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Aftershocks and Earthquake Statistics (II)

—— Further Investigation of Aftershocks and Other Earthquake Sequences Based on a New Classification of Earthquake Sequences ——

Tokuji UTSU

(Received Aug. 25, 1970)

Abstract

Earthquake sequences are usually classified into three types, 1: main shock — aftershock sequence, 2: foreshock sequence — main shock — aftershock sequence, and 3: earthquake swarm. However, it is pointed out in this paper that not a few sequences are multiple, i.e., they are characterized by the sequential occurrence of several simple sequences as illustrated by hourly frequency diagrams for many sequences in Japan. Taking such multiplicity into consideration, a new classification of earthquake sequences is proposed. According to this classification, it is seen that each of the terms "foreshocks", "aftershocks", and "earthquake swarm" used hitherto does not necessarily indicate only one type of earthquake groups. For example, earthquake swarms can be classified into two types. The first type, in which the activity is not triggered by large shocks, occurs mostly in volcanic areas and the largest magnitude rarely exceeds 6. The second type, which is a multiple occurrence of aftershock sequences triggered by several large shocks of comparable magnitudes, is found both in volcanic and non-volcanic areas and the largest shock in non-volcanic areas occasionally reaches magnitude 8. The geographical distribution of the two types of swarms and the similar type of multiple sequences shows some characteristic features. The b values for some of the second type swarms are very small ($b \fallingdotseq 0.5$). This is explained by the multiplicity of the sequence. The small b values reported for some foreshock sequences may be due to the structure of the sequence similar to the second type swarms.

The mutual dependence among aftershocks in the same sequence is investigated by several statistical methods. It may be concluded that there is a noticeable tendency for aftershocks to cluster in relatively short intervals of time, but this tendency is not so remarkable that most aftershocks can be practically regarded as mutually independent events. In a multiple sequence, if the main shock of the earliest constitutent sequence is the largest, all later constituent sequences are usually included in the aftershock sequence of the initial main shock. For such sequences parameters D_1 and b are rather small and paramters c and $A/\overline{A}$ are rather large. However, these parameters estimated for each constituent sequences do not seem to deviate systematically from their standard values, though only limited data are available to confirm this.

The theories or models for the occurrence of aftershocks, most of which are concerned with the temporal distribution of aftershocks, are reviewed and some modifications based on new ideas are suggested. Delayed fracture of rocks in the soruce regions may explain some important features of aftershock sequences and multiple sequences, though many problems still remain unsolved.

In previous chapters, a general statistical survey of the occurrence of aftershocks was made using data mainly from the region of Japan. The chief concerns were with the formulas representing the distribution of aftershocks in time, space, and magnitude, together with the values of some parameters characterizing an aftershock sequence. In the following five chapters, further discussions will be made on the "structure" of the distribution of shocks in various types of earthquake sequences. In Chapter 12 a review of several theories for aftershock occurrence will be presented referring to the results obtained in earlier chapters.

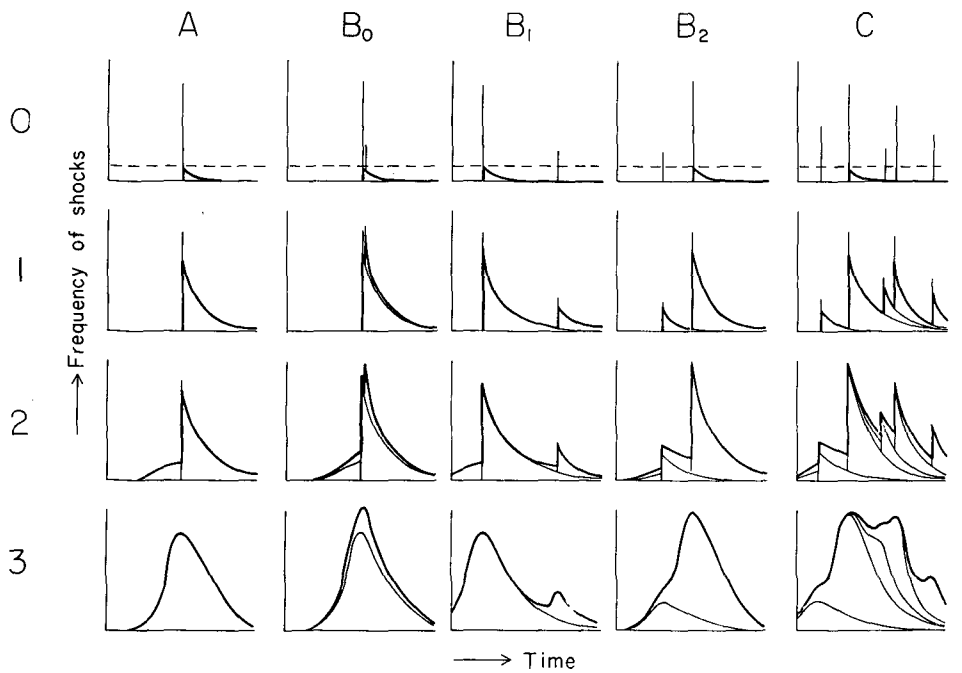
7. A classification of earthquake sequences according to patterns of temporal distributions

7.1 *Classification*

Mogi[14] in 1963 classified earthquake sequences into following three types according to patterns of temporal variation in activity. Type 1: main shock —aftershock sequence, Type 2: foreshock sequence—main shock—aftershock sequence, Type 3: earthquake swarm. These three types are schematically represented by graphs 1–A, 2–A, 3–A in Figure 75. Mogi,[14),194)] on the basis of the results from laboratory experiments on fracturing of rocks and the geographical distribution of each type of earthquake sequences in Japan, stated that the difference among the three types is due to the difference in degree of heterogeneity or fracturing of rocks and stress concentration in the focal region.

In the present study this classification is extended to include the multiple occurrence of simple sequences and single events. In Figure 75 curves indicate temporal variations in the rate of occurrence of shocks, and each vertical bar refers to the time and magnitude of a "main shock" which may trigger a series of aftershocks.

In the case of Type 0 (0-A, 0-B, and 0-C in Figure 75), aftershock activities following the main shocks are very weak or absent, therefore a small number of aftershocks are observed only for a small proportion of the main shocks under favorable conditions. However two or more earthquakes occur in groups



	A	B_0	B_1	B_2	C
0	Single event *	(Doublet)*			Multiplet *
1	Main shock – aftershock series				#
2	Foreshock - main shock - aftershock series				#
3	Swarm				

Fig. 75. Types of earthquake sequences. Length of the vertical bar represents the size of the "main shock".
* When the events above the broken line only are observed.
♯ If one main shock has exceedingly large magnitude, the sequence is no more called swarm.

as shown in graphs 0-B and 0-C in a different manner from ordinary aftershock sequences. Such grouping is sometimes observed even in intermediate and deep earthquakes (e.g., Isacks et al.,[16] Utsu,[17] Santo,[195),196] and Oike[197]).

In the case of Type 1, the main shocks are followed by considerable

aftershock activities but not preceded by foreshocks. In Type 1-B (1-B_1, 1-B_2, and 1-B_3) two main shocks accompanied by aftershock sequences occur in succession. When the earlier one has magnitude smaller than the later, the earlier one and its aftershocks have been called a foreshock sequence, and in the reversed case the later one is usually called a large aftershock accompanied by a series of secondary aftershocks. Type 1-C, in which several main shocks with aftershock sequences occur in a cluster, has been regarded as a swarm unless one of the main shocks has by far the larger magnitude than others.

In the case of Type 2, main shocks are preceded by foreshock sequences whose pattern is considerably different from that of Type 1-B_2. Earthquake swarms of Type 3 (3-A, 3-B, and 3-C) differ from those of Types 1-C and 2-C in having no apparent main shocks in the sequences. According to Mogi's original classification, Type 1-B_2 is included in Type 2, and swarms of Types 1-C and 2-C are included in Type 3. Recognition of the difference between these types is a point of this paper. Mogi[198] mentioned that the time distribution of foreshocks is classified into two types, C (continuous type — gradually increasing activity toward the main shock) and D (discontinuous type — sudden increase of activity followed by a decrease before the main shock), which approximately correspond to Types 2-A and 1-B_2 respectively.

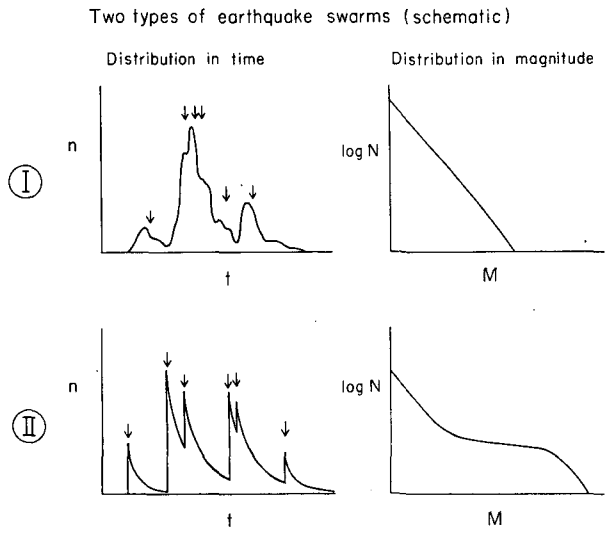


Fig. 76. Two types of earthquake swarms. I: swarm of the first kind, II: swarm of the second kind. Arrows indicate the occurrence of large shocks.

Since earthquake swarms can be classified into two types, i.e., Type 3 and Types 1-C and 2-C, we shall refer to these as swarms of the first kind (Type 3) and the second kind (Types 1-C and 2-C). In Figure 76 some features of the two kinds of swarms are exhibited. Swarms of the first kind are usually observed in volcanic regions, whereas some swarms of the second kind occur in non-volcanic regions such as off the Pacific coast of north-eastern Japan.

A classification of earthquake sequences into four classes 0, 1, 2, and 3, each of which has three types A (single sequence), B (double sequence), and C (multiple sequence), involves some uncertainty because of the transitional character of the sequences. Nevertheless this classification seems very useful to describe and interpret some statistical properties of earthquake sequences.

7.2 *Examples*

Examples of each type of sequences occurring in Japan are presented in the following.

i) *Single and multiple occurrence of earthquakes accompanied by no aftershocks (Type 0)*

Few deep earthquakes in the region of Japan are accompanied by observable aftershocks (cf. Chapter 2). Some shallow earthquakes of considerable magnitude occurring in the same region have no aftershocks detected by the JMA network of seismic stations. However some of these earthquakes occur in clusters within relatively short intervals of time and space in a different manner from ordinary foreshock or aftershock sequences.

Type 0-A: A deep earthquake (h=260 km) off the northeastern coast of Hokkaido on January 19, 1969, M=6.4. No earthquakes were recorded by JMA within 100 km and three months from its origin. However sensitive seismographs at the Urakawa Seismological Observatory recorded at least four aftershocks, largest of which had magnitude about $3\,^1/_2$ unit smaller than the main shock.

Type 0-B (Doublet): Two shallow earthquakes in the southwestern part of the Japan Sea on September 6 and 7, 1963, M=6.0 and 6.2, 19.2 hours apart. No aftershocks or foreshocks of these earthquakes were detected by JMA. Earthquakes with magnitude larger than about 4 could have been registered by JMA.

Type 0-C (Multiplet): Table 9 lists three clusters of intermidiate and deep earthquakes in and near Japan. The data for the first two clusters are

Table 9. Some examples of intermidiate and deep earthquake clusters in and near Japan.

Date and time (GMT) d h m	Epicenter °N	°E	Depth km	M
1948 Aug. 26 20:39	27.4	141.0	300	$6^1/_4$
26 20:56	27.7	141.2	300	6
27 00:21	27.3	140.7	300	$5^1/_2$-$5^3/_4$
1957 Apr. 9 00:24	$30^3/_4$	$138^3/_4$	450	$6^1/_2$
9 10:35	$30^3/_4$	$138^1/_2$	450	$5^1/_2$-$5^3/_4$
July 18 12:06	$30^1/_4$	139	400	6-$6^1/_4$
Aug. 02 17:53	29.8	139.8	400	$5^1/_2$-$5^3/_4$
28 23:14	$29^3/_4$	$139^1/_4$	450	5?
Sept. 17 18:44	30	$139^1/_2$	450	$5^1/_2$-$5^3/_4$
28 00:27	31	138	450	6
Nov. 17 17:55	$30^1/_2$	$138^3/_4$	450	6-$6^1/_4$
1966 Dec. 1 18:56	41.5	140.1	160	5.4
1967 Feb. 2 16:25	41.5	140.0	180	5.4
20 00:35	41.4	140.3	180	4.5

taken from Katsumata's table (1958).[199]

ii) *Single and multiple occurrence of "main shock — aftershock sequence" (Type 1)*

Since large shallow earthquakes are usually followed by many aftershocks but not usually preceded by foreshocks, there are many examples of Type 1 sequences. However, upon closer examination of the time distribution, it has turned out that not a few sequences are multiple, and single sequences are rather few. It should be mentioned here that aftershocks treated in the previous chapters were not restricted to single sequences, and both single and multiple sequences participated in the statistical results obtained hitherto.

Type 1-A: Variation of the hourly frequency of aftershocks of an earthquake of magnitude 6.7 which occurred near the Urakawa Seismological Observatory on January 20, 1970 is shown in Figure 83. Owing to high sensitivities of seismographs in use, lower limit of magnitude chosen for counting can be set at 0.7, which is smaller by 6 magnitude unit than the main shock. The largest aftershock (M=4.8) which occurred about 19 hours after the main shock and other large aftershocks were followed by no appreciable aftershock activities of their own. The hourly frequency variation is well represented by the modified Omori formula with p=1.1 and c= 0.01 day.

Other examples are also shown in Figures 77 to 82. It is to be noted

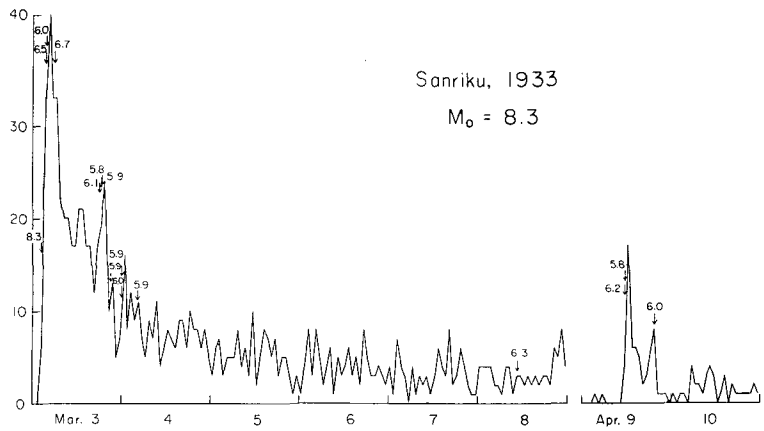


Fig. 77. Variation of the hourly frequency of shocks. Arrows with figures indicate the time and magnitude of large shocks. Dates are based on JST. (This caption is effective for Figs. 77–103.)

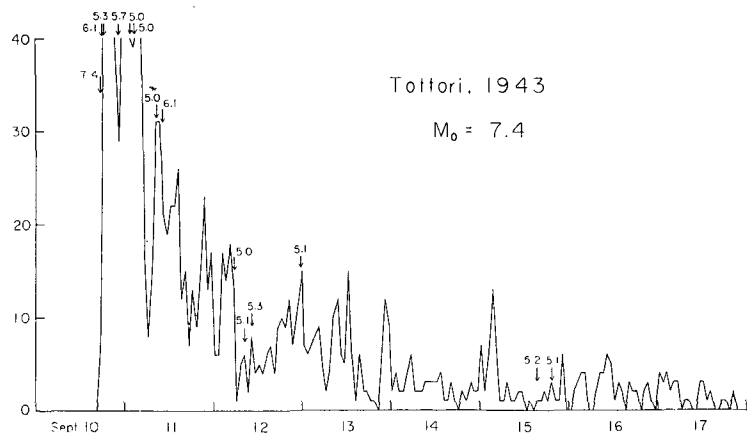


Fig. 78.

that large aftershocks (marked with arrows with magnitude values) initiate no appreciable activities in these sequences, even the magnitude 7 aftershock on March 10 in the Tokachi sequence of 1952 (Figure 79). An exception is the magnitude 6.2 shock on April 9 in the Sanriku sequence of 1933 which triggered the secondary aftershock sequence as can been seen from Figure 77. Slight increase of activity was also observed after the magnitude 6.4 shock on April

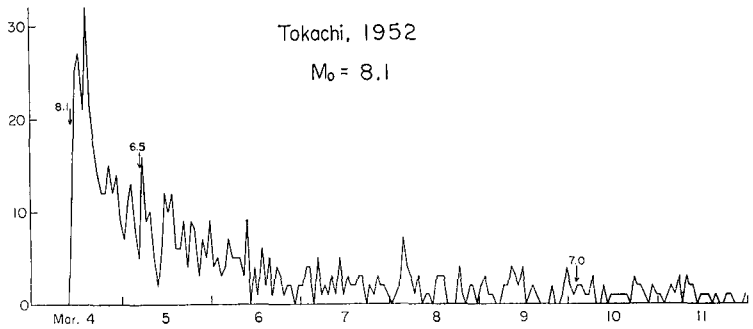


Fig. 79.

23. However no increase of activity was observed after other large aftershocks in the same sequence, such as those on March 8 (M=6.3), on July 10 (M=6.3), on July 21 (M=6.2), etc. The largest aftershock (M=6.1) of the Tottori earthquake shown in Figure 78 also initiated no marked secondary activities, in contrast with two earthquakes of the same magnitude which occurred in the same region about half a year before and accompanied by many aftershocks (Figure 85). The three largest aftershocks (M=6.1) of the Niigata earthquake occurred in the first four hours after the main shock when the activity was very high. The fourth largest aftershock (M=6.0) occurred about four weeks after the main shock but no marked increase of activity was observed by JMA stations. Temporal stations at Oguni and Awashima recognized an increase of aftershock frequency after this earthquake.[52),53)]

Type 1-B: Two earthquakes with magnitude 6.1 in Tottori Prefecture on March 4, 1943 were followed by aftershocks as shown in Figure 85. In this case two main shocks have the same magnitude, therefore one of which can not

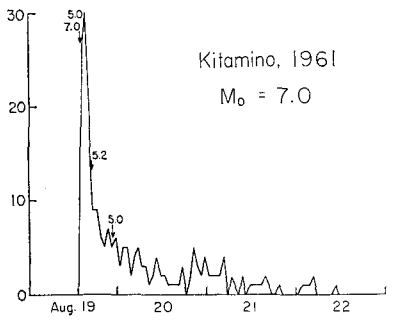


Fig. 80.

