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PAPER Special Section on Mathematical Systems Science and its Applications

LMI-Based Design of Output Feedback Controllers with Decentralized Event-Triggering

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SUMMARY In this paper, event-triggered control over a sensor network is studied as one of the control methods of cyber-physical systems. Event-triggered control is a method that communications occur only when the measured value is widely changed. In the proposed method, by solving an LMI (Linear Matrix Inequality) feasibility problem, an event-triggered output feedback controller such that the closed-loop system is asymptotically stable is derived. First, the problem formulation is given. Next, the control problem is reduced to an LMI feasibility problem. Finally, the proposed method is demonstrated by a numerical example.

key words: *cyber-physical systems, event-triggered control, LMI (Linear Matrix Inequality), output feedback*

1. Introduction

A cyber-physical system (CPS) is a system where physical and software components are deeply intertwined through communication networks. A CPS has several applications such as smart grid, medical monitoring, and automobile systems. In order to realize these applications, it is important to develop a control method of CPSs. In CPSs, event-triggered control is well known as one of the important control methods [5]–[7], [9], [12], [16], [20]–[23]. Event-triggered control is a method that communications occur only when the measured signal is widely changed (i.e., an event occurs). One of the typical event-triggering conditions is to evaluate the difference between the current measured signal and the past one that was recently sent to the controller. By appropriately choosing a threshold, we can consider both the control performance such as the transient response and the communication load from sensors (controllers) to controllers (actuators).

In large-scale CPSs, a sensor network plays an important role, and has several applications such as distributed robotic systems. In sensor networks, sensors are located in a distributed way. In event-triggered control over sensor networks, an event-triggering condition is assigned to each sensor, that is, event-triggering conditions are decentralized. Such control method is called a decentralized event-triggered method (see, e.g., [3], [6], [7], [13]–[15], [18]). If at least one of event-triggering conditions is satisfied, then all sensors send the measured values to the controller. The controller generates the control input based on all measured values. The controller may not be necessarily decentralized. We may use the centralized controller.

In this paper, for discrete-time linear systems, we consider the problem of finding an event-triggered output feedback controller such that the closed-loop system is asymptotically stable. In the event-triggering condition of each sensor, we evaluate the difference between the current measured output and the past one that was recently sent to the controller. The form of the controller is given by a discrete-time linear system. The coefficient matrices in the controller can be derived by solving an LMI (Linear Matrix Inequality) feasibility problem. Design methods of event-triggered output feedback have been studied so far (see, e.g., [3], [6], [7]). To the best of our knowledge, a design method of the dynamic controller via LMI has not been studied in the framework of decentralized event-triggered control. In [8], LMI-based simultaneous design of a static output feedback controller and a filter has been proposed. However, a dynamic controller is not considered. In [19], LMI-based design of event-triggering conditions has been proposed, where the controller is given in advance. Thus, the proposed method provides a new LMI-based design method of event-triggered dynamic controllers.

This paper is organized as follows. In Sect. 2, the control problem studied in this paper is formulated. In Sect. 3, the problem is reduced to an LMI feasibility problem. In Sect. 4, a numerical example is presented to demonstrated the proposed method. In Sect. 5, we conclude this paper.

Notation: Let $\mathcal{R}$ denote the set of real numbers. Let I_n and $0_{m\times n}$ denote the $n\times n$ identity matrix and the $m\times n$ zero matrix, respectively. For simplicity, we sometimes use the symbol 0 instead of $0_{m\times n}$, and the symbol I instead of I_n. Let $M \succ 0$ ($M \succeq 0$) denote that the matrix M is positive-definite (positive-semidefinite). For the vector x, let x_i denote the i-th element of x. For the vector $x = [x_1 \ \ x_2 \ \ \cdots \ \ x_n]^\top$ and the index set $\mathcal{I} = \{i_1, i_2, \ldots, i_m\} \subseteq \{1, 2, \ldots, n\}$, define $[x_i]_{i\in\mathcal{I}} := [x_{i_1} \ \ x_{i_2} \ \ \cdots \ \ x_{i_m}]^\top$. For the vector x, let $\|x\|$ denote the Euclidean norm of x. For scalars $a_1, a_2, \ldots, a_n$, let $\mathrm{diag}(a_1, a_2, \ldots, a_n)$ denote the diagonal matrix. The symmetric matrix $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix}$ is denoted by $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$.

