in Fig. 2.3. Here, one considers a situation in which two adjoining planes with a depth offset Δz are illuminated by two light sheets with different colors, and the particle images in the two planes are correctly separated into foreside I_f and backside images I_b in advance. If these illuminated planes are sufficiently large, a particle with a considerable out-of-plane velocity appearing on one plane should also appear on the other plane. The idea of ensemble correlation in SP-PIV thus can be applied to the temporal brightness signals across the two planes. Consider a situation in which a particle passes a position $\mathbf{x}_0 = (x_0, y_0)$ in one plane at frame $t = t_0$. If the out-of-plane velocity at $\mathbf{x}_0$ is not negligible, this particle appears at $\mathbf{x}_\zeta$ in the other plane before or after $t = t_0$ at a different frame $t = t_\zeta = t_0 + \zeta \Delta t$ depending on the direction of the out-of-plane velocity. Here, ζ is an integer $\zeta = \pm 1, \pm 2, \dots$ representing the frame offset and Δt is the time interval of the image acquisition. That is, one can prepare associated pairs of brightness signals extracted from sequential images in the two planes, i.e., $I_f(\mathbf{x}_0, t)$ and $I_b(\mathbf{x}_\zeta, t)$, with known frame offset ζ .

The dual-plane ensemble correlation can be calculated using these signals, $I_f(\mathbf{x}_0, t)$ and $I_b(\mathbf{x}_\zeta, t)$, as

$$R_{fb}(\zeta) = \frac{\sigma_{fb}(\zeta)}{\sigma_f(0) \sigma_b(\zeta)}. \quad (2.9)$$

Here, $\sigma_{fb}(\zeta)$ represents the covariance of the two brightness signals from the two different planes with a frame offset of ζ . It is defined as

$$\sigma_{fb}(\zeta) = \sum_{k=0}^{N-1} [I_f(\mathbf{x}_0, t_k) - \bar{I}_f] [I_b(\mathbf{x}_\zeta, t_{k+\zeta}) - \bar{I}_b]. \quad (2.10)$$

In addition, $\sigma_f(0)$ and $\sigma_b(\zeta)$ are the standard deviations of each signal,

$$\sigma_f(0) = \sqrt{\sum_{k=0}^{N-1} [I_f(\mathbf{x}_0, t_k) - \bar{I}_f]^2} \quad (2.11)$$

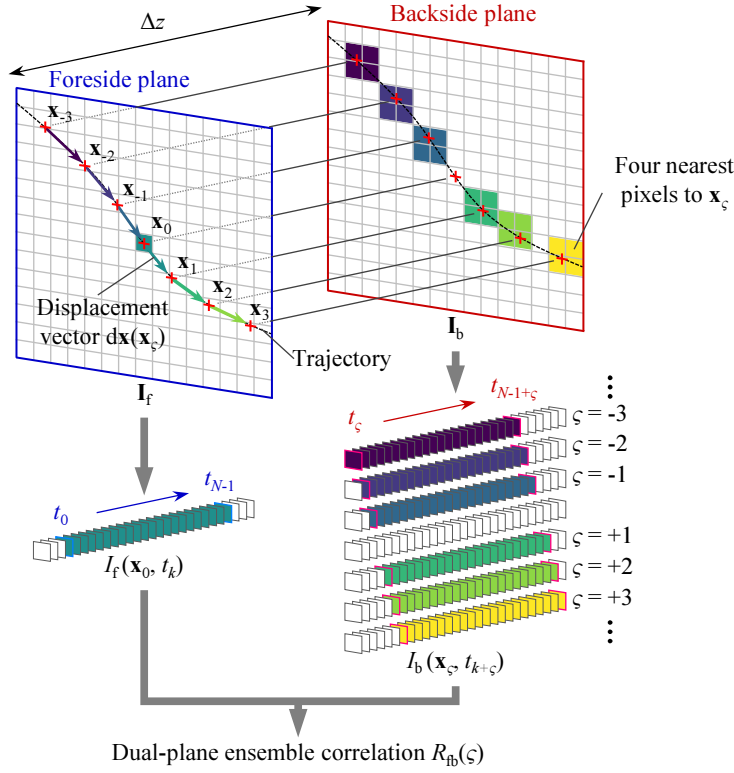


Figure 2.3: Conceptual image of the dual-plane ensemble correlation.

and

$$\sigma_b(\zeta) = \sqrt{\sum_{k=0}^{N-1} [I_b(\mathbf{x}_\zeta, t_{k+\zeta}) - \bar{I}_b]^2}. \quad (2.12)$$

Here, N is the number of the frames, and $\bar{I}_f$ and $\bar{I}_b$ are the mean values of each signal. Using the dual-plane ensemble correlation for positive and negative frame offsets, one can determine the optimal frame offset, i.e., how many frames before or after the current frame does the particle appear in the other plane. That is, the optimal frame offset field Δf_f is determined pixelwise as $\Delta f_f = \zeta_{\max}$, where $\zeta_{\max}$ is the frame offset yielding the maximum R_{fb} . The same procedure can be conducted for R_{bf} , and thus Δf_b is obtained.

The out-of-plane velocity fields $w(\mathbf{x})$ can be determined using Δf as

$$w_f(\mathbf{x}) = \frac{\Delta z}{\Delta t} \Delta f_f(\mathbf{x})^{-1} \quad \text{and} \quad w_b(\mathbf{x}) = -\frac{\Delta z}{\Delta t} \Delta f_b(\mathbf{x})^{-1} \quad (2.13)$$

for the foreside and backside planes, respectively. Because the dual-plane ensemble correlation allows pixelwise frame offset fields Δf to be measured, the out-of-plane velocity fields w can be measured up to a maximum velocity of $\pm|\Delta z/\Delta t|$ with single-pixel resolution.

2.3 Implementation and experiment

2.3.1 Experiment

To examine the algorithm of dual-plane ensemble correlation, a test experiment was conducted and the setup is schematically illustrated in Fig. 2.4. The photograph of the actual setup in the laboratory is also shown in Fig. 2.5. A rotating flow driven by an impeller mounting a mixer was used as the measurement target. An acrylic cylinder with a diameter of 200 mm was used as the fluid vessel and inserted into a rectangular vessel to act as a water jacket. Water was used as the test fluid and filled the vessel up to a height of 150 mm. Particles consisting of silver-coated hollow glass spheres (mean diameter 10 μm , density 1.4 g/cm^3 , *Dantec Dynamics*) were seeded into the fluid as tracer particles. The impeller, comprising four wings with an attack angle of 30° , was located at the center of the cylinder, and the bottom of it was placed at half the height of the fluid ($y = 75 \text{ mm}$). The impeller rotated at a constant angular velocity of 30 rpm in the clockwise direction, which was sufficiently fast to induce three-dimensional flows while keeping the fluid surface flat.

Two adjoining laser sheets, a continuous wave (CW) blue laser (wavelength 450 nm) and a CW red laser (wavelength 640 nm), illuminated two parallel planes with a depth offset of $\Delta z = 1 \text{ mm}$. Each of the two lasers formed a 1-mm-thick laser sheet using a cylindrical lens, and they illuminated vertical cross-sections at $z = -40 \pm 1 \text{ mm}$ without overlapping. The depth offset and the thicknesses of the two laser sheets were carefully adjusted to vary less than 10% throughout the field of view in the present study, as these mechanical settings directly affect the accuracy of the out-of-plane velocity. A CMOS color camera with a resolution of 0.1 mm/pixel was placed perpendicular to the planes. Continuous image acquisition was performed at 90 fps for 200 s, and 18,000 images were recorded as color images $\mathbf{I}_{\text{org}}$.

2.3.2 Pre-processing

The particles appearing in the original images $\mathbf{I}_{\text{org}}$ [Fig. 2.6(a)] first need to be separated by color into foreside $\mathbf{I}_{\text{f}}$ and backside images $\mathbf{I}_{\text{b}}$. Because of the colors of the laser sheets, blue-dominant and red-dominant pixels are regarded as indicating the particles existing in each plane, and are hence assigned to $\mathbf{I}_{\text{f}}$ and $\mathbf{I}_{\text{b}}$. This separation is formulated pixelwise as

$$I_{\text{f}}(x, y) = \begin{cases} I_{\text{org}}(x, y) & \text{if } I_{\text{org}}(x, y) = B_{\text{org}}(x, y) \\ 0 & \text{otherwise} \end{cases} \quad (2.14)$$

and

$$I_{\text{b}}(x, y) = \begin{cases} I_{\text{org}}(x, y) & \text{if } I_{\text{org}}(x, y) = R_{\text{org}}(x, y) \\ 0 & \text{otherwise} \end{cases}. \quad (2.15)$$

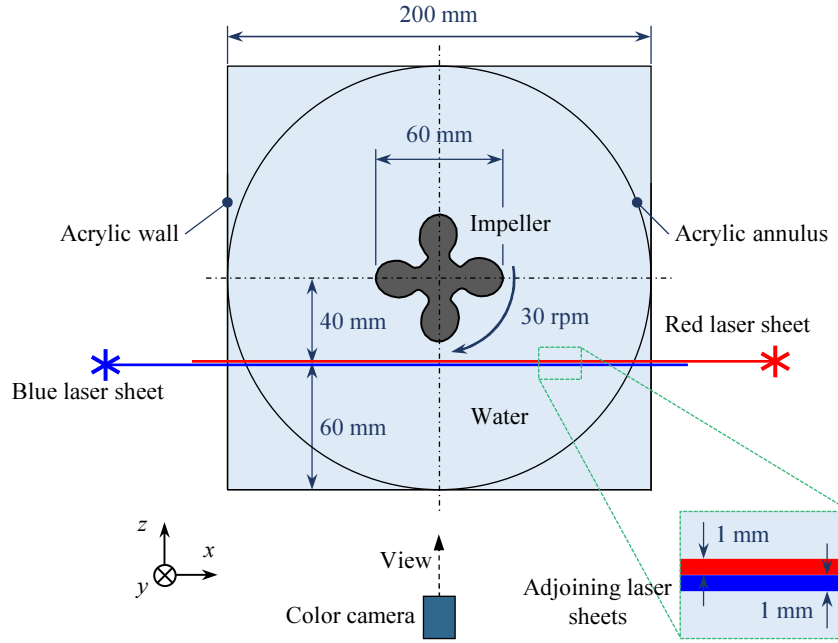


Figure 2.4: Schematic illustration of the top view of the experimental setup.

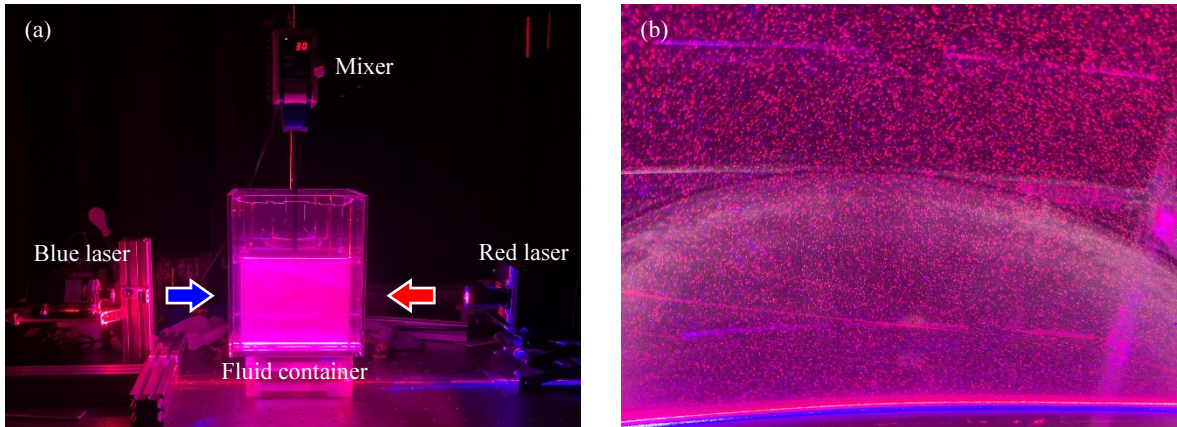


Figure 2.5: Photograph of the experimental setup: (a) overview and (b) close-up view of the measurement planes. The case of $\Delta z = 2$ mm is shown for visibility.

Here, I_{org} represents the brightness value at (x, y) , defined as the maximum of the three brightness values of red (R_{org}), green (G_{org}), and blue (B_{org}). The backside image $\mathbf{I}_b$ contains defective pixels due to particles assigned to the foreside image $\mathbf{I}_f$ since the two planes were visualized by a single camera. Such defects were therefore padded by Gaussian blurring after background subtraction; the results are shown in Fig. 2.6(b).

SP-PIV analysis [208] is then performed on the sets of $\mathbf{I}_f$ and $\mathbf{I}_b$ individually to obtain the in-plane displacement vector fields, $\mathbf{dx}_f$ and $\mathbf{dx}_b$, at the two planes. According to the earlier works [120, 130, 208], SP-PIV requires a number of sequential images to achieve statistically converged velocity fields, while the number depends on the measurement conditions; it is typically, from a few thousand to ten thousand. In the present study, similar to the previous studies, at least a few thousands of sequential images were required for the convergence in the present study, and 16,000 consecutive images were used for the ensemble correlation to acquire fully converged velocity fields $\mathbf{dx}_f$ and $\mathbf{dx}_b$ with single-pixel resolution. The accuracy of the displacements was improved to sub-pixel level by fitting Gaussian-peaks to the correlation maps. The results are shown in Fig. 2.6(c). The two flow fields show similar patterns but with considerable differences due to the small differences in positions relative to the impeller, placed at $y = 750$ pixel. Please note that the accuracy of the in-plane displacement can be improved by using several time steps for SP-PIV, however, only one frame offset was adopted at this moment to match the frame offset used in the subsequent analysis for the sake of simplicity.

2.3.3 Out-of-plane velocity determination

Assuming that the in-plane velocity hardly changes between the two planes, a position $\mathbf{x}_\zeta$ on one plane is determined by tracking the particle trajectory on the other plane as

$$\mathbf{x}_\zeta = \mathbf{x}_0 + \text{sgn}(\zeta) \sum_{l=1}^{|\zeta|} \mathbf{dx}(\mathbf{x}_{\text{sgn}(\zeta)(l-1)}), \quad (2.16)$$

where $\text{sgn}(\cdot)$ denotes the signum function. The conceptual image for determining the dual-plane correspondence for the dual-plane ensemble correlation is illustrated in Fig. 2.7. The illustrated case focuses on the foreside plane. The temporal brightness signals in the foreside at $t = t_0$ and those in the backside at $t = t_{\pm 1}, t_{\pm 2}, t_{\pm 3}, t_{\pm 4} \dots$ indicated by the yellow circles are utilized for the dual-plane ensemble correlation. As $\mathbf{x}_\zeta$ is determined with sub-pixel accuracy, the weighted averages of the displacement vectors $\mathbf{dx}$ and brightness values I of the four nearest pixels are used as the representative values at position $\mathbf{x}_\zeta$, as shown in Fig. 2.3. The dual-plane ensemble correlation is calculated only along the particle trajectory, and thus the computational time for searching ± 50 frames is almost equivalent to that required for SP-PIV searching 10×10 pixels.

Results of the most likely frame offset fields and out-of-plane velocity fields are shown in Fig. 2.8(a) and (b), respectively. Here, ten thousands of sequential images were required at

most for the convergence of the frame offset fields, which is a larger number than that required for the convergence of the in-plane velocity fields, typically a few thousand. The frame offset fields in Fig. 2.8(a) have an approximately inverse sign relationship. The resultant out-of-plane velocity fields in Fig. 2.8(b) are quite similar to each other, as the two planes are adjoining in the present study with a depth offset of $\Delta z = 1$ mm. For comparison, the mean out-of-plane velocity field measured by the conventional PIV employing the direct cross-correlation method is shown in Fig. 2.8(c). This was obtained by stacking the mean velocity fields in the x - z planes at different y positions every 10 mm from $y = 5$ to 145 mm. The spatial resolutions in the x and y directions are 20 pixels (= 2 mm) and 100 pixels (= 10 mm), respectively. The velocity fields in Fig. 2.8(b) show good accordance with those in Fig. 2.8(c) in terms of the outline and magnitude, while retaining single-pixel resolution.

2.4 Summary and outlook

A novel method determining the out-of-plane velocity component has been developed as an extension of SP-PIV. The proposed method, called dual-plane ensemble correlation, uses the ensemble correlation of SP-PIV across two planes illuminated by two differently colored laser sheets. This enables the most likely frame offset and the out-of-plane velocity field to be determined. In addition to the in-plane velocity fields in the two planes acquired by the color image separation and SP-PIV, the out-of-plane velocity fields can be determined with single-pixel resolution using only a single-color camera. The method was successfully applied to a test experiment consisting of a rotating flow induced by an impeller and validated with conventional PIV.

In contrast to conventional PIV, ensemble correlation does not require spatial correlation, and thus it may be a robust approach in multiphase flows containing additional dispersed phases, e.g., gas and solid fractions. Because the 3C velocity fields at the two adjoining cross-sections can be quantified by the present method, velocity gradients in both the depth and in-plane directions can be computed. That is, all nine components of the velocity gradients can be computed, pixel by pixel.

Although the ability of the proposed method to make 3C velocity field measurements was demonstrated, some options for further refinements exist. For instance, the maximum magnitude of the out-of-plane velocity fields in Figs. 2.8(a) and b seem to be overestimated. This overestimation originates from the velocity resolution of the proposed method. Because velocity fields w are inversely proportional to frame offset Δf , the velocity resolution decreases as the out-of-plane velocity increases. In the present case, the frame offsets $\Delta f = 1, 2, 3 \dots 50$ correspond with out-of-plane velocities of $w = 90, 45, 22.5 \dots 1.8$ mm/s. The velocity resolution could tentatively be improved by adopting a larger depth offset; however, resolution in frame offset needs to be improved using methods such as Gaussian-peak fitting to determine in-plane

displacements with sub-pixel accuracy in planar PIV. Iterative optimization by searching for sub-frame shifts around the optimal frame offset may solve this problem. Alternatively, an increase in the number of visualized planes may allow accurate frame offsets to be estimated more flexibly because the combinations of planes that could be used to compute dual-plane ensemble correlation would increase. In mechanical aspects, it is necessary to determine optimal settings of laser sheets which may depend on measurement conditions in terms of beam shape, thickness, and offset.

For an implementation, one needs to pay attention to the size of particles on the digital images. Since the proposed method requires an image separation based on the color values, the particle colors are necessary to be identified correctly. Single sensor CMOS or CCD camera, however, requires color interpolation (demosaicing) after image acquisitions. Thus at least a few pixels in diameter of a single particle are essential for the color interpolation of the particle images. Otherwise, the color image separation cannot be performed properly. As long as the colors of the multiple laser sheets scattered by the particles are successfully separable on the digital images, one can increase the number of illumination planes. Increase in the number of the illuminated planes will increase the complexity of the post-processing. For this case, the dual-plane ensemble correlation proposed in this chapter may be extended to the multiple-plane ensemble correlation and more accurate and volumetric 3C velocity field measurements becomes possible.

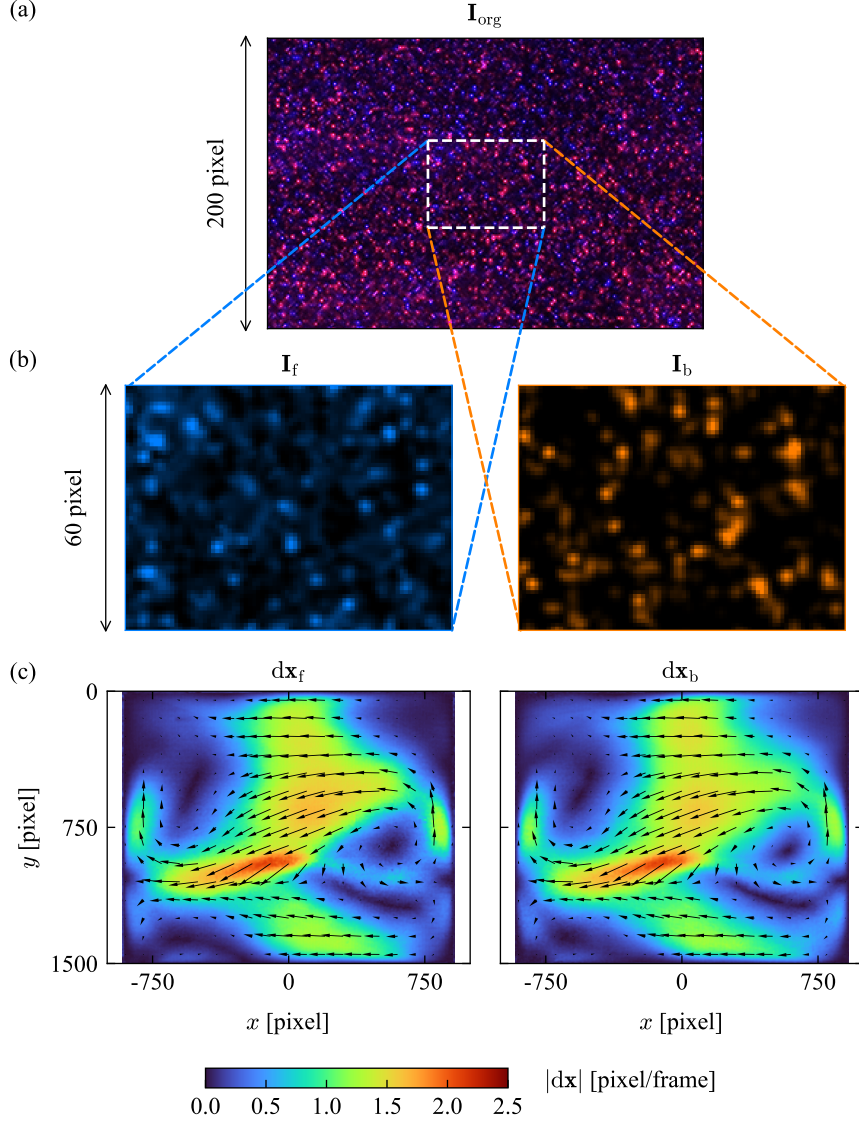


Figure 2.6: Pre-processing to measure the in-plane displacement vector fields at the two planes: (a) original color image; (b) foreside (left) and backside images (right) corresponding to the enclosed region of (a); (c) overall views of displacement vector fields dx_f (left) and dx_b (right). (a) and (b) show cropped regions of 300×200 pixels and 80×60 pixels, respectively. The images in (b) are colored for better visibility. The color contours of (c) show the absolute displacements $|dx|$, and every hundredth vector in the x and y directions is superposed as a black arrow.

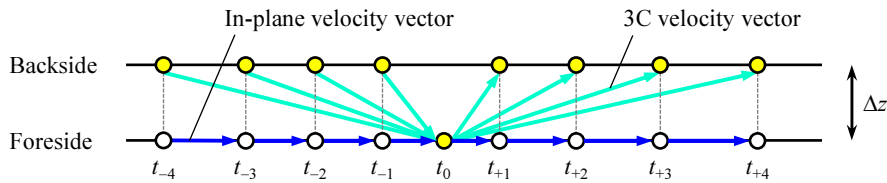


Figure 2.7: Conceptual image of the dual-plane correspondence obtained from the in-plane velocity vectors and trajectories. For the dual-plane ensemble correlation, the temporal brightness signals in the foreside at $t = t_0$ and those in the backside at $t = t_{\pm 1}, t_{\pm 2}, t_{\pm 3}, t_{\pm 4} \dots$ are utilized.

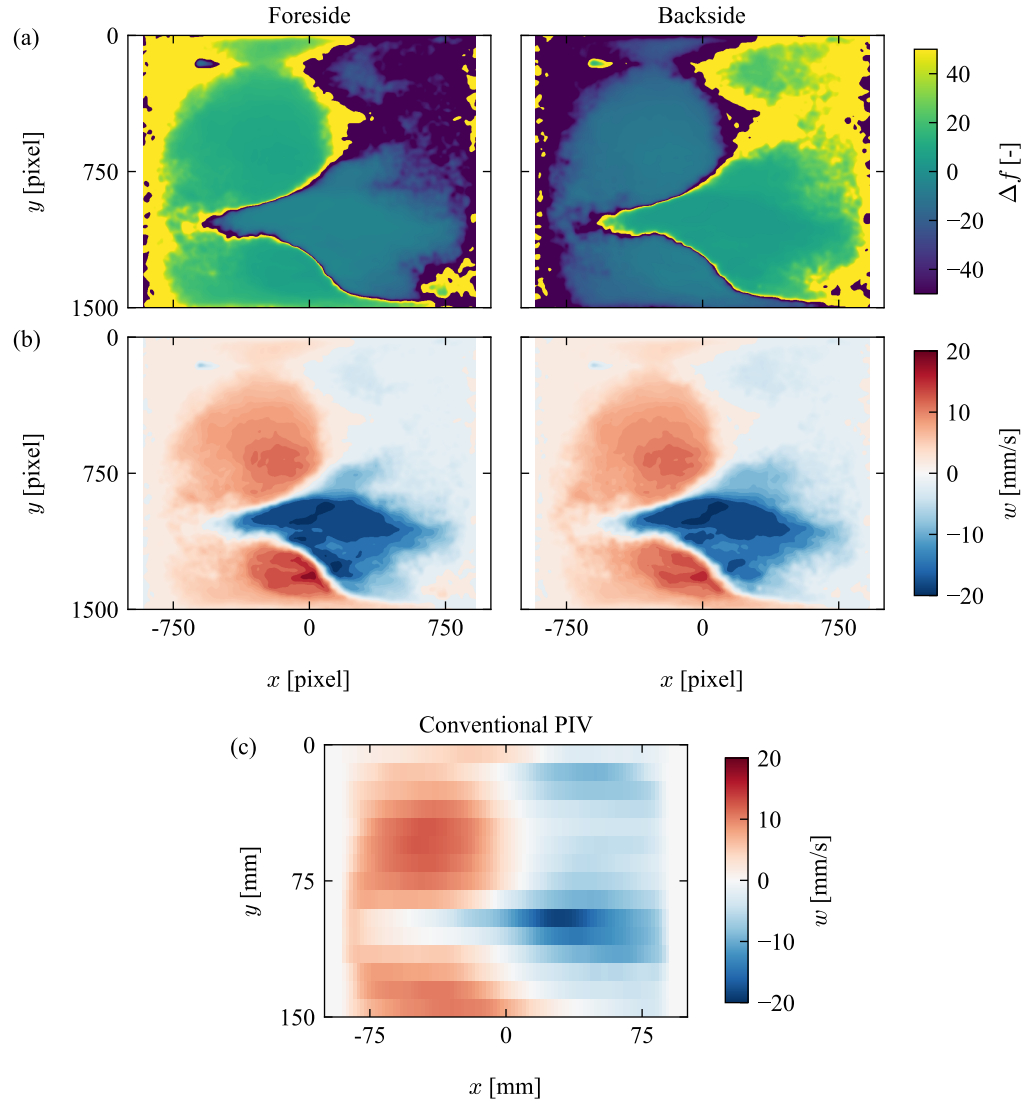


Figure 2.8: (a) Frame offset fields of the foreside (left) and backside (right) planes, and (b) the corresponding out-of-plane velocity fields. (c) Out-of-plane velocity fields measured by conventional PIV at different horizontal planes.

DEVELOPMENT OF IN SITU COLOR-TO-DEPTH CALIBRATION METHOD TOWARD PRACTICAL 3D COLOR PTV

Preface

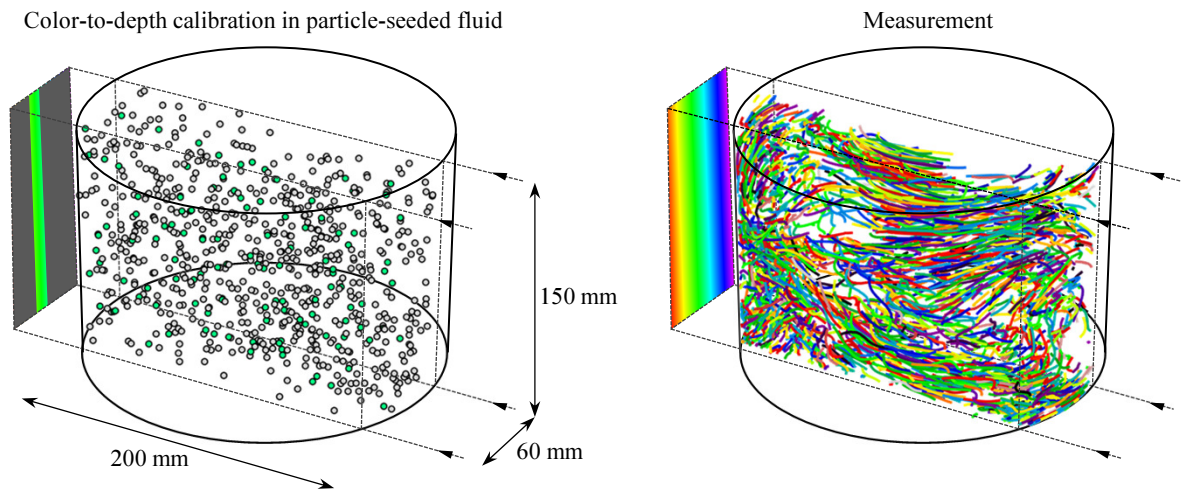


Figure 3.1: Graphical abstract of this chapter.

In Chapter 1 and 2, the color information has been utilized to separate two planes at different depth positions. That is, classification problems have been solved through the color-based image separation. Since the digitally recorded color information can be recognizable in 2^{24} levels in 24-bit format color image, such binarization problem cannot be regarded to maximize the available color information. In principle, spatial positions in the depth direction can be separated into 2^{24} levels, if ideal illumination and color extraction are possible. Considering the requirement in flow field measurement, 2^{24} levels may be considered to be infinity compared to

the in-plane resolution, which is determined by the pixel resolutions of the image sensors and it is typically ~ 1000 pixels in a megapixel camera. Thus, even if the color information cannot be utilized in the full 2^{24} levels and the ‘color resolution’ drops to 2^{10} , depth positions can be determined at comparable accuracy with the in-plane positions. Incorporating these discussions, to maximize the available color information for the determination of depth positions, stepless color identification is necessary. This approach will be a regression problem for associating color information with depth positions.

In Chapter 3, a seeding particle-based color-to-depth calibration methodology is proposed for three-dimensional color particle tracking velocimetry (3D color PTV) using a single camera and volumetric rainbow gradient illumination. The use of sheet-color illumination from a liquid crystal display projector enables in situ calibration, namely the color-to-depth relationships of particles seeded in a fluid are determined without inserting any calibration equipment or taking a different optical setup. That is, in this methodology, the calibration and application can be performed using the same optical configuration, and only the digital illumination patterns need to be changed. Adopting this calibration allows evaluating actual color-to-depth relationships of the particles in measurements. The calibration is conducted regarding the relationship between spatially distributed particle colors and their depth coordinates by support of an artificial neural network. By combining conventional PTV with the depth estimated by the color, particle trajectories in 3D real space can be reconstructed from the calibration. The performance of the proposed method was evaluated using a rotating flow in a cylindrical tank by comparing its results with the flow fields measured by conventional particle image velocimetry. Good accordance in the comparison at the highly 3D flow suggests the applicability of the present methodology for various flow configurations.

The results shown in Chapter 3 were published as Noto et al. [126] and the details are below.

Author Daisuke Noto, Yuji Tasaka, and Yuichi Murai

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3.1 Introduction

The range of applications of volumetric flow field measurement through tomographic particle image velocimetry (PIV) [e.g. 46, 209] is increasing. The shake-the-box method [167] greatly reduced ghost particle problem arising in tomographic PIV, yielding dense Lagrangian particle tracking. These techniques measuring three-dimensional, three-component (3D-3C) velocity fields are now regarded as standard techniques in fluid flow measurements. However, these require careful optical arrangements of at least four cameras to acquire reliable particle displacements, and thus the cost of a tomographic system may not be affordable for many applications.

To tackle the complexity arising in such optical settings as well as their high cost, some alternatives using only a single camera have been proposed in the last few decades. Especially under the limited measurement environments of microfluidics, the necessity of implementations on microscopes has motivated developments of such single-view approaches as advanced methods of micro-PIV. Single-view approaches require to record particle depth positions as additional information to planer images; astigmatism particle tracking velocimetry (PTV) utilizes particle distortions [e.g. 32, 33], and three-pinhole aperture PIV uses geometric relationships of triplet particle images [e.g. 139, 192]. These techniques yielded a particle image density of $O(10^{-4})$ particles per pixel (ppp) with a measurable depth of $O(10\text{--}100\text{ }\mu\text{m})$. For larger measurable depth without microscopes, holographic PIV increased the particle image density to $O(10^{-3}\text{--}10^{-2}\text{ ppp})$ as well as the measurable depth to $O(1\text{--}10\text{ mm})$ [e.g. 166, 172]. Recently, light-field PIV has been introduced as a new type of technique [e.g. 175] and allows a rather easier implementation than the techniques described above with yielding a higher particle image density of $O(10^{-2}\text{ ppp})$ with the measurable depth of $O(10\text{ mm})$. These various volumetric measurement techniques using a single camera, however, often require additional special optics.

The simplest solution for locating particles seeded in a 3D space without special optics may be the use of color gradient information imposed along the depth (view) direction of a single camera [17, 106, 108, 134, 151, 164, 200, 214, 215]. This technique is called color PTV or rainbow PTV. Recently, Aguirre-Pablo et al. [6] proposed a method called intensity PTV using a structured monochromatic illumination. The only required optical equipment is a single camera and a light source that can emit light of multiple wavelengths such as a liquid crystal display (LCD) projector. The optical arrangements are thus substantially less complex than those used in tomographic PIV and other single-view methods. A conceptual image of color PTV is shown in Fig. 3.2. An LCD projector displays a rainbow gradient, changing the color in the depth direction (the y -direction in the illustration) of the camera. Because the LCD projector is originally designed to illuminate a large space, the light emitted from the LCD projector diverges. Hence, the light is collimated by a collimation lens (a linear Fresnel lens is used in the present study) to form a parallel color beam. This ensures that the relationship between the color and

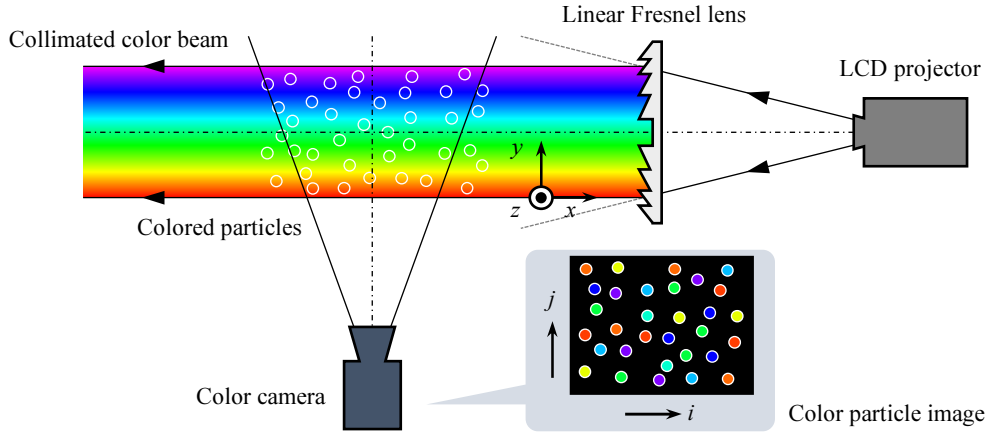


Figure 3.2: Conceptual image of the optical setting for LCD-based-color PTV.

depth coordinate is fixed. The color beam illuminates particles seeded into a test fluid, and a camera captures the particle images. If light with a color gradient is used, as shown in Fig. 3.2, the particles scatter light, and the color of that light reflects the depth. The particle positions in the in-plane coordinates (x and z) and out-of-plane coordinate (y) thus can be determined from the image coordinates (i and j) and the color, respectively. The concept of color PTV is simple and allows a large measurable depth of $O(\geq 10 \text{ mm})$ and the particle image density of $O(10^{-2} \text{ ppp})$. However, many issues must be addressed to ensure reasonable accuracy in the practical application of this technique.

To practically apply color PTV in actual flow field measurements, a feasible method for calibrating the color-to-depth relationship is required. [108] utilized a microscope slide containing a low density chalk dust having the same size as the particles for the calibration as shown in Fig. 3.3(a). By placing the slide obliquely in the illuminated region in the x - y plane, color changes in the y direction could be tracked in the x direction of the image plane. [6] used a 3D laser-engraved calibration cube that contains micro-cracks that mimic spatially distributed particles as illustrated in Fig. 3.3(b). The known 3D crack positions inside the cube can be used for calibration. Note that they used a monochromatic illumination instead of rainbow illumination, while Fig. 3.3(b) illustrates rainbow colors for easier comparison with the others. They demonstrated successful results for a rotating flow of index-matched fluid in an open tank. The need to use a calibration cube, however, may not be user-friendly and is far from wide applications, as the calibration cube needs to be index-matched with the test fluid. It is therefore not realistic to produce a unique calibration cube for a variety of flow conditions. The light scattering characteristics of seeded particles, however, are not necessarily the same as those of such chalk dust and glass cracks. This is the difficulty in considering the particle color or intensity as additional information to the conventional calibration adopted in the stereoscopic and tomographic PIV, requiring only the relationship between the real and image coordinates.

Watanabe et al. [200] tried to calibrate the color-to-depth relationship in a particle-seeded fluid using a rainbow gradient illumination within the image plane as shown in Fig. 3.3(c). In

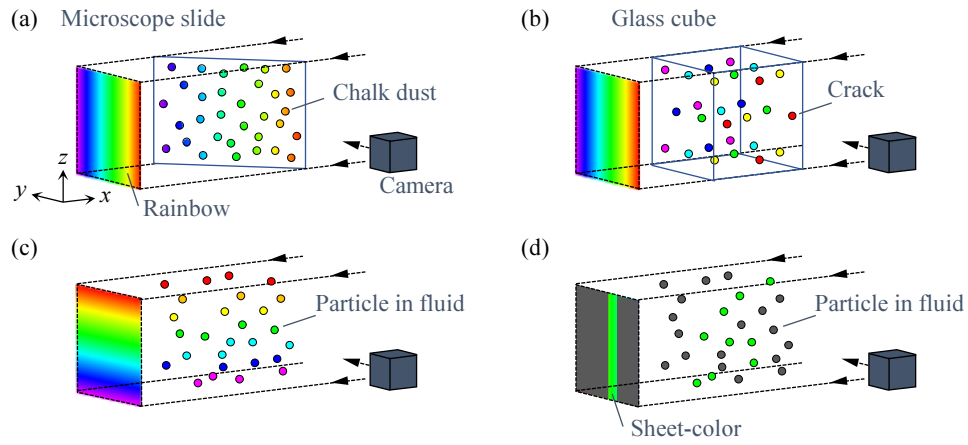


Figure 3.3: Calibration methods for the color PTV: (a) oblique slide [108]; (b) calibration cube [6]; (c) in-plane rainbow [200]; and (d) sheet-color (the present study).

the calibration step, the particle images were obtained in the x - z plane. The color-to-in-plane relationships obtained in the special optical arrangement only for calibration are used to compute the calibration curves. The curves then are used to convert particle colors to depth positions in the actual measurements which illuminates a rainbow gradient in the depth direction. This calibration, however, requires a strong and unreasonable assumption that the colors scattered by the particles do not change in the two different optical arrangements.

These calibration methods proposed earlier cannot guarantee the actual accuracy in detecting the depth positions of particles seeded in the fluids, because the actual color-to-depth relationships of the particles are not directly used. For these reasons, there are two requirements for an in situ method for color-to-depth calibration as follows. One is to obtain the actual relationships between particle depth positions and their scattered colors in the flow conditions to be measured. Another is to conduct the calibration in the same optical arrangement as the measurements. That is, the color-to-depth calibration needs to be conducted in the particle-seeded fluid used for the actual measurements to satisfy these requirements. The aim of the present study is to establish such a reliable calibration methodology for color PTV that solves these issues. To do this, three different illumination measurements, full-color, sheet-color, and dark illumination, are adopted. The conceptual image of the proposed calibration method is shown in Fig. 3.3(d) for comparison with the others. The use of an LCD projector allows such flexible illumination (within the resolution of the LCD projector) without changing the optical arrangement. In this chapter, then a color-to-depth calibration method for particle-seeded fluids is proposed with series of supporting image processing and artificial neural network (ANN) to solve the multi-regression problem on the calibration. The method was tested in a highly 3D rotating flow generated by an impeller and was validated through comparison with conventional PIV to show reasonable accuracy of the proposed method.

3.2 Methodology

For the calibration and application of color PTV, all measurements need to be performed using an identical experimental setup. Only the illumination light is changed for the calibration step, as detailed in Sec. 3.2.2. Thus, the measurement target is used in the first step of the measurements (Sec. 3.2.1). The methodology comprises four parts: three different illumination measurements (Sec. 3.2.2), a series of image processing operations (Sec. 3.2.3), color-to-depth calibration supported by ANN (Sec. 3.2.4), and particle tracking in the 3D real space (Sec. 3.2.5). A flow chart of the proposed method is shown in Fig. 3.4 for the convenience of readers to facilitate their understanding of the subsequent discussion.

3.2.1 Experimental design

Here a rotating flow in a cylindrical fluid layer driven by an impeller was employed as the measurement target, as schematically illustrated in Fig. 3.5. Water was used as the test fluid, and a rectangular vessel was filled to a height of 150 mm. An annulus with an inner diameter of 200 mm was inserted into the rectangular vessel to form a cylindrical fluid layer. Both the rectangular vessel and annulus were filled with water to reduce unwanted optical refraction. Tracer particles (HP20, mean diameter 0.5 mm, mean density 1.01 g/cm³, *Mitsubishi Chemical Co.*) were seeded in the fluid only inside the annulus. The fluid in the annulus was driven by a propeller-type-impeller with a diameter of 60 mm comprising four small wings with an attack angle of 30° and rotated constantly at $\omega = 30$ rpm in a clockwise direction. The rotational axis of the impeller was located at the center of the annulus ($x = y = 0$) and the bottom of the impeller was placed at half the height of the fluid, $z = 75$ mm.

A rainbow gradient pattern was projected from an LCD projector (3,000 lm, 1,280 × 800 pixels, EB-W420, *Seiko Epson Co.*) and collimated by a linear Fresnel lens (focal length 150 mm) set in front of the projector. The collimated volumetric beam had a thickness of $\delta y = 60$ mm in the depth (y) direction. A color CMOS camera (DFK33UP5000, *The Imaging Source Co.*) equipped with a lens (focal length 35 mm, aperture F#8, estimated depth of field ~ 80 mm, Nikkor, *Nikon*) was set perpendicular to the collimated beam and acquired particle images in the x - z plane. The original images had Bayer patterns and were processed by an advanced linear interpolation method for demosaicing [103]. The illuminated volume was $200 \times 60 \times 150$ mm³ in the x - y - z directions. The measurement volume in the annulus was slightly smaller than this and is shown as the region enclosed by red dashed lines in Fig. 3.5. The arrangement illustrated in Fig. 3.5 shows a scheme in which the collimated beam illuminates the fluid layer in the range of $y = -50$ mm to 10 mm.

Various illumination patterns have been proposed for single-camera 3D PTV based on an LCD projector. For example, Watamura et al. [200] used an illumination that cycles through multiple hues over time and Aguirre-Pablo et al. [6] demonstrated the efficiency of a brightness-

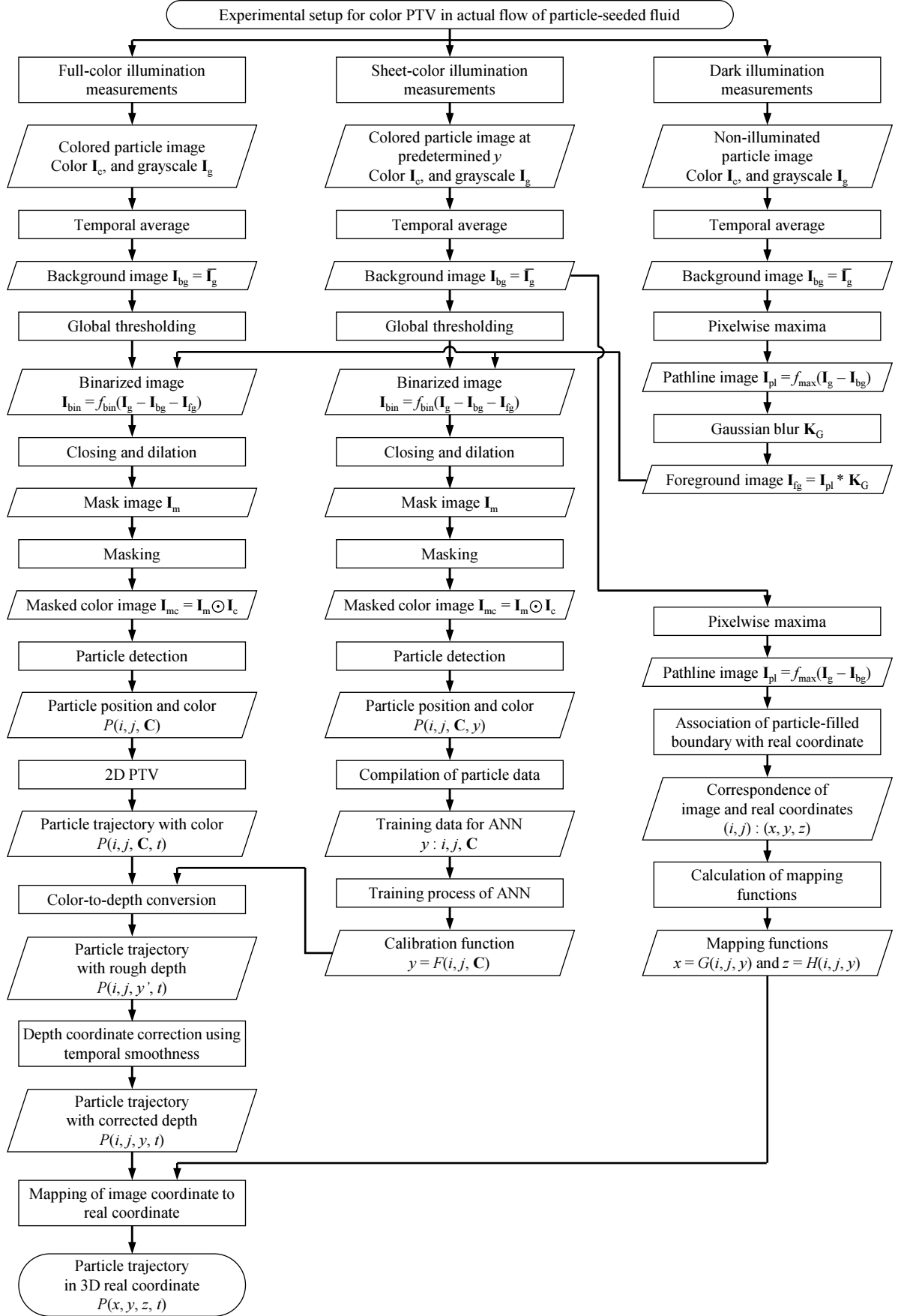


Figure 3.4: Flow chart of the proposed method.

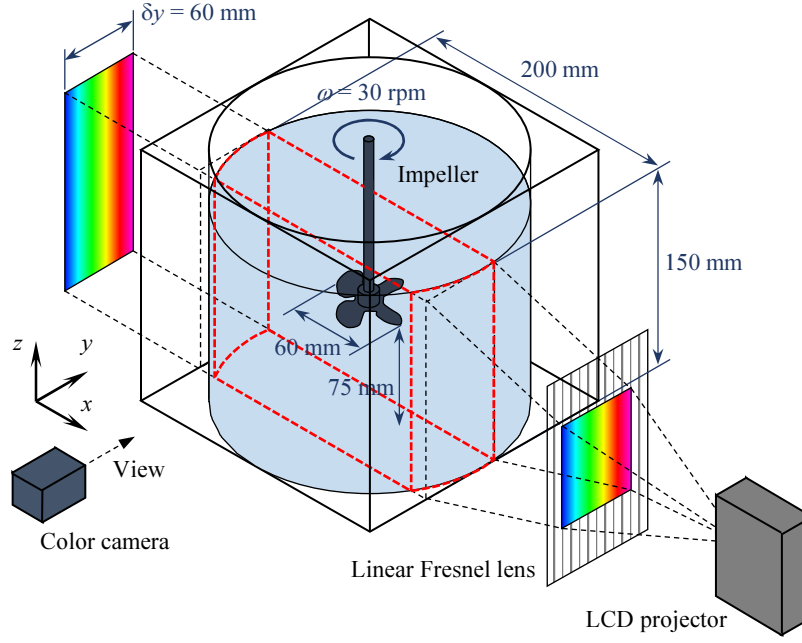


Figure 3.5: Schematic of the experimental setup for the rotating flow measurements.

changing animation illumination. The employment of such animation effects in the illumination patterns may increase the flexibility of the post-processing, improving the accuracy of depth coordinate estimation. In contrast, the quality of an animation projected by an LCD projector may strongly depend on its specifications, such as refresh rate or time required for color stabilization (transition time), and these mechanical restrictions limit the measurement targets to slowly moving fluid flows, as discussed in Aguirre-Pablo et al. [6]. Thus, a fixed rainbow illumination during measurements was utilized so that the generalization of the color PTV method does not depend on the LCD projector. This approach also reduces the complexity arising in the post-processing steps. The rainbow gradient pattern used as an input to the LCD projector in the present study is presented in Fig. 3.6. The color pattern shown in Fig. 3.6(a) is projected by the LCD projector to change color in the depth direction, and the corresponding brightness values of three color channels, R , G , and B , each in 8-bit format, are shown in Fig. 3.6(b). The maximum resolution of the projector, 800 pixels, corresponds to the thickness of the collimated beam $\delta y = 60$ mm. The brightness values in the RGB domain are shown in Fig. 3.6(c). The solid lines shown in Figs. 3.6(a) and b represent the used color values. It is clear that the employed rainbow gradient can be well separated in the 3D- RGB domain, and the depth coordinates of the seeded particles should be determined using all three brightness values. Details of the color-to-depth calibration method are addressed in Sec. 3.2.4.

3.2.2 Measurement procedure

The measurement procedure comprises measurements under three different illuminations: full-color, sheet-color, and dark illumination. These measurements are used for the subsequent image

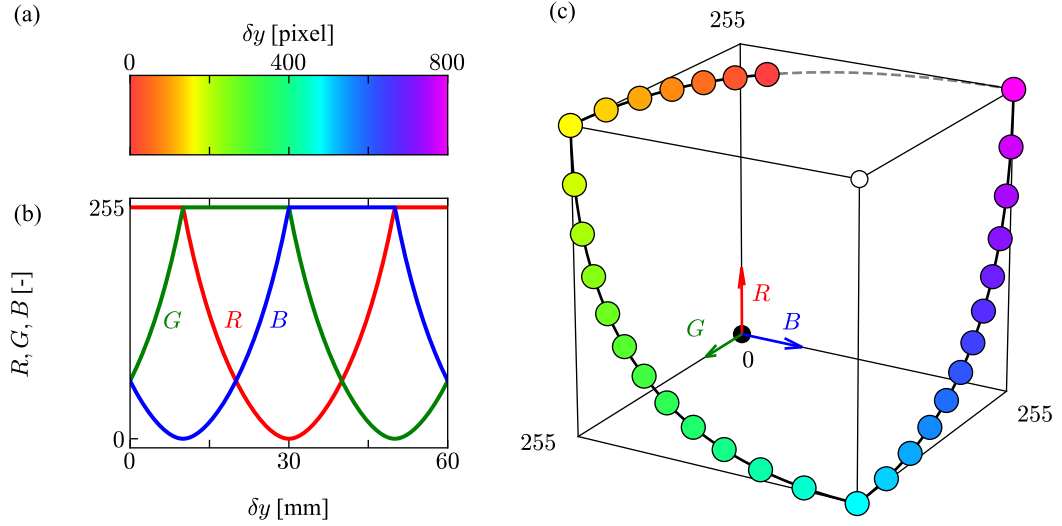


Figure 3.6: Details of the projected color pattern for full-color illumination measurements. (a) Original color pattern projected by the LCD projector, (b) brightness values of each RGB channel in 8-bit format, and (c) 3D-Cartesian plots of the illumination colors. The maximum height in pixels of the LCD projector $\delta y = 800$ pixels corresponds to the thickness of the collimated color beam $\delta y = 60$ mm. The solid line in (c) represents the used colors; the colored circles are plotted every 2 mm for reference.

analyses. During these three measurements, only the projection images are changed while the optical arrangements of the experimental equipment are fixed, and all the measurements can be conducted in the particle-seeded fluid.

3.2.2.1 Full-color illumination measurements

The full-color illumination pattern, as illustrated in Fig. 3.5, is the main type of measurement for investigating flow fields. During the measurements, the full-color pattern was projected continuously by the LCD projector. Because the LCD projector illuminates particles seeded in water with a full-color pattern having a thickness of $\delta y = 60$ mm, the depth coordinates of the particles y within the thickness are represented by the colors scattered by the particles in the acquired color images. The total depth of the illuminated volume varied less than 1 mm ($< 2\%$) across the field of view. In the present study, rotating flows provoked by the impeller set in the middle of the cylindrical tank were recorded as consecutive color images with 90 fps for 2 minutes (60 rotations of the impeller).

3.2.2.2 Sheet-color illumination measurements

To obtain the relationships between the particle color and depth information for color-to-depth calibration, it is necessary to predetermine these relationships under known conditions. To do this, the particle-seeded fluid should be also illuminated by sheet-color patterns, as illustrated in Fig. 3.7. The digitally segmented sheet-color patterns illuminate particles at known depth positions without changing the optical arrangements. This reduces any mechanical error that

could arise in the traversal of the calibration plates. In the present study, the full-color projection pattern used in the actual measurements was segmented into 40 sheet-color patterns by placing black digital slits in front of the full-color pattern. Each sheet possessed a thickness of 20 pixels in the projected pattern resulting in a 1.5-mm-thick sheet beam in the real dimensions, as the thickness of the full-color pattern was $\delta y = 800 \text{ pixels} = 60 \text{ mm}$. Each sheet yielded a slight color change within the thin thickness. The regions outside the sheet in the sheet-color patterns were filled with black ($R = G = B = 0$), so as not to illuminate particles outside the sheet region. In principle, particles illuminated by a certain sheet-color pattern exist within a thin sheet region at a known depth coordinate. For all the sheet-color patterns, the sheet thickness varied much less than 5% across the field of view.

In this study, large particles with a mean diameter of 0.5 mm were utilized for better color scattering. Suppose that the particles scatter colors when a part of the particle enters the sheet illumination. Then, the depth positions of particles are detected with an accuracy of $\pm(0.75 + 0.25) \text{ mm} = \pm 1.0 \text{ mm}$, which is half of the sheet thickness and particle radius. By recording particle images illuminated by the 40 sheet-color patterns, particle colors can be identified at different 40 depth levels with at least this accuracy of $\pm 1.0 \text{ mm}$. In the color-to-depth calibration step detailed in Sec. 3.2.4, this original accuracy given by the system is the target value for the optimization. For each illumination, more than 1,000 images were recorded to obtain a sufficient number of colored particles present in the thin sheet regions. Note that these sheet-color illumination measurements using 40 patterns can be fully automated by projecting a slowly changing animation of the sheet-color patterns.

Thinner sheet-color patterns can obtain more accurate relationships between the colors and the known depth coordinates. In principle, the thickness of a sheet-color pattern can be as thin as 1 pixel, corresponding to 0.075 mm in the present condition, if the brightness of the colored particles can be distinguished from those of the non-illuminated particles. At this time, the diameter of the seeded particles also becomes important when capturing scattered color information from the seeded particles, and this needs to be considered when selecting tracer particles.

3.2.2.3 Dark illumination measurements

Ideally, the full-color and sheet-color illumination measurements only record particles illuminated by the color beams. However, the LCD projector illuminates the space corresponding with the black pixel regions of the sheet-color patterns with weak light intensity. Particles in the black regions are hence recorded on particle images as spots with low brightness and without a strong saturation, as illustrated in Fig. 3.7. Moreover, such dark particles appear on particle images recorded during the full-color illumination measurements, as the projector leaks light with weak light intensity at the peripherals of the illumination volume. It may not be possible to fully exclude such low-brightness particles during the image acquisition steps. For these reasons,

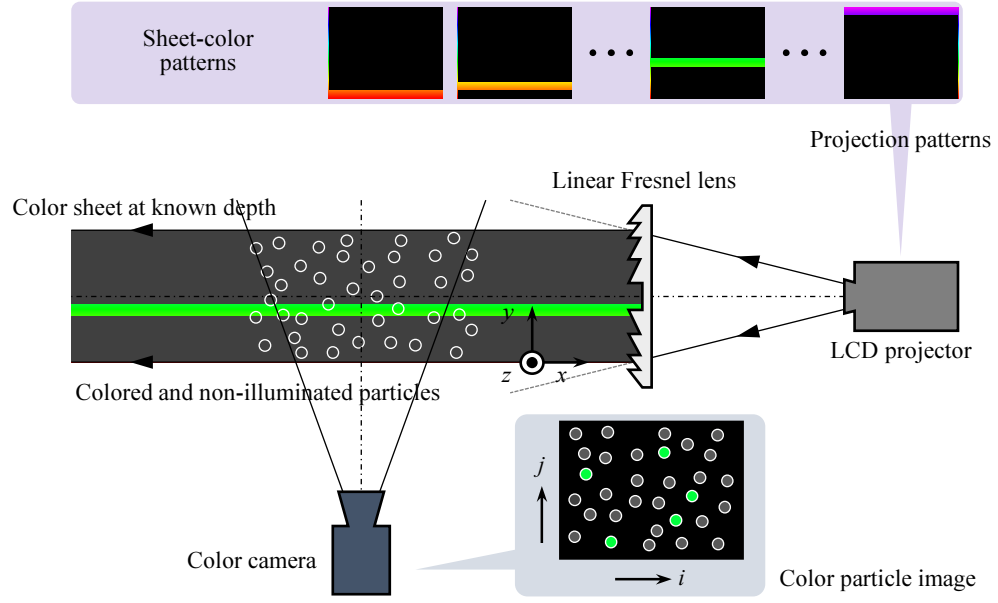


Figure 3.7: Sheet-color illumination measurement for color-to-depth calibration.

non-illuminated particles should be also recorded by projecting a fully black pattern of the same size as the full-color image. During this dark illumination measurement, more than 1000 images were recorded as in the sheet-color illumination measurements, to obtain a sufficient number of non-illuminated particles.

3.2.3 Image processing for colored-particle detection

In the three measurements using different projection patterns described in Sec. 3.2.2, color images I_c were originally acquired in 24-bit format (8 bits for each R , G , and B channel). To track particle motions in the 3D real space, the seeded particles must be detected along with their color information with excluding non-illuminated particles. A series of image processing steps to detect colored-particles for combining particle images obtained in the three measurements and standard image processing techniques is provided. Examples of these procedures are shown in Fig. 3.8. The same image processing scheme for the full-color (Sec. 3.2.2.1) and sheet-color (Sec. 3.2.2.2) illumination measurements enables the detection of colored particles, as shown in Figs. 3.8(a) and (b), respectively. Color patterns projected for the full-color and sheet-color illumination measurements are shown in the upper row of Fig. 3.8, and these have resolutions of $1,280 \times 800$ pixels, which corresponds to the resolution of the LCD projector. When these illumination patterns were adopted, original particle images were recorded as color images I_c , as shown in the first column of Fig. 3.8. Here, the particle images are magnified views of small 100×100 pixel regions so that the particles are easily visible. The details of these procedures for colored-particle detection are presented in the following.

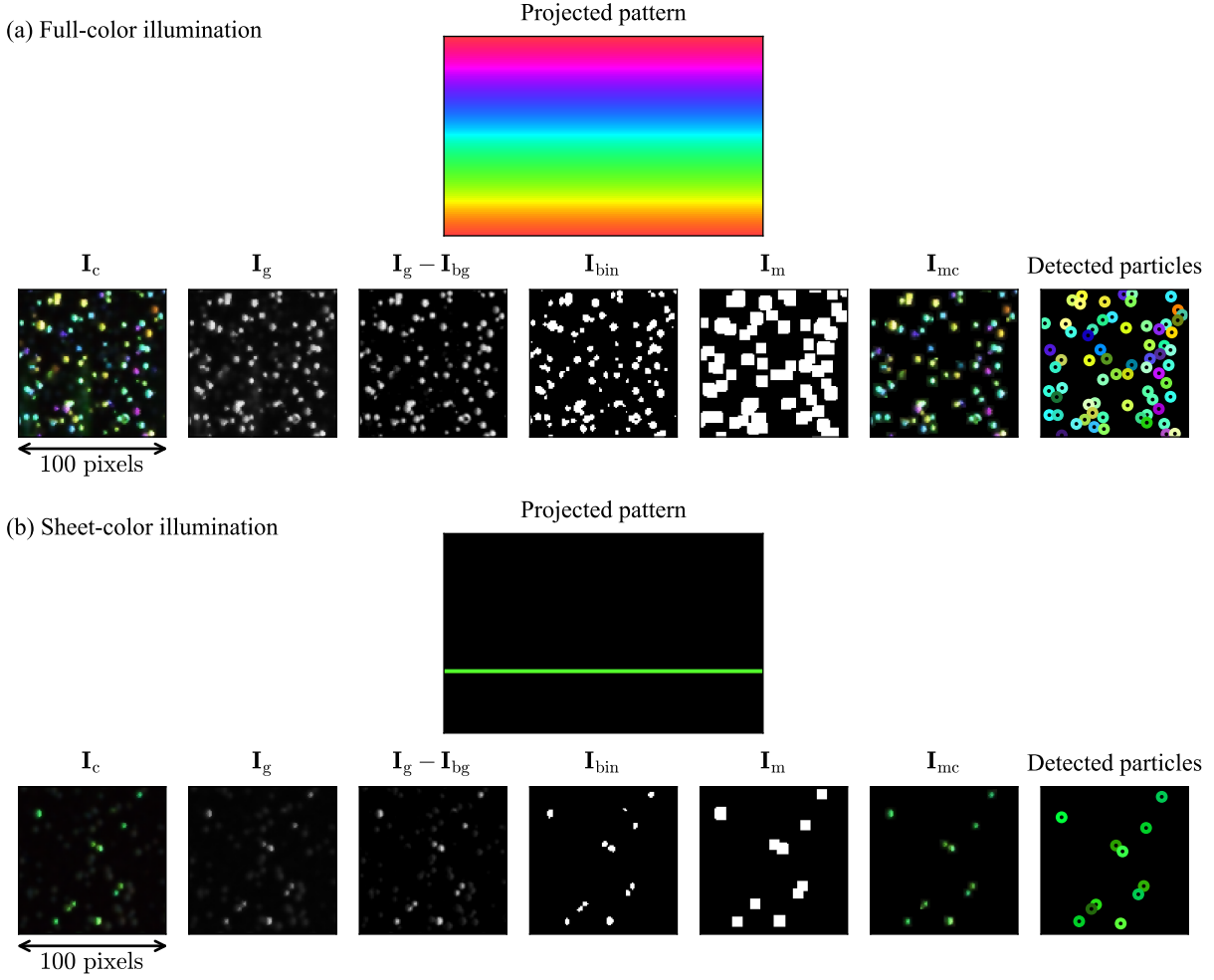


Figure 3.8: Series of image processing steps for colored-particle detection, performed for (a) full-color illumination and (b) sheet-color illumination measurements. Images in the first column represent the projection color patterns ($1, 280 \times 800$ pixels) used for the two measurements. Images from the second to seventh columns are magnified views of 100×100 pixel regions of original color images, grayscale images, background subtracted images, binarized images after background and foreground subtraction, mask images, and masked color images, respectively. Detected particle positions are indicated by circles with central dots in the far-right column, and each circle is colored by the averaged particle color within a region of 3×3 pixels. Particle images in (a) show the case of ~ 0.007 ppp.

3.2.3.1 Pre-processing

First, color images I_c are converted to grayscale images I_g , as shown in the second column of Fig. 3.8. Because the original images I_c or I_g contain static artifacts such as light scattered from the tank, background subtraction is performed for grayscale images I_g to identify only the particles. Background images I_{bg} are generated by temporally averaging the grayscale images in each measurement, which is formulated as $I_{bg} = \overline{I_g}$. A background-subtracted image is then obtained as $I_g - I_{bg}$, as shown in the third column of Fig. 3.8. Only the seeded particles are now visible in the background-subtracted images; however, some particles with small brightness and hue values also exist. These uncolored particles should not be included in the particle detection because they do not provide informative color values and should be excluded from the subsequent

analyses. Therefore, it is necessary to predetermine a threshold brightness value for excluding non-illuminated particles.

In the dark illumination measurement (Sec. 3.2.2.3), non-illuminated particle images have already been obtained. For these images, the background subtraction is performed in the manner described above. To determine thresholds for excluding non-illuminated particles, a pathline image I_{pl} of the non-illuminated particles is compiled by taking the pixelwise maxima of stacked a number of background-subtracted images. This operation is formulated as $I_{pl} = f_{\max}(I_g - I_{bg})$, where $f_{\max}$ represents a pixelwise operation for taking the maximum values of a stacked lot of background-subtracted images. Since the pathline image only contains the non-illuminated particles, it is considered to represent the pixelwise brightness values for the non-illuminated particles in the full-color and sheet-color illumination measurements, which are typically much darker than the colored particles. To ensure spatially homogeneous distribution of the pathline brightness, a Gaussian blur with a kernel size of 15×15 pixels (K_G) is performed. The blurred pathline image of the non-illuminated particles is called a foreground image $I_{fg} = I_{pl} * K_G$ hereafter, where $*$ represents a convolution operation.

The common foreground image I_{fg} is subtracted from the background-subtracted images of the full-color and sheet-color illumination measurements. The image is then binarized by a global threshold value after the foreground subtraction to filter out the non-illuminated particles. This is written as $I_{bin} = f_{bin}(I_g - I_{bg} - I_{fg})$, where f_{bin} is the step function for binarization. The binarized images I_{bin} are shown in the fourth column of Fig. 3.8. The regions of the colored particles remain after this binarization because they are typically brighter than those of the non-illuminated particles. Tiny defects in I_{bin} are padded by a closing process (kernel size 3×3 pixels), and each particle region is dilated by a kernel of 5×5 pixels to ensure the regions are large enough for the subsequent particle detection step. After these procedures, one obtains mask images I_m as shown in the fifth column of Fig. 3.8. The mask images are used for masking the original color image I_c to extract only the colored-particles. Finally one obtains masked color images $I_{mc} = I_m \odot I_c$, where $\odot$ represents the Hadamard product, as shown in the sixth column of Fig. 3.8.

Comparing the original color image I_c (the first column) and the masked color image I_{mc} (the sixth column), only colored particles are successfully identified in both the full-color and the sheet-color illumination measurements through identical image processing steps. Particle colors at the same depth positions obtained in the full-color and the sheet-color illumination measurements were regarded as statistically the same because the masking procedure fully filtered out particles with insignificant colors.

3.2.3.2 Colored-particle detection

Particle detection is performed on the masked color images I_{mc} through various methods, as I_{mc} only contains the colored-particle regions. Here, the particle mask correlation method [190]

using a Gaussian brightness distribution as a template was utilized for the particle detection. This method can detect particles allowing for slight particle overlaps and determines particle positions $P(i, j)$ at sub-pixel level.

Color extraction from the detected particles may be performed by a variety of methods. [200] proposed seven algorithms for determining the hue values of colored particle images, such as the average hue or saturation-weighted hue of a particle region. [134] attempted to identify representative particle colors from intentionally defocused images to minimize color deviations within the particle regions. The best values may change in each experimental condition, and it is necessary to select the optimal one through case studies. However, use of a representative color value may not fully represent the color information originally recorded in 24-bit format. In the present study, no reduction of the color information to hue values is allowed, but instead use all the RGB values at the central pixel (the closest pixel to the sub-pixel center) and its surrounding eight pixels. Thus, the RGB values in the 3×3 pixels, resulting in 27 brightness values for a single particle are utilized to determine its depth coordinate. For convenience, this $3 \times 3 \times 3$ matrix is represented as a color matrix $\mathbf{C}$ hereafter. This systematic color identification will provide the robustness of the proposed method by excluding empirical decisions required for seeking an optimal color identification scheme unique to the measurement environment. With this, the particle position in the masked color image $\mathbf{I}_{mc}$ is represented as $P(i, j, \mathbf{C})$, where $\mathbf{C}$ should correspond to depth coordinate y .

In the far right column of Fig. 3.8, the positions of the detected colored particles are plotted by circles with central dots. Each circle is colored by the average color of color matrix $\mathbf{C}$. The colored particles are successfully detected, even those that slightly overlap, by the particle mask correlation. Each color is thus well extracted for both the full-color and the sheet-color illumination measurements.

3.2.4 Color-to-depth calibration based on an artificial neural network

Using the particle images obtained in the sheet-color illumination measurements (Sec. 3.2.2.2) and subsequent image processing presented in Sec. 3.2.3, particle colors $\mathbf{C}$ at a certain depth coordinate y are obtained. These known color-to-depth relationships are used to obtain the color-to-depth calibration. It is essential to use the color-to-depth relationship obtained for the actual particles seeded in the fluid. This is because the color values of the illuminated pattern given as input to the LCD projector and the scattered light from the seeded particles should be different. The discrepancies between the ideal and the actual color values are inevitable in the measurement environment due to various factors such as the light emission characteristics of the LCD projector, the light scattering characteristics of the particles, the light receiving characteristics of the color camera, the internal setting of the color camera. Accordingly, the direct use of the particle color for the color-to-depth calibration is essential to consider such inevitable discrepancies. In other words, the original spectra of the illuminated pattern are not

important for in situ color-to-depth calibration, and the calibration becomes possible as long as the color change in the depth direction changes smoothly in the digital color space, the RGB domain. As the color matrix of a particle $\mathbf{C}$ possess 27 values, this calibration procedure becomes a multi-regression problem, and thus an artificial neural network (ANN) was employed to solve this problem. Similar efforts to solve multi-regression problems have been performed recently in the color-to-temperature calibration of thermochromic liquid crystals [13, 111].

3.2.4.1 Construction and training of an ANN

First, the known relationships between the particle position $P(i, j, \mathbf{C})$ and the depth position y are summarized for all images obtained in the 40 sheet-color illumination measurements. For each sheet position, 5,000 colored-particles are randomly selected, and 200,000 ($= 5,000 \times 40$) items of particle information in total are compiled as training data for the ANN calibration. In addition to the training data, the same number of colored-particle data are stored in the same manner as the test data for later evaluation. Here, one considers both the image coordinates (i, j) and color matrix at position $\mathbf{C}_{i,j}$ as input values for the ANN calibration. This is because the particle color may depend on the position in the image plane, as the color beam from the LCD projector attenuates in the x -direction and diverges to the z -direction in real coordinates corresponding to the i - and j -directions of the image coordinates. Further, color divergence (unwanted mixing of colors) leading to positional dependence of colors may occur as the color beam from the LCD projector was not fully focused across the field of view. Thus, 29 values $(i, j, \mathbf{C}_{i,j})$ are provided for an input of the ANN to determine a single output y , and this depth coordinate estimation based on the ANN is conceptually formulated as $y = \mathcal{F}(i, j, \mathbf{C}_{i,j})$. Various ANN structures, e.g., the number of nodes and number of hidden layers, were tested in preliminary trials. Throughout the trials, an ANN structure with two hidden layers, each containing 800 and 400 nodes, was determined as empirically optimal with respect to accuracy for estimating the depth coordinate from the input values. The training process (optimization of the weight matrices) of the constructed ANN is conducted using the adaptive momentum estimation proposed by [90] with mini-batch training. For the activation function, the rectified linear unit (ReLU) was employed.

In Fig. 3.9, the root-mean-squared error $RMSE$ for the training and test data during the learning process is illustrated. Here, the $RMSE$ value is defined as

$$RMSE = \sqrt{\frac{1}{N} \sum_k^N (y_{\text{pred}} - y_{\text{true}})^2}, \quad (3.1)$$

where N , y_{pred} , and y_{true} represent the number of data ($N = 200,000$), the depth coordinate predicted by the ANN, and the true depth coordinate (the known y coordinate in the sheet-color illumination measurements), respectively. To construct an optimal ANN system, the performance

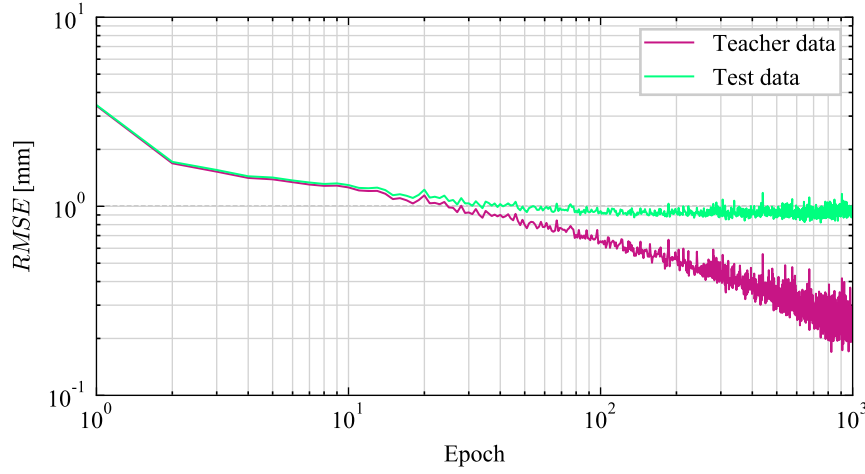


Figure 3.9: Training progress of the constructed ANN according to the $RMSE$ values for the training and test data.

should be evaluated on the test data. Otherwise, the ANN fits only the training data, which is the so-called over-learning problem. Whereas the $RMSE$ for the training data decreases over 1,000 epochs, that for the test data stays almost constant, at slightly below 1 mm. This indicates that the ANN may have over-learned the training data. Therefore, the weight matrices obtained after 100 epochs were chosen as the optimal ones for the constructed ANN system. This selection keeps accuracy $RMSE < 1$ mm sufficiently high because the original accuracy of this experimental configuration is ± 1 mm (half of the sheet thickness and particle radius), as mentioned in Sec. 3.2.2.2.

3.2.4.2 Evaluation of depth coordinate estimation

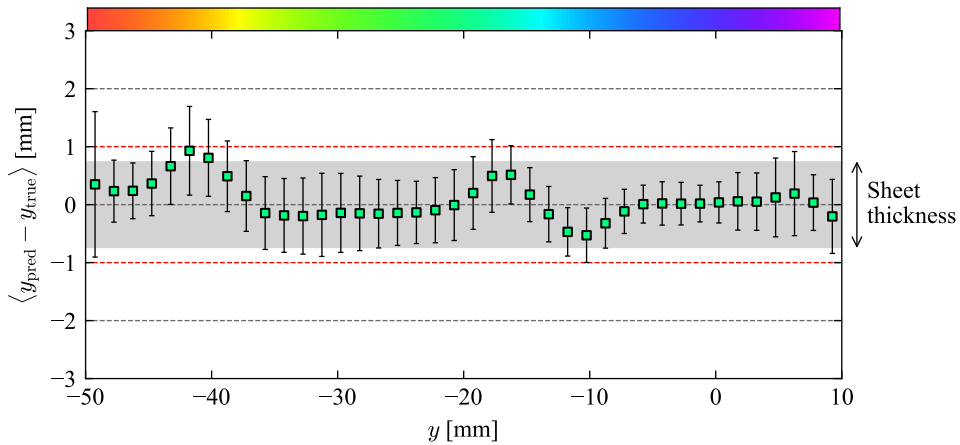


Figure 3.10: Mean deviations at each depth coordinate. Error bars indicate the standard deviations for each plot. The projected full-color image is shown above for reference. The target accuracy ± 1 mm is indicated by the red dashed lines, and the thickness of the sheet-color pattern ± 0.75 mm is shown as gray-shaded region.

Performance of the calibration supported by ANN was evaluated using the test data which has a known relation between the actual depth coordinates and colors. This thus reflects actual

accuracy for determining the depth coordinates of the particles seeded in the fluids. The *RMSE* values shown in Fig. 3.9 represent a global trend for the whole measurement volume $\delta y = 60$ mm. Furthermore, the depth-dependent deviation between the estimated depth and the true depth at each reference point is required to evaluate the depth coordinate estimation. In Fig. 3.10, the mean deviation $\langle y_{\text{pred}} - y_{\text{true}} \rangle$ for the test data at each reference point is shown with the corresponding one standard deviation range as an error bar. The projected color is shown above for reference. Overall, the mean deviation stays within -1.0 to 1.0 mm, surpassing the target accuracy ± 1 mm, indicated by red dashed lines in Fig. 3.10. Considering this, the mean deviation for each reference point is low enough to ensure sufficient accuracy on depth coordinate estimation. Although the other regions show good agreement with the true values, with deviations of less than 0.5 mm, there are two error-prone regions in the projected color, at $y \sim -45$ to -38 mm (orange) and $y \sim -20$ to -7 mm (cyan). Similarly, earlier works performed by [200] and [134] showed error-prone regions in the hue-to-depth calibration curves. This may occur for various reasons, such as the quality of the color illumination projected by the LCD projector, the light receiving characteristics of the camera, or the light scattering characteristics of the particles. In addition, relatively large deviations in the depth estimation are located in the corner regions of the 3D-*RGB* domain of illumination colors, as shown in Fig. 3.6(c). The changes in colors of the rainbow gradient in these regions may be smaller than for colors in other regions. It might be better to exclude the error-prone regions from the projected pattern, however, the validated accuracy, less than ± 1 mm, is sufficiently close to the existing deviation ± 1 mm in the present system discussed in Sec. 3.2.2.2. It is possible to conclude that the color-to-depth calibration works well for depth coordinate estimation over the whole illuminated volume.

3.2.5 Particle tracking in 3D real space

Tracking particles in 3D real space (x, y, z) , as in the tomographic PIV technique [46, 167], is a straightforward approach to 3D velocity field measurements. However, it is necessary to consider the accuracy of particle coordinate estimation before the 3D tracking. The depth coordinates of the particles determined from the color information have an accuracy of around ± 1 mm in the present case. This is much lower than the sub-pixel accuracy $O(0.01 \text{ mm})$ of the in-plane coordinates. Accordingly, particle tracking in 3D real space may not be feasible, as the particle displacements in the y direction cannot be treated like as those in the x and z directions. To overcome this problem, a slightly more complicated 3D particle tracking must be followed, and an analytic scheme to reconstruct 3D particle trajectories is proposed in this section.

3.2.5.1 Two-dimensional particle tracking in the image plane

It is possible to track particle trajectories in the an image plane projecting the 3D space by adopting the conventional 2D PTV technique for the detected particles. In the present study, an

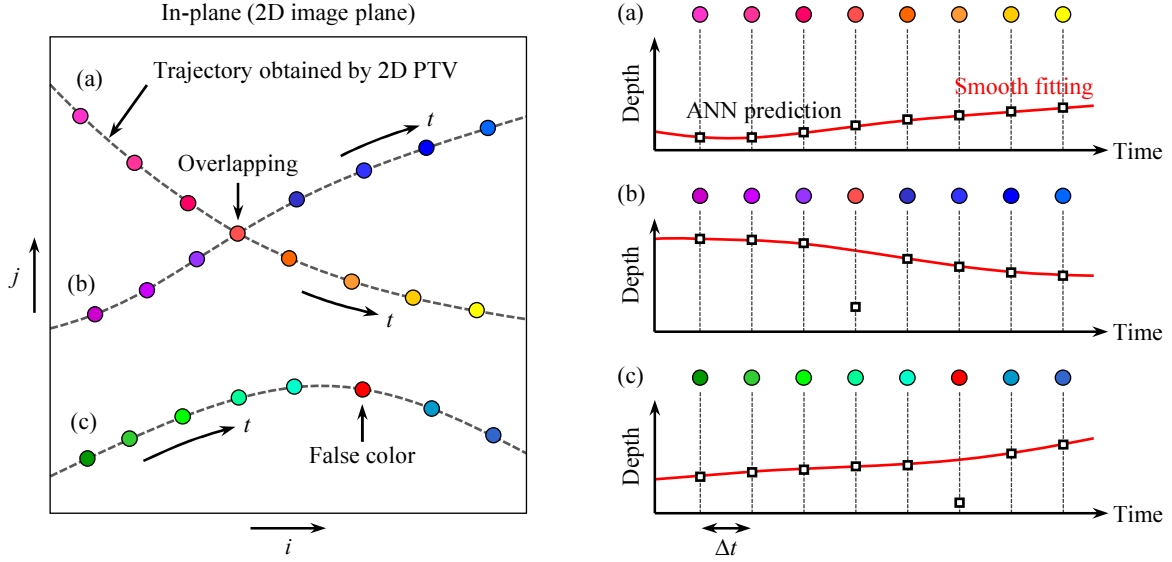


Figure 3.11: Schematic illustration of the particle trajectories and the color change (left). Particles (a), (b), and (c) are the particles with continuous color change, with discontinuous color change due to overlapping, and false color, respectively. Depth estimation and fitting are also schematically illustrated in the right.

in-house 2D PTV code was adopted. The 2D PTV code simply links the particle displacements by the nearest-neighbor method [138], and outlier displacements are detected by the universal outlier detection [207]. The outlier displacements are then associated with other close particles, and these procedures are iterated until the convergence. It is worth computing the 2D trajectories for the purpose of labeling each particle in the image plane prior to 3D tracking. A 2D trajectory for a single particle is composed of temporally continuous image coordinates (i, j) along with color matrix $C_{i,j}$. Conceptually, the particle motion in the 3D real space is smooth, and the recorded trajectory $(i, j, C_{i,j})$ also provides a smooth change over time. Thus, tracking particles in the 2D image plane enables the temporal smoothness of the particle trajectory to be used to correct unreasonable deviations in the depth coordinate estimation.

3.2.5.2 Correction of the depth coordinates

As noted above, the color-to-depth calibration provides a probable estimation of depth coordinates from the color information of the particles. This initial estimation by the ANN does not ensure temporal smoothness of the trajectories in the depth direction. Thus, the trajectories in the depth direction need to be corrected to ensure temporal smoothness. A similar approach was already performed by [6]. The reasons for the non-smoothness are originated from various issues. One is the overlapping of the particles. Since the present study is based on the single-view approach, the particles can be overlapped on the 2D image plane. This issue is schematically illustrated in the left of Fig. 3.11. Particle (a) and (b) are occasionally overlapping, as the far particle cannot be recorded if the near particle is recorded at the same image positions (i, j) . The other for the non-smoothness is the false color, which is illustrated in the left of Fig. 3.11. The false color can be generated by various reasons, such as the imperfection of the color interpolation

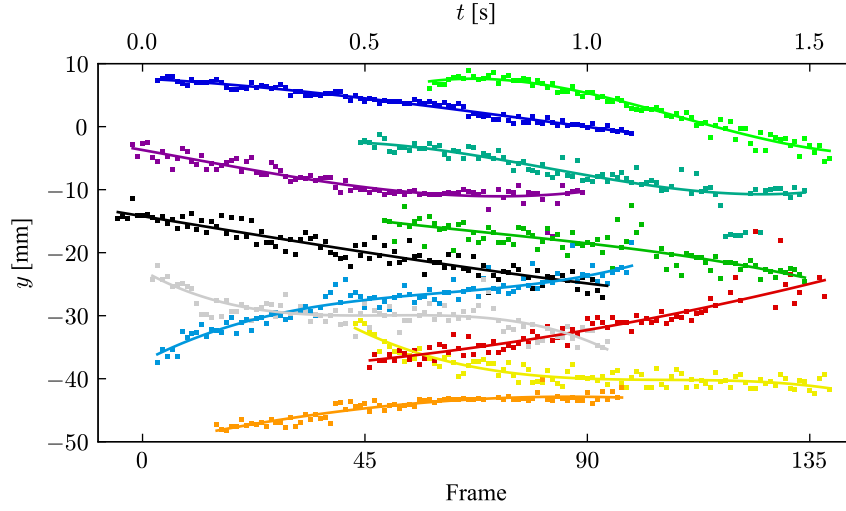


Figure 3.12: Particle trajectories with respect to the depth coordinates. Particle positions originally estimated by the ANN are represented by squares, and fitted curves for each particle trajectory are shown as solid lines. The squares and lines are colored uniquely for each particle.

(demosaiicing) scheme, the inaccurate particle detection, the light scattering characteristics of the particles. Even if such discontinuities in particle colors cannot be eliminated, it is possible to track particles in the 2D image plane by adopting the standard 2D PTV. The depth fitting can be achieved by tracking the particle in the image plane as illustrated in the right of Fig. 3.11. As long as the depth positions of a single particle predicted by the ANN system are roughly tracked, the discontinuity in the depth estimation can be filtered out by the fitting for the temporal depth signals.

In Fig. 3.12, examples of the depth coordinates estimated by the ANN-based calibration are plotted as uniquely colored squares for each particle. Only 80 to 100 successive frames of eleven trajectories are shown for visibility. As shown in Fig. 3.12, the depth coordinates estimated by the ANN fluctuate over time, and these fluctuations increase when the particles enter the error-prone region ($y \sim -45$ to -38 mm and $y \sim -20$ to -7 mm), as shown in Fig. 3.10. The deviating particle trajectories are fitted by the robust estimation method proposed by [73], and the fitted curves are drawn by solid lines. For these fluctuating plots, the robust estimation successfully draws smooth curves while excluding the outlier points that may originate from deviations in the colors of the particles. The fitted curves ensure temporally smooth displacements of the particles in the depth direction, and thus these corrected depths can be used as the measured depth coordinates. Please note that the deviations in Fig. 3.12 and the error-prone regions in Fig. 3.10 do not correspond because the trajectories shown in Fig. 3.12 are extracted from the whole region of the measurement domain and the regions close to the impeller originally have deviations due to the vortices induced around the impeller.

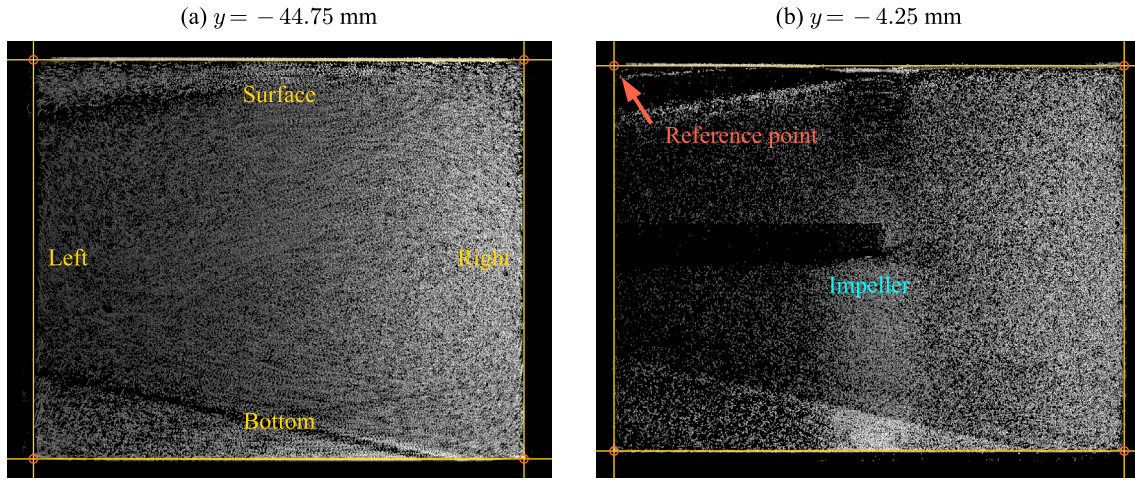


Figure 3.13: Long-term pathline images at different cross-sections, (a) $y = -44.75$ mm and (b) $y = -4.25$ mm. Four boundaries are indicated by solid lines, and reference points of four corners are indicated by circles.

3.2.5.3 Mapping image coordinates to real coordinates

Image coordinates (i, j) cannot be directly converted to real coordinates (x, z) because the magnification rate differs with respect to depth coordinate y . Thus, after obtaining the corrected depth coordinate, the in-plane real coordinates (x, y) need to be calculated through mapping functions $x = \mathcal{G}(i, j, y)$ and $z = \mathcal{H}(i, j, y)$. The mapping function is best calculated during the 3D calibration process employed in the stereoscopic PIV measurement, which mechanically traverses a calibration plate with dotted or grid patterns in micro-stages [e.g. 181, 194]. In the present study, a method that requires only particle images to create mapping functions is proposed. The mapping function can be obtained only from the particle images, assuming that the lens aberrations and other optical refraction are negligible. If a sufficient number of tracer particles are seeded in the fluid, the colored particles should appear over the whole illuminated regions during long-term observation. Accordingly, long-term pathline images $\mathbf{I}_{pl}$ compiled using the masked color images of the sheet-color illumination measurements provide particle regions in each thin sheet region. Here, $\mathbf{I}_{pl}$ is newly obtained for all the sheet positions through the same procedure described in Section 3.2.3, resulting in a brighter image compared to the foreground image $\mathbf{I}_{fg}$. Examples of the long-term pathline images are shown in Fig. 3.13. From the long-term pathline images, the boundaries between the fluid and sidewalls or fluid surface become apparent, as the images possess bright pixels only in the fluid region seeded with the tracer particles. Accordingly, the real coordinates (x, y, z) of the four corners are known, and the corresponding image coordinates (i, j) are also identifiable from the long-term pathline image at each cross-section. In the present study, it was possible to obtain 160 reference points (four corners of 40 cross-sections).

Since the reference points at the four corners of different depth positions are aligned along the circumferences of the cylindrical vessel, these move in a bit complicated manner within the image planes over the different depth. Here, a third-degree polynomial fitting was employed to

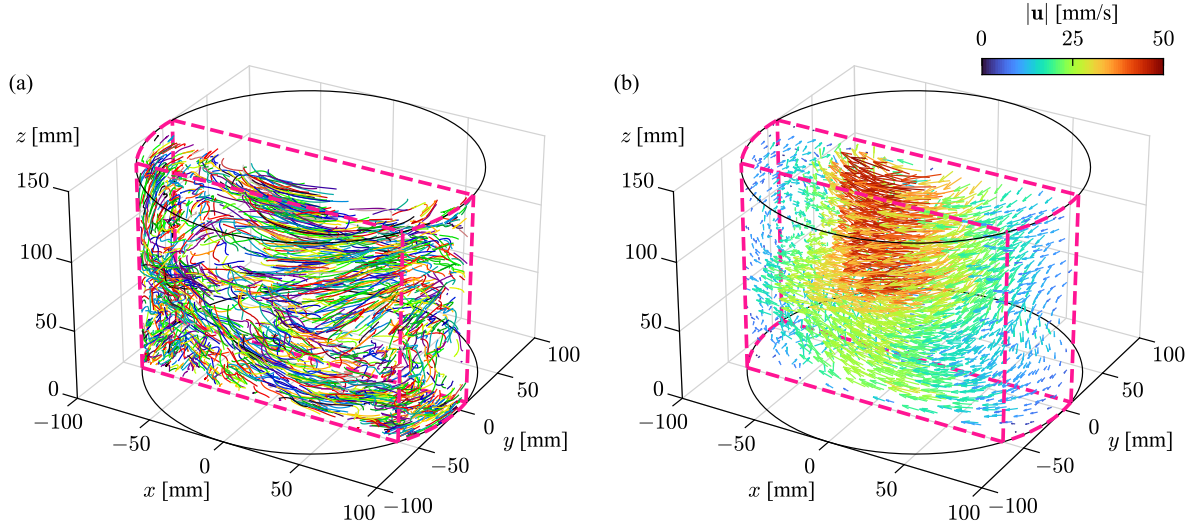


Figure 3.14: Reconstructed 3D flow field in a rotating flow induced by an impeller: (a) 3D particle trajectories of 2,000 particles and (b) mean velocity field interpolated on a regular grid during 30 s (fifteen cycles of the impeller). The trajectories in (a) are colored uniquely to identify each particle. The velocity vectors in (b) are colored by the velocity magnitude $|\mathbf{u}|$, and only 1% of the interpolated vectors are shown for clearer visibility. The measurement volume of the color PTV is enclosed by pink dashed lines.

obtain the mapping function $\mathcal{G}$ for the x coordinate, and this is described as

$$\begin{aligned}
 x &= \mathcal{G}(i, j, y) \\
 &= a_0 + a_1 i + a_2 j + a_3 y + a_4 i^2 + a_5 j^2 + a_6 y^2 + a_7 i j \\
 &\quad + a_8 j y + a_9 y i^2 + a_{10} i^3 + a_{11} i^3 + a_{12} i j^2 + a_{13} i^2 j \\
 &\quad + a_{14} j y^2 + a_{15} j^2 y + a_{16} y^2 i + a_{17} i y^2 + a_{18} i j y.
 \end{aligned} \tag{3.2}$$

Here, a_0 to a_{18} are constants obtained by the least-squares method. In the same manner, the mapping function $\mathcal{H}(i, j, y)$ for the z coordinate can be computed. [6] employed second-order regression for their cubic geometry, however, the best fitting function may differ depending on the spatial distribution of the reference points. Using these mapping functions, the final in-plane coordinates x and z are obtained, and the particle trajectories in 3D real space $P(x, y, z, t)$ are determined.

3.3 Validation and perspective of the methodology

3.3.1 Validation of the methodology in a rotating flow

The methodology established in Section 3.2 was validated in a rotating flow experiment in a cylindrical tank, details of which are given in Sec. 3.2.1. In the presented case, the typical particle image density was ~ 0.01 ppp resulting in the tracking of approximately 20,000 particles simultaneously. The reconstructed 3D particle trajectories are shown in Fig. 3.14(a).

The trajectories of 2,000 particles are shown in Fig. 3.14(a), and colored uniquely to identify each particle. The measurement volume of the color PTV is indicated by the pink dashed lines. A rotating flow structure within the measurement volume is qualitatively shown by the trajectories.

To validate the performance of the color PTV quantitatively, a temporal mean velocity field in 3D space was computed using the tracking result. For this, the average of all the particle trajectories measured during 30 s (fifteen rotations of the impeller) was taken, and velocity vectors on a regular grid with 2 mm distancing in any direction were interpolated by Shepard's method [173]. For this interpolation, only the velocity vectors within ± 0.75 mm from each grid were used, and the weighted average values were interpolated on each grid using the squares of the inverse distances between the grid and the particles as the weight. The interpolated velocity field is shown in Fig. 3.14(b) with the velocity vectors colored by the magnitudes $|\mathbf{u}|$.

The interpolated velocity field must be compared with the flow field measured by the conventional PIV. For the PIV measurements, a 1.5-mm-thick red laser (DPRLu-5W, *Japan Laser Co.*) was used for sheet illumination at arbitrary cross-sections. In principle, any cross-sections can be measured by the PIV because the fluid vessel is fully transparent. The direct cross-correlation method was utilized in the PIV measurement. Mean velocity fields at a vertical cross-section of $y = -30$ mm measured by conventional PIV and color PTV are shown in Figs. 3.15(a) and (b). Here, the velocity field measured by the conventional PIV [Fig. 3.15(a)] was obtained by taking the temporal average for the same duration as that of color PTV, and that by color PTV [Fig. 3.15(a)] was extracted from the interpolated velocity fields shown in Fig. 3.14(b). The color contours of Fig. 3.15(a) and (b) represent the magnitude of the in-plane velocity $\sqrt{u^2 + w^2}$, and the in-plane velocity vectors are shown by white arrows. The mean velocity field measured by color PTV seems to be more blurred than that measured by conventional PIV. However, the result of color PTV is found to reconstruct the smallest toroidal vortex structures present in the system. This performance may be sufficiently high in terms of practical applications. Velocity profiles at fixed horizontal lines of $z = 20, 74$, and 130 mm are shown in Fig. 3.15(c), and the profiles measured by conventional PIV and color PTV are shown by different lines and symbols, respectively. The corresponding horizontal lines are represented by dashed lines in Figs. 3.15(a) and (b). Overall, the velocity profiles measured by the two methods show the same trend with respect to magnitudes and gradients. The velocity profiles at $z = 74$ mm, however, show discrepancies especially at the vicinity of the side wall $|x| > 70$ mm. These discrepancies may originate in low particle density around the region and the grid interpolation scheme, which requires spatial averaging adapted to the data obtained by color PTV. Thus, an increase in the particle image density may solve this problem, and that can be ideally increased up to ~ 0.02 ppp in the present configuration, as 7×7 pixels were required to reconstruct the color matrix $\mathbf{C}$ with 3×3 pixels through color demosaicing. Apart from these steep changes around the edge regions, color PTV reconstructs the velocity field well.

In the same manner, the mean velocity fields at a horizontal cross-section of $z = 25$ mm

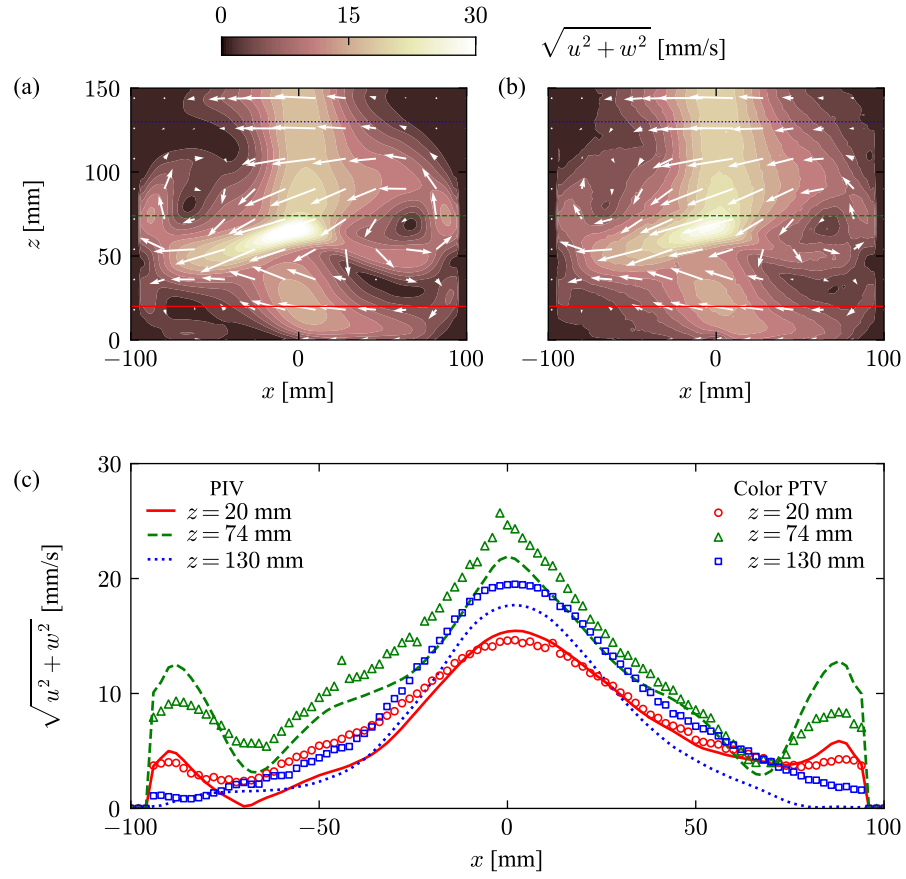


Figure 3.15: Mean velocity fields measured by 2D PIV and color PTV at a vertical cross-section of $y = -30$ mm: Velocity fields measured by (a) conventional PIV and (b) color PTV. (c) Velocity profiles at fixed horizontal lines of $z = 20$, 74, and 130 mm.

measured by the two methods are shown in Figs. 3.16(a) and (b). The color contours of Figs. 3.16(a) and (b) represent the magnitude of the in-plane velocity $\sqrt{u^2 + v^2}$, and the in-plane velocity vectors are shown by white arrows. Velocity profiles at fixed horizontal lines of $y = -40$, -20 , and 0 mm are shown in Fig. 3.16(c), and the profiles measured by conventional PIV and color PTV are shown by different lines and symbols, respectively. Similar to the results shown in Fig. 3.15, the mean velocity field measured by color PTV is blurred, and observable steep changes, especially around the center of the cylinder, are not fully reconstructed. However, color PTV well represents the circulating fluid motion induced by the impeller in any 3D direction, and the order of the velocity magnitude matches that measured by conventional PIV.

For further validations, the mean absolute deviations MAD and the cross-correlations of the velocity fields measured through the two different methods are computed. Here, the MAD is defined as

$$MAD = \frac{1}{N} \sum |u_{PIV} - u_{CPTV}|, \quad (3.3)$$

where the subscripts PIV and CPTV correspond to the velocity values measured by the standard PIV and the 3D color PTV. The results for the velocity fields shown in Fig. 3.15 and 3.16 are summarized in Table 3.1. For all the velocity components, the MAD values record ~ 0.5 mm/s,

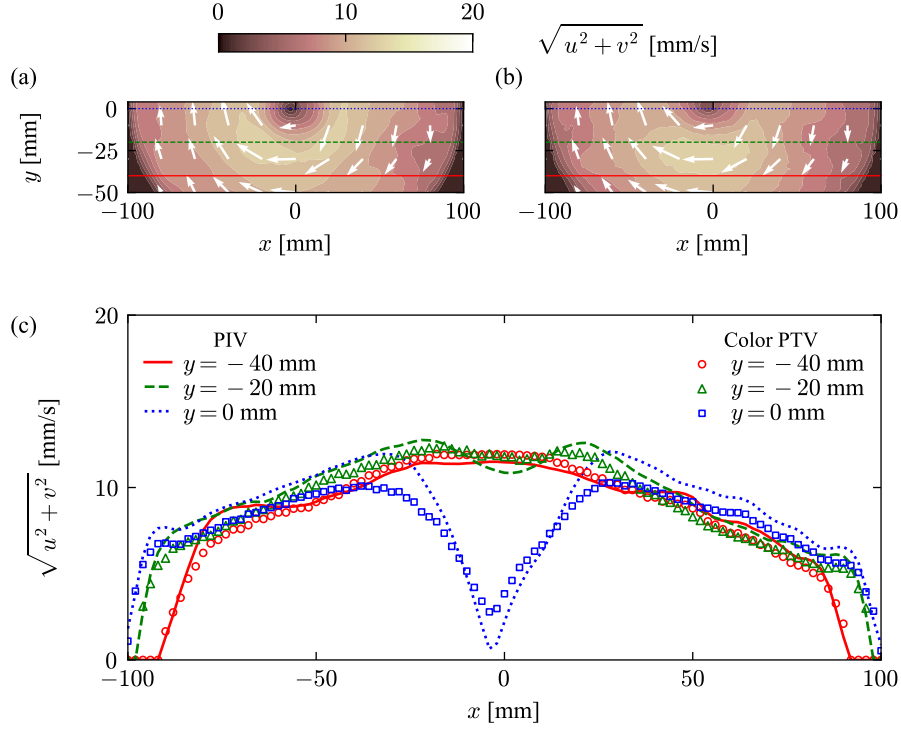


Figure 3.16: Mean velocity fields measured by 2D PIV and color PTV at a horizontal cross-section of $z = 25$ mm: Velocity fields measured by (a) conventional PIV and (b) color PTV. (c) Velocity profiles at fixed horizontal lines of $y = -40, -20, \text{ and } 0$ mm.

Table 3.1: Validation of the velocity fields measured by the 3D color PTV.

Quantity	MAD [mm/s]	Cross-correlation [-]
$u(x, y = -30 \text{ mm}, z)$	0.627	0.997
$w(x, y = -30 \text{ mm}, z)$	0.419	0.988
$u(x, y, z = 25 \text{ mm})$	0.531	0.999
$v(x, y, z = 25 \text{ mm})$	0.470	0.998

and these values may be sufficiently small compared to the velocity range of the presently focusing flow fields, $O(10^1 \text{ mm/s})$. The cross-correlations are very close to unity for all the cases, meaning that the 3D color PTV reconstructed well the mean velocity fields at least for the present experiments.

Incorporating all the discussions, it is possible to conclude that color PTV can measure the 3D-3C velocity fields through the methodology proposed in Sec. 3.2 without changing optical arrangements. In the present study, the upper limit of the particle image density was 0.02 ppp. Ideally, this can be increased up to 0.05 ppp if a 3-CCD-camera is employed, because it does not require color demosaicing restricting the minimum number of pixels comprising a single particle. This maximum particle image density is equivalent to that of the 2D PTV [49], while the shake-the-box method yields 0.125 ppp at the maximum through the multiple camera approach [167]. The measurable spatial resolution, however, may depend on the measurement

depth since the particles distributing in the 3D domain are projected to the image plane. In the present setting, the number of detected particles in the full-color illumination measurement indicates a particle number density of $\sim 0.01 \text{ mm}^{-3}$ per frame with the measurement depth of $\delta y = 60 \text{ mm}$. To calculate the mean velocity field, the particle number density becomes $\sim 27 \text{ mm}^{-3}$ (90 f.p.s for 30 s). This was sufficient to resolve the smallest flow structures $O(10 \text{ mm})$ of the toroidal vortices, while the steep velocity gradients inside the vortices shown in Fig. 3.15(a) were underestimated.

3.3.2 Perspective of the methodology

So far the performance of the proposed 3D color PTV has been validated, and its reasonable performance was justified. There are other various single-view method to measure 3D-3C velocity fields, and these techniques are summarized in Table 3.2. Here, only the measurable depth and the particle image density in terms of the particle per pixels (ppp) are paid attention. It is noteworthy that the present method and other color PTV approaches take advantage in the measurable depth (volume) and the particle image density. The color PTV has been established as an advanced method which extends the measurement ability of the conventional PIV/PTV, and thus it can be applied without special optics. This is the reason why the color PTV yields a large particle image density. Since the camera and lens used for the color PTV are standard, the measurable depth can be also increased by changing the depth of field. In the present study, the resultant depth of field was $\sim 80 \text{ mm}$, however, it can be increased by increasing the aperture or the focal length of the lens.

The particle image density is a kind of value important in image processing, and it does not represent the actual measurable spatial resolution in the velocity fields. To consider the application of the color PTV for measurements of some target phenomena, the particle number density in the unit of mm^{-3} is significant for judging whether the method can resolve the phenomena or not. As above, the particle number density changes with the variations in the measurement depth (volume). In Fig. 3.17, relationship between the particle image density and the measurement depth are shown. The displayed example is the case using a 1-Mega-pixel camera (1024×1024 pixels) with a spatial resolution of 0.1 mm/pixel . The result of the present study shown in Sec. 3.3.1 corresponds to the star symbol, 0.01 ppp and $\delta y = 60 \text{ mm}$. The limitations in the particle image density for the present method is shown by the vertical dashed line. This graph can be compiled only if the measurement volume and the specifications of the optics are determined before the measurements. Thus one should consider both the particle number density achievable in their measurement environments and the required spatial resolution to reveal the phenomena.

Apart from the requirements discussed above, the particle size on the digital image will be another constraint for the color PTV. This is because at least a few pixels should be occupied by a single particle on the digital image to extract informative color. In particular, single-tip

Table 3.2: Comparison of typical abilities of single-view 3D-3C velocity field measurement techniques.

Author	Technique	Measurable depth	Particle per pixel (ppp)
Cierpka et al. [32]	Astigmatism PIV	90 μm	0.0005
Cierpka et al. [33]	Astigmatism PIV	100 μm	0.00006
Pereira et al. [139]	Three-pinhole aperture PIV	160 μm	$O(0.0001)$ (not described)
Tien et al. [192]	Three-pinhole aperture PIV	600 μm	0.0004
Salazar et al. [166]	Holographic PIV	10 mm	0.0015
Sheng et al. [172]	Holographic PIV	1.5 mm	0.014
Shi et al. [175]	Light-field PIV	10 mm	0.0118
Watamura et al. [200]	Color PTV	50 mm	0.000125
Aguirre-Pablo et al. [6]	Monochromatic (color) PTV	60 mm	0.002 (up to 0.02)
Present study [126]	Color PTV	60 mm	~ 0.01 (up to 0.02)

CMOS or CCD cameras employ the Bayer pattern to acquire color images, the three brightness values RGB on a single pixel are interpolated after image acquisition. Accordingly, a small particle only occupying less than 3 pixels in diameter never returns correct color, since the color interpolation (demosaicing) cannot be performed properly. However, this consideration is simply required only for the implementation of the color PTV, and it is sufficient that one pays attention in the measurement steps. The maximum particle image density 0.02 ppp indicates that a single particle occupies at least 50 pixels. That is, as long as the particles are recorded to spread more than 3 pixels in diameter, the size of the particle will not be a bottleneck in the tracking algorithm of the 2D PTV.

Another issue which should be addressed is arising from the use of the LCD projector, the refresh rate of the LCD projector. In general, the LCD projector overwrites the projection pattern with a certain refresh rate. It is typically $O(10 \text{ kHz})$ and $O(10 \text{ Hz})$ in the horizontal and the vertical directions. Even if the full-color illumination projects a rainbow light continuously, the LCD projector always overwrites the light with the predetermined refresh rate. Since there is a transition time for the liquid crystals of the LCD projector, a projected color at a certain position changes with time in the frequency corresponding to the refresh rate. Thus the depth estimation only from an instantaneous image will be affected by this issue, especially in the high-speed image acquisitions faster than the refresh rate. This issue is inevitable as long as the LCD projector is adopted as the light source. The 2D tracking and subsequent depth correction proposed in the present method can reduce the instantaneous erroneous depth estimation by considering the temporal smoothness. From this perspective, the presently established method

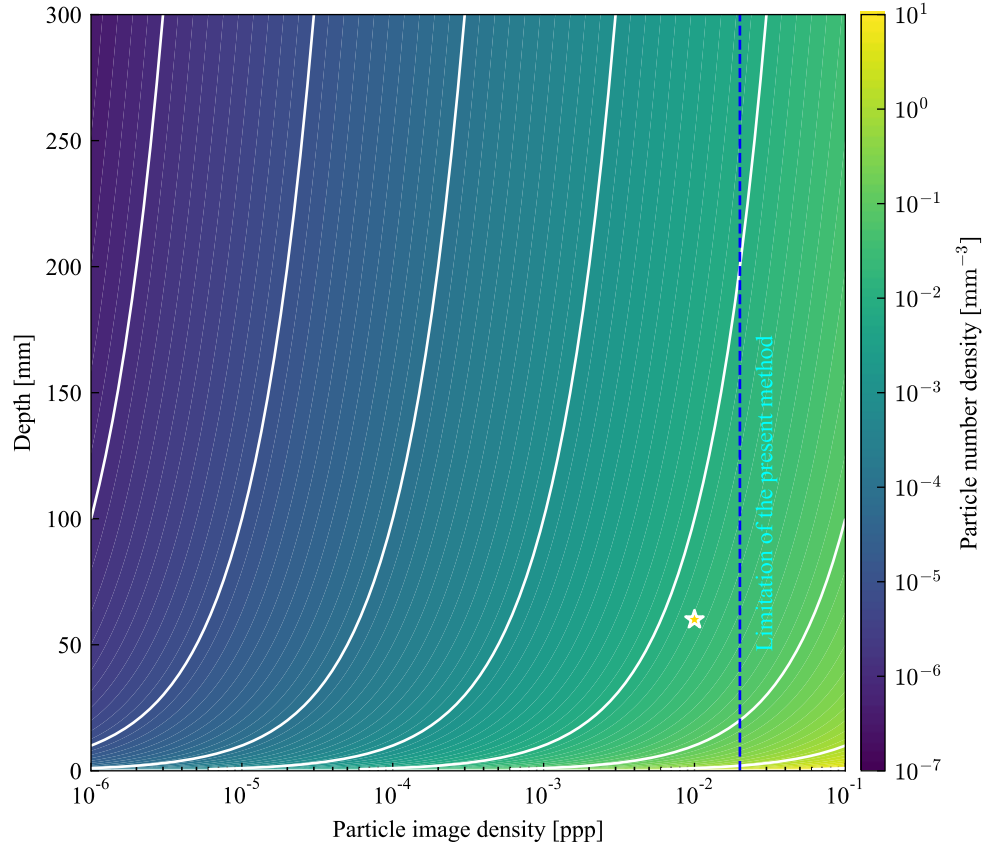


Figure 3.17: Relationship between the particle image density and the measurement depth for the case using an 1-Mega-pixel camera with a spatial resolution of 0.1 mm/pixel. The vertical dashed line indicates the limitations in the particle image density for the present method. The star symbol corresponds to the case shown in Sec. 3.3.1.

deals well various issues arising in the LCD projector-based color PTV.

3.4 Concluding remarks

In this chapter, a color-to-depth calibration methodology for 3D color PTV and performed a direct accuracy evaluation was proposed. The in situ method does not require calibration plates or cubes for associating color-to-depth relationships and is directly applicable to the particle-seeded fluids. Unreasonable assumptions, which have been conventionally taken, arising from the different optical arrangements between the measurements and calibration thus are not required. The use of a consumer-grade LCD projector reduces the cost and complexity of the 3D flow field measurements as the color PTV requires only a single camera and a volumetric rainbow gradient illumination. The proposed method consists of three different illumination measurements and a series of standard image processing steps for enabling conventional 2D PTV. The accuracy of the depth estimation was around 1 mm out of the full thickness of the volumetric illumination of 60 mm. The performance of the method was evaluated quantitatively by comparing the flow fields measured by the proposed method with those measured by the conventional PIV, and good accordance with the PIV with respect to the outline of the rotating flow was quantitatively

confirmed. This result promises that the method can reconstruct 3D flow fields only with the particle-seeded fluid, as long as a sufficient number of colored-particles are detectable to resolve target flow fields.

The present study formulated robust procedures from experimental design to 3D-3C flow field measurements. Each process, however, can be further optimized in terms of selection of illumination color pattern, color identification scheme, color-to-depth regression scheme, 2D PTV algorithm, and so on. For instance, the selection of the projected illumination patterns and the methods to calibrate the color-to-depth relationships may not be critical to the actual implementation of general color PTV, including this study, as long as a sufficiently high accuracy is ensured to resolve measurement targets. This is because variety of procedures have previously been proposed [6, 106, 108, 134, 200, 215], and each has both advantages and disadvantages. Moreover, the best approach may change depending on the conditions in the experimental environments, such as light source, camera, and the materials to be illuminated.

To implement the proposed method in 3D flow measurements, one needs to solve the limitations as follows. First, the actual size of particles should be large enough to scatter informative colors. Also, the size of tracer particles on the image planes needs to be large enough to extract reliable color information, namely more than 3 pixels in the diameter would be better in the present condition. The measurement volume is determined by the camera resolution and mounting lens specification, to solve the particle size limitation as above. The illumination intensity of the LCD projector limits the dynamic range of the velocity, in terms of the shutter speed of the camera. Under these limitations, the particle number density needs to be as much as high to reconstruct fine flow structures. As long as these limitations are properly considered, the proposed method can measure various flows as 3D-3C velocity fields in practical use.

APPLICABILITY EVALUATION OF THE STRESS-OPTIC LAW IN NEWTONIAN FLUIDS TOWARD STRESS FIELD MEASUREMENTS

Preface

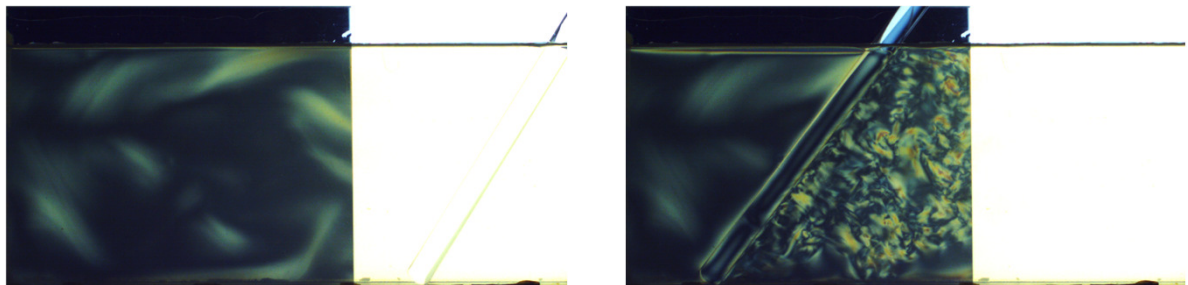


Figure 4.1: Graphical abstract of this chapter.

So far in Chapter 1, 2, and 3, color information has been utilized for determination of the out-of-plane positions. These techniques are a kind of optional ways of other advanced PIV methods, stereoscopic PIV and tomographic PIV. As the optical configurations of the proposed methods are simpler than those in such advanced PIV methods, wider and easier implementations in various measurement environments are expected. The proposed methods utilized colored illumination, which intentionally coloring particles seeded in the test fluids, by contrivances of light sources. Unlike such artificial coloration techniques, natural coloration mechanism may be also possible to be utilized in flow measurements.

The famous coloration found in fluid flows may be the interference color in soap liquids. With varying the thickness of the thin films of the soap liquids, beautiful rainbow-like coloration appears. Such interference colors can be found in the observation of compositions of materials of minerals, and internal stress fields of plastic materials. In Chapter 4, a possibility of the use of interference colors in fluid flow measurements are sought. This will be a completely novel approach to measure internal stress (fluid deformation) fields of Newtonian fluid flows through the photoelasticity based on the stress-optic law.

A novel rheoscopic fluid displaying coloration due to flow-induced birefringence is investigated through laboratory experiments to ascertain if it satisfies the stress-optic law in fluids in direct stress field measurements of Newtonian fluids. To validate this experimentally, two types of spin-up experiments ensuring two-dimensional flows were performed: color visualization using the rheoscopic fluid and velocity field measurement by particle image velocimetry. Using a wavenumber analysis, the orientation angle of the birefringence was determined from a single color snapshot in axisymmetric and unidirectional flows. Direct comparisons between color and shear stress values made it possible to establish a unique relation between the two, verifying that the stress-optic law can be applied in the fluid. The results of the present study suggest a promising avenue toward enabling temporally and spatially resolved stress field measurements in Newtonian fluids.

The results shown in Chapter 4 were published as Noto et al. [124] and the details are below.

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4.1 Introduction

Direct measurements of stress fields in fluid flows are useful to understand details of fluid physics/mechanics. For Newtonian fluids, such as water and air, the stress tensor $\boldsymbol{\tau}$ is linearly proportional to the strain rate $\nabla \mathbf{u}$ with a constant viscosity μ as $\boldsymbol{\tau} = \mu \nabla \mathbf{u}$. However, measurements of the fields of the spatially resolved velocity gradient tensor $\nabla \mathbf{u}$ in fluid flows is not straightforward, because in practical flows, the accessible value is the velocity field, not the gradient of the field.

Particle image velocimetry (PIV) makes it possible to measure flow fields in cross-sections of flows with simple planar PIV [5, 154]. Then by determining derivatives of the velocity vector fields, physical quantities, including the vorticity, shear rate, and other parameters, can be calculated in post-processing. The qualities of the derivative values are directly affected by both the dynamic range and the spatial resolution of the velocity vector fields. The velocity vectors are determined by identifying dominant displacements of particles in minute regions of two images recorded at short time intervals, and the resulting velocity vectors are smoothed within the interrogation region, limiting the dynamic range of the PIV. Additionally, the spatial resolution of PIV is typically orders of magnitude poorer than that of the original image, and will be too small for the acquisition of the fine details of derivatives of velocity fields. Efforts to measure the derivatives of velocity fields have been made [84], but currently PIV does not satisfy the desired needs for direct fluid deformation measurements, as far as the derivatives cannot be measured as the first observation values.

An alternative to direct measurements of fluid deformation, molecular tagging velocimetry (MTV) is summarized by Tropea and Yarin [193]. In MTV, numerous “tags” are created in fluorescent fluids using a laser beam lattice. Velocity vectors are quantified by tracking the centroids of the tags without needing the seeding with tracer particles. Shear rates can be obtained from the deformation of the shape of the tags without knowing the derivatives of the velocity vectors even in multiphase turbulent flows [68, 69]. The established work has focused on turbulent flows and was applied to obtain statistics of the turbulent flows, and the spatial resolution of the shear rate measurements obtained in this manner is not applicable to field measurements as the spatial resolution of MTV depends on the distances between the beams forming the beam lattice, and is equivalent to or poorer than PIV.

In different research fields, such as geology and material science, stress or strain field measurements utilizing the stress-optic law have been conducted for several decades as reviewed by Patterson [135]. The stress-optic law is written as

$$\mathbf{n}_{ij} = C \boldsymbol{\tau}_{ij}, \quad (4.1)$$

where, $\mathbf{n}_{ij}$, $\boldsymbol{\tau}_{ij}$, and C are the refractive index tensor, the stress tensor, and the photoelastic (stress-optical) coefficient, respectively. The internal stress of a solid material induces birefrin-

gence, and a light ray passing through the material is affected by showing changes in coloration. Backlight projection with optical filters enables observations of the internal stress distributions of solid materials like in plastic resins even with irregular geometries. This technique is termed photoelasticity (photoelastic analysis), and various methods to quantify the birefringence observed here have been developed as summarized by Ramesh et al. [156].

The photoelastic analysis in fluid flows has also been reported [136, 140, 141]. These reports investigated the stress-optic law as it is manifested in flows of polymer solutions (i.e., non-Newtonian fluids). The birefringence is induced by fluid deformations in such non-Newtonian fluids by analogy with solid materials, and termed a flow-induced birefringence. The anisotropy originating in the polymer structures provides the rheological properties of the fluids, and the stress-optic law in fluids is applied in the field of rheology and known as rheo-optics. For Newtonian fluids, however, it is not directly possible to show the flow-induced birefringence because the fluid is isotropic. Classically, rheoscopic fluids containing small flake particles, such as aluminum flakes and mica, have been used for qualitative visualization of Newtonian fluid flows with coherent flow patterns [105]. The flake particles are considered to preferentially align in the principal shear direction in the flow, even though actual particle motions are much more complicated [3, 23, 55]. A recent report seeking an alternative to classical rheoscopic fluids by Borrero-Echeverry et al. [21] introduced the use of shaving cream containing stearic acid to provide anisotropy. Such anisotropic additives are, however, too large to affect light rays.

The flow-induced birefringence was utilized to a Newtonian fluid by Hu et al. [71, 72], where wakes around a swimming fish were visualized by the tobacco mosaic virus (rod-shaped particles). Laminar Sciences Corporation [1] has produced a novel rheoscopic fluid named KaleidoFlow™ as a birefringent Newtonian fluid composed of tiny flake particles and deionized water (as detailed in Sec. 4.2.1). This fluid shows local fluid deformations by displaying multiple colors as shown in Fig. 4.2; the fluid was poured into a 10-mm-thick rectangular vessel, and disturbed by a moving glass stick. In Fig. 4.2, the left half of the vessel is sandwiched by two orthogonally arranged polarization filters, the so-called crossed-Nicol method, for simultaneous visualizations of backlight without polarizer (the right half) and the crossed-Nicol images. Complicated color patterns generated by the flow-induced birefringence can be seen surrounding the glass stick in the left half, while no flow structures are observed in the right half.

As detailed in Hu et al. [72], the birefringence in these Newtonian fluids is elicited by the preferential alignments of the anisotropic particles in the shear direction, as similar to the classical rheoscopic fluids. The cumulative effect of particle orientation along the optical axis deviates the polarization plane of the light emerging from the polarizer, thus generating a bright coloration. By increasing the shear rate, the particles become more and more oriented in the shear direction until a saturation corresponding to a plateau of birefringence, a particle orientation perfectly in the shear direction. Hence, it is expected that the stress-optic law in the fluids may be valid until certain high values of shear rate, if the stress-optic law is applicable for the photoelasticity of the

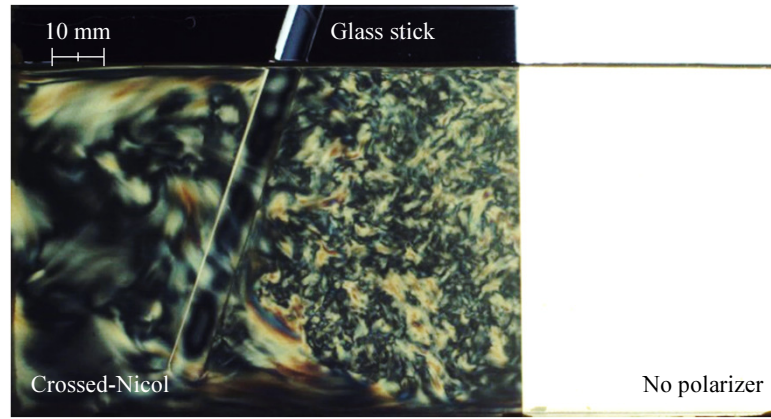


Figure 4.2: An example of flow visualization using the rheoscopic fluid with the crossed-Nicol method (left half) and without polarizers (right half). The fluid was disturbed by a moving glass stick.

Newtonian rheoscopic fluid.

The present study aims to investigate if the stress-optic law holds in the Newtonian rheoscopic fluid, and to explore a doable way of conducting photoelastic analysis or stress field measurement in experimental investigations. First, the coloration mechanism of the rheoscopic fluid and the applicable flow conditions for photoelastic analysis in fluid flows will be discussed in Sec. 4.2. Second, a detailed investigation of laboratory experiments is conducted under conditions fulfilling the limitations will be discussed in Sec. 4.3. In Sec. 4.4, the stress-optic law in the fluid will be evaluated using the experimental results. Finally, a summary of the present study and plausible ways to conduct future stress field measurements in Newtonian fluids will be provided in Sec. 4.5.

4.2 Photoelasticity in a Newtonian fluid

4.2.1 Flow visualizer

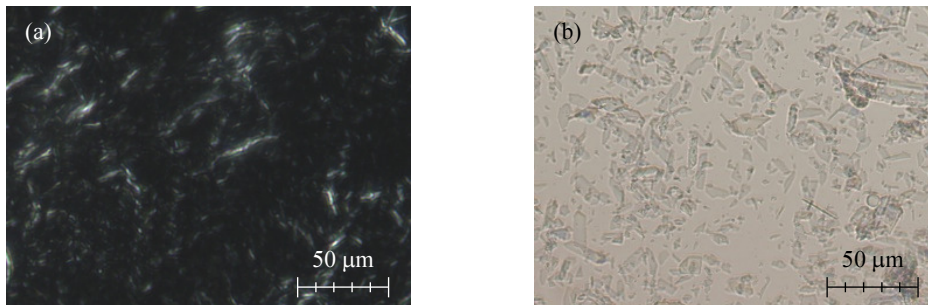


Figure 4.3: Microscope photos of two different rheoscopic fluids; (a) the rheoscopic fluid used in this study, KaleidoFlow™ (Laminar Science Corp. [1]) recorded with a crossed-Nicol method, and (b) the classical rheoscopic fluid, AQ-1000 (Kalliroscope®) taken with an opened-Nicol method.

Specifications of the novel rheoscopic fluid, KaleidoFlow™, are available in patents by Laminar Sciences Corporation [19, 20]. According to the documents, a small amount of flake-

or plate-structured vermiculite particles is dispersed into deionized water. The typical particle dimensions are $O(10\ \mu\text{m})$ in longitudinal directions and $O(10\ \text{\AA})$ in thickness, and the specific gravity is ~ 2.65 . The particles are homogeneously dispersed in the deionized solvent, and sedimentation of the particles occurs finally after standing the fluid for a week. For a wide color spectrum coloration, the electrical conductivity of the solvent needs to be very small. In our preliminary test for diluting the rheoscopic fluid, the use of tap water for diluting induced the flocculation of the vermiculite particles. A microscope photo of the rheoscopic fluid taken with a crossed-Nicol method is shown in Fig. 4.3(a). For comparison, a microscope photo of the classical rheoscopic fluid, Kalliroscope[®][105], taken with a single linear polarizer (opened-Nicol method) is shown in Fig. 4.3(b). Flake-structure particles of the Kalliroscope[®], length scale $O(10^{-6} - 10^{-5}\ \text{m})$ can be observed in Fig. 4.3(b). Different from this, no clear shapes of the KaleidoFlowTM are visible at the same magnification in Fig. 4.3(a). These photos suggest that the size of the particles in the KaleidoFlowTM are much smaller than those of the Kalliroscope[®], and it is concluded that the novel rheoscopic fluid affect visible light rays directly by optical anisotropy like liquid crystals, and displays coloration due to flow-induced birefringence as shown in Fig. 4.2, despite the medium being a Newtonian fluid. As it was not possible to recognize flake particles in the diluted rheoscopic fluid with naked eyes, the expected spatial resolution of the visualization can be deduced to be similar to or better than $O(10\ \mu\text{m})$. This fine spatial resolution is much better than the general PIV technique that requiring seeding tracer particles of $O(10 - 100\ \mu\text{m})$ in diameter.

4.2.2 Mechanism of coloration

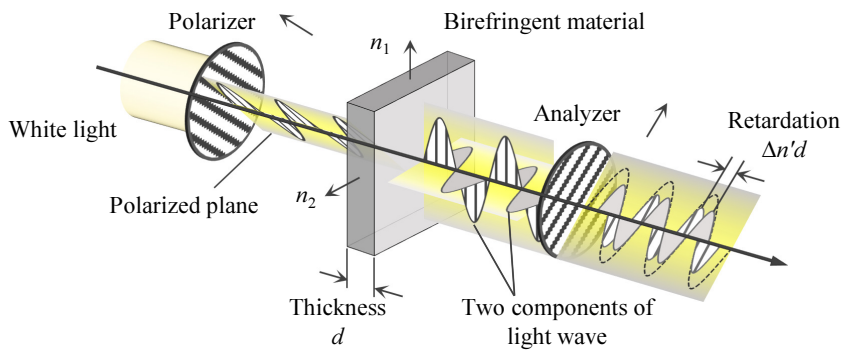


Figure 4.4: Schematic illustration of the coloration mechanism caused by a birefringent material.