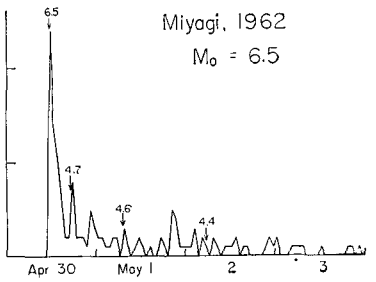


Fig. 81

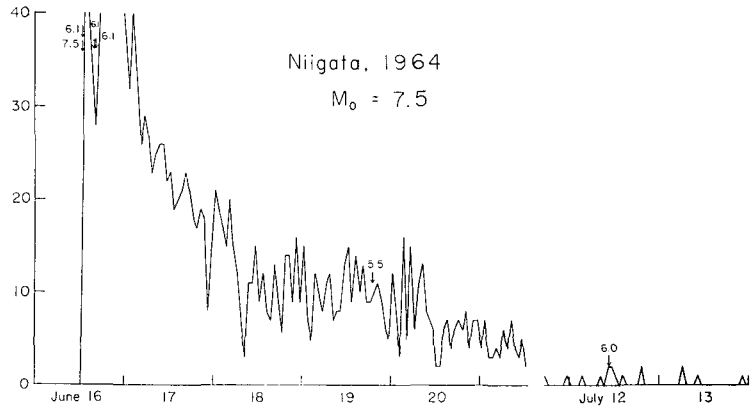


Fig. 82.

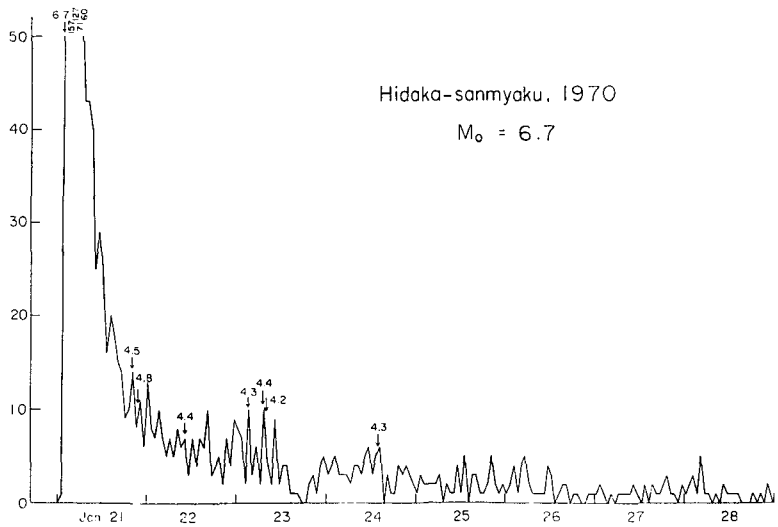


Fig. 83.

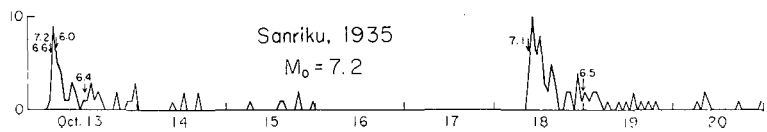


Fig. 84.

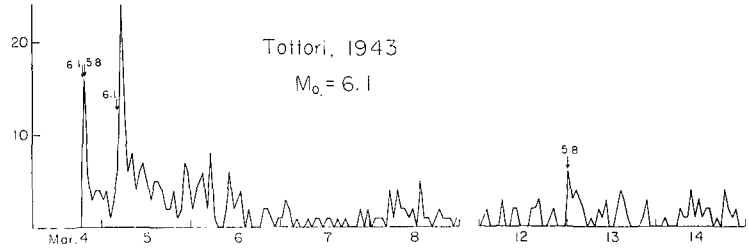


Fig. 85.

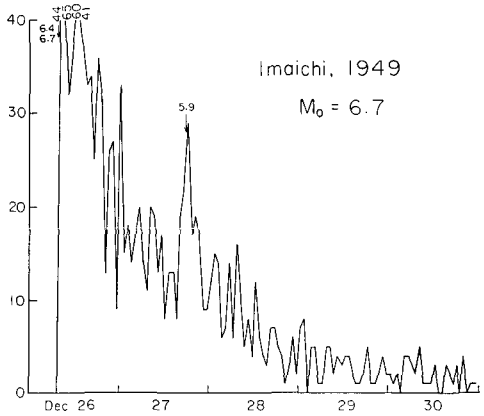


Fig. 86.

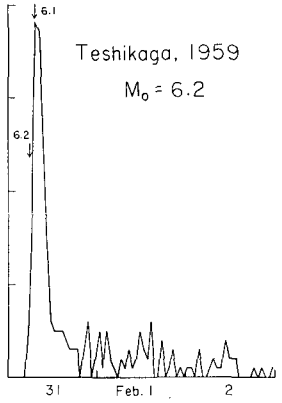


Fig. 87.

be considered as a foreshock or an aftershock of the other. The Sanriku earthquakes of 1935 shown in Figure 84 are of the same character. If two main shocks occur at a short interval of time, the two aftershock sequences can not be separated as in the cases of the Imaichi earthquakes of 1949 (M=6.4 and 6.7, 8 minutes apart, see Figure 86), the Teshikaga earthquakes of January 31, 1959 (M=6.2 and 6.1, 1.6 hours apart, see Figure 87), the Oga erathquakes of May 1, 1939 (M=7.0 and 6.7, two minutes apart), etc.

When two or more earthquakes occur at very short intervals, say ten seconds, they are usually regarded as a single event, though the seismograms of such a event are more or less complicated. Such a phenomenon has been reported in some large earthquakes.[200)-209)]. In connection with this, the term "twin earthquake" has been used by some seismologists.

It is remarked here that typical examples of Type I-B sequences are few. Most of the sequences consisting of two remarkable secondary sequences are

accompanied with other minor secondary sequences (see Figures 85 and 86 in which earthquakes of magnitude 5.8 and 5.9 are accompanied by secondary activities). If these minor sequences are included, the sequences must be regarded as those of Type 1-C.

A good example of Type 1-B_1 sequences is the Tango sequence which began with the main shock of magnitude 7.5 on March 7, 1927. A large aftershock of magnitude 6.2 occurred on March 31 which was accompanied with many secondary aftershocks[38),39),69)] (see also pages 78 to 80 of Matuzawa's book[210)]). There are few good examples of Type 1-B_2 sequence. The first

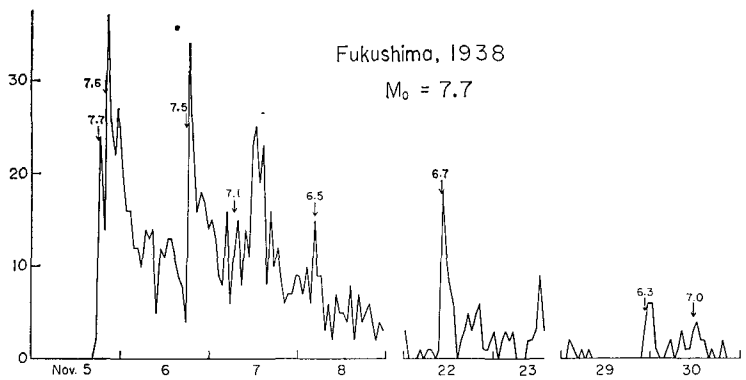


Fig. 88.

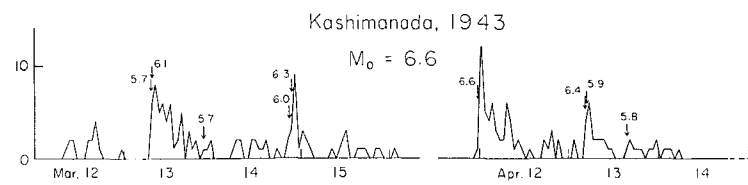


Fig. 89.

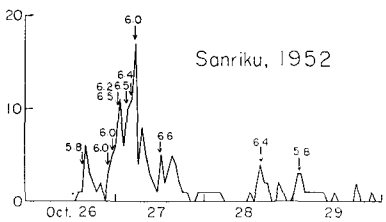


Fig. 90.

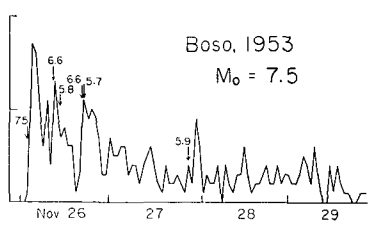


Fig. 91.

Table 10. A large earthquake swarm in Japan which occurred off the Pacific coast of northeastern Japan (near 143°E, 39°N). Earthquakes with $M \geq 6.5$ are listed.

Date and time (GMT)	M	m	M_R	Remarks
1896 June 15 10:33	7.6*	8?		Great tsunami, 27,122 killed
15 23:01	7.6			
Aug. 5 14:07	7.2			
1897 Feb. 7 07:35	7.1**	$7^3/_4\pm$	8.3±	
19 20:50	7.8	$7^3/_4\pm$	8.3±	Moderate tsunami Minor damage
19 23:47	7.8	$7^3/_4\pm$	8.3±	
Mar. 27 10:49	6.9			
May 18 01:10	7.1			
23 12:23	7.5			
Aug. 5 00:10	7.7	8±	8.7±	Moderate tsunami Minor damage
5 23:48	7.4			
16 07:51	7.4	$7^1/_2\pm$	7.9±	
Sept. 21 00:02	6.8			
Oct. 2 12:45	7.7			Minor damage
Dec. 4 00:18	7.4			
1898 Jan. 12 23:16	6.6			
Apr. 22 23:37	7.8	$7^3/_4\pm$	8.3±	Minor damage
July 5 09:57	6.9			

M: magnitude by Kawasumi,[213] m: unified magnitude by Gutenberg,[214] M_R: magnitude by Richter[6] (converted from m).

* This estimate seems too small considering the wave hight and the source area of tsunami (Hatori[215]). However macroseismic effects on land were rather small, suggesting the seismic body wave energy hence the stress drop for this earthquake is comparatively small.

** The epicenter of this shock was located on land, but it was probably off-shore since no damage was reported.

week of the Southern Kurile Islands sequence of 1963 (Figure 94) shows a pattern of Type 1-B_2.

Type 1-C: As illustrated in Figures 88–95, many earthquake sequences accompanying large shallow earthquakes occurring off the Pacific coast of northeastern Japan belong to this type. When the largest shock in a sequence is preceded by no earthquakes with comparable magnitude, it is usually called the main shock of the whole sequence and all succeeding shocks are called aftershocks, even if some of them have comparable magnitudes to the main shock, as in the cases of the Fukushima earthquake of 1938 (Figure 88), the Kashimanada earthquake of 1961 (Figure 93), and the Tokachi earthquake of

1968 (Figure 95). In these sequences it is clearly seen that several large aftershocks triggered their own aftershock activities.

When the largest shock is preceded by a sequence initiated by a comparatively small earthquake, this sequence is called the foreshock sequence, as in the case of the southern Kurile earthquake of 1963 (Figure 94). Another exmaple of such foreshock sequences is a series of small earthquakes recorded at the Urakawa seismological observatory accompanying an earthquake of magnitude 5.3 on May 1, 1968 in the region of the Tokachi earthquake of May 16, 1968.[211)-213)]

When several shocks of comparable magnitudes to the largest one occur in a sequence and the first shock is not the largest one, the sequence is called an earthquake swarm (of the second kind). The Kashimanada swarm of 1943 (Figure 89) and the Sanriku swarm of 1952 come under this category. However there seems to be no essential difference between such swarms and other Type 1-C sequences. The largest known swarm of the second kind in Japan is the Sanriku earthquakes of 1896–1898 shown in Table 10. It is not impossible to consider the remarkable activities of 1944–1948 in central Japan including two magnitude 8 earthquakes and three magnitude 7 ones (Table 17) as a swarm of this kind, but they are treated separately in this paper.

iii) *Single and multiple occurrence of "foreshocks — main shock — aftershock sequence" (Type 2)*

Temporal distributions of foreshocks of this type are usually irregular. The foreshock sequences of the Mikawa and Nii-jima[107)] earthquakes shown in Figures 96 and 100 are in themselves like swarms. The duration of foreshock sequence varies widely from 1 hour or less to more than ten days. Watanabe and Kuroiso[217)] reported that the Wachi, Kyoto Prefecture, earthquake of August 18, 1968 (M=5.6) was preceded by hundreds of microearthquakes. This foreshock activity started with a magnitude 4.5 earthquake on February 14, 1968 about half a year before the main shock. On the other hand, according to the observation by Suyehiro et al.[147)] the foreshock sequence of a small earthquake (M=3.3) in Nagano Prefecture on January 22, 1964 was preceded by a foreshock sequence which lasted about four hours. The foreshock sequence of the magnitude 7.8 earthquake off the east coast of Hokkaido on August 12, 1969 continued for only half an hour.[218)]

As mentioned before, some sequences of Type 2-C belong to the earthquake swarm of second kind. The Oshima sequence of 1964 shown in Figure 102 can be classified into this type, since this swarm includes at least four Type

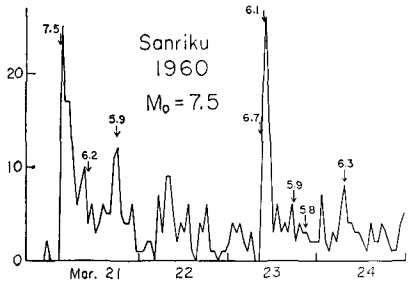


Fig. 92.

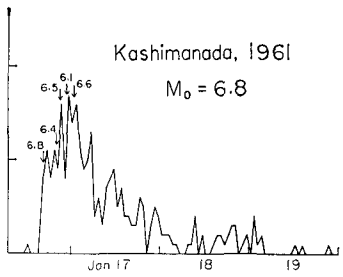


Fig. 93.

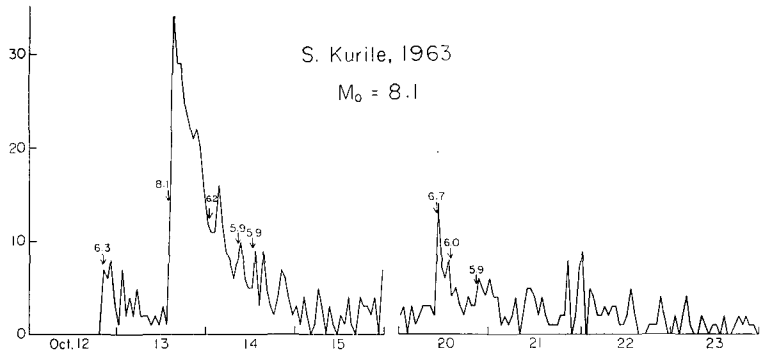


Fig. 94.

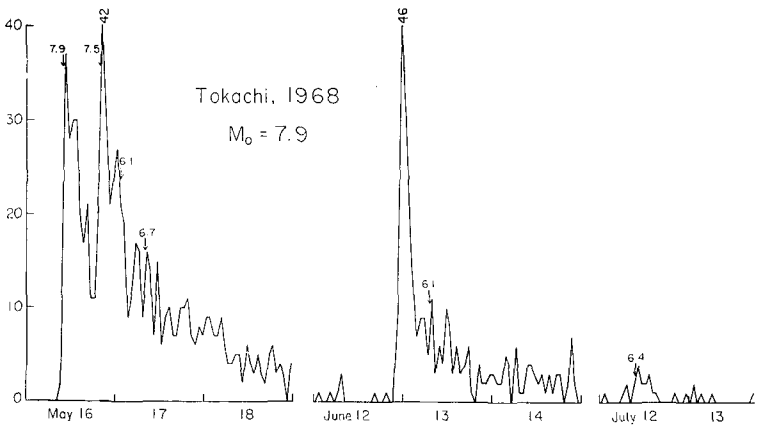


Fig. 95.

2-A (or 1-A) sequences with main shocks of magnitude 5.8, 5.2, 5.3, and 5.5. The Matsushiro earthquake swarm starting in August, 1965 is very complex. Some parts of this swarm may be regarded as a swarm of the second kind as exhibited in Figure 103, because of the sudden increase of activity after the occurrence of large shocks (see also the reports by the members of ERI[149)–151)]). The Ebino earthquakes of February—March, 1968[219)–220)] also form a swarm of the second kind with five main shocks of magnitude 5.7, 6.1, 5.6, 5.7, and 5.4, each of which was preceded by some foreshocks and followed by many aftershocks.

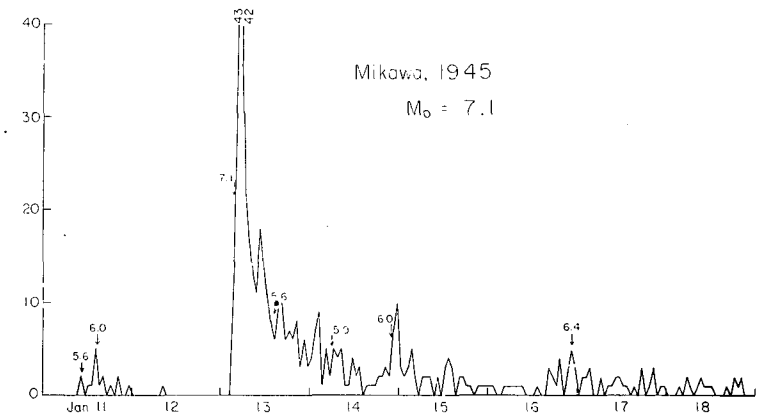


Fig. 96

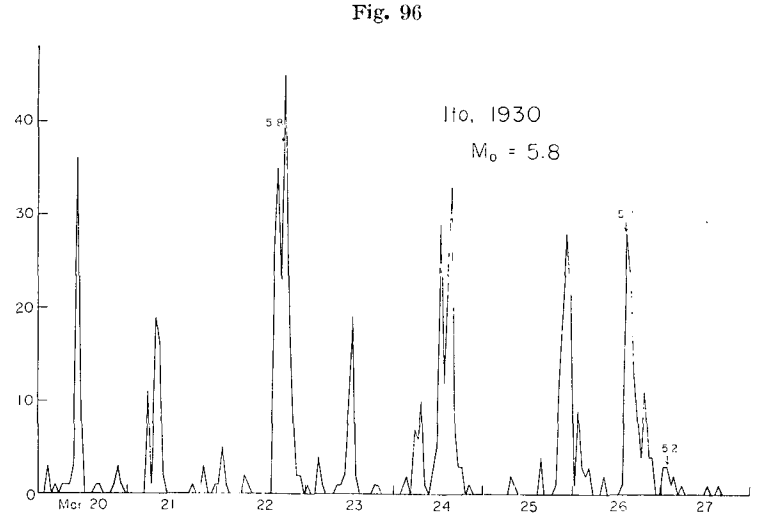


Fig. 97.