2. Problem Formulation

As a plant, consider the following discrete-time linear system:

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$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state, $u(k) \in \mathcal{R}^m$ is the control input, $y(k) \in \mathcal{R}^p$ is the measured output, $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, and $C \in \mathcal{R}^{p\times n}$ are given matrices, and $k \in \{0, 1, 2, \dots\}$ is the discrete time. We suppose that the number of sensors is p (the number of outputs), and the sensor $i \in \{1, 2, \dots, p\}$ measures the i-th element of the output. For simplicity of discussion, we assume that C is given by

$$C = [I_p \;\; 0_{p\times(n-p)}].$$

In other words, the i-th sensor measures the i-th element of the state (i.e., $y_i = x_i$, $i \in \{1, 2, \dots, p\}$).

As a controller, we consider the following output-feedback controller:

$$\begin{cases} \hat{x}(k+1) = A_c\hat{x}(k) + B_c\hat{y}(k), \\ u(k) = C_c\hat{x}(k), \end{cases} \tag{2}$$

where $\hat{x}(k) \in \mathcal{R}^n$ is the state of the controller, and $A_c \in \mathcal{R}^{n\times n}$, $B_c \in \mathcal{R}^{n\times m}$, and $C_c \in \mathcal{R}^{p\times n}$ are design parameters. The vector $\hat{y}(k) \in \mathcal{R}^n$ is the input to the controller, and is defined by

$$\hat{y}(k) := \begin{cases} y(k) & \text{if } y(k) \text{ is updated,} \\ \hat{y}(k-1) & \text{if } y(k) \text{ is not updated.} \end{cases} \tag{3}$$

For the sensor $i \in \{1, 2, \dots, p\}$, consider the following event-triggering condition:

$$(\hat{y}_i(k-1) - y_i(k))^2 > \sigma_i^2 y_i^2(k), \tag{4}$$

where $\sigma_i > 0$ is a parameter given in advance. When the condition (4) is satisfied, we say that the event occurs. Then, the measured output is updated. We remark here that the control input may be updated even if the event does not occur. Using (4), $\hat{y}(k)$ of (3) can be rewritten as

$$\hat{y}(k) := \begin{cases} y(k) & \text{if (4) holds for some } i, \\ \hat{y}(k-1) & \text{otherwise.} \end{cases} \tag{5}$$

Equation (5) implies that when in at least one sensor, the measured output satisfies (4), this sensor sends this fact to other sensors, after that, all sensors send the measured value to the controller. Then, it is difficult to implement such aggregation in a completely decentralized manner. Instead of (5), we may consider the following event-triggering condition:

$$\hat{y}_i(k) := \begin{cases} y_i(k) & \text{if (4) holds,} \\ \hat{y}_i(k-1) & \text{otherwise.} \end{cases} \tag{6}$$

In this case, the control performance may become deteriorated, but the controller does not need to aggregate all measured signals. Implementation in a decentralized manner will be relatively easy. Depending on the control specification and the communication environment, we can choose either (5) or (6). From (2), (4), and (5) (or (2), (4), and (6)), we see that the event-triggering condition is decentralized, but the controller is centralized. Equation (5) (or (6)) yields the following inequality:

$$(\hat{y}_i(k) - y_i(k))^2 \leq \sigma_i^2 y_i^2(k), \quad i \in \{1, 2, \dots, p\}. \tag{7}$$

Under the above preparations, the design problem of decentralized event-triggered control via output feedback is formulated as follows.

Problem 1: For the system (1), suppose that σ_i in (4) is given. Then, find an event-triggered output feedback controller (2) such that the closed-loop system is asymptotically stable.

