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The Dynamics of Takeovers through Exchange Offers in the Presence of Competition

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Abstract

While a stock offer is outstanding, the bidder must commit real valuation, legal, and regulatory resources to support its equity as acquisition currency. We model this friction as an acquisition-currency verification cost in a continuous-time stochastic game between two bidders and a target with correlated geometric Brownian asset values, in which offer selection, target acceptance, and merger completion are separate decisions. In equilibrium only the selected bidder bears it until acceptance. A selected offer is an American exchange option written by the bidder; homogeneity reduces every stopping problem to a threshold in the bidder–target value ratio. Under a strong-rival condition and a positive verification-cost rate, we characterize a stationary Markov-perfect equilibrium in threshold strategies: the less efficient bidder initiates the contest with a break-even offer at a closed-form challenge threshold, and the more efficient bidder's counteroffer solves a bounded one-dimensional problem. Neither bidder preempts: the target can select no offer and preserve the future contest's continuation value, and verification costs make earlier selected offers strictly less profitable. Competition therefore separates the three stages; a higher verification-cost rate induces a weakly more target-favorable offer and weakly earlier acceptance without moving the merger-completion boundary. Relative to the efficient bidder's single-offer benchmark, competition lowers its incremental takeover value and raises the target's; joint value rises or falls according as the timing gain from earlier access to the efficient merger option exceeds the verification loss. Numerically, this loss is non-monotone in the cost rate: high costs induce terms that collapse selection and acceptance.

Keywords Mergers and acquisitions · Real options games · Optimal stopping · Stochastic game · Exchange offer · Acquisition-currency verification

JEL Classification C61 · C73 · G34

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1 Introduction

An exchange-offer takeover is not a single decision. A bidder chooses when to submit a stock offer and what fraction of the combined firm to retain; a rival may respond; the target selects an offer, which then binds the proposing bidder, and chooses whether and when to accept it; and, once acceptance transfers control, the acquiring bidder chooses when to complete the back-end merger. These decisions need not be made at the same time. Their temporal separation determines how long a transaction remains pending and, together with the offer terms and state dynamics, shapes how takeover gains are allocated and whether an initiated transaction is ultimately completed.

This paper develops a stochastic dynamic game linking these decisions in a competitive stock-financed takeover. Stock consideration is particularly important in large takeovers, for which all-cash financing may be difficult. Yet the real resources a bidder must devote to supporting that equity as acquisition currency while an offer is pending have received little formal attention. Before target shareholders can accept a stock offer, they must evaluate the offered equity itself, and the bidder commits valuation, legal, accounting, regulatory, and managerial resources to sustaining the credibility of that equity as acquisition currency. We represent these real bidder-side resources by an acquisition-currency verification cost incurred while the bidder's own offer is outstanding. In equilibrium, only the selected bidder bears the cost during the interval from offer selection to target acceptance. It is an incremental bidder-side cost, not a transfer to the target or a loss of standalone payouts.

The model has two potential bidders and one target; the bidders differ in integration costs, so one creates a larger merger surplus and is called the more efficient bidder. Asset values follow correlated geometric Brownian motions, and a change of numeraire together with homogeneity reduces the payoff-relevant state to the ratio of the bidder's to the target's asset value, as in two-factor real-option models of investment under correlated sources of uncertainty (e.g., Kijima and Tian 2013). Offers are non-exclusive, and an offer that is not selected may be withdrawn. Only credible offers—those giving the proposer a nonnegative value if selected—are admissible. The target may select one standing offer or none; by selecting none, it preserves the continuation value of future competitive bidding. A selected offer is irrevocable and binds the bidder until acceptance, so the target chooses when to accept, and the acquiring bidder when to complete the merger. We solve the relevant continuation game by backward induction and, under a strong-rival condition, a positive verification-cost rate, and the tie-breaking rules specified below, characterize a stationary Markov-perfect equilibrium with threshold strategies for initial states below the endogenous challenge state. The strong-rival condition keeps the selected response on the branch where acceptance can precede completion.

The analysis yields four main results. First, the bidder that activates the contest need not be the bidder whose offer is selected. In the single-bidder benchmark, proposal, acceptance, and completion occur together, so the verification cost is off the equilibrium path; competition can separate these decisions. Under the stated tie-breaking rules, the less efficient bidder submits an immediately acceptable break-even offer at a closed-form challenge state, and the more efficient bidder then responds. Its offer is selected because that bidder can at least match the value delivered by the challenge while retaining a positive surplus. Thus the less efficient bidder

initiates a contest in which the more efficient bidder's offer is selected, and the response reduces to a scalar participation boundary and a bounded one-dimensional maximization.

Second, if an offer is made before the challenge state, the target can select no offer and preserve the continuation value generated by the future challenge–response equilibrium. Any preemptive offer must therefore be credible and deliver at least this value. With a positive verification-cost rate, the less efficient bidder has no credible target-selectable early offer, and every target-selectable early offer by the more efficient bidder is strictly less profitable than preserving the contest. Hence neither bidder preempts.

Third, competition can separate the stages of the takeover. The contest begins at the less efficient bidder's challenge state, the more efficient bidder's counteroffer may remain outstanding until the target's acceptance boundary, and the merger can complete only upon reaching a later boundary. A higher verification-cost rate induces a weakly more target-favorable offer and weakly earlier acceptance but does not move the completion boundary: the selected offer's terms not only determine the sharing rule but also govern the length of the pending-offer period.

Fourth, competition has clear distributional but ambiguous joint-value effects. Relative to the more efficient bidder's single-offer benchmark, it lowers that bidder's incremental takeover value and raises the target's incremental takeover value; the transfer from a more target-favorable sharing rule cancels in the sum. The remaining comparison balances a nonnegative timing gain from potentially earlier access to the more efficient bidder's merger option against the real verification resources consumed by a pending offer, so competition can raise or lower joint takeover value depending on whether the timing gain dominates the verification loss. The numerical analysis illustrates these effects and shows that the more efficient bidder's counteroffer problem is a closed-interval offer-design problem: a lower-corner response collapses offer selection and target acceptance when verification costs are high, so the realized verification loss is non-monotone in the cost rate.

The paper contributes to the literature on real-option games and the dynamics of mergers and acquisitions. In option-exercise games, the timing of an irreversible decision is determined in equilibrium by the value of waiting and the threat of preemption (Grenadier 2002; Lambrecht and Perraudin 2003); Azevedo and Paxson (2014) and Trigeorgis and Tsekrekos (2018) survey this literature. Applied to takeovers, real-option and stochastic-game models endogenize the timing, terms, and competitive structure of mergers (Lambrecht 2004; Morellec and Zhdanov 2005; Hackbarth and Morellec 2008; Thijssen 2008; Hackbarth and Miao 2012), and game-theoretic option models have been used to study acquisition contracts, bargaining, and risk sharing between merging parties (Lukas et al. 2012; Avinadav et al. 2017). Leung and Kwok (2011, 2016) analyze real-option games between asymmetric firms; the latter characterizes sequential, preemptive, and simultaneous Markov-perfect equilibria and identifies conditions under which preemption does not occur. Relative to this literature, we distinguish offer selection, target acceptance, and back-end merger completion within one equilibrium game between two bidders and a target, and show how the target's continuation value and the acquisition-currency verification cost jointly determine preemption, offer terms, and the timing from selection to acceptance. Our no-preemption result also differs in its mechanism from real-option contests in which even a small threat of

being preempted induces investment at a break-even point (e.g., Hsu and Lambrecht 2007) and from the second-mover advantage that removes preemption in Leung and Kwok (2016): here the less efficient bidder's break-even challenge arises from the credible-offer frontier, proposing early confers no advantage because selection rests with the target, and a positive verification-cost rate makes any earlier selected offer strictly less profitable. Because the challenge state is in closed form and the counteroffer problem is a bounded one-dimensional maximization, the equilibrium and all value comparisons are directly computable.

The paper also relates to research on acquisition currency and information in stock-financed acquisitions. Classic models study how the medium of exchange allocates valuation risk and interacts with asymmetric information and preemptive bidding (Hansen 1987; Fishman 1989; Eckbo et al. 1990), while recent empirical work emphasizes bidder-stock liquidity and target investors' information processing (Huang et al. 2024; Kim et al. 2025). The closest dynamic model is that of Leung and Kwok (2018), who treat the restructuring opportunity as an exchange option between a single acquirer and a passive target and let the target observe the acquirer's synergy only imperfectly, so that the terms and timing of the offer are shaped by signaling. We retain the exchange-option structure but study competition between two bidders under complete information: we do not model hidden bidder types or Bayesian learning; instead we isolate the real resources required while bidder equity is evaluated as acquisition currency and show how this reduced-form friction changes takeover timing, terms, and the allocation of gains.

The remainder of the paper is organized as follows. Section 2 introduces the model, payoffs, verification cost, and protocol; Section 3 characterizes the single-offer benchmark and the competitive equilibrium; Section 4 derives timing, distributional, and joint-value implications; Section 5 presents the numerical analysis; and Section 6 concludes. Essential equilibrium proofs are collected in Appendix A; standard and branch-specific derivations are retained in the Online Resource (Sections S.1–S.8), submitted as Supplementary Information.

2 Model and Game

An exchange-offer takeover is modeled as a sequence of economically distinct decisions. A bidder chooses whether to submit a stock offer and the fraction of the combined firm that it will retain. A rival may respond with its own offer, and an offer that is not selected may be withdrawn. The target selects at most one standing offer, which thereby becomes irrevocable, and chooses when, if ever, to accept it; once acceptance transfers control, the acquiring bidder chooses when to complete the back-end merger. The acquisition-currency verification cost is incurred while an offer is outstanding, from proposal until withdrawal or, once selected, until acceptance.

This section specifies the primitives and institutional rules needed to construct the competitive equilibrium. Subsection 2.1 introduces the firms, merger surplus, and one-dimensional state reduction. Subsection 2.2 defines the exchange-offer payoffs and verification cost, and Subsection 2.3 specifies the competitive bidding protocol. Standard valuation arguments and branch-specific calculations are deferred to the Online Resource.

2.1 Firms, merger surplus, and state reduction

We first describe a generic bidder and the target, suppressing the bidder index until the competitive environment is introduced in Subsection 2.3. Following Hackbarth and Morellec (2008), both firms are financed entirely by equity, and the unit values of capital held by the bidder and the target, denoted by X_t and Y_t, follow geometric Brownian motions under the risk-neutral measure $\mathbb{Q}$:

$$\mathrm{d}X_t = \mu_X X_t \,\mathrm{d}t + \sigma_X X_t \,\mathrm{d}W_t^X, \qquad X_0 = x, \tag{1a}$$

$$\mathrm{d}Y_t = \mu_Y Y_t \,\mathrm{d}t + \sigma_Y Y_t \,\mathrm{d}W_t^Y, \qquad Y_0 = y. \tag{1b}$$

The constants satisfy $\mu_X < r$, $\mu_Y < r$, $\sigma_X > 0$, and $\sigma_Y > 0$, where r is the risk-free rate. The Brownian motions have constant correlation $\rho \in (-1, 1)$. Let $(\mathcal{F}_t)_{t\geq 0}$ be their augmented natural filtration and $\mathcal{T}$ the set of $\mathcal{F}_t$-stopping times.

The bidder and the target hold strictly positive capital stocks $k_X > 0$ and $k_Y > 0$, respectively. Completion of the takeover is irreversible and generates the merger surplus

$$G_M(x, y) = k_Y\{\alpha(x - y) - \varphi y\}, \tag{2}$$

where $\alpha > 0$ governs the operating gain from combining the firms and $\varphi \geq 0$ is the integration-cost parameter. Acceptance of the exchange offer transfers control to the bidder, but the merger surplus is realized only when the bidder subsequently completes the back-end merger.[1]

All payoffs and costs introduced below are homogeneous of degree one in (x, y). We therefore use the target value as the scaling variable and define

$$Z_t := \frac{X_t}{Y_t}, \qquad Z_0 = z := \frac{x}{y}. \tag{3}$$

For a homogeneous payoff G, write $g(z) := G(z, 1)$. Let $\mathbb{Q}^{(Y)}$ denote the equivalent measure defined in Online Resource S.1. For any bounded stopping time $\tau \in \mathcal{T}$, and by localization for the hitting times used below,

$$\mathbb{E}^{\mathbb{Q}}_{x,y}\left[\mathrm{e}^{-r\tau} G(X_\tau, Y_\tau)\right] = y\, \mathbb{E}^{\mathbb{Q}^{(Y)}}_{z}\left[\mathrm{e}^{-(r-\mu_Y)\tau} g(Z_\tau)\right]. \tag{4}$$

Thus every value function can be written as $V(x, y) = y\, v(x/y)$.

Under $\mathbb{Q}^{(Y)}$, the ratio follows

$$\frac{\mathrm{d}Z_t}{Z_t} = (\mu_X - \mu_Y)\,\mathrm{d}t + \sigma_Z \,\mathrm{d}W_t^{(Y)}, \qquad \sigma_Z^2 := \sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2. \tag{5}$$

Because $\mu_X < r$, the discounted ratio $\mathrm{e}^{-(r-\mu_Y)t} Z_t$ is a supermartingale under $\mathbb{Q}^{(Y)}$. Let $\theta > 1$

[1] An exchange offer need not acquire all outstanding target shares. After obtaining control, the bidder can often eliminate the remaining minority shareholders through a squeeze-out or cash-out merger, subject to applicable corporate law. We treat that step as part of the back-end merger. Its separate cost, delay, and execution risk could be modeled but are outside the present analysis.

be the positive root of

$$\frac{1}{2}\sigma_Z^2 m(m-1) + (\mu_X - \mu_Y)m - (r - \mu_Y) = 0. \tag{6}$$

For an upper boundary $a > z$ and $\tau_a := \inf\{t \geq 0 : Z_t \geq a\}$, the standard perpetual hitting-time identity is

$$\mathbb{E}_z^{\mathbb{Q}^{(Y)}}\left[\mathrm{e}^{-(r-\mu_Y)\tau_a};\, \tau_a < \infty\right] = \left(\frac{z}{a}\right)^{\theta}. \tag{7}$$

Accordingly, all stopping rules below can be expressed as thresholds in the single state variable Z_t. The change of measure, supermartingale calculation, and optional-sampling details are collected in Online Resource S.1.

