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# The Growth of Graupel

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**Abstract**

Based on the observed results for embryos and formation mechanisms of graupel, the growth process from snow crystal embryos and frozen drop embryos to graupel particles was studied by numerical calculations. The main results are as follows. (1) The growth rate ($dD/dt$) of graupel particles is faster as the temperature decreases and they grow to a diameter of about 8 mm within 30 minutes. (2) As the occasion demands, it has been possible to obtain the bulk density of graupel particles with each size for various kinds of meteorological conditions from the calculated results. (3) The bulk densities of graupel particles approach the bulk densities of hailstones at temperatures over approximately −10°C under the condition of 1 g/m$^3$ in liquid water content.

It was shown that the bulk densities of graupel particles calculated in this paper were in good agreement with those of natural graupel particles and wind-tunnel-grown graupel particles.

## 1. Introduction

In previous papers (Harimaya, 1976, 1977), the embryo and formation mechanism of graupel were studied. As a result, it was concluded that both snow crystals and frozen drops can become graupel embryos. On the other hand, the formation mechanisms of three types of graupel were explained by the combination of a falling behavior and embryo. Based on these results, it is possible to numerically study the growth process of graupel.

The bulk density of graupel is necessary in order to calculate the falling velocity and growth rate of graupel. On the other hand, hailstone embryos are graupel in most cases (List, 1960; Knight and Knight; 1970). But the bulk density of graupel is smaller than 0.4 g/cm$^3$ and that of hailstones is about 0.9 g/cm$^3$ (e.g. Harimaya, 1974). Accordingly the bulk density of graupel must become larger for graupel to grow into hailstones. Therefore, it is important for the discussion of hailstone generation to study the meteorological condition under which graupel particles grow into hailstones. In this paper, the growth of graupel is studied with the focus of attention on the bulk density of graupel.

## 2. Equations and methods of numerical calculations

It is considered that the embryo of a graupel particle falls through a supercooled cloud and grows simultaneously by sublimation process and accretion process. If the graupel particle is larger than a certain critical value, the accretion growth rate is faster and the rise of surface temperature of the graupel particle by release of latent heat of freezing is higher. As a result, the growth rate by the sublimation process is negative, that is to say, the graupel particle evaporates partially, although the total mass of the graupel particle continues to increase by the accretion process.

In this calculation, the cloud is as follows. The cloud was assumed to be always saturated with respect to liquid water. The meteorological conditions in the cloud were assumed to be constant with time.

In the cloud, the growth rate of a graupel particle is given by

$$\frac{dM}{dt} = \frac{dM_1}{dt} + \frac{dM_2}{dt}, \tag{1}$$

where $M$, $M_1$ and $M_2$ are the total mass of the graupel particle, the masses which the graupel particle gains through sublimation and accretion of cloud droplets, respectively. $M_1$ and $M_2$ are given as follows, respectively.

$$\frac{dM_1}{dt} = 4\pi CDF(\rho - \rho_s), \tag{2}$$

$$\frac{dM_2}{dt} = \pi a^2 EL(V - v), \tag{3}$$

where,

$C$ : electrostatic capacity of a graupel particle,
$D$: coefficient of diffusion of water vapor in air,
$F$ : ventilation factor of a graupel particle during fall,
$\rho$ : ambient vapor density,
$\rho_s$: saturation vapor density over ice at the surface of a graupel particle,
$a$ : major semi-axis of a graupel particle,
$E$ : collection efficiency of cloud droplets by a graupel particle,
$L$ : liquid water content of cloud droplets,
$V$: terminal falling velocity of a graupel particle,
$v$ : terminal falling velocity of accreted cloud droplets.

