



Title	Simultaneous analysis of slope instabilities on a small catchment-scale using coupled surface and subsurface flows
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1 **Simultaneous Analysis of Slope Instabilities on a Small Catchment-scale using Coupled Surface**  
2 **and Subsurface Flows**

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27

## Highlights

28

29 1. A coupled model of surface flow, subsurface flow, and soil mechanics is proposed.

30 2. Slope instabilities are analyzed on a small catchment-scale.

31 3. Runoff increases the possibility of embankment collapse at the exit of the gully.

32

## 33 **Abstract**

34 High-velocity runoff generated in hillslopes during heavy rainfall caused by typhoon increases  
35 the instability of the embankment slope at the exit of the gully. Such effects of high-velocity  
36 runoff are usually neglected in conventional rainfall-induced slope failure analysis. In order to  
37 consider the effects of runoff on the slope instability, this study attempts to simulate the runoff,  
38 infiltration, seepage, and slope instabilities on a small catchment-scale simultaneously. For this  
39 purpose, this study firstly proposes a coupled model of surface flow, subsurface flow, and soil  
40 mechanics based on shallow water equations, Richards's equation, Green-Ampt infiltration  
41 capacity model, and local factor of safety (LFS) approach. Next, to make the proposed coupled  
42 model effective in the practical analysis of runoff, a diffusion wave approximation of shallow  
43 water equations is validated by numerical simulations, and then it is used to replace shallow  
44 water equations in the proposed coupled model. Finally, the proposed coupled model is verified  
45 by Abdul and Gillham system and applied to a natural slope in Hokkaido, Japan. The numerical  
46 results highlight the influences of runoff from upstream on the embankment slope failure at the  
47 exit of the gully. Furthermore, the small catchment-scale slope instabilities assessment  
48 approach proposed in this study provides an effective approach for simulating heavy rainfall  
49 induced runoff and slope instabilities. The distribution map of the factor of safety (FOS) has  
50 significant implications for precisely determining the dangerous spots (instead of areas) on a  
51 small catchment-scale and accurately releasing the warning information to these dangerous  
52 spots.

53 **Keywords:** Surface flow; Subsurface flow; Slope instabilities; Local factor of safety

## 54 **1 Introduction**

55 The rainstorms and unexpected typhoons cause sediment-related disasters threatening  
56 the lives and public property in many parts of the world, especially in rainy mountainous

57 terrains. During a rainstorm, the infiltration capacity of the slope is not enough to absorb all  
58 the rainwater into the soil, resulting in the rainwater that cannot infiltrate into the soil flows in  
59 the form of runoff on the slope surface (Cuomo and Della Sala, 2013; Kean et al., 2013; Wei  
60 et al., 2017; Van Asch et al., 2018). Both rainwater infiltration and runoff could deteriorate the  
61 slope stability. The rainwater infiltration causes a decrease in the suction in the unsaturated  
62 zone and an increase in positive pore pressure in the saturated zone due to groundwater, which  
63 eventually induces the occurrence of landslide/slope failure (Chowdhury and Flentje, 2002;  
64 Rahardjo et al., 2005; Acharya et al., 2009; Zhang et al., 2014). On the other hand, the runoff,  
65 i.e. fast-flowing surface water, may cause erosion of the slope surface or pond in the concave  
66 areas increasing the possibility of slope failure at some locations such as a road embankment  
67 which crosses a gully. For example, during the summer season (August-September), the  
68 Japanese archipelago is often struck by violent typhoons with extremely intense rainfalls,  
69 which cause a large number of disasters, e.g. floods, debris flows, and landslides (Wang and  
70 Sassa, 2003; Fujisawa et al., 2010). According to the statistics of the Ministry of Land,  
71 Infrastructure, Transport, and Tourism (MLIT), there are 200,000 dangerous valleys and slopes  
72 in Japan, and about 1,000 landslide disasters reported annually (Osanai et al., 2010).

73 Both on catchment-scale and slope-scale, when analyzing the unsaturated soil slope  
74 instability under rainwater infiltration, for simplification, the influences of runoff are usually  
75 neglected (Liu et al., 2017; Chiu et al., 2019). In this case, two assumptions of the rainfall  
76 infiltration are generally used. One is that the rainfall infiltration is equal to the rainfall intensity.  
77 Another is that rainfall infiltration is equal to the component of rainfall intensity perpendicular  
78 to the boundary. Obviously, the above assumptions cannot fully reproduce the actual processes  
79 of rainfall/runoff infiltration, especially under heavy rainfall conditions (rainfall intensity is  
80 much larger than the infiltration capacity). Therefore, earlier studies made many attempts to

81 develop models that can describe the behavior of surface and subsurface flows. [Tian and Liu](#)  
82 [\(2011\)](#) coupled two-dimensional (2D) Saint Venant equations and three-dimensional (3D)  
83 Richards's equation in an Integrated Surface Water-Ground water Model (ISWGM).  
84 [Fernández-Pato et al. \(2016\)](#) combined 2D shallow water equations with two infiltration  
85 models, Horton model and Green-Ampt model, for estimating the runoff and infiltration in a  
86 watershed. However, there are no simplified methods or numerical models developed to  
87 simulate the slope instability by coupled surface and subsurface flows during heavy rainfall.

88 From the view of slope stability analysis, the limit-equilibrium method (LEM) and  
89 shear strength reduction technique (SSRT) are commonly used in some general commercial  
90 software packages e.g. GeoStudio ([GEO-SLOPE International, 2007](#)) and FLAC<sup>3D</sup> ([Itasca,](#)  
91 [2012](#)). The LEM discretizes the mass of a potential failure slope into smaller vertical slices and  
92 assesses the ratio of shear strength to shear stress for all slices as the factor of safety (FOS) of  
93 an identified or assumed potential failure surface ([Bishop, 1955](#); [Morgenstern and Price, 1965](#)).  
94 While the determination of where the failure initiates or the ultimate geometry and position of  
95 a landslide failure surface is one of the fundamental challenges when using LEM ([Lu et al.,](#)  
96 [2012](#)). Unlike conventional LEM, it is not necessary to specify the shape of the failure surface  
97 in advance when using the SSRT ([Sciarra et al., 2017](#); [Pasculli et al., 2018](#)). The FOS is defined  
98 as the ratio of the real shear strength to the reduced shear strength of the soil. The failure surface  
99 is determined by reducing the shear strength parameters (cohesion and friction angle) of the  
100 soil until the slope becomes unstable ([Farshidfar and Nayeri, 2015](#)). However, it is worth noting  
101 that LEM and SSRT are effective for the stability analysis of a single slope but not feasible for  
102 the analysis of slope instabilities on a catchment-scale. [Lu et al. \(2012\)](#) proposed an approach,  
103 i.e. local factor of safety (LFS) approach to quantify the FOS of a slope under rainwater  
104 infiltration, and verified that the assessment of the LFS approach is consistent with the LEM.

105 Furthermore, the LFS approach has the potential to overcome several major limitations in the  
106 classical FOS methodologies, such as the initiation and evolution of instability with changes in  
107 pore water pressure, and the inherent underestimation of slope instability (Lu et al., 2012).  
108 Besides, when analyzing the stability of multi slopes (more than one slope), LEM and SSRT  
109 need to analyze the slopes one by one. However, the LFS approach also has the potential to  
110 give the distribution map of FOS. Therefore, in this study, the LFS approach is used to assess  
111 the slope instabilities on a small catchment-scale.

112 Accordingly, the objectives of this study are to (1) develop a coupled model of surface  
113 flow and subsurface flow to simulate the relationship between rainfall, runoff, and infiltration  
114 under heavy rainfall conditions; (2) model the rainfall/runoff induced slope instabilities to  
115 determine the dangerous spots on a small catchment-scale. For these purposes, this study firstly  
116 proposes a coupled model of surface flow, subsurface flow, and soil mechanics to  
117 simultaneously simulate runoff, infiltration, seepage, and slope instabilities on a small  
118 catchment-scale. Surface flow is governed by 2D shallow water equations. Subsurface flow is  
119 governed by 3D Richards's equation. Two well-known models, namely Horton model (Horton,  
120 1933) and Green-Ampt model (Green and Ampt, 1911), are commonly used to estimate soil  
121 infiltration capacity. The parameters in the Horton model have no clear physical basis and must  
122 be estimated from the experimental data, while the parameters in the Green-Ampt model have  
123 physical meaning and can be estimated from soil properties (Fernández-Pato et al., 2016).  
124 Therefore, the Green-Ampt model is used in this study for estimating the infiltration capacity  
125 of the ground surface, which could be used to determine the boundary conditions of subsurface  
126 flow analysis. The LFS approach is used to assess the slope instabilities on a small catchment-  
127 scale. Next, to make the proposed coupled model effective in the practical analysis of runoff,  
128 a diffusion wave approximation of shallow water equations is validated based on numerical

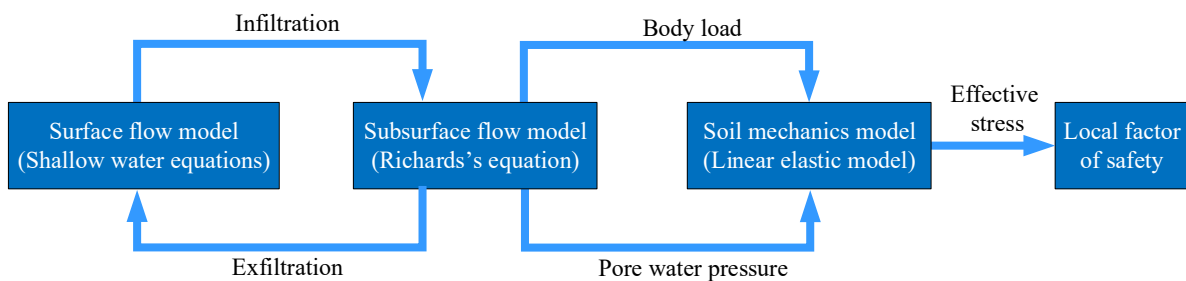
129 simulations. Afterward, the diffusion wave approximation is used to replace shallow water  
130 equations in the proposed coupled model. Then, the proposed coupled model is verified by  
131 Abdul and Gillham system ([Abdul and Gillham, 1984](#)). Finally, the simulation of surface flow,  
132 subsurface flow, and slope instabilities for a natural mountain area in Hokkaido, Japan is  
133 performed by using the proposed coupled model.