2.2 Exchange offers and acquisition-currency verification

An exchange offer specifies a retained share $\xi \in (0,1)$: after acceptance, the bidder holds the fraction ξ of the post-exchange block, while the target shareholders receive the fraction $1-\xi$. Because a selected offer binds the bidder while the target chooses whether and when to accept, the bidder is the writer and the target shareholders are the holders of the resulting option. The stock-swap component of the transaction is

$$G_E(x, y, \xi) = (1-\xi)k_X x - \xi k_Y y, \tag{8a}$$

$$g_E(z, \xi) = (1-\xi)k_X z - \xi k_Y. \tag{8b}$$

The associated pure-exchange threshold is

$$z_E(\xi) = \frac{\theta}{\theta - 1}\frac{\xi k_Y}{(1-\xi)k_X}. \tag{9}$$

The reduced merger surplus and the value of the back-end merger option are

$$g_M(z) = k_Y\{\alpha(z-1) - \varphi\}, \tag{10a}$$

$$z_M = \frac{\theta}{\theta - 1}\frac{\alpha + \varphi}{\alpha}, \tag{10b}$$

$$v_M(z) = \begin{cases} g_M(z_M)\left(\dfrac{z}{z_M}\right)^{\theta}, & 0 < z < z_M, \\ g_M(z), & z \geq z_M. \end{cases} \tag{10c}$$

Thus $V_M(x, y) = y\, v_M(x/y)$.

To describe how the post-exchange block is allocated between the parties, define its value by

$$\mathcal{B}(x, y) := k_X x + k_Y y + V_M(x, y). \tag{11}$$

At acceptance, the target and bidder obtain the incremental gains

$$G_A(x,y,\xi) := (1-\xi)\mathcal{B}(x,y) - k_Y y = (1-\xi)V_M(x,y) + G_E(x,y,\xi), \tag{12a}$$

$$G_P(x,y,\xi) := \xi\mathcal{B}(x,y) - k_X x = \xi V_M(x,y) - G_E(x,y,\xi). \tag{12b}$$

When acceptance and the back-end merger occur simultaneously, their reduced payoffs become

$$g_M^Y(z,\xi) := (1-\xi)g_M(z) + g_E(z,\xi), \tag{13a}$$

$$g_M^X(z,\xi) := \xi g_M(z) - g_E(z,\xi). \tag{13b}$$

Throughout, we assume $\alpha+\varphi>1$ and restrict attention to bidder-viable retained shares

$$\xi_{\min} := \frac{k_X}{\alpha k_Y + k_X}, \qquad \Xi := (\xi_{\min}, 1). \tag{14}$$

For $\xi \in \Xi$, both simultaneous-exercise payoffs have positive call-type exercise values. Their thresholds are

$$z_M^Y(\xi) = \frac{\theta}{\theta-1}\frac{k_Y\{(1-\xi)(\alpha+\varphi)+\xi\}}{(1-\xi)(\alpha k_Y + k_X)}, \tag{15a}$$

$$z_M^X(\xi) = \frac{\theta}{\theta-1}\frac{\xi k_Y(\alpha+\varphi-1)}{\xi\alpha k_Y - (1-\xi)k_X}. \tag{15b}$$

The critical retained share

$$\widetilde{\xi} := \frac{k_X(\alpha+\varphi)}{k_X(\alpha+\varphi)+\alpha k_Y} \tag{16}$$

satisfies $\xi_{\min} < \widetilde{\xi} < 1$ and aligns all four primitive thresholds:

$$z_E(\widetilde{\xi}) = z_M^Y(\widetilde{\xi}) = z_M = z_M^X(\widetilde{\xi}). \tag{17}$$

For $\xi \le \widetilde{\xi}$, the exchange threshold lies below the merger threshold and acceptance can precede completion; for $\xi > \widetilde{\xi}$, the target waits until acceptance and completion can occur together. Online Resource S.2 gives the complete ordering and its proof.

The target's acceptance problem. Conditional on a selected offer $\xi \in \Xi$, the target chooses when to realize G_A. Its reduced value is

$$v_A(z,\xi) := \sup_{\tau_A \in \mathcal{T}} \mathbb{E}_z^{\mathbb{Q}^{(Y)}}\left[\mathrm{e}^{-(r-\mu_Y)\tau_A}\{(1-\xi)v_M(Z_{\tau_A}) + g_E(Z_{\tau_A},\xi)\}\right], \tag{18}$$

where the option of never accepting yields zero. The optimal acceptance boundary is

$$z_A(\xi) = z_E(\xi) \wedge z_M^Y(\xi) = \begin{cases} z_E(\xi), & \xi_{\min} < \xi \le \widetilde{\xi}, \\ z_M^Y(\xi), & \widetilde{\xi} < \xi < 1. \end{cases} \tag{19}$$

Accordingly, $\tau_A^\xi := \inf\{t \geq 0 : Z_t \geq z_A(\xi)\}$. We call the first case in (19), where $\xi \leq \widetilde{\xi}$ and $z_A(\xi) = z_E(\xi)$, the *exchange-threshold branch*, and the second, where $\xi > \widetilde{\xi}$ and $z_A(\xi) = z_M^Y(\xi)$, the *simultaneous-exercise branch*. The competitive equilibrium uses the exchange-threshold branch, on which

$$v_A(z,\xi) = \begin{cases} (1-\xi)v_M(z) + g_E(z_E(\xi),\xi)\left(\dfrac{z}{z_E(\xi)}\right)^\theta, & 0 < z < z_E(\xi), \\ (1-\xi)v_M(z) + g_E(z,\xi), & z_E(\xi) \leq z < z_M, \\ g_M^Y(z,\xi), & z \geq z_M. \end{cases} \tag{20}$$

The simultaneous-exercise branch and the verification of (19) are given in Online Resource S.3.

Verification cost and the bidder's proposal payoff. Suppose that an offer is proposed at τ_P and, if selected, remains outstanding until the target accepts it at $\tau_A \geq \tau_P$. While it is outstanding, the proposing bidder uses real resources at the flow rate

$$\mathrm{d}C_t^V = \lambda k_X X_t \,\mathrm{d}t, \qquad \tau_P \leq t < \tau_A, \tag{21}$$

where $\lambda \geq 0$; an offer that is withdrawn before being selected accrues this cost only until it is withdrawn, whereas a selected offer is irrevocable and accrues it until acceptance. This is an incremental bidder-side verification cost, not a payment to the target, not a transfer between the parties, and not a loss of ordinary standalone payouts. If an offer is made at $z < a$ and accepted when Z first reaches a, its reduced expected cost is

$$v_C(z,a) = \frac{\lambda k_X}{r-\mu_X}\left\{z - a\left(\frac{z}{a}\right)^\theta\right\}, \qquad 0 < z < a, \tag{22}$$

with $v_C(z,a) = 0$ for $z \geq a$. Online Resource S.1 derives this identity.

The bidder's ordinary standalone payouts are not part of this cost: since $k_X X_t$ is an ex-payout value with payout yield $\delta_X = r - \mu_X$, those payouts are fully credited over any stopping time (Online Resource S.1). Using (12b), the reduced payoff from proposing immediately at state z is

$$\Pi(z,\xi) = \mathbb{E}_z^{\mathbb{Q}^{(Y)}}\left[\mathrm{e}^{-(r-\mu_Y)\tau_A^\xi}\{\xi v_M(Z_{\tau_A^\xi}) - g_E(Z_{\tau_A^\xi},\xi)\}\right] - v_C(z, z_A(\xi)). \tag{23}$$

On the exchange-threshold branch, the immediate-proposal payoff is

$$\Pi(z,\xi) = \begin{cases} \{\xi v_M(z_E(\xi)) - g_E(z_E(\xi),\xi)\}\left(\dfrac{z}{z_E(\xi)}\right)^\theta - v_C(z, z_E(\xi)), & 0 < z < z_E(\xi), \\ \xi v_M(z) - g_E(z,\xi), & z_E(\xi) \leq z < z_M, \\ g_M^X(z,\xi), & z \geq z_M. \end{cases} \tag{24}$$

Finally, for a fixed retained share, the bidder's option to choose the proposal time is

$$v_P(z,\xi) := \sup_{\tau_P \in \mathcal{T}} \mathbb{E}_z^{\mathbb{Q}^{(Y)}} \left[\mathrm{e}^{-(r-\mu_Y)\tau_P} \Pi(Z_{\tau_P}, \xi) \right]. \tag{25}$$

The full piecewise formulas and standard optimal-stopping verification are in Online Resource S.3. The verification cost has a key timing implication. For a fixed offer that is selected upon proposal, proposing it strictly before its acceptance boundary starts verification earlier without changing the eventual acceptance payoff. Thus, when $\lambda > 0$, early proposal is strictly dominated by waiting, whereas at $\lambda = 0$ the bidder is indifferent. Appendix A.1 formalizes this result; the same timing logic underlies the no-preemption result in Subsection 3.3.

2.3 Competitive bidding protocol

There are two potential bidders, indexed by $i \in \{w, \ell\}$. They have the same standalone asset dynamics, capital stock k_X, and operating-gain parameter α, but differ in the integration cost:

$$\alpha_w = \alpha_\ell =: \alpha > 0, \qquad 0 \le \varphi_w < \varphi_\ell, \qquad \alpha + \varphi_w > 1. \tag{26}$$

It follows that $\alpha + \varphi_\ell > 1$ and

$$g_{M,w}(z) > g_{M,\ell}(z) \qquad \text{for every } z > 0.$$

We call w the more efficient bidder and ℓ the less efficient bidder. These labels refer only to merger surplus; which bidder initiates the contest and which bidder's offer is selected are equilibrium outcomes. Conditional on acceptance, the selected bidder becomes the acquirer.

We append the bidder index to every merger-dependent object introduced in Subsection 2.2. Thus $v_{M,i}$, $z_{M,i}$, $z^Y_{M,i}$, $z^X_{M,i}$, $\widetilde{\xi}_i$, $z_{A,i}$, $v_{A,i}$, Π_i, $v_{P,i}$, and $z_{P,i}$ are obtained from their generic counterparts by using φ_i. Because the bidders share α, k_X, and the standalone process X, the feasible set Ξ, the exchange component g_E, its threshold z_E, and the verification-cost function v_C are common to both bidders.

Credible offers. At any state z, bidder i, provided it has no standing offer, may propose an exchange offer specifying a retained share $\xi_i \in \Xi$. Because a selected offer binds the proposing bidder until the target accepts, and the target accepts optimally, we use *credible* in the bidder-rational sense: an offer is credible if the bidder's value from having it selected is nonnegative, and we restrict bidder i's admissible offers to

$$\mathcal{C}_i(z) := \{\xi \in \Xi : \Pi_i(z,\xi) \ge 0\}. \tag{27}$$

If a credible offer $\xi_i \in \mathcal{C}_i(z)$ is selected at z, the proposing bidder's reduced incremental continuation value is $\Pi_i(z,\xi_i)$ and the target's acceptance-option value is $v_{A,i}(z,\xi_i)$; both are evaluated at the selection date, whereas acceptance itself occurs at the boundary $z_{A,i}(\xi_i)$, either immediately or at a later date. Condition (27) rules out an offer that would serve as a costless threat but

would not be bidder-rational once selected.

Bidding protocol. The competitive protocol has three rules.

(P1) *Proposal.* At any state either bidder may propose a credible offer. Proposals are non-exclusive: both bidders' offers may stand at the same time.

(P2) *Selection, irrevocability, and withdrawal.* At any state the target may select one standing offer or select none. A selected offer is irrevocable: it binds the proposing bidder until the target accepts. An offer that has not been selected may be withdrawn by its bidder at any time; a withdrawn offer accrues no further verification cost.

(P3) *Acceptance and completion.* Once an offer is selected, the bidding phase closes for the remainder of the game, so neither bidder may submit another offer. The target chooses when to accept the selected offer; acceptance transfers control, after which the acquiring bidder chooses when to complete the back-end merger.

A bidder incurs the acquisition-currency verification cost while its own offer is outstanding, that is, from the proposal time until the offer is withdrawn or, once the offer is selected, until the target accepts it. Selection and acceptance are distinct: selection makes the offer binding, whereas acceptance is the target's optimal exercise of the resulting option and occurs at the acceptance boundary $z_{A,i}(\xi)$. Irrevocability after selection is what makes $v_{A,i}(z, \xi)$ the value of an American-style exchange option written by the bidder. Were a selected offer revocable, the target would instead hold a cancellable, game-type option, and the bidder could abandon a pending offer once the state deteriorated, so the verification friction could be undone by abandonment rather than borne until acceptance.

Within a bidding round, proposal, any response, and the target's selection decision occur sequentially, but no calendar time elapses; hence Z remains unchanged throughout the round.

Nothing in (P1)–(P3) depends on the pre-selection history. Before any offer is selected, a withdrawn offer may in principle be proposed again at a later no-offer state. If the target selects no offer, the standing offers do not disappear automatically; each remains outstanding until its bidder withdraws it. Only after all unselected offers have been withdrawn does the game return to the same no-offer state. In the equilibrium characterized below, these withdrawals occur immediately. We identify an instantaneous no-selection round followed by those withdrawals with inactivity, thereby ruling out repeated zero-duration rounds. We therefore restrict attention to stationary strategies as functions of the payoff-relevant state.

Tie-breaking. Each player first maximizes its primary continuation payoff. Among effective proposal times yielding the same maximal payoff, a bidder chooses the earliest one; a proposal time is effective if the resulting instantaneous bidding round ends with a selected offer. Among offer terms yielding the same bidder payoff, the bidder chooses the one that gives the target the largest acceptance value. An instantaneous round in which no offer is selected and all standing offers are immediately withdrawn by their bidders is identified with inactivity, and a proposal or counteroffer that cannot affect the target's current selection and is immediately withdrawn is

identified with not proposing. If both bidders act at the same state, bidder ℓ's offer is processed first and bidder w retains the immediate counter-opportunity. If the target is indifferent between two standing offers, it selects bidder w's offer; if it is indifferent between selecting the best standing offer and selecting none, it selects the best standing offer.