In the standard approach, first, we design a controller, and next, determine the threshold of an event-triggering condition (see, e.g., [3], [4]). In the above problem, we consider finding a controller under the assumption where the threshold of an event-triggering condition is given. Such approaches have also been studied in e.g., [8], [10], [15], [16], [20]–[23]. Since in event-triggered control, there are uncertainties in the measured value managed in a controller, it may be appropriate to design a controller considering these uncertainties together with model uncertainties and disturbances. Moreover, the threshold of an event-triggering condition may be determined in advance based on the accuracy requirement of measurement. Thus, in this paper, we consider the above problem. In the above problem, it is easy to add the effect of disturbances including the quantized error (see, e.g., [21], [22]).

Remark 1: For simplicity of discussion, we suppose that each element of the measured output is assigned to each individual sensor. Each sensor may measure multiple elements of the measured output. In this case, we suppose that the sensor $j \in \{1, 2, \dots, q\}$ $(q < p)$ measures $[y_i(k)]_{i\in\mathcal{I}_j}$, $\mathcal{I}_j \subseteq \{1, 2, \dots, p\}$, where $\cup_{j=1}^q \mathcal{I}_j = \{1, 2, \dots, p\}$ and $\cap_{j=1}^q \mathcal{I}_j = \emptyset$ hold. Then, instead of (4), the event-triggering condition is given by $\|[\hat{y}_i(k-1)]_{i\in\mathcal{I}_j} - [y_i(k)]_{i\in\mathcal{I}_j}\|^2 > \sigma_j^2\|[y_i(k)]_{i\in\mathcal{I}_j}\|^2$. The main result in the next section can be applied to this case by minor modifications.

3. Main Result

In this section, an LMI-based solution method for Problem 1 is proposed. As the main result, Problem 1 is rewritten as an LMI feasibility problem.

As a preparation, we define the error variable as follows:

$$e(k) := \hat{y}(k) - y(k).$$

From this definition, (7) is replaced with

$$e_i^2(k) \leq \sigma_i^2 y_i^2(k). \tag{8}$$

From the system (1) and the controller (2), the closed-loop system is given by

$$\bar{x}(k+1) = \bar{A}x(k) + \bar{B}e(k), \tag{9}$$

where

$$\bar{x} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \ \bar{A} = \begin{bmatrix} A & BC_c \\ B_cC & A_c \end{bmatrix}, \ \bar{B} = \begin{bmatrix} 0 \\ B_c \end{bmatrix}.$$

First, we introduce the following quadratic Lyapunov function:

$$V(k) = \bar{x}^\top(k)P\bar{x}(k), \tag{10}$$

where $P = P^\top \in \mathcal{R}^{2n\times 2n}$ is a positive-definite matrix. Here, consider the problem of finding a controller such that

$$V(k+1) - V(k) < -\beta V(k), \tag{11}$$

where $\beta \in [0, 1)$ is a given parameter.

Then, the following lemma can be obtained.

Lemma 1: Equation (11) holds if the following condition holds:

$$P_1 - \sum_{i=1}^{p} \tau_i P_{2,i} > 0, \tag{12}$$

where P_1 and $P_{2,i}$ are given by

$$P_1 = \begin{bmatrix} \bar{\beta}P - \bar{A}^\top P\bar{A} & * \\ -\bar{B}^\top P\bar{A} & -\bar{B}^\top P\bar{B} \end{bmatrix},$$

$$P_{2,i} = \begin{bmatrix} \sigma_i^2 E_i & * & * \\ 0 & 0_{(2n-p)\times(2n-p)} & * \\ 0 & 0 & -E_i \end{bmatrix},$$

where $\bar{\beta} := 1 - \beta$, and $\tau_1, \tau_2, \ldots, \tau_p > 0$ are design parameters. In addition, $E_i \in \mathcal{R}^{p\times p}$ denotes the matrix in which the (i, i)-th element is 1, and other elements are 0.