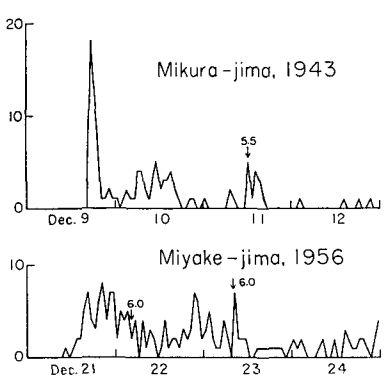


Fig. 98-99.

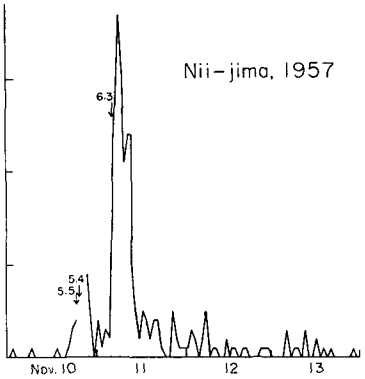


Fig. 100.

iv) *Earthquake swarms of the first kind (Type 3)*

Swarms of the first kind differ from the second kind in having *no* prominent main shocks accompanied by their own aftershock activities. The Ito swarm of March—May, 1930[220] and the Miyake-jima swarm of August—September, 1962 have many peaks of activities as partly shown in Figures 97 and 101. Some of these peak periods include large shocks, but the peaks themsevles are not initiated by the large shocks. For example the largest shock in the Ito swarm on March 22 ($M=5.8$) and the two largest shocks in the Miyake-jima swarm on August 26 ($M=5.9$) and August 30 ($M=5.8$) seem to have triggered no appreciable activities, though these large shocks occurred in periods of increased activities.

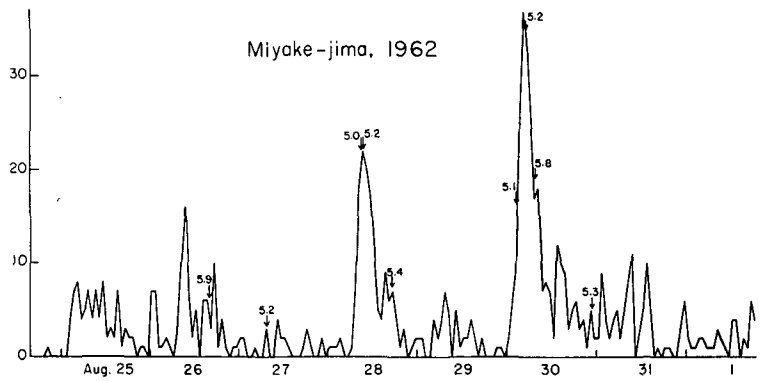


Fig. 101.

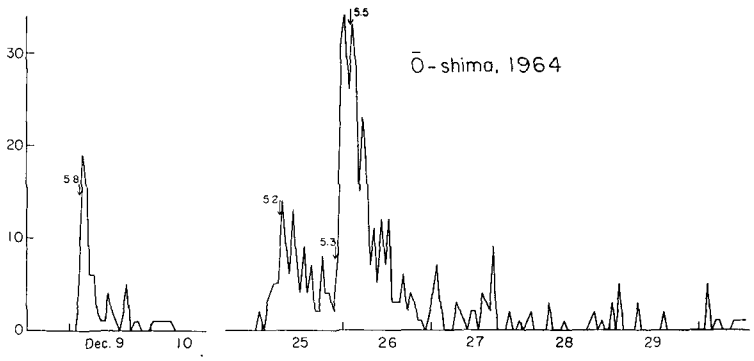


Fig. 102.

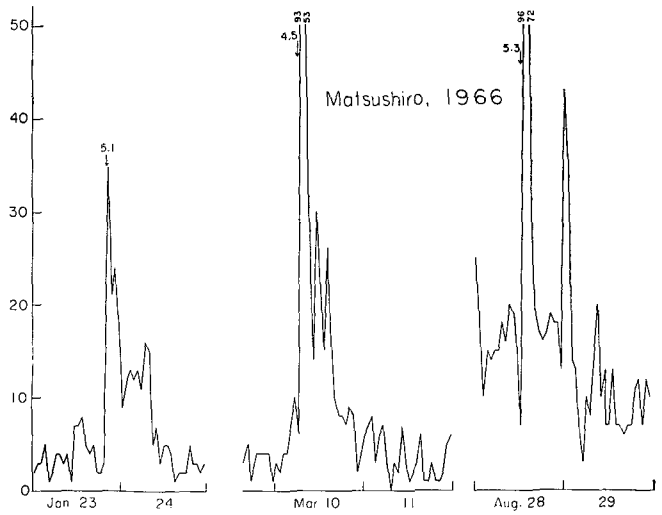


Fig. 103.

It is said that the Kita-Izu earthquake of November 25, 1930 ($M=7.0$) was accompanied by many foreshocks and aftershocks. However the variation of the frequency of shocks with time before and after this large earthquake (see Mogi,[14], p. 638 and Matuzawa,[210] p. 63) indicates that no ordinary aftershock sequence (which fits the modified Omori formula) was generated. It is rather reasonable to consider the whole sequence as a swarm of the first kind with one exceedingly large shock included.

v) *Mixed type*

The earthquake sequence in Nagano Prefecture during February—March, 1963 studied by Hagiwara et al.[115] consists of two separate sequences of Type 2-A and Type 3-A as shown in Figure 104. Other examples of such a mixed type sequence, a Type 1-A (or 2-A) sequence followed or preceded by a swarm of the first kind, are the Kumaishi, Hokkaido, sequence (a swarm in

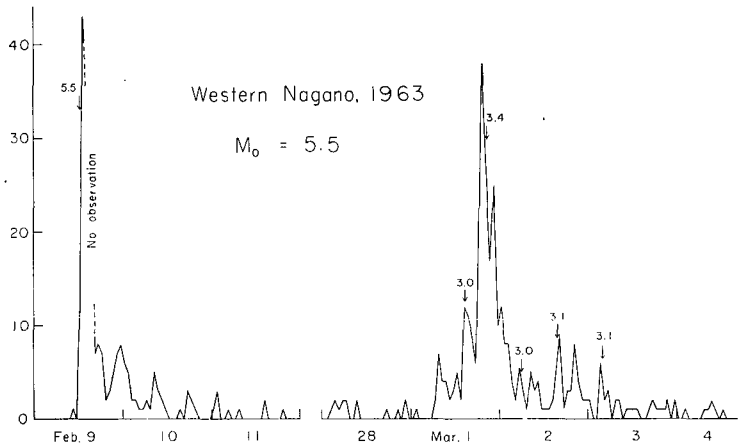


Fig. 104. Variation of the hourly frequency of shocks recorded at Kurobe IV Dam with S−P between 2 and 4 sec (after Hagiwara et al.[115]).

May 1953 followed by a Type 1-A sequence with the main shock of magnitude 5.4 on July 21, 1953) and the Nii-jima sequence of July, 1960 (a Type 1-A sequence with the main shock of magnitude 5.5 at 20 h 28 m, 13th followed by a swarm during 09 h—17 h, 15 th).

8. Further investigation into temporal distribution of aftershocks

8.1 *p and c values for constituent sequences in a multiple aftershock sequence*

In Chapter 3 it has been reconfirmed that the frequency of aftershocks decays usually in accordance with the modified Omori formula. However many of the aftershock sequences treated there and in a previous paper[1] are multiple ones. Nevertheless the modified Omori formula fits fairly well to the frequency vs time plots on a log-log scale for most of these multiple sequences, and the p and c values have been determined. If the data for a multiple sequence are plotted on a linear scale as shown schematically in Figure 105, the modified Omori formula fitting the data best will be represented

by a broken line in the same figure. Therefore it is very likely that p and c values determined for each constituent sequence such as 1, 2, 3,.... in Figure 105 differ from those determined for the whole sequence. Although it is difficult to obtain accurate estimates of p and c for constituent sequences as will be seen from the plots in Figures 106–109, some determinations of these values are listed in Table 11. To construct such figures as Figures 106–109, it is necessary to exclude the activities due to other constituent sequences.

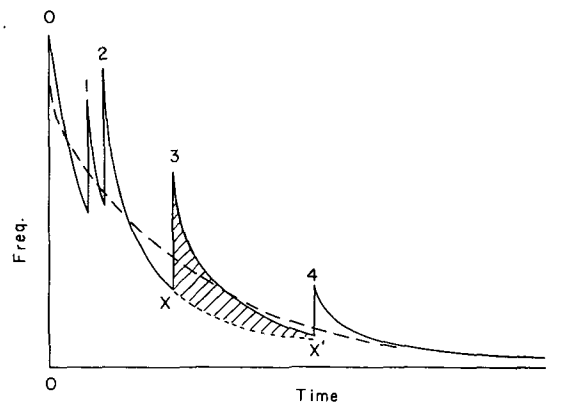


Fig. 105. A schematic graph of Type 1-C aftershock sequence. The borken line represents the curve for the modified Omori formula fitting the whole sequence. The shaded area indicate the secondary aftershocks triggered by shock 3.

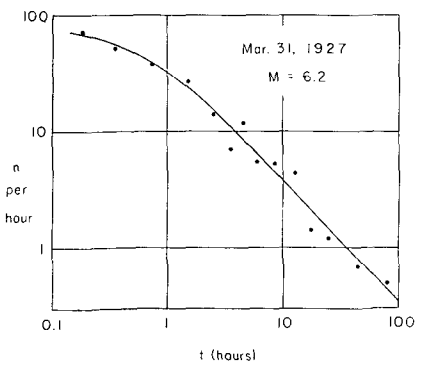


Fig. 106.

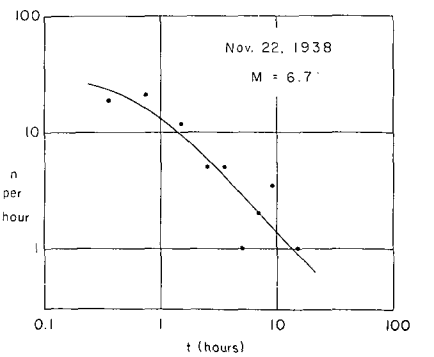


Fig. 107.

Fig. 106. Secondary aftershock sequence following an aftershock of the Tango earthquake of Mar. 7, 1927.

Fig. 107. Secondary aftershock sequence following an aftershock of the Fukushima earthquake of Nov. 5, 1938.

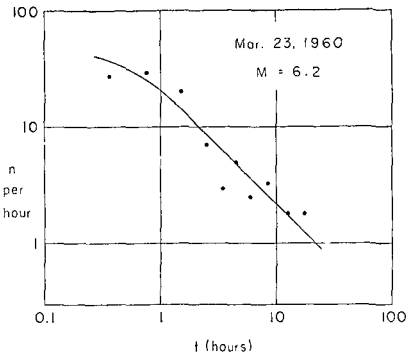


Fig. 108.

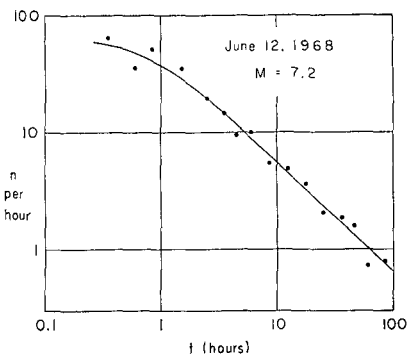


Fig. 109.

Fig. 108. Secondary aftershock sequence following an aftershock of the Sanriku earthquake of Mar. 20, 1960.

Fig. 109. Secondary aftershock sequence following an aftershock of the Tokachi earthquake of May 16, 1968.

Table 11. p and c values for some secondary aftershock sequences observed in Japan.

Date and time (GMT)	M	p	c (days)	Date and name of the main shock
1927 Mar. 31 21:07	6.2	1.1	0.025±	1927 Mar. 7, Tango
1938 Nov. 6 08:54	7.5	1±	0.04 ±	1938 Nov. 5, Off Fukushima Pref.
Nov. 22 01:14	6.7	1±	0.02 ±	〃
1949 Dec. 25 08:56	5.9	1±	0.05 ±	1949 Dec. 25, Imaichi
1960 Mar. 23 00:39	6.7	1±	0.02 ±	1960 Mar. 20, Off Sanriku
1968 May 16 00:39	7.5	1±	0.05 ±	1968 May 16, Off Tokachi
June 12 13:41	7.2	1.0	0.02 ±	〃

For example, for sequence 3 in Figure 105 the number of shocks falling in the shaded part only should be counted. If all shocks above the abscissa are included, somewhat smaller p values will be obtained.

The data in Table 11, though limited in number, suggest that c values for constituent sequences are smaller than its median (c=0.3 day) for the whole sequence (Table 4). Large c values for some aftershock sequences listed in Table 4 (e.g., the Fukushima sequence of 1938 (c=1.5 days), the Boso sequence of 1953 (c=1.5 days), and the Tokachi sequence of 1968 (c=0.5 day)) may be explained, at least partially, by the multiplicity of the sequences. This may correspond to Yamakawa's suggestion[54] that the c value represents the degree of complexity of fracturing in the focal region.

8.2 *Clustering of shocks in an aftershock sequence*

The second problem to be discussed in this chapter is whether or not the occurrence of an aftershock is influenced by the occurrence of other shocks in the same sequence. Of course all aftershocks are dependent on the main shock, but evidence has been presented to indicate the random occurrence of aftershocks.[39),45),57),62),222),223)] On the other hand, clustering of shocks has been found in some aftershock sequences, chiefly due to secondary aftershocks.

The secondary aftershock sequence is not a rare phenomenon, but there are many instances that large aftershocks are accompanied by no observable secondary aftershock sequences as mentioned in Chapter 7 and in other papers.[1),49),62)] Sagisaka[33)] in 1927 pointed out that no changes in aftershock activity were evident at the time of large aftershocks of the Tango earthquake of 1927 except a shock on March 31, which might be considered as an independent shock. Richter[6)] described that late large aftershocks are apt to be accompanied by secondary aftershocks and they are dynamically independent of the main shock.

It is well known that if the earthquake occurrence is stationary and random in respect to time, the number n of shocks occurring in a period of length Δt has a Poisson distribution

$$p(n) = (\nu \Delta t)^n e^{-\nu \Delta t}/n ! \tag{39}$$

where ν is the average number of shocks per unit time interval, and the time interval τ between two successive shocks has a negative exponential distribution

$$\phi(\tau) = \nu e^{-\nu \tau} . \tag{40}$$

It is also known that if the whole period T under consideration consists of N periods of length Δt ($T = N\Delta t$), the ratio of the variance of n to its mean multipled by N, i.e.,

$$\chi_0^2 = NV(n)/E(n) = \sum_{i=1}^{N} (n_i - \bar{n})^2/\bar{n} \tag{41}$$

where n_i is the number of shocks in the i th period and $\bar{n} = E(n) = \sum_{i=1}^{N} n_i/N = \nu \Delta t$, is distirbuted as χ^2 with N degrees of freedom. Statistical tests for the hypothesis of stationarity *and* randomness of the earthquake occurrence can be made using above three distributions. The procedure using the last equation may be the simplest. Of course there are other independent methods of testing the hypothesis.[224)–225)]

In the case of a non-stationary earthquake sequence, if the whole sequence is divided into shorter periods in which the occurrence of shocks can be regarded as approximately stationary, the above tests may be applied to each period. Especially the distributions of n or τ during relatively short periods have been investigated for various sequences, comparing with the theoretical distribution function (39) or (40). For example Senshu[222] showed in 1959 that the time interval distribution of aftershocks for each short period in the Sanriku sequence of 1933 was approximately represented by a negative exponential function. In a few cases of the sequences studied by Yamakawa,[61),226] Hamada,[151] and Page[62] disagreement with a negative exponential

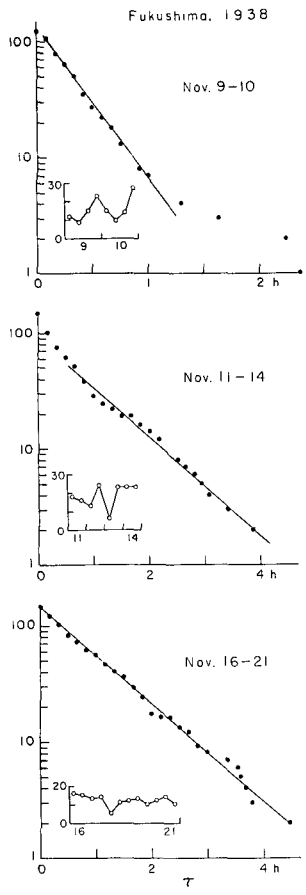


Fig. 110.

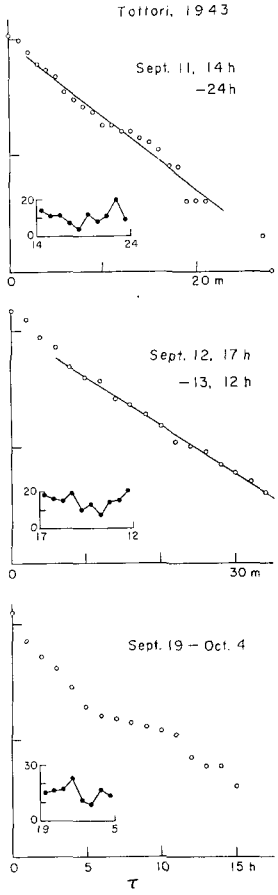


Fig. 111.

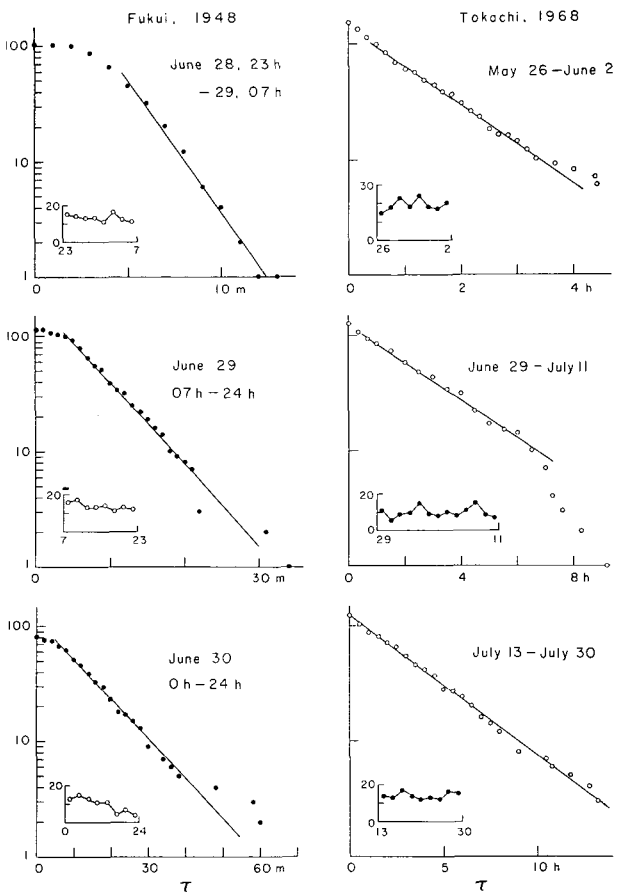


Fig. 112. Fig. 113.