## 134 **2 Numerical Modeling Strategy**

135 The surface and subsurface flows are complex environmental systems that often behave  
136 in a coupled manner. In this study, a coupled model of surface flow, subsurface flow, and soil  
137 mechanics is proposed by using a finite element software, COMSOL Multiphysics ([COMSOL  
138 Multiphysics, 2018](#)). In the coupled model, the 3D soil mechanics model (linear elastic model)  
139 is established by the solid mechanics module of COMSOL. The 2D surface flow model  
140 (shallow water equations) and 3D subsurface flow model (Richards's equations) are established  
141 by the PDEs (partial differential equations) module of COMSOL. The surface flow model and  
142 subsurface flow model are coupled through infiltration and exfiltration. The subsurface flow  
143 model and soil mechanics model are coupled in two ways: (1) the body load function that  
144 depends on the volumetric water content is applied to a linear elastic soil mechanics model to  
145 manifest the effect of moisture variation on the self-weight and stress distribution, and (2) the  
146 effect of volumetric water content variation on the pore water pressure (suction) is considered  
147 for evaluating effective stress. Accordingly, the local factor of safety can be calculated by using  
148 effective stress as shown in Fig. 1.

149 During torrential rain, rainwater infiltration is a two-stage process, i.e. rainfall  
150 infiltration (rainfall derived infiltration) in the early stage of the rainfall event, and runoff  
151 infiltration (runoff derived infiltration, in this case, the infiltration is controlled by the pressure  
152 gradient rather than the rainfall intensity) in the later stage of the rainfall event. Therefore, the

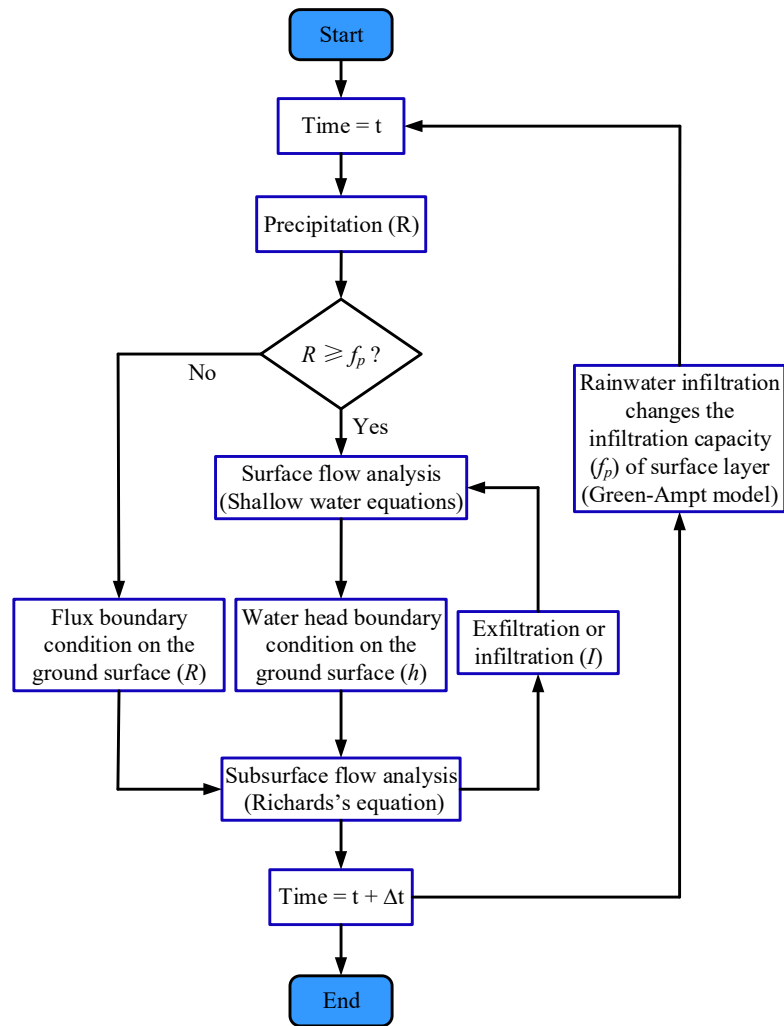
153 Green-Ampt model is used for estimating the infiltration capacity ( $f_p$ ) of the ground surface to  
 154 determine the boundary conditions of subsurface flow analysis. That is in the early stage of the  
 155 rainfall event, as the rainfall intensity ( $R$ ) is usually weak and less than the infiltration capacity  
 156 ( $f_p$ ) of the ground surface, the Richards's equation is directly solved with a flux boundary  
 157 condition at the surface. If the rainfall intensity exceeds the infiltration capacity ( $f_p$ ), part of  
 158 rainwater infiltrates into the ground, and the rest generates runoff on the ground surface. On  
 159 the other hand, the exfiltration (positive value) and infiltration (negative value) calculated from  
 160 the subsurface flow model is added to the surface flow model as a source and sink item. Then,  
 161 the infiltrated rainwater causes the decrease of the infiltration capacity ( $f_p$ ) of the ground surface,  
 162 and the calculation of the next timestep will be carried out. This study proposes an iterative  
 163 cross-coupled surface and subsurface flows model to simulate this process. The flowchart of  
 164 the time-marching scheme in iterative cross-coupled surface and subsurface flows model is  
 165 shown in Fig. 2.



166

167

**Fig. 1.** Scheme of the proposed coupled hydrological and slope stability model.



168

169

**Fig. 2.** Flowchart of the time-marching scheme in iterative cross-coupled surface and

170

subsurface flows model.

171

### 3 Governing Equations

172

The governing equations in the proposed coupled model are represented in this part.

173

The surface flow is governed by the 2D shallow water equations, and the subsurface flow is

174

governed by the 3D Richards's equation. The soil infiltration capacity is estimated by the

175

Green-Ampt model, and the slope instabilities are assessed by the LFS approach.

176 3.1 Governing equation for surface flow

177 Surface flow is calculated by 2D shallow water equations which can be expressed as  
 178 follows (Murillo et al., 2007).

179 Equation of continuity:

$$180 \quad \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = R - I \quad (1)$$

181 Equations of motion:

$$182 \quad \frac{\partial(uh)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -hg \frac{\partial H}{\partial x} - hgS_{fx} + D_x \quad (2)$$

$$183 \quad \frac{\partial(vh)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -hg \frac{\partial H}{\partial y} - hgS_{fy} + D_y \quad (3)$$

184 where,  $h$  is water depth (m);  $u$ ,  $v$  is water velocity in the  $x$  and  $y$  direction (m/s);  $R$  is rainfall  
 185 intensity (m/s);  $I$  is infiltration rate (m/s);  $g$  is the gravitational acceleration ( $m/s^2$ );  $H$  is water  
 186 surface elevation (m);  $t$  is time (s);  $D_x$ ,  $D_y$  is advection term in the  $x$  and  $y$  direction;  $S_{fx}$ ,  $S_{fy}$  is  
 187 the friction slope in the  $x$  and  $y$  direction respectively, usually written in term of the Manning's  
 188 roughness coefficient  $n_m$  ( $s/m^{1/3}$ ).

$$189 \quad S_{fx} = n_m^2 u \sqrt{u^2 + v^2} / h^{4/3}, S_{fy} = n_m^2 v \sqrt{u^2 + v^2} / h^{4/3} \quad (4)$$

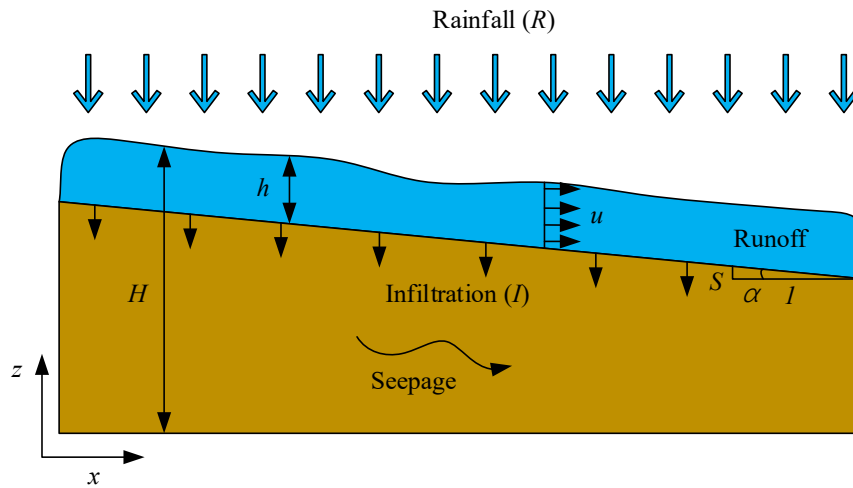
190 The first terms on the right-hand side of the equations of motion, Eq. (2) and Eq. (3),  
 191 represent the driving forces from the slope gradient and water depth gradient. The second terms  
 192 represent the drag forces due to friction (friction loss gradient). The third terms are advection  
 193 terms, and they can be assumed by the following conditions.

$$194 \quad D_x = \frac{\partial}{\partial x} \left[ v_t \frac{\partial(uh)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ v_t \frac{\partial(uh)}{\partial y} \right], \quad D_y = \frac{\partial}{\partial x} \left[ v_t \frac{\partial(vh)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ v_t \frac{\partial(vh)}{\partial y} \right] \quad (5)$$

195 In which,  $v_t$  is eddy viscosity coefficient ( $m^2/s$ ), and it can be assumed as follows (Zeng  
 196 et al., 2010).

197 
$$v_t = \lambda h U^* \text{ and } U^* = \sqrt{ghS} \tag{6}$$

198 where,  $U^*$  is the frictional velocity (m/s);  $\lambda$  is dimensionless eddy viscosity, and its standard  
 199 value for an infinitely wide channel is 0.067 (Zeng et al., 2010);  $S$  is the gradient of water  
 200 surface. As the water depth gradient is much smaller than the slope gradient, it can be assumed  
 201 that  $S$  is equal to the slope gradient (Weill et al., 2009) as shown in Fig. 3.