During the growth of the graupel particle, the release of latent heat by sublimation and freezing of the accreted droplets causes the surface temperature

of the graupel particle to rise above that of its environment. When the latent heat is dispersed by conduction and forced convection from such a ventilated graupel particle, we have

$$L_s \frac{dM_1}{dt} + \{L_c + c_w(T-T_m) + c_i(T_m-T_s)\} \frac{dM_2}{dt} = 4\pi CKF(T_s-T), \quad (4)$$

where,

$L_s$ : latent heat of sublimation,
$L_c$ : latent heat of freezing,
$c_w$ : specific heat of water,
$c_i$ : specific heat of ice,
$T$ : ambient temperature,
$T_m$: melting temperature of ice,
$T_s$ : surface temperature of a graupel particle,
$K$ : thermal conductivity of air.

From equations (2) and (4), we have

$$\frac{dM_1}{dt} = \frac{4\pi CF \cdot \dfrac{P-P_e(T)}{P_e(T)} - \dfrac{JL_sL_c + JL_s(c_w-c_i)(T-T_m)}{R_wT^2\left(K + \dfrac{c_i}{4\pi CF} \cdot \dfrac{dM_2}{dt}\right)} \cdot \dfrac{dM_2}{dt}}{\dfrac{JL_s^2}{R_wT^2\left(K + \dfrac{c_i}{4\pi CF} \cdot \dfrac{dM_2}{dt}\right)} + \dfrac{R_wT}{DP_e(T)}}, \quad (5)$$

where,

$P$ : ambient vapor pressure,
$P_e(T)$: saturation vapor pressure over ice at the ambient temperature $T$,
$J$ : mechanical equivalent of heat,
$R_w$ : gas constant of water vapor.

Numerical calculations were performed by using equations (3) and (5) as follows. In order to express graupel embryos and graupel particles as a single shape, the shape of graupel particles was treated as oblate spheroids. The falling velocity of graupel particles was calculated by McDonald's method (1960). In the calculation the author used the values which took into consideration the axial ratios of spheres obtained by Wieselsberger (1922) and circular disks obtained by Schmiedel (1928) and Wieselsberger (1922) as the drag coefficients of oblate spheroids. Further, as the collection efficiencies of oblate spheroids, the values which took into consideration the axial ratios of

spheres obtained by Langmuir (1948) and circular disks obtained by Ranz and Wong (1952) were used. As the ventilation factor, the empirical results obtained by Kinzer and Gunn (1951) were adopted.

The distribution of growth rates along a-axis and c-axis of a graupel particle was assumed as shown in Fig. 1, where $\Delta a_1$ and $\Delta a_2$ are the growth rate along a-axis by sublimation and accretion processes, respectively, and $\Delta c_1$ and $\Delta c_2$ are the growth rate along c-axis by sublimation and accretion processes, respectively. The author considered that the growth rates were as follows. When the growth rate of the sublimation process is positive ($\Delta M_1 \geqq 0$) and exceeds that of the accretion process ($\Delta M_1 \geqq \Delta M_2$), the growth of the sublimation process obeys the empirical results of axial ratio of snow crystals by Ono (1969) and the growth of the accretion process is such that the growth along a-axis

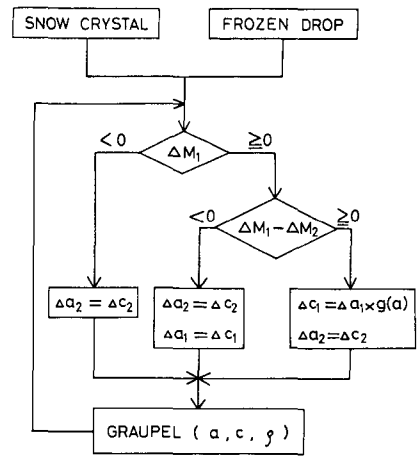


Fig. 1 Distribution of growth rate along a-axis and c-axis of a graupel particle. $\Delta a$ and $\Delta c$ show the growth rate along a-axis and c-axis, respectively. Suffixes 1 and 2 show sublimation and accretion processes, respectively. $g(a)$ shows the empirical result of the axial ratio of a snow crystal by Ono (1969).