Fig. 110. — Fig. 113.
Cumulative frequency distributions of time intervals between two successive aftershocks. Inserted small graphs show the temporal variations of aftershock frequencies.

distribution or a Poisson distribution has been obtained. The disagreement suggests the clustering of aftershocks. In Figures 110–113, distributions of time interval τ are plotted for several periods in the aftershock sequences of the Fukushima (1938), Tottori (1943), Fukui (1948), and Tokachi (1968) earthquakes. The periods have been so chosen that no remarkable changes in activities are included. The straight lines are those fitted to the points

and not those calculated from equation (40). In some periods of three sequences other than the Fukui sequence, a slight tendency to increase in the slope in ranges of small τ is recognized, but in some periods the fit of the plotted points to the straight lines is satisfactory in wide ranges of τ. The reason for a remarkable decrease in the slope in the range of $\tau<5$ min as observed in the Fukui sequence remains unexplained, though Senshu[221] has already treated this problem.

For a relatively long period in a sequence, it is a matter of course that the observed distribution of n or τ does not fit equation (39) or (40). All studies along this line have reached this conclusion.[49),51),61),66),104),109),148),162),183),227)-229)] However it is not acceptable to conclude that such disagreement with equation (39) or (40) is due to a lack of randomness and the dependence among shocks such as clustering of aftershocks as described in some papers. Such an effect many partly be responsible for the disagreement, but the chief reason for the disagreement is the non-stationarity of the sequence. In some sequences the distribution of τ is well represented by an equation

$$\phi(\tau) = k\tau^{-q} \tag{42}$$

which was first proposed by Tomoda[183] in 1954. However this equation seems to have no primary importance, as discussed by Senshu[222], Utsu,[228] Mogi,[230] and a later part of this chapter.

The distribution of the number n of shocks per time interval Δt for all shocks occurring in a long period from t_1 to t_2 during which the rate of occurrence $n(t)$ changes considerably is given by

$$p(n) = \int_{t_1}^{t_2} \frac{\{n(t)\,\Delta t\}^n}{n!}\, e^{-n(t)\Delta t}\, dt/(t_2-t_1) \,. \tag{43}$$

For an aftershock sequence to which the modified Omori formula $n(t)=K/(t+c)^p$ is applicable, the above equation takes the form

$$p(n) = A\Gamma\left(n-\frac{1}{p}\right)\Big/\Gamma(n+1)-B(n)-C(n) \tag{44}$$

where

$$A = (K\Delta t)^{1/p}/\{p(t_2-t_1)\} \,, \tag{45}$$

$$B(n) = \frac{A}{n!}\int_{K(t_1+c)^{-p}\Delta t}^{\infty} x^{n-(1/p)-1}\, e^{-x}\, dx \,, \tag{46}$$

$$C(n) = \frac{A}{n!} \int_0^{K(t_2+c)^{-\hat{p}}\Delta t} x^{n-(1/\hat{p})-1} e^{-x}\, dx \ . \tag{47}$$

If $c \to 0$ and $t_1 \to 0$, $B(n) \to 0$. In addition, if $t_2 \to \infty$, $C(n) \to 0$.

Consequently,

$$p(n) \to A\Gamma\left(n - \frac{1}{\hat{p}}\right) \Big/ \Gamma(n+1) \ . \tag{48}$$

(A tends to zero more slowly than $B(n)$ and $C(n)$ do.) For large n ($n \gtrsim 4$), the equation

$$p(n) = An^{-r} \tag{49}$$

where

$$r = 1 + \frac{1}{\hat{p}} \ , \tag{50}$$

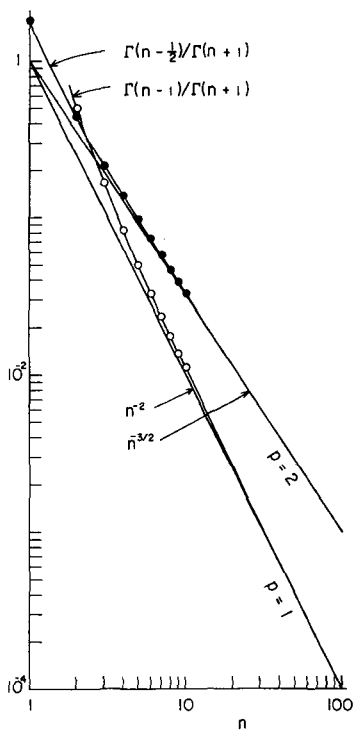


Fig. 114. Relation between $\Gamma(n-(1/\hat{p}))/\Gamma(n-1)$ and n^{-r} ($r=(1+(1/\hat{p}))$ for $\hat{p}=1$ and 2.

gives a good approximation of the right side of equation (48) for normal values of p as can be seen from Figure 114.

The distribution of time interval τ between two successive shocks for a non-stationary earthquake series between t_1 and t_2 is expressed by

$$\phi(\tau) = \int_{t_1}^{t_2} \{n(t)\}^2 e^{-n(t)\tau}\, dt \Big/ \int_{t_1}^{t_2} n(t)\, dt \,. \tag{51}$$

If $n(t)$ is replaced by the modified Omori formula, the above equation becomes

$$\phi(\tau) = A' \Gamma\left(2-\frac{1}{p}\right) \tau^{-(2-(1/p))} - B(\tau) - C(\tau) \,, \tag{52}$$

where

$$A' = K^{-(1(-1/p))}\, c^{p-1} \left(1-\frac{1}{p}\right) \Big/ D \,, \tag{53}$$

$$B(\tau) = A' \int_{K(t_1+c)^{-p}}^{\infty} x^{1-(1/p)}\, e^{-\tau x}\, dx \,, \tag{54}$$

$$C(\tau) = A' \int_{0}^{K(t_2+c)^{-p}} x^{1-(1/p)}\, e^{-\tau x}\, dx \,, \tag{55}$$

$$D = c^{p-1} \{(t_1+c)^{1-p} - (t_2+c)^{1-p}\} \,. \tag{56}$$

If $t_1 \to 0$ and $t_2 \to \infty$, $D \to 1$ and $C(\tau) \to 0$. In addition, if $c \to 0$, $B(\tau) \to 0$. Consequently for the entire period of an aftershock sequence (0 to ∞) with $c=0$,

$$\phi(\tau) = A' \Gamma\left(2-\frac{1}{p}\right) \tau^{-q} \tag{57}$$

where

$$q = 2-\frac{1}{p} \,. \tag{58}$$

Comparing with equation (50), we obtain

$$q+r = 3 \,. \tag{59}$$

The right side of equation (58), i.e., the distribution of time interval τ for an aftershock sequence of the type $n(t)=Kt^{-p}$, has already derived by Senshu.[222] Suzuki and Suzuki[231] described that the two expressions $p(n) \propto n^{-r}$ and $\phi(\tau) \propto \tau^{-q}$ are equivalent and the relation (59) exists between the two indexes. (They introduced the above two expressions in a study of the spatial distribution of epicenters, but it is essentially the same as the distribu-

tion of origin times on a time axis.) However, the derivation of the two expressions (49) and (57) in this paper starts from the equation $n(t)=Kt^{-p}$. The two expressions must not be equivalent unconditionally. It is easy to show examples of series of events having a τ^{-1} type distribution for τ but the distribution of n is quite different from an n^{-r} type.[224)]

It is a remarkable fact that the distributions of n and τ, as well as the distribution of origin time t, can be represented at least approximately by inverse power type formulas. The distribution of τ for an aftershock sequence plotted on a log-log scale fits a straight line with a slope of around 1.5 farily closely. Table 12 lists the values for slope q_0 determined from such diagrams for various aftershock sequences together with the calculated values

Table 12. Comparison between observed and calculated indexes q_c and q_0 $(=2-(1/p))$ in the inverse power type distribution function for time intervals between two successive aftershocks.

Date (GMT)	Name of the main shock	p	q_c	q_0	Period of investigation	Reference
1927 Mar. 7	Tango	1.1*	1.0_9	1.23	Mar. 7-Aug. 30	183)
1931 Sept. 21	Saitama	1.3*	1.2_3	1.7	No description	221)
1933 Mar. 2	Off Sanriku	1.5*	1.3_3	1.5	Mar. 2-Mar. 30 ?	221)
1938 Nov. 5	Off Fukushima	1.2*	1.1_7	1.43	Nov. 5-Apr. 30, 1939	Fig. 115
1943 Sept. 10	Tottori	1.2*	1.1_7	1.27	Sept. 10-July 31, 1944	Fig. 116
1948 June 28	Fukui	1.3*	1.2_3	1.48	June 28-July 28	183)
1952 Mar. 4	Off Tokachi	1.1*	1.0_9	1.8	No description	221)
1964 May 7	Off Oga Pen.	1.34	1.25	1.39	May 7-May 31	51)
1968 Jan. 29	Off Shikotan Is.	1.3	1.2_3	1.3	Jan. 29-Mar. 31	49)
1968 May 16	Off Tokachi	1.0	1.0_0	1.08	May 16-Dec. 31	Fig. 117
1958 July 10	Southeast Alaska	1.13	1.12	1.37	July 10-Nov. 20	227)
1963 Oct. 13	S. Kurile Is.	1.1	1.0_9	1.25	Oct. 13-Sept. 30, 1964	61)
1966 Feb. 5	Cremasta, Greece	0.78	(0.72)	0.81	Feb. 5-Nov. 30	66)

* Estimated by Utsu[1)].

q_c from equation (58). It is obvious that in every sequence q_0 is somewhat larger than q_c. The reason for the disagreement between observed and theoretical distributions may be explained by the following.

(1) The constant c in the modified Omori formula is not zero.

(2) The data immediately after the main shock are more or less incomplete.

(3) Errors in the estimates of p and q_0.

(4) The period from which the data are taken does not cover the entire aftershock sequence.

(5) The assumption that the aftershock occurrence in each short period of time is random, i.e., aftershocks are mutually independent (except their common dependence on the main shock) is not valid. Secondary aftershock sequences or other types of clustering actually occur in an aftershock sequence.

The first two matters (1) and (2) are responsible for a decrease in slope near the left end (small τ) of the distribution curve. However the last two ones (4) and (5) cause an increase in slope in wide ranges of τ. Figure 116 shows how the length of period affects the distribution curve of τ for the Tottori aftershock sequence. The slope -1.27 determined from the curve for a period of about ten months may be regarded as the final one, though a rapid decrease of frequency is observed near the right end of the curve. (This part of the curve may be modified, if the data for later period are added.)

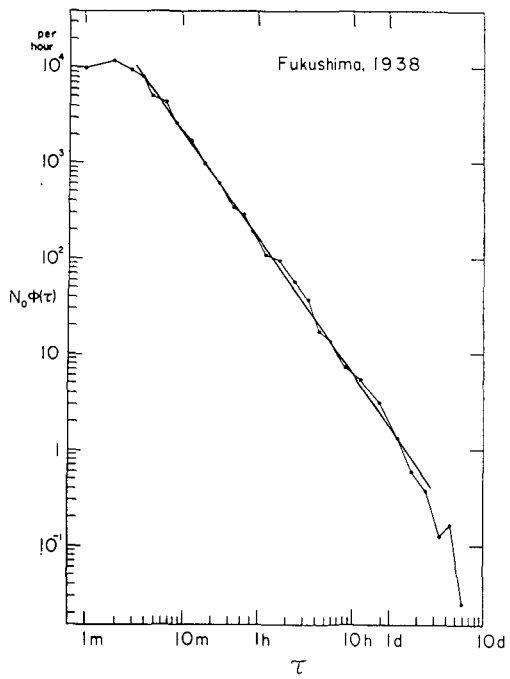


Fig. 115. Frequency distribution of time intervals between two successive aftershocks of the Fukushima earthquake of 1938 for the period from the main shock to the end of April 1939.

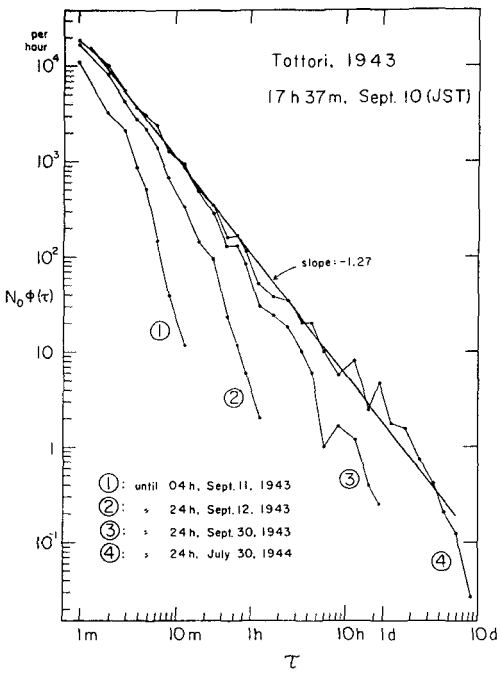


Fig. 116. Frequency distribution of time intervals between two successive aftershocks of the Tottori earthquake of 1943 for each period indicated in the figure.

Since the temporal distribution of aftershocks of this sequence is represented by the modified Omori formula for more than twenty years,[46] the p value, accordingly the q_c value, can be determined fairly accurately. Therefore the small difference between q_0 and q_c may be attributed to a weak tendency of clustering in the sequence. For other sequences whose q_0 values were determined from the data taken from a long period of time (say more than six months from the main shock), the chief reason for $q_c<q_0$ is probably the same as the Tottori sequence.

It appears that the analysis of distributions of time intervals is not always an effective method for investigating the clustering of aftershocks, then some other methods will be tried below. The theory of runs has been applied to earthquakes in order to test the hypothesis of stationary random occurrence.[17),225),232)] Here the same theory is applied to some aftershock sequences.

The epicenters of aftershocks of the Tokachi earthquake of May 16, 1968

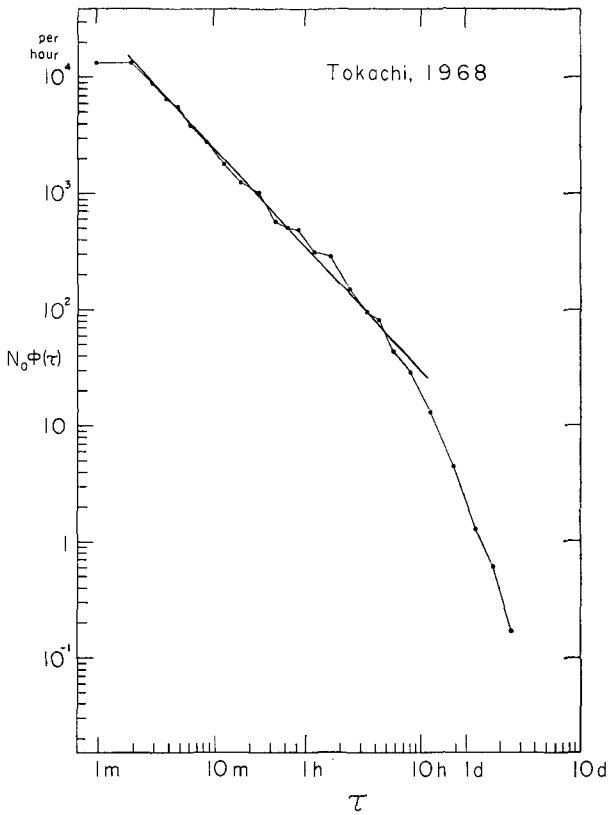


Fig. 117. Frequency distribution of time intervals between two successive aftershocks of the Tokachi earthquake of 1968 for the period from the main shock to the end of 1968.

were distributed from 39°N to 42°N as seen from Figure 42. A total of 211 aftershocks in a late stage of the sequence (November, 1968 through April, 1970) whose epicenters were located by JMA have been classified into three groups A (41°N–42°N), B (40°N—41°N), and C (39°N–40°N) according to the latitude of epicenters. (Thirteen shocks in a swarm of August, 1969 on the eastern side of the aftershock region are excluded.) Then the aftershocks listed in order of time of occurrence can be transformed into an arrangement such as

April, 1969

$$\cdots\cdots \mathrm{B\,B\,B\,B\,B\,B\,A\,A\,B\,A\,B\,C\,C\,C\,C} \cdots\cdots$$

If every C is replaced by a + sign and every A and B by a − sign, and the

number of runs R is counted, we obtain $R=63$. For a random arrangement the expectency of R is given by

$$E(R) = \frac{2n_+n_-}{N} + 1 \tag{60}$$

where n_+ and n_- represent the numbers of + and − signs respectively, and $N = n_+ + n_-$. If R is significantly smaller than $E(R)$, the aftershocks, especially those in group C can be considered as having a tendency of clustering. The test of significance can be made using that R is approximately normally distributed with a variance of

$$V(R) = \frac{2n_+n_-(2n_+n_- - N)}{N^2(N-1)}. \tag{61}$$

For the present case, $E(R)=75.2$, and $\sigma(R)=\sqrt{V(R)}=4.9$, therefore the probability that $R \leq 63$ is about 0.007, which is sufficiently small to reject the hypothesis of the random occurrence of aftershocks. Table 13 includes results for other combinations of A, B, and C. It is not necessary to take account of a gradual decrease in aftershock activity during the 18 month period tested,

Table 13. Test of randomness for aftershocks in the later stage of the Tokachi sequence from November 1968 through April 1970.

+	−	n_+	n_-	R	$E(R)$	$\sigma(R)$	$\frac{E(R)-R}{\sigma(R)}$	Prob.
A	B&C	80	132	94	101.1	6.9	1.0	0.15
B	C&A	83	128	78	101.7	6.9	3.4	0.0003
C	A&B	48	163	63	75.2	4.9	2.5	0.007

if it is assumed that the rate of decrease is the same for the three groups. However this method is applicable only to such cases that a fairly large number of shocks can be divided into groups according to the location of epicenters. The use of runs can be found in other types of arrangements of shocks (cf. Table 14).

A simple measure for grouping of events used by Vinogradov and Mirzoyev[233] is given by

$$u = N_G/N_0 \tag{62}$$

where N_0 is the total number of events and N_G is the number of events which belong to "groups". The "group" is defined here as two or more events separated at time intervals less than $\eta\tau$, where τ is the average time interval

Table 14. Results of the three tests for four case. $\alpha, \alpha', \alpha''$ are the signfiicance means that the hypothesis can not

Case	Aftershock sequence	Period	τ	η
I	Fukushima 1968	Nov. 11–14	39.5 m	0.5
II	Tottori 1943	Sept. 12, 17h –13, 12h	7.70 m	0.584
III	Tokachi 1968	May 26- June 2	56.0 m	0.5
IV	〃	July 13- July 30	3.40 h	0.5

between two successive events and $\eta=0.5$ in their paper. This measure is meaningful only when the sequence is stationary. It is easy to show that in the case of a stationary random occurrence, the expectency of u is given by

$$E(u) = 1-e^{-2\eta} \tag{63}$$

and if $\eta=0.5$, $E(u)=0.632$. If N_G for observational data is significantly larger than $N_0E(u)$ calculated from equation (63), the events can be regarded as having a tendency of clustering. The significance test may be performed by using that N_G has a binominal distribution. In Table 14 u and $E(u)$ are obtained for parts of some aftershock sequences In cases II, III, and IV, u is not significantly different from $E(u)$, whereas in case I the hypothesis of $u = E(u)$ can be rejected at a significance level α of about 0.08.