As the mechanism of the flow-induced birefringence utilizing anisotropic particles is detailed in Hu et al. [72], a general coloration mechanism is provided in this section. Coloration like in Fig. 4.2 emerges when an optically anisotropic (birefringent) material is placed between two orthogonally arranged linear polarizers and lit up from behind by a white light. A schematic illustration of this coloration mechanism simplified into a two-dimensional system is shown in Fig. 4.4. A white light placed in front of (left in the figure) the first polarization filter

in front of the birefringent component with thickness d , termed the polarizer, vibrates in all directions perpendicular to the traveling direction of the light. After the polarizer, only the light ray component vibrating in the direction of the polarizer passes through the polarizer and this polarized light enters the birefringent material. The polarized light is doubly refracted and separated into two components vibrating along the optical axes of the material perpendicular to each other, noted as n_1 and n_2 in Fig. 4.4. Then, the polarized light rays pass through the material and enter the second polarization filter, termed the analyzer. Due to the velocity difference between the two light rays passing through the material, the light rays are elliptically polarized, and the analyzer lets only the light ray component parallel to itself pass.

The light intensity I transmitted from the white light source irradiated from the back through these optical settings is theoretically expressed as

$$I = I_0 \sin^2 2\chi \sin^2 \frac{\pi\Gamma}{\lambda}, \quad (4.2)$$

where I_0 , χ ($0 \leq \chi \leq \pi/4$), Γ , and λ represent the original light intensity, the angular difference between the optical axis of the material and the polarizer axes, the so called orientation angle, the retardation (optical path difference), and the wavelength of the light, respectively. $\pi\Gamma/\lambda$ has been termed the phase difference of the two light rays in some reports. The retardation Γ has a length scale of $O(10^{-6} \text{ m})$, and is defined as

$$\Gamma = |\Delta n'|d = |n_1 - n_2|d, \quad (4.3)$$

where $|\Delta n'|$ is the difference between the two refractive indices, the larger one n_1 and the smaller one n_2 , termed the birefringence of the material, and d denotes the length of the optical path (equivalent to the material thickness) as shown in Fig. 4.4. Substituting Eq. 4.3 into Eq. 4.2, one obtains

$$I = I_0 \sin^2 2\chi \sin^2 \frac{\pi\Delta n'd}{\lambda}. \quad (4.4)$$

Here $|\Delta n'|$ is defined as $\Delta n'$ (≥ 0) to enhance readability. In practice, I_0 is the intensity of the original light source and is a specific function of λ with a fixed light source. Further, d is a constant at a set material thickness, and thus Γ is a scaled value of $\Delta n'$. Finally only two variables, χ and $\Delta n'$, determine the behavior of I displaying a specific color fringe pattern, termed the “*Michel–Lévy interference color chart*”.

A detailed development process for the color chart was reported by Sørensen [182]. The color chart was obtained numerically by using the spectrum of the light used in the present study with a fixed material thickness ($d = 20 \text{ mm}$), and shown in Fig. 4.5. As a result of the I transmitted by the birefringent material, the coloration is recorded as *RGB* brightness values in digital images. Please note that the color chart is a function of both $\Delta n'$ (or Γ with a set d) in the horizontal axis, and χ in the vertical axis of Fig. 4.5(a). With increases in $\Delta n'$, the color chart

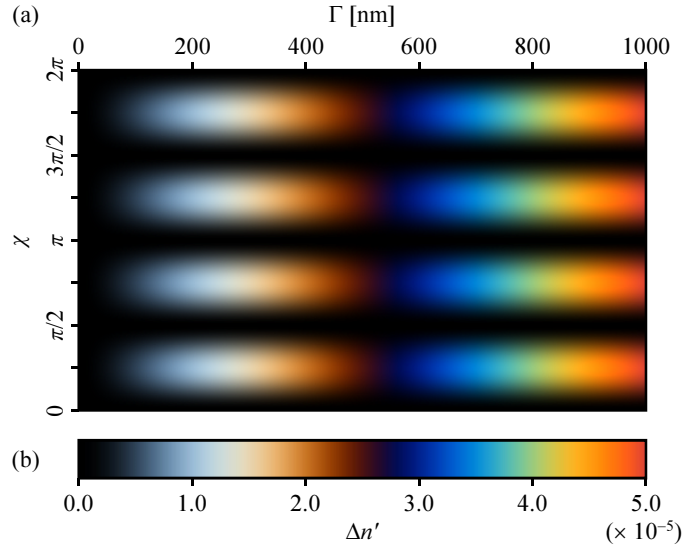


Figure 4.5: Michel–Lévy interference color charts obtained in the present system at a fixed $d = 20$ mm: (a) a full color chart with both $\Delta n'$ and χ as variables, and (b) extracted brightest coloration at $\chi = \pi/4$.

displays an increasingly vivid coloration. Contrary to this, four same color cycles appear with variations of χ ; dark at $\chi = m\pi/2$ and bright at $(2m - 1)\pi/4$ with arbitrary integer $m \in [1, 4]$. This color cycle is consistent with Eq. 4.4 as $I \propto \sin^2 2\chi$. For a detailed view of the brightest coloration, this color at $\chi = \pi/4$ is extracted in Fig. 4.5(b) where there are two dark colored sectors at $\Delta n' = 0.0$ and $\Delta n' \sim 2.8 \times 10^{-5}$. The color chart continues at further larger $\Delta n'$ regions, which is not shown in the figure. From the color chart, it is clear that both χ and $\Delta n'$ are necessary to determine a unique color in the chart.

4.2.3 Applicable flow conditions

The most significant outstanding issue is whether the stress-optic law holds in the flow-induced birefringence of the rheoscopic fluid considered here. To determine this, flow-induced birefringence $\Delta n'$ and orientation angle χ must be associated uniquely with the stress in a specific flow by laboratory experiments.

The photoelastic analysis requires backlight illumination, and light rays transmitted through a birefringent material are recorded as the color information. This makes the coloration the result of an integration of the material state over the depth (in the light ray direction). For solid materials, this need not be a significant problem because thin structures are required for the backlight illumination, ensuring that one-dimensional (1D) or two-dimensional (2D) structures are acceptable, and that the material state can be regarded as identical over the full depth of the sample. Fluid flows easily form three-dimensional (3D) structures, and to evaluate a proper $\Delta n'$ using a single optical setting, the flows are required to be confined in thin fluid vessels like in Hele-Shaw cells to ensure the minimum length scale of the flow to be equivalent to or larger than the thickness of the fluid vessel. Otherwise, 1D or 2D fluid flows must be able to be assumed by

the presence of external forces like rotation.

4.2.4 Requirements for fluid flow measurement

To surpass the limitations of the conventional methods like PIV and MTV, the photoelastic analysis in the fluid flows is required to possess high temporal and spatial resolutions.

In the photoelastic analysis, determination of χ , the vertical axis of Fig. 4.5(a), could present a bottleneck. For solid materials, χ is easily obtained by rotating one polarizer or the materials, because the transmitted color repeats bright and dark cycles as shown in Fig. 4.5(a). To ensure a correct determination of χ , at least four images taken with two linear polarizers aligned at different angles are required [129]. This would be efficient because the solid materials do not change the states during the rotation of the polarizer, i.e. solid materials may be regarded as offering a steady state under fixed externally imposed conditions, and no time-lag will be required in the determination of χ . A fluid in the rotating vessel however will present changes in the flow state during rotation. This makes it necessary for χ to be quantified instantaneously or within a very short period of time. Without near instantaneous temporal resolution measurement targets for the photoelastic analysis in fluid flows become limited to steady flows.

As introduced in Sec. 4.1, methods of analysis other than photoelastic stress field measurements are possible with other conventional methods, such as PIV and MTV. The spatial resolution of these conventional methods, however, are not fully detailed full-field measurements, and in practice they offer ten times poorer spatial resolution than that of the image resolution, and details of small scale structures and the large velocity gradients generated in turbulent flows are not resolved. To get around this limitation in spatial resolution, the use of pixelwise information could be useful, and photoelastic analysis offers advantages for achieving this, and would make it possible to quantify internal stress distributions also in irregularly shaped solid materials.

The present investigation attempts to verify the validity of the stress-optic law in rheoscopic fluid flows by using laboratory experiments to achieve results similar to those for solid materials immediately above.

4.3 Flow visualization in spin-up fluid flow

4.3.1 Spin-up experiment

As a first step in evaluating the validity of the stress-optic law in 1D or 2D fluid flows, spin-up flow was chosen as the measurement target. Spin-up flows have a long history of investigations (e.g. [18, 60, 203, 205]), and here the experiments ensure axisymmetry, and unidirectionality in the azimuthal direction, also during the transient state. Thus, the limitations of steady flow measurements can be overcome.

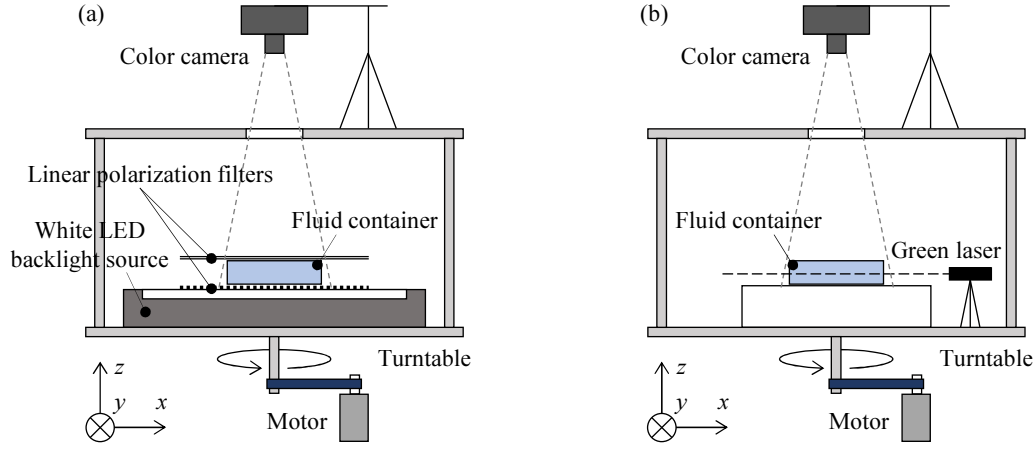


Figure 4.6: Schematic diagrams of the experimental setup for (a) color visualization using the rheoscopic fluid and (b) velocity field measurement (PIV) using water.

In the present study, two series of spin-up experiments were conducted by confining two different fluids in a small cylindrical vessel with depth $d = 20$ mm and inner diameter $D = 80$ mm. One is a color visualization experiment using the rheoscopic fluid diluted with the same volume of distilled water; the physical properties of the diluted rheoscopic fluid are not different from water, because the fluid is composed of deionized water with a tiny amount of additives. The second series is a velocity field measurement using distilled water containing small amounts of porous resin particles (mean diameter $d_p = 90$ μm , mean density $\rho_p = 1.02 \times 10^3$ kg/m^3 , *HP20SS, Mitsubishi Chemical Co.*) as tracers for PIV. Schematic diagrams of the experimental setups are shown in Fig. 4.6. The image acquisition for both was made with a digital camera (*D3400, Nikon*) with a lens (*Nikkor 35 mm, Nikon*), fixed above the fluid vessel and recording color images at 25 f.p.s. with shutter speed of $1/500$ s, aperture of $f/2.8$, and a spatial resolution of 0.1 mm/pixel. As discussed in Sec. 4.2.1, the expected spatial resolution of the flow-induced birefringence is $O(10$ $\mu\text{m})$, each pixel represents cumulative values within the area, and pixelwise information can be used in this camera configuration. For the color visualization, the fluid container with the rheoscopic fluid was sandwiched between two orthogonally arranged linear polarizers, placed on an LED white backlight source (maximum brightness $21,000$ cd/m^2 , *HDSBPA, Hayashi-Repic*). Please note that the selection of the white light source does not affect the outline of analysis procedure and discussions going to be presented in this study. Here, the polarization axes of the polarizer and the analyzer are parallel to the y - and x -axes of the image, respectively. For the velocity measurements, the image acquisition was conducted at the same camera settings by placing the container on a stage, and a horizontal cross section at $z = 0.5d$ was illuminated by a 2-W green laser sheet (2 mm in thickness, *MGL-F-532, Changchun New Industries Optoelectronics Technology Co., Ltd.*). As suggested in Fig. 4.6, all of the equipment was placed on the turn table with two stainless steel stages (1-m diameter) and rotated together. The observed recorded images are taken in the rotating system, and the center of the cylindrical vessel and the camera axis coincided with the rotational axis of the turn table.

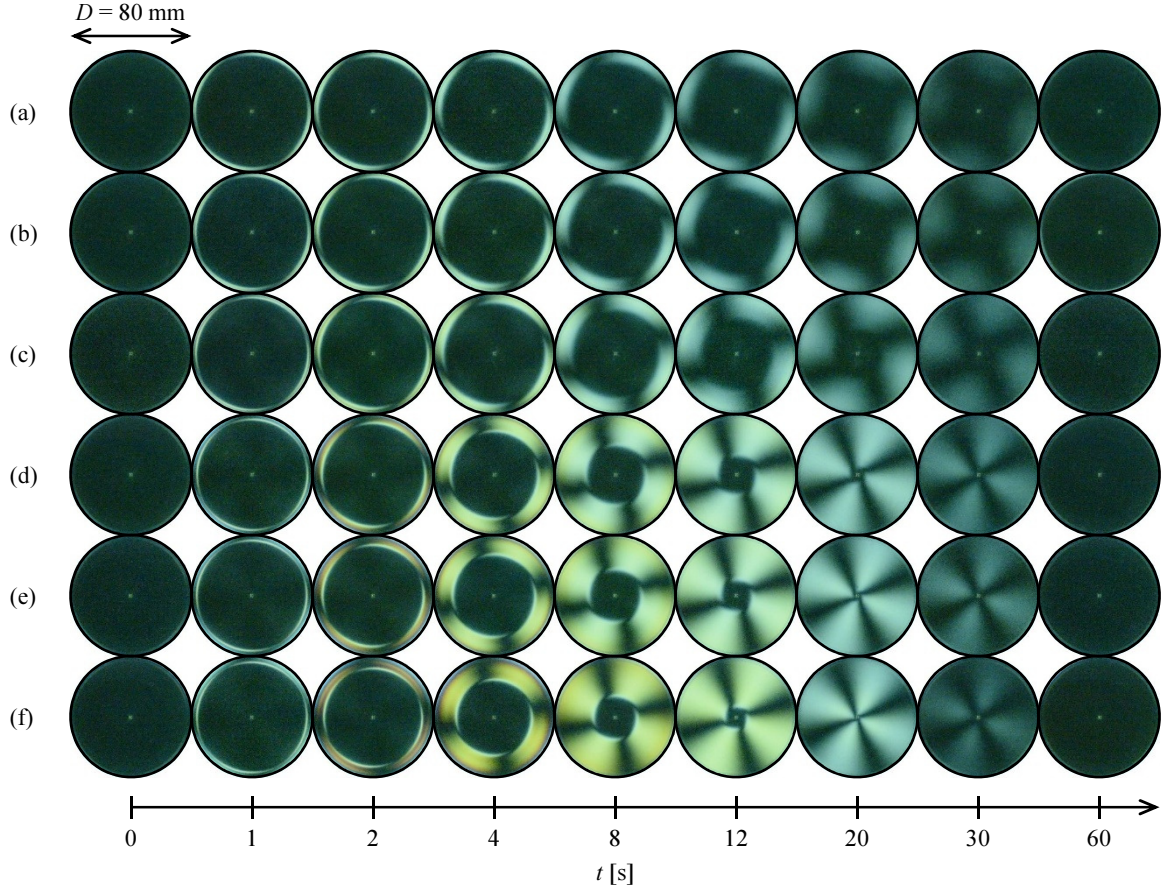


Figure 4.7: Snapshots recorded as color images of the rheoscopic fluid during spin-up: (a) $Ek = 1.0 \times 10^{-2}$ ($\omega = 0.126$ rad/s), (b) 7.0×10^{-3} (0.179 rad/s), (c) 4.0×10^{-3} (0.314 rad/s), (d) 1.0×10^{-3} (1.26 rad/s), (e) 7.0×10^{-4} (1.79 rad/s), and (f) 4.0×10^{-4} (3.14 rad/s). Time elapses from left to right.

The turn table can rotate with a set angular velocities ω in the range of 0.0 to 5.0 rad/s, and the experiments used six angular velocities: $\omega = 0.126, 0.179, 0.314, 1.26, 1.79$, and 3.14 rad/s. As a controlling parameter, the Ekman number Ek is defined as

$$Ek = \frac{\nu}{2\omega d^2}, \quad (4.5)$$

where ν is the kinematic viscosity of the fluid. With the ω values set in the present study, the Ek ranged from 4.0×10^{-4} to 1.0×10^{-2} . Further, the typical spin-up time, the Ekman timescale t_E [61], can be estimated as

$$t_E = \frac{d}{\sqrt{\nu\omega}}, \quad (4.6)$$

and distributes in the range of $11.3 \text{ s} \leq t_E \leq 56.3 \text{ s}$ here. Rotation in the counter clockwise direction was imposed on a quiescent fluid and the camera recorded images during 60 s of the spin-up toward rigid-body rotation, where the fluid rotates together with the rotating vessel and the angular velocity becomes $v_\theta = r\omega$.

4.3.2 Color visualization utilizing the rheoscopic fluid

Color snapshots recorded with the rotational system during spin-up of the rheoscopic fluid are shown in Fig. 4.7. The time $t = 0$ s corresponds to the rest states, and are shown at the left end of each of the (a) to (f) cases. At the stationary state ($t = 0$ s), no color emerges in the images in any of the experiments. This is because the white light emitted from the backlight cannot penetrate the orthogonally arranged two polarization filters. This condition shows that without shear in the flow, the flake particles align randomly and there is no flow-induced birefringence. Coloration appears from the sidewall (at $t = 1$ s) and propagates gradually toward the center of the cylinder, and multiple colors distribute with the increase in ω . With time, the coloration darkens, from $t = 12$ s, and at $t = 60$ s the visualized images are restored to the original dark images. In the present experimental system, the camera was rotating with the fluid container, and no fluid deformation (shear) can be observed with the camera when the flow has reached the rigid-body rotation. This is the reason why the coloration completely disappears at $t = 60$ s. Considering these observed behaviors suggests the conclusion that the coloration observed in Fig. 4.7 originates in the flow-induced birefringence as a result of the rheoscopic fluid flows.

4.3.3 Velocity field measurements by PIV

Horizontal velocity fields measured by PIV are shown in Fig. 4.8 to compare it with the color visualizations (Fig. 4.7). The spin-up flows are fully reproducible and the color images in Fig. 4.7 and 4.8 can be directly compared at the same exact times. The grayscale in Fig. 4.8 shows the magnitude of the horizontal velocity $|\mathbf{u}| = \sqrt{u^2 + v^2}$ normalized by the maximum azimuthal velocity at the sidewall $U_{\text{wall}} = D\omega/2$ and it darkens with increases in the magnitude of the relative velocity. Stationary, at $t = 0$ s the left of the (a) to (f) panels, where the figure is uniformly white showing a zero velocity throughout the vessel. During the time when the coloration emerges in Fig. 4.7 ($t = 1 - 30$ s), there is a momentum transfer from the side walls together with the development of the velocity fields. During this period, larger velocity magnitudes are shown by the more variety in the coloration. After the flow states reach rigid body rotation, the velocity distributions in Fig. 4.8 show uniformly white again, because the camera rotates together with the fluid container. Considering the direction of the velocity vectors, the flow is regarded as axisymmetric and unidirectional in the azimuthal direction during spin-up. Comparing Fig. 4.7 and Fig. 4.8, the coloration of the flow-induced birefringence and the flow field may be assumed to be closely related to each other. Hence, it is possible to simplify the spin-up flow as a shear induced, axisymmetric unidirectional flow for considering the flow-induced birefringence hereafter.

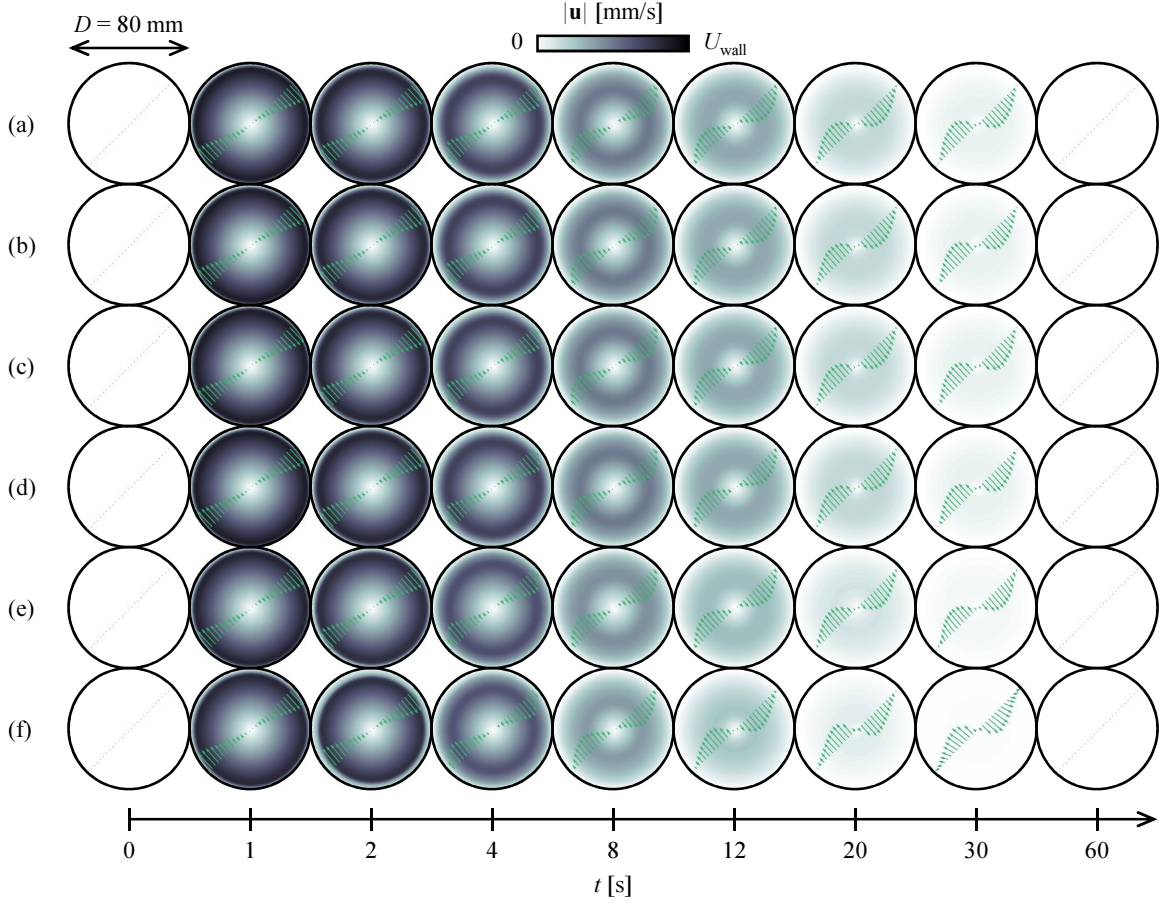


Figure 4.8: Velocity fields at $z = 0.5d$ measured by PIV during the spin-up of water: (a) $Ek = 1.0 \times 10^{-2}$ ($\omega = 0.126$ rad/s), (b) 7.0×10^{-3} (0.179 rad/s), (c) 4.0×10^{-3} (0.314 rad/s), (d) 1.0×10^{-3} (1.26 rad/s), (e) 7.0×10^{-4} (1.79 rad/s), and (f) 4.0×10^{-4} (3.14 rad/s). Time elapses from left to right. The grayscale indicates the magnitude of the in-plane velocity $|\mathbf{u}| = \sqrt{u^2 + v^2}$ obtained in the rotating frame. Velocity vectors are plotted as radial profile with arbitrary scaling for visibility because of azimuthally symmetric flow.

4.4 Evalutation of the stress-optic law in Newtonian fluids

For a flow during spin-up that is axisymmetric and unidirectional in the azimuthal direction, the only velocity gradient is that in the radial direction r in polar coordinates (r - θ domain). This allows the principal stresses τ_1 and τ_2 to be disregarded, and the stress-optic law (Eq. 4.1) is rewritten as

$$\tau_\theta(r, t) = \frac{1}{2C} \Delta n'(r, t) \sin 2\chi(r, t), \quad (4.7)$$

according to the findings in [78, 176]. In Eq. 4.7, τ_θ is the azimuthal shear stress and $\Delta n'$ and χ become functions of only r and t . Further, the photoelastic coefficient C is a physical property of the rheoscopic fluid and can be regarded as constant if the flake particles distribute uniformly in the fluid. Assuming that the stress-optic law (Eq. 4.7) is established in the rheoscopic fluid, the combination of $\Delta n'$ and χ must be uniquely related to τ_θ , and the following will attempt to explain details of the relation that ensure the establishment of the stress-optic law.

4.4.1 Wavenumber analysis for quantification of the orientation angle χ

The color chart (Fig. 4.5) involves two variables χ and $\Delta n'$, and at least one of these two variables needs to be determined in advance to quantify the other. Considering the axisymmetry of the spin-up flow, χ would be elucidated. The dark crosses, representing isogyres [88, 136], are always observed during the spin-up (see Fig. 4.7), and four dark regions appear along the azimuthal direction at any radial position r where the fluid is deformed. The azimuthal positions of the dark regions correspond to the position where χ coincides with the optical axis of the polarizer or the analyzer (the second polarizer), i.e. the extinction angles with $\chi = m\pi/2$. The presence of the dark crosses is consistent with Eq. 4.4, and thus χ can be evaluated by the inclined angle of the crosses to the x and y coordinates.

There is also a dark area at the center of the fluid container in Fig. 4.7, as there is no deformation of the fluid here; the spin-up flow does not reach this area and it remains stagnant, $\Delta n' = 0$. Expressed differently, the appearance of the isogyres at the radial position is evidence that the fluid deformation due to the spin-up propagates from the wall to the radial position. To distinguish the extinction angles ($\Delta n' > 0$ and $\chi = m\pi/2$, $m \in [1, 4]$) from the non-deformed regions ($\Delta n' = 0$), the azimuthal profiles of the brightness value Y would enable to identify the inclination of the dark crosses from in the system coordinate, and can be simply calculated as

$$Y = \frac{R + G + B}{3}, \quad (4.8)$$

where R , G , and B are three brightness values of RGB (red, green, and blue) recorded for each pixel in a 24-bit format (8-bit for each color). An example, extracting the azimuthal profiles of Y from an instantaneous image of $t = 5$ s with $Ek = 4.0 \times 10^{-4}$ is shown in Fig. 4.9(a). As indicated by the dashed lines in Fig. 4.9(a), Y values are extracted along the θ direction at each r position, and the corresponding Y profiles are plotted in Fig. 4.9(b). The Y profiles at $r \gtrsim 0.25D$ appear in periodic four cycles ($\pi/2$) along the θ direction, while there is no periodicity in Y at $r \leq 0.15D$.

The azimuthal locations of the four peaks of the Y profiles shown in Fig. 4.9(b) are slightly different at the different radial positions, the profile at $r = 0.25D$ is shifted towards higher angles (the $+\theta$ direction) from those at $r = 0.35D$, and $0.45D$. This suggests that χ is a function of only r , i.e. it can be represented by $\chi(r)$, because the flow is regarded as axisymmetric. To determine $\chi(r)$, a discrete Fourier transform (DFT) was applied to the azimuthal profiles of $Y(r, \theta)$. An azimuthally extended brightness image of $Y(r, \theta)$ compiled from the image shown in Fig. 4.9(a) is shown in Fig. 4.10(a), and the corresponding power spectra $\hat{Y}(r, k_\theta)$ are also shown in Fig. 4.10. The vertical axes of the graphs are the radial position r and the horizontal axis of Fig. 4.10(a) is θ , and that of Fig. 4.10(b) is the azimuthal wavenumber k_θ . There are four brightness peaks at active regions, where the radial positions $r \gtrsim D/4$ present fluid deformations, in Fig. 4.10(a). The distinct peaks in the spectra appear at the wavenumber of $k_\theta = 4$, excluding

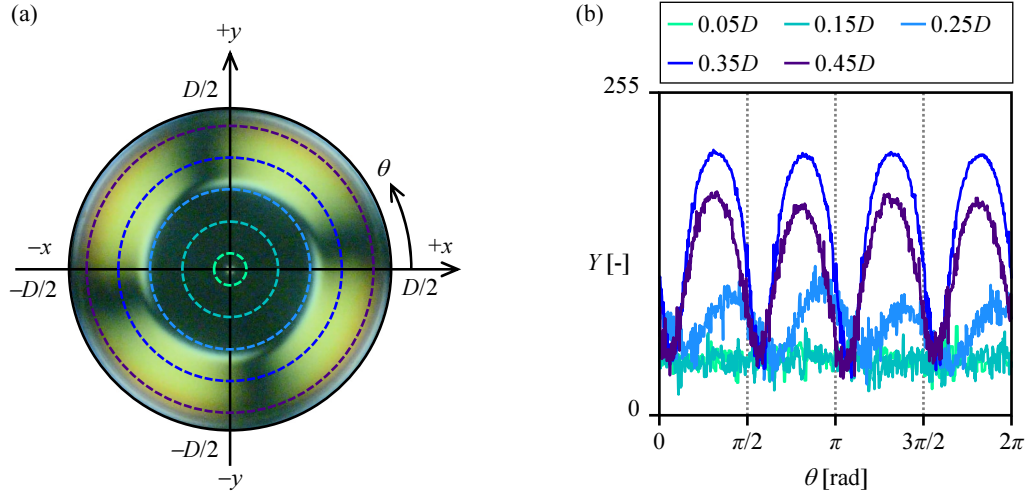


Figure 4.9: Plot of azimuthal profiles of brightness Y : (a) original image at $t = 5$ s in the case of $Ek = 4.0 \times 10^{-4}$, and (b) brightness profiles Y in θ at different radial positions (0.1, 0.3, 0.5, 0.7, and 0.9 D). Each profile corresponds to the dashed circles drawn on (a) with the same color.

the direct current component $k_\theta = 0$ in Fig. 4.10(b). This makes it possible to determine the effective radius r_{eff} separating the non-deformed region and the active region using the maximum peak wavenumber. The figure shows that if the maximum peak wavenumber is at $k_\theta = 4$, the corresponding radial position is active by the shear-induced fluid flow. The minimum radial position r showing a distinct wavenumber peak at $k_\theta = 4$ is defined as the effective radius r_{eff} dividing the non-deformed ($r < r_{\text{eff}}$) and active regions ($r \geq r_{\text{eff}}$), and is shown by the purple dashed line in Fig. 4.10.

Here Y , corresponding to the intensity I , are at the maxima and minima at which $\chi = (2m - 1)\pi/4$ and $\chi = m\pi/2$ with arbitrary integers of $m \in [1, 4]$. The brightest angle $\theta = \phi_m(r)$ on the image can be established by the arguments of the complex number $\hat{Y}$, $\Re[\hat{Y}(r, k_\theta)]$ and $\Im[\hat{Y}(r, k_\theta)]$, with a fixed wavenumber $k_\theta = 4$, and expressed as

$$\phi_m(r) = \frac{m\pi}{2} - \frac{1}{4} \tan^{-1} \left\{ \frac{\Im[\hat{Y}(r, k_\theta = 4)]}{\Re[\hat{Y}(r, k_\theta = 4)]} \right\}. \quad (4.9)$$

The minimum ϕ_m is placed as ϕ_1 and is in the first quadrant ($0 \leq \theta < \pi/2$). The darkest angle $\phi'_m(r)$, corresponding to the isogyres, can be calculated as $\phi'_m(r) = \phi_m(r) + \pi/4$.

The χ is the angle relative to the polarization axes of the linear polarizers (x - and y -axes corresponding to $\theta = m\pi/2$), and it is defined in a range of $0 \leq \chi \leq \pi/4$. Its general form is written as

$$\chi_m(r, \theta) = f(r, \Theta_m(r, \theta)), \quad (4.10)$$

where f is a function expressing the spatial distribution of χ . The Θ_m is the angle from the

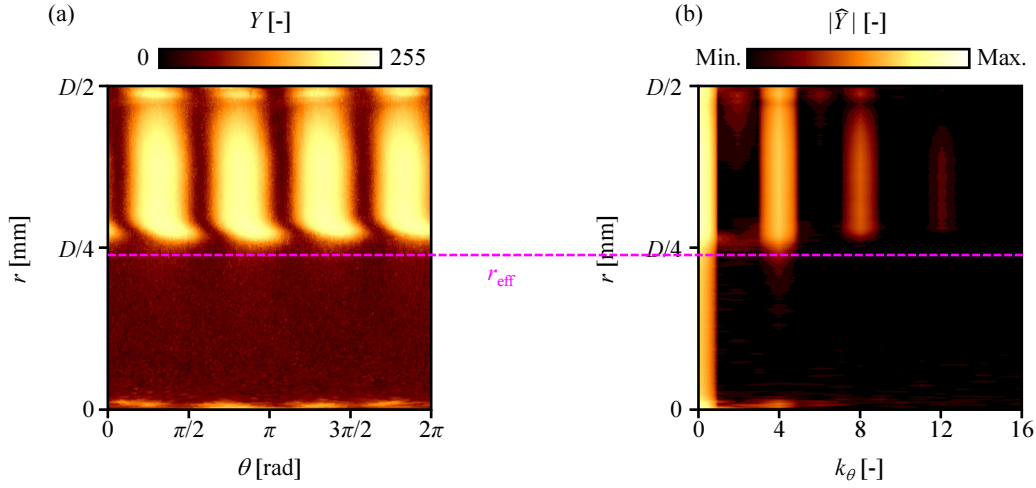


Figure 4.10: (a) Pseudo-grayscale (brightness Y) image extended on θ and (b) power of brightness $|\hat{Y}|$ in wavenumber space k_θ . Please note that the wavenumber k_θ is shown only in a range of $0 \leq k_\theta \leq 16$ for visibility. The color contour of (a) is in $0 \leq Y \leq 255$, and (b) is $|\hat{Y}|$ in logarithmic scale and normalized with the maximum and the minimum values. The displayed example is corresponding to Fig. 4.9 ($Ek = 4.0 \times 10^{-4}$ at $t = 5$ s). Purple dashed line is the effective radius at this moment $r_{\text{eff}} = 0.245D$ (19.6 mm).

darkest position ϕ'_m and defined as

$$\Theta_m(r, \theta) = \theta - \phi'_m(r). \quad (4.11)$$

Here, Θ_m is a cyclic value and is defined in the range of $0 \leq \Theta_m < \pi/2$. In an axisymmetric flow, the linear change of χ_m may be postulated along the θ directions at any r position. Thus, χ_m can be expressed as

$$\chi_m(r, \theta) = \frac{\pi}{4} - \frac{1}{2}\Theta_m(r, \theta), \quad (4.12)$$

and, the spatial distribution of the orientation angle $\chi(r, \theta)$ is obtained.

An example determining the spatial distribution of χ is shown in Fig. 4.11. Fig. 4.11(a) is the original image taken at $t = 5$ s and the processed result is presented in Fig. 4.11(b). In Fig. 4.11(b), the border of the central non-deformed and the active regions at $r = r_{\text{eff}}$ is indicated as the purple circle. The brightest positions $\phi_m(r)$, and the darkest positions $\phi'_m(r)$ are drawn as red and blue solid lines, and the color contour shows the orientation angle $\chi(r, \theta) \in [0, \pi/4]$. The spatial distribution of χ shown in Fig. 4.11(b) matches well with the brightness pattern of the original image shown in Fig. 4.11(a).

Incorporating all the results in this section, in the case of an axisymmetric flow, a determination of the spatial distribution of χ becomes possible from a single image with satisfying both the limitation of 2D flow and requirements of temporal and spatial resolutions stated in Sec. 4.2. That is, $\chi(r, t)$ is now accessible in Eq. 4.7.

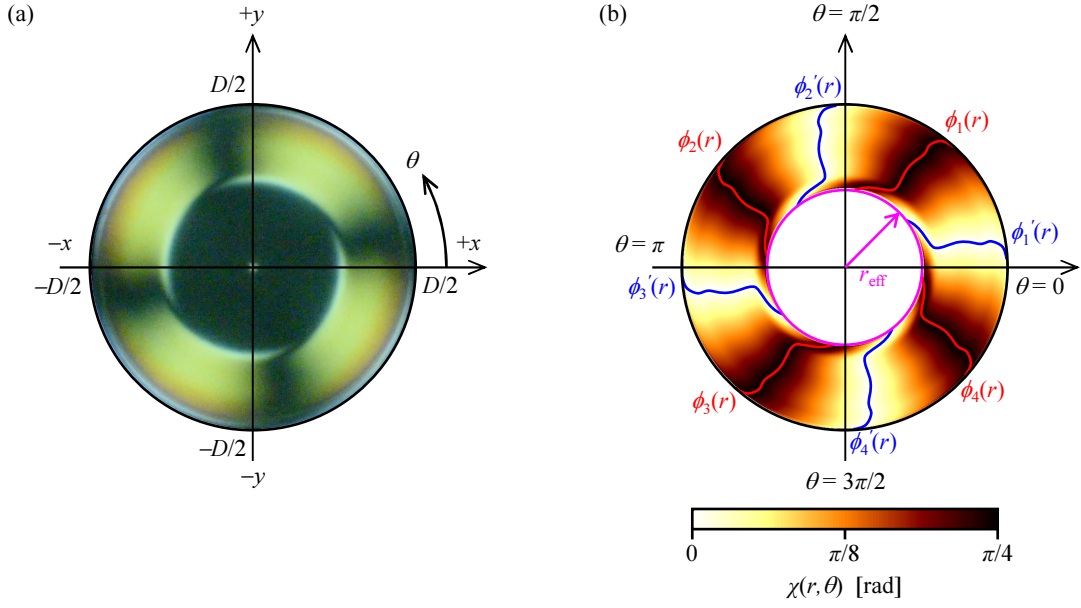


Figure 4.11: An example quantifying χ with $Ek = 4.0 \times 10^{-4}$ at $t = 5$ s (same conditions as those of Fig. 4.9 and 4.10): (a) original color image, and (b) processed result showing the effective radius r_{eff} (purple circle), the brightest positions $\phi_m(r)$ (red lines), the darkest positions $\phi'_m(r)$ (blue lines), and the orientation angle distribution $\chi(r, \theta)$ (color contours), respectively.

4.4.2 Association of color with shear stress τ_θ

The flow-induced birefringence $\Delta n'$ remains as an unknown variable in Eq. 4.7. To ensure the applicability of the stress-optic law, however, $\Delta n'$ need not be obtained because χ is predetermined in Sec. 4.4.1. At the brightest positions ($\theta = \phi_m$), the reduced stress-optic law in Eq. 4.7 can be simplified further as

$$\tau_\theta(r, t) = \frac{1}{2C} \Delta n'(r, t), \quad (4.13)$$

as for $\theta = \phi_m$, “ $\sin 2\chi$ ” in Eq. 4.7 is unity with $\chi = m\pi/4$. In addition to the reduced stress-optic law, the transmitted light intensity in Eq. 4.4 can also be simplified at $\theta = \phi_m$ as

$$I = I_0 \sin^2 \frac{\pi \Delta n' d}{\lambda}. \quad (4.14)$$

Substituting Eq. 4.13 into Eq. 4.14, one obtains

$$I = I_0 \sin^2 \frac{2\pi C d \tau_\theta}{\lambda}. \quad (4.15)$$

Here $\Delta n'$ disappears from the equations. As it is clear in Eq. 4.15, as long as χ is predetermined, I and τ_θ relate uniquely to each other, and it is possible to establish a unique relation between I (or RGB values on digital images) and τ_θ is evidence ensuring that the stress-optic law holds in the rheoscopic fluid.

The coloration for $\chi = m\pi/4$ is the most vivid and the corresponding RGB values change

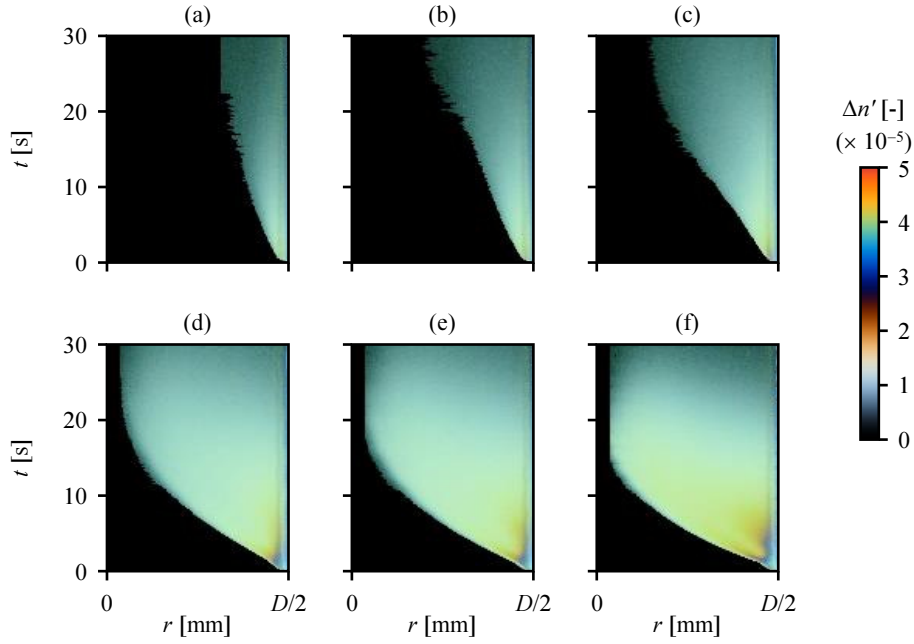


Figure 4.12: Timeline images at the brightest angle ($\theta = \phi_1(r)$) compiled from the snapshots (Fig. 4.7): (a) $Ek = 1.0 \times 10^{-2}$ ($\omega = 0.126$ rad/s), (b) 7.0×10^{-3} (0.179 rad/s), (c) 4.0×10^{-3} (0.314 rad/s), (d) 1.0×10^{-3} (1.26 rad/s), (e) 7.0×10^{-4} (1.79 rad/s), and (f) 4.0×10^{-4} (3.14 rad/s). Time elapses from bottom to top in $0 \leq t \leq 30$ s. The no deformation region ($r < r_{\text{eff}}$) appears as black. The coloration qualitatively corresponds to the birefringence of $\Delta n'$ expressed by the color chart at $\chi = m\pi/4$ displayed at the bottom of the panels.

a lot compared to other χ conditions as shown in Fig. 4.5. Fig. 4.12 shows timeline images compiled by extracting RGB values of the brightest position $\theta = \phi_1(r)$ of the rheoscopic fluid visualizations (Fig. 4.7). The horizontal axis depicts r and the vertical axis shows t . In Fig. 4.12, the non-deformed regions $r < r_{\text{eff}}$ are displayed as black, and the active regions ($r \geq r_{\text{eff}}$) show colors extracted from the original images. Simply, the wall-to-center progression of r_{eff} may be described by the development of the side wall viscous layer $\delta_\nu \propto \sqrt{\nu t}$, and $D/2 - r_{\text{eff}}$ corresponds to the δ_ν . The resultant timeline images qualitatively support this, as shown by the parabolic shapes that can be seen in each panel of Fig. 4.12.

Using the velocity fields measured by PIV, τ_θ can be estimated. The velocity vector in Cartesian coordinates $\mathbf{u}(x, y) = [u, v]$ can be converted to that in the polar coordinates $\mathbf{u}(r, \theta) = [v_r, v_\theta]$, as

$$\begin{cases} v_r = u \cos \theta + v \sin \theta \\ v_\theta = -u \sin \theta + v \cos \theta \end{cases}, \quad (4.16)$$

by placing the origin at the axis of rotation (center of the cylindrical vessel). In an axisymmetric flow, v_r is negligible compared to v_θ as apparent from Fig. 4.8, and thus τ_θ is written as,

$$\tau_\theta(r, t) = \mu \left[\frac{\partial \langle v_\theta(r, t) \rangle_\theta}{\partial r} - \frac{\langle v_\theta(r, t) \rangle_\theta}{r} \right]. \quad (4.17)$$

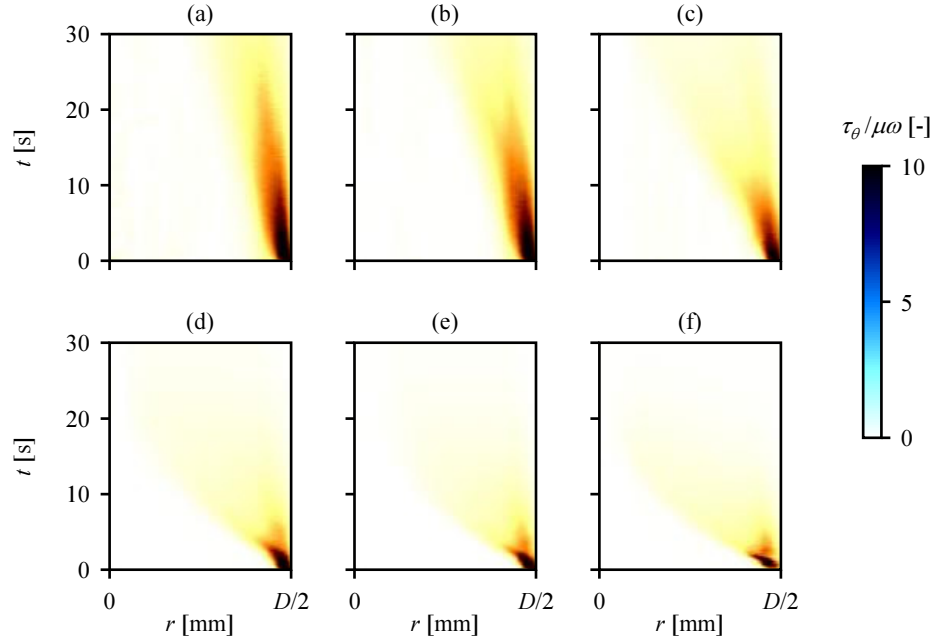


Figure 4.13: Timeline images (from bottom to top, $0 \leq t \leq 30$ s) of the azimuthal shear stress τ_θ : (a) $Ek = 1.0 \times 10^{-2}$ ($\omega = 0.126$ rad/s), (b) 7.0×10^{-3} (0.179 rad/s), (c) 4.0×10^{-3} (0.314 rad/s), (d) 1.0×10^{-3} (1.26 rad/s), (e) 7.0×10^{-4} (1.79 rad/s), and (f) 4.0×10^{-4} (3.14 rad/s). Color contours of shear stress τ_θ are scaled by $\mu\omega$.

Here, the azimuthal velocity $\langle v_\theta(r, t) \rangle_\theta$ is the averaged value of v_θ along the θ direction at a specific radius derived as

$$\langle v_\theta(r, t) \rangle_\theta = \frac{1}{2\pi} \int_0^{2\pi} v_\theta(r, \theta, t) d\theta. \quad (4.18)$$

This manipulation using the azimuthal average is possible in an axisymmetric flow, and minimizes measurement noise in the calculation of the derivative in Eq. 4.17. The estimated shear stress fields extended on the time axes are shown in Fig. 4.13. Here, τ_θ of each case are scaled by $\mu\omega$ to enhance visibility. Commonly, intense shear stress regions indicated as dark colors gradually propagate to the center of the fluid container. The faster rotation (smaller Ek) cases show faster propagation of the shear stress. This is apparent in the Ekman timescale t_E , which is proportional to $\omega^{-1/2}$, and easily understood from the development of the side wall viscous layer as $\delta_\nu \sim \sqrt{\nu t}$. The active regions shown in Fig. 4.12 correspond well to the high τ_θ regions in Fig. 4.13.

To compare the recorded color information with the theoretical color chart shown in Fig. 4.5, the colors in Fig. 4.12(f) are projected into the α - β domain (the polar chromaticity coordinate) as shown in Fig. 4.14. Here the α - β domain is a color expression method obtained from RGB values by projecting the RGB Cartesian cube to a 2D domain of α - β , and α and β are derived respectively as

$$\alpha = \frac{1}{2} (2R - G - B), \quad (4.19)$$

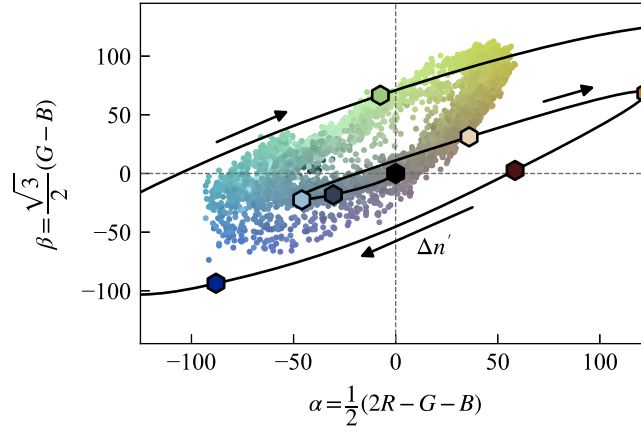


Figure 4.14: Plot of the recorded and the theoretical color values compared in the α - β domain. Solid line shows the theoretical values accessible from the color chart in a range from $\Delta n' = 0$ to 5.0×10^{-5} . Hexagonal symbols are plotted every 5.0×10^{-6} from $\Delta n' = 0$ on the line, and colored in the same color to the color chart.

and

$$\beta = \frac{\sqrt{3}}{2} (G - B). \quad (4.20)$$

In Fig. 4.14, the scatter plot is colored with the actually recorded colors shown in Fig. 4.12(f). Distribution of the plotted data forms a stretched elliptical shape rising to the right, regardless of brightness or vividness. The solid line, swirling from $(\alpha, \beta) = (0, 0)$ outward with increases in $\Delta n'$, shows the theoretical values accessible from the color chart in Fig. 4.5(b). The hexagonal symbols are plotted at every 5.0×10^{-6} from $\Delta n' = 0$, and colored in the same color to the color chart. The elliptical distributions are similar to each other, while there is a considerable discrepancy, which may originate in the non-ideal optical environment. However, it is possible to qualitatively consider the coloration of the rheoscopic fluid as the interference color from the plot in Fig. 4.14.

The spin-up is fully reproducible, and the spatial and temporal distributions of the RGB values in Fig. 4.12 and τ_θ in Fig. 4.13 can be compared. The correspondence between the τ_θ and color values with the case of $Ek = 4.0 \times 10^{-4}$ is shown in Fig. 4.15 as the RGB domain of (a), the RB domain of (b), the BG domain of (c), and the RG domain of (d). Here the timeline images of the four brightest angles ($\theta = \phi_1, \phi_2, \phi_3$ and ϕ_4) are averaged to reduce spiky noises originating from the ISO sensitivity of the camera. The spatiotemporal distribution of τ_θ is interpolated in the r direction using the third-order spline interpolation to fit the number of data points of τ_θ with that of the pixel RGB values. Conventionally, the color information is expressed as projected values such as α - β or hue (the angular component in the α - β domain), to simplify the manipulations of the color information reducing the original information. In Fig. 4.14 and Fig. 4.15(b-d), however, the unique relation between τ_θ and the color values cannot be simply discriminated in any of the 2D color domains. In the three-dimensional domain of RGB shown in Fig. 4.15(a), τ_θ can be identified uniquely, even though the deviations in the color increases with the increase in the τ_θ . The deviations might be unique in the present spin-up

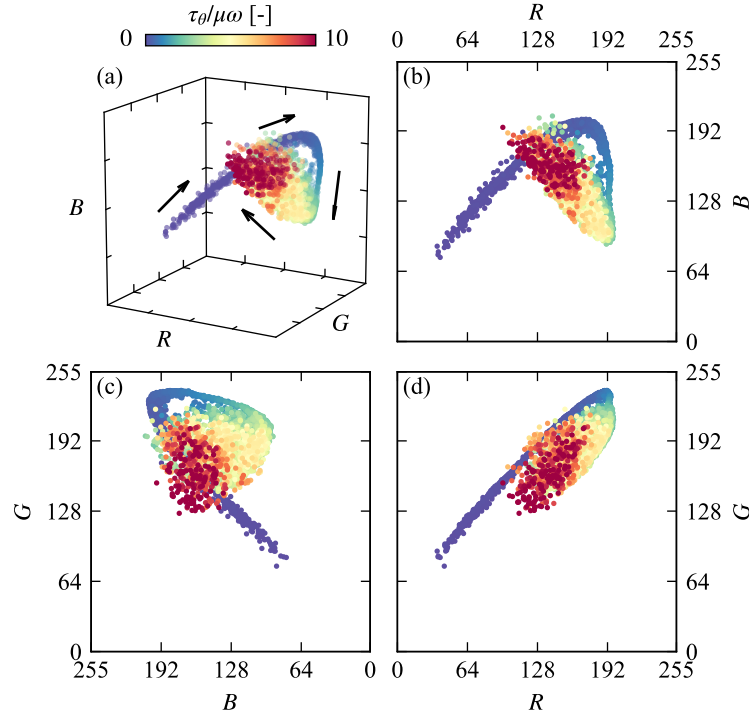


Figure 4.15: The τ_θ and RGB values plotted in different domains: (a) RGB domain, (b) RB domain, (c) BG domain, and (d) RG domain. Plots are colored with τ_θ normalized by $\mu\omega$. Black arrows in (a) guide the increase in τ_θ .

experiment which induced high shear stress regions at the sidewall corresponding to the edge of the visualized images. Overall, τ_θ is successfully associated with the RGB values without reducing the color information, even with such deviations. In conclusion, it is verified that the stress-optic law can be considered to be established in the rheoscopic fluid as Eq. 4.15 is shown to hold here.

As the validity of the stress-optic law in the rheoscopic fluid is established, the magnitude of the photoelastic coefficient $|C|$ of the rheoscopic fluid can be estimated by using representative values of $\Delta n'$ and τ_θ . In Fig. 4.12(f), the maximum value of $\Delta n'$ is approximated as $\Delta n'_{\max} \sim 4 \times 10^{-5}$ from the coloration. Similarly, the maximum value of τ_θ takes $\tau_{\theta \max} \sim 5 \times 10^{-3}$ Pa in Fig. 4.13(f). Using Eq. 4.13, $|C|$ is estimated as $4 \times 10^{-3} = O(10^{-3} \text{ Pa}^{-1})$. In comparison, the order of $|C|$ was measured as $O(10^{-7} \text{ Pa}^{-1})$ in wormlike micellar solutions [78, 176]. For the solid materials, the $|C|$ values are frequently expressed in the unit of Brewster ($= 10^{-12} \text{ Pa}^{-1}$). Considering these, the flow-induced birefringence in the rheoscopic fluid is very sensitive to stress and may display colors even with weak stress field imposed.

4.5 Summary and outlook

To establish the stress-optic law as being held in Newtonian fluids, the present study investigated a rheoscopic fluid showing the flow-induced birefringence in a direct stress field measurement

of fluid flows. Two types of spin-up experiments ensuring 2D flows were conducted to fulfill the requirements of high temporal and spatial resolutions for the photoelastic analysis in fluid flows. One experiment was a color visualization using the rheoscopic fluid, and another was velocity field measurements by PIV using water. The wavenumber analysis allowed a successful quantification of the spatial distribution of the orientation angle χ from an instantaneous color image by the results of the axisymmetric flow measurements. Subsequent direct comparisons of the RGB values from the color visualization and the τ_θ estimated from the velocity fields measured by PIV elucidated a unique relation between these two parameters, verifying the validity of the stress-optic law in the fluid.

Detailed discussion of the quantification method of the stress field is not attempted here, because the aim of the present study set out to ensure the validity of the stress-optic law with the rheoscopic fluid. To utilize the stress-optic law in a straightforward way, $\Delta n'$ is required to be determined in addition to χ . There have been reports of a determination of $\Delta n'$ elsewhere (e.g. [9, 135, 156]). To fulfill the requirements mentioned in Sec. 4.2.4, the so-called RGB photoelasticity (e.g. [7–9]) and the colorimetry-based retardation measurement method (e.g. [113]) would seem to offer plausible solutions because they use a single color camera and relatively simple optical settings. These methods measure $\Delta n'$ or Γ from the color information, and the stress-optic law can be utilized, if the photoelastic coefficient C is predetermined. As an alternative, direct association between stress values and colors would offer a plausible way to quantify the stress fields by performing a reference (calibration) experiment, because $\Delta n'$ (or C) is not necessarily required in practice as shown by Eq. 4.15. The present case successfully identified τ_θ by using all the RGB values in the 3D domain. To solve this kind of multiple input regression problems, an artificial neural network (ANN) would be a practical solution. The application of ANN to a similar color fluid measurement as the one reported here can be seen in liquid crystal thermometry (e.g. [13]). Although the applicable range of the stress field measurements needs more consideration in terms of the flow intensity, incorporating these various methods, temporally and spatially resolved stress field measurements in Newtonian fluids utilizing the stress-optic law would become possible in future research.

Part III

Study on thermal convection utilizing thermochromic liquid crystals

VISUALIZATION OF THERMAL CONVECTION UTILIZING THERMOCHROMIC LIQUID CRYSTALS

Preface

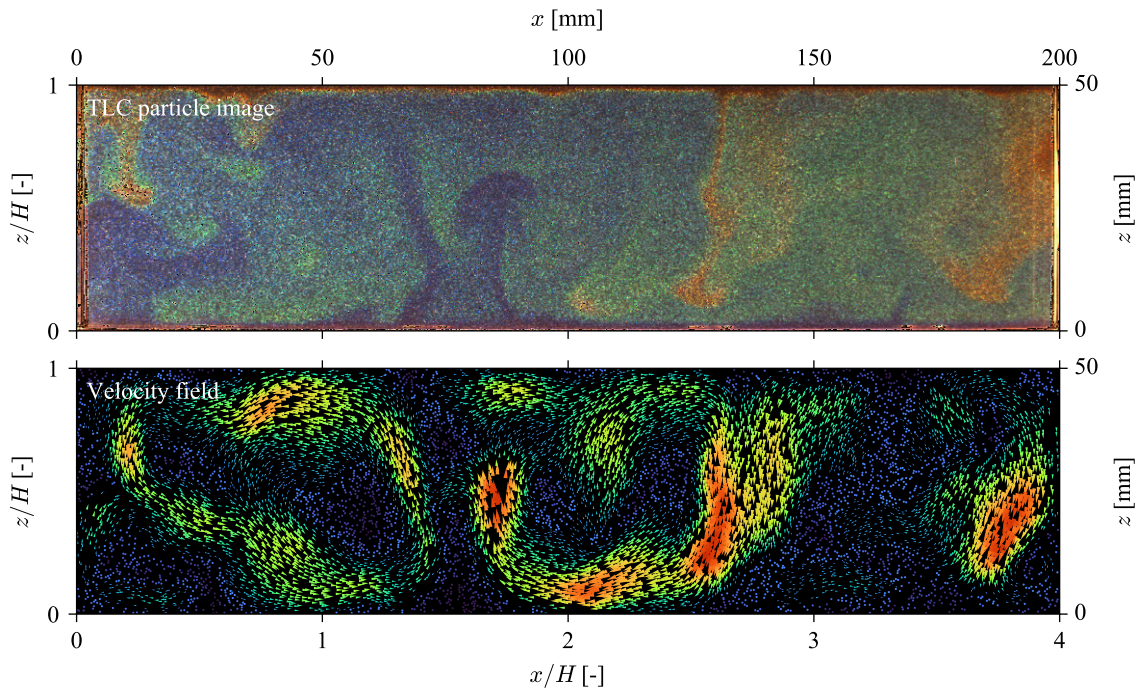


Figure 1.1: Graphical abstract of this chapter.

In Part III, physics of thermal convection are studied mainly through optical visualization utilizing thermochromic liquid crystals (TLC). In particular, horizontal convection and rotating convection are studied in detail. As discussed in Part I, thermal convection is originally driven by

the density inhomogeneity produced by the temperature difference within the fluids. In general, and it may be needless to say, temperature itself is not visible, unlike velocity presented by the motions of particles seeded into the fluids. The TLC is one of the solutions to make temperature visible as optical information. The TLC scatter colored lights corresponding to temperatures as red, green, and blue, from lower to higher temperatures. Incorporating the knowledge collected so far in Part II, the coloration of the TLC is also possible to be utilized for the temperature measurements.

Compared to the standard tracer particles used for the standard PIV measurements, the TLC encapsulated particles require slightly different experimental techniques. In Chapter 1, details of visualization of thermal convection utilizing the TLC particles are presented. For the measurements of thermal convection to pursue its comprehensive understanding, multi-timescale physics of thermal convection should be covered in the measurements. Typically, the timescale of thermal convection in laboratory experiments lies from $O(1 \text{ s})$ to $O(10 \text{ hour})$. Thus, optical measurements are sometimes required to continue such long timescales. In Chapter 1, the feasibility of the use of the TLC particles in the measurements of the long timescale thermal convection is examined in detail, especially in terms of the traceability and dispersibility. Image processing schemes that are specifically required for the visualized images of the TLC particles are also established for the subsequent study on horizontal convection and rotating convection discussed in Chapter 2, 3, and 4.

1.1 Introduction

To understand details of flow structures, optical visualization for quantifying velocity fields has been regarded as one of the most straightforward ways, after the establishment of PIV/PTV. The requirement for flow measurements through the PIV/PTV is that the seeding of optimal tracer particles, namely, particles perfectly follow the fluid flows. However, it is almost impossible to find perfect tracer particles matched to the fluids in actual experiments, and instead, reasonable tracer particles should be selected depending on the objectives of the measurements. This problem becomes prominent especially in the measurements of thermal convection in which the density of the fluid changes with the change of temperature.

As above, the crucial issue arising in the optical visualization of thermal convection problems is the selection of the fluid tracers. This is because the flow speed in thermal convection is, in principle, quite slow even if it is in a turbulent state. In general, the Navier–Stokes (momentum) equation for the Boussinesq fluid is given as

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_0} \nabla p^* + \nu \nabla^2 \mathbf{u} - \beta(T - T_0)g\hat{\mathbf{z}}, \quad (1.1)$$

where ρ_0 and T_0 are the reference density and temperature, respectively, and p^* is the modified pressure after the subtraction of hydrostatic pressure. Considering only the z -component, one obtains

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_r} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - g\beta(T - T_r). \quad (1.2)$$

Suppose that the system is steady, one-dimensional in z direction, and the viscous force can be ignored, one obtains

$$w \frac{\partial w}{\partial z} \sim g\beta(T - T_r). \quad (1.3)$$

Let the representative vertical velocity, height, and temperature difference be W , H , and ΔT , this formula is rewritten as

$$W \frac{W}{H} = g\beta\Delta T. \quad (1.4)$$

Accordingly, the representative velocity W can be estimated as

$$W = \sqrt{g\beta\Delta TH} = U_f. \quad (1.5)$$

This representative velocity W is termed the Prandtl's free-fall velocity U_f . As above derivation, U_f ignores the influence of viscosity and may represent the maximum vertical velocity potentially realized adopting the thermal buoyancy produced by the vertical temperature difference ΔT . If the experiments are conducted using a table-top convection cell with a height of $H = O(10^2 \text{ mm})$ and a vertical temperature gap of $\Delta T = O(10^1 \text{ K})$, the characteristic flow speed becomes

$U_f = O(10^1 \text{ mm/s})$ as the maximum in water. The actual speed in a thermal turbulence may be rather slower than this estimated U_f .

Traceability of seeding particles, flow tracer fidelity, is measured using the Stokes number St expressed as

$$St = \frac{U_c t_r}{l_c}, \quad (1.6)$$

where U_c , t_r , and l_c are respectively characteristic velocity, relaxation time of the particle, and characteristic length. In cases of particles with a small St are used as a tracer, the particles follow streamlines of the flow while particles with a large St do not follow the streamlines due to their large inertia. The relaxation time t_r needs to be short enough compared to the smallest timescale of the flow in order to ensure the traceability of the particles. In the cases of Stokes flow, where the advective inertial forces are negligibly small compared to the viscous forces, t_r is estimated as

$$t_r = \frac{\rho_p d_p^2}{18\mu}, \quad (1.7)$$

where ρ_p , d_p , and μ are respectively the particle density, the particle diameter, and the fluid viscosity. In general, particles with $St \ll 1$ are required for acceptable traceability. According to the book by Tropea and Yarin [193], tracing accuracy will be below 1% when particles with $St < 0.1$ are used for the tracer.

In experimental measurements of thermal convection, this requirement for St hardly becomes a bottleneck. If the maximum velocity U_f and fluid layer depth H are used for the characteristic velocity U_c and length l_c , St becomes

$$St = \frac{\rho_p d_p^2 U_f}{18\mu H} = \frac{\rho_p d_p^2}{18\mu} \sqrt{\frac{g\beta\Delta T}{H}}. \quad (1.8)$$

Particles used for flow tracer generally have a diameter of $O(10^0\text{--}10^2 \text{ }\mu\text{m})$ and specific gravity of 0.9–1.4, for the conventional planar PIV analysis. Considering the condition that a large and heavy particle with diameter of $d_p = 500 \text{ }\mu\text{m}$ and density of 1.4 g/cm^3 is seeded into thermal convection of water with $\Delta T = 50 \text{ K}$ and $H = 40 \text{ mm}$, St is calculated as $St \sim 0.03$. This condition may be the worst case that can be realized in laboratory experiments, and even in this condition the traceability of particles is assured enough as $St \ll 1$.

Incorporating above discussions, any tracer particles seem to be possible to employ as tracer particles for visualization of thermal convection. However, the selection of tracer particles needs to be carefully carried out in terms of dispersibility of the particles. That is, the particle needs to be homogeneously distributed into the fluid even the fluid is quiescent. Even if the seeded particle is neutrally buoyant, that is the density difference between the particles and test fluid is negligible, the particle finally floats to the top or sediments to the bottom of the vessel. This is because the density of the fluid changes owing to the temperature variations within the imposed temperature difference ΔT . For instance, water is used as the test fluid and the temperatures

at the top and bottom boundaries are set at $T = 10$ and 30°C , the density of the fluid varies from $\rho = 999.7$ to 995.7 kg/m^3 . Thus, fully neutrally-buoyant particles never exist in thermal convection measurements unless the particles change the density along with the density change of the test fluids. The variation of the density is only $\sim 0.4\%$ at maximum in this condition, however, this becomes a severe problem as experimental measurements of thermal convection often require long-time observations for $O(1 \text{ hour})$. This is because the characteristic timescale of thermal convection is expressed using the thermal diffusion timescale

$$\tau_\kappa = \frac{H^2}{\kappa}. \quad (1.9)$$

When water is confined in a 50-mm-height vessel, τ_κ becomes 5 hours. Flotation and sedimentation of the particles mean the decrease in the number of tracer particles in the bulk resulting in a decrease in the spatial resolution of the quantified flow fields. A number of tracer particles are thus required to stay in the test fluid at least during this long-time duration to conduct later PIV/PTV analysis.

Moreover, the dispersibility of particles is important in terms of the reproducibility of the convection phenomena. Once the particles float or sediment to the top or bottom of the vessel, they hardly return to the flows as the particles are in the boundary layers. In other words, the particles are trapped within the boundary layers covering the walls on which temperature conditions are imposed due to flotation and sedimentation. Obviously, these prevent intended heat supply and removal, and may change the target phenomena. Incorporating all the discussions above, the selection of optimal tracer particles in thermal convection is not an *a priori* issue as it cannot be predetermined adopting only St .

The present study utilizes micro-particles of encapsulated thermochromic liquid crystals (TLC) as tracer particles for the visualization of thermal convection. Use of the TLC particles allows to visualize temperature fields as different scattering colors by seeding into the fluids. The details of this characteristic can be found in extensive review papers, for instance, by Dabiri and Gharib [38] and Abdullah et al. [4] and references therein. Recent works utilized the TLC particles for the measurements of thermal convection are available in [13, 111]. Detail descriptions on the TLC, in particular about the mechanism of coloration, are available in Appendix A. This special type of tracer particles yields to acquire temperature information of thermal convection as coloration, as well as velocity information represented by the particle displacements. That is, use of the TLC particles for visualization of thermal convection provides more informative visualization images than those using normal types of tracer particles. In this chapter, the plausibility in the use of TLC particles for visualization of thermal convection is examined in detail, in terms of traceability and dispersibility. Furthermore, an overview of visualization of thermal convection, involving experimental procedures, image processing, and quantification of flow fields, is presented as technical notes. Details of the experimental setup for

this preliminary test are described in Section 1.2. A series of image processing is demonstrated in the experiments, and dispersibility of the TLC particles is tested in Section 1.3. Discussions held in this chapter are summarized in Section 1.4.

1.2 Experiment

1.2.1 Experimental fluid vessel

A rectangular box with dimensions of $200 \times 30 \times 50 \text{ mm}^3$ in x - y - z directions is employed as the fluid container for the preliminary tests conducted in this chapter. This box is schematically illustrated as shown in Fig. 1.2. The sidewalls made of acrylic plates are fully transparent to allow optical visualization. The bottom of the box is a copper plate embedding a ceramic heater to supply heat to the fluid layer. The temperature at the bottom copper plate can be fixed at constant value by controlling the electric current for the heater adopting a temperature controller. The top of the box is closed by a cooling device consisting of a copper plate, a rubber sheet, and an acrylic plate. A meandering internal channel is formed by graving both the copper plate and the rubber sheet, and a temperature controlled refrigerant from a thermostatic bath is allowed to circulate inside this channel. With this, the temperature at the copper plate can be also controlled by changing the temperature of the circulating refrigerant.

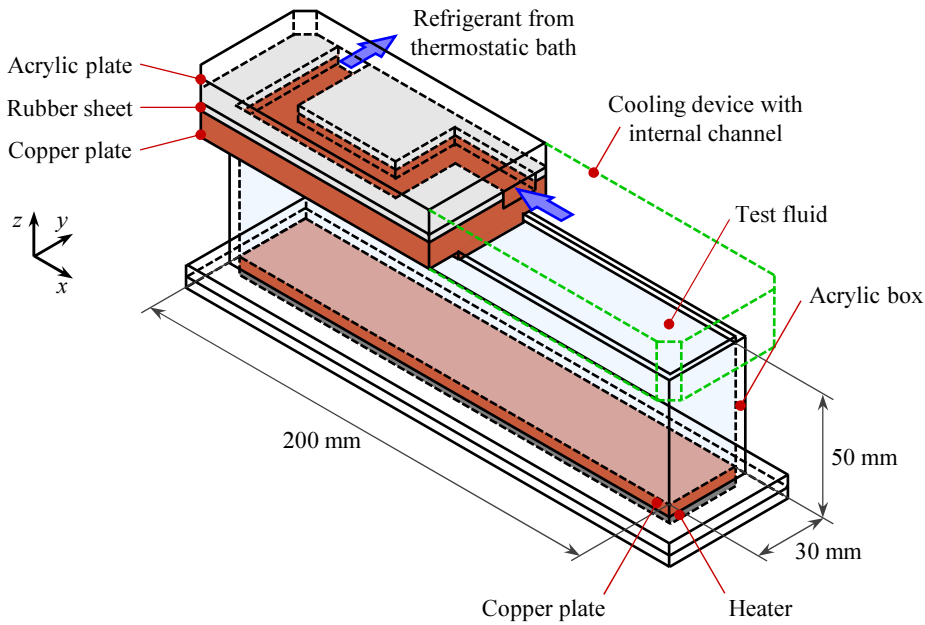


Figure 1.2: Schematic of the experimental apparatus used in this chapter.

Table 1.1: Details of the tracer particles used in the preliminary experiments. Values of St are calculated from Eq. 1.8 using $H = 50$ mm and $\Delta T = 5$ K at room temperature $T = 20^\circ\text{C}$.

Particle type	Product name (manufacturer)	Diameter d_p [m]	Density ρ_p [kg/m ³]	Stokes number St [—]
TLC particle	KWN2025 (Japan Capsular Co.)	20×10^{-6}	1.0×10^3	2.5×10^{-6}
Fluorescent particle	FLUOSTAR (EBM Co.)	15×10^{-6}	1.1×10^3	6.2×10^{-6}
Resin particle	SP20SS (Mitsubishi Chemical Co.)	70×10^{-6}	1.0×10^3	1.2×10^{-4}

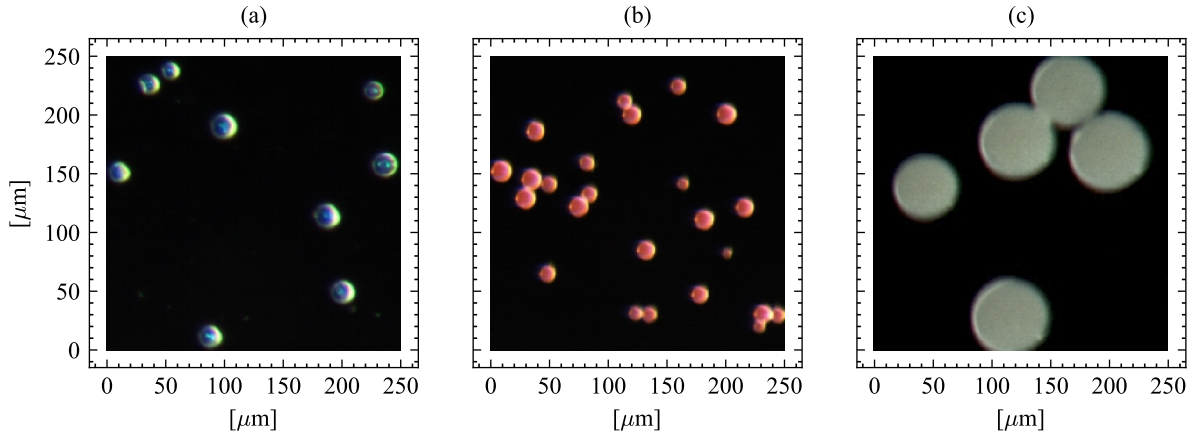


Figure 1.3: Photographs of the three types of tracer particles taken using a microscope: (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS).

1.2.2 Tracer particles

To test the ability of various tracer particles, three types of particles, TLC, fluorescent, and spherical particles, are examined in the identical experimental environment. The details of these three types of particles are listed in Table 1.1, and photographs of the three particles taken by a microscope are shown in Fig. 1.3.

TLC particles — The TLC particles, *KWN2025* manufactured by Japan Capsular Co., scatter colors corresponding to the fluid temperature in an approximate range of $20^\circ\text{C} \lesssim T \lesssim 25^\circ\text{C}$. The particles encapsulate the TLC by urea resin, and the photograph recorded by the microscope is shown in Fig. 1.3(a). The mean diameter of this TLC particle is $d_p = 20 \times 10^{-6}$ m, and the density is $\rho_p = 1.0 \times 10^3$ kg/m³. The temperature range where the TLC effectively scatters colors changes depending on optical environments such as lighting and observation angles. In general, the temperature range offered by the manufacturer of the TLC is measured in a different lighting condition; a white light illuminates the TLC on a plane from the back of an observer. This observation condition does not match the use in flow visualizations, as these

require orthogonal relation between the lighting and view. Under this condition, the temperature range of the TLC particles is expected to shift to a lower temperature range. The detail of lighting condition is described in Section 1.2.3.

Fluorescent particles — The fluorescent particles, *FLUOSTAR*, manufactured by EBM Co. were used for the second experiments. These are polymer particles encapsulating the Rhodamine-B, fluorescent dye, which emits strong orange-colored light when illuminated by a green laser. The microsphere is composed of carboxy-modified acrylate resin, and the photograph recorded by the microscope is shown in Fig. 1.3(b). The mean diameter of the fluorescent particle is $d_p = 15 \times 10^{-6}$ m, and the density is $\rho_p = 1.1 \times 10^3$ kg/m³.

Resin particles — The resin particles, *SP20SS*, manufactured by Mitsubishi Chemical Co. are used as representative of normal-type particles. This particle scatters incident light due to the Mie scattering, and thus the color of the lighting is directly scattered by the particles. This product is originally an adsorbent ion-exchange resin, and the photograph recorded by the microscope is shown in Fig. 1.3(c). The mean diameter of this particle is $d_p = 70 \times 10^{-6}$ m, and the density is $\rho_p = 1.0 \times 10^3$ kg/m³.

1.2.3 Optical equipment

It is necessary to select effective illumination methods matching each particle. To illuminate the particles seeded in the fluid, two light sources were employed. One is a white light generated by a halogen lamp (*MHAB-150W-100V*, MORITEX Co.). The white light is collimated by a fiber linear light guide and a linear cylindrical lens to form a light sheet with a thickness of ~ 2 mm. The other is a 2-W green laser (wavelength $\lambda = 532 \pm 1$ nm, *MGL-F-532*, Changchun New Industries Optoelectronics Technology Co., Ltd.). The green laser was collimated by a cylindrical lens to form a laser sheet with a thickness of ~ 1.5 mm. The white light sheet was used when the TLC particles were seeded in the test fluid, while the green laser sheet was used to illuminate the fluorescent or resin particles. These light sheets illuminated the whole x - z plane, $0 \text{ mm} \leq x \leq 200 \text{ mm}$ and $0 \text{ mm} \leq z \leq 50 \text{ mm}$, at $y = 15 \text{ mm}$.

Particle motions were captured by a complementary metal-oxide-semiconductor (CMOS) color camera (the maximum resolution of 2592×2048 pixels, *DFK33UP5000*, The Imaging Source Co.) mounting a lens (aperture F8, focal length 35 mm, *Nikkor*, Nikon Co.) set perpendicular to the illuminated cross-section. Continuous image acquisitions were performed with 20 frames per second (f.p.s.) for 1 min at 5, 10, 30, 60, 120, and 180 min from the start of experiments. The shutter speeds were set less than $1/40$ s so as to freeze the moving particles in the images. The camera originally records the Bayer pattern in 8 bit, and the images are required to debayer (demosaic) as a post-processing. The details of the image processing are explained in Section 1.3.1.

1.2.4 Experimental condition

All the experiments seeding different tracer particles are conducted in the identical experimental condition. The temperature at the bottom was set at $T = 22^\circ\text{C}$, and that at the top was set at $T = 17^\circ\text{C}$. These resulted a vertical temperature difference of $\Delta T = 5^\circ\text{C}$. In this condition, the Rayleigh number became

$$Ra = \frac{g\beta\Delta TH^3}{\kappa\nu} = 8.7 \times 10^6, \quad (1.10)$$

with the Prandtl number of $Pr = \nu/\kappa \sim 7$. The flows driven by thermal convection are expected to be the thermal turbulence (soft turbulence) according to the work by Castaing et al. [25]. The Stokes number St of the three experiments under this condition is also listed in Table 1.1, and all of them are much less than unity, $St = O(10^{-6} - 10^{-4})$.

1.3 Visualization of thermal convection

1.3.1 Image pre-processing

To perform PIV/PTV analysis for quantification of flow fields, originally recorded images need to be pre-processed properly in advance. In this section, a series of image pre-processing that is common in the three experiments is described. The pre-processing consists of three steps as follows:

- Step 1 Color demosaicing;
- Step 2 Background subtraction;
- Step 3 Identification of rigid boundaries.

These steps are required to be followed in order for later PIV/PTV analysis, and details are explained below.

Step 1. Color demosaicing — As above, the camera originally records digital images with the Bayer pattern in 8 bit for each pixel. The Bayer image is a raw image recording light passing through color filter arrays by the CMOS sensor. The raw image has a checker-board pattern, as each pixel can record only one of the three brightness values of RGB owing to the color filter arrays. Thus, the Bayer image is originally not appropriate for the subsequent PIV/PTV analysis, and requires color demosaicing (debayering) to reconstruct the corresponding grayscale images as well as color image. A number of color demosaicing methods has been proposed and the selection of method, of course, needs to be carefully carried out, if the color information is important in subsequent analyses. In this chapter, the bilinear interpolation method is adopted to allow the fastest reconstruction of color images, as only the brightness information is necessary in the PIV/PTV analysis. Example of color demosaicing process adopting the bilinear interpolation is shown in Fig. 1.4. The images in Fig. 1.4(a) are the region of 36×36 pixels acquired in

the TLC particle experiment, and those in Fig. 1.4(b) are the magnified view of the region of 6×6 pixels indicated by the yellow solid squares in Fig. 1.4(a). The original Bayer image $\mathbf{I}_{\text{org}}$ is shown in the left end of Fig. 1.4. The image shown in the second column of Fig. 1.4 is a pseudo-color image of the Bayer image $\mathbf{I}'_{\text{org}}$ to show the pattern of the color filter array. Each pixel has a single brightness value in one of the three brightness of RGB . The pattern of the color filter is predetermined by the manufacturer in the combinations of four pixels, one R , two G , and one B pixel. These four pixels are aligned in 2×2 pixels to form a single unit, and a number of the unit is aligned in the horizontal and vertical directions within the CMOS sensor. In the second column of Fig. 1.4(b), 9 pairs (3×3 pairs) of the unit with $RGGB$ pattern are visible. The pixel at $(3, 2)$ in Fig. 1.4(b), for instance, only has a green brightness, and red and blue brightness values are empty (zero). The bilinear interpolation method reconstructs these empty color information using brightness values of surrounding pixels. Here, the missing (empty) R and B brightness values on the green pixel $G_{3,2}$ in Fig. 1.4(b) is calculated as

$$R_{3,2} = \frac{R_{2,2} + R_{4,2}}{2} \quad \text{and} \quad B_{3,2} = \frac{B_{2,1} + B_{2,3}}{2}. \quad (1.11)$$

Similarly, the missing R and G brightness values on the blue pixel $B_{3,1}$ is calculated as

$$R_{3,1} = \frac{R_{2,0} + R_{2,2} + R_{4,0} + R_{4,4}}{4} \quad \text{and} \quad G_{3,1} = \frac{G_{2,1} + G_{3,0} + G_{3,2} + G_{4,1}}{4}, \quad (1.12)$$

and in the same manner, the empty G and B brightness values on the blue pixel $R_{2,2}$ is calculated as

$$G_{2,2} = \frac{G_{1,2} + G_{2,1} + G_{2,3} + G_{3,2}}{4} \quad \text{and} \quad B_{2,2} = \frac{B_{1,1} + B_{1,3} + B_{3,1} + B_{3,3}}{4}. \quad (1.13)$$

Adopting these calculations to all the pixels, a full color image $\mathbf{I}_c$ in 24-bit format can be reconstructed as shown in the third column of Fig. 1.4. A gradual change from red, green, to blue is visible from the upper to lower in $\mathbf{I}_c$ of Fig. 1.4(a), and the enclosed region show a complicated color pattern as shown in Fig. 1.4(b). This color pattern may involve artificial false colors which originating from the color demosaicing method, and can be reduced by applying a more sophisticated method, e.g. Chung and Chan [31], Guarnera et al. [64], Malvar et al. [103], Menon et al. [109]. The color image can be converted to a grayscale image $\mathbf{I}_g$ by adopting a pixelwise calculation

$$I_{i,j} = \max(R_{i,j}, G_{i,j}, B_{i,j}), \quad (1.14)$$

where I is the brightness value of the corresponding pixel and $\max(\cdot)$ is a function returns the maximum brightness value of the three RGB values. The grayscale image is shown in the right end column of Fig. 1.4. Particle outlines are clear in the color and grayscale images displayed in the third and right end columns of Fig. 1.4, compared to the Bayer images shown in the first and second columns of Fig. 1.4.

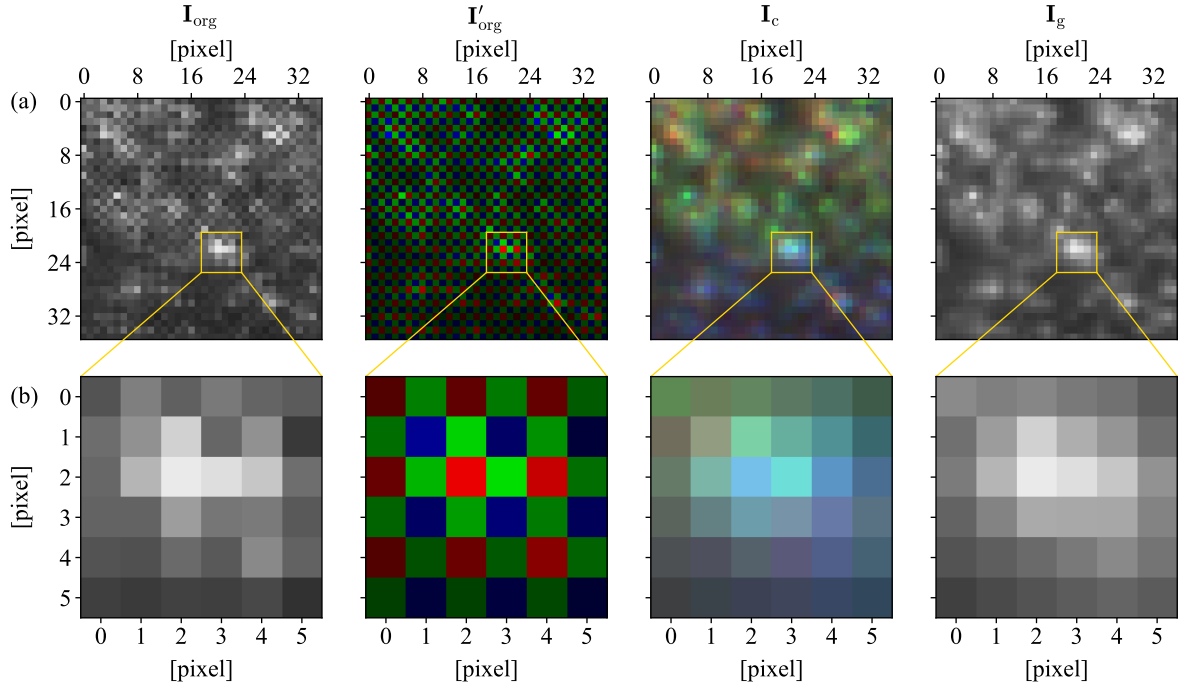


Figure 1.4: Color demosaicing process adopting the bilinear interpolation method: The region of 36×36 pixels of the visualized image in the TLC measurements is shown in (a), and the magnified views of the region of 6×6 pixels enclosed by yellow solid lines in (a) are shown in (b). Original Bayer pattern image I_{org} (8 bit), pseudo-color image I'_{org} (24 bit), color image I_c (24 bit), and grayscale image I_g (8 bit) are show respectively from the left to right.

Background subtraction — To analyze particle images quantitatively, it is necessary to perform a series of image processing for the grayscale images properly. This is because an image of the fluid vessel is recorded in addition to a number of seeded particles in the original images. Even if the fluid vessel is made of transparent materials like acrylic or glass, the illumination light reflects at the rigid boundaries. These reflection and halation of the incident light at the rigid boundaries of the vessel are recorded as static brightness distributions in the image. Furthermore, spatially homogeneous illumination throughout the whole measurement region may not be possible to realize in actual laboratory environments, due to problems of divergence or attenuation of the light sheet. This inhomogeneity in lighting also generates a uniqueness in the visualized images that is always present during the measurements. In the PIV analysis, these static images will induce erroneous velocity vectors, as spatial particle distributions within a small interrogation window need to be tracked. Such static brightness distributions, therefore, are apparently not desired in the particle images to be analyzed by PIV, and need to be eliminated from the particle images. As it turns out, the static brightness distributions, background of the particle images, can be eliminated successfully through the background subtraction method. There are three schemes to compile background images, and the simplest one is to capture a background image directly. That is, a background image is captured as an image of fluid container filled with the test fluids without tracer particles. This method might be the simplest, however, the optical configuration for the background image acquisition and the measurements should be completely identical without allowing any shift of the fluid container or the camera in the physical space. In

an opened system, this may be possible. In a closed system, like Rayleigh–Bénard convection, this limitation will be a bottleneck, as it requires closing of the fluid container after seeding tracer particles; experimentalists are required to touch the fixed optical configurations. Apart from this simplest method, there are two other schemes to compile background images, which can be implemented as in situ methods, as below.

One is based on the assumption that a single original grayscale image $\mathbf{I}_g$ recorded instantaneously can be decomposed to an average and a fluctuation component. Under this assumption, we have a relation

$$\mathbf{I}_g = \bar{\mathbf{I}} + \mathbf{I}', \quad (1.15)$$

similar to the Reynolds decomposition. Here, $\bar{\mathbf{I}}$ and $\mathbf{I}'$ are the temporal average and the instantaneous fluctuation of the original image $\mathbf{I}_{\text{org}}$, and thus $\overline{\mathbf{I}'} = \mathbf{0}$. By taking temporal average of Eq. 1.15, the average image $\bar{\mathbf{I}}$ can be obtained as

$$\bar{\mathbf{I}} = \overline{\mathbf{I}_g} - \overline{\mathbf{I}'} = \overline{\mathbf{I}_g}. \quad (1.16)$$

The average image $\bar{\mathbf{I}}$ means the background image of the continuously acquired images, as it is static with time. That is, the background image $\mathbf{I}_{\text{bg}}$ can be obtained simply by taking temporal average of the continuously acquired grayscale images as

$$\mathbf{I}_{\text{bg}} = \bar{\mathbf{I}} = \frac{1}{N} \sum_{k=1}^N \mathbf{I}_g(t_k), \quad (1.17)$$

where t_k is a time step at which the k -th image is recorded. The seeded particles are moving during the measurements, and these particles are involved in the fluctuation image $\mathbf{I}'$. Using the background image $\mathbf{I}_{\text{bg}}$, the particle image $\mathbf{I}_p$ can be obtained as

$$\mathbf{I}_p(t_k) \approx \mathbf{I}'(t_k) = \mathbf{I}_g(t_k) - \mathbf{I}_{\text{bg}}. \quad (1.18)$$

Note that $\mathbf{I}_p(t_k)$ is not fully equal to the fluctuation images $\mathbf{I}'(t_k)$ here, as the brightness signals from only the seeded particles should be larger than 0, and $\bar{\mathbf{I}}_p \neq \mathbf{0}$, under a standard lighting condition. In other words, this background subtraction is based on an assumption that the seeded particles scatter brighter lights compared to the background. Accordingly, it is not possible to extract particles superposed on too bright background regions. That is, pixels with negative brightness values in $\mathbf{I}_p(t_k)$ had better be filled by 0 (black) after the background subtraction; otherwise, inverted particles may appear in the particle images due to the fluctuation of the background, and these also leading erroneous vectors in PIV analysis. Taking care of this consideration, instantaneous particle image $\mathbf{I}_p(t_k)$ can be extracted from the original image by adopting this background subtraction.

Examples of the background subtraction through the mean image method adopted to the

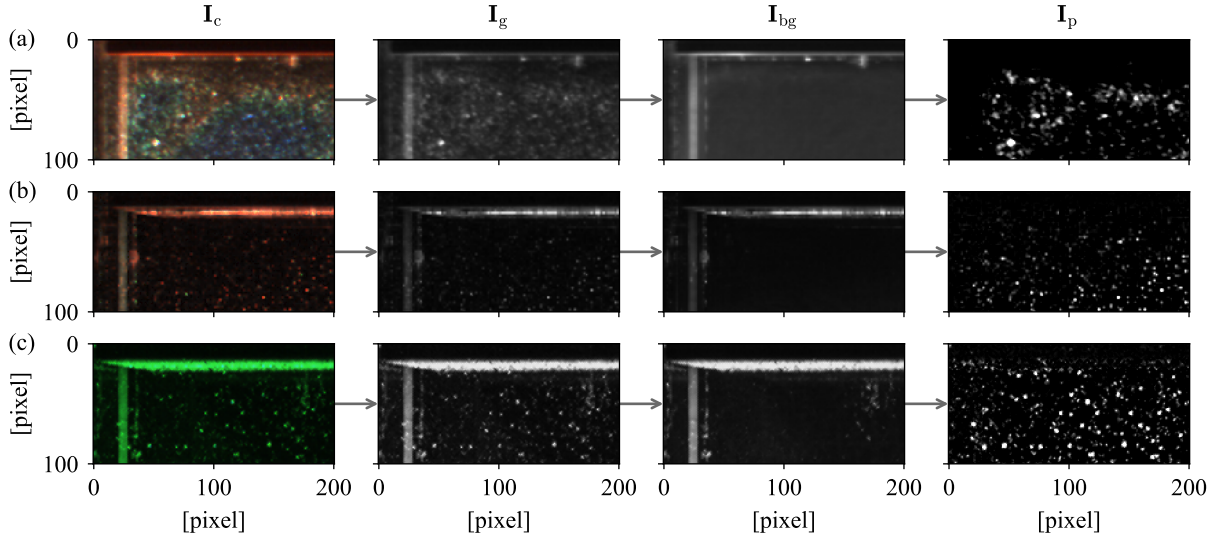


Figure 1.5: Examples of background subtraction process using the mean image method applied to the three particle images: (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS). The background subtraction proceeds from left to right as indicated by arrows; The original color image I_c is converted to the grayscale image I_g . The background image I_{bg} is compiled taking a temporal average of I_g , and is subtracted from I_g to obtain the particle image I_p . Contrast of I_p is adjusted for better visibility.

experiments seeding three types of particles are shown in Fig. 1.5: the TLC, fluorescent, and resin particle images are shown in Fig. 1.5(a), (b), and (c), respectively. A region of 200×100 pixels at the upper left of the fluid vessel is shown. The background subtraction proceeds from the left to right. In the left end of Fig. 1.5, original color images I_c after the color demosaicing are shown. These contain bright lines, which correspond to reflection and halation of the incident light, at the rigid boundaries. These color images are converted to grayscale images I_g adopting Eq. 1.14. For each experimental condition, all the grayscale images recorded during the measurement duration are compiled to compute a temporally-averaged image I_{bg} as a background image. In the displayed conditions, 1200 images in total are used for temporal averaging. No point-spread brightness distribution is observed in I_{bg} as shown in the third column of Fig. 1.5. Finally, particle images I_p are obtained by subtracting I_{bg} from I_g . Contrast of the particle images are enhanced in Fig. 1.5 for a better visibility. It is found that only particle outlines are present in I_p at the right end of Fig. 1.5. Here, the same image processing procedures are done for the images recorded in the three different experiments, and particle images can be successfully extracted from the original images containing static backgrounds.

The other scheme to compile the background image is based on an assumption that the background does not deviate at all during the measurements. If there are no particles in the fluid, the original image I_g does not contain bright signals, apart from the rigid boundaries. This dark image is the background image, however, tracer particles scatter lights brighter than the background in practice. In other words, background image I_{bg} can be extracted from the particle images I_g by eliminating such bright signals scattered by the particles. The potentially darkest image can be paraphrased as the minimum brightness image I_{min} , and thus the background image

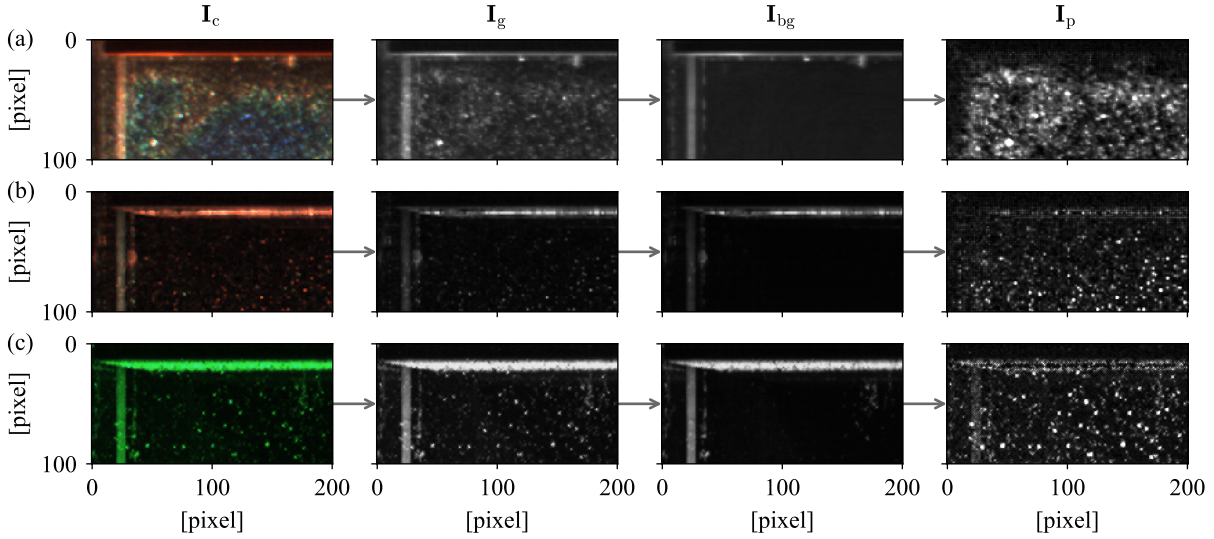


Figure 1.6: Examples of background subtraction process using the minimum image method applied to the three particle images: (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS). The background subtraction proceeds from left to right as indicated by arrows; The original color image I_c is converted to the grayscale image I_g . The background image I_{bg} is compiled taking a temporal average of I_g , and is subtracted from I_g to obtain the particle image I_p . Contrast of I_p is adjusted for better visibility.

can be obtained as

$$I_{bg} = I_{\min}. \quad (1.19)$$

The $I_{\min}$ can be computed by taking a pixelwise minimum intensity as

$$I_{\min} = \min(I_g(t_k)) \quad \text{with } k \in [1, N]. \quad (1.20)$$

Using the background image I_{bg} , the particle image I_p can be obtained as

$$I_p(t_k) = I_g(t_k) - I_{bg}. \quad (1.21)$$

Examples of the background subtraction through the minimum image method adopted to the experiments seeding three types of particles are shown in Fig. 1.6: the TLC, fluorescent, and resin particle images are shown in Fig. 1.6(a), (b), and (c), respectively. Overall, the resultant particle images I_p in Fig. 1.6 are similar to those in Fig. 1.5. In particular, the particle image from the TLC experiment shown in Fig. 1.6(a) contains larger number of particles compared to that in Fig. 1.5(a), while excluding the static bright regions. In contrast, the particle images from the fluorescent and resin particle experiments shown in Fig. 1.6(b) and (c) contain the static bright regions, even though these are darker than the original images.

The results shown in Fig. 1.5 and 1.6 suggest that the appropriate background subtraction method differ depending on the features of the seeding particles, such as size, and optical characteristics. In the present study, the mean image method is optimal for the particle images acquired in the experiments seeding the fluorescent and resin particles. This is because that such

standard tracer particles were illuminated by a intense laser light sheet, and the light scattered by such particles were always strong. Thus, the darker background was successfully eliminated from the original images. In contrast, the minimum image method effectively extract particles in the images acquired in the experiment seeding the TLC particles. This is because the intensity of the light scattered by the TLC particles depend on the its temperature. The mean image sometimes becomes brighter than the low-intensity light scattered by the TLC particles, while such low-intensity lights can be recognized in the original images. The selection of optimal background subtraction method will greatly increase the ease of subsequent image analysis involving PIV/PTV. In the present study, these two different background subtraction methods are utilized depending on the seeding particles afterward.

Identification of rigid boundaries — Another issue arising in the quantification of flow fields through digital image processing is the identification of the rigid boundaries of the fluid vessel. To give a proper length scale for the visualized images, a calibration plate is used in general precise measurements. By plotting a number of markers with pre-known coordinates on the calibration plate, it is possible to associate the real coordinates (x, z) to the image coordinates (i, j) . This calibration procedure is often employed in stereoscopic PIV and tomographic PIV measurements [e.g. 46, 150]. However, it is necessary to insert a calibration plate into the fluid vessel filled by the test fluid. That is, the optical arrangement in the calibration process needs to be identical to the actual measurement conditions, apart from the seeding particles. A slight difference in the optical arrangements of the calibration and measurements will induce significant errors in the association of the real and image coordinates. Unlike in the measurements in an opened vessel, this severe requirement for the repeatability in the optical arrangement becomes a bottleneck especially in the measurements in a closed fluid vessel like the setup of this chapter. Even if the repeatability is ensured, it is difficult to find out the rigid boundaries of the vessel using a calibration plate, as the markers on the plate cannot be placed at the boundaries. Accordingly, a feasible method to identify the rigid boundaries on the image is required.

The rigid boundaries dividing the region of the fluid and vessel can be identified using particle images. In the test fluid, a number of tracer particles are seeded, and, of course, the particles should exist within the fluid. The particles move over the whole fluid region, if one waits a sufficiently long time. In other words, the region, where the brightness signals of the particles appear, is the fluid region. Assuming that the brightness signals of the particles appear on the whole fluid region at least during the measurement duration, the particle-existing region can be found using the particle images I_p obtained above. To consider all the particle images recorded at once, a pathline image needs to be compiled adopting a pixelwise function as

$$I_{pl} = \max(I_p(t_k)) \quad \text{with } k \in [1, N], \quad (1.22)$$

where I_{pl} and I_p are brightness values of a specific pixel of the pathline image I_{pl} and that of

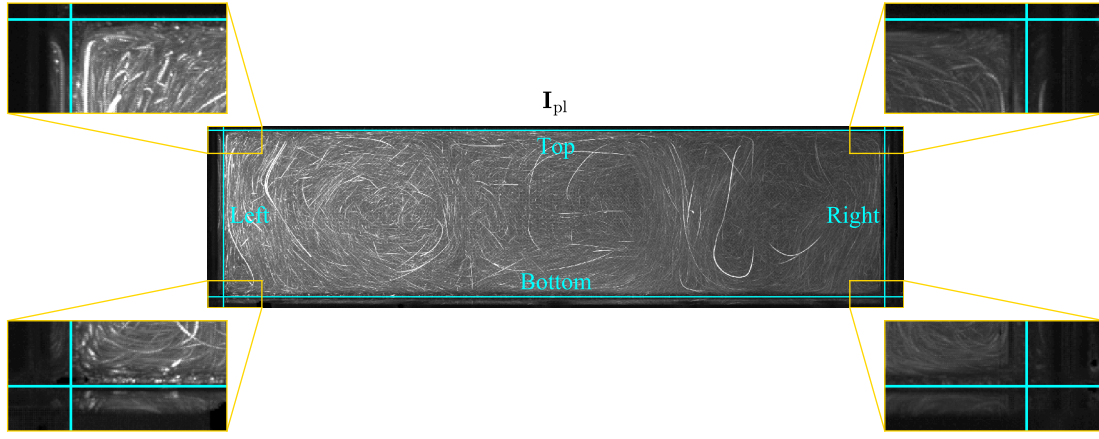


Figure 1.7: Example of identification of rigid boundaries for the pathline image I_{pl} recorded in the TLC experiment. Four boundaries at the top, bottom, left and right are indicated by cyan solid lines. Magnified views of the four corners enclosed by yellow solid squares are displayed around the overview located in the center for the sake of detailed observation.

the particle image I_p recorded at a time step t_k . The function $\max(\cdot)$ returns the maximum brightness value of all the time steps at the pixel. As the particle images I_p contain only brightness distributions of the particles, this pathline image I_{pl} also has the brightness distributions of the particles. An example of the pathline image obtained from the visualization using the TLC particles is shown in Fig. 1.7. The overview of the I_{pl} with 2592×676 pixels is shown at the center, and magnified views of the four corner regions enclosed by the yellow squares with 200×100 pixels are displayed around the overview. The particle-existing regions show bright signals compared to the rigid regions. To identify the boundaries between the fluid and rigid walls precisely, the corner regions need to be carefully observed. As the rigid walls of the vessel are smooth, a mirror image of the particle pathlines appears at the walls because in general the transparent walls reflect lights scattered by the tracer particles. This mirroring effect is useful to identify the rigid boundaries, and the boundaries can be drawn at the center of the line-symmetric image. In Fig. 1.7, the four boundary lines at the top, bottom, left and right are drawn by the cyan solid lines. The region enclosed by these four lines is the particle-existing region, i.e., fluid region. In this condition, the width and height of the fluid region become $W_{\text{image}} = 2464$ pixels and $H_{\text{image}} = 620$ pixels.

As long as the lens aberration and other optical issues are negligible, the calibration step is not necessarily required. In such a case, the conversion of length scale from the image to real coordinate can be simply given as

$$x = \frac{W_{\text{real}}}{W_{\text{image}}}i \quad \text{and} \quad z = \frac{H_{\text{real}}}{H_{\text{image}}}j, \quad (1.23)$$

where W_{real} and H_{real} are the width and height in the real dimensions corresponding to the image width W_{image} and height H_{image} . The fluid vessel has an internal length of $W_{\text{real}} = 200$ mm and height of $H_{\text{real}} = 50$ mm, and thus the resolution of the visualized image becomes $\approx$

0.081 mm/pixel in x and z directions.

The three steps presented above, color demosaicing, background subtraction, and identification of rigid boundaries, must be done absolutely as a series of pre-processing to measure reliable flow fields in the subsequent PIV/PTV analysis. After these pre-processing, we can finally proceed to the quantification of flow fields.

1.3.2 Dispersibility of the tracer particles

In Section 1.3.1, a series of image pre-processing has been proposed, and the procedure can be applied to all the images recorded in the different experiments seeding different types of particles in the same manner. As listed in Table 1.1, the St values of the three particles satisfy the requirement $St \ll 1$ addressed in Section 2.1. To verify the usefulness of these tracer particles, temporal changes of the particle images I_p are shown in Fig. 1.8. The pathline images recorded in the experiments using the TLC, fluorescent, and resin particles are respectively shown in Fig. 1.8(a), (b), and (c). The time from the beginning of each experiment is noted above the top panels, and the time elapses from the left to right. At 5 min, a number of particles can be seen in all the experiments. However, the number of particles in the experiments using the fluorescent and resin particles decreases with time, while that in the experiment using the TLC particles seems to be same for 180 min. The decrease in the number of seeded particles is apparent in naked eyes even during the experiments.

To quantitatively discuss these temporal changes in the number of particles, the number of particles is counted by detecting particles. Particle detection is done adopting the particle mask correlation method [190]. On the particle images, each particle is considered to have a point-spread brightness distribution like a 2D Gaussian distribution. Thus, a 2D Gaussian distribution is considered as a template (mask) mimicking a brightness distribution of a single particle in the particle mask correlation. The template is scanned in the whole particle image to calculate cross-correlation values. Pixels with high cross-correlation values are considered as the positions where particles exist. The cross-correlation values are calculated pixelwisely, and thus sub-pixel peak fitting can be performed for the local maxima. In this study, the Gaussian-sub-pixel peak fitting is employed, and particle positions can be detected with accuracy of ≈ 0.1 pixel ($= 0.08$ mm).

Adopting the particle mask correlation method, the number of seeded particles is counted in the particle images I_p . Temporal changes of the number of particles are shown in Fig. 1.9. Here, the number of particles is normalized by that at $t = 5$ min to understand the existence ratio of the particles to the beginning of the experiments. Each plot represents the averaged value of 1200 images recorded for 1 min, and the deviations in the number of detected particles during this period are small enough not to appear as error bars. At $t = 5$ min, more than 10000 particles are detected within the thickness of the light sheets for all the three conditions. In the experiment

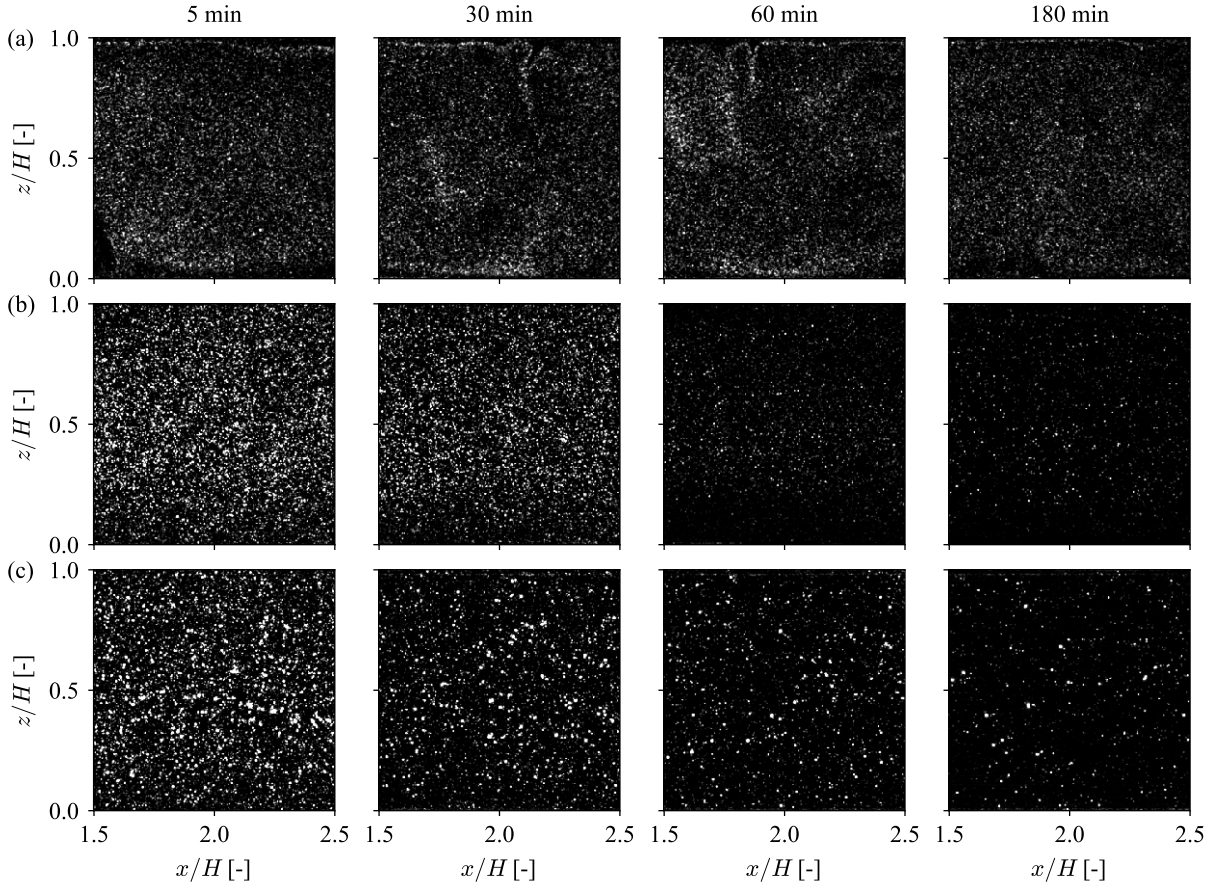


Figure 1.8: Temporal changes of the particle images I_p : (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS). The images show the central region of the fluid vessel, $1.5 \leq x/H \leq 2.5$ and $0.0 \leq z/H \leq 1.0$. The time elapsed from the start of experiments is noted above the top panels.

seeding the TLC particles, the existence ratio at $t = 180$ min is 0.94. Only 6% of the seeded TLC particles sediment or float during the 180-min measurements. By contrast, the existence ratios in the experiments using the fluorescent and resin particles decrease monotonically, and those at $t = 180$ min become 0.42 and 0.38. In these conditions, around 60% of the seeded particles sediment or float after the 180-min measurements.

These incredible decrease in the number of particles may induce severe problem on the thermal convection as the target phenomena. Suppose that all the particles uniformly sediment to the bottom of the fluid vessel, approximately 6000 particles cover the illuminated region of the bottom plate on which a heat input is imposed. In the experiment using green laser sheet, the area of the illuminated region becomes $200 \text{ mm} \times 1.5 \text{ mm} = 300 \text{ mm}^2$ (horizontal dimension times laser sheet thickness). The total area of the sediment particles within the sheet thickness becomes $\pi d_p^2/4 \times 6000$. From these calculation, 0.35% of the bottom plate is covered by the particles in the experiment using the fluorescent particles. This covering ratio in the experiment using the resin particles becomes 7.7% of the bottom plate. Obviously, this covering ratio cannot be ignored in considering uniform heat input to the fluid for keeping a target temperature gap during the measurements. Furthermore, the decrease of particle number means the decrease of

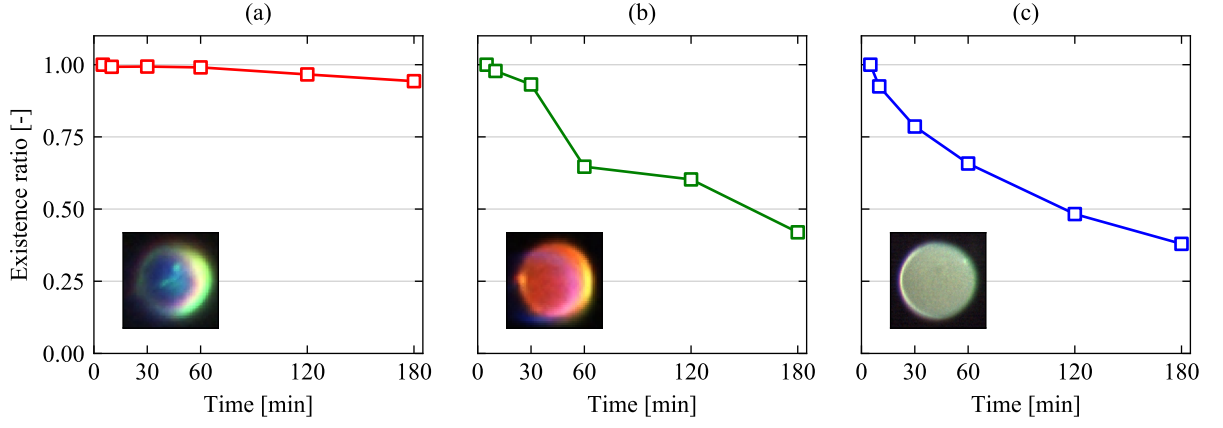


Figure 1.9: Existence ratio of the seeded particles with time: (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS). Photo of each particle is shown in inset of each panel.

the spatial resolution in the subsequent PIV/PTV analysis, as these analyses require information about the spatial distribution of particles. At least two or three particles are required to be present within the flow structures which need to be resolved. If the number of particles keep 10000 during the measurements, the particle density on the image becomes 1 mm^2 , and this allows to resolve flow structures with $2 - 3 \text{ mm}^2$ at the maximum. However, the particle density on the image becomes 0.4 mm^2 with 4000 particles, and the maximum spatial resolution of the flow structures becomes $5 - 7.5 \text{ mm}^2$. This decrease in the spatial resolution may be a bottleneck in the measurements of turbulent flows.

Incorporating all the discussions above, the dispersibility of the tracer particles is a highly important criterion as well as the Stokes number St . However, this dispersibility cannot be estimated in advance of the measurements, the selection of optimal tracer particles relies on the empirical considerations of experimentalists.

Using consecutive particle images, flow fields can be quantified via PIV or PTV analysis. The PIV analysis was performed adopting in-house algorithm to the particle images. Direct cross-correlation is employed to compute maps of cross-correlation within each search region. Recursive algorithm is also adopted, and interrogation windows decrease the size with the iterations of cross-correlation calculation. To identify outlier vectors, the universal outlier detection method [207] is used. The grid points from which the vectors are eliminated by the outlier detection are interpolated by the methods termed the Laplace equation rearrangement (LER) and the biquadratic ellipsoidal equation rearrangement (BER), proposed by Ido and Murai [75], Ido et al. [76].

To investigate the dispersibility of the tracer particles in terms of the flow fields, the number of erroneous vectors with time is shown in Fig. 1.10. Here the ordinate represents the ratio of the grids with erroneous vectors identified by the universal outlier detection to the total number of the grids ($N_x \times N_z = 201 \times 51 = 10251$). The ratio of erroneous vectors in the experiments seeding the TLC particles stays less than 1% after the 180-min measurements as shown in Fig. 1.10.

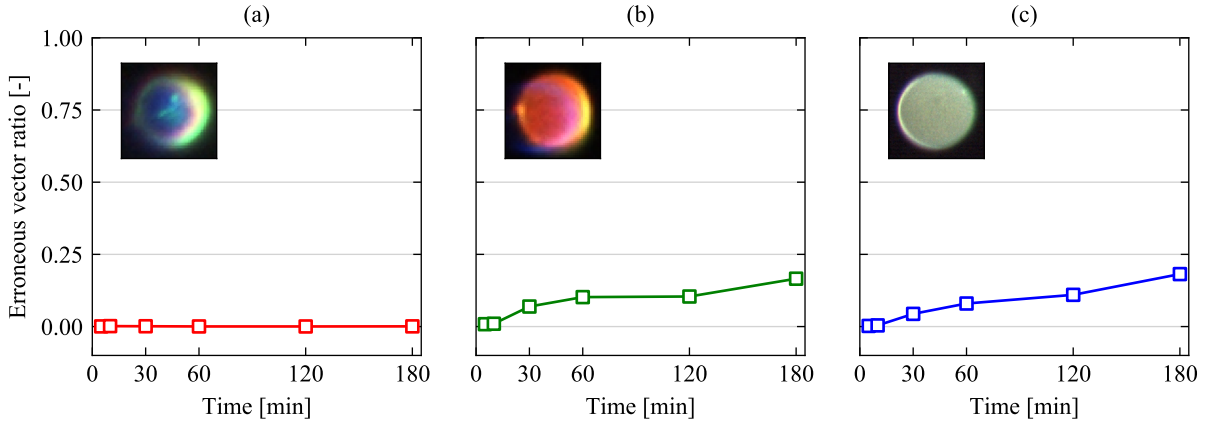


Figure 1.10: Ratio of erroneous vectors with time: (a) TLC particle (KWN2025); (b) fluorescent particle (FLU-OSTAR); and (c) resin particle (SP20SS). Photo of each particle is shown in inset of each panel.

That in the experiments seeding fluorescent and resin particles becomes 16.6% and 18.2% at $t = 180$ min, respectively. These increase in erroneous vector ratios seem to be inversely proportional to the existence ratios of the tracer particles shown in Fig. 1.9. The erroneous vectors can be interpolated using the methods mentioned above, however, these interpolated vectors cannot be treated fully equally to the original vectors.

To compare the results of PIV analysis, mean flow fields measured in the three experiments are shown in Fig. 1.11: the results of the experiments seeding the TLC, fluorescent, and resin particles are shown respectively in Fig. 1.11(a), (b), and (c). The instantaneous velocity fields are computed by the same PIV configuration for all the conditions. For each mean flow field, 1200 instantaneous velocity fields recorded during 1 min are used to take temporal averages. Time elapses from the top to bottom of Fig. 1.11, and noted above the left panels in each row. Mean velocity vectors (U, W) are drawn as black arrows, and these are superposed on the color contours representing the speed of velocity vectors. The speed is normalized by the free-fall velocity U_f , and the maximum mean velocity is $\sim 0.15U_f$ under the condition of $Ra = 8.7 \times 10^6$. For all the experiments, two to four cell-like structures can be seen. These cell-like structures are considered as large scale circulations (LSCs) formed by a number of thermal plumes [e.g. 153, 212]. The number and size of the LSCs change with time for all the three conditions. In addition, flow reversals, changes of the circulating directions of the LSCs, can be seen due to the growth of the small circulating flows at the corners. However, it may be difficult to identify systematical differences among the three results, as these are the temporal averages. However, it is possible to see smooth color contours in Fig. 1.11(a) during the whole measurement duration, while those at $t = 180$ min in Fig. 1.11(b) and (c) contain rough color changes at slow speed regions. These discontinuities appeared in the velocity fields originate from the erroneous vectors due to lack of tracer particles, as the PIV configuration is common for all the conditions. In other words, the later moments showing the discontinuities should not be analyzed by the same PIV configuration to obtain reliable flow fields, and the interrogation window needs to be

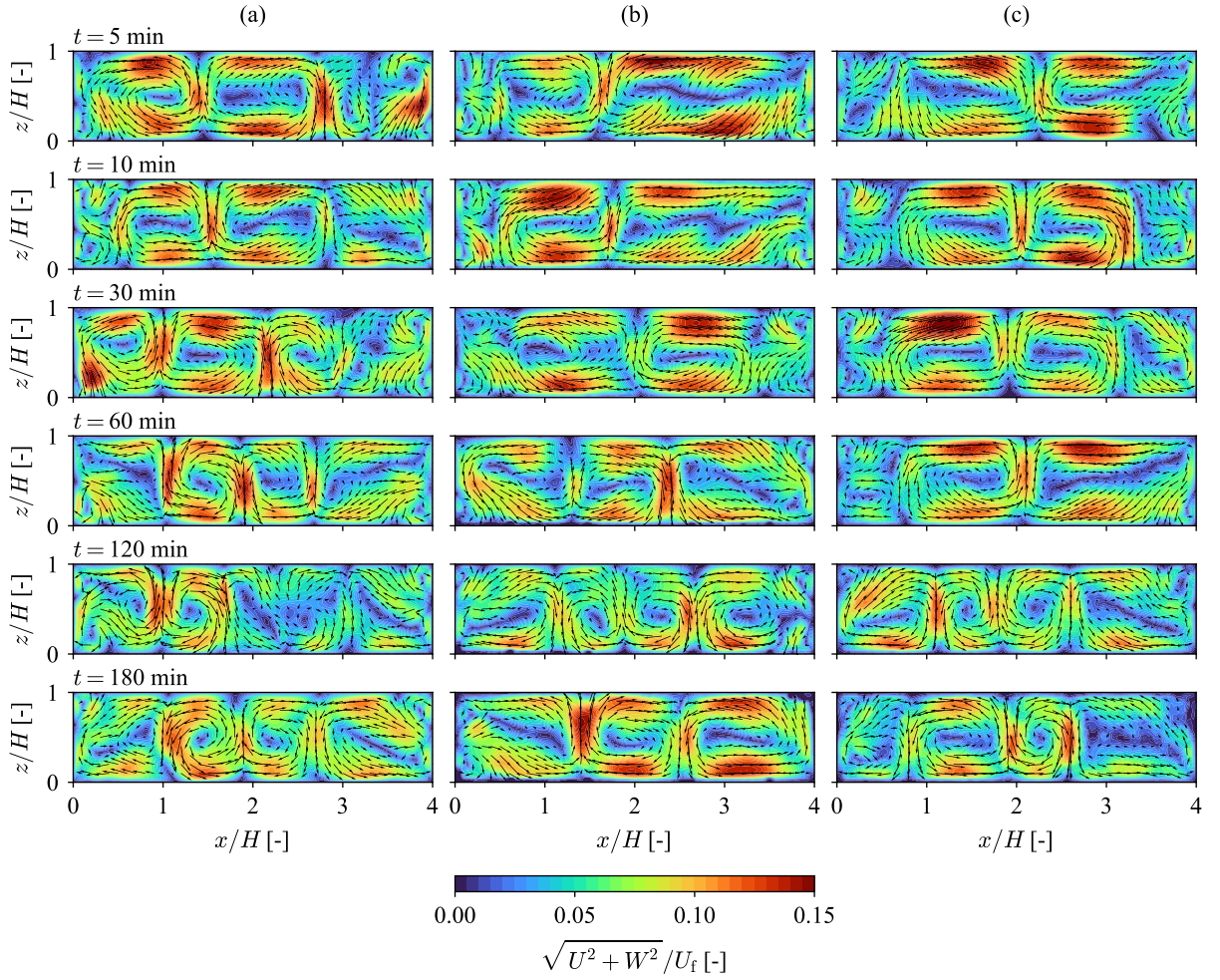


Figure 1.11: Temporal changes of mean flow fields under the condition of $Ra = 8.7 \times 10^6$ measured in the experiments using different tracer particles: (a) TLC particle (KWN2025); (b) fluorescent particle (FLUOSTAR); and (c) resin particle (SP20SS). In-plane velocity vectors (U , W) represented by the arrow are superposed on the color contours representing the speed. The speed is normalized by the free-fall velocity U_f . One of five vectors is plotted in x and z directions for the sake of clarity. Time elapses from top to bottom as noted above each panel of (a).

set larger than those in the earlier moments. That is, velocity fields cannot be treated equally through the measurement duration, if the particle dispersibility is not sufficient for such long-time measurements.

Incorporating all the discussions above, the TLC particles are the best selection of the three types of tracer particles investigated in this chapter, if long time measurements of $O(1 \text{ hour})$ are required. As described earlier, thermal convection often requires such long time measurements comparable to the thermal diffusion timescale τ_κ . That is, the TLC particles can be regarded as the best tracer particles for observations and measurements of thermal convection. The present study thus utilizes the TLC particles as the main tracer particles for detailed studies on advanced thermal convection problems mentioned in Chapter 2, 3, and 3.

1.3.3 Exemplary visualization of Rayleigh–Bénard convection using TLC particles

In this section, examples of visualization and quantification of RBC are introduced. The condition of $Ra = 8.7 \times 10^6$ is considered as the soft turbulence, which consists of numerous thermal plumes. The thermal plumes are the cold or hot blob detached from the thermal boundary layers on the top and bottom surfaces.

Separations of thermal boundary layers as cold plumes are shown in Fig. 1.12. Original color images I_c of the TLC particles are shown in Fig. 1.12(a). The TLC particles scatter color corresponding to the fluid temperature as red, green, and blue, from the lower to higher temperatures. Note that this color-to-temperature relation is inverse from the human's intuitive understanding, red is hot and blue is cold. In Fig. 1.12(a), a thin red-colored region can be seen immediately beneath the top surface on which imposed the low temperature. This thin region is considered to be the thermal boundary layer at the top surface. By contrast, a thin blue-colored region appears at the bottom surface, showing the thermal boundary layer at the bottom. With time, the thermal boundary layer at the top detaches from the wall and generates thermal plumes as shown in Fig. 1.12(a). This separation of thermal boundary layer as a cold plume is shown by the instantaneous velocity fields measured by PIV as Fig. 1.12(b). Here, the velocity vectors (u, w) are represented by the black arrows, and superposed on the color contours showing the speed. The speed is normalized by the free-fall velocity U_f . Compared to the mean flow fields shown in Fig. 1.11(a), the maximum velocity records $\sim 0.25U_f$ in the instantaneous velocity fields shown in Fig. 1.12(b). Such high-speed regions coincide with the down-welling flows originating from the cold plumes. The cold plumes firstly orient to the lower left at $t = 0 - 10$ s. The additional separation of the top thermal boundary layer changes the direction of the down-welling flow at $t = 15$ s toward the lower right of the fluid layer. This change in the flow direction is originated from a balance of numerous and intermittent emissions of thermal plumes. And thus, vertical down-welling and up-welling flows construct the cell-like structures of the LSCs in the mean flow fields shown in Fig. 1.11(a). The instantaneous velocity fields measured by PTV analysis are also shown in Fig. 1.12(c). Here, the velocity vectors are colored by the speed. The PTV analysis directly tracks particle displacements, and thus it is possible to resolve fine flow structures compared to the PIV. Qualitatively, the flow fields measured by the PIV and PTV are quite similar to each other, however, the results obtained by the PTV show sharper velocity changes than those from the PIV. This is natural that PTV does not require any spatial blurring effects that are inevitable in PIV analysis.

Similarly, separations of thermal boundary layers as the hot plume emission from the bottom boundary are shown in Fig. 1.13: The original color images I_c are shown in Fig. 1.13(a), and instantaneous velocity fields measured by the PIV and PTV are shown in Fig. 1.13(b) and (c), respectively. As the TLC particles scatter blue colors at higher temperatures, the hot plumes can

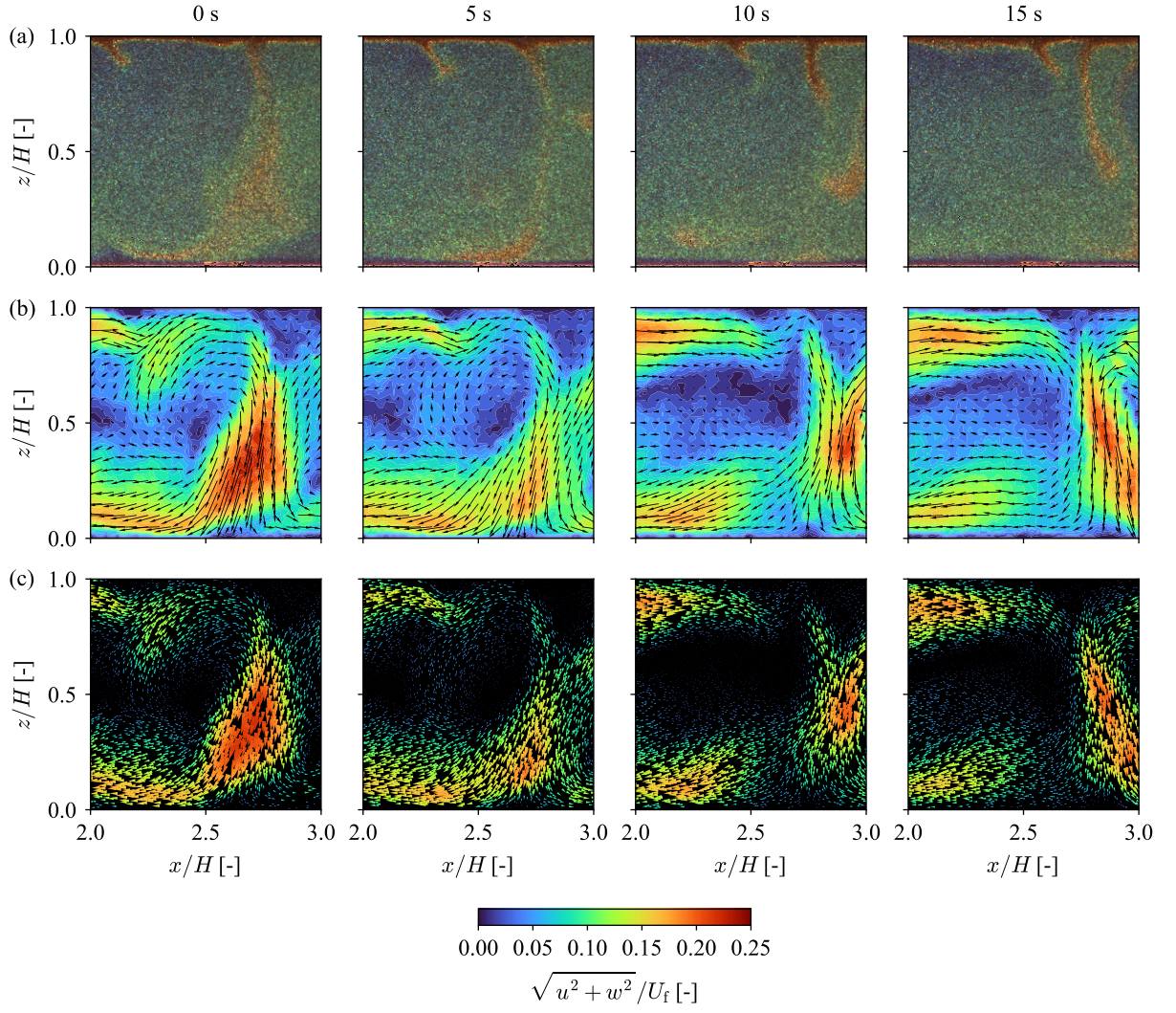


Figure 1.12: Separations of the top thermal boundary layers as cold plumes observed in the experiments seeding the TLC particles under the condition of $Ra = 8.7 \times 10^6$: (a) color images $\mathbf{I}_c$; (b) instantaneous velocity fields measured by PIV; and (c) instantaneous velocity fields measured by PTV. In-plane velocity vectors (u, w) represented by the arrow are superposed on the color contours representing the speed in (b), and those in (c) are represented by the arrows colored by the speed. The speed is normalized by the free-fall velocity U_f . Only 50% of the detected vectors are shown in (c) for a clearer visibility. Time elapses from top to bottom as noted above each panel of (a).

be identified as blue blobs (mushroom-like shapes) emitted from the bottom boundary as shown in Fig. 1.13(a). Two thermal plumes generated at the left and right are getting close to each other with time, and merge into a single up-welling flow at $t = 15$ s. This merging event of thermal plumes are more visible in the instantaneous velocity fields shown in Fig. 1.13(b) and (c).