is equivalent to that along c-axis. When the growth rate of the sublimation process is smaller than that of the accretion process ($\Delta M_1 < \Delta M_2$), the growth rates of the sublimation and accretion processes are such that the growth along a-axis is equivalent to that along c-axis in the respective process. On the other hand, when the growth rate of the sublimation process is negative ($\Delta M_1 < 0$), the graupel particle grows as the same extent as that of the difference between the growth of the accretion process and evaporation in such a way

that the growth along a-axis is equivalent to that along c-axis.

The density of the portion of growth obtained during each calculation step was estimated as follows. In the case of the sublimation process, the author used the relation between the diameter and density which was obtained from the relation between the diameter and mass obtained by Nakaya and Terada (1935) and the relation between the diameter and thickness obtained by Ono (1969). In the calculation of growth, however the density was used only in the initial stage of growth, because the sublimation process acts as evaporation in the region of larger graupel particles. In the case of the accretion process, the author used the empirical results of riming in the laboratory experiments by Macklin (1962), where the density of rime is determined by the surface temperature of rime, the impact velocity and the size of cloud droplets. Then, the surface temperature of graupel particles was obtained from equation (4) and the impact velocity of cloud droplets was obtained from the falling velocity difference between graupel particles and cloud droplets by using McDonald's method (1960). The bulk densities of graupel particles which are the final results are given from the mass and volume of a graupel particle in each calculation step. One calculation step was 10 cm for the fall distance in the initial growth and 1 m for other times.

## 3. Results of numerical calculations

Fig. 2 shows the growth of a graupel particle for each temperature, when the graupel particle grows at a liquid water content of 1 g/m$^3$ and at the size distribution ($X$) of actually observed cloud droplets (Sasyo et al., 1967). Solid and broken curves indicate the growth curves from a snow crystal with a diameter of 1.5 mm and a frozen drop with a diameter of 140 $\mu$m to a large graupel particle, respectively. Temperatures in clouds are shown at the right end of each curve and the elapsed times in growth are shown along each curve.

It is seen that the growth rate ($dD/dt$) is larger as the temperature decreases in both cases of snow crystal embryos and frozen drop embryos. Further, the graupel particles grow to a diameter of about 8 mm within 30 minutes under each temperature, except for the case of frozen drop embryo at temperature of −5°C.

In case of frozen drop embryos, the growth of the graupel particle is at first accompanied by a very rapid decrease in bulk density and then, the bulk density of the graupel particle becomes larger as its size grows larger.

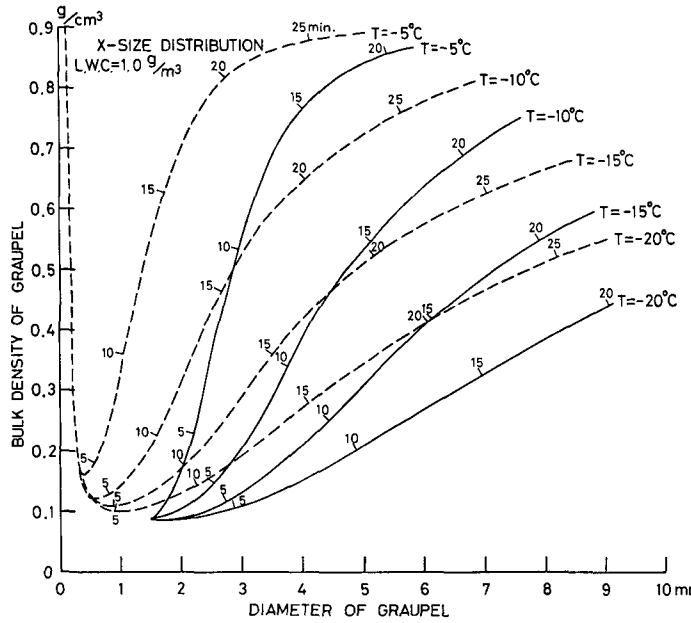


Fig. 2 Growth curve of a graupel particle growing at a liquid water content of 1 g/m³ and at the size distribution ($X$) of actually observed cloud droplets (Sasyo et al., 1967) for each temperature. Solid and broken curves indicate cases of snow crystal embryos and frozen drop embryos, respectively.