Values of α'' in Table 14 indicate significance levels at which a χ^2 test rejects the hypothesis that the distribution of τ fit a negative exponential distribution. R is the number of runs for arrangements of + and − signs assigned to each time interval between successive shocks according as it is larger than or smaller than $\eta\tau$ respectively. In case I, R is smaller than its expectency $E(R)$ for stationary random occurrence at a significance level of about 0.07. For other three cases no significant difference between R and $E(R)$ is found. Thus the use of u and R gives similar results.

As discussed in this section, there are various methods for testing the hypothesis of stationary random occurrence of earthquakes. Each method is based on a different statistical property of stationary random events, therefore it is quite possible that one method can reject the hypothesis at a high significance level while another can not reject the hypothesis even at low significance levels. It is impossible to designate generally which is the most

stationary random occurrence of aftershocks in levels at which the hypothesis is rejected. A cricle be rejected even at the signficance level of 0.1.

N_0	N_G	u	$E(u)$	α	R	$E(R)$	α'	α''
145	105	0.724	0.632	0.08	60	68.0	0.07	0.001
148	97	0.656	0.689	○	72	77.0	○	0.02
153	102	0.667	0.632	○	72	71.1	○	○
127	73	0.570	0.632	○	58	59.1	○	○

sensitive method. If the hypothesis can not be rejected by a method, it never means that the hypothesis is accepted. Moreover, if the hypothesis is rejected, it can not be decided which is responsible for it, a lack of stationarity, or a lack of randomness, or the both.

Discussions in this section, together with those in other papers, indicate that there is some tendency to clustering in aftershock sequences, though it is also concluded that most aftershocks in a sequence are independent one another. Which of these two contradictory properties of aftershocks is revealed more prominently depends largely on the data selection.

8.3 *Frequency of aftershocks for a standard aftershock sequence*

In the above two sections, temporal properties of aftershock occurrence departing from a simple decay law such as the modified Omori formula have been treated. However the modified Omori formula is still a most adequate one for representing general characteristics of the distribution of aftershocks in time. In studying seismicity problems it is sometimes necessary to know the average rate of the occurrence of aftershocks having magnitude M_s and larger t days after a large shallow earthquake of magnitude M_0. For this purpose the following equation is employed to represent a standard rate of occurrence of aftershocks per day.

$$n(t) = \frac{10^{0.85(M_0-M_s)-1.83}}{(t+0.3)^{1.3}} . \tag{64}$$

The values of $p=1.3$, $c=0.3$ day, and $b=0.85$ used in this equation are the medians for sequences listed in Table 4 ($M_0 \geq 5.5$), and a constant 1.83 is chosen to obtain a best fit to the observed frequency of shocks in these sequences at

t=1 day and 100 days. Since the fluctuations of p, c, b, etc. from sequence to sequence are not small, the actual frequency for each sequence may differ considerably from that given by equation (64). In Figure 118 a graph for finding $n(t)$ from M_0-M_s and t is shown. This figure shows, for exmaple, that the rate of microaftershocks of $M_s \geq 0$ ten years after a magnitude 7 earthquake is about 100 per year, etc. The frequency at such late stages largely depends on the value of p. Figure 119 is a graph used in estimating the frequency at late stages of the sequence from the frequency at t=100 days. For example, if the felt aftershocks at a certain place is once a day at t=100 days, the rate of felt aftershocks there at t=100 years is once a year when p=1.0, but it is once per five years when p=1.3. Figures 118–119 will be useful in discussing such problems as the relation between microseismicity and aftershocks.

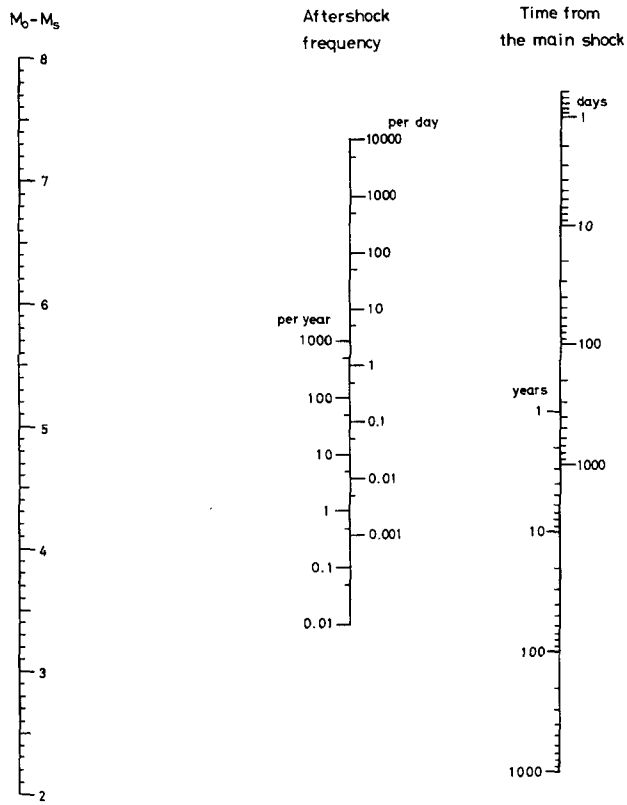


Fig. 118. A nomogram for calculating the frequency of aftershocks with magnitude M_s and larger following a main shock of magnitude M_0 on the basis of a standard aftershock sequence expressed by equation (64).

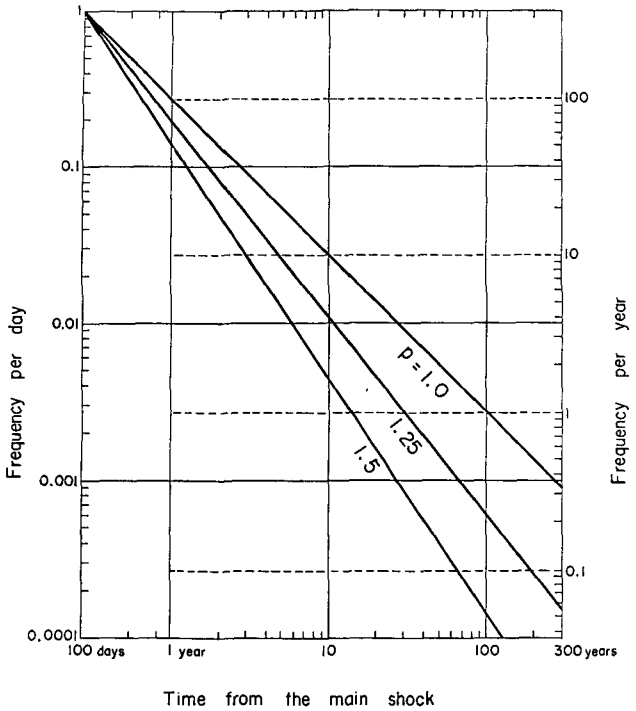


Fig. 119. A graph for estimating the frequency of aftershocks from the frequency 100 days after the main shock.

9. Magnitude-frequency relation for various types of earthquake sequences

Although it is generally accepted that the distribution of earthquakes in respect to magnitude is represented by Gutenberg-Richter's equation (1), there are characteristic differences in the magnitude-frequency distribution among various types of earthquake sequences introduced in Chapter 7. The standard magnitude-frequency distribution for aftershocks in a sequence of Type 1-A is schematically represented by Figure 120 (upper). As mentioned in Chapter 2, the main shock (a solid circle in Figure 120) usually has too large magnitude to be considered as a sample from the same distribution as the aftershocks. This is the reason why no main shocks have been included in the magnitude-cumutative frequency plots for aftershocks shown in Figures 48–68 (except Figure 56 in which the largest shock in the swarm is included).

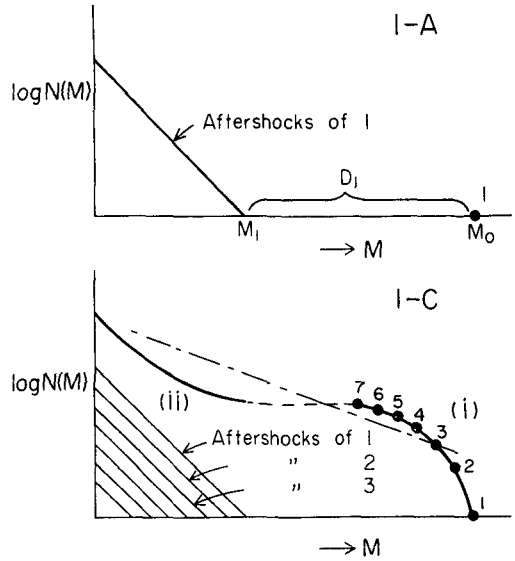


Fig. 120. Magnitude vs cumulative frequency distributions for sequences of Type 1-A and Type 1-C.

The magnitude-frequency distribution of earthquakes in a multiple sequence such as Type 1-C depends on both the magnitude distribution for the main shocks in the sequence and that for the aftershocks triggered by each main shock, as schematically represented in Figure 120 (lower). In this figure solid circles indicate the cumulative frequency distribution for seven main shocks. Each main shock is accompanied by a series of aftershocks whose cumulative frequency distribution is represented by each of the seven parallel straight lines. Accordingly the cumulative frequency distribution for the whole sequence is represented by the thick curve (i) ~ (ii). If this curve is approximated by a straight line, its slope (b value) will be considerably smaller than the slope for each aftershock series. This provides an explanation for the particularly small b values obtained for some swarms of the second kind (Type 1-C) occurring off the Pacific coast of northeastern Japan as illustrated in Figure 72. To demonstrate the remarkable difference in magnitude-frequency relation between the two types of sequences, the Sanriku sequences of 1933 (Type 1-A, M_0=8.3) and 1952 (Type 1-C, M_0=6.6) are compared in Figure 121. The 1952 sequence occurred in a part of the aftershock region of the 1933 earthquake. The number of shocks with

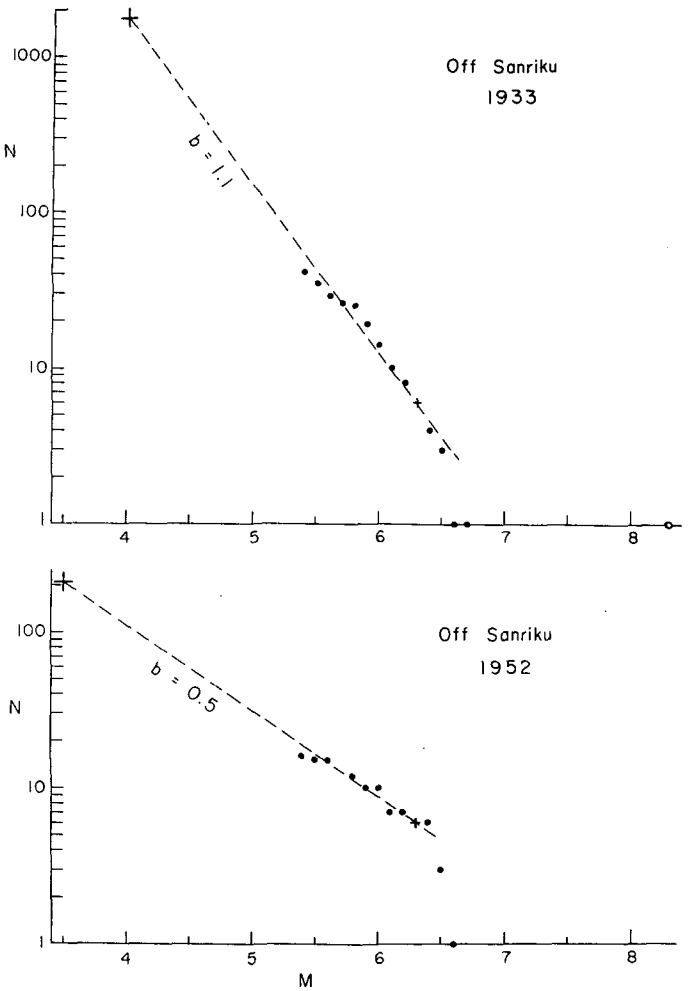


Fig. 121. Magnitude vs cumulative frequency distributions for the aftershock sequence of the Sanriku earthquake of 1933 and for the earthquake swarm of the second kind off Sanriku in October 1952.

magnitude 6.3 and above is the same (six) for both sequences, while the number of shocks detected by the JMA network of stations is quite different (1,800 vs 210).

It is to be remarked here that not all the sequences of Type I-C have small b values. In some cases the two parts of the curve (i) and (ii) in Figure 120 are so close that the curve is almost a straight line with the same slope as

each aftershock sequence. The Tokachi earthquake of 1968 (Figure 66) is an example of such cases.

The magnitude distribution for a swarm of the first kind, including the largest shock in it, usually fits Gutenberg-Richter's formula with a normal or somewhat high value of b.

Suyehiro et al.,[147] and Suyehiro[130),234] have pointed out that the b values for some foreshock sequences are considerably small as compared with those for aftershock sequences. The similar property has also been noticed in laboratory experiments on fracturing of rock samples by Vinogradov,[235] Mogi,[188] and Scholz,[236] and in the case of rockbursts in mines.[237] Although the difference in b value between foreshocks and aftershocks seems to be statistically significant, the present author suggests that the apparent small b values for some foreshock sequences can be explained in the same way as in the case of sequences of Type 1-C, i.e., some foreshock sequences having small b values are multiple sequences like Type 1-C or 2-C. Foreshock sequences in

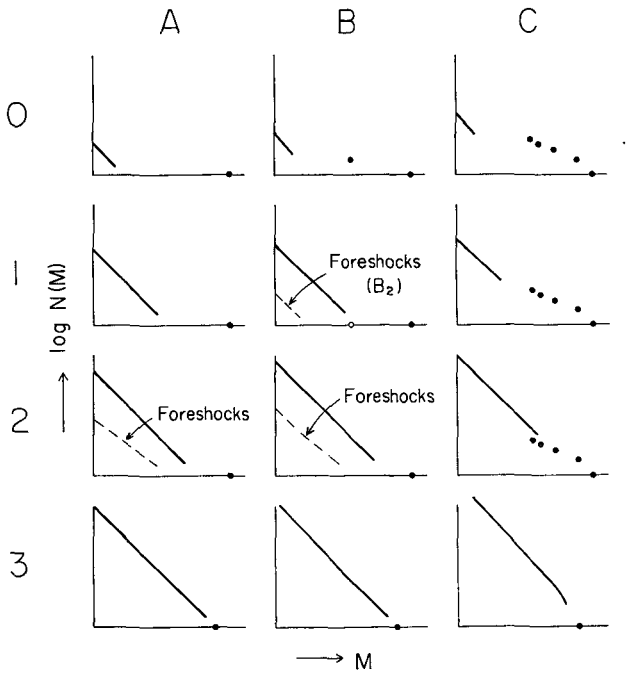


Fig. 122. Magnitude vs cumulative frequency distribution patterns for various types of earthquake sequences.

Type 2-A sequences, which are not triggered by large events, have more close analogy with foreshocks observed in laboratory experiments. The earthquake in Kyoto Prefecture on August 18, 1968 (the Wachi earthquake) may belong to this type (b=0.59 for foreshocks and b=0.80 for aftershocks, after Watanabe and Kuroiso[217]).

According to this idea, foreshock sequences of Type 1-A have normal b values provided that the largest foreshock is exlcuded. Motoya[211] reported that the b value for a series of earthquakes starting on May 1, 1968 in the region of the Tokachi earthquake of May 16, 1968 have almost the same b value as that for the aftershock sequence. The foreshock sequence of the southern Kurile Islands earthquake of 1963 also has a b value approximately equal to that for the aftershocks.

On the basis of the idea expressed above, the magnitude distribution for each type of the earthquake sequences shown in Figure 75 can be represented schematically in Figure 122 in the form of magnitude vs logarithm of cumulative frequency.

10. Magnitude and time distribution of main shocks in multiple sequences

Groups of shallow earthquakes in and near Japan during the years from 1926 through 1969, which occurred successively within relatively short intervals of time and space are listed in Table 15. Each line of the table corresponds to one group. A criterion similar to that explained in Chapter 2 is adopted to distinguish such groups. Moreover these groups have been chosen under the conditions that the largest shock in each group has a magnitude 5.0 or more, and the magnitude of each shock in a group does not differ by more than 0.4 from another shock in the same group. If earthquakes in a group are arranged in order of magnitude and the ith magnitude is denoted by M_{i-1}, magnitudes listed in the third column "Magnitudes of large shocks" are those satisfying the condition M_{i-1}-$M_i \leq 0.4$ (i=1, 2,....). (There are several exceptional cases, which are indicated in the table.) If six or more shocks satisfying this condition are found in the same group, the largest five shocks are listed in the third column and the sixth magnitude is indicated in the column "M of the next shock" with a mark of C. A mark $\times$ in this column means that no shocks belonging to the same group was observed by the JMA network. Although the selection of earthquake groups is somewhat tentative, it may be recognized from this table that the sequential occurrence of a few

Table 15. Swarms and swarmlike groups of shallow earthquakes in and near Japan $M_0 \geq 5.0$, 1926–1969.

