学位論文内容の要旨

博士の専攻分野の名称：博士（経済学）　　氏名：米永 航志朗

学位論文題名

# Functionals of a Wishart matrix and a normal vector and its application to linear discriminant analysis

# （ウィッシャート行列と正規ベクトルの関数と線形判別分析への応用）

[要旨]
In this dissertation, we investigate some functionals of a Wishart matrix and a normal vector and discuss the application to linear discriminant analysis in a Bayesian framework.

In section 2, we consider the distribution of the product of a Wishart matrix and a normal vector which are independently distributed. We derive the stochastic representation of the product which is used to derive the density function and higher order moments of the product. Based on the higher order moments of the product, we further present an Edgeworth type expansion for the product. In addition, it turns out that the obtained stochastic representation, density function and moments of the product remain valid for the product of a singular Wishart matrix and a normal vector.

In section 3, we consider the distribution of the product of a Wishart matrix and a conditional normal vector given a Wishart matrix. This type of the product plays an important role in Bayesian analysis of the optimal portfolio. We derive the novel stochastic representation for the product and observe from the stochastic representation that the distribution of the product is closed under conditioning, marginalization, and affine transformations. Moreover, the formulae for the first four moments, density function and an Edgeworth type expansion are explicitly presented.

In section 4, we consider discriminant analysis in the case of two multivariate normal populations with different means and common covariance matrices. We derive the posterior predictive density function and the first four moments of the population linear discriminant function under some prior distributions. Based on the derived posterior predictive density function,

we consider the Bayesian estimation for the misclassification rate associated with a population linear discriminant function, referred to as the optimal error rate. We obtain an explicit expression of the Bayes estimator of the optimal error rate. Although the Bayes estimator of the optimal error rate is expressed by the infinite sums and special functions in general, it is simply expressed under some conditions. In addition, an Edgeworth type expansion for the Bayes estimator is suggested based on the approximate posterior predictive distribution of the population linear discriminant function.