202

203 **Fig. 3.** Conceptual schematic of the surface flow model.

204 In principle, by simultaneously solving the equations of continuity and motion, the  
 205 behavior of the surface runoff can be tracked, and the water depth and velocity can be obtained  
 206 at any interesting location. However, the timesteps need to be set very small, or simulations  
 207 only can be performed under relatively flat terrain conditions. It causes the low calculation  
 208 efficiency of shallow water equations when performing the practical runoff analysis in vast  
 209 mountainous areas (Rengers et al., 2016). The main reason can be considered as that some  
 210 insignificant terms in the equation of motion, Eq. (2) and Eq. (3), significantly increase the  
 211 calculation time and decrease convergence. Therefore, to ignore the insignificant terms in the  
 212 equations of motion, the contribution of each term is examined by performing numerical  
 213 simulations using seven surface flow models with different slope angles ( $<1^\circ$ ,  $5^\circ$ ,  $10^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,

214 45°, and 60°). In the surface flow model, Manning's coefficient value is 0.3 s/m<sup>1/3</sup>. A fixed  
 215 water head boundary condition ( $h=0.001$  m) is applied to the left side and zero gradient  
 216 boundary condition ( $\partial h/\partial x=0$ ) is applied on the right side. The simulation time is 10 s with the  
 217 timestep of  $1.0 \times 10^{-4}$  s. Table 1 shows the simulation results, i.e. the contribution of each term  
 218 in Eq. (2) as a fraction of 100 % of the total for surface water flow under different slope angles.

219 **Table 1** Contribution of each term in the equations of motion for surface water flow.

Slope angle (°)	Inertia term $\frac{\partial(uh)}{\partial t}$ (%)	Velocity term $\frac{\partial(hu^2)}{\partial x}$ (%)	Driving force term $-hg\frac{\partial H}{\partial x}$ (%)	Friction term $-hgS_{fx}$ (%)	Advection term $D_x$ (%)
<1	<0.001	<0.001	50.038	49.961	<0.001
5	<0.001	0.005	49.584	50.410	<0.001
10	<0.001	0.011	49.595	50.393	<0.001
15	0.003	0.015	49.535	50.446	0.001
30	0.024	0.219	52.537	47.200	0.020
45	0.051	0.850	44.260	54.730	0.109
60	0.159	0.327	50.808	48.679	0.027

220 From the results, it is clear that the sum of the driving force term and friction term  
 221 accounts for over 99% for all types, while the total contribution of the inertia term, advection  
 222 term, and velocity term is less than 1% together. Therefore, the following diffusion wave  
 223 approximation can be typically used in practice.

224 
$$S_{fx} + \frac{\partial H}{\partial x} = 0 \quad (7)$$

225 
$$S_{fy} + \frac{\partial H}{\partial y} = 0 \quad (8)$$

226 After substituting Eq. (4) into Eq. (7) and Eq. (8), the expressions for the components  
 227 of the velocity vector can be obtained as follows.

228 
$$u = -\frac{h^{2/3}}{n_m\sqrt{|S|}}\nabla_x(H), \quad v = -\frac{h^{2/3}}{n_m\sqrt{|S|}}\nabla_y(H) \quad (9)$$

229 Finally, Eq. (1) can be written as follows, and then it is used to replace shallow water  
230 equations in the proposed coupled model.

231 
$$\frac{\partial h}{\partial t} - \nabla \left( \frac{h^{5/3}}{n_m\sqrt{|S|}}\nabla(H) \right) = R - I \quad (10)$$

### 232 3.2 Governing equation for subsurface flow

233 When performing subsurface flow analysis in unsaturated soils, two well-known  
234 methods are commonly used: Richards's equations and two-phase Darcy's law method.  
235 Compared with the two-phase Darcy's law method, Richards's equation is simple in form and  
236 the physical meaning of each parameter is relatively clear. Therefore, the subsurface flow is  
237 governed by 3D Richards's equation (Richards, 1931).

238 
$$\nabla \cdot [k_s k_r \cdot \nabla(H_p + z)] + Q_w = [C_m + S_e S_c] \frac{\partial H_p}{\partial t} \quad (11)$$

239 where,  $C_m$  is specific moisture capacity ( $\text{m}^{-1}$ );  $S_c$  is specific storage coefficient ( $\text{m}^{-1}$ );  $S_e$  is the  
240 effective degree of saturation;  $H_p$  is pressure head ( $H_p$  is negative in unsaturated soil, and  
241 positive in saturated soil) (m);  $z$  is elevation (m);  $k_r$  is relative hydraulic conductivity;  $k_s$  is  
242 saturated hydraulic conductivity (m/s);  $Q_w$  is sink and source of water ( $\text{s}^{-1}$ ).

243 van Genuchten (1980) proposed a Soil Water Characteristic Curve (SWCC) to describe  
244 the relationship in  $C_m$ ,  $S_e$ ,  $k_r$ ,  $\theta$ , and  $H_p$  in unsaturated soil. As shown in Eq. (12) to Eq. (15),  
245 these parameters could be specified by the saturated and residual volumetric water content  $\theta_s$   
246 and  $\theta_r$ , as well as constants of  $a$ ,  $n$ ,  $m$ , and  $l$ .

247 
$$\theta = \theta_r + S_e(\theta_s - \theta_r) \quad (12)$$

248 
$$S_e = \frac{1}{[1+(aH_p)^n]^m}, \quad m = 1 - \frac{1}{n} \quad (13)$$

249 
$$C_m = \frac{am}{1-m} (\theta_s - \theta_r) S_e^{\frac{1}{m}} (1 - S_e^{\frac{1}{m}})^m \quad (14)$$

250 
$$k_r = S_e^l \left[ 1 - (1 - S_e^{\frac{1}{m}})^m \right]^2 \quad (15)$$

251 **3.3 Soil infiltration capacity model**

252 The Green-Ampt model (Green and Ampt, 1911) assumes that the soil infiltration  
 253 capacity is governed by the soil properties and rainfall conditions. The soil infiltration capacity,  
 254  $f_p$ , can be approximated as follows.

255 
$$f_p = k_s \left( 1 + \frac{\Psi \Delta \theta}{F} \right) \quad (16)$$

256 where,  $f_p$  is infiltration capacity (m/s);  $\psi$  is the average suction head at the wetting front (m),  
 257 and  $\Delta \theta$  is the difference between the saturated volumetric water content,  $\theta_s$ , and the initial  
 258 volumetric water content,  $\theta_i$ , ( $\Delta \theta = \theta_s - \theta_i$ ) ( $\text{m}^3/\text{m}^3$ ).  $F$  is the cumulative infiltration (m).

259 
$$F = \int_0^t I dt \quad (17)$$

260 In this study, it is considered that the Green-Ampt model is only used to estimate the  
 261 infiltration capacity ( $f_p$ ) during the rainfall infiltration stage and determine when the runoff  
 262 generate. At the beginning of the simulation, a profile with initial moisture content is used to  
 263 determine the initial infiltration capacity of the ground surface. With infiltration of the  
 264 rainwater, the Green-Ampt model is running and infiltration capacity will be redistributed at  
 265 each timestep according to Eq. 16. Afterward, the rainfall intensity at the next timestep will be  
 266 compared with the updated infiltration capacity. If the rainfall intensity exceeds the infiltration  
 267 capacity of the ground surface, the water is ponding on the ground surface. The infiltration rate  
 268 will be determined by pressure head at the surface (zero or higher depending on the increasing  
 269 runoff  $h$ ) and the pressure head in the cell below. A part of the not infiltrated water and the  
 270 exfiltrated water from the underground will then be applied to the runoff simulation as source  
 271 item in the next timestep.

### 272 3.4 3D soil mechanics model (LFS approach)

273 [Lu et al. \(2012\)](#) proposed the LFS approach to quantify the factor of safety (FOS) of a  
274 point based on the current state of stress and the change in the suction ( $u_a - u_w$ ) caused by  
275 rainwater infiltration. The pore water pressure,  $u_w$ , can be calculated by the pressure head,  $H_p$ ,  
276 ( $u_w = \rho_w g H_p$ ,  $\rho_w$  is the density of water with the value of  $1000 \text{ kg/m}^3$ ). Afterward, the 3D  
277 distribution of soil moisture and related  $u_w$  in the saturated and unsaturated zone during a  
278 rainfall event is coupled with the 3D soil mechanics model in two ways. One is that the  
279 volumetric water content,  $\theta$ , is applied to the 3D soil mechanics model as body load to manifest  
280 the effect of moisture variation on the self-weight and stress distribution as follows.

$$281 \quad \nabla \cdot (\boldsymbol{\sigma}) + \gamma(\theta)\mathbf{b} = 0 \quad (18)$$

282 where,  $\boldsymbol{\sigma}$  is the stress tensor (kPa);  $\mathbf{b}$  is the unit vector of body forces, and  $\gamma$  is the bulk unit  
283 weight ( $\text{N/m}^3$ ), which is a function of the volumetric water content  $\theta$ .