In cases of both snow crystal embryos and frozen drop embryos, it is seen that the bulk densities of graupel particles approach the bulk densities of hailstones at temperature above about −10°C.

Next, in order to examine the effect of liquid water content, numerical calculations were performed under three kinds of liquid water contents. The results are shown in Fig. 3. Meteorological conditions are the same as in Fig. 2 except for liquid water content. Liquid water contents in clouds are shown at the right end of each curve.

It is seen that the pattern of growth curves is similar to Fig. 2. The growth rate ($dD/dt$) grows larger as the liquid water content increases in both cases of snow crystal embryos and frozen drop embryos. The bulk densities of graupel particles approach the bulk densities of hailstones at liquid water content of more than 1 g/m³ under the condition of −10°C in temperature.

Lastly, in order to examine the contribution of each cloud droplet size to the bulk density of a graupel particle, the growth of graupel particles was

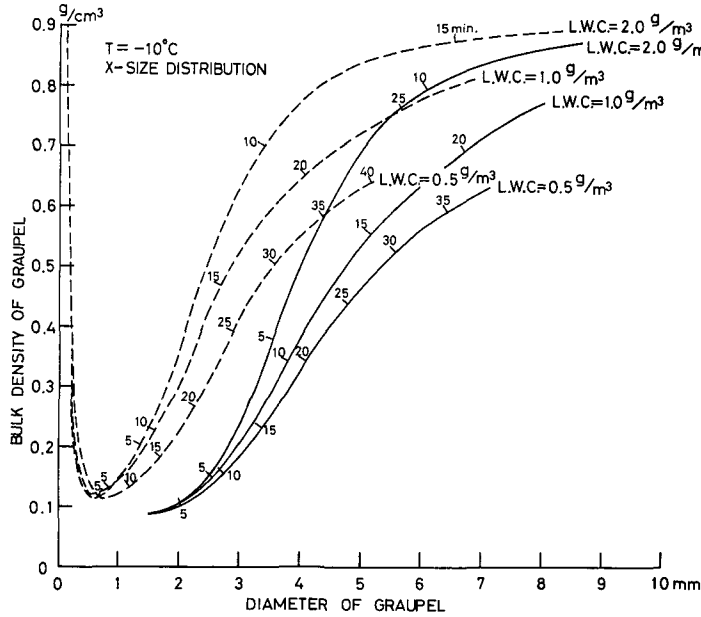


Fig. 3 As in Fig. 2 except for liquid water content.

calculated under conditions of the cloud droplets with uniform sizes of 10, 15 and 20 $\mu$m in radius. The results are shown in Fig. 4. Meteorological conditions are the same as in Fig. 2 except for each size of cloud droplets. Sizes of cloud droplets are shown at the right end of each curve. And $X$ shows the results under a given size distribution of cloud droplets.

As a result, a clear difference in the growth rate ($dD/dt$) is not found under conditions where cloud droplets are of different sizes. It is seen that in the case of frozen drop embryos the bulk densities of graupel particles approach the bulk densities of hailstones under cloud droplets with radii over approximately 10 $\mu$m. The size distribution ($X$) of cloud droplets used in the previous calculation corresponds to cloud droplets with the uniform size of 15 $\mu$m. On the other hand, in the case of snow crystal embryos, the bulk densities of graupel particles approach the bulk densities of hailstones under cloud droplets of radii above about 15 $\mu$m and snow crystals can not grow to graupel particles under cloud droplets of radii below approximately 10 $\mu$m.