The payoff-relevant state observed by all players consists of the ratio Z_t and the current phase of the takeover: a competitive no-offer phase, a phase in which one or more offers stand and may be countered, selected, or withdrawn, a selected offer awaiting acceptance, or an accepted offer awaiting merger completion. The phase records the identity and terms of any standing or selected offer and, after acceptance, the identity of the acquiring bidder; every restriction in (P1)–(P3) is carried by this state rather than by the path leading to it.

A stationary Markov strategy specifies each bidder's proposal timing and offer terms, its withdrawal rule, the target's selection and acceptance rules, and the acquiring bidder's merger-completion rule. The equilibrium theorem below is stated for initial states below the endogenous challenge state characterized in Proposition 2. The relevant continuation domain also includes the off-path no-offer states needed to evaluate delayed challenges by bidder ℓ; Appendix A.4 specifies the corresponding responses. The equilibrium is Markov perfect on this relevant continuation game; a global characterization of histories in which players disregard the tie-breaking refinement indefinitely is not claimed. Online Resource S.5 records the formal phase and strategy notation, and the equilibrium-specific off-path actions and the one-deviation verification are given in Appendix A.4.

3 Competitive Equilibrium

Section 2 separated the post-selection stopping problems from the rules of the bidding game. We now solve the relevant continuation game by backward induction. Subsection 3.1 first reduces the no-competition problem to a benchmark. Subsection 3.2 then identifies the less efficient bidder's earliest credible challenge and the more efficient bidder's optimal counteroffer. Finally, Subsection 3.3 constructs the target's continuation value from selecting no earlier offer, uses that value to rule out preemption, and states the competitive equilibrium.

The benchmark is useful because it isolates what competition changes. With only one bidder, optimal proposal timing leaves no interval between proposal, acceptance, and merger completion, so the acquisition-currency verification cost is not incurred on the equilibrium path. Competition can instead induce bidder w's offer to be selected at a state below its single-offer boundary. The selected offer may then remain outstanding, making both the sharing rule and the verification cost relevant for equilibrium timing.

3.1 Single-offer benchmark

Consider bidder $i \in \{w, \ell\}$ in isolation. The target's acceptance problem and bidder i's fixed-share proposal problem are those defined in Subsection 2.2, with the merger-dependent objects evaluated at φ_i. The following proposition collects the threshold and share-choice results needed later; the standard stopping calculations are retained in Sections S.3 and S.4 of the Online

Resource.

Proposition 1 (Single-offer benchmark)**.** *Fix bidder $i \in \{w, \ell\}$.*

(i) Fixed retained share. *For every $\xi \in \Xi$, the target accepts at*

$$z_{A,i}(\xi) = z_E(\xi) \wedge z^Y_{M,i}(\xi) = \begin{cases} z_E(\xi), & \xi_{\min} < \xi \leq \widetilde{\xi}_i, \\ z^Y_{M,i}(\xi), & \widetilde{\xi}_i < \xi < 1. \end{cases} \tag{28}$$

An optimal proposal policy is the first hitting time of

$$z_{P,i}(\xi) = z_{A,i}(\xi) \vee z^X_{M,i}(\xi) = \begin{cases} z^X_{M,i}(\xi), & \xi_{\min} < \xi \leq \widetilde{\xi}_i, \\ z^Y_{M,i}(\xi), & \widetilde{\xi}_i < \xi < 1. \end{cases} \tag{29}$$

In particular, $z_{P,i}(\xi) \geq z_{M,i}$ and $z_{P,i}(\xi) \geq z_{A,i}(\xi)$. When bidder i proposes at this boundary, the target accepts immediately and bidder i completes the back-end merger at the same state. The selected offer therefore remains outstanding for zero time and incurs no verification cost.

(ii) Optimal retained share. *Fix an initial state $0 < z < z_{M,i}$ and define*

$$\Xi^*_i(z) := \arg\max_{\xi \in \Xi} v_{P,i}(z, \xi). \tag{30}$$

Then $\Xi^*_i(z)$ is nonempty and compact, and

$$\Xi^*_i(z) \subset [\widetilde{\xi}_i, 1). \tag{31}$$

Consistently with the target-favorable tie-breaking rule, define the selected monopoly share by

$$\xi^*_i(z) := \min \Xi^*_i(z), \tag{32}$$

and suppress its argument when the initial state is fixed. Because the continuation value factors as z^θ for every $0 < z < z_{M,i}$, the maximizing set and the selected share are independent of the particular initial state in this region. Let

$$z_{W,i} := z_{P,i}(\xi^*_i) = z_{A,i}(\xi^*_i) \geq z_{M,i}. \tag{33}$$

A single bidder optimally waits until

$$\tau_{W,i} := \inf\{t \geq 0 : Z_t \geq z_{W,i}\}, \tag{34}$$

then proposes ξ^*_i; proposal, target acceptance, and merger completion occur at $\tau_{W,i}$ whenever it is finite. The maximizing set, the selected share, its attained boundary, and the attained value are independent of λ. If $\lambda > 0$, proposing the same selected offer strictly below its acceptance boundary is dominated by waiting; when $\lambda = 0$, such an earlier proposal can be payoff-equivalent.

The benchmark does not render the sequential structure redundant. Rather, it shows that the stages collapse only because a lone bidder can wait for its privately optimal exercise state.

Competition will force bidder w to compare this option with responding to a credible challenge from a rival at an earlier state. That comparison reopens the interval between selection and acceptance and makes the verification friction economically active.

3.2 Credible challenge and optimal counteroffer

We solve the instantaneous contest at the earliest state at which the less efficient bidder can submit an immediately acceptable credible challenge. We first identify bidder ℓ's break-even offer and then solve bidder w's response, using the value delivered by that challenge as the target's participation constraint.

Proposition 2 (Earliest credible challenge)**.** *There is a unique retained share $\xi_\ell^0 \in \Xi$ for which bidder ℓ breaks even when its offer is made at its own acceptance boundary. It lies on the exchange-threshold branch and determines*

$$\xi_\ell^0 \in (\xi_{\min}, \widetilde{\xi}_\ell), \qquad z_L := z_E(\xi_\ell^0) = z_{A,\ell}(\xi_\ell^0), \tag{35a}$$

$$z_L = z_{M,\ell}(\alpha + \varphi_\ell)^{-1/\theta} = \frac{\theta}{\theta - 1} \frac{(\alpha + \varphi_\ell)^{1-1/\theta}}{\alpha}, \tag{35b}$$

$$\xi_\ell^0 = \frac{(\theta - 1) k_X z_L}{(\theta - 1) k_X z_L + \theta k_Y}. \tag{35c}$$

At this pair,

$$\Pi_\ell(z_L, \xi_\ell^0) = 0, \qquad \{\xi_\ell^0\} = \arg\max_{\xi \in \mathcal{C}_\ell(z_L)} v_{A,\ell}(z_L, \xi). \tag{36}$$

The offer is accepted immediately, incurs no verification cost, and gives the target

$$A_L := v_{A,\ell}(z_L, \xi_\ell^0) = v_{M,\ell}(z_L) = \frac{k_Y}{\theta - 1}. \tag{37}$$

Moreover, z_L is the earliest state at which bidder ℓ can submit an immediately acceptable credible offer:

$$z_L = \inf \left\{ p > 0 : \begin{array}{l} \text{there exists } \xi \in \Xi \text{ such that } z_{A,\ell}(\xi) \le p, \\ \text{and } \Pi_\ell(p, \xi) \ge 0 \end{array} \right\}. \tag{38}$$

Proposition 2 identifies the first state at which bidder ℓ can submit an immediately acceptable credible challenge. It does not by itself select the equilibrium contest date: this requires ruling out both an earlier target-selectable offer and a gain from delaying the challenge. The former is resolved in Subsection 3.3; the latter follows from the next comparison. In the closed forms φ_ℓ shifts the challenge state and its terms, while A_L is pinned down by bidder ℓ's break-even condition.

Delayed challenges. For every $p \ge z_L$, define bidder ℓ's immediately acceptable break-even share by

$$\widehat{\xi}_\ell(p) := \frac{k_X p}{k_X p + k_Y + v_{M,\ell}(p)}. \tag{39}$$

Then $\widehat{\xi}_\ell(z_L) = \xi_\ell^0$ and

$$\Pi_\ell\big(p, \widehat{\xi}_\ell(p)\big) = 0, \qquad \max_{\xi \in \mathcal{C}_\ell(p)} v_{A,\ell}(p, \xi) = v_{M,\ell}(p). \tag{40}$$

Consequently,

$$\frac{1}{p^\theta} \max_{\xi \in \mathcal{C}_\ell(p)} v_{A,\ell}(p, \xi) = \frac{v_{M,\ell}(p)}{p^\theta} \leq \frac{A_L}{z_L^\theta}, \tag{41}$$

with equality for $z_L \leq p \leq z_{M,\ell}$ and strict inequality for $p > z_{M,\ell}$. Moreover, if bidder w submits the same retained share, the offer is credible and gives the target a strictly larger value:

$$\Pi_w\big(p, \widehat{\xi}_\ell(p)\big) = \widehat{\xi}_\ell(p)\{v_{M,w}(p) - v_{M,\ell}(p)\} > 0, \tag{42}$$

$$v_{A,w}\big(p, \widehat{\xi}_\ell(p)\big) - v_{A,\ell}\big(p, \widehat{\xi}_\ell(p)\big) = \{1 - \widehat{\xi}_\ell(p)\}\{v_{M,w}(p) - v_{M,\ell}(p)\} > 0. \tag{43}$$

Thus every credible challenge by bidder ℓ at $p \geq z_L$ can be displaced by a target-preferred credible response from bidder w, and delaying the challenge cannot raise bidder ℓ's primary takeover payoff above zero. Appendix A.2 proves these statements.

Conditional on the challenge (z_L, ξ_ℓ^0), bidder w solves

$$\max_{\xi \in \Xi} \Pi_w(z_L, \xi) \quad \text{subject to} \quad v_{A,w}(z_L, \xi) \geq A_L, \qquad \Pi_w(z_L, \xi) \geq 0.$$

The first constraint makes the counteroffer weakly preferable to the initial offer; the second is bidder w's participation, or credibility, constraint. We focus on the regime in which all relevant responses induce acceptance before the back-end merger.

Assumption 1 (Strong-rival regime)**.** *The challenge by bidder ℓ is sufficiently valuable to the target that*

$$A_L > v_{A,w}(z_L, \widetilde{\xi}_w). \tag{44}$$

Because $v_{A,w}(z_L, \xi)$ is strictly decreasing in ξ, Assumption 1 places the target-participation boundary strictly below $\widetilde{\xi}_w$. Hence the relevant counteroffers remain on the exchange-threshold branch. Online Resource S.6 gives a primitive equivalent of the assumption.

For $\xi \geq \xi_\ell^0$, define the discount factor from the acceptance boundary to the challenge state by

$$H_L(\xi) := \left(\frac{z_L}{z_E(\xi)}\right)^\theta = \left\{\frac{\xi_\ell^0(1-\xi)}{(1-\xi_\ell^0)\xi}\right\}^\theta, \tag{45}$$

and separate the unit cost from the expected verification kernel:

$$c_L(\xi) := \frac{k_X}{r - \mu_X}\{z_L - z_E(\xi)H_L(\xi)\}, \qquad v_C(z_L, z_E(\xi)) = \lambda c_L(\xi). \tag{46}$$

Proposition 3 (Optimal counteroffer)**.** *Suppose bidder ℓ submits the challenge in Proposition 2, and maintain Assumption 1.*

(i) Target-participation boundary. *There is a unique $\bar{\xi}_w$ satisfying*

$$v_{A,w}(z_L, \bar{\xi}_w) = A_L, \qquad \xi_\ell^0 < \bar{\xi}_w < \widetilde{\xi}_w. \tag{47}$$

Moreover,

$$v_{A,w}(z_L, \xi) \geq A_L \quad \Longleftrightarrow \quad \xi \leq \bar{\xi}_w. \tag{48}$$

(ii) One-dimensional response problem. *For* $\xi \in [\xi^0_\ell, \bar{\xi}_w]$,

$$v_{A,w}(z_L, \xi) = (1-\xi)v_{M,w}(z_L) + \xi A_L H_L(\xi), \tag{49}$$

$$\psi_w(\xi; \lambda) := \Pi_w(z_L, \xi) = \xi v_{M,w}(z_L) - \xi A_L H_L(\xi) - \lambda c_L(\xi). \tag{50}$$

Thus

$$\mathcal{R}_w(\lambda) := \arg\max_{\xi \in [\xi^0_\ell, \bar{\xi}_w]} \psi_w(\xi; \lambda) \tag{51}$$

is nonempty and compact. The tie-breaking rule selects

$$\xi^R_w(\lambda) := \min \mathcal{R}_w(\lambda). \tag{52}$$

Bidder w's participation constraint is slack at every maximizer, and

$$\xi^0_\ell \leq \xi^R_w(\lambda) \leq \bar{\xi}_w < \widetilde{\xi}_w. \tag{53}$$

The target therefore selects bidder w's counteroffer whenever the two offers are compared; if the two target values coincide, the tie-breaking rule selects bidder w.