284 Another is that the negative pore water pressure (suction) in the unsaturated zone will  
285 increase the effective stress of the soil, while the positive pore water pressure in the saturated  
286 zone due to groundwater will decrease the effective stress of the soil. The influence of the  
287 suction on the effective stress is evaluated with Bishop's effective stress ([Bishop, 1954](#)).

$$288 \quad \sigma' = (\sigma - u_a) + \chi(u_a - u_w), \quad \chi = \frac{S_e - S_r}{1 - S_r} \quad (19)$$

289 where,  $\sigma'$  is effective stress (kPa);  $u_a$  is the pore air pressure (kPa);  $u_w$  is the pore water pressure  
290 (kPa), and  $\chi$  is the matrix suction coefficient which varies from 0 to 1 depending on the degree  
291 of saturation.  $S_r$  is the residual degree of saturation. Finally, the local factor of safety ( $F_{LFS}$ ) at  
292 each point within a hillslope can be defined as follows.

$$293 \quad F_{LFS} = \frac{2 \cdot \cos \phi'}{\sigma'_1 - \sigma'_3} \left[ c' + \frac{\sigma'_1 + \sigma'_3}{2} \tan \phi' \right] \quad (20)$$

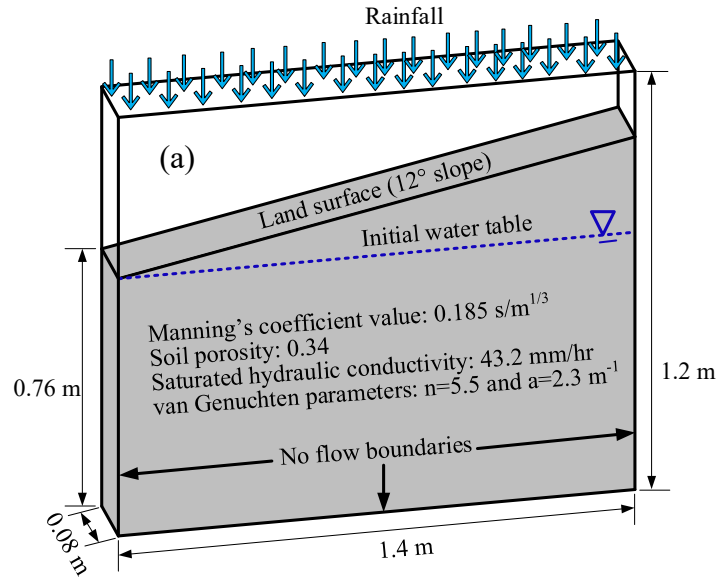
294 where,  $c'$  is the effective cohesion (kPa);  $\phi'$  is the effective friction angle ( $^{\circ}$ );  $\sigma_1'$  and  $\sigma_3'$  are  
295 the maximum and minimum principal stress for the unsaturated soil (kPa).

#### 296 **4 Validation of the Iterative Cross-coupled Surface and Subsurface Flows Model**

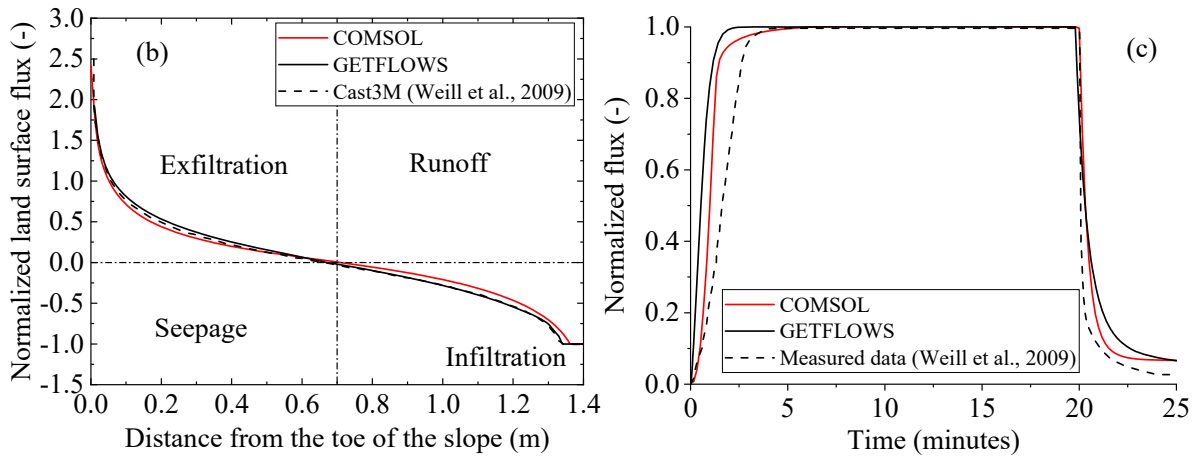
297 In this part, the iterative cross-coupled surface and subsurface flows model proposed in  
298 chapter 2 is verified by the experimental system presented by [Abdul and Gillham \(1984\)](#). The  
299 experimental system is composed of a 1.4 m×1.2 m×0.08 m Plexiglas box filled with medium-  
300 fine sand as illustrated in Fig. 4(a). The free water is drained off at the toe of the slope, and the  
301 initial water table is located at the toe of the slope. The soil properties are shown in Fig. 4(a).  
302 A rainfall rate of 43.2 mm/hr is applied on the whole surface domain in the first 20 minutes of  
303 a total time of 25 minutes. To verify the simulation results of the proposed coupled model, the  
304 experimental system is also simulated by a commercial software, GETFLOWS ([GETFLOWS,](#)  
305 [2014](#)), which is a finite difference fluid flow numerical simulator. [Kitamura et al. \(2016\)](#) and  
306 [Malow et al. \(2017\)](#) validated the applicability of GETFLOWS for simulating the surface flow  
307 and subsurface flow process by comparing the simulation results of GETFLOWS and  
308 measurements of river water levels in the area of eastern Fukushima Prefecture in Japan and  
309 the area of Kourtimalei in Djibouti, respectively. The difference between GETFLOWS and the  
310 proposed model in this study in terms of theory and governing equations is that GETFLOWS  
311 simulate surface and subsurface flows in a fully coupled way by using air and water two-phase  
312 flows, and the governing equation of mass conservation is expressed as follows ([Mori et al.,](#)  
313 [2015](#)).

$$314 \quad \frac{\partial(\phi S_p)}{\partial t} - \nabla \cdot (u_p) = q_p, \quad p = (\text{water, air}) \quad (21)$$

315 where, subscript  $p$  indicates fluid phase, water ( $w$ ) or air ( $a$ );  $\phi$  is the effective porosity ( $\text{m}^3/\text{m}^3$ );  
316  $S_p$  is fluid saturation of  $p$  phase, ( $S_w + S_a = 1$ );  $u_p$  is the fluid flow velocity of  $p$  phase (Pa);  $q_p$  is  
317 the volumetric flux of sink and source of  $p$  phase ( $\text{m}^3/\text{m}^3/\text{s}$ ).



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**Fig. 4.** (a) Abdul and Gillham system; (b) Comparison of calculated results of normalized flux along the land surface at the 19 minutes after rain; (c) Comparison of calculated results and measured data of normalized flux of discharge at the toe of the slope.

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Fig. 4(b) and Fig. 4(c) plot the results calculated by COMSOL and GETFLOWS compared with the results calculated by Cast3M (a finite element code that was used by Weill et al. (2009) for modeling surface/subsurface flow in a fully integrated way, which is similar to GETFLOWS), and measured data referred from Weill et al. (2009). Fig. 4(b) shows the fluxes along the land surface, and all the fluxes are normalized by the rainfall flux imposed at the land surface (entering fluxes are negative by convention). The results imply that the models

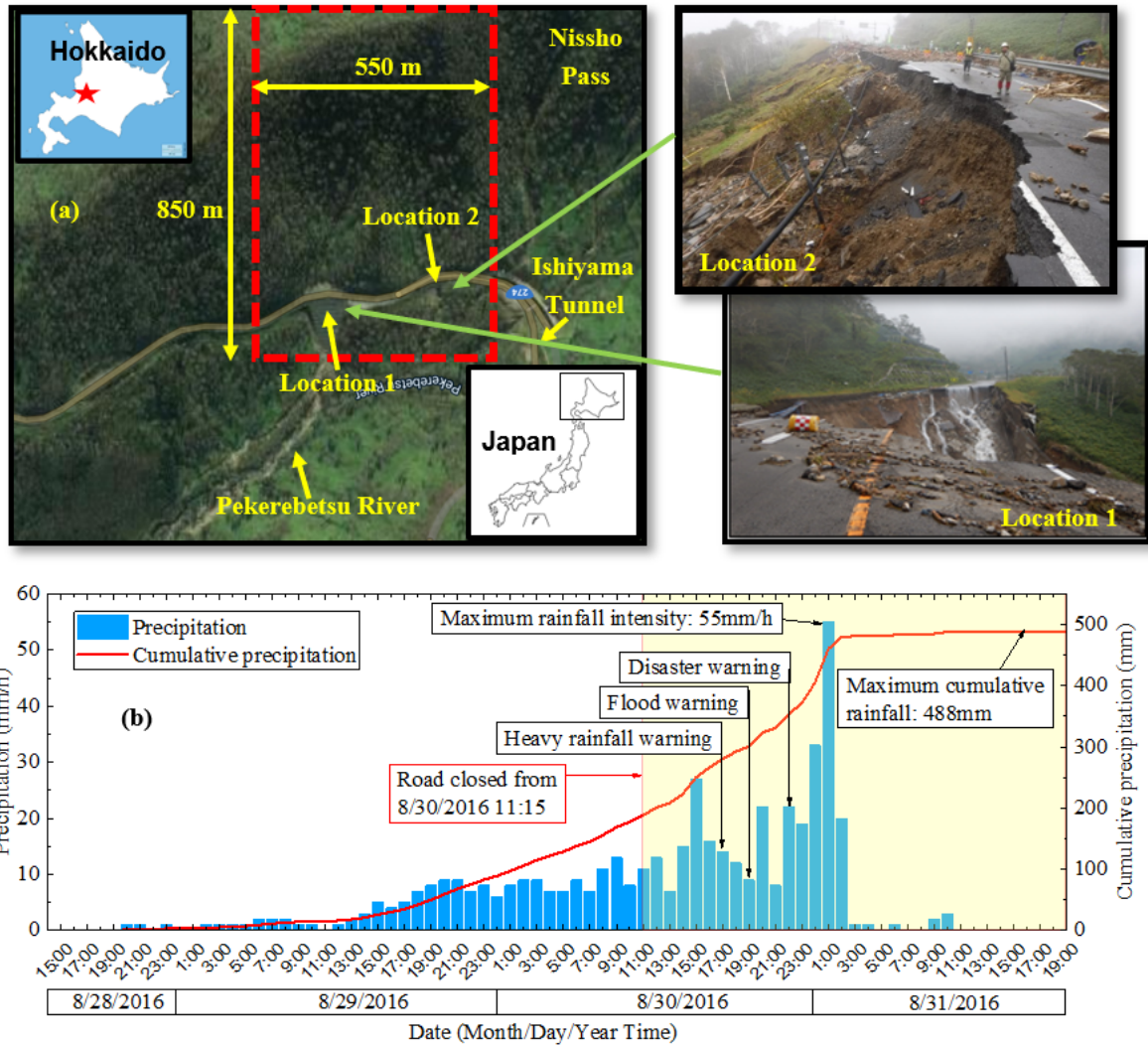
329 implemented in COMSOL and GETFLOWS are able to describe the three surface regimes  
330 (infiltration, runoff, and exfiltration) along the land surface: in a small area at the top of the  
331 slope, all rainwater infiltrates into the soil (normalized flux equals -1); in the upper half of the  
332 slope, part of the rainwater infiltrates, and the rest flows in the form of runoff on the land  
333 surface (normalized flux between -1 and 0); at the lower half of the slope, groundwater  
334 exfiltrates to the land surface and flows out with the runoff from the upper part (normalized  
335 flux positive). Fig. 4(c) displays the normalized flux of discharge at the toe of the slope  
336 calculated by COMSOL and GETFLOWS compared with the data measured by [Weill et al.](#)  
337 [\(2009\)](#). It shows that the calculated results agree well with the measured data, though the  
338 simulated time to reach the steady state of overland water and groundwater exchange is shorter  
339 than the experimental one. The presence of air could significantly slow down the infiltration  
340 process ([Weill et al., 2009](#)), and the inconsideration of this effect in the modeling approach  
341 could be responsible for that.