As above, use of the TLC particles as tracer particles for visualization and quantification of thermal convection works well at least under the condition of $Ra = 8.7 \times 10^6$, the soft turbulence. Since the PIV and PTV could resolve the finest flow structures, thermal plumes, in Fig. 1.12 and 1.13, much higher Ra conditions, $Ra = O(\geq 10^7)$, are expected to be resolved in the same manner.

The difference between the results from the PIV and PTV may be apparent in the fields of velocity derivatives, i.e., vorticity, divergence, shear rate, and convective acceleration. For

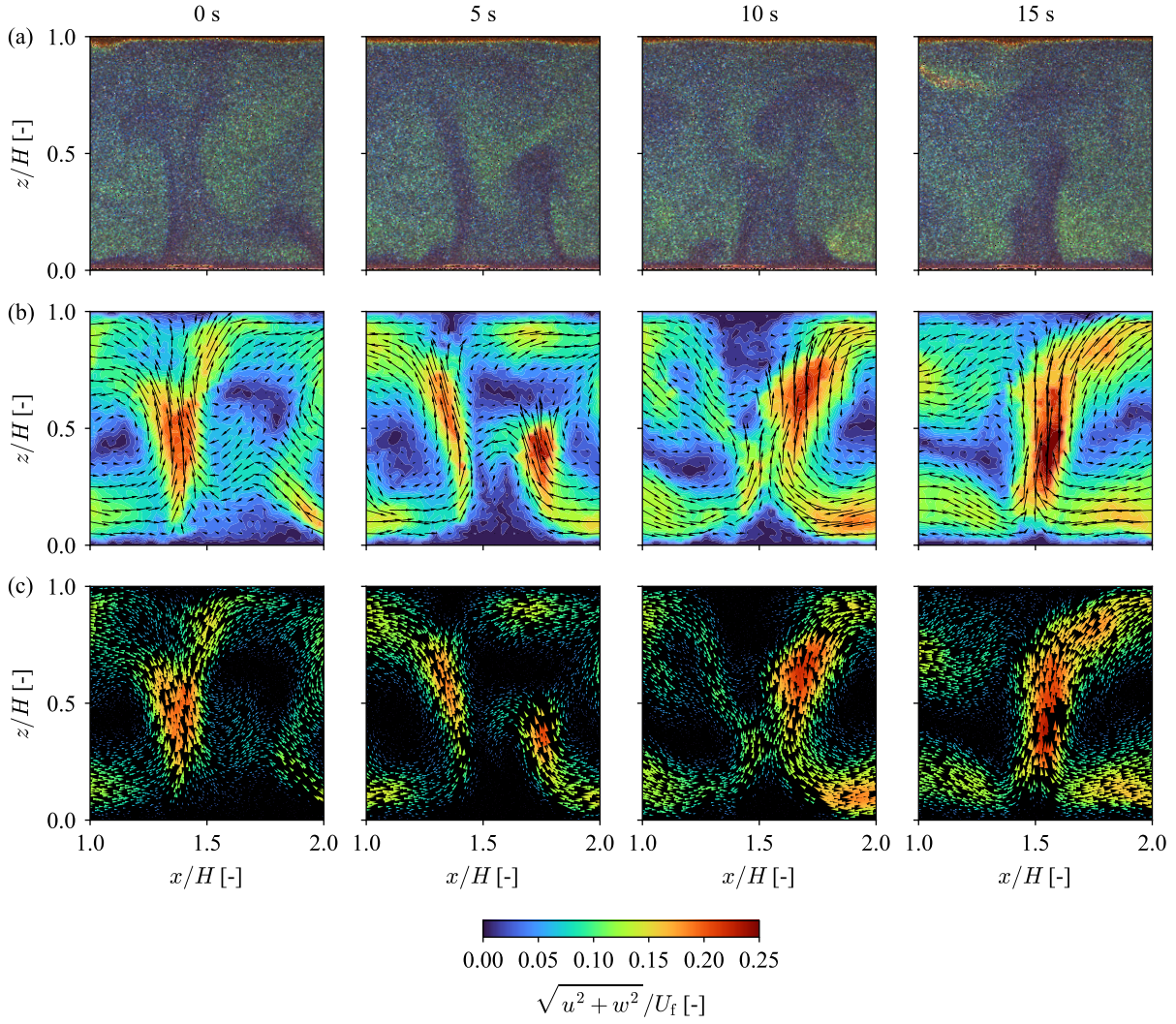


Figure 1.13: Separations of bottom thermal boundary layers as hot plumes observed in the experiments seeding the TLC particles under the condition of $Ra = 8.7 \times 10^6$: (a) color images $\mathbf{I}_c$; (b) instantaneous velocity fields measured by PIV; and (c) instantaneous velocity fields measured by PTV. In-plane velocity vectors (u, w) represented by the arrow are superposed on the color contours representing the speed in (b), and those in (c) are represented by the arrows colored by the speed. The speed is normalized by the free-fall velocity U_f . Only 50% of the detected vectors are shown in (c) for a clearer visibility. Time elapses from top to bottom as noted above each panel of (a).

instance, vorticity is calculated as the curl of the velocity vector expressed as

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}. \quad (1.24)$$

In the present measurements, only y -component vorticity can be derived as

$$\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}. \quad (1.25)$$

For the PIV data aligned in the regular grid, the second-order central-difference scheme is used for deriving the spatial derivatives. For the PTV data irregularly distributed in the space, the moving least squares (MLS) method [96] is used to obtain the spatial derivatives [91, 191]. The

MLS method is also known as the local polynomial regression, and calculate optimal planes fitted to the locally scattered data. In this method, a velocity vector at an arbitrary coordinate (x, z) is assumed to be represented by the 2D curved surface,

$$\mathbf{u}(x, y) = \begin{cases} u(x, z) = a_0^u x^2 + a_1^u xz + a_2^u z^2 + a_3^u x + a_4^u z + a_5^u \\ w(x, z) = a_0^w x^2 + a_1^w xz + a_2^w z^2 + a_3^w x + a_4^w z + a_5^w \end{cases}, \quad (1.26)$$

where $(a_0^u, a_1^u, a_2^u, a_3^u, a_4^u, a_5^u)$ and $(a_0^w, a_1^w, a_2^w, a_3^w, a_4^w, a_5^w)$ are the factors obtained by the least squares fit using scattered velocity vectors within a small local region. Substituting Eq. 1.26 to Eq. 1.25, the vorticity at the coordinate (x, z) is given as

$$\omega_y = (a_1^u x + 2a_2^u z + a_4^u) - (2a_0^w x + a_1^w z + a_3^w). \quad (1.27)$$

The merit of this method is that the derivatives can be directly obtained using the scattered velocity vectors measured by PTV without grid interpolation, and thus the errors originated from the standard grid interpolation methods [e.g. 76] can be minimized.

To compare the performance of PIV and PTV in terms of the velocity derivatives, instantaneous vorticity fields are shown in Fig. 1.14. The original color image shown in Fig. 1.14(a) contains two hot plumes; plume 1 is large and 2 is small. The instantaneous vorticity fields quantified from the PIV and PTV results are shown in Fig. 1.14(b) and (c), respectively. Here, the color contours represent the vorticity normalized by the buoyancy timescale $\tau_b = H/U_f$. Global trends of the two vorticity fields are identical in that the thermal plumes are emitted upwards with accompanying strong vorticity regions. The result from the PIV data seem to be rough compared to that from the PTV data. This trend is prominent especially around low vorticity regions, corresponding to low-speed regions. As the velocity gradients of PIV are computed using the differential scheme, the resolution of such velocity derivatives becomes poor. By contrast, the vorticity field of PTV is smooth even in the low-speed regions as shown in Fig. 1.14(c). Moreover, the borders of positive and negative vorticities are clearer than those in the PIV.

However, the computational cost for the MLS method is much larger than the differential scheme used in PIV data. In a sense, the vorticity field of PIV is sufficiently high-resolution and reliable to discuss the physics in the displayed case. This is because sufficient numbers of the tracer particles, the TLC particles, are dispersed in the fluid and high particle density images could be acquired in the measurements. It is necessary to select optimal methods satisfying the measurement requirements to reveal target phenomena. Incorporating all the results presented in this section, the use of the TLC particles is durable for various quantification methods to measure thermal convection even if it is in a turbulent state. For velocity fields, both the PIV and PTV can be applied, and for the derivation of spatial gradients of velocity fields, such as vorticity fields, grid-based discretized derivatives and grid-less methods like the MLS method are available.

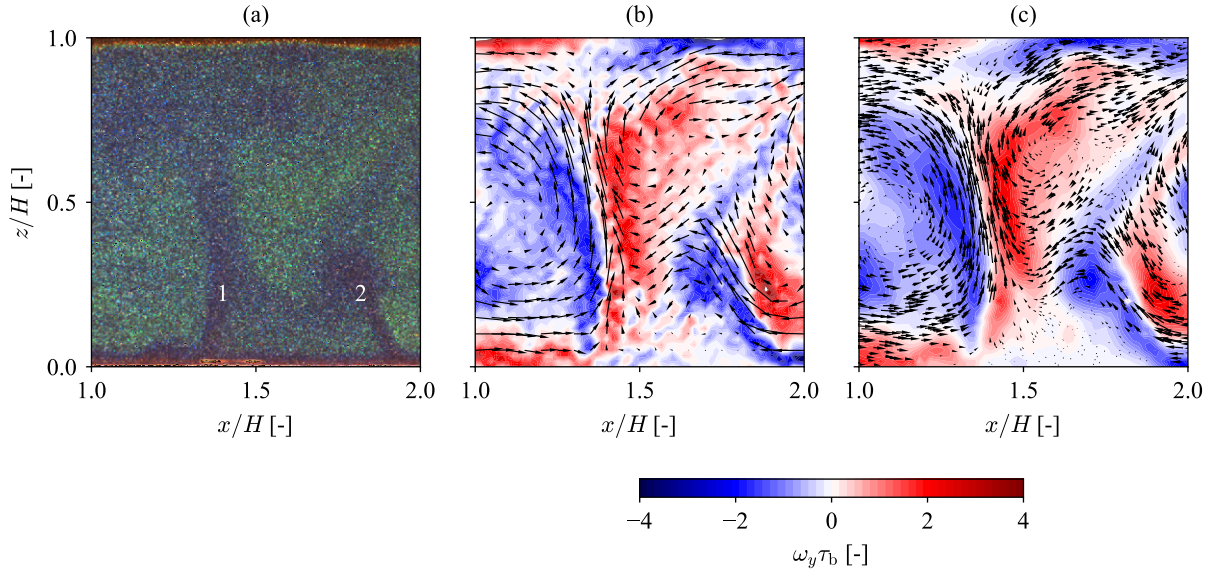


Figure 1.14: Vorticity fields around two thermal plumes: (a) original color image $\mathbf{I}_c$ containing two hot plumes, and vorticity fields measure by (b) PIV and (c) PTV analysis. Color contours represent the vorticity ω_y , and velocity vectors are superposed as black arrows. The vorticity is normalized by the buoyancy timescale τ_b . The velocity vectors are reduced in these plots for the sake of clarity.

1.4 Summary

Optical visualization of thermal convection is suffered by a number of problems arising in laboratory environments. In this chapter, one of the fundamental issues, selection of optimal tracer particles was examined in terms of the traceability and dispersibility of the particles. Three types of tracer particles, TLC, fluorescent, and resin particles, with each St is much less than unity, were investigated in thermal turbulent state of RBC ($Ra = 8.7 \times 10^7$). A series of image pre-processing that is required for the subsequent PIV/PTV analysis was explained in detail. The dispersibility of the tracer particles was examined via 180-min measurements performed under practical RBC experiments by counting the number of particles dispersed into the fluid. The TLC particles showed the best traceability that more than 90% of the seeded particles were present after the 180-min measurements. On the contrary, approximately 40% of the fluorescent and resin particles sediment after the 190-min measurements, even though the St numbers of these are $O(10^{-6})$ and $O(10^{-4})$, respectively. These results suggest that the optimal selection of tracer particles in thermal convection experiments became an empirical problem, and the TLC particles are highly useful for long-time measurements of thermal convection. Exemplary visualization of RBC using the TLC particles was presented. Thermal plumes emitted from the thermal boundary layers at the top and bottom of the fluid vessel could be visualized by color images due to the TLC particles. From the color images, qualitative temperature fields could be understood from the coloration of the scattered lights of TLC particles. Subsequent PIV and PTV analysis successfully resolved such thermal plumes in the velocity fields of the thermal turbulent state, as the particle images extracted from the TLC images contained sufficient number

of particles. Incorporating all the results shown in this chapter, the TLC particles are expected to be utilized in various flow conditions of thermal convection, involving laminar and turbulent states containing multi-timescale physics of $O(1 \text{ s})$ to $O(10 \text{ hour})$. Further, the TLC particles are durable for long-time measurements equivalent to the thermal diffusion timescale $O(10 \text{ hour})$.

EFFECT OF STABLE TEMPERATURE STRATIFICATION ON DEVELOPING HORIZONTAL CONVECTION

Preface

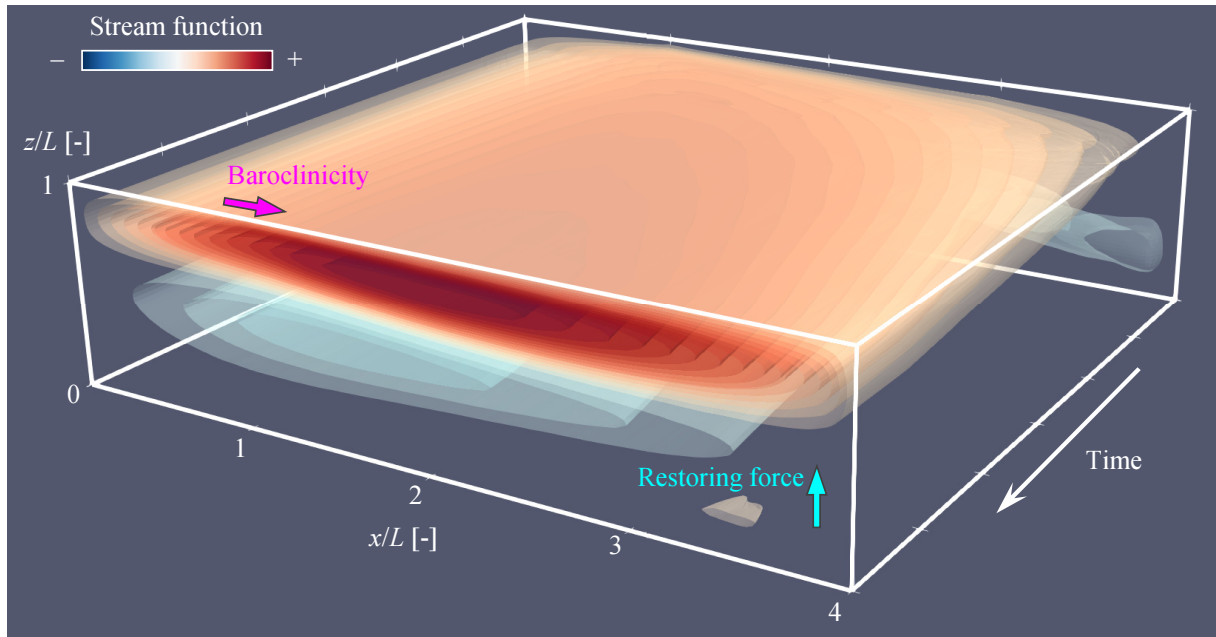


Figure 2.1: Graphical abstract of this chapter.

In Chapter 1, the applicability of the TLC particles to thermal convection was studied, and the TLC particles were found to be the most optimal tracer for optical measurements of thermal convection. The most notable advantage is its dispersibility as well as the traceability, since thermal convection often involves multiple timescales, for instance, buoyancy, viscous, and

thermal diffusion timescales.

As the TLC particles are highly efficient in slow flow fields with a long timescale physics, in Chapter 2, developing stages of horizontal convection are studied as one of the slowest flows. Horizontal convection is provoked by the horizontal temperature gradient along a horizontal surface at the same level. Use of the TLC particles will help a lot in the experimental visualization of such incredibly slow flows, which will develop together with the thermal diffusion.

The effect of background stable temperature stratification in the developing stage of horizontal convection is studied by conducting laboratory experiments. By imposing horizontally differential heating at the top of a layer of low-temperature water, both vertical and horizontal temperature differences are explicitly defined. In developing horizontal convection of the present study, the flow structures are dominated by the balance of baroclinic torque produced by the horizontal temperature difference and restoring force of stable temperature stratification. A novel nondimensional stratification parameter was thus defined, and it represents the restoring force of the stable temperature stratification acting against the baroclinic torque, in addition to the Rayleigh number. Various features of the flow structures, maximum velocity, stream function, roll thickness, circulation of the roll, total kinetic energy, and Reynolds number, which are quantified via particle tracking velocimetry, are summarized in spaces of the two nondimensional parameters. In the developing HC, the quantified flow features are well organized by power laws of the nondimensional parameter. The finite domain of the fluid container augments the effect of the restoring force, and the bulk quantities of the roll structures are suppressed by the stable temperature stratification. These results are the evidences for the significance of the nondimensional stratification parameter in the developing horizontal convection, unlike thermally-equilibrated horizontal convection conventionally considering destabilizing thermal buoyancy as the primary driving force.

The results shown in Chapter 2 were accepted as a journal paper as below.

Author Daisuke Noto, Tomomi Terada, Takatoshi Yanagisawa, Takehiro Miyagoshi, and Yuji Tasaka

Title Developing horizontal convection against stable temperature stratification in a rectangular container

Journal *Physical Review Fluids* (in press)

2.1 Introduction

Horizontal convection (HC), driven by a temperature gradient along a horizontal surface at the same level, is a fundamental topic describing fluid motions with various length scales in geophysical flows and engineering processes. Flows originating from HC are universal and have thus interest in research on meridional overturning circulation in ocean [51, 77, 162, 163, 197, 199, 211], land- and sea-breeze circulation, heat-/cool-island circulation [99, 100, 114, 119], and glass-melting furnaces [29, 56]. Extensive reviews and detailed discussions on HC can be read in a paper written by Hughes and Griffiths [74] and references therein.

Conventionally, HC has been studied in its thermally-equilibrated state as illustrated in Fig. 2.2(a). Here, a situation is considered that different temperatures, a high-temperature T_h and a low-temperature T_l , are imposed on the top surface of the fluid layer with an internal temperature T_i confined by adiabatic walls, resulting in $T_l < T_i < T_h$. One-half of the top surfaces act as heating and the other as cooling boundaries to the fluid layer; the heat flux to the fluid is namely balanced in the thermally-equilibrated state [70, 161, 177, 179, 196]. The two horizontal boundaries with different temperatures are thus respectively worked as ‘stabilizing source’ and ‘destabilizing source’ [74] to the internal fluid of T_i . Convective motion is then driven by both baroclinic torque produced by the horizontal temperature difference and thermal buoyancy at the destabilizing source, and a clockwise circulation in Fig. 2.2(a) is produced. In addition to these two effects facilitating the convective motion, the restoring force due to the stable temperature stratification at the stabilizing source decelerates the clockwise circulation. As a result, an asymmetric convection structure is generated. For most cases, a single overturning convection cell penetrating full height of the fluid layer is formed as illustrated by arrows in Fig. 2.2(a). Since thermal buoyancy explicitly facilitates this thermally-equilibrated HC, various characteristic quantities, such as features of convection cell and heat transport in terms of the Nusselt number, have been described using the Rayleigh number, like those in Rayleigh–Bénard convection [e.g. 29, 77, 132, 162, 170, 177]. Here, the Rayleigh number meaning a ratio of thermal buoyancy potentially existing in the bulk to the viscous and thermal dissipation is

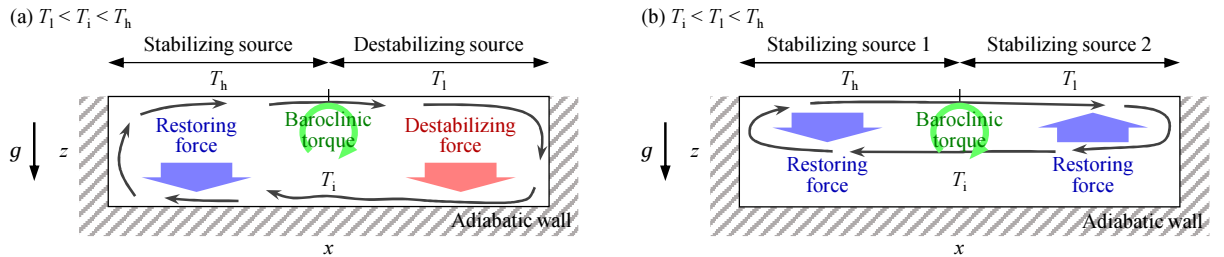


Figure 2.2: Conceptual diagrams of two different problem settings for HC: (a) thermally-equilibrated HC and (b) developing HC. T_i denotes the interior temperature and T_h and T_l are the temperatures set at the same horizontal plane, $T_h > T_l$.

represented as

$$Ra = \frac{g\beta\Delta\theta W^3}{\kappa\nu}, \quad (2.1)$$

where g , β , ν , κ , $\Delta\theta$, and W are respectively the gravity acceleration, thermal expansion coefficient, kinematic viscosity, thermal diffusivity, horizontal temperature difference, and horizontal dimension of the fluid layer. It is worth noting that the Ra involves two meanings simultaneously. One is the strength of the baroclinic torque provided by the horizontal temperature difference. The other is the thermal buoyancy generated due to the vertical temperature gradient of the unstable temperature stratification at the destabilizing source, as $T_l < T_i < T_h = T_l + \Delta\theta$. The presence of the horizontal temperature difference in the thermally-equilibrated HC originally produces the two effects, the baroclinic torque and the destabilizing thermal buoyancy, and thus it is inevitable to involve both of the two effects in the Ra for the thermally-equilibrated HC. In addition, the contribution of the baroclinic torque alone to the convective motion is considered much smaller than that of the destabilizing buoyancy as evident in Coman et al. [36]; they showed that the circulation strength produced in the thermally-equilibrated HC is 10–20% of the situation when the same temperature difference is imposed on the vertical direction, i.e., Rayleigh–Bénard convection. Consequently, Ra is regarded as the dominant parameter in the thermally-equilibrated HC, and this is quite convenient as it is possible to describe analogy to Rayleigh–Bénard convection.

From the aspects of geophysical and astrophysical interests, transient states of HC developing toward the thermally-equilibrated HC are worth discussing, as the timescale of the development is extremely long. Here, the problem setting of the developing stage of HC is considered as illustrated in Fig. 2.2(b). Both of the surface temperatures T_h and T_l are set higher than the interior temperature T_i , resulting in $T_i < T_l < T_h$, hence they act as stabilizing sources to the internal fluid in a global sense. Only the temperature difference between T_h and T_l generating a baroclinic torque drives the flow. Initial transitions from a quiescent fluid to thermally-equilibrated states have been mentioned previously [29, 115, 199], and reported that those take quite a long time equivalent to or longer than the thermal diffusion timescale due to the absence of destabilizing source. This kind of flow developments is totally different from that in the thermally-equilibrated HC. Similarly, Rossby [163] and Griffiths et al. [62] paid attention to transient processes of thermally-equilibrated HC by providing a sudden temperature change to one of the surfaces imposed a temperature difference leading to a heat flux imbalance. They found that transient adjustments to equilibrium states take a longer time when the temperature of the destabilizing source is weakened, and those can be achieved faster when that of the stabilizing source is weakened. This is because the destabilizing thermal buoyancy is the primary factor to determine the convection structure in thermally-equilibrated HC, and the weakening of the destabilizing force sustains only the baroclinic torque while it negates the destabilizing thermal buoyancy. Further, the weakening of the destabilizing force yielded the formation of secondary convection roll due to the friction of the fluid viscosity, as thermal plumes emitted from the weakened

destabilizing source cannot penetrate full height of the domain. In such transient evolution toward thermally-equilibrated states, it is not possible to describe flow structures by only the Ra defined in Eq. 2.1 showing the strength of the bulk buoyancy. Instead, the baroclinic torque and the restoring force of the stable temperature stratification become prominent, and these are required to consider as the primary parameters.

Studies on transient processes of developing HC appeared less frequent. Even in the studies, thermal buoyancy destabilizing fluid layer has been always treated as an important parameter. For instance, a balanced situation of convection driven by both baroclinic torque and thermal buoyancy was investigated experimentally by Lu et al. [99, 100] and numerically by Niino et al. [119] from the perspective of the temporal evolution problem for urban heat-island circulation. These analogs of heat-island circulation suggest regime transitions occur due to the balancing of baroclinic torque and thermal buoyancy. Some previous studies [29, 115, 199], are only a few cases dealing with the developing HC, i.e., the initial transitions without destabilizing source from one thermally-equilibrated HC to the other thermally-equilibrated HC. However, they did not describe details of flow developments in the transitions, as they focused on thermally-equilibrated states at which the destabilizing thermal buoyancy is dominant. Flow structures in developing HC without destabilizing source has only been addressed in Mori and Niino [114] from their theoretical and numerical approaches. In their work, linear stable temperature stratification was considered for base states, and heat removal was imposed at a part of the bottom. The formulation of the problem is given as a two-dimensional vorticity equation,

$$\frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} \frac{\partial \psi}{\partial z} - \frac{\partial \eta}{\partial z} \frac{\partial \psi}{\partial x} = g\beta \frac{\partial T}{\partial x} + \nu \nabla^2 \eta, \quad (2.2)$$

and a thermodynamic equation,

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} \frac{\partial \psi}{\partial z} - \frac{\partial T}{\partial z} \frac{\partial \psi}{\partial x} = \gamma \frac{\partial \psi}{\partial x} + \kappa \nabla^2 T, \quad (2.3)$$

where ψ , η , T , and γ are respectively the stream function, y -component of vorticity, temperature, and the vertical temperature gradient of the base temperature field, $T(z) = T_0 + \gamma z$. Under this condition, the first term on the right-hand side of Eq. 2.2 is the baroclinic vorticity production term, which is the main driving force of convective motion. The first term on the right-hand side of Eq. 2.3 is the restoring force of stable temperature stratification, which stabilizes the convective motion. With these formulations explicitly denoting the effect of base stable stratification as γ , Mori and Niino [114] suggested three flow regimes based on the time and degree of the stratification, namely diffusion, gravity current, and gravity wave regimes, and found self-similar solutions for each regime in laterally infinite domains. These flow regimes are totally different from those in thermally-equilibrated states [e.g. 29, 162]. To incorporate the baroclinicity and the restoring force of the stable temperature stratification, Mori and Niino [114] introduced the nondimensional stratification parameter γ' . It is a ratio of temperature gradients arising from the

restoring force of the stable temperature stratification γ and the baroclinic vorticity ξ denoted as

$$\gamma' = \frac{\gamma}{\xi} = \gamma \frac{\left(\frac{\kappa^2}{g\beta\Delta\theta}\right)^{1/3}}{\Delta\theta} = g\beta\gamma \left(\frac{\sqrt{\kappa}}{g\beta\Delta\theta}\right)^{4/3}. \quad (2.4)$$

It is difficult to define the length scale for the baroclinic torque, however, Mori and Niino [114] employed $[\kappa^2/(g\beta\Delta\theta)]^{1/3}$ as the length scale for the baroclinic torque. This length represents the distance that the heated fluids at the stabilizing source can possess sufficient thermal buoyancy compared to the surroundings even with losing their buoyancy by thermal diffusion, i.e., the length that the baroclinic torque can effectively act against the thermal diffusion. The importance of the work of Mori and Niino [114] is the explicit consideration of the restoring force due to the stable temperature stratification. The baroclinicity driving the convective motion and restoring force of the stable temperature stratification are taken into account by defining γ' as the primary factor in the developing HC. In contrast, γ' is not exposed in thermally-equilibrated HC as γ becomes zero.

Although Mori and Niino [114] showed the importance of γ' in the developing HC, there are two points expected to be addressed to fully understand the physics in developing HC. One is the influence of side walls; a laterally infinite domain was only investigated, while thermally-equilibrated HC have been examined in finite domains. It is obvious that the self-similar solutions found by Mori and Niino [114] cannot be held in finite domains due to its geometric restriction. The contribution of γ' to the developing HC in a finite domain is worth discussing from the perspective of transient adjustment processes of thermally-equilibrated HC [62, 163], since the developing HC finally reaches the thermally-equilibrated HC after a sufficiently long time due to thermal diffusion. Another is that γ' was considered as a given parameter irrespective of flow structures. As the convective motion transports heat from the horizontal surface, the degree of stable temperature stratification should be modified with time. In particular, this issue becomes prominent for considering a finite domain. These issues should be addressed to understand developing HC as a transient process arising in thermally-equilibrated HC.

In the present study, developing HC at which no destabilizing source exists was investigated through experimental studies. Using a rectangular container as the fluid layer, convective flow structures developed in a finite domain are clarified. Further, the effect of time relaxation of the stable temperature stratification on flow structures of the developing HC was focused on. These conditions are realized in laboratory experiments by imposing differential heating at a top of a cold fluid confined within thermally insulated walls. Transient processes developing toward thermally-equilibrated HC are quantified by adopting optical visualization techniques. Section 2.2 presents the experimental setup, procedures, and a series of data analyses for acquiring flow characteristics. Temporal evolution of flow structures and the effect of stable stratification on HC are explained in Section 2.3 and discussed in detail in Section 2.4. Summary

of the findings of the present study will be provided in Section 2.5.

2.2 Experiment

2.2.1 Experimental setup

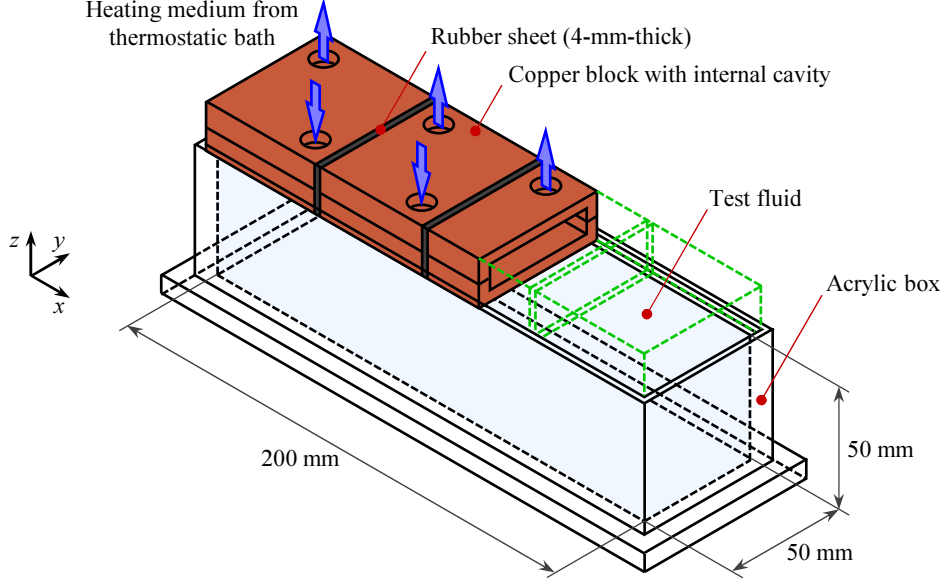


Figure 2.3: Schematic of the rectangular fluid container.

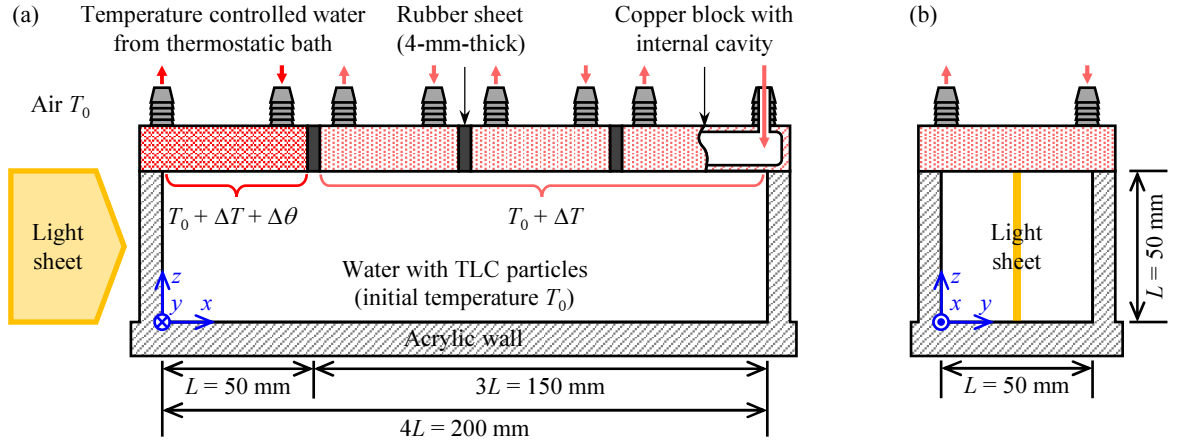


Figure 2.4: Schematic of the experimental setup: (a) front view and (b) side view.

A rectangular container with internal dimensions of 200 mm ($4L$) $\times$ 50 mm (L) $\times$ 50 mm (L) in x , y , and z directions was used for the experimental investigations. The fluid container is illustrated in Fig. 2.3, and details of experimentally imposed conditions are shown in Fig. 2.4. The sides and bottom of the container were consisted of acrylic walls thicker than 10 mm. The top of the fluid layer was closed by four copper blocks, with each block having an internal

cavity allowing the circulation of water from thermostatic baths. The copper blocks were aligned and isolated by 4-mm-thick rubber sheets to prevent heat exchange between adjacent blocks and allow the control of the temperatures of the blocks individually. Degassed distilled water (with the Prandtl number of $Pr = \nu/\kappa \sim 7$ at $T = 20^\circ\text{C}$) was used as the test fluid, and two types of thermochromic liquid crystal (TLC) particles (having a mean diameter $\sim 15\ \mu\text{m}$, mean density $1.02 \times 10^{-3}\ \text{kg}/\text{m}^3$; KWN-2025 and KWN-2530, *Japan Capsular Products Inc.*) were seeded for flow visualization. The approximate temperature range was 5 K for the two types of TLC particles; one type of TLC particle showed coloration at $20 - 25^\circ\text{C}$ and the other showed coloration at $25 - 30^\circ\text{C}$. Accordingly, two color cycles of the TLC particles were expected to be observed from 20 to 30°C . The thermal conductivity of water, $\sim 0.6\ \text{W}/(\text{m} \cdot \text{K})$, is three times larger than that of acrylic, $\sim 0.2\ \text{W}/(\text{m} \cdot \text{K})$. The thermal condition at the side and bottom walls thus cannot be considered fully adiabatic, and relatively small temperature difference between the bulk of the fluid and the ambient temperature. In the preliminary tests, however, it is confirmed that the flows on the y - z plane were negligibly slow, and therefore, the flow fields explored in the present study can be regarded as (quasi-) two-dimensional as expected in the work by Mori and Niino [114]. The vertical cross-section at $y = 0.5L$ was visualized using a white light sheet generated by a cylindrical lens and a halogen lamp for quantitative measurements. With these optical configurations, the TLC particles allowed the visualization of temperature fields as variations in color as red, green, and blue for lower to higher temperature in the specific ranges. In addition, particle motions can be traced adopting well-known techniques such as particle image velocimetry (PIV) and particle tracking velocimetry (PTV), and flow fields can be measured as detailed in Section 2.2.3. The use of the TLC particles for the purpose of PIV was introduced in our earlier works [13, 127]. A CMOS color camera (DFK33UP5000, *The Imaging Source Co.*) was set in front of the fluid container to record particle motions in the illuminated x - z plane at $y = 0.5L$. The image resolution was $2400\ \text{pixel} \times 600\ \text{pixel}$ to $200\ \text{mm}$ ($4L$) $\times$ $50\ \text{mm}$ (L). For direct measurements of the temperature of the top surface on which a horizontal temperature difference was imposed, seven horizontally aligned thermistor sensors having a diameter of 1.5 mm were inserted into the fluid layer and fixed close to the top surface at $z \sim 0.98L$. The thermistor sensors measured the temperature with accuracy better than $\pm 0.1\ \text{K}$ and precision of less than $\pm 0.02\ \text{K}$. The details of temperature measurements are described in Section 2.2.3.

2.2.2 Experimental procedure

To systematically investigate the flow structures observed in developing HC through laboratory experiments, two variable experimental parameters, namely the vertical temperature difference ΔT and horizontal temperature difference $\Delta\theta$ were controlled. Experimental measurements were made in a room at a controlled temperature of $T_0 = 20^\circ\text{C}$. Initially, the fluid container was filled with a quiescent fluid at a set temperature T_0 , and temperatures of the four copper blocks at

the top of the fluid layer were maintained also at T_0 . At $t = 0$, the set temperature of one of the thermostatic baths connected to the left-end copper block was increased to $T = T_0 + \Delta T + \Delta\theta$ and that of the other thermostatic bath connected to the right three copper blocks was increased to $T_0 + \Delta T$. Here, ΔT is a positive temperature difference imposed at the three right blocks relative to the initial temperature T_0 to generate stable temperature stratification at $L \lesssim x \leq 4L$. $\Delta\theta$ is a positive temperature difference imposed at the left-end copper block relative to the three right copper blocks to realize stable temperature stratification at $0 \leq x \lesssim L$. Note that the thermostats typically required a few minutes to increase the temperature to the set values, and the initial transient flows are thus excluded from the following discussions. For each condition, continuous image acquisitions were made at a frequency of 10 Hz for the first 30 min and for 1 min at every 30 min from the initiation of the heating up to 3 hours, corresponding to 0.6 in the thermal diffusion timescale $\tau_\kappa = L^2/\kappa$.

With the explicitly defined temperature differences, $\Delta\theta$ and ΔT , it is possible calculate the nondimensional stratification parameter γ' , denoting the balance of temperature gradients arising from the restoring force of the stable temperature stratification and the baroclinicity. It is convenient if the stratification parameter is given *a priori* as fixed values, and it is thus considered here as the linearly stratified condition with a mean temperature gradient of

$$\gamma_{\text{lin}} = \frac{T_{\text{wall}} - T_0}{L}, \quad (2.5)$$

where, T_{wall} is the mean temperature of the heating copper blocks simply given as,

$$T_{\text{wall}} = \frac{1}{4} (T_0 + \Delta T + \Delta\theta) + \frac{3}{4} (T_0 + \Delta T). \quad (2.6)$$

Accordingly, the mean vertical temperature gradient is expressed as

$$\gamma_{\text{lin}} = \frac{4\Delta T + \Delta\theta}{4L}. \quad (2.7)$$

With these notations, the nondimensional stratification parameter in the linearly stratified case γ'_{lin} is expressed as

$$\gamma'_{\text{lin}} = \frac{\gamma_{\text{lin}}}{\xi} = \frac{g\beta(4\Delta T + \Delta\theta)}{4L} \left(\frac{\sqrt{\kappa}}{g\beta\Delta\theta} \right)^{4/3}. \quad (2.8)$$

For reference, conventionally used Ra is also calculated from Eq. 2.1 using $W = 4L$ as the horizontal length scale. Please note that Ra corresponds to the maximum buoyancy potentially existing in the system under thermally-equilibrated states. Thus, Ra calculated here is the equivalent Rayleigh number realized when the same horizontal temperature difference is imposed in thermally-equilibrated HC.

In the present study, the imposed temperature differences were chosen within feasible ranges in the laboratory environments with yielding convective motions. For both $\Delta\theta$ and ΔT , four

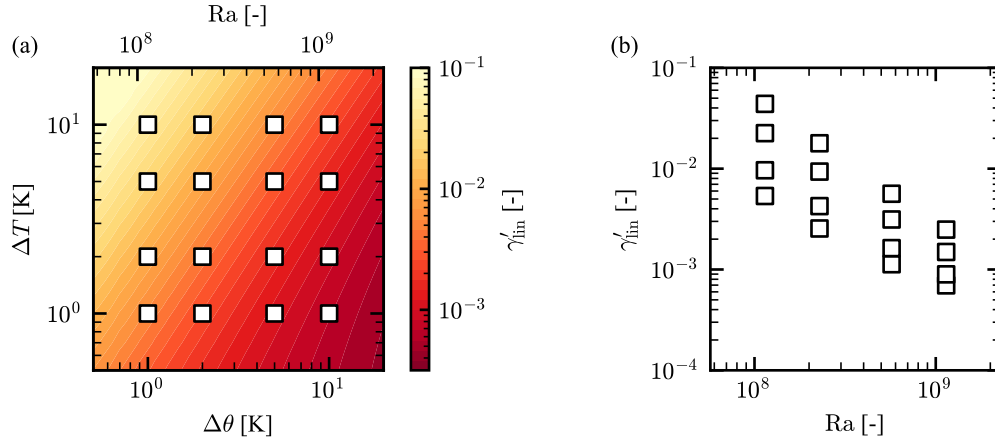


Figure 2.5: Parameters explored in the present study: (a) plots of experimentally controllable parameters, horizontal temperature difference $\Delta\theta$, and vertical temperature difference ΔT and (b) plots of nondimensional parameters in the $Ra-\gamma'_{lin}$ domain. Color contours in (a) represent γ'_{lin} values.

different temperature differences were imposed resulting in 16 parameters, and the explored parameters are summarized in Table 2.1 and plotted in Fig. 2.5; Fig. 2.5(a) shows plots in the domain of experimentally controllable parameters ($\Delta\theta-\Delta T$ with a unit of K) while plots in the domain of nondimensional parameters ($Ra-\gamma'_{lin}$) is shown in Fig. 2.5(b). The color contours in Fig. 2.5(a) represent the nondimensional stratification parameter γ'_{lin} . According to Mori and Niino [114], flow regimes in a laterally infinite system can be categorized into three regimes, namely diffusion, gravity current, and gravity wave regimes depending on γ'_{lin} and time from heating. The diffusion regime may not be observed in the experiments, because the transition from the regime will be completed on a timescale $O(< 10^0 \text{ s})$ in cases that water is used as the test fluid. In the present study, the initial temperature of the fluid is T_0 at $t = 0$, and the dimensional stratification parameter γ thus asymptotes from ∞ to 0 with the growth of thermal boundary layers at the top, which defines the length scale of the stratification. Most of the presently explored parameters $\gamma'_{lin} = O(10^{-4} - 10^{-1})$ are expected to be in the gravity current regime after a sufficiently long time for thermal diffusion according to the condition $\gamma'_{lin} \ll 1$, if the system is horizontally infinite. In the present study, a rectangular container for the fluid layer was used, and the horizontal length scale is regulated up to $4L$. This geometric constraint should affect the flow regimes observed in the present study. Suppose the same $\Delta\theta$ is imposed in thermally-equilibrated HC, the resultant Ra ranges 10^8-10^9 . These parameters are sufficiently strong to form a single overturning convection cell penetrating the full depth of the domain in a thermally-equilibrated state.

2.2.3 Data acquisition methodology

In Fig. 2.6, an example of flow visualization using the two types of TLC particles with the conditions $\Delta T = 2 \text{ K}$ and $\Delta\theta = 5 \text{ K}$ taken at $t = 30 \text{ min}$ is shown as a color image. Colors of the TLC particles can be qualitatively understood as the isotherms. The first color cycle of one

Table 2.1: Experimental parameters explored in the present study. For all the conditions, the initial fluid temperature was $T_0 = 20^\circ\text{C}$.

Case	$\Delta\theta$ [K]	Ra [-]	ΔT [K]	γ'_{lin} [-]
1	1	1.1×10^8	1	5.4×10^{-3}
2			2	9.6×10^{-3}
3			5	2.3×10^{-2}
4			10	4.4×10^{-2}
5	2	2.3×10^8	1	2.6×10^{-3}
6			2	4.3×10^{-3}
7			5	9.4×10^{-3}
8			10	1.8×10^{-2}
9	5	5.7×10^8	1	1.1×10^{-3}
10			2	1.6×10^{-3}
11			5	3.1×10^{-3}
12			10	5.6×10^{-3}
13	10	1.1×10^9	1	7.0×10^{-4}
14			2	9.0×10^{-4}
15			5	1.5×10^{-3}
16			10	2.5×10^{-3}

type of TLC particle can be seen as red, green, and blue regions from the bottom to top of the fluid layer, and this gradual color change corresponds to an increase in temperature, showing stable temperature stratification. Please note that the color change of the TLC particle is not linear, and the temperature range from red to green is narrow < 1 K out of 5 K. The second color cycle of the other type of TLC particle appears within a shallow region immediately beneath the left-end copper block ($0 \leq x \lesssim L$) meaning a steep temperature gradient beneath the block; a magnified view of this second cycle in the region enclosed by the solid square ($0.2L \times 0.1L$) is shown in the inset. The colors blue, green, and red appear in order from upper left to lower right. Considering the color changes in the magnified region, isotherms are qualitatively drawn as the dotted lines. Nearly vertical isotherms are identified at $x \sim L$ and may produce baroclinic vorticity. Overall, except for the magnified region, the test fluid in the rectangular container is found to form stable temperature stratification with different degrees of stratification along the horizontal direction. With stable temperature stratification, a clockwise circulating motion forms beneath the top copper blocks owing to the baroclinic torque as indicated by the yellow arrows. This circulation is measured quantitatively adopting image processing techniques in the following.

In advance of the quantification of flow fields, the source of horizontal circulation is examined using the measurement of temperature. To confirm the vertically aligned isotherms for generating baroclinic vorticity qualitatively illustrated in the magnified inset of Fig. 2.6, horizon-

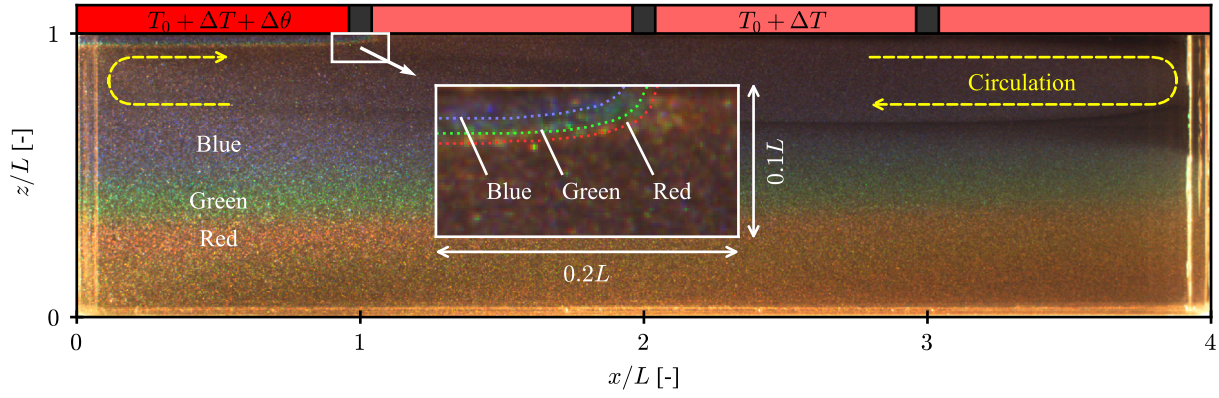


Figure 2.6: Example of flow visualization using the TLC particles of two different temperature ranges at $t = 30$ min with the condition of $\Delta T = 2$ K and $\Delta\theta = 5$ K. Red and black panels above the fluid layer indicate the locations of the heating copper blocks and insulating rubber sheets. A magnified view of the region enclosed by the solid square of $0.2L \times 0.1L$ is presented in the inset, and qualitative isotherms are indicated by dotted lines. Colors of the TLC particles change as red, green, and blue according to the increase in fluid temperature. The color contrast is adjusted to enhance visibility.

tal temperature profiles beneath the copper blocks are directly measured using the thermistor sensors. Measurement points of the seven sensors were set immediately beneath the copper blocks at $z \sim 0.98L$ and $x = L, L \pm 0.1L, L \pm 0.2L$, and $L \pm 0.5L$, which cover the region around $x = L$ where the horizontal temperature difference $\Delta\theta$ was imposed. An example of temperature measurements as a function of the x -position for $\Delta T = 5$ K and $\Delta\theta = 5$ K is shown in Fig. 2.7. For clarity, temperature profiles measured every 20 min are plotted. The squares show the temperatures measured by the thermistor sensors and are colored according to the time elapsed from the initiation of heating. The horizontal dotted lines are the set temperatures of $T = T_0, T_0 + \Delta T$, and $T_0 + \Delta T + \Delta\theta$ from bottom to top. At $t = 0$ min, the temperatures of all sensors record $T \simeq T_0$ because the initial temperature of the test fluid is T_0 . At $t \geq 20$ min, the measured temperatures increase, and they do not change with time until $t = 180$ min. Since the temperatures at the top surfaces were kept constantly, the measured temperatures at different time are fully overlying in Fig. 2.7(a) and hard to be distinguished. The measured temperatures are slightly different from the set temperature of the copper blocks as the sensors measure the fluid temperature at $z \sim 0.98L$. These horizontal temperature profiles measured by the sensors can be expressed as,

$$T(x) = \frac{A}{\pi} \tan^{-1} [B(x - x_c)] + C, \quad (2.9)$$

where A , B , and C are constants obtained by least-squares fitting. Additionally, x_c denotes the x position at which the maximum horizontal temperature gradient is imposed. The fitting curves are drawn as solid lines colored by the elapsed time in Fig. 2.7(a). Using the fitting curve in Eq. 2.9, the horizontal temperature gradient $\partial T / \partial x$ is derived as

$$\frac{\partial T(x)}{\partial x} = \frac{AB}{\pi \{ [B(x - x_c)]^2 + 1 \}}. \quad (2.10)$$

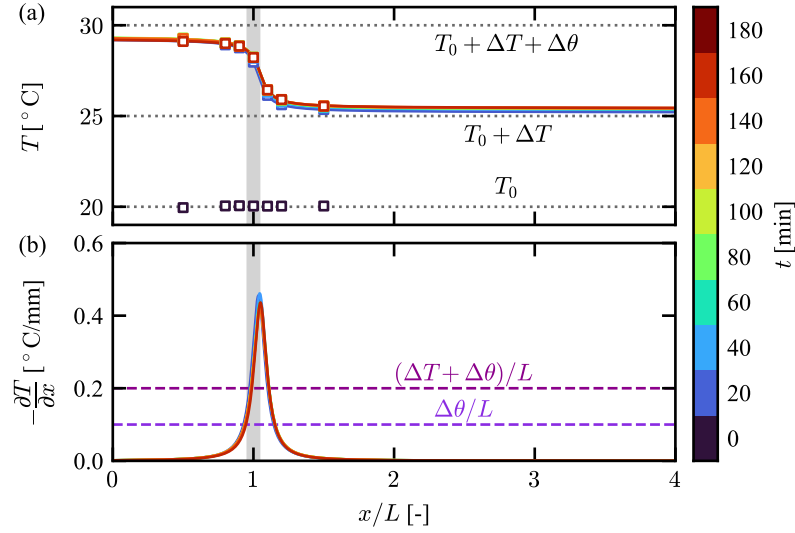


Figure 2.7: Example of temperature measurements made beneath the copper blocks at $z \sim 0.98L$ under the conditions $\Delta T = 5$ K and $\Delta\theta = 5$ K: (a) temperatures measured by the thermistor sensors (squares) and corresponding fitting curves and (b) horizontal temperature gradients derived from the fitting curves. Each plot and curve is colored according to the time elapsed from the initiation of heating. Horizontal dotted lines in (a) show the set temperatures of $T = T_0$, $T_0 + \Delta T$, and $T_0 + \Delta T + \Delta\theta$ from bottom to top. Horizontal dashed lines indicate typical temperature gradients obtained using the set constants. Regions shaded gray show horizontal positions of the insulating rubber sheet, at which the horizontal temperature difference $\Delta\theta$ is imposed.

In Fig. 2.7(b), the absolute values of horizontal temperature gradients at each moment are drawn as solid lines colored according to the elapsed time. For reference, typical temperature gradients obtained using characteristic temperature differences and lengths are drawn as horizontal dashed lines. One peak in $-\partial T/\partial x$ stands slightly downstream from $x = L$ for each moment after $t = 20$ min owing to heat transport from the left-end copper block toward the downstream, and the shape of the curves hardly changes with time. In the vorticity equation given as Eq. 2.2, the horizontal temperature gradient corresponds to the baroclinic vorticity production term. Accordingly, the peak in Fig. 2.7(b) is the source of the circulating motions localized in the upper region of the fluid layer, and the driving force in the system is thus considered always constant once the temperatures of the copper blocks are fixed. Meanwhile, the intensity of the stratification may relax with time owing to the thermal insulating condition at the side and bottom walls, and the restoring force of stable stratification thus weakens with time. The circulating motions indicated in Fig. 2.6 therefore show temporal developments, which will be detailed in Section 2.3.

The circulating motions illustrated qualitatively in Fig. 2.6 can be quantified through particle image analyses like PIV for the visualized images containing the TLC particles. In most cases investigated in the present study, the circulating motions are confined in the narrow upper region of the fluid layer as shown in Fig. 2.6, and there may be large fluid deformation at the rotational axis of roll structures. Thus, velocity field measurements made adopting standard PIV algorithms, direct cross-correlation and fast-Fourier-transform (FFT) correlation methods may not be appropriate owing to the requirement of interrogation windows. The image resolution

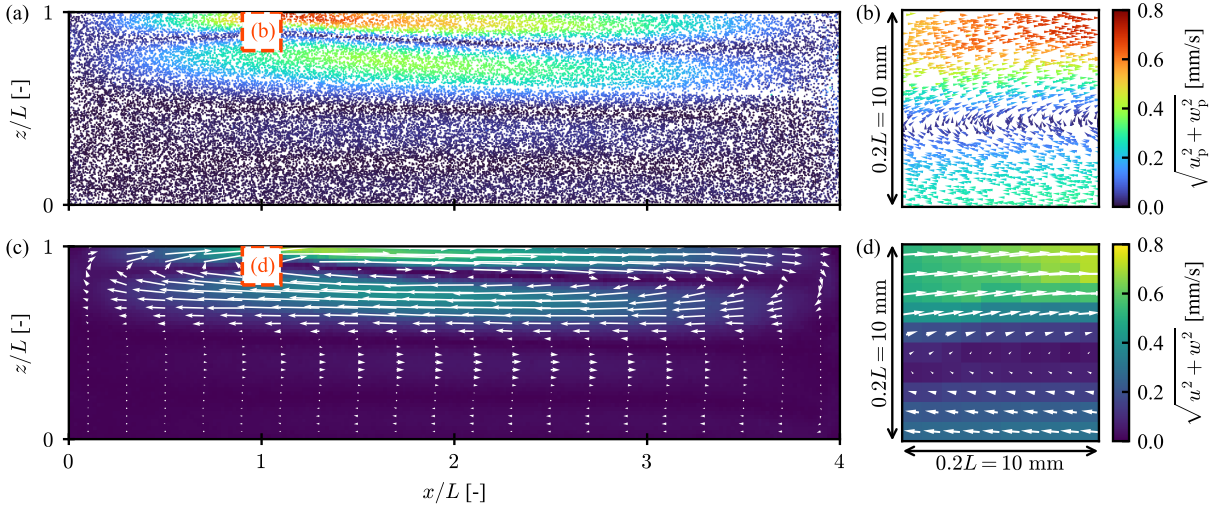


Figure 2.8: Example of velocity field measurements at $t = 90$ min under the conditions of $\Delta T = 2$ K and $\Delta\theta = 5$ K: (a) particles tracked through PTV analysis indicated by dots colored according to speed; (b) magnified view of the particle vectors in the region enclosed by the dashed square in (a); (c) overview of the interpolated velocity fields; and (d) magnified view. Only 1% and 4% of the tracked particles are displayed in (a) and (b), respectively. Interpolated velocity vectors are superposed on the color contours of speeds as white arrows in (c) and (d), and only one out of ten and one out of two vectors are plotted in x and z directions respectively for the sake of clarity in (c).

of the present optical setting is better than 0.1 mm/pixel (i.e., 12 pixels for 1 mm), and it is possible to identify each TLC particle as the particle image spreads across two or three pixels on the visualized image after background subtraction. Therefore, PTV analysis based on the nearest-neighbor method was adopted to measure velocity fields. An example of tracked particles is shown in Fig. 2.8(a) as dots colored according to speed. More than 40 million particles were tracked in total during 1 min in the case presented in Fig. 2.8(a), where only 1% are plotted as dots. Particle velocity vectors in the region enclosed by the dashed square in Fig. 2.8(a) are shown in Fig. 2.8(b) as arrows colored according to speed. Only 4% of the tracked vectors are shown for clarity in Fig. 2.8(b). The TLC particles are homogeneously distributed in space, and dense particle vectors are thus obtained. A high-speed region is localized immediately beneath the copper blocks imposed with a horizontal temperature difference $\Delta\theta$ [Fig. 2.8(a)], and a clockwise circulating motion is recognized [Fig. 2.8(b)].

To assist the series of analyses conducted afterward, velocity vectors $\mathbf{u} = [u, w]$ on regular grids, which have a resolution of 1 mm in x and z directions, are calculated using all particle vectors $\mathbf{u}_p = [u_p, w_p]$ measured during a period of 1 min, as the flow fields hardly change during this short time. The Shepard's method [173] was employed for the grid interpolation, and the squares of the inverse distances between the regular cells and the particle positions are used as the weight for the interpolation. That is, u and w are calculated as

$$u = \frac{\sum_i |\mathbf{r}_i|^{-2} u_{pi}}{\sum_i |\mathbf{r}_i|^{-2}}, \quad w = \frac{\sum_i |\mathbf{r}_i|^{-2} w_{pi}}{\sum_i |\mathbf{r}_i|^{-2}}, \quad (2.11)$$

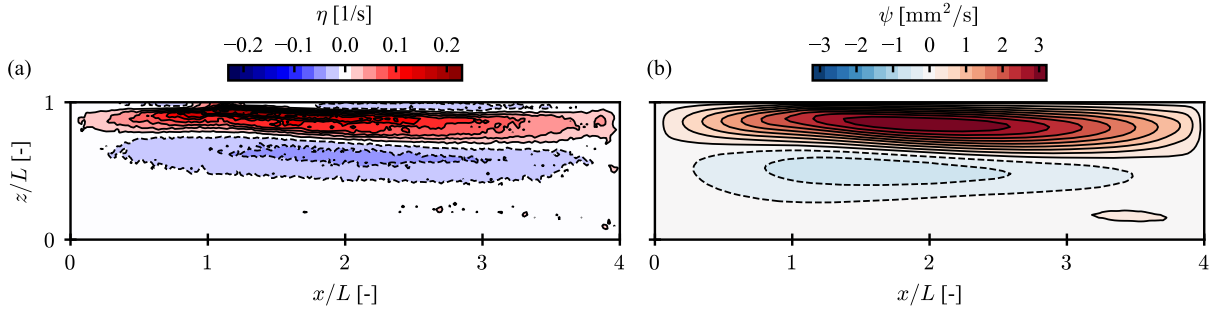


Figure 2.9: Example of vorticity and stream function fields at $t = 150$ min under the conditions $\Delta T = 2$ K and $\Delta\theta = 5$ K: (a) vorticity η field and (b) stream function ψ field. Positive and negative contours are drawn using solid and dashed lines.

where $\mathbf{r}_i = \mathbf{x} - \mathbf{x}_{pi}$ is the Euclidean distance between the cells $\mathbf{x}$ and the particle positions $\mathbf{x}_{pi}$. As all particle vectors measured during 1 min are used in the interpolation, the interpolated velocity $\mathbf{u}$ gives the temporal averaged velocity, while the flow field hardly changes. An example of the interpolated velocity field is shown in Fig. 2.8(c) and a magnified view of the region enclosed by the dashed square in Fig. 2.8(c) is shown in Fig. 2.8(d). Interpolated vectors represented by white arrows are superposed on the color contours of the speeds in Fig. 2.8(c) and (d). Only one out of ten and one out of two vectors are plotted in x and z directions, respectively, for the sake of clarity, in Fig. 2.8(c). The horizontal flows driven in the region where a horizontal temperature difference is imposed impinge on the sidewall at the right end and turn toward the $-z$ direction. The turned flow penetrates the quiescent fluid layer up to certain depths and turns again toward the $-x$ direction to generate circulating motion. The clockwise circulation occupying the upper half of the fluid layer is well reconstructed in the interpolated velocity fields.

To directly compare the governing equations given as Eq. 2.2, the out-of-plane vorticity η and the stream function ψ are calculated from the interpolated velocity vector fields $\mathbf{u}(x, y)$. Here, the out-of-plane vorticity η is expressed as

$$\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad (2.12)$$

and the stream function ψ is expressed as

$$\nabla^2 \psi = \eta. \quad (2.13)$$

The vorticity distributions are calculated using the second-order central-difference scheme for the interpolated velocity fields according to Eq. 2.12. The stream function distribution can be computed by solving the Poisson equation (Eq. 2.13) adopting the successive over-relaxation (SOR) method [50] with the boundary conditions of $\psi = 0$ at the solid walls. Examples of the vorticity and stream function fields are shown in Fig. 2.9. In Fig. 2.9(a), the vorticity field at $t = 150$ min under the conditions $\Delta T = 2$ K and $\Delta\theta = 5$ K is shown as color contours while the corresponding stream function field is shown as color contours in Fig. 2.9(b). Positive and

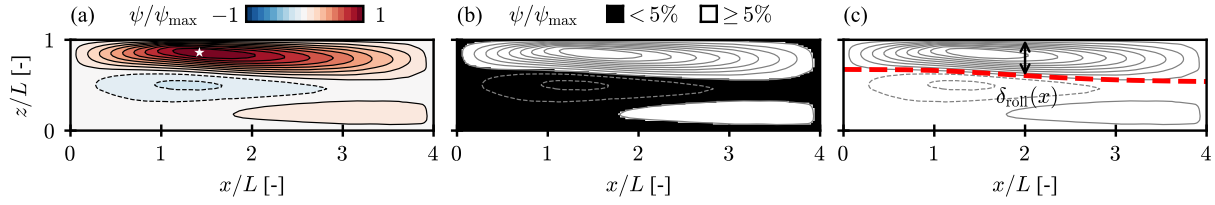


Figure 2.10: Extraction of the roll region according to the intensity of the stream function field at $t = 180$ min under the conditions $\Delta T = 2$ K and $\Delta \theta = 2$ K: (a) stream function field normalized by the maximum value $\psi_{\max}$; (b) binarized stream function field with a threshold at 5% intensity of the maximum value; and (c) bottom line of the roll structure fitted by the Gompertz function, showing thickness of the roll structure $\delta_{\text{roll}}(x)$. The position of $\psi = \psi_{\max}$ is indicated by a white star in (a). Contour lines of $\psi/\psi_{\max}$ are superposed on (b) and (c) as gray solid (positive) and dashed (negative) lines.

negative contours are respectively drawn using solid and dashed lines, and the contour lines in Fig. 2.9(b) coincide with the streamlines of the flow. The order of ψ , a few to 10 times larger than the thermal diffusivity κ , is consistent with the results of previous work [77]. The clockwise circulation beneath the top copper blocks is now visible both in vorticity and stream function fields as large positive regions in the upper half of the fluid layer. In addition, the second negative and third positive roll structures become visible beneath the first roll structure in Fig. 2.9(b). Compared with the relatively noisy vorticity distributions in Fig. 2.9(a), the stream function field is smooth as the iterations of the SOR method provide smooth stream function distributions.

The depths of the circulating flow, that is the thickness of the roll structures δ_{roll} , can be measured using the stream function fields, as the lower boundary of the roll is clearer in Fig. 2.9(b) than in Fig. 2.9(a). Here, roll regions are defined as coherent regions having more than 5% intensity of the maximum stream function value $\psi_{\max}$ throughout the domain. The bottom of the roll structures are slightly inclined to the lower right as shown in Fig. 2.9(b). This inclined roll boundary is considered to originate from the horizontally differing degree of stable stratification γ ; the left side has strong and the right side has weak stable stratification as the imposed heating is higher on the left side. This trend is identical to all the explored parameters. The roll thickness at any horizontal position $\delta_{\text{roll}}(x)$ is then fitted by an asymmetric function, the Gompertz function, in the form

$$\delta_{\text{roll}}(x) = A \exp[-B \exp(-Cx)] + D \quad (2.14)$$

where A , B , C , and D are constants obtained by least-squares fitting, with B and C being positive. From this fitting, the roll region is identified as the closed region of $0 \leq x \leq 4L$ and $L - \delta_{\text{roll}}(x) \leq z \leq L$. A series of analyses for computing the roll thickness δ_{roll} is presented in Fig. 2.10. The original stream function field is shown in Fig. 2.10(a), and the position at which $\psi = \psi_{\max}$ is indicated by a white star. The position is located downstream from $x = L$, and its horizontal ordinate roughly coincides with that at which $u = u_{\max}$ as shown in Fig. 2.8. The stream function field is then binarized with a threshold of 5% intensity of $\psi_{\max}$. Regions exceeding the threshold are shown as white in Fig. 2.10(b). From the binarized stream function field shown in Fig. 2.10(b), the level corresponding to the bottom of the extracted roll region

immediately beneath the top surface is fitted using Eq. 2.14, and the fitted curve is drawn as a red dashed line in Fig. 2.10(c).

2.3 Results

2.3.1 Temporal evolution of horizontal convection

The present study imposes a horizontal temperature difference $\Delta\theta$ on a quiescent fluid, and the induced flow structures changed with time. The developing structures are illustrated as 3D isosurfaces of stream function in the spatio-temporal domain under the conditions of $\Delta T = 1, 2, 5$, and 10 K, respectively in Fig. 2.11, 2.12, 2.13, and 2.14. All the isosurfaces are represented using the same color contours in a range from $\psi = -3$ to 3 mm²/s with a step of 0.3 mm²/s. The initial transient structures disappear in a short time of $O(1 \text{ min})$ after the initiation of heating. Entirely, larger $\Delta\theta$ yields larger values of ψ for each ΔT condition, meaning that stronger baroclinic torque drives faster circulating motions of roll structures. Comparing results among Fig. 2.11, 2.12, 2.13, and 2.14, the roll thickness becomes thinner for larger ΔT .

During the transient process toward thermally-equilibrated state, the convection structure formed beneath the heating blocks develops with time. To investigate details of the flow structures of developing HC, the temporal evolution of the flow fields under fixed conditions $\Delta T = 2$ K and $\Delta\theta = 2$ K is shown in Fig. 2.15. Figure 2.15(a–h) shows the overall view of the stream function fields (color contours) and the lower-ends of the roll structures corresponding to the roll thicknesses (red dashed lines), at the moments noted above each subfigure. Each color contour is scaled by the common color bar, which is roughly adjusted to the maximum value at $t = 180$ min [Fig. 2.15(h)] and shown above the stream function fields. At $t = 10$ min as shown in Fig. 2.15(a), the stream function field is weak; however, the roll structure in the upper region already extends horizontally to $\sim 4L$, the width of the fluid container, with very small thickness $\sim 0.3L$. With time, the stream function within the upper roll region increases, representing the strengthening of the roll structure. Additionally, the roll region increases with the strengthening of the roll structure, and the roll thickness increases as the roll structure is able to grow only in the $-z$ -direction owing to the horizontal confinement.

Adopting the data acquisition methods described in Section 2.2.3, various flow characteristics of horizontal convection can be quantified. In Fig. 2.15(i), changes in the maximum stream function $\psi_{\max}$ with time are plotted as circles, and changes in the roll thickness δ_{roll} are shown. δ_{roll} changes with the x -position, being shallower on the left and deeper on the right, as shown in Fig. 2.9 and 2.10. Here, the mean roll thickness $\overline{\delta_{\text{roll}}}$ along the x -direction is plotted as squares, and the roll thicknesses at the left-end ($x = 0$) and right-end $x = 4L$ are represented as left- and right-pointing triangles. Both $\psi_{\max}$ and $\overline{\delta_{\text{roll}}}$ increase with time, and these increasing trends can be approximated adopting least-squares fitting. In the case of the displayed condition,

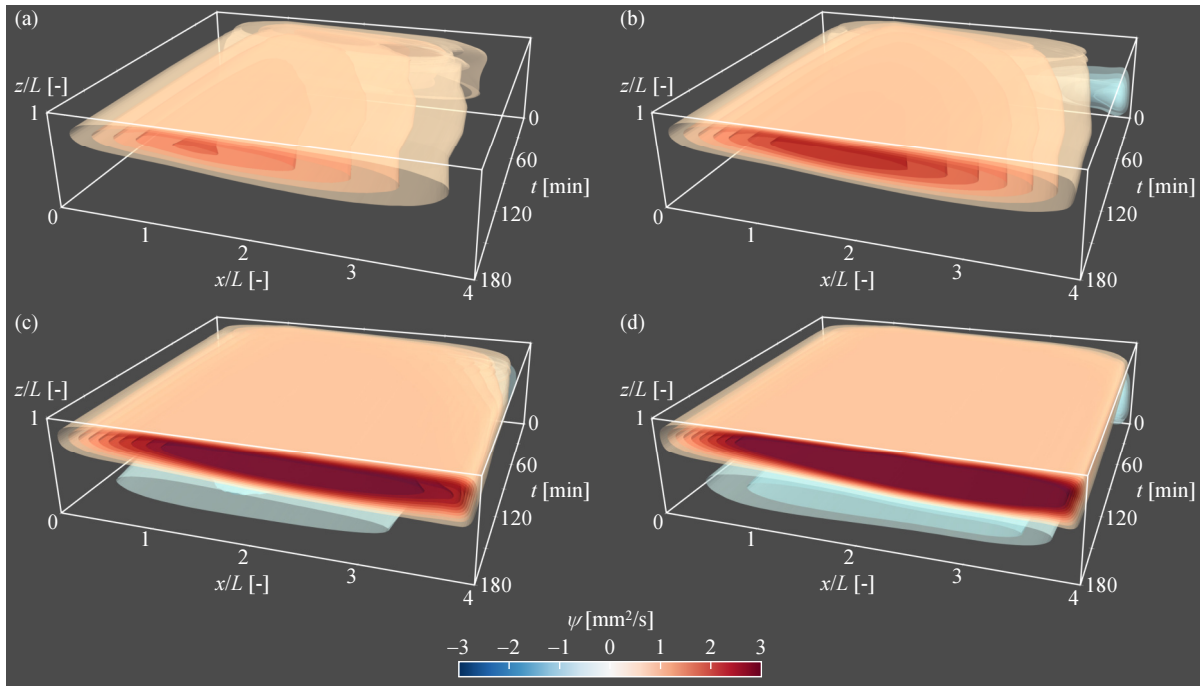


Figure 2.11: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 1$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

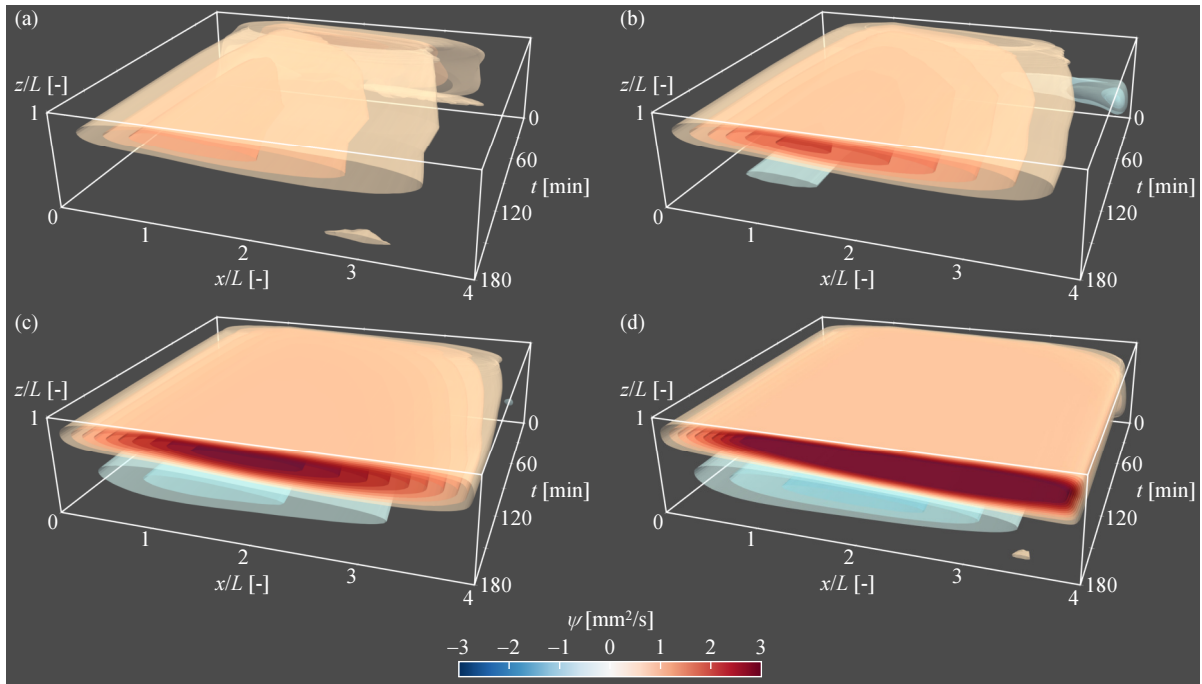


Figure 2.12: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 2$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

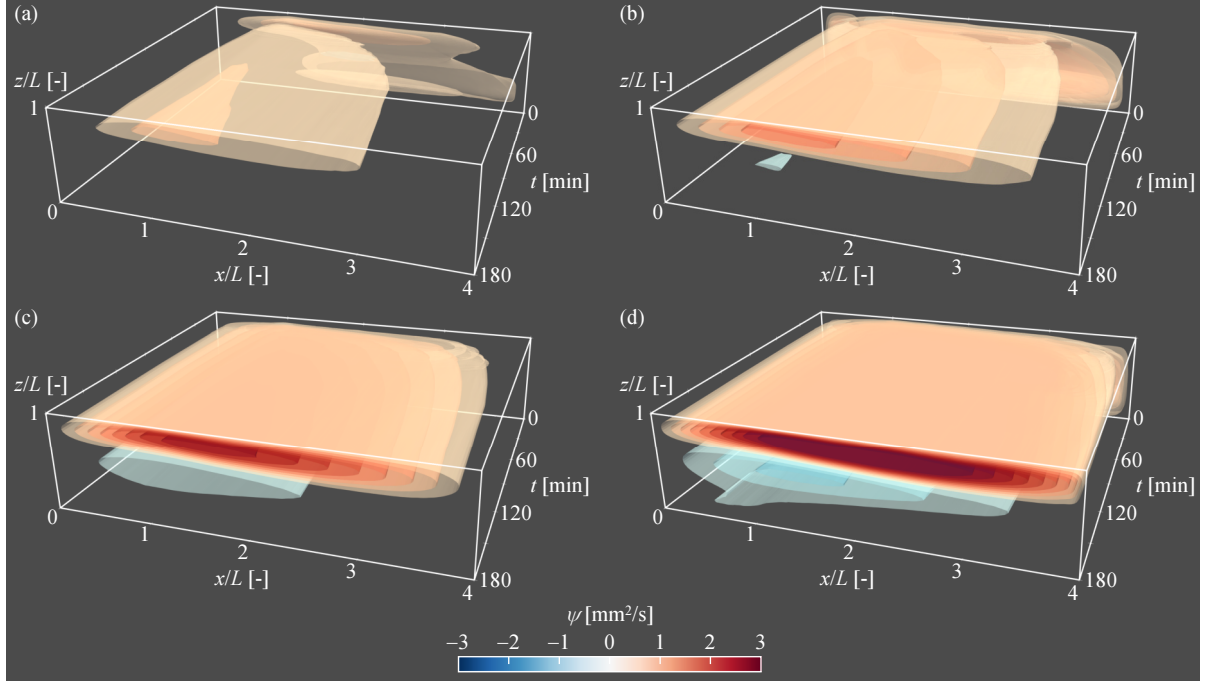


Figure 2.13: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 5$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

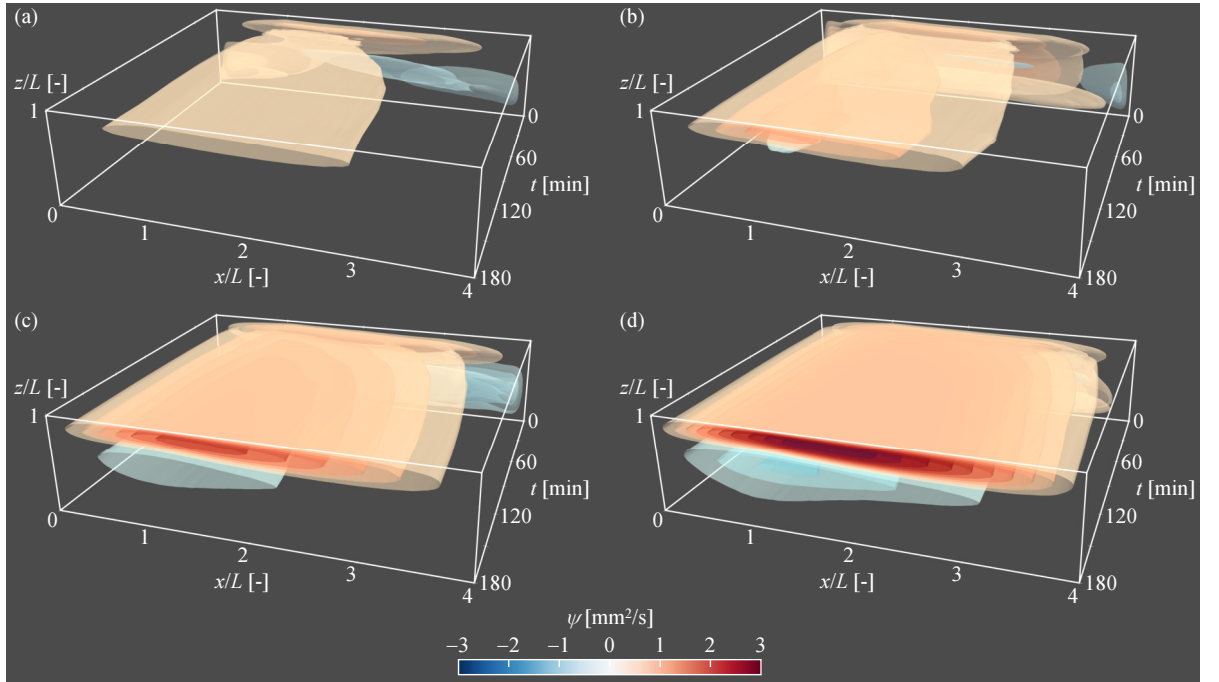


Figure 2.14: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 10$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

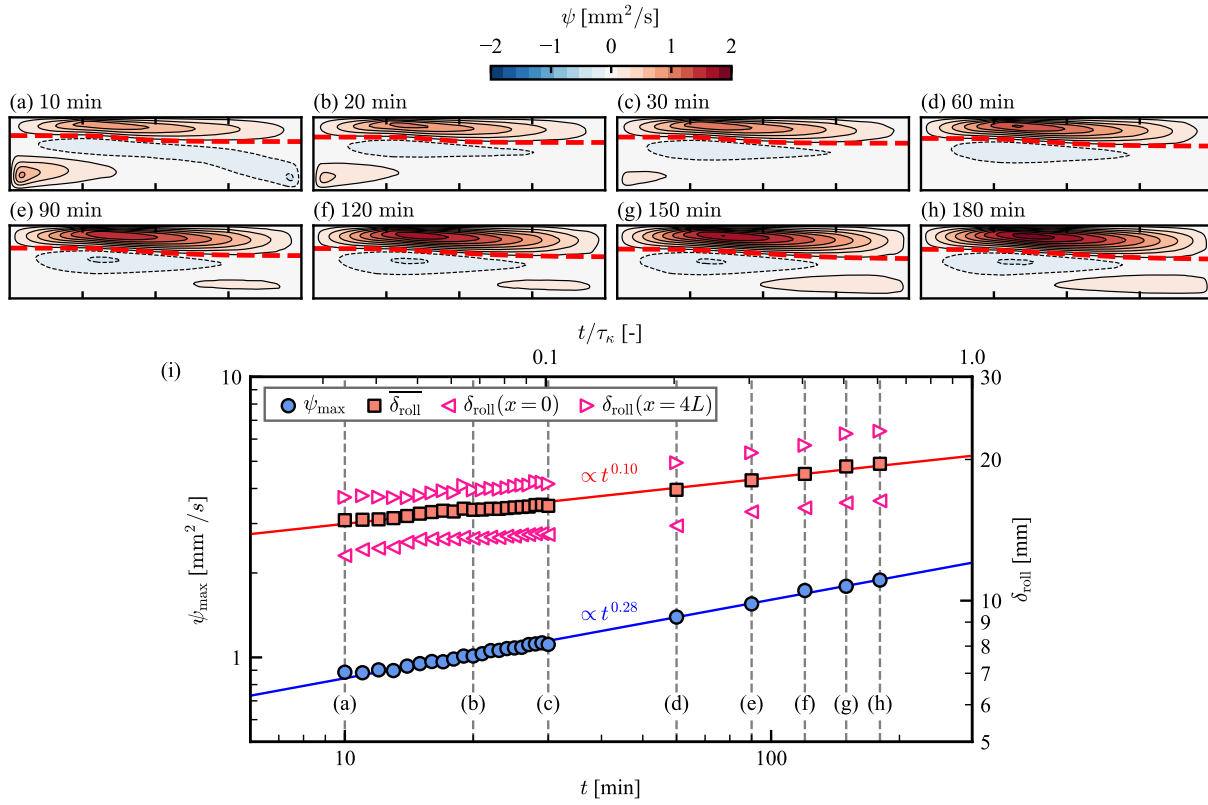


Figure 2.15: Temporal evolution of roll structures at $t \geq 10$ min under the conditions $\Delta T = 2$ K and $\Delta\theta = 2$ K observed in (a-h) stream function fields and (i) characteristic values, ψ_{max} and δ_{roll} . Color contours shown in (a) to (h) represent the overall view of stream function fields and red dashed lines indicate the roll thickness δ_{roll} . Circles and squares in (i) show changes in ψ_{max} and δ_{roll} with time. Left- and right-pointing triangles are changes in δ_{roll} at $x = 0$ and $x = 4L$, respectively. Blue and red solid lines are least-squares fits for ψ_{max} and δ_{roll} . Vertical dashed lines in (i) correspond to the flow fields shown in (a) to (h).

fitting curves of $\psi_{\text{max}} = 0.44t^{0.28}$ and $\delta_{\text{roll}} = 11.6t^{0.10}$ were obtained in dimensional units. For reference, the maximum horizontal velocity u_{max} increases with time with $u_{\text{max}} \propto t^{0.14}$ for this condition, while this is not displayed in Fig. 2.15.

Under the condition of larger Ra , $\Delta T = 2$ K and $\Delta\theta = 10$ K, the temporal evolution of the values are obtained as $u_{\text{max}} \propto t^{0.03}$, $\psi_{\text{max}} \propto t^{0.10}$, and $\delta_{\text{roll}} \propto t^{0.10}$. The temporal dependencies of u_{max} and ψ_{max} are weaker than those in the case that $\Delta T = 2$ K and $\Delta\theta = 2$ K, shown in Fig. 2.15. Considering that the roll thickness increases with the same temporal dependence, the roll thickness is not affected by the strength of the roll structures, and other factors may therefore be dominant in determining the roll thickness.

Mori and Niino [114] suggested self-similar solutions for determining the horizontal and vertical length scales of developing nonlinear HC in a laterally and vertically infinite system. The vertical length scale δ is given as the thermal diffusion length scale $\delta_{\kappa} \propto t^{1/2}$. The horizontal length scale ℓ is given as the integration of the speed of gravity current, $U_{\text{gc}} \propto \sqrt{g\beta\Delta\theta\delta_{\kappa}} \propto t^{1/4}$, and thus $\ell \propto t^{5/4}$. Making the same assumptions adopted in Mori and Niino [114], the stream function also changes with time as $\psi \propto t^{3/4}$. Such self-similar solutions, however, may easily collapse in the case of a finite domain, and the length scale is immediately determined by the

geometry as $\ell \sim 4L$ in the present study. Thus, the temporal evolution of the two quantities, $\delta_{\text{roll}} \propto t^{0.10}$, may be slow compared to those obtained theoretically in the infinite system by Mori and Niino [114]. Such slow evolution may originate in the finite system, which potentially increases the effect of the restoring force of the stable stratification by forcing the flow directions. The temporal evolution of the roll structures thus may not have a universal trend even in the same finite domain of the fluid container.

2.3.2 Effect of stable temperature stratification

For all the explored cases, the roll structures are confined in the upper regions as shown in Fig. 2.15(a-h). Mullarney et al. [115] also showed a similar flow structure confined in a shallow region during their transient process to a thermally-equilibrated state. This type of flow structure is superficially similar to the so-called boundary-layer regime in thermally-equilibrated HC [29]; a shallow circulating region is formed owing to strong advection, and the circulation allows temperature variations only in the circulating region. The boundary-layer regime also allows presence of thermal plumes at the end of destabilizing surface, and thus isotherms are not fully closed within the circulating region but connected throughout the domain height. In this sense, the flow structures realized in this study are different from those in the boundary-layer regime, because the roll structures are always regulated by the restoring force of stable temperature stratification over the whole horizontal position, meaning isotherms closed within the roll region.

Since γ'_{lin} is considered as the primary factor, it is worth summarizing γ'_{lin} dependence of various flow characteristics. The maximum horizontal velocity u_{max} , the maximum stream function ψ_{max} , and the mean roll thickness $\overline{\delta_{\text{roll}}}$ are shown for all the explored parameters in Fig. 2.16. These characteristic quantities are plotted in γ'_{lin} domain as shown in Fig. 2.16(a), (c), and (e), and also plotted in the equivalent Ra domain as shown in Fig. 2.16(b), (d), and (f). These are shown for reference to compare the previous studies conducted in thermally-equilibrated HC [29, 77, 162]. Ra here simply represents the horizontal temperature difference $\Delta\theta$. Here, all data recorded after $t = 30$ min are plotted so as not to include the first transient states. As found in Fig. 2.15, the flow structures develop with time during the measurements, and each condition thus has temporal deviations. The temporal evolution is indicated by blue arrows in Fig. 2.16(a), (c), and (e), while such evolution may not be apparent in Fig. 2.16(b), (d), and (f) as the plots are overlapping.

In Fig. 2.16(a) and (b), u_{max} for various explored parameters is plotted in comparison with γ'_{lin} or Ra . Each plot is colored by the values of Ra and γ'_{lin} respectively. The dependence of u_{max} on γ'_{lin} is shown in Fig. 2.16(a). Values of u_{max} in strongly stratified cases are lower than those in weakly stratified cases. The present system is horizontally finite, and the gravity current is forced to turn to the lower fluid layer against the stratification as shown in Fig. 2.8(c). This geometric confinement may work as a braking force for the circulation, and u_{max} is considered to be smaller in the strongly stratified cases. In the case of fixed Ra ($\Delta\theta$) conditions, u_{max} has

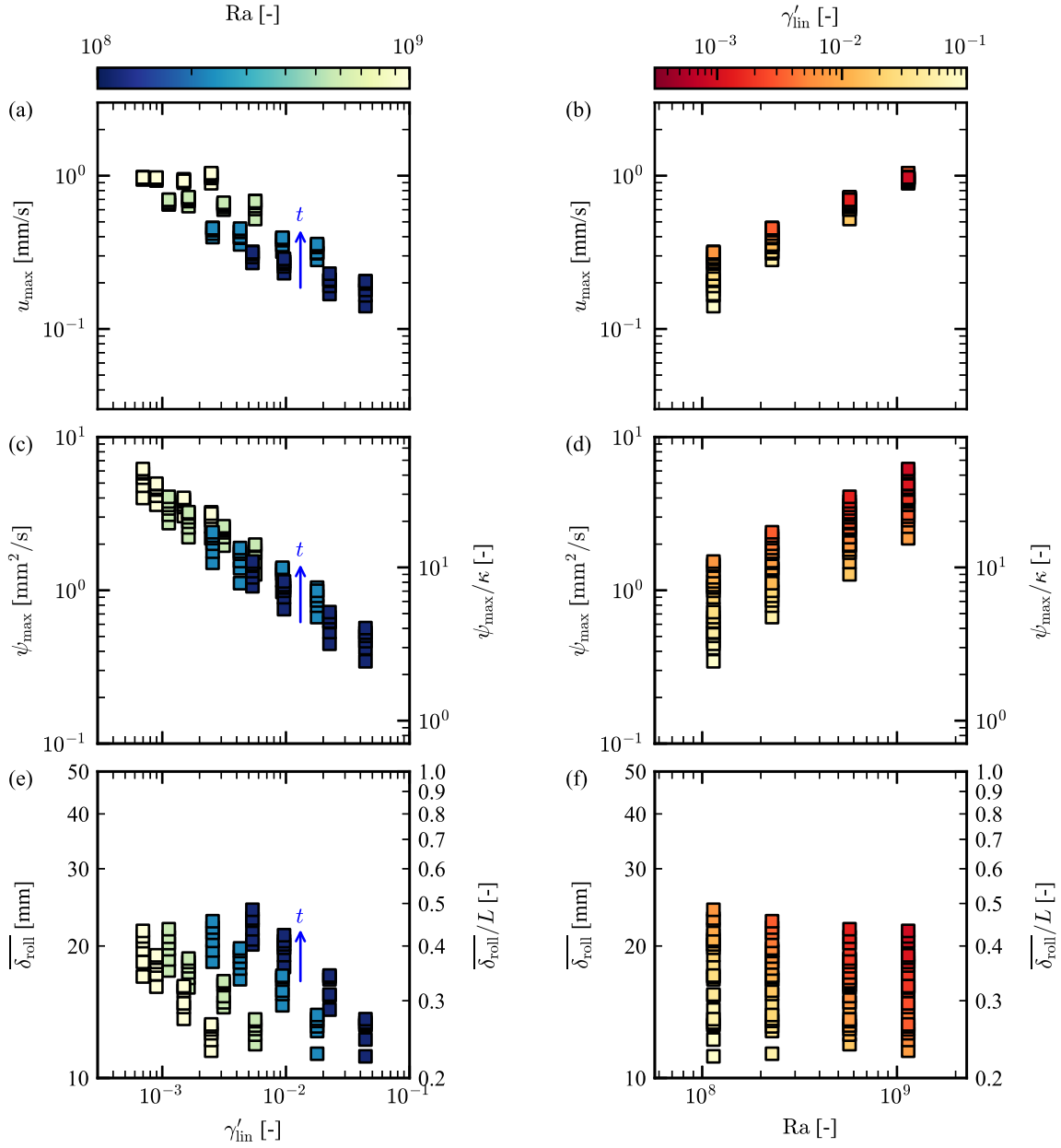


Figure 2.16: Plots of characteristic values of horizontal convection for all the explored parameters measured at $t \geq 30$ min in Ra or γ'_{lin} domains: (a) and (b) the maximum horizontal velocity u_{max} ; (c) and (d) maximum stream function values; and (e) and (f) roll thickness $\overline{\delta_{roll}}$ averaged over the x -direction. Colors of plots in (a), (c), and (e) represent the Ra values while those in (b), (d), and (f) are the stratification parameter γ'_{lin} . Blue arrows in (a), (c), and (e) guide the temporal evolution.

almost the same values especially under the conditions of larger Ra , regardless of the γ'_{lin} values. Sufficiently strong advection within the roll region may weaken the influence of such braking effects. Decreasing in Ra , in turn, means to approach to the gravity wave regime, and thus the γ'_{lin} dependence gradually becomes prominent with Ra decreasing. From these plots, the maximum horizontal velocity is found to be determined roughly by Ra ($\Delta\theta$) values, if the braking force of the stable stratification is weak compared with the advection induced by baroclinic torque.

In Fig. 2.16(c) and (d), the maximum stream function ψ_{max} is plotted as functions of γ'_{lin} and Ra . The right axes of the two plots are the maximum stream function ψ_{max} normalized by the thermal diffusion coefficient κ . The ψ_{max} values plotted in the γ'_{lin} domain [Fig. 2.16(c)] show clear dependence on the γ'_{lin} for fixed Ra conditions. As in the example illustrated in Fig. 2.15(i), the values of ψ_{max} increase with time for each initial condition as indicated by the blue arrow. In contrast to relevant dependence on γ'_{lin} shown in Fig. 2.16(c), the plots are widely distributed in Fig. 2.16(d).

The mean roll thickness $\overline{\delta_{\text{roll}}}$ for the explored parameters is plotted in Fig. 2.16(e) and (f). In the case of the present parameters, the roll structures penetrated the fluid layer up to $\sim 50\%$ of the total height L . For fixed Ra conditions, $\overline{\delta_{\text{roll}}}$ strongly depends on γ'_{lin} as shown in Fig. 2.16(e), while showing the growth with time under each initial condition is indicated by the blue arrow. Similar to the plots of ψ_{max} in Fig. 2.16(d), $\overline{\delta_{\text{roll}}}$ does not have a clear dependence on Ra as shown in Fig. 2.16(f).

Incorporating all the results shown in Fig. 2.16, the stratification parameter γ'_{lin} seems to dominate the characteristic quantities of the developing HC. While γ'_{lin} has importance, the plots for each condition have temporal changes as shown by the blue arrows in Fig. 2.16(a), (c), and (e). The above discussion is based on constant stratification parameters γ'_{lin} defined in Eq. 2.7 using a fixed vertical temperature gradient γ_{lin} . Considering γ'_{lin} as constants may not be appropriate for describing all the trends shown in Fig. 2.16, as γ and γ' should originally be functions of both the horizontal coordinate x and time t . In the present system, the fluid layer is always heated from the top and other walls are quasi-adiabatic. Similarly, the stable temperature stratification relaxes, and γ and γ' may change from ∞ to 0 with time, without equilibrium states being realized during the measurements. That is, the γ' characterizing the flow structure requires the consideration of the temporal evolution. This will be discussed in the following section.

2.3.3 Effect of relaxing stable temperature stratification

As shown in Fig. 2.15, the roll structure and its characteristics change with time in developing HC. It is thus natural to replace the vertical length scale L for the stratification parameter γ'_{lin} (Eq. 2.7) by another length scale considering temporal evolution of the developing HC. Mori and Niino [114] explained this temporal evolution can be described together with the thermal diffusion from the top boundary, because the convective heat transport by the circulating motion is small. In the developing HC, the vertical temperature gradient γ relaxes with time owing to

both convective heat transport and thermal diffusion from the heating copper blocks at the top. From the results shown in Fig. 2.8 and Fig. 2.16(a) and (b), the characteristic velocity scale is $O(0.1 \text{ mm/s})$. The horizontal dimension of the domain is $O(100 \text{ mm})$, and the overturning timescale of the convective motions is thus estimated as $O(10^3 \text{ s})$. This is comparable to or longer than the thermal diffusion timescale within the roll region $\tau_{\kappa, \text{roll}} = \delta_{\text{roll}}^2 / \kappa = O(10^2 - 10^3 \text{ s})$. These discussions can be justified in terms of the Peclet number, $Pe = \psi / \kappa$. As shown in Fig. 2.16(c) and (d), the Pe defined using the maximum stream function ψ_{max} ranges from ~ 2 to ~ 50 . Thus the Pe defined using the mean ψ values within the roll region is $O(< 10^0)$, and this implies the less contribution of the convective transport on the heat transport compared to the diffusion. Accordingly, the thermal diffusion timescale may be dominant in the present setting, and the heating of the fluid layer is mainly carried out by thermal diffusion rather than convective heat transport.

Considering the discussion above, it is reasonable to include the relaxation process of the stable temperature stratification to the stratification parameter γ' , in terms of the thermal diffusion. As the convective heat transport is trivial in this setting, the thermal diffusion process is supposed to be one-dimensional heating from the top; the fluid with initial temperature T_0 is heated to $T_{\text{wall}} > T_0$ from the top copper plate at $z = L$. The temperature profile along the z -axis can be regarded as an unsteady thermal conduction solution in the form

$$T(z, t) = T_0 + (T_{\text{wall}} - T_0) \operatorname{erfc} \left(\frac{L - z}{2\sqrt{\kappa t}} \right), \quad (2.15)$$

where erfc is the complementary error function. The vertical temperature gradient at the top is then derived as

$$\left. \frac{\partial T}{\partial z} \right|_{z=L} = \frac{T_{\text{wall}} - T_0}{\sqrt{\pi \kappa t}} \propto \frac{T_{\text{wall}} - T_0}{\delta_{\kappa}}. \quad (2.16)$$

By replacing the dimensional stratification parameter γ with the vertical temperature gradient at the top (Eq. 2.16) as $\gamma = (T_{\text{wall}} - T_0) / \delta_{\kappa}$, the nondimensional stratification parameter γ' with consideration of the temporal evolution can be defined. Here, the temperatures at the top copper blocks differ as $T_{\text{wall}} = T_0 + \Delta T + \Delta\theta$ ($0 \leq x \lesssim L$) and $T_{\text{wall}} = T_0 + \Delta T$ ($L \lesssim x \leq 4L$), and the mean wall temperature Eq. 2.6 is used. The mean vertical temperature gradient γ is simply calculated similarly to the calculation in Eq. 2.7 as

$$\gamma = \frac{4\Delta T + \Delta\theta}{4\sqrt{\kappa t}}. \quad (2.17)$$

Accordingly, the nondimensional stratification parameter γ' considering temporal evolution is written as

$$\gamma' = \frac{g\beta(4\Delta T + \Delta\theta)}{4\sqrt{\kappa t}} \left(\frac{\sqrt{\kappa}}{g\beta\Delta\theta} \right)^{4/3}, \quad (2.18)$$

where the length scale L in Eq. 2.8 is simply replaced by δ_{κ} . Here it is considered that the

degree of stable temperature stratification within the thermal boundary layer will be relaxed with $\gamma' \propto t^{-1/2}$.

The flow characteristics shown in Fig. 2.16 are considered again while considering the relaxing stable stratification. The various flow characteristics compared in γ'_{lin} and Ra domains (Fig. 2.16) are re-plotted in γ' and Ra domains as shown in Fig. 2.17. In Fig. 2.17(a), (c), and (e), the horizontal axes give γ' , the plots in Fig. 2.17(b), (d), and (f) are colored by γ' .

As for the maximum horizontal velocity u_{max} , plotted in Fig. 2.17(a), the temporal deviations, which exist in the u_{max} axis of Fig. 2.16(a), are no longer visible, and all the data for a fixed Ra value align along a single line. The γ' dependence can be approximated as $u_{\text{max}} \propto \gamma'^{-2/7}$ in the cases of lowest Ra , while the cases of largest Ra have almost constant u_{max} , and these are indicated by the black dashed lines drawn in Fig. 2.17(a). In contrast, it may be difficult to find appreciable differences between the plots in Fig. 2.17(b) and Fig. 2.16(b). On the basis of the flow regimes characterized according to γ' [114], the case investigated in the present study is in the gravity current regime, $\gamma'_{\text{lin}} \ll O(1)$. In this regime, the characteristic horizontal velocity can be estimated as the propagation speed of the gravity current, $U_{\text{gc}} = \sqrt{g\beta\Delta\theta\delta_{\kappa}} \propto \Delta\theta^{1/2} \propto Ra^{1/2}$, where the thermal diffusion length scale $\delta_{\kappa} = \sqrt{\kappa t}$ is considered [114, 119]. This Ra dependence is shown as the red solid line in Fig. 2.17(b), and roughly matches the measured results.

The values of the maximum stream function ψ_{max} are shown in Fig. 2.17(c) and (e). The ψ_{max} plots in the γ' domain [Fig. 2.17(c)] are well organized regardless of the time elapsed from heating at the top, and the plots are aligned along a single line for each Ra condition. This line can be approximated as $\psi_{\text{max}} \propto \gamma'^{-1/2}$ as shown by the black dashed line in Fig. 2.17(c). In contrast, similar to the case for u_{max} , differences in Fig. 2.17(d) compared with Fig. 2.16(d) are difficult to recognize. In the boundary-layer regime of thermally-equilibrated HC, it is possible to find a scaling for the maximum stream function as

$$\psi_{\text{max}} \sim Ra^{1/5}, \quad (2.19)$$

and this scaling has been justified in many previous works [29, 77, 162]. In the developing HC, the 1/5-power law (Eq. 2.19) should not be useful, however, it is indicated for reference as the red solid line in Fig. 2.17(d). As expected, it is not possible to find certain scaling of Eq. 2.19.

The mean roll thicknesses $\overline{\delta_{\text{roll}}}$ are plotted in Fig. 2.17(e) and (f). The γ' dependence is clear for $\overline{\delta_{\text{roll}}}$ as shown in Fig. 2.17(e). Largely deviating plots due to the temporal evolution in Fig. 2.16(e) now lie on a line for each Ra condition. This γ' dependence of $\overline{\delta_{\text{roll}}}$ is approximated as $\overline{\delta_{\text{roll}}} \propto \gamma'^{-1/4}$ as indicated by the black dashed line in Fig. 2.17(e). In the boundary-layer regime, the scaling for the roll thickness is derived as

$$\delta_{\text{roll}} \sim Ra^{-1/5}, \quad (2.20)$$

together with Eq. 2.19 [29, 77, 162]. This scaling Eq. 2.20 is also not necessarily held in the

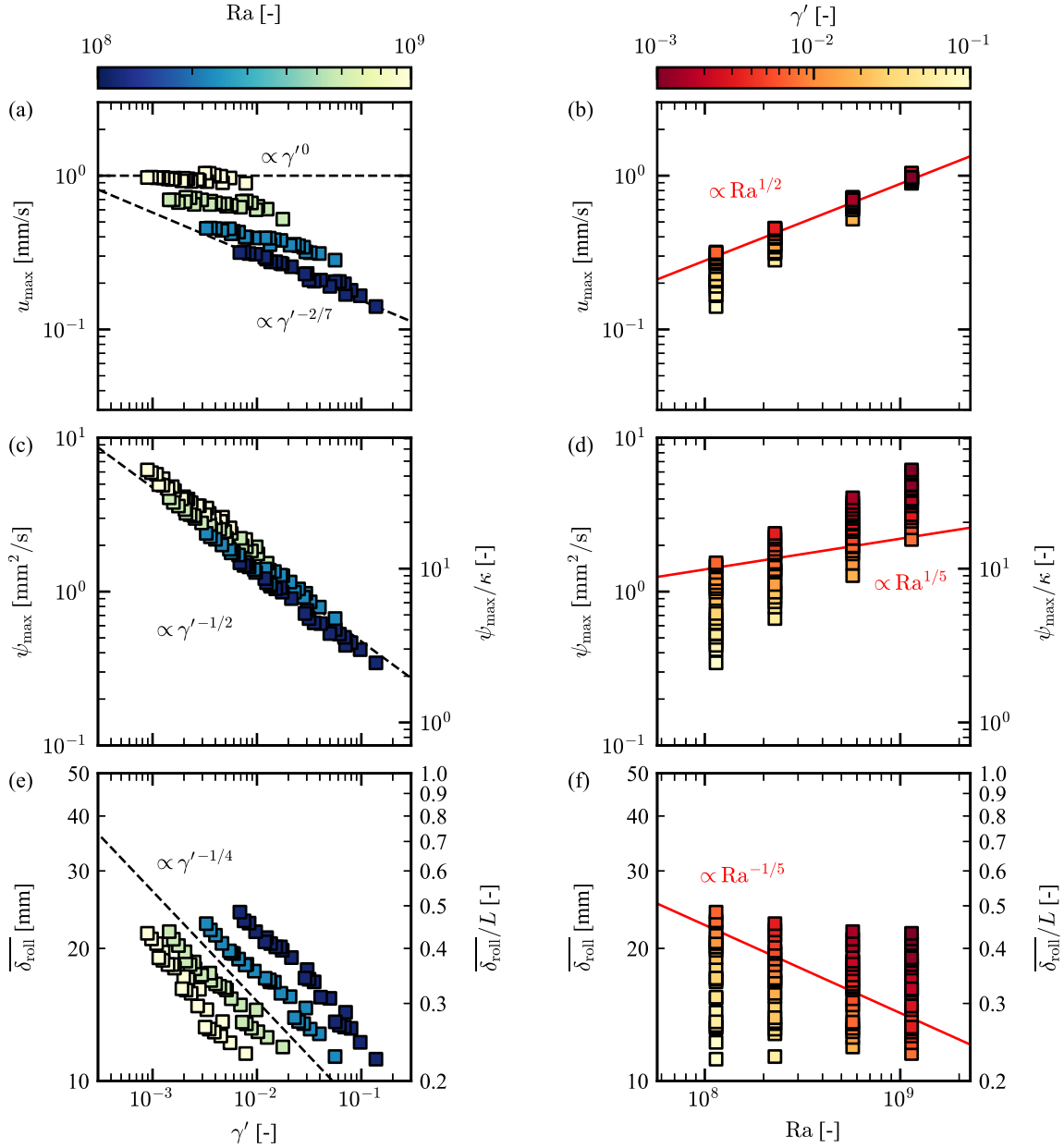


Figure 2.17: Plots of characteristic values of horizontal convection for all the explored parameters after $t \geq 30$ min in Ra or γ' domains: (a) and (b) the maximum horizontal velocity $u_{\max}$; (c) and (d) maximum stream function values; (e) and (f) roll thickness $\overline{\delta_{\text{roll}}}$ averaged over the x -direction. Colors of plots in (a), (c), and (e) represent the Ra values while those in (b), (d), and (f) are the stratification parameter γ' . Black dashed lines in (a), (c), and (e) are power laws that seemingly fit the present data, and red solid lines in (b), (d), and (f) are power laws predicted in previous studies.

developing HC, and the corresponding line is indicated for a reference by the red solid line in Fig. 2.17(f). For $\overline{\delta_{\text{roll}}}$, it is also not possible to find the Ra scaling previously claimed in thermally-equilibrated HC (Eq. 2.20).

Incorporating all the discussions above, the developing stratification parameter γ' defined in Eq. 2.18 describes well the flow characteristics of the developing HC for each Ra condition. Additionally, the braking force due to the stable stratification represented by γ' is considered to depend on the geometrical conditions and not only the temperature imposed at the top. If a horizontally smaller fluid container is employed while keeping the temperature difference at the top, the circulation distance will be shortened while the same baroclinic vorticity is produced. This condition will produce a braking force stronger than that in the case of the larger aspect ratio. It is therefore necessary to evaluate the roll structures using bulk characteristic values that consider the geometric environment.

2.3.4 Bulk characteristics of developing horizontal convection

To take the geometrical dependence into account, bulk quantities that consider the whole fluid layer may be good indicators with which to characterize the roll structures. Note that the present system can be regarded as two-dimensional flows, and the bulk quantities that will be discussed in the following are measured in the visualized cross-section $0 \leq x \leq 4L$ and $0 \leq z \leq L$ at $y = 0.5L$. The maximum stream function ψ_{max} represents the flow rate of the roll structure, which does not consider the geometric constraints, and this is not appropriate to characterize the strength of the roll structures. Instead of ψ_{max} , the circulation of the roll region Γ_c is computed as

$$\Gamma_c = \int_S \omega \cdot d\mathbf{S} = \int_{S_{\text{roll}}} \eta dS, \quad (2.21)$$

where S_{roll} is the area of the closed roll region that has been determined using Eq. 2.14 and is calculated as

$$S_{\text{roll}} = \int_0^{4L} \delta_{\text{roll}}(x) dx = 4L \overline{\delta_{\text{roll}}}. \quad (2.22)$$

In considering the effect of the geometric environment, Γ_c may be more important than ψ_{max} , since Γ_c can consider the strength of the rotation due to the roll structures originating in the baroclinic vorticity imposed at the top. Therefore, Γ_c is a better indicator with which to observe the balance of the baroclinic vorticity and the braking force due to the stable stratification during the developing HC. In Fig. 2.18(a) and (b), the circulation Γ_c is plotted as functions of γ' and Ra , respectively. A clear γ' dependence of Γ_c appears in Fig. 2.18(a) and is approximated as $\Gamma_c \propto \gamma'^{-1/4}$; this is weaker than the γ' dependence of ψ_{max} ($\psi_{\text{max}} \propto \gamma'^{-1/2}$). Under a fixed Ra condition, Γ_c is only determined by γ' , similar to the case for ψ_{max} values shown in Fig. 2.17(c). Meanwhile, the Γ_c increases with Ra as shown in Fig. 2.18(b).

The flow fields shown in Fig. 2.9 and 2.15 suggest the existence of weak second and third

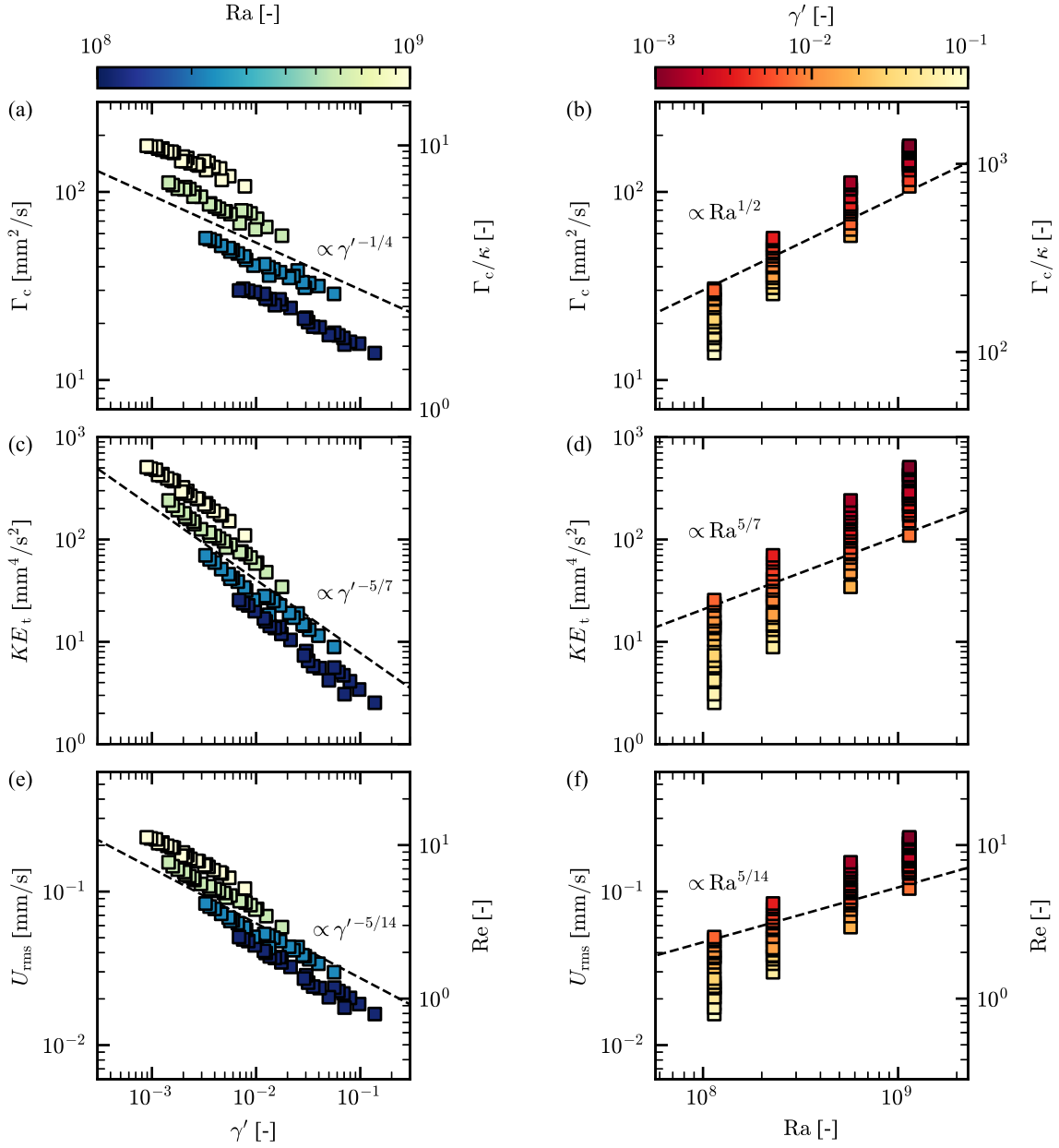


Figure 2.18: Plots of bulk representative values of horizontal convection after $t \geq 30$ min as functions of Ra and γ' : (a) and (b) the circulation of the roll region Γ_c ; (c) and (d) total kinetic energy KE_t ; and (e) and (f) root-mean-squared velocity U_{rms} (left axis) and Reynolds number Re (right axis); Colors of plots in (a), (c), and (e) represent the Ra values while those in (b), (d), and (f) are the stratification parameter γ' . Black dashed lines in (a), (c), and (e) are power laws of γ' while those in (b), (d), and (f) are power laws of Ra , seemingly fitting the present data.

roll structures beneath the first roll structure in the upper half of the fluid container. The layered second and third rolls are anticipated to be driven by the viscous force of the first roll structure and not directly by the baroclinic torque, and counter-clockwise and clockwise circulations are thus generated in order beneath the first clockwise circulation. Similar flow structures were observed in the transient process from one thermally-equilibrated HC to another when the destabilizing source was weakened [62, 163]. Accordingly, the circulations of the second and third rolls are expected to be strong when the circulation of the first roll is strong. To simultaneously consider all the roll structures, the bulk total kinetic energy KE_t is computed as

$$KE_t = \int \frac{1}{2} (u^2 + w^2) dA = \int_0^L \int_0^{4L} \frac{1}{2} (u^2 + w^2) dx dz, \quad (2.23)$$

where KE_t has a unit of mm^4/s^2 . In Fig. 2.18(c) and (d), KE_t values are plotted as functions of γ' and Ra , respectively. The γ' dependence appears clearly as shown in Fig. 2.18(c), suggesting a scaling of $KE_t \propto \gamma'^{-5/7}$ as drawn by the black dashed line. As expected, conditions of larger Ra result in larger KE_t values overall in Fig. 2.18(d).

In addition to the total kinetic energy KE_t , the Reynolds number Re may be another important quantity. It is defined as $Re = U_{\text{rms}}L/\nu$, where U_{rms} is the root-mean-squared velocity computed as

$$U_{\text{rms}} = \sqrt{\frac{KE_t}{4L^2}}. \quad (2.24)$$

In Fig. 2.18(e) and (f), U_{rms} (Re) values are plotted as functions of γ' and Ra . The computed Re values are distributed in the range of $O(10^0-10^1)$. As the Re values are in the relation $Re \propto \sqrt{KE_t}$, the overall trends are similar to those in Fig. 2.18(c) and (d), and γ' dependency appears as $Re \propto \gamma'^{-5/14}$, as shown by the black dashed lines.

The bulk quantities shown in Fig. 2.18 also suggest dependencies on γ' , similar to the characteristic quantities shown in Fig. 2.17. Details of these parameter dependencies are discussed in the following section.

2.4 Discussion

In summarizing the results presented in Section 2.3, the various characteristic quantities of developing HC shown in Fig. 2.17 and 2.18 seem to depend on γ' . While the presence of destabilizing source is not allowed in the present study, it may be possible to find Ra dependence of these characteristic quantities, because Ra represents strength of baroclinic torque. Assuming that these dependencies can be expressed by certain power law relations like Eq. 2.19 and 2.20, one has

$$\phi \propto \gamma'^p Ra^q, \quad (2.25)$$

where ϕ takes any characteristic quantities that are plotted in Fig. 2.17 and 2.18, and p and q are respectively arbitrary scaling factors for γ' and Ra . The optimal scaling factors are simultaneously computed for the measured quantities shown in Fig. 2.17 and 2.18 adopting singular value decomposition (SVD). Please note that Ra considered here does not represent the buoyancy produced at the destabilizing source, and represents the strength of baroclinic torque alone. In Fig. 2.19, the characteristic quantities (a) $u_{\max}$, (b) $\psi_{\max}$, (c) $\overline{\delta_{\text{roll}}}$, (d) Γ_c , (e) KE_t , and (f) Re are plotted as functions of both γ' and Ra . The colors of the plots represent the measured quantities shown in the color bar above each panel. The colored contour lines drawn behind the plots are the optimal planes determined adopting the SVD method, and the scaling factors are noted on each panel. Overall, the colors of the plots that show the measured values are similar to those of the computed planes, and this suggests that the six quantities shown in Fig. 2.19 can be represented by the power laws for the two nondimensional parameters γ' and Ra . Large γ' values represent a strong restoring force of the stable temperature stratification against the baroclinic circulation, and large Ra values mean that strong baroclinic vorticities are imposed on the fluid layer.

The scaling for $u_{\max}$ is computed as $u_{\max} \propto \gamma'^{-0.15} Ra^{0.45}$, as suggested in Fig. 2.19(a). The $u_{\max}$ values have a weak dependence on γ' . The scaling factor for Ra can be considered to represent the 1/2-power law that may originate from the propagation speed of the gravity current $U_{gc} \propto Ra^{1/2} \propto \Delta\theta^{1/2}$ as discussed in Section 2.3.2. From the γ' dependence of $u_{\max}$, the present flow regime can be considered to lie between the gravity current and the gravity wave regimes [114] even though γ' is sufficiently small. In general, the gravity wave regime appears when the restoring force of the stable temperature stratification is much stronger than the baroclinic torque. For the smallest $\Delta\theta$, $\Delta\theta = 1$ K, the gravity wave regime requires $\Delta T \sim 250$ K to be achieved with $\gamma' \sim 1$, and of course, this is not realistic. That is, the finite domain of the fluid container increases the apparent restoring force by restricting the flow direction as discussed in Section 2.3.1.

As shown in Fig. 2.19(b), the optimal scaling for $\psi_{\max}$ is found to be $\psi_{\max} \propto \gamma'^{-0.46} Ra^{0.20}$. A strong γ' dependence is found to be $\gamma' \sim -1/2$. Wang and Huang [199] showed the scaling, $\psi_{\max} \propto Ra_{\delta}^{1/2}$ by defining the Rayleigh number within the velocity boundary layer Ra_{δ} . The thickness of the boundary layer is, however, not a predetermined value and is determined by the flow structure in a thermally-equilibrated HC. Thus, they claimed that the layer thickness was difficult to use in practice. This 1/2-power law found in Wang and Huang [199] seems to correspond to the presently obtained scaling $\psi_{\max} \propto \gamma'^{-1/2}$, as the thickness of the velocity boundary layer δ can be considered to well represent the braking force due to the stable temperature stratification; δ becomes thicker as γ' becomes smaller. A point of this scaling is that the previously suggested scaling of $\psi_{\max} \propto Ra^{1/5}$ in thermally-equilibrated HC (Eq. 2.19) is found even in the developing HC. The maximum stream function $\psi_{\max}$ is just a local maximum representing the strength of circulation induced by the baroclinic torque, and thus this may be

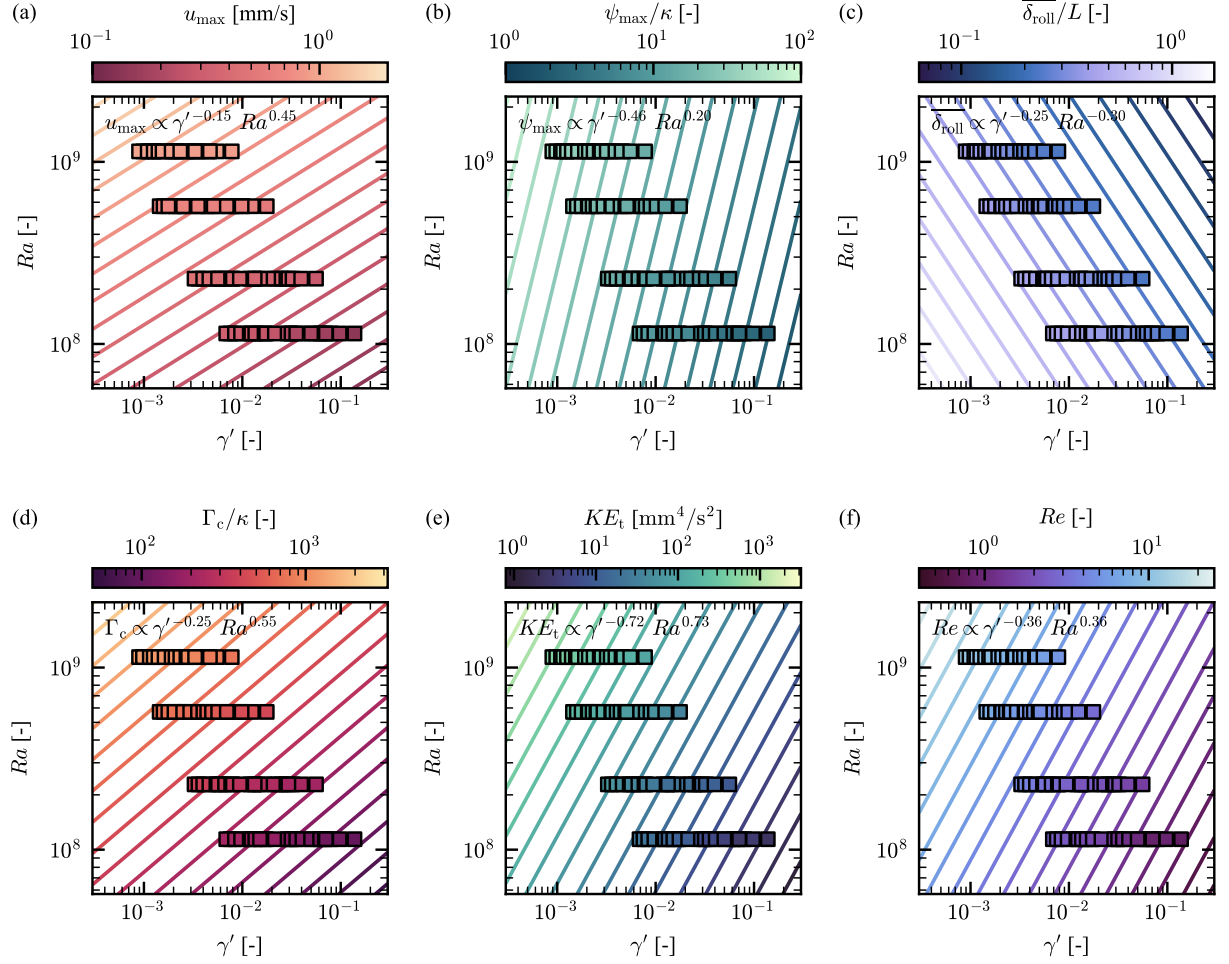


Figure 2.19: Various quantities plotted in the γ' – Ra domain; (a) maximum horizontal velocity $u_{\max}$, (b) maximum stream function $\psi_{\max}$, (c) average roll thickness $\overline{\delta_{\text{roll}}}$, (d) circulation of the roll region Γ_c , (e) total kinetic energy KE_t , and (f) the Reynolds number Re . Colors of the plots represent the quantities shown above each panel. Colored contour lines shown behind the plots are the planes fitted in the SVD method, and the scaling factors computed in the method are noted at the upper left of each panel.

determined by only the horizontal temperature difference $\Delta\theta$. The traditional scaling in Eq. 2.19 is therefore universal in the developing HC.

The scaling for $\overline{\delta_{\text{roll}}}$ is obtained as $\overline{\delta_{\text{roll}}} \propto \gamma'^{-0.25} Ra^{-0.30}$ [Fig. 2.19(c)]. Unlike the case of ψ_{max} , the roll thickness is strongly affected by the restoring force of the stable temperature stratification and the geometric environment in the developing HC. This is because the roll structure can penetrate only the stratified fluid relaxed together with the thermal diffusion. The γ' dependence appears as $\sim -1/4$, and γ' seems to be important in determining the roll thickness. The roll thickness decreases for γ' values meaning strongly stratified situations. The scaling factor for Ra indicates a Ra dependence slightly stronger than the suggested one, $\delta_{\text{roll}} \propto Ra^{-1/5}$ (Eq. 2.20); however, a similar decreasing trend is obtained. As the destabilizing source does not exist, Ra only represents the strength of baroclinic torque. The scaling in Eq. 2.20 may therefore not be necessarily acquired in the developing HC.

In Fig. 2.19(d), Γ_c values are plotted, and the scaling factors are computed as $\Gamma_c \propto \gamma'^{-0.25} Ra^{0.55}$. The γ' dependence appears as a power to $\sim -1/4$. The circulation may be strongly affected by geometric confinements like that of the aspect ratio. However, it becomes large with a decrease in γ' , similar to the case for ψ_{max} . On the basis of Eq. 2.21 and 2.22, the total vorticity produced in the roll region η_{roll} is expressed as

$$\eta_{\text{roll}} = \frac{1}{S_{\text{roll}}} \int_{S_{\text{roll}}} \eta dS = \frac{\Gamma_c}{S_{\text{roll}}} = \frac{\Gamma_c}{4L\overline{\delta_{\text{roll}}}}. \quad (2.26)$$

Using the scalings computed in Fig. 2.19(c) and (d), the scaling for η_{roll} is obtained as $\eta_{\text{roll}} \propto \gamma'^{0.00} Ra^{0.85}$. This indicates the total vorticity produced by the horizontal temperature difference is only determined by Ra with no contribution made by γ' . This result is consistent with the temperature gradient profiles shown in Fig. 2.7(b), in that the baroclinic vorticity production is only determined by the horizontal temperature difference $\Delta\theta \propto Ra$. The exponent 0.85 is anticipated to asymptote to unity if the system is in full equilibrium like in the case of conventionally studied horizontal convection or there is no dissipation effect due to stable stratification and the finite fluid container.

The total kinetic energy KE_t and Re as functions of γ' and Ra are shown in Fig. 2.19(e) and (f), respectively. For KE_t , the optimal scaling factors are obtained as $KE_t \propto \gamma'^{-0.72} Ra^{0.73}$. This scaling suggests that the restoring force of the stable stratification works at the same degree of the baroclinic torque. The restoring force acting against the baroclinic torque suppresses the enhancement of the circulating motions, as well as the viscous dissipation. In addition, because the finite fluid container may augment the restoring force as the main flow structure, the first roll formed beneath the copper blocks is confined in a shallow region. The lower quiescent fluid is thus driven by the first roll owing to the fluid viscosity and forms the second and third rolls [62, 163]. This viscous dissipation may appear as the negative dependence of KE_t on γ' . As the Re is proportional to the square of KE_t , the exponents for Re become half those for KE_t ;

these are computed as $Re \propto \gamma'^{-0.36} Ra^{0.36}$. From these exponents, the total kinetic energy, as well as the Reynolds number, becomes large when the imposed baroclinic vorticity is strong and the restoring force of the stable temperature stratification is weak.

The γ' dependencies of these characteristic quantities are modified by giving different geometrical conditions. As discussed in Section 2.3.1, the laterally finite domain regulates the circulating motion of the roll structures, and the restoring force of the stable temperature stratification strengthens as a result. This augmentation of the apparent restoring force may be prominent when a fluid container having a small aspect ratio is used for the fluid container. In this case, the γ' dependencies increase from those in the present case shown in Fig. 2.19. Meanwhile, a fluid container having a large aspect ratio or being laterally infinite may reduce the effect of the stable temperature stratification, and the γ' dependencies also decrease. When the γ' dependence becomes negligibly small, the developing HC is characterized only by Ra like in the case of thermally-equilibrated HC.

2.5 Conclusions

Transient process of developing horizontal convection (HC), in which baroclinic torque is the primary factor determining flow structures, was investigated experimentally in detail. Unlike thermally-equilibrated HC, which has been studied previously, the developing HC excludes the presence of ‘destabilizing source’ at the boundary. Water was used as the test fluid and a differential horizontal heating was imposed at the top of a quiescent cold water confined in a rectangular fluid container. Seeding of particles of thermochromic liquid crystals allowed optical visualization for the purpose of particle tracking velocimetry. As destabilizing source does not exist, the flow structures were determined by only the balance of baroclinic torque and restoring force of stable temperature stratification. Therefore, the nondimensional stratification parameter γ' , which expresses the restoring force of the stable temperature stratification acting against the baroclinic torque was defined, in addition to the Rayleigh number which may be dominant in thermally-equilibrated HC.