(iii) Verification-cost comparative statics. *For* $0 \leq \lambda_1 < \lambda_2$,

$$\xi^R_w(\lambda_2) \leq \xi^R_w(\lambda_1), \tag{54}$$

and

$$\xi^R_w(0) = \bar{\xi}_w, \qquad \lim_{\lambda \to \infty} \xi^R_w(\lambda) = \xi^0_\ell. \tag{55}$$

The upper boundary $\bar{\xi}_w$ matches the challenger's target value but need not maximize bidder w's payoff. Equation (50) displays the trade-off: a larger retained share extracts more merger value but raises the acceptance boundary and expected verification expenditure, so, since c_L is strictly increasing, a higher unit cost induces a weakly smaller, more target-favorable share. Propositions 2 and 3 then give

$$z_L = z_E(\xi^0_\ell) \leq z_E(\xi^R_w) < z_{M,w}, \tag{56}$$

where we suppress the argument of ξ^R_w when $\lambda > 0$ is fixed. The target may therefore accept after selection but before merger completion.

3.3 Competitive continuation, no-preemption, and equilibrium

Propositions 2 and 3 solve the bidding round conditional on reaching z_L. To close the game, we compute the continuation value available to the target from selecting no earlier offer. At z_L, bidder ℓ challenges and bidder w responds; the relevant target value is therefore the acceptance-option value of bidder w's selected counteroffer. We discount this value below z_L and use it to

verify that neither bidder acts earlier.

At the challenge state, define

$$A_C(\lambda) := v_{A,w}\big(z_L, \xi_w^R(\lambda)\big). \tag{57}$$

Thus A_C is the target's acceptance-option value after bidder w's equilibrium counteroffer is selected; A_L is the value delivered by bidder ℓ's break-even challenge before the response. For the maintained $\lambda > 0$, write $A_C := A_C(\lambda)$, and let

$$\tau_L := \inf\{t \geq 0 : Z_t \geq z_L\}. \tag{58}$$

The candidate target continuation value below the challenge state is

$$v_A^C(z) := \left(\frac{z}{z_L}\right)^\theta A_C, \qquad 0 < z \leq z_L. \tag{59}$$

An offer made at $p < z_L$ can be selected only if it is credible and gives the target at least this continuation value. Define

$$\mathcal{S}_i(p) := \{\xi \in \mathcal{C}_i(p) : v_{A,i}(p, \xi) \geq v_A^C(p)\}, \qquad i \in \{w, \ell\}. \tag{60}$$

Finally, let

$$R_w := \Pi_w(z_L, \xi_w^R) > 0, \qquad v_w^C(z) := \left(\frac{z}{z_L}\right)^\theta R_w. \tag{61}$$

Bidder w obtains $v_w^C(z)$ by preserving the contest and responding at z_L; bidder ℓ's corresponding continuation payoff is zero.

Proposition 4 (Competitive continuation and no-preemption)**.** *Maintain Assumption 1.*

(i) Competitive continuation. *The target value at the challenge state is*

$$A_C = (1 - \xi_w^R) v_{M,w}(z_L) + \xi_w^R A_L H_L(\xi_w^R), \tag{62}$$

and satisfies

$$A_L \leq A_C \leq v_{A,w}(z_L, \xi_\ell^0) = A_L + (1 - \xi_\ell^0)\{v_{M,w}(z_L) - v_{M,\ell}(z_L)\}. \tag{63}$$

Moreover,

$$A_C > A_L \quad \Longleftrightarrow \quad \xi_w^R < \bar{\xi}_w. \tag{64}$$

The map $A_C(\lambda)$ is weakly increasing, with

$$A_C(0) = A_L, \qquad \lim_{\lambda \to \infty} A_C(\lambda) = v_{A,w}(z_L, \xi_\ell^0) > A_L. \tag{65}$$

The value in (59) is the target's continuation value from selecting no current offer and waiting

for the future challenge–response subgame. For every $0 < z < z_L$,

$$v_{A,w}(z, \xi_w^R) = v_A^C(z), \tag{66}$$

$$v_{A,\ell}(z, \xi_\ell^0) = \left(\frac{z}{z_L}\right)^\theta A_L \leq v_A^C(z). \tag{67}$$

Consequently,

$$v_{A,w}(z, \xi) \geq v_A^C(z) \quad \Longleftrightarrow \quad \xi \leq \xi_w^R, \tag{68}$$

$$v_{A,\ell}(z, \xi) \geq v_A^C(z) \quad \Longrightarrow \quad \xi \leq \xi_\ell^0. \tag{69}$$

The last inequality is strict whenever $A_C > A_L$.

(ii) No preemption. *Suppose* $\lambda > 0$. *For every* $0 < p < z_L$,

$$\mathcal{S}_\ell(p) = \varnothing. \tag{70}$$

For bidder w, *every target-selectable offer is strictly dominated by preserving the contest:*

$$\Pi_w(p, \xi) < \left(\frac{p}{z_L}\right)^\theta \Pi_w(z_L, \xi_w^R) = v_w^C(p), \qquad \xi \in \mathcal{S}_w(p). \tag{71}$$

More generally, for every $0 < z < z_L$,

$$\sup_{\substack{z<p<z_L \\ \xi \in \mathcal{S}_w(p)}} \left(\frac{z}{p}\right)^\theta \Pi_w(p, \xi) = \left(\frac{z}{z_L}\right)^\theta R_w, \tag{72}$$

but no $p < z_L$ *attains the supremum.*

At any $p < z_L$, an early offer by bidder w must deliver at least $v_A^C(p)$, the value of waiting for the equilibrium response at z_L. This limits the retained share to $\xi \leq \xi_w^R$; among such offers, positive verification costs make early selection strictly less profitable than waiting.

The proof uses a fixed-offer timing lemma—after normalizing by the hitting-time factor, delaying a selected stock offer strictly raises the bidder's payoff when $\lambda > 0$. Bidder ℓ faces a tighter restriction: the terms needed to reach the target's continuation value are not credible before z_L.

Remark 1 (Timing of contest initiation)**.** *Proposition 4 rules out an effective challenge before* z_L *when* $\lambda > 0$. *At* z_L, *bidder* ℓ*'s challenge is displaced by bidder* w, *so bidder* ℓ *obtains zero. Equations* (40)–(43) *show that every later credible challenge is likewise displaced and gives bidder* ℓ *the same primary payoff. Primary payoffs therefore do not select among the effective challenge times at and above* z_L. *The earliest-effective-proposal tie-breaking rule selects* z_L. *This is an equilibrium-selection refinement, not a priority right and not a forfeiture of future proposal opportunities.*

Remark 2 (Zero verification cost)**.** *When* $\lambda = 0$, *Proposition 3 gives* $\xi_w^R(0) = \bar{\xi}_w$. *For every*

$0 < p < z_L$, *the fixed-offer payoff associated with these terms satisfies*

$$\frac{\Pi_w(p, \xi_w^R(0))}{p^\theta} = \frac{\Pi_w(z_L, \xi_w^R(0))}{z_L^\theta}. \tag{73}$$

Thus early proposal and waiting are payoff-equivalent in the frictionless benchmark. The target's continuation-value constraint alone therefore does not rule out an early selected offer that delivers the same target value. Any positive verification-cost rate, however small, breaks this indifference and makes the inequality in (71) *strict.*

Theorem 1 (Competitive takeover equilibrium)**.** *Suppose that Assumption 1 holds,* $\lambda > 0$*, the initial state satisfies* $0 < z < z_L$*, and the earliest-effective-proposal tie-break in Subsection 2.3 is applied. The relevant continuation game admits a stationary Markov-perfect equilibrium with the following on-path outcome.*

1. *No bidder initiates while* $Z_t < z_L$*. The first bidding round occurs at* τ_L *in* (58)*.*
2. *At* τ_L*, bidder* ℓ *submits the break-even offer* ξ_ℓ^0*. Bidder* w *responds immediately with* ξ_w^R*; the target selects bidder* w*'s offer, which becomes irrevocable, and bidder* ℓ *withdraws its unselected offer.*
3. *After selection, the target accepts at*

$$\tau_A^w := \inf\{t \geq \tau_L : Z_t \geq z_{A,w}(\xi_w^R)\}, \tag{74}$$

and bidder w *completes the back-end merger at*

$$\tau_M^w := \inf\{t \geq \tau_A^w : Z_t \geq z_{M,w}\}, \tag{75}$$

provided that the respective hitting times are finite.

Before the contest, the reduced continuation values of bidder w*, bidder* ℓ*, and the target are* $v_w^C(z)$*, zero, and* $v_A^C(z)$*, respectively. Under the tie-breaking rules in Subsection 2.3, the on-path challenge state, initial offer, selected counteroffer, and selected bidder are unique within stationary threshold strategies. Conditional on acceptance, the selected bidder is also the acquirer. Without the earliest-effective-proposal tie-break, primary takeover payoffs do not uniquely select among bidder* ℓ*'s zero-payoff effective challenge times at and above* z_L*; the contest date and the corresponding challenge terms therefore need not be unique, although any such challenge is displaced by bidder* w*.*

Appendices A.3 and A.4 prove Proposition 4 and verify Theorem 1. The equilibrium closes two loops: the target's pre-contest continuation value is generated by the future challenge–response equilibrium and bidder w cannot profitably bypass that equilibrium by proposing earlier. The credible-offer frontier determines the earliest possible contest date; under the timing tie-break, bidder ℓ initiates there, while bidder w determines the offer ultimately selected.

4 Analytical Implications

Section 3 determines when the contest begins, which offer is selected, and why neither bidder preempts the challenge state. We now translate that equilibrium into implications for the subsequent path and value of the takeover. The bidding round itself is instantaneous, but the selected exchange offer can remain outstanding before acceptance, and acceptance can in turn precede completion of the back-end merger. Subsection 4.1 characterizes these stages. Subsection 4.2 compares the private and joint values generated by competition with bidder w's single-offer benchmark.

4.1 Offer timing and acceptance

Condition on $\tau_L < \infty$, so the equilibrium contest occurs at $Z_{\tau_L} = z_L$ and bidder w's counteroffer is selected. To display the dependence on the verification-cost rate, define

$$b_A(\lambda) := z_E\big(\xi_w^R(\lambda)\big), \qquad b_M := z_{M,w}. \tag{76}$$

For the maintained $\lambda > 0$, we write $b_A := b_A(\lambda)$. The first boundary governs target acceptance of the selected offer; the second governs completion of the back-end merger.

Proposition 5 (Post-selection timing)**.** *Maintain Assumption 1 and condition on $\tau_L < \infty$.*

(i) Separation of stages. *The stopping times in (74)–(75) are the first post-selection hitting times of b_A and b_M, respectively, and*

$$z_L = z_E(\xi_\ell^0) \le b_A < b_M = z_{M,w}. \tag{77}$$

Consequently, on $\{\tau_M^w < \infty\}$, $\tau_L \le \tau_A^w < \tau_M^w$.

(ii) Acceptance delay and verification expenditure. *Acceptance is immediate precisely at the lower-corner response:*

$$\tau_A^w = \tau_L \quad \Longleftrightarrow \quad b_A = z_L \quad \Longleftrightarrow \quad \xi_w^R = \xi_\ell^0. \tag{78}$$

If $b_A > z_L$, acceptance occurs strictly after the challenge if the acceptance boundary is reached; if the boundary is never reached, the selected offer is never accepted. The reduced expected verification expenditure incurred between selection and acceptance is

$$C_V^R := v_C(z_L, b_A) = \frac{\lambda k_X}{r - \mu_X}\left\{ z_L - b_A \left(\frac{z_L}{b_A}\right)^{\theta} \right\}. \tag{79}$$

For $\lambda > 0$,

$$C_V^R = 0 \quad \Longleftrightarrow \quad b_A = z_L \quad \Longleftrightarrow \quad \xi_w^R = \xi_\ell^0. \tag{80}$$

(iii) Verification-cost comparative statics. *For $0 \le \lambda_1 < \lambda_2$,*

$$b_A(\lambda_2) \le b_A(\lambda_1), \tag{81}$$

and

$$b_A(0) = z_E(\bar{\xi}_w), \qquad \lim_{\lambda\to\infty} b_A(\lambda) = z_L. \tag{82}$$

The completion boundary b_M is independent of λ.

A higher verification-cost rate changes the transaction path by inducing the bidder to retain a smaller share. It does not affect the back-end merger boundary, which is pinned down by the merger-surplus parameters. Bidder w offers the target a weakly larger fraction of the combined firm, which weakly lowers b_A and transfers control no later, while b_M remains fixed. The value $b_A(0)$ in (82) describes the conditional challenge–response benchmark at zero cost; as noted in Remark 2, the complete timing game need not have a unique contest date when $\lambda = 0$.

The proof of Proposition 5 is in Online Resource S.7.

4.2 Distributional and joint-value effects

We finally compare the competitive equilibrium with bidder w's single-offer policy. Fix an initial state $0 < z < z_L$. Let $\xi_w^* := \min \Xi_w^*(z)$ be the target-favorable monopoly-optimal retained share selected in Proposition 1, and write

$$z_W := z_{W,w} = z_{P,w}(\xi_w^*) = z_{A,w}(\xi_w^*) \geq z_{M,w}. \tag{83}$$

The competitive values are $v_w^C(z)$ and $v_A^C(z)$ from (61) and (59). Their monopoly counterparts are

$$v_w^M(z) := v_{P,w}(z, \xi_w^*), \qquad v_A^M(z) := \left(\frac{z}{z_W}\right)^\theta v_{A,w}(z_W, \xi_w^*). \tag{84}$$

Thus all comparisons below concern incremental takeover value; the firms' standalone values have already been removed.

Define the efficient bidder's loss and the target's gain by

$$\Delta_w(z) := v_w^M(z) - v_w^C(z), \qquad \Delta_A(z) := v_A^C(z) - v_A^M(z). \tag{85}$$

For the aggregate comparison, let

$$J^C(z) := v_w^C(z) + v_A^C(z), \qquad J^M(z) := v_w^M(z) + v_A^M(z). \tag{86}$$

The next proposition separates redistribution from the real-value effect of competition.