The largest shock					Magnitude of large shocks* (in order of time of occurrence)	M of the next shock	Type
Date (GMT)			Epicenter °N	°E			
1926	Jan.	10	36.3	141.4	**5.4** 5.0		IIa
	May	22	42.4	145.0	**5.3 5.3**		II
	June	6	41.9	145.4	5.6 5.4 5.3 5.3 **5.9**	5.0	II
	Oct.	3	37.0	143.0	**6.2** 5.2 6.0 5.6 5.7		IIa
1927	Jan.	10	40.5	143.5	**5.5** 5.1		IIa
	June	9	38.5	141.8	5.5 5.1 **5.6**		II
	Dec.	4	32.6	129.9	5.1 **5.4** 5.3		II
1928	Mar.	31	42.5	145.5	5.1 **5.4**		II
	May	27	40.0	143.2	**7.0** 6.9 6.9	6.0	IIa
	June	3	31.7	128.8	**6.4** 5.9 5.4 #		IIa
	July	7	42.4	144.4	**6.0** 5.6 5.5	5.0	IIa
1929	Jan.	1	33.1	130.9	4.9 4.7 5.2 **5.4** 4.9		II
	Feb.	25	42.2	143.1	**5.1** 4.7	×	IIa
	Mar.	31	39.0	144.2	5.8 5.9 5.3 5.4 **6.3**		II
	Apr.	16	36.3	141.3	**6.0** 5.7		IIa
	June	24	37.2	141.3	5.0 5.3 5.3 5.6 **5.7**	4.8	II
1930	Mar.	22	34.8	139.1	5.4 **5.8** 5.1 5.2 5.4	5.0	I
	Dec.	20	35.0	132.9	**6.0** 5.7	5.1	IIa
1931	June	23	36.3	141.2	6.1 6.3 **6.7** 5.9	4.7	II
	Nov.	2	32.2	132.1	6.3 **6.6** 6.2	5.7	II
	Dec.	22	32.6	130.5	5.4 **5.6 5.6**		II
1933	Jan.	7	40.0	144.5	6.4 **6.8** 6.2	5.5	II
1934	Feb.	22	36.2	141.3	5.0 5.2 4.8 **5.3**		II
	Oct.	5	41.5	142.8	**6.1** 5.6 #		IIa
1935	June	28	34.7	140.2	6.0 **6.1**	5.2	II
	Oct.	12	40.0	143.6	**7.2** 7.1	6.6	IIa
1937	Jan.	27	32.8	130.8	5.0 **5.3**		II
	Feb.	14	33.3	132.1	5.0 **5.1** 4.9	×	II
1938	Sept.	12	39.2	143.0	5.4 **5.5**		II
	Nov.	5	37.1	141.6	**7.7** 7.6 7.5 7.1 7.0	6.7C	IIa
	Dec.	14	38.4	143.1	6.1 **6.3** 6.0		II
1939	May	1	39.9	139.8	**7.0** 6.7 6.4	5.7	IIa
	June	17	38.6	142.5	**5.4** 5.0		IIa
	Oct.	10	38.4	143.0	**6.8** 6.3 #		IIa
1940	June	12	35.3	141.0	**6.1** 5.9	5.2	IIa
	Aug.	25	36.1	140.0	4.9 **5.4** 5.1		II
1941	Feb.	27	40.7	142.4	4.8 **5.1**		II
	Mar.	12	39.5	143.5	6.0 **6.3 6.3**	5.6	II
	May	9	36.0	142.3	6.1 **6.2**		II
1942	Feb.	14	42.0	145.0	**6.1** 5.7		IIa

* Boldfaced figures indicate the magnitude of the largest shock in each group.

\# The case of $M_0 - M_1 = 0.5$. This is included if "M of the next shock" was so small that it was not determined from observations by JMA.

× No shock was recorded by JMA.

Table 15. Continued

The largest shock					Magnitude of large shocks* (in order of time of occurrence)	*M* of the next shock	Type
Date (GMT)			Epicenter °N	°E			
	Feb.	18	35.5	141.0	**6.0** 5.8		IIa
	Aug.	22	32.2	132.3	**6.2** 6.1	5.3	IIa
	Sept.	8	36.5	141.3	5.9 **6.0**		II
1943	Jan.	19	36.1	140.5	**5.8** 5.5		IIa
	Mar.	4	35.6	134.2	**6.1** 5.8 **6.1** 5.8	4.2	IIa
	Apr.	11	36.2	141.2	6.1 6.0 6.3 **6.6** 6.4	5.9C	II
	June	13	41.1	142.7	**7.1** 6.3 6.5 6.2 6.8	6.1C	IIa
1944	Mar.	21	40.9	143.1	6.1 **6.2**	5.0	II
	Dec.	8	34.4	139.5	**5.7** 5.5		Im
1946	May	1	32.0	132.0	**5.4** 5.2		IIa
	Dec.	1	35.7	140.4	**5.1** 4.7 4.5	×	IIa
1947	May	9	33.3	131.1	**5.5** **5.5**		II
	Oct.	10	31.0	131.4	**6.0** 5.7		IIa
1948	May	12	37.8	142.3	**6.6** 6.3 6.3	5.5	IIa
	Dec.	16	34.7	139.4	**5.0** 4.6		Im
1949	Aug.	9	35.3	135.6	4.5 **5.1** 4.7 4.5		II
	Dec.	26	36.7	139.7	6.4 **6.7**	5.9	II
1950	Aug.	22	35.2	132.7	**5.3** 5.0		IIa
	Dec.	24	31.8	132.0	**5.8** 5.5		IIa
1952	June	15	39.0	143.0	5.8 5.4 **5.9**		II
	Oct.	27	39.4	143.4	6.5 6.5 6.4 **6.6** 6.4	6.4C	II
1953	Oct.	28	31.8	129.3	**5.7** 5.4		IIa
1954	May	12	41.5	140.6	**5.2** 4.8 4.4		IIa
	May	16	35.2	132.8	4.9 **5.3**		II
1955	May	1	39.7	143.8	5.6 **5.9** 5.7 5.6 5.6		II
	June	4	40.2	143.0	**5.8** 5.6	5.0	IIa
1956	Dec.	22	33.7	139.5	**6.0** **6.0**	4.6	I
1957	Aug.	30	37.4	141.5	4.8 5.0 5.0 5.0 **5.1**	4.6C	II
1958	Apr.	8	38.2	143.7	**6.5** 6.2 6.2 6.1 6.4	5.6	IIa
	July	23	31.0	142.0	6.1 **6.5**	5.2	II
	Sept.	29	39.6	143.4	**5.6** 5.3	5.2	IIa
	Dec.	11	30.5	140.2	5.5 **5.7**		Im
1959	Jan.	30	43.4	144.4	5.7 **6.2** 6.1	5.0	II
	Apr.	28	31.9	129.5	5.2 **5.3** 4.9	×	II
	May	25	40.8	143.4	**5.1** 4.8		IIa
	Sept	28	43.2	146.6	5.0 **5.5** 5.3		II
1960	Feb.	5	38.6	143.2	**6.1** 5.7	4.6	IIa
	Apr.	15	41.5	144.8	**5.5** 5.0 #		IIa
	July	30	40.2	142.6	6.0 **6.7** 6.2 #	5.2	II
	Oct.	3	36.4	140.8	5.0 4.8 **5.1** 4.5		II
	Nov.	7	32.4	132.1	5.1 5.6 **5.8** #		II
1961	Jan.	16	36.0	142.3	**6.8** 6.4 6.5 6.1 6.6	5.7C	IIa
	Feb.	12	43.2	147.9	5.8 **6.7** 6.3 6.1 6.3	5.3	II
	Mar.	15	32.0	130.7	**5.5** **5.5**	5.6	II
	June	19	39.1	143.6	5.6 **5.8** 5.5	4.5	II

Table 15 Continued

The largest shock				Magnitude of large shocks* (in order of time of occurrence)	M of the next shock	Type
Date (GMT)			Epicenter °N °E			
	Aug.	11	42.8 145.6	**7.2** 6.9	6.1	IIa
1962	Apr.	12	38.0 142.5	**6.8** 6.4	5.8	IIa
	Aug.	29	34.0 139.3	**5.9** 5.2 5.4 5.8 5.3	5.2C	I
1963	Mar.	31	35.1 132.4	**5.1** 5.0		IIa
	Sept.	7	36.7 130.7	6.0 **6.2**	×	II
	Oct.	3	31.9 132.2	**6.3** 5.8 ♯		IIa
1964	May	7	40.6 139.0	**6.9** 6.5	5.2	IIa
	June	1	43.2 147.0	4.9 **5.0** 4.6	×	II
	Dec.	20	37.2 141.8	5.0 4.6 **5.3**		II
	Dec.	8	34.6 139.3	**5.8** 5.2 5.3 5.5 5.3	5.1C	IIa
1965	Mar.	17	40.7 143.2	**6.4 6.4**	5.4	II
	Apr.	6	36.0 139.9	**5.5** 5.1	4.5	IIa
	May	18	43.3 146.9	5.1 **5.5** 5.1		II
	Aug.	3	34.3 139.3	**5.0** 4.6 4.7	3.9	IIa
	Aug.	31	43.5 144.4	**5.1** 5.0 **5.1**		II
	Sept.	25	39.5 143.7	**5.6** 5.5 **5.6**		II
	Nov.	6	34.1 139.0	4.9 5.2 4.8 4.7 **5.6**	4.6C	Im
1966	Jan.	11	33.6 137.2	5.4 **5.9**	4.6	II
	Apr.	5	36.6 138.3	**5.4** 5.3 5.3 5.3 5.3	5.1C	Im
	Apr.	21	35.5 142.3	**5.8** 5.5	×	IIa
	May	15	34.1 139.0	5.4 **5.5**		II
	Oct.	29	41.6 144.4	4.5 4.6 **5.1** ♯		II
1967	Apr.	6	34.2 139.1	**5.3** 5.2 4.9 5.2	4.3	Im
	Apr.	29	35.8 140.9	**5.1** 4.9	4.3	IIa
	May	18	41.7 145.2	5.2 4.7 **5.4** 5.1	4.1	II
	July	17	38.2 142.2	4.5 **5.0** ♯		II
1968	Feb.	21	32.0 130.7	4.7 **6.1** 5.6 5.7 5.4	4.7	II
	Feb.	25	34.1 139.2	**5.0 5.0** 4.9 4.9 4.6	4.2C	I?
	May	16	40.7 143.6	**7.9** 7.5 7.2	6.7	IIa
	May	19	35.4 142.4	**5.8** 5.5	×	IIa
	Aug.	18	35.2 135.4	**5.6** 5.2	4.5	IIa
	Sept.	21	36.8 138.3	**5.3** 4.6 4.9 4.6	4.1	IIa
1969	July	23	37.2 141.8	5.2 **5.5**	4.5	II
	Sept.	2	36.2 137.7	4.7 4.4 **5.0**	3.9	II
	Sept.	17	30.9 131.7	**5.9** 5.5		IIa
	Nov.	29	33.3 132.4	**5.1 5.1**		II

shocks with magnitude not very much different from the largest shock in a sequence is not a rare phenomenon.

The groups listed in the table should correspond to one of the types of earthquake sequences shown in Figure 75 (except O-A). Only three groups indicated by a mark I in the last column can be regarded as swarms of the

first kind. Six groups are classified as a "mixed type" of a swarm of the first kind and other type of sequences, and a mark Im is given in the last column. All other groups (marked with II or IIa) may be regarded as swarms of the second kind in a broad sense. As mention in Chapter 7, if the first shock is the largest, the group is usually called "main shock —aftershock sequence". A mark IIa is given for these groups. If few shocks are observed other than the

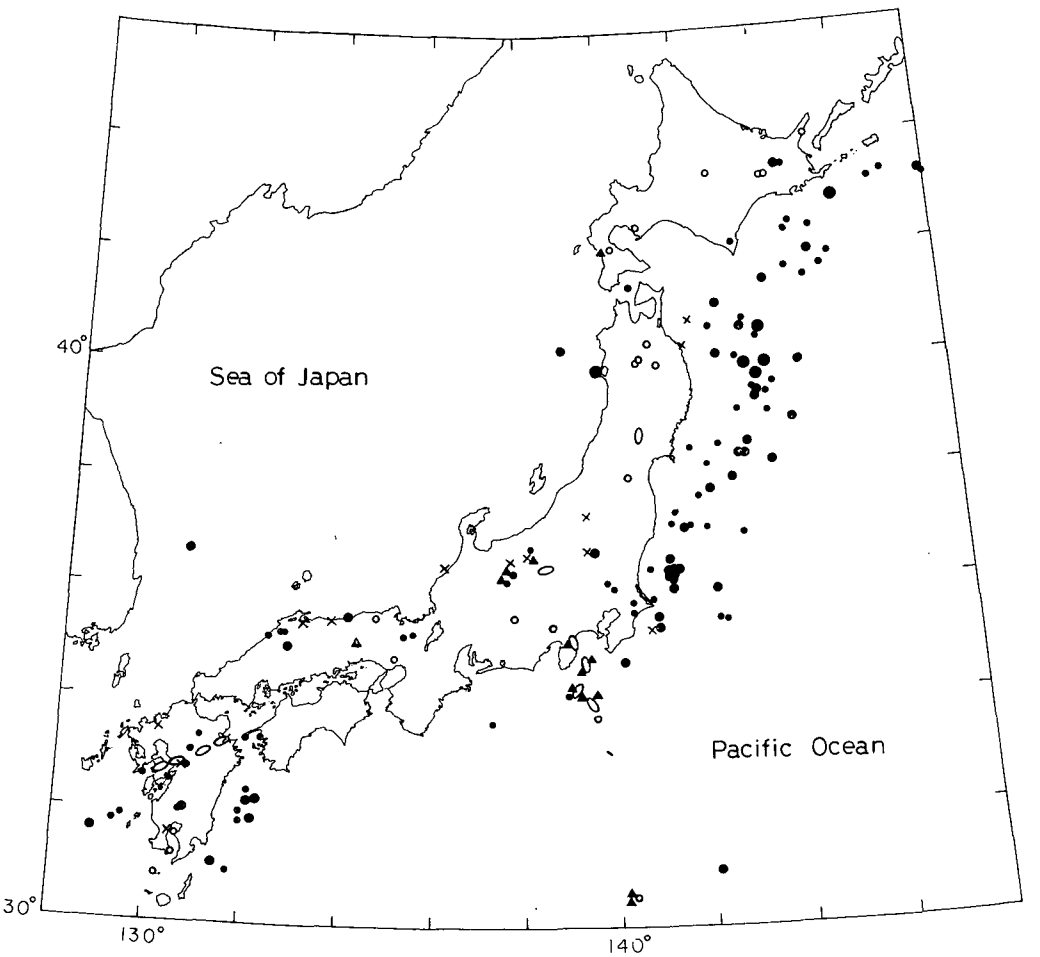


Fig. 123. Locations of the swarm of the first kind listed in Table 16 (open circles and ellipses) and the swarms of the second kind and the similar type sequences listed in Table 15 (solid circles). Three sizes of cricles indicate that the largest mangitude M_0 is $5 \leq M_0 < 6$, $6 \leq M_0 < 7$, and $7 < M_0$ respectively. Triangles indicate mixed types.

Table 16. Earthquake swarms of the first kind involving more than 30 shocks recorded in Japan during 1926–1969. (Earthquakes originating near Wakayama, Sakura-jima, and Asama-yama are not listed.

Period		Location	M_0	Reference
1927	Apr.-May	Near Meakan-dake		238)
	June	Near Oita		
1929	Dec.-1930 Feb.	Near Unzen-dake		
1930	Feb.-May	Near Ito	5.8	221), 239)
1931	Apr.	Near Kuju-zan, Oita Pref.		
	July	Arima, Hyogo Pref.		
1933	Mar.	Iwagasaki, Miyagi Pref.		
	Mar.-Apr.	Miyakawa, Akita Pref.		
	Aug.-Sept.	Okunakayama, Iwate Pref.		210)
1934	Aug.	Near Iida, Nagano Pref.		210)
	Sept.	Near Iwo-jima, Kagoshima Pref. #		240)
1935	June-July	Near Zao-zan		241), 242)
	July-Aug.	Near Iida, Nagano Pref.		
	Aug.-Sept.	Near Miyake-jima		
	Sept.	Near Unzen-dake		
1936	Nov.-Dec.	Hanawa, Akita Pref.		243)
1939	Sept.-Oct.	Near Oshima		
	Oct.	Near Fuji-san		
	Dec.	Near Oshima		244)
1940	Mar.-Apr.	Near Oshima		
	May	Near Unzen-dake		
1941	Sept.-Oct.	Near Oshima		
1942	Apr.-May	Narugo, Miyagi Pref.		245)
	Nov.	Near Oshima*		
1943	Apr.	Hakone-yama		
	June	Near Kinpo-zan	4.0	
	Dec. 1944 Apr.	Near Hakone-yama*	4.0	
	Dec.	Near Mikura-jima		Fig. 98
	Dec.-1945 Oct.	Near Usu-san #		246)-248)
1944	Feb.-May	Near Oshima		
	Aug.-Nov.	Near Aso-san	3.8	
1946	Dec.	Near Kuju-zan, Oita Pref.		
	Dec.	Near Kinpo-zan	5.1	
1948	Dec.	Near Oshima*	5.0	
1949	Aug.	Near Oshima		
1950	Apr.	Shirakawa, Gifu Pref.		
	Aug.-Sept.	Near Oshima		
	Sept.	Onikobe, Miyagi Pref.		
1951	Aug.	Unzen-dake		
1952	Mar.	Near Meakan-dake		249)
	Apr.-May	Near Tori-shima		250)
	Nov.	Near Hakone-yama		251)
	Dec.	Near Oshima		
1954	Mar.-Aug.	Near Lake Towada	3.9	252)
	May-Aug.	Near Kumaishi, Hokkaido*	5.3	253)

Table 16 Continued

Period	Location	M_0	Reference
1955 Sept.	Near Oshima		
1956 Mar.	Near Kumamoto		
Apr.-May	Near Oshima*		
Oct.	Near Unzen-dake		
Dec.	Near Miyake-jima	6.0	Fig. 99
1957 Jan.	Near Oshima	4.2	
Feb.-Mar.	Near Kinpo-zan		
1958 June-Sept.	Near Yake-dake, Nagano Pref.*	3.8	254)
Sept.	Near Unzen-dake		
Dec.	Near Torishima*	5.7	
1959 June	Near Oshima		
July	Near Tori-shima		
Sept.-1960 Mar.	Near Hakone-yama	4.0	255)
Dec.	Near Oshima		
1960 Apr.	Near Oshima		
1961 July	Near Oshima	4.6	
Sept.	Near Tottori		
1962 Apr.	Near Unzendake		113), 114)
Aug.-Oct.	Near Miyake-jima	6.0	Fig. 101
1963 Feb.-Mar.	Nagano Pref.*	5.5	115)
1964 Jan.-Feb.	Rausu, Hokkaido	4.6	144)
Apr.	Near Unzen-dake		
1965 Feb.	Near Oshima	4.8	
Oct.-Nov.	Near Kozu-shima*	5.6	256)
Nov.	Near Tori-shima*	6.5	
1966 May-June	Namarikawa, Hokkaido	3.7	253)
July	Near Unzen-dake		
Aug.	Near Kozu-shima*	4.2	
1967 Apr.	Near Kozu-shima*	5.3	257)
Aug.	Near Ibusuki, Kagoshima Pref.	3.9	143)
1968 Feb.	Near Kozu-shima	5.0	
May	Near Tokachi-dake		118)

* Mixed type. ♯ Accompanying volcanic eruption.

shocks included in the column "Magnitude of large shocks", the group is classified as a Type O-B or Type O-C sequence. Table 15 includes several such groups.

Since the earthquake groups listed in Table 15 are classified as swarms of the first kind, or swarms of the second kind in a broad sense, or mixed types, the geographical distribution of these groups is shown in Figure 123 using open circles, solid circles, and triangles for respective types. There are many other swarms of the first kind with the largest shocks of magnitude less than 5.0 occurring in Japan during the years from 1926 through 1969. These are

listed in Table 16. These are also indicated by open circles or triangles (mixed type) in Figure 123. If three or more such swarms occurred in this period in almost the same place, they are represented by an open ellipese.