During the developing stage of HC, the baroclinic torque becomes dominant due to the absence of destabilizing source. A single convection roll develops within a shallow region immediately beneath the heating boundary, and its motion was very slow meaning small contribution of convective heat transport. The convection roll grows with time due to the relaxation of stable temperature stratification by the thermal diffusion; the roll structure gradually penetrate the relaxed stratified fluid deeper with time. The temporal evolution and γ' or Ra dependencies of the flow structures, in terms of maximum velocity, maximum stream function, roll thickness, circulation, total kinetic energy, and Reynolds number, were evaluated for the flow fields. The nondimensional stratification parameter γ' , considering the thermal diffusion, was a good indicator to represent the flow characteristics. For the explored parameter ranges, $\gamma' = O(10^{-4}$ –

10^{-2}) and $Ra = O(10^8-10^9)$, the flow characteristics were well described by power laws in the form of $\gamma'^p Ra^q$, and the scaling factors were determined for each measured quantity. The characteristic quantities showed relevant dependence on γ' , even γ' changed with time together with the relaxing of the stable temperature stratification. In addition, the finite domain of the fluid container seems to augment the effect of the restoring force. The bulk quantities of the roll structures are suppressed by stable temperature stratification. These results suggest that the balance of the restoring force of stable temperature stratification and the baroclinic torque is a primary factor characterizing temporally developing HC, while this may not be apparent in thermally-equilibrated HC.

CHAPTER 3

STEPWISE TRANSITIONS DURING SPIN-UP OF ROTATING RAYLEIGH–BÉNARD CONVECTION

Preface

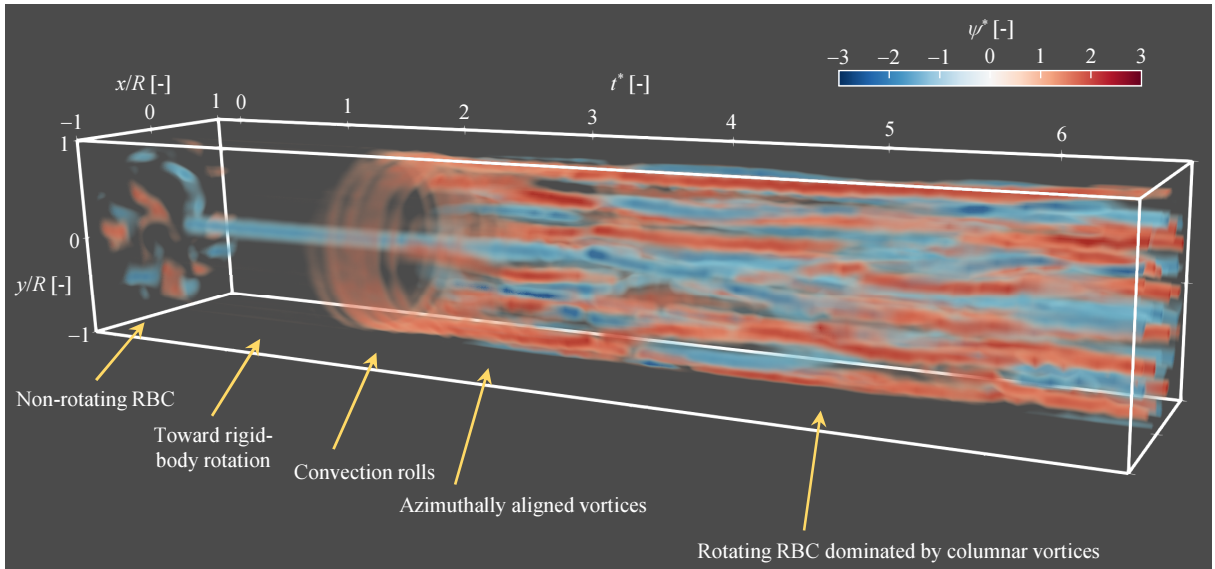


Figure 3.1: Graphical abstract.

In Chapter 2, very slow dynamics of horizontal convection with a timescale of $O(1 \text{ hour})$ have been studied using the TLC particles. As the TLC particles well represent various timescale physics of thermal convection as shown in Chapter 1, more complicated multi-timescale physics are worth studying.

In Chapter 3, nonlinear dynamics undergoing on spin-up of rotating Rayleigh–Bénard con-

vection (RBC) is investigated in detail via optical visualization utilizing the TLC particles. Horizontal and vertical velocity fields were measured by particle image velocimetry with water as the test fluid. Varying the aspect ratio, Rayleigh number, and Taylor number, a total of twenty parameters were systematically explored. Toroidal and spiral rolls were formed when the flow reached the rigid-body rotation state, and creation of these structures propagated from the rim towards the internal regions together with the development of the spin-up. Alternate alignments of rolls with opposite meridional circulation transported azimuthal momentum in the rigid-body rotation, and a meandering velocity profile in the radial direction induced Kelvin–Helmholtz (KH) instability generating azimuthally aligned vortices. The vortices progressively decreased in horizontal dimensions with the wall-to-center propagation of the vortex formation, but the vortical structures remain larger than the columnar vortices formed in the equilibrium state of a rotating RBC. At intermediate-radius of the fluid layer, the wall-to-center propagation of the roll formation was overtaken by that of the KH vortex formation. Further into the interior region, thermal plumes forming columnar vortices were generated as separations of the thermal boundary layers, and the system reached an equilibrium state of rotating RBC dominated by columnar vortices. Use of a fluid vessel with a moderate aspect ratio clarified these transitions to occur in a stepwise fashion, and a spin-up timescale unique in the rotating RBC was found to be from a few to ten times of the Ekman timescale.

The results shown in Chapter 3 were published as Noto et al. [127] and the details are below.

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Title Stepwise transitions in spin-up of rotating Rayleigh–Bénard convection

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3.1 Introduction

Thermal convection occurring between two horizontal plates with a vertical temperature gradient, i.e. Rayleigh–Bénard convection (RBC), is a fundamental system arising in fluid mechanics involving large-scale fluid motion. The RBC system imposed a background rotation, the so-called rotating RBC, has attracted much interest and has been applied to geophysical or astrophysical fluid flows [e.g. 57, 80]. Flow morphologies observed in rotating RBC with respect to the heat transfer (the Nusselt number) variations have been detailed elsewhere, and a rotating RBC can be described by dimensionless parameters as follows: The Prandtl number defined as the ratio of the kinematic viscosity ν and the thermal diffusivity κ as

$$Pr = \frac{\nu}{\kappa} \quad (3.1)$$

reflects the relation of the thickness of the viscous and thermal boundary layers. The Rayleigh number denotes the thermal buoyancy effect against restrictions of fluids due to the viscous and thermal diffusions as

$$Ra = \frac{g\beta\Delta TH^3}{\kappa\nu}, \quad (3.2)$$

where g , β , ΔT , and H denote the gravitational acceleration, the thermal expansion coefficient, the temperature difference between top and bottom of the fluid layer, and the height of the fluid layer, respectively. The influence of the background rotation is found to be dictated by the Taylor number, defined as the ratio of the Coriolis force to the viscous force and expressed by

$$Ta = \left(\frac{2\Omega H^2}{\nu} \right)^2, \quad (3.3)$$

where Ω is the rotational speed of the background rotation. Expressed differently, the rotational effect is often expressed by the Ekman number in the form $Ek = \nu / (\Omega H^2) = 2Ta^{-1/2}$. To include both thermal and rotational forces, the convective Rossby number

$$Ro = \frac{1}{2\Omega} \sqrt{\frac{g\beta\Delta T}{H}} = \sqrt{\frac{Ra}{Pr Ta}} \quad (3.4)$$

is frequently used in addition to the above. With decreasing Ro values, the flow structures in an equilibrium state of a rotating RBC with $Pr > 0.2$ may be categorized into four regimes: Three-dimensional turbulence, vortical plume, convective Taylor column (CTC), and cellular regimes [82, 98, 189, 206]. Recent experimental work to elucidate the regime transitions is summarized in Cheng et al. [28]. In the vortical plume and the CTC regimes, columnar vortices, originating from thermal plumes stretched along a vertical (rotational) axis, dominate the flow fields. Interest in the heat transfer characteristics and flow morphologies have so far been investigated in equilibrium states of rotating RBC, i.e., fully developed states of both rotation and

thermal convection. As established in Noto et al. [123] the equilibrium state of a rotating RBC dominated by columnar vortices is described for various timescales, even with steady rotation.

In many actual phenomena, system rotation is not precisely steady but accompanies fluctuations, and transient behaviors associated with changes in the rotation speed may radicalize (dull) a part of the multi-timescale phenomena described above. As a typical example of transient phenomena on rotating flows, a problem of spin-up induced by a rapid change in the rotation speed has long research history and rich knowledge. With respect to rotating convection, however, detailed investigations on transient states during spin-up from an RBC at rest towards a fully developed rotating RBC are fewer and not frequently discussed except in some experimental work [e.g. 22, 198, 220] and a numerical work [158]. It is worth investigating the transient behaviors observed during the spin-up process from a non-rotating RBC to a rotating RBC as a part of the multi-timescale phenomena in rotating RBC. The earlier work imposed a rotation to a fully developed thermal convection [22, 198] or evaporative fluids at a rest [158, 220]. This procedure is equivalent to providing a thermal convection as an initial state for the classical problem of a spin-up of fluid from rest as summarized in Greenspan [60], as well as imposing thermal boundary conditions at the top and bottom flat boundaries.

Here, results of preliminary tests are shown in Fig. 3.2 to confirm these phenomena explained so far. These photographs were taken using an open-top cylinder with a diameter of 200 mm and the depth of fluid was 20 mm. Water suspended flake particles were used as the test fluid. The corresponding Ro values of Fig. 3.2(a), (b), and (c) are $Ro \sim 0.63$, 0.31 , and 0.16 , respectively. For each condition, convection structures without rotation at $t = 0$ s changed to flow structures dominated by columnar vortices at $t = 120$ s via some different flow states. These transient flow states were reported earlier in Vorobieff and Ecke [198] and Zhong et al. [220]. These preliminary tests successfully reproduced transient behaviors during spin-up, however, the boundary conditions and dimensionless parameters are hard to control precisely in this open-top system. Therefore, a more controllable system, i.e., RBC should be employed for measurement targets so as to simplify the problems.

The previous work have reported similar transient behaviors during the spin-up of thermal convection, even with the geometries and boundary conditions of each are not identical: After imposing rotation, ring-shaped convection rolls are formed, and a shear-induced Kelvin–Helmholtz instability breaks up the rolls to vortices. These transient states evolve to the equilibrium states of rotating convection which is dominated by a number of columnar vortices after a sufficiently long time. Apart from the limited investigations of spin-up of thermal convection, there is a large literature on spin-up of rotating fluids [e.g. 18, 60, 203, 205]. Considering the spin-up of convection, this problem can be regarded as an important phenomenon as a sort of classical spin-up problem with a special boundary condition.

The objective of this study is to provide a comprehensive explanation for the transient behaviors reported in the previous work [22, 158, 198, 220], associating with both of the rotating

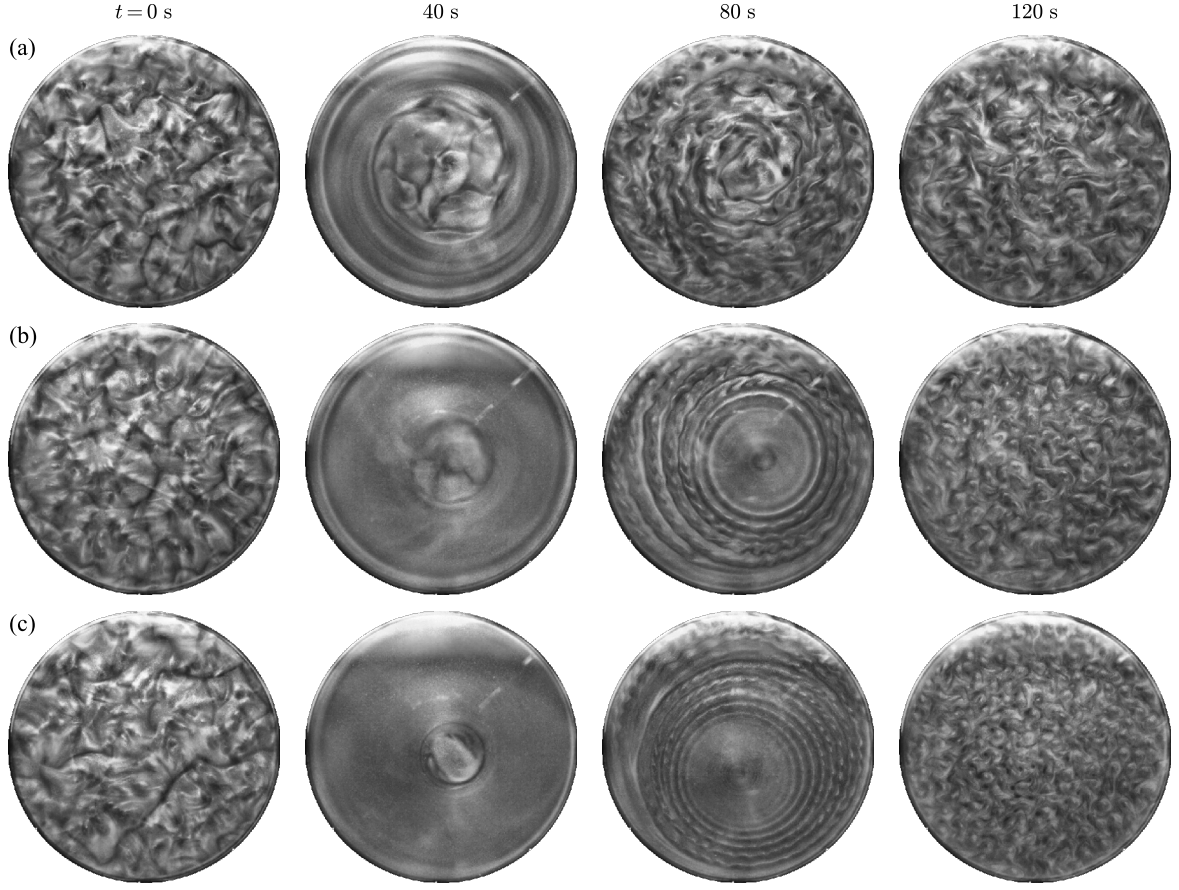


Figure 3.2: Transient behaviors in spin-up of rotating open-top thermal convection visualized using flake particles at different Ro conditions: (a) $Ro \sim 0.63$, (b) $Ro \sim 0.31$, and (c) $Ro \sim 0.16$. Time elapses from left to right as noted above (a). The contrast was adjusted for a better visibility.

convection and the classical rotating fluid dynamics. To investigate the transient behaviors during spin-up of rotating RBC systematically, this study focused on imposing a rotation instantaneously to a fully developed RBC as following the former work. The parameter ranges for the vortical plumes or the CTC regime as equilibrium states are examined, $Ra = 2.0 \times 10^6$ or 1.0×10^7 and $3.0 \times 10^6 \leq Ta \leq 2.0 \times 10^8$ corresponding to $0.0378 \leq Ro \leq 0.690$. For each condition, the velocity and temperature fields in the horizontal and vertical cross-sections in fluid layers of two different aspect ratios were examined. First the mechanism of formation of toroidal convection rolls initially appearing during the spin-up in thermal convection as described earlier is investigated. Second the mechanism of formation of vortex structures originating in convection rolls are quantified. To sum up all of the measured results, a comprehensive explanation for the transient behaviors, and the essential spin-up timescale unique in rotating RBC will be provided.

3.2 Overview of laboratory experiments

3.2.1 Experimental setup

The fluid container is schematically illustrated in Fig. 3.3. The experimental facilities used in the present study were detailed elsewhere [50, 122, 123] and they are shown in the schematic diagrams in figure 3.4. A rectangular vessel composed of 15-mm-thick acrylic walls with inner geometries of $200 \times 200 \times 40 \text{ mm}^3$ was used. Cylindrical fluid layers of the height of the vessel, $H = 40 \text{ mm}$, were formed by placing one of two different inner radius cylinders $R = 40$ and 95 mm in the rectangular vessel. The resulting aspect ratios of the cylindrical fluid layers are $\Gamma = 2R/H = 2.00$ and 4.75 . The bottom of the fluid layer was confined and heated by a 10-mm-thick copper plate heater, and the top of the fluid was confined and cooled by a transparent cooling device. This cooling device has an internal refrigerant chamber letting cooled water from a thermostat circulate inside the device, cooling the fluid layer through a 3-mm-thick glass plate. With this, optical visualization from the top of the fluid layer is possible through the transparent cooling device. The top and bottom temperatures were maintained constant, and monitored with thermistors throughout a single experimental run. The convective fluid motion realized in the present arrangement was described and explicated in earlier reports [122, 123]. Degassed distilled water ($Pr \sim 7$ at 20°C) was used as the test fluid and in the cylindrical fluid layers. The gap between the rectangle walls and the cylinders were also filled with the test fluid to act as a water jacket, as well as to reduce unwanted optical distortion.

The present study conducted two independent series of visualization experiments by altering the optical equipment; horizontal cross-section ($z = 0.5H$) and vertical cross-section ($y = 0$) as illustrated in Fig. 3.4(b) and (c). For the horizontal cross-section visualization, a tiny amount of thermochromic liquid crystal (TLC) particles (mean diameter $d_p = 15 \text{ }\mu\text{m}$, mean density $\rho_p = 1.02 \times 10^3 \text{ kg/m}^3$, approximate dynamic range $20\text{--}25^\circ\text{C}$, Japan Capsular Products Inc.) were suspended in the test fluid. As shown in Fig. 3.4(a), a white light sheet ($\sim 5\text{-mm-thick}$) generated by a cylindrical lens and a halogen lamp illuminated the half height ($z = 0.5H = 20 \text{ mm}$) of the fluid layer. A CMOS color camera (DFK33UP5000, The Imaging Source Co.) was mounted above the fluid layer perpendicular to the light sheet for continuous optical observations. To conduct the PIV analysis, the images were acquired at 25 or 40 frames per second (f.p.s.) to suit the frame rate for the PIV. For the vertical cross-section visualization, porous resin particles (mean diameter $d_p = 120 \text{ }\mu\text{m}$, mean density $\rho_p = 1.02 \times 10^3 \text{ kg/m}^3$, CHP20P, Mitsubishi Chemical Co.) were suspended. A 2 W green laser sheet ($< 3\text{-mm-thick}$) illuminated the vertical cross-section from above at the middle position ($y = 0$) coinciding with the rotational axis. Image acquisition at 25 or 40 f.p.s. was conducted by the same camera as above set at the side of the fluid layer to be perpendicular to the laser sheet.

As shown in Fig. 3.4(a), all the equipment was mounted and fixed on a turntable which

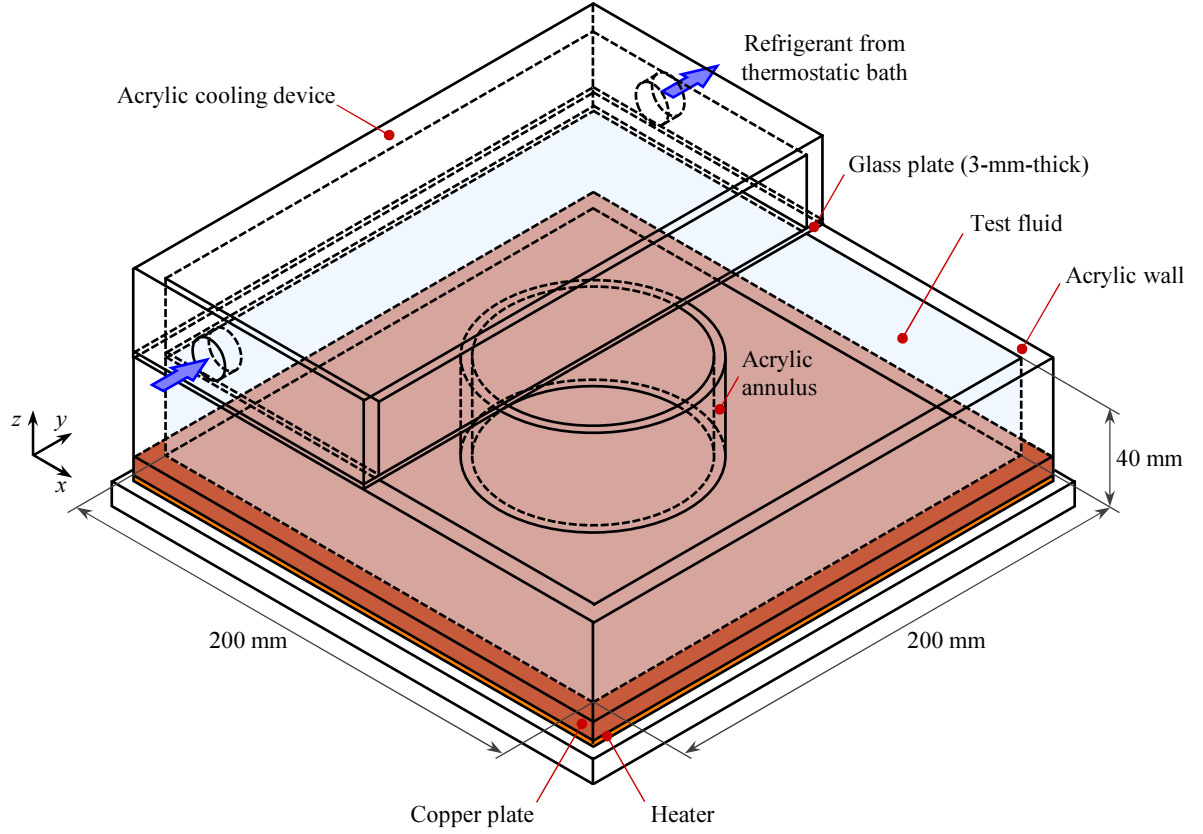


Figure 3.3: Schematic illustration of the fluid container.

can rotate at the constant rotational speed of $\Omega = 0.0\text{--}5.0$ rad/s and the corresponding Ta is $0.0\text{--}2.8 \times 10^8$ for the present conditions. Here the rotational axis coincides with the center of the cylindrical fluid layers and the optical axis of the camera rotating together with the turntable, the resulting visualization images were recorded in the rotating frame.

3.2.2 Experimental procedures and conditions

To investigate the temporal evolution of the transient behaviors during the spin-up for the RBC in a rest state towards a rotating RBC, fully developed states of non-rotating RBC were realized first. The non-rotating RBC was maintained for more than 30 min with one temperature difference between the top and bottom boundaries. A rotation was imposed to this fully developed RBC, and the image acquisition was initiated at this time. Here the time constant of the rotational acceleration of the turntable was 1 s at any rotational speed Ω , and thus it is possible to consider that the rotation was provided like in a step function.

The experimental parameters explored in this study are detailed in Table 3.1. For each of the two cylinders, two different Ra values were examined at five different Ta values, and 20 parameters were explored in total. Since the horizontal and the vertical cross-section visualizations could not be conducted simultaneously, two independent experiments were conducted for each parameter, giving a total of 40 experimental runs. In a non-rotating field at the explored

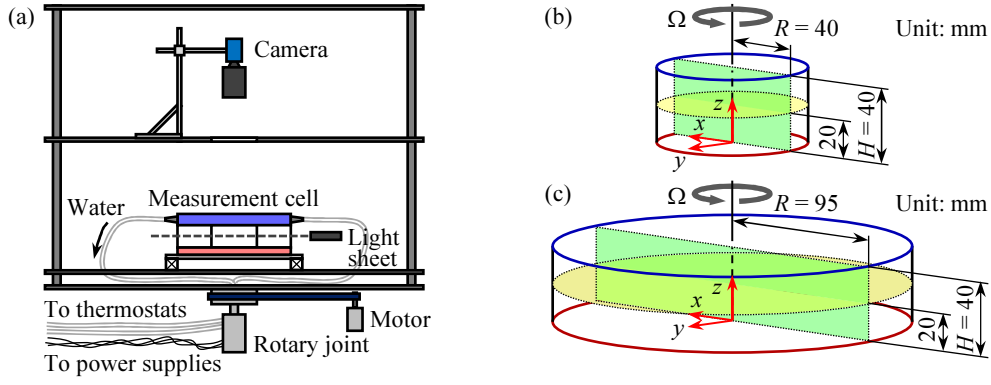


Figure 3.4: (a) Side and oblique views of the turntable schematically illustrated together with the measurement cell and optical devices for the horizontal cross-section visualization and geometries of two different radii cylindrical fluid layers: (b) $\Gamma = 2.00$ and (c) $\Gamma = 4.75$. Visualized cross-sections in a horizontal and a vertical plane are colored yellow and green, respectively. Gray arrows indicate the direction of rotation.

Ra values, $Ra = 2.0 \times 10^6$ and 1.0×10^7 , the flow fields are expected to be dominated by thermally coherent structures such as thermal plumes or large-scale structures. If a sufficiently long period elapses after providing a rotation of $3.0 \times 10^6 \leq Ta \leq 2.0 \times 10^8$, the flow regimes in the present conditions lie in the vortical plume or the CTC regimes [50, 122, 123]. That is, the flow structures dominated by a number of columnar vortices are expected at an equilibrium state of the rotating RBC. In addition to the principal parameters of the rotating RBC, Pr , Ra , and Ta , the influence of the centrifugal force is often considered. Here, defined as the ratio of the centrifugal force to gravity,

$$Fr = \frac{R\Omega^2}{g}, \quad (3.5)$$

the Froude number is conventionally used. Also, the spin-up of a fluid at rest is characterized by the Ekman timescale τ_E [61], which is defined as

$$\tau_E = \frac{H}{\sqrt{\nu\Omega}} = Ek^{\frac{1}{2}}\tau_\nu, \quad (3.6)$$

where $\tau_\nu = H^2/\nu$ expresses the viscous diffusion timescale. As may be considered from this formula, rigid-body rotation can be achieved at $O(10^1 \text{ s})$, which is much faster than would be expected from the viscous diffusion timescale of $O(10^3 \text{ s})$. Considering the rotation timescale of the system as $\tau_\Omega = 1/(2\Omega) = O(10^{-1} \text{ s})$, these three timescales are related as $\tau_\Omega \ll \tau_E \ll \tau_\nu$ [205]. To characterize the timescale of these phenomena, the time t elapsed from imposing the rotation is normalized by the widely used Ekman timescale τ_E , and will be expressed as $t^*(= t/\tau_E)$ below.

3.2.3 Qualitative observation of transient behaviors

To understand the transition of fluid motion from a non-rotating RBC to a rotating RBC, first, visualization results from the TLC particle experiments for the horizontal cross-section will be

Table 3.1: Experimental parameters explored in the present study. For each parameter, both the horizontal and the vertical cross-section visualization were conducted independently, resulting in conducting 40 experimental runs.

Pr	Γ	Ra	Ta	Ro	Fr	τ_E (s)
~ 7 (at 20°C)	2.00	2.0×10^6	3.0×10^6	0.308	0.0012	54.3
			1.0×10^7	0.169	0.00402	40.1
			3.0×10^7	0.0975	0.01205	30.4
			1.0×10^8	0.0534	0.04015	22.5
			2.0×10^8	0.0378	0.0803	18.9
		1.0×10^7	3.0×10^6	0.690	0.0012	54.3
			1.0×10^7	0.378	0.00402	40.1
			3.0×10^7	0.218	0.01205	30.4
			1.0×10^8	0.119	0.04015	22.5
			2.0×10^8	0.0845	0.0803	18.9
	4.75	2.0×10^6	3.0×10^6	0.308	0.00286	54.3
			1.0×10^7	0.169	0.00954	40.1
			3.0×10^7	0.0975	0.02861	30.4
			1.0×10^8	0.0534	0.09536	22.5
			2.0×10^8	0.0378	0.19072	18.9
		1.0×10^7	3.0×10^6	0.690	0.00286	54.3
			1.0×10^7	0.378	0.00954	40.1
			3.0×10^7	0.218	0.02861	30.4
			1.0×10^8	0.119	0.09536	22.5
			2.0×10^8	0.0845	0.19072	18.9

discussed qualitatively in this section 3.2.3 before introducing quantitative considerations. As mentioned above, a rotation is induced in a fully developed thermally turbulent state ($Ra > 1.0 \times 10^6$) at rest. In the laboratory, thermally turbulent states dominated by thermal plumes were maintained for more than 30 min before imposing the rotation. Typical snapshots of non-rotating RBC states with different Ra values in the smaller cylinder ($\Gamma = 2.00$) are shown in Fig. 3.5. In Fig. 3.5, TLC particles provide colors corresponding to the temperature, red, green and blue as temperature increases, with the smaller Ra condition [Fig. 3.5(a)] showing relatively larger coherent structures than those for the larger Ra case [Fig. 3.5(b)]. As well as the size of the thermally coherent structures, a widely distributed green color can be seen at the smaller Ra value because the temperature difference is smaller than that with the larger Ra value [Fig. 3.5(b)]. For both cases, flow structures are governed by a number of thermal plumes which are visible as red and blue spots in both panels of Fig. 3.5, the realized flow structures can be regarded as fully developed thermal turbulent structures.

The temporal evolution of the spin-up state after the rotation starts in the case of $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$ with $\Gamma = 2.00$ is shown in Fig. 3.6. Time elapses from the upper left to the lower right. Soon after imposing the rotation in the flow structures of a non-rotating RBC like in Fig. 3.5 at $t^* = 0.0$, thermally coherent structures such as thermal plumes and

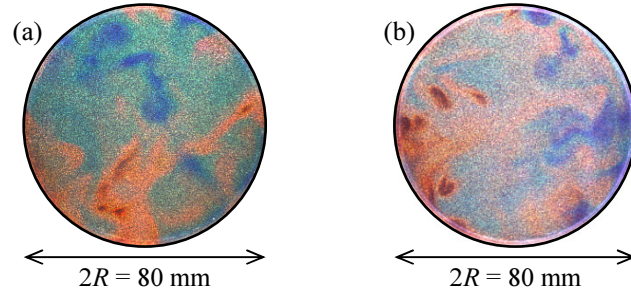


Figure 3.5: Snapshots of horizontal cross-sections at half height ($z = 0.5H$) of non-rotating thermal convection in the smaller cylinder $\Gamma = 2.00$ visualized by the TLC particles: (a) $Ra = 2.0 \times 10^6$ and (b) $Ra = 1.0 \times 10^7$. Red and blue show cold and hot regions qualitatively because of the TLC coloration characteristics.

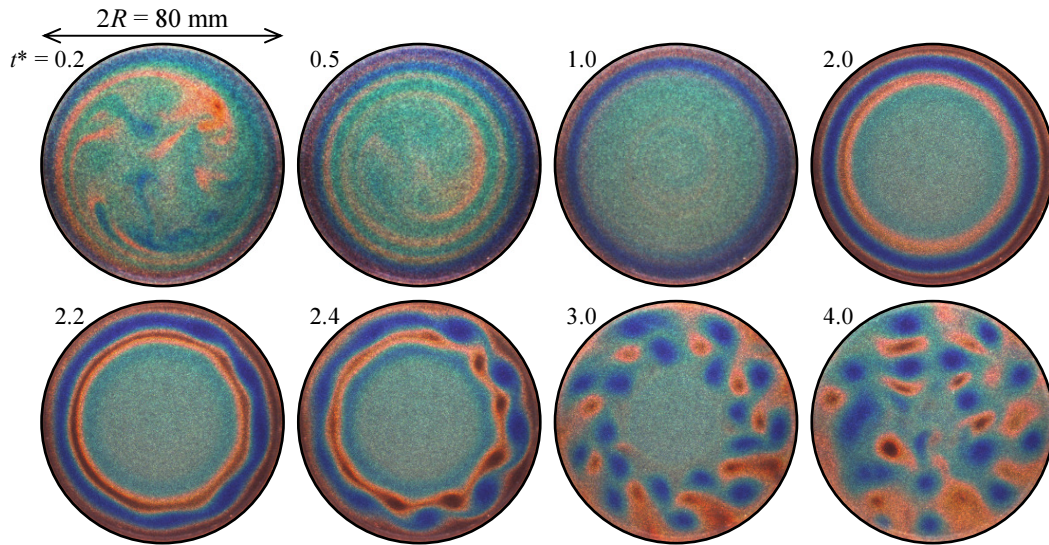


Figure 3.6: Temporal evolution of spin-up state after the initiation of the rotation with the condition of $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$ in the $\Gamma = 2.00$ cylinder. Each snapshot is a horizontal cross-section at $z = 0.5H$ visualized by the TLC particles. Time elapses from top left to bottom right, and the corresponding t^* values are at the upper left of each snapshot. Please note that time intervals between the snapshots are not the same. The corresponding movie is available from the supplementary material Movie 1.

large-scale structures disappear and the fluid layer is nearly fully mixed. This would occur as an intense shear is suddenly induced at the vicinity of both the top and the bottom walls as well as at the sidewall. In addition, the centrifugal force drives cold (hot) fluid radially outward (inward) due to the mixed density differences in the fluid layer. Accordingly, in Fig. 3.6, the thermal plumes present in the non-rotating state swirl from the center to the wall, and as a result a coherent color pattern extends into the azimuthal direction and can be seen as alternative red and blue lines ($t^* = 0.2 - 0.5$). After the fluid is completely mixed, there is an almost uniform coloration at $t^* = 1.0$, with narrow red and blue rings appearing only at the vicinity of the sidewall. These rings eventually grow wider and the ring formation progresses from the sidewall toward the center of the fluid layer. At $t^* = 2.0$ such the rings are more clearly visible than at $t^* = 1.0$. From the coloration sequence of the rings, red, blue, and red again from the wall towards the center of the fluid layer, it may be assumed that the corresponding temperatures are cold, hot, and cold again as visualized by the TLC particles. Considering these colorations as a temperature distributions, these rings may be regarded as horizontal slices of toroidal convection rolls; respectively down-welling, up-welling, down-welling flows progressing from the wall to the center. The emergence of such rolls has previously been reported by other researchers [22, 158, 198, 220].

After the emergence of the concentric convection rolls, the rings start to fluctuate and meander in the azimuthal direction displaying wavy patterns at $t^* = 2.2$ of Fig. 3.6. Like the ring formations, this fluctuation was reported by earlier works [22, 198, 220]. The fluctuations gradually increase in amplitude in the radial directions and finally form azimuthal vortex trails like at $t^* = 2.4$ of Fig. 3.6, where there are clearly separated red (cold), blue (hot), and red (cold) vortices aligned inwards from the wall toward the center. Expressed differently, the vortical structures form along the boundaries of the radially aligned concentric convection rolls. With time, the vortical structures grow and the formation of these progressively develop towards the center at $t^* = 3.0$ of figure 3.6. However, it remains possible to distinguish the regularly aligned red and blue circles in places. The circles can be considered as the down-welling and up-welling vortices, and they are moving very slowly throughout the fluid layer as addressed by Noto et al. [123]. At $t^* = 4.0$ of Fig. 3.6, the fluid layers are fully governed by the vortical structures which are randomly distributed throughout the fluid layer. The final states look quite similar to the equilibrium state of a fully developed rotating RBC.

Another example with $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$ with $\Gamma = 4.75$ is shown in Fig. 3.7. Here, the radius of the large fluid layer is more than double of that of the small one, even though the image size in figure 3.6 and 3.7 are identical. Thermally coherent structures disintegrate, and concentric convection rolls gradually form from the sidewall at $t^* = 0.2-2.0$. The number of rolls is larger than that in Fig. 3.6. The convection rolls subsequently fluctuate and form azimuthally aligned vortical structures at $t^* = 2.4 - 3.0$ even in the large radius fluid layer. This vortex formation progresses from the sidewall towards the center of the fluid layer,

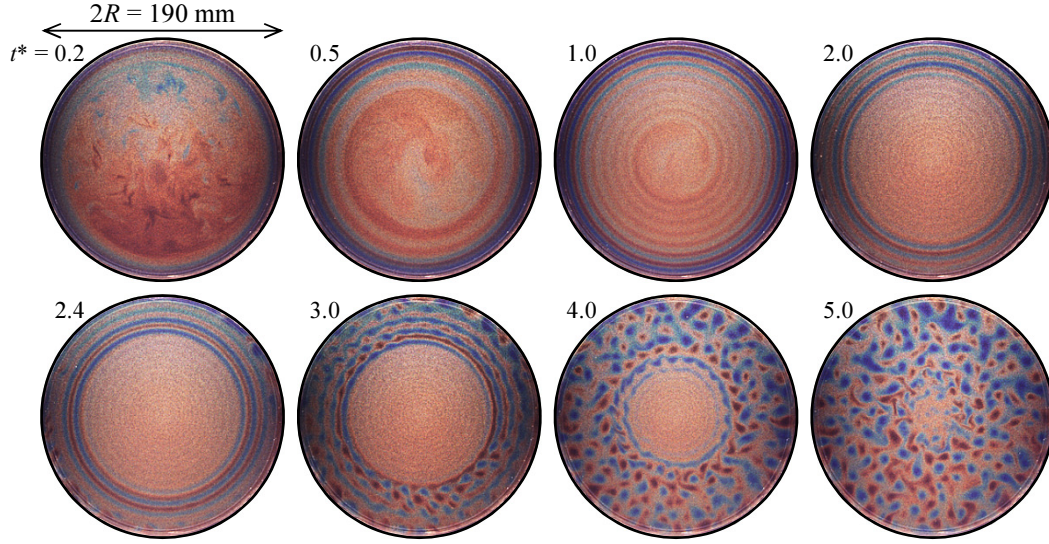


Figure 3.7: Temporal evolution of the spin-up state after the initiation of the rotation with the condition of $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$ in the $\Gamma = 4.75$ cylinder. Each snapshot is a horizontal cross-section at $z = 0.5H$ visualized by the TLC particles. Time elapses from upper left to lower right, and the corresponding t^* values are noted at top left of each snapshot. Please note that time intervals between the snapshots are not constant. The corresponding movie is available from the supplementary material Movie 2.

and the final state at $t^* = 6.0$ also looks similar to the equilibrium state of a fully developed rotating RBC. While some differences can be seen between Fig. 3.6 and 3.7, the displayed cases show basically similar transitions from non-rotating to rotating RBC.

Transient behaviors, which have been mentioned in earlier works [22, 198, 220], were successfully observed in the rotating RBC system of the present study. First the structures of the non-rotating RBC disintegrated and subsequently toroidal convection rolls form rings at the vicinity of the sidewall. The rings start to fluctuate in the azimuthal direction and develop into vortical structures and finally the fluid layer is dominated by numerous vortices. These are the stages of a time transition that develops progressively from a non-rotating to a rotating RBC. Each of the transient behaviors will be investigated by a quantitative approach afterwards.

3.3 Emergence of concentric convection roll structures

3.3.1 Types of convection roll structures

Imposing a rotation to an RBC flow, thermally coherent structures like thermal plumes as appeared in Fig. 3.6 and 3.7 disintegrate due to the intense viscous dissipation of the rotation and the centrifugal forces. Mixing of the fluid in the azimuthal direction is shown as red and blue swirls which represent thermal plumes. The flow state can be considered as that of a stage in the spin-up of a fluid at rest without any thermal force imposed. Toroidal convection rolls appear progressively from the sidewall as shown in Fig. 3.6 and 3.7. The emergence of these ring structures was previously reported in the works [22, 158, 198, 220]. Considering these earlier

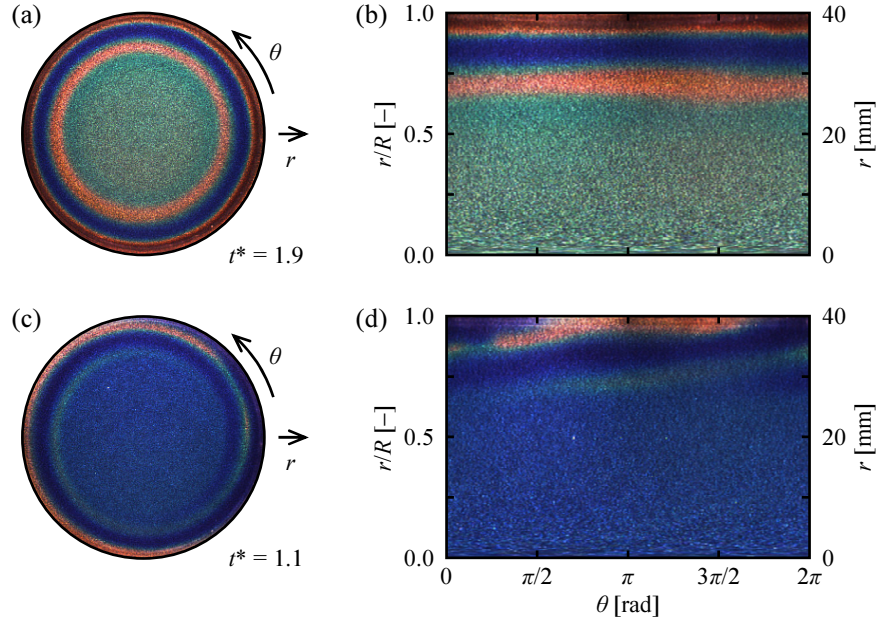


Figure 3.8: Two types of concentric roll structures observed in the smaller cylinder $\Gamma = 2.00$ at $Ra = 2.0 \times 10^6$: (a) ring type ($Ta = 3.0 \times 10^7$) and (b) its azimuthally extended image, and (c) spiral type ($Ta = 1.0 \times 10^7$) and (d) its azimuthally extended image.

reports, the ring structures appear regardless of the boundary conditions, namely, both in open to the air (slip) and enclosed (no-slip) vessels. In addition, the vessel geometry is not significant for the ring formation as the ring formations were observed both in rectangular [22, 158, 220], and cylindrical vessels [22, 198] and the present study. The fluid vessels used in the earlier work were narrower ($0.656 \leq \Gamma \leq 2.00$) than those of the present study ($\Gamma = 2.00$ and 4.75), however similar transient events were observed in the present conditions.

Vertical fluid motions are restricted in the spin-up process, except for the Stewartson layer formed at the sidewall and the center of the fluid layer. The convection rolls may be induced by the vertical fluid transports carried out in the Stewartson layers at the sidewalls, as the layers transport hot and cold fluids at the bottom and top boundaries. Further, the centrifugal force may influence the roll formation, as lower density (hot) and higher density (cold) fluids are transported towards the center and the sidewall due to centrifugal effects.

In addition to the ring structures shown in Fig. 3.6, another type of concentric convection roll, a spiral structure, was identified. For a comparison of these two types of roll structures, Fig. 3.8 shows original images of ring (a) and spiral (c) structures, and the corresponding azimuthally extended images are in (b) and (d). The spiral-type rolls have not been reported in the previous work except for the earlier work by Boubnov and Golitsyn [22]; they noted that various roll patterns such as spirals or ‘8-like’ structures were formed in eccentric arrangements between the center of the fluid layer and the rotational axis, even though they did not provide any clear explanations. The two types of roll structures are formed in the same smaller cylinder $\Gamma = 2.00$ at the same Ra number $Ra = 2.0 \times 10^6$ with a slight difference in Ta values $Ta = 1.0 \times 10^7$

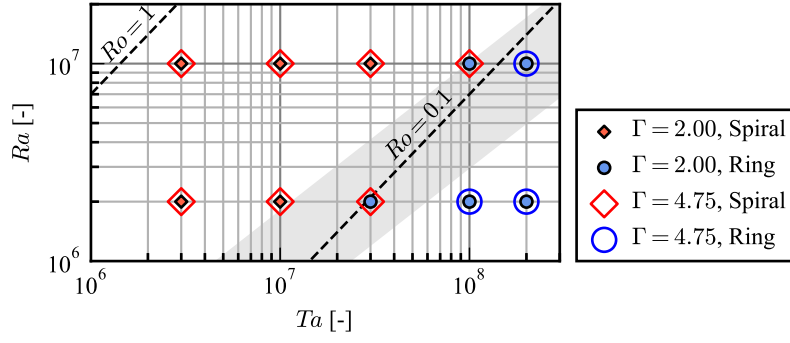


Figure 3.9: Regime diagram showing the types of convection roll structures which emerged soon after imposing the rotation. Small symbols indicate the plots for $\Gamma = 2.00$ and large symbols indicate the plots for $\Gamma = 4.75$. Spiral types and ring types are represented by red diamonds and blue circles. Dashed lines indicate $Ro = 1$ and 0.1 with $Pr = 7$. Gray shaded region is the cross-over range of the boundary layer thicknesses predicted by [89].

and 3.0×10^7 . The types of rolls at all the explored parameters are summarized and detailed in Fig. 3.9. All the rolls are categorized into the ring and spiral types, and plotted with circles and diamonds, and for the smaller Γ and the larger Γ cases plotted as filled and open markers, respectively. Ring structures have a propensity to be formed at the lower Ra values and relatively higher Ta values. In addition, they are easily formed in the smaller cylinder $\Gamma = 2.00$ when compared with the larger cylinder $\Gamma = 4.75$.

Suppose the present system is a vertical symmetry, it may be not possible to observe the spiral-type rolls, as the centrifugal forces work as radially isotropic forces, and this situation always yields down-welling flows at the sidewall. Accordingly, a vertical asymmetry that provides an unbalance in the fluid deformation for forming the rolls, can be considered as one of the possible mechanisms to explain the presence of the spiral-type rolls. The present study performs experimental investigations, and it is possible to deduce that there is non-Oberbeck-Boussinesq (NOB) effect to generate a vertical asymmetry, as the effect may appear in the set temperature differences of $\Delta T = 2.2$ and 11.1 K for $Ra = 2.0 \times 10^6$ and 1.0×10^7 [59]. Horn and Shishkina [67] provided direct numerical simulations for rotating RBC in water with NOB condition and showed that the thicknesses of the thermal boundary layers at the top and bottom change drastically at $Ro \sim 0.14$, due to a cross-over in the thicknesses of the thermal and viscous boundary layers. For $Ro \gtrsim 0.14$, the thermal boundary layer at the top is always thicker than that at the bottom, and this relation reverses for $Ro \lesssim 0.14$. For the larger Ro conditions, the thermal boundary layer at the bottom is easily unstabilized compared to the smaller Ro conditions and the convection flows can develop from both the top and bottom plates, whereas the thermal boundary layer at the top is easily unstabilized for the smaller Ro conditions. King et al. [89] observed that the cross-over in the thicknesses of the boundary layers occur somewhere between $6 \lesssim RaEk^{3/2} \lesssim 20$, or expressed in terms of Ta , it should exist in the range $3 \lesssim RaTa^{-3/4} \lesssim 10$. The predicted range is shaded as gray in Fig. 3.8, and it is noteworthy that the range roughly agrees with the border for separating the two types of rolls at $Ro \sim 0.1$, indicated by a dashed line in the figure.

Further, the aspect ratio of the fluid layer Γ is another factor for determining the roll type. The present study uses the cylindrical vessel of $\Gamma = 2.00$ and 4.75 , which is larger than the one ($\Gamma = 1$) used by Vorobieff and Ecke [198]. The larger aspect ratio may easily provide a vertical asymmetry to the system, as the longer circumference of the vessel is hard to be synchronized in the fluid motions even with a strong centrifugal force. The dependence on the aspect ratio might not possible to ignore and rather significant than the NOB effect for considering this kind of pattern formation problems. Incorporating the discussions above, the balance of the centrifugal force and the degree of vertical symmetry of the top and bottom boundary layers are considered as the determinant of the roll types: The conditions with sufficiently strong centrifugal forces compared to the vertical disturbances originating from the vertical asymmetry of the boundary layers can form the ring-type rolls.

3.3.2 Characteristics of the concentric roll structures

To observe the formation of concentric convection rolls quantitatively, flow fields were obtained by PIV analysis for both the horizontal and vertical visualizations experiments. As well as for the vertical cross-section measurements, in the present case PIV analysis is possible for particle images obtained by the horizontal TLC measurements because of the sufficiently small and well distributed TLC particles and the high resolution digital images with 2000 pixels to the fluid layer diameter of $2R = 80$ or 190 mm. Examples of the flow fields in the situation where the roll formation has taken place in the horizontal ($z = 0.5H$) and the vertical ($y = 0.0$) cross-sections are shown in Fig. 3.10. For the PIV analysis, velocity vectors are obtained at every 1 mm in both directions of the Cartesian coordinates, and one of two vectors (black arrows) are displayed in any directions in Fig. 3.10. The arrow sizes are adjusted in each panel regardless of the magnitude of the velocity for clearer visibility. The velocity fields in the horizontal cross-section [Fig. 3.10(a)] and those in the vertical cross-section [Fig. 3.10(b)], are obtained with data for two different individual experimental runs, and thus the extracted different snapshots of the flow fields are strictly speaking different. The color contours in Fig. 3.10 show the out-of-plane vorticities; the vorticity in z -direction $\omega_z = \partial u_y / \partial x - \partial u_x / \partial y$ [Fig. 3.10(a)], and that in y -direction $\omega_y = \partial u_x / \partial z - \partial u_z / \partial x$ [Fig. 3.10(b)]. At $t^* = 1.00$ the ω_z in the horizontal plane is large due to momentum transfer from the sidewall. In the vertical plane, the interior core region is highly disorganized with small coherent structures of ω_y , while there are boundary layers at the sidewall. At $t^* = 1.50$ in the horizontal plane, the core region with negative ω_z values is smaller, and a prominent positive ω_z ring appears at the vicinity of the sidewall. Correspondingly, in the vertical plane the roll structures expressed as vorticity distributions organized in the z -direction emerge at the vicinity of the wall, while the core region is still disorganized. At $t^* = 2.00$ formation of convection rolls has progressed away from the wall towards the center of the fluid layer. This is clearly the case both in the horizontal and vertical planes as shown by the alignment of alternate positive and negative vorticity in the radial directions.

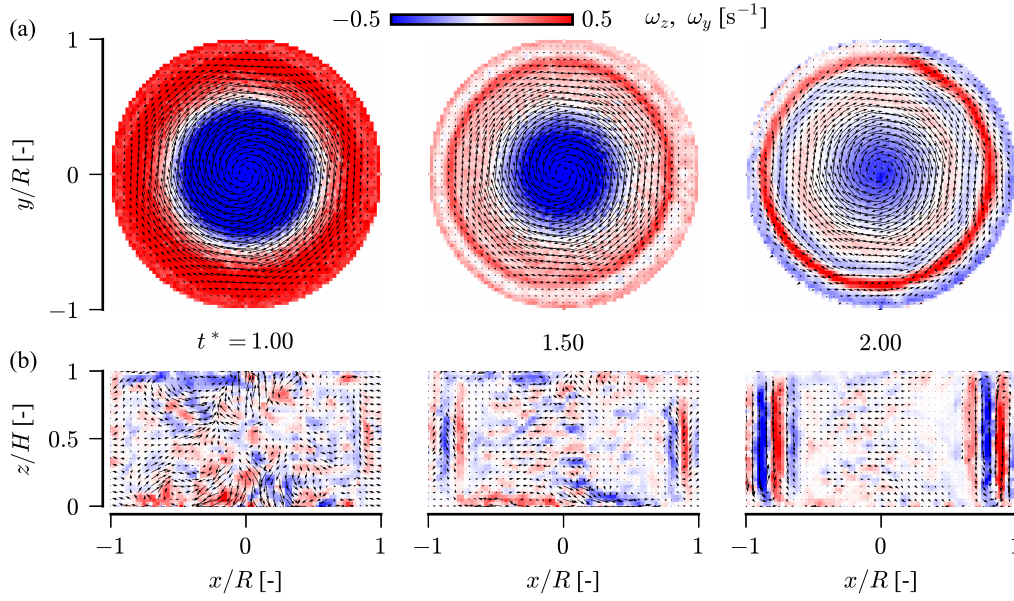


Figure 3.10: Instantaneous velocity and vorticity fields obtained by PIV analysis for $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$, and $Ta = 3.0 \times 10^7$ in the situation of roll formation: (a) u_x – u_y vectors (black arrows) plotted on the out-of-plane vorticity ω_z (colored contours) at $z = 0.5H$. (b) u_x – u_z vectors (black arrows) plotted on the out-of-plane vorticity ω_y (colored contours) at $y = 0.0$. Time advances from left to right and is indicated between the top and bottom panels. One of two vectors is plotted in any directions for visibility and the vector sizes are adjusted in each panel to enhance clarity.

To investigate the concentric convection roll structures quantitatively, each roll structure is necessary to be fully established from the velocity fields. Roll boundaries can be identified by the directions of vertical flows u_z , as developed thermal convection should have vertical flows in one direction along with the fluid layer depth (z -direction). For this reason the vertically-averaged vertical velocity $\langle u_z \rangle_z$ is a useful indicator to identify the vertical flows, and it is calculated as

$$\langle u_z \rangle_z = \frac{1}{H} \int_0^H u_z(x, z) dz, \quad (3.7)$$

in the PIV results of the vertical cross-section shown in Fig. 3.10. An example of the radial profile of a calculated $\langle u_z \rangle_z$ for the case of $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$, and $Ta = 3.0 \times 10^7$ is shown in Fig. 3.11. Here $\langle u_z \rangle_z$ is normalized by the free-fall velocity $U_f = \sqrt{g\beta\Delta TH}$. The time evolution of the profile is shown using color coding. Clear wavy patterns can be seen extending radially outward ($|x| \gtrsim R/2$) and there are some zero-crossing points at which $\langle u_z \rangle_z = 0$ as indicated by the diamonds in the figure. These zero-crossings are regarded as the centers of the roll structures, and the local maxima and local minima of $\langle u_z \rangle_z$ correspond to the roll boundaries. The zero-crossing points and the local maxima/minima are determined by the spline interpolation on the profile of $\langle u_z \rangle_z$. Boundary layers at the two sidewalls are marked in the darkest gray. Each roll structure and the corresponding thickness δ_{roll} may be considered as the distance between the local maxima and local minima, and are identified individually by the painted regions with different lighter grays inward in Fig. 3.11. The thickness of rolls are numbered in order from

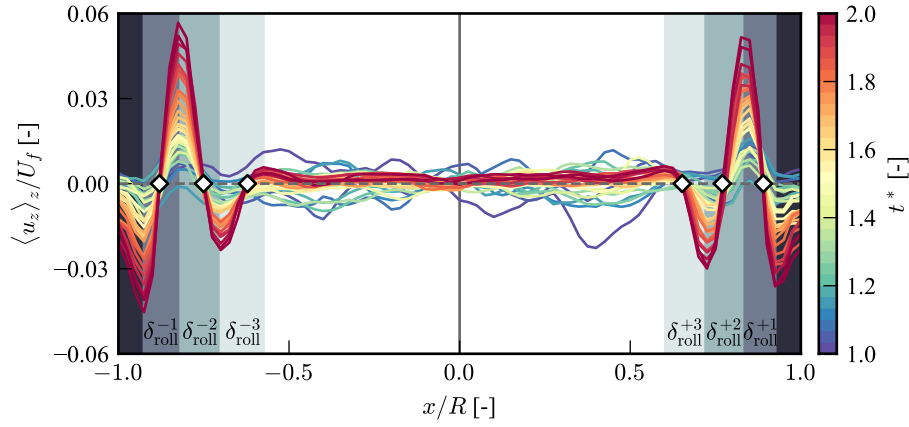


Figure 3.11: Horizontal profiles of the vertically-averaged vertical velocity $\langle u_z \rangle_z$ in the vertical cross-section $y = 0.0$ with $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$. Here $\langle u_z \rangle_z$ is normalized by the free-fall velocity U_f . Time evolution of the profiles is displayed by the color coding. Zero-crossing points of the profiles are marked by diamonds. The regions nearby two sidewalls painted in the darkest gray are the sidewall boundary layers. Other regions painted in gray being gradually lighter inward indicate the thickness of individual roll structures δ_{roll} which can be determined by the distance between the local maxima and minima of $\langle u_z \rangle_z$.

the wall towards the center as $\pm 1, \pm 2, \pm 3$ and these are indicated at the bottom in Fig. 3.11. The result shown in Fig. 3.11 is the case for the ring-type rolls, however, the cases for the spiral-type rolls are also possible to analyze in the same manner. The zero-crossing points and the local minima/maxima seem to be fixed at least during $t^* = 1.0 - 2.0$ and behave as nodes and antinodes in Fig. 3.11. Expressed differently, the roll structures do not widen in thickness or move radially as time elapses from they form and until they collapse into azimuthally aligned vortical structures as shown in Fig. 3.6 and 3.7.

Considering the negligibly small time variations of the roll structures, the roll thickness δ_{roll} can be estimated from the distance between the local maxima/minima on temporally averaged profiles of $\langle u_z \rangle_z$ for a specific duration. In Fig. 3.12(a), the estimated δ_{roll} including $\delta_{\text{roll}}^{\pm 1}$, $\delta_{\text{roll}}^{\pm 2}$, $\delta_{\text{roll}}^{\pm 3} \dots$, for all the explored parameters are summarized. The roll thicknesses δ_{roll} normalized by the fluid layer depth H distribute around 0.1–0.2, showing the rolls to be narrow and tall structures. There appear to be no regular changes in δ_{roll} for the Ra and Ta values in Fig. 3.12(a).

As δ_{roll} is not clearly described by either of the Ra or Ta values, another determinant for δ_{roll} would exist, one that is independent of thermal buoyancy and Coriolis forces. The centrifugal force may be a possible principal influence since it is a radially isotropic force forming the concentric structures as discussed in section 3.2.3. One candidate for understanding the effect of the centrifugal force could be the Froude number Fr . The effective Froude number is defined as $Fr_{\text{eff}} = r_{\text{roll}} \Omega^2 / g$, where r_{roll} indicates the radial position of the roll center (zero-crossing point of $\langle u_z \rangle_z$). In Fig. 3.12(b), δ_{roll} is plotted with Fr_{eff} . With the increase of Fr_{eff} (the centrifugal effect), δ_{roll} gradually decreases, and the deviations of δ_{roll} with smaller Fr_{eff} are larger. Further δ_{roll} for the same Ta conditions decreases with increases in Fr_{eff} . Taken together, the roll thickness becomes narrower as the roll forms radially outward. For comparison, the critical wavelength at the onset of the rotating RBC, $2.4HTa^{-1/6}$, derived by Cheng et al. [28] using the

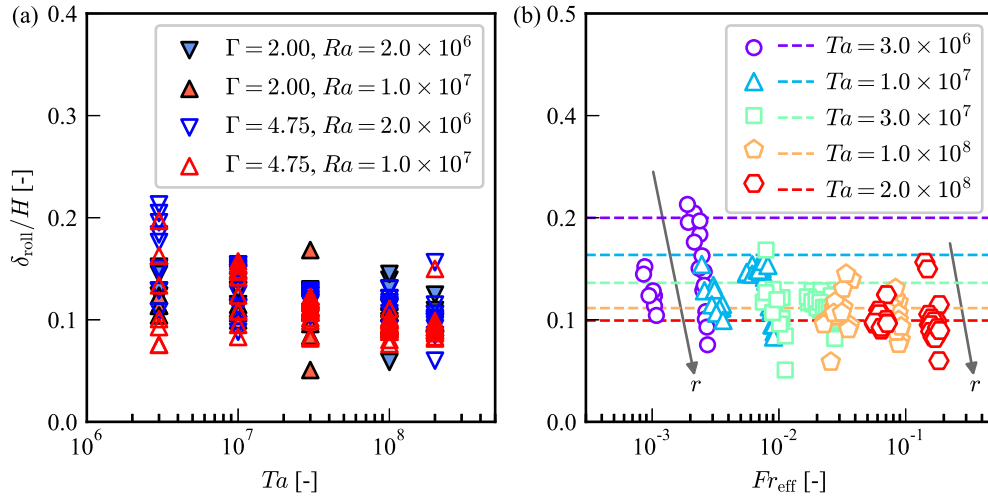


Figure 3.12: Roll thickness δ_{roll} measured for all the explored parameters: (a) δ_{roll}/H plotted with Ta values. Filled and open symbols indicate the case with $\Gamma = 2.00$, and with $\Gamma = 4.75$, respectively. Inverse and regular triangles are for $Ra = 2.0 \times 10^6$, and $Ra = 1.0 \times 10^7$, respectively. (b) δ_{roll}/H plotted with effective Fr values Fr_{eff} . Circles, triangles, squares, pentagons, and hexagons indicate $Ta = 3.0 \times 10^6$, 1.0×10^7 , 3.0×10^7 , 1.0×10^8 , and 2.0×10^8 , respectively. The horizontal dashed lines are the theoretical values $2.4HTa^{-1/6}$ estimated by Cheng et al. [28]. The markers and dashed lines are individually color coded with Ta values. Gray arrows are approximate guides indicating the radial position of each convection roll.

linear stability analysis are plotted with the dashed horizontal lines. In particular the measured δ_{roll} for the larger Ta cases are slightly smaller than what would be expected from the previous reports.

Differences in r_{roll} result in quite large deviations in δ_{roll} as guided by the gray arrows in Fig. 3.12(b); δ_{roll} is wider at the interior of the fluid layer (see Fig. 3.11). The relation between δ_{roll} and r_{roll} is plotted in figure 3.13(a). Here the radial positions r_{roll} are normalized by the fluid layer radius R , and results in both of the fluid layers of $\Gamma = 2.00$ and 4.75 are plotted. Regardless of Γ and Ra values, δ_{roll} increases as roll structures form at the interior of the fluid layer, even with large deviations exist especially for the smaller Ta conditions. This trend can be fitted roughly by the linear regressions in the form of $\delta_{\text{roll}}/H = ar_{\text{roll}}/R + b$ individually for each Ta condition, and is shown as dashed lines in Fig. 3.13(a). The slopes are steeper for smaller Ta values, and gradually become flatter with increases in Ta values, even with the large deviations due to a fixed observation cross-section at $y = 0$. This Ta dependence of the absolute values of the slopes $|a|$ is shown in Fig. 3.13(b). Standard deviations for $|a|$ in the linear regressions of Fig. 3.13(a) are shown as error bars. For the explored Ta parameters, $|a|$ is proportional to $Ta^{-0.7 \pm 0.01}$ as indicated by the dashed line. Hence, δ_{roll} can be approximately estimated from only the Ta values and the formation radius r_{roll} . Vorobieff and Ecke [198] mentioned that the radial positions of the first hot rings (up-welling streams), r_{roll} in the present notation, are always seen at $r_{\text{roll}} \sim 3R/4$ in the cylinder with $\Gamma = 1$ regardless of the experimental parameters. The first position of roll formation is, however, not essential because this is automatically determined by the roll thickness δ_{roll} first formed at the vicinity of the sidewall. And thus the present study using rather larger Γ than that of Vorobieff and Ecke [198], shows larger radial positions

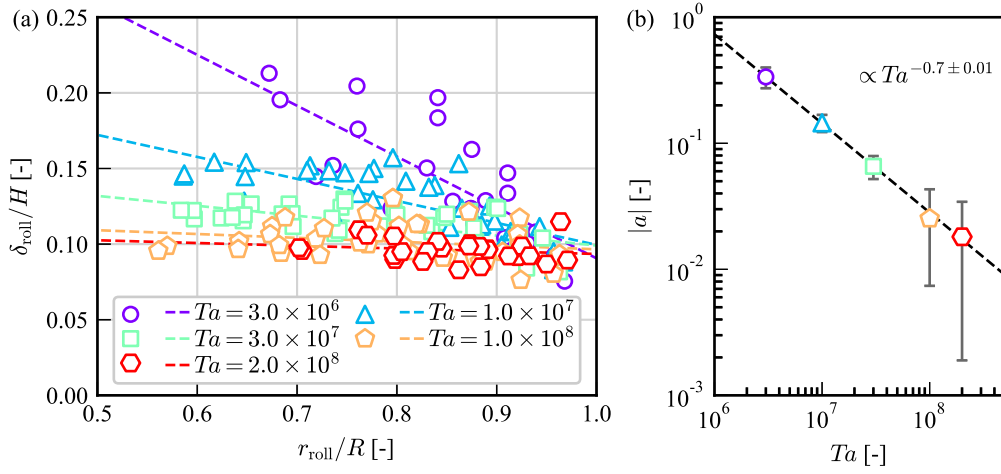


Figure 3.13: (a) Relation between the roll thickness δ_{roll} and the corresponding radial position. Circles, triangles, squares, pentagons, and hexagons indicate $Ta = 3.0 \times 10^6$, 1.0×10^7 , 3.0×10^7 , 1.0×10^8 , and 2.0×10^8 , respectively. Dashed lines show the least-squares fits of linear regressions ($\delta_{\text{roll}}/H = ar_{\text{roll}}/R + b$) for each Ta condition. The markers and dashed lines are individually color coded with Ta values. (b) Ta dependence of the absolute value of slopes to the fitting lines $|a|$. The slopes in (a) become flatter with increase in Ta proportional to $Ta^{-0.7 \pm 0.01}$ as guided by the dashed line. Standard deviations for $|a|$ in the fitting of (a) are shown as error bars.

($r_{\text{roll}}/R > 0.75$) at which the first roll is formed.

3.3.3 Wall-to-center propagation of convection roll formation

Convection rolls remain stable from their emergence till the appearance of azimuthal distortions as detailed in section 3.4. In this section, the progress of roll generation from the sidewall towards the center of the fluid layer shown in Fig. 3.6, 3.7 and 3.10 is investigated in detail. Spatiotemporal extended images of radial profiles of $\langle u_z \rangle_z$ at $Ra = 1.0 \times 10^7$ and $Ta = 3.0 \times 10^7$ with changing Γ are shown in Fig. 3.14 to better understand the temporal evolution of the roll formation. The color contour shows values of $\langle u_z \rangle_z$ normalized by the free-fall velocity U_f . The result of $\Gamma = 2.00$ is shown in Fig. 3.14(a) and that of $\Gamma = 4.75$ in Fig. 3.14(b). Positive and negative values of $\langle u_z \rangle_z$ appear from the sidewall ($|x|/R \sim 1$) alternately and are shown as red and blue organized columns. These columns in Fig. 3.14 qualitatively express the convection roll structures. Considering the emergence points of the organized columns, it is obvious that the rolls form in order from the sidewall and migrate towards the center of the fluid layer. As a reason for this wall-to-center progression of the roll formation, it is plausible to consider the development of accelerated region from the sidewall due to the fluid viscosity. For the discussion here the increase of the thickness of accelerated region at sidewall δ_ν is simply assumed to be described by that of the two-dimensional Rayleigh problem (Stokes first problem); the accelerated region develops as $\delta_\nu \propto \sqrt{\nu t}$. For reference, the time variations of δ_ν with arbitrary scaling factors is plotted as black dashed lines in Fig. 3.14. The time of appearance of the roll structures qualitatively match the development of the accelerated region. That would allow the assumption that the roll structures form in the regions where rigid-body rotation has been achieved.

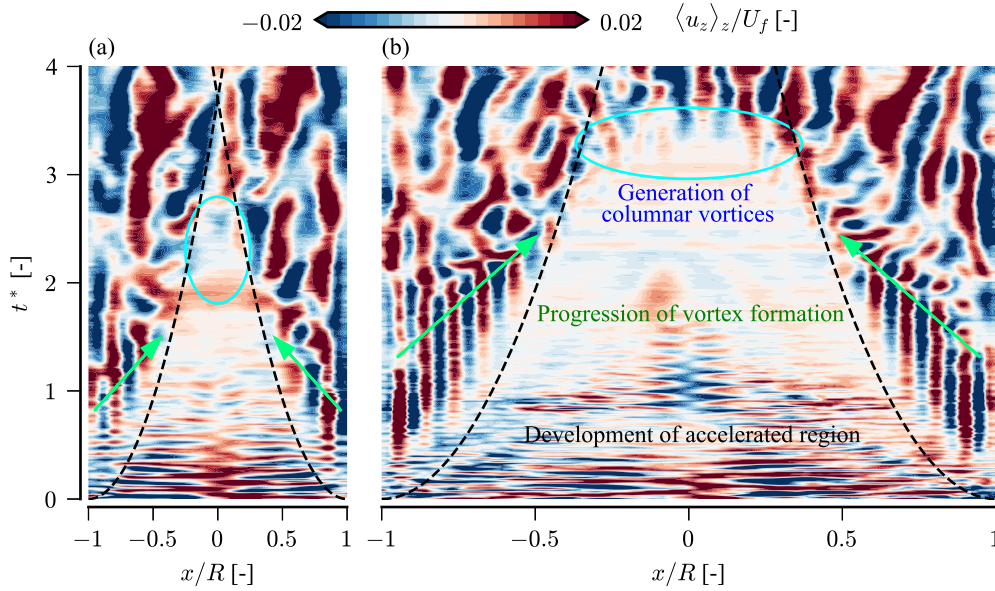


Figure 3.14: Time evolution of the vertically averaged vertical velocity $\langle u_z \rangle_z$ for $Ra = 1.0 \times 10^7$ and $Ta = 3.0 \times 10^7$ at different aspect ratios: (a) $\Gamma = 2.00$ and (b) $\Gamma = 4.75$. Time elapses from bottom to top. Dashed lines indicate the development of accelerated region due to the fluid viscosity from the sidewall expressed as $\delta_\nu \propto \sqrt{\nu t}$ with arbitrary scaling factors. Arrows from the wall to the center indicate the progression of the vortex formation induced by the KH instability to be discussed in section 3.4. Regions enclosed by ellipses indicate the generation of the columnar vortices due to separation of the thermal boundary layer.

Some time after the roll formation, the red and blue columns in Fig. 3.14 start to fluctuate and become disorganized, which is apparent from the discontinuity of the columns along the time (vertical) axis. As detailed in section 3.3.2, the convection roll thickness and positions change little until their collapse (at $t^* \sim 1.2$ for the outer rolls in the displayed cases). In Fig. 3.10 and 3.14, the inner region where no rolls form appear to be organized by small, turbulent structures during the wall-to-center progression of the roll formation. Viscous dissipation in the interior region, at which the rigid-body rotation has not been achieved, can be considered as too strong for the separation of the thermal boundary layer. This results in the creation of convection rolls being suppressed, and only few organized colored columns are seen at the interior region in Fig. 3.14.

3.4 Formation of azimuthally aligned vortices

3.4.1 Instability to induce vortical structures

As reported elsewhere, concentric roll structures, rings or spirals, formed at the early stage of the spin-up of the thermal convection subsequently collapses into vortical structures as shown in Fig. 3.6 and 3.7. Boubnov and Golitsyn [22] indicated the cause of the emergence of these azimuthally aligned vortices as instances of Kelvin–Helmholtz (KH) instability. Vorobieff and Ecke [198] showed this by PIV analysis. Zhong et al. [220] also conducted PIV analysis to

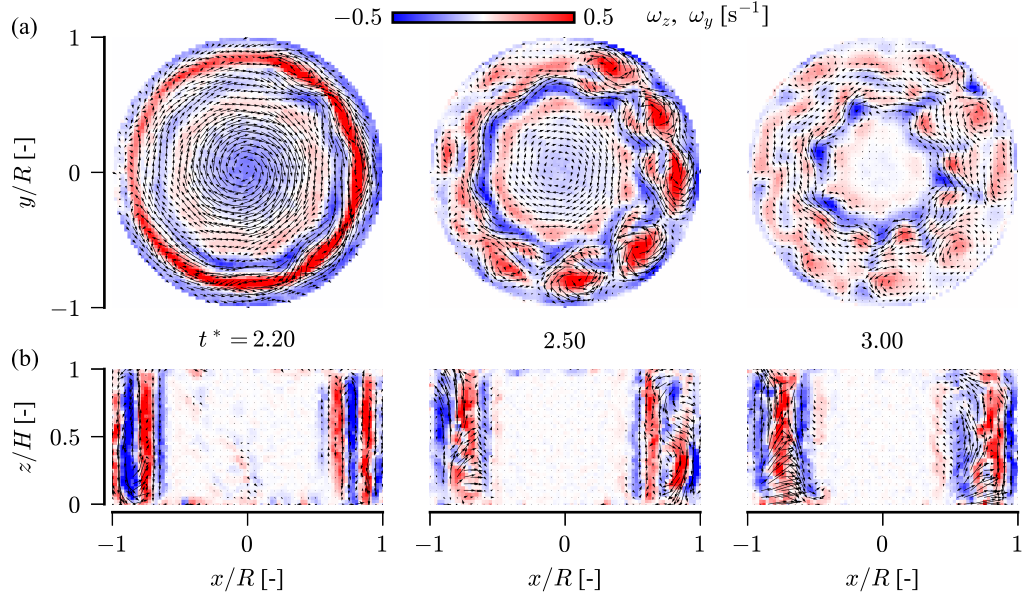


Figure 3.15: Velocity and vorticity fields obtained by PIV analysis for $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$, and $Ta = 3.0 \times 10^7$ in the situation with vortex formation: (a) u_x-u_y vectors (black arrows) plotted on the out-of-plane vorticity ω_z (colored contours) at $z = 0.5H$. (b) u_x-u_z vectors (black arrows) plotted on the out-of-plane vorticity ω_y (colored contours) at $y = 0.0$. Time elapses from left to right and is indicated between the top and bottom panels. One of two vectors is plotted in any directions for visibility and the vector sizes are adjusted in each panel to enhance clarity.

provide a quantitative explanation of the emergence of the KH instability. Intuitively these explanations are plausible, however, it is necessary to show proof of the KH instability such as in the existence of inflection points on the radial profiles of the azimuthal velocity, something which has not been reported as yet. In the present study the spatial resolution of the PIV grids is sufficiently fine (1 mm) to resolve the vortical structures induced by the KH instability $O(10^1 \text{ mm})$. Examples of velocity vector fields both in the horizontal and the vertical slices at the time around the emergence of the vortical structures are shown in Fig. 3.15. As in Fig. 3.15(a), rings of different sign vorticity are aligned alternately from the sidewall to the center at $t^* = 2.20$. The fluctuations of the rings increase with radial amplitudes and grow gradually into vortical structures at $t^* = 2.20$ and 2.50. This fluctuation pattern propagates in the azimuthal direction from the first fluctuating position (right side in the figure) and eventually azimuthally aligned vortices are formed at $t^* = 2.50$ and 3.00. In the vertical cross-section, it is possible to see vertically organized patterns of the vorticity ω_y at $t^* = 2.20$. Away from the outer regions forming the convection rolls, fluid is stagnant in the interior region. With time elapsed, the flow directions vary within the rolls expressed by the same signed vorticity columns. These results indicate that the convection roll structures collapse into vortices aligned in the azimuthal direction.

To prove the occurrence of the KH instability, radial inflection points $\tilde{r}$ of azimuthal velocity $u_\theta = -u_x \sin \theta + u_y \cos \theta$, where $\partial^2 u_\theta / \partial r^2 = 0$, should be explored quantitatively. For this, using the azimuthal velocity in an absolute system $U_\theta = u_\theta + r\Omega$ is appropriate. This is because

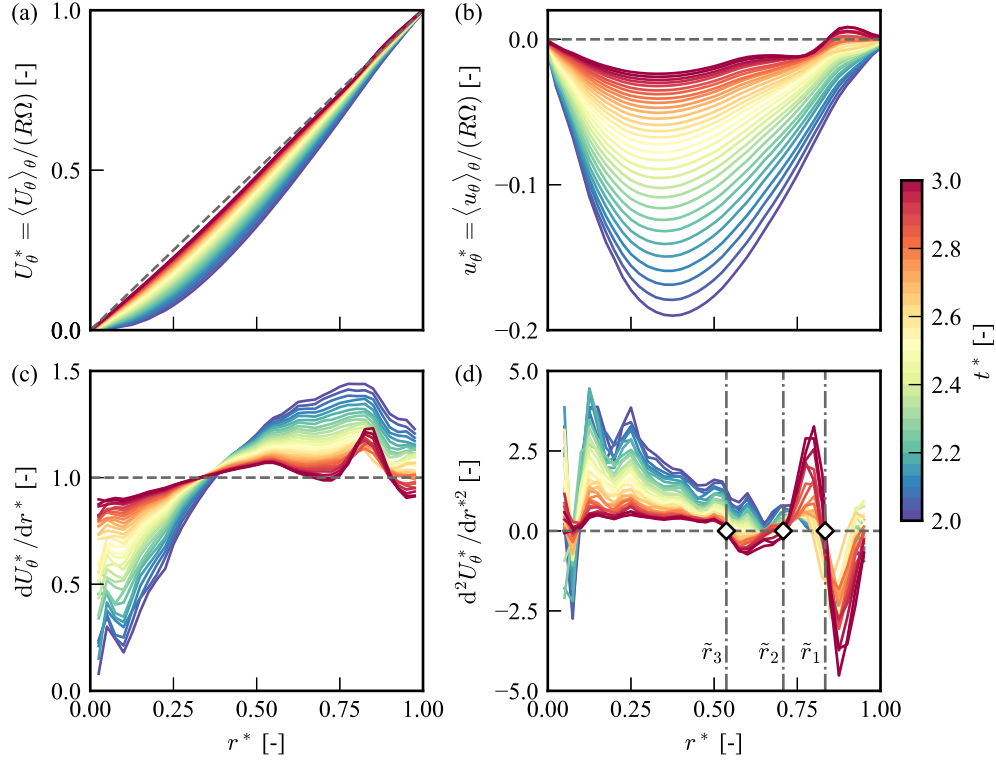


Figure 3.16: Radial profiles of velocity and its derivatives obtained by PIV analysis for $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$ and $Ta = 3.0 \times 10^7$ at $z = 0.5H$: (a) the averaged azimuthal velocity U_θ^* in the absolute coordinate system plotted with the rigid-body rotation velocity profile (dashed gray line); (b) the averaged azimuthal velocity u_θ^* in the rotational (PIV) coordinate system plotted with the rigid-body rotation velocity profile (dashed gray line); (c) the first derivative of U_θ^* differentiated by the nondimensional radius r^* indicating the slope of (a) plotted with the system angular velocity (dashed gray line); (d) the second derivative of U_θ^* differentiated by r^* . Zero-crossing points $\tilde{r}$ indicated as diamonds correspond to the inflection points of U_θ^* . Three inflection points indicated by the vertical dashed-dotted lines are numbered in order from the sidewall. Time evolution is displayed using color coding.

the KH instability occurs in the absolute system with a small perturbation affecting the rigid-body rotation. In the horizontal cross-section, azimuthally-averaged profiles of the azimuthal velocity $\langle U_\theta \rangle_\theta$ is calculated according to,

$$\langle U_\theta \rangle_\theta = \frac{1}{2\pi} \int_0^{2\pi} U_\theta(r, \theta) d\theta, \quad (3.8)$$

and shown in Fig. 3.16(a). In the calculation of U_θ , the velocity vectors in the Cartesian grid are assigned to the nearest one-dimensional radial array made every 1 mm from $r = 0$ to R . This conversion involving the azimuthal averaging improves the apparent velocity resolution of the PIV, and makes it possible to obtain the second derivative of $\langle U_\theta \rangle$ for indicating inflection points. Further, slight differences in velocity profiles originating from the two different roll types, ring and spiral, become negligible by this azimuthal averaging, and the two types can be treated equally in the subsequent analysis. In Fig. 3.16(a) $\langle U_\theta \rangle_\theta$ is normalized by the sidewall velocity $R\Omega$ and represented as U_θ^* . Also, the radial coordinate r is normalized by the cylinder radius R and represented as r^* . The curves of $\langle U_\theta \rangle_\theta$ in the figure eventually approach linearity with a slope of Ω (indicated as the dashed gray line), and thus the flow can be considered to approach rigid-body rotation. Especially at the vicinity of the sidewall $r^* \sim 1$, U_θ^* matches well with the velocity of the rigid-body rotation at $t^* = 3.0$. For comparison, azimuthally-averaged velocity profiles in the rotational system $\langle u_\theta \rangle_\theta = \langle U_\theta \rangle_\theta - r\Omega$ are also shown in Fig. 3.16(b). Here a similar normalization to U_θ^* is adopted for $\langle u_\theta \rangle_\theta$ and represented as u_θ^* . Unlike in Fig. 3.16(a), the curves in Fig. 3.16(b) start to fluctuate especially around the sidewall when the time approaches $t^* \sim 3.0$. By differentiating U_θ^* by r^* , it is possible to obtain the slope of U_θ^* , as shown in Fig. 3.16(c). The dashed gray line in the figure shows $dU_\theta^*/dr^* = 1$ representing $U_\theta/r = \Omega$. Further, the second derivative of U_θ^* with r^* displayed in Fig. 3.16(d) proves that there are inflection points as indicated by the diamonds, where $d^2U_\theta^*/dr^{*2} = 0$. In the displayed condition in Fig. 3.16, there are three inflection points, and the radial positions of the inflection points $\tilde{r}$ are found to correspond to the positions of up-welling and down-welling flows, i.e. the roll boundaries. These results provide proof for the occurrence of the KH instability from the typical characteristics of the velocity profiles.

3.4.2 Wavenumbers and wavelengths of vortical structures

Because the vortical structures originating from the KH instability are generated along the boundaries of concentric rolls, the wavenumber of the KH instability can be determined from the number of vortices formed along the circumferences on the inflection points $r = \tilde{r}$. As shown in Fig. 3.15(a), a vortex can be identified from the spatial distributions of ω_z showing azimuthally aligned patterns at $t^* = 2.50$ and 3.00 . To count the number of vortices, a fast Fourier transform (FFT) analysis for the azimuthal profiles of ω_z along the circumferences at $r = \tilde{r}$ is useful, because the prominent power spectrum is expected to appear at the wavenumber corresponding

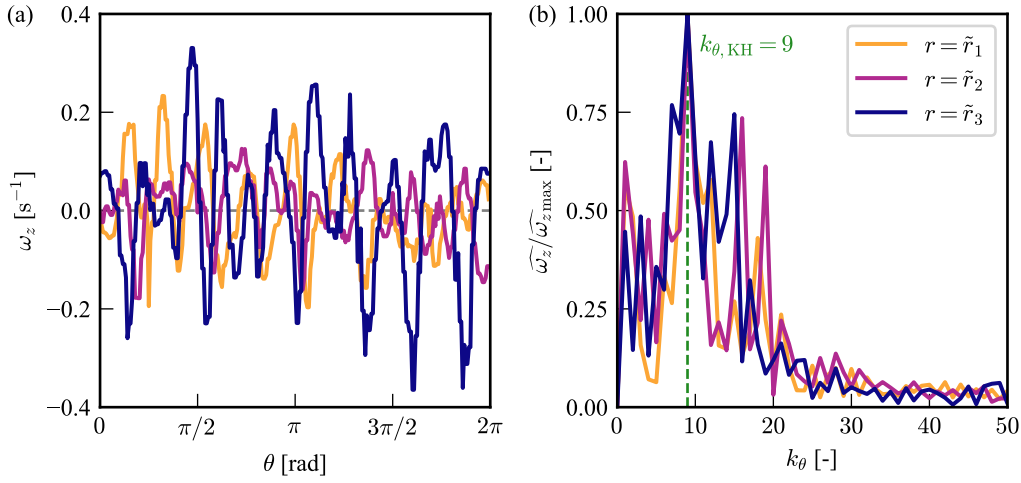


Figure 3.17: (a) Azimuthal profiles of ω_z along the circumferences at the three inflection points $r = \tilde{r}_1, \tilde{r}_2$ and $\tilde{r}_3$, and (b) the corresponding power spectra $\widehat{\omega}_z$ obtained by the FFT analysis with $\Gamma = 2.00$, $Ra = 2.0 \times 10^6$, and $Ta = 3.0 \times 10^7$. The power spectra are normalized by the maximum values of each power spectrum $\widehat{\omega}_{z,\max}$ for better visibility. Vertical dashed line is the azimuthal wavenumber $k_\theta = 9$ showing the maximum values of each power spectrum $\widehat{\omega}_{z,\max}$, and is regarded as the wavenumber of the KH instability $k_{\theta,KH}$.

to the number of vortices. In Fig. 3.16(d), three inflection points are identified as indicated by diamonds, and the radial positions of the inflection points are labeled in order as $\tilde{r}_1, \tilde{r}_2$, and $\tilde{r}_3$ from the wall towards the center. The three azimuthal profiles of ω_z along the circumferences at $r = \tilde{r}$ are shown in Fig. 3.17(a), and the corresponding power spectra $\widehat{\omega}_z$ obtained by an FFT analysis are shown in Fig. 3.17(b). In Fig. 3.17(b) the power spectra are normalized by the various maximum values $\widehat{\omega}_{z,\max}$ for better visibility. The azimuthal wavenumbers k_θ which show $\widehat{\omega}_{z,\max}$ can be regarded as the wavenumber of the KH instability $k_{\theta,KH}$, and are identical to $k_\theta = 9$ for all the inflection points.

The wavenumbers of the KH instability $k_{\theta,KH}$ for all the explored parameters are shown in Fig. 3.18(a). The larger aspect ratio $\Gamma = 4.75$ yields larger values of $k_{\theta,KH}$ than those in $\Gamma = 2.00$. For both the Γ values, $k_{\theta,KH}$ decreases monotonically with increases in the Ro values. The decreasing tendencies were approximated by the least-squares method of $k_{\theta,KH} = 4.4Ro^{-0.35}$ ($\Gamma = 2.00$) and $k_{\theta,KH} = 13.2Ro^{-0.37}$ ($\Gamma = 4.75$), and are shown by the dashed and dashed-dotted lines respectively in Fig. 3.18(a). These approximations suggest that $k_{\theta,KH}$ depends on only the Ro values, regardless of the radial positions and thicknesses of the rolls.

Next a reason for the Ro dependence of $k_{\theta,KH}$ will be considered. According to Drazin and Reid [42], in the classical problem setting of the KH instability considering vertically stratified fluids with different densities under the influence of gravity, the wavenumber k_{KH} of the KH instability is expressed as

$$k_{KH} = \frac{g(\rho_1^2 - \rho_2^2)}{\rho_1 \rho_2 (U_1 - U_2)^2}, \quad (3.9)$$

with U_1 and U_2 the horizontal velocities at the density interface of two stratified fluids with densities of ρ_1 and ρ_2 . For the discussion on the present experimental results it is simply assumed

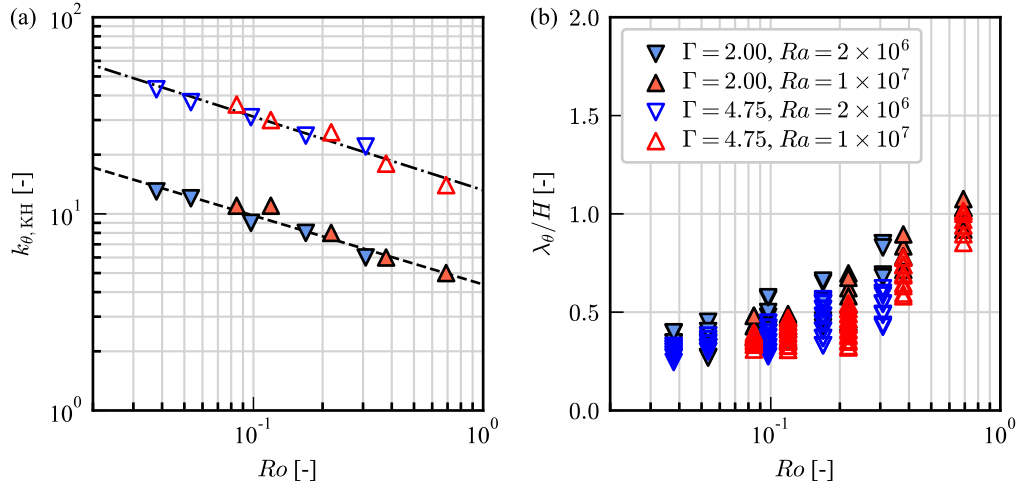


Figure 3.18: (a) Wavenumber of the KH instability $k_{\theta, \text{KH}}$ and (b) the azimuthal wavelength of the KH instability λ_{θ} for all the inflection points. Filled and open symbols indicate the case with $\Gamma = 2.00$, and with $\Gamma = 4.75$, respectively. Inverse and regular triangles are for $Ra = 2.0 \times 10^6$, and $Ra = 1.0 \times 10^7$, respectively. The dashed and the dashed-dotted lines displayed in (a) are the least-squares fits of $k_{\theta, \text{KH}} = 4.4Ro^{-0.35}$ and $k_{\theta, \text{KH}} = 13.2Ro^{-0.37}$, respectively for $\Gamma = 2.00$ and 4.75 .

that the KH instability happens in radially stratified fluids with different densities under the influence of centrifugal forces, the gravitational acceleration g in the classical KH problem can be replaced by the centrifugal acceleration $r\Omega^2$. With this, the azimuthal wavenumber $k_{\theta, \text{KH}}$ of the KH instability can be written as

$$\frac{k_{\theta, \text{KH}}}{2\pi r} = \frac{r\Omega^2 (\rho_1^2 - \rho_2^2)}{\rho_1 \rho_2 (U_{\theta 1} - U_{\theta 2})^2}, \quad (3.10)$$

here $U_{\theta 1}$ and $U_{\theta 2}$ are the azimuthal velocities in each layer of the two fluids with densities of ρ_1 and ρ_2 . Under the Boussinesq approximation, the density difference in the radial direction can be represented by the temperature difference ΔT in the radial direction as $|\rho_1 - \rho_2| = \Delta\rho = \rho_0\beta\Delta T$, where ρ_0 is the reference density. Considering the meridional circulation of convection rolls, ΔT can be replaced by an Ra value because the temperature difference in the radial direction and that in the vertical direction are identical. This allows the part related to the density in Eq. 3.10 to be expressed by an arbitrary function of Ra as $\mathcal{F}(Ra)$. The azimuthal velocity difference can be also rewritten as $|U_{\theta 1} - U_{\theta 2}| = |(r + \Delta r)\Omega - r\Omega| = \Delta r\Omega$ because the instability happens in the situation of the rigid-body rotation. Accordingly, suppose Δr is equivalent to the roll thickness δ_{roll} , and Eq. 3.10 can be modified to

$$k_{\theta, \text{KH}} \propto r^2 \mathcal{F}(Ra) \delta_{\text{roll}}^{-2}. \quad (3.11)$$

As mentioned in section 3.3.2, δ_{roll} is approximated by $\delta_{\text{roll}} \sim -Ta^{\alpha}r$, where α is a scaling index [$\alpha \sim -0.7$ in the present case as indicated in Fig. 3.12(b)]. Incorporating all of the assumptions,

the wavenumber is simply written as

$$k_{\theta, \text{KH}} \propto r^2 \mathcal{F}(Ra) (Ta^\alpha r)^{-2} = \mathcal{G}(Ra, Ta). \quad (3.12)$$

The dependence of $k_{\theta, \text{KH}}$ on the radial position is canceled out, and $k_{\theta, \text{KH}}$ can be expressed as an arbitrary function $\mathcal{G}$ of Ra and Ta . For a specific Pr value, the Ra and Ta values can be replaced by the Ro value as shown in Fig. 3.18(a).

The azimuthal wavelength of the KH instability λ_θ , the horizontal scale of the vortical structures, can be calculated at each inflection point $\tilde{r}$ using $k_{\theta, \text{KH}}$ as

$$\lambda_\theta = \frac{2\pi\tilde{r}}{k_{\theta, \text{KH}}}, \quad (3.13)$$

and this is plotted in Fig. 3.18(b). In the figure, λ_θ increases as Ro values increase with considerable deviations however. As mentioned above, $k_{\theta, \text{KH}}$ has a unique value independent of the radial position, and thus λ_θ decreases when moving radially inward into the fluid layer.

The deviations in each Ro value in Fig. 3.18(b) indicate the dependence of λ_θ on the radial position as shown in Fig. 3.19. Here the λ_θ values are normalized by the horizontal scale of the columnar vortices λ_{th} in an equilibrium state of the rotating RBC estimated theoretically by Sakai [165]. There λ_{th} is derived as

$$\lambda_{\text{th}} = 4.6H \frac{\delta_t^2}{2\delta_t^2 + 2\delta_t\delta_E + \delta_E^2} Ra^{1/3} Ta^{-3/8}, \quad (3.14)$$

where δ_t and δ_E are the thickness of the thermal boundary layer and the Ekman layer, respectively. The applicability of λ_{th} in describing the experimental data has been confirmed previously in [50, 122, 123, 165]. In Fig. 3.19, λ_θ takes smaller values at more inner positions, but does not exceed λ_{th} even with the smallest cases around $r^* = 0.5$. The markers in Fig. 3.19 disappear at $r^* < 0.5$, at the intermediate-radius of the fluid layer, as no distinct inflection points could be observed at $r^* < 0.5$. This is also apparent in Fig. 3.14 where the formations of convection rolls shown as the organized vertical columns is not present at $|x|/R < 0.5$. Here the λ_θ represents the length in the azimuthal direction regardless of that in the radial direction. As observed in Fig. 3.6, 3.7 and 3.15, the shapes of the vortices induced by the KH instability are elliptic at the vicinity of the sidewalls and closer to circular in radially further inward positions of the fluid layer. This suggests that the wavelength of a KH instability is always larger than that of the columnar vortices formed in an equilibrium state of rotating RBC till the instability expires for $r^* < 0.5$.

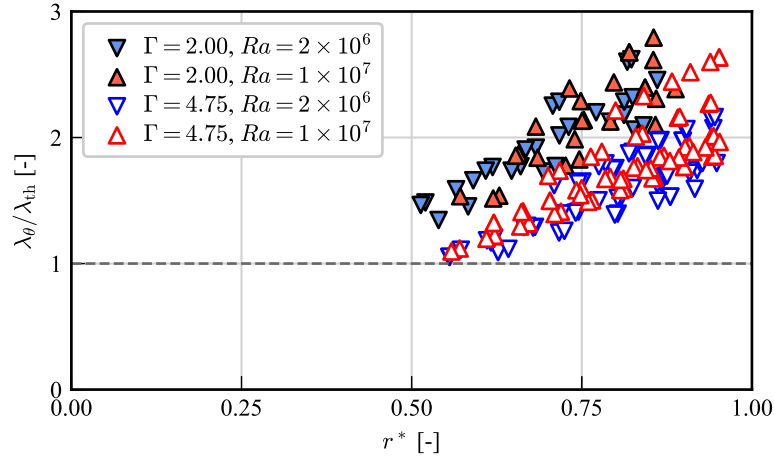


Figure 3.19: Radial distributions of the horizontal scale of the vortical structures as the azimuthal wavelength of the KH instability λ_θ normalized by the theoretically derived horizontal scale of the convection cell λ_{th} under an equilibrium state of rotating RBC by [165]. Filled and opened symbols are for the case with $\Gamma = 2.00$, and with $\Gamma = 4.75$, respectively. Inverse and regular triangles indicate $Ra = 2.0 \times 10^6$, and $Ra = 1.0 \times 10^7$, respectively.

3.4.3 Wall-to-center propagation of vortex formation

The convection rolls maintain their structures for certain periods in the spin-up process and are broken because of the onset of the KH instability. This is expressed by the disappearance of the organized columns representing the convection rolls in Fig. 3.14. The onset of the KH instability and subsequent formation of the vortical structures progressively propagate from the sidewall towards the center as indicated by green arrows in Fig. 3.14, and this trend is similar to the wall-to-center progression of the roll formations detailed in section 3.3.3. As expressed by the shorter lengths of the organized columns at the internal positions in Fig. 3.14, the lifetime of convection rolls becomes shorter as the positions of the convection rolls place radially inward. Considering these results, it may be conducted that the vortex formation progresses faster than that of the convection roll formation.

This difference in the speed of progression of vortex and roll could be explained by the growth rate of the KH instability. The azimuthal wavenumber $k_{\theta, KH}$ of the KH instability is determined only by Ra and Ta regardless of the radial positions as mentioned in section 3.4.2. The growth rate of the classical KH instability is defined by the wavelength; perturbations with shorter wavelength grow more rapidly [42]. In cylindrical coordinates, the wavelength is calculated using the corresponding length of the circumference at the radial position as $\lambda_\theta = 2\pi r / k_{\theta, KH}$. That is, for a fixed azimuthal wavenumber, the growth rates of the KH instability accelerate as the radial positions locate increasingly inward in the particular system.

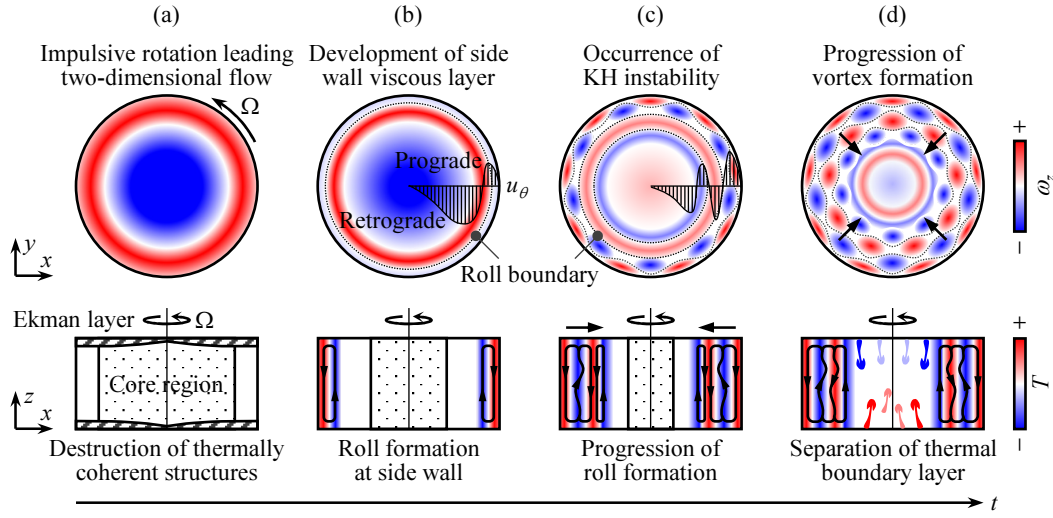


Figure 3.20: Schematic illustrations of transient behaviors during spin-up of rotating RBC, where time elapse from left to right: (a) Soon after imposing the rotation; (b) roll formation at which the rigid-body rotation achieved; (c) vortex formation induced by the KH instability and progression of roll formation towards the center; (d) progression of vortex formation and separation of thermal boundary layer forming columnar vortices in the interior. Each moment is illustrated as top views (top row) and side views (bottom row). Some expressions are exaggerated to enhance understanding.

3.5 Stepwise transitions from non-rotating to rotating RBC

3.5.1 Comprehensive explanation for transient behaviors

Transient behaviors during the spin-up for RBC have been quantified above by PIV analyses of both horizontal and vertical cross-sections. Incorporating all of the discussion, the mechanisms of the transient behaviors may be summarized and detailed as illustrated in Fig. 3.20.

In Fig. 3.20(a), soon after imposing rotation on a fully developed RBC not so far subject to background rotation, the existing thermal turbulent flow structures are disturbed due to the rotation. All presently existing thermally coherent structures like thermal plumes and large-scale structures disintegrate and all of the fluid is mixed. The whole of the fluid layer responds to the spin-up of any part of the fluid regardless of the initial state. After the first few revolutions of the fluid container, the Ekman boundary layers are formed at the top and bottom walls [60, 203], making the flow structure throughout very similar to a two-dimensional structure, excluding the boundary layers.

Following this [Fig. 3.20(b)], the accelerated region at the sidewall due to the fluid viscosity progressively grows towards the center of the fluid layer decreasing the size of the core region. The thickness of the accelerated region δ_v grows with time as $\delta_v \propto \sqrt{\nu t}$, and the azimuthal velocity approaches that of the rigid-body rotation velocity $U_\theta = r\Omega$. The Stewartson layers formed at the sidewall enhance the vertical fluid transports that leading onset of convection at the accelerated region, and toroidal convection rolls emerge at the sidewall. Together with the development of the accelerated region, the generation of the convection rolls progresses from the

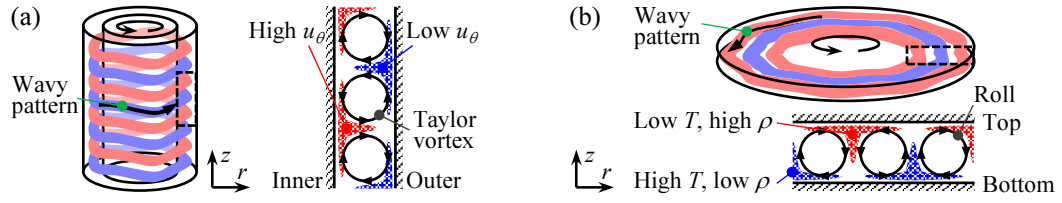


Figure 3.21: Schematic illustrations of mechanisms for inducing KH instability in different systems: (a) wavy Taylor vortex in Taylor–Couette flow and (b) rotating RBC (present study).

sidewall towards the center of the fluid layer as illustrated in Fig. 3.20(c).

The KH instability is induced at the roll boundaries after the emergence of the rolls and azimuthally aligned vortical structures are formed, as illustrated in the top view of Fig. 3.20(c). A similar mechanism can be seen in the generation of wavy vortex flows in a Taylor–Couette flow driven by a rotating inner cylinder mentioned elsewhere [10, 41, 79, 104].

The vortex formations propagate progressively from the sidewall towards the center of the fluid layer as illustrated in Fig. 3.20(d). As mentioned in section 3.4.2, the number of the azimuthally aligned vortices is independent of the radial positions. Accordingly, the azimuthal wavelength of the vortices decreases gradually as they are formed at radially internal regions of the fluid layer. The wall-to-center progression of the vortex formation is eventually accelerated because fluctuations with smaller wavelengths grow faster than larger ones. When the development of the accelerated region from the sidewall is overtaken by the progression of the vortex formation, any more convection rolls do not emerge. This happens at $r \sim 0.5R$ for all the explored cases as presented in Fig. 3.19.

Once large velocity differences between the internal region away from the walls $r < 0.5R$ and the top and bottom boundaries are relaxed because of the progression of the spin-up, the thermal boundary layers at the top and bottom boundaries yield to separation into thermal plumes. These thermal plumes are developed into columnar vortices due to the Coriolis force. Accordingly, after the formation of the convection rolls has terminated, the internal region away from the walls without rolls are dominated by the columnar vortices originating in the thermal plumes. The columnar vortices provide an efficient heat transfer throughout the vortices via the Ekman pumping. The size of the internal columnar vortices is smaller than that of the outer vortices induced by the KH instability as shown in Fig. 3.19, resulting in the outer vortices gradually altering their structure to become narrower for optimal heat transfer, as suggested by Sakai [165].

3.5.2 Spin-up timescale unique in rotating RBC

As summarized in section 3.5.1, the transient behaviors in the spin-up of RBC emerge in order, and it may be assumed that correspondingly several timescales are embedded in the spin-up process.

The fastest timescale is the system rotation timescale $\tau_\Omega = O(10^{-1} \text{ s})$ in which the Ekman boundary layers are formed at the top and bottom boundaries [see Fig. 3.20(a)]. Subsequently the flow approaches the rigid-body rotation in the Ekman spin-up timescale $\tau_E = O(10^1 \text{ s})$. The accelerated region at the sidewall develops towards the center, and the thickness of the region can be estimated as $\delta_\nu \propto \sqrt{\nu t} \propto t^{1/2}$. As the accelerated region is sufficiently calm for yielding the onset of convection, toroidal convection rolls are formed at the region. In the present study, the growth of the thermal boundary layers is slower than that of the Ekman boundary layers, because the Pr number is larger than unity ($Pr = \nu/\kappa > 1$) with water as the test fluid. Accordingly, the formation of convection rolls propagates radially inwards together with the growth of the accelerated region at the sidewall. Examples for showing more prominent results can be seen in experiments performed by Koschmieder [93] used a 100 cSt silicon oil with $Pr = O(10^2)$. In his experiments, the formation of the axisymmetric convection rolls have propagated into the center of the cylindrical fluid layer with no azimuthal distortion, unlike the present study, as a high viscous fluid allowed fast propagation of the rigid-body rotation. Contrary to this, suppose a low Pr fluid is used as the test fluid, then a well-regulated progression of roll formation may not be observed, and instead oscillatory convection or the occurrence of turbulent convection dominated by thermal plumes could be expected to be initiated from the beginning [35, 95].

Vortical structures are generated at the roll boundaries by the KH instability, and the formation of these progresses from the sidewall towards the center. The wall-to-center progression of the vortex formation is faster than that of the roll formation because the instability grows faster as it happens at the radially internal positions. In the further internal region $r < 0.5R$, no convection rolls form because the wall-to-center progression of the roll formation is overtaken by that of the vortex formation.

Instead of roll formation, thermal plumes forming columnar vortices are generated as there is separation of the thermal boundary layers in the internal region. As the system is initially unstable, the local Ra values within the thermal boundary layers are larger than the Ra_c values. However, large velocity differences between the core region at which not accelerated by the viscosity and the top and bottom boundary layers suppress the separation of the thermal boundary layers, and the boundary layers may be super-critical states. Once the velocity differences are sufficiently relaxed together with the development of the accelerated region, the separation of the thermal boundary layers forms thermal plumes. That is, the onset of rotating convection is abruptly realized with time. As if to back up this simple explanation, the columnar vortices suddenly appear in the internal region ($r < 0.5R$) as indicated by the ellipses in Fig. 3.14, once after the strong 2D flow in the core region becomes negligible. In particular, the sudden appearance of the columnar vortices is more visible in the larger Γ case [Fig. 3.14(b)]. This plume formation in the interior region may be possible to observe when thermal forces are imposed on a rigidly rotating fluid.

Comparing Fig. 3.14(a) and (b), the whole of the stepwise transitions discussed above are

relatively ambiguously appearing in the smaller Γ cylinder. Regarding the aspect ratios of the fluid vessels, $0.656 \leq \Gamma \leq 2.00$, used elsewhere [22, 198, 220], the stepwise transitions might be difficult to observe. That is, the transient behaviors that occur in a stepwise fashion during the spin-up of rotating RBC are established because of the use of the fluid layer with the moderate aspect ratio, $\Gamma = 4.75$ in the present study. Incorporating all the transient behaviors occurring in order together with the development of the accelerated region from the sidewall, the spin-up timescale unique to rotating RBC should be a few to ten times longer than the Ekman spin-up timescale τ_E .

3.6 Concluding remarks

To elucidate details of transient behaviors during spin-up of rotating RBC, systematic laboratory experiments using water as the test fluid were conducted by imposing a rotation on a fully developed thermal convection with Rayleigh numbers $Ra = 2.0 \times 10^6$ and 1.0×10^7 . The Taylor number $3.0 \times 10^6 \leq Ta \leq 2.0 \times 10^8$ corresponding to the convective Rossby number $0.0378 \leq Ro \leq 0.690$ was given. Stepwise transitions from an RBC not so far subject to background rotation to a rotating RBC were observed through individually performed visualization experiments in horizontal and vertical cross-sections of two different aspect ratio cylinders. By means of PIV both the horizontal and vertical flow fields were measured and a quantitative understanding was made possible. Soon after imposing the rotation, all thermal plumes or larger coherent structures appearing as a result of thermal convection disintegrated due to the intense viscous dissipation of the rotation. The resulting flow state became identical to that of spin-up from rest without thermal forcing. Together with the growth of accelerated region due to the fluid viscosity, formation of the convection rolls forming ring or spiral structures propagate from the sidewall towards the center coinciding with the rotational axis of the fluid layer. Meridional circulations of the convection rolls transport low and high density fluid from the bottom and top boundaries to the interior. As a result, high, but opposite azimuthal momentum is aligned alternately in the radial direction, which develops prograde and retrograde fluid motions in the rotating system. Moreover, alternate radial density distributions are generated due to the existence of the toroidal convection rolls. Accordingly the KH instability evolves gradually in the azimuthal direction with radial amplitude at which the vertical flows, roll boundaries, exist and led to the formation of azimuthally aligned vortices along the roll boundaries. Wavenumbers equivalent to the number of vortices, is identical for a fixed experimental condition regardless of the radial positions since the wavenumber of the KH instability only depends on Ro . The vortex size decreases with the progression of the instability towards the center. The minimum size of the vortex induced by the KH instability does not exceed the size of the columnar vortices formed in an equilibrium state of a rotating RBC. Wall-to-center progression of the roll formation is always caught up around the intermediate-radius of the cylinder by that of vortex

formation. At radially internal region of the fluid layer where no vortices are present due to the KH instability, separation of the thermal boundary layers occurs and thermal plumes are emitted to form columnar vortices. Vortices induced by the KH instability alter the horizontal size to become narrower so as to carry optimal heat transfer of an equilibrium state of rotating RBC. Finally the timescales dominating all the transient behaviors during the spin-up of rotating RBC were found to be explained by the timescales of each stepwise phenomenon.

VORTEX DYNAMICS IN ROTATING RAYLEIGH–BÉNARD CONVECTION

Preface

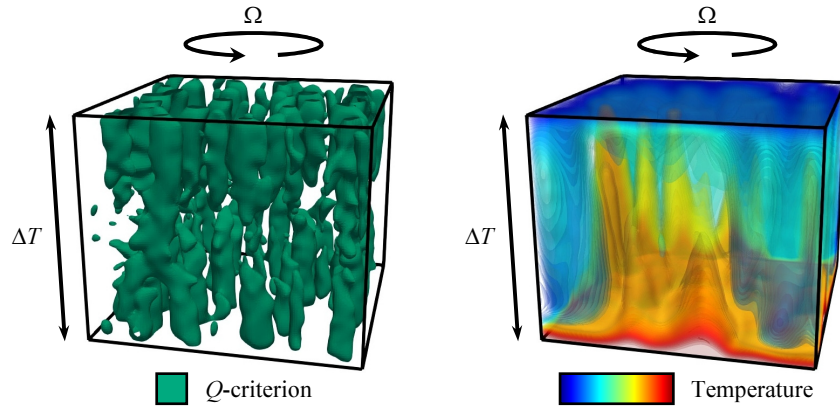


Figure 4.1: Graphical abstract of this chapter.

In Chapter 2 and 3, the TLC particles have been successfully utilized as a flow visualizer to capture fluid motions of thermal convection entailing various timescale physics. Even though the coloration of the TLC particles is only utilized as supporting information about temperature, it is highly informative to consider the heat transport together with the quantified velocity fields.

In Chapter 4, the color optical measurements cultivated so far in Part II are sublimated as a temperature measurement method. That is, the color information scattered by the TLC particles dispersed into the fluid is utilized to quantify the temperature fields of the fluid layer. The target phenomenon dealt with in Chapter 4 is rotating Rayleigh–Bénard convection (RBC). It involves various timescale physics, and has been studied from the perspective of fluid dynamics, engineering, geo-/astrophysics, and so on. Conventionally, rotating RBC has been studied to

characterize the heat transport characteristics. The heat transport, however, has often been considered as a bulk trend of the whole system, and the local effects have been omitted. Under a condition at which the thermal and rotational forces are balanced, rotating RBC is dominated by interesting flow structures, so called columnar vortices. The vortices possess a vertical coherent structure stretched along the rotational axis of the system, and sometimes maintains their structure for a long time of $O(10^2 \text{ s})$. Accordingly, the heat transport carried out by such a long-living columnar vortex should produce a local trend in the global heat transport. In addition, the columnar vortices tend to move in the horizontal extent in a random-walk-like fashion. However, such a local heat transport characteristic of a single columnar vortex has not been focused, and worth investigating. To experimentally observe the vortex dynamics and its heat transport characteristic, simultaneous 3D-3C velocity and 3D temperature field measurements are performed. Experiments were held in a rectangular fluid vessel using water as the test fluids. A scanning system for stereoscopic PIV and liquid crystal thermometry (LCT) was constructed. The constructed system yields to simultaneously measure quasi-instantaneous 3D velocity and temperature fields by a single scan with a spatial resolution of 1 mm in all the directions and a temporal resolution of 22 s. For the LCT, the artificial neural network was utilized to associate the multiple color information taken by the two color cameras and the temperature. As a result, the temperature fields could be measured with an accuracy better than $\pm 0.2 \text{ K}$ in the whole measurement domain and for the temperature range of 5 K.

The constructed system successfully reconstructed the 3D velocity and the temperature fields of rotating RBC; with increasing the rotational force, the vertically coherent structures of the columnar vortices stood out. The measured quantities were validated by the statistical analysis. From the quantified flow fields, the columnar vortices were identified. The detected vortices were tracked in the horizontal domain, and the random-walk-like motions of the columnar vortices were clarified. The heat transport characteristics carried by the single columnar vortices were also studied, and the vertical heat transport by the Ekman pumping was identified as the important factor. The vortex dynamics and the heat transport characteristics were then associated by computing the ensemble average of the temporal signals of the vortex advection speed and the Nusselt number carried by the vortex. From the cross-correlation of these two signals, the heat transport carried by the vortex was found to decrease immediately after the large displacement. Incorporating all the results, the random-walk-like motions of the columnar vortices were explained by the balance of the two opposing effects related to the thermal boundary layers. One is the suppression of the growth of the thermal boundary layer at the no-vortex regions, as the Ekman pumping at the vortex-occupying region sucks the heated and cooled fluids from the surrounding no-vortex regions. The other is growth of the thermal boundary layer due to the constant heating and cooling at the bottom and the top boundaries. The mechanism of the vortex dynamics was partly explained by investigating both the dynamics and the heat transport characteristics of the single columnar vortices.

4.1 Introduction

Rotating Rayleigh–Bénard convection (RBC), which is often considered as a system imposed a background rotation along the vertical axis, has been one of the most active research topic in the field of fluid dynamics. In particular, rotating RBC is a key phenomenon to understand large-scale fluid motions on the Earth, e.g. oceanic, atmospheric flows, liquid metal convection inside the outer core of the Earth [57, 80]. Rotating RBC has attracted a number of researchers all over the worlds especially from the perspectives of heat transport characteristics and flow morphology. One of the interesting phenomenon of rotating RBC is the change of heat transport efficiency with change of rotational speed of the system. It is natural and imaginable that the rotation suppresses the vertical heat transport due to its horizontal constraint, and the onset of convection itself becomes hard to be reached due to the stabilizing effect of rotation [27]. However, by imposing a moderate rotation, the Ekman pumping facilitates vertical flows and accompanying heat transports [89, 98, 188, 204]. For instance, the experimental and numerical results by Stevens et al. [188] suggested that the enhancement of heat transport becomes up to $\sim 20\%$ in the case of water compared to non-rotating RBC. Further increase in rotational speed will weaken the convection itself, and finally the convection will cease with a too fast rotation. The other fascinating feature of rotating RBC is its flow morphology, and various flow structures appear together with the change in the heat transport characteristics described above [28]. For the cases of water, the flow structures are categorized into four regimes: with increasing the rotational speed, 3D turbulence, vortical plumes, convective Taylor columns (CTCs), and cellular regimes are identified by the variations in heat transport characteristics [98, 189, 206]. Especially under the conditions at which the thermal and rotational forces are balanced, i.e., sufficiently rapid rotation is imposed, a number of columnar vortices called vortical plumes or CTCs are formed in the whole fluid layer due to the Taylor–Proudman constraint [63, 82, 118]. The columnar vortices possess vertically coherent structures, which are stretched along the vertical (rotational) axis of the system.

The columnar vortices formed in rotating RBC carry vertical heat transports, since these structures are originated from thermal plumes detached from the thermal boundary layers. These vortices show horizontal translational motions with keeping their vertically coherent structures. These motions were considered to be chaotic motions in the numerical studies by Julien et al. [81]. Quantitative studies on the vortex motions have been left until the experimental studies in Noto et al. [122, 123]. These studies found that the motions of the columnar vortices can be described as the diffusive motions, i.e., random-walk-like or Brownian motions. The diffusive motions are regarded as anomalous diffusive motions, which showing different kinds of diffusive motions in different timescales. This is because of the merging and repulsion of multiple columnar vortices. The vortices can be categorized into two types; one is up-welling vortices with high internal temperature and the other is down-welling vortices with low internal temperature. The two

types of vortices rotate in counter directions to each other at any horizontal cross-sections. The same type of vortices can merge to when their meet in the fluid layer. On the other hand, the different types of vortices cannot get merged, but repulse to each other. No split of vortices has been observed. Even though the merging does not influence the statistics of the diffusive motions, the repulsion highly restricts the normal diffusive motions. Accordingly, the vortices show sub-diffusive motions in the timescale at which the repulsion of multiple vortices occurs, $O(10^1 \text{ s})$. In addition, the significance of the experimental work conducted by Noto et al. [123] is that the influence of the centrifugal force on the vortex motions was firstly found in their long-time observations of $O(10^3 \text{ s})$. The up-welling vortices move toward the center of rotation with their smaller density, and the down-welling vortices move toward the rim of the fluid layer with their larger density. These tendencies are superficially not obvious, however, the long-time observations lead to a question against the empirical criterion for neglecting the contribution of the centrifugal force. That is, the centrifugal force should be considered appropriately in the discussion of long timescale physics. A similar research was performed in Chong et al. [30], and explored ballistic and diffusive behavior of columnar vortices in shorter timescales, $O(< 10^2 \text{ s})$.

The heat transport characteristics have been discussed as a global characteristic in terms of the Nusselt number Nu [e.g. 89, 189], i.e., the spatiotemporal averaged value. However, the heat transport should be carried out by each columnar vortex, as they originate from the thermal plumes detached from the thermal boundary layers at the horizontal boundaries. As above, the columnar vortices maintain their structures for long timescale, which sometimes becomes $O(10^2 \text{ s})$. The presence of such long-living vortices will provide local trends in heat transport characteristics of rotating RBC. Therefore, the local heat transport characteristics of individual vortices and the corresponding diffusive motions are worth studying. The present study aims to clarify the relationship between the vortex dynamics and the local heat transport characteristics via experimental investigations. To quantify both the vortex dynamics and the accompanying heat transport characteristics, 3D-3C velocity and 3D temperature fields will be measured simultaneously. As revealed in Noto et al. [122, 123], the advection speed of the columnar vortices is very slow, $O(10^{-1} \text{ mm/s})$. Accordingly, it is possible to measure quasi-instantaneous flow fields by a mechanically scanning system, without implementing volumetric measurement techniques like tomographic PIV [46] and 3D color PTV [126]. The implementation of a scanning system will generate a number of planar (2D) flow fields piled on the scanning direction. Thus, the present study develops a scanning stereoscopic PIV system to measure 3D-3C velocity fields, and scanning liquid crystal thermometry (LCT) to measure 3D temperature fields by utilizing the thermochromic liquid crystal (TLC) particles. Using the constructed measurement system, the vortex dynamics and the heat transport characteristics are elucidated.

4.2 Experiment

All the equipment, the fluid layer and the scanning system comprised of the optical device and the linear slider, was mounted on a turntable. The turntable used in this chapter is the same configuration as that used in Chapter 3. Here, the different points are mentioned as below.

4.2.1 Fluid layer

Experiments were conducted using a rectangular fluid layer with dimensions of $120 \times 120 \times 100 \text{ mm}^3$ in the x , y , and z directions as illustrated in Fig. 4.2. The sidewalls were made of acrylic plates with a thickness of 15 mm. The top of the fluid layer was closed by a cooling device with an internal water chamber yielding circulations of refrigerant supplied from an external thermostatic bath. Cooling of the fluid layer was achieved through a 1.1-mm-thick glass plate mounted at the bottom of the cooling device. The bottom of the fluid layer was closed by a black-chromate-coated copper plate with a thickness of 10 mm. Heating the fluid layer was done by placing a silicone-rubber heater beneath the copper block. With the sufficient thickness of the copper block, homogeneous heating of the fluid layer was expected. Both the cooling refrigerant supplied to the cooling device and the heater were maintained each at a constant temperature during the measurement. As a result, the temperature difference imposed vertically was maintained constantly during each measurement.

Deaerated water was employed as the test fluid, and the thermochromic liquid crystal (TLC) particles were seeded into the fluid. The TLC particles showed colors in a temperature range of 25 to 30°C according to the manufacturer. The actual coloration range, however, depends on the optical configurations, and thus *in situ* calibration is required.

4.2.2 Scanning system and optical equipment

To simultaneously measure 3D-3C velocity and 3D temperature fields, the present study employed a scanning system for the purpose of scanning stereoscopic PIV and scanning LCT. The scanning system was comprised of a linear slider driven by a stepper motor connected to a ball screw, and the motor was controlled by a micro-computer. Schematic of the scanning system is illustrated in Fig. 4.3. A cylindrical lens emitting white light sheet for illuminating the x - z plane was mounted on the scanning system. The cylindrical lens was connected to a halogen lamp to generate a white light sheet. Two CMOS color cameras were also mounted on the system to capture particle images in the illuminated plane. The two cameras were fixed with a relative angle of $\sim 40^\circ$. For each camera, the same lens with an aperture of F#2.8, a focal length of 8 mm, and a depth of field of $\sim 80 \text{ mm}$ was mounted. As the cylindrical lens and the two cameras were fully fixed on the scanning system, the relative distance between the illuminated plane and the imaging planes were always the same, $\sim 250 \text{ mm}$. The scanning system enabled translational

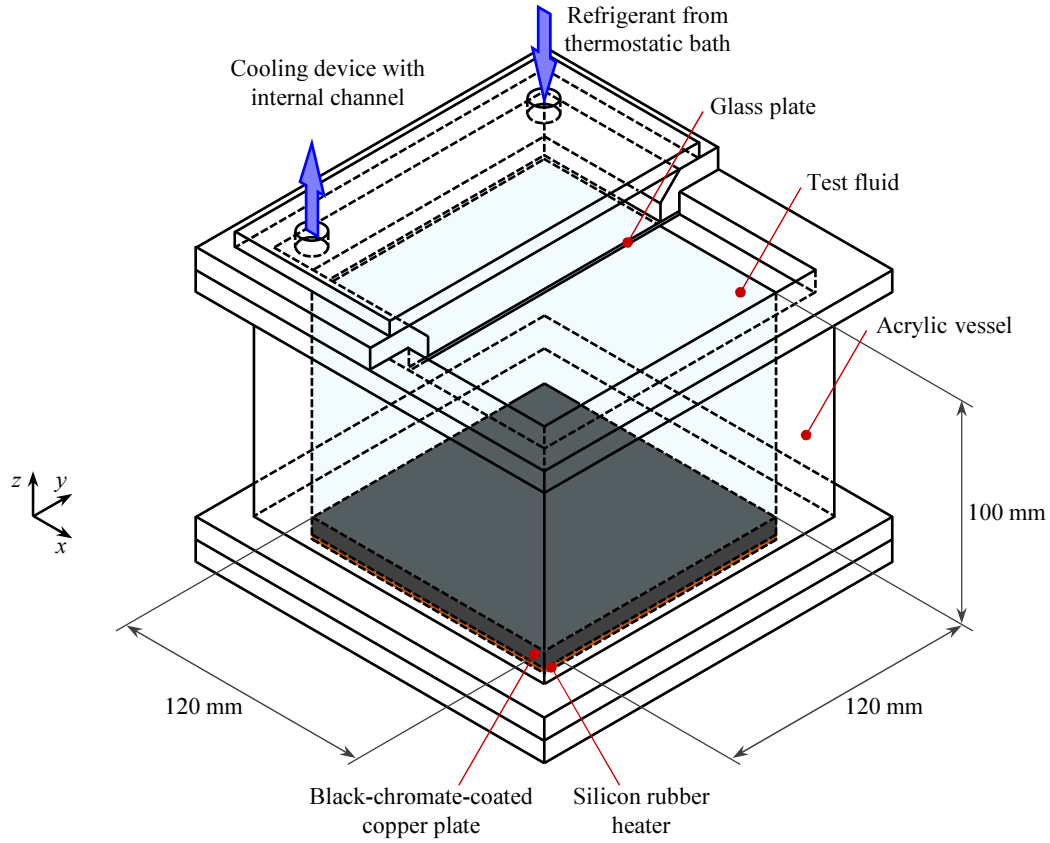


Figure 4.2: Schematic illustration of the fluid layer.

motions in the y -axis. To minimize undesirable vibrations of the two cameras during the scans, the horizontal beam mounting the cameras were supported by an additional column with a ball caster as illustrated in the lower light of Fig. 4.3.

The control strategy of the scanning system is shown in Fig. 4.4. The scanning system was controlled to be reciprocated by the micro-computer in the range of $y = -2$ mm to 122 mm, while the measurement domain lied in $y = 0$ to 120 mm. An electric signal for the rotating direction of the motor is shown in Fig. 4.4(a). Here, the signal shows a duration of two reciprocating motions. Step frequency of the motor is shown in Fig. 4.4(b). To avoid the motor stepping out due to insufficient torque, the step frequency of the motor was controlled via a trapezoidal velocity control strategy. For the forward motion, the step frequency is linearly accelerated up to 320 s^{-1} , resulting in the scanning speed of $V_{\text{scan}} = 8 \text{ mm/s}$ with the present setting. For the backward motion, the step frequency is linearly accelerated up to 1000 s^{-1} , resulting in the scanning speed of -25 mm/s , so as to reset the system as fast as possible. The sheet position driven by these control strategy is shown in Fig. 4.4(c). The image acquisitions were conducted during the forward scans. The acceleration and deceleration regions for the forward motion corresponded to the regions out of the measurement domain, $y = -2$ to 0 mm and $y = 120$ to 122 mm. The measurement domain, $y = 0$ to 120 mm, was scanned at the constant speed of $V_{\text{scan}} = 8 \text{ mm/s}$. It took 15 s for a single forward scan, and additional

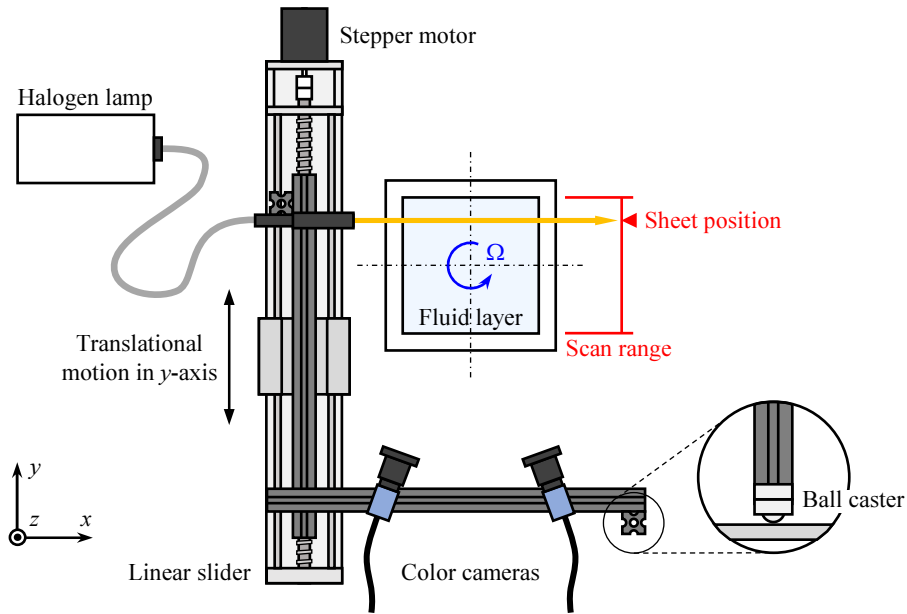


Figure 4.3: Schematic illustration of the scanning system and mounted optical equipment.

7 s was required to reset the scanning system to the initial position; 22 s was required for a single reciprocating motion. Thus, the time interval between the scans for measuring the quasi-instantaneous flow fields is $\Delta t = 22$ s. This time interval is the temporal resolution for tracking vortex motions over different quasi-instantaneous flow fields. Note that all the optical equipment and the fluid layer were mounted on the turntable, and thus the obtained flow fields are in the rotational system.

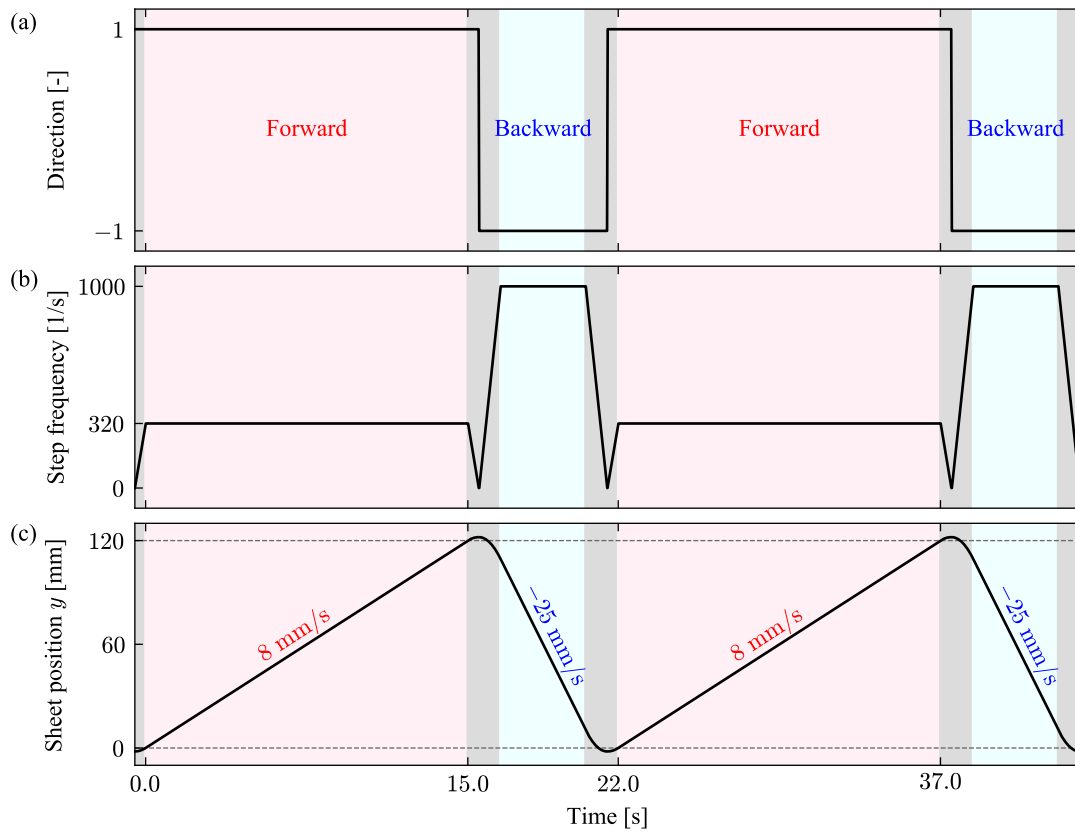


Figure 4.4: Schematic diagram of the control strategy of the linear slider: (a) Rotational direction of the stepper motor, (b) step frequency of the stepper motor with trapezoidal velocity control, and (c) resultant motion of the light sheet mounted on the linear slider. Two reciprocating motions are shown.

