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# Distributed Estimation Based on Weighted Data Aggregation over Delayed Sensor Networks

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## Abstract

In this paper, data aggregation laws and distributed observers over delayed sensor networks with any topology are proposed. In the proposed method, each node compensates communication delays of received data. For the delay compensation, each node predicts the future output based on state space models. To stabilize the aggregation data in any networks, the received data are multiplied by weight coefficients before the aggregation. The stability condition of the weighted aggregation laws is expressed by a weighted adjacency matrix. The aggregated value of the measurements at each node is expressed by a linear time-varying function of the current state. To estimate the state, we utilize the Kalman filters as the distributed observers. The effectiveness of the proposed method is confirmed by a numerical simulation.

*Key words:* Distributed Estimation; Sensor Network.

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## 1 Introduction

Internet of Things (IoT) is a concept which means that any things, e.g. machines, sensors, actuators and storage, are connected by the internet. In the context of control systems, we can regard networked control systems and cyber physical systems as the IoT systems. Wireless sensor networks, which are systems where sensors are connected by a wireless network, are considered as IoT-based observation systems. Compared with the observation via legacy field buses, the sensor network handles a lot of measurements from many sensors. Therefore, the sensor network has the aspect of large observation systems. In addition, we can build a wireless network more easily than wired networks. The establishment of frameworks by which one can effectively utilize the sensor network, contributes to the realization of smart cities [4].

For the wireless sensor network, we wish to obtain a large amount of information with no-delay and reliable communications. However, the actual nodes are often built by small computers which have insufficient computational capabilities to handle a lot of data gathered from

the large-scale network. In addition, we can not ignore the communication delays on the wireless networks unlike in the cases of wired networks. Furthermore, there are some incidents which prohibit the real-time communication on the wireless networks, for example packet drops or failures. Therefore, we should consider these technical issues to realize the sensor networks.

If the communication satisfies the scalability, amounts of communications in each node is independent of the number of nodes. Scalable algorithms are required for the networked information system with a large-scale network and small computers. In the past studies, some scalable laws, which communicate the measurement in the sensor networks, were proposed, for example, consensus filter methods [8,7,10,12,9], gossip algorithm [3,6] and data aggregation [11]. However, these works do not provide real-time synchronization of the measurements over delayed sensor networks.

For the real-time synchronization over delayed sensor networks, the authors have proposed data aggregation laws with delay compensations [2,1]. In these method, each node compensates communication delays of the received data by the prediction based on the delay compensated observer proposed in [14]. In [2], all nodes can estimate the state values by the distributed observers designed by the finite pole assignment technique, where

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the infinite poles caused by delays are hidden. However, to satisfy the stability condition, in the previously proposed data aggregation laws, the graph topology is limited to tree networks. In the tree network, there is no redundant path which connects a node to others. Therefore, the tree sensor networks do not provide reliable communications against failures.

In this paper, to extend the result of [2] to any graph topology, a weighted data aggregation is proposed. The data aggregation over any graph may diverge regardless of the measurement values because old messages continue remaining on a cyclic part of the network. Therefore, in the proposed method, each node multiplies some weight coefficients and the received data with delay compensation before the aggregation. The weights reduce the effects of the old measurements propagating through the networks, and thus prevent the divergence of the aggregated values. The stability condition of the proposed communication law is expressed by the weighted adjacency matrix. We also propose the distributed observer for the estimation from the aggregated measurements.

The conference paper [2] is a preliminary version of this paper. In this paper, we provide further detailed explanations about the design of weights and practical effectiveness. In [2], a design method for weights is future work. Therefore, a new lemma (Lemma 3 in Section 3.2) is added for determining the weight in this paper. In the numerical simulation of [2], a toy system is utilized. To show practical effectiveness, a quadruple tank system [5] is utilized in this paper.

This paper is organized as follows. Section 2 indicates the modeling of systems and sensor networks as well as the problem formulations. The weighted aggregation laws and their stability are shown in Section 3. The distributed observer at each node is proposed in Section 4. Section 5 shows the effectiveness of the proposed methods by a numerical simulation. Finally, Section 6 concludes this paper.

## 2 Modeling and Problem Formulation

### 2.1 Notation

Let  $\lfloor \cdot \rfloor : \mathbb{R} \rightarrow \mathbb{N}$  be a floor function such that

$$\lfloor x \rfloor = \max_{n \in \mathbb{N}} \{n; n \leq x\}.$$

If  $S$  is a set,  $|S|$  denotes the number of elements in  $S$ . For a matrix  $M \in \mathbb{R}^{n \times q}$ , the  $k$ -th row of  $M$  is denoted by  $\text{row}(M)_k \in \mathbb{R}^{1 \times q}$ .

### 2.2 Modeling of Systems

Consider a linear time-invariant plant whose actuators and sensors are connected by communication networks

like Fig. 1. We define the dynamics of the plant as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned} \quad (1)$$

where  $x \in \mathbb{R}^n$  denotes the state,  $u \in \mathbb{R}^m$  is the input, and  $y \in \mathbb{R}^p$  is the output of the system. The output  $y$  consists of all redundant raw measurements. Therefore