## 342 **5 Case Study of Typhoon Induced Embankment Slope Failures**

### 343 5.1 Outline of disasters

344 In 2016, from the Pacific, Typhoon No.10 (Lionrock) landed on Hokkaido, Japan on  
345 August 29<sup>th</sup>-31<sup>st</sup>, and the sediments eroded and transported from slopes and banks during the  
346 event were estimated to be approximately  $3.7 \times 10^5 \text{ m}^3$  within the Pekerebetsu catchment in  
347 Hokkaido, Japan ([Furuichi et al., 2018](#)). Near to Nissho Pass along the National Highway Route  
348 274 in Hokkaido, Japan, Typhoon No.10 triggered intense landslides, embankment collapses,  
349 and debris flows as shown in Fig. 5(a). According to the rainfall records obtained from  
350 Automated Meteorological Data Acquisition System (AMeDAS), the observed cumulative  
351 rainfall that fell from 19:00 on August 28<sup>th</sup> to 10:00 on August 31<sup>st</sup> was 488 mm with the peak  
352 value of 55 mm at 01:00 on August 31<sup>st</sup> as shown in Fig. 5(b), which is the highest rainfall ever

353 recorded in that area. Based on the prediction of the occurrence of sediment-related disasters  
 354 by the Japanese early warning system, the road was closed from 11:15 on August 30<sup>th</sup>, and  
 355 Hokkaido government released heavy rainfall warning information, flood warning information,  
 356 and disaster warning information at 17:00, 19:00, and 22:00 on August 30<sup>th</sup>, respectively.

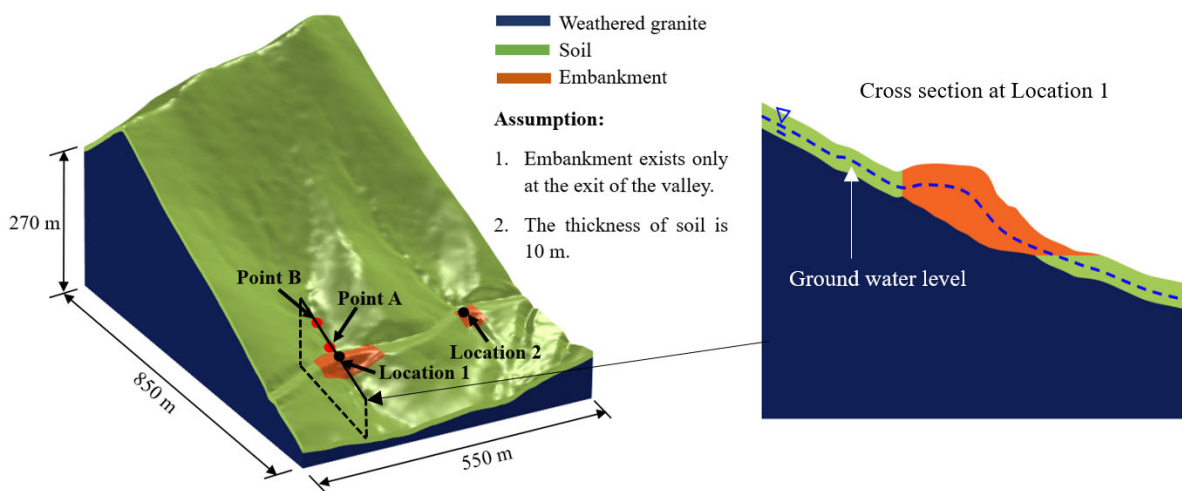


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 358 **Fig. 5.** (a) Locations of slope failures induced by Typhoon No.10 along National  
 359 Highway Route 274; (b) Rainfall recorded during Typhoon No.10 at Nissho Pass.

360 5.2 Simulation of surface and subsurface flows for a natural mountain area

361 The surface flow and subsurface flow analysis at Nissho Pass are performed by the  
 362 proposed coupled model. Based on the digital elevation model (DEM) produced from airborne

363 laser scanning (1m resolution), a 3D model for a natural mountain area (surrounded by the red  
 364 dashed box in Fig. 5(a)) for slope instabilities assessment with surface and subsurface flows  
 365 analysis is built as shown in Fig. 6. The model is composed of three parts: weathered granite,  
 366 soil, and embankment. Soil properties are listed in Table 2. The parameters, i.e. dry density  
 367 ( $\rho_s$ ), saturated hydraulic conductivity ( $k_s$ ), saturated volumetric water content ( $\theta_s$ ), effective  
 368 cohesion ( $c'$ ), and effective friction angle ( $\phi'$ ), have been obtained from laboratory element  
 369 tests (Sato et al., 2017). The parameters for which no results of laboratory tests are available,  
 370 i.e. residual volumetric water content ( $\theta_r$ ) and van Genuchten parameters ( $\alpha$  and  $m$ ), were  
 371 estimated based on the grain size curve of soil (SoilVision, 2018). Although there were three  
 372 other typhoons before Typhoon No.10, since they were at least one week apart from Typhoon  
 373 No.10, they had little effect on the groundwater level during Typhoon No.10. Therefore, the  
 374 initial groundwater level is set to -5.5m from the ground surface according to the historical  
 375 measured average value in the same period of previous years. Manning's coefficient value is  
 376  $0.3 \text{ s/m}^{1/3}$  for the slope, which is the recommended value of Japan Institute of Country-ology  
 377 and Engineering (JICE) for mountain grassland. The simulation time is from 19:00 on August  
 378 28<sup>th</sup>, 2016 to 17:00 on August 31<sup>st</sup>, 2016 for a total of 70 hours with the timestep of 1 hour.



380 **Fig. 6.** Three-dimensional numerical model of a natural mountain area at Nissho Pass.

381

**Table 2** Soil properties used for the simulation of the natural mountain area.

Parameters	Embankment	Soil	Weathered granite
Dry density, $\rho_s$ (kg/m <sup>3</sup> )	1695	1020	2000
Effective cohesion, $c'$ (kPa)	0	0	37
Effective friction angle, $\phi'$ (°)	37	35	21
Saturated hydraulic conductivity, $k_s$ (m/s)	$1.12 \times 10^{-5}$	$1.4 \times 10^{-6}$	$3.47 \times 10^{-9}$
Saturated volumetric water content, $\theta_s$ (m <sup>3</sup> /m <sup>3</sup> )	0.36	0.63	0.48
Residual volumetric water content, $\theta_r$ (m <sup>3</sup> /m <sup>3</sup> )	0.035	0.19	0.008
van Genuchten parameter, $\alpha$ (1/m)	0.538	0.810	0.012
van Genuchten parameter, $m$	0.468	0.437	0.246

382

As there are no measurement data at the locations where the slope failures occurred,

383

the simulated results in this study are theoretically verified by a physical runoff model (Tank

384

model) proposed by [Sugawara et al. \(1974\)](#), which uses multi-layered tanks to simulate

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rainwater infiltration and surface runoff. The Tank model has been proved that it is effective

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for describing the outflow in the watershed ([Hong et al., 2015](#)). The calculation of runoff is

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based on the generic value of the parameters suggested by [Okada \(2001\)](#) as listed in Fig. 7. The

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water storage depth (mm) for each tank is calculated based on the equations below.

389

$$\frac{dH_1}{dt} = R - I_1 - q_{11} - q_{12} \quad (22)$$

390

$$\frac{dH_2}{dt} = I_1 - q_2 - I_2 \quad (23)$$

391

$$\frac{dH_3}{dt} = I_2 - q_3 - I_3 \quad (24)$$

392

The outflow rate from each outlet of the model and infiltration rate from the upper tank

393

to the lower tank is calculated based on the equations below.