Proposition 6 (Distributional and joint-value effects)**.** *Maintain Assumption 1, let $\lambda > 0$, and fix $0 < z < z_L$.*

(i) Distributional effects. *Competition strictly reduces the efficient bidder's value and strictly increases the target's value:*

$$\Delta_w(z) > 0, \qquad \Delta_A(z) > 0.$$

In particular, the target gain can be written as

$$\Delta_A(z) = \left(\frac{z}{z_L}\right)^\theta \left\{v_{A,w}(z_L, \xi_w^R) - v_{A,w}(z_L, \xi_w^*)\right\} > 0. \tag{87}$$

(ii) Joint value. *The competitive and monopoly joint values satisfy*

$$J^C(z) = \left(\frac{z}{z_L}\right)^{\theta} \{v_{M,w}(z_L) - v_C(z_L, b_A)\}, \tag{88}$$

$$J^M(z) = \left(\frac{z}{z_W}\right)^{\theta} v_{M,w}(z_W). \tag{89}$$

Define the gross gain from earlier access to the efficient merger option and the verification-resource loss by

$$\Gamma_J(z) := \left(\frac{z}{z_L}\right)^{\theta} v_{M,w}(z_L) - \left(\frac{z}{z_W}\right)^{\theta} v_{M,w}(z_W), \tag{90}$$

$$\Delta_V(z;\lambda) := \left(\frac{z}{z_L}\right)^{\theta} v_C(z_L, b_A(\lambda)). \tag{91}$$

Then

$$\Gamma_J(z) \geq 0, \qquad \Gamma_J(z) > 0 \iff z_W > z_{M,w} \iff \xi_w^* > \widetilde{\xi}_w, \tag{92}$$

and

$$J^C(z) - J^M(z) = \Gamma_J(z) - \Delta_V(z;\lambda). \tag{93}$$

Equivalently,

$$\Delta_w(z) - \Delta_A(z) = J^M(z) - J^C(z) = \Delta_V(z;\lambda) - \Gamma_J(z). \tag{94}$$

Hence competition raises, leaves unchanged, or lowers joint takeover value according as $\Gamma_J(z)$ is greater than, equal to, or smaller than $\Delta_V(z;\lambda)$.

(iii) Endogenous verification loss. *Using (46),*

$$\Delta_V(z;\lambda) = \left(\frac{z}{z_L}\right)^{\theta} \lambda c_L\big(\xi_w^R(\lambda)\big). \tag{95}$$

It satisfies

$$\Delta_V(z;0) = 0, \qquad \lim_{\lambda\to\infty} \Delta_V(z;\lambda) = 0, \tag{96}$$

and there exists $\varepsilon > 0$ such that

$$\Delta_V(z;\lambda) > 0 \qquad \text{for every } 0 < \lambda < \varepsilon. \tag{97}$$

Thus the realized resource loss is non-monotone in the unit cost: a larger λ makes a given pending offer more expensive but also induces a more target-favorable offer and shortens the period over which verification is required.

The distributional conclusion is unambiguous, but it does not determine efficiency. A more target-favorable retained-share rule transfers value from bidder w to the target and therefore cancels from the joint sum. What remains is the comparison in (93): competition weakly advances access to bidder w's merger option, but a selected stock offer that remains pending consumes real resources. The zero-cost endpoint in (96) refers to the conditional challenge–response benchmark;

Table 1: Benchmark primitive parameters. HM denotes Hackbarth and Morellec (2008).

Symbol	Description	Value	Source/role
r	Risk-free rate	0.0600	Hackbarth–Morellec
δ_X	Bidder payout yield	0.0200	HM average
δ_Y	Target payout yield	0.0200	HM average
μ_X	Bidder risk-neutral drift	0.0400	implied
μ_Y	Target risk-neutral drift	0.0400	implied
σ_X	Bidder asset volatility	0.2000	Hackbarth–Morellec
σ_Y	Target asset volatility	0.2000	Hackbarth–Morellec
ρ	Correlation	0.9000	industry peers
k_X	Bidder capital stock	1.0000	normalization
k_Y	Target capital stock	1.0000	normalization
α	Operating-gain parameter	1.5000	$\alpha/\varphi_w = 1$
φ_w	Efficient bidder integration cost	1.5000	benchmark
φ_ℓ	Less efficient bidder integration cost	2.0000	less efficient rival
Λ	$\lambda/(r-\mu_X)$	0.2000	verification intensity
λ	Verification-cost rate	0.0040	implied

as explained in Remark 2, the complete timing game requires an additional selection rule at $\lambda = 0$. The detailed decomposition of Δ_w into share-concession, timing, and verification components is retained in Online Resource S.8. Appendix A.5 proves Proposition 6.

5 Numerical Analysis

5.1 Computational procedure and benchmark parameters

We compute the equilibrium objects from the closed-form expressions of Sections 2–4. Given σ_Z^2 and the root $\theta > 1$ in (6), Proposition 2 delivers the challenge state z_L, retained share ξ_ℓ^0, and target value A_L in closed form. We then solve the scalar participation equation $v_{A,w}(z_L, \xi) = A_L$ for $\bar{\xi}_w$ and the bounded one-dimensional problem $\xi_w^R(\lambda) \in \arg\max_{\xi \in [\xi_\ell^0, \bar{\xi}_w]} \psi_w(\xi; \lambda)$. Because the maximization is over a closed interval, the optimum need not be interior: it may be the most bidder-favorable offer meeting the target's participation constraint or, at high verification cost, the lower-corner offer. The boundaries b_A, b_M, the expenditure C_V^R, the monopoly objects (ξ_w^*, z_W), and all value comparisons then follow by substitution.

Table 1 reports the benchmark primitive parameters. Following Hackbarth and Morellec (2008), we set $r = 0.06$, $\sigma_X = \sigma_Y = 0.20$, unit capital stocks, and equal payout yields $\delta_X = \delta_Y = 0.020$ (which removes a built-in drift in the ratio), with correlation $\rho = 0.90$ for firms in related businesses; hence $\theta = 2.7913$ and $\theta/(\theta-1) \approx 1.56$. The merger parameters $\alpha = 1.5$, $\varphi_w = 1.5$, $\varphi_\ell = 2.0$ preserve the efficient bidder's ratio $\alpha/\varphi_w = 1$ (NPV break-even ratio $1 + \varphi_w/\alpha = 2$); the common scale of α and φ_w makes the merger surplus quantitatively material relative to the exchange component, and the less efficient bidder ($\varphi_\ell = 2.0$) remains competitive enough to generate the strong-rival regime.

The benchmark satisfies Assumption 1, with $A_L - v_{A,w}(z_L, \widetilde{\xi}_w) = 0.1495 > 0$, and the relevant response stays on the exchange-threshold branch. The three post-contest stages separate,

$z_L = 2.3211 < b_A = 2.5113 < b_M = 3.1165$, and at the benchmark intensity $\Lambda = \lambda/(r-\mu_X) = 0.2$ the selected response is the upper participation boundary, $\xi_w^R = \bar{\xi}_w = 0.6171$: at this cost the efficient bidder concedes no more than the target's continuation-value constraint requires. The comparative statics below vary Λ.

5.2 Verification cost, offer terms, and takeover timing

We vary the normalized intensity $\Lambda := \lambda/(r-\mu_X)$, holding other parameters fixed, and for each Λ recompute $\xi_w^R(\lambda)$ from (51)–(52), together with $b_A(\lambda) = z_E(\xi_w^R(\lambda))$ and $C_V^R(\lambda) = v_C(z_L, b_A(\lambda))$, while $b_M = z_{M,w}$. The point $\Lambda = 0$ is the conditional challenge–response benchmark; as in Remark 2, the full timing game at $\lambda = 0$ needs an additional selection convention.

Figure 1 plots the response payoff $\Pi_w(z_L, \xi)$ over $[\xi_\ell^0, \bar{\xi}_w]$ for three intensities, illustrating the three solution types of the closed-interval problem: an increasing payoff selects the upper boundary $\bar{\xi}_w$, a decreasing payoff the lower corner ξ_ℓ^0, and an intermediate cost an interior optimum.

The transition between the two endpoint responses is narrow: the upper boundary ceases to be selected near $\Lambda \approx 0.233$ and the lower corner takes over by $\Lambda \approx 0.235$, with an interior response only in between. In the benchmark, this sharp transition reflects the economically relevant trade-off between the two endpoint offers—the upper boundary, which minimizes the target's share subject to participation, and the lower corner, which eliminates the verification interval.

Figure 2 shows the induced comparative statics. Panel (a) confirms Proposition 3: $\xi_w^R(\lambda)$ is weakly decreasing in Λ, staying at $\bar{\xi}_w$ for low costs and switching to ξ_ℓ^0 once costs are high enough. Panel (b), which illustrates the three-stage mechanism, translates this into timing. At low cost the offer is selected at z_L but accepted only at $b_A(\lambda) = z_E(\bar{\xi}_w) > z_L$, and the merger completes still later at $b_M = z_{M,w} > b_A$: the selected offer is outstanding over $[z_L, b_A)$ and the accepted transaction incomplete over $[b_A, b_M)$, so competition separates selection, acceptance, and completion. As Λ rises, $b_A(\lambda)$ falls toward z_L while b_M is fixed; once the lower corner is selected, $b_A = z_E(\xi_\ell^0) = z_L$, selection and acceptance collapse, and only completion remains delayed. The friction thus does not move the merger boundary—it compresses the interval between selection and acceptance.

The expected verification expenditure $C_V^R(\lambda) = v_C(z_L, b_A(\lambda))$ is non-monotone in the unit cost. While the selected share remains at $\bar{\xi}_w$, the acceptance boundary $b_A(\lambda)$ is fixed and the expenditure rises with Λ; once bidder w switches to the lower endpoint, the acceptance boundary collapses to z_L, the selected offer is accepted immediately, and the verification interval vanishes, so $C_V^R(\lambda) = 0$. The $\widehat{\Delta}_V$ row of Table 2 reports this quantity at the three representative intensities, the interior peak lying between the first two.

The bidder's response is therefore also a choice of transaction architecture. At low verification cost it can hold a bidder-favorable offer and tolerate a genuinely staged transaction (selection at z_L, acceptance at b_A, completion at b_M); at high cost it optimally concedes more of the combined firm to collapse selection and acceptance and remove the costly pending-offer interval. A more target-favorable stock offer is thus a device for compressing the takeover timeline, not merely a

Table 2: Distributional and joint-value effects

Object	$\Lambda = 0.200$ (upper)	$\Lambda = 0.234$ (interior)	$\Lambda = 0.300$ (lower)
$\widehat{\Delta}_w$	0.3633	0.3735	0.3735
$\widehat{\Delta}_A$	0.3921	0.4456	0.4634
$\widehat{\Gamma}_J$	0.0899	0.0899	0.0899
$\widehat{\Delta}_V$	0.0611	0.0178	0.0000
$\widehat{J}^C - \widehat{J}^M$	0.0288	0.0721	0.0899

transfer of surplus to the target.

5.3 Distributional effects and joint value

We finally quantify the value effects. Since all comparisons in Proposition 6 share the factor $(z/z_L)^\theta$, we report challenge-state normalized values $\widehat{\Delta}_w$, $\widehat{\Delta}_A$, $\widehat{\Gamma}_J$, $\widehat{\Delta}_V$, and $\widehat{J}^C - \widehat{J}^M$ (each object divided by $(z/z_L)^\theta$), which removes the arbitrary initial state $z < z_L$.

Table 2 reports these for the same three intensities as the response type moves from the upper boundary through an interior response to the lower corner. In every case, relative to bidder w's single-offer benchmark, competition lowers bidder w's normalized incremental takeover value and raises the target's, $\widehat{\Delta}_w > 0$ and $\widehat{\Delta}_A > 0$, confirming Proposition 6; the target gain grows with Λ as bidder w shifts toward more target-favorable terms to shorten the pending-offer interval.

The last three rows decompose the joint-value effect. The gross timing gain $\widehat{\Gamma}_J$ is independent of Λ—it is fixed by the earlier access to bidder w's merger option through z_L and z_W—whereas the realized verification loss $\widehat{\Delta}_V$ is endogenous, positive while the offer remains outstanding after the challenge and falling to zero once the lower corner is selected and $b_A = z_L$. In the benchmark the timing gain dominates for all three intensities, so $\widehat{J}^C - \widehat{J}^M = \widehat{\Gamma}_J - \widehat{\Delta}_V > 0$ and competition raises joint value. This is not a general efficiency result: by Proposition 6 the sign turns on Γ_J versus Δ_V, and a verification loss exceeding the timing gain would lower joint value while still benefiting the target. The same accounting identity $\Delta_w(z) - \Delta_A(z) = \Delta_V(z;\lambda) - \Gamma_J(z)$ implies $\widehat{\Delta}_A > \widehat{\Delta}_w$ exactly when competition raises joint value: here the target's gain is not pure redistribution but carries a positive net timing gain after netting out verification expenditure.

6 Conclusion

This paper developed a stochastic dynamic game of a stock-financed takeover in which offer selection, target acceptance, and back-end merger completion are separate decisions. The key friction is an acquisition-currency verification cost incurred while a selected stock offer remains outstanding. The model shows how this friction, together with the continuation value available when the target selects no offer, changes both the timing and the terms of a competitive exchange offer.

Under the stated tie-breaking rules, the equilibrium characterization identifies a regime in which the less efficient rival activates the contest but the more efficient bidder responds and

has its offer selected. The less efficient bidder's challenge is obtained in closed form, and the more efficient bidder's counteroffer is reduced to a scalar participation boundary and a bounded one-dimensional response problem. If an offer is made before the challenge state, the target can select no offer and preserve the continuation value generated by the future challenge–response equilibrium. With a positive verification-cost rate, the less efficient bidder has no credible target-selectable early offer, and every target-selectable early offer by the more efficient bidder is strictly less profitable than preserving the contest. Hence neither bidder preempts.

Competition can separate offer selection, target acceptance, and back-end merger completion. A higher verification-cost rate induces a weakly more target-favorable selected offer and weakly earlier target acceptance, but it does not move the merger-completion boundary. The numerical analysis illustrates this mechanism as a closed-interval offer-design problem: low verification costs favor the most bidder-favorable offer satisfying the target's participation constraint, whereas high verification costs can make it optimal to choose terms that collapse offer selection and target acceptance.