As the occasion demands, it is possible to obtain the bulk density of the

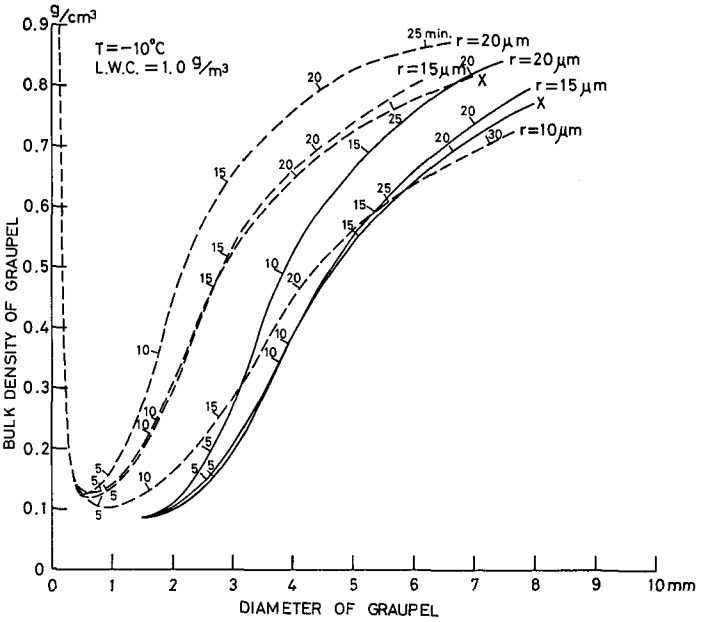


Fig. 4 As in Fig. 2 except for size of cloud droplets.

graupel particle with each size for each temperature, liquid water content and size of cloud droplets from Figs. 2, 3 and 4.

## 4. Considerations

In a previous section, the growth of graupel particles was examined by numerical calculations under various kinds of meteorological conditions. Next, these results must be verified. In the same manner as in the previous section, the bulk densities of graupel particles were calculated under the same meteorological condition as that when the bulk densities were observed and further both values of bulk densities obtained by calculation and observation were compared.

The bulk densities of graupel particles were measured by the same method used by Kajikawa (1976), where their volumes were obtained from the photographs taken by a close-up camera and their masses were obtained by the filter paper method, after which the bulk densities were calculated as the quotients of the masses to the volumes. The results are shown in Fig. 5. Since numerical calculations of the bulk densities were performed only under a

condition of no-melting, the observed values under the condition of no-melting alone are plotted in Fig. 5. It is seen that each value is included in a certain region, although it is scattered. The bulk densities become larger as the size of graupel particles grow larger except for the results of Nakaya and Terada (1935). The tendency is in good agreement with that of the numerical calculation.

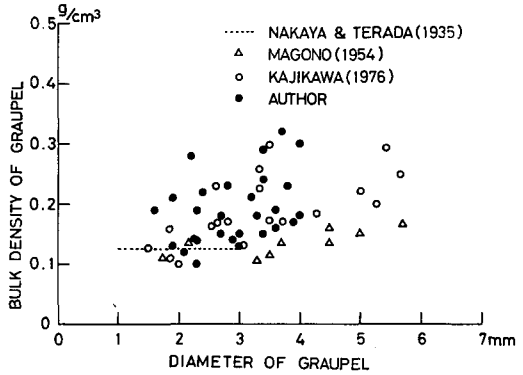


Fig. 5 Observed bulk densities of graupel particles.