394

$$q_{11} = \begin{cases} \alpha_{11} \times (H_1 - L_{11}) & , \text{if } H_1 > L_{11} \\ 0 & , \text{if } H_1 \leq L_{11} \end{cases} \quad (25)$$

395 
$$q_{12} = \begin{cases} \alpha_{12} \times (H_1 - L_{12}) & , \text{if } H_1 > L_{12} \\ 0 & , \text{if } H_1 \leq L_{12} \end{cases} \quad (26)$$

396 
$$q_2 = \begin{cases} \alpha_2 \times (H_2 - L_2) & , \text{if } H_2 > L_2 \\ 0 & , \text{if } H_2 \leq L_2 \end{cases} \quad (27)$$

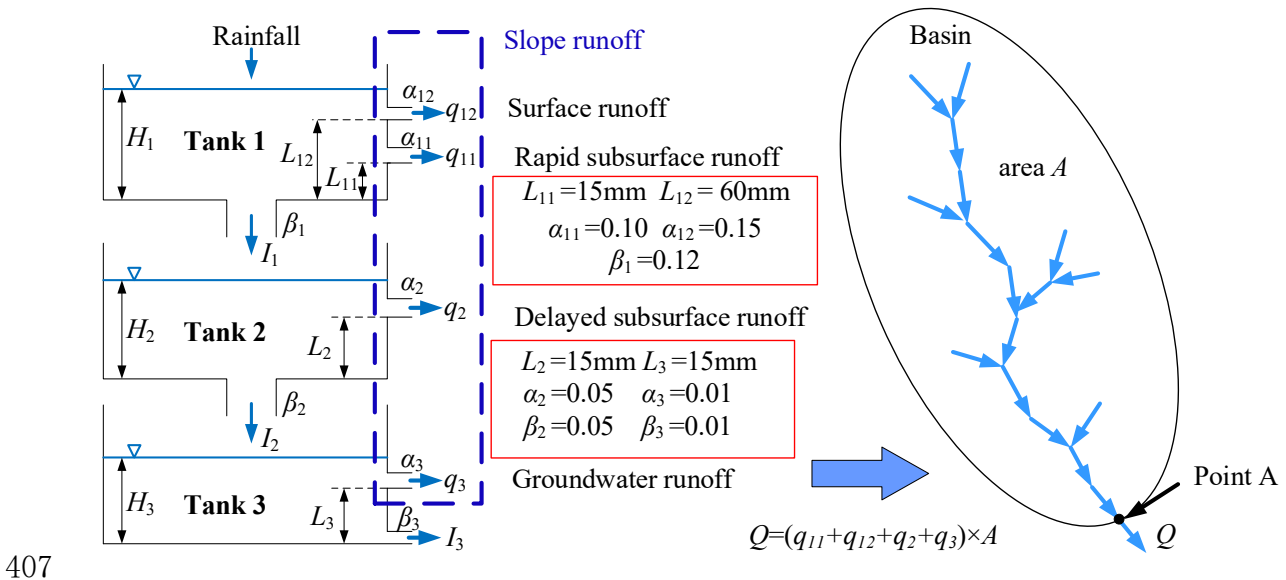
397 
$$q_3 = \begin{cases} \alpha_3 \times (H_3 - L_3) & , \text{if } H_3 > L_3 \\ 0 & , \text{if } H_3 \leq L_3 \end{cases} \quad (28)$$

398 
$$I_1 = \beta_1 \times H_1, \quad I_2 = \beta_2 \times H_2, \quad I_3 = \beta_3 \times H_3 \quad (29)$$

399 The total outflow rate of the basin is represented as follows.

400 
$$Q = (q_{11} + q_{12} + q_2 + q_3) \times A \quad (30)$$

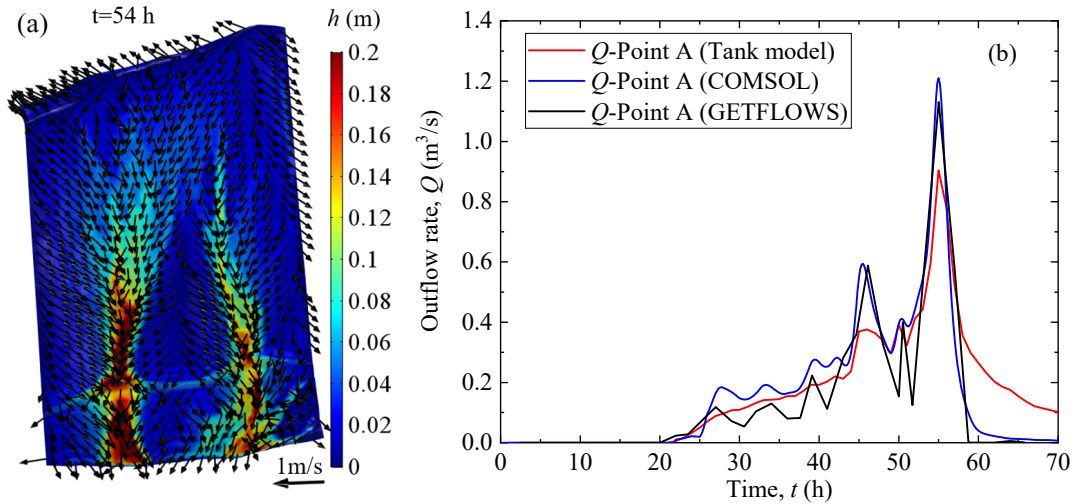
401 where,  $I_1$ ,  $I_2$ , and  $I_3$  are the infiltration rate from the upper tank to the lower tank (mm/h);  $q_{11}$ ,  
 402  $q_{12}$ ,  $q_2$ , and  $q_3$  are the outflow rate (mm/h) for each outlet of sidewall;  $L_{11}$ ,  $L_{12}$ ,  $L_2$ , and  $L_3$   
 403 represent each outlet height (mm);  $H_1$ ,  $H_2$ , and  $H_3$  are the water storage depth (mm) in each  
 404 layer;  $\alpha_{11}$ ,  $\alpha_{12}$ ,  $\alpha_2$ , and  $\alpha_3$  are the outflow coefficient (1/h) for each outlet;  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are the  
 405 coefficients of permeability (1/h) from the bottom hole of each tank;  $Q$  is total of outflow rate  
 406 of the basin ( $\text{m}^3/\text{s}$ ), and  $A$  is the area of the basin ( $\text{km}^2$ ).



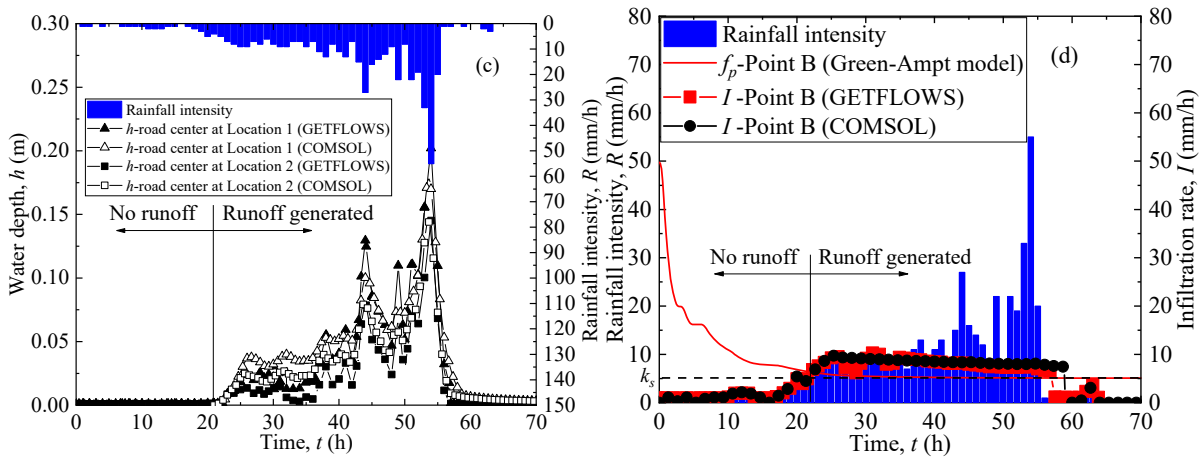
408 **Fig. 7.** Schematic diagram of the three-layer Tank model.

409 Fig. 8(a) displays the distribution of water depth and the vector of flow velocity  
410 calculated by COMSOL. From Fig. 8(a), it can be seen that a large amount of water from the  
411 upstream is gathered at Location 1 and Location 2, which allows more water to infiltrate into  
412 the embankment. The outflow rates of the catchment area at Location 1 (Point A in Fig. 6 and  
413 Fig. 7) calculated by the Tank model, COMSOL, and GETFLOWS are shown in Fig. 8(b). To  
414 avoid the calculation errors caused by the water coming from Location 2 along the road (see  
415 Fig. 5(a) and Fig. 8(a)), Point A is located on the edge of the embankment on the side of the  
416 mountain. From Fig. 8(b), it is recognized that the outflow rates calculated by the Tank model,  
417 COMSOL, and GETFLOWS are quite similar suggesting that the numerical results are reliable.  
418 Tank model assumes that the slope runoff flows out of the catchment area according to a certain  
419 percentage of water storage depth in each tank. Accordingly, the Tank model does not consider  
420 the impact of slope angle, which causes the outflow rate calculated by the Tank model is  
421 smaller at the peak value, and larger at the end of the rainfall event than that calculated by  
422 COMSOL and GETFLOWS. Fig. 8(c) plots the surface water depth located at the road center  
423 (where water comes from the upstream catchment and road) at Location 1 and Location 2. To  
424 discuss the two-stage process of rainwater infiltration, a representative point on the hillside  
425 slope (Point B in Fig. 6, located in the gully upstream of Location 1) is selected to display the  
426 relationship between infiltration rate and rainfall intensity on the hillside slope as shown in Fig.  
427 8(d). The results shown in Fig. 8(d) suggest that at the beginning of a rainfall event, the  
428 infiltration rate is equal to the rainfall intensity (all rainwater infiltrates into the soil). From Fig.  
429 8(c), it is recognized that the runoff simulated by COMSOL and GETFLOWS is generated  
430 from 22 hours after the rainfall event happens, which is consistent with the results calculated  
431 by the Tank model shown in Fig. 8(b). Nearly at the same time, rainfall intensity exceeds the  
432 infiltration capacity ( $f_p$ ) of the ground surface in Fig. 8(d).

433



434



435

**Fig. 8.** (a) Distribution of water depth and the vector of flow velocity calculated by

436

COMSOL; (b) Comparison of outflow rate ( $Q$ ) calculated by each approach; (c) Surface

437

water depth ( $h$ ) at Location 1 and Location 2; (d) Infiltration capacity ( $f_p$ ) and infiltration rate

438

( $I$ ).