4.2.3 Experimental condition

To perform scanning stereoscopic PIV and scanning LCT, three measurement series were conducted; the stereoscopic calibration measurements, the temperature calibration measurements, and the rotating measurements were conducted with the same optical setup. The details of the two calibration experiments will be addressed later. Here, the details of the rotating measurements, which are the main experiments, are explained. Rotating RBC is often described by the following nondimensional parameters. The Prandtl number Pr is given as the physical property of the test fluid in the form

$$Pr = \frac{\nu}{\kappa}, \quad (4.1)$$

where ν and κ are the kinematic viscosity and the thermal diffusivity. The Rayleigh number Ra , the ratio of the thermal buoyancy to the viscous and thermal diffusion, is defined as

$$Ra = \frac{g\beta\Delta TL^3}{\kappa\nu}, \quad (4.2)$$

where g , β , ΔT , L are the gravity acceleration, the thermal expansion coefficient, the vertical temperature difference imposed on the fluid layer, and the height of the fluid layer, respectively. The effect of rotation is often described by the Taylor number Ta , the ratio of the Coriolis force to the viscous force, and it is expressed as

$$Ta = \left(\frac{2\Omega L^2}{\nu} \right)^2, \quad (4.3)$$

where Ω is the angular velocity of the imposed rotation. The effect of rotation is sometimes expressed by the Ekman number Ek as

$$Ek = \frac{\nu}{2\Omega L^2} = Ta^{-1/2}. \quad (4.4)$$

To incorporate both the thermal and rotational forces, the Rossby number Ro is defined as

$$Ro = \frac{1}{2\Omega} \sqrt{\frac{g\beta\Delta T}{L}} = \sqrt{\frac{Ra}{Pr Ta}}. \quad (4.5)$$

Apart from the Coriolis force, the centrifugal force potentially produced by the rotation is expressed as the ratio of the centrifugal force and the gravitational force and represented by the Froude number Fr as

$$Fr = \frac{W\Omega^2}{2g}. \quad (4.6)$$

According to the earlier works [27, 82, 118, 155], the supercriticality of rotating RBC is expressed by the reduced Rayleigh number $\widetilde{Ra}$ defined as

$$\widetilde{Ra} = RaEk^{3/4} = RaTa^{-3/2}. \quad (4.7)$$

The critical value for the onset of convection is considered to be $\widetilde{Ra}_c \simeq 8.6956$ [27]; the convective motion do not occur at $\widetilde{Ra}$ below this critical value.

Experimental parameters explored in the present study are shown in Fig. 4.5, and summarized in Table 4.1. Two Ra values, $Ra = 1.9 \times 10^7$ ($\sim 2 \times 10^7$) and 9.7×10^7 ($\sim 1 \times 10^8$), were examined by providing vertical temperature differences of $\Delta T = 1$ and 5 K. These parameters were considered to be sufficiently large to yield thermal turbulence dominated by a number of thermal plume emissions [25]. Five different Ta values from $Ta = 3.0 \times 10^7$ to 3.0×10^9 were examined for each Ra value. As a result, ten parameters were examined systematically following the same experimental procedure. The corresponding Ro and Fr values are also summarized in Table 4.1. The Ro values were from $Ro = 0.0326$ to 0.728, and these were expected to cover the vortical plume, convective Taylor column, and cellular regimes. The Fr values were less than 0.1 for all the conditions, and thus the effect of the centrifugal forces was negligible according to the conventional criteria [28]. According to the $\widetilde{Ra}$, the flow regimes of rotating RBC can be roughly categorized, and the predicted flow regimes after Julien et al. [82], Nieves et al. [118] are also summarized in the right-end column of Table 4.1.

For all the experimental conditions, at least 30 min were spent for ensuring sufficiently developed thermal turbulence before imposing rotation. The image acquisitions of the two cameras were initiated at non-rotating states, and the rotation was given after a single scan of the scanning system in non-rotating state. In total, the image acquisitions were conducted continuously for 150 min = 9000 s, which involving spin-up states from non-rotating to equilibrated rotating RBC. This involves ~ 400 scans of the scanning system. The image acquisitions of the two cameras were performed at 16 frames per second (f.p.s.) with a shutter speed of 1/20 s.

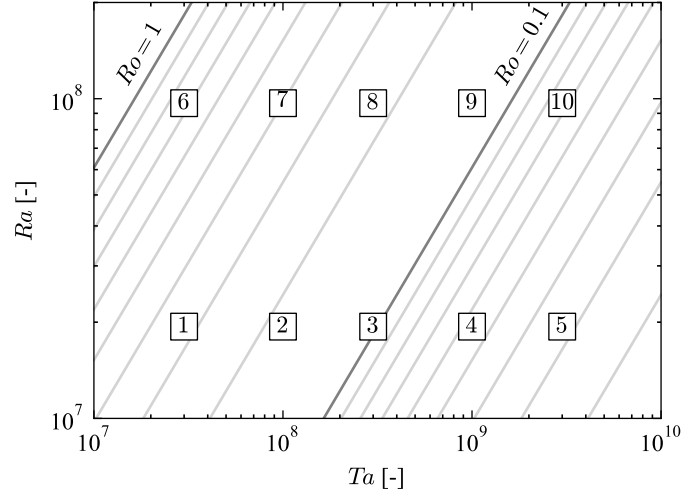


Figure 4.5: Experimental parameters explored in the present study. Labeled numbers are corresponding to the experimental conditions listed in Table 4.1.

Table 4.1: Experimental parameters explored in the rotating measurements. Fluid properties at $T = 25^\circ\text{C}$ are used for the calculations, resulting in $Pr \sim 6$. Categorization of the flow regimes is after Julien et al. [82], Nieves et al. [118]; “Turbulence”, “Plume”, “Column”, and “Cellular” correspond to the geostrophic turbulence, the vortical plumes, the convective Taylor columns, and the cellular regimes.

Case	Ra	Ta	Ek	$\widetilde{Ra}$	Ro	Fr	Regime
1	1.9×10^7	3.0×10^7	1.8×10^{-4}	197	0.326	0.000364	Plume
2		1.0×10^8	1.0×10^{-4}	88	0.178	0.00121	Plume
3		3.0×10^8	5.8×10^{-5}	42	0.103	0.00364	Column
4		1.0×10^9	3.2×10^{-5}	19	0.0564	0.0121	Cellular
5		3.0×10^9	1.8×10^{-5}	9	0.0326	0.0364	Cellular
6	9.7×10^7	3.0×10^7	1.8×10^{-4}	1005	0.728	0.000364	Turbulence
7		1.0×10^8	1.0×10^{-4}	450	0.399	0.00121	Turbulence
8		3.0×10^8	5.8×10^{-5}	216	0.230	0.00364	Plume
9		1.0×10^9	3.2×10^{-5}	97	0.126	0.0121	Plume
10		3.0×10^9	1.8×10^{-5}	47	0.0728	0.0364	Column

4.3 Calibration experiments

In this study, quasi-instantaneous 3D-3C velocity and temperature fields are measured through scanning stereoscopic PIV and LCT. The former requires stereo calibration associating real space coordinates with image coordinates, and the latter requires temperature calibration associating color with temperature of the TLC particles. In this section, details of these two calibration procedures essential for the subsequent quantitative analyses are presented.

4.3.1 Calibration for stereoscopic PIV

Stereoscopic calibration is necessary to associate 2D image coordinates $\mathbf{X} = (X, Z) = (i, j)$ with 3D physical coordinates $\mathbf{x} = (x, y, z)$. The relationship between the two coordinates is linked via a mapping function F and written as

$$\mathbf{X}^{(c)} = F^{(c)}(\mathbf{x}), \quad (4.8)$$

where $c = 1$ and 2 for the left and right cameras, respectively.

Stereoscopic calibration was conducted using a calibration plate. The calibration plate was comprised of an acrylic plate with a thickness of 1.5 mm and a vinyl-filmed paper. Predetermined patterns were printed on the paper, and the calibration pattern is shown in Fig. 4.6. In the calibration pattern, two sets of reference points were provided; one is regular dot patterns printed from $(x, z) = (10 \text{ mm}, 10 \text{ mm})$ to $(110 \text{ mm}, 90 \text{ mm})$ with 10-mm distancing, and the other is regular grid patterns (crossed points) printed from $(x, z) = (5 \text{ mm}, 5 \text{ mm})$ to $(115 \text{ mm}, 95 \text{ mm})$ with 10-mm distancing. From the calibration pattern, 219 reference points in total, 99 points from the dots and 120 points from the grids, can be utilized for the stereoscopic calibration. These reference points can be identified from binarized images via labeling and Harris's corner detection [66].

In the present study, the optical system was always actuated in the y -direction. Accordingly, the calibration should be done for all the available light sheet positions (also referred to as measurement planes). This is because the optical path through the test fluid differs in each measurement plane, and the resultant visualized images for the calibration plate are different in each plane even though the distance between the light sheet and camera positions are fixed. However, it is not realistic (or exhausting work) to conduct the stereoscopic calibration at all the available measurement planes, since the number of the planes are 241 in total with the present photographing condition (16 f.p.s. for the scanning duration of 15 s). Therefore, the mapping functions for each measurement plane are interpolated (extrapolated) after computing the mapping functions at some reference planes. The sets of calibration photographing was performed at 11 different measurement planes from $y = 10 \text{ mm}$ to 110 mm with 10-mm spacing. For each measurement plane position y_{sheet} , the relative distance of optics to the fluid vessel was

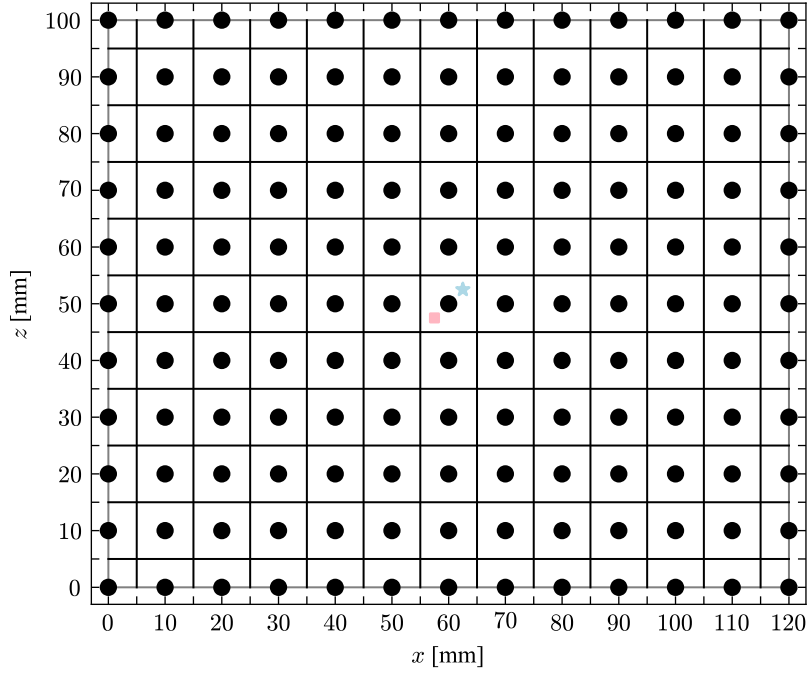


Figure 4.6: Calibration pattern printed on the calibration plate. Dots and grids, printed every 10 mm in the x and z directions, were used as the reference points. As a result, coordinates of 99 dots and 120 grids excluding the rim of the pattern were used as the reference points. The star and square symbols were printed only to identify the central dot at $(x, z) = (60 \text{ mm}, 50 \text{ mm})$.

fixed, and the printed side of the calibration plate was placed at $y = y_{\text{meas}}, y_{\text{meas}} \pm 2, y_{\text{meas}} \pm 4 \text{ mm}$, as illustrated in Fig. 4.7. That is, calibration images at five planes at different known depths were captured for each position of the measurement plane. The mapping function was then computed independently in each position of the measurement plane using second-order polynomial function [181] as

$$F^{(c)}(x, y, z) = a_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2 + a_7xy + a_8yz + a_9zx, \quad (4.9)$$

where a_i ($i = 0, 1, \dots, 9$) represents optimal factors computed by the least-squares method. This mapping function was computed for X (i) and Z (j) positions of images captured by the camera 1 and those of camera 2. The computed factors at different measurement planes are shown in Fig. 4.8. Each factor of $F^{(c)}$ changes smoothly in the depth (y -) direction. Using the computed factors from the calibration plates, their depth dependence was fitted as third-order polynomial adopting the least-squares method. The fitted results are also shown as lines in Fig. 4.8. The fitting curves seem to represent the missing y positions. Thus, the fitted mapping functions are used for the subsequent stereoscopic PIV analysis. Equation 4.9 is thus rewritten by considering the y dependence as

$$\begin{aligned} F^{(c)}(x, y, z) &= a_0(y) + a_1(y)x + a_2(y)y + a_3(y)z + a_4(y)x^2 \\ &+ a_5(y)y^2 + a_6(y)z^2 + a_7(y)xy + a_8(y)yz + a_9(y)zx. \end{aligned} \quad (4.10)$$

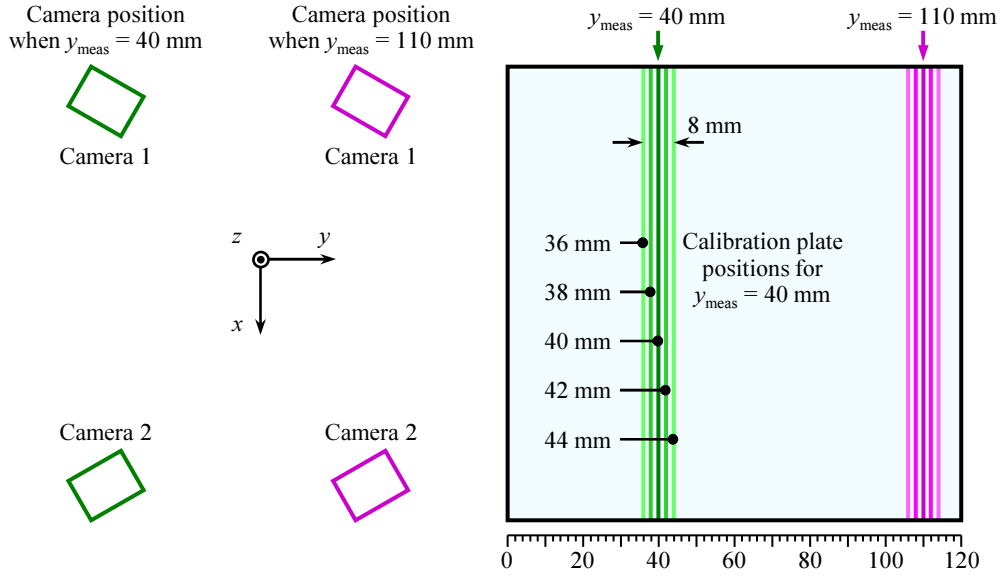


Figure 4.7: Conceptual image of stereoscopic calibration.

The mapping function $F^{(c)}$ needs to be obtained for the horizontal X (i) and the vertical Z (j) positions in the images captured by each camera. In total, four mapping functions, $F_X^{(1)}$ and $F_Z^{(1)}$ for the camera 1, and $F_X^{(2)}$ and $F_Z^{(2)}$ for the camera 2, are computed through the least-squares method from the reference points.

In Fig. 4.9, examples of backward projection (dewarping) at $y_{\text{meas}} = 40$ mm are shown. Here, the fitted mapping functions are used for associating the physical coordinates and the image coordinates. Original calibration images shown in Fig. 4.9(a) and (b) are tilted and distorted due to the angle offset camera configuration. The identified reference points, 99 dots and 220 grids, are indicated by red and blue circles in Fig. 4.9(a) and (b). The backward-projected (dewarped) images are shown in Fig. 4.9(c) and (d). Compared to the original images in Fig. 4.9(a) and (b), these dewarped images successfully show regular dots and grid patterns through the stereoscopic calibration. In Fig. 4.10, examples of backward projection at $y_{\text{meas}} = 110$ mm are shown in the same manner as Fig. 4.9. The calibration plate in the original images shown in Fig. 4.10(a) and (b) is recorded larger than that in Fig. 4.9(a) and (b), due to the different optical path-length inside the fluid. However, these original images with different size are successfully dewarped by the fitted mapping functions as shown in Fig. 4.10(c) and (d).

The stereoscopic calibration performed so far seems to successfully associate the image coordinates to the physical coordinates. These calibration results are used for the subsequent stereoscopic PIV analysis.

Assuming that the obtained mapping functions are independent on time, the in-plane dis-

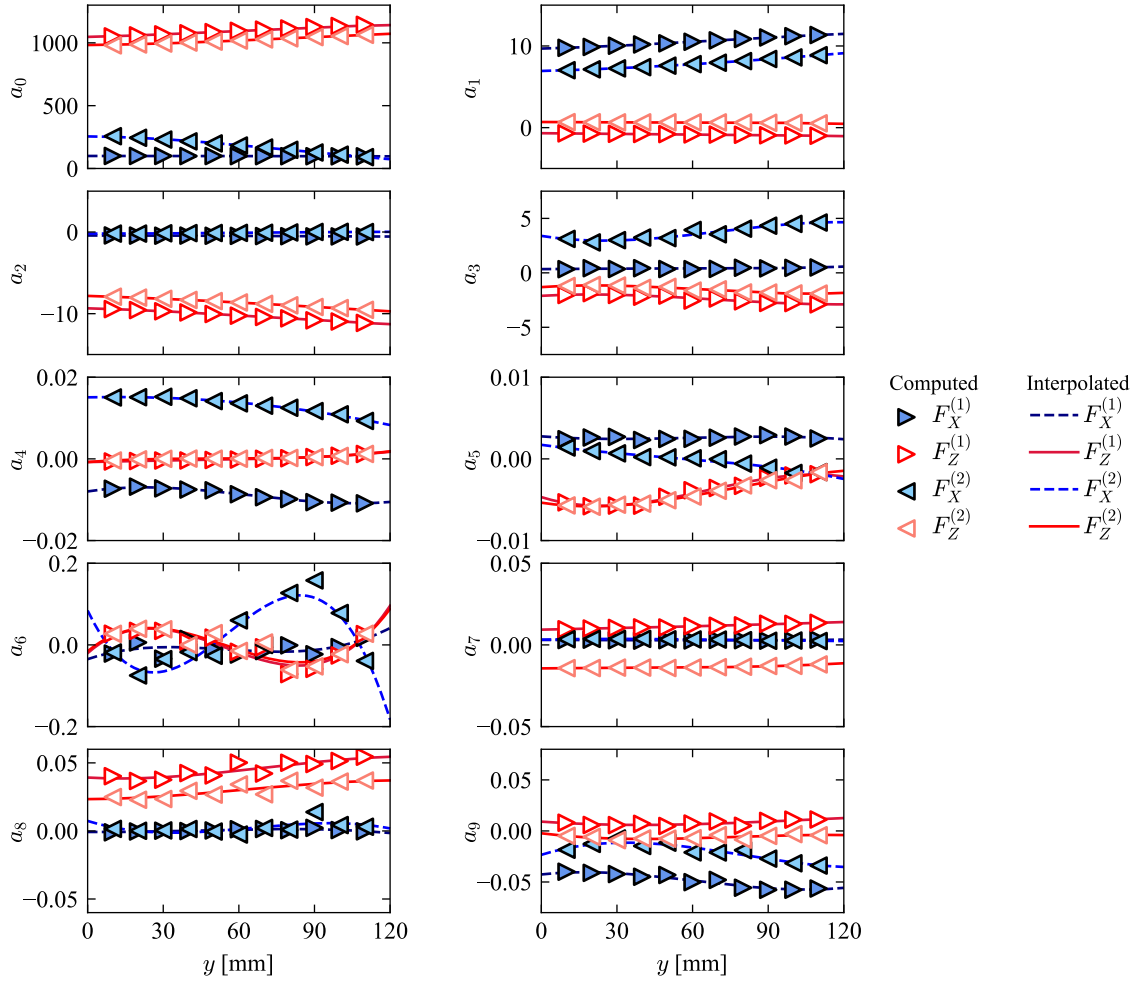


Figure 4.8: Factors of the mapping functions computed at each measurement plane (scattered plots) and fitting curves (lines).

placements in camera c , $U^{(c)}$ and $W^{(c)}$, are calculated by deriving Eq. 4.8 as

$$\begin{aligned}
 U^{(c)} \equiv \frac{\partial X^{(c)}}{\partial t} &= \frac{\partial F_X^{(c)}}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial F_X^{(c)}}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial F_X^{(c)}}{\partial z} \frac{\partial z}{\partial t} \\
 &= \frac{\partial F_X^{(c)}}{\partial x} u + \frac{\partial F_X^{(c)}}{\partial y} v + \frac{\partial F_X^{(c)}}{\partial z} w,
 \end{aligned} \tag{4.11}$$

and

$$\begin{aligned}
 W^{(c)} \equiv \frac{\partial Z^{(c)}}{\partial t} &= \frac{\partial F_Z^{(c)}}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial F_Z^{(c)}}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial F_Z^{(c)}}{\partial z} \frac{\partial z}{\partial t} \\
 &= \frac{\partial F_Z^{(c)}}{\partial x} u + \frac{\partial F_Z^{(c)}}{\partial y} v + \frac{\partial F_Z^{(c)}}{\partial z} w,
 \end{aligned} \tag{4.12}$$

where (u, v, w) is the velocity in the physical space. From these equations, the relationship

between (U, W) and (u, v, w) can be expressed in a matrix equation as

$$\begin{bmatrix} U^{(1)} \\ W^{(1)} \\ U^{(2)} \\ W^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_X^{(1)}}{\partial x} & \frac{\partial F_X^{(1)}}{\partial y} & \frac{\partial F_X^{(1)}}{\partial z} \\ \frac{\partial F_Z^{(1)}}{\partial x} & \frac{\partial F_Z^{(1)}}{\partial y} & \frac{\partial F_Z^{(1)}}{\partial z} \\ \frac{\partial F_X^{(2)}}{\partial x} & \frac{\partial F_X^{(2)}}{\partial y} & \frac{\partial F_X^{(2)}}{\partial z} \\ \frac{\partial F_Z^{(2)}}{\partial x} & \frac{\partial F_Z^{(2)}}{\partial y} & \frac{\partial F_Z^{(2)}}{\partial z} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}. \quad (4.13)$$

This equation can be expressed simply as

$$\mathbf{U} = \nabla F \mathbf{u}, \quad (4.14)$$

where $\mathbf{U} = [U^{(1)}, W^{(1)}, U^{(2)}, W^{(2)}]^\top$ and $\mathbf{u} = [u, v, w]^\top$. As the final step, the velocity vector with three components $\mathbf{u}$ is acquired as

$$\mathbf{u} = [\nabla F^\top \nabla F]^{-1} [\nabla F^\top \mathbf{U}]. \quad (4.15)$$

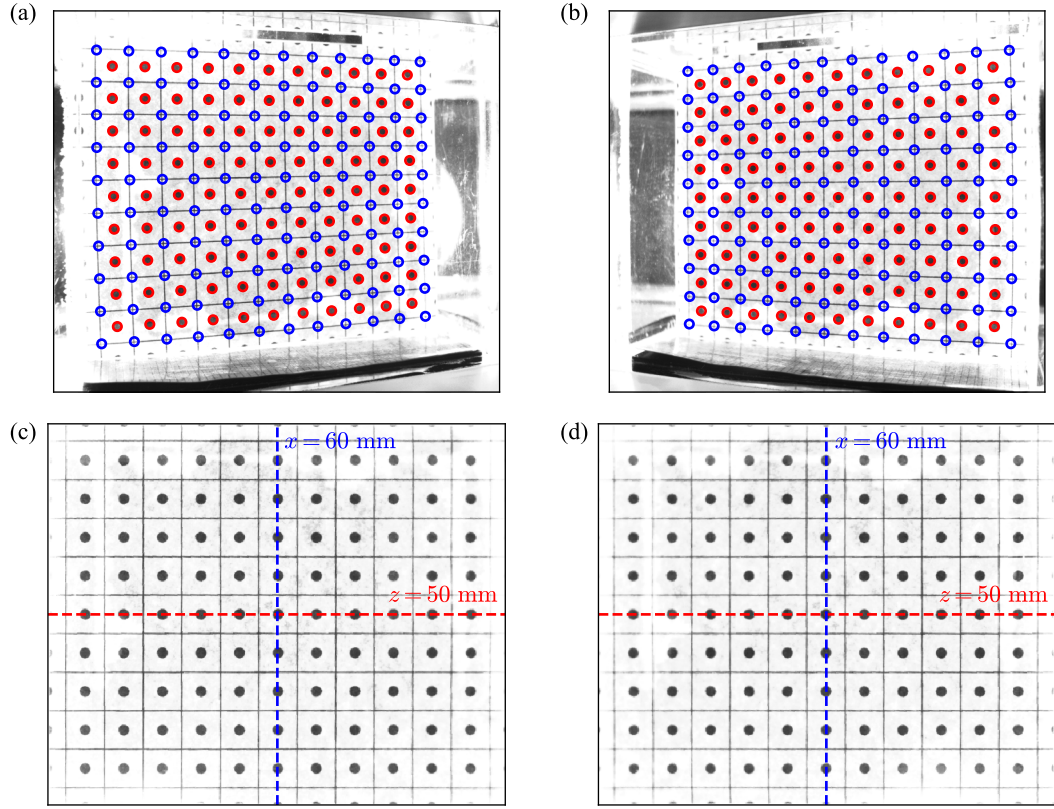


Figure 4.9: Example of backward projection (dewarping) at $y_{\text{meas}} = 40 \text{ mm}$: (a) and (b) are original images taken using camera 1 and 2 in the calibration measurement. (c) and (d) are the corresponding dewarped images. Identified reference points for the dot and grid patterns are indicated by red and blue circles in (a) and (b).

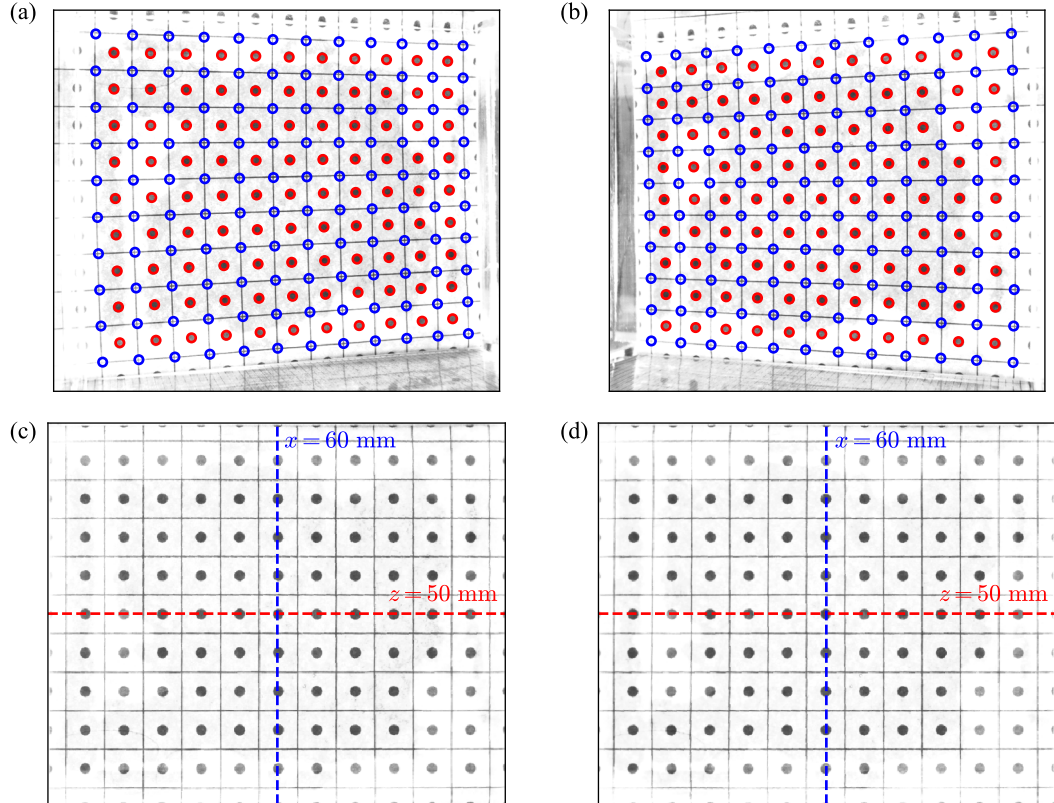


Figure 4.10: Example of backward projection (dewarping) at $y_{\text{meas}} = 110 \text{ mm}$. Detailed descriptions are the same as those in Fig. 4.9.

4.3.2 Calibration for liquid crystal thermometry

To measure 3D temperature fields via LCT, color calibration is necessary. A number of temperature calibration scheme has been proposed.

The color images were originally recorded as a Bayer pattern in 8-bit format. These raw data were then converted to color images in 24-bit format via a specific color demosaicing (debayering) method. In this study, an adaptive color demosaicing method proposed by Malvar et al. [103] was employed. For the temperature measurements, digitally recorded color information as 24-bit format needs to be utilized for temperature calibration. Anders et al. [13] utilized all the color information, three brightness of RGB values, and the spatial coordinates for the temperature calibration. That is, they considered the temperature calibration as a multi-regression problem taking two spatial coordinates and three color values as inputs, in their 2D system. Similar to the work by Anders et al. [13], both the spatial coordinates and full color information are utilized for the temperature calibration. The detail of the calibration scheme as the post-processing will be addressed in later. Even in any calibration schemes, predetermined relationships between the temperature and color as well as the spatial positions need to be prepared in advance.

4.3.2.1 Temperature calibration measurements

The color scattered by the TLC particles depends on its spatial position, since it is sensitive to the optical configurations such as observation angle, attenuation of light sheet (light intensity). Accordingly, it is better to acquire color images of fluids at globally constant temperatures not to involve color differences arising from differences in temperatures. In the present study, the image acquisitions for the temperature calibration were performed for the fluid at a number of constant temperatures. By actuating the scanning system with acquiring images continuously, it is possible to obtain digital color information (R, G, B) by associating these with the corresponding spatial positions (x, y, z) inside the full measurement domain.

The temperature of the fluid needs to be adjusted to a specific constant temperature. Here two different temperature adjustment methods, referred to as the thermal diffusion method and the thermal convection method, are schematically illustrated in Fig. 4.11(a) and (b), respectively. The solid lines represent the vertical profiles of the horizontally averaged temperatures. The thermal diffusion method illustrated in Fig. 4.11(a) is the most straightforward method without needing special considerations. The fluid container had a volume of $120 \times 120 \times 100 \text{ mm}^3$, and the internal temperatures were only controlled by the top and bottom surfaces. Once a reference temperature T_{ref} used for the temperature calibration is determined, both the top and bottom temperatures are adjusted to $T_{\text{top}} = T_{\text{bot}} = T_{\text{ref}}$. After a sufficiently long time, it is possible to obtain fluid at a constant temperature T_{ref} thanks to thermal diffusion. It takes a very long time to adjust the initial temperature T_{int} to the reference temperature T_{ref} in general, as it can be estimated as the thermal diffusion timescale $\tau_{\kappa} = L^2/\kappa \approx 6.8 \times 10^4 \text{ s} \approx 19 \text{ hours}$. As

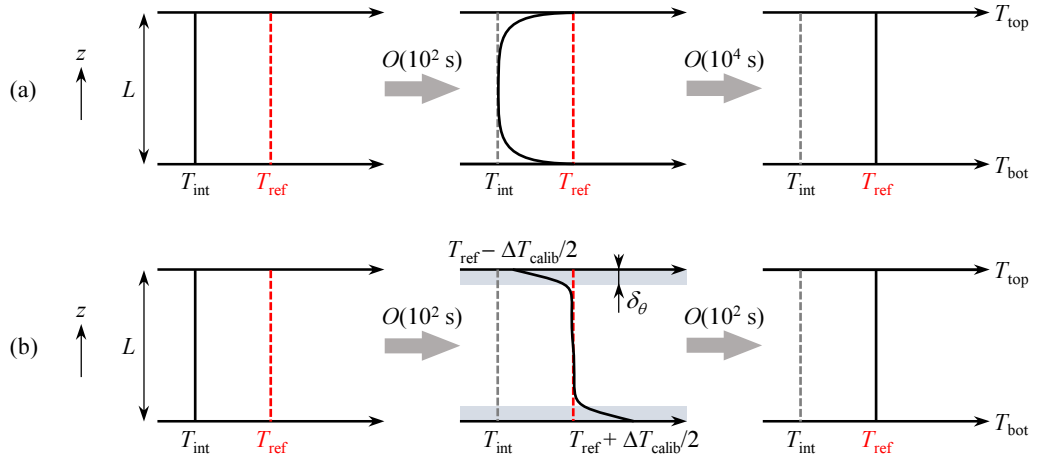


Figure 4.11: Schematic of the vertical profiles of the horizontally averaged temperatures in the two temperature adjustment methods: (a) thermal diffusion method and (b) thermal convection method.

the temperature calibration requires a number of reference temperatures, this thermal diffusion method is straightforward, but not realistic considering the dispersion time of the TLC particles.

To solve this problem, a more tricky method, thermal convection method is chosen in the present study, as illustrated in Fig. 4.11(b). First, a reference temperature T_{ref} was predetermined. Second, the top and bottom temperatures were set at $T_{\text{top}} = T_{\text{ref}} - \Delta T_{\text{calib}}/2$ and $T_{\text{bot}} = T_{\text{ref}} + \Delta T_{\text{calib}}/2$, respectively. Here ΔT_{calib} represents the vertical temperature difference, and $\Delta T_{\text{calib}} = 4 \text{ K}$ was set in the present study. This temperature setting yielded turbulent thermal convection with $Ra = 7.8 \times 10^7$, and the bulk mean temperature becomes $T = T_{\text{ref}}$ apart from the thermal boundary layers at the top and bottom surfaces. The circulation timescale τ_{circ} of the large scale circulation of the turbulent thermal convection can be estimated by the free-fall velocity U_f as

$$\tau_{\text{circ}} = \frac{L_{\text{total}}}{U_f} \sim \frac{L_{\text{total}}}{\sqrt{g\beta\Delta T_{\text{calib}}L}}, \quad (4.16)$$

where L_{total} is the total length of the peripherals of the fluid layer, and $L_{\text{total}} = 4.4L$ in the present study. It is calculated as $\tau_{\text{circ}} \approx 14 \text{ s}$. From this result, the bulk mean temperature apart from the thermal boundary layers can be adjusted to T_{ref} immediately once after the convective motion begins thanks to the convective heat transport. As it takes some rotations of the large scale circulation so as to adjust the bulk mean temperature to T_{ref} , some minutes of waiting time are required. In this study, at least 30 min was spent for the waiting time. After this duration, the top and bottom temperatures were set to $T_{\text{top}} = T_{\text{bot}} = T_{\text{ref}}$ as the final step for the temperature adjustment. At this moment, only the temperatures within thermal boundary layers are far from T_{ref} . The thickness of the thermal boundary layers δ_θ can be estimated by the theory of Prandtl–Blasius boundary layers, and it is confirmed by some earlier works [e.g. 43, 180]. For a similar condition to the present work, $Ra = 8 \times 10^8$ and $Pr = 6$, they found $\delta_\theta \approx 4 \text{ mm}$. Accordingly, the thermal diffusion time which is required to adjust the boundary layer temperatures to the reference temperature is estimated as $\tau_\kappa = \delta_\theta^2/\kappa \approx 100 \text{ s}$. Thus, some

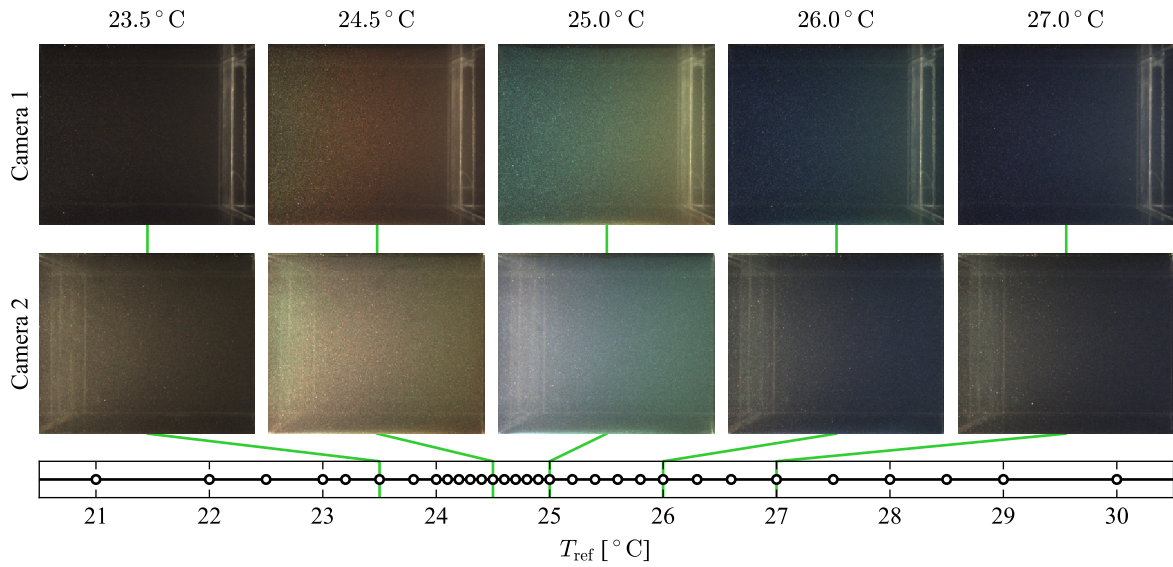


Figure 4.12: Examples of color images acquired in the temperature calibration measurements at $y = 60$ mm (the central cross-section of the fluid layer). Images in the top row are from the camera 1 and those in the bottom row are from the camera 2, and top and bottom images are the corresponding to each other. The images are dewarped by the mapping function and the resolution is 1200×1000 pixels.

minutes of waiting time are sufficient to adjust temperatures in the thermal boundary layers to the reference temperature. In this study, at least 30 min was spent for the waiting time. This second waiting time is also required to cease the convective motions of the thermal turbulence by both the viscous and the thermal diffusion. Following the procedures detailed above, it is possible to greatly reduce the time required for setting the fluid at a constant temperature throughout the fluid layer. Note that the fluid motions cannot be fully ceased even both in the two different methods. This is because the thermal conditions at the sidewalls are not regarded as perfect insulating conditions, and heat loss or input at the sidewalls always drive weak thermal convection. Such unwanted convective motions are inevitable in the actual experimental environment, however, the influence can be considered negligible as no distinctive color variation is visible.

The effective temperature range of the TLC particles ensured by the manufacturer was from 25°C to 30°C . However, this nominal temperature range largely changes depending on the observation condition, especially on the observation angle. The present study set two cameras not perpendicular to the light sheet, and thus the angle dependence of the TLC particles becomes prominent. To cover the temperature range showing informative coloration under the present measurement environment, a wide temperature range was investigated in the calibration step. As long as the digitally recorded color values are separable, the corresponding temperature can be utilized for the color-to-temperature calibration, even if the color is hard to recognize by the human eye. In the present study, $N_{\text{ref}} = 31$ reference temperatures were selected from $T_{\text{ref}} = 21.0^{\circ}\text{C}$ to 30.0°C . Examples of the color images are shown in Fig. 4.12. The reference temperatures were sampled unevenly from the examined temperature range. This is because the color change is not proportional to the corresponding temperature change. The temperature

ranges with large color changes were sampled frequently compared to those with blunt color changes. In particular, the reference temperatures were sampled every 0.1 K in the range of $24^\circ\text{C} \leq T_{\text{ref}} \leq 25^\circ\text{C}$, as the coloration changed drastically in the range. All the images in Fig. 4.12 are the results from $y = 60$ mm, and dewarped by the mapping functions obtained in Sec. 4.3.1. The spatial resolution of the dewarped images is 1200×1000 pixels. Only the images at five reference temperatures are shown in Fig. 4.12. In general, the images change the colors as red, green, blue, and dark blue from the lower to higher temperature. However, the coloration obtained at the same temperature by the two cameras are totally different to each other. In addition, color gradations can be recognized even in a single visualized image capturing the constant temperature fluid. For each reference temperature, the scanning image acquisitions were performed $N_{\text{scan}} = 10$ times.

4.3.2.2 Image processing for temperature calibration

First, the original color images $\mathbf{I}_{\text{org}}$ which were acquired in the temperature calibration measurements need pre-processing in order to obtain calibration images $\mathbf{I}_{\text{calib}}$. This is because the $\mathbf{I}_{\text{org}}$ contains lots of undesired information such as background information, and these should be fully excluded before the color-to-temperature calibration. Here, the $\mathbf{I}$ is comprised as a 3D stack of three 2D matrices, $\mathbf{R}$, $\mathbf{G}$, and $\mathbf{B}$, corresponding to the RGB channels each with 8-bit format, i.e. $\mathbf{I} = [\mathbf{R}, \mathbf{G}, \mathbf{B}]$. For all the images acquired in the temperature calibration measurements, the minimum brightness images $\mathbf{I}_{\text{min}} = [\mathbf{R}_{\text{min}}, \mathbf{G}_{\text{min}}, \mathbf{B}_{\text{min}}]$ can be computed using

$$\mathbf{R}_{\text{min}} = \min(\mathbf{R}\mathbf{s}_{\text{org}}), \quad \mathbf{G}_{\text{min}} = \min(\mathbf{G}\mathbf{s}_{\text{org}}), \quad \text{and} \quad \mathbf{B}_{\text{min}} = \min(\mathbf{B}\mathbf{s}_{\text{org}}). \quad (4.17)$$

Here, $\mathbf{R}\mathbf{s}_{\text{org}}$, $\mathbf{G}\mathbf{s}_{\text{org}}$, and $\mathbf{B}\mathbf{s}_{\text{org}}$ are the 3D stacks of each channel represented as, for example, $\mathbf{R}\mathbf{s}_{\text{org}} = [\mathbf{R}_{\text{org}}^1, \mathbf{R}_{\text{org}}^2, \dots, \mathbf{R}_{\text{org}}^N]$ with N as the total number of the original images acquired at the same y position, and $N = N_{\text{scan}} \times N_{\text{ref}} = 10 \times 31$ (10 scans for 31 reference temperatures). The function $\min(\cdot)$ represents a pixelwise operator to find the minimum values of each pixel position from the stacked matrices. Since the calibration measurements covered a wider temperature range than that of the TLC coloration, $\mathbf{I}_{\text{min}}$ is considered to represent the background image under the circumstances with seeding the TLC particles, i.e. the background image can be acquired as $\mathbf{I}_{\text{back}} = \mathbf{I}_{\text{min}}$. The background images $\mathbf{I}_{\text{back}}$ are compiled at all the y positions. The background-subtracted image $\mathbf{I}_{\text{bs}}$ can be computed as

$$\mathbf{I}_{\text{bs}} = \mathbf{I}_{\text{org}} - \mathbf{I}_{\text{back}}. \quad (4.18)$$

The series of image pre-processing about the background subtraction process is shown in Fig. 4.13.

The background-subtracted images $\mathbf{I}_{\text{bs}}$ does not contain background information, however, dark regions are now apparent. The dark regions appear due to the strong brightness intensity

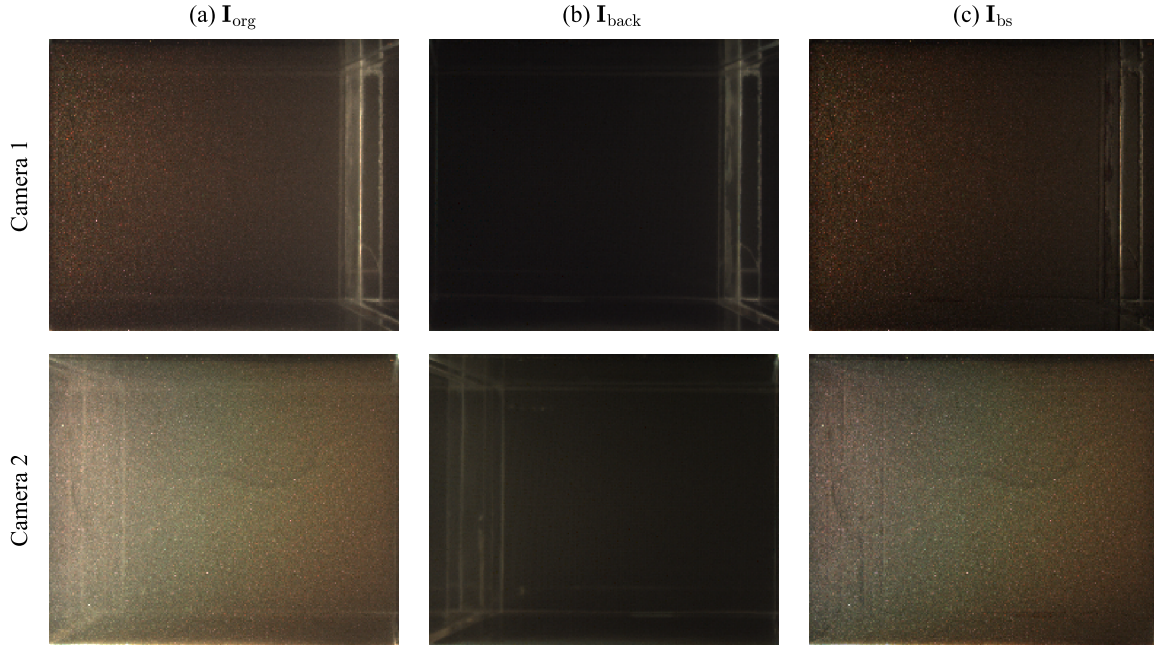


Figure 4.13: Background subtraction applied to the image acquired at $y = 60$ mm with $T_{\text{ref}} = 24.4^\circ\text{C}$: (a) the original color image $\mathbf{I}_{\text{org}}$, (b) the background (minimum) image $\mathbf{I}_{\text{back}}$, and (c) the background-subtracted image $\mathbf{I}_{\text{bs}}$. Images in the top and bottom rows correspond to the images obtained by the camera 1 and camera 2, respectively.

of the background image corresponding to the connecting parts of the acrylic walls composing the fluid vessel. It is not possible to fully extract effective color information from such regions strongly affected by the background. Thus, these regions should be filtered out in advance to the color-to-temperature calibration by a masking procedure. The mask image $\mathbf{I}_{\text{mask}}$ can be easily created using the background image $\mathbf{I}_{\text{back}}$. First, a grayscale background image is obtained by taking pixelwise maxima of $\mathbf{I}_{\text{back}}$, as $\mathbf{V}_{\text{back}} = \max(\mathbf{I}_{\text{back}})$. A global thresholding for $\mathbf{V}_{\text{back}}$ is applied and a binarized image is obtained as a result. The binarized image is then inverted to acquire the mask image $\mathbf{M}$. The masking procedure for the background subtracted image to obtain the masked image $\mathbf{I}_{\text{mbs}}$ is then computed as

$$\mathbf{R}_{\text{mbs}} = \mathbf{M} \odot \mathbf{R}_{\text{bs}}, \quad \mathbf{G}_{\text{mbs}} = \mathbf{M} \odot \mathbf{G}_{\text{bs}}, \quad \text{and} \quad \mathbf{B}_{\text{mbs}} = \mathbf{M} \odot \mathbf{B}_{\text{bs}}, \quad (4.19)$$

where $\odot$ represents the Hadamard product operator. The masking procedure is shown in Fig. 4.14.

The coloration recorded as the digital images is the lights originally scattered by the TLC particles dispersed into the fluid. The brightness distribution scattered by the particles is regarded as the Gaussian shape, and thus the roughness (discontinuity) in the spatial distribution of colors is an inevitable issue in the TLC images; in other words, this is the reason why the PIV analysis can be adopted for the TLC images. Such roughness in colors directly results in the roughness in temperature fields. Accordingly, pixelwise temperature field measurements are not possible in principle, as spatial smoothing is required for the visualized images or the converted temperature fields. In this study, the roughness in colors is eliminated in advance to the color-to-temperature

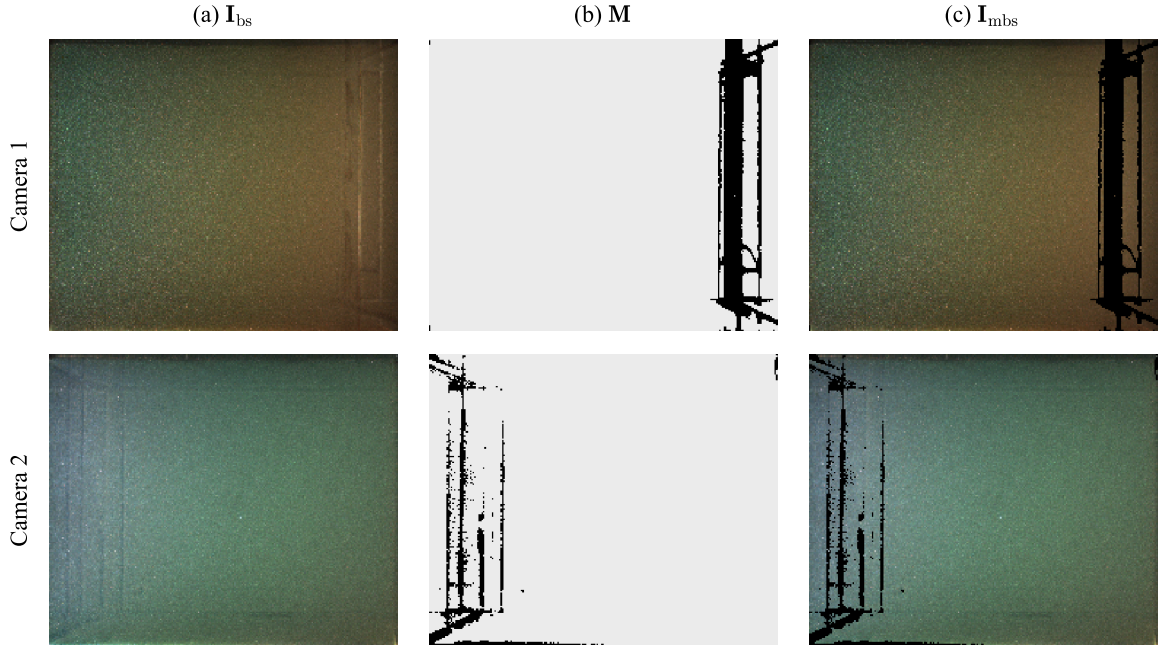


Figure 4.14: Masking procedure applied to the image acquired at $y = 60$ mm with $T_{\text{ref}} = 24.8^\circ\text{C}$: (a) the background-subtracted image $\mathbf{I}_{\text{bs}}$, (b) the mask image $\mathbf{M}$, and (c) the masked image $\mathbf{I}_{\text{mbs}}$. Images in the top and bottom rows correspond to the images obtained by the camera 1 and camera 2, respectively. Non-mask regions of $\mathbf{M}$ are colored as gray for visibility.

calibration, not after the temperature conversion. The wavelength of the original fluctuations caused by the inhomogeneous scattered lights of the TLC particles are approximately less than 5 pixels corresponding to 0.5 mm in the physical space. Adoption of spatial smoothing within 5×5 pixels does not influence the subsequent analysis, since the spatial resolution of 0.5 mm for the temperature measurements is sufficiently high compared to that of the velocity field measurements, 1 mm. Here, the masked image $\mathbf{I}_{\text{mbs}}$ is coarse-grained by adopting a median filtering with a kernel size of 5×5 pixels. The original shape 1200×1000 pixels of $\mathbf{I}_{\text{mbs}}$ is then converted to a coarse-grained image $\mathbf{I}_{\text{cg}}$ with a shape of 241×201 pixels. The masked pixels are excluded from the calculation of the median filtering. If the number of masked pixels exceeds the half number of pixels within a single kernel, the corresponding pixel in $\mathbf{I}_{\text{cg}}$ is discarded for the subsequent analysis. The conceptual image of the coarse-graining is illustrated in Fig. 4.15. The rim regions of the $\mathbf{I}_{\text{mbs}}$ are coarse-grained by small kernels, 2×2 pixels (at the corners) and 5×2 (2×5) pixels (at the other rim regions), while the interior regions are coarse-grained using the kernel with a size of 5×5 pixels. Each kernel is enclosed by red lines in Fig. 4.15.

The results of the coarse-graining are shown in Fig. 4.16. The masked images $\mathbf{I}_{\text{mbs}}$ applied the coarse-graining are shown in Fig. 4.16(a), and the results are shown in Fig. 4.16(b). Rough brightness distributions in Fig. 4.16(a) are suppressed in Fig. 4.16(b). For reference, the brightness profiles along x -axis at $z = 50$ mm are shown in Fig. 4.16(c). Fluctuations with large wavenumbers of R_{mbs} , G_{mbs} , and B_{mbs} drawn by the light colors are smoothed in the brightness profiles of R_{cg} , G_{cg} , and B_{cg} drawn by the dark colors. Missing values correspond to the masked

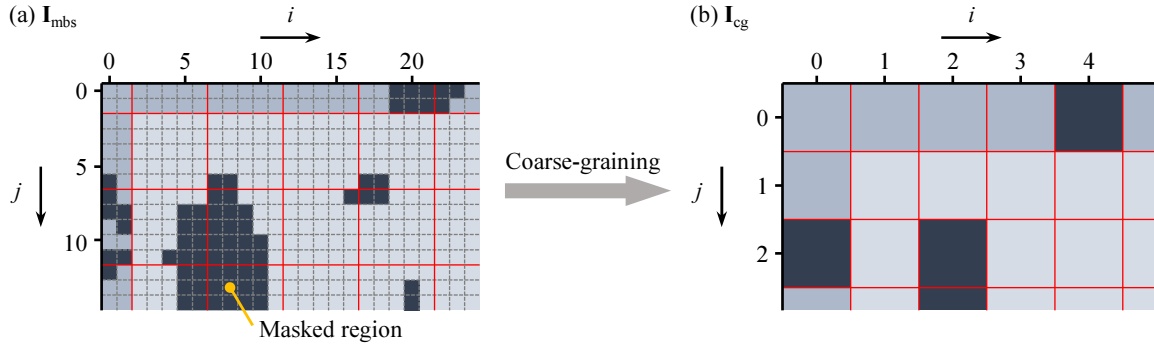


Figure 4.15: Conceptual image of the coarse-graining: (a) the masked image $\mathbf{I}_{\text{mbs}}$ and (b) the coarse-grained image $\mathbf{I}_{\text{cg}}$.

regions.

A series of image pre-processing has been established so far, and the resultant coarse-gained images $\mathbf{I}_{\text{cg}}$ are used as the calibration images $\mathbf{I}_{\text{calib}} = \mathbf{I}_{\text{cg}}$. Since the calibration images are acquired at known y positions and known temperature T_{ref} , the sets of the relationships between the temperatures T and the colors (R, G, B) as well as the positions (x, y, z) are prepared. The color values RGB originally recorded in 24-bit format is not appropriate to the color-to-temperature calibration. Such absolute brightness values strongly depends on the seeding density of the TLC particles, and the seeding density cannot be fully controlled among different experimental runs. Thus, the color values cannot be directly utilized for the color-to-temperature calibration. For a robust color-to-temperature calibration, the color values should be normalized. Anders et al. [13] used normalized RGB values $r^*g^*b^*$ defined as

$$r^* = \frac{R}{R + G + B}, \quad g^* = \frac{G}{R + G + B}, \quad \text{and} \quad b^* = \frac{B}{R + G + B}. \quad (4.20)$$

In Fig. 4.17, the color variation with temperature change is plotted in the $r^*g^*b^*$ space defined in Eq. 4.20. The color of each plots represents the reference temperature T_{ref} . The dynamic range of the $r^*g^*b^*$ defined in Eq. 4.20 is small, and typically from 0.2 to 0.4. In this study, a different definition is utilized to improve the dynamic range of the $r^*g^*b^*$ values as

$$r^* = \frac{R}{\sqrt{R^2 + G^2 + B^2}}, \quad g^* = \frac{G}{\sqrt{R^2 + G^2 + B^2}}, \quad \text{and} \quad b^* = \frac{B}{\sqrt{R^2 + G^2 + B^2}}. \quad (4.21)$$

Here, RGB values are from the calibrated image $\mathbf{I}_{\text{calib}}$. In Fig. 4.18, the color variation with temperature change is plotted in the $r^*g^*b^*$ space in Eq. 4.21. The dynamic range is improved compared to the results in Fig. 4.17, and typically from 0.3 to 0.8. Incorporating these results, the temperature can be identified using all the $r^*g^*b^*$ values defined in Eq. 4.21.

In Fig. 4.18, the color-to-temperature relationship only at $(x, y, z) = (60 \text{ mm}, 60 \text{ mm}, 50 \text{ mm})$ is shown. However, as described above, the coloration of the scattered lights from the TLC particles strongly depends on the spatial positions. The spatial dependence of the coloration

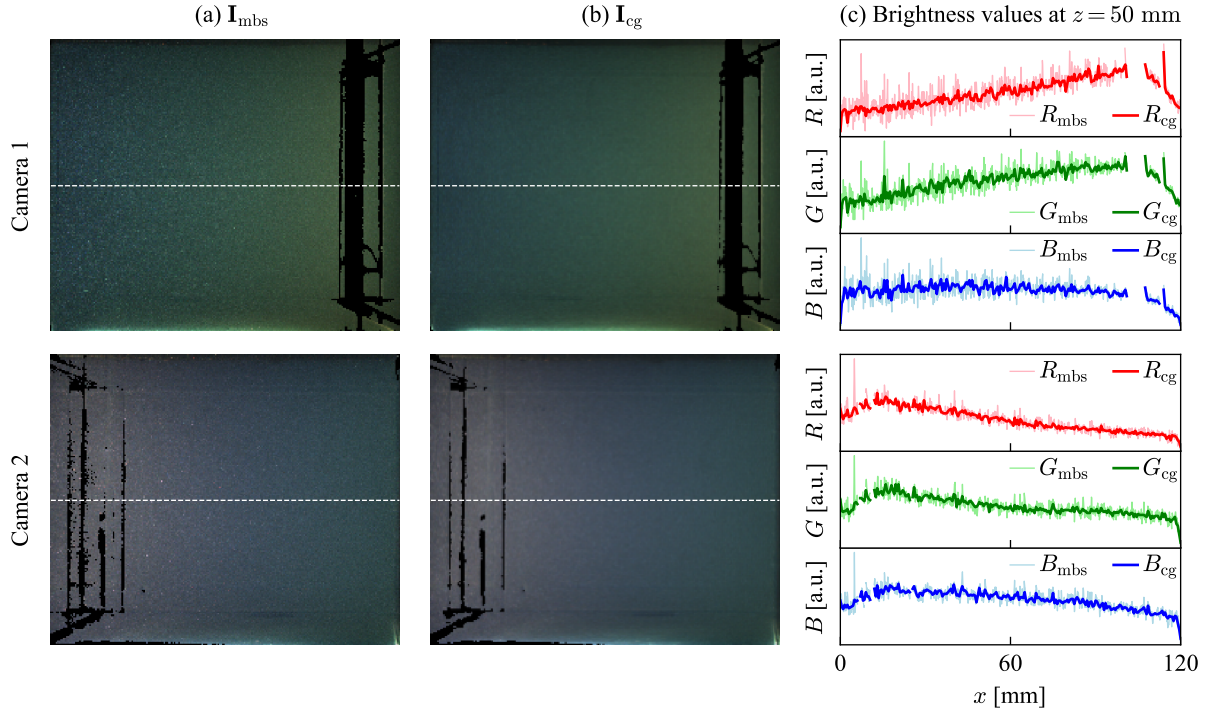


Figure 4.16: Coarse-graining applied to the image acquired at $y = 60$ mm with $T_{\text{ref}} = 25.4^\circ\text{C}$: (a) the masked image $\mathbf{I}_{\text{mbs}}$, (b) the coarse-grained image $\mathbf{I}_{\text{cg}}$, and (c) the brightness profiles at the same lines of $z = 50$ mm. Images in the top and bottom rows correspond to the images obtained by the camera 1 and camera 2, respectively.

is shown in Fig. 4.19, 4.20, and 4.21. The color contours represent r^* , g^* , and b^* values from the left to the right column, and (a) and (b) are the results from the camera 1 and camera 2. In particular, the x -position dependence shown in Fig. 4.19 is the largest. This is because the light sheet illuminates the measurement planes from the $-x$ to the $+x$ direction, and the cameras are angled to the x -direction. From these results, such spatial dependence is necessary to take into account for the color-to-temperature calibration.

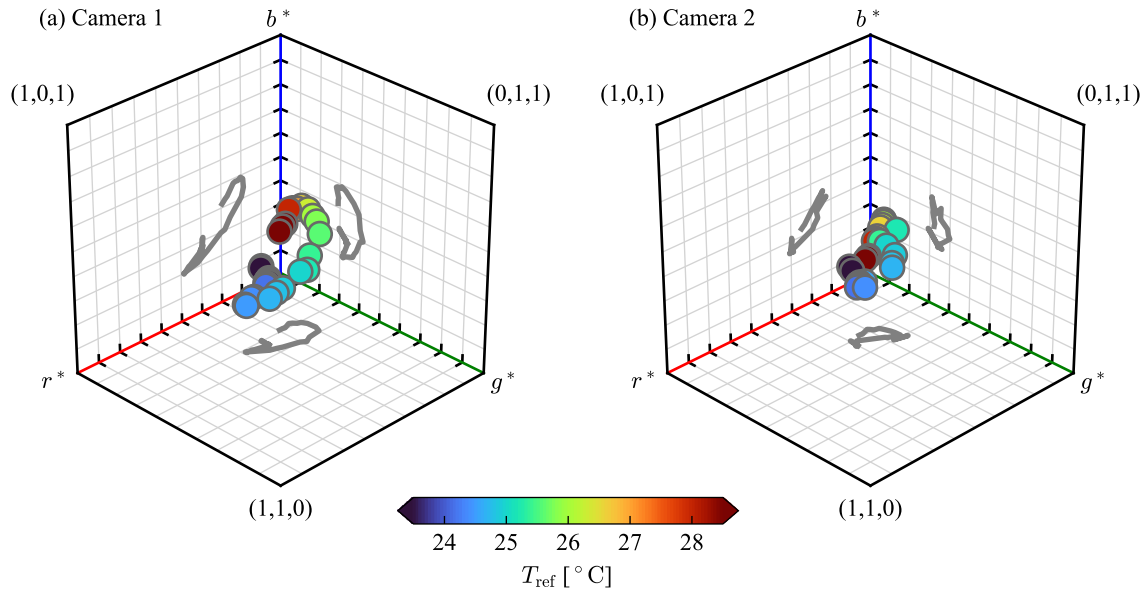


Figure 4.17: Color variation with temperature change expressed in the $r^*g^*b^*$ space at $(x, y, z) = (60 \text{ mm}, 60 \text{ mm}, 50 \text{ mm})$, according to Eq. 4.20 proposed in Anders et al. [13]. Plots in (a) and (b) correspond to the results obtained by the camera 1 and 2, respectively.

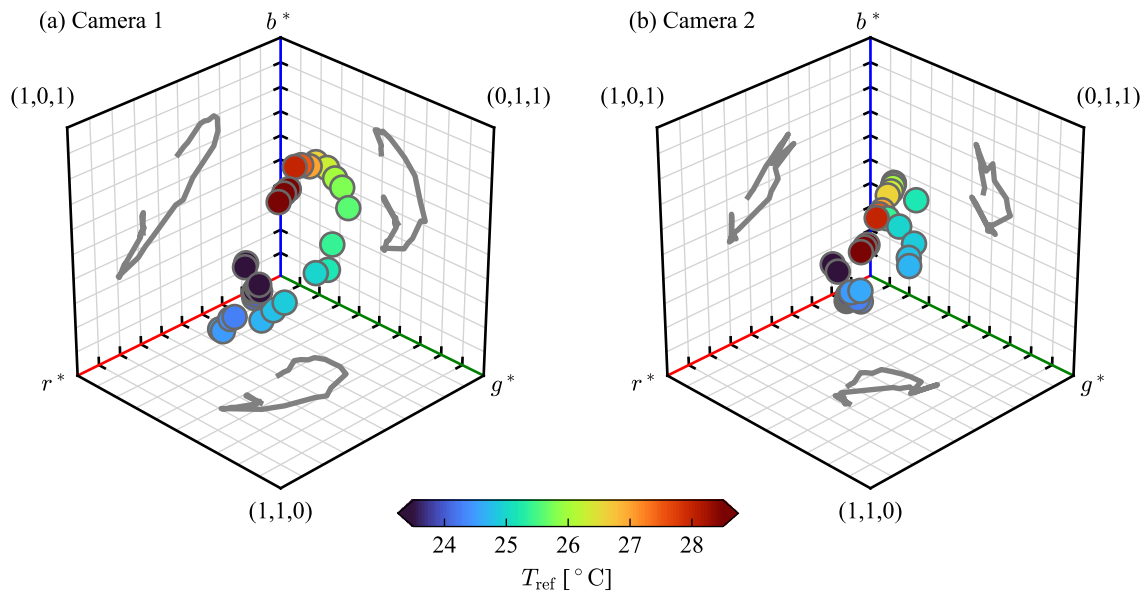


Figure 4.18: Color variation with temperature change expressed in the $r^*g^*b^*$ space at $(x, y, z) = (60 \text{ mm}, 60 \text{ mm}, 50 \text{ mm})$, according to Eq. 4.21 proposed in this study. Plots in (a) and (b) correspond to the results obtained by the camera 1 and 2, respectively.

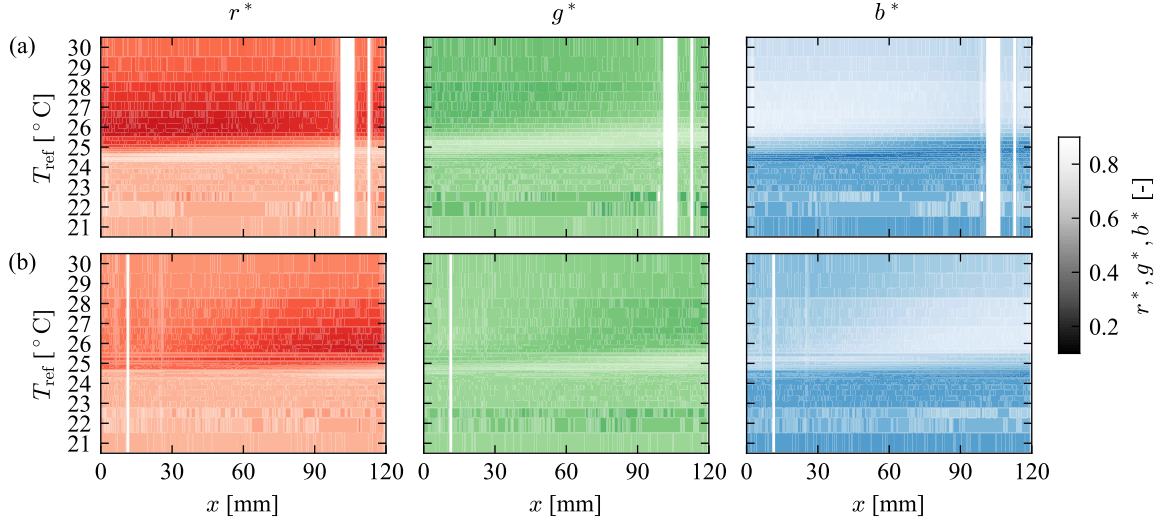


Figure 4.19: Color change in the x direction at $(y, z) = (60 \text{ mm}, 50 \text{ mm})$: (a) camera 1 and (b) camera 2.

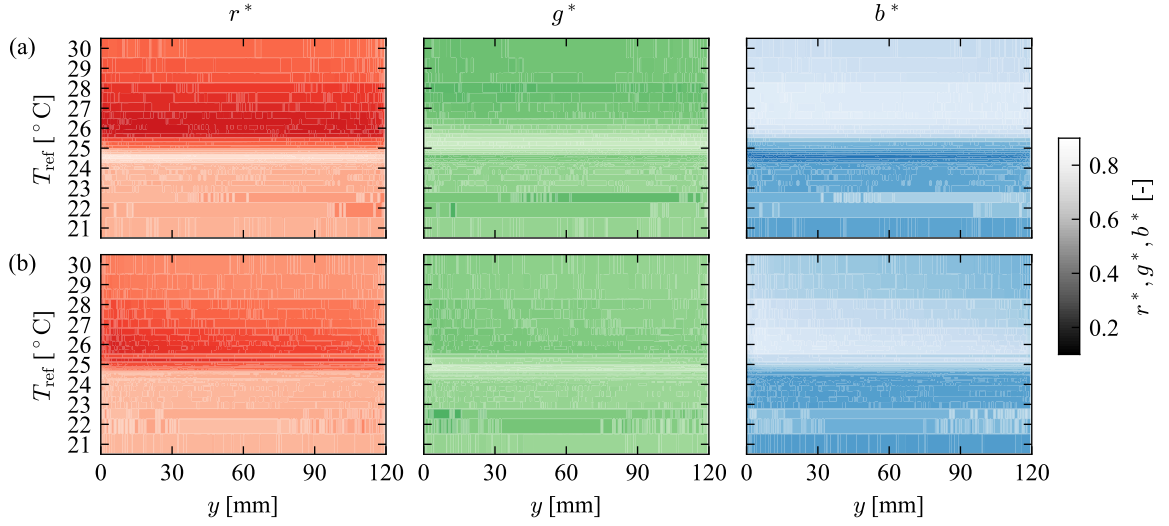


Figure 4.20: Color change in the y direction at $(x, z) = (60 \text{ mm}, 50 \text{ mm})$: (a) camera 1 and (b) camera 2.

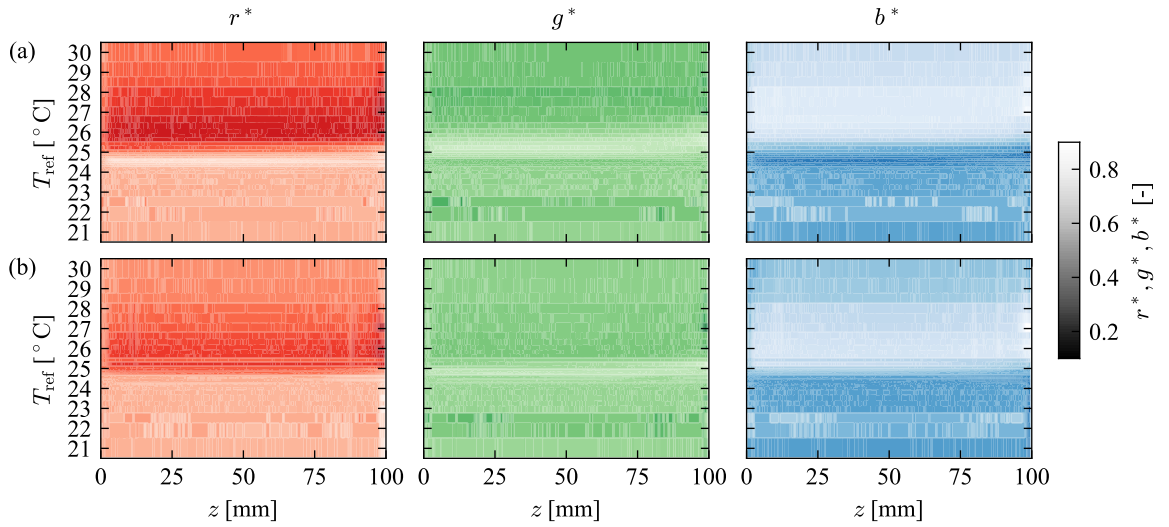


Figure 4.21: Color change in the z direction at $(x, y) = (60 \text{ mm}, 60 \text{ mm})$: (a) camera 1 and (b) camera 2.

4.3.2.3 Temperature calibration procedure

Incorporating all the results shown in Fig. 4.18, 4.19, 4.20, and 4.21, both the normalized color information (r^*, g^*, b^*) and the spatial position (x, y, z) are required to be utilized for the temperature calibration. Since two cameras are employed in this study, the normalized colors (r_1^*, g_1^*, b_1^*) and (r_2^*, g_2^*, b_2^*) from the two cameras can be utilized. In summary, the temperature calibration can be interpret as a multi-regression problem for acquiring the following function $\mathcal{F}$,

$$\zeta = \mathcal{F}(\xi). \quad (4.22)$$

Here, ξ and ζ are the input and output vectors. For the input vector ξ , the spatial position $\mathbf{x}$ and the corresponding colors $\mathbf{c}_1(\mathbf{x})$ and $\mathbf{c}_2(\mathbf{x})$ are stacked; $\mathbf{x}$ is the normalized position vector $\mathbf{x} = [x/W, y/W, z/L]^\top = [x^*, y^*, z^*]^\top$ and $\mathbf{c}(\mathbf{x})$ is the corresponding normalized color vector $\mathbf{c}_1(\mathbf{x}) = [r_1^*(\mathbf{x}), g_1^*(\mathbf{x}), b_1^*(\mathbf{x})]^\top$ and $\mathbf{c}_2(\mathbf{x}) = [r_2^*(\mathbf{x}), g_2^*(\mathbf{x}), b_2^*(\mathbf{x})]^\top$ obtained from the calibration image $\mathbf{I}_{\text{calib}}$. Note that all the normalized values are defined from 0 to 1. Only the temperature T at the position $\mathbf{x}$ is assigned to as the output vector ζ . That is, the function $\mathcal{F}$ takes an input vector with 9 components and returns an output vector with a single value (scalar) T at the position $\mathbf{x}$. As shown in Fig. 4.18, 4.19, 4.20, and 4.21, it may be difficult to model an optimal regression curve in advance, as all the nine variables intricately contribute to the resultant temperature. To reduce empirical decisions to find optimal regression curves, an artificial neural network (ANN) is employed to find the optimal regression curve, the function $\mathcal{F}$, without needing any models for the curves in the present study. The fundamental procedure for constructing the ANN system follows the procedure taken in Anders et al. [13].

First, a set of known relationships between the output vectors $\zeta(\mathbf{x}) = T_{\text{ref}}(\mathbf{x})$ and the input vectors $\xi(\mathbf{x}) = [\mathbf{x}, \mathbf{c}_1(\mathbf{x}), \mathbf{c}_2(\mathbf{x})]^\top$ needs to be prepared as a teacher dataset. In order to utilize the effective temperature ranges at which the TLC particles scatter informative coloration for the color calibration, the color values are plotted against the reference temperatures in Fig. 4.22. In Fig. 4.22(a) and (b), the normalized color values (r^*, g^*, b^*) from the camera 1 and 2 are plotted, respectively. In Fig. 4.22(c) and (d), the corresponding color values from HSV -color space, hue h , saturation s , and brightness value v , from the camera 1 and 2 are plotted, respectively. Here, h , s , and v are defined as

$$h = \frac{1}{2\pi} \tan^{-1} \left[\frac{\sqrt{3}(G - B)}{2R - G - B} \right], \quad (4.23)$$

$$s = \frac{\max(R, G, B) - \min(R, G, B)}{\max(R, G, B)}, \quad (4.24)$$

$$v = \frac{\max(R, G, B)}{255}, \quad (4.25)$$

and all these values are defined in $[0, 1]$. In particular, the s value represents the vividness of the color, and is a good indicator to determine the effective temperature range at which shows

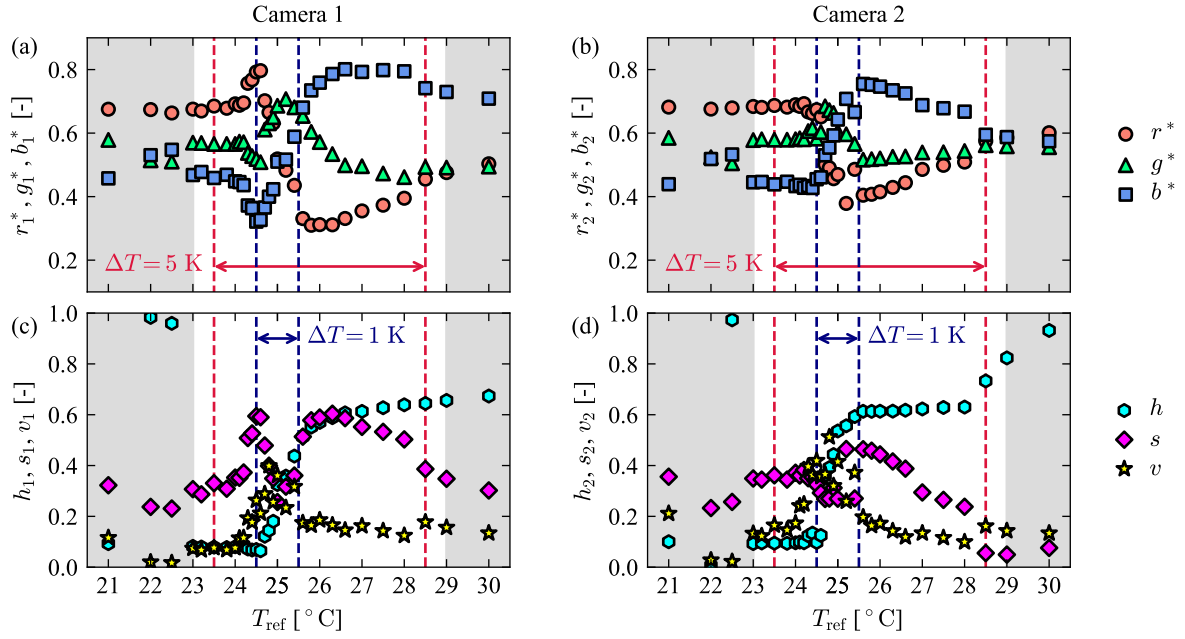


Figure 4.22: Plots of color values against the reference temperature T_{ref} at $(x, y, z) = (60 \text{ mm}, 60 \text{ mm}, 50 \text{ mm})$. The normalized color values (r^*, g^*, b^*) from the camera 1 and 2 are plotted in (a) and (b). The corresponding hsv (hue, saturation, brightness value) values are plotted in (c) and (d). The temperature ranges discarded from the color-to-temperature calibration are painted by gray. The temperature ranges set for the subsequent measurements, $\Delta T = 1$ and 5 K, are indicated by the vertical dashed lines.

informative coloration. In this study, the temperature range $23.0^{\circ}\text{C} \leq T \leq 29.0^{\circ}\text{C}$ is utilized for the color-to-temperature calibration. The rest of the data, painted by gray in Fig. 4.22, is discarded from the calibration, since it is hard to distinguish unique coloration, combination of (r^*, g^*, b^*) . As a result, $N_{\text{ref}} = 27$ reference temperatures are utilized for the calibration. Considering the configuration of the calibration measurements, there are $N_{\text{scan}} \times N_{\text{ref}} \times N_x \times N_y \times N_z = 10 \times 27 \times 241 \times 241 \times 201 \approx 3.15 \times 10^9$ known relationships between the input and output values. If all the relationships are utilized as the teacher dataset, the ANN system will cause an over-learning issue limiting the robustness of the system. Thus, some of the relationships need to be sampled as the teacher dataset. Here, approximately 0.2% of all the relationships was randomly selected as the teacher dataset. The teacher data was sampled uniformly in the physical space for each reference temperatures. During the random sampling process, the masked pixels of $\mathbf{I}_{\text{calib}}$ were excluded from the sampling. In the same manner, a test dataset is compiled for the later validation of the ANN system, not to include the same data in the teacher dataset.

The ANN system is comprised by four layers, input layer, two hidden layers, and output layers, as schematically illustrated in Fig. 4.23. The input layer possesses 9 nodes corresponding to the 9 input values of the input vector $\xi = [a_1, a_2, \dots, a_9]^{\top}$ and one bias. Weighted summations of all the nodes are connected to the nodes composing the first hidden layer, and this input value

for the nodes in the hidden layer can be written as

$$c_j = \sum_i^{N_{\text{in}}} w_{ji} a_i + b_{\text{in}}, \quad (4.26)$$

where a , b_{in} , w_{ji} , and c_j are the input values from the nodes in the input layer, the bias at the input layer, the weight value connecting the nodes between the input and the first hidden layer, and the input value for the nodes in the first hidden layer as the summation. Thus, i and j are defined in the ranges of $i \in [1, N_{\text{in}}]$ and $j \in [1, N_{\text{hid1}}]$, where N_{in} and N_{hid1} are the number of nodes in the input and the first hidden layers. The summation value c_j is then processed by an activation function ϕ to obtain the subsequent input value a_j for the second hidden layer as

$$a_j = \varphi(c_j) = \begin{cases} 1 & \text{if } c_j > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (4.27)$$

Here, the activation function φ is called the rectified linear unit (ReLU) or ramp function. The ReLU is considered to be the best activation function as reported in LeCun et al. [97]. In the same manner, the connection between the first and the second hidden layers can be written as

$$c_k = \sum_j^{N_{\text{hid1}}} w_{kj} a_j + b_{\text{hid1}}, \quad (4.28)$$

with $k \in [1, N_{\text{hid2}}]$ and the summation c_k is then processed by the ReLU as

$$a_k = \varphi(c_k) = \begin{cases} 1 & \text{if } c_k > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (4.29)$$

Finally, the obtained a_k are given as the input values to the output layer as

$$c_l = \sum_k^{N_{\text{hid2}}} w_{lk} a_k + b_{\text{hid2}}, \quad (4.30)$$

with $l \in [1, N_{\text{out}}]$. The final output value c_l is directly obtained as the output value of the ANN system without any activation functions, that is,

$$\zeta = c_l. \quad (4.31)$$

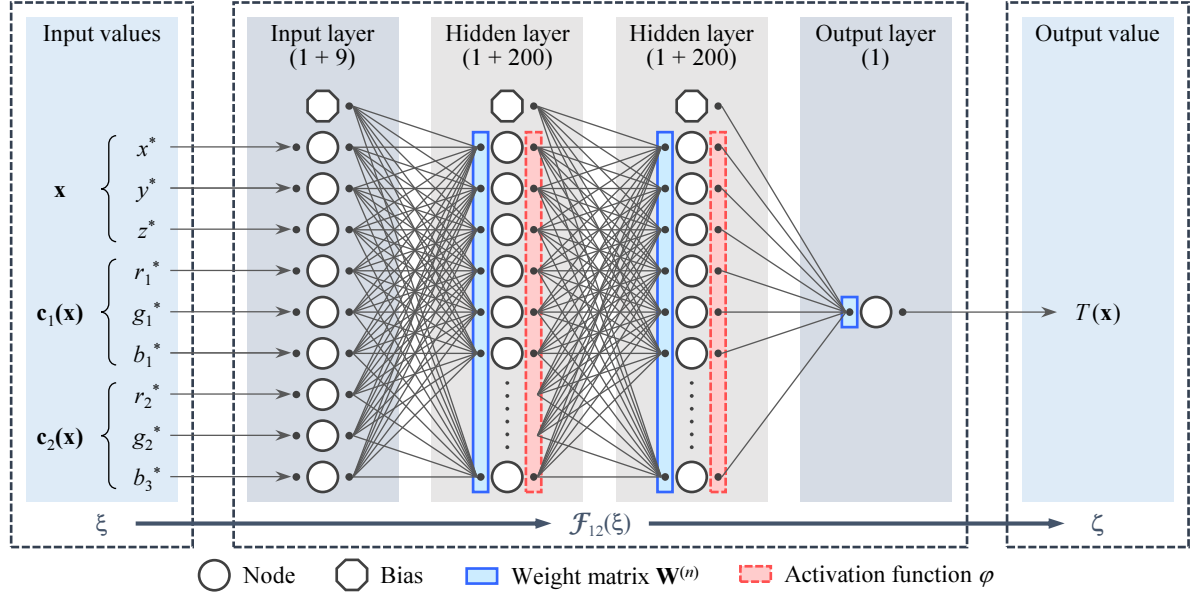


Figure 4.23: Conceptual image of the constructed ANN system with four layers. The case for the $\mathcal{F}_{12}$ is shown.

In a vector notation, these ANN system can be summarized as

$$\zeta(\mathbf{x}) = \mathbf{W}^{\text{out}} \left[\varphi \left(\mathbf{W}^{\text{hid2}} \left[\varphi \left(\mathbf{W}^{\text{hid1}} \left[\begin{matrix} 1 \\ \xi(\mathbf{x}) \end{matrix} \right] \right) \right] \right) \right], \quad (4.32)$$

where $\mathbf{W}^{\text{in}}$, $\mathbf{W}^{\text{hid1}}$, and $\mathbf{W}^{\text{hid2}}$ are the weight matrices with shapes of $N_{\text{hid1}} \times (N_{\text{in}} + 1)$, $N_{\text{hid2}} \times (N_{\text{hid1}} + 1)$, and $N_{\text{out}} \times (N_{\text{hid2}} + 1)$ in rows and columns, respectively. The function φ represents the activation function working to all the components of the argument vectors individually. In this study, the numbers of nodes in each layer are fixed to $N_{\text{in}} = 9$, $N_{\text{hid1}} = 200$, $N_{\text{hid2}} = 200$, and $N_{\text{out}} = 1$ for the ANN system $\mathcal{F}_{12}$.

Learning procedure of the ANN system is an optimization problem for determining the best weight matrices $\mathbf{W}$ to minimize a set loss function. In this study, the mean-squared error is used for the loss function and it is defined as,

$$MSE = \frac{1}{2} \sum \|\zeta_p - \zeta_t\|^2. \quad (4.33)$$

Here, ζ_p and ζ_t are the predicted and teacher value in the unit of $^{\circ}\text{C}$. The weight matrices $\mathbf{W}$ are optimized using the adaptive momentum gradient method, the so-called Adam, proposed by Kingma and Ba [90]. Various optimization methods have been proposed frequently [e.g. 58, 101, 160]. The efficacy of the Adam was proved in the work by Anders et al. [13], and thus the present study also utilizes the Adam for the optimization method.

The ANN system has been optimized and the regression function is determined so far.

However, some of the pixels in $\mathbf{I}_{\text{calib}}$ are masked due to the intense background information. Accordingly, the computed temperatures $T(\mathbf{x})$ are valid only at the spatial positions without lacking of the color vectors, $\mathbf{c}_1(\mathbf{x}) \neq \mathbf{0}$ or $\mathbf{c}_2(\mathbf{x}) \neq \mathbf{0}$. Obviously, the position with $\mathbf{c}_1(\mathbf{x}) \neq \mathbf{0}$ and $\mathbf{c}_2(\mathbf{x}) \neq \mathbf{0}$ cannot be utilized for the color-to-temperature conversion. However, if either of the color vectors is valid, the corresponding position can be utilized for the color-to-temperature calibration via different ANN systems. For convenience, three flag matrices are defined according to the mask matrices $\mathbf{M}_1$ and $\mathbf{M}_2$ compiled by stacking the mask image in the y direction as follows,

$$\mathbf{F}_{12}(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{M}_1(\mathbf{x}) = 1 \text{ and } \mathbf{M}_2(\mathbf{x}) = 1 \\ 0 & \text{otherwise} \end{cases}, \quad (4.34)$$

$$\mathbf{F}_1(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{M}_1(\mathbf{x}) = 1 \text{ and } \mathbf{M}_2(\mathbf{x}) = 0 \\ 0 & \text{otherwise} \end{cases}, \quad (4.35)$$

and

$$\mathbf{F}_2(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{M}_1(\mathbf{x}) = 0 \text{ and } \mathbf{M}_2(\mathbf{x}) = 1 \\ 0 & \text{otherwise} \end{cases}. \quad (4.36)$$

Here, $\mathbf{F}_{12}$, $\mathbf{F}_1$, and $\mathbf{F}_2$ are the step functions correspond to the positions with valid $\mathbf{c}_1(\mathbf{x})$ and $\mathbf{c}_2(\mathbf{x})$, valid $\mathbf{c}_1(\mathbf{x})$ and invalid $\mathbf{c}_2(\mathbf{x})$, and invalid $\mathbf{c}_2(\mathbf{x})$ and valid $\mathbf{c}_1(\mathbf{x})$. In summary, the input vectors $\boldsymbol{\xi}$ should be prepared according to the flag matrices as

$$\boldsymbol{\xi}(\mathbf{x}) = \begin{cases} \boldsymbol{\xi}_{12}(\mathbf{x}) = [\mathbf{x}, \mathbf{c}_1(\mathbf{x}), \mathbf{c}_2(\mathbf{x})]^\top & \text{if } \mathbf{F}_{12}(\mathbf{x}) = 1 \\ \boldsymbol{\xi}_1(\mathbf{x}) = [\mathbf{x}, \mathbf{c}_1(\mathbf{x})]^\top & \text{if } \mathbf{F}_1(\mathbf{x}) = 1 \\ \boldsymbol{\xi}_2(\mathbf{x}) = [\mathbf{x}, \mathbf{c}_2(\mathbf{x})]^\top & \text{if } \mathbf{F}_2(\mathbf{x}) = 1 \end{cases}. \quad (4.37)$$

In the same manner, the additional ANN systems should be also constructed for the two different input vectors $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$. Writing the ANN system for the $\boldsymbol{\xi}_{12}$ as $\zeta_{12} = \mathcal{F}_{12}(\boldsymbol{\xi}_{12})$, the two additional ANN systems can be written as $\zeta_1 = \mathcal{F}_1(\boldsymbol{\xi}_1)$ and $\zeta_2 = \mathcal{F}_2(\boldsymbol{\xi}_2)$. Apart from the difference in the numbers of the input vectors, the two additional ANN systems are constructed in the same configuration to the $\mathcal{F}_{12}$, and the same optimization procedures are conducted. That is, the numbers of nodes in each layer are fixed to $N_{\text{in}} = 6$, $N_{\text{hid1}} = 200$, $N_{\text{hid2}} = 200$, and $N_{\text{out}} = 1$ for the ANN systems $\mathcal{F}_1$ and $\mathcal{F}_2$.

4.3.2.4 Validation of the temperature calibration systems

The constructed ANN systems are validated through two different approaches. One is the performance test during the learning process, and the other is that for the optimized systems. While the optimization is conducted using the MSE value, the residuals during the learning process is easily understood in the unit of K. Here, the root-mean-squared error $RMSE$ is

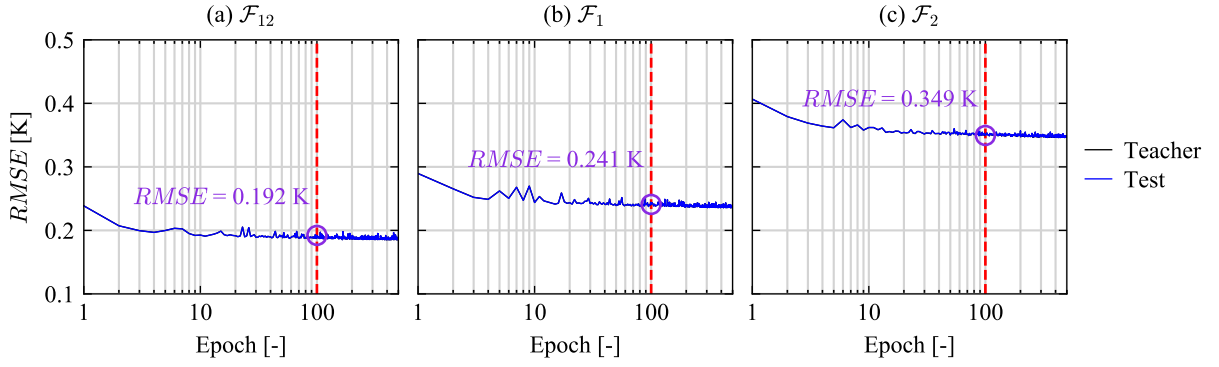


Figure 4.24: Change of the $RMSE$ values over the learning epoch.

defined as

$$RMSE = \sqrt{\frac{1}{N} \sum \|\zeta_p - \zeta_t\|^2}, \quad (4.38)$$

where N is the number of teacher or test data. The optimization process in terms of the change in the $RMSE$ is shown in Fig. 4.24. For all the three ANN systems, $\mathcal{F}_{12}$, $\mathcal{F}_1$, and $\mathcal{F}_2$, the $RMSE$ values decrease with the progress of the learning epoch. The plots of the $RMSE$ for the teacher dataset and the test dataset almost coincide with each other, and thus the over-learning problem is considered to be successfully avoided in all the three systems. The ANN system $\mathcal{F}_{12}$ with an input vector with 9 components outperforms the others. The ANN system $\mathcal{F}_1$ is the second best system, and the $\mathcal{F}_2$ is the worst in the three. From the curve of the $RMSE$ values, it is possible to regard that the learning process saturate after ~ 100 epoch or earlier. In this study, thus the weight matrices at the epoch number 100 is utilized as the best epoch. The corresponding $RMSE$ values are shown in Fig. 4.24. The performance of the $\mathcal{F}_{12}$ is the best, showing the $RMSE$ better than 0.2 K. This is because of the numbers of components of the input vectors ξ_{12} are larger than the others ξ_1 and ξ_2 , and it may be easy to identify temperatures using such rich information. The second best was the performance of the system $\mathcal{F}_1$, $RMSE < 0.25$ K, and the reason is considered as that the coloration of obtained from camera 1 is more vivid than that from camera 2 as shown in Fig. 4.18; the temperature in the (r_1^*, g_1^*, b_1^*) space is easier to be distinguished than that in the (r_2^*, g_2^*, b_2^*) space. Please note that the $RMSE$ values shown here are calculated using all the data containing the reference temperatures of $N_{\text{ref}} = 27$, and thus the performances are underestimated.

Using the optimized weight matrices at the learning epoch of 100, finally the 3D temperature fields $\mathbf{T}$ can be compiled as

$$\mathbf{T} = \mathbf{F}_{12} \odot \mathbf{T}_{12} + \mathbf{F}_1 \odot \mathbf{T}_1 + \mathbf{F}_2 \odot \mathbf{T}_2, \quad (4.39)$$

where $\mathbf{T}_{12}$, $\mathbf{T}_1$, and $\mathbf{T}_2$ are the 3D temperature fields obtained through the three different ANN systems. Here, the performance of the optimized color-to-temperature calibration system is validated using the $\mathbf{I}_{\text{calib}}$ obtained in the calibration experiments. The bulk mean deviations,

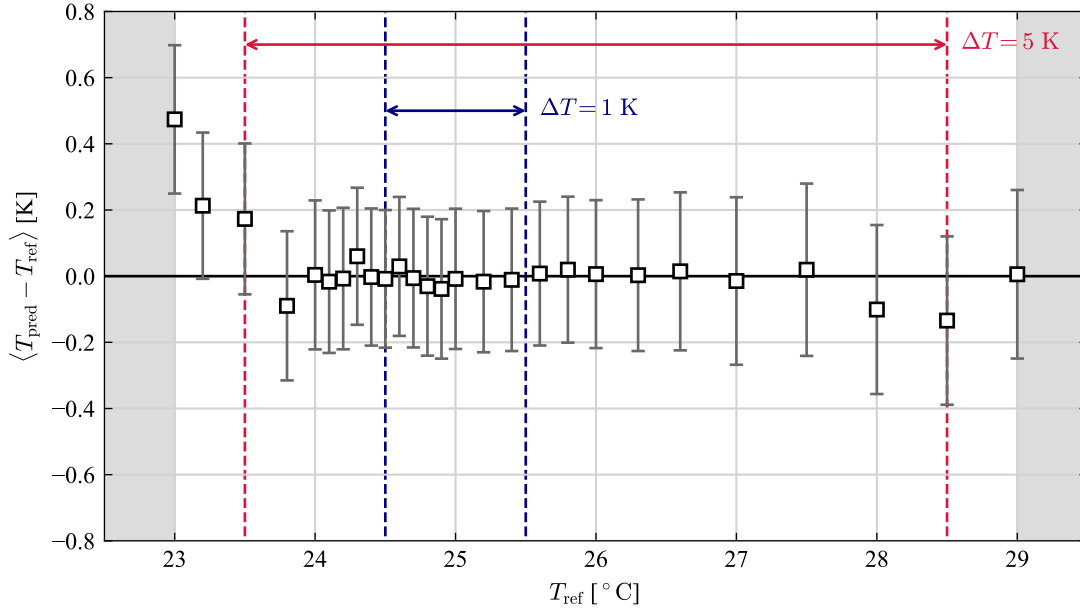


Figure 4.25: Bulk mean deviations of the optimized ANN system. The error-bars represent the standard deviations for each plot. The vertical dashed lines show the vertical temperature differences ΔT imposed in the subsequent convection experiments.

$\langle T_{\text{pred}} - T_{\text{ref}} \rangle$ where T_{pred} is the predicted temperature through the ANN system and $\langle \cdot \rangle$ is the volumetric average, are calculated for each reference temperature T_{ref} , and shown in Fig. 4.25. Here, the bulk mean deviations represent the precision of the temperature measurement system. The bulk mean deviations are less than 0.5 K for all the reference temperatures utilized for the learning process. In particular, the temperature ranges for the subsequent convection experiments, $\Delta T = 1$ and 5 K indicated by the vertical dashed lines, the bulk mean temperature deviations are less than 0.2 K. The standard deviations for each reference temperature, meaning the accuracy, stay $\lesssim 0.2$ K, and shown as the error-bars in Fig. 4.25. Mean absolute deviations MAD within the ranges for ΔT imposed in the subsequent convection experiments are then calculated as

$$MAD = \frac{1}{N'_{\text{ref}}} \sum_n^{N'_{\text{ref}}} |\langle T_{\text{pred}} - T_{\text{ref}} \rangle|_n \quad (4.40)$$

where, N'_{ref} is the number of reference temperatures within the ranges of imposed ΔT . The MAD values for $\Delta T = 1$ and 5 K are 0.017 K and 0.034 K, respectively.

Considering that these performances are evaluated using the full volume of the measurement domain, the constructed ANN system is regarded to have sufficiently high performance to reconstruct 3D temperature fields. The temperature fields shown in the subsequent sections are thus computed using the optimized ANN system in this section.

4.4 Performance of the scanning stereoscopic PIV and LCT

In Section 4.4, the performance of the constructed scanning PIV and LCT methods are examined.

4.4.1 Qualitative descriptions

To qualitatively observe flow structures, especially the vortical structures, the Q value is computed from the measured 3D-3C velocity fields. The Q value, the second invariant of the velocity gradient tensor $\nabla \mathbf{u}$, is a widely used quantity for the vortex detection [155] and calculated as

$$Q = \frac{1}{2} (\|\boldsymbol{\Omega}\|^2 - \|\mathbf{S}\|^2), \quad (4.41)$$

where $\boldsymbol{\Omega}$ and $\mathbf{S}$ are the asymmetric and symmetric parts of the velocity gradient tensor $\nabla \mathbf{u} = \boldsymbol{\Omega} + \mathbf{S}$. That is, the $\boldsymbol{\Omega}$ is the vorticity tensor defined as

$$\boldsymbol{\Omega} = \frac{1}{2} \begin{pmatrix} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} & 0 \end{pmatrix}, \quad (4.42)$$

and the $\mathbf{S}$ is the strain-rate tensor defined as

$$\mathbf{S} = \frac{1}{2} \begin{pmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} & 2\frac{\partial w}{\partial z} \end{pmatrix}. \quad (4.43)$$

The operator $\|\mathbf{A}\|$ is the Euclidean norm

$$\|\mathbf{A}\| = \sqrt{\text{Tr}(\mathbf{A}\mathbf{A}^\top)}. \quad (4.44)$$

The reconstructed flow fields are shown as isosurfaces of the Q value and the temperature T in Fig. 4.26 and 4.27. In the left column, the snapshots (quasi-instantaneous fields obtained by a single scan for 15 s) of the isosurface of the Q value are shown, and those of the temperature T are shown in the right column. The Q values are normalized by the volume root-mean-squared value Q_{rms} , and the isosurface of $Q = 0.3Q_{\text{rms}}$ are shown for visibility. Rajaei et al. [155] used $Q = 0.5Q_{\text{rms}}$ to show vortical structures reconstructed in their 3D PTV measurements. The

temperature fields are also normalized by the imposed vertical temperature difference ΔT as $(T - T_m)/\Delta T \in [-0.5, 0.5]$, and represented by the color contours. In Fig. 4.26 and Fig. 4.27, the results of the different Ra conditions, $Ra = 2 \times 10^7$ and 1×10^8 , are shown separately.

For the non-rotating cases shown Fig. 4.26(a) and Fig. 4.27(a), the flow structures are considered to be dominated by multiple thermal plumes, and these plumes form a global overturning structure called large-scale circulation (LSC). The agglomerated fluctuations of the thermal boundary layers are converged as sheet-like structures [178], and these are visible in the temperature fields. From these structures, the realized flow structures are qualitatively considered to be in the thermal turbulent states.

Increase in Ta values, $Ta = 3 \times 10^7$, 1×10^8 , 3×10^8 , 1×10^9 , and 3×10^9 , for $Ra = 2 \times 10^7$ is shown respectively in Fig. 4.26(b), (c), (d), (e), and (f). With increasing Ta , the disorganized flow structures represented by the Q values gradually organized along the z -axis (the rotational axis), meaning the emergence of columnar vortices. Together with the organization of the vertically coherent vortical structures, the temperature isosurfaces become prominent at the vortex regions. However, at the largest Ta case shown in Fig. 4.26(f), such columnar vortices are no longer visible in the Q field. The corresponding temperature field suggests the presence of a certain wall mode. The supercriticality for the condition shown in Fig. 4.26(f) is $\widetilde{Ra} \sim 9$, and this is very close to the critical value $\widetilde{Ra}_c \simeq 8.656$ [27]. Thus the flows are very slow and the velocity fields might not be possible to be measured by the constructed scanning stereoscopic PIV system due to the limitation in the dynamic range.

In the same manner, increase in Ta values for the condition of large Ra , $Ra = 1 \times 10^8$, is shown in Fig. 4.27(b), (c), (d), (e), and (f). Similar to the results show in Fig. 4.26, the disorganized vortical structures formed inside the fluid layer gradually show vertical coherence along the z -axis. This trend is also apparent in the corresponding temperature fields. The number of the columnar vortices seems to increase with increasing Ta values. Unlike the case for $Ra = 2 \times 10^7$ shown in Fig. 4.26(f), the fastest rotation case for $Ra = 1 \times 10^8$ shown in Fig. 4.27(f) show the presence of multiple columnar vortices, meaning the production of a sufficiently strong thermal force to drive convective motions. This is obvious from the supercriticality, $\widetilde{Ra} \sim 47$, of this condition, and this exceeds the critical values. Considering the supercriticality $\widetilde{Ra}$ or the Rossby number Ro , similar flow structures may be observed in the pairs of Fig. 4.26(b) ($\widetilde{Ra} \sim 88$, $Ro \sim 0.18$) and Fig. 4.27(e) ($\widetilde{Ra} \sim 97$, $Ro \sim 0.13$), and Fig. 4.26(c) ($\widetilde{Ra} \sim 42$, $Ro \sim 0.10$) and Fig. 4.27(f) ($\widetilde{Ra} \sim 47$, $Ro \sim 0.07$).

The flow fields shown in Fig. 4.26 and 4.27 seem to be reasonable from quantitative perspectives as above. Accordingly, the measurement system, scanning stereoscopic PIV and scanning LCT constructed in Sec. 4.3, can be considered to successfully measure the flow fields of various flow regimes, turbulence, vortical plumes, CTCs, and cellular, of rotating RBC. The quantitative evaluation will be described in the following section.

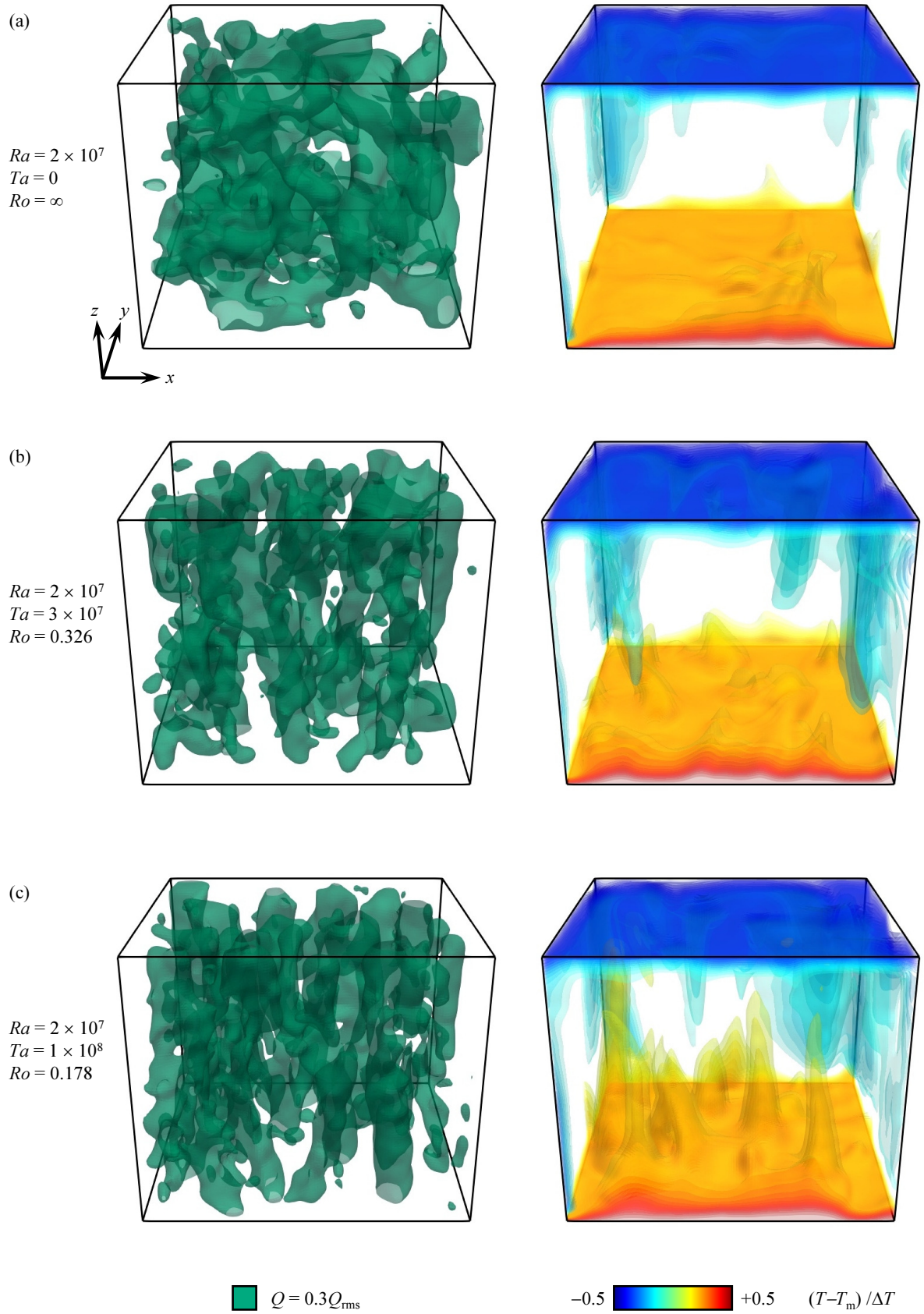


Figure 4.26: Snapshots of the isosurfaces of Q -criterion (left column) and normalized temperature $(T - T_m) / \Delta T$ (right column) with the condition of $Ra = 2 \times 10^7$: (a) $Ta = 0$, (b) $Ta = 3 \times 10^7$, and (c) $Ta = 1 \times 10^8$. Isosurface at $Q = 0.3Q_{rms}$ is shown in the left column.

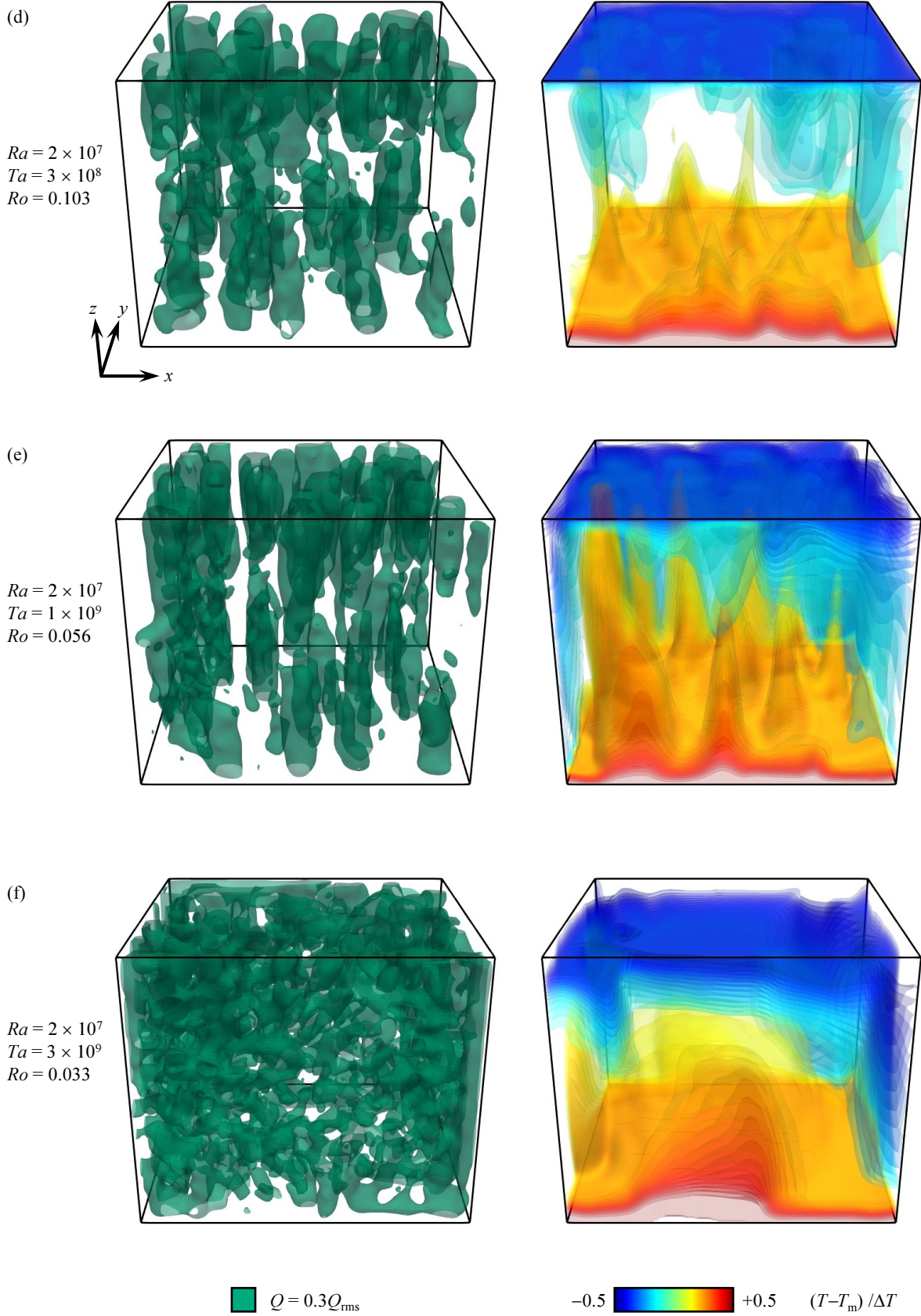


Figure 4.26: (continued.): (d) $Ta = 3 \times 10^8$, (e) $Ta = 1 \times 10^9$, and (f) $Ta = 3 \times 10^9$.

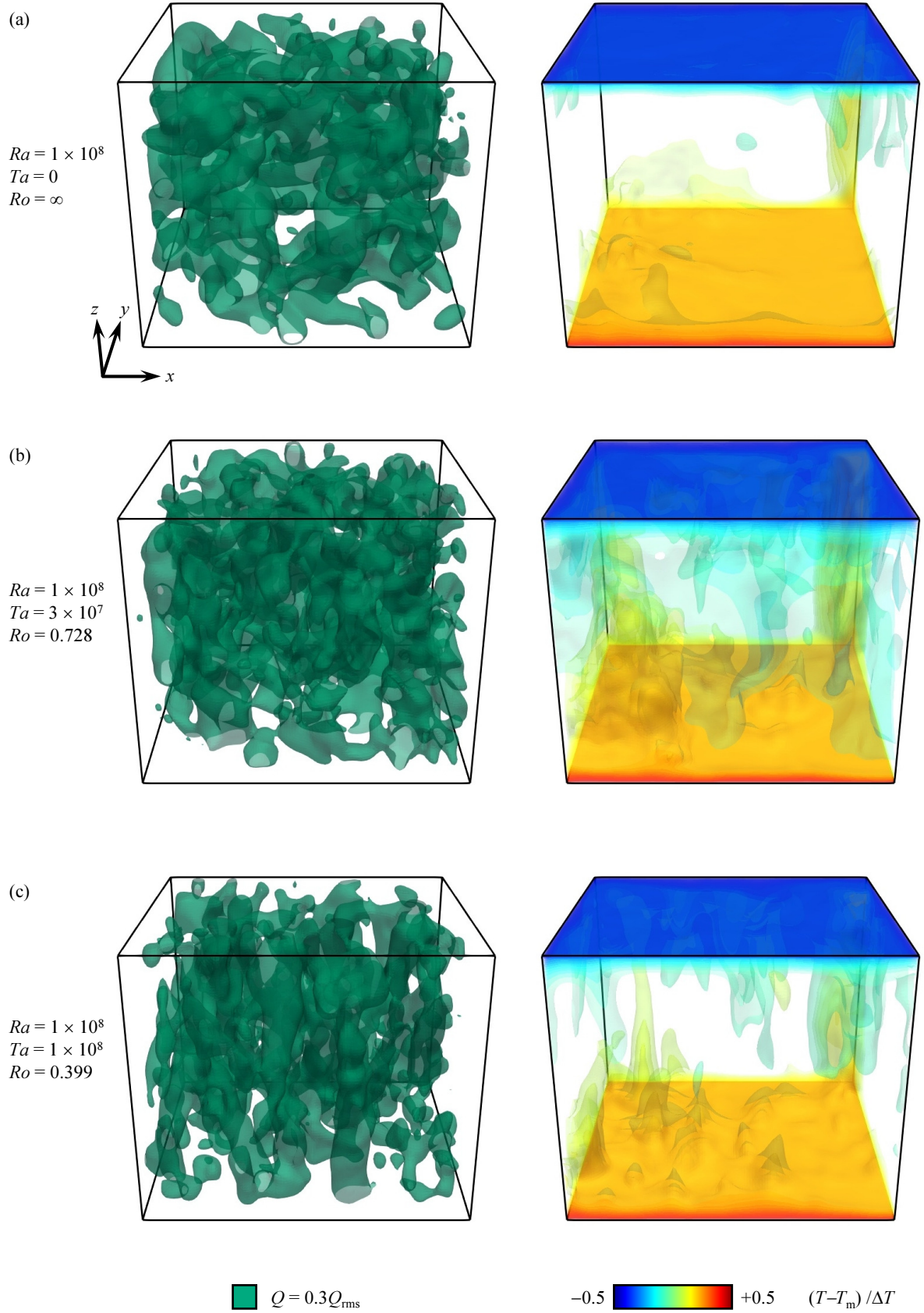


Figure 4.27: Snapshots of the isosurfaces of Q -criterion (left column) and normalized temperature $(T - T_m)/\Delta T$ (right column) with the condition of $Ra = 1 \times 10^8$: (a) $Ta = 0$, (b) $Ta = 3 \times 10^7$, and (c) $Ta = 1 \times 10^8$. Isosurface at $Q = 0.3Q_{rms}$ is shown in the left column.

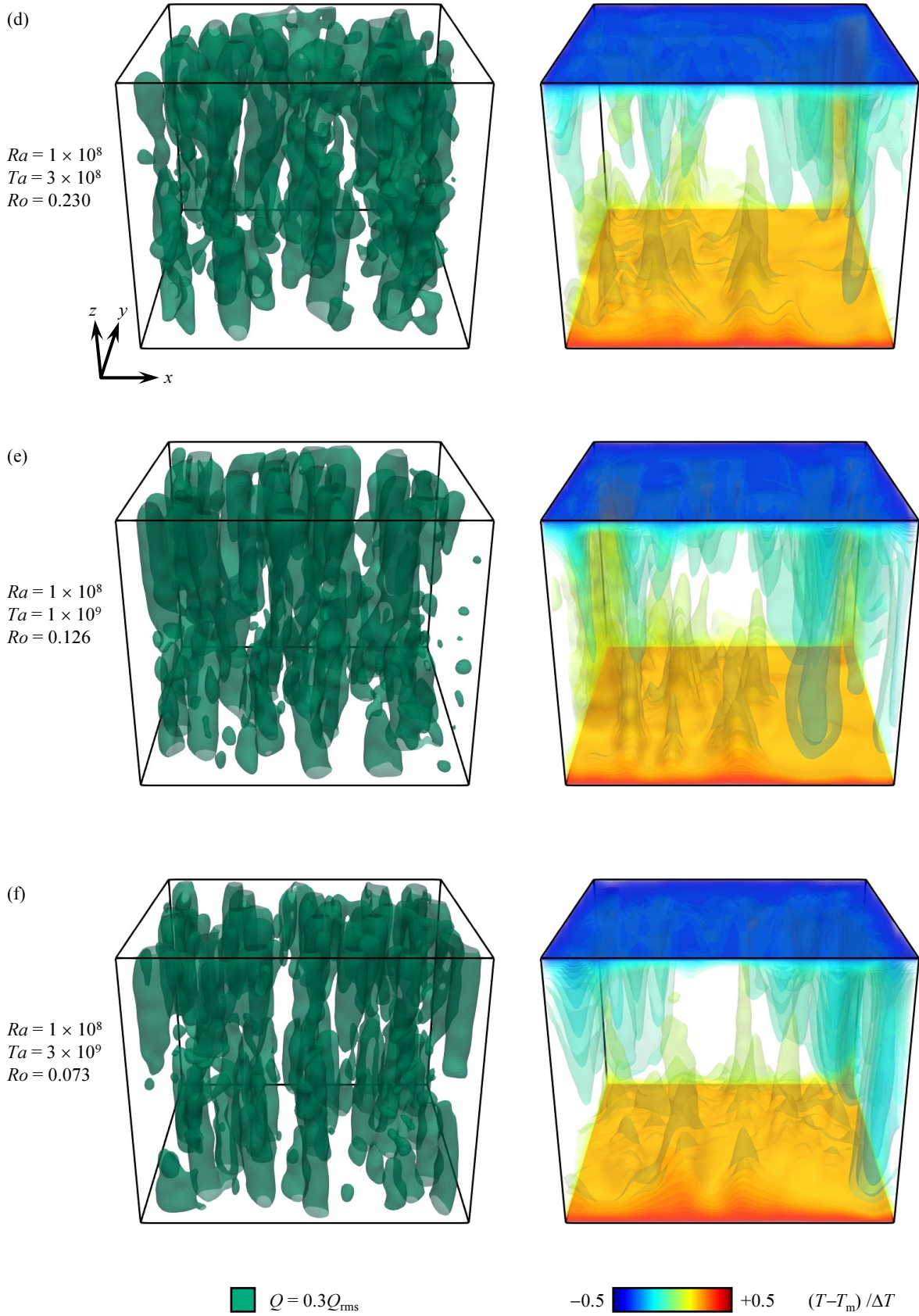


Figure 4.27: (continued.): (d) $Ta = 3 \times 10^8$, (e) $Ta = 1 \times 10^9$, and (f) $Ta = 3 \times 10^9$.

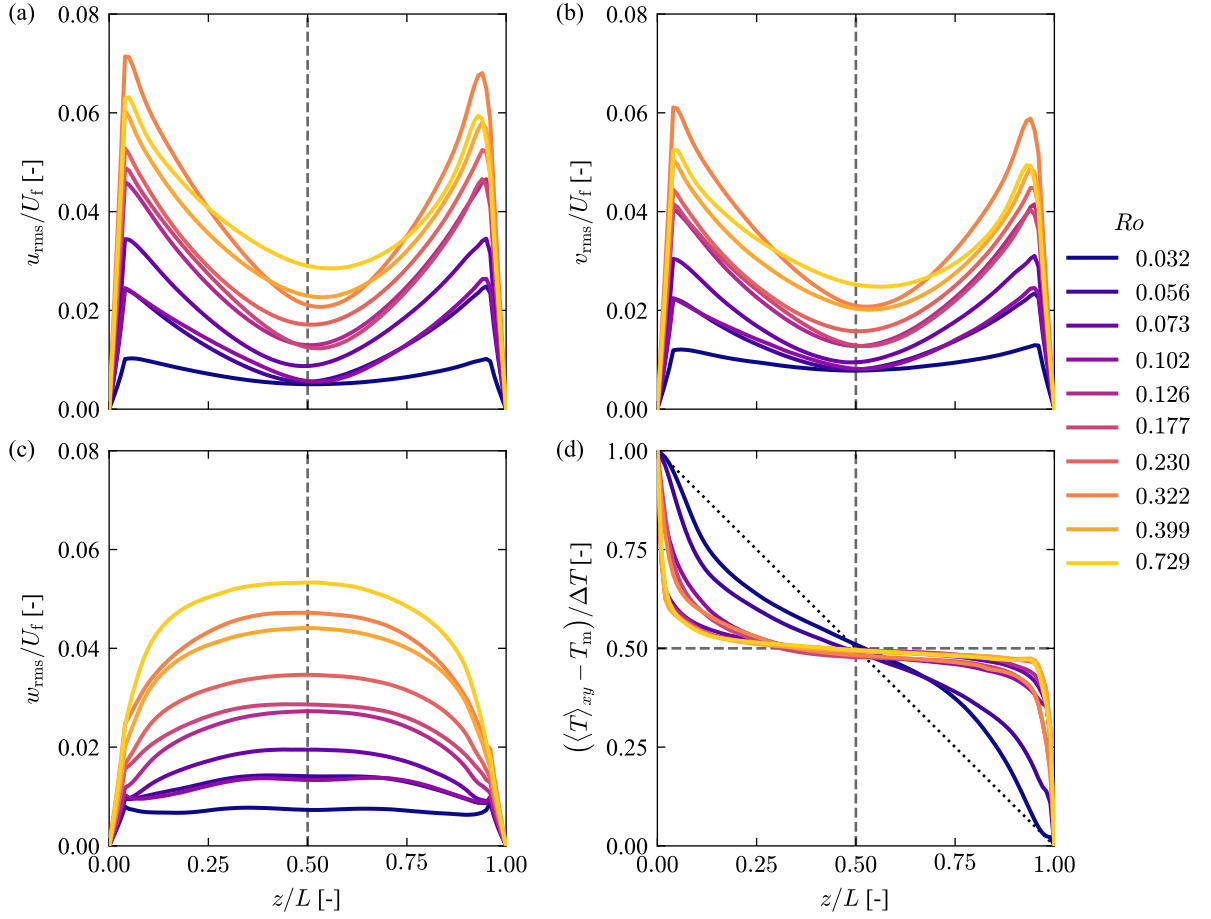


Figure 4.28: Vertical profiles of the horizontal root-mean-squared velocity: (a) u_{rms} , (b) v_{rms} , and (c) w_{rms} . (d) Vertical profiles of the horizontally-averaged temperature $\langle T \rangle_{xy}$. Lines are colored by the values of Ro .

4.4.2 Quantitative descriptions

4.4.2.1 Statistics of measured quantities

To quantitatively examine the performance of the measurement system, the statistics of the measured quantities are mentioned. In Fig. 4.28, the horizontal root-mean-squared (RMS) velocity and horizontally-averaged temperature are shown as vertical profiles. The RMS velocities are normalized by the free-fall velocity U_f , and the averaged temperatures are normalized by ΔT . The lines are colored by the values of Ro . For the horizontal velocities, u_{rms} and v_{rms} , shown in Fig. 4.28(a) and (b), the large RMS values are observed near the top and bottom boundaries. The RMS values gradually decrease with leaving from the boundaries, and record the minima roughly at the mid-height $z \sim 0.5L$ indicated by the vertical lines. The profiles of v_{rms} seem to be slightly blunt about the steep changes compared to those of the u_{rms} . This trend may originate from the measurement system, as the u values are measured as the in-plane velocity while the v values are measured as the out-of-plane velocity. In contrast to the profiles of the u_{rms} and v_{rms} , the w_{rms} profiles shown in Fig. 4.28(c) show a different trend. The RMS values increase monotonically toward the internal region, and record the maxima at the mid-height $z \sim 0.5L$.

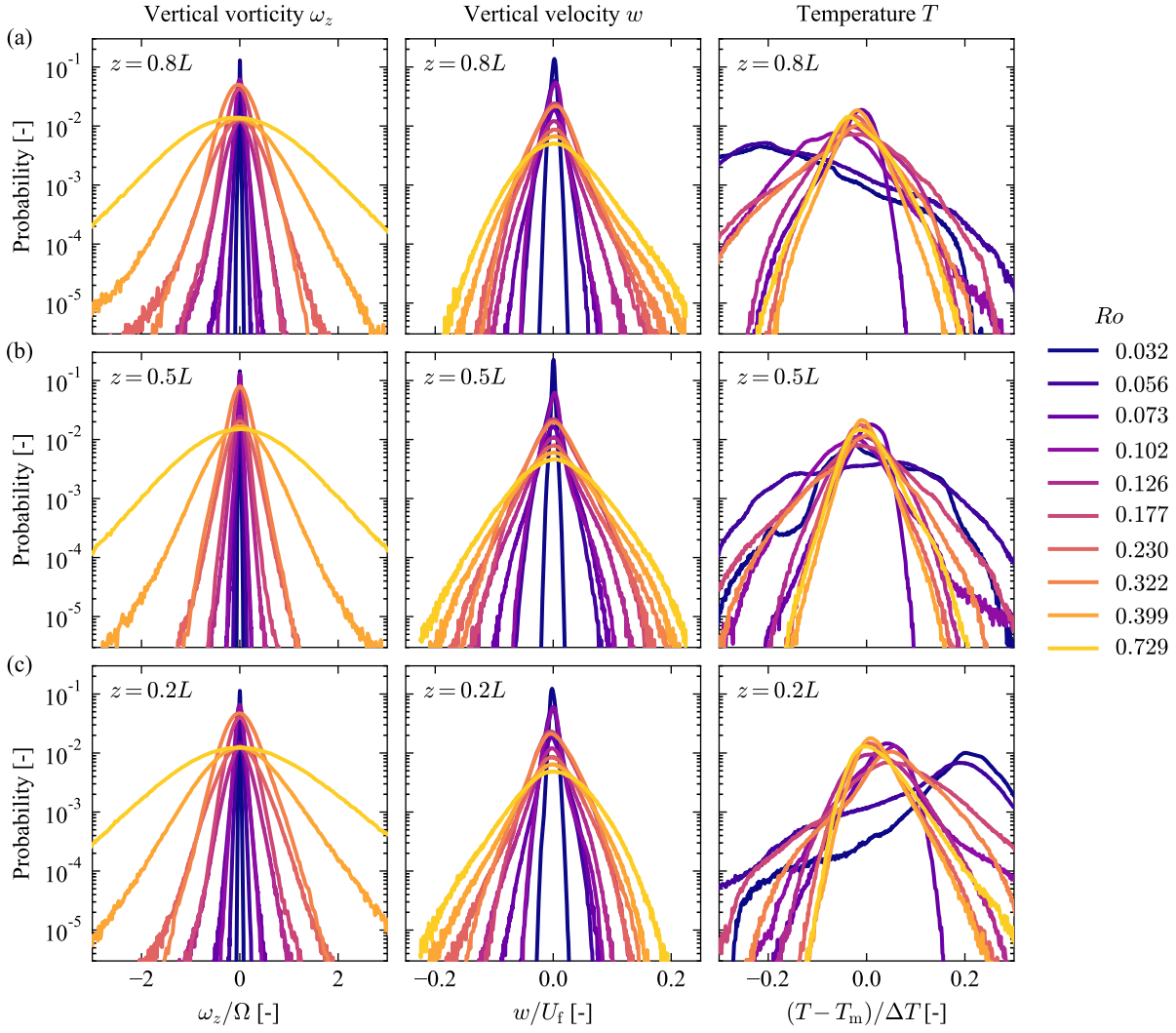


Figure 4.29: Probability density distributions of (left) the vertical vorticity ω_z , (center) the vertical velocity w , and (right) the temperature T taken at the different height (a) $z = 0.8L$, (b) $z = 0.5L$, and (c) $z = 0.2L$. The three quantities, ω_z , w , and T , are normalized by the angular velocity of the system Ω , the free-fall velocity U_f , and the vertical temperature difference ΔT , respectively. Lines are colored by the values of Ro .

As a global fashion, the smaller Ro conditions suggest the smaller RMS values. This indicates the suppression of the active convection motions by the rotation due to its 2D constraint, i.e., the Taylor–Proudman constraint. These RMS velocity profiles are quite similar to those explored earlier [e.g. 47]. The profiles of the horizontally-averaged temperature $\langle T \rangle_{xy}$ are shown in Fig. 4.28(d). With increasing the Ro values, the averaged temperature profiles move further away from the conduction temperature profile drawn by the dotted line. In particular, the regions of the steep vertical temperature gradients are formed at the top and the bottom boundaries. These steep temperature gradients indicate the presence of the thermal boundary layers. Unlike the proximity of the boundaries, the averaged temperatures at the internal regions are almost constant values, meaning the core regions. These temperature profiles are quite similar to the earlier reports [e.g. 81]. Incorporating these results, the constructed measurement system successfully quantified characteristic flow features of rotating RBC.