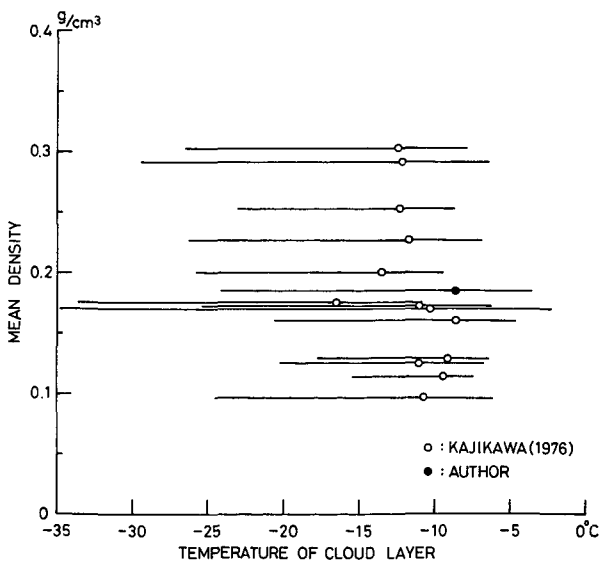


Fig. 6 Temperature ranges of cloud layer when the bulk densities of graupel particles were observed. A solid circle and open circles show the mean temperatures of growth layers.

Next, the observed bulk densities were compared with the values calculated under the same meteorological conditions. Fig. 6 shows the temperature ranges of cloud layers when the bulk densities of graupel particles were observed. Solid lines with a solid circle and open circles show the author's result and Kajikawa's results (1976), respectively. Since the positions where the graupel particles grew could not be determined exactly, it was assumed that the growth layers of graupel particles were in the lower halves of cloud layers. Mean temperatures of the growth layers are shown by a solid circle and open circles. Their values are between –8.5°C and –16.5°C and are adopted in the following numerical calculation. Since the liquid water content in the winter clouds over Hokkaido is below 1 g/m$^3$, the bulk densities were calculated under both meteorological conditions with the temperature of –8.5°C and liquid water content of 1 g/m$^3$, and with the temperature of –16.5°C and liquid water content of 0.5 g/m$^3$ so that the maximum and the minimum values are obtained in the numerical calculations. The same values as used in a previous section were adopted as the size distribution of cloud droplets and initial diameters of embryos. The calculated results are shown with the

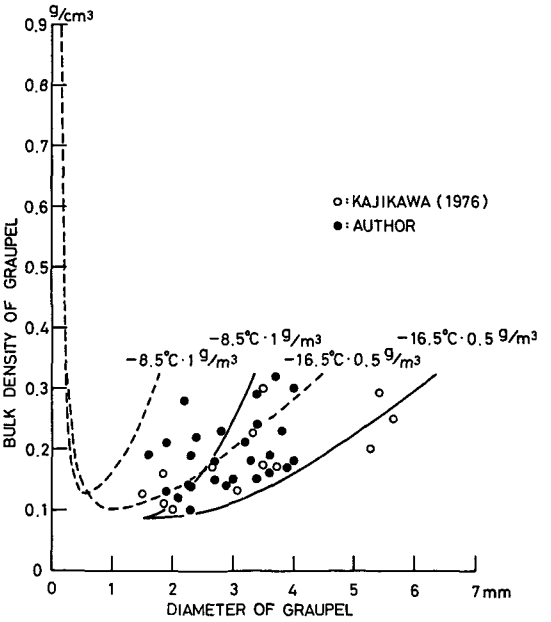


Fig. 7 Observed and calculated bulk densities of graupel particles.

observed results in Fig. 7. Solid and broken lines show the cases of the snow crystal embryos and frozen drop embryos, respectively. The calculated maximum and minimum values are in good agreement with the observed maximum and minimum values, respectively.