439

After rainfall intensity exceeds the infiltration capacity ( $f_p$ ), i.e. the runoff is generated,

440

the infiltration rate is no longer equal to rainfall intensity. At this time, the infiltration rate is

441

governed by the pressure head gradient. The pressure head gradient is controlled by the water

442

depth and the pressure head in the cell below. With the rainwater infiltration, the increase of

443

the pressure head in the cell below is more significant than the increase of the water depth. It

444

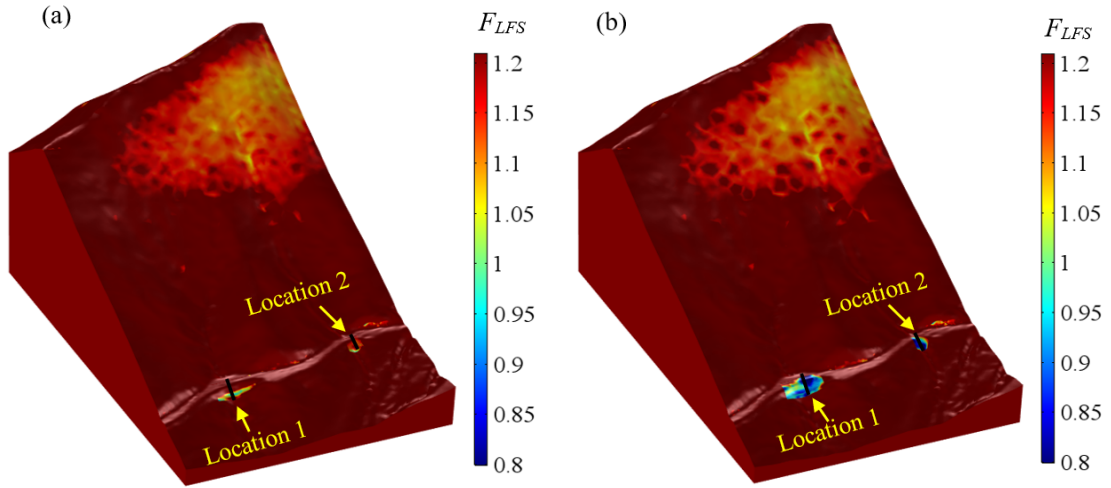
means that the pressure head gradient will become smaller compared with when the runoff is

445 just generated, i.e., the infiltration rate decreases with time during runoff as shown in Fig. 8(d).  
446 Furthermore, the surface water depth (Fig. 8(c)) and infiltration rate (Fig. 8(d)) calculated by  
447 GETFLOWS agree well with the results calculated by COMSOL at both Location 1 and  
448 Location 2, meaning that the two software can be mutually verified and the coupled surface  
449 and subsurface flows model is more reliable than Tank model.

### 450 5.3 Slope instabilities assessment along the highway on a small catchment-scale

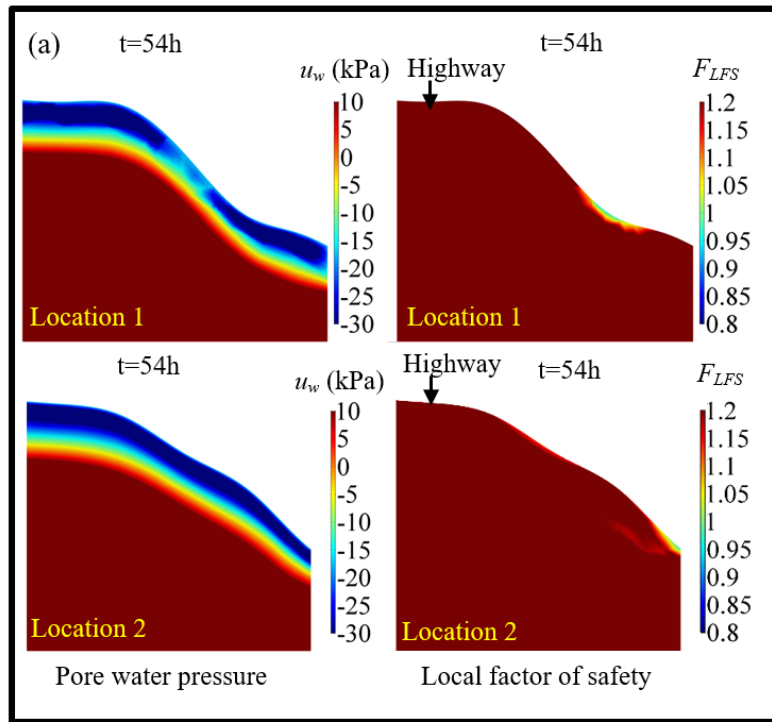
451 In order to investigate the effect of runoff and subsurface flow on the slope stability  
452 during Typhoon No.10, the slope instabilities on a small catchment-scale (including two  
453 embankment slopes at Location 1 and Location 2, respectively) are analyzed by using the  
454 proposed coupled model. The effects of runoff on the infiltration and subsurface flow are  
455 considered by the coupled surface and subsurface flows model proposed in chapter 2. By  
456 incorporating the body load (volumetric water content) and pore water pressure ( $u_w$ ) calculated  
457 by the coupled surface and subsurface flows model into the soil mechanics model, two cases  
458 are studied, i.e., slope instabilities analysis with considering runoff and without. Fig. 9 shows  
459 the distribution of the local factor of safety ( $F_{LFS}$ ) in the two cases. From Fig. 9(b), it is  
460 recognized that with considering runoff, the safe area and two dangerous spots are identified  
461 in a 550m wide and 850m long area during the heavy rainfall, i.e., the slope failure occurred  
462 ( $F_{LFS} < 1.0$ ) at Location 1 and Location 2 are successfully reproduced. Conversely, without  
463 considering runoff, slope failure only occurred in a very small area at Location 1 as shown in  
464 Fig. 9(a). Fig. 10 shows the distributed pore water pressure ( $u_w$ ) and local factor of safety ( $F_{LFS}$ )  
465 at Location 1 and Location 2 under the two study cases (with considering runoff and without).  
466 From Fig. 10, it can be seen that the presence of runoff leads to a more significant increase in  
467 pore water pressure. The negative pore water pressure increases to zero or even becomes  
468 positive pore water pressure in the surface layer of the soil, which causes the occurrence of the

469 slope failure. From Fig. 10(b), it can be seen that the simulated slip surface (red line in Fig.  
470 10(b)) is slightly shallower than the actual slip surface (blue line in Fig. 10(b)) at Location 2.  
471 The main reason why the simulated slip surface is slightly shallower than the actual slip surface  
472 is that in the actual process of slope failure, the collapsed part moved downstream, and runoff  
473 further eroded the newly exposed soil and caused further damage of the embankment.  
474 Therefore, it implies that runoff has significant effects on the embankment slope failures  
475 especially at the exit of the gully. On the other hand, by using the LFS approach, slope  
476 instabilities analysis can be performed on a small catchment-scale, which has significant  
477 advantages as compared with other methods to analyze the stability of a single slope. Moreover,  
478 the distribution map of FOS conduces to determine the dangerous spots in the target area. This  
479 has significant implications for precisely determining the dangerous spots (instead of areas) on  
480 a small catchment-scale and accurately releasing warning information to the dangerous spots.  
481 For example, in Japan, the disaster warning information is released to a 5 km×5 km area  
482 according to the national early warning system. Based on the early warning system, the  
483 occurrence time of slope failures can be roughly estimated, while it is difficult to determine the  
484 specific number and location of slope failures. Therefore, the coupled model of surface flow,  
485 subsurface flow, and soil mechanics proposed in this study provides an effective way for  
486 simulating heavy rainfall-induced runoff and slope instabilities in the target area. By expanding  
487 the size of the coupled model proposed in this paper to 5 km×5 km, the occurrence of slope  
488 failures can be roughly predicted by the Japan early warning system, and the dangerous spots  
489 can be identified in this wide area by the numerical results. It means that combining the  
490 numerical simulation results with the prediction results of an early warning system, warning  
491 information will be accurately released to the dangerous spots instead of broader areas.

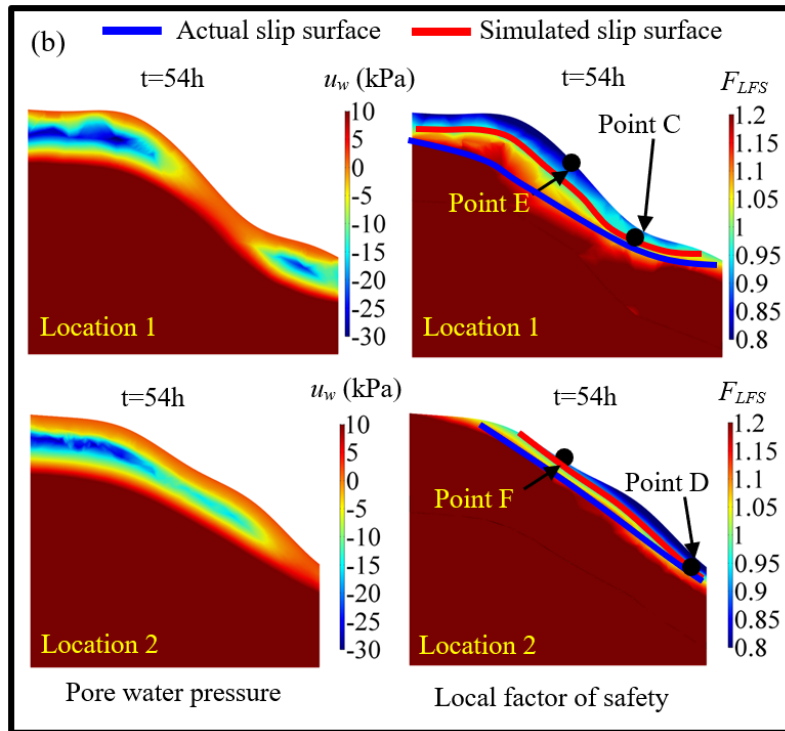


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**Fig. 9.** Distribution map of FOS on the small catchment-scale during Typhoon No.10. (a) Without considering runoff; (b) With considering runoff.