For a further validation, the probability density distributions of the measured quantities are also shown in Fig. 4.29. The probability density distributions are taken at different height, $z = 0.8L$ (close to the top boundary), $z = 0.5L$ (the mid-height), and $z = 0.2L$ (close to the bottom boundary), and shown respectively in Fig. 4.29(a), (b), and (c). In the left column, the probability density distributions of the vertical vorticity $\omega_z = \partial v/\partial x - \partial u/\partial y$ normalized by the angular velocity of the system are shown, and the corresponding skewness and kurtosis are listed in Table 4.2. Here, the skewness α_3 is the third standardized moment defined as

$$\alpha_3 = \frac{E[(X - \mu)^3]}{(E[(X - \mu)^2])^{3/2}}, \quad (4.45)$$

and the kurtosis α_4 is the fourth standardized moment defined as

$$\alpha_4 = \frac{E[(X - \mu)^4]}{(E[(X - \mu)^2])^2}. \quad (4.46)$$

Here, X , μ , and $E(\cdot)$ are the variables (ω_z , w , or T), the mean of the variables, and the expectation operator, respectively. As the mid-height $z = 0.5L$, the probability density distributions of ω_z become fatter resulting in the decrease in the kurtosis with increasing the Ro values. The distributions near the top and the bottom show asymmetry, the large skewness; the top yields larger frequency at the negative, while the frequency at the positive becomes larger near the bottom. In the central column, the probability density distributions of the vertical velocity w normalized by the free-fall velocity U_f are shown, and the corresponding skewness and kurtosis are listed in Table 4.3. Similar to the plots in Fig. 4.29(a), the distributions become fatter with increasing of the Ro , and the smaller Ro conditions show narrower distributions. The asymmetry in the distributions is also observed in the upper and the lower regions like ω_z shown in Fig. 4.29(a). In the right column, the probability density distributions of the temperature T normalized by the vertical temperature difference ΔT are shown, and the corresponding skewness and kurtosis are listed in Table 4.4. In general, the distributions of T approach to the Gaussian shape $\alpha_4 = 3$ or exponential one $\alpha_4 = 6$, if the convective motions are characterized as soft turbulence or hard turbulence. The present conditions are categorized in both regimes if the rotation is not imposed, as the soft turbulence is realized for $2 \times 10^5 < Ra < 4 \times 10^7$ and the hard turbulence is identified in $4 \times 10^7 < Ra < 6 \times 10^{12}$ [25]. However, considering that the convection influenced by the rotation is suppressed and the supercriticality is small, the flow regimes may be categorized into the soft turbulence or non-turbulent states. For the larger Ro conditions at which the rotation influences weakly, it is possible to find the intermediate states between the Gaussian and the exponential $3 < \alpha < 6$. In contrast, the smaller Ro conditions show wider distributions. Similar to the other two quantities, the skewness of the distributions increase in the upper and the lower regions.

Table 4.2: Statistical quantities of the probability density distributions of the vertical vorticity ω_z , corresponding to the left column of Fig. 4.29.

Case	Ro	Skewness α_3			Kurtosis α_4		
		$z = 0.8L$	$z = 0.5L$	$z = 0.2L$	$z = 0.8L$	$z = 0.5L$	$z = 0.2L$
5	0.0326	−0.66	−0.26	0.12	6.87	6.11	4.38
4	0.0564	−0.13	−0.35	0.04	3.97	6.02	3.90
10	0.0728	−0.18	−0.11	−0.01	4.20	5.29	3.93
3	0.103	−0.14	−0.40	0.17	4.04	5.87	4.35
9	0.126	−0.17	−0.13	−0.01	4.28	4.27	4.11
2	0.178	−0.09	−0.05	0.20	3.94	4.86	4.00
8	0.230	−0.16	−0.03	0.05	4.68	4.09	4.11
1	0.326	0.00	−0.05	0.28	3.89	4.41	4.04
7	0.399	−0.10	0.00	0.13	4.56	4.71	4.84
6	0.728	−0.06	0.06	0.27	4.60	4.67	5.07

Table 4.3: Statistical quantities of the probability density distributions of the vertical velocity w , corresponding to the central column of Fig. 4.29.

Case	Ro	Skewness α_3			Kurtosis α_4		
		$z = 0.8L$	$z = 0.5L$	$z = 0.2L$	$z = 0.8L$	$z = 0.5L$	$z = 0.2L$
5	0.0326	−0.31	−0.18	−1.82	3.00	3.17	9.03
4	0.0564	−0.18	0.22	−0.89	3.55	2.67	4.57
10	0.0728	−0.19	0.05	−0.70	5.62	4.48	4.19
3	0.103	−0.21	0.43	0.85	3.69	3.72	8.55
9	0.126	0.12	0.10	0.35	3.20	3.22	3.43
2	0.178	0.17	0.30	0.02	4.18	4.20	4.02
8	0.230	0.22	0.06	0.65	4.24	3.63	4.55
1	0.326	0.22	0.16	0.00	4.40	4.53	4.36
7	0.399	0.36	0.12	0.85	4.41	3.90	5.20
6	0.728	0.30	0.05	1.05	3.79	3.45	5.19

Table 4.4: Statistical quantities of the probability density distributions of the temperature T , corresponding to the right column of Fig. 4.29.

Case	Ro	Skewness α_3			Kurtosis α_4		
		$z = 0.8L$	$z = 0.5L$	$z = 0.2L$	$z = 0.8L$	$z = 0.5L$	$z = 0.2L$
5	0.0326	0.48	0.54	0.12	6.87	6.11	4.38
4	0.0564	0.71	−0.24	0.04	3.97	6.02	3.90
10	0.0728	−1.21	−0.68	−0.01	4.20	5.29	3.93
3	0.103	−0.06	−1.18	0.17	4.04	5.87	4.35
9	0.126	0.15	−0.14	−0.01	4.28	4.27	4.11
2	0.178	−0.52	−0.43	0.20	3.94	4.86	4.00
8	0.230	0.33	−0.61	0.05	4.68	4.09	4.11
1	0.326	−0.70	−0.93	0.28	3.89	4.41	4.04
7	0.399	0.01	−0.60	0.13	4.56	4.71	4.84
6	0.728	0.38	−0.33	0.27	4.60	4.67	5.07

Entirely, the trends shown in Fig. 4.29 are quite similar to those from the numerical simulations reported earlier by Julien et al. [81]. In addition, the statistical quantities, α_3 and α_4 , represent well the characteristics of rotating RBC. From these quantitative aspects, the presently constructed system can be considered to measure the physics of rotating RBC properly.

4.4.2.2 Heat transport

Since both the velocity and the temperature fields are measured simultaneously, the bulk heat transport in terms of the Nu is also examined to evaluate the performance of the measurement system. There are some definitions of the Nu values [89, 112, 131]. The Nu value is defined as

$$Nu = \frac{\Phi_{\text{cond}} + \Phi_{\text{conv}}}{\Phi_{\text{cond}}} = 1 + \frac{\Phi_{\text{conv}}}{\Phi_{\text{cond}}}, \quad (4.47)$$

where Φ_{cond} and Φ_{conv} are the conductive and convective heat transports. It is often described by using the nondimensionalization as L for the length, κ/L for the velocity, and ΔT for the temperature scales, and Eq. 4.47 is rewritten as

$$Nu = 1 + \langle w' \delta T' \rangle_{V,t}, \quad (4.48)$$

where $\langle \cdot \rangle_{V,t}$ represents the spatiotemporal average. Here w' is the nondimensionalized vertical velocity simply calculated as

$$w' = \frac{L}{\kappa} w, \quad (4.49)$$

and $\delta T'$ is the nondimensionalized temperature deviation from the lowest temperature at the top boundary T_{top} defined as

$$\delta T' = \frac{T - T_{\text{top}}}{\Delta T}. \quad (4.50)$$

Using these nondimensionalizations, the Nu value is calculated as

$$Nu = 1 + \left\langle \left(\frac{L}{\kappa} w \right) \left(\frac{T - T_{\text{top}}}{\Delta T} \right) \right\rangle_{V,t} = 1 + \frac{L}{\kappa \Delta T} \langle w(T - T_{\text{top}}) \rangle_{V,t}. \quad (4.51)$$

That is, the Nu value can be calculated by the spatiotemporal average of the vertical heat flux $j_z = w(T - T_{\text{top}})$.

In Fig. 4.30, the computed Nu values are shown. The instantaneous Nu values computed by taking only volume average are shown as temporal signals in Fig. 4.30(a). The lines are colored by the Ro values. The time is normalized by the thermal diffusion timescale $\tau_\kappa = L^2/\kappa \sim 6.8 \times 10^4 \text{ s} \sim 19 \text{ hour}$. Entirely, the temporal signals fluctuate larger in the larger Ro conditions. This may be natural because the large Ro conditions yield disorganized structures close to the thermal turbulence, while the flow structures formed in the small Ro conditions are more organized and may not change largely with time. The bulk Nusselt numbers Nu

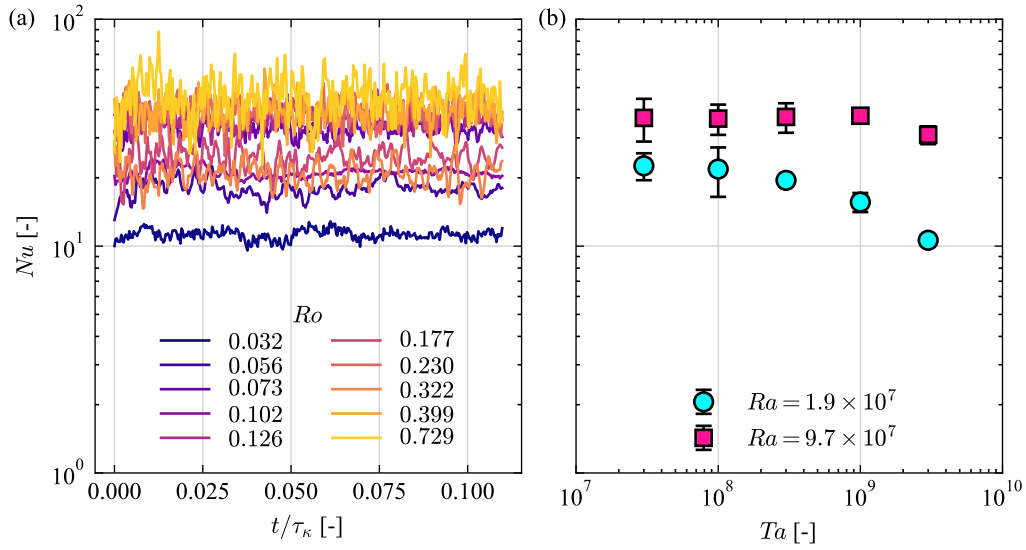


Figure 4.30: (a) Temporal changes of instantaneous Nusselt number Nu , and (b) the averaged Nu over the measurement duration. The error-bars in (b) indicated the standard deviations for the temporal signals.

computed by taking the temporal average of the signals shown in Fig. 4.30(a) are also plotted in Fig. 4.30(b). The abscissa is the Ta value, and two different symbols are used for the different Ra conditions. The error-bars represent the standard deviations of the temporal signals. The order of the measured Nu values matches well to the values reported earlier [e.g. 81, 89, 189, 219]. In addition, it is possible to see an increase in Nu value with increasing Ta values for $Ra = 9.7 \times 10^7$, the maximum Nu is recorded at $Ta = 1 \times 10^9$. Unlike the results for the large Ra , the results for $Ra = 1.9 \times 10^7$ do not present such increasing trend in the Nu value, and the Nu decreases monotonically with increase in the Ta . Considering that the large error-bar for $Ra = 1.9 \times 10^7$ and $Ta = 1 \times 10^8$, the Nu value might record the maximum at this condition. It may be not possible to measure the Nu value precisely for the smaller Ra conditions, as the imposed vertical temperature difference is only $\Delta T = 1$ K. Even some limitations in the measurements exist, however, the global trend in the heat transport characteristics of rotating RBC seems to be successfully measured by the constructed measurement methods.

Incorporating the results shown in Sec. 4.4.2, the measurement systems, scanning stereoscopic PIV and scanning LCT, can measure the intermediate to long timescale physics of the rotating RBC of $O(\geq 10^1 \text{ s})$ successfully, in terms of the flow morphology and the heat transport characteristics.

4.5 Results and discussion

The main objective of Chapter 4 is to elucidate the dynamics of the columnar vortices and the entailing heat transport characteristics through experimental study. Utilizing the constructed measurement systems in Sec. 4.3, these open issues are addressed in Sec. 4.5.

4.5.1 Extraction of vortex region

First, the columnar vortices are required to be detected from the measured flow fields. To examine the horizontal motions of columnar vortices and the heat transport characteristics, the volumetric vortex regions are also required to be extracted.

4.5.1.1 Q value method

The most straightforward way may be the utilization of the Q value, as it is often utilized to identify vortical structures as positive values of the Q indicate vortex-dominant regions. Thus, the all the grid points in the measurement domain are binarized using the Q fields; grid points with $Q \geq 0$ are vortex-dominant and those with $Q < 0$ are strain-dominant. This binarized field is expressed with P_v defined as

$$P_v(x, y, z) = \begin{cases} 1 & \text{if } Q(x, y, z) \geq Q_{th} \\ 0 & \text{otherwise} \end{cases}, \quad (4.52)$$

where Q_{th} is a threshold value for the Q value, and Q_{th} should be larger than 0 for extracting the vortex-dominant regions. The original Q fields contain spatial discontinuities, since the Q value is calculated using the spatial gradient of velocity fields measured by the scanning stereoscopic PIV. To smooth the discontinuities in the Q -criterion fields, a Gaussian kernel was convolved to the original velocity fields for the purpose of performing spatial averaging. The vortex-dominant regions are then labeled in the voxel space of the P_v using the six neighboring voxels.

The result of vortex detection using the Q value adopting a global threshold of $Q_{th} = 0.15Q_{rms}$ is shown in Fig. 4.31. Each labeled vortex region is colored uniquely for visibility. The columnar vortices should possess vertically coherent structures, and the labeled vortex regions should be connected vertically. The results shown in Fig. 4.31, however, display broken structures at $z \sim 0.5L$. These disconnections are natural, as the rotational direction inside each columnar vortex changes at $z \sim 0.5L$ [165], and the strength of the vortex represented by the Q value at the height approaches to zero or negative. If a smaller threshold Q_{th} is adopted, unwanted connections of columnar vortices in the horizontal domain happen. Considering these results, it is not realistic to identify columnar vortices by the Q -criterion as the columnar vortices are not fully separable.

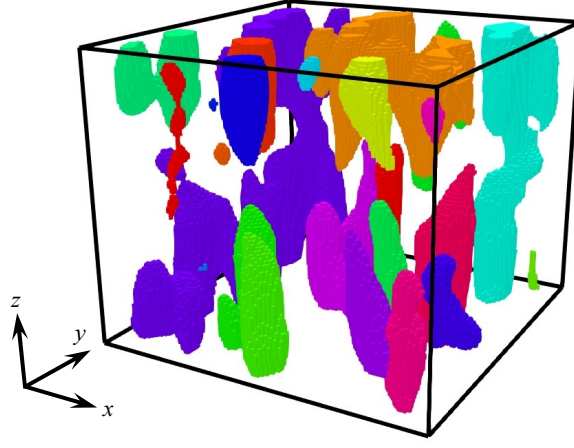


Figure 4.31: Labeled vortex regions identified by the Q -criterion for $Ra = 2 \times 10^7$ and $Ta = 3 \times 10^8$ ($Ro \sim 0.10$, $\widetilde{Ra} = 42$). Each columnar vortex is colored uniquely.

4.5.1.2 Template matching method

Another approach to identify the positions of the columnar vortices is utilization of the concept of template matching. In horizontal cross-sections, which are close to the top or bottom boundaries, for instance, at $z = 0.8L$ or $z = 0.2L$, each columnar vortex can be recognized as a large undulation on temperature fields [122]. The method proposed earlier by Noto et al. [122] utilized a 2D Gaussian distribution to express the undulations in the horizontal temperature fields. The 2D Gaussian distribution $G(x, y)$ has a point-spread shape and defined as

$$G(x, y) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \quad (4.53)$$

where σ is the standard deviation of the Gaussian. The horizontal scale of the columnar vortices, i.e., the diameter of the vortex D_v , was predicted earlier by Sakai [165], and it is derived as

$$D_v = 4.6L \frac{\delta_t^2}{2\delta_t^2 + \delta_t\delta_E + \delta_E^2} Ra^{1/3} Ta^{-3/8}. \quad (4.54)$$

Here, the δ_t and δ_E are the thicknesses of the thermal and the Ekman boundary layers, respectively, and these are estimated as

$$\delta_t = 3.8LRa^{-1/3}, \quad (4.55)$$

and

$$\delta_E = \sqrt{2}LTa^{-1/4}. \quad (4.56)$$

The theoretically estimated vortex diameter D_v has been confirmed to match well the actual horizontal scale of vortices in the earlier experimental works by Noto et al. [122, 123], Sakai [165]. Noto et al. [122] then tried to identify the horizontal vortex positions by the concept of template matching utilizing the Gaussian as the template. To model the temperature undulations

by the 2D Gaussian, the size of the template is limited to the vortex diameter D_v , and the half of the quarter size of the D_v was used for the standard deviation, $\sigma = D_v/4$. That is, the vortex-mimicking Gaussian distribution $G_v(x, y)$ is written as

$$\begin{aligned} G_v(x, y) &= \frac{1}{\sqrt{\pi D_v^2/8}} \exp\left(-\frac{x^2 + y^2}{D_v^2/8}\right) \\ &= \sqrt{\frac{2}{S_v}} \exp\left(-\frac{x^2 + y^2}{R_v^2/2}\right), \quad -R_v \leq x, y \leq R_v, \end{aligned} \quad (4.57)$$

where S_v is the area of the horizontal cross-section of the columnar vortex, $S_v = \pi D_v^2/4$, and R_v is the radius of the vortex, $R_v = D_v/2$. To find the vortex positions, at which a horizontal temperature distribution $T(x, y, z = z_0)$ at a certain height $z = z_0$ is similar to the $G_v(x, y)$, the cross-correlation coefficient distribution $C_{TG}(x, y) \in [-1, 1]$ is calculated as

$$C_{TG}(x, y, z_0) = \frac{\sum_{\xi=-R_v}^{R_v} \sum_{\eta=-R_v}^{R_v} [T(x + \xi, y + \eta, z_0) - \bar{T}] [G_v(\xi, \eta) - \bar{G}_v]}{\sqrt{\sum_{\xi=-R_v}^{R_v} \sum_{\eta=-R_v}^{R_v} [T(x + \xi, y + \eta, z_0) - \bar{T}]^2 \sum_{\xi=-R_v}^{R_v} \sum_{\eta=-R_v}^{R_v} [G_v(\xi, \eta) - \bar{G}_v]^2}}, \quad (4.58)$$

where $\bar{T}$ and $\bar{G}_v$ are the mean values within the kernel. In this study, the horizontal temperature distribution at $z_0 = 0.2L$ is used for identifying the up-welling vortices, and that at $z_0 = 0.8L$ is used for the down-welling vortex. For the calculation of the $C_{TG}(x, y, 0.8L)$, the G_v is inverted, i.e., $-G_v$ is used. This is because the temperature distribution inside an up-welling vortex is convex upward, while that inside a down-welling vortex is convex downward. By seeking local maxima from the C_{TG} distribution, the horizontal vortex positions can be identified. In addition, a threshold $C_{TG} = 0.5$ is imposed for identifying the vortex positions to eliminate non-significant peaks of the C_{TG} distributions. The vortex positions are determined in the sub-grid level by adopting the Gaussian peak fit for the C_{TG} distribution.

Example of vortex detection utilizing the template matching method described above is shown in Fig. 4.32. The horizontal temperature fields at $z = 0.8L$ (top) and $0.2L$ (bottom) are shown as color contours in Fig. 4.32(a). Here the temperature value is normalized as $(T - T_m)/\Delta T$. The corresponding cross-correlation coefficient distributions C_{TG} are shown in Fig. 4.32(b). The vortex-mimicking Gaussian template $\pm G_v$ are also illustrated between Fig. 4.32(a) and (b). It is found that the columnar vortices in the temperature fields possess similar horizontal scales to the template made of the theoretically estimated diameter D_v . The detected vortex positions after adopting the local maximum filter and a threshold are plotted in Fig. 4.32(c). The columnar vortices distribute randomly in the horizontal domain.

Since the horizontal scale of the columnar vortices are well represented by Eq. 4.54, the vortex-occupying regions in the horizontal domain can be defined by the circular regions around

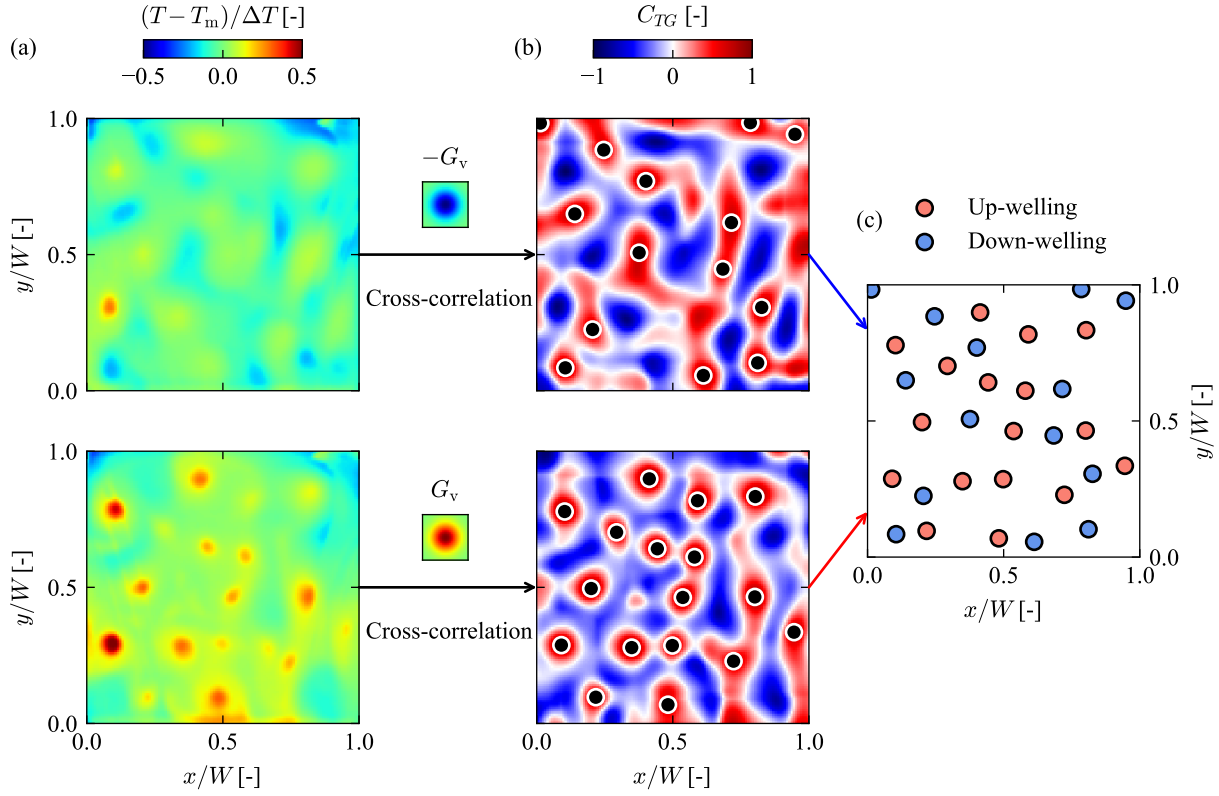


Figure 4.32: Example of vortex detection utilizing the template matching for the horizontal temperature fields for $Ra = 2 \times 10^7$ and $Ta = 3 \times 10^8$ ($Ro \sim 0.10$, $\widetilde{Ra} = 42$): (a) horizontal temperature fields $(T - T_m)/\Delta T$ at $z = 0.8L$ (top) and $z = 0.2L$ (bottom), (b) cross-correlation distributions C_{TG} at $z = 0.8L$ (top) and $z = 0.2L$ (bottom), and (c) detected vortex positions. The Gaussian templates G_v are also shown between (a) and (b).

the detected vortex positions with the diameter of D_v . The volumetric regions of the columnar vortices are thus simply defined as the cylindrical region having the axis in the z -direction. That is, the volume of the columnar vortex is computed as $V_v = S_v L$. The oblique view of the labeled vortex regions identified by the template matching for the temperature field is shown in Fig. 4.33. Each columnar vortex is colored uniquely for visibility. Individual columnar vortices are well defined as the cylindrical regions in the 3D domain.

4.5.2 Vortex dynamics

To study the horizontal motions of columnar vortices, the detected vortex positions in the horizontal domain are tracked. Since the vortex motions are considered to be diffusive motions, i.e., random-walk-like motions of Brownian particles [30, 123], the nearest neighbor method is adopted to link the same vortex over time. Examples of the tracked trajectories of the columnar vortices are illustrated in Fig. 4.34. The trajectories of the up-welling and down-welling vortices are drawn by the dashed and the solid lines, and the appearance and disappearance positions are also indicated by the symbols. The trajectories of the vortices generated within 5 scans, for 110 s, are shown. For the larger $\widetilde{Ra}$ condition shown in Fig. 4.34(a), the flow regime is assigned to the vortical plumes, and the number of vortices are small and move largely between each scan.

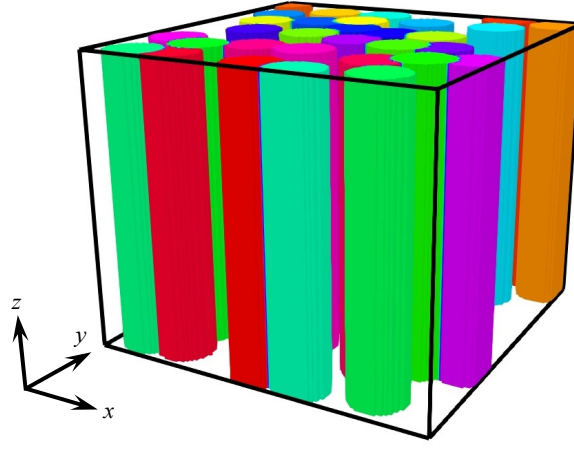


Figure 4.33: Labeled vortex regions identified by the template matching for the temperature field for $Ra = 2 \times 10^7$ and $Ta = 3 \times 10^8$ ($Ro \sim 0.10$, $\widetilde{Ra} = 42$). Each columnar vortex is colored uniquely.

For the smaller $\widetilde{Ra}$ condition shown in Fig. 4.34(b), the flow regime is assigned to the convective Taylor columns (CTCs), and the number of vortices are large and the displacements between the scans are smaller than those in Fig. 4.34(a). For both cases, the vortex motions seem to be random, and no significant trend is observed.

To quantify the random-walk-like vortex motions in the horizontal domain, the advection speed $v_{adv} = \sqrt{v_x^2 + v_y^2}$ is studied. Here, v_x and v_y are the advection velocities in the x and y -directions. In Fig. 4.35, the probability distributions of the v_{adv} are plotted for all the measured conditions. The horizontal advection speed v_{adv} is very small, $O(10^{-1} \text{ mm/s})$, while the characteristic velocity in the flow fields is $O(10^0 \text{ mm/s})$. Note that the speed of the scanning system is $V_{scan} = 8 \text{ mm/s}$, and thus the columnar vortices hardly move within a single scan. The probability density distributions of v_{adv} localized in the slower speed regions (smaller energy regions) for the smaller Ro conditions. In contrast, the larger Ro show fatter distributions, meaning the presence of high-speed vortices. The probability distributions are qualitatively similar to the Maxwell (Boltzmann) distribution as earlier reported by Noto et al. [123]. The Maxwell distribution assumes a probability density function of speed of molecules in a thermally equilibrium state, and written in a simplified form as

$$P_M = A v_{adv} \exp\left(-\frac{v_{adv}^2}{B}\right) \quad (4.59)$$

where, A and B are constant values. The probability distributions for the moderate $\widetilde{Ra}$ conditions shown in Fig. 4.35(b) are fitted by Eq. 4.59, and the results are illustrated by the dotted lines. The fitting curves largely coincide with the original plots, however, discrepancies exist especially at the higher energy regions, i.e., the probabilities for the high speed vortices are underestimated. The same trend was earlier reported in rotating RBC systems by Noto et al. [123]. This trend, the distorted Maxwell distribution, was earlier found in a system yielding inelastic collisions, unlike perfectly elastic collisions of Brownian particles [24, 152]. That is, the collisions of multiple

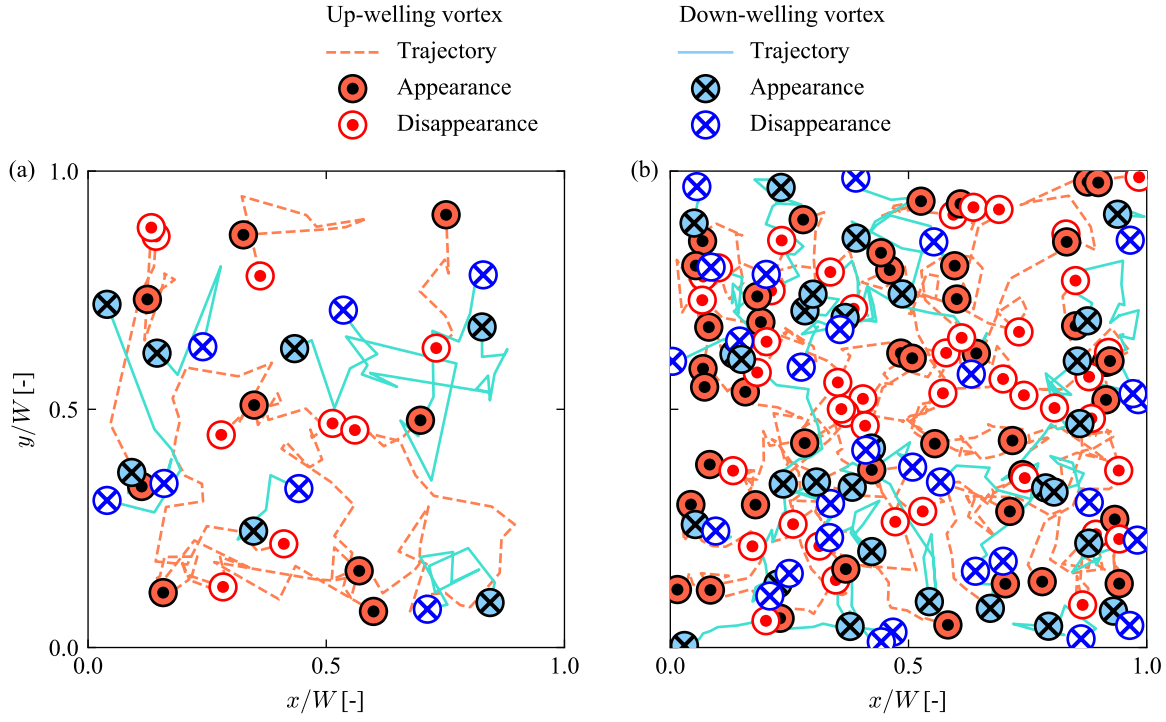


Figure 4.34: Trajectories of horizontal vortex motions: (a) vortical plumes for $Ra = 2 \times 10^7$, $Ta = 1 \times 10^8$ ($Ro \sim 0.18$, $\widehat{Ra} = 88$) and (b) convective Taylor columns for $Ra = 1 \times 10^8$, $Ta = 3 \times 10^9$ ($Ro \sim 0.07$, $\widehat{Ra} = 47$). The trajectories are drawn by lines, and the appearance and disappearance positions are indicated by the symbols.

vortices may be also described as an inelastic collision process; the collisions of the multiple vortices are damped by the viscous force. In addition, Chong et al. [30] reported the presence of historical force in the motions of the columnar vortices. The historical force was observed in very short timescale of $O(< 10^0 \text{ s})$, since the inertia of the columnar vortices is too large to be ignored like that in the classical Brownian system. To fit the distorted Maxwell distributions, the modified Maxwell distribution is newly defined as

$$P_{\text{MM}} = A v_{\text{adv}}^B \exp \left(-\frac{v_{\text{adv}}^C}{D} \right), \quad (4.60)$$

where A , B , C and D are constant values. The fitting results are also drawn by the solid lines in Fig. 4.35(b). Compared to the original Maxwell fitting, the modified ones match well to the plots.

Form the fitting by the modified Maxwell distribution, the most probable speed v_{mp} can be computed. By differentiating Eq. 4.60 by v_{adv} , one obtains

$$P'_{\text{MM}} = \frac{A}{D} v_{\text{adv}}^{B-1} (BD - C v_{\text{adv}}^C) \exp \left(-\frac{v_{\text{adv}}^C}{D} \right). \quad (4.61)$$

Thus the most probable speed (peak speed) v_{mp} can be identified as the v_{adv} value satisfying

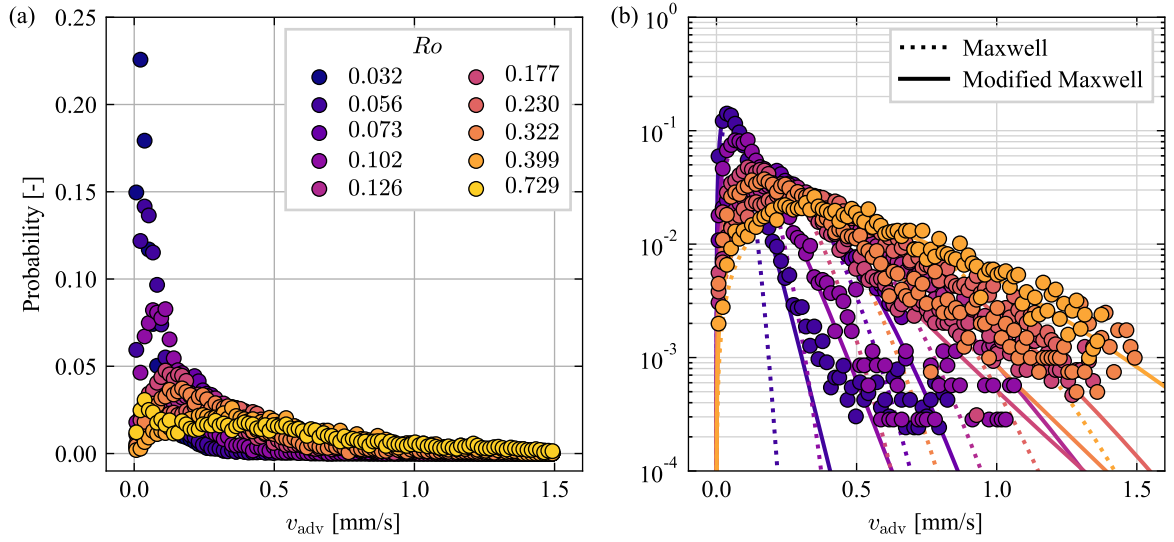


Figure 4.35: Probability distributions of horizontal advection speed of the columnar vortices v_{adv} : (a) plots for all the conditions in the linear domain, and (b) plots for the moderate rotation cases, $0.073 \lesssim Ro \lesssim 0.399$, in the logarithmic domain. Fitting curves by the Maxwell distribution (Eq. 4.59) and the modified Maxwell distribution (Eq. 4.60) are shown by the dotted and solid lines. The plots and the lines are colored by the Ro values.

$P'_{MM} = 0$, and expressed as

$$v_{mp} = \left(\frac{BD}{C} \right)^{1/C}. \quad (4.62)$$

In Fig. 4.36(a), the values of v_{mp} are plotted over the Ta values. The different symbols are used to identify different Ra conditions. The v_{mp} values decrease monotonically with increasing of the Ta values for the fixed Ra conditions within the range of $O(10^{-2} - 10^{-1} \text{ mm/s})$; the minimum value is $v_{mp} = 0.038 \text{ mm/s}$ for $Ra = 1.9 \times 10^7$ and $Ta = 1 \times 10^9$, and the maximum value is $v_{mp} = 0.30 \text{ mm/s}$ for $Ra = 9.7 \times 10^7$ and $Ta = 1 \times 10^8$. Entirely, the large Ra conditions yield the large v_{mp} values even for the same Ta conditions. Thus, the v_{mp} values are normalized by the free-fall velocity U_f , and plotted in the Ro space as shown in Fig. 4.36(b). The plots collapse into a line, $v_{mp}/U_f \propto Ro^{1/2}$, except for the smallest Ro condition, as indicated by the solid line. The supercriticality of the smallest Ro condition is $\widetilde{Ra} = 19$, and thus the flow regime is assigned to as the cellular regime rather than the CTCs regime. In the cellular regime, the columnar vortices are localized and the vortex motions will stop [28, 82]. The discrepancy for the smallest Ro is thus originated from the difference in the flow regime. However, most of the explored conditions are well described by the fitting utilizing the free-fall velocity U_f . This suggests the importance of the thermal buoyancy on the random-walk-like vortex motions in the CTCs and the vortical plume regimes.

The horizontal diffusive motions of the columnar vortices are further justified by determining the mean-squared displacement (MSD). The MSD is defined as

$$\text{MSD} = \langle \delta \mathbf{r}(t)^2 \rangle = \frac{1}{N} \sum_n^N [\mathbf{r}_n(t) - \mathbf{r}_n(t = t_0)]^2, \quad (4.63)$$

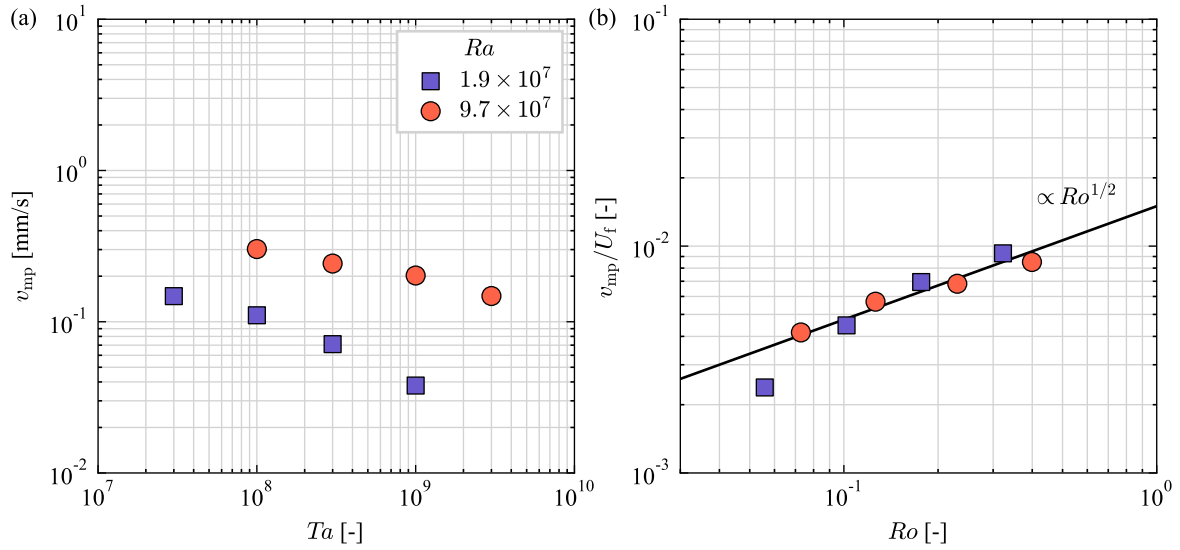


Figure 4.36: Most probable speed v_{mp} plotted in (a) Ta space and (b) Ro space with normalization by U_f . The solid line drawn in (b) indicates $v_{mp}/U_f \propto Ro^{1/2}$.

where $\delta \mathbf{r}(t)$ is the displacement vector from the initial vortex position $\mathbf{r}(t = t_0)$ emerged at $t = t_0$, and N is the number of tracked vortices. The temporal dependence of the MSD for a diffusion process is provided as

$$\langle \delta \mathbf{r}(t)^2 \rangle \propto t^\gamma, \quad \gamma \in [0, 2]. \quad (4.64)$$

For the normal diffusion process, Brownian motion, the γ becomes unity. If $\gamma \neq 1$, the diffusion process will be categorized into anomalous diffusion; the case with $0 < \gamma < 1$ represents the sub-diffusive motion, and that with $1 < \gamma < 2$ is the super-diffusive motions. The instance with $\gamma = 0$ means no motion, and that with $\gamma = 2$ corresponds to the ballistic motion driven without drag. In Fig. 4.37(a), the MSD values are plotted. The symbols are colored by the Ro values, and the minimum and the maximum Ro conditions are excluded from the plots as these conditions do not yield presence of the columnar vortices. For reference, two dashed lines representing the normal diffusion, $\langle \delta \mathbf{r}(t)^2 \rangle \propto t^\gamma$ with $\gamma = 1$ are shown. Most of the plots seem to show trends proportional to t , meaning the normal diffusion (Brownian motion). In long timescale of $O(\geq 10^2 \text{ s})$, it is not possible to find clear dependence on time due to the lack of data for the ensemble averaging of Eq. 4.63. To quantify the anomalies in the diffusive motions, γ is obtained by the least-squares fits for the MSD data in $t < 200 \text{ s}$ using the function

$$\langle \delta \mathbf{r}(t)^2 \rangle = 2Dt^\gamma. \quad (4.65)$$

Here, D represents the diffusion coefficient in the unit of mm^2/s . The obtained values of D and γ are plotted in Fig. 4.37(b). The diffusion coefficients D increases with increase in Ro , meaning that the strong rotation suppresses the horizontal advection of columnar vortices. This can be explained by the horizontal scales of columnar vortices; the horizontal scale decreases

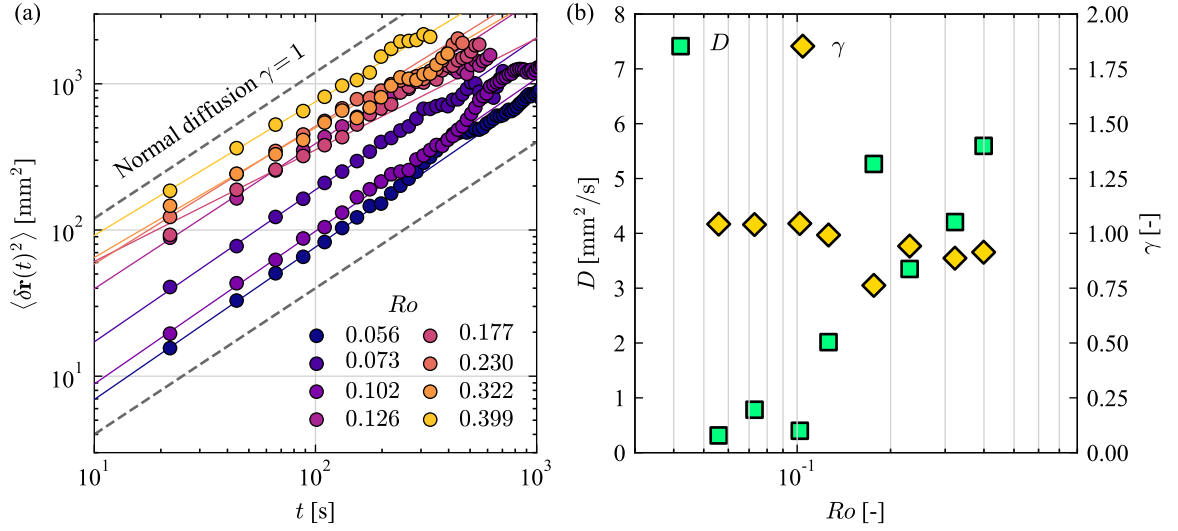


Figure 4.37: (a) Mean-squared displacement (MSD) of the columnar vortices $\langle \delta \mathbf{r}(t)^2 \rangle$. Symbols are colored by the Ro values. The dashed lines represent the normal diffusion, $\langle \delta \mathbf{r}(t)^2 \rangle \propto t^\gamma$ with $\gamma = 1$. Solid lines are the least-squares fits for the MSD data in $t < 200$ s. (b) Diffusion coefficients D and anomalies γ obtained by the least-squares fits.

with increasing the rotation as $D_v \propto Ta^{-3/8}$ (Eq. 4.54). The number of columnar vortices also increase with increase in the Ta values, and each columnar vortex is forced to localize by the surrounding vortices. The values of D are comparable to or larger than the viscous diffusion $\nu = 1 \text{ mm}^2/\text{s}$. The anomalies γ are close to unity for the most of the measured cases, meaning that the diffusive motions observed in the timescale of $O(\leq 10^2 \text{ s})$ are considered as normal diffusion, as expected in the earlier works [30, 123].

4.5.3 Heat transport characteristics of columnar vortex

Since the columnar vortices are extracted from the 3D domain, heat transport characteristic of each columnar vortex can be quantified. Here, the studied parameters are limited to only the ranges for ensuring the presence of columnar vortices, the vortical plumes and the CTCs. Thus, the cases 5 and 6 of Table 4.1, are excluded as they show the wall mode and turbulent structures in Fig. 4.26 and 4.27; the range of $0.056 \lesssim Ro \lesssim 0.399$ ($42 \leq \widetilde{Ra} \leq 450$) are studied in detail. This is because the vertically coherent structures of columnar vortices, penetrating from the top to bottom of the fluid layer, are not ensured in the smallest and the largest Ro conditions. To study details of the role of the columnar vortices on the heat transport, conditional sampling is performed for the columnar vortices extracted by the method mentioned in Sec. 4.5.1. A similar analysis was reported by Pieri et al. [143] for their direct numerical simulations, and they discussed heat transport characteristics of thermal plumes of rotating RBC weakly affected by rotation. However, detailed studies on the lower Ro conditions, ensuring the presence of the columnar vortices, are remained as an open issue.

The vertical velocity w inside a single columnar vortex is calculated as a conditional hori-

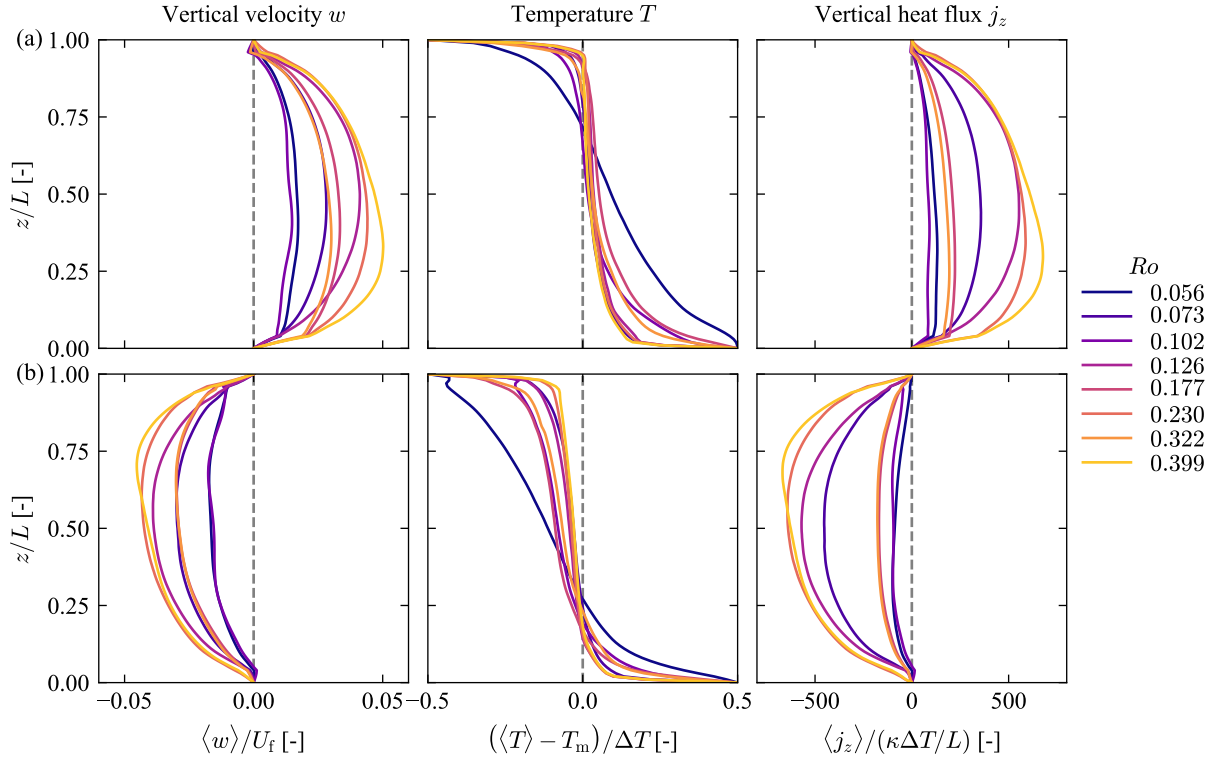


Figure 4.38: Conditional averaged values of (left) vertical velocity w , (center) temperature T , and (right) vertical heat flux j_z inside the columnar vortex as a function of the z . (a) and (b) are the results for the up-welling and the down-welling columnar vortices. The lines are colored by the Ro values.

zonal averages $\langle w \rangle$ as functions of the z . The results for both the up-welling and down-welling vortices are shown in the left column of Fig. 4.38(a) and (b), respectively. For most cases, the w profiles are vertically symmetric, while the larger Ro conditions show asymmetry. For the up-welling vortices, the w values take the maxima at $z \sim 0.25L$, and the down-welling vortices show opposite trends recording the maxima at $z \sim 0.75L$. The vertical asymmetry in $\langle w \rangle$ directly indicate the vertical asymmetry of the flow structures. Considering that such asymmetry is observed at the larger Ro conditions, these conditions do not yield emergence of the columnar vortices and produce thermal plumes, as the columnar vortices should have vertically symmetric structures. Similarly, the conditional horizontal averages of the temperature inside the vortices $\langle T \rangle$ are computed and shown in the central column Fig. 4.38. The internal temperatures of the up-welling vortices shown in Fig. 4.38(a) are slightly shifted to the positive side, while those of the down-welling vortices shown in Fig. 4.38(b) are shifted to the negative side. In particular, for the smallest Ro condition, the internal temperature of the up-welling columnar vortices largely shifts to the positive side in $0 \leq z \lesssim 0.75L$, and that of the down-welling vortices is negative from the top to $z \sim 0.25L$. The intermediate Ro conditions show steeper vertical temperature gradients. The heat transport carried by the columnar vortices are examined in the form of conditional average of the vertical heat flux $j_z = w\delta T$ as a function of z , and shown in the left column of Fig. 4.38. Entirely, it is slightly difficult to see a clear trend in the change of the Ro values, however, the profiles suggest vertical symmetry in the Ro conditions considered to form

the columnar vortices. Interestingly, the condition $Ro = 0.126$ show large j_z values close to those in the larger Ro conditions. The condition $Ro = 0.126$, $Ra = 1.9 \times 10^7$ and $Ta = 1 \times 10^9$, is the case that the bulk Nusselt number Nu recorded the maximum value. From this result, the heat transport carried by a single columnar vortex increases with increasing of the bulk Nu value. In other words, the bulk heat transport is enhanced by augmentation of heat transport carried by the columnar vortices. The results are consistent with the explanations on the basis of the Ekman pumping [e.g. 89, 143, 189]. The Ekman pumping directly enhances the vertical flow. The heated and cooled fluids at the bottom and the top boundaries are carried by the Ekman pumping. This can be identified by the large shift of the internal temperature from the mean temperature T_m for the smaller Ro conditions.

4.5.4 Relationship between vortex dynamics and heat transport

The dynamics of horizontal diffusive motions of the columnar vortices and the heat transport characteristics have been clarified and discussed so far separately. These were discussed as the temporal averages or the ensemble averages of physical quantities of the multiple columnar vortices. The flow states and the entailing heat transport of the columnar vortices, however, should change from time to time. The constructed measurement systems can quantify the dynamics and the heat transport by tracking individual vortices, a more dynamical approach to characterize the relationship between the vortex dynamics and the heat transport characteristics can be provided.

To observe such unsteady physical quantities of the moving columnar vortices, temporal signals of various quantities are plotted simultaneously in Fig. 4.39. The three top rows show the horizontal positions, x , y , and r (radial position). The three middle rows show the advection velocities, v_x , v_y , and v_{adv} . The three bottom rows are the mean vertical velocity $\langle w \rangle$, the mean temperature $\langle T \rangle$, and the instantaneous local Nusselt number carried by the single vortex Nu_v . For the sake of clarity, the temporal signals of four different long-living up-welling vortices are shown in Fig. 4.39(a), and those of four different down-welling vortices are shown in Fig. 4.39(b). The signals of the different vortices are colored differently. Here, the instantaneous local Nusselt number Nu_v of a single columnar vortex can be computed [112] by modifying Eq. 4.51 as

$$Nu_v(t) = 1 + \frac{L}{\kappa \Delta T} \langle j_z(t) \rangle_{V_v} = 1 + \frac{L}{\kappa \Delta T} \langle w(t) \delta T(t) \rangle_{V_v}, \quad (4.66)$$

where $\langle \cdot \rangle_{V_v}$ is the spatial average taken within the corresponding volumetric vortex region, the cylindrical region extracted in Sec. 4.5.1. The temperature deviation δT is defined differently for the up-welling vortices as $\delta T = T - T_{top}$ and for the down-welling vortices as $\delta T = T - T_{bottom}$. As discussed in Sec. 4.5.2, the horizontal motions of the columnar vortices are random-walk-like motions. It is apparent from the temporal signals of the horizontal positions, x , y , and r . The corresponding advection velocities v_x and v_y fluctuate around zero, and the values of v_{adv} show

small values for most time and show large values sometimes. The quantities computed within the columnar vortices also change with time. The values of Nu_v are very large compared to the bulk Nu , as these are computed only internal regions of the columnar vortices. Since these three quantities are computed within the vortex region, these have similarities to each other.

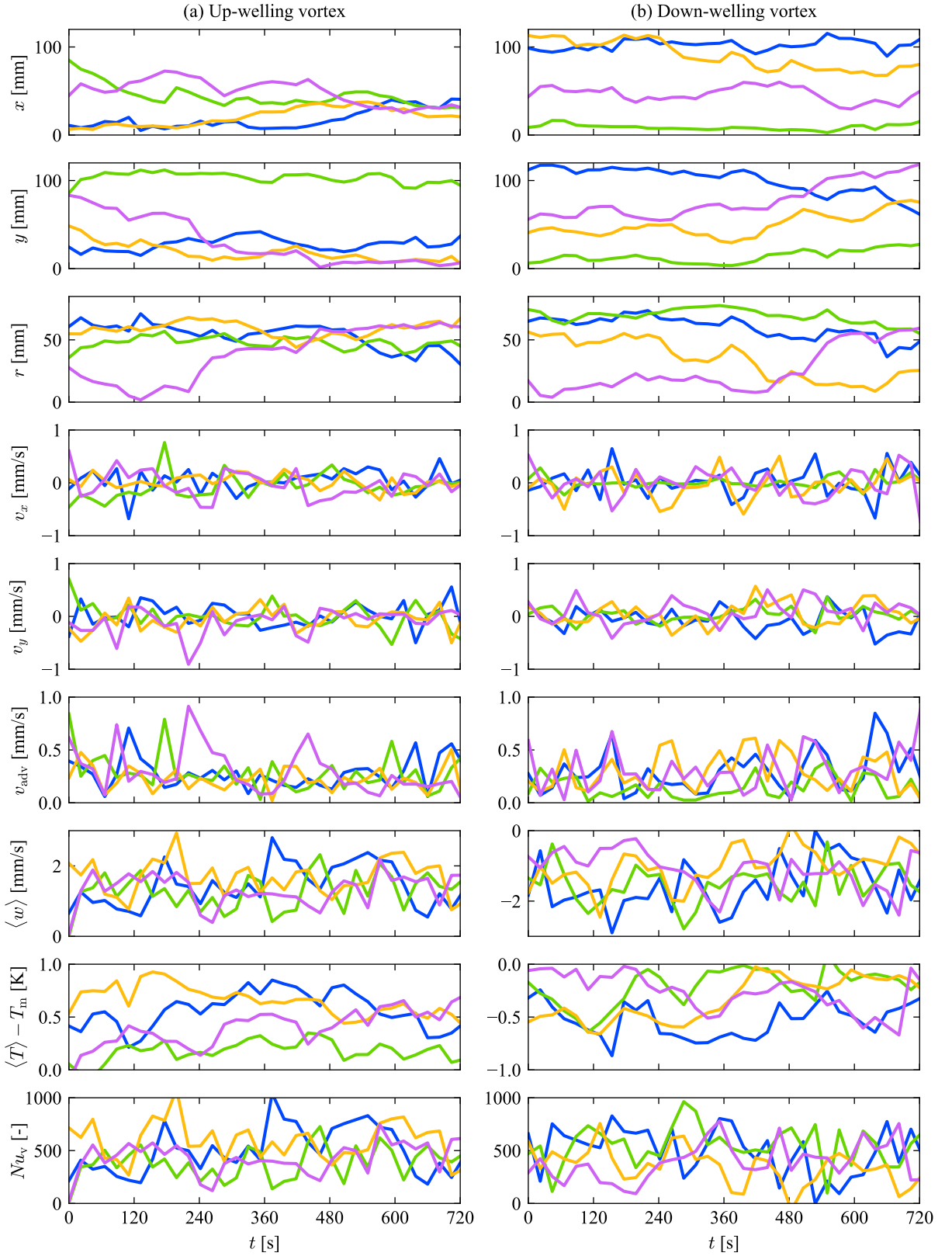


Figure 4.39: Examples of the temporal signals of various physical quantities of the columnar vortices for the case $Ra = 1 \times 10^8$ and $Ta = 1 \times 10^9$ ($Ro \sim 0.126$ and $\bar{Ra} = 97$): (a) up-welling and (b) down-welling vortices. From the top to the bottom, horizontal positions of the vortices, x , y , and r , advection velocities, v_x , v_y , and v_{adv} , the internal vertical velocity $\langle w \rangle$, the internal temperature $\langle T \rangle$, and the local Nusselt number Nu_v are shown respectively. For both types of the vortices, only four differently tracked vortices are shown, and colored uniquely for the sake of clarity.

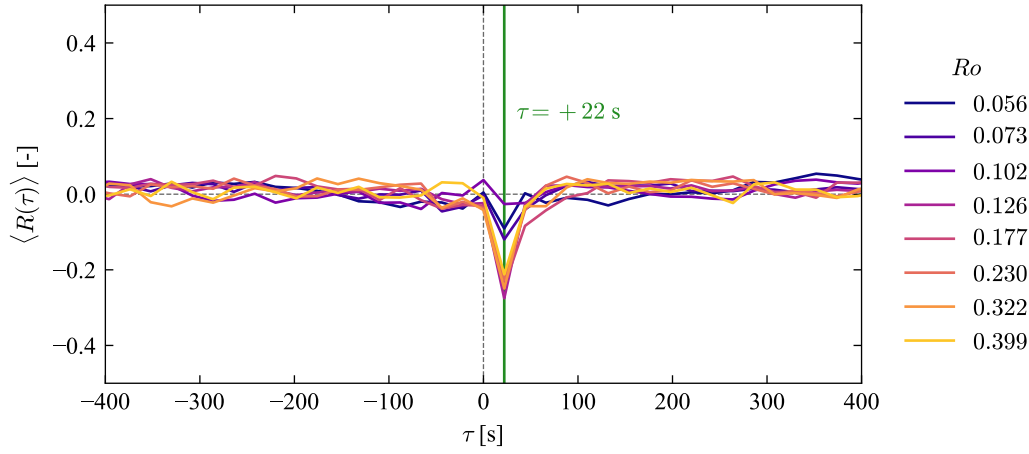


Figure 4.40: Ensemble average cross-correlations $\langle R \rangle$ with the time-lag of Nu_v relative to v_{adv} . The lines are colored by the Ro values. The distinct time-lag $\tau = +22$ s is indicated by the vertical solid line.

From the raw data shown in Fig. 4.39, any clear trends in the relationship between the vortex dynamics and the heat transport characteristics cannot be recognized. To characterize the relationship between the horizontal vortex motions and the heat transport, here the two quantities, v_{adv} and Nu_v , are paid attention. To find out relationship between the temporal signals of the v_{adv} corresponding to the displacement of the columnar vortices, and the Nu_v , the cross-correlation is computed. Here, the cross-correlations R of the temporal signals of two physical quantities, Nu_v and v_{adv} , are defined as

$$R(\tau) = \frac{\sum_{t=t_0}^T [Nu_v(t) - \overline{Nu_v}] [v_{adv}(t + \tau) - \overline{v_{adv}}]}{\sqrt{\sum_{t=t_0}^T [Nu_v(t) - \overline{Nu_v}]^2 \sum_{t=t_0}^T [v_{adv}(t + \tau) - \overline{v_{adv}}]^2}}, \quad (4.67)$$

where τ is the time-lag of the Nu_v relative to the v_{adv} , and the over-line represents the temporal average. Since the cross-correlation is normalized, it is defined in $R \in [-1, 1]$. In Fig. 4.40, the ensemble averaged cross-correlations $\langle R \rangle$ of multiple vortices are shown. The abscissa is the time-lag of Nu_v relative to v_{adv} . The lines are colored by the values of Ro . For most time-lag, there is no distinct peak meaning no significant relationship between the vortex motion and the heat transport. Only at the time-lag of $\tau = +22$ s, a distinct negative peak is found for most of the Ro conditions, as indicated by the vertical solid line.

For a further detailed observation, the probability density distributions of the R at different time-lags are shown in Fig. 4.41. In Fig. 4.41(a), the probability density distributions of R without time-lag, $\tau = 0$ s are shown, and those with a time-lag of $\tau = +22$ s are shown in Fig. 4.41(b). For the case with $\tau = 0$ s, the probability density distributions are symmetry, like a Gaussian shape, and this indicates that the two signals of Nu_v and v_{adv} are related randomly. In contrast, for the case with $\tau = +22$ s, the distributions are shifted to the negative side, meaning

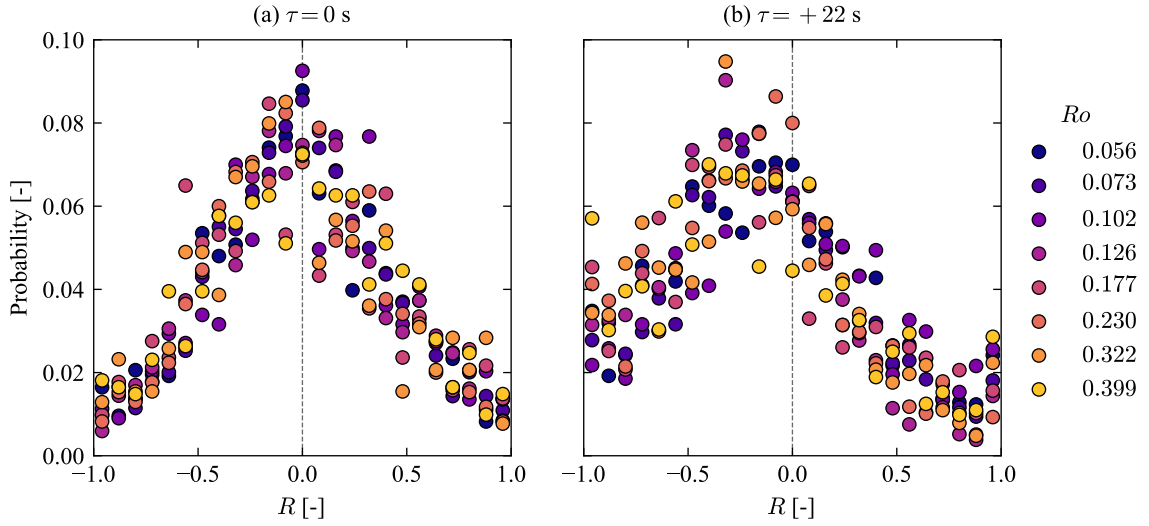


Figure 4.41: Probability density distributions of the cross-correlation R between the Nu_v and v_{adv} : (a) no time-lag $\tau = 0$ s and (b) the peak time-lag $\tau = +22$ s.

the negative correlation.

The results shown in Fig. 4.40 and 4.41 suggest a clear trend of the negative correlation between the heat transport Nu_v and the advection speed v_{adv} with a time-lag of $\tau = +22$ s, corresponding to the temporal resolution of the measurement system, a single scan. That is, the heat transport carried by a single columnar vortex decreases immediately after the large displacement of the vortex. The decrease in Nu_v indicates the inefficient heat transport of the columnar vortex. The heat transport carried by the single columnar vortex is strongly related to the Ekman pumping. The Ekman pumping transports fluids vertically from the Ekman boundary layer formed on the top and bottom walls. Thus, the Ekman pumping potentially transport the heated or cooled fluids localized at the bottom or the top boundary layers efficiently depending on the physical properties of the fluid, i.e., the Pr . The schematic illustrations of the Ekman pumping on the bottom wall are shown in Fig. 4.42. For high Pr conditions, the Ekman boundary layer is much thicker than the thermal boundary layer. The thermal diffusion is less efficient, however, the vertical heat transport from the thermal boundary layer is also less efficient due to the thin thermal boundary layer. For the case of intermediate Pr conditions, the thicknesses of the Ekman and the thermal boundary layers are comparable, and this case correspond to the present study using water, $Pr \sim 6$. The vertical heat transport is effectively done by the Ekman pumping. For low Pr conditions, the thermal diffusion is dominant because the thermal boundary layer is thicker than the Ekman boundary layer, and the vertical heat transport by the Ekman pumping is weak.

Considering the mechanism of the heat transport by the Ekman pumping explained above, it may be possible to explain the decrease in Nu_v after the large displacement of the columnar vortices. The thickness of the thermal boundary layer is, of course, not constant even for the fixed Ra condition, and it should changes with time and space. For the case of water, $Pr \sim 6$,

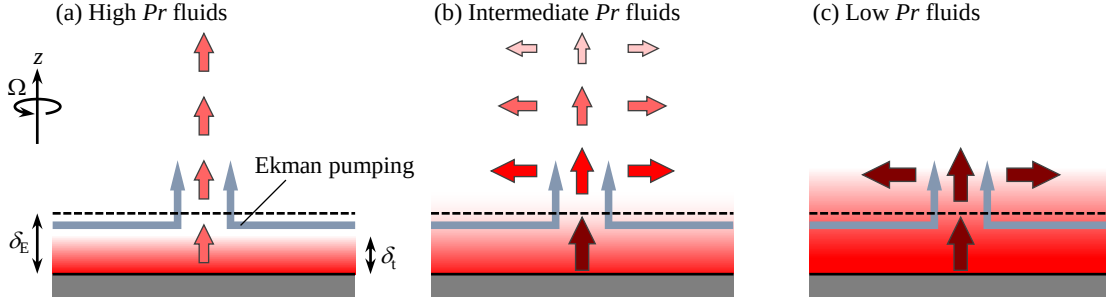


Figure 4.42: Schematic illustrations of the Ekman pumping at the bottom wall in side views: (a) high Pr , (b) intermediate Pr , and (c) low Pr fluids.

the thicker thermal boundary layer yields the larger heat transport by the Ekman pumping. Conversely, the thinner thermal boundary layer allows the smaller heat transport. The decrease in the Nu_v immediately after the large displacement is thus considered that the columnar vortex locates the horizontal position with a thin thermal boundary layer. In other words, the columnar vortices move toward positions with thin thermal boundary layers. The thickness of the thermal boundary layer is sustained locally by the presence of the columnar vortex. At the positions without the columnar vortex, the thickness of the thermal boundary layer is suppressed as the heated or the cooled fluids within the boundary layers are sucked by the lower pressure regions corresponding to the columnar vortices. As an opposing effect, the thickness of the thermal boundary layer will increase at no vortex-occupying regions due to the absence of detachment of thermal blobs, since the constant heating and cooling at the boundaries are sustained in the present configuration. As discussed in Sec. 4.5.2, the most probable speed of the horizontal advection of the columnar vortices v_{mp} is strongly related to the free-fall velocity U_f . The growth of the thermal boundary layers is also explained by the intensity of the thermal forcing, and thus the discussion above is also consistent with the result of the v_{mp} .

Incorporating all the discussions, it can be concluded that the balance of these two opposing effects, the suction of thermal blobs from the non-vortex regions to the vortex-occupying regions and the growth of the thermal boundary layers, is one of the key factors to determine the relationship between the random-walk-like vortex dynamics and the heat transport carried by the single columnar vortices. These facts can be firstly found by the analysis for the individual vortex motions and the instantaneous heat transport characteristics. Due to the limitations of the measurement system especially in terms of the temporal resolutions, the full explanations on the mechanism of the vortex dynamics cannot be provided. However, the simultaneous measurements of the 3D-3C velocity and the 3D temperature fields allowed to give the new insight for explaining a part of the mechanism as above.

4.6 Concluding remarks

To elucidate the dynamics of the columnar vortices formed in rotating Rayleigh–Bénard convection (RBC) and its local and instantaneous heat transport characteristics, experimental investigations using water as the test fluid were performed. For the purpose of the heat transport measurements, 3D-3C velocity and 3D temperature fields were simultaneously measured by constructing a scanning system. The scanning system was comprised of a linear slider and optical equipment, a white light sheet and two color cameras. By seeding thermochromic liquid crystals (TLC) particles, the 3D-3C velocity fields were quantified by scanning stereoscopic PIV, and the 3D temperature fields were measured by scanning LCT. The coloration of the TLC particles were utilized for the temperature measurements. The relationship between the coloration of the TLC particles and the temperature was complex, and thus the color-to-temperature calibration was achieved by utilizing the artificial neural network (ANN); the calibration was considered as a multi-regression problem. As a result, the temperature was measured with accuracy better than ± 0.2 K for most of the temperature range showing effective coloration. This accuracy was ensured at all the positions of the whole 3D measurement domain. The velocity and temperature fields were measured with a spatial resolution of 1 mm in all the directions. The constructed scanning measurement systems were validated in actual flow fields from the aspects of the qualitative observations of the flow fields; with increasing the rotational speed, the turbulent structures weakly affected by rotation, vortical plumes, convective Taylor columns, and cellular regimes formed in rotating RBC were identified successfully both in the Q -criterion and temperature fields. Further, the systems were validated by the quantitative statistics for various physical quantities, velocity, vorticity, temperature, and heat transport.

Adopting the constructed systems, the dynamics of the columnar vortices were studied. The columnar vortices were successfully extracted from the 3D domain by applying the Gaussian template matching method for the horizontal temperature distributions established in Noto et al. [122]. The identified vortices were then tracked in the horizontal domain. As expected in the earlier works [30, 123], the random-walk-like vortex motions were found. Such horizontal motions can be described as diffusive motions, and the diffusivity was justified from the Maxwellian probability density distributions of the advection speed. From the mean-squared displacement, the diffusive motions were revealed as the normal diffusion. The very slow advection speed of the columnar vortices $O(10^{-1} \text{ mm/s})$ yielded the diffusion coefficient of $O(1 \text{ mm}^2/\text{s})$, comparable to the viscous diffusion. Utilizing the cylindrical vortex regions detected by the template matching method, the heat transport characteristics of a single columnar vortex can be derived by computing heat flux within the internal regions of the vortices. From the conditional averages of the vertical heat flux, it is found that the smaller Ro conditions sustain large heat flux at the internal regions of the fluid layer. This suggests the importance of the Ekman pumping in the vertical heat transport of the columnar vortices. The random-walk-like horizontal motions of the

columnar vortices and the heat transport characteristics carried by the moving columnar vortices are studied by analyzing the temporal signals of the instantaneous local Nusselt number Nu_v and the advection speed v_{adv} . The ensemble average of the cross-correlations of the two signals showed a distinct negative peak at a set time-lag of 22 s, corresponding to the temporal resolution of the measurement system. This time-lag suggests that the heat transport carried by a single columnar vortex decreases immediately after a large horizontal displacement. Considering the fact that the vertical heat transport is dominated by the Ekman pumping, the strong relationship between the thickness of the thermal boundary layer and the vortex motion is clarified. The Ekman pumping at the columnar vortices sucks the surrounding heated or cooled fluids within the thermal boundary layer. This leads to the suppression of the growth of the thermal boundary layer at no-vortex regions. As an opposing effect, the thermal boundary layer at no-vortex regions should grow due to the constant cooling and heating at the boundaries. The balance of these two effects is considered to be one of the key factors for determining the random-walk-like motions of the columnar vortices.

The results shown in Chapter 4 could not provide a full explanation of the mechanism of the random-walk-like vortex dynamics due to the measurement limitations of the scanning system. However, the measurement system allowed to simultaneously understand the vortex dynamics and the heat transport characteristics. The relationship between the dynamics and the heat transport provided important insight into the mechanism of the diffusive motions of the columnar vortices, which had not been mentioned elsewhere.

Part IV

Summary and perspective

SUMMARY

In Part [IV](#), the summary of this thesis is provided. In this thesis, two main objectives have been studied; one is the advancement of color-based optical measurements, and the other is study on thermal convection utilizing thermochromic liquid crystals.

Advancement of color-based measurements — In Part [II](#), toward advancement of color-based flow measurement techniques surpassing the conventional methods, various approaches to utilize color information have been studied.

The first attempt shown in Chapter [1](#) was the image separation technique based on color information. Particles seeded into the two immiscible fluids were illuminated individually by the two differently colored laser sheets at different depth positions. The two differently colored particles existing in the different depth positions were recorded (projected) on the same image plane of the color images. From the color information of the scattered lights of the particles, all the particles could be categorized (reprojected) into the two different depth positions. After the color-based image separation, the standard PIV (stereoscopic PIV) was applied to image pairs of each depth position. That is, the method was developed as a pre-processing which is adopted in advance to the standard techniques, and no adjustment for the algorithms of the standard techniques is required. As a result, 2D-3C velocity fields at two different planes were successfully measured. The required optical equipment was two color cameras and two different color laser sheets. With the developed color-based method, coupling structures of two-layer convection have been studied in detail. The method was effective to identify fluid motions in the two immiscible fluids, and the 3D coupling structures were quantified as the 2D interfacial interactions of the convection cells formed in each fluid layer.

The second developed technique shown in Chapter [2](#) was also utilized the image separation technique based on color information. The developed method, dual-plane ensemble correlation, enables to measure the out-of-plane velocity only using a single color camera by analyzing the brightness signals across the two planes illuminated by two different laser sheets. This method is

a fully novel technique, which was invented as an application of the earlier proposed techniques, single-pixel PIV and dual-plane PIV. Such an advanced algorithm developed for the proposed method yields reduction of optical equipment to measure 2D-3C velocity fields, as only a single camera and two different color laser sheets are required. The performance of the proposed method was validated in highly 3D flow fields, and the method outperformed the standard PIV in terms of the spatial resolution.

As a further advancement in the utilization of color information for the depth position estimation, 3D color PTV was developed in Chapter 3. In the 3D color PTV, color information was utilized to the color-to-depth calibration for determining the depth positions steplessly. In situ color-to-depth calibration scheme was developed, which can be employed in various measurement conditions, toward the generalization of the color PTV. The required optical equipment is only a single color camera and a single liquid crystal display projector. Thus, the developed 3D color PTV can greatly reduce the complexities in the measurement environments of the standard tomographic PIV to measure 3D-3C velocity fields. The breakthrough was that the developed method utilized full color information, RGB values, recorded on the color images to estimate the particle depth positions. Different from the classification problems dealt with in Chapter 1 and 2, the stepless color-to-depth calibration became a multi-regression problem. That is, the advancement in the analytic procedure reduced the complexities in the measurements. The developed measurement procedure was validated in highly 3D flow fields, and its performance was ensured by comparing the standard PIV.

The color information has been successfully utilized to determine the out-of-plane positions of particles seeded into the fluids as addressed in Chapter 1, 2, and 3. In Chapter 4, color information was utilized to estimate other physical quantities of the flow fields. Unlike artificial coloration created by intentional illuminations employed in Chapter 1, 2, and 3, a natural coloration mechanism is utilized in Chapter 4; interference colors originate from the birefringence induced by fluid deformation were studied. Since the birefringence appeared only by the additives of tiny flake particles to water, the possibility of the photoelastic analysis in Newtonian fluids were examined. Finally, the unique relationship between color and internal stress, meaning the establishment of the stress-optic law in Newtonian fluids, was proved by the experiments. This was a promising result in the utilization of color information toward direct measurements of fluid deformations, which have been conventionally acquired as spatial gradients of velocity fields measured by the standard methods like PIV.

As above, various color-based optical measurement techniques have been developed in Part II. The utilization of intentional color illumination can be regarded as a plausible approach to reduce the complexities in the standard PIV methods, stereoscopic and tomographic PIV. In contrast, utilization of natural coloration mechanism will be a fully novel method enabling to measure of physical quantities which cannot be directly accessed by the standard methods.

Study on thermal convection — In Part III, physics of thermal convection has been studied by

utilizing the thermochromic liquid crystals (TLC) particles as the flow visualizer.

In Chapter 1, the usefulness of the TLC particles in measurements of thermal convection was examined in detail. The TLC particles outperformed other conventional tracer particles in terms of dispersibility; the TLC particles remained inside the convective flows for a long timescale of $O(1 \text{ hour})$ or more, without settling or floating to the bottom or top of the vessel. This characteristic of the TLC particles is essentially important, since thermal convection is often discussed with a long timescale of thermal diffusion, and temperature boundary conditions determine the fluid motions. Details of data processing schemes for the color images of the TLC particles were also presented for the later analysis.

As the first application of the visualization using the TLC particles, horizontal convection (HC) was studied in Chapter 2. Conventionally, the thermally-equilibrated states have been studied, and the flow structures are determined by the balance of baroclinic torque, thermal forcing produced at the destabilizing source, and the restoring force produced at the stabilizing source. In contrast, the flow structures of the developing HC studied in Chapter 2 are determined by the balance of the baroclinic torque and the restoring force due to the absence of the destabilizing source. Under the condition without thermal forces from the destabilizing source, the convective flows are purely driven by the baroclinicity. The flows are quite slow, and thus the heat transport carried by the convective flows is less efficient than the thermal diffusion. Thus the characteristic timescale is only described by the thermal diffusion timescale of $O(1 \text{ hour})$. Taking the thermal diffusion length scale, which develops with time, as the representative length scale, a nondimensional parameter was defined as the ratio of restoring force of the stable temperature stratification and the baroclinic torque. From the experimental results, the parameter was found to successfully describe characteristic quantities of the developing HC. Together with the velocity information measured by the particle displacements, the coloration of the TLC particles provided supporting information about temperature distributions. Such very slow physics in developing HC were elucidated by taking advantages of the visualization utilizing the TLC particles.

In Chapter 3, transient fluid motions observed during spin-up of rotating Rayleigh–Bénard convection (RBC) were studied. Rotating RBC originally entails multi-timescale physics involving the Coriolis, buoyancy, viscous, thermal diffusion timescales, and the spin-up process from non-rotating to rotating RBC was more complicated phenomenon. The velocity fields were measured by the standard PIV applied to the TLC particles, and temperature fields were qualitatively observed from the coloration of the TLC particles. Incorporating these information, stepwise transitions in the spin-up process were revealed. First disordered structures of turbulent convection formed by a number of thermal plumes were disrupted by the sudden start of the rotation. Second, concentric convection rolls formed at the vicinity of the sidewall at which the viscous boundary layer developed together with the propagation of the rigid-body rotation. Third, vertical momentum transports by the roll structures yielded generations of inflection

points in azimuthal velocity profiles, and azimuthally aligned vortices formed along the roll boundaries due to the Kelvin–Helmholtz instability. At further internal regions at which neither the rolls or vortices formed, thermal plumes were emitted from the thermal boundary layers and formed columnar vortices, and finally the equilibrium states of rotating RBC were achieved. Both velocity and temperature information effectively shown by the TLC particles helped to understand these stepwise transitions.

As an accumulation of all the knowledge in the color-based measurements explored in Part II those in the study of thermal convection, the equilibrium states of rotating RBC were studied in detail in Chapter 4. The coloration of the TLC particles was utilized to measure temperature fields. In order to study the dynamics and the heat transport characteristic of the columnar vortices formed in rotating RBC, a scanning optic system was constructed to conduct 3D scanning stereoscopic PIV and liquid crystal thermometry (LCT) simultaneously. The constructed system successfully measured simultaneous 3D-3C velocity and 3D temperature fields after the careful calibrations for the stereoscopic PIV and the LCT. The color information recorded by the two color cameras was processed by the artificial neural network, and the multi-regression to the temperature was done. As a result, the temperature fields could be measured with an accuracy better than ± 0.2 K for the whole measurement domain and for the temperature range of 5 K. From the quantified flow fields, the columnar vortices were extracted, and tracked for a long time up to $O(10^3 \text{ s})$. The horizontal motions of the columnar vortices were elucidated as the diffusive motions, i.e., random-walk-like motions. The heat transport carried by each columnar vortex was also studied by taking conditional averages of multiple-sampled vortices. The contribution of the Ekman pumping becomes larger with increasing the rotational speed of the system. The temporal signals of the dynamics and the heat transport characteristics of the columnar vortices were associated as the cross-correlations. The cross-correlations suggested that the decreasing trend in the heat transport carried by the single columnar vortex immediately after the large horizontal motion. From this result, the thickness of the thermal boundary layer was found to be one of the determinant of the diffusive vortex motions.

As above, two different problem settings of advanced thermal convection were studied utilizing the TLC particles. The particles allowed to provide temperature information as colors, and thus both velocity and temperature fields could be visualized. In addition to this advantage, the high dispersibility of the TLC particles allowed measuring physics of thermal convection in various timescales of $O(10^{-1} - 10^3 \text{ s})$.

PERSPECTIVES

This thesis has mainly developed color-based optical measurement methods, as additional functions to the conventional planar PIV/PTV. All the examples shown in this thesis, depth estimation, fluid deformation measurements, and temperature measurements, successfully reveal the physics to be studied. As a summary, perspectives of color-based flow measurements and experimental study on thermal convection in future are addressed.

Categories of color-based measurements — The first procedure of the color-based flow measurements is, needless to say, the determination of what physical quantities are measured by the color information. In Part II, Chapter 1, 2, and 3, the color information was utilized to estimate the depth (out-of-plane) positions of the particles seeded into the fluids. In Part II Chapter 4, the interference colors produced by the flow-induced birefringence were utilized to measure the fluid deformations. In Part III, the colors of the TLC particles were utilized to observe/measure the temperature fields. As another option, which had not been mentioned in this thesis, the pressure-sensitive paints are also available to measure surface pressure fields. By adopting appropriate color-based measurements, at least the physical quantities, $\mathbf{x}$, $\mathbf{u}$, $\nabla \mathbf{u}$, T , and p can be measured through the color information. The utilization of the color information can be categorized by the mechanism of coloration, i.e., the type of the spectra of the light used for visualization.

One of the categories is a classification problem, when colored lights, which are separable in their spectrum spaces, are employed for visualization. The schematic illustration of this classification problem in color-based optical measurements is shown in Fig. 2.1. When the number of the peaks in the spectra of the light source $n = 1$, it will be a standard visualization method like the conventional PIV/PTV. For this case, of course, the visualized images can be directly used for the subsequent analysis. When there are two distinct peaks in the spectra, $n = 2$, the color information is utilized for the binarization, i.e., classification for the two types, as explored in Part II, Chapter 1 and 2. Such classification problems arise when the wavelength of

the incident lights are limited to very small regions, and two wavelengths can be distinguished to each other. For instance, the two wavelengths of the incident lights can be fully separated only by the binarization of the digital color images. When three different wavelengths are utilized, $n = 3$, the classification will simply be a ternarization problem. Further, multiple (n) wavelengths which can be still separable in the spectrum are utilized, the classification becomes simply a multi-leveling problem. As long as the multiple peaks in the spectra with different wavelengths can be separated by adopting certain thresholdings for the color values, the color-based optical measurement method can be categorized as the classification problem.

The other category of the color-based measurements is a regression problem. This category may be beyond the limitations of the classification problem. The light source for visualization is required to involve spectra with continuous wavelengths, i.e., a white light source is utilized. Schematic diagram of general spectra of white lights are illustrated in Fig. 2.2. The light-emitting diode (LED) and the fluorescent lamp shown in Fig. 2.2(a) and (b) have stronger spectra in the smaller wavelength, corresponding to blue colors. In contrast, the halogen lamp shown in Fig. 2.2(c) has stronger spectra at the larger wavelength region, meaning a red-dominant light. In Part II Chapter 3, an artificial rainbow light illuminating volumetric regions in the physical space was utilized. This approach corresponds to the extension of the classification problems mentioned above, i.e., $n \rightarrow \infty$. The depth positions of the particles were estimated steplessly through the artificial neural network solving the multi-regression problem associating between the colors and the depth positions. In Part II, Chapter 4, the interference colors displayed by the flow-induced birefringence were evaluated as measurable quantities to determine the fluid deformations directly. Unlike the structured light, the mechanism of the coloration is not artificial, and the color image processing should be dealt with as the regression problem. In Part III Chapter 4, the coloration of the TLC particles was utilized to measure the temperature fields. Similar to the coloration mechanism of the flow-induced birefringence, the coloration of the TLC is not artificial and thus required to be dealt with as a regression problem.

Practical implementation of color-based measurements — The color information recorded on the digital images is a result of incorporating various factors in the actual experimental environment. For instance, the spectra of the light source, the spectral characteristics of the illuminated materials are the inevitable factors strongly depending on the experimental facilities. The specifications and the optical setting of the color camera are also important factors, which can be determined by the experimentalists. For these reasons, it is almost impossible to control the coloration actually recorded on the visualized images, and ideal situations may not be realized. In considering the implementation of the color-based optical measurements, the color information is thus required to be properly processed on the visualized color images.

For a practical implementation of the classification problem, it is easy to use artificially structured lighting, such as different laser with different wavelengths, multiple light-emitting diodes (LED), wavelength filters for a white light source. These light sources with multiple

wavelengths enable to illuminate various planned spatial positions easily. The implementation and the image processing for the image separation is rather simple, as the colors can be easily separable in the *RGB* space of the recorded images even if the planned colors are not fully realized. This is because only the requirement is the multi-leveling of the colors into the number of the light sources with different wavelengths. However, such a classification type only enables to measure multiple planes. For the case of PIV/PTV, it will just reveal 2D-2C velocity fields at the multiple planes. The physical quantities across the multiple planes cannot be measured or strongly limited, in principle. If possible, the displacements of particles across the multiple planes are characterized as step functions.

The implementation of the regression problem is much difficult, however, will provide more sophisticated measurements. The light sources should have a wide spectra containing various wavelengths as shown in Fig. 2.2. Since the digital color camera records such spectra of the lights as three different brightness, *R*, *G*, and *B*, mimicking the three stimuli received by the human's eye. Accordingly, the original spectra are digitized as the *RGB* signals. The coloration representing certain physical quantities should be identified in the *RGB* domain. It is the most straightforward to utilize all the *RGB* values to determine the corresponding physical quantities. Of course, depending on the measurement targets, it may be possible to reduce the *RGB* values to the other color representations such as hue value. In principle, however, such color conversion will reduce the original rich color information and may not produce benefit more than the simplification of the color calibration scheme. In addition, the actual coloration on the visualized images strongly depends on the measurement environment, and thus the formula of the color calibration will also change depending on the environment. In other words, it is not possible to generalize the color calibration formula, and a proposal of such formula does not ensure the quality (accuracy) of the color-based measurements. The regression-type color-based measurements thus always require in situ calibration. For a robust implementation, the formalization of the color calibration is better left as an unknown factor. The approach utilizing the artificial neural network (ANN) takes an advantage for these problems. There is no need to formalize the color conversion only by changing the configurations of the ANN. Further, multiple color values can be dealt with at once, and thus the color information recorded on the images can be fully utilized without reducing the original rich information due to the color conversion.

As summarized above, the color-based measurements can be categorized into two, and each requires careful color handling for a proper quantification. However, only if a careful treatment of color information is possible, the color-based measurements greatly increase the degree of freedom in the fluid flow measurements. In addition to the standard method like the planer PIV/PTV, other physical quantities can be simultaneously obtained from the additional color information. Thus the implementation of the color-based measurements will expand in the future together with the advancement of the optical equipment and the saturation in the development of the standard measurement techniques.

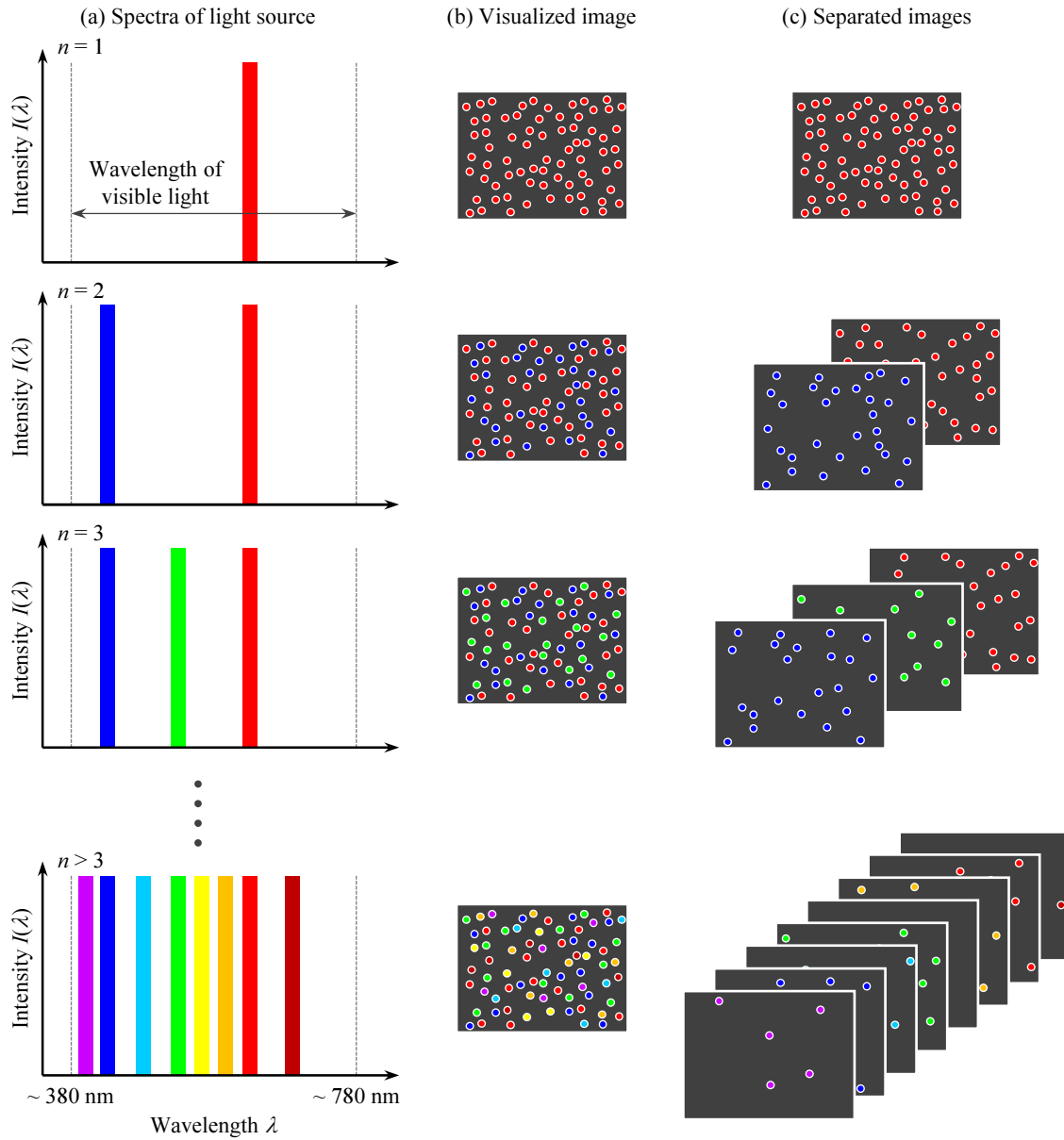


Figure 2.1: Conditional averaged values of (left) vertical velocity w , (center) temperature T , and (right) vertical heat flux j_z inside the columnar vortex as a function of the z . (a) and (b) are the results for the up-welling and the down-welling columnar vortices. The lines are colored by the Ro values.

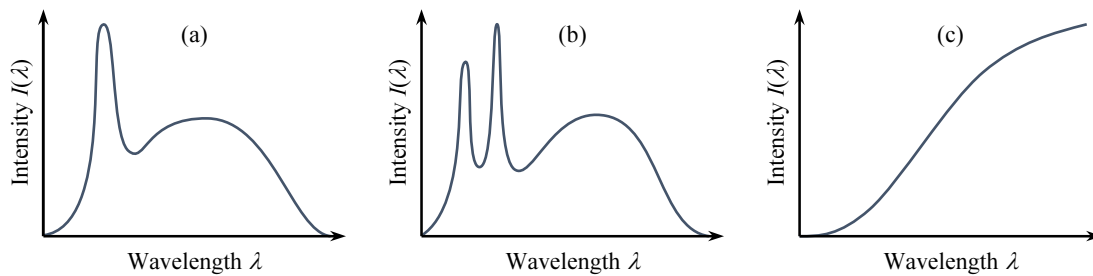


Figure 2.2: Schematic diagram of spectra of various white light source: (a) light-emitting diode (LED), (b) fluorescent lamp, and (c) halogen lamp.

Experimental study on thermal convection — As concluding remarks, the perspectives of the future experimental study on thermal convection are provided. Since the study by Bénard [16] and Lord Rayleigh [159], thermal convection has been studied theoretically, experimentally, and numerically. The importance of the experimental studies has been exceeded gradually by the numerical studies together with the advancement of the computational resources. Most of the classical convection problems have become possible to be measured (realized) even by the direct numerical simulations. Due to the limitations in the experimental techniques, the experimental investigations have become just validations of the numerical results.

These situations are very regrettable for experimentalists. And thus, experimentalists are required to study complicated or unknown problem settings that cannot be formulated or directly computed by the numerical approach. Otherwise, experimentalists need to advance the measurement techniques so as to measure physical quantities not inferior to the numerical results. The former approach will be the easiest way, as it may be sometimes very difficult to simulate thermal convection involving such as temperature dependence of the physical properties, porous media, phase transitions leading to multiphase flows, and non-Newtonian effects. Eventually, however, experimental study on such complicated problem settings will be overtaken by the numerical approach due to great advancement in computational resources. Experimentalists should understand the advantage unique in the experiments themselves; experiments can realize actual physics properly. Thus the advancement of measurement techniques should be continued.

The presented technique, liquid crystal thermometry (LCT), is one of the advancements that can exceed the limitations of classical methods like PIV/PTV. In addition to the velocity information characterized by the displacement of the tracer particles, the coloration of the TLC particles provides temperature information. Obviously, the heat transport is inevitable to describe the physics of thermal convection, and thus temperature measurements are essential. The experimental measurements of thermal convection should conquer both the velocity and temperature fields. The non-invasive and field measurements of temperature utilizing the color images of the TLC particles greatly surpass the limitations of the mechanical temperature sensors. However, the implementation of the LCT has been limited and not standard in measurements of thermal convection. This may be because of the difficulties in the color-to-temperature calibration. As above, in situ color-to-temperature calibration is required and it will be a multi-regression problem. Slight rich knowledge about optics and post-processing are essential. Once if these requirements are satisfied, the implementation of the LCT in measurements of thermal convection will spread rapidly.

Experimental study on thermal convection is originally difficult due to the presence of heat (temperature) compared to the flow without temperature change. Therefore, thermal convection is an optimal measurement target of the color-based optical measurement techniques like the LCT can take an advantage. The advancement in measurement techniques will advance the understanding of unknown physics.

THERMOCHROMIC LIQUID CRYSTALS

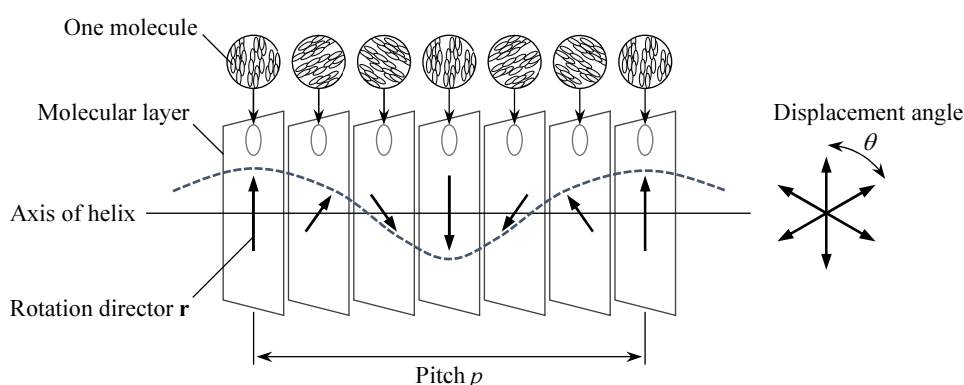


Figure A.1: Structure of the cholesteric liquid crystal.

In Appendix A, general descriptions of thermochromic liquid crystals (TLC) are provided to enhance understanding of readers. There are a number of review papers on TLC [e.g. 4, 133, 187] in variety of research fields. Here, only a general explanation of the mechanism of TLC is presented.

There are three distinctive types of TLC, smectic, nematic, and cholesteric, and these show different optical properties and possess different structures. The TLC used in this study is classified as a kind of cholesteric (chiral nematic) crystals, and the structure is schematically illustrated in Fig. A.1. A single molecule of the liquid crystal has a rod-like structure under cholesteric mesophases. A number of molecules comprises a thin layer, and the molecules in each layer has a preferential orientation. The orientation of each layer is referred to as rotation director $\mathbf{r}$. These layers are stacked along a longitudinal axis, and the rotation director displays a continuous change (twist) along the axis. The twist angle of a single layer to the adjacent is referred to as displacement angle θ . As a result, the rotation director $\mathbf{r}$ traces out a helix, as indicated by the dashed line in Fig. A.1. The longitudinal length of the aggregated layers are of the order of the wavelength of visible light $O(100 \text{ nm})$. The degree of the twist is represented by pitch p ; the pitch p is the longitudinal length of the helical structure of the molecular layers

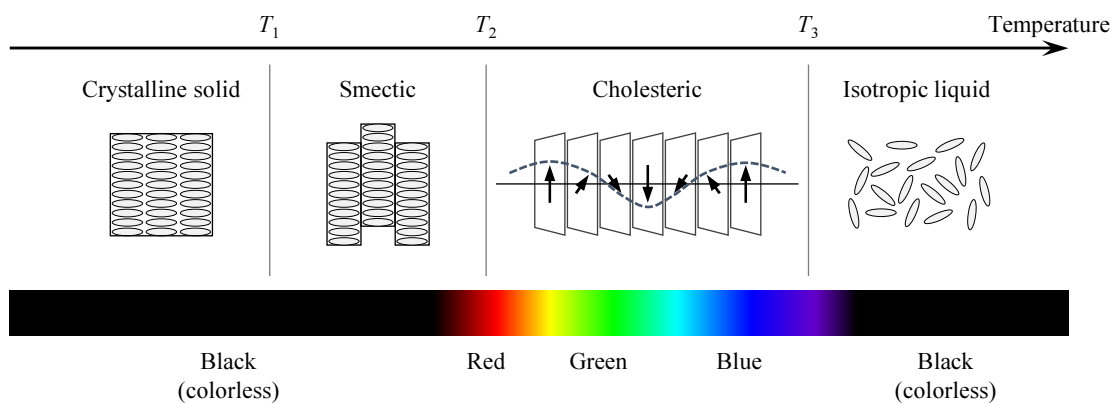


Figure A.2: Schematic representation of the phase transitions of TLC and corresponding color change.

show one complete revolution 360° . The pitch p is thus determined by the distance between the layers d and the displacement angle θ . The wavelength of the incident light close to the pitch p is influenced by the optical property of the TLC detailed later.

The TLC can stably take two states, the focal-conic phase and the planar phase, and this is referred to as bistability. If the helices of the TLC are at 90° to the incident light, it is called the focal-conic phase and results in birefringence. In the focal-conic phase, the TLC scatters diffusive light and no distinctive coloration appears. In the planar phase, the axes of the helices are aligned with the axes parallel to the incident light. As a result, a unique optical effect, selective reflection of white light, appears to show vivid coloration due to circular dichroism. Basically, the mechanism of the coloration is the same as that of normal liquid crystal displays, while the electric current for imposing voltage is not required for the TLC.

The TLC originally take various phases with changes of temperature, crystalline solid, smectic, cholesteric, and isotropic liquid phases, as illustrated in Fig. A.2. Each phase is separated at transitional temperature T_1 , T_2 , and T_3 , and all the phases are thermally reversible. Since only the cholesteric phase shows optical anisotropy described above, coloration appears under the cholesteric phase realized in a predetermined temperature range, and the other phases do not show any coloration.

Increase in temperature results in active thermal motions of molecules leading to increase in molar volume. As a result, the following effects for the helical structures of TLC are led in two opposing ways as illustrated in Fig. A.3. One is the increase in the distance d between the layers comprising the helices along the helical axes. The pitch p is increased by this effect. The other is the increase in the displacement angle θ between the adjacent layers piled along the helical axes, and the pitch p is decreased by this effect. In general, the latter effect is dominant and thus the resultant pitch p decreases with increase in temperature. As described above, the pitch p directly affects the selectivity of the reflection wavelength, and a continuous color change can be observed by the change of temperature.

In Fig. A.4, the typical reflection wavelength of TLC over the change of temperature is shown after Stasiek et al. [187]. With increasing temperature, the reflection wavelength decreases like

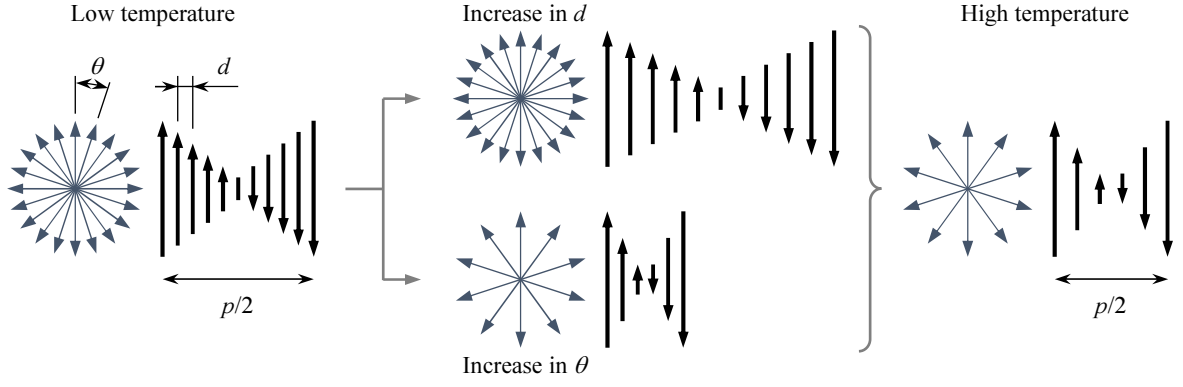


Figure A.3: Two opposing effects of temperature on pitch p .

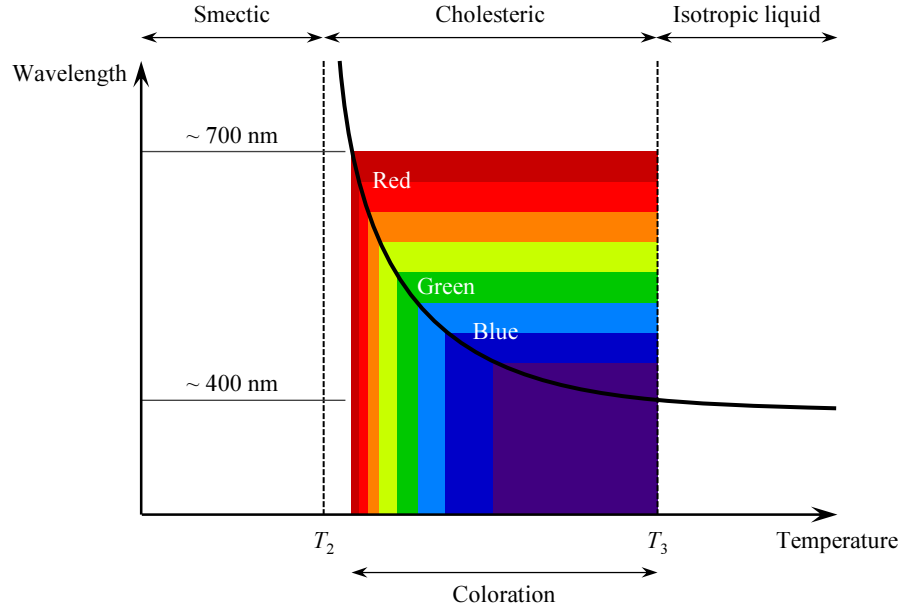


Figure A.4: Typical reflection wavelength over temperature of TLC (modified from [187]).

in an inverse proportional fashion as indicated by the black solid line. The coloration appears in the wavelength range of visible light $\sim 400 \text{ nm} \simeq \lambda \simeq 700 \text{ nm}$, as red, green, and blue from lower to higher temperature. Since the inverse proportional fashion of the reflection wavelength, the temperature range for red color is narrower than that for blue.

This study utilized TLC particles, which is made by encapsulating the TLC in urea resin films. As the TLC is encapsulated, the particles can be used as tracer particles to visualize fluid flows. In Fig. A.5, an example of TLC visualization is shown. The photo in Fig. A.5 captured a stable temperature stratification of water illuminated by a white LED light sheet. The bottom and top of the fluid layer were fixed at 19°C and 24°C . As the fluid layer was linearly stratified, the temperature changes with a vertical temperature gradient as $\partial T/\partial z = \text{const}$. The color values, RGB brightness and hue h values over the temperature are shown. The color values strongly depend on the observation environment, e.g. observation angle and spectrum of incident light, and the displayed results are not universal. It is, however, possible to see continuous color

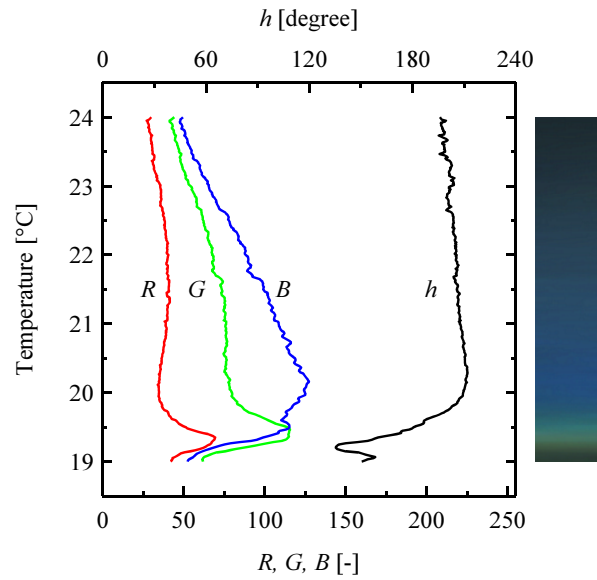


Figure A.5: Relationship between color, R , G , B , and h , and temperature of the TLC particles dispersed into stable temperature stratification of water. The image shown in the right is the corresponding visualized image.

changes over the change of temperature.

In practice, an experimental contrivance for generating more ideal color profiles, such as monotonically changing hue h value, over temperature is not required. This is because the coloration recorded as RGB values can successfully determines a unique temperature by utilizing all the RGB values. Thus the color-to-temperature conversion can be realized by solving a multi-regression problem unique to the experimental (optical) condition. As investigated in detail in Part III Chapter 1, the TLC particles are very useful (excellent) tracers for visualization of thermal convection.

NUMERICAL SIMULATION OF HORIZONTAL CONVECTION

Developing horizontal convection (HC) was studied in Part III Chapter 2. The experimental study successfully found the dominant parameter in the developing stage of HC. In Appendix B, the experimental results are justified through numerical simulation.

B.1 Governing equation and nondimensionalization

The governing equations for the problem can be based on the Boussinesq approximation, and the 2D equations are given as follows;

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (\text{B.1})$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right), \quad (\text{B.2})$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + g\beta T', \quad (\text{B.3})$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (\text{B.4})$$

As the flow fields realized in the experiments can be considered as 2D flows, the governing equations for the problem can be written in the stream function-vorticity formulations. By taking

rotation of the Navier–Stokes equation, we obtain

$$\frac{\partial}{\partial t} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + u \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + w \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \quad (\text{B.5})$$

$$+ \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = g\beta \frac{\partial T}{\partial x} + \nu \nabla^2 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right). \quad (\text{B.6})$$

Here, the y -component of vorticity η is defined as

$$\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad (\text{B.7})$$

and thus, we can derive the vorticity equation as

$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} + w \frac{\partial \eta}{\partial z} = g\beta \frac{\partial T}{\partial x} + \nu \nabla^2 \eta. \quad (\text{B.8})$$

Representing u and w by the stream function ψ as

$$u = \frac{\partial \psi}{\partial z} \text{ and } w = -\frac{\partial \psi}{\partial x}, \quad (\text{B.9})$$

the vorticity equation can be expressed only by ψ and η as

$$\frac{\partial \eta}{\partial t} + \frac{\partial \psi}{\partial z} \frac{\partial \eta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \eta}{\partial z} = g\beta \frac{\partial T}{\partial x} + \nu \nabla^2 \eta. \quad (\text{B.10})$$

Similarly, the thermodynamic equation is rewritten using only ψ and T as

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} = \kappa \nabla^2 T. \quad (\text{B.11})$$

Please note that the use of ψ automatically satisfies the continuity equation as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \frac{\partial^2 \psi}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial x \partial z} = 0. \quad (\text{B.12})$$

The governing equations need to be nondimensionalized for the computation. Here, we perform the following nondimensionalization;

$$x = Lx', \quad z = Lz'$$

$$t = \frac{L^2}{\kappa} t'$$

$$u = \frac{\kappa}{L} u', \quad w = \frac{\kappa}{L} w'$$

$$\eta = \frac{\kappa}{L^2} \eta'$$

$$\psi = \kappa\psi'$$

$$T = \alpha LT' + T_r = (T_0 - T_1)T' - T_r = \Delta TT' - T_r$$

$$Pr = \frac{\nu}{\kappa}$$

$$Ra = \frac{g\beta\alpha L^4}{\kappa\nu} = \frac{g\beta(T_0 - T_1)L^3}{\kappa\nu} = \frac{g\beta\Delta TL^3}{\kappa\nu}$$

Using these nondimensional variables, the nondimensionalized governing equations can be written as

$$\frac{\partial\eta}{\partial t} + \frac{\partial\psi}{\partial z}\frac{\partial\eta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\eta}{\partial z} = PrRa\frac{\partial T}{\partial x} + Pr\nabla^2\eta, \quad (\text{B.13})$$

$$\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z} = \nabla^2 T, \quad (\text{B.14})$$

$$\nabla^2\psi = \eta. \quad (\text{B.15})$$

Please note that the prime symbols for each variable are omitted from these equations for readability.

B.1.1 Differential approximations of the governing equations

To numerically solve the nondimensionalized governing equations, the spatial differentiation and time integration methods need to be explicitly defined. For simplicity, the Cartesian (regular) grid is employed for the simulation, and its distance and time interval are expressed by Δd and Δt hereafter.

The vorticity equation is transformed for the differential approximation as

$$\frac{\partial\eta}{\partial t} = - \left(\frac{\partial\psi}{\partial z}\frac{\partial\eta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\eta}{\partial z} \right) + PrRa\frac{\partial T}{\partial x} + Pr\nabla^2\eta. \quad (\text{B.16})$$

For the nonlinear advection term (the first term of the right-hand side), the Arakawa Jacobian's method is employed, and it is expressed as

$$\frac{\partial\psi}{\partial z}\frac{\partial\eta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\eta}{\partial z} = J_a = \frac{1}{3}(J_1 + J_2 + J_3), \quad (\text{B.17})$$

where,

$$J_1 = \frac{\partial \psi}{\partial z} \frac{\partial \eta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \eta}{\partial z}, \quad (\text{B.18})$$

$$J_2 = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial z} \eta \right) - \frac{\partial}{\partial z} \left(\frac{\partial \psi}{\partial x} \eta \right), \quad (\text{B.19})$$

$$J_3 = \frac{\partial}{\partial z} \left(\psi \frac{\partial \eta}{\partial x} \right) - \frac{\partial}{\partial x} \left(\psi \frac{\partial \eta}{\partial z} \right). \quad (\text{B.20})$$

Employment of the Arakawa Jacobian J_a ensures the conservation of the vorticity (J_1), the kinetic energy (J_2), and the enstrophy (J_3), and thus the simulation becomes durable for a long-term temporal integration. Using the second-order central-difference scheme for these three Jacobians, the discretized equations become

$$J_{1 \ i,j} \simeq \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta d} \frac{\eta_{i+1,j} - \eta_{i-1,j}}{2\Delta d} - \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta d} \frac{\eta_{i,j+1} - \eta_{i,j-1}}{2\Delta d}, \quad (\text{B.21})$$

$$J_{2 \ i,j} \simeq \left(\frac{\psi_{i+1,j+1} - \psi_{i+1,j-1}}{2\Delta d} \eta_{i+1,j} - \frac{\psi_{i-1,j+1} - \psi_{i-1,j-1}}{2\Delta d} \eta_{i-1,j} \right) / (2\Delta d) \quad (\text{B.22})$$

$$- \left(\frac{\psi_{i+1,j+1} - \psi_{i-1,j+1}}{2\Delta d} \eta_{i,j+1} - \frac{\psi_{i+1,j-1} - \psi_{i-1,j-1}}{2\Delta d} \eta_{i,j-1} \right) / (2\Delta d), \quad (\text{B.23})$$

$$J_{3 \ i,j} \simeq \left(\psi_{i,j+1} \frac{\eta_{i+1,j+1} - \eta_{i-1,j+1}}{2\Delta d} - \psi_{i,j-1} \frac{\eta_{i+1,j-1} - \eta_{i-1,j-1}}{2\Delta d} \right) / (2\Delta d) \quad (\text{B.24})$$

$$- \left(\psi_{i+1,j} \frac{\eta_{i+1,j+1} - \eta_{i+1,j-1}}{2\Delta d} - \psi_{i-1,j} \frac{\eta_{i-1,j+1} - \eta_{i-1,j-1}}{2\Delta d} \right) / (2\Delta d). \quad (\text{B.25})$$

The buoyancy term (the second term of the right-hand side) is discretized as

$$\frac{\partial T}{\partial x} \simeq \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta d}. \quad (\text{B.26})$$

The diffusion term (the third term of the right-hand side) is discretized as

$$\nabla^2 \eta = \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial z^2} = \frac{\partial}{\partial x} \left(\frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial \eta}{\partial z} \right) \quad (\text{B.27})$$

$$\simeq \left(\frac{\eta_{i+1,j} - \eta_{i,j}}{\Delta d} - \frac{\eta_{i,j} - \eta_{i-1,j}}{\Delta d} \right) / \Delta d \quad (\text{B.28})$$

$$+ \left(\frac{\eta_{i,j+1} - \eta_{i,j}}{\Delta d} - \frac{\eta_{i,j} - \eta_{i,j-1}}{\Delta d} \right) / \Delta d \quad (\text{B.29})$$

$$= \frac{\eta_{i+1,j} + \eta_{i-1,j} + \eta_{i,j+1} + \eta_{i,j-1} - 4\eta_{i,j}}{\Delta d^2}. \quad (\text{B.30})$$

The right-hand side of the vorticity equation at a certain time step k is then represented as

$$f_{i,j}^k = -\frac{1}{3} (J_{1\ i,j} + J_{2\ i,j} + J_{3\ i,j}) \quad (\text{B.31})$$

$$+ PrRa \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta d} \quad (\text{B.32})$$

$$+ Pr \frac{\eta_{i+1,j} + \eta_{i-1,j} + \eta_{i,j+1} + \eta_{i,j-1} - 4\eta_{i,j}}{\Delta d^2}. \quad (\text{B.33})$$

For the temporal integration of the first step, the Euler scheme is employed, while the Adams–Bashforth scheme is used after the first step. Thus, the temporal integration of the vorticity equation can be written as

$$\eta_{i,j}^{k+1} = \begin{cases} \eta_{i,j}^k + f_{i,j}^k \Delta t & (k = 0) \\ \eta_{i,j}^k + \left(\frac{3}{2} f_{i,j}^k - \frac{1}{2} f_{i,j}^{k-1} \right) \Delta t & (k \geq 1) \end{cases} \quad (\text{B.34})$$

In the same manner, the differential approximation for the thermodynamic equation are performed, and the equation can be transformed in the form

$$\frac{\partial T}{\partial t} = - \left(\frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} \right) + \nabla^2 T. \quad (\text{B.35})$$

For the nonlinear advection term (the first term of the right-hand side) is obtained using the Arakawa's Jacobian J_a as

$$\frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} = J_a = \frac{1}{3} (J_1 + J_2 + J_3), \quad (\text{B.36})$$

where,

$$J_1 = \frac{\partial \psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z}, \quad (\text{B.37})$$

$$J_2 = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial z} T \right) - \frac{\partial}{\partial z} \left(\frac{\partial \psi}{\partial x} T \right), \quad (\text{B.38})$$

$$J_3 = \frac{\partial}{\partial z} \left(\psi \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial x} \left(\psi \frac{\partial T}{\partial z} \right). \quad (\text{B.39})$$

The discretized forms of these Jacobians are written as

$$J_{1\ i,j} \simeq \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta d} \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta d} - \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta d} \frac{T_{i,j+1} - T_{i,j-1}}{2\Delta d}, \quad (\text{B.40})$$

$$J_{2\ i,j} \simeq \left(\frac{\psi_{i+1,j+1} - \psi_{i+1,j-1}}{2\Delta d} T_{i+1,j} - \frac{\psi_{i-1,j+1} - \psi_{i-1,j-1}}{2\Delta d} T_{i-1,j} \right) / (2\Delta d) \quad (\text{B.41})$$

$$- \left(\frac{\psi_{i+1,j+1} - \psi_{i-1,j+1}}{2\Delta d} T_{i,j+1} - \frac{\psi_{i+1,j-1} - \psi_{i-1,j-1}}{2\Delta d} T_{i,j-1} \right) / (2\Delta d), \quad (\text{B.42})$$

$$J_{3\ i,j} \simeq \left(\psi_{i,j+1} \frac{T_{i+1,j+1} - T_{i-1,j+1}}{2\Delta d} - \psi_{i,j-1} \frac{T_{i+1,j-1} - T_{i-1,j-1}}{2\Delta d} \right) / (2\Delta d) \quad (\text{B.43})$$

$$- \left(\psi_{i+1,j} \frac{T_{i+1,j+1} - T_{i+1,j-1}}{2\Delta d} - \psi_{i-1,j} \frac{T_{i-1,j+1} - T_{i-1,j-1}}{2\Delta d} \right) / (2\Delta d). \quad (\text{B.44})$$

The diffusion term (the second term of the right-hand side) is discretized as

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T}{\partial z} \right) \quad (\text{B.45})$$

$$\simeq \left(\frac{T_{i+1,j} - T_{i,j}}{\Delta d} - \frac{T_{i,j} - T_{i-1,j}}{\Delta d} \right) / \Delta d \quad (\text{B.46})$$

$$+ \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta d} - \frac{T_{i,j} - T_{i,j-1}}{\Delta d} \right) / \Delta d \quad (\text{B.47})$$

$$= \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j}}{\Delta d^2}. \quad (\text{B.48})$$

The right-hand side of the thermodynamic equation at a certain time step k is then represented as

$$g_{i,j}^k = -\frac{1}{3} (J_{1\ i,j} + J_{2\ i,j} + J_{3\ i,j}) \quad (\text{B.49})$$

$$+ \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j}}{\Delta d^2}. \quad (\text{B.50})$$

Thus, the temporal integration of the thermodynamic equation can be written as

$$T_{i,j}^{k+1} = \begin{cases} T_{i,j}^k + f_{i,j}^k \Delta t & (k = 0) \\ T_{i,j}^k + \left(\frac{3}{2} g_{i,j}^k - \frac{1}{2} g_{i,j}^{k-1} \right) \Delta t & (k \geq 1) \end{cases} \quad (\text{B.51})$$

The stream function ψ for each time step is computed from the following Poisson equation

$$\eta = \nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}. \quad (\text{B.52})$$

Using the second-order-central-difference scheme for the discretization, this is rewritten as

$$\eta_{i,j} = \frac{\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1} - 4\psi_{i,j}}{\Delta d^2}. \quad (\text{B.53})$$

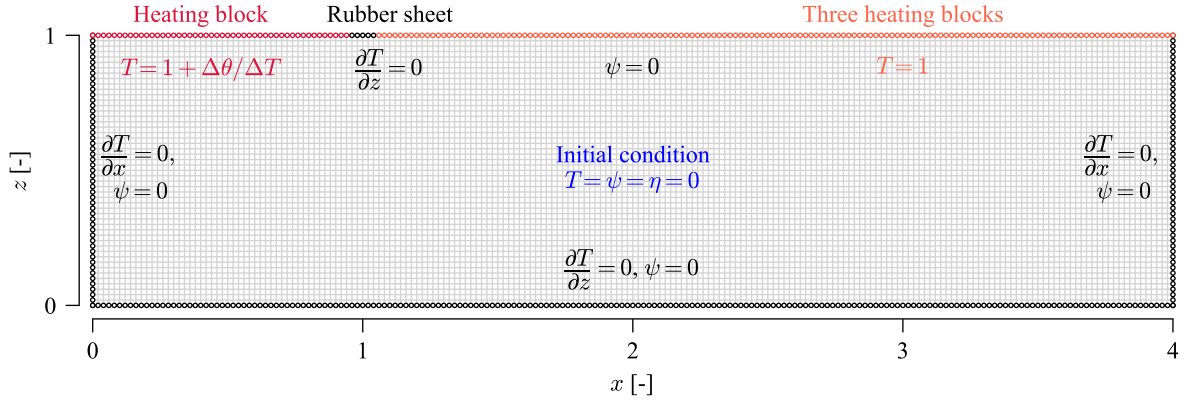


Figure B.1: Schematic of the computation domain for the numerical simulation of developing horizontal convection. Circles forming the rectangular computation domain represent the grid points at the boundaries, and imposed boundary conditions for the temperature T and stream function ψ are noted.

That is,

$$\psi_{i,j} = \frac{\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1} - (\Delta d^2)\eta_{i,j}}{4}. \quad (\text{B.54})$$

From this, it is found that the ψ value at (i, j) is determined by the surrounding four ψ values and the η value at the grid point. This Poisson equation can be solved by the iterations of the successive-over-relaxation (SOR) method, the same approach to the ψ computation for the experimental data.

B.2 Initial and boundary conditions

The initial and the boundary conditions are set to imitate the experimental conditions in the numerical simulation. The aspect ratio of the computation domain is set to 4, which is the same to the finite vessel used in the experiments, and thus the ranges for the nondimensionalized x and z are $0 \leq x \leq 4$ and $0 \leq z \leq 1$. The number of grids are then given as $4N + 1$ and $N + 1$ for the x and z directions, at which N represents the number of intervals of the grids, and the discretized coordinates are expressed as $i \in [0, 4N]$ and $j \in [0, N]$. Accordingly, the grid interval Δd is given as $\Delta d = 1/N$, and in this study $N = 50$ and $\Delta d = 0.02$, corresponding to 1 mm in the dimensional unit.

A schematic of the computation domain and the initial and boundary conditions are shown in Fig. B.1. The fluid filled in the domain is initially quiescent and at constant temperature. Thus the vorticity and stream function are $\eta = \psi = 0$, and the temperature is $T = T_0 = 0$ for the whole domain, except for the boundaries.

For the temperature conditions at the top boundary, the Dirichlet boundary conditions (constant temperatures) are given for the regions of the heating copper blocks, and the Neumann

boundary condition (adiabatic $\partial T/\partial z = 0$) are given. These are formalized as

$$T_{i,N} = \begin{cases} 1 + \Delta\theta/\Delta T & 0 \leq x < 0.96 \\ T_{i,N-1} & 0.96 \leq x \leq 1.04 \\ 1 & 1.04 < x \leq 4 \end{cases} \quad (\text{B.55})$$

where $0 \leq x < 0.96$ is the region heated by the left-end copper block with $T = T_0 + \Delta T + \Delta\theta$, $0.96 \leq x \leq 1.04$ is the region of the rubber sheet padding the space between the copper blocks, and $1.04 < x \leq 4$ is the region heated by the right three copper blocks. Here the two rubber sheets padded in the spaces between the right three copper blocks are considered to be imposed the same constant temperature condition to the copper blocks. For the side and bottom walls where the acrylic walls place, adiabatic conditions, $\partial T/\partial x = 0$ or $\partial T/\partial z = 0$, are considered, and these are formalized as

$$T_{0,j} = T_{1,j}, \quad T_{4N,j} = T_{4N-1,j}, \quad \text{and} \quad T_{i,0} = T_{i,1}. \quad (\text{B.56})$$

All the rigid boundaries are considered as non-slip conditions, $u = w = 0$, and accordingly $\psi = 0$. This is written as

$$\psi_{0,j} = \psi_{4N-1,j} = \psi_{i,0} = \psi_{i,N} = 0. \quad (\text{B.57})$$

The vorticity at the rigid boundaries are then obtained as

$$\eta_{0,j} = -\frac{2\psi_{1,j}}{\Delta d^2}, \quad \eta_{4N,j} = -\frac{2\psi_{4N-1,j}}{\Delta d^2}, \quad \eta_{i,0} = -\frac{2\psi_{i,1}}{\Delta d^2}, \quad \text{and} \quad \eta_{i,N} = -\frac{2\psi_{i,N-1}}{\Delta d^2}, \quad (\text{B.58})$$

from the computation of the second-order-central-difference scheme.

No initial perturbation is required for the computation, as the horizontal temperature difference $\Delta\theta$ imposed at the top boundary always produces the baroclinic vorticities. The same temperature boundary conditions as the experiments were set, i.e., 16 conditions changing $\Delta T = 1, 2, 5$, and 10 K and $\Delta\theta = 1, 2, 5$, and 10 K were studied. Time integration of the 16 conditions were conducted for 180 min ($\sim 0.6\tau_K$) to match the duration of the experiments.

B.3 Results and discussions

Results of numerical simulation are shown as isosurfaces of stream function in Fig. B.2, B.3, B.4, and B.5. These figures correspond to those in Fig. 2.11, 2.12, 2.13, and 2.14 shown in Part III Chapter 2. All the isosurfaces are represented using the same color contours in a range from $\psi = -21.3$ to 21.3 (-3 to 3 mm²/s). Entirely, the trend of the flow developments is quite similar to those in the experiments. That is, larger $\Delta\theta$ yields larger values of ψ for each

ΔT condition, meaning that stronger baroclinic torque drives faster circulating motions of roll structures. Comparing results among Fig. 2.11, 2.12, 2.13, and 2.14 shown in Part III Chapter 2, the roll thickness becomes thinner for larger ΔT . A significant difference from the experiments can be seen as oscillatory structures appear under the conditions with strong baroclinic torque against weak restoring forces, shown in Fig. B.2(c) and (d) and Fig. B.3(d).

The temporal evolution of the flow fields in terms of the stream function and the temperature fields for the condition of $\Delta\theta = 2$ K and $\Delta T = 2$ K are shown in Fig. B.6. The time elapsed from the start of heating are noted above each panel. The color contours for the stream function fields shown in the left column of Fig. B.6 are scaled by the maximum stream function $\psi_{\max}$ recorded at $t = 180$ min.

Similar to the experimental results shown in Fig. 2.17 (Part III Chapter 2), the roll characteristics, the maximum horizontal velocity $u_{\max}$, the maximum stream function $\psi_{\max}$, and the roll thickness $\overline{\delta_{\text{roll}}}$, are shown in Fig. B.7. The characteristic values plotted for the γ' shown in Fig. B.7(a), (c), and (e) align along lines for each Ra condition with similar scaling factors to those in Fig. 2.17(a), (c), and (e) shown in Part III Chapter 2. Here, the values are dimensionalized to easily compare the results shown in Fig. 2.17. In contrast to the plots for the γ' , those for the Ra shown in Fig. B.7(b), (d), and (f) suggest that the conventional Ra scalings are not applicable for the developing horizontal convection.

In addition, bulk characteristics values, the circulation of roll Γ_c , the total kinetic energy KE_t , and the Reynolds number Re , are plotted in Fig. B.8. Here, the values are dimensionalized to easily compare the results shown in Fig. 2.18 (Part III Chapter 2). Dependence on the Ra shown in Fig. B.8(a), (c), and (e) is hard to observe similar to those shown in Fig. 2.18(a), (c), and (e). In contrast, distinctive γ' dependence can be recognized in Fig. B.8(b), (d), and (f). However, the γ' dependence of KE_t and Re becomes stronger as shown by steeper purple lines drawn in Fig. B.8(d) and (f) than the gray lines after the experimental results. These differences may originate from the non-ideal conditions of the experiments, as it is not possible to produce fully adiabatic boundary conditions. In addition, the experimental measurements were conducted in a rectangular box, while the numerical simulation was performed in a 2D domain. While yielding these slight differences, the three bulk characteristics are represented well by the γ' in the simulation.

The γ' and Ra dependence found in the numerical simulation are also summarized in the 2D domain of γ' - Ra as shown in Fig. B.9. As the results shown in Fig. B.7 and B.8 suggest that the characteristics values of the developing horizontal convection measured via the numerical simulation can be represented by both the Ra and γ' , the same analysis, the SVD method, can be applied for acquiring fitted planes. The scaling factors for $\phi \propto \gamma'^p Ra^q$ obtained from the experiments and simulations are summarized in Table. B.1. The local characteristic values shown in Fig. B.9(a), (b), and (c) suggest quite similar dependence on Ra and γ' to those shown in Fig. 2.19(a), (b), and (c). However, the bulk characteristic values shown in Fig. B.9(d), (e), and

Table B.1: Scaling factors p and q for the roll characteristics with the relationship $\phi \propto \gamma'^p Ra^q$.

	Experiment	Numerical simulation
$u_{\max}$	$p = -0.15, q = 0.45$	$p = -0.22, q = 0.34$
$\psi_{\max}$	$p = -0.46, q = 0.20$	$p = -0.49, q = 0.09$
δ_{roll}	$p = -0.25, q = -0.30$	$p = -0.09, q = -0.07$
Γ_c	$p = -0.25, q = 0.55$	$p = -0.32, q = 0.42$
KE_t	$p = -0.72, q = 0.73$	$p = -1.02, q = 0.35$
Re	$p = -0.36, q = 0.36$	$p = -0.51, q = 0.18$

(f) show stronger dependence on γ' than those in Fig. 2.19(d), (e), and (f). As above, these differences may originate in the non-ideal temperature boundary conditions in the experiments, and 2D and 3D flow developments of the simulation and experiments.