Relative to the more efficient bidder's single-offer benchmark, competition lowers that bidder's incremental takeover value and raises the target's, but the sign of its joint-value effect turns on an explicit comparison. That comparison balances the nonnegative timing gain from potentially earlier access to the more efficient bidder's merger option against the real verification resources consumed while a stock offer remains pending. Despite its stylized structure, the model isolates a tractable mechanism: the competitive game determines which bidder's offer is selected, while competition and verification costs together shape the selected terms, target-acceptance timing, and the allocation of takeover gains; the merger-completion boundary remains pinned down by merger fundamentals.

Statements and Declarations

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Author contributions All authors contributed to the model formulation and analysis. R. Miyata and T. Suzuki developed the equilibrium characterization; K. Yagi and T. Suzuki carried out the numerical analysis. All authors read and approved the final manuscript.

Data availability No empirical datasets were used in this study. The numerical results were generated from the theoretical model. The MATLAB code used to reproduce the numerical results is available from the corresponding author upon reasonable request.

Supplementary information The article is accompanied by an Online Resource, *Technical Proofs and Additional Results* (Sections S.1–S.8), which contains the standard valuation arguments, the branch-specific derivations, the formal strategy space, and the additional value decompositions referred to in the text.

Use of generative AI During the preparation of this work, the authors used ChatGPT and Claude to verify intermediate mathematical derivations and check numerical results. After using these tools, the authors thoroughly reviewed, verified, and edited the resulting material and take full responsibility for the theoretical formulations, proof validity, and the entire content of the publication.

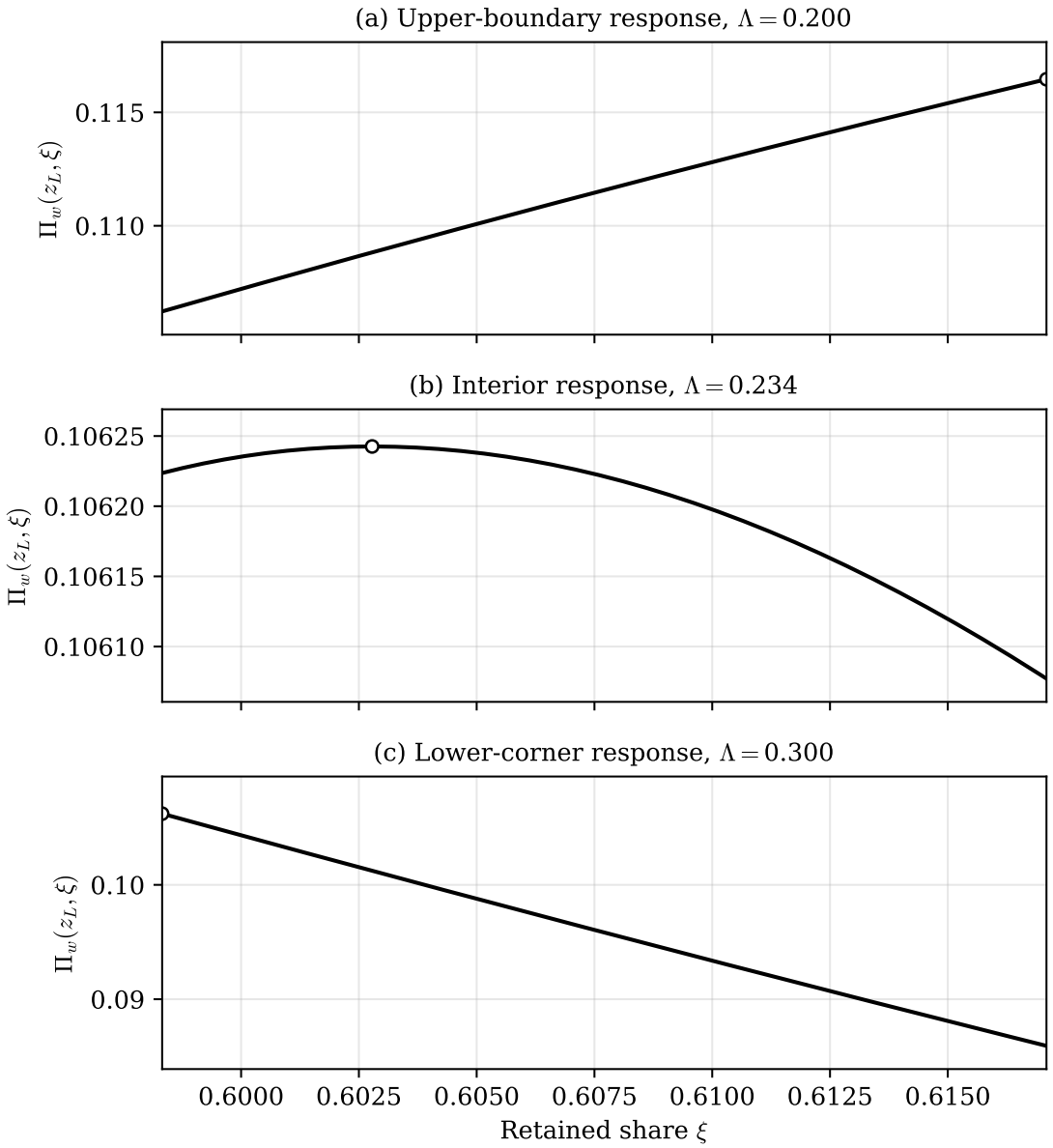


Figure 1: Response payoff over the feasible response interval. For each verification-cost intensity, the figure plots $\Pi_w(z_L, \xi)$ over $[\xi_\ell^0, \bar{\xi}_w]$; the marker indicates the selected response. Low costs select the upper participation boundary, intermediate costs an interior response, and high costs the lower corner.

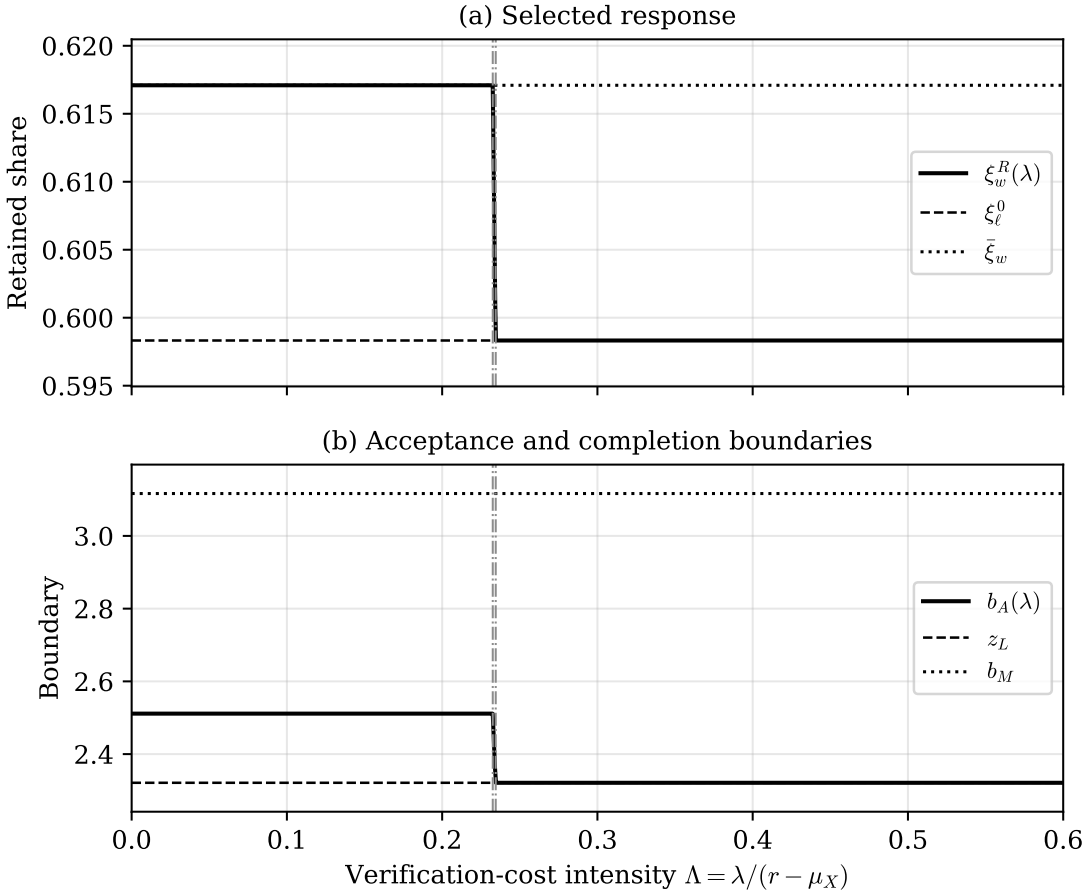


Figure 2: Verification cost, selected response, and takeover boundaries. Panel (a) plots the selected retained share $\xi_w^R(\lambda)$ with the lower endpoint ξ_ℓ^0 and the participation boundary $\bar{\xi}_w$. Panel (b) plots the acceptance boundary $b_A(\lambda) = z_E(\xi_w^R(\lambda))$, the challenge state z_L, and the completion boundary $b_M = z_{M,w}$. For low costs $z_L < b_A(\lambda) < b_M$, so selection, acceptance, and completion separate; for high costs $b_A(\lambda) = z_L$, so selection and acceptance collapse while completion remains delayed.

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A Essential Proofs

This appendix contains only the proofs needed to establish the paper's novel economic mechanism and the equilibrium itself. Standard perpetual-option calculations and repetitive branch verification are placed in the Online Resource.

A.1 Single-offer timing and the verification-cost effect

Online Resource S.3 gives the complete fixed-share majorant verification, and Online Resource S.4 establishes existence and location of a monopoly-optimal retained share. We retain here the strict timing inequality generated by the acquisition-currency verification cost and then assemble the proof of Proposition 1.

Lemma 1 (Cost of a premature proposal)**.** *Fix $\xi \in \Xi$ and let $a := z_A(\xi)$. Suppose $\lambda > 0$ and $0 < z < p < a$. If the bidder waits until Z first reaches p and then proposes the same offer, its value at z exceeds the payoff from proposing immediately at z by*

$$\left(\frac{z}{p}\right)^{\theta} \Pi(p,\xi) - \Pi(z,\xi) = \frac{\lambda k_X}{r-\mu_X} z \left\{1 - \left(\frac{z}{p}\right)^{\theta-1}\right\} > 0. \tag{A1}$$

In particular, waiting until the acceptance boundary and then proposing yields

$$\left(\frac{z}{a}\right)^{\theta} \{\xi v_M(a) - g_E(a,\xi)\} - \Pi(z,\xi) = v_C(z,a) > 0. \tag{A2}$$

When $\lambda = 0$, both differences are zero.

Proof. For every $u < a$, equation (23) and the hitting-time identity give

$$\Pi(u, \xi) = \{\xi v_M(a) - g_E(a, \xi)\} \left(\frac{u}{a}\right)^{\theta} - v_C(u, a). \tag{A3}$$

Discounting the payoff at p back to z and subtracting (A3) at $u = z$ cancels the common acceptance payoff. Using (22),

$$\begin{aligned}\left(\frac{z}{p}\right)^{\theta} \Pi(p, \xi) - \Pi(z, \xi) &= v_C(z, a) - \left(\frac{z}{p}\right)^{\theta} v_C(p, a) \\ &= \frac{\lambda k_X}{r - \mu_X}\{z - z^{\theta} p^{1-\theta}\},\end{aligned}$$

which is (A1). Letting $p \uparrow a$ gives (A2). The ordinary payout identity in Online Resource S.1 has already credited all standalone payouts; the strict difference is therefore exactly the verification expenditure avoided by delay. □

Proof of Proposition 1. Applying Lemma S.3 to bidder i's merger surplus gives $z_{P,i}(\xi) = z_{A,i}(\xi) \vee z^X_{M,i}(\xi)$ and the two branches in (29); (28) is the bidder-specific form of (19), and Lemma S.1 gives $z_{P,i}(\xi) \geq z_{M,i} \vee z_{A,i}(\xi)$. Hence proposal at $z_{P,i}(\xi)$ is accepted immediately, the back-end merger is already exercisable, and $v_C(z_{P,i}(\xi), z_{A,i}(\xi)) = 0$.

For the share choice, Lemma S.5 with $\varphi = \varphi_i$ establishes compactness and nonemptiness of (30) and the inclusion (31), and strict decrease of the target value in ξ makes the minimum in (32) target-favorable. For $\xi_i^* \geq \widetilde{\xi}_i$, equations (28)–(29) give $z_{P,i}(\xi_i^*) = z_{A,i}(\xi_i^*) = z^Y_{M,i}(\xi_i^*) \geq z_{M,i}$, proving (33), and Lemma S.3 makes (34) attain the value. Since the verification interval is empty at the exercise state, the proposal value in Lemma S.3 contains no λ, so the maximizing set, boundary, and value are independent of the verification-cost rate; finally, Lemma 1 gives the strict timing comparison below the acceptance boundary when $\lambda > 0$ and indifference when $\lambda = 0$. □

A.2 Credible challenge and optimal response

We first prove the closed-form challenge and then the reduction and comparative statics of the response problem.