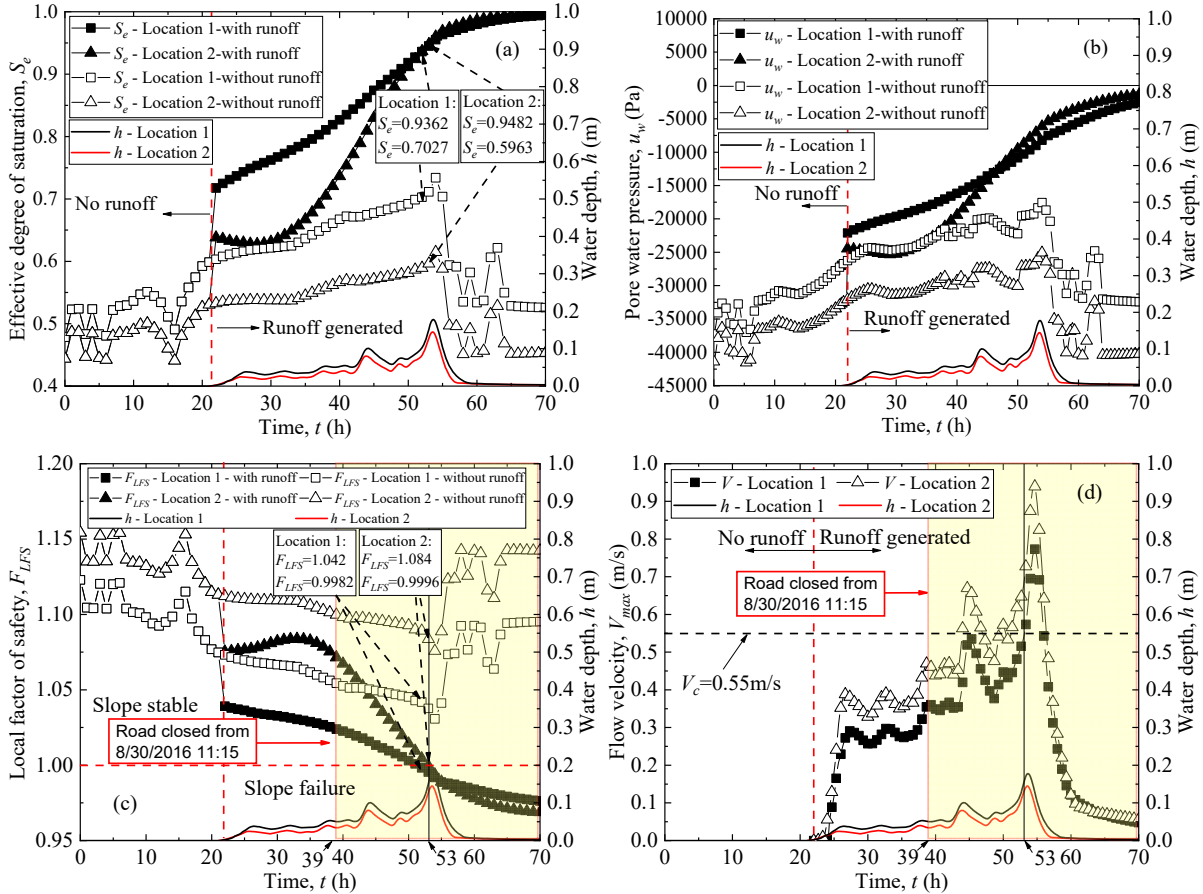
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496  
 497 **Fig. 10.** Distribution of pore water pressure ( $u_w$ ) and local factor of safety ( $F_{LFS}$ ). (a)  
 498 Without considering runoff; (b) With considering runoff.

499 To further analyze the effect of runoff on slope stability, Fig. 11(a) to Fig. 11(c)  
 500 illustrates the time-dependent effective degree of saturation ( $S_e$ ),  $u_w$ , and  $F_{LFS}$  at Location 1 and  
 501 Location 2 (Point C and Point D in Fig. 10, located near the ultimate slip surface as shown by  
 502 the red lines in Fig. 10(b)). It is recognized that the infiltrated rainwater causes the increase of  
 503 the  $S_e$  as shown in Fig. 11(a), which causes an increase in the  $u_w$  as shown in Fig. 11(b), thereby  
 504 causing a decrease in the suction of unsaturated soil, and eventually decreases the  $F_{LFS}$  as shown  
 505 in Fig. 11(c). It is worth noting that the increase of  $S_e$  and  $u_w$ , and the decrease of  $F_{LFS}$  are more  
 506 significant when the runoff is considered. After runoff is generated,  $S_e$  and  $u_w$  have a steep  
 507 increase and  $F_{LFS}$  sharp declines, meaning that the runoff from upstream allows more water to  
 508 infiltrate into the embankment, thereby causing a more significant decrease in the suction of  
 509 the embankment and the possibility of embankment slope failure at the exit of the gully to be  
 510 much greater than other locations along the highway. On the other hand, the excessive high-

511 velocity runoff could allow more water to infiltrate into the embankment and eventually trigger  
512 slope failure. Therefore, the runoff is a key factor causing the embankment slope failures, and  
513 its effects cannot be neglected. Erosion might occur on the slope surface when the flow velocity  
514 exceeds a critical value, called critical erosional velocity,  $V_c$  (Blais and McGinn, 2011). The  
515 critical erosional velocity is 0.55m/s at Location 1 and Location 2, which is calculated by the  
516 critical erosion velocity estimation model ( $V_c=0.33D_{50}^{0.47}$ ) proposed by Bogardi (1978).  $V_c$  is  
517 critical erosion velocity measured in m/s.  $D_{50}$  represents the median diameter of the sediment  
518 material in mm, and its value is 3.0 mm for the embankment at Nissho Pass (Kawamura and  
519 Miura, 2018). To discuss the effects of the runoff on the erosion of the embankment slope  
520 surface, Fig. 11(d) shows the flow velocity ( $V$ ) on the embankment slope at Location 1 and  
521 Location 2 (Point E and Point F in Fig. 9, located on the embankment slope surface). In Fig.  
522 11(d), it can be identified that the flow velocity exceeds the critical erosion velocity,  $V_c$  (0.55  
523 m/s), and even close to 1.0 m/s at peak both at Location 1 and Location 2. Therefore, it can be  
524 considered as the excessive high-velocity runoff and could cause severe erosion of the  
525 embankment slope. Moreover, from Fig. 11(c) and Fig. 11(d), it is recognized that the  $F_{LFS}$   
526 becomes less than 1.0 and the flow velocity exceeds  $V_c$  at 00:00 on August 31<sup>st</sup>, 2016 after the  
527 road was closed from 11:15 on August 30<sup>th</sup>, 2016 (the yellow period in Fig. 11(c) and Fig.  
528 11(d)). It means that the simulation results are reliable. However, the runoff induced erosion  
529 of the embankment slope is not considered in the numerical simulation of this study. The  
530 discussion in the effects of runoff on the erosion of the embankment slope surface and the  
531 influences of the erosion on the slope instability is a future assignment in this study.



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## 6 Discussions and Conclusions

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**Fig. 11.** Time-dependent effective degree of saturation ( $S_e$ ), pore water pressure ( $u_w$ ), local factor of safety ( $F_{LFS}$ ), and flow velocity ( $V$ ) at Location 1 and Location 2. (a)  $S_e$  vs. time; (b)  $u_w$  vs. time; (c)  $F_{LFS}$  vs. time; (d)  $V$  vs. time.

This study attempts to propose a numerical model that is applicable for simulating heavy rainfall induced runoff and slope instabilities on a small catchment-scale to determine the danger points (instead of areas) in the target area and accurately release warning information. At present, full shallow water equations are mainly used to simulate erosion and flooding. Due to the long calculation time of full shallow water equations, it cannot quickly obtain the calculation result and be applied to practice. Therefore, this study firstly simplified the shallow water equations by ignoring the insignificant terms in the equations of motion. Afterward, as

545 the general commercial software used for stability analysis of slopes can hardly simulate runoff,  
546 for example, GeoStudio and FLAC<sup>3D</sup>. Ignoring runoff is still the main calculation assumptions  
547 of surface hydrology in these software. Therefore, this study proposes a coupled model of  
548 surface flow, subsurface flow, and soil mechanics, which provides an effective way for  
549 simulating heavy rainfall-induced runoff and slope instabilities on a small catchment-scale. In  
550 addition, combining the numerical simulation results of the coupled model proposed in this  
551 paper with the prediction results of the Japanese early warning system in the target area (e.g. 5  
552 km×5 km), warning information will be accurately released to the dangerous spots instead of  
553 areas. The findings from this study can be outlined as follows:

554 1. In the equations of motion of shallow water equations, the driving force term and friction  
555 term are the main contribution terms, while the total contribution of the inertia term, advection  
556 term, and velocity term is less than 1% together. Therefore, the diffusion wave approximation  
557 that simplifies the equations of motion by considering only the driving force term and friction  
558 term is applicable to the practical runoff analysis.

559 2. The coupled model of surface flow, subsurface flow, and soil mechanics proposed in this  
560 study can reflect the two-stage process of rainwater infiltration, i.e. rainfall infiltration in the  
561 early stage of rainfall event and runoff infiltration in the later stage of the rainfall event, and it  
562 is also applicable to simulate runoff, infiltration, seepage, and slope instabilities on a small  
563 catchment-scale.

564 3. Excessive high-velocity runoff is a key factor causing the embankment slope failures at the  
565 exit of the gully, and its effects cannot be neglected. In this study, runoff and slope stability  
566 with two watersheds (each including an embankment slope) are successfully simulated by the  
567 coupled model of surface flow, subsurface flow, and soil mechanics proposed in this study.

568 The effects of runoff on the slope stability are successfully taken into consideration and the  
569 slope failures caused by runoff are reproduced through numerical simulation.

570 4. The LFS approach is more efficient to predict the rainfall-induced slope failures in the target  
571 area by simulating slope instabilities on a small catchment-scale, which has significant  
572 advantages as compared with other methods to analyze the stability of a single slope.  
573 Furthermore, the distribution map of FOS on the small catchment-scale conduces to determine  
574 the dangerous spots in the target area.

575 The research findings of this study are expected to help improve the numerical model  
576 of heavy rainfall-induced surface flow and slope failure, and benefit the prediction of the  
577 occurrence of heavy rainfall-induced disasters in the future. Expanding the coupled model  
578 proposed in this study to a larger area with more watersheds and combining the numerical  
579 results with the prediction results of the Japanese early warning system, as well as the  
580 discussion in the effects of runoff on the erosion of the embankment slope surface and the  
581 influences of the erosion on the slope instability are future assignments in this study.

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## 587 **Data Availability**

588 Weather station data and terrain information used in this research are publicly available at the  
589 website of Japan Meteorological Agency (<http://www.data.jma.go.jp/gmd/risk/obsdl/index.php>)  
590 and Geospatial Information Authority of Japan (<https://www.gsi.go.jp/top.html>). The results

591 data obtained in this research are available at the website  
592 (<https://data.mendeley.com/datasets/2n9zyyyfyz/1>).

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