Proof : Substituting (9) and (10) into (11), the following inequality can be obtained:

$$(\bar{A}x(k) + \bar{B}e(k))^\top P(\bar{A}x(k) + \bar{B}e(k)) - \bar{x}^\top(k)P\bar{x}(k) < -\beta\bar{x}^\top(k)P\bar{x}(k).$$

From this inequality, we can obtain

$$\begin{bmatrix} \bar{x}(k) \\ e(k) \end{bmatrix}^\top P_1 \begin{bmatrix} \bar{x}(k) \\ e(k) \end{bmatrix} > 0. \tag{13}$$

Equation (8) can be rewritten as

$$\begin{bmatrix} \bar{x}(k) \\ e(k) \end{bmatrix}^\top P_{2,i} \begin{bmatrix} \bar{x}(k) \\ e(k) \end{bmatrix} \geq 0, \ i \in \{1, 2, \ldots, p\}. \tag{14}$$

Finally, we have (12) by applying the $\mathcal{S}$-procedure [2] to (13) and (14). □

From Lemma 1, we have the following theorem.

Theorem 1: Problem 1 is reduced to the following LMI feasibility problem:

Problem 2:

find $X > 0, Y > 0, L > 0, W_1, W_2, W_3$
subject to

$$\begin{bmatrix} \bar{\beta}X & * & * & * & * & * \\ \bar{\beta}I & \bar{\beta}Y & * & * & * & * \\ 0 & 0 & L & * & * & * \\ XA + W_1C & W_3 & W_1 & X & * & * \\ A & AY - BW_2 & 0 & I & Y & * \\ M & MY & 0 & 0 & 0 & 2I - L \end{bmatrix} > 0, \tag{15}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{16}$$

where $M = [\mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_p) \ \ 0_{p\times(n-p)}]$. The matrices $X, Y \in \mathcal{R}^{n\times n}$ and $L = \mathrm{diag}(\tau_1, \tau_2, \ldots, \tau_p) \in \mathcal{R}^{p\times p}$ are positive-definite, and $W_1 \in \mathcal{R}^{n\times p}$, $W_2 \in \mathcal{R}^{m\times n}$, and $W_3 \in \mathcal{R}^{n\times n}$ are unconstrained.

Using the solution for Problem 2, the matrices A_c, B_c, and C_c in the controller (2) are derived as

$$A_c = Z^{-1}(XAY + ZB_cCY - XBC_cY - W_3)Y^{-1},$$
$$B_c = Z^{-1}W_1,$$
$$C_c = W_2Y^{-1},$$

respectively.

Proof : First, without of loss of generality, the positive definite matrix P can be replaced with

$$P = \begin{bmatrix} X & Z \\ Z & Z \end{bmatrix}$$

(see [11]). By applying the Schur complement [2] to $P > 0$, we can obtain $X - Z > 0$. Then, the matrix $Y := (X - Z)^{-1} > 0$ is defined.

Next, the condition (12) can be rewritten as

$$\Theta_1 - \Theta_2^\top\Theta_3^{-1}\Theta_2 > 0, \tag{17}$$

where

$$\Theta_1 = \begin{bmatrix} \bar{\beta}P & 0 \\ 0 & L \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} P\bar{A} & P\bar{B} \\ \bar{C} & 0 \end{bmatrix},$$

$$\Theta_3 = \begin{bmatrix} P & 0 \\ 0 & L^{-1} \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} M & 0_{p\times n} \end{bmatrix}.$$

By applying the Schur complement to (17), we can obtain

$$\begin{bmatrix} \bar{\beta}P & * & * & * \\ 0 & L & * & * \\ P\bar{A} & P\bar{B} & P & * \\ \bar{C} & 0 & 0 & L^{-1} \end{bmatrix} > 0.$$

We define the matrix T as follows:

$$T := \begin{bmatrix} I_n & 0 \\ Y & -Y \end{bmatrix}.$$

Pre-multiplying by block-diag(T, I, T, I), and post-multiplying by block-diag$(T^\top, I, T^\top, I)$, we can obtain

$$\begin{bmatrix} \bar{\beta}TPT^\top & * & * & * \\ 0 & L & * & * \\ TP\bar{A}T^\top & TP\bar{B} & TPT^\top & * \\ \bar{C}T^\top & 0 & 0 & L^{-1} \end{bmatrix} \succ 0. \quad (18)$$