Proof of Proposition 2. By Lemma S.4, bidder ℓ's payoff on the exchange-threshold branch is

$$\Pi_\ell(z_E(\xi), \xi) = \xi \left\{ v_{M,\ell}(z_E(\xi)) - \frac{k_Y}{\theta - 1} \right\}. \tag{A4}$$

Let $q_\ell := (\alpha + \varphi_\ell)^{-1} \in (0, 1)$. For $z < z_{M,\ell}$,

$$v_{M,\ell}(z) = \frac{k_Y(\alpha + \varphi_\ell)}{\theta - 1} \left(\frac{z}{z_{M,\ell}}\right)^{\theta}. \tag{A5}$$

Hence the unique state below $z_{M,\ell}$ at which this value equals $k_Y/(\theta-1)$ is

$$\widehat{z}_\ell := z_{M,\ell} q_\ell^{1/\theta}. \tag{A6}$$

The definitions of $\xi_{\min}$ and $\widetilde{\xi}_\ell$ imply

$$z_E(\xi_{\min}) = z_{M,\ell} q_\ell < \widehat{z}_\ell < z_{M,\ell} = z_E(\widetilde{\xi}_\ell).$$

Since z_E is strictly increasing, there is a unique $\xi_\ell^0 \in (\xi_{\min}, \widetilde{\xi}_\ell)$ with $z_E(\xi_\ell^0) = \widehat{z}_\ell$. Equations (A4)–(A6) give $\Pi_\ell(z_E(\xi_\ell^0), \xi_\ell^0) = 0$, and the exchange-branch acceptance rule gives $z_L = z_E(\xi_\ell^0) = z_{A,\ell}(\xi_\ell^0)$. Substitution of $z_{M,\ell}$ and inversion of z_E yield (35b) and (35c).

There is no second solution on the simultaneous-exercise branch. If $\xi > \widetilde{\xi}_\ell$, then Lemma S.1 gives

$$z_{M,\ell}^X(\xi) < z_{A,\ell}(\xi) = z_{M,\ell}^Y(\xi).$$

The affine payoff $g_{M,\ell}^X(\cdot, \xi)$ is strictly increasing and is already positive at its own call threshold. Therefore

$$\Pi_\ell(z_{A,\ell}(\xi), \xi) = g_{M,\ell}^X(z_{M,\ell}^Y(\xi), \xi) > 0,$$

which proves uniqueness on all of Ξ.

At (z_L, ξ_ℓ^0), acceptance is immediate and $z_L < z_{M,\ell}$. Adding the bidder and target payoffs cancels the exchange transfer and gives

$$v_{A,\ell}(z_L, \xi_\ell^0) + \Pi_\ell(z_L, \xi_\ell^0) = v_{M,\ell}(z_L).$$

The bidder payoff is zero and the construction of z_L gives $v_{M,\ell}(z_L) = k_Y/(\theta-1)$, proving (37). Since acceptance is immediate, $v_C(z_L, z_L) = 0$.

For every $\xi < \xi_\ell^0$, the offer is accepted immediately at z_L, and the marginal-share monotonicity in Lemma S.4 implies that $\Pi_\ell(z_L, \xi)$ is strictly increasing in ξ. Since it equals zero at ξ_ℓ^0, such a lower share is not credible. Hence $\mathcal{C}_\ell(z_L) \subset [\xi_\ell^0, 1)$, while $\xi_\ell^0 \in \mathcal{C}_\ell(z_L)$. The strict decrease of $v_{A,\ell}(z_L, \xi)$ in ξ then proves the argmax statement in (36).

Finally, suppose $p < z_L$ and $z_{A,\ell}(\xi) \le p$. Then $\xi < \xi_\ell^0$ and $z_{A,\ell}(\xi) = z_E(\xi)$. Let $\xi_E(p)$ satisfy $z_E(\xi_E(p)) = p$. Since the immediate-acceptance payoff is increasing in the retained share,

$$\begin{aligned}\Pi_\ell(p, \xi) &\le \Pi_\ell(p, \xi_E(p)) \\ &= \xi_E(p) \left\{ v_{M,\ell}(p) - \frac{k_Y}{\theta - 1} \right\} < 0,\end{aligned}$$

where the last inequality follows from $p < z_L$ and (A5). At $p = z_L$, the share ξ_ℓ^0 is immediately accepted and breaks even, proving (38). More generally, combining the same argument with the pre-acceptance branch of the complete proposal payoff in Online Resource S.3 yields

$$p \vee z_{A,\ell}(\xi) < z_L \quad \Longrightarrow \quad \Pi_\ell(p, \xi) < 0.$$

□

Proof of the delayed-challenge statements. Fix $p \geq z_L$ and $\xi \in \mathcal{C}_\ell(p)$. Adding the target and bidder values at the target's optimal acceptance time τ_A^ξ cancels the exchange transfer and gives

$$\begin{aligned} v_{A,\ell}(p,\xi) + \Pi_\ell(p,\xi) &= \mathbb{E}_p^{\mathbb{Q}^{(Y)}}\left[\mathrm{e}^{-(r-\mu_Y)\tau_A^\xi} v_{M,\ell}(Z_{\tau_A^\xi})\right] - v_C\big(p, z_{A,\ell}(\xi)\big) \\ &\leq v_{M,\ell}(p). \end{aligned} \tag{A7}$$

The inequality follows because $v_{M,\ell}$ is the value of the merger stopping problem, so its discounted Snell envelope is a supermartingale, and because the verification cost is nonnegative. Credibility implies $\Pi_\ell(p,\xi) \geq 0$, and hence

$$v_{A,\ell}(p,\xi) \leq v_{M,\ell}(p). \tag{A8}$$

Now let $\widehat{\xi}_\ell(p)$ be given by (39). It belongs to Ξ. To see this, note that $\widehat{\xi}_\ell(z_L) = \xi_\ell^0 > \xi_{\min}$ and that

$$\widehat{\xi}_\ell(p) > \xi_{\min} \quad \Longleftrightarrow \quad \alpha k_Y p > k_Y + v_{M,\ell}(p).$$

Below $z_{M,\ell}$ the difference on the right has derivative $\alpha k_Y - v'_{M,\ell}(p) > 0$ and is positive at z_L; at and above $z_{M,\ell}$ it equals $k_Y(\alpha + \varphi_\ell - 1) > 0$. Moreover, since $v_{M,\ell}(p) \geq v_{M,\ell}(z_L) = A_L$,

$$\frac{z_E(\widehat{\xi}_\ell(p))}{p} = \frac{\theta k_Y}{(\theta-1)\{k_Y + v_{M,\ell}(p)\}} \leq 1.$$

Thus the offer is accepted immediately and incurs no verification cost. Direct substitution gives

$$\Pi_\ell\big(p, \widehat{\xi}_\ell(p)\big) = 0, \qquad v_{A,\ell}\big(p, \widehat{\xi}_\ell(p)\big) = v_{M,\ell}(p),$$

which, together with (A8), proves (40). The ratio $v_{M,\ell}(p)/p^\theta$ is constant up to $z_{M,\ell}$ and strictly decreasing thereafter, proving (41).

Because z_E is common to the two bidders, bidder w's offer with the same share is also accepted immediately. The exchange component cancels when the two bidder payoffs or the two target payoffs are compared, yielding (42)–(43). Hence the set of credible responses that weakly beat any standing offer by bidder ℓ is nonempty, and every payoff-maximizing response gives bidder w a strictly positive payoff. □

Proof of Proposition 3. Since $\varphi_w < \varphi_\ell$, $v_{M,w}(u) > v_{M,\ell}(u)$ for every $u > 0$. At (z_L, ξ_ℓ^0) both offers are accepted immediately, so

$$v_{A,w}(z_L,\xi_\ell^0) \geq (1-\xi_\ell^0) v_{M,w}(z_L) + g_E(z_L,\xi_\ell^0) > (1-\xi_\ell^0) v_{M,\ell}(z_L) + g_E(z_L,\xi_\ell^0) = A_L.$$

With Assumption 1, continuity and strict decrease of $v_{A,w}(z_L,\cdot)$ give a unique $\bar{\xi}_w \in (\xi_\ell^0, \widetilde{\xi}_w)$ satisfying (47), and the same monotonicity gives (48).

Shares below ξ_ℓ^0 are accepted immediately and, by Lemma S.4, give bidder w a strictly smaller payoff, while the lower endpoint is strictly profitable because $\Pi_w(z_L,\xi_\ell^0) - \Pi_\ell(z_L,\xi_\ell^0) = \xi_\ell^0\{v_{M,w}(z_L) - v_{M,\ell}(z_L)\} > 0$. The response problem therefore reduces to the compact interval $[\xi_\ell^0, \bar{\xi}_w]$, on which every maximizer earns a strictly positive payoff; the argmax is nonempty and compact, and the tie-break selects its smallest element. On this interval $z_L \leq z_E(\xi) < z_{M,w}$, so

the exchange-threshold formulas give (49)–(50) with c_L strictly increasing, and the comparative-statics claims (54)–(55) follow from strict monotonicity of c_L. The supporting algebra, the derivative $c_L' > 0$, and the endpoint arguments are recorded in Online Resource S.6. □

A.3 Competitive continuation and no-preemption

This subsection proves Proposition 4. The proof first constructs the target's continuation value. The fixed-offer timing comparison used in the no-preemption step is stated separately because it is a technical lemma rather than an independent economic implication.

Lemma 2 (Fixed-offer timing)**.** *Fix bidder $i \in \{w, \ell\}$ and $\xi \in (\xi_{\min}, \widetilde{\xi}_i)$. If $\lambda > 0$, then*

$$p \longmapsto \frac{\Pi_i(p, \xi)}{p^\theta} \tag{A9}$$

is continuous and strictly increasing on $(0, z_{M,i})$. When $\lambda = 0$, it is nondecreasing and is constant below the offer's acceptance boundary.

Proof. Fix $i \in \{w, \ell\}$ and $\xi \in (\xi_{\min}, \widetilde{\xi}_i)$, and write $a := z_{A,i}(\xi) = z_E(\xi) < z_{M,i}$. Set $\widehat{\Pi}_i(p, \xi) := \Pi_i(p, \xi)/p^\theta$. For $0 < p < a$, equations (24) and (22) yield

$$\widehat{\Pi}_i(p, \xi) = \frac{\xi v_{M,i}(a) - g_E(a, \xi)}{a^\theta} - \frac{\lambda k_X}{r - \mu_X}\{p^{1-\theta} - a^{1-\theta}\}.$$

Hence

$$\frac{\partial \widehat{\Pi}_i(p, \xi)}{\partial p} = \frac{\lambda k_X(\theta - 1)}{r - \mu_X} p^{-\theta}. \tag{A10}$$

For $a < p < z_{M,i}$, the target accepts immediately and $\Pi_i(p, \xi) = \xi v_{M,i}(p) - g_E(p, \xi)$. Since $v_{M,i}(p)/p^\theta$ is constant below $z_{M,i}$,

$$\begin{aligned} \frac{\partial \widehat{\Pi}_i(p, \xi)}{\partial p} &= \frac{(\theta - 1)(1 - \xi)k_X p - \theta \xi k_Y}{p^{\theta+1}} \\ &= \frac{(\theta - 1)(1 - \xi)k_X}{p^{\theta+1}}\{p - z_E(\xi)\} > 0. \end{aligned} \tag{A11}$$

The two expressions agree at $p = a$ because $v_C(a, a) = 0$. Therefore (A9) is continuous and strictly increasing when $\lambda > 0$. When $\lambda = 0$, (A10) is zero and (A11) remains positive. □

Proof of Proposition 4. Step 1: the target's competitive continuation value. Evaluating (49) at $\xi = \xi_w^R$ gives (62). Proposition 3 implies

$$\xi_\ell^0 \le \xi_w^R \le \bar{\xi}_w.$$

By Lemma S.4, $v_{A,w}(z_L, \xi)$ is strictly decreasing in ξ, while (47) gives $v_{A,w}(z_L, \bar{\xi}_w) = A_L$. Hence

$$A_L \le A_C \le v_{A,w}(z_L, \xi_\ell^0).$$

At (z_L, ξ_ℓ^0) both offers are accepted immediately, and Proposition 2 gives

$$v_{A,w}(z_L, \xi_\ell^0) - A_L = (1 - \xi_\ell^0)\{v_{M,w}(z_L) - v_{M,\ell}(z_L)\} > 0.$$

This proves (63); strict monotonicity proves (64). Proposition 3 also shows that $\xi_w^R(\lambda)$ is weakly decreasing in λ. Composing this result with the strict decrease of the target value proves that $A_C(\lambda)$ is weakly increasing. The limits in (55) then yield (65).

For $0 < z < z_L$, the hitting-time identity (7) gives

$$\mathbb{E}_z^{\mathbb{Q}^{(Y)}}\left[\mathrm{e}^{-(r-\mu_Y)\tau_L}; \tau_L < \infty\right] = \left(\frac{z}{z_L}\right)^\theta.$$

Multiplication by A_C proves that (59) is the value of waiting for the challenge-response subgame.

Next, (56) gives

$$z_L \le z_E(\xi_w^R) < z_{M,w}.$$

Thus z and z_L lie in the continuation region of the same target acceptance problem. Equation (20), together with the continuation-region formula for $v_{M,w}$, implies

$$v_{A,w}(z, \xi_w^R) = \left(\frac{z}{z_L}\right)^\theta v_{A,w}(z_L, \xi_w^R) = v_A^C(z),$$

which proves (66). Similarly, $z_L = z_E(\xi_\ell^0) < z_{M,\ell}$ and $v_{A,\ell}(z_L, \xi_\ell^0) = A_L$, so

$$v_{A,\ell}(z, \xi_\ell^0) = \left(\frac{z}{z_L}\right)^\theta A_L \le v_A^C(z).$$

This is (67). Finally, Lemma S.4 shows that each target value is strictly decreasing in the bidder's retained share. The two matching relations therefore give (68) and (69); the latter is strict when $A_C > A_L$.

Step 2: bidder ℓ cannot preempt. Fix $0 < p < z_L$ and suppose that an offer ξ satisfies the target constraint in (60). Equation (69) implies $\xi \le \xi_\ell^0 < \widetilde{\xi}_\ell$. At z_L, such an offer is accepted immediately, and Lemma S.4 gives

$$\Pi_\ell(z_L, \xi) \le \Pi_\ell(z_L, \xi_\ell^0) = 0.$$

Because $p < z_L < z_{M,\ell}$, Lemma 2 implies

$$\frac{\Pi_\ell(p, \xi)}{p^\theta} < \frac{\Pi_\ell(z_L, \xi)}{z_L^\theta} \le 0.$$

Thus $\Pi_\ell(p, \xi) < 0$, contradicting credibility. This proves (70).