B.4 Summary

In Appendix B, experimental results discussed in Chapter 2 of Part III have been compared to the numerical simulation based on the Boussinesq approximation. As the developing horizontal convection examined possesses strong two-dimensionality, it was possible to model the flow using the stream function and vorticity equation. The 2D simulation seemed to realize the transient flow states observed during differential heating of the top of low-temperature fluid. From the rich physical quantities accessible in the numerical results, the flow structures of the developing HC were justified to grow with the heat diffusion of the heating surface. Even the slight differences on temperature boundary conditions and geometry of fluid layer allow to produce different dependence on Ra and γ' , entire trends of the developing flow structures were reproduced successfully by the simple numerical approach.

One different feature observed in the numerical results compared to the experimental results is the emergence of the fluctuating motion. When the driving force originated in the baroclinic torque is sufficiently stronger than the braking force of the stable temperature stratification, the circulating flows start to fluctuate with a certain wavelength. This may be generated because of shear-based instability, such as Kelvin–Helmholtz instability, as the horizontal advection velocity immediately beneath the top surface is fast. This instability was not able to observe in the experimental results, and thus it is possible to deduce that such fluctuations can be generated in a fluid container with a larger length in the y -direction. In a 3D fluid container, the 2D fluctuation will be emerged as 3D flow structures, and such flow structures not described in Chapter 2 of Part III are worth investigating. An exemplary result is shown in Appendix C.

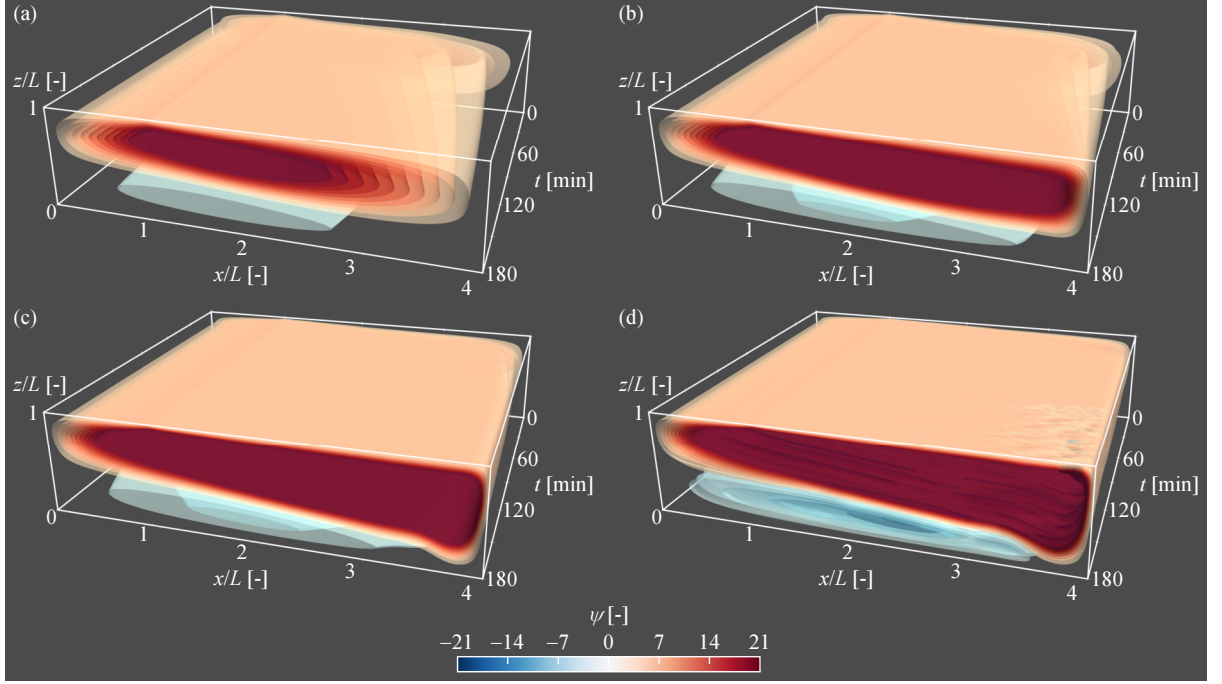


Figure B.2: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 1$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

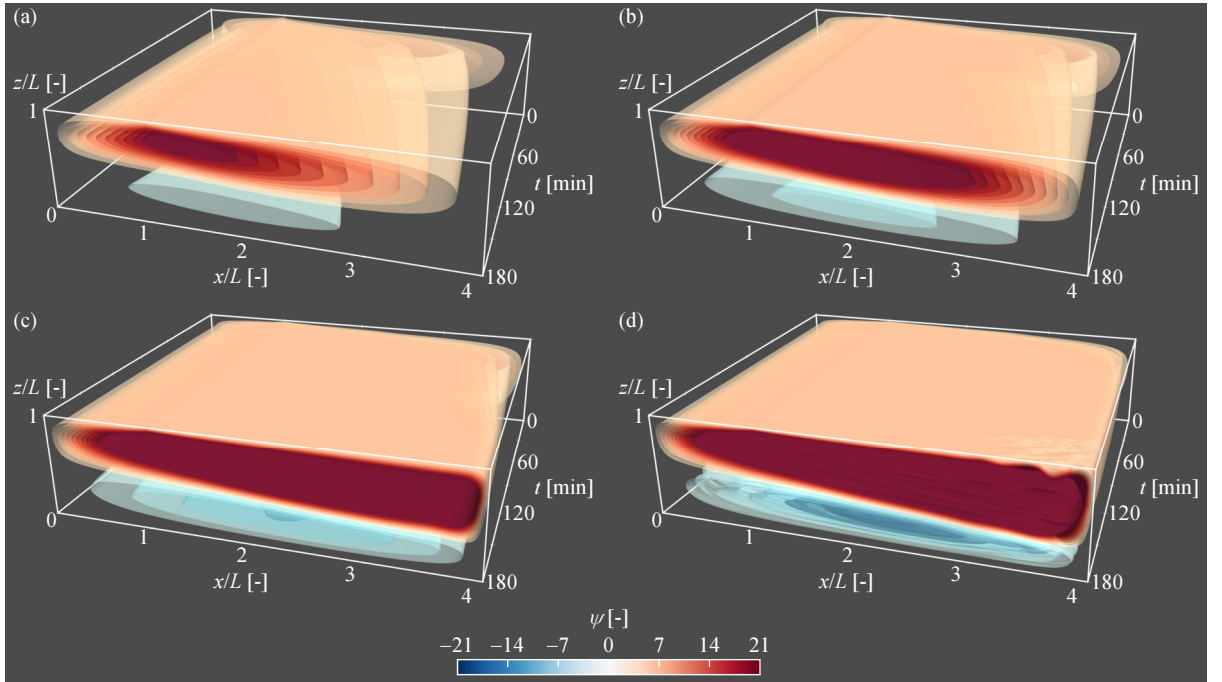


Figure B.3: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 2$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

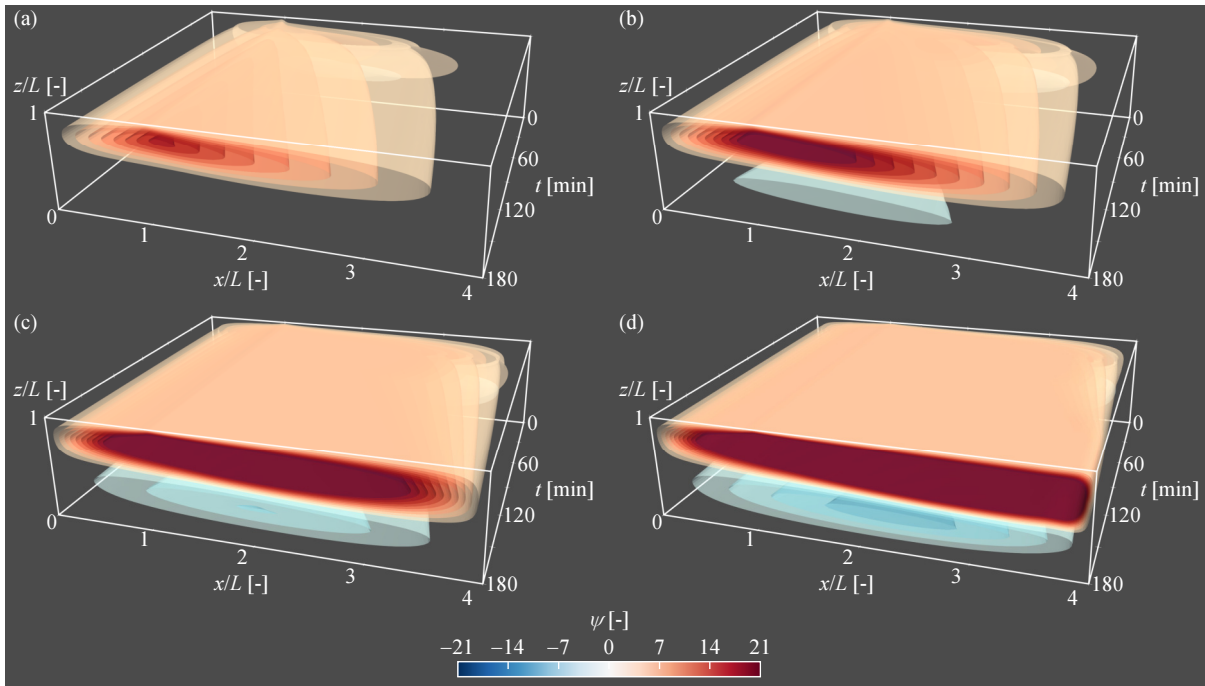


Figure B.4: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 5$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

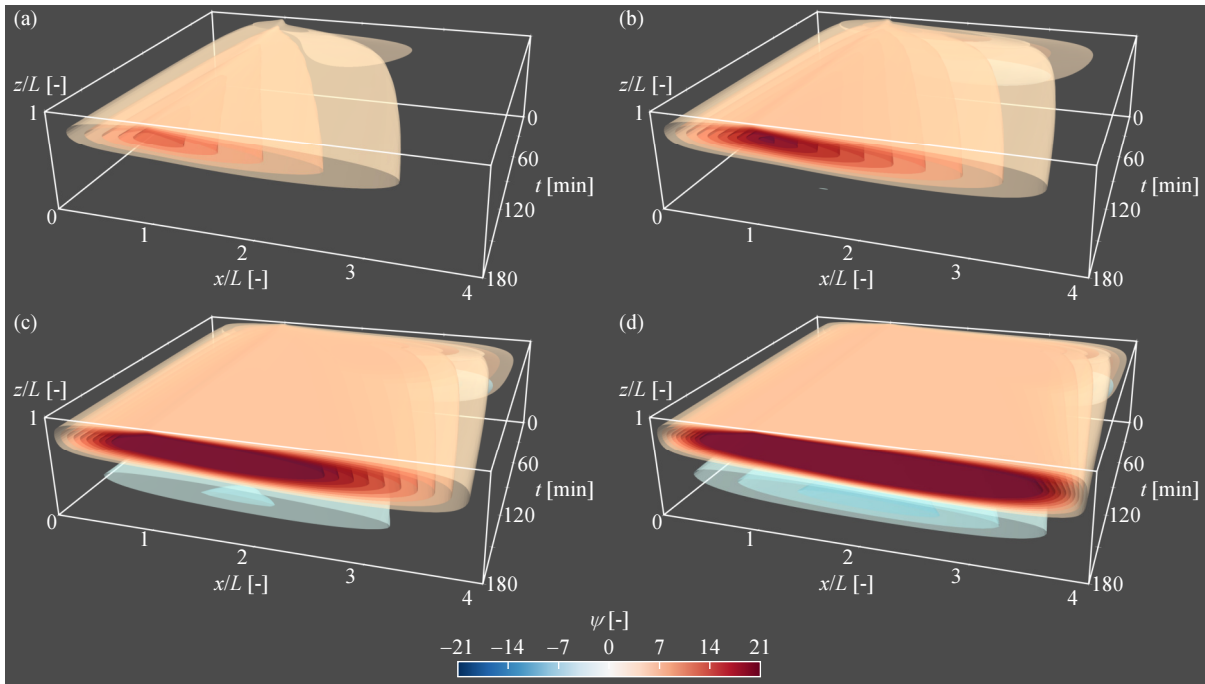


Figure B.5: Isosurfaces of stream function in the spatio-temporal domain under the condition of $\Delta T = 10$ K: (a) $\Delta\theta = 1$ K, (b) 2 K, (c) 5 K, and (d) 10 K.

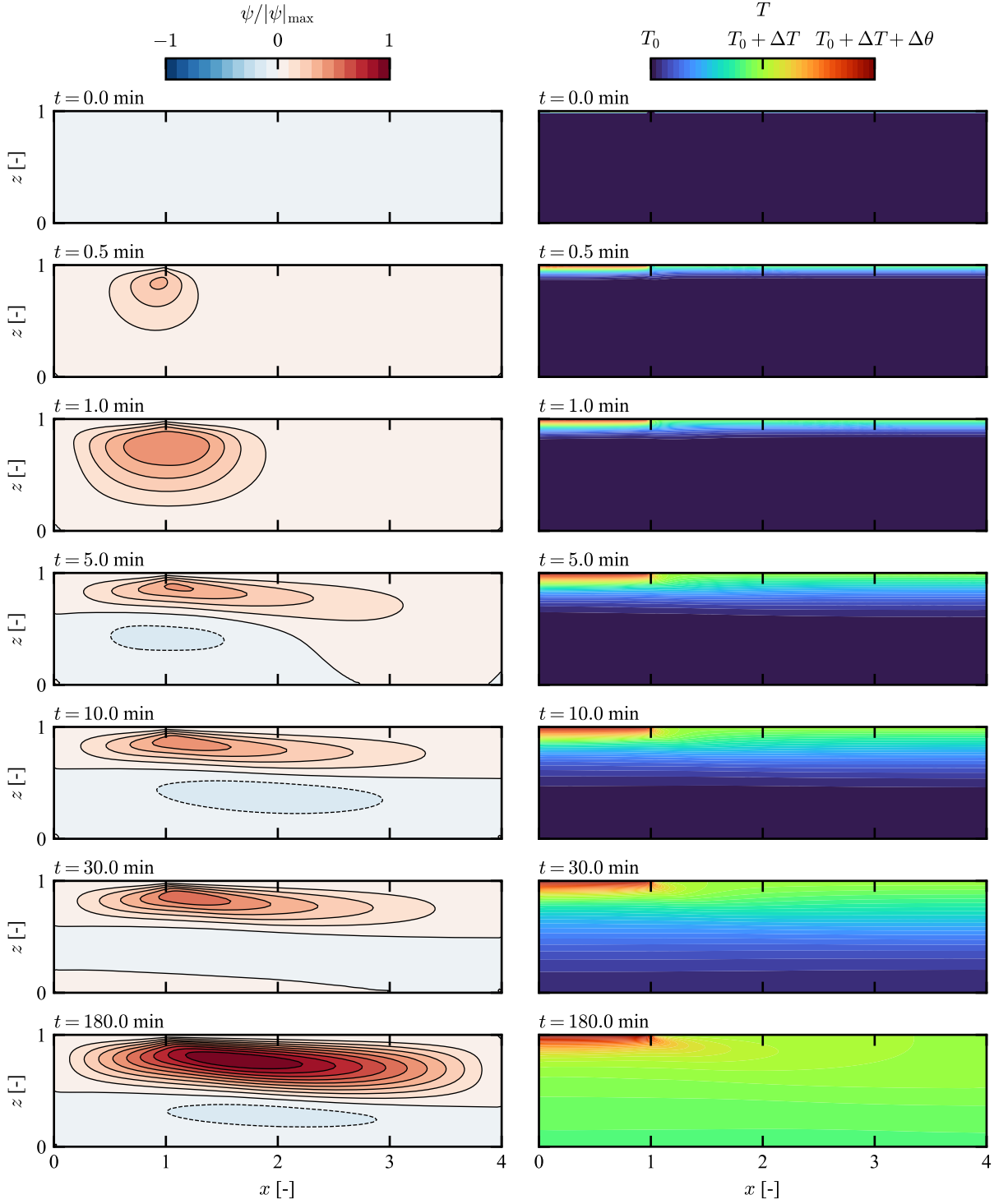


Figure B.6: Temporal evolution of stream function fields (left) and temperature fields (right) for the condition of $\Delta\theta = 2$ K and $\Delta T = 2$ K. Time elapsed from the start of heating are noted above each panel. Color contours for the stream function fields are normalized by the maximum value obtained at $t = 180$ min. The results shown here correspond to the condition shown in Fig. B.3(b).

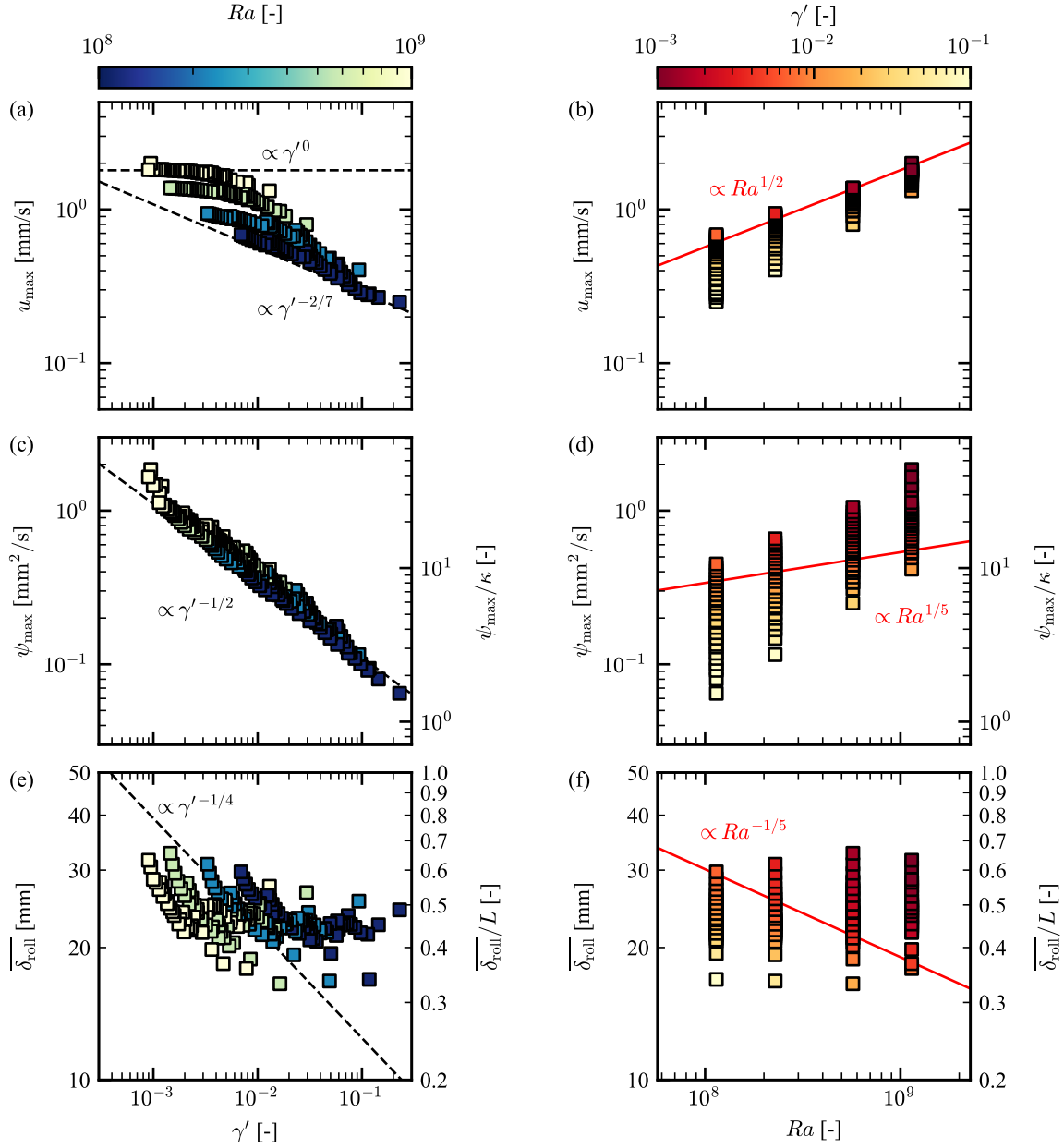


Figure B.7: Schematic of the computation domain for the numerical simulation of developing horizontal convection. Circles forming the rectangular computation domain represent the grid points at the boundaries, and imposed boundary conditions for the temperature T and stream function ψ are noted.

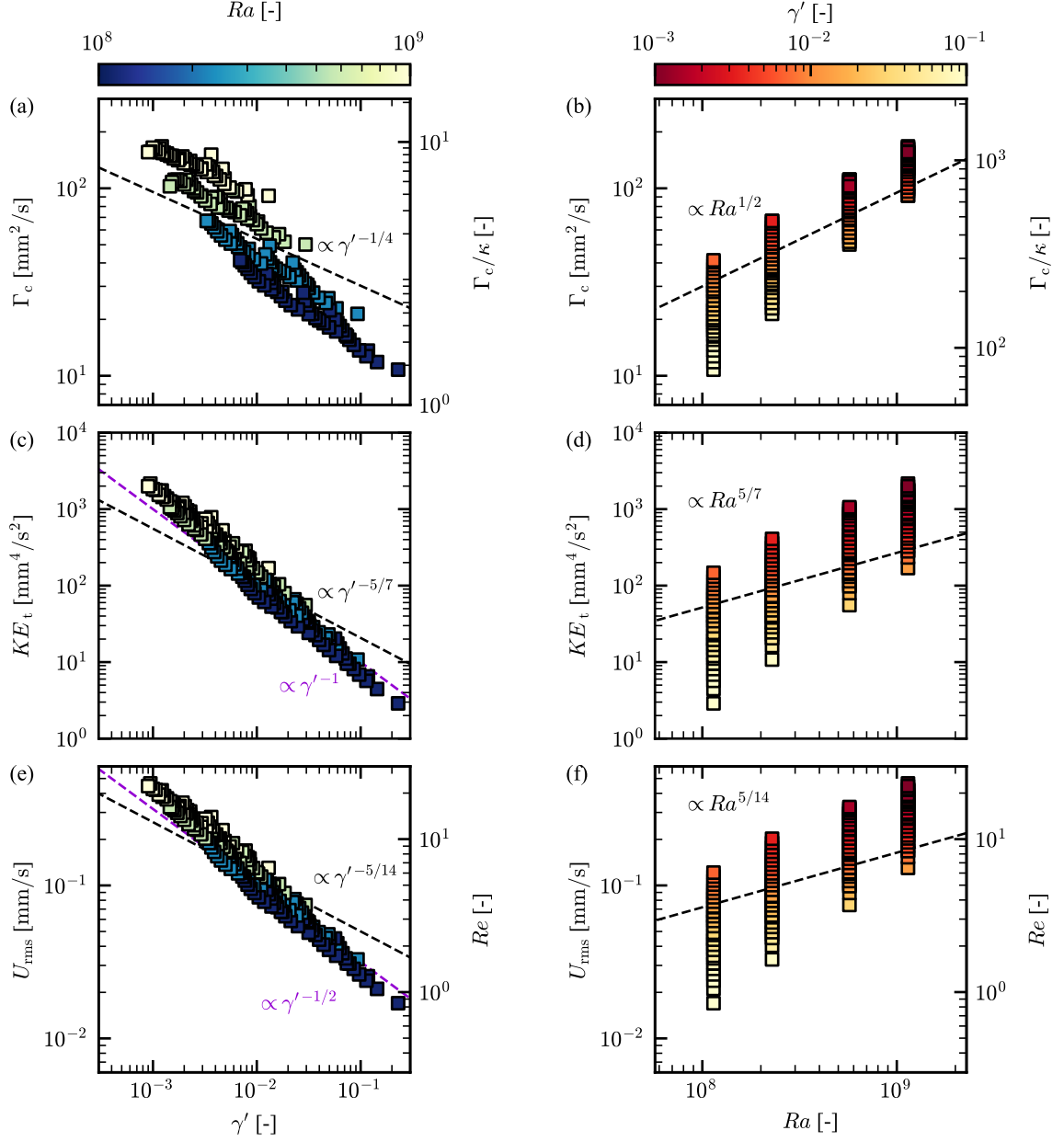


Figure B.8: Schematic of the computation domain for the numerical simulation of developing horizontal convection. Circles forming the rectangular computation domain represent the grid points at the boundaries, and imposed boundary conditions for the temperature T and stream function ψ are noted.

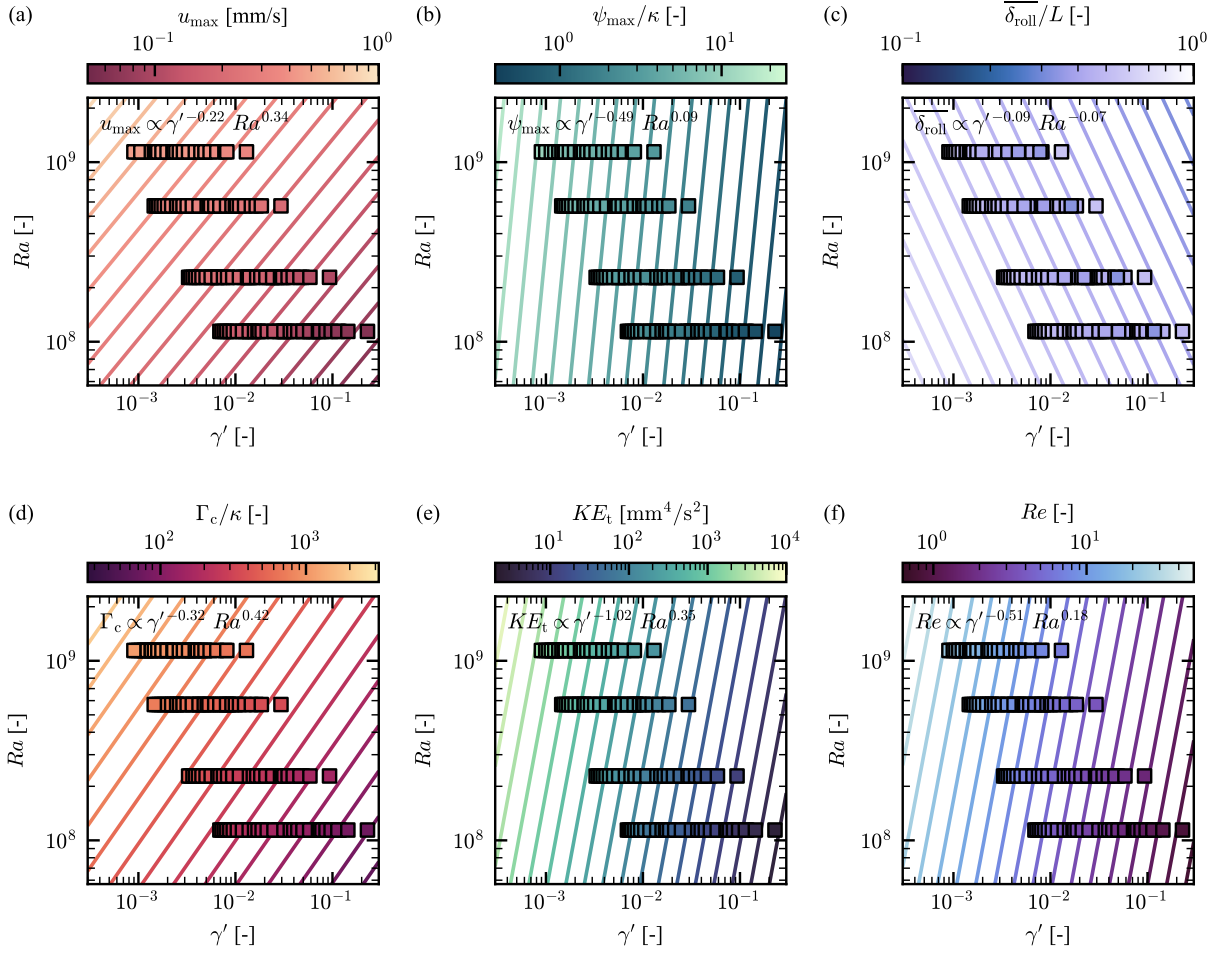


Figure B.9: Schematic of the computation domain for the numerical simulation of developing horizontal convection. Circles forming the rectangular computation domain represent the grid points at the boundaries, and imposed boundary conditions for the temperature T and stream function ψ are noted.

THREE DIMENSIONAL DEVELOPMENT OF HORIZONTAL CONVECTION

In Part III Chapter 2, the developing stage of horizontal convection (HC) was studied in detail. Most of the resultant flow structures can be considered as two dimensional (2D), however, some conditions show three-dimensionality which had not been described so far. In Appendix C, a brief explanation on the three dimensional (3D) development of HC is provided.

The experiments were conducted in the same setup as that used in Part III Chapter 2. For the condition with the largest horizontal temperature difference $\Delta\theta = 10$ K and the smallest vertical temperature difference $\Delta T = 1$ K, a different cross-section in the y - z plane at $x = 2.5L$ was visualized. To understand flow structures in the cross-section qualitatively, pathline images are compiled as shown in Fig. C.1. The pathline images obtained for different time, $t = 10, 20, 30$, and 180 min, from the start of the heating are shown. With time, it becomes possible to identify small circulating flows immediately beneath the top boundary, and the circulations at the bottom of the fluid layer originating from the initial states gradually are weakened. The small circulations first appear at the corners $y/L \sim 0$ and 1 and the center $y/L \sim 0.5$ of the cross-section at $t = 20$ min. At $t = 180$ min, eight roll structures were formed. Since the main stream of the developing HC is in the x -direction, these small circulating motions are generated as secondary flows.

For a detailed investigation, the timeline image of the isosurfaces of the stream function fields at this cross-section is compiled and shown in Fig. C.2. It is also possible to identify circulating motions align alternately in the y direction after ceasing the initial flow structures. The corresponding isosurfaces of the stream function in the x - z plane are shown in Fig. 2.11(d). Comparing the values of the stream function of the circulation in the x - z plane and in those in the y - z plane, the latter is much smaller than the former one. From this fact, the strength of the secondary circulation is less than 10% of the main circulation in the x - z direction.

From the results shown so far, it is possible to deduce the mechanism of the secondary

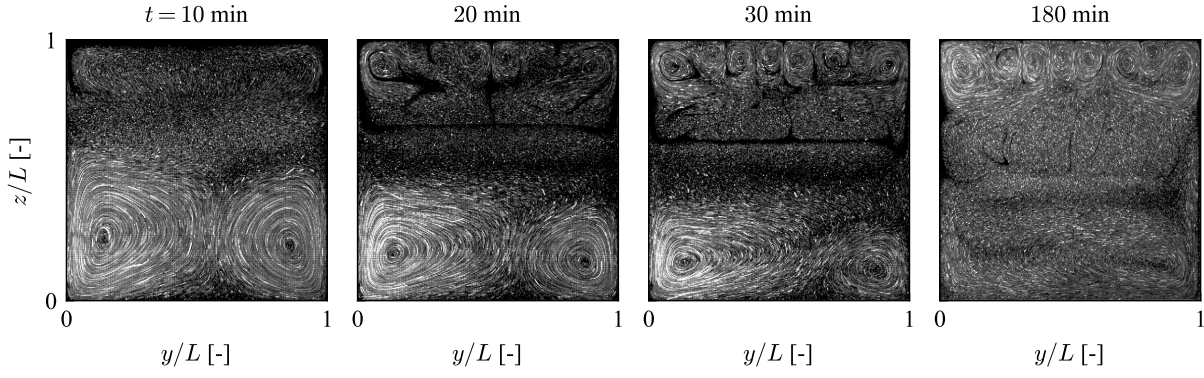


Figure C.1: Pathline images in the y - z plane at $x = 2.5L$. Time elapsed from the start of the heating is noted above each image.

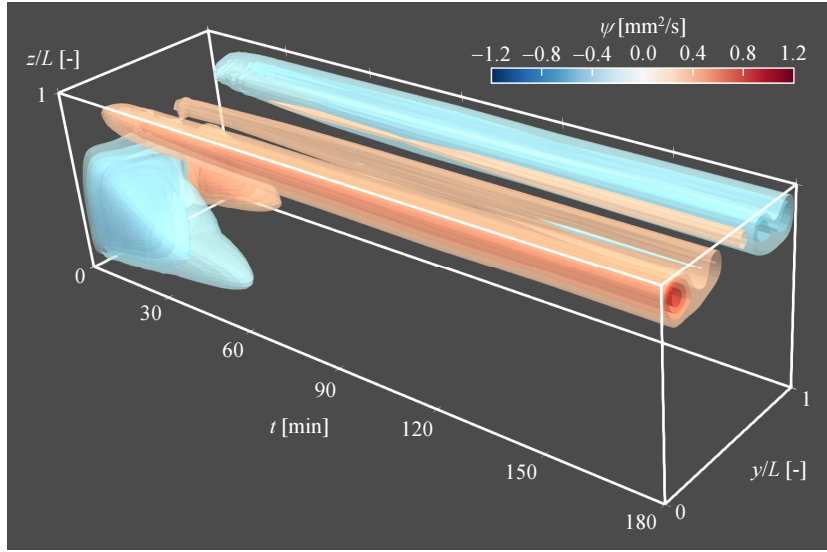


Figure C.2: Isosurfaces of stream function ψ at $x = 125 \text{ mm} = 2.5L$ in the spatio-temporal domain under the condition of $\Delta T = 1 \text{ K}$ and $\Delta\theta = 10 \text{ K}$.

circulations in the y - z plane. The condition focused here, $\Delta\theta = 10 \text{ K}$ and $\Delta T = 1 \text{ K}$, has a potential for efficient convective heat transport as well as the thermal diffusion. This is because the horizontal velocity u in the x -direction is determined by the speed of the gravity current, $U_{gc} \propto \sqrt{g\beta\Delta\theta\delta_\kappa}$. Thus the heated fluid at the hottest top boundary with $T = T_0 + \Delta T + \Delta\theta$ can be transported far from the hottest region without decreasing temperature. This is schematically illustrated in Fig. C.3. As a result, the heated fluid go under the relatively cold fluid at the boundary with $T = T_0 + \Delta T$ at a certain depth of δ . The local Rayleigh number for the depth δ is then defined as

$$Ra_\delta = \frac{g\beta\Delta\theta\delta^3}{\kappa\nu}, \quad (\text{C.1})$$

and local thermal convection reaches onset when Ra_δ exceeds the critical value. Practically, it is possible to replace δ by $\delta_{\text{roll}}/2$ as only the flow in the positive x -direction is sufficient to be considered. For the present condition, δ is thus typically $\delta \sim 10 \text{ mm}$. The resultant Ra_δ is $Ra_\delta \approx 1.4 \times 10^5$, and this is sufficient for the onset of thermal convection. From these

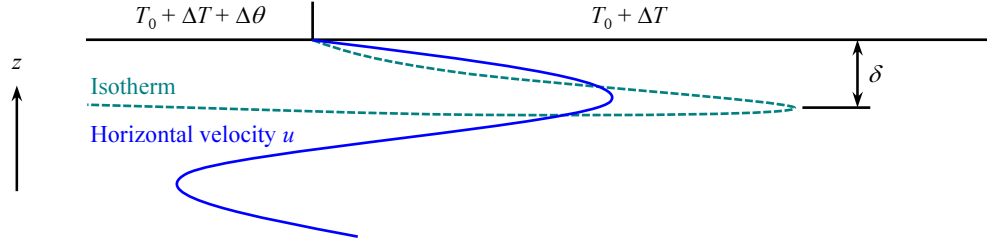


Figure C.3: Schematic illustration immediately beneath the top boundary.

discussions, it is possible to conclude that the secondary circulations are the convective roll structures having their longitudinal axes in the x -direction.

Such roll structures have been also known as the horizontal convective rolls. Most of the studies on the horizontal convective rolls were motivated for investigating the cloud pattern formed at the sea-breeze front in the atmospheric flows, and based on the observation facts [39, 110, 137, 201, 202]. An experimental study can be found in the work by Vila et al. [196], and they realized similar horizontal convective rolls in a closed container. Although detailed discussions on the horizontal convective rolls are not held in this thesis, it is worth investigating the mechanism and the conditions of the formation of the horizontal convective rolls through experimental studies.

COLLECTION OF VISUALIZATION OF THERMAL CONVECTION

In Appendix D, various visualization results of thermal convection are presented. This appendix is assembled as a collection of visualized images of various thermal convection utilizing the TLC particles. Even if the coloration of the TLC particles is not quantified as temperature, use of the TLC particles provides lots of information both in qualitative and quantitative manners. Thus, no quantified result will be provided, while the quantification is possible like the main contents discussed in Part III. Seeding of the TLC particles sometimes requires special cares according to the test fluids. However, as long as the test fluids are transparent, the TLC particles show informative coloration, while the coloring temperature range shifts from the originally expected one. Since various transparent fluids have been employed as the test fluids, technical notes are also provided.

D.1 Rayleigh–Bénard convection

The first example is Rayleigh–Bénard convection (RBC), which is the most classical system of thermal convection, confined in rectangular containers. In this chapter, three different fluid containers with different geometries are employed as the fluid layers¹. One of the fluid layers has a geometry of $200 \times 30 \times 50 \text{ mm}^3$ as illustrated in Fig. D.1. The temperature at the bottom can be maintained at a constant value by controlling electric current provided to a heater. That of the top can be also maintained at a constant value by circulating water from a thermostatic bath. Glycerol solutions (GS), 80-vol.% and 40-vol.%, were employed as the test fluids, and the corresponding Prandtl number $Pr = \nu/\kappa$ is estimated as $Pr \approx 677$ and 29, respectively. The

¹The results shown in this chapter were obtained in cooperation with Dr. T. Yanagisawa, Japan Agency for Marine–Earth Science and Technology (JAMSTEC). The experiments were conducted in 2019–2020. The results were originally acquired for compiling educational materials to demonstrate beautiful visualization of thermal convection, and thus the fluid containers employed here were also provided by Dr. T. Yanagisawa.

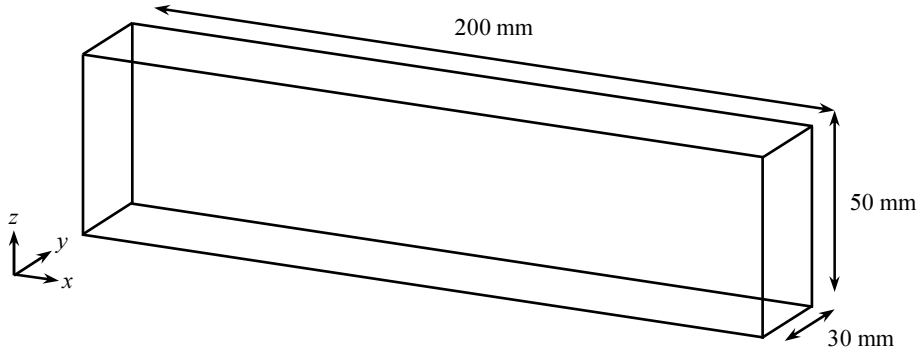


Figure D.1: Geometry of the fluid layer with a size of $200 \times 30 \times 50 \text{ mm}^3$.

TLC particles with a nominal coloring temperature range of $20\text{--}30^\circ\text{C}$ were seeded as the tracer. It is worth noting that the coloring temperature range of the TLC particles significantly change according to the test fluids. The coloration itself also changes with the selection of the test fluids. In general, it is possible to visualize vivid coloration by using GSs as the test fluids compared to water, and the coloring temperature range shifts to the lower temperature. For the present study, the actual coloring temperature range is roughly in $18\text{--}28^\circ\text{C}$. A vertical cross-section at the mid-depth, $y = 15 \text{ mm}$, was illuminated by a white light sheet generated by a halogen lamp and a cylindrical lens connected to a linear light guide. For all the cases, the lighting was performed from the left side ($x = 0 \text{ mm}$).

For the 80-vol.% GS ($Pr \approx 677$), two distinctive results are shown. One is the case imposed a vertical temperature difference of $\Delta T = 2 \text{ K}$ by setting the temperature at the top $T_{\text{top}} = 18^\circ\text{C}$ and that at the bottom $T_{\text{btm}} = 20^\circ\text{C}$. The Rayleigh number, defined as $Ra = g\beta\Delta TH^3/(\kappa\nu)$, is estimated as $Ra \approx 2.0 \times 10^5$. The temporal evolution for 10 min is shown in Fig. D.2. This case formed quasi-steady convection rolls, which hardly changes with time. Four or five roll structures were formed, and each roll rotates rigidly. The temperature range imposed across the fluid layer match perfectly to that showing effective color changes of the TLC particles.

The other case imposed a vertical temperature difference of $\Delta T = 10 \text{ K}$ by setting the temperature at the top $T_{\text{top}} = 15^\circ\text{C}$ and that at the bottom $T_{\text{btm}} = 25^\circ\text{C}$, resulting in $Ra \approx 1.0 \times 10^6$. The temporal evolution for 10 min is shown in Fig. D.3. The coloration is not as vivid as the case shown in Fig. D.2, however, it is possible to recognize thermal boundary layers formed at the top and the bottom walls. From the fluctuating boundary layers, the number of roll structures was found to increase with the larger ΔT . Similar to the case of $Ra \approx 2.0 \times 10^5$, the roll structures hardly change with time at least during 10 min, and thus these rolls can be also considered as quasi-steady rolls. One failure in visualization for this case is the presence of bubbles with a size of $O(1 \text{ mm})$. These bubbles are obvious especially in the left half of the fluid layer. Tips for eliminating such bubbles will be provided later.

For the 40-vol.% GS ($Pr \approx 29$), three distinctive results are shown. In Fig. D.4, the case imposed a vertical temperature difference of $\Delta T = 2 \text{ K}$ by setting the temperature at the top $T_{\text{top}} = 18^\circ\text{C}$ and that at the bottom $T_{\text{btm}} = 20^\circ\text{C}$. The Ra is estimated as $Ra \approx 1.6 \times 10^6$.

It is possible to observe regularly aligned roll structures, apart from the side walls. From the fluctuations of the thermal boundary layers, these roll structures are found to slightly oscillate in the x -direction. Thus, these roll structures can be recognized as unsteady roll structures. Even though the temperature ranges adopted for the cases shown in Fig. D.2 and D.4 are the same, the coloration is different from each other. This is because of the coloration characteristics of the TLC particles as detailed in Appendix A.

A larger Ra condition, $Ra \approx 6.0 \times 10^6$, is shown in Fig. D.5. The temperature condition was $T_{\text{top}} = 15^\circ\text{C}$ and $T_{\text{btm}} = 22.5^\circ\text{C}$, resulting in $\Delta T = 7.5$ K. The flow fields shown in Fig. D.5 are fully different from those shown in Fig. D.2, D.5, and D.4. Thermal plumes are formed as detachments of the thermal boundary layers, and cold plumes and hot plumes are identified as red blobs and blue blobs shaping mushroom-like structures in Fig. D.5. According to Castaing et al. [25], thermal convection with a large Ra value, $Ra > 1.0 \times 10^6$ is categorized as the (soft-)thermal turbulence, and the results are consistent with the categorization.

An extreme case utilizing 40-vol.% GS is shown in Fig. D.6. The imposed temperature difference was $\Delta T = 26$ K, by setting the temperatures as $T_{\text{top}} = 5^\circ\text{C}$ and $T_{\text{btm}} = 31^\circ\text{C}$, resulting in $Ra \approx 2.1 \times 10^7$. Since the vertical temperature difference is very large, it may not be possible to assume Boussinesq condition, however, non-Oberbeck-Boussinesq effect [59] is not focused here. The number of thermal plumes emitted from the thermal boundary layers greatly increased compared to the case shown in Fig. D.5. The flow fields are locally unsteady by the numerous plume emissions, however, the plumes drive a global circulation trend called large scale circulation (LSC). For the present case, two or three LSCs are formed due to the fluid container with a large aspect ratio, 4.

The other examples employing a different fluid layer are also provided. The height of the fluid layer employed here can be adjusted by changing positions of the top wall. Two different heights, 25 mm and 85 mm, are demonstrated, and the dimensions in the x and y -directions are fixed at 200 mm and 25 mm, respectively. The geometries of the fluid layers are shown in Fig. D.7 and D.8. The temperatures at the bottom and the top can be controlled individually by circulating water supplied from two different thermostatic baths. The lighting for visualization was performed at the center of the fluid layers, $y = 12.5$ mm, by using the white light sheet generated by the halogen lamp and the cylindrical lens connected by the linear light guide, and the fluid layers were illuminated from the left side. The test fluids, the height of the fluid layer, and temperature conditions are different from each other, and the conditions are summarized in Table D.1. In Table D.1, the cases shown in Fig. D.2 to D.16 are also summarized. The results of visualization utilizing the TLC particles are shown in Fig. D.9 to D.16. Since the realized flow fields are very typical, detailed explanations for them are omitted here.

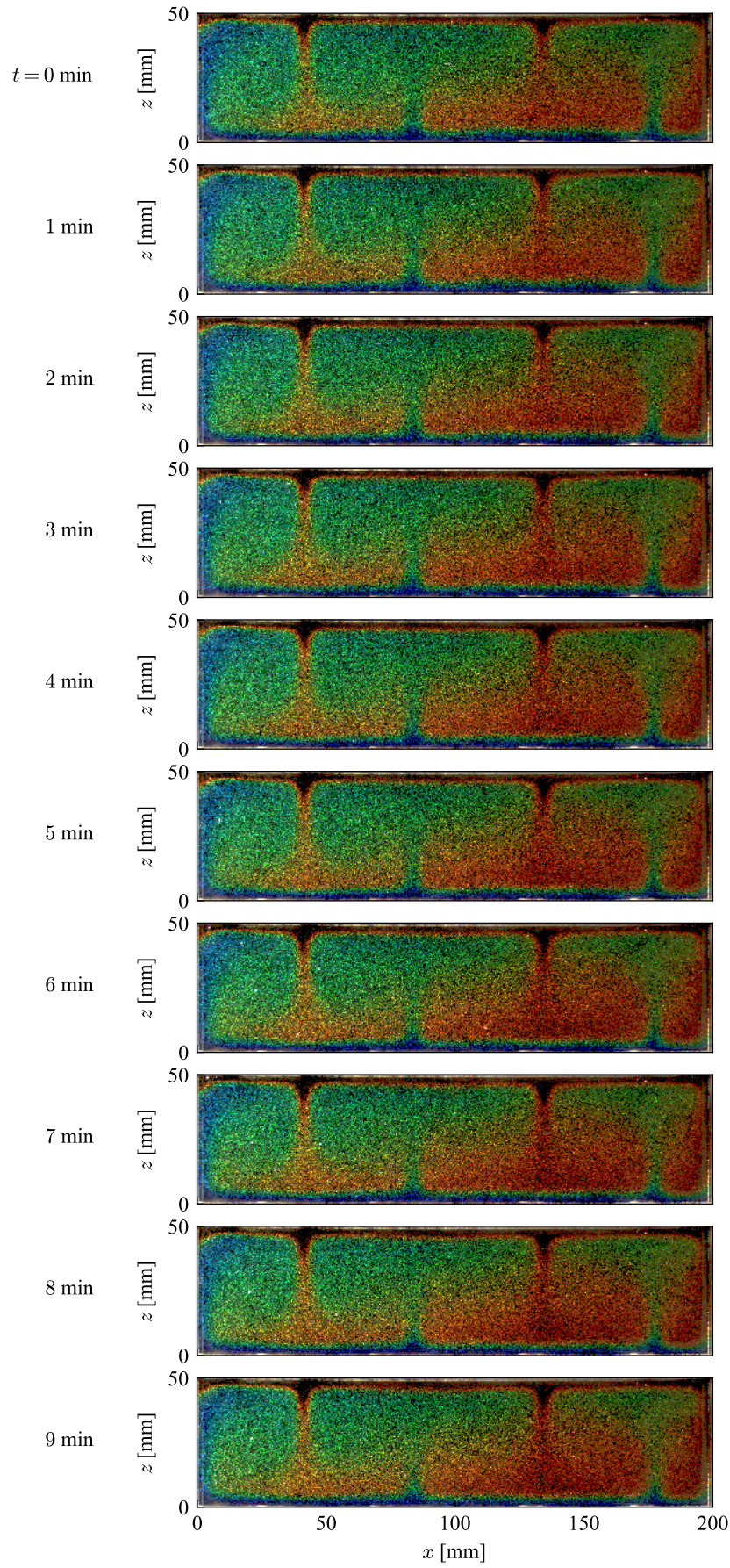


Figure D.2: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 2.0 \times 10^5$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

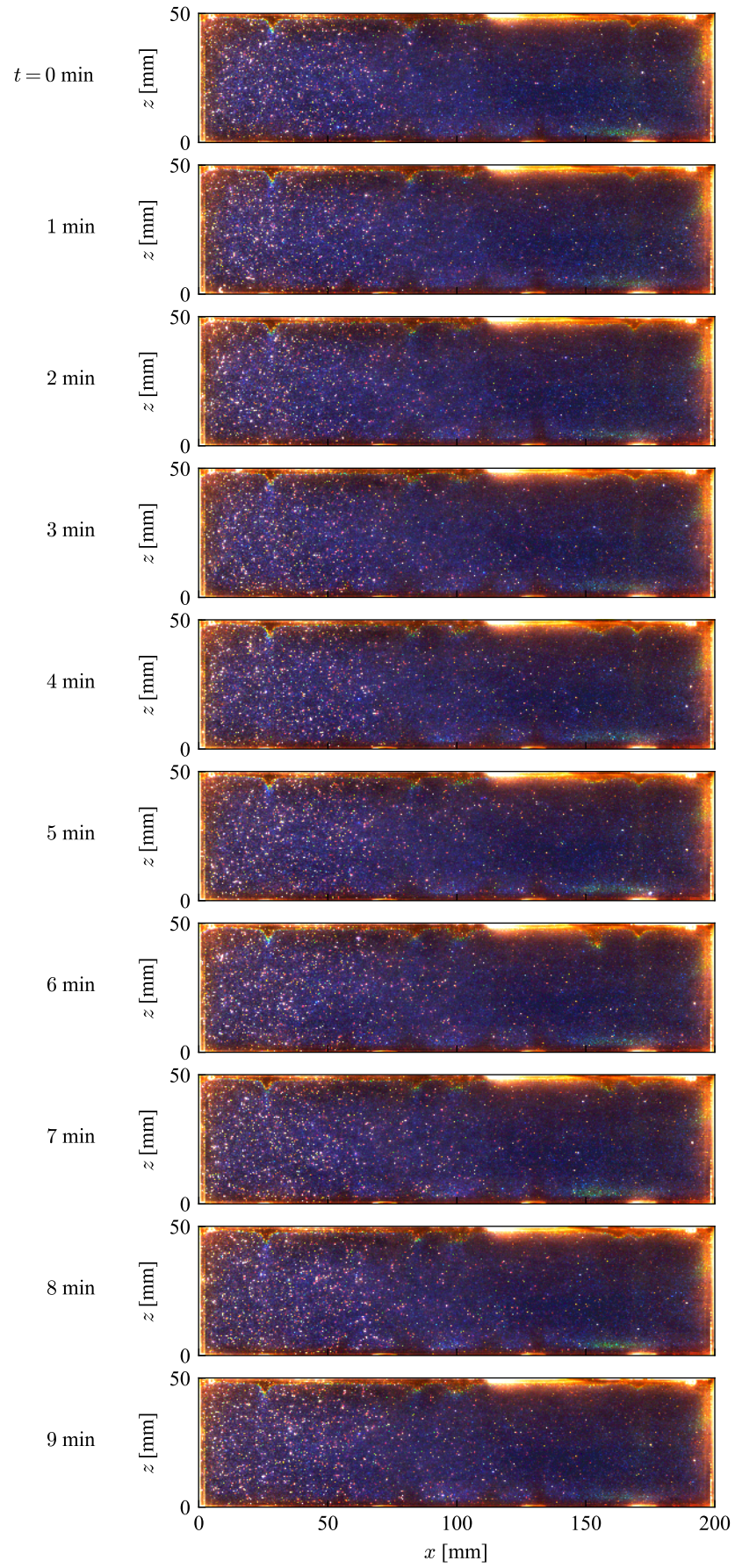


Figure D.3: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 1.0 \times 10^6$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

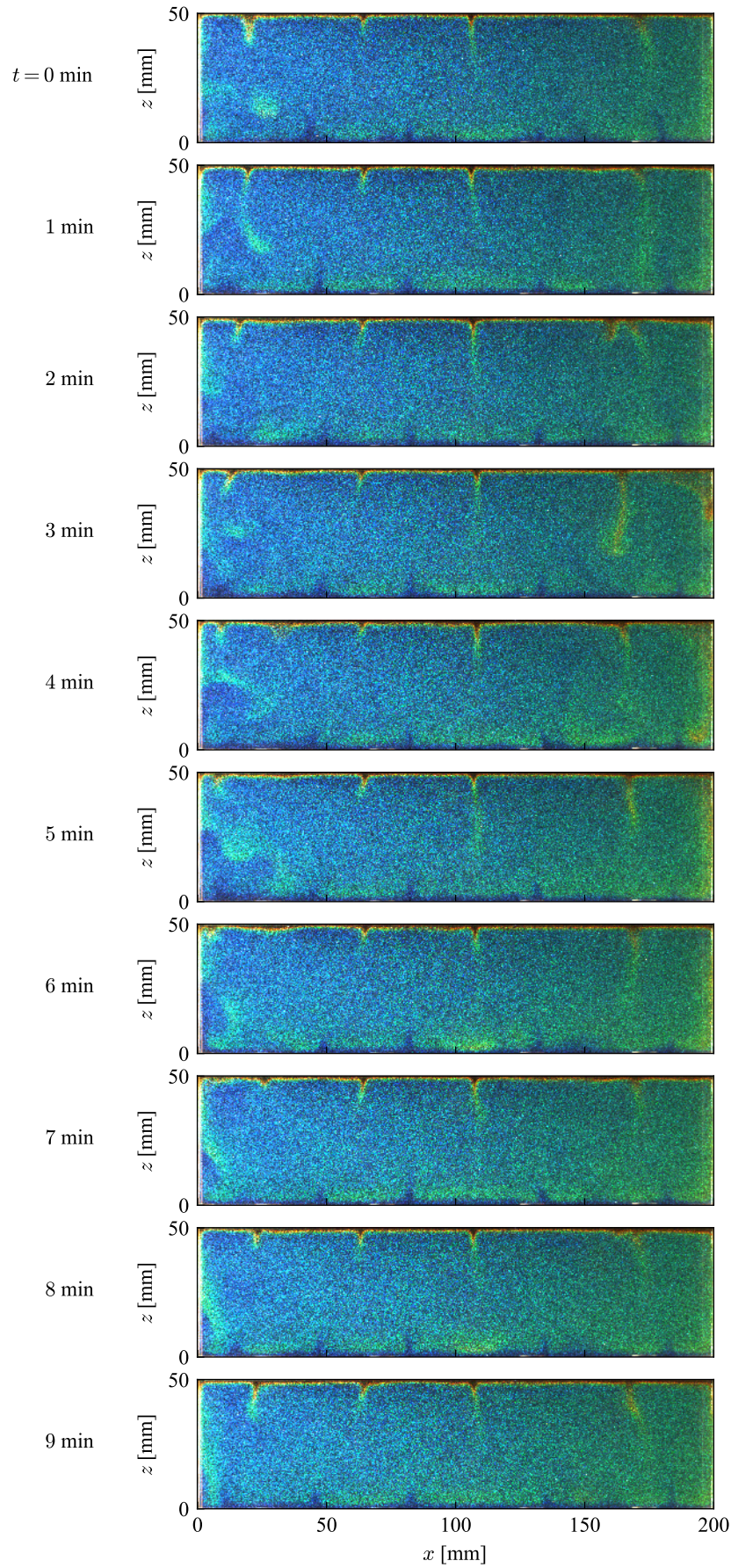


Figure D.4: Temporal evolution of RBC for the case of $Pr \approx 29$ and $Ra \approx 1.6 \times 10^6$ utilizing 40-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

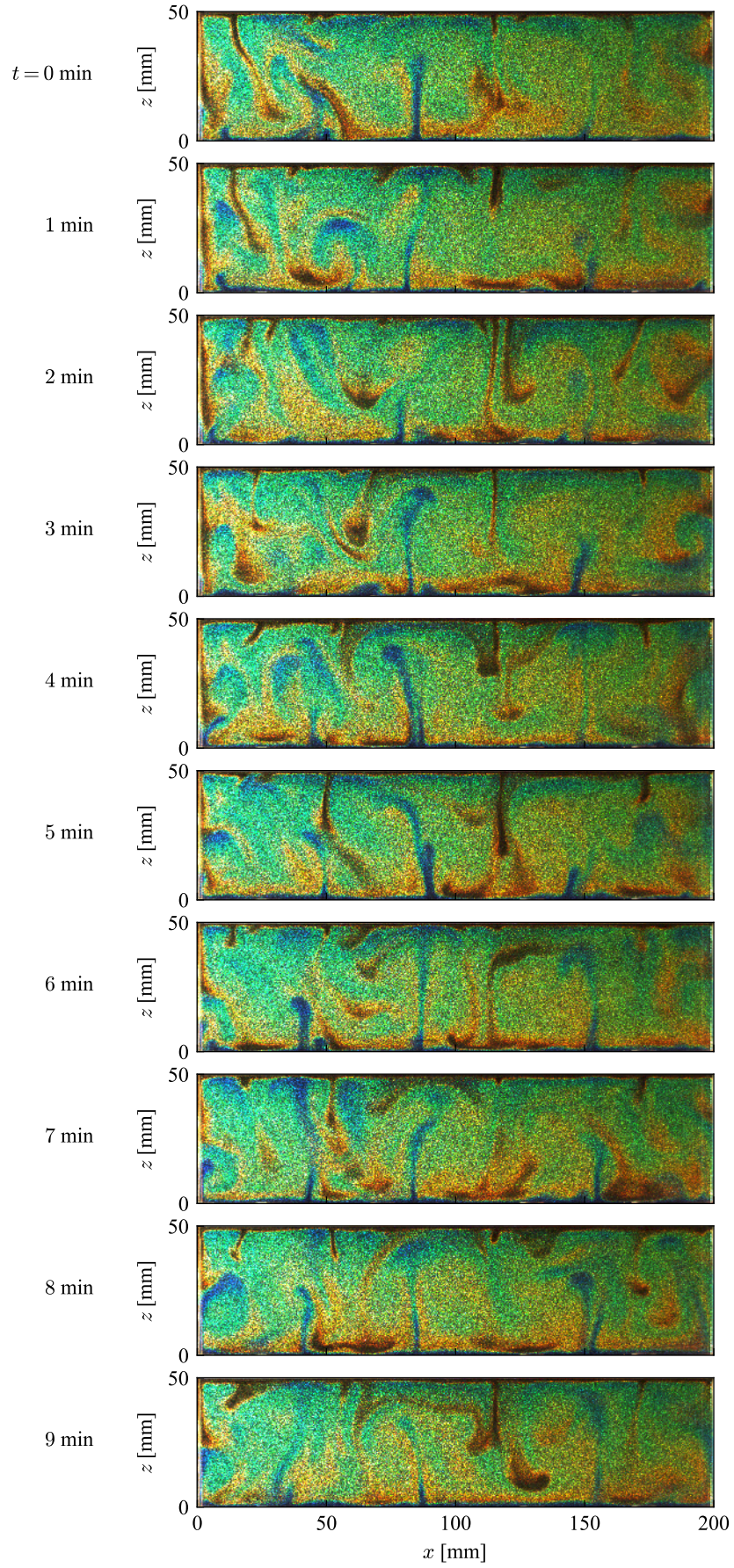


Figure D.5: Temporal evolution of RBC for the case of $Pr \approx 29$ and $Ra \approx 6.0 \times 10^6$ utilizing 40-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

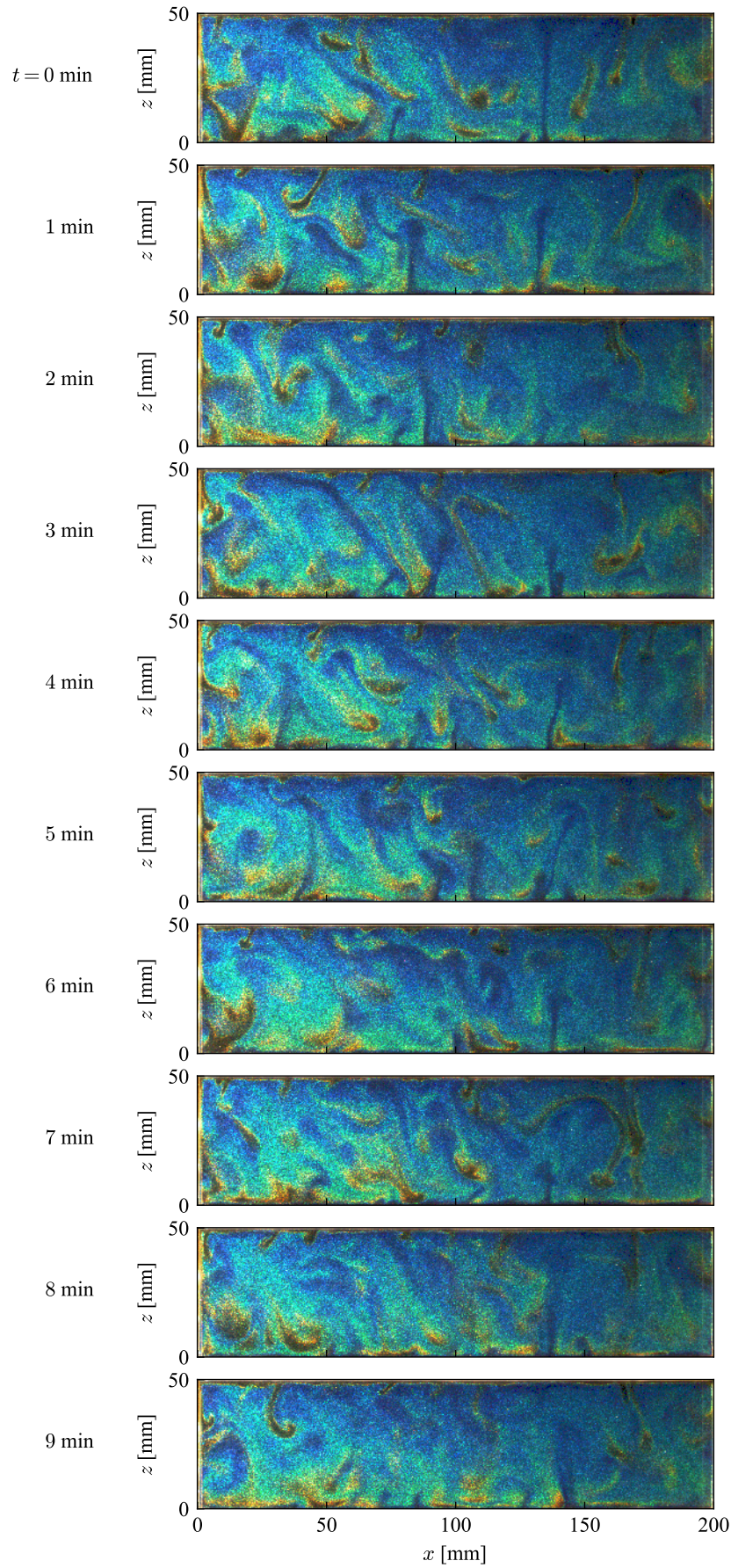
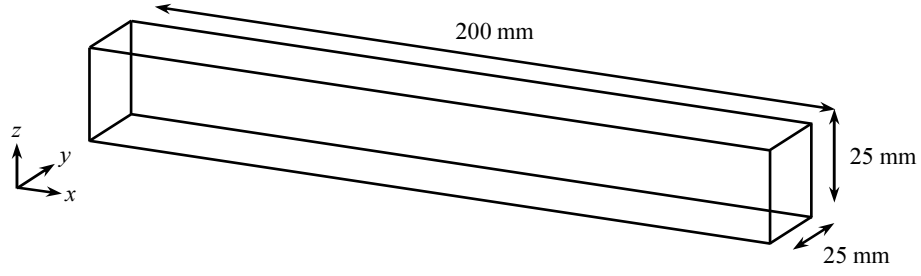
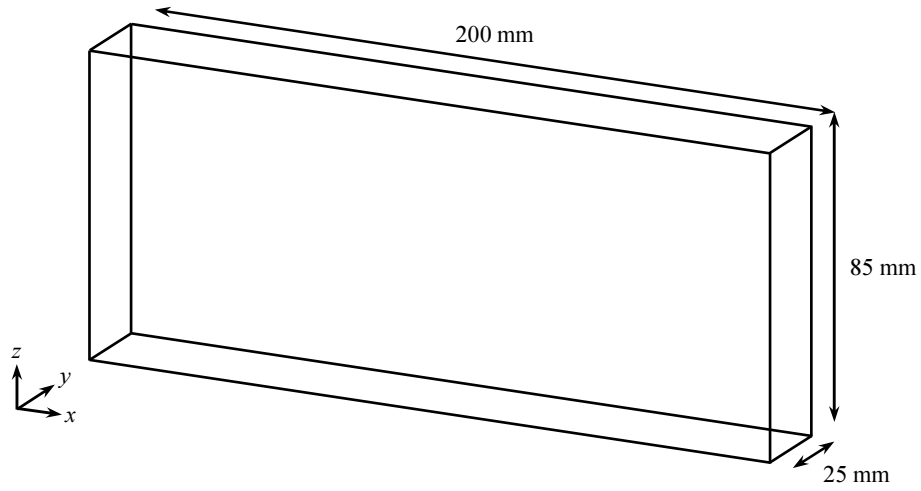


Figure D.6: Temporal evolution of RBC for the case of $Pr \approx 29$ and $Ra \approx 2.1 \times 10^7$ utilizing 40-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

Table D.1: Experimental conditions for visualization of RBC utilizing the TLC particles.

Figure	Geometry [mm ³]	Test fluid	Pr [-]	T_{top} [°C]	T_{btm} [°C]	ΔT [K]	Ra [-]
D.2	$200 \times 30 \times 50$	80-vol.% GS	677	18	20	2	2.0×10^5
D.3	$200 \times 30 \times 50$	80-vol.% GS	677	15	20	5	1.0×10^6
D.4	$200 \times 30 \times 50$	40-vol.% GS	29	18	20	2	1.6×10^6
D.5	$200 \times 30 \times 50$	40-vol.% GS	29	15	22.5	7.5	6.0×10^6
D.6	$200 \times 30 \times 50$	40-vol.% GS	29	5	31	26	2.1×10^7
D.9	$200 \times 25 \times 25$	80-vol.% GS	677	19.1	20.1	1	1.5×10^4
D.10	$200 \times 25 \times 25$	80-vol.% GS	677	10	27	17	2.6×10^5
D.11	$200 \times 25 \times 25$	Water	7	19	20	1	2.2×10^5
D.12	$200 \times 25 \times 25$	Water	7	15	25	10	2.2×10^6
D.13	$200 \times 25 \times 85$	80-vol.% GS	677	19	21	2	1.2×10^6
D.14	$200 \times 25 \times 85$	80-vol.% GS	677	10	27	17	1.0×10^7
D.15	$200 \times 25 \times 85$	Water	7	18	20	2	1.7×10^7
D.16	$200 \times 25 \times 85$	Water	7	10	30	20	1.7×10^8


Figure D.7: Geometry of the fluid layer with a size of $200 \times 25 \times 25$ mm³.

Figure D.8: Geometry of the fluid layer with a size of $200 \times 25 \times 85$ mm³.

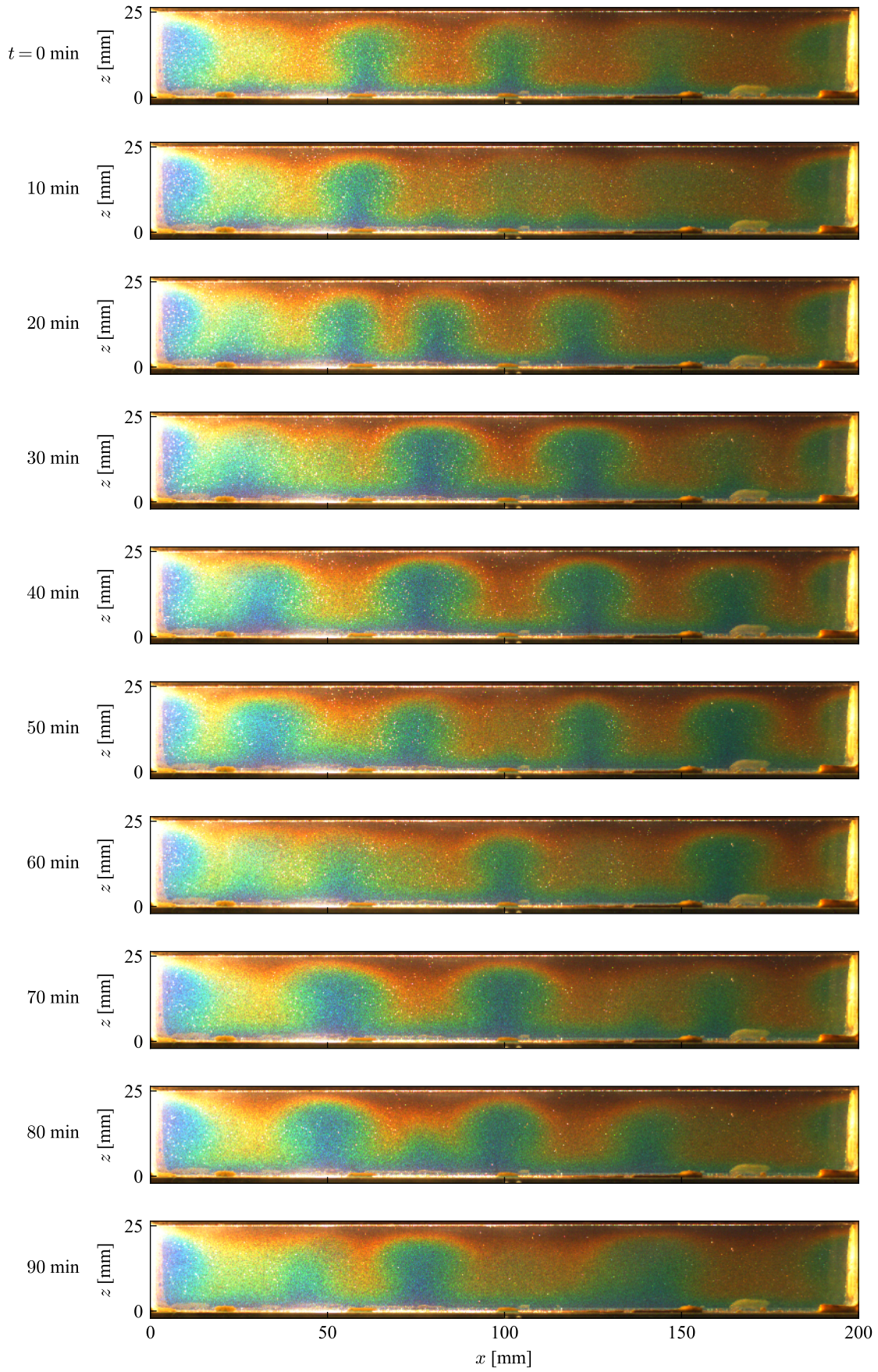


Figure D.9: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 1.5 \times 10^4$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

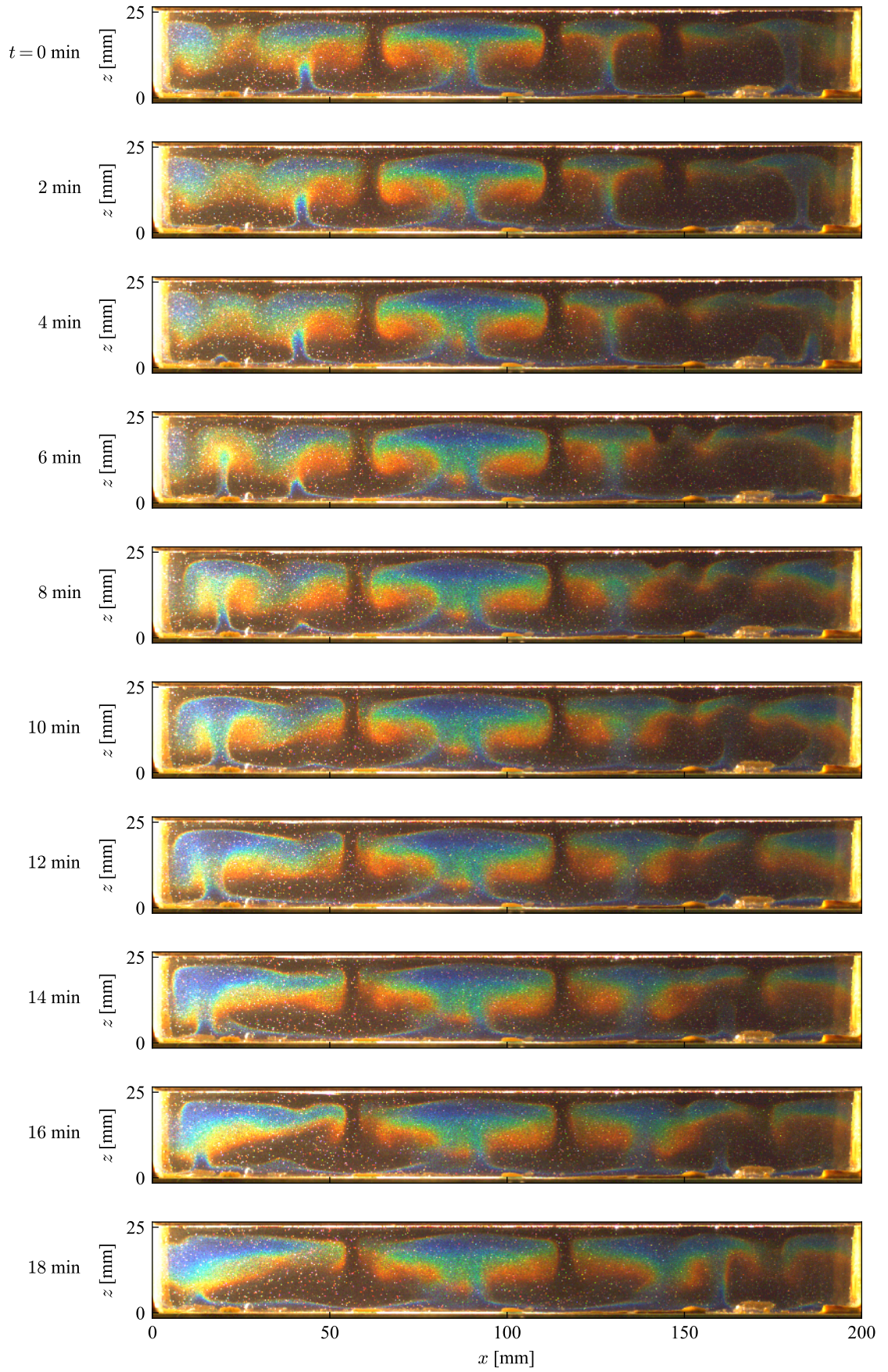


Figure D.10: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 2.6 \times 10^5$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

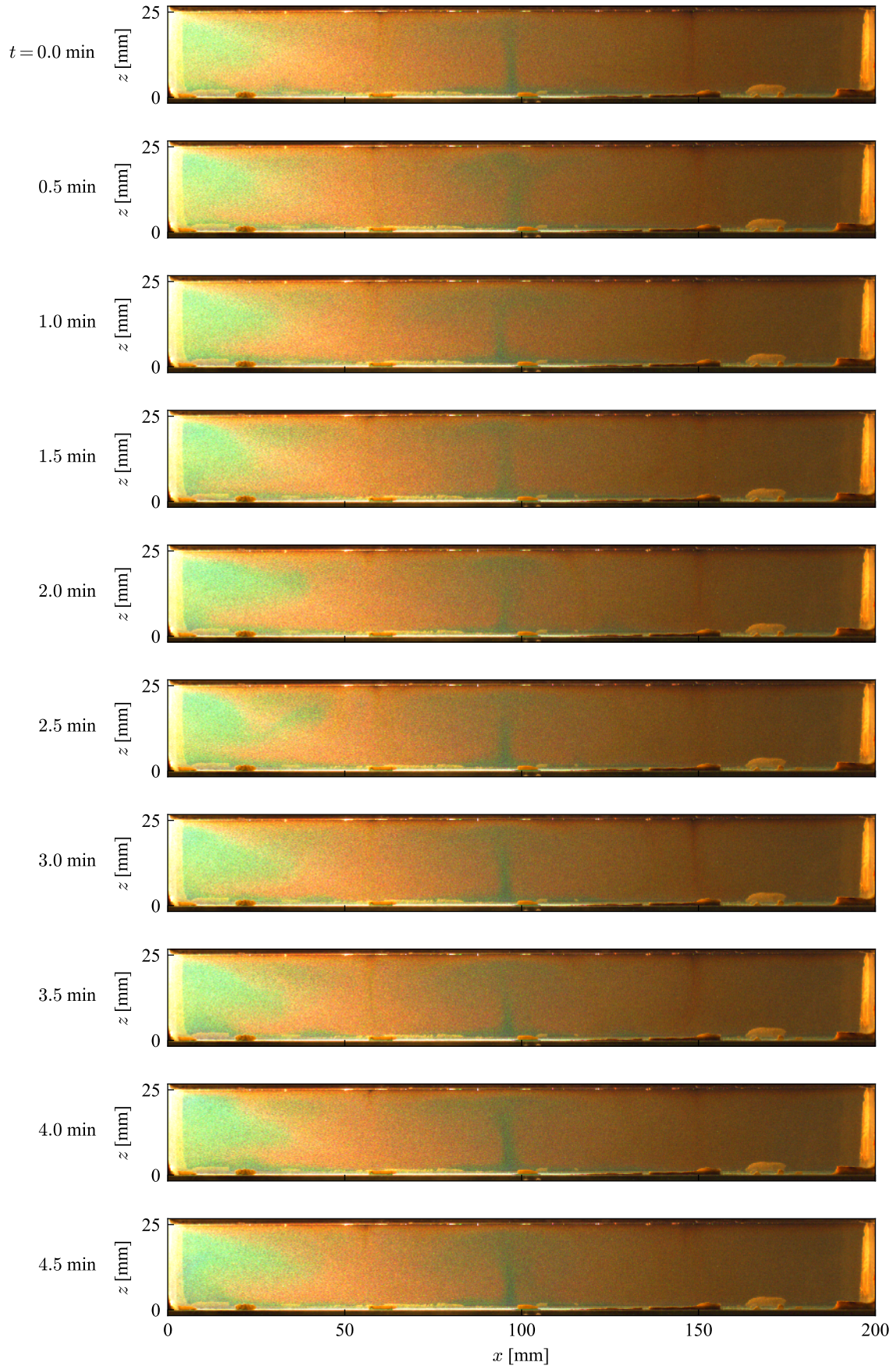


Figure D.11: Temporal evolution of RBC for the case of $Pr \approx 7$ and $Ra \approx 2.2 \times 10^5$ utilizing water as the test fluid. Color contrast is adjusted for the sake of visibility.

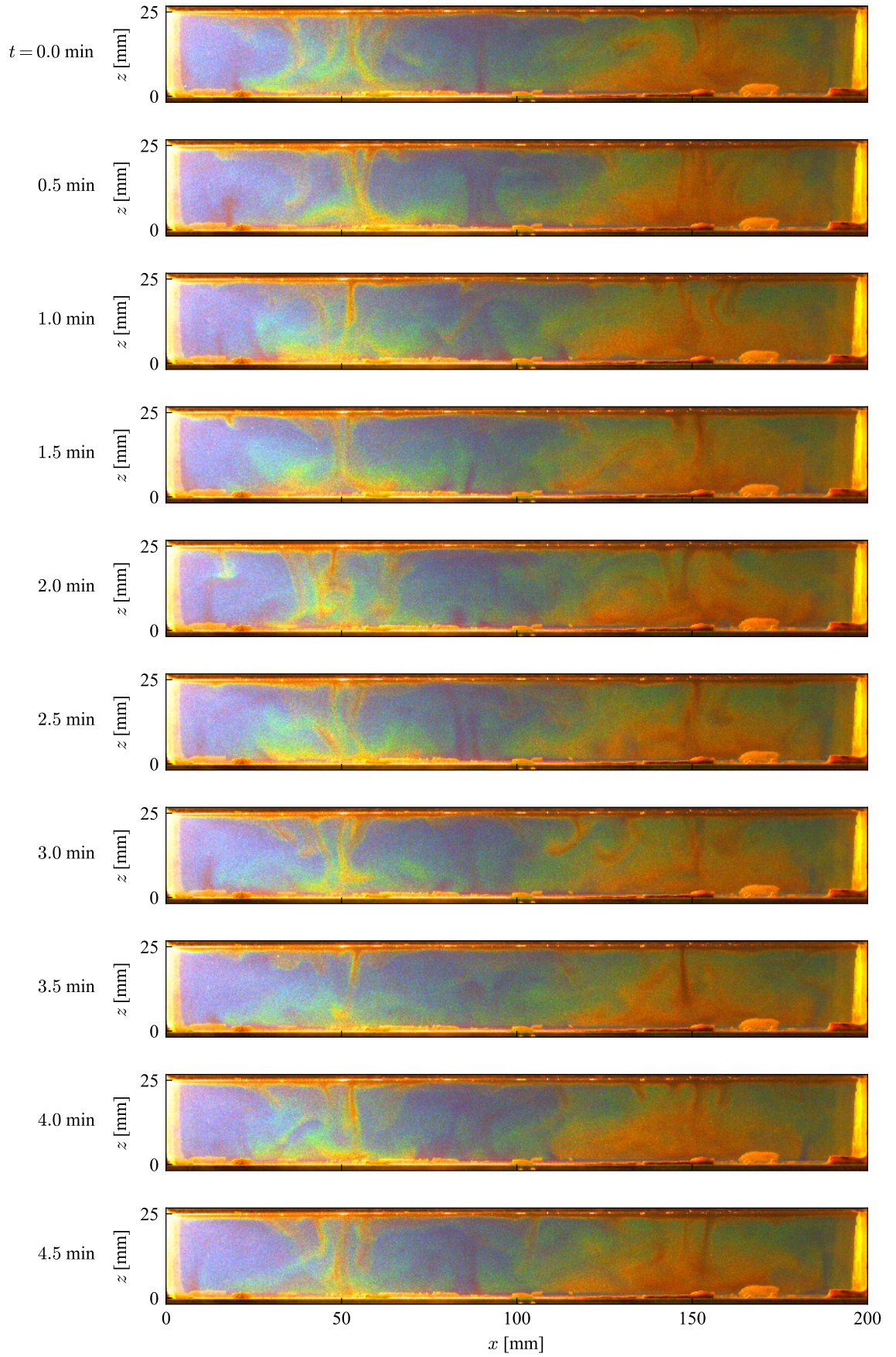


Figure D.12: Temporal evolution of RBC for the case of $Pr \approx 7$ and $Ra \approx 2.2 \times 10^6$ utilizing water as the test fluid. Color contrast is adjusted for the sake of visibility.

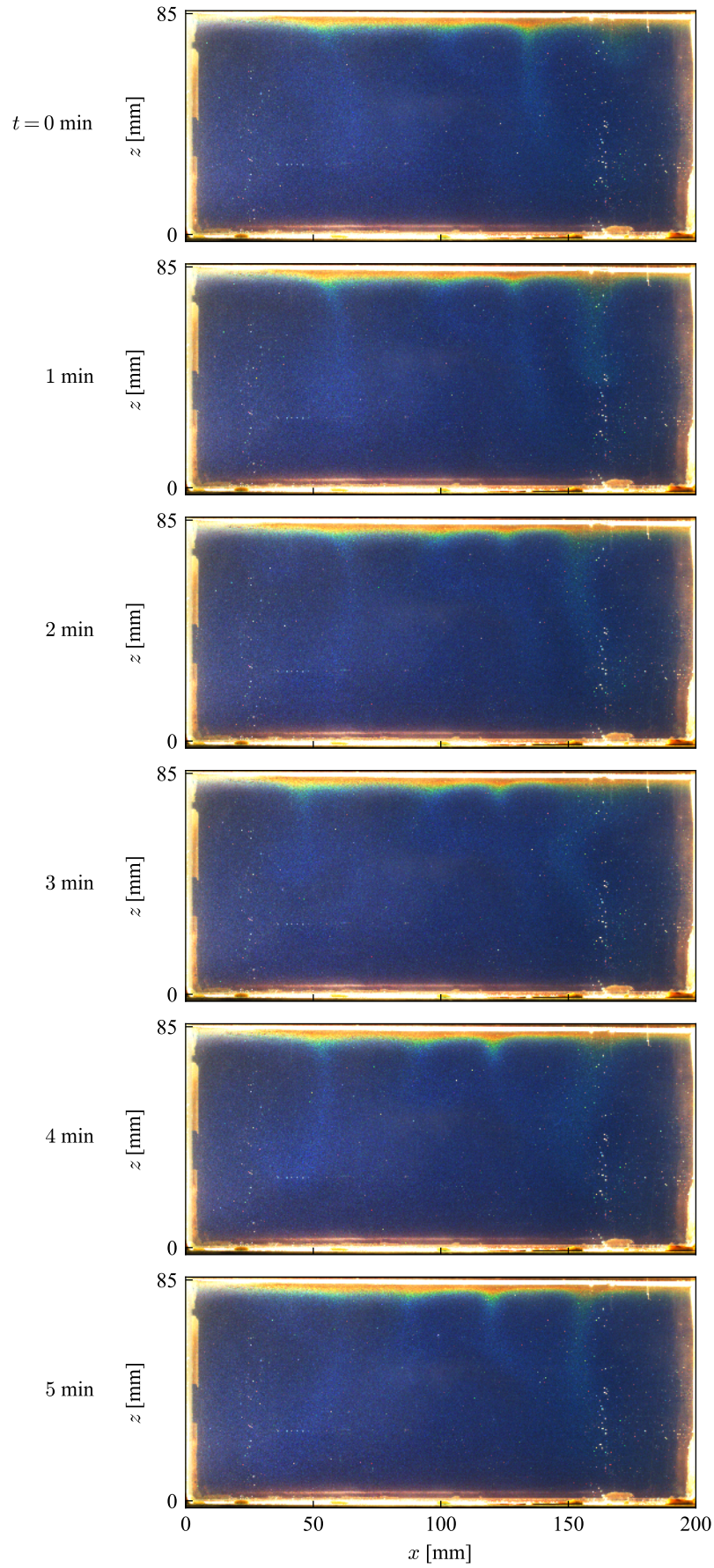


Figure D.13: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 1.2 \times 10^6$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

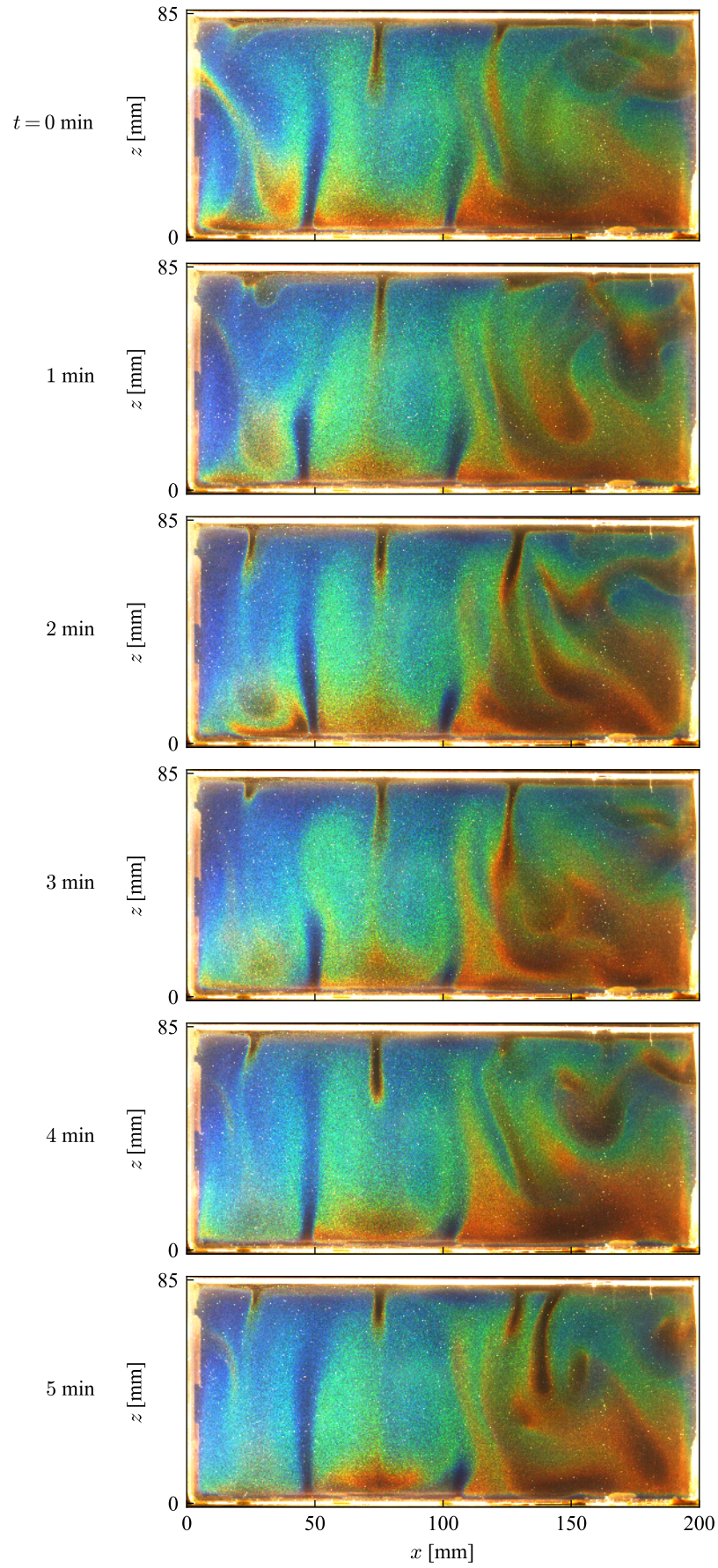


Figure D.14: Temporal evolution of RBC for the case of $Pr \approx 677$ and $Ra \approx 1.0 \times 10^7$ utilizing 80-vol.% GS as the test fluid. Color contrast is adjusted for the sake of visibility.

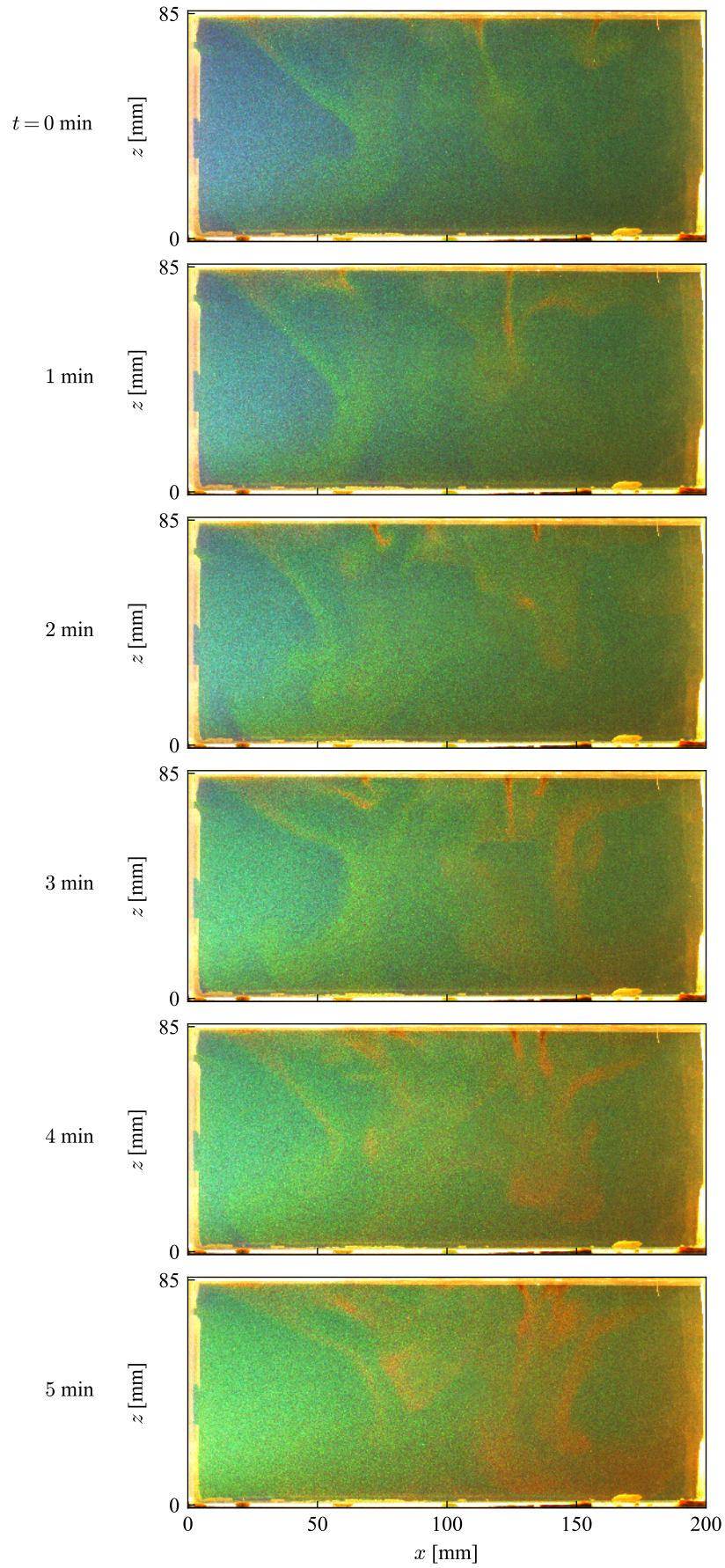


Figure D.15: Temporal evolution of RBC for the case of $Pr \approx 7$ and $Ra \approx 1.7 \times 10^7$ utilizing water as the test fluid. Color contrast is adjusted for the sake of visibility.

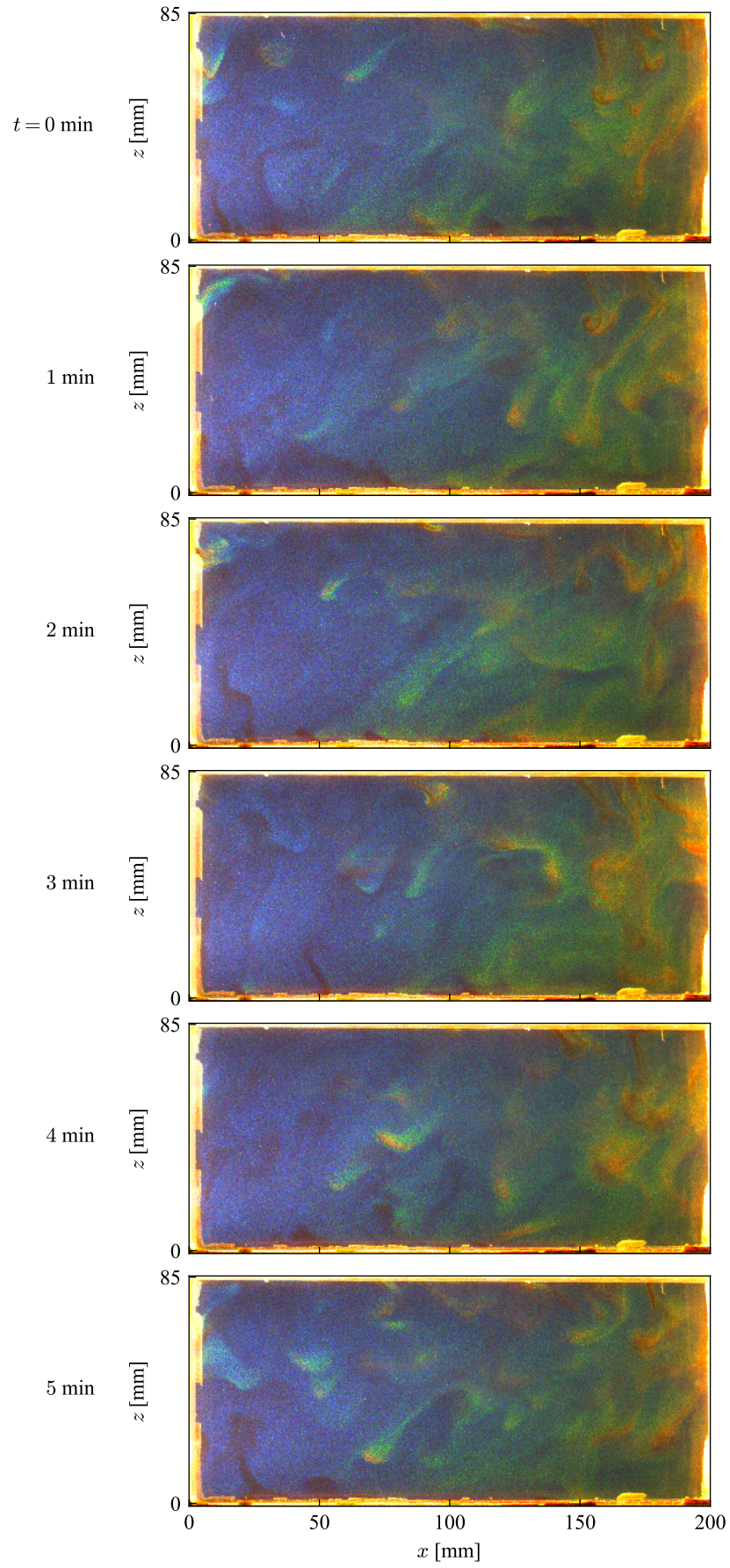


Figure D.16: Temporal evolution of RBC for the case of $Pr \approx 7$ and $Ra \approx 1.7 \times 10^8$ utilizing water as the test fluid. Color contrast is adjusted for the sake of visibility.

As technical notes, tips for the visualization of GS with the TLC particles are provided. For the case of water employed as the test fluid, deaeration is a must process before measurements. This is because air dissolved into water gradually appears as bubbles due to the temperature change. The emergence of bubble disturbs both optical visualization and proper heat transport from the top and bottom walls on which a temperature difference is imposed. The deaeration of water can be achieved easily by boiling water, however, GSs cannot be boiled in a standard laboratory environment. Thus, deaeration of GS should be achieved by decompression with using a vacuum compressor and a vacuum vessel. A point that one needs to take special care of is the seeding process of the TLC particles. The TLC particles are slurry, and thus need to be stirred in the test fluids to disperse homogeneously into the fluids. If the seeding is done for the GSs, bubbles will be generated by entraining air from the gas-liquid interface due to the stir. These bubbles hardly evacuate from the fluids due to the viscosity, and necessarily to be eliminated after the seeding process. For the case of water, this process does not require any special cares since water is not viscous enough to capture such bubbles, and the TLC particles are easily dispersed homogeneously. Incorporating these facts, one is required to seed the TLC particles into deaerated water first. Then, the GSs should be prepared using the water and pure glycerol. As the final step, the GS seeded the TLC particles should be deaerated using the vacuum pump.

D.2 Rotating Rayleigh–Bénard convection

The second example is rotating RBC which has been discussed in Chapter 3 and 4 in Part III. The fluid container for generating RBC utilized here is the same as that used in Chapter 4 in Part III. The top of the fluid layer can be cooled by circulating water supplied from a thermostatic bath. The bottom of the fluid layer can be heated by supplying electric current to the silicon rubber heater embedded at the bottom through a copper plate. The fluid layer and optical equipment were fixed on a turntable, and photographing in the rotational system can be performed. The dimension of the rectangular fluid container is $120 \times 120 \times 100 \text{ mm}^3$ as illustrated in Fig. D.17. In addition to the rectangular type, two different cylindrical geometries were studied here. One has a diameter of 100 mm and the other has a diameter of 50 mm, and these dimensions are illustrated in Fig. D.18 and D.19. These two different cylindrical fluid layers were formed by inserting acrylic annulus into the rectangular vessel. Thus, the height of these three fluid layers was common, $H = 100 \text{ mm}$. For convenience, the two cylindrical fluid layers are called fat cylinder and slender cylinder according to the different diameters. Please note that the rotational axis coincided to the center of the fluid layers, $(x, y) = (0 \text{ mm}, 0 \text{ mm})$.

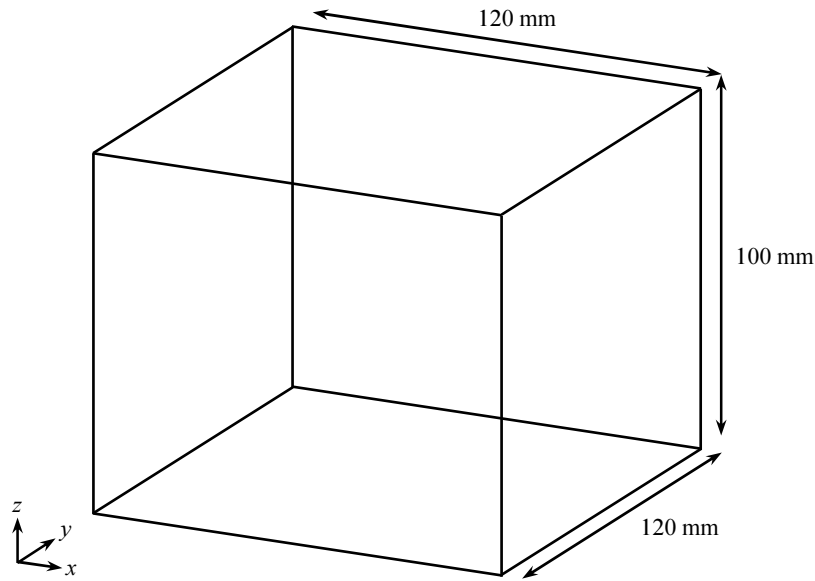


Figure D.17: Geometry of the rectangular fluid layer with a size of $120 \times 120 \times 100 \text{ mm}^3$.

Experiments were done by employing water as the test fluid. For visualization, the TLC particles with an effective coloring temperature range of $20\text{--}25^\circ\text{C}$ were seeded into deaerated water. For all the fluid layers, three different experimental conditions were performed, and visualization was conducted at two different cross-sections for each experimental condition. The conditions are summarized in Table D.2. The top and bottom temperatures were fixed for all the experimental conditions at $T_{\text{top}} = 18^\circ\text{C}$ and $T_{\text{btm}} = 23^\circ\text{C}$ resulting in a vertical temperature difference of $\Delta T = 5 \text{ K}$ with the Prandtl number of $Pr \sim 7$. The Ra number corresponded to $Ra = 7.0 \times 10^7$. For all the fluid layers, non-rotating case and two rotating cases with the Taylor

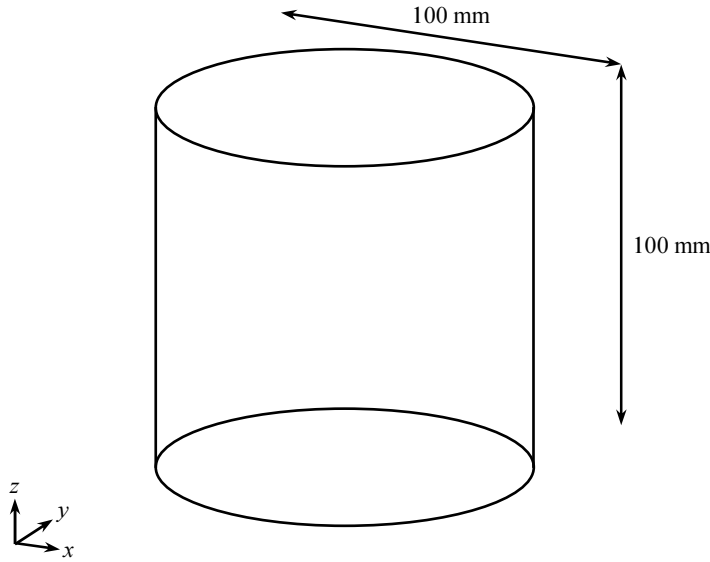


Figure D.18: Geometry of the fat cylindrical fluid layer with a size of 100 mm in diameter and 100 mm in height.

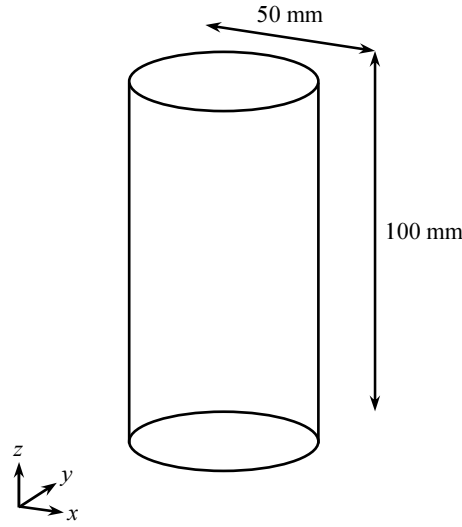


Figure D.19: Geometry of the slender cylindrical fluid layer with a size of 50 mm in diameter and 100 mm in height.

numbers $Ta = (2\Omega H^2/\nu)^2$ of $Ta = 1.0 \times 10^8$ and 1.0×10^9 were visualized. The convective Rossby numbers, $Ro = \sqrt{Ra/(Pr Ta)}$, are $Ro = \infty, 0.32$, and 0.10 , respectively. Lighting was performed using the white light sheet generated by the halogen lamp and the cylindrical lens connected by the linear light guide, as similar to the cases shown in Appendix D.1. The visualized cross-sections were the x - y plane at $z = 95$ mm (close to the top surface) and the x - z plane at $y = 0$ mm (center). Because of the mechanical limitations in installation of the optical equipment, the two different visualization planes for each condition were performed individually. The photographing and rotation was initiated at the equilibrated states of non-rotating RBC.

The results of the visualization experiments are shown in Fig. D.20 to D.37. The time elapsed from the start of rotation are denoted above each subfigure. The experimental conditions

Table D.2: Experimental conditions for visualization of rotating RBC utilizing the TLC particles.

Figure	Geometry	Visualization	Ra [-]	Ta [-]	Ro [-]
D.20	Rectangular	x - y plane at $z = 95$ mm	7.0×10^7	0	∞
D.21	Rectangular	x - z plane at $y = 0$ mm	7.0×10^7	0	∞
D.22	Rectangular	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^8	0.32
D.23	Rectangular	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^8	0.32
D.24	Rectangular	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^9	0.10
D.25	Rectangular	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^9	0.10
D.26	Fat cylinder	x - y plane at $z = 95$ mm	7.0×10^7	0	∞
D.27	Fat cylinder	x - z plane at $y = 0$ mm	7.0×10^7	0	∞
D.28	Fat cylinder	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^8	0.32
D.29	Fat cylinder	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^8	0.32
D.30	Fat cylinder	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^9	0.10
D.31	Fat cylinder	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^9	0.10
D.32	Slender cylinder	x - y plane at $z = 95$ mm	7.0×10^7	0	∞
D.33	Slender cylinder	x - z plane at $y = 0$ mm	7.0×10^7	0	∞
D.34	Slender cylinder	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^8	0.32
D.35	Slender cylinder	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^8	0.32
D.36	Slender cylinder	x - y plane at $z = 95$ mm	7.0×10^7	1.0×10^9	0.10
D.37	Slender cylinder	x - z plane at $y = 0$ mm	7.0×10^7	1.0×10^9	0.10

corresponding to these figures are also listed in Table D.2. The images shown in these figures are color pathline compiled by adopting a special image processing scheme as follows. First, background subtraction is performed in the Bayer image format before color demosaicing. The background subtracted images are then stacked pixelwisely along time axis for a short time duration (e.g., 1 s). Then the pixelwise maxima were computed, and the maxima were utilized as the values of the Bayer image of the color pathline. Finally, the color pathline image can be obtained by adopting arbitrary color demosaicing methods to the Bayer image. With these procedures, it is possible to visualize particle streaks representing flow directions and the color pattern of the TLC particles simultaneously, as if the images were taken with a long exposure time.

For non-rotating cases shown in Fig D.20, D.26, and D.32, the sheet-like plume structures can be observed, and the LSC structures can be recognized in Fig D.21, D.27, and D.33. Since the visualization was initiated at the same time as the start of rotation, it is possible to observe both spin-up processes and equilibrated states in Fig. D.22 to D.25, Fig. D.28 to D.31, and Fig. D.34 to D.37. Details of the mechanism of the stepwise transitions observed during spin-up of rotating RBC have been discussed in part III, Chapter 3, and only visualized images are shown here. At the equilibrated states, columnar vortices, the vortical plumes and the convective Taylor columns, dominate the whole region of the fluid layers as expected from the Ro values.

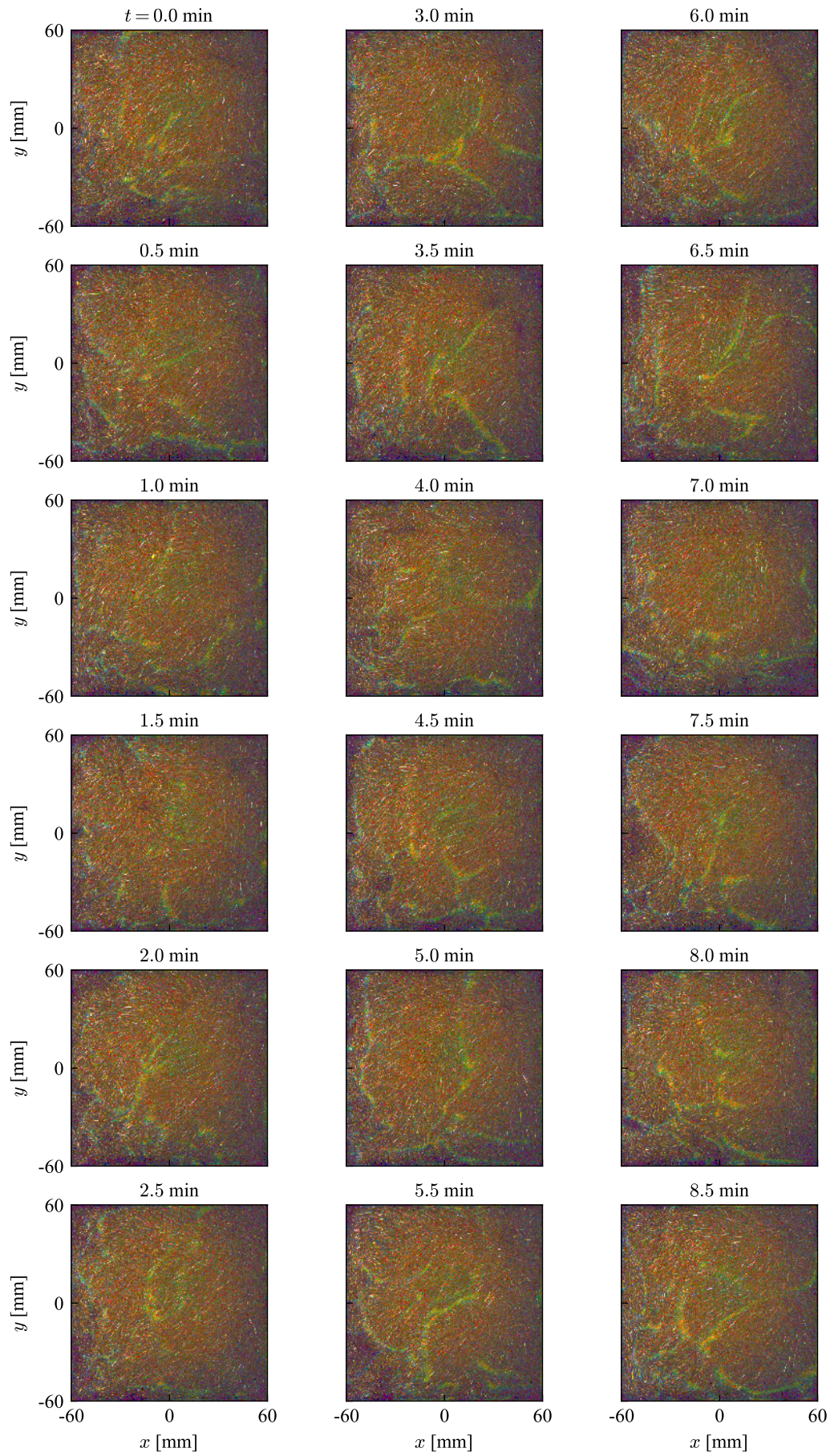


Figure D.20: Temporal evolution of non-rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

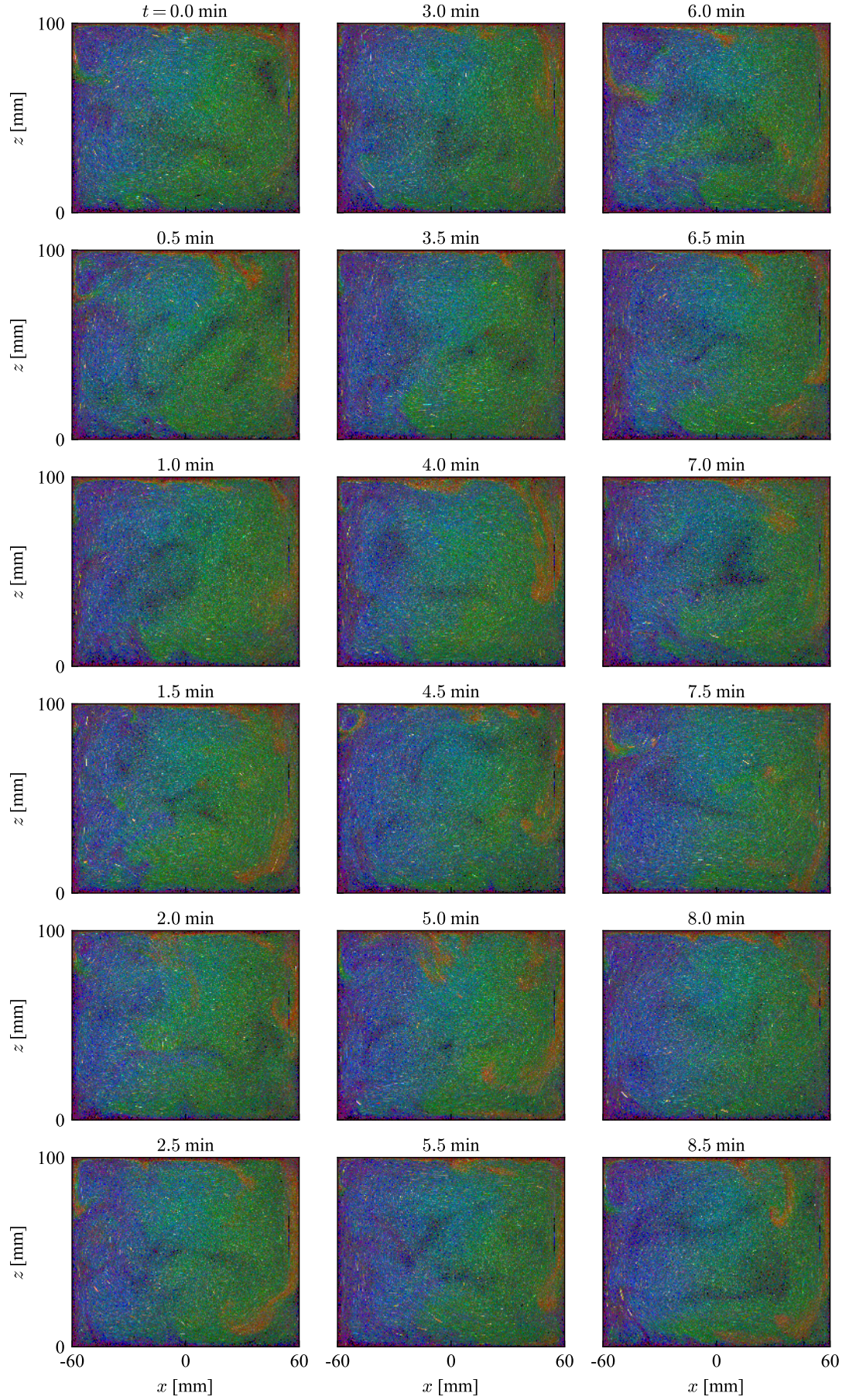


Figure D.21: Temporal evolution of non-rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

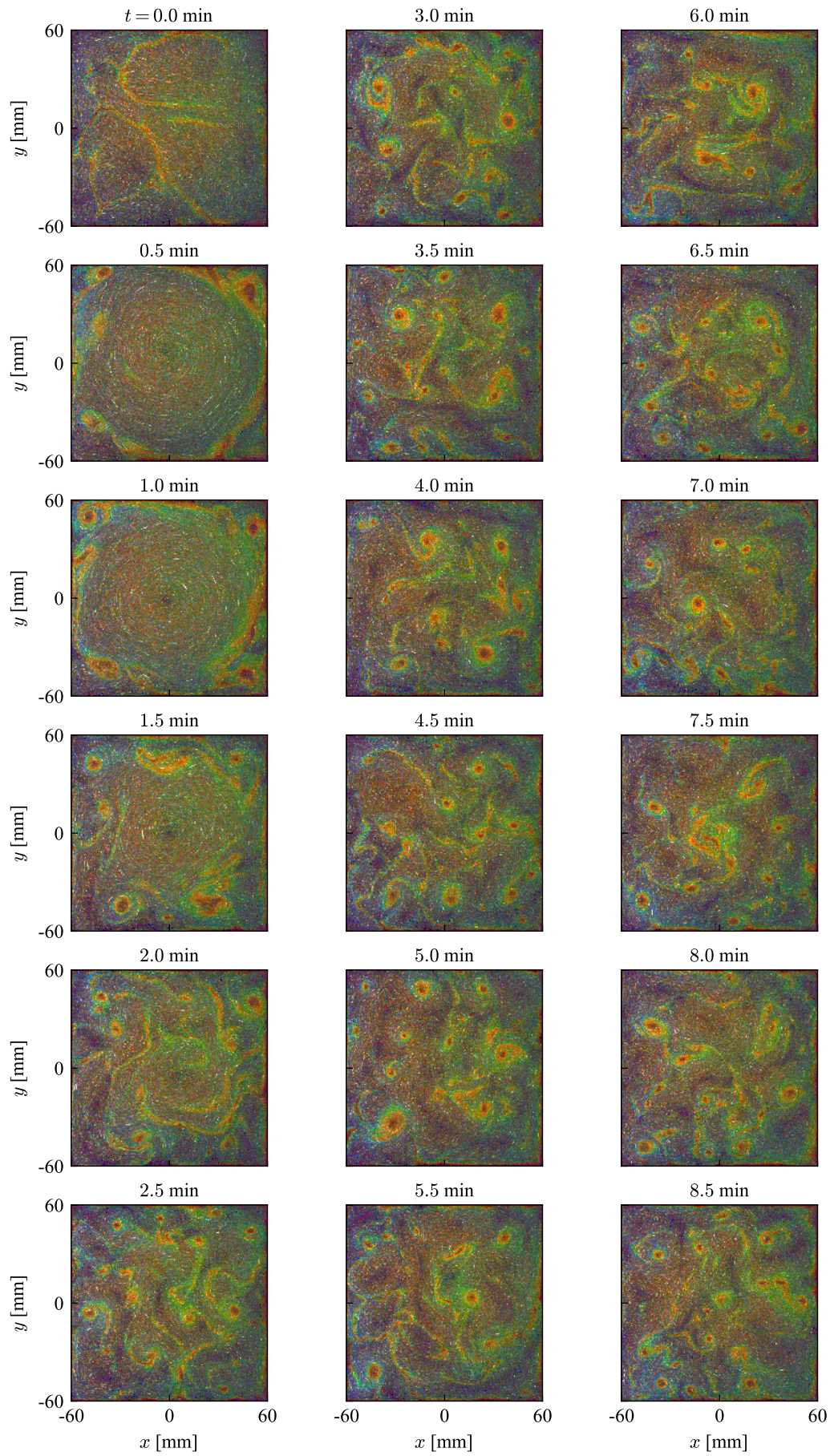


Figure D.22: Temporal evolution of rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

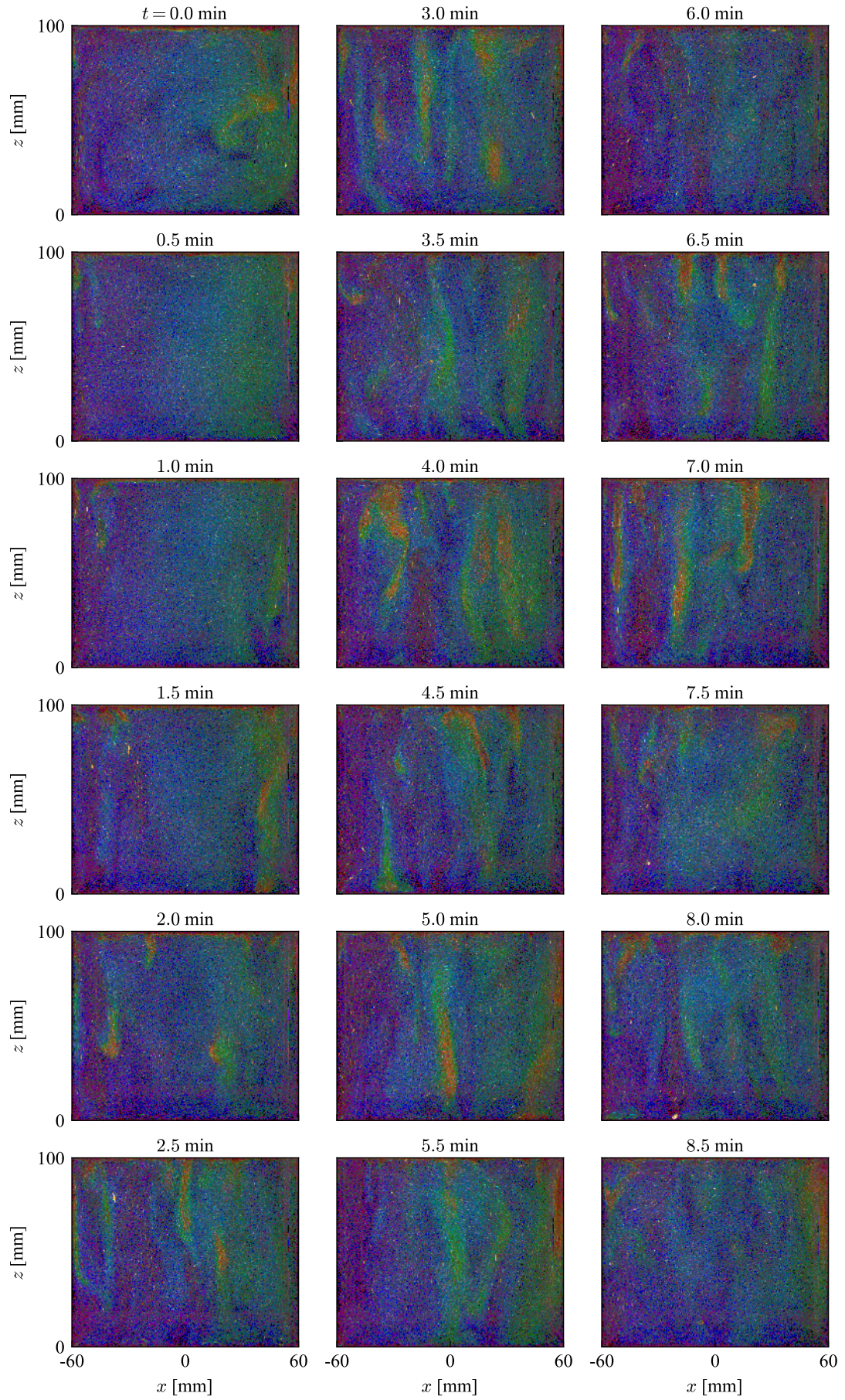


Figure D.23: Temporal evolution of rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

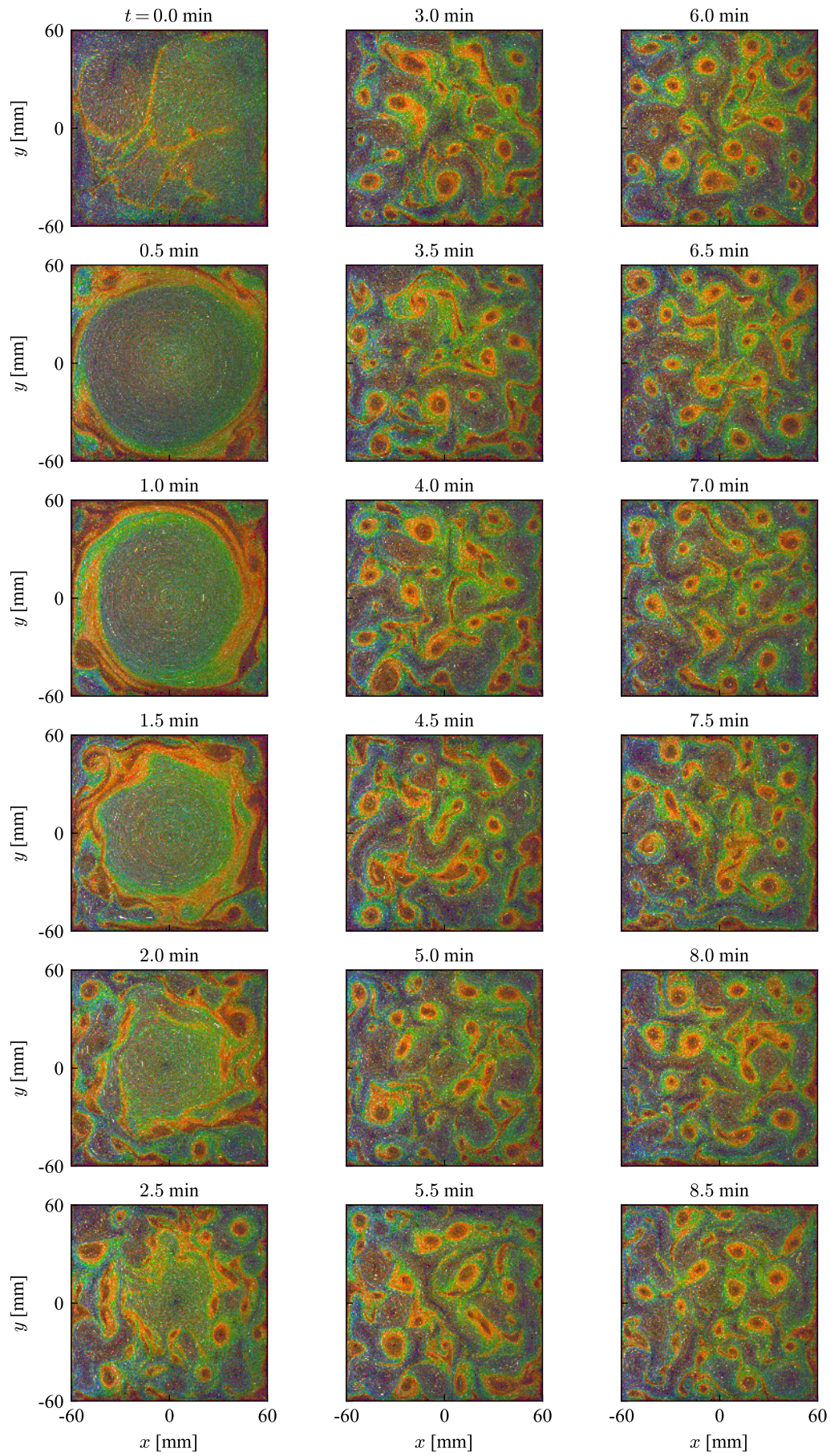


Figure D.24: Temporal evolution of rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

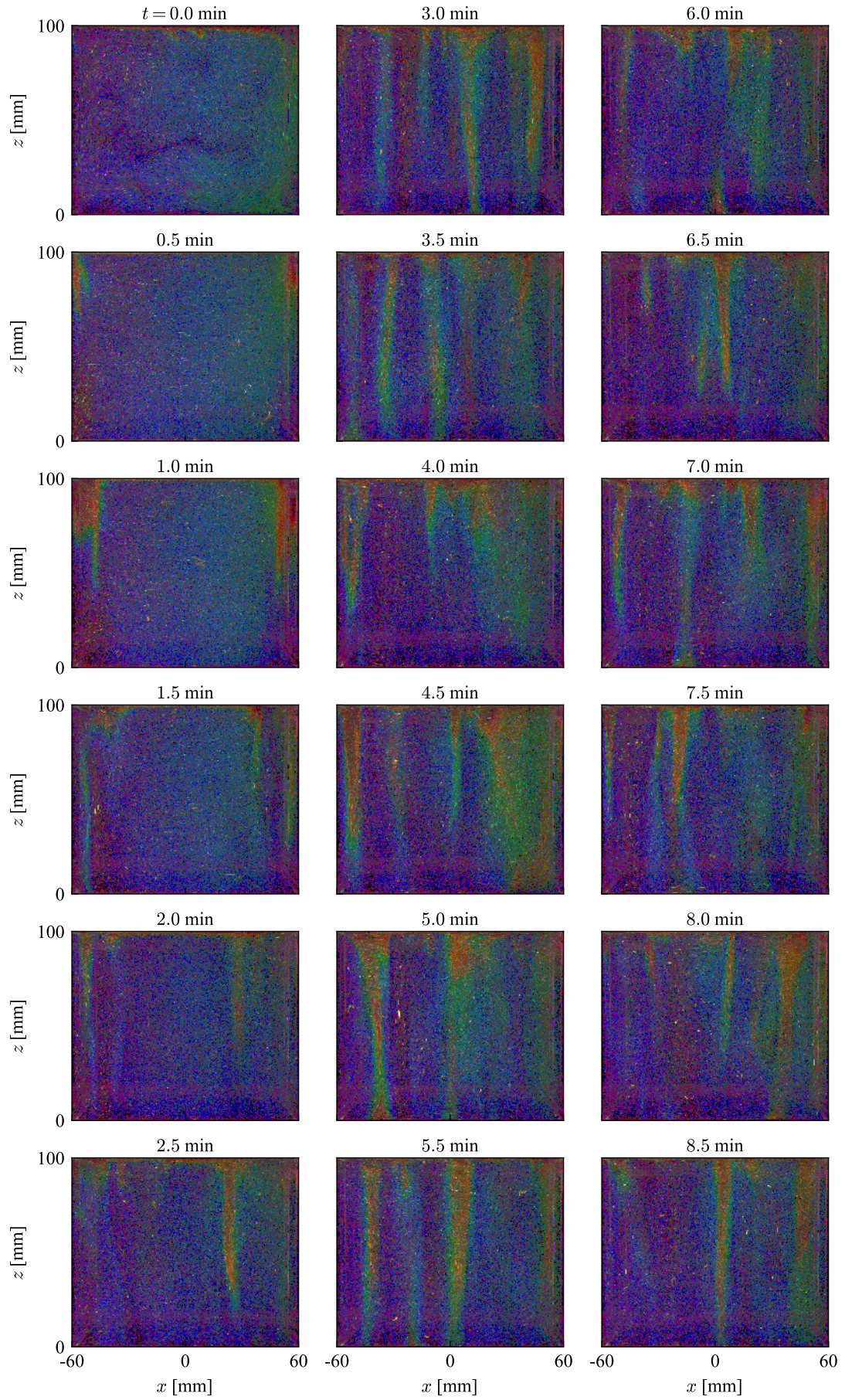


Figure D.25: Temporal evolution of rotating RBC in the rectangular fluid layer for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