It is readily apparent from this figure that the distribution of earthquake swarms and swarmlike multiple sequences in and near Japan is fairly systematic. A comparison with seismicity maps of Japan (e.g., Usami et al.[258] and Katsumata[259]) shows that the region where swarms and multiple sequences occur roughly corresponds to the region where very shallow earthquakes (focal depths less than 30 km) occur. Matushima[18] mentioned that most earthquake clusters are distributed at depths of 20 km to 80 km. This is not confirmed by the present data. Most of the swarms of the first kind are located in volcanic regions (some of which accompanied volcanic eruptions), whereas many swarms of the second kind and multiple sequences of the similar nature occur in non-volcanic regions such as off the Pacific coast of northeastern Japan just west of the axis of Japan trench, in Kashimanada (near 36°N, 141°E), a part of Hyuganada (near $32^1/_2$°N, $132^1/_2$°E), etc. Matushima[260] noted the close correlation between distributions of earthquake swarms and hot springs in Japan. It is also recognized from Tables 15 and 16 that the magnitude of the largest shock in a swarm of the first kind rarely exceeds 6, while the magnitudes of large shocks in some swarms of the second kind reach about 8 as already mentioned in Chapter 7. Mogi[14),22)] showed a map of geographical distribution of earthquake swarms in Japan and remarked that swarms occur in the fracture zone, though he did not classified swarms as is done in the present paper. Mogi[14)] tabulated 133 swarms in Japan, but only a half of these are included in Table 16, which lists the cases including more than 30 shocks recorded. Small earthquake swarms of the second kind are also observed occasionally. The Hamasaka swarm of 1964[145)] (M 3.6, 3.1, ···), the Yakedake swarm of 1968[261)] (M 3.4, 3.3, 3.1, ···), and the Kutsugahara, Hiroshima Prefecture, earthquakes of 1970[262)] (M 4.6, 4.3, 4.3, ···), are examples of such swarms. (In 1969 a larger swarm occurred near Yakedake.[263)] This is included in Table 15.)

It is sometimes announced by authorities shortly after a destructive earthquake when people are in panic that there is no necessity for fearing a large earthquake to come, because aftershocks are as a rule smaller than the main shock. Seismologically this is not always true, especially for earthquakes originating in regions where multiple sequences are apt to occur. Omori[264),265)] reported several cases of successive occurrence of destructive

earthquakes from almost the same sources. Matuzawa[266] cited 28 such cases involving 60 destructive earthquakes in Japan. Referring to Omori's and Matuzawa's papers, the places where two destructive earthquakes with magnitude difference 0.5 or less occurred successively before 1926 are indicated by $\times$ marks in Figure 123. However successive occurrence of great earthquakes of magnitude about 8 off the Pacific coast of central and western Japan (off Nankaido, Tonankaido, and Tokaido) which was observed several time in historical times are not indicated in Figure 123. This figure may be used in considering the possibility that a large shallow earthquake in a certain region is followed by another large earthquake of approximately equal magnitude.

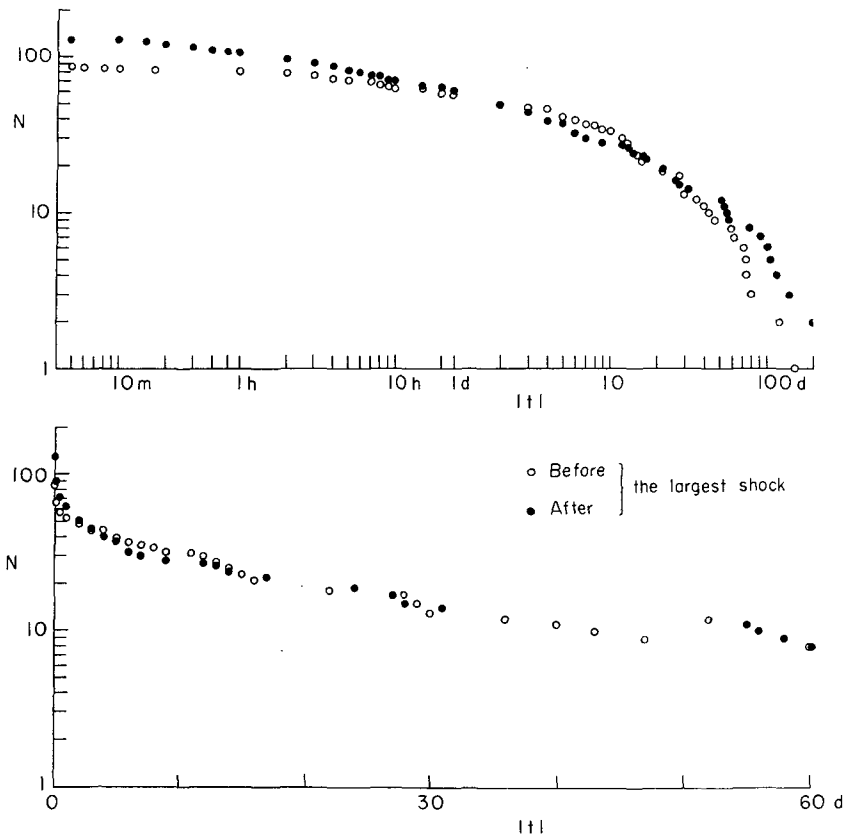


Fig. 124. Time distribution of shocks in the groups listed in Table 15. The origin time t is measured from the time of the largest shock in each group. The graph shows the number of shocks with origin times $|t|$ and larger.

It is somewhat difficult to represent adequately the temporal distribution of large earthquakes in each group listed in Table 15. Figure 124 shows the distribution of the number of shocks in the third column of Table 15 with absolute values of origin time larger than $|t|$ measured from the time of the largest shock in each group. Open and closed cricles refer to shocks occurring before and after the largest shock respectively. If two largest shocks have the same magnitude, the earlier shock is tentatively regarded as the largest one. It is seen from this figure that the distribution has an intermidiate nature between $|t|^{-p}$ type and $e^{-\alpha|t|}$ type distributions. Since Figure 124 is a superposition of many groups, it may not represent the temporal character of individual groups.

11. Some spatial characteristics of earthquake sequences

It seems quite possible that the pattern of spatial distribution of earthquakes depends on the type of earthquake sequences exhibited in Figure 75. However spatial characteristics peculiar to each type of sequences are poorly known, partly because of the complexity of the pattern and partly because of the inaccuracy of hypocenter locations. In an earthquake sequence accompanying a great earthquake with magnitude about 8 or more, the errors in the location of individual earthquakes are much less important since the earthquakes are distributed in large areas. In such a case, spatial characteristics of the sequence are investigated more easily.

Figure 125 represents schematically the spatical distribution pattern of a typical large earthquake sequence. Region A is the source region of the main (largest) shock of the sequence. Primary aftershocks occur there, with a frequency decay curve shown in a graph marked by A_0. If no other remarkable activities associated with the main shock are found, this is the simplest aftershock sequence of Type 1-A. As explained in Chapter 7, the Sanriku earthquake of 1933 (Figure 77) can be classified as Type 1-A as far as the early stage of the sequence is concerned. According to the definition of aftershock area adopted in Chapter 4, its aftershock area is about a half of that calculated from equation (14). In the later stage of the sequence the activity spread to neighboring areas.[169] The most remarkable activity in the neighboring areas is that associated with a magnitude 7.1 earthquake of June 19 off Kinkazan.

Such spreads of activity over the neighboring areas (regions B, C,...., S, etc. in Figure 125) have been observed in many cases.[169] If at some time

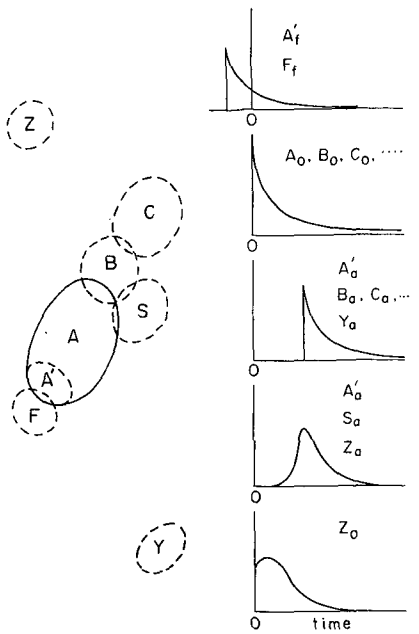


Fig. 125. A schematic representation of the spatial distribution of the source regions of the constitutent sequences F, A, B, C, S,.... in a multiple sequence. Separate regions Y, Z,.... are activated by the main shock in region A. The graphs on the right shows temporal variation of activity in each source region.

after the main shock a large earthquake occurs in region B which initiates a new series of aftershocks in this region as illustrated in graph B_a in Figure 125, the sequence is classified as Type 1-B. Similarly if the region C is activated by another large earthquake, the sequence becomes Type 1-C. It may be possible that the activity revives in region A′ which is a part of region A. If the activities in regions B, C,.... start within one month from the main shocks, the area A+B+C+.... is regarded as the aftershock area of the main shock of region A according to the definition of aftershock area adopted in Chapter 4. In this case the aftershock area is larger than that calculated from equation (14), though the areas of A, B, C,.... are compaable to or smaller than those estimated from equation (14) using the magnitude of each main shock as M_0. The Tokachi earthquake of 1968 (Figures 42 and 95) was followed by an earthquake of magnitude 7.2 within one month from the main shock. It occurred south of the primary aftershock area and

accompanied by many secondary aftershocks. The secondary aftershock area is about 2.7×10^3km^2, which is a little smaller than that calculated from equation (14). A large aftershock of May 16 ($M=7.5$) also activated the area of about 3.3×10^3 km^2 which is again slightly smaller than that calculated from equation (14). However the whole aftershock area of the Tokachi earthquake, according to the definition in Chapter 4, is about three times as large as that calculated from equation (14).

In some cases a swarm of the first kind occur in region S or A'. When the swarm is remarkable, the sequence may be called a mixed type one. The occurrence of such swarms or many small size B_a-type activities may be recognized as a gradual spread of aftershock area. The activity in April 1952 off Urakawa (near 42°N, 143°E) after the Tokachi earthquake of March 4, 1952 may provide an example of such swarmlike activities.

In some cases the main shock in region A is preceded by a sequence of Type 1-A occurring in region F or A'. The foreshock sequence of the southern Kurile Islands earthquake of 1963 (Figure 94) occur in an area of about 3.2×10^3km^2 near the west end of the aftershock area of the main shock according to the data from USCGS. This foreshock area is less than one-tenth of the aftershock area, but it is about ten times as large as the calculated value from equation (14) using the magnitude of 6.3 for the largest foreshock. The Sanriku earthquake of 1933 also preceded by a series of earthquakes occurring north of its source region. The area of this foreshock sequence was also considerably large (cf. Table 4). These two foreshock sequences are in themselves nearly multiple sequences.

It has been pointed out by some investigators that there were instances that the seismic activity started immediately or shortly after the occurrence of a large earthquake in some separated regions from the source region of the main shock such as regions Y and Z in Figure 125. Three earthquake swarms in 1933 occurring in Tohoku district listed in Table 16 may have some connection with the Sanriku earthquake of 1933. Likewise, earthquake swarms occurred near Oshima after the Tonankai earthquake of 1944, near Kuju-zan, Kyushu, after the Nankaido earthquake of 1946, near Meakandake immidiately after the Tokachi earthquake of 1968, etc. The Teshikaga earthquakes and the Rausu swarm occurred about three months after the southern Kurile Islands earthquakes of 1958 and 1963 respectively. Hamaguchi et al.[267]. reported an activity in northern Iwate after the Tokachi earthquake of 1968.

Table 17 summarizes the above mentioned successive occurrences of

Table 17. Seismic activities associated with several great earthquakes in and near Japan.

Name of the main shock	Activity		
	Type*	Time (GMT)	Remarks
Sanriku, 1933	F_f	Jan. 7, 1933	$M=6.8$
	A_o	Mar. 2, 1933	$M_0=\mathbf{8.3}$
	Z_a	Mar., 1933	Iwagasaki swarm
	Z_a	Mar.-Apr., 1933	Miyakawa swarm
	Z_a	Aug.-Sept., 1933	Okunakyama swarm
	B_a	June 18, 1933	$M=7.1$ (Off Kinkazan)
Tonankai, 1944	A_o	Dec. 7, 1944	$M=\mathbf{8.0}$
	B_a	Jan. 12, 1945	$M=7.1$
	Z_a	Dec., 1944	Oshima swarm
Nankaido, 1946	A_o	Dec. 20, 1946	$M_0=\mathbf{8.1}$
	B_a	June 15, 1948	$M=7.0$
	Z_a	Dec., 1946	Kuju-zan swarm
	Y_a?	June 28, 1948	$M=7.2$ Fukui)
Tokachi, 1952	A_f?	Mar. 1, 1952	$M=6.0$
	A_o	Mar. 4, 1952	$M_0=\mathbf{8.1}$
	S_a	Mar.-Apr., 1952	$M=6.3$, 6.2,....Off Urakawa
	Z_o	Mar., 1952	Meakan rumblings
S. Kurile Is., 1958	A_o	Nov. 6, 1958	$M_0=\mathbf{8.2}$
	Y_a	Jan. 30, 1959	$M=6.2$ (Teshikaga)
S. Kurile Is., 1963	F_f or A'_f	Oct. 12, 1963	$M=6.3$
	A_o	Oct. 13, 1963	$M_0=\mathbf{8.1}$
	A'?	Oct. 20, 1963	$M=6.7$
	Z_a	Jan., 1964	Rausu swarm
Tokachi, 1968	A'_f or F_f	May, 1, 1968	$M=5.3$
	A_o	May 16, 1968	$M_0=\mathbf{7.9}$
	A'ä	May 16, 1968	$M=7.5$
	B_a	June 12, 1968	$M=7.2$
	Y_a(or Y_o)	May, 1968	Northern Iwate
	Z_o	May, 1968	Tokachi-dake

* Cf. Figure 125.

earthquake activities for seven great earthquakes in and near Japan.

As explained above, multiple sequences usually have larger aftershock areas than those estimated from equation (14). However a few sequences in Table 4 (Nos. 15, 57, and 62) have relatively small aftershock areas, though they are apparently multiple sequences. This may imply the repeated occurrence of large shocks of comparable size from the same source region.

12. Models for aftershock occurrence

The most remarkable property of the phenomenon of aftershocks is their frequency decay law represented by the Omori (or modified Omori) formula. It is substantially different from the negative exponential formula appearing in most decay laws in physics. It is natural that most theories of aftershocks attempted to explain the Omori formula. Most theories or models for aftershock occurrence have been developed from the following three different points of view.

(1) Energy release in the whole aftershock region.
(2) Probability of fracture in source regions of individual aftershocks.
(3) Rheological behavior of rocks in the source region.

This classification is of course tentative and other different views (e.g., the dislocation theory) are possible. "Reservoir type model" and "prepackaged model" named by Vere-Jones[268] may correspond to (1) and (2) respectively.

12.1 *Energy release in the whole aftershock region.*

The temporal variation of the potential energy E stored in the whole aftershock region which is to be released in aftershocks will be discussed here. The energy εdt released in the time interval between t and $t+dt$ is considered as a function of E. In the simplest case in which ε is proporitional to E,[33]

$$\varepsilon\, dt = -dE = \alpha E\, dt\,. \tag{65}$$

It follows that

$$E = E_0\, e^{-\alpha t} \tag{66}$$

and

$$\varepsilon = \alpha E_0\, e^{-\alpha t} \tag{67}$$

where E_0 is a constant (total energy released in aftershocks). In some studies, the relation between the number of shocks ndt and the energy release εdt during the time interval dt has been assumed to be proportional, i.e.,

$$n = k\,\varepsilon\,. \tag{68}$$

Under this assumption equation (67) leads to the result that the frequency of aftershocks also decreases exponentially. Sagisaka[33] who studied the energy release in aftershocks of the Tango earthquake of 1927 indicated that the frequency of aftershocks decayed more slowly than the energy, and the exponential formula did not fit the frequency vs time curve.

In the case that ε is proportional to E^2 (or $\varepsilon/E \propto E$),

$$\varepsilon\, dt = -a\, E = a\, E^2\, dt \tag{69}$$

then,

$$E = \frac{1}{a(t+c)} \tag{70}$$

and

$$\varepsilon = \frac{a}{(t+c)^2} \tag{71}$$

where c is a constant. In order to derive the Omori formula under this assumption, it is necessary to assume that

$$n = kE \tag{72}$$

i.e.,

$$n = k\sqrt{\varepsilon/a}\,. \tag{73}$$

In this case, we obtain

$$n = \frac{K}{t+c} \tag{74}$$

where

$$K = k/a\,. \tag{75}$$

Enya's theory of 1901[269] which was probably the first attempt to explain the Omori formula was constructed along this line, though he did not use the term "energy". Arakawa's theory[270] which employs a certain type of energy distribution law among aftershocks is essentially the same as the above derivation as far as the temporal variation of frequency is concerned. Equations (69) to (71) were described by Tomoda[271] in a discussion of Enya's original theory.

The modified Omori formula $n(t)=K/(t+c)^p$ can be obtained, if it is assumed that

$$n = k\, E^p \tag{76}$$

i.e.,

$$n = k\, a^{-p/2}\, \varepsilon^{p/2}\,. \tag{77}$$

The same formula can also be derived from a more general assumptions

$$\varepsilon = a\, E^q \qquad (q > 1) \tag{78}$$

and

$$n = k\, E^r\,, \tag{79}$$

i.e.,

$$n = k\,\alpha^{-r/q}\,\varepsilon^{r/q} \tag{80}$$

In this case the exponent p in the Omori formula is expressed by

$$p = r/(q-1)\ . \tag{81}$$

In connection with these assumptions, it has been verified[1] theoretically and empirically that the total energy ε for random samples of size n from a population of earthquakes whose magnitudes are distributed according to Gutenberg-Richter's formula (1) is approximately proportional to the (β/b)th power of n, where β is the coefficient in the magnitude-energy relationship

$$\log E = \alpha + \beta M\ . \tag{82}$$

Comparing equation (77) with the above mentioned relation $\varepsilon \propto n^{\beta/b}$, Utsu[1] obtained the relation

$$p = 2b/\beta\ . \tag{83}$$

This equation gives a reasonable p value on the average, since p varies from 1 to 2 as β/b varies from 2 to 1, and for standard values of β and b ($\beta=1.5$, $b=0.85$), $p=1.2$. However, the points showing the relation between p and b in Figure 70 are too much scattered to give either support or opposition to equation (83).