From $L \succ 0$ and $L^{-1} \succ 0$, the following inequality holds (see, e.g., [21]):

$$(I - L)L^{-1}(I - L) \geq 0,$$

which can be rewritten as

$$L^{-1} - (2I - L) \geq 0.$$

Applying this inequality to (18), the following condition can be obtained as a sufficient condition of (18):

$$\begin{bmatrix} \bar{\beta}TPT^\top & * & * & * \\ 0 & L & * & * \\ TP\bar{A}T^\top & TP\bar{B} & TPT^\top & * \\ \bar{C}T^\top & 0 & 0 & 2I - L \end{bmatrix} \succ 0. \quad (19)$$

Finally, from the definition of T, we can obtain

$$\begin{aligned} TPT^\top &= \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \\ TP\bar{B} &= \begin{bmatrix} W_1 \\ 0 \end{bmatrix}, \\ TP\bar{A}T^\top &= \begin{bmatrix} XA + W_1C & W_3 \\ A & AY - BW_2 \end{bmatrix}, \\ \bar{C}T^\top &= \begin{bmatrix} M & MY \end{bmatrix}, \\ W_1 &= ZB_c, \\ W_2 &= C_cY, \\ W_3 &= XAY + ZB_cCY - XBC_cY - ZA_cY. \end{aligned}$$

Applying these relations to (19), we have (15). In addition, from $P \succ 0$, i.e., $TPT^\top \succ 0$, we have (16). The matrices A_c, B_c, and C_c in the controller (2) can be obtained from the above W_1, W_2, and W_3. This completes the proof. □

An LMI feasibility problem can be easily solved by using e.g., MATLAB.

In the problem formulation of this paper, we focus on only the asymptotic stability, and we do not consider the transient response. We can consider a kind of the optimality by combining the proposed method with the linear quadratic regulator (LQR). The design problem of event-triggered LQR is reduced to an LMI optimization problem [15], [20]. The control performance about the transient response can be evaluated by adding an appropriate objective function to Problem 2. A detail is one of the future efforts.

4. Numerical Example

To demonstrate the proposed method, a numerical example is presented. As a plant, consider the discrete-time linear system obtained from the linearized fourth-order inverted pendulum system [20], where A, B, and C in (1) are given by

$$A = \begin{bmatrix} 1.0000 & -0.0006 & 0.0488 & 0 \\ 0 & 1.0378 & 0.0036 & 0.0506 \\ 0 & -0.0253 & 0.9512 & -0.0006 \\ 0 & 1.5195 & 0.1440 & 1.0378 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1040 \\ -0.3045 \\ 4.1260 \\ -12.1545 \end{bmatrix},$$

$$C = \begin{bmatrix} I_2 & 0_{2,2} \end{bmatrix}$$

(i.e., $n = 4$, $m = 1$, and $p = 2$). In the state $x = [x_1 \ \ x_2 \ \ x_3 \ \ x_4]^\top$, x_1, x_2, x_3, and x_4 correspond to the cart position, the angle of the pendulum, the cart velocity, and the angular velocity of the pendulum, respectively. In this example, (5) is used considering the control performance. The parameters σ_1 and σ_2 in the event-triggering condition (4) are given by $\sigma_1 = \sigma_2 = 0.1$, respectively. The parameter β in (11) is given by $\beta = 0.06$.

By solving Problem 2 (i.e., the LMI feasibility problem), we can obtain X, Y, L, W_1, W_2, and W_3 as follows:

$$X = 10^4 \times \begin{bmatrix} 0.0004 & -0.0007 & 0.0007 & 0.0001 \\ -0.0007 & 0.9046 & 0.3029 & -0.1612 \\ 0.0007 & 0.3029 & 1.6837 & -0.0311 \\ 0.0001 & -0.1612 & -0.0311 & 0.0291 \end{bmatrix},$$

$$Y = 10^3 \times \begin{bmatrix} 0.0147 & -0.0026 & -0.0253 & 0.0202 \\ -0.0026 & 0.0073 & -0.0001 & -0.0200 \\ -0.0253 & -0.0001 & 0.1866 & -0.3209 \\ 0.0202 & -0.0200 & -0.3209 & 0.7907 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.2216 & 0 \\ 0 & 1.5758 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.2216 & -0.0002 \\ 0.0945 & -0.9342 \\ 0.0129 & 0.1718 \\ -0.0173 & -0.0460 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.9326 & -2.2800 & 12.8993 & -33.6139 \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 0.8571 & 0.0149 & 0.1424 & -0.1147 \\ 0.0563 & 0.8193 & 0.4719 & -2.2305 \\ -0.0399 & -0.0099 & 0.7737 & 0.4679 \\ -0.0017 & -0.0432 & -0.0852 & 1.1964 \end{bmatrix}.$$