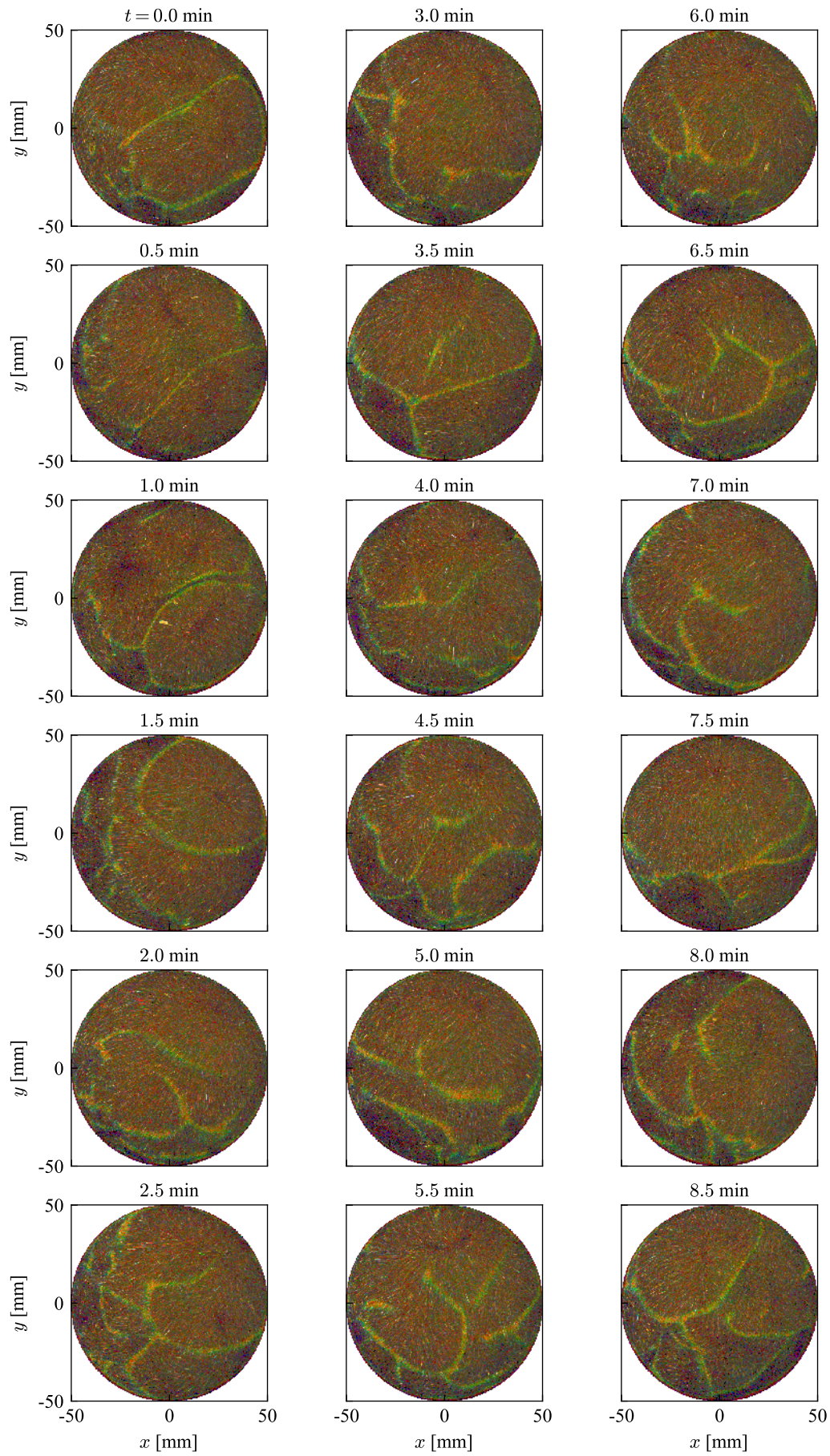


Figure D.26: Temporal evolution of non-rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

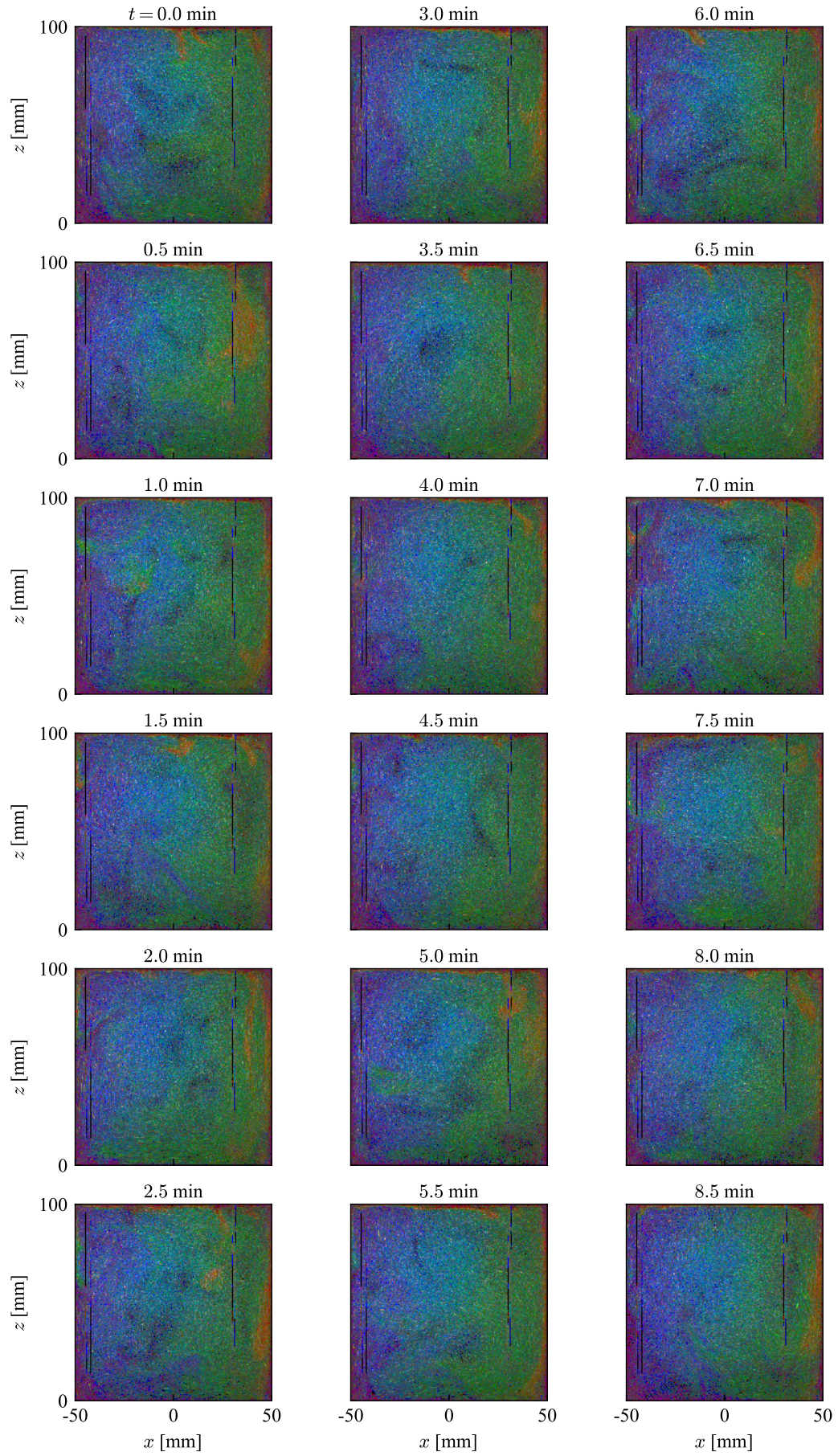


Figure D.27: Temporal evolution of non-rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

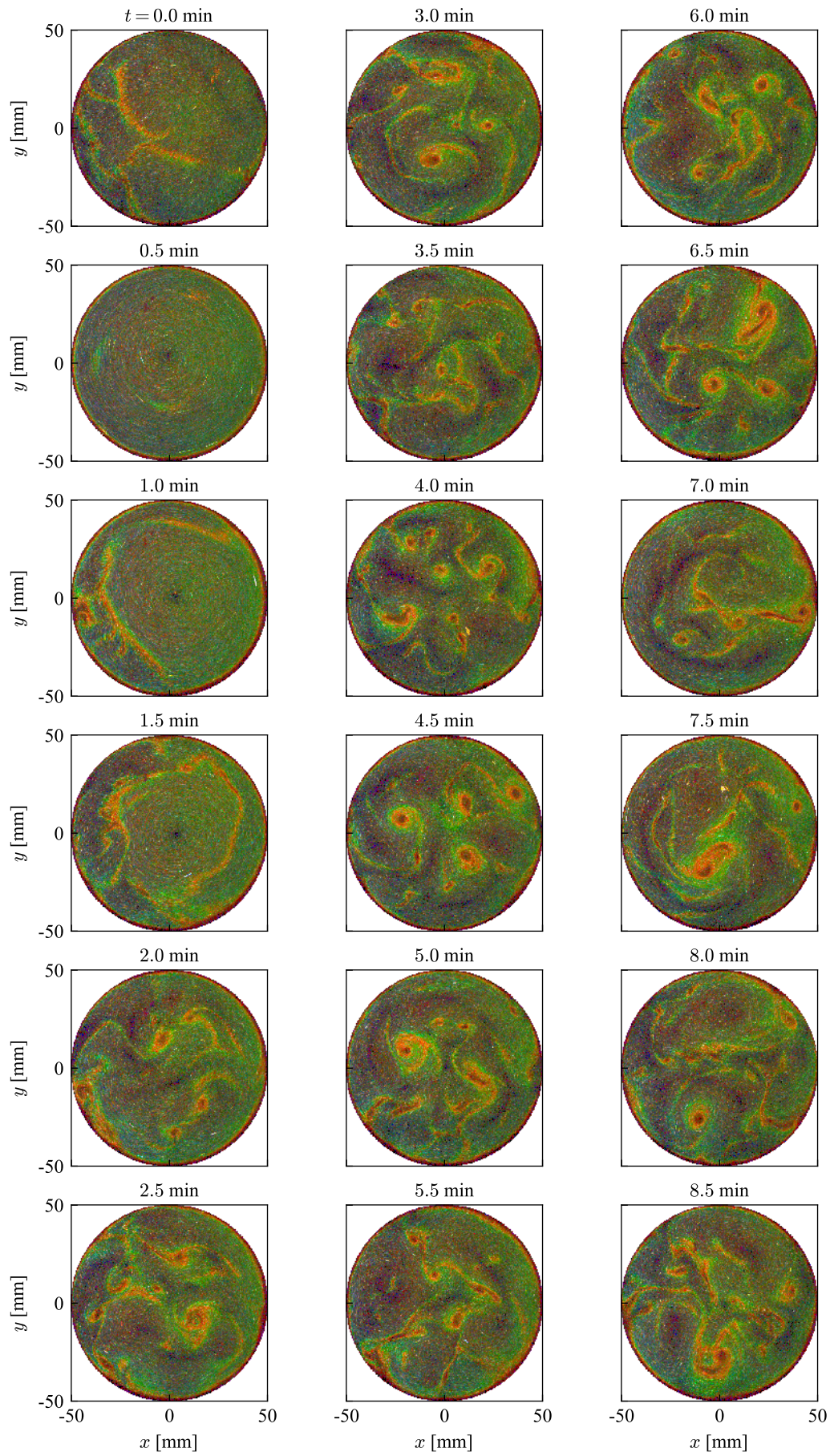


Figure D.28: Temporal evolution of rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

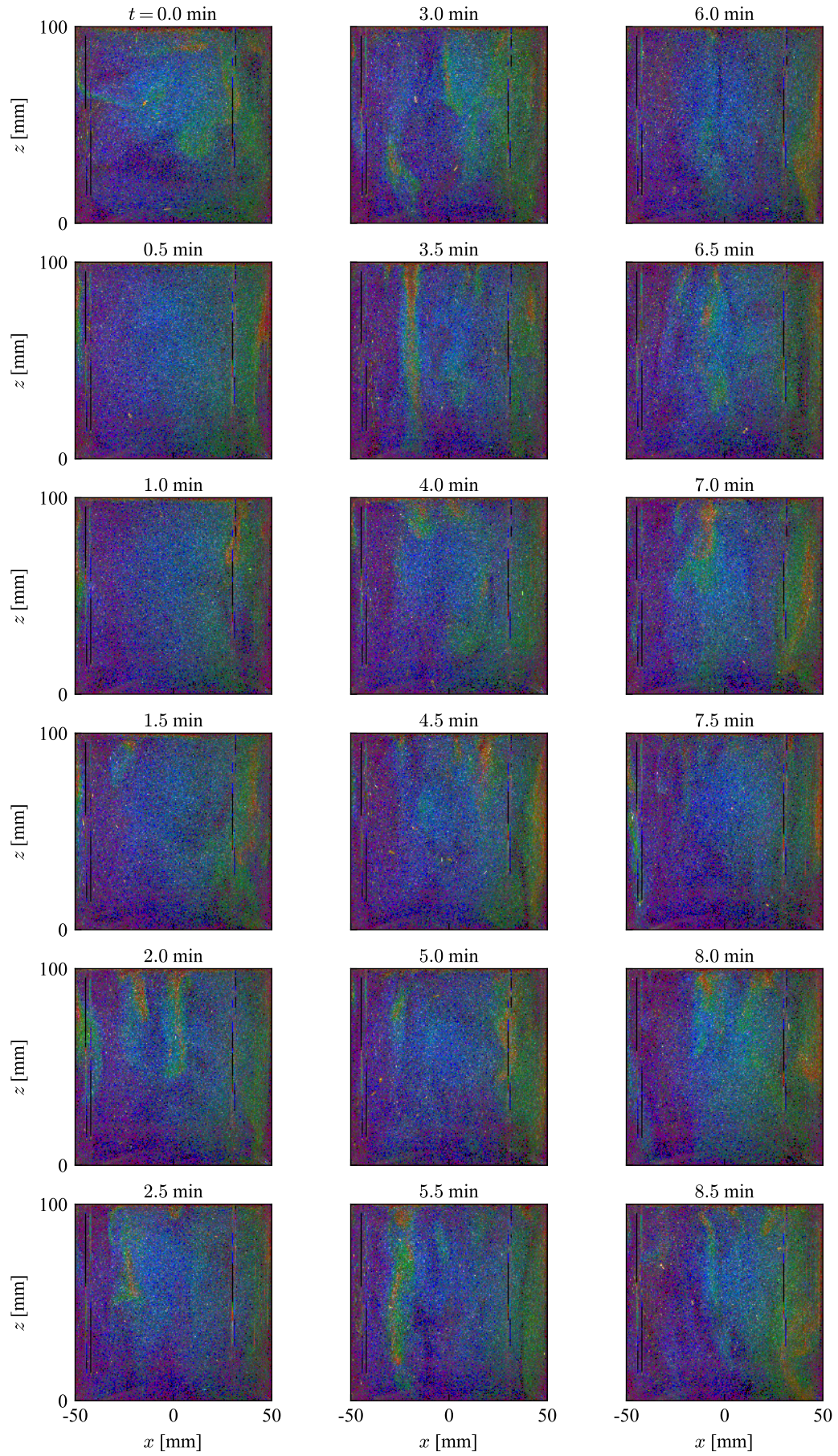


Figure D.29: Temporal evolution of rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

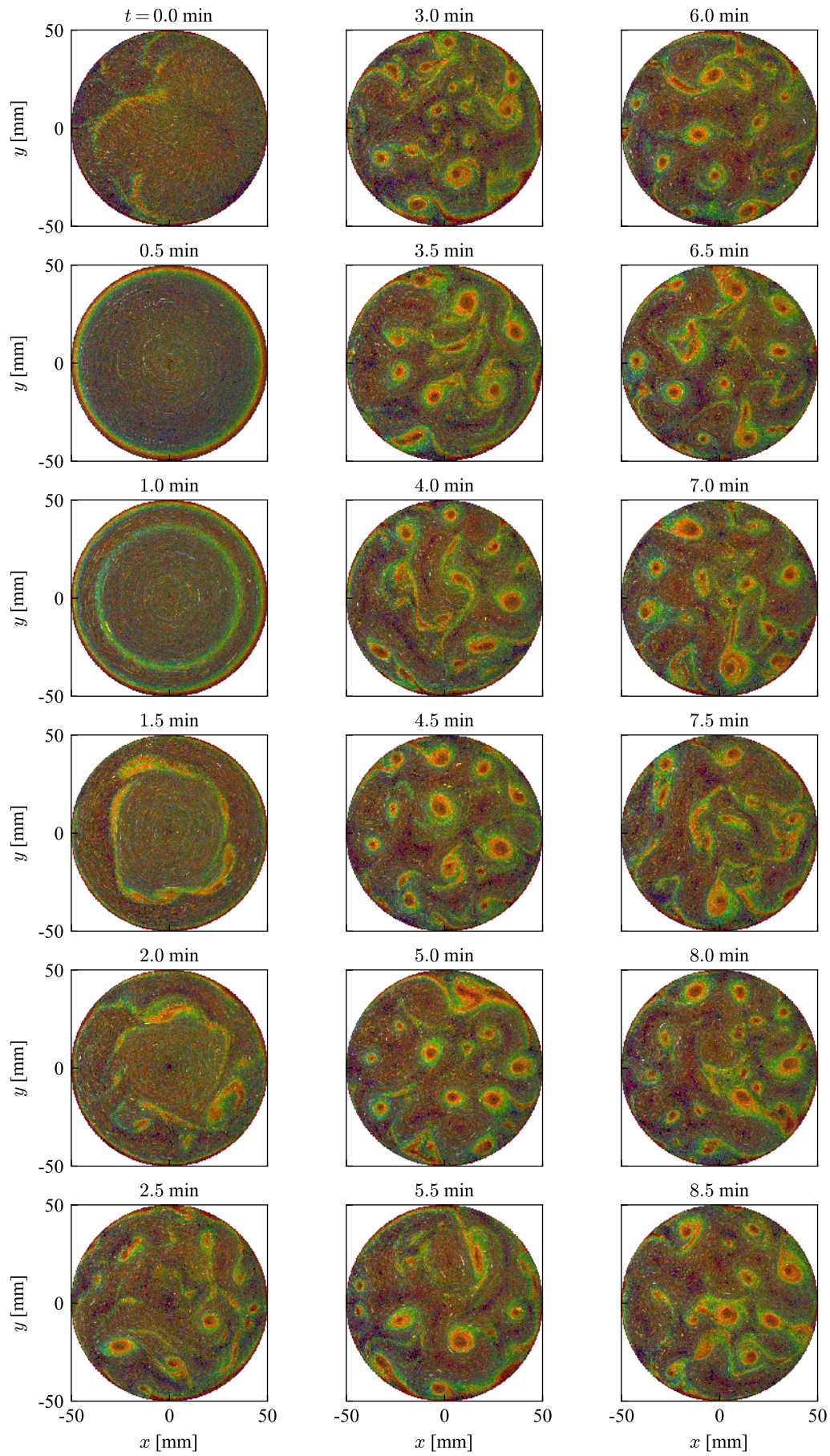


Figure D.30: Temporal evolution of rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

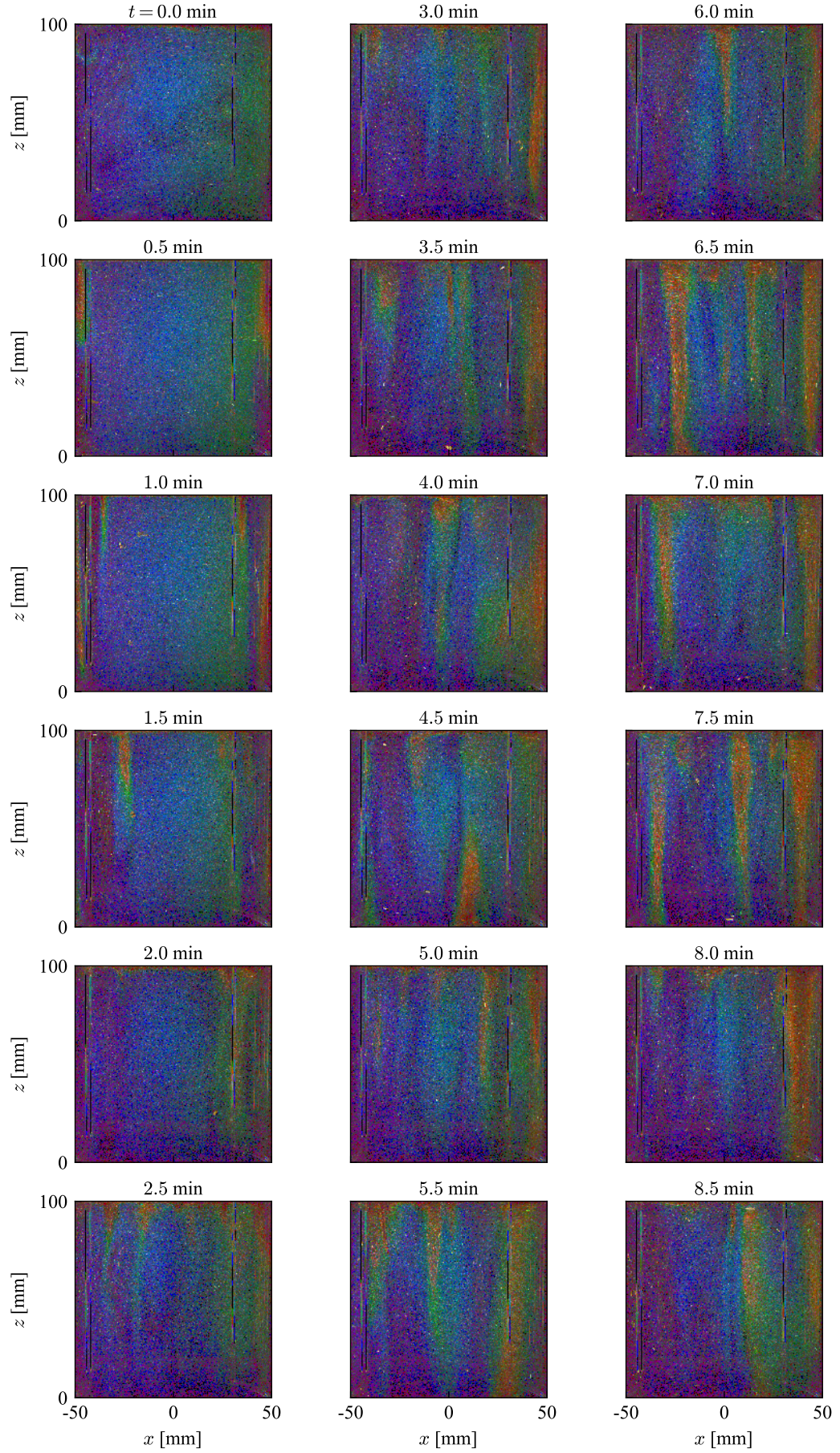


Figure D.31: Temporal evolution of rotating RBC in the fat cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

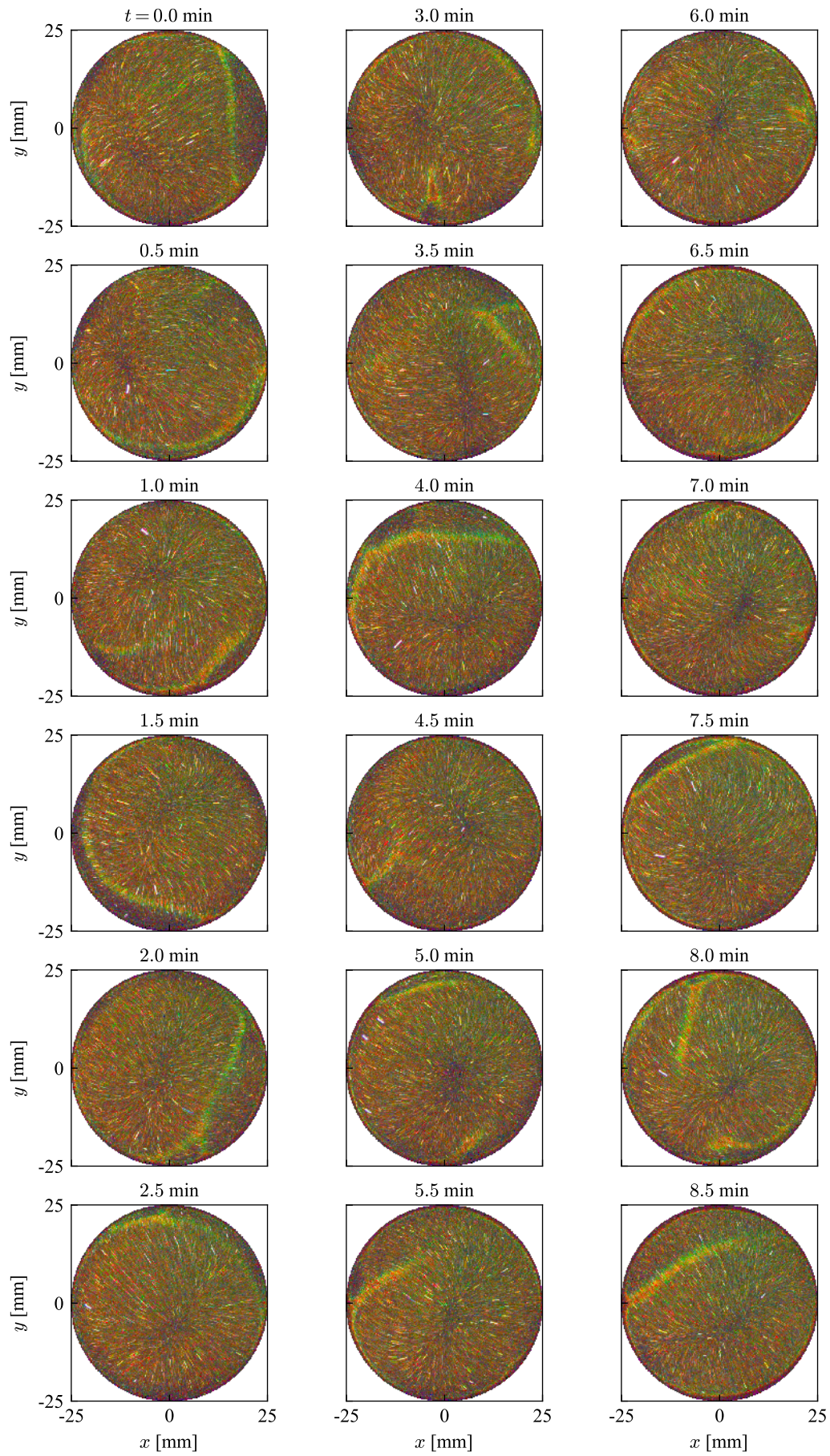


Figure D.32: Temporal evolution of non-rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

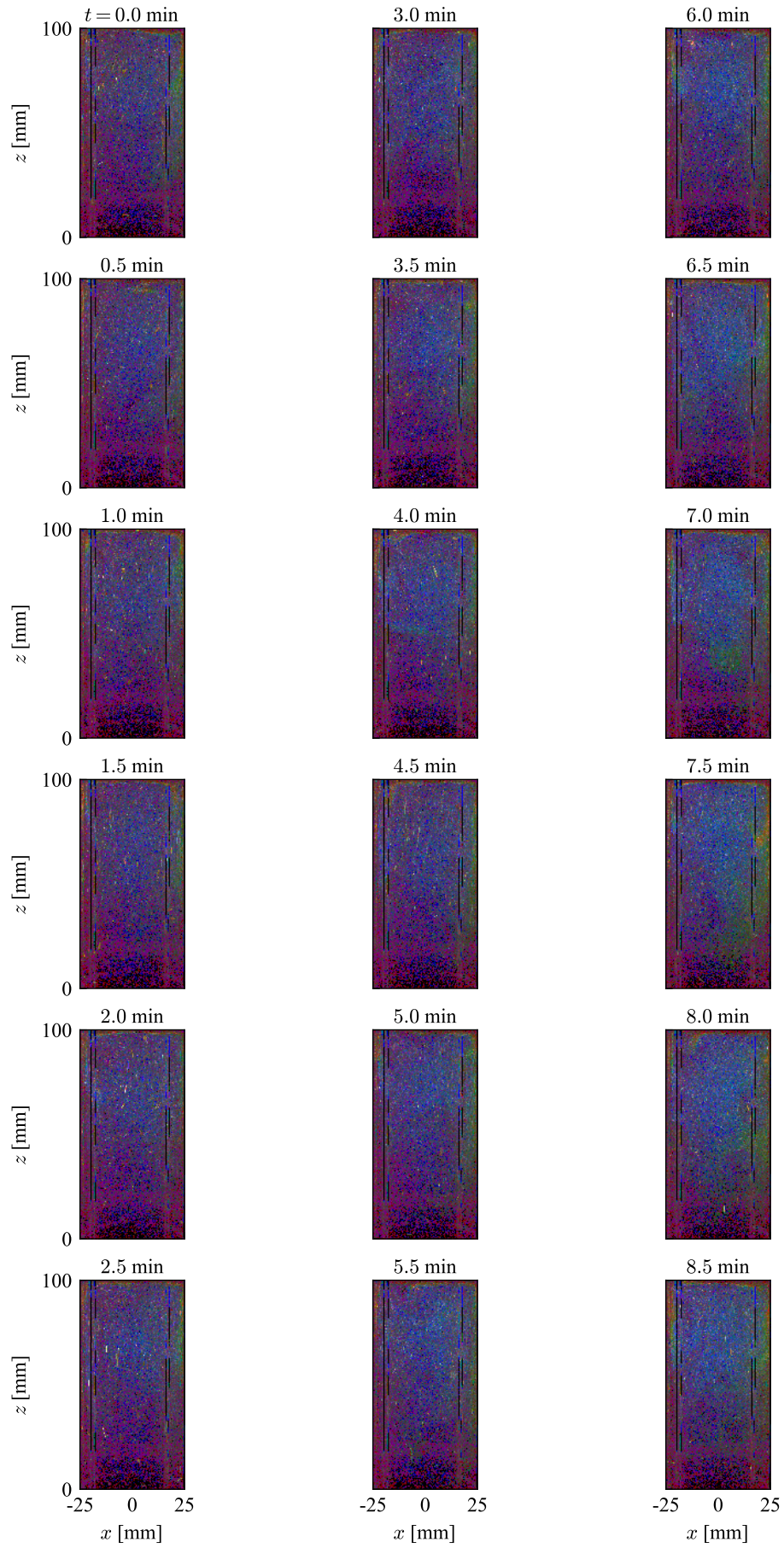


Figure D.33: Temporal evolution of non-rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 0$ ($Ro = \infty$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

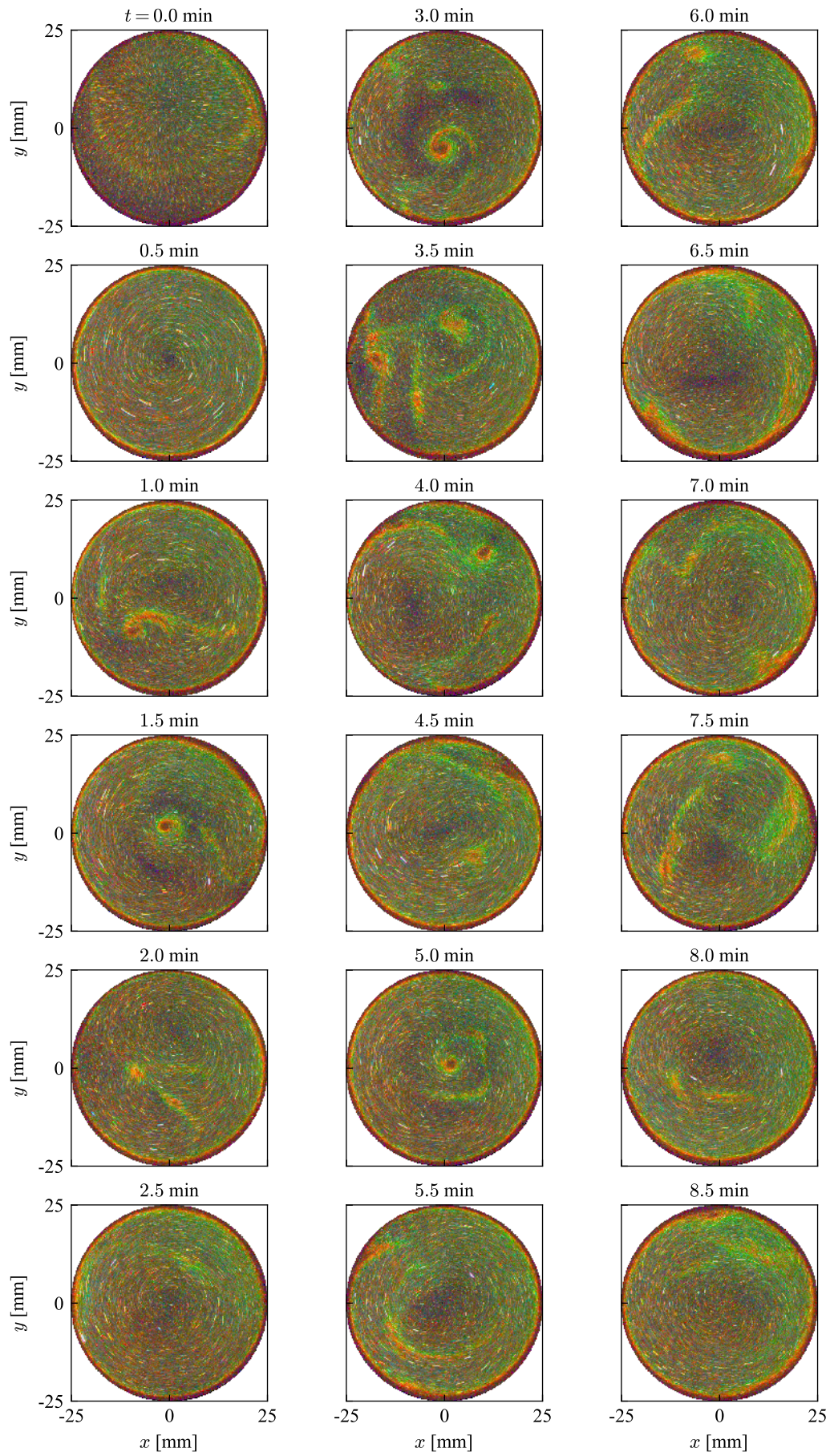


Figure D.34: Temporal evolution of rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

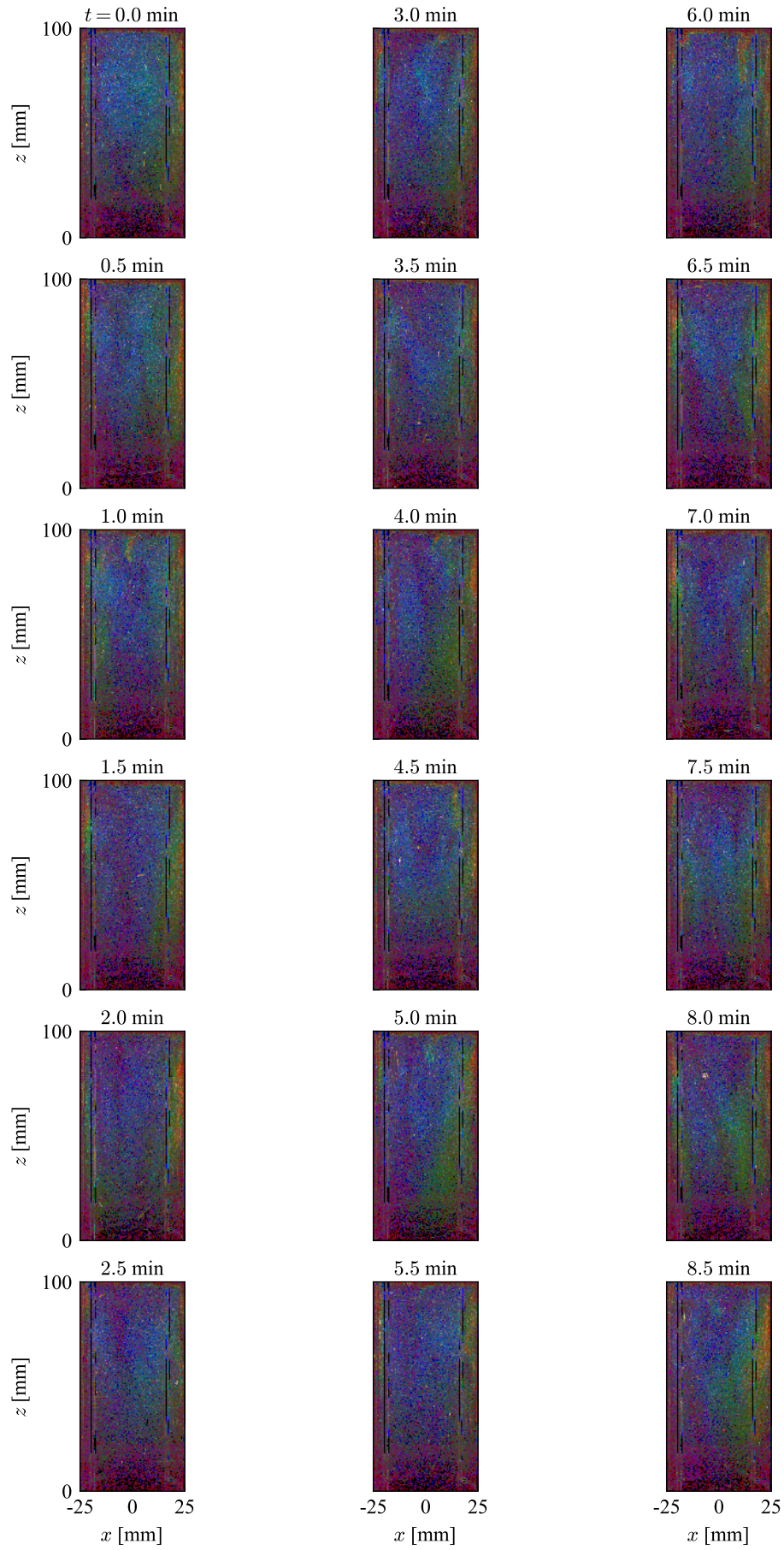


Figure D.35: Temporal evolution of rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.32$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

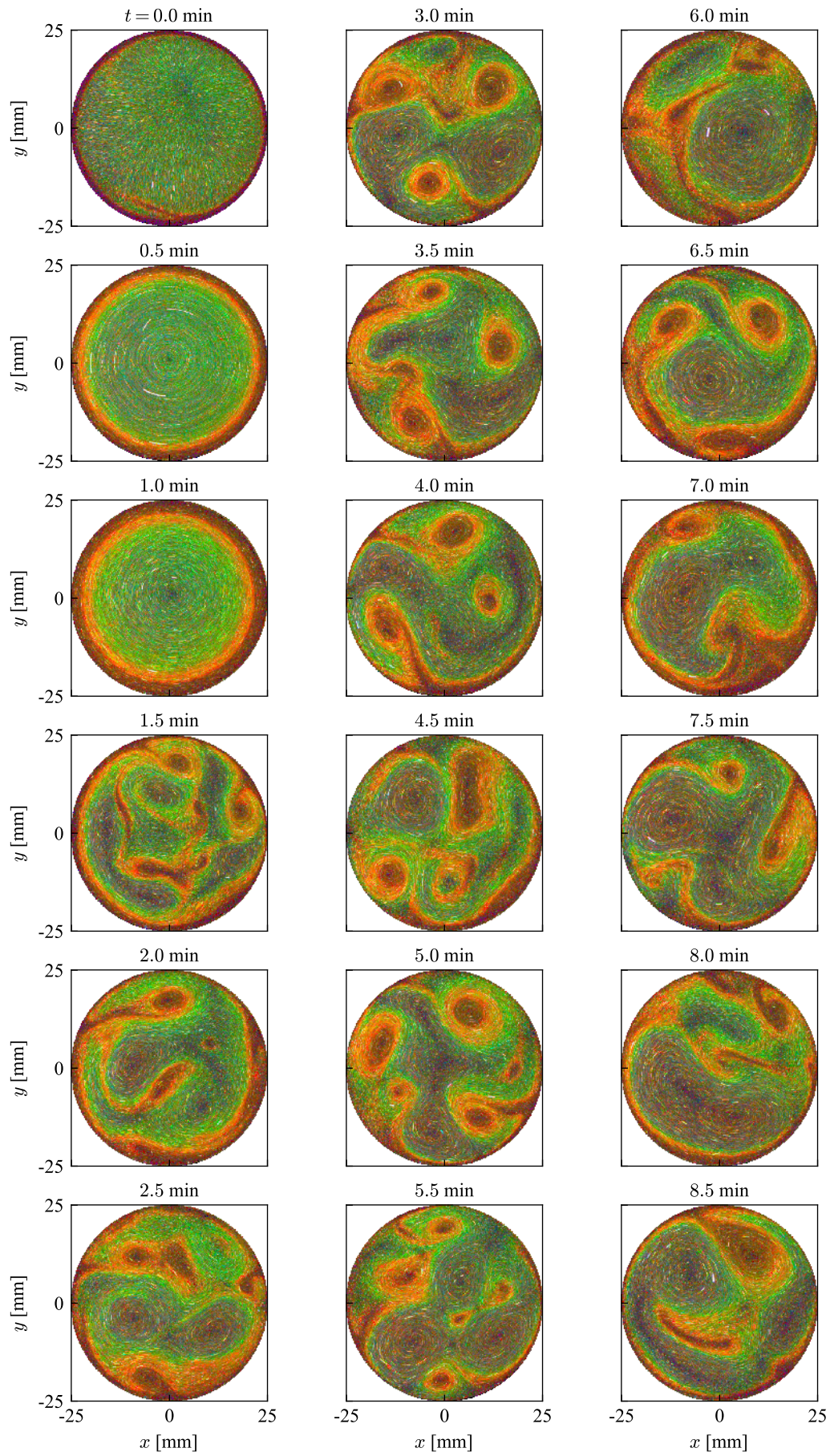


Figure D.36: Temporal evolution of rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $z = 95$ mm. The color contrast is adjusted to enhance visibility.

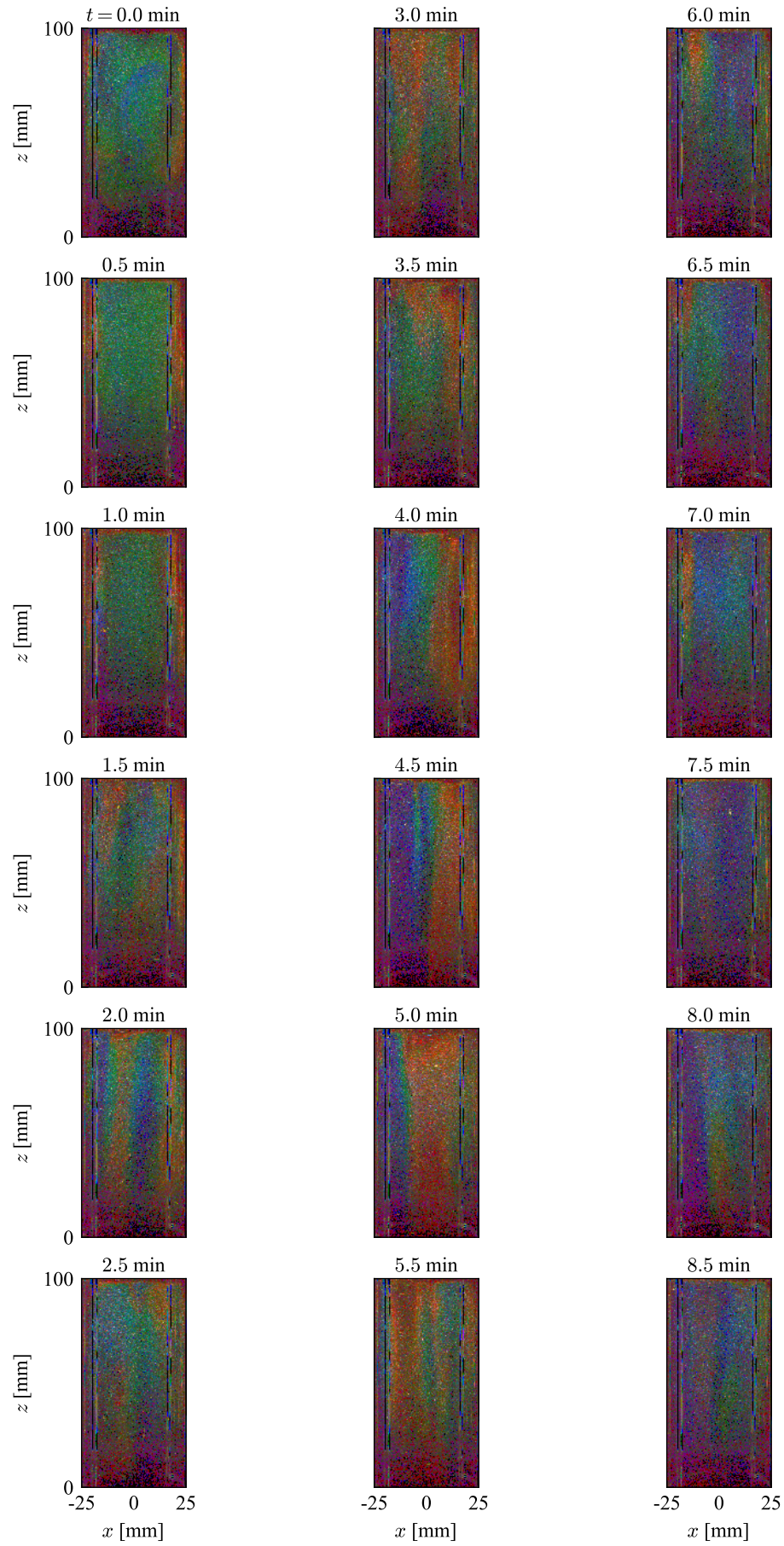


Figure D.37: Temporal evolution of rotating RBC in the slender cylinder for the case with $Ra = 7.0 \times 10^7$ and $Ta = 1.0 \times 10^8$ ($Ro = 0.10$) visualized at $y = 0$ mm. The color contrast is adjusted to enhance visibility.

D.3 Double diffusive convection with crystallization

The third example is double diffusive convection with crystallization. This topic is not directly related to the main contents, however, some typical visualization results are displayed as examples of challenging measurement targets. This project is originally motivated by seeking the source of the magnetic field of Ganymede, the third moon of Jupiter, and has been tacked on as collaborative research with Dr. S. Anders and Dr. S. Eckert in Helmholtz-Zentrum Dresden-Rossendorf, Germany.

The experiments were performed as analogues of the iron-snow regime, which has been considered as the most plausible mechanism for sustaining the magnetic field of Ganymede, by using aqueous ammonium-chloride solution as the test fluid. A Hele-Shaw cell comprised of acrylic walls with a dimension of $200 \times 10 \times 100 \text{ mm}^3$ illustrated in Fig. D.38 was utilized as the fluid container. Both of the top and the bottom walls are composed of copper blocks with internal fluid chamber. By circulating refrigerant supplied from two thermostatic baths in the chamber, the temperatures at the top and the bottom can be controlled individually. The TLC particles with an effective coloring temperature range of $19\text{--}24^\circ\text{C}$ were seeded into 28.8-wt.% ammonium chloride solution with a saturation temperature at $T_{\text{sat}} = 28^\circ\text{C}$. The lighting was performed using a white light sheet generated by an LED light source and a cylindrical lens connected by a linear light guide.

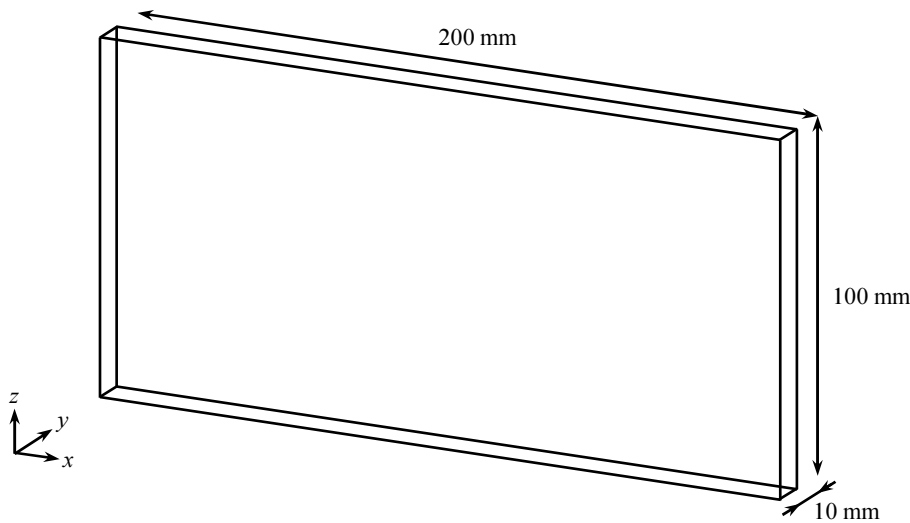


Figure D.38: Geometry of the fluid layer (Hele-Shaw cell) with a size of $200 \times 10 \times 100 \text{ mm}^3$.

The experimental conditions are summarized in Table D.3. The double diffusive convection observed during cooling process of the fluid layer is the measurement target. Initially, the temperatures of the fluid and the walls were set higher than $T_{\text{sat}} = 28^\circ\text{C}$, and thus the initial states were fully melted ammonium chloride solutions without crystals. The top and the bottom temperatures were then cooled at the constant cooling rate $\dot{T}$, e.g., $\dot{T} = -1 \text{ K/min}$. The cooling was kept until the bulk temperature $T_{\text{bulk}} = (T_{\text{top}} + T_{\text{btm}})/2$ reached a set bulk temperature

Table D.3: Experimental conditions for visualization of double diffusive convection with crystallization.

Figure	Experimental type	T_{top} [°C]	T_{btm} [°C]	ΔT [K]	$\dot{T}$ [K/min]	T_{sup} [K]
D.39	Thermally neutral	37.5 → 17.5	37.5 → 17.5	0	−1	10.5
D.40	Thermally neutral	37.5 → 17.5	37.5 → 17.5	0	−3	10.5
D.41	Thermally neutral	32.5 → 12.5	32.5 → 12.5	0	−1	15.5
D.42	Thermally stable	40.0 → 20.0	35.0 → 15.0	−5	−1	10.5
D.43	Thermally stable	42.5 → 22.5	32.5 → 12.5	−10	−1	10.5
D.43	Thermally unstable	35.0 → 15.0	40.0 → 20.0	5	−1	10.5
D.43	Thermally unstable	32.5 → 12.5	42.5 → 22.5	10	−1	10.5

lower than T_{sat} ; the final bulk set temperature was set at $T_{\text{bulk}} = T_{\text{sat}} - T_{\text{sup}}$. Here, T_{sup} is the super-cooling temperature. By following the procedure above, three different experimental types were studied by controlling the top and the bottom temperatures individually; namely, thermally neutral, thermally stable, and thermally unstable conditions were studied. The thermally neutral conditions were performed by imposing the same temperature at the top and the bottom walls, resulting in the vertical temperature difference of $\Delta T = T_{\text{btm}} - T_{\text{top}} = 0$ K. The thermally stable conditions were realized by keeping the vertical temperature difference of $\Delta T = T_{\text{btm}} - T_{\text{top}} < 0$ K for all the time, resulting in a stable temperature stratification. Meanwhile, the thermally unstable conditions were realized by imposing the vertical temperature difference of $\Delta T = T_{\text{btm}} - T_{\text{top}} > 0$ K for all the time, resulting in an unstable temperature stratification. Both of the temperatures were controlled at once, and ΔT was kept during the measurements. For all the three different conditions, double diffusive convection can be realized. Since the thermally neutral and the thermally stable conditions did not yield unstable temperature stratification, only the difference in the concentration due to the crystallization and accompanying latent heat were expected to drive compositional and thermal convective motions. In the thermally unstable conditions, the background unstable temperature stratification originally drove thermal convection and compositional convection was driven by crystallization.

In Fig. D.39, temporal evolution of thermally neutral condition with $\dot{T} = -1$ K/min and $T_{\text{sup}} = 10.5$ K is shown. The time $t = 0$ min corresponds to the time at which the top and the bottom boundaries were cooled with the constant cooling rate $\dot{T}$. This condition will be a benchmark for the other conditions. Six bright objects aligned vertically at $x = 100$ mm are the thermistor sensors immersed into the fluid layer. Due to the cooling of the top and the bottom boundaries, unstable temperature stratification was realized immediately beneath the top boundary leading to thermal convection. The convective motions facilitated the cooling process of the bulk fluid. After the bulk temperature T_{bulk} became lower than the saturation temperature $T_{\text{sat}} = 28^\circ\text{C}$, equiaxed crystals were suddenly generated in the fluid. At almost the same time, columnar crystallization started at both the top and the bottom boundaries. Since the equiaxed crystals precipitated to the bottom due to the larger density than the fluid, mushy

crystal layers at the top and the bottom boundaries were different in those thicknesses; the mushy layers at the bottom were thicker than that at the top. The mushy layers at the top and the bottom boundaries grew homogeneously over the x -direction, meaning that the flows originating in the compositional convection homogenized the concentration of ammonium-chloride without any spatial trends.

The effect of the cooling rate $\dot{T}$ on the convective motions is examined by providing a faster cooling $\dot{T} = -3$ K/min, and the results are shown in Fig. D.40. The other conditions are the same as those set for the case shown in Fig. D.39. Major trend is identical to that shown in Fig. D.39, however, it is possible to identify a difference in the growth of the columnar crystals. Compared to the columnar crystals formed in Fig. D.39, those in Fig. D.40 are narrower and the number of them is larger. As the final set temperatures were the same, the total volume of the solid phase is expected to be the same, however, the fast cooling yielded such fine crystal growth.

The third case is a demonstration showing the effect of the super-cooling temperature T_{sup} by imposing $T_{\text{sup}} = 15.5$ K. The other conditions are the same as those set for the case shown in Fig. D.39. As easily imagined, the crystallization became prominent compared to that in the case shown in Fig. D.39. With the decrease in the bulk temperature, the temperature distribution inside the fluid region became apparent compared to those in Fig. D.39 and D.40. Lower temperature fluids located the upper region around the columnar crystals and the lower region above the lower mushy layer, and these can be identified from the coloration of the TLC particles. The main driving force of the convective flows is the solute plumes generated inside the mushy layer at the bottom. Within the mushy layer, the ammonium-chloride solution decrease its concentration due to the crystallization, and the light fluid can be emitted as solute plumes from the mushy layer. The solute plumes were cooled by the bottom boundary, and thus cold, however, dilute solute plumes were generated, and these are recognizable as green vertical lines in Fig. D.41.

The thermally neutral conditions are typical examples for proving the emergence of strong convective flows only by the cooling of the fluids. To simplify the understanding of such complicated phenomena, the thermally stable conditions are studied. By explicitly imposing background stable temperature stratification, convective flows originated in thermal convection can be reduced, and only the contribution of the compositional convection may be extracted. In Fig. D.42, the temporal evolution of the thermally stable condition with the vertical temperature difference of $\Delta T = -5$ K is shown. The notable difference from the case shown in Fig. D.39 is the asymmetric columnar crystal growth at the top and the bottom boundaries. As the super-cooling temperature at the top was smaller than that at the bottom, the columnar crystals hardly grew at the top boundary, and the precipitation of the equiaxed crystals further promoted the asymmetry. In addition, the number of the equiaxed crystals was larger than those in the case shown in Fig. D.39. This indicates that the equiaxed crystals in the thermally stable condition can float longer time than the thermally neutral conditions.

The stronger stable temperature stratification case is shown in Fig. D.43. This is the case

imposed the vertical temperature difference of $\Delta T = -10$ K. Major trend followed that in Fig. D.42, however, the vertical asymmetry became larger. The columnar crystals at the top boundary were less, and only the mushy layer at the bottom grew. A number of solute plumes can be identified as numerous bright blue lines aligned horizontally above the mush layer.

The most complicated, however, the most significant examples are the thermally unstable temperature stratification conditions. This condition has been considered as the actual flow structures inside the Ganymede. The thermally unstable condition with $\Delta T = 5$ K is shown in Fig. D.44. Due to the unstable temperature stratification, thermal convection always drove full-layer convective motions. The equiaxed crystals formed in the fluid had finer structures and intense precipitation could be observed. Even with the sedimentation of the equiaxed crystals, the thickness of the mush layer at the bottom was thinner than that at the top boundary, as the local super-cooling temperature at the top exceeded that at the bottom. The columnar crystals formed at the top boundary had fragile structures with hourglass shapes. With the background thermal convection, the sedimentation of the equiaxed crystals to the bottom was uneven, and thicker regions are identified at the side walls. The internal temperature distribution was similar to those shown in the other conditions.

A larger vertical temperature difference, $\Delta T = 10$ K, was imposed, and the results are shown in Fig. D.45. The precipitation of the equiaxed crystals became very intense. Thus thermal convection originating in the background unstable temperature stratification and numerous floating equiaxed crystals drove turbulent flows. The flows can be roughly divided into two regions, i.e., the left side and the right side. The two large scale circulations were observed as temporal averages, and as a result, the equiaxed crystals formed drifts at the corner regions. The fragile columnar crystals were broken by the shear of the turbulent flows, and fell on the mushy layer at the bottom.

In Appendix D.3, only qualitative explanations were provided so far. In order to optically measure the complicated flow fields in double diffusive convection with crystallization, measurement techniques combining multiple image processing schemes have been established, and were published as two journal papers Anders et al. [12] and Anders et al. [13]. The details of these two papers are below.

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Title Spectral random masking: a novel dynamic masking technique for PIV in multiphase flows

Journal *Experiments in Fluids*, Vol. 60, No. 68 (2019)

DOI <https://doi.org/10.1007/s00348-019-2703-8>

Author Sten Anders, Daisuke Noto, Yuji Tasaka, and Sven Eckert

Title Simultaneous optical measurement of temperature and velocity fields in solidifying liquids

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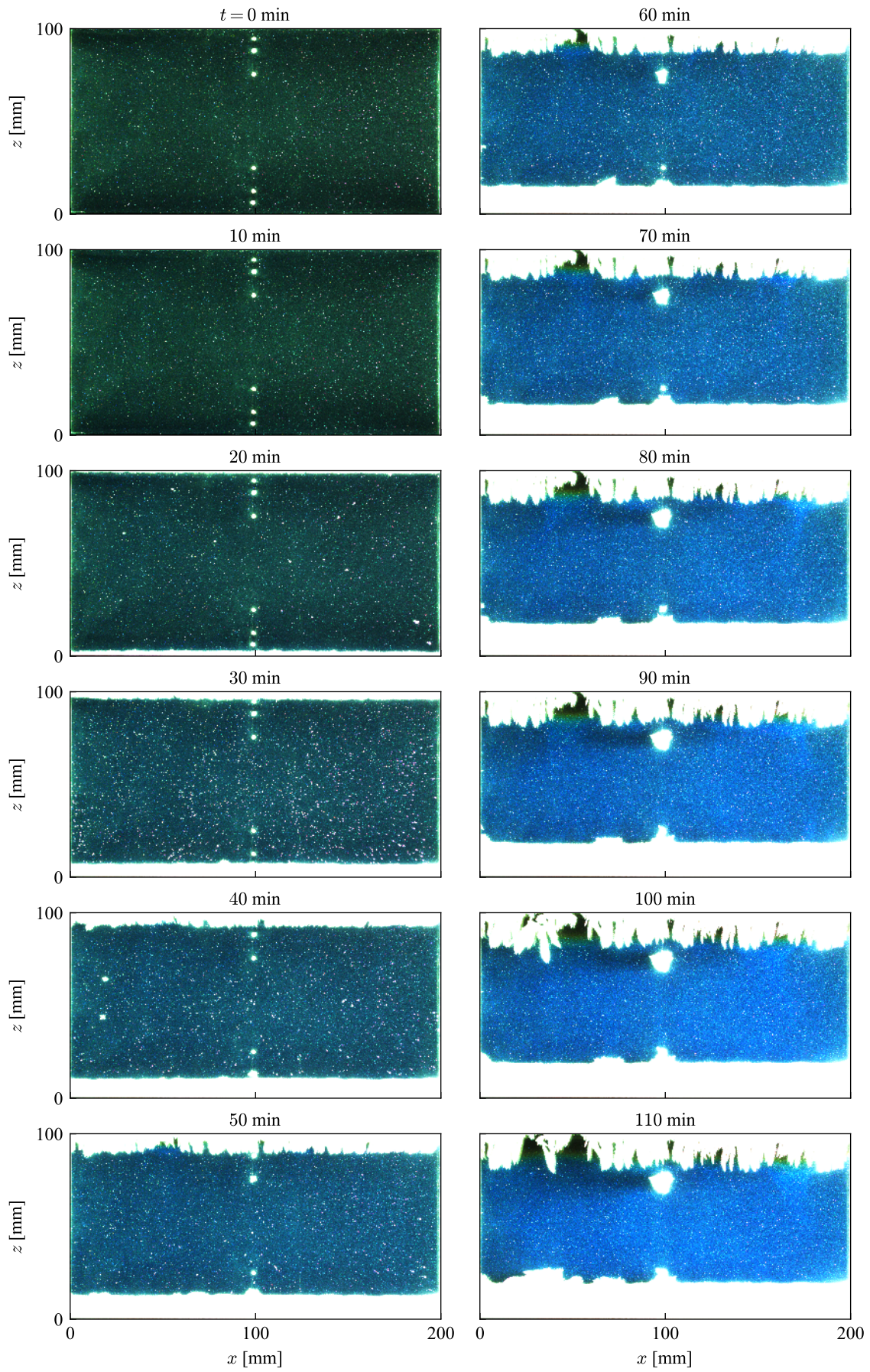


Figure D.39: Temporal evolution of double diffusive convection with crystallization for the thermally neutral condition with $\Delta T = 0$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

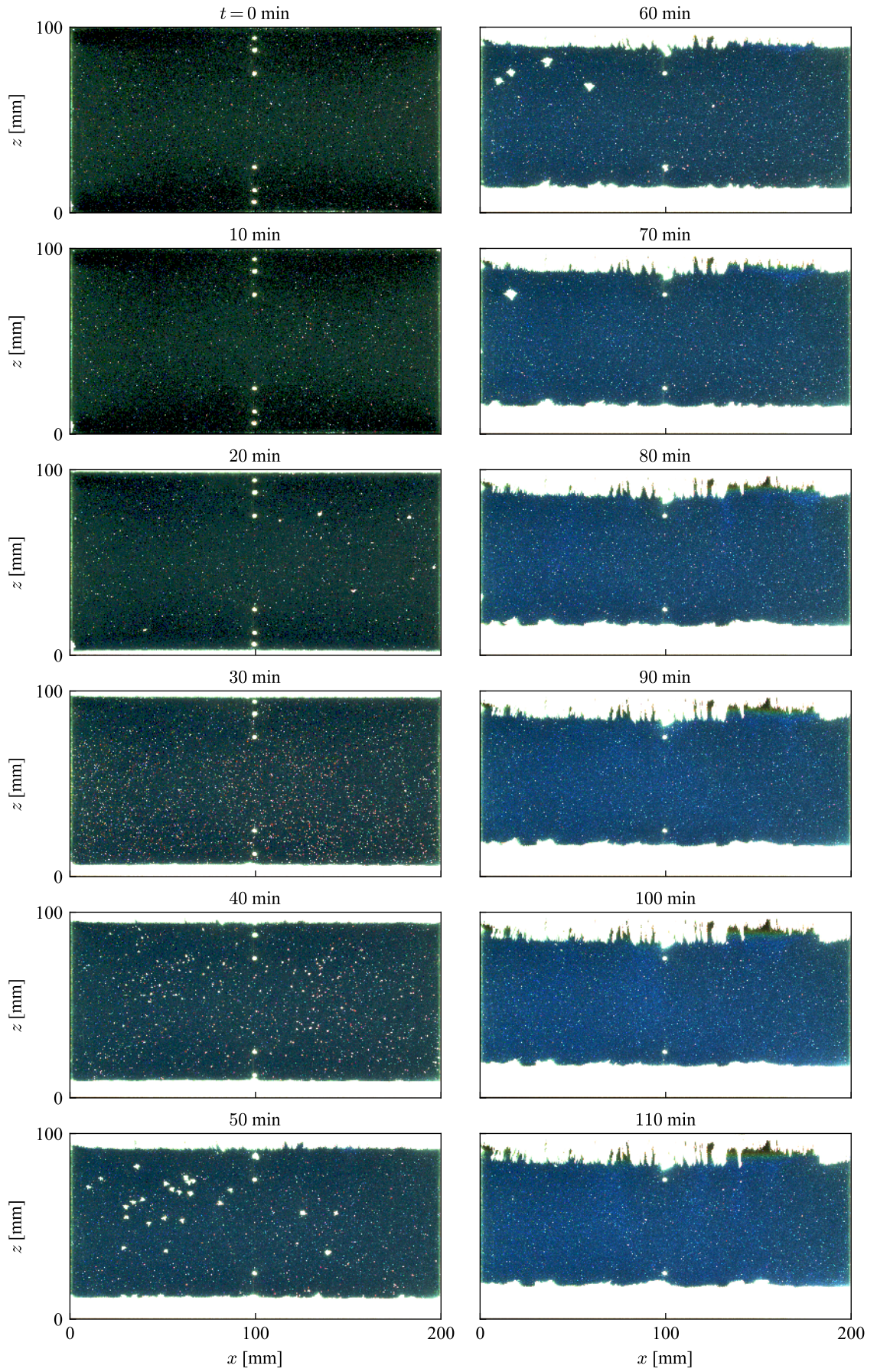


Figure D.40: Temporal evolution of double diffusive convection with crystallization for the thermally neutral condition with $\Delta T = 0$ K, $\dot{T} = -3$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

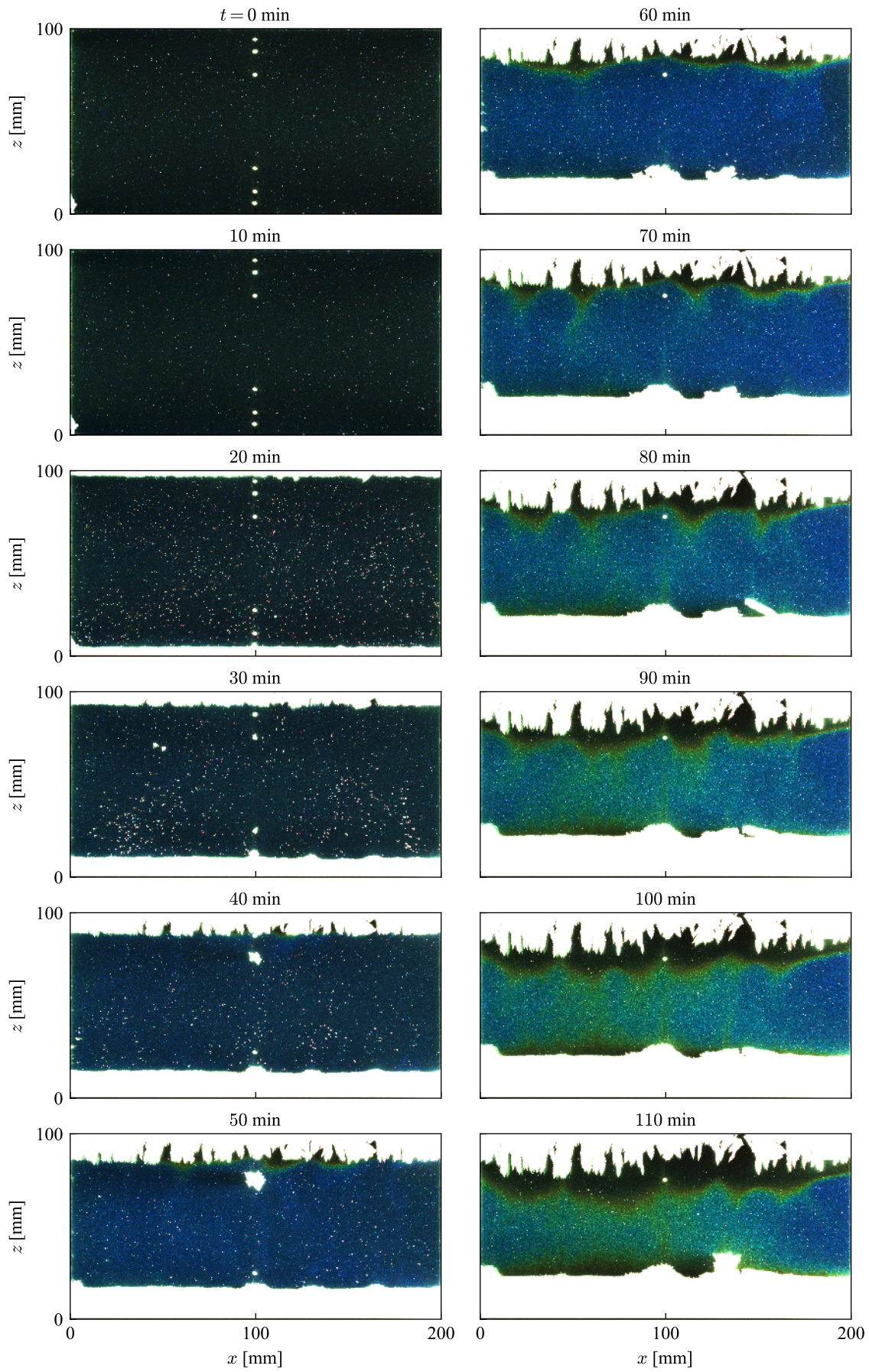


Figure D.41: Temporal evolution of double diffusive convection with crystallization for the thermally neutral condition with $\Delta T = 0$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 15.5$ K. The color contrast is adjusted for visibility.

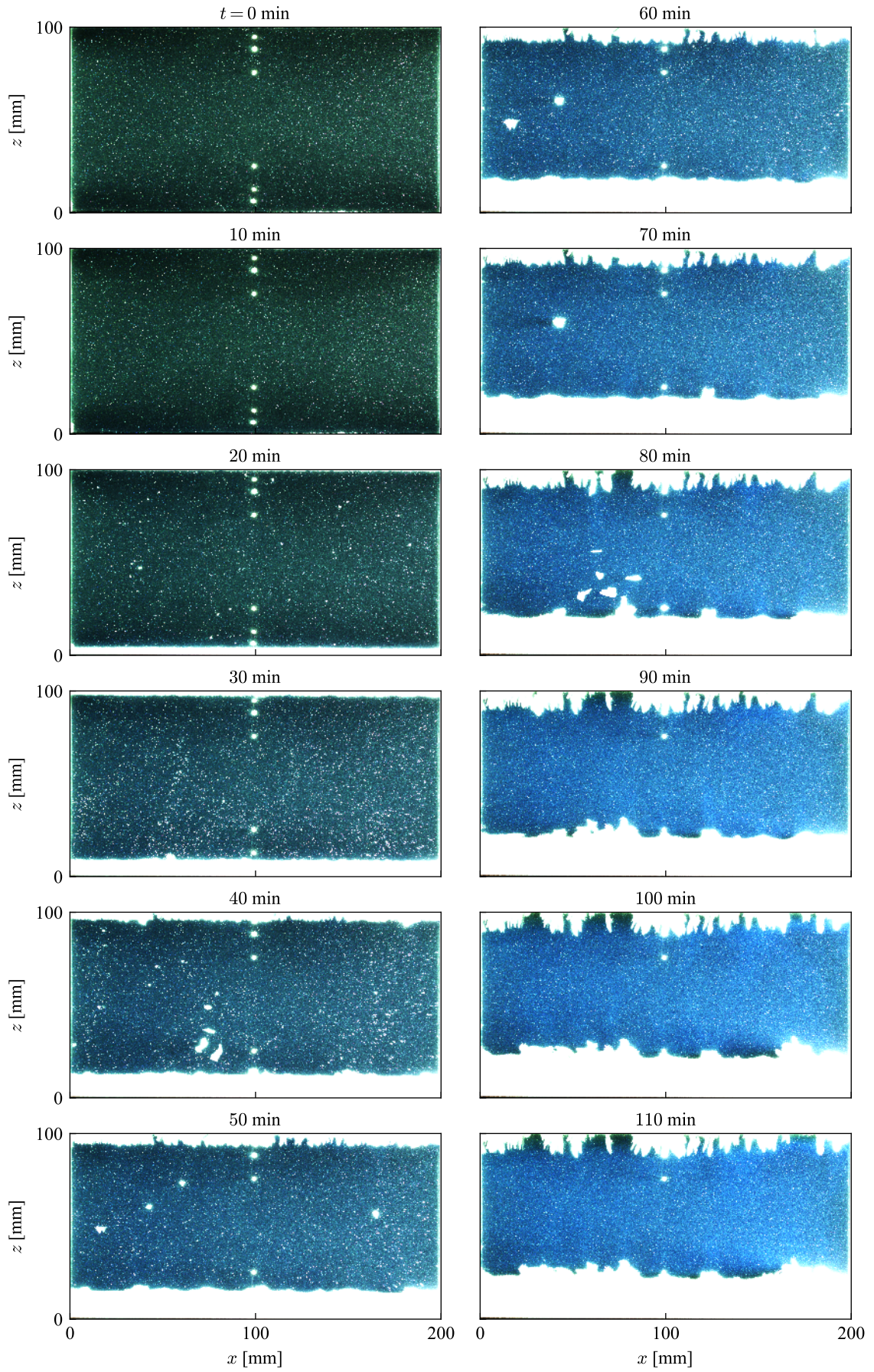


Figure D.42: Temporal evolution of double diffusive convection with crystallization for the thermally stable condition with $\Delta T = -5$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

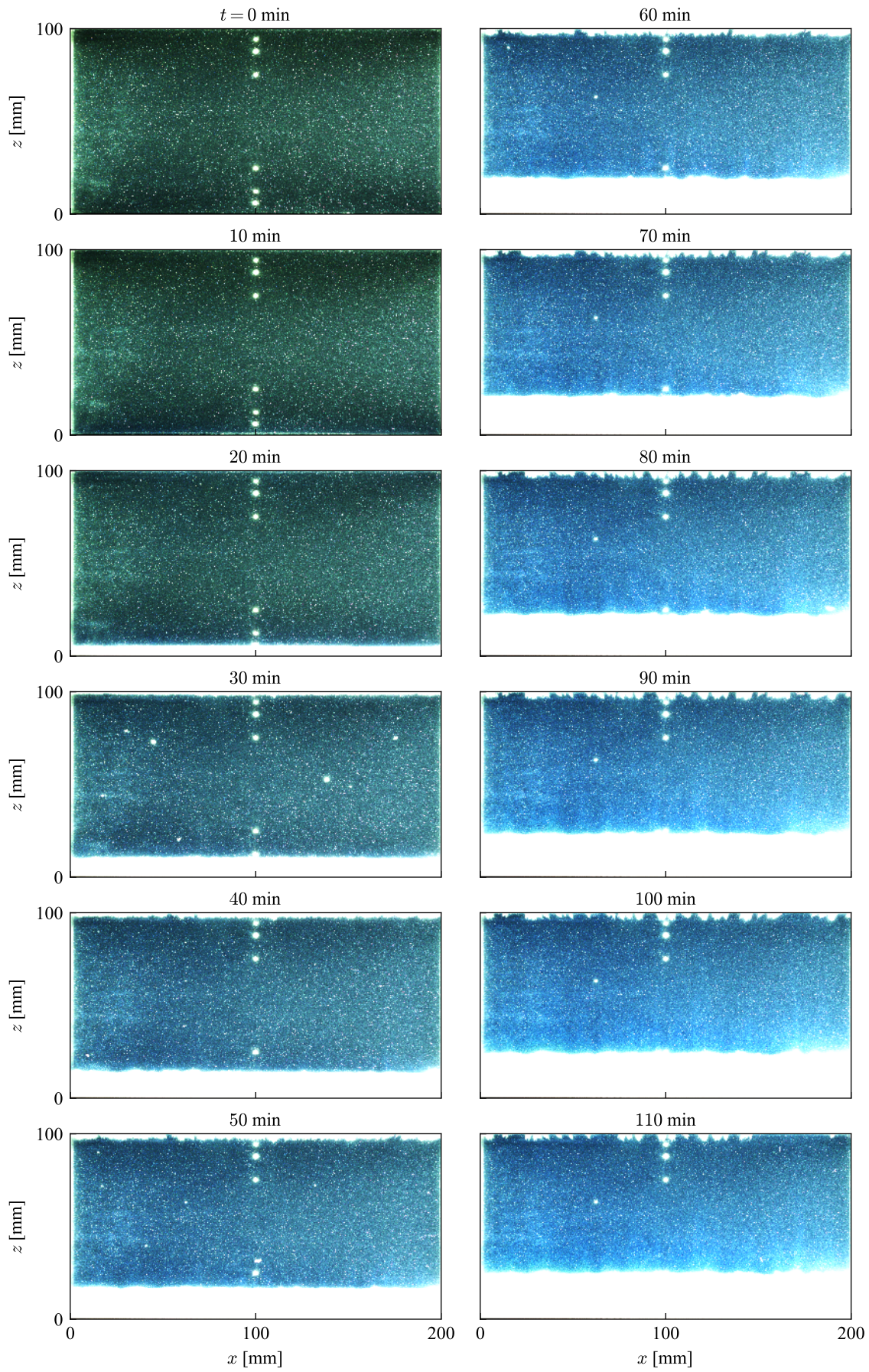


Figure D.43: Temporal evolution of double diffusive convection with crystallization for the thermally stable condition with $\Delta T = -10$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

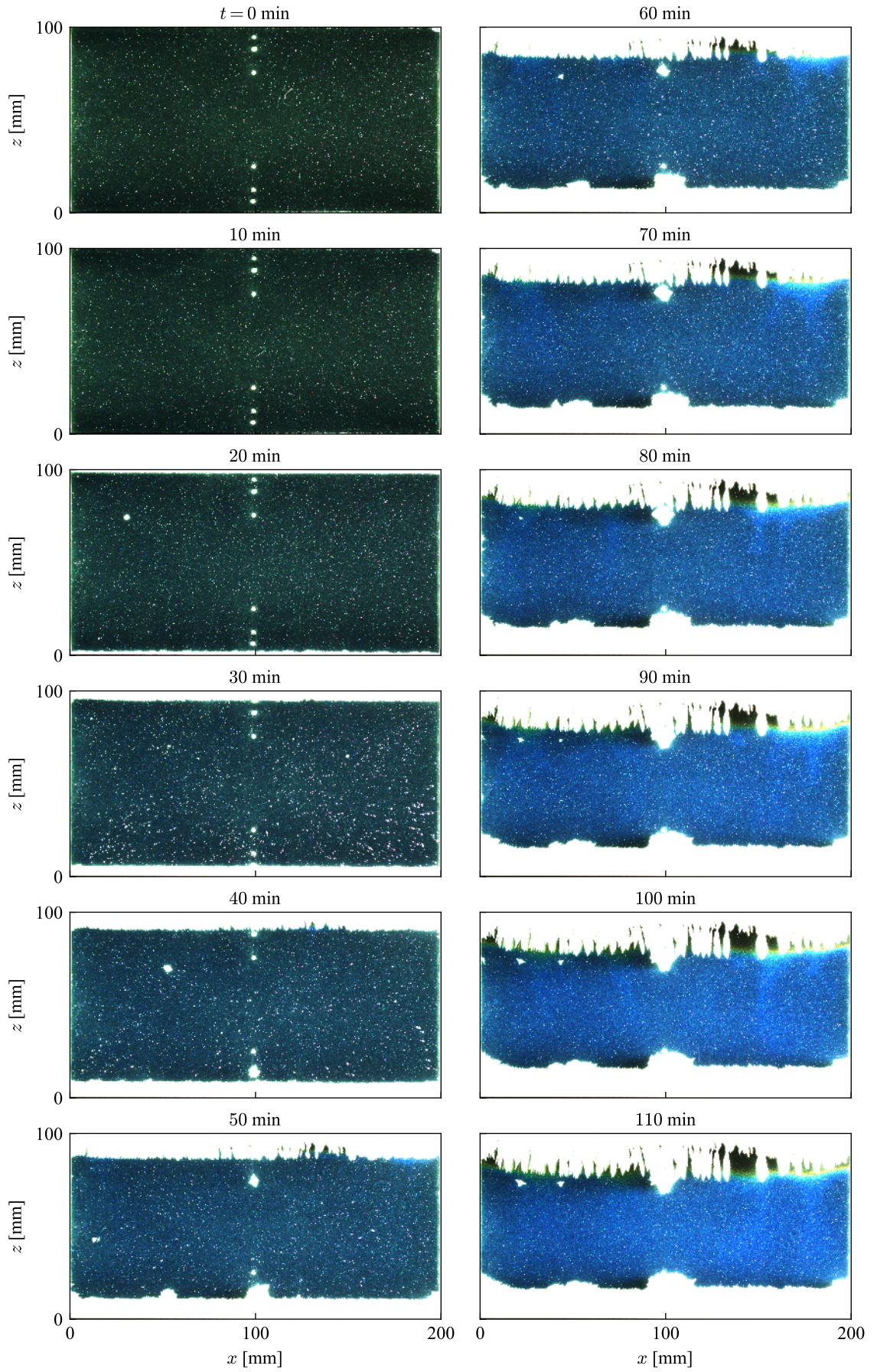


Figure D.44: Temporal evolution of double diffusive convection with crystallization for the thermally unstable condition with $\Delta T = 5$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

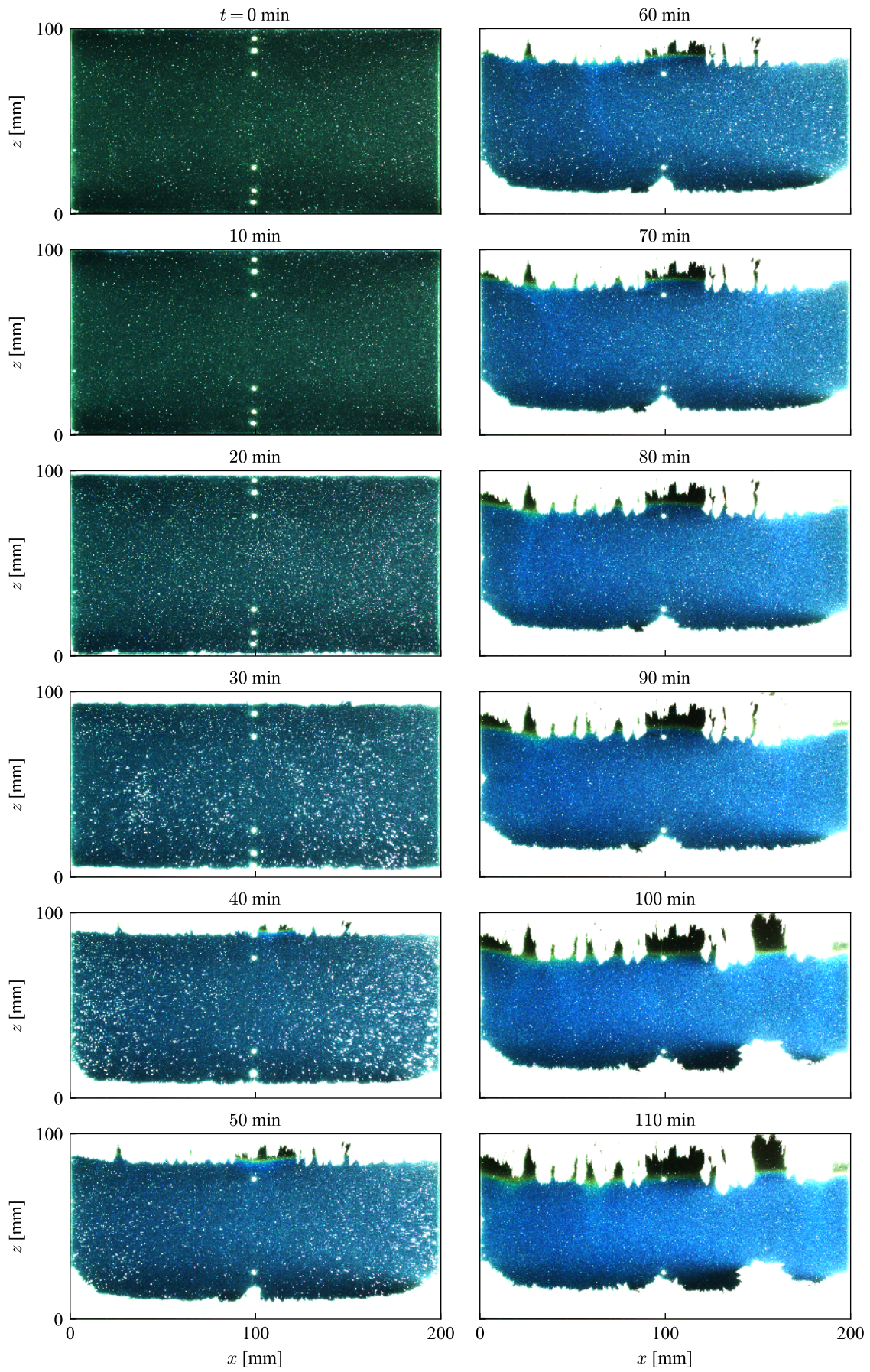


Figure D.45: Temporal evolution of double diffusive convection with crystallization for the thermally unstable condition with $\Delta T = 10$ K, $\dot{T} = -1$ K/min, and $T_{\text{sup}} = 10.5$ K. The color contrast is adjusted for visibility.

BIBLIOGRAPHY

- [1] Laminar Science Corporation. <http://www.laminarsciences.com/>. Accessed: August 24, 2021.
- [2] *Physical Properties of Glycerine and Its Solutions*. Glycerine Producers' Association, 1963.
- [3] N. Abcha, N. Latrache, F. Dumouchel, and I. Mutabazi. Qualitative relation between reflected light intensity by Kalliroscope flakes and velocity field in the Couette–Taylor flow system. *Experiments in Fluids*, 45(1):85–94, 2008.
- [4] N. Abdullah, A. R. A. Talib, A. A. Jaafar, M. A. M. Salleh, and W. T. Chong. The basics and issues of thermochromic liquid crystal calibrations. *Experimental Thermal and Fluid Science*, 34(8):1089–1121, 2010.
- [5] R. J. Adrian. Twenty years of particle image velocimetry. *Experiments in Fluids*, 39(2): 159–169, 2005.
- [6] A. A. Aguirre-Pablo, A. B. Aljedaani, J. Xiong, R. Idoughi, W. Heidrich, and S. T. Thoroddsen. Single-camera 3D PTV using particle intensities and structured light. *Experiments in Fluids*, 60(2):25, 2019.
- [7] A. Ajovalasit, S. Barone, and G. Petrucci. Towards RGB photoelasticity: full-field automated photoelasticity in white light. *Experimental Mechanics*, 35(3):193–200, 1995.
- [8] A. Ajovalasit, G. Petrucci, and M. Scafidi. RGB photoelasticity: review and improvements. *Strain*, 46(2):137–147, 2010.
- [9] A. Ajovalasit, G. Petrucci, and M. Scafidi. Review of RGB photoelasticity. *Optics and Lasers in Engineering*, 68:58–73, 2015.
- [10] A. Akonur and R. M. Lueptow. Three-dimensional velocity field for wavy Taylor–Couette flow. *Physics of Fluids*, 15(4):947–960, 2003.

- [11] C. D. Andereck, P. W. Colovas, M. M. Degen, and Y. Y. Renardy. Instabilities in two layer Rayleigh–Bénard convection: overview and outlook. *International Journal of Engineering Science*, 36(12-14):1451–1470, 1998.
- [12] S. Anders, D. Noto, M. Seilmayer, and S. Eckert. Spectral random masking: a novel dynamic masking technique for PIV in multiphase flows. *Experiments in Fluids*, 60(4): 1–6, 2019.
- [13] S. Anders, D. Noto, Y. Tasaka, and S. Eckert. Simultaneous optical measurement of temperature and velocity fields in solidifying liquids. *Experiments in Fluids*, 61:1–19, 2020.
- [14] T. Arita. Fundamental study on rollover of two-density-stratified layers of liquids. *ASME Heat Transfer Division*, 1992, 198:67–72, 1992.
- [15] S. Bates and D. S. Morrison. Modelling the behaviour of stratified liquid natural gas in storage tanks: a study of the rollover phenomenon. *International Journal of Heat and Mass Transfer*, 40(8):1875–1884, 1997.
- [16] H. Bénard. Les Tourbillons Cellulaires dans une Nappe Liquide Transportant de la Chaleur par Convection en Régime Permanent. *Annales de Chimie et de Physique*, 23(192):62–144, 1901.
- [17] C. Bendicks, D. Tarlet, C. Roloff, R. Bordás, B. Wunderlich, B. Michaelis, and D. Thévenin. Improved 3-D particle tracking velocimetry with colored particles. *Journal of Signal and Information Processing*, 2(02):59, 2011.
- [18] E. R. Benton and A. Clark Jr. Spin-up. *Annual Review of Fluid Mechanics*, 6(1):257–280, 1974.
- [19] R. K. Blackstock. Decorative optical display, May 1 2007. US Patent 7,210,809.
- [20] R. K. Blackstock. Decorative optical display, February 26 2008. US Patent 7,334,910.
- [21] D. Borrero-Echeverry, C. J. Crowley, and T. P. Riddick. Rheoscopic fluids in a post-Kalliroscope world. *Physics of Fluids*, 30(8):087103, 2018.
- [22] B. M. Boubnov and G. S. Golitsyn. Experimental study of convective structures in rotating fluids. *Journal of Fluid Mechanics*, 167:503–531, 1986.
- [23] M. Byron, J. Einarsson, K. Gustavsson, G. Voth, B. Mehlig, and E. Variano. Shape-dependence of particle rotation in isotropic turbulence. *Physics of Fluids*, 27(3):035101, 2015.

-
- [24] A. T. Cadillo-Martínez and R. Sánchez. Experimental velocity distributions in a granular submonolayer. *Physica A: Statistical Mechanics and its Applications*, 465:221–228, 2017.
- [25] B. Castaing, G. Gunaratne, L. Kadanoff, A. Libchaber, and F. Heslot. Scaling of hard thermal turbulence in Rayleigh-Bénard convection. *Journal of Fluid Mechanics*, 204(1): 1–30, 1989.
- [26] A. Cenedese and A. Paglialunga. A new technique for the determination of the third velocity component with PIV. *Experiments in Fluids*, 8(3):228–230, 1989.
- [27] S. Chandrasekhar. *Hydrodynamic and Hydromagnetic stability*. Oxford University Press, 1961.
- [28] J. S. Cheng, J. M. Aurnou, K. Julien, and R. P. J. Kunnen. A heuristic framework for next-generation models of geostrophic convective turbulence. *Geophysical & Astrophysical Fluid Dynamics*, 112(4):277–300, 2018.
- [29] S. Chiu-Webster, E. J. Hinch, and J. R. Lister. Very viscous horizontal convection. *Journal of Fluid Mechanics*, 611:395–426, 2008.
- [30] K. L. Chong, J.-Q. Shi, G.-Y. Ding, S.-S. Ding, H.-Y. Lu, J.-Q. Zhong, and K.-Q. Xia. Vortices as Brownian particles in turbulent flows. *Science Advances*, 6(34):eaaz1110, 2020.
- [31] K.-H. Chung and Y.-H. Chan. Low-complexity color demosaicing algorithm based on integrated gradients. *Journal of Electronic Imaging*, 19(2):021104, 2010.
- [32] C. Cierpka, M. Rossi, R. Segura, and C. J. Kähler. On the calibration of astigmatism particle tracking velocimetry for microflows. *Measurement Science and Technology*, 22(1):015401, 2010.
- [33] C. Cierpka, R. Segura, R. Hain, and C. J. Kähler. A simple single camera 3C3D velocity measurement technique without errors due to depth of correlation and spatial averaging for microfluidics. *Measurement Science and Technology*, 21(4):045401, 2010.
- [34] M. Ciofalo, M. Signorino, and M. Simiano. Tomographic particle-image velocimetry and thermography in Rayleigh-Bénard convection using suspended thermochromic liquid crystals and digital image processing. *Experiments in Fluids*, 34(2):156–172, 2003.
- [35] R. M. Clever and F. H. Busse. Nonlinear oscillatory convection. *Journal of Fluid Mechanics*, 176:403–417, 1987.
- [36] M. A. Coman, R. W. Griffiths, and G. O. Hughes. Sandström’s experiments revisited. *Journal of Marine Research*, 64(6):783–796, 2006.
-

- [37] L.-A. Couston, D. Lecoanet, B. Favier, and M. Le Bars. Dynamics of mixed convective–stably-stratified fluids. *Physical Review Fluids*, 2(9):094804, 2017.
- [38] D. Dabiri and M. Gharib. Digital particle image thermometry: the method and implementation. *Experiments in Fluids*, 11(2-3):77–86, 1991.
- [39] P. S. Dailey and R. G. Fovell. Numerical simulation of the interaction between the sea-breeze front and horizontal convective rolls. Part I: Offshore ambient flow. *Monthly Weather Review*, 127(5):858–878, 1999.
- [40] A. Davaille. Two-layer thermal convection in miscible viscous fluids. *Journal of Fluid Mechanics*, 379:223–253, 1999.
- [41] A. Davey, R. C. Di Prima, and J. T. Stuart. On the instability of Taylor vortices. *Journal of Fluid Mechanics*, 31(1):17–52, 1968.
- [42] P. G. Drazin and W. H. Reid. *Hydrodynamic stability*. Cambridge University Press, 2004.
- [43] R. du Puits, C. Resagk, A. Tilgner, F. H. Busse, and A. Thess. Structure of thermal boundary layers in turbulent Rayleigh–Bénard convection. *Journal of Fluid Mechanics*, 572:231–254, 2007.
- [44] J. M. Dudley, V. Sarano, and F. Dias. On Hokusai’s Great wave off Kanagawa: localization, linearity and a rogue wave in sub-Antarctic waters. *Notes and Records of the Royal Society*, 67(2):159–164, 2013.
- [45] K. Ellsworth and G. Schubert. Numerical models of thermally and mechanically coupled two-layer convection of highly viscous fluids. *Geophysical Journal International*, 93(2):347–363, 1988.
- [46] G. E. Elsinga, F. Scarano, B. Wieneke, and B. W. van Oudheusden. Tomographic particle image velocimetry. *Experiments in Fluids*, 41(6):933–947, 2006.
- [47] H. J. S. Fernando, R.-R. Chen, and D. L. Boyer. Effects of rotation on convective turbulence. *Journal of Fluid Mechanics*, 228:513–547, 1991.
- [48] A. Forsey and S. Gungor. Demosaicing images from colour cameras for digital image correlation. *Optics and Lasers in Engineering*, 86:20–28, 2016.
- [49] T. Fuchs, R. Hain, and C. J. Kähler. Non-iterative double-frame 2D/3D particle tracking velocimetry. *Experiments in Fluids*, 58(9):1–5, 2017.
- [50] K. Fujita, Y. Tasaka, T. Yanagisawa, D. Noto, and Y. Murai. Three-dimensional visualization of columnar vortices in rotating Rayleigh–Bénard convection. *Journal of Visualization*, 23:635–647, 2020.

-
- [51] B. Gayen, R. W. Griffiths, and G. O. Hughes. Stability transitions and turbulence in horizontal convection. *Journal of Fluid Mechanics*, 751:698, 2014.
- [52] M. Gharib, M. Raffel, O. Ronneberger, and J. Kompenhans. Feasibility study of three-dimensional PIV by correlating images of particles within parallel light sheet planes. *Experiments in Fluids*, 19(2):69–77, 1995.
- [53] S. Gogineni, L. Goss, D. Pestian, and R. Rivir. Two-color digital PIV employing a single CCD camera. *Experiments in Fluids*, 25(4):320–328, 1998.
- [54] L. P. Goss, M. E. Post, D. D. Trump, and B. Sarka. Two-Color Particle-Imaging Velocimetry. *Journal of Laser Applications*, 3(1):36–42, 1991.
- [55] S. Goto, S. Kida, and S. Fujiwara. Flow visualization using reflective flakes. *Journal of Fluid Mechanics*, 683:417–429, 2011.
- [56] H. J. J. Gramberg, P. D. Howell, and J. R. Ockendon. Convection by a horizontal thermal gradient. *Journal of Fluid Mechanics*, 586:41, 2007.
- [57] A. M. Grannan, B. Favier, M. Le Bars, and J. M. Aurnou. Tidally forced turbulence in planetary interiors. *Geophysical Journal International*, 208(3):1690–1703, 2016.
- [58] A. Graves. Generating sequences with recurrent neural networks. *arXiv preprint arXiv:1308.0850*, 2013.
- [59] D. D. Gray and A. Giorgini. The validity of the Boussinesq approximation for liquids and gases. *International Journal of Heat Mass Transfer*, 19(5):545–551, 1976.
- [60] H. P. Greenspan. *The theory of rotating fluids*. Cambridge University Press, 1968.
- [61] H. P. Greenspan and L. N. Howard. On a time-dependent motion of a rotating fluid. *Journal of Fluid Mechanics*, 17(3):385–404, 1963.
- [62] R. W. Griffiths, G. O. Hughes, and B. Gayen. Horizontal convection dynamics: insights from transient adjustment. *Journal of Fluid Mechanics*, 726:559–595, 2013.
- [63] I. Grooms, K. Julien, J. B. Weiss, and E. Knobloch. Model of convective Taylor columns in rotating Rayleigh-Bénard convection. *Physical Review Letters*, 104(22):224501, 2010.
- [64] M. Guarnera, G. Messina, and V. Tomaselli. Adaptive color demosaicing and false color removal. *Journal of Electronic Imaging*, 19(2):021105, 2010.
- [65] A. Günther and P. R. von Rohr. Structure of the temperature field in a flow over heated waves. *Experiments in Fluids*, 33(6):920–930, 2002.
-

- [66] C. G. Harris and M. Stephens. A combined corner and edge detector. In *Alvey Vision Conference*, volume 15, pages 10–5244. Citeseer, 1988.
- [67] S. Horn and O. Shishkina. Rotating non-Oberbeck–Boussinesq Rayleigh–Bénard convection in water. *Physics of Fluids*, 26(5):055111, 2014.
- [68] S. Hosokawa, T. Fukunaga, and A. Tomiyama. Application of photobleaching molecular tagging velocimetry to turbulent bubbly flow in a square duct. *Experiments in Fluids*, 47(4-5):745, 2009.
- [69] S. Hosokawa, T. Suzuki, and A. Tomiyama. Turbulence kinetic energy budget in bubbly flows in a vertical duct. *Experiments in Fluids*, 52(3):719–728, 2012.
- [70] S. Hossain, T. Vo, and G. J. Sheard. Horizontal convection in shallow enclosures scales with height, not length, at low Rayleigh numbers. *International Communications in Heat and Mass Transfer*, 109:104308, 2019.
- [71] D. L. Hu, L. Mendel, B. Chan, T. Goreau, and J. W. M. Bush. Visualization of a fish wake using tobacco mosaic virus. *Physics of Fluids*, 17(9):091103, 2005.
- [72] D. L. Hu, T. J. Goreau, and J. W. M. Bush. Flow visualization using tobacco mosaic virus. *Experiments in Fluids*, 46(3):477–484, 2009.
- [73] P. J. Huber. Robust Estimation of a Location Parameter. *Annals of Mathematical Statistics*, 35(1):73–101, 03 1964.
- [74] G. O. Hughes and R. W. Griffiths. Horizontal convection. *Annual Review of Fluid Mechanics*, 40:185–208, 2008.
- [75] T. Ido and Y. Murai. A recursive interpolation algorithm for particle tracking velocimetry. *Flow Measurement and Instrumentation*, 17(5):267–275, 2006.
- [76] T. Ido, Y. Murai, and F. Yamamoto. Postprocessing algorithm for particle-tracking velocimetry based on ellipsoidal equations. *Experiments in Fluids*, 32(3):326–336, 2002.
- [77] M. Ilıcak and G. K. Vallis. Simulations and scaling of horizontal convection. *Tellus A*, 64(1):18377, 2012.
- [78] M. Ito, Y. Yoshitake, and T. Takahashi. Shear-induced structure change in shear-banding of a wormlike micellar solution in concentric cylinder flow. *Journal of Rheology*, 60(5):1019–1029, 2016.
- [79] C. A. Jones. The transition to wavy Taylor vortices. *Journal of Fluid Mechanics*, 157:135–162, 1985.

-
- [80] C. A. Jones. Planetary magnetic fields and fluid dynamos. *Annual Review of Fluid Mechanics*, 43:583–614, 2011.
- [81] K. Julien, S. Legg, J. McWilliams, and J. Werner. Rapidly rotating turbulent Rayleigh-Bénard convection. *Journal of Fluid Mechanics*, 322:243–273, 1996.
- [82] K. Julien, A. M. Rubio, I. Grooms, and E. Knobloch. Statistical and physical balances in low Rossby number Rayleigh-bénard convection. *Geophysical & Astrophysical Fluid Dynamics*, 106(4-5):392–428, 2012.
- [83] C. J. Kähler, U. Scholz, and J. Ortmanns. Wall-shear-stress and near-wall turbulence measurements up to single pixel resolution by means of long-distance micro-PIV. *Experiments in Fluids*, 41(2):327–341, 2006.
- [84] C. J. Kähler, T. Astarita, P. P. Vlachos, J. Sakakibara, R. Hain, S. Discetti, R. La Foy, and C. Cierpka. Main results of the 4th international PIV challenge. *Experiments in Fluids*, 57(6):97, 2016.
- [85] V. U. Kakade, G. D. Lock, M. Wilson, J. M. Owen, and J. E. Mayhew. Accurate heat transfer measurements using thermochromic liquid crystal. Part 1: Calibration and characteristics of crystals. *International Journal of Heat and Fluid Flow*, 30(5):939–949, 2009.
- [86] V. U. Kakade, G. D. Lock, M. Wilson, J. M. Owen, and J. E. Mayhew. Accurate heat transfer measurements using thermochromic liquid crystal. Part 2: Application to a rotating disc. *International Journal of Heat and Fluid Flow*, 30(5):950–959, 2009.
- [87] M. Kemp. Leonardo da Vinci’s laboratory: studies in flow. *Nature*, 571(7765):322–324, 2019.
- [88] Y.-K. Kim, B. Senyuk, and O. D. Lavrentovich. Molecular reorientation of a nematic liquid crystal by thermal expansion. *Nature Communications*, 3:1133, 2012.
- [89] E. M. King, S. Stellmach, and J. M. Aurnou. Heat transfer by rapidly rotating Rayleigh-Bénard convection. *Journal of Fluid Mechanics*, 691:568–582, 2012.
- [90] D. P. Kingma and J. Ba. Adam: A Method for Stochastic Optimization, 2017.
- [91] J. Kolaas, A. Jensen, and M. Mielnik. Visualization and measurements of flows in micro silicon Y-channels. *The European Physical Journal E*, 36(2):1–11, 2013.
- [92] M. Kono and P. H. Roberts. Recent geodynamo simulations and observations of the geomagnetic field. *Reviews of Geophysics*, 40(4):4–1, 2002.
-

- [93] E. L. Koschmieder. Convection on a non-uniformly heated, rotating plane. *Journal of Fluid Mechanics*, 33(3):515–527, 1968.
- [94] E. L. Koschmieder and S. G. Pallas. Heat transfer through a shallow, horizontal convecting fluid layer. *International Journal of Heat and Mass Transfer*, 17(9):991–1002, 1974.
- [95] R. Krishnamurti. Some further studies on the transition to turbulent convection. *Journal of Fluid Mechanics*, 60(2):285–303, 1973.
- [96] P. Lancaster and K. Salkauskas. Surfaces generated by moving least squares methods. *Mathematics of Computation*, 37(155):141–158, 1981.
- [97] Y. LeCun, Y. Bengio, and G. Hinton. Deep learning. *Nature*, 521(7553):436–444, 2015.
- [98] Y. Liu and R. E. Ecke. Heat transport measurements in turbulent rotating Rayleigh-Bénard convection. *Physical Review E*, 80(3):036314, 2009.
- [99] J. Lu, S. P. Arya, W. H. Snyder, and R. E. Lawson Jr. A laboratory study of the urban heat island in a calm and stably stratified environment. Part I: Temperature field. *Journal of Applied Meteorology*, 36(10):1377–1391, 1997.
- [100] J. Lu, S. P. Arya, W. H. Snyder, and R. E. Lawson Jr. A laboratory study of the urban heat island in a calm and stably stratified environment. Part II: Velocity field. *Journal of Applied Meteorology*, 36(10):1392–1402, 1997.
- [101] L. Luo, Y. Xiong, Y. Liu, and X. Sun. Adaptive gradient methods with dynamic bound of learning rate. *arXiv preprint arXiv:1902.09843*, 2019.
- [102] W. V. R. Malkus and G. Veronis. Finite amplitude cellular convection. *Journal of Fluid Mechanics*, 4(3):225–260, 1958.
- [103] H. S. Malvar, L.-W. He, and R. Cutler. High-quality linear interpolation for demosaicing of Bayer-patterned color images. In *2004 IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume 3, pages iii–485. IEEE, 2004.
- [104] D. Martinand, E. Serre, and R. M. Lueptow. Mechanisms for the transition to waviness for Taylor vortices. *Physics of Fluids*, 26(9):094102, 2014.
- [105] P. Matisse and M. Gorman. Neutrally buoyant anisotropic particles for flow visualization. *Physics of Fluids*, 27(4):759–760, 1984.
- [106] H. Matsushita, T. Mochizuki, and N. Kaji. Calibration scheme for three-dimensional particle tracking with a prismatic light. *Review of Scientific Instruments*, 75(2):541–545, 2004.

-
- [107] G. B. McFadden and S. R. Coriell. Onset of oscillatory convection in two liquid layers with phase change. *Physics of Fluids*, 21(3):034101, 2009.
- [108] T. J. McGregor, D. J. Spence, and D. W. Coutts. Laser-based volumetric colour-coded three-dimensional particle velocimetry. *Optics and Lasers in Engineering*, 45(8):882–889, 2007.
- [109] D. Menon, S. Andriani, and G. Calvagno. Demosaicing with directional filtering and a posteriori decision. *IEEE Transactions on Image Processing*, 16(1):132–141, 2006.
- [110] S. Miao and F. Chen. Formation of horizontal convective rolls in urban areas. *Atmospheric Research*, 89(3):298–304, 2008.
- [111] S. Moller, C. Resagk, and C. Cierpka. On the application of neural networks for temperature field measurements using thermochromic liquid crystals. *Experiments in Fluids*, 61: 1–21, 2020.
- [112] S. Moller, C. Resagk, and C. Cierpka. Long-time experimental investigation of turbulent superstructures in Rayleigh–bénard convection by noninvasive simultaneous measurements of temperature and velocity fields. *Experiments in Fluids*, 62(4):1–18, 2021.
- [113] C. C. Montarou, T. K. Gaylord, R. A. Villalaz, and E. N. Glytsis. Colorimetry-based retardation measurement method with white-light interference. *Applied Optics*, 41(25): 5290–5297, 2002.
- [114] A. Mori and H. Niino. Time evolution of nonlinear horizontal convection: its flow regimes and self-similar solutions. *Journal of the Atmospheric Sciences*, 59(11):1841–1856, 2002.
- [115] J. C. Mullarney, R. W. Griffiths, and G. O. Hughes. Convection driven by differential heating at a horizontal boundary. *Journal of Fluid Mechanics*, 516:181–209, 2004.
- [116] T. Munakata and I. Tanasawa. Numerical Study on Effect of Initial Concentration Difference on Onset of Rollover. *Transactions of the Japan Society of Mechanical Engineers Series B*, 60(578):3512–3518, 1994.
- [117] I. Mutabazi, J. E. Wesfreid, and E. Guyon. *Dynamics of spatio-temporal cellular structures: Henri Bénard centenary review*, volume 207. Springer, 2010.
- [118] D. Nieves, A. M. Rubio, and K. Julien. Statistical classification of flow morphology in rapidly rotating Rayleigh–Bénard convection. *Physics of Fluids*, 26(8):086602, 2014.
- [119] H. Niino, A. Mori, T. Satomura, and S. Akiba. Flow regimes of nonlinear heat island circulation. *Journal of the Atmospheric Sciences*, 63(5):1538–1547, 2006.
-

- [120] T. Nonomura, Y. Ozawa, T. Ibuki, K. Nankai, A. Komuro, H. Nishida, K. Marios, N. Kubo, and H. Kawabata. Single-pixel particle image velocimetry for characterization of dielectric barrier discharge plasma actuators. *AIAA journal*, 58(11):4952–4957, 2020.
- [121] D. Noto and Y. Tasaka. Dual-plane ensemble correlation for pixelwise 2D-3C velocity field measurements using a single camera. *Experiments in Fluids*, 62(5):1–5, 2021.
- [122] D. Noto, Y. Tasaka, T. Yanagisawa, H. J. Park, and Y. Murai. Vortex tracking on visualized temperature fields in a rotating Rayleigh–Bénard convection. *Journal of Visualization*, 21(6):987–998, 2018.
- [123] D. Noto, Y. Tasaka, T. Yanagisawa, and Y. Murai. Horizontal diffusive motion of columnar vortices in rotating Rayleigh–Bénard convection. *Journal of Fluid Mechanics*, 871:401–426, 2019.
- [124] D. Noto, Y. Tasaka, J. Hitomi, and Y. Murai. Applicability evaluation of the stress-optic law in Newtonian fluids toward stress field measurements. *Physical Review Research*, 2(4):043111, 2020.
- [125] D. Noto, Y. Tasaka, and Y. Murai. Coupling structures of two-layer natural convection in a cylindrical vessel investigated by simultaneous two-layer visualization. *Experimental Thermal and Fluid Science*, page 110394, 2021.
- [126] D. Noto, Y. Tasaka, and Murai Y. In situ color-to-depth calibration: toward practical three-dimensional color particle tracking velocimetry. *Experiments in Fluids*, 62(131):1–13, 2021.
- [127] D. Noto, Y. Tasaka, T. Yanagisawa, and T. Miyagoshi. Stepwise transitions in spin-up of rotating Rayleigh–Bénard convection. *Journal of Fluid Mechanics*, 911:A43, 2021.
- [128] K. Okamoto, S. Nishio, T. Saga, and T. Kobayashi. Standard images for particle-image velocimetry. *Measurement Science and Technology*, 11(6):685, 2000.
- [129] T. Onuma and Y. Otani. A development of two-dimensional birefringence distribution measurement system with a sampling rate of 1.3 MHz. *Optics Communications*, 315:69–73, 2014.
- [130] Y. Ozawa, T. Ibuki, T. Nonomura, K. Suzuki, A. Komuro, A. Ando, and K. Asai. Single-pixel resolution velocity/convection velocity field of a supersonic jet measured by particle/schlieren image velocimetry. *Experiments in Fluids*, 61:1–18, 2020.
- [131] A. Pandey, J. D. Scheel, and J. Schumacher. Turbulent superstructures in Rayleigh–Bénard convection. *Nature Communications*, 9(1):1–11, 2018.

-
- [132] F. Paparella and W. R. Young. Horizontal convection is non-turbulent. *Journal of Fluid Mechanics*, 466:205–214, 2002.
- [133] H. G. Park, D. Dabiri, and M. Gharib. Digital particle image velocimetry/thermometry and application to the wake of a heated circular cylinder. *Experiments in Fluids*, 30(3):327–338, 2001.
- [134] H. J. Park, S. Yamagishi, S. Osuka, Y. Tasaka, and Y. Murai. Development of multi-cycle rainbow particle tracking velocimetry improved by particle defocusing technique and an example of its application on twisted Savonius turbine. *Experiments in Fluids*, 62(4):1–15, 2021.
- [135] E. A. Patterson. Digital photoelasticity: principles, practice and potential: measurements lecture. *Strain*, 38(1):27–39, 2002.
- [136] D. S. Pearson, A. D. Kiss, L. J. Fetters, and M. Doi. Flow-Induced Birefringence of Concentrated Polyisoprene Solutions. *Journal of Rheology*, 33(3):517–535, 1989.
- [137] S. E. Peckham, R. B. Wilhelmson, L. J. Wicker, and C. L. Ziegler. Numerical simulation of the interaction between the dryline and horizontal convective rolls. *Monthly Weather Review*, 132(7):1792–1812, 2004.
- [138] F. Pereira, H. Stürer, E. C. Graff, and M. Gharib. Two-frame 3D particle tracking. *Measurement Science and Technology*, 17(7):1680, 2006.
- [139] F. Pereira, J. Lu, E. Castano-Graff, and M. Gharib. Microscale 3D flow mapping with μ ddpiv. *Experiments in Fluids*, 42(4):589–599, 2007.
- [140] A. Peterlin. Optical effects in flow. *Annual Review of Fluid Mechanics*, 8(1):35–55, 1976.
- [141] W. Philippoff. Flow birefringence and stress. *Journal of Applied Physics*, 27(9):984–989, 1956.
- [142] S. Pick and F.-O. Lehmann. Stereoscopic PIV on multiple color-coded light sheets and its application to axial flow in flapping robotic insect wings. *Experiments in Fluids*, 47(6):1009–1023, 2009.
- [143] A. B. Pieri, F. Falasca, J. von Hardenberg, and A. Provenzale. Plume dynamics in rotating Rayleigh–Bénard convection. *Physics Letters A*, 380(14–15):1363–1367, 2016.
- [144] T. S. Pottebaum and M. Gharib. The pinch-off process in a starting buoyant plume. *Experiments in Fluids*, 37(1):87–94, 2004.
-

- [145] A. Prakash and J. N. Koster. Convection in multiple layers of immiscible liquids in a shallow cavity–I. Steady natural convection. *International Journal of Multiphase Flow*, 20(2):383–396, 1994.
- [146] A. Prakash and J. N. Koster. Convection in multiple layers of immiscible liquids in a shallow cavity–II. Steady thermocapillary convection. *International Journal of Multiphase Flow*, 20(2):397–414, 1994.
- [147] A. Prakash and J. N. Koster. Steady natural convection in a two-layer system of immiscible liquids. *International Journal of Heat and Mass Transfer*, 40(12):2799–2812, 1997.
- [148] A. Prakash, K. Yasuda, F. Otsubo, K. Kuwahara, and T. Doi. Flow coupling mechanisms in two-layer Rayleigh–Bénard convection. *Experiments in Fluids*, 23(3):252–261, 1997.
- [149] A. K. Prasad. Stereoscopic particle image velocimetry. *Experiments in Fluids*, 29(2):103–116, 2000.
- [150] A. K. Prasad and R. J. Adrian. Stereoscopic particle image velocimetry applied to liquid flows. *Experiments in Fluids*, 15(1):49–60, 1993.
- [151] J. P. Prenel and Y. Bailly. Recent evolutions of imagery in fluid mechanics: From standard tomographic visualization to 3D volumic velocimetry. *Optics and Lasers in Engineering*, 44(3-4):321–334, 2006.
- [152] A. Puglisi, V. Loreto, U. M. B. Marconi, A. Petri, and A. Vulpiani. Clustering and non-gaussian behavior in granular matter. *Physical Review Letters*, 81(18):3848, 1998.
- [153] X.-L. Qiu and P. Tong. Large-scale velocity structures in turbulent thermal convection. *Physical Review E*, 64(3):036304, 2001.
- [154] M. Raffel, C. E. Willert, F. Scarano, C. J. Kähler, S. T. Wereley, and J. Kompenhans. *Particle image velocimetry: a practical guide*. Springer, 2018.
- [155] H. Rajaei, R. P. J. Kunnen, and H. J. H. Clercx. Exploring the geostrophic regime of rapidly rotating convection with experiments. *Physics of Fluids*, 29(4):045105, 2017.
- [156] K. Ramesh, T. Kasimayan, and B. Neethi Simon. Digital photoelasticity–A comprehensive review. *The Journal of Strain Analysis for Engineering Design*, 46(4):245–266, 2011.
- [157] S. Rasenat, F. H. Busse, and I. Rehberg. A theoretical and experimental study of double-layer convection. *Journal of Fluid Mechanics*, 199:519–540, 1989.
- [158] S. Ravichandran and J. S. Wettlaufer. Transient convective spin-up dynamics. *Journal of Fluid Mechanics*, 897:A24, 2020.

-
- [159] Lord Rayleigh. LIX. On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 32(192):529–546, 1916.
- [160] S. J. Reddi, S. Kale, and S. Kumar. On the convergence of adam and beyond. *arXiv preprint arXiv:1904.09237*, 2019.
- [161] M. G. Rosevear, B. Gayen, and R. W. Griffiths. Turbulent horizontal convection under spatially periodic forcing: a regime governed by interior inertia. *Journal of Fluid Mechanics*, 831:491–523, 2017.
- [162] H. T. Rossby. On thermal convection driven by non-uniform heating from below: an experimental study. In *Deep-Sea Research*, volume 12, pages 9–16. Elsevier, 1965.
- [163] T. Rossby. Numerical experiments with a fluid heated non-uniformly from below. *Tellus A*, 50(2):242–257, 1998.
- [164] B. Ruck. Colour-coded tomography in fluid mechanics. *Optics & Laser Technology*, 43(2):375–380, 2011.
- [165] S. Sakai. The horizontal scale of rotating convection in the geostrophic regime. *Journal of Fluid Mechanics*, 333:85–95, 1997.
- [166] J. P. L. C. Salazar, J. De Jong, L. Cao, S. H. Woodward, H. Meng, and L. R. Collins. Experimental and numerical investigation of inertial particle clustering in isotropic turbulence. *Journal of Fluid Mechanics*, 600:245, 2008.
- [167] D. Schanz, S. Gesemann, and A. Schröder. Shake-The-Box: Lagrangian particle tracking at high particle image densities. *Experiments in Fluids*, 57(5):70, 2016.
- [168] S. Scharnowski, R. Hain, and C. J. Kähler. Reynolds stress estimation up to single-pixel resolution using PIV-measurements. *Experiments in Fluids*, 52(4):985–1002, 2012.
- [169] J. Schumacher and K. R. Sreenivasan. Colloquium: Unusual dynamics of convection in the sun. *Reviews of Modern Physics*, 92(4):041001, 2020.
- [170] A. Scotti and B. White. Is horizontal convection really “non-turbulent?”. *Geophysical Research Letters*, 38(21), 2011.
- [171] L. A. Segel. The non-linear interaction of a finite number of disturbances to a layer of fluid heated from below. *Journal of Fluid Mechanics*, 21(2):359–384, 1965.
- [172] J. Sheng, E. Malkiel, and J. Katz. Using digital holographic microscopy for simultaneous measurements of 3D near wall velocity and wall shear stress in a turbulent boundary layer. *Experiments in Fluids*, 45(6):1023–1035, 2008.
-

- [173] D. Shepard. A two-dimensional interpolation function for irregularly-spaced data. In *Proceedings of the 1968 23rd ACM National Conference*, pages 517–524, New York, NY, USA, 1968. Association for Computing Machinery.
- [174] J. Q. Shi, C. Beduz, and R. G. Scurlock. Numerical modelling and flow visualization of mixing of stratified layers and rollover in LNG. *Cryogenics*, 33(12):1116–1124, 1993.
- [175] S. Shi, J. Ding, C. Atkinson, J. Soria, and T. H. New. A detailed comparison of single-camera light-field PIV and tomographic PIV. *Experiments in Fluids*, 59(3):1–13, 2018.
- [176] T. Shikata, S. J. Dahman, and D. S. Pearson. Rheo-optical behavior of wormlike micelles. *Langmuir*, 10(10):3470–3476, 1994.
- [177] O. Shishkina. Mean flow structure in horizontal convection. *Journal of Fluid Mechanics*, 812:525–540, 2017.
- [178] O. Shishkina and C. Wagner. Analysis of sheet-like thermal plumes in turbulent Rayleigh-Bénard convection. *Journal of Fluid Mechanics*, 599:383, 2008.
- [179] O. Shishkina and S. Wagner. Prandtl-number dependence of heat transport in laminar horizontal convection. *Physical Review Letters*, 116(2):024302, 2016.
- [180] O. Shishkina, R. J. A. M. Stevens, S. Grossmann, and D. Lohse. Boundary layer structure in turbulent thermal convection and its consequences for the required numerical resolution. *New Journal of Physics*, 12(7):075022, 2010.
- [181] S. M. Soloff, R. J. Adrian, and Z.-C. Liu. Distortion compensation for generalized stereoscopic particle image velocimetry. *Measurement Science and Technology*, 8(12):1441, 1997.
- [182] B. Sørensen. A revised Michel-Lévy interference colour chart based on first-principles calculations. *European Journal of Mineralogy*, 25(1):5–10, 2012.
- [183] T. Spohn and G. Schubert. Modes of mantle convection and the removal of heat from the Earth’s interior. *Journal of Geophysical Research: Solid Earth*, 87(B6):4682–4696, 1982.
- [184] M. Stanislas, K. Okamoto, and C. Kähler. Main results of the first international PIV challenge. *Measurement Science and Technology*, 14(10):R63, 2003.
- [185] M. Stanislas, K. Okamoto, C. J. Kähler, and J. Westerweel. Main results of the second international PIV challenge. *Experiments in Fluids*, 39(2):170–191, 2005.
- [186] M. Stanislas, K. Okamoto, C. J. Kähler, J. Westerweel, and F. Scarano. Main results of the third international PIV challenge. *Experiments in Fluids*, 45(1):27–71, 2008.

-
- [187] J. Stasiek, M. Jewartowski, and T. A. Kowalewski. The use of liquid crystal thermography in selected technical and medical applications – recent development. *Journal of Crystallization Process and Technology*, 2014, 2014.
- [188] R. J. A. M. Stevens, J. Q. Zhong, H. J. H. Clercx, G. Ahlers, and D. Lohse. Transitions between turbulent states in rotating Rayleigh–Bénard convection. *Physical Review Letters*, 103(2):024503, 2009.
- [189] R. J. A. M. Stevens, H. J. H. Clercx, and D. Lohse. Heat transport and flow structure in rotating Rayleigh–Bénard convection. *European Journal of Mechanics - B/Fluids*, 40: 41–49, 2013.
- [190] K. Takehara and T. Etoh. A study on particle identification in PTV particle mask correlation method. *Journal of Visualization*, 1(3):313–323, 1998.
- [191] K. Takehara and T. G. Etoh. Accurate estimation of vorticity from PTV velocity distribution. In *Proceedings of 8th International Symposium on Particle Image Velocimetry-PIV09*, volume 185, 2009.
- [192] W.-H. Tien, D. Dabiri, and J. R. Hove. Color-coded three-dimensional micro particle tracking velocimetry and application to micro backward-facing step flows. *Experiments in Fluids*, 55(3):1–14, 2014.
- [193] C. Tropea and A. L. Yarin. *Springer Handbook of Experimental Fluid Mechanics*. Springer Science & Business Media, 2007.
- [194] C. W. H. van Doorne and J. Westerweel. Measurement of laminar, transitional and turbulent pipe flow using stereoscopic-PIV. *Experiments in Fluids*, 42(2):259–279, 2007.
- [195] M. Van Dyke. *An album of fluid motion*, volume 176. Parabolic press Stanford, 1982.
- [196] C. S. Vila, S. Discetti, G. M. Carlomagno, T. Astarita, and A. Ianiro. On the onset of horizontal convection. *International Journal of Thermal Sciences*, 110:96–108, 2016.
- [197] N. A. Vinnichenko, A. V. Pushtaev, Y. Y. Plaksina, Y. K. Rudenko, and A. V. Uvarov. Horizontal convection driven by nonuniform radiative heating in liquids with different surface behavior. *International Journal of Heat and Mass Transfer*, 126:400–410, 2018.
- [198] P. Vorobieff and R. E. Ecke. Transient states during spin-up of a Rayleigh–Bénard cell. *Physics of Fluids*, 10(10):2525–2538, 1998.
- [199] W. Wang and R. X. Huang. An experimental study on thermal circulation driven by horizontal differential heating. *Journal of Fluid Mechanics*, 540:49–73, 2005.
-

- [200] T. Watamura, Y. Tasaka, and Y. Murai. LCD-projector-based 3D color PTV. *Experimental Thermal and Fluid Science*, 47:68–80, 2013.
- [201] T. M. Weckwerth, J. W. Wilson, R. M. Wakimoto, and N. A. Crook. Horizontal convective rolls: Determining the environmental conditions supporting their existence and characteristics. *Monthly Weather Review*, 125(4):505–526, 1997.
- [202] T. M. Weckwerth, T. W. Horst, and J. W. Wilson. An observational study of the evolution of horizontal convective rolls. *Monthly Weather Review*, 127(9):2160–2179, 1999.
- [203] E. H. Wedemeyer. The unsteady flow within a spinning cylinder. *Journal of Fluid Mechanics*, 20(3):383–399, 1964.
- [204] P. Wei, S. Weiss, and G. Ahlers. Multiple transitions in rotating turbulent Rayleigh-Bénard convection. *Physical Review Letters*, 114(11):114506, 2015.
- [205] P. D. Weidman. On the spin-up and spin-down of a rotating fluid. Part 1. Extending the Wedemeyer model. *Journal of Fluid Mechanics*, 77(4):685–708, 1976.
- [206] S. Weiss and G. Ahlers. Heat transport by turbulent rotating Rayleigh-Bénard convection and its dependence on the aspect ratio. *Journal of Fluid Mechanics*, 684:407, 2011.
- [207] J. Westerweel and F. Scarano. Universal outlier detection for PIV data. *Experiments in Fluids*, 39(6):1096–1100, 2005.
- [208] J. Westerweel, P. F. Geelhoed, and R. Lindken. Single-pixel resolution ensemble correlation for micro-PIV applications. *Experiments in Fluids*, 37(3):375–384, 2004.
- [209] J. Westerweel, G. E. Elsinga, and R. J. Adrian. Particle image velocimetry for complex and turbulent flows. *Annual Review of Fluid Mechanics*, 45:409–436, 2013.
- [210] R. Wiberg and N. Lior. Errors in thermochromic liquid crystal thermometry. *Review of Scientific Instruments*, 75(9):2985–2994, 2004.
- [211] C. Wunsch and R. Ferrari. Vertical mixing, energy, and the general circulation of the oceans. *Annual Review of Fluid Mechanics*, 36:281–314, 2004.
- [212] H.-D. Xi, S. Lam, and K.-Q. Xia. From laminar plumes to organized flows: the onset of large-scale circulation in turbulent thermal convection. *Journal of Fluid Mechanics*, 503:47, 2004.
- [213] Y.-C. Xie and K.-Q. Xia. Dynamics and flow coupling in two-layer turbulent thermal convection. *Journal of Fluid Mechanics*, 728:R1, 2013.

- [214] J. Xiong, R. Idoughi, A. A. Aguirre-Pablo, A. B. Aljedaani, X. Dun, Q. Fu, S. T. Thoroddsen, and W. Heidrich. Rainbow particle imaging velocimetry for dense 3D fluid velocity imaging. *ACM Transactions on Graphics*, 36(4):1–14, 2017.
- [215] J. Xiong, Q. Fu, R. Idoughi, and W. Heidrich. Reconfigurable rainbow PIV for 3D flow measurement. In *2018 IEEE International Conference on Computational Photography*, pages 1–9. IEEE, 2018.
- [216] M. Yoshida and Y. Hamano. Numerical studies on the dynamics of two-layer Rayleigh-Bénard convection with an infinite Prandtl number and large viscosity contrasts. *Physics of Fluids*, 28(11):116601, 2016.
- [217] M. Yoshida, H. Iwamori, Y. Hamano, and D. Suetsugu. Heat transport and coupling modes in Rayleigh-Bénard convection occurring between two layers with largely different viscosities. *Physics of Fluids*, 29(9):096602, 2017.
- [218] Z. Zakaria, M. S. O. Baslasl, A. Samsuri, I. Ismail, A. Supee, and N. B. Haladin. Rollover phenomenon in liquefied natural gas storage tank. *Journal of Failure Analysis and Prevention*, 19(5):1439–1447, 2019.
- [219] F. Zhong, R. E. Ecke, and V. Steinberg. Rotating Rayleigh-Bénard convection: asymmetric modes and vortex states. *Journal of Fluid Mechanics*, 249:135–159, 1993.
- [220] J.-Q. Zhong, M. D. Patterson, and J. S. Wettlaufer. Streaks to rings to vortex grids: generic patterns in transient convective spin up of an evaporating fluid. *Physical Review Letters*, 105(4):044504, 2010.
- [221] W. B. Zimmerman and J. M. Rees. Rollover instability due to double diffusion in a stably stratified cylindrical tank. *Physics of Fluids*, 19(12):123604, 2007.
- [222] G. Zocchi, E. Moses, and A. Libchaber. Coherent structures in turbulent convection, an experimental study. *Physica A: Statistical Mechanics and its Applications*, 166(3):387–407, 1990.