The application of the theory of Markov process has been put forward by Aki[272] and Vere-Jones.[273] These authors used the number of aftershocks N_t which occurred until time t or the energy of the system E_t at time t as the random variable governing the process. The state of the process is represented by the distribution function $P(N, t)$ or $P(E, t)$ which denotes the probability of $N_t=N$ or $E_t=E$ at time t respectively. To solve the problem one must know the functional form of $\lambda\,(N, t)$, or $\lambda\,(E, t)$ and $F\,(H, E)$, where $\lambda\,(N, t)dt$ and $\lambda\,(E, t)\,dt$ are the probabilities that an aftershock occur during the time interval between t and $t+dt$ when $N_t=N$ and $E_t=E$ respectively, and $F(H, E)$ is the probability that the value of E_t jumps from H to a certain value equal to or less than E by the occurrence of a shock at time t. Under the assumption that

$$\lambda(N, t) = h\,e^{-kN}\ , \tag{84}$$

or

$$\lambda(E, t) = k\,E^{\alpha} \tag{85}$$

and

$$F(H, E) = (E/H)^{\beta}\ , \tag{86}$$

Aki and Vere-Jones succeeded in approximate derivations of the Omori formula

using the basic equations for Markov process.

The derivations of Omori (or modified Omori) formula described in this section are based on some assumptions. Most of these assumptions do not seem unreasonable, but none of them have been varified theoretically or experimentally.

One difficulty, among others, of the theories based on the energy equation for the whole aftershock region described in this section is that no sudden decrease in aftershock activity is observed just after the occurrence of large aftershocks. A large aftershock releases a considerable portion of (in some cases more than a half of) the total energy of the aftershock sequence, therefore the energy of the aftershock region E drops considerably at the time of the large aftershock, thus the frequency of aftershocks n must drop considerably if n is directly related to E by equation (72) or (76) or (79). Although such effects of large aftershocks have rarely been observed,[212),213)] the fact that the dependence between aftershocks is not so strong as described in a previous chapter rather supports the fracture models described in the next section

12.2 *Probability of fracture in each elementary region*

In this section the whole aftershock region is considered to be made up of numerous elementary regions, each of which corresponds to the source region of a single aftershock. At the time of the main shock each elementary region come to have potentiality of producing one aftershock by fracturing owing to the redistribution of stress and the decrease in strength.

If the probability that an elementary region which has not yet fractured at time t will fracture in the time interval between t and $t+dt$ is denoted by $q(t)\,dt$, the probability that the fracture takes place at a time later than t is expressed by

$$p(t) = \int_t^\infty q(t)\,dt\,. \tag{87}$$

The function $q(t)$ is represented by

$$q(t) = n(t)/N_0 \tag{88}$$

where N_0 is the total number of aftershocks, i.e., $N_0 = \int_0^\infty n(t)dt$. It is also expressed by

$$q(t) = p(t)\,\mu(t) \tag{89}$$

where $\mu(t)$ is a function of t which may be called the fracture rate. It follows that

$$q(t) = -dp(t)/dt\,, \tag{90}$$

therefore

$$\mu(t)\,dt = dp(t)/p(t) = -d\ln p(t)\,. \tag{91}$$

If $\mu(t)$ is independent of t, i.e., $\mu(t)=\mu$ (constant),

$$p(t) = e^{-\mu t} \tag{92}$$

and

$$n(t) = N_0\mu\, e^{-\mu t}\,. \tag{93}$$

This is the same consideration as in the theory of radioactive disintegration. The above treatment is essentially the same as that given by Watanabe[274] in 1936 in a discussion on the time interval distribution between successive earthquakes. The present problem is closely connected with the reliability theory, in which $p(t)$ and $\mu(t)$ are called reliability function and hazard function respectively.

To derive the modified Omori formula from equation (91), it is necessary to ssume either a certain functional form of $\mu(t)$ common to all elementary regions or a certain distribution function of the number of elementary regions with respect to μ which is independent of time but varies from region to region.

For any $\mu(t)$

$$q(t) = \mu(t)\exp\Big(-\int_0^t \mu(t)\,dt\Big)\,, \tag{94}$$

and

$$p(t) = \exp\Big(-\int_0^t \mu(t)\,dt\Big)\,. \tag{95}$$

On the other hand, for sequences to which the modified Omori formula is applicable,

$$p(t) = \frac{1}{N_0}\int_t^\infty n(t)\,dt = \frac{1}{N_0}\int_t^\infty \frac{K}{(t+c)^p}\,dt = \frac{c^{p-1}}{(t+c)^{p-1}}\,. \tag{96}$$

Therefore

$$\mu(t) = -\frac{d\ln p(t)}{dt} = \frac{p-1}{t+c}\,. \tag{97}$$

This means that $\mu(t)$ is a hypabolically decreasing function of time.

According to experimental studies on fracture of some brittle materials, the time to fracture t after the application of a stress σ is a random variable having a negative exponential distribution

$$s(t) = \mu e^{-\mu t} , \tag{98}$$

and μ is related to the stress σ as

$$\mu = A e^{\beta\sigma} \tag{99}$$

where A and β are constants depending on the material, size and shape of the sample, and the type of the stress (see Mogi[230] and Scholz[275] for further references).

Mogi[45),230)] in 1962 considered that the departure of the frequency decay curve of aftershocks from an exponential one was due to a relaxation of the stress in the focal region. He suggested the time dependent stress in the form

$$\sigma(t) = \sigma_0 + \sigma_1 e^{-\gamma t} . \tag{100}$$

On the other hand, combining equations (97) and (99), we obtain

$$\sigma(t) = \frac{1}{\beta} \ln \frac{p-1}{A(t+c)} . \tag{101}$$

This is a strange result, since $|\sigma(t)|$ increases infinitely as t tends to infinity. Of course the relation (99) is valid in a limited range of stress, and it is not reasonable to equate (97) to (99) outside of this range.

Another derivation of the modified Omori formula introduced by Utsu[57)] in 1962 is based on the distribution of μ values. Here each elementary region has a time invariant fracture rate μ, but the fracture rate varies from region to region. If the number of elementary region having fracture rate between μ and $\mu+d\mu$ is denoted by $N_0 f(\mu)d\mu$, and if it is assumed that each elementary region fractures only once, we obtain

$$n(t) = N_0 \int_0^\infty \mu e^{-\mu t} f(\mu)\, d\mu = \frac{N_0 (p-1)\, c^{p-1}}{(t+c)^p} . \tag{103}$$

It easy to show that

$$f(\mu) = \frac{(p-1)\, c^{p-1} e^{-c\mu}}{\Gamma(p)\, \mu^{2-p}} \tag{104}$$

satisfies the above equation. $f(\mu)$ is a montonically decreasing function of μ.

According to this idea, the distributoin of fracture rates for unfractured regions at time t is expressed by

$$f_t(\mu) = f(\mu)\, e^{-\mu t} . \tag{105}$$

Therefore the average fracture rate for all elementary regions not fractured until time t is given by

$$\bar{\mu}(t) = \int_0^\infty \mu f(\mu)\, e^{-\mu t}\, d\mu \Big/ \int_0^\infty f(\mu)\, e^{-\mu t}\, d\mu . \tag{106}$$

Using equation (103) this becomes

$$\bar{\mu}(t) = \frac{p-1}{t+c} . \tag{107}$$

This is the same one as equation (97). Thus the two approaches are closely related. If A and β in equation (99) are constants common to all elementary regions, the average stress responsible for fracturing decrease with time as $\mu(t)$ decreases. If so, the stress drop associated with aftershocks tends to decrease with time. However, no observational evidence to indicate this effect has been reported. A and β in equation (99) which are related to the strength may vary from region to region, therefore the average stress does not simply decrease with time.

Scholz's derivation[276] of the hypabolic relationship

$$n(t) = K/t \tag{108}$$

can be regarded as a special case ($c=0$, $p=1$) of the above discussion, although he used different terms and notations. He adopted an equation similar to (99) for the relation between μ and σ, and also assumed that the distribution of σ is uniform in a certain range of stress. This assumption is equivalent to

$$f(\mu) = K/\mu \tag{109}$$

for a range of $\mu_1 \leq \mu \leq \mu_2$ (otherwise $f(\mu)=0$), since $d\mu \propto \mu d\sigma$. Substituting $f(\mu)$ in equation (103) by the above, we arrive at equation (108) if $\mu_1 \to 0$ and $\mu_2 \to \infty$ (otherwise equation (108) hold for a limited range of time). It seems to the author that no definite basis for supporting the conditions $c=0$ and $p=1$.

In this section we have seen that the modified Omori formula is derived from a distribution of fracture rate μ either in time or in space. The fracture rate μ is primarily dependent on the stress and the strength. It is likely that there are temporal and/or spatial variations of stress and/or strength relevant

to the distribution of μ, but it has not been confirmed directly by theory or observation.

12.3 *Rheological behavior of rocks in the source region*

The first attempt to interpret the Omori formula in terms of creep characteristics of rocks as observed in laboratory experiments was published in 1904 by Kusakabe.[277] Considering the aftershock frequency to be proportional to the rate of strain recovery in rock $d\xi/dt$, he derived the Omori formula as an approximation.

If the activity of shocks is proportional to the inelastic strain rate as observed in some laboratory experiments,[278)-281)] the following empirical creep formulas are relevant to the Omori (or modified Omori) formula.

$$\xi = \alpha + \beta \log t \,, \tag{110}$$

$$\xi = \alpha + \beta \log (1+\gamma t) \,, \tag{111}$$

and

$$\xi = \alpha\{(1+\gamma t)^{\beta} - 1\} \,. \tag{112}$$

References to the above and other formulas for creep of rocks and other materials are found in Benioff's[282),283)] and Iida's[284)] articles.

Benioff[282),285)] in 1951 published a creep mechanism of aftershocks on the basis of the similarity between so-called strain release curves for aftershock sequences and the emprical creep curves. In this theory, strain release x in an aftershock is connected with the energy of the seismic waves E by the equation

$$E = \frac{1}{2} \eta \mu V x^2 \tag{113}$$

where μ is the shear modulus, V is the source volume, and η is the efficiency factor (seismic wave energy divided by strain energy). Aftershocks have been considered to be associated with the same fault that moved during the main shock, and the source volume is common to all aftershocks and the main shock. The mechanical model proposed by him is shown in Figure 126 (upper diagram). Thus under the assumption of independence of η on E, the cumulative strain or displacement on the fault is proportional to the sum of the square roots of individual energy E which is calculated by an equation of the type (82). Benioff observed the two kinds of strain release curves in several aftershock sequences which is expressed by

$$s = A + B \log t \tag{114}$$

and

$$s = A + B(1 - e^{-c\sqrt{t - t_1}}) \,. \qquad (t_1 > 0) \tag{115}$$

He identified these curves with those for the creep recovery as observed in laboratory experiments.

After Benioff's work, such strain release curves have been constructed for various aftershock sequences and other earthquake sequences.[10),21),64),65),122),125)–127),128),135),179),216),278),285)–297)] Strain release maps have also been published by many investigators. Popov[298)] discussed the generation of heat in Benioff's model. However, this idea of accumulating square roots of energy is subject to criticism[1)], since $\sum \sqrt{E}$ depends critically on the lower limit of magnitude chosen for summation, if we admit the magnitude-frequency relation for aftershocks represented by Gutenberg-Richter's formula (1) and the energy-magnitude relation (80) in which $\beta > b$. The form of a strain release curve is largely influenced by the lower limit of magnitude, and generally speaking, a curve constructed by using a relatively large number of shocks is smoother and closer to the curve of cumultative frequency than that constructed from a small number of larger shocks. This nature of strain release curves is evidently recognized by comparing many strain release curves for aftershock sequences and other earthquake sequences published hitherto.

Pshennikov[299)] assumed that the stress at time t is calculated by

$$\sigma(t) = (\sum \sqrt{E})_\infty - (\sum \sqrt{E})_t \tag{116}$$

where $(\sum \sqrt{E})_\infty$ and $(\sum \sqrt{E})_t$ are the sums of energy E for all aftershocks and for aftershocks which occurred until time t respectively. He found the empirical relation of the following form (see also Purcaru[21)] and Papazachos et al.[9)]).

$$\sigma(t) = \sigma_0 \exp\left(-\frac{t}{a + bt}\right). \tag{117}$$

Noticing the difficulties in summing up $\sqrt{E}$ for the strain release in an aftershock region, Båth and Duda[162)] considered that the sum of the deformation D for individual aftershocks represents that deformation characteristics of the whole aftershock region. The deformation is defined by

$$D = x\, V \,. \tag{118}$$

Since the strain x is almost independent of the energy E, and the energy is nearly proportional to the value V, the deformation D is nearly proportional

to E, therefore the deformation charactierstics of an aftershock sequence is almost the same as the energy release characteristics. No difficulty due to the divergence for small magnitude events arises. Some investigators fitted the curves representing equations (114) and (115) to the deformation curves of several aftershock sequences[9),66),162),300)] However, considering the scatter of the data, it is not certain that equation (115) fits better than equation (112).

Rheological behavior of earth's materials has been discussed by many geophysicists, and it is an established fact that the earth has rheological properties. Since gradual crustal movements with decreasing speed with time have been observed after some large earthquakes (e.g., Tango,[301)] Tottori,[302)] Nankaido,[303)] Niigata,[304)] and Parkfield,[305)]), it seems certain that the deformation similar to the creep of rocks observed in laboratory occurs in the source region. However, it is not a self-evident matter that the occurrence of an earthquake sequence is directly related to the rheological behavior of the rocks in the source region. Some theory or observational evidence is necessary to connect the occurrence of fractures with the inelastic deformation of rocks. Scholz[274)] derived a creep law in a similar way to his derivation of the aftershock frequency law on the basis that the creep in brittle rock is due to the delayed microfracturing (see also Scholz[306)]).

Nagumo[307)] assumed that the number of small shocks in a visco-elastic medium under initial stress is proportional to the density of dislocations, which is again proportional to the curvature of the plastic deformation. Then the frequency of shocks is proportional to the speed of plastic deformation $dv(t)/dt$. He obtained the relation between the deformation $v(t)$ and the internal force $q(t)$, and in the case that

$$q(t) = q_0(a+1)\frac{\sqrt{t}}{t+a}, \tag{119}$$

$dv(t)/dt$ is given by

$$\frac{dv(t)}{dt} \fallingdotseq c\left[\left(\frac{a}{t+a}\right)^{3/2} + c'\left(1-\sqrt{\frac{a}{t+a}}\right)\right]. \tag{120}$$

For small a (e.g., $a=0.1$) this equation corresponds approximately to the modified Omori formula with $p=1.5$.

12.4 *Discussions*

It is now widely accepted that most earthquakes, including aftershocks,

are caused by the fracturing of rock within the earth. Remarkable similaities in many aspects have been discovered between earthquakes and fractures in laboratory experiments and rockbursts in mines. The aftershock phenomenon has been observed in experiments on fracture of brittle materials.[188),308),309)] Keeping these things in mind, the occurrence of afterhocks can be interpreted as follows. When a large earthquake associated with a rupture along a large fault F_1 occurs, elastic strain energy stored in the source region G_1 is converted into seismic wave energy, frictional heat at the main fault F_1, formation of new faults around the main fault, etc. Although the average stress in the whole source region drops considerably, stresses at many particular regions g_{11}, g_{12}, ... may increase instantaneously owing to structural irregularities such as pre-

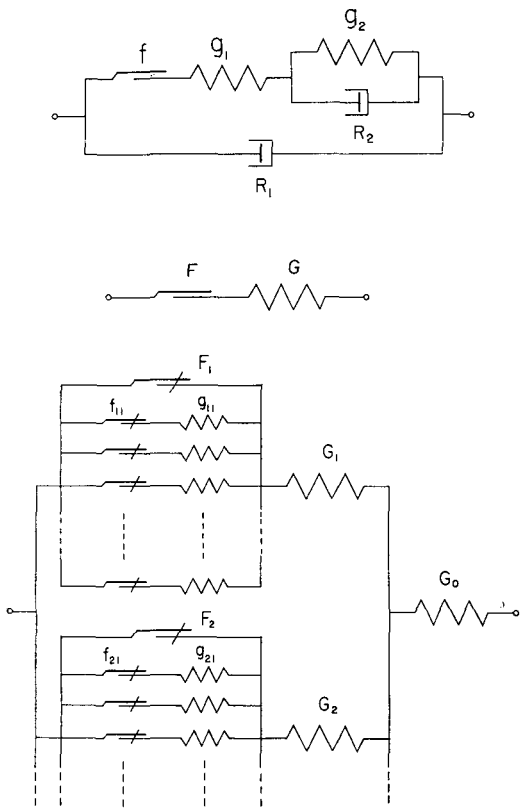


Fig. 126. Mechanical models for earthquake sequences. Upper: Benioff's model of aftershocks, lower: an aftershock model in which every slider has delay characteristics.

existing and newly formed faults. Such redistributed stresses cause ruptures at faults f_{11}, f_{12},.... These ruptures must have the similar character to the delayed fracture (sometimes called the static fatigue) observed in laboratory experiments. The rate of the occurrence of fault rupture, i.e. the occurrence of aftershocks, depends on the stress and the strength (or friction) on each fault. If the stress relaxes with time as Mogi[45] suggested (changes of the strength with time may also be possible), or the fracture rate varies regionally due to the variation of the stress (variations of the strength may also be possible) as suggested by Utsu[57] and Scholz,[275] it is possible that the frequency of aftershocks decreases according to the modified Omori formula as described in Section 12.2.

The above mechanism of aftershocks may be represented by a model in Figure 126 (lower diagram), though it is too simple to represent all the features of aftershock sequences. The spatial coupling between numerous secondary faults f_{11}, f_{12},.... may be much more complicated. The diagram represents the case that all aftershocks are mutually independent. The sliders in this model are assumed to have time-dependent characteristics which represent the delayed fracture, and the system is subject to such a compressional stress that the two terminals draw near at a constant speed.

The model consists of a spring G and a slider F (Figure 126, center) is very simple, but it represents some important features of the earthquake occurrence such as the intermittent occurrence of great earthquakes as observed on the Pacific coast side of Japan. Of course this simple model can not generate aftershocks. Burridge and Knopoff[310] was able to generate aftershock sequences using a somewhat elaborated model of a different type.

The occurrence of multiple sequences can be represented by a parallel connection of two or more systems G_1-F_1, G_2-F_2,.... as shown in Figure 126. The slip of F_1 results in a decrease of stress in G_0, and G_1 and an increase of stress in G_2, G_3,...., then the subsequent slip of F_i (i=2, 3,....) will occur if the stress in G_i before the stress redistribution was already near the strength of the fault F_i. Such a parallel connection of the system consisting of a spring and a slider is the same as Kasahara's model[311], if all the secondary elements for aftershocks are omitted.

The phenomenon of aftershocks can be interpreted by the delayed fracture of rocks in the source region. However its physical mechanism is far from being completely understood. The inelastic deformation without accompanying fracture may also affect the stress condition in the source

region. The mechanisms of fracture and inelastic deformation of heterogeneous rock under high pressure and temperature must be studied more intensively.

(to be continued)

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