Using there matrices, A_c, B_c, and C_c in the controller (2) can be obtained as follows:

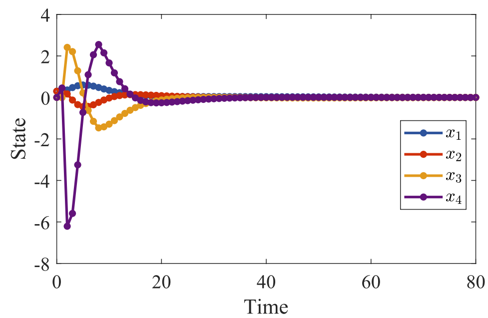

Fig. 1 Time response of the state in the plant.

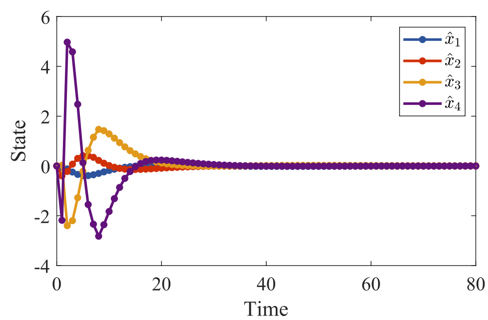

Fig. 2 Time response of the state in the controller.

$$A_c = \begin{bmatrix} 0.9545 & -0.0297 & 0.0680 & 0.0137 \\ -0.0834 & -0.5007 & -0.0525 & 0.0108 \\ 0.8530 & 3.1549 & 1.7113 & 0.5395 \\ -2.6198 & -14.7044 & -2.0952 & -0.5532 \end{bmatrix},$$

$$B_c = \begin{bmatrix} -0.0670 & -0.1067 \\ -0.0205 & -1.3115 \\ 0.0017 & 0.1038 \\ -0.1119 & -7.1616 \end{bmatrix},$$

$$C_c = \begin{bmatrix} -0.2063 & -0.7456 & -0.1842 & -0.1309 \end{bmatrix}.$$

Next, we present the result of numerical simulation. The initial states of the plant and the controller are given by $x(0) = [0.3\ \ 0.3\ \ 0\ \ 0]^\top$ and $\hat{x}(0) = [0\ \ 0\ \ 0\ \ 0]^\top$, respectively. Figure 1 shows the time response of the state in the plant. Figure 2 shows the time response of the state in the controller. Figure 3 shows the time response of the control input. Figure 4 shows the inter-event time. Figure 5 shows the time response of $x + \hat{x}$.

From Fig. 1 and Fig. 2, we see that both the states in the plant and the controller converge to the origin. From Fig. 3 and Fig. 4, we see that even if the event does not occur, the control input may be changed. Because the controller has the dynamics.

From Fig. 1 and Fig. 2, we see that the controller does not seem to estimate the state of the plant. On the other hand, from Fig. 5, we see that the controller estimates $-x$. Thus, in this example, the state equation $-\hat{x}(k+1) = A_c \cdot (-\hat{x}(k)) + (-B_c)\hat{y}(k)$ and the output equation $u(k) = (-C_c) \cdot (-\hat{x}(k))$ of the controller correspond to the full-order state observer and the state-feedback controller using the estimated state, respectively.

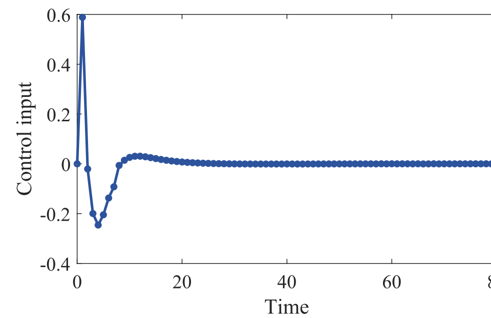

Fig. 3 Time response of the control input.