Step 3: bidder w cannot preempt. First observe that, for every $\xi \le \xi_w^R$,

$$\Pi_w(z_L, \xi) \le \Pi_w(z_L, \xi_w^R) = R_w. \tag{A12}$$

If $\xi \in [\xi_\ell^0, \xi_w^R]$, this follows from the optimality of ξ_w^R in (51). If $\xi < \xi_\ell^0$, the offer is accepted immediately at z_L, and Lemma S.4 implies

$$\Pi_w(z_L, \xi) < \Pi_w(z_L, \xi_\ell^0) \leq R_w.$$

Now fix $\xi \in \mathcal{S}_w(p)$. Equation (68) gives $\xi \leq \xi_w^R < \widetilde{\xi}_w$, and (56) gives $z_L < z_{M,w}$. Applying Lemma 2 and then (A12),

$$\frac{\Pi_w(p, \xi)}{p^\theta} < \frac{\Pi_w(z_L, \xi)}{z_L^\theta} \leq \frac{R_w}{z_L^\theta}.$$

Multiplication by p^θ proves (71).

For the supremum statement, the preceding inequality gives the upper bound in (72). Choose $p_n \uparrow z_L$ and keep the response terms $\xi = \xi_w^R$. Equation (66) makes the target constraint bind at every p_n. Since $R_w > 0$ and $\Pi_w(\cdot, \xi_w^R)$ is continuous, the offer is credible for all sufficiently large n. Therefore

$$\lim_{n\to\infty} \left(\frac{z}{p_n}\right)^\theta \Pi_w(p_n, \xi_w^R) = \left(\frac{z}{z_L}\right)^\theta R_w.$$

No $p < z_L$ attains the bound because the pointwise inequality is strict. This proves (72) and completes the proof. □

A.4 Equilibrium verification

We record the off-path actions suppressed from Theorem 1. The phase notation is formalized in Online Resource S.5. The relevant continuation domain includes delayed challenges by bidder ℓ at no-offer states $p \geq z_L$, but the theorem does not characterize histories in which both bidders disregard the tie-breaking refinement indefinitely.

For every $p \geq z_L$, equations (42)–(43) show that $\widehat{\xi}_\ell(p)$ gives bidder w a strictly positive payoff and gives the target more than $v_{M,\ell}(p)$. Since $v_{A,w}(p, \xi)$ is continuous and strictly decreasing in ξ, there is a unique upper response share $\bar{\xi}_w^D(p) > \widehat{\xi}_\ell(p)$ satisfying

$$v_{A,w}\big(p, \bar{\xi}_w^D(p)\big) = v_{M,\ell}(p). \tag{A13}$$

Shares below $\widehat{\xi}_\ell(p)$ are accepted immediately and give bidder w a strictly smaller payoff than $\widehat{\xi}_\ell(p)$. Hence bidder w's delayed-challenge response problem reduces to the compact interval

$$\mathcal{D}_w(p) := [\widehat{\xi}_\ell(p), \bar{\xi}_w^D(p)], \tag{A14}$$

and the target-favorable payoff-maximizing response

$$\xi_w^D(p) := \min \arg\max_{\xi \in \mathcal{D}_w(p)} \Pi_w(p, \xi) \tag{A15}$$

exists and earns a strictly positive payoff. Define the associated target value by

$$A_C^D(p) := v_{A,w}\big(p, \xi_w^D(p)\big) \geq v_{M,\ell}(p). \tag{A16}$$

At $p = z_L$, one has $\bar{\xi}_w^D(z_L) = \bar{\xi}_w$, $\xi_w^D(z_L) = \xi_w^R$, and $A_C^D(z_L) = A_C$.

Stationary strategy profile on the relevant continuation domain. At a no-offer state $0 < p < z_L$, neither bidder initiates. If bidder ℓ nevertheless submits a credible initial offer, bidder w passes and the target selects no offer. If bidder w submits a credible initial offer, bidder ℓ passes; the target selects bidder w's offer if and only if its acceptance value is at least $v_A^C(p)$, and otherwise selects none. Whenever the target selects no offer, each bidder immediately withdraws its unselected standing offer. For $\lambda > 0$, keeping an unselected offer outstanding over a positive interval is strictly dominated by withdrawing it and, if desired, re-proposing the same terms later: withdrawal preserves the future proposal opportunity while avoiding the intervening verification cost.

At a no-offer state $p \geq z_L$ reached by delaying the challenge, the earliest-effective-proposal and target-favorable offer tie-breaks prescribe that bidder ℓ submits $\widehat{\xi}_\ell(p)$. Bidder w responds with $\xi_w^D(p)$, the target selects bidder w's offer, and bidder ℓ withdraws its unselected offer. After any other credible initial offer by bidder ℓ at such a state, bidder w uses the same response $\xi_w^D(p)$ whenever doing so affects the target's choice. By (40), no offer by bidder ℓ gives the target more than $v_{M,\ell}(p) \leq A_C^D(p)$; if the target selects none, the prescribed immediate withdrawals return the game to the same no-offer state, whose continuation value is $A_C^D(p)$. The target therefore selects bidder w's offer under the selection tie-break. At $p = z_L$, these prescriptions reduce to the on-path offers ξ_ℓ^0 and ξ_w^R. After any offer (i, ξ) is selected, the target uses $z_{A,i}(\xi)$ and bidder i uses $z_{M,i}$, with no further bidding.

Proof of Theorem 1. We verify sequential rationality in four steps.

Step 1: premature bidding rounds. At $0 < p < z_L$, Proposition 4 gives continuation values $v_A^C(p)$, $v_w^C(p)$, and zero for the target, bidder w, and bidder ℓ. If bidder ℓ initiates, no credible offer reaches the target's continuation value by (70), and any counteroffer bidder w would have had selected yields less than $v_w^C(p)$ by (71), so passing and selecting no offer are optimal. If bidder w initiates, bidder ℓ has no selectable credible response; an offer below $v_A^C(p)$ is not selected and one reaching it is selected but, by (71), pays strictly less than $v_w^C(p)$, so bidder w does not initiate. Whenever no offer is selected, immediate withdrawal of every unselected offer is optimal by the preceding cost argument. Bidder ℓ therefore obtains zero both from inactivity and from an initial offer that is not selected and is immediately withdrawn; the latter no-selection-and-withdrawal round is identified with inactivity.

Step 2: bidder w's selected offer at z_L. Selecting no offer, followed by the prescribed immediate withdrawals, gives the target $A_C = v_{A,w}(z_L, \xi_w^R)$, and strict decrease of $v_{A,w}(z_L, \cdot)$ makes $v_{A,w}(z_L, \xi) \geq A_C \Leftrightarrow \xi \leq \xi_w^R$. For $\xi \in [\xi_\ell^0, \xi_w^R]$ response optimality (51) gives $\Pi_w(z_L, \xi) \leq R_w$, and for $\xi < \xi_\ell^0$, Lemma S.4 gives $\Pi_w(z_L, \xi) < \Pi_w(z_L, \xi_\ell^0) \leq R_w$. Hence no target-selectable offer at z_L gives bidder w more than R_w, and ξ_w^R is its target-favorable payoff-maximizing offer.

Step 3: challenge timing and the challenge–response round. Proposition 4 rules out an effective challenge before z_L. At any $p \geq z_L$, (40) shows that bidder ℓ's target-favorable credible offer is $\widehat{\xi}_\ell(p)$ and gives bidder ℓ zero if selected. Equations (42)–(43) and the response rule (A15) imply that bidder w responds with a target-preferred credible offer and earns a strictly positive

payoff, so bidder ℓ's offer is not selected and its primary payoff remains zero. Delaying the challenge therefore cannot improve bidder ℓ's payoff. The earliest-effective-proposal tie-break selects $p = z_L$, and the target-favorable offer tie-break selects $\widehat{\xi}_\ell(z_L) = \xi^0_\ell$.

At z_L, bidder w earns $R_w > 0$ from the prescribed response ξ^R_w, which passing cannot improve, so it responds immediately. The target's value is A_C under bidder w, at most A_L under bidder ℓ, and A_C if it selects none and the standing offers are then immediately withdrawn, so it selects bidder w's offer; the stated tie-breaks resolve the remaining ties.

Step 4: post-selection continuation, values, and uniqueness. After (i, ξ) becomes binding, Proposition 1 and Online Resource S.3 give the acceptance and merger rules (74)–(75), with the verification expenditure already in Π_i and no further bidding. From $0 < z < z_L$, continuity of Z makes the first on-path round occur at τ_L, and discounting R_w, zero, and A_C by (7) gives $v^C_w(z)$, zero, and $v^C_A(z)$. Strict no-preemption and delayed-challenge dominance fix the selected boundary at z_L, and Propositions 2 and 3 with the tie-breaks select ξ^0_ℓ, ξ^R_w, and bidder w. Hence the on-path objects are unique within stationary threshold strategies under the stated tie-breaking rules. All prescribed actions depend only on the ratio and phase, and the one-deviation principle establishes the Markov-perfect equilibrium on the relevant continuation domain. Without the earliest-effective-proposal tie-break, the timing of bidder ℓ's zero-payoff challenge is not uniquely selected. □

A.5 Distributional and joint-value effects

Proof of Proposition 6. For brevity write $\xi := \xi^R_w$, $a := b_A = z_E(\xi)$, and $p^* := z_{P,w}(\xi)$. Propositions 1, 3, and 5 with the threshold ordering in Online Resource S.2 give

$$\xi^0_\ell \le \xi < \widetilde{\xi}_w \le \xi^*_w, \qquad z_L \le a < z_{M,w} < p^* = z^X_{M,w}(\xi). \tag{A17}$$

Step 1: private values. Holding ξ fixed, Lemma 2 makes $p \mapsto \Pi_w(p, \xi)/p^\theta$ strictly increasing up to $z_{M,w}$, and on $[z_{M,w}, p^*)$, writing $g^X_{M,w}(p, \xi) = \beta p - \gamma$ with $\beta, \gamma > 0$ and $p^* = \theta\gamma/\{(\theta-1)\beta\}$,

$$\frac{\partial}{\partial p}\left\{\frac{g^X_{M,w}(p,\xi)}{p^\theta}\right\} = \frac{\theta\gamma - (\theta-1)\beta p}{p^{\theta+1}} > 0 \qquad (z_{M,w} \le p < p^*). \tag{A18}$$

Continuity at $z_{M,w}$ gives

$$v^C_w(z) = (z/z_L)^\theta \Pi_w(z_L, \xi) < v_{P,w}(z, \xi).$$

The proof of Lemma S.5 also shows that $v_{P,w}(z, \cdot)$ is strictly increasing on $(\xi_{\min}, \widetilde{\xi}_w]$. Therefore, (A17) yields

$$v_{P,w}(z, \xi) < v_{P,w}(z, \widetilde{\xi}_w) \le v_{P,w}(z, \xi^*_w) = v^M_w(z),$$

which proves $\Delta_w(z) > 0$. Online Resource S.8 refines this loss into its share, timing, and verification components.

For the target, $z_L < z_{M,w} \le z_W$ places the monopoly offer in its continuation region at z_L, so $v_{A,w}(z_L, \xi^*_w) = (z_L/z_W)^\theta v_{A,w}(z_W, \xi^*_w)$. Combined with (84), (59), and $A_C = v_{A,w}(z_L, \xi^R_w)$, this

gives (87). Strict decrease of the target value in the retained share (Lemma S.4), together with $\xi_w^R < \widetilde{\xi}_w \leq \xi_w^*$, gives its final inequality.

Step 2: joint value. On the exchange-threshold branch the bidder and target values at z_L are $\{\xi v_{M,w}(a) - g_E(a,\xi)\}(z_L/a)^\theta - v_C(z_L,a)$ and $\{(1-\xi)v_{M,w}(a) + g_E(a,\xi)\}(z_L/a)^\theta$; adding them cancels the exchange transfer, and $v_{M,w}(a)(z_L/a)^\theta = v_{M,w}(z_L)$ since $a < z_{M,w}$, so discounting to z proves (88). Under the monopoly policy proposal, acceptance, and completion occur together at z_W with no verification expenditure and exercise payoffs summing to $v_{M,w}(z_W)$, which is (89). Since $p \mapsto v_{M,w}(p)/p^\theta$ is constant on $(0, z_{M,w}]$ and strictly decreasing beyond, $z_L < z_{M,w} \leq z_W$ gives $v_{M,w}(z_W)/z_W^\theta \leq v_{M,w}(z_L)/z_L^\theta$, strict exactly when $z_W > z_{M,w}$; with $z_W = z_{A,w}(\xi_w^*)$, $z_{A,w}$ strictly increasing, and $z_{A,w}(\widetilde{\xi}_w) = z_{M,w}$ this proves (92). Subtracting (89) from (88) yields (93) and hence the accounting identity (94).

Step 3: endpoint behavior of the verification loss. Equation (46) gives (95), so the value at $\lambda = 0$ is zero. With $B_w(\xi) := \xi v_{M,w}(z_L) - \xi A_L H_L(\xi)$, optimality of $\xi_w^R(\lambda)$ relative to ξ_ℓ^0 and $c_L(\xi_\ell^0) = 0$ give $0 \leq \lambda c_L(\xi_w^R(\lambda)) \leq B_w(\xi_w^R(\lambda)) - B_w(\xi_\ell^0)$; since $\xi_w^R(\lambda) \to \xi_\ell^0$ by Proposition 3 and B_w is continuous, the right side vanishes, proving the second limit in (96). Finally $\xi_w^R(0) = \bar{\xi}_w > \xi_\ell^0$, so with $d := B_w(\bar{\xi}_w) - B_w(\xi_\ell^0) > 0$ and $\varepsilon := d/c_L(\bar{\xi}_w)$, for $0 < \lambda < \varepsilon$ one has $\psi_w(\bar{\xi}_w;\lambda) > B_w(\xi_\ell^0) = \psi_w(\xi_\ell^0;\lambda)$, so $\xi_w^R(\lambda) > \xi_\ell^0$; strict increase of z_E and positivity of $v_C(z_L, b_A)$ for $b_A > z_L$ prove (97). □