In the case of natural graupel particles, it is difficult to determine exactly the meteorological conditions along their trajectories in clouds. So the calculated results can not be compared with the observed results in a strict sense. On the other hand, in laboratory experiments it is possible to grow graupel particles under constant meteorological conditions controlled completely. Pflaum et al. (1978) succeeded in the growth from frozen drop embryos to graupel particles using a vertical wind tunnel where temperature and liquid water content can be accurately controlled. In their experiments, frozen drops with the initial diameter of 470 $\mu$m grew to graupel particles with the mean bulk density of 0.21 g/cm$^3$ under the meteorological conditions with a temperature of −15°C, liquid water content of 2 g/m$^3$ and cloud droplet radius of 7 $\mu$m. Fig. 8 shows the experimental result (+mark) and the result (broken line) calculated under the same meteorological condition as in their

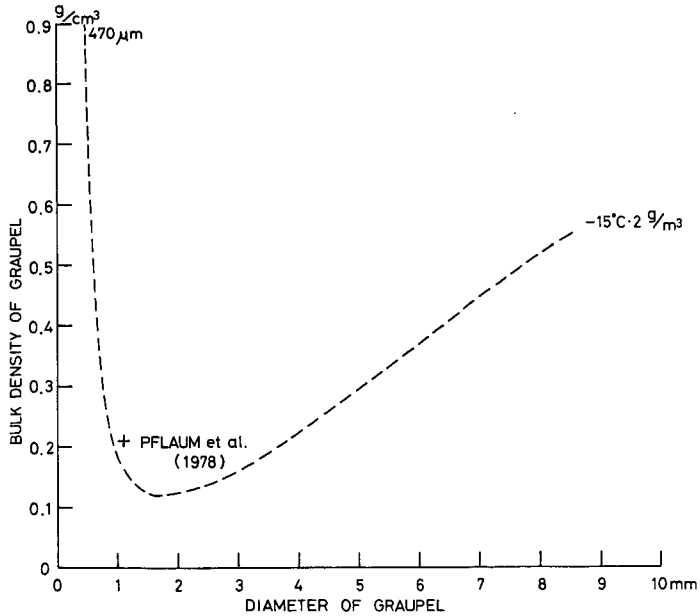


Fig. 8 Experimental (+mark) and calculated (broken line) bulk densities of graupel particles under the same meteorological condition.

experiment. It is seen that the calculated value is in good agreement with the experimental value.

## 5. Conclusions

Based on the observed results for embryos and formation mechanisms of graupel, the growth process from snow crystal embryos and frozen drop embryos to graupel particles was studied by numerical calculations. As a result, it is seen that the growth rate ($dD/dt$) of graupel particles is faster as the temperature decreases and liquid water content increases. Under the meteorological conditions generally observed in clouds over Hokkaido, graupel particles grow to a diameter of about 8 mm within 30 minutes.

Since the bulk density of graupel particles is necessary in order to calculate the falling velocity and growth rate of graupel particles, the values for each size of graupel particles was calculated under various meteorological conditions. As the occasion demands, it has been possible to obtain the bulk density of graupel particles with each size for various meteorological conditions from the calculated results. The bulk density of graupel particles growing from snow crystal embryos becomes larger as their sizes grow larger, but in the case of frozen drop embryos the bulk density first is accompanied by a very rapid decrease and then, becomes larger as the size grows larger. Therefore, both snow crystal embryos and frozen drop embryos can grow to graupel particles of 1–2 mm diameter which have a small bulk density of 0.1–0.2 g/cm$^3$ in agreement with observed results.

It is seen that the bulk densities of graupel particles approach the bulk densities of hailstones at temperatures over approximately −10°C under the condition of 1 g/m$^3$ in liquid water content. Therefore graupel particles can become the embryos of hailstones.

The results calculated in this paper were verified as follows. At first the bulk density of graupel particles was observed. Then, the values were compared with the values calculated under the same meteorological conditions as under observation. It is seen in the comparison that the calculated maximum and minimum values are in good agreement with the observed maximum and minimum values, respectively. Next, in order to verify the calculated results in detail, the calculated values were compared with the results of laboratory experiments under constant meteorological conditions controlled completely. It is seen in the comparison that the calculated values are in good agreement with the experimental values. Therefore it

is considered that the values calculated in this paper are reasonable.

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