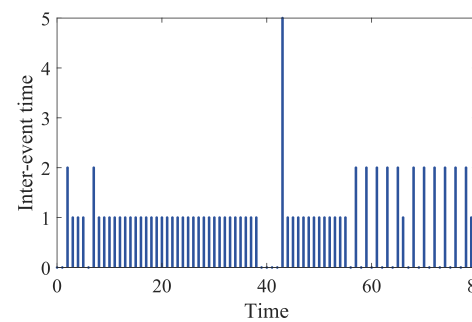

Fig. 4 Inter-event time.

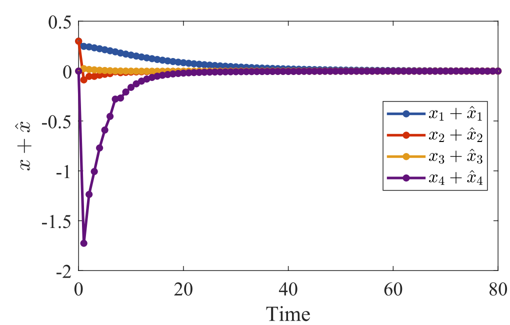

Fig. 5 Time response of $x + \hat{x}$.

Third, we discuss the computation results with different parameters σ_i. In comparison, we introduce two performance indices $p_s := \sum_{k=0}^{80} \sum_{i=1}^{4} |x_i(k)|$ and p_e. The index p_e is defined by the number of times that the event occurs. Consider three cases ($\sigma_i = 0.02, 0.1, 0.18$). The computation results are presented as follows:

$$\begin{aligned} \sigma_i = 0.02: &\quad p_s = 59.8178,\ p_e = 79, \\ \sigma_i = 0.1: &\quad p_s = 61.5684,\ p_e = 63, \\ \sigma_i = 0.18: &\quad p_s = 72.7385,\ p_e = 55. \end{aligned}$$

In the case of $\sigma_i = 0.02$, the event occurs at almost every time. From these results, we see that for a larger σ_i, p_e becomes smaller, but the performance index p_s on the state become deteriorated.

Finally, we compare the proposed method with the standard method. In the standard method, we calculate a controller satisfying only (11). In other words, we derive a conventional stabilizing controller. For the obtained controller, the event-triggering condition (4) is applied. Consider three cases ($\sigma_i = 0.02, 0.1, 0.18$). In all cases, the state of the closed-loop systems converges to the origin. The performance indices p_s and p_e are presented as follows:

$$\sigma_i = 0.02 : \quad p_s = 89.1035, \; p_e = 79,$$
$$\sigma_i = 0.1 : \quad p_s = 91.8776, \; p_e = 63,$$
$$\sigma_i = 0.18 : \quad p_s = 97.1998, \; p_e = 50.$$

From these results, we see that in this example, the performance of the proposed method is better than that of the standard method. Since the optimality is not considered, such results may be obtained. For general cases, further discussions are required.

5. Conclusion

In this paper, we proposed the LMI-based design method of event-triggered output feedback controllers for discrete-time linear systems, where sensors are located in a distributed way. Using the proposed method, unnecessary communications can be excluded. The design problem of the controller is reduced to an LMI feasibility problem, which can be easily solved by using e.g., MATLAB. The proposed method is demonstrated by a numerical example.

There are several future efforts. First, in this paper, we consider only communications from the sensors to the controller. It is one of the future efforts to consider reducing communications from the controller to the actuators. Next, since we consider the asymptotic stability, communications always occur in the steady state. To avoid such communications, it is important to use the notion of uniformly ultimate boundedness [1], [10], [17], [21]. Third, in this paper, the parameter σ_i in the event-triggering condition (4) is given in advance. It is important to consider the simultaneous design of σ_i and the controller. Fourth, it is also important to clarify the meaning of the state in the controller (2). In the numerical example of Sect. 4, the state $\hat{x}$ in the controller corresponds to $-x$. General cases should be analyzed theoretically. Finally, to show the effectiveness of the proposed method from the practical viewpoint, it is significant to apply the proposed method to more practical and large-scale systems such as air conditioning systems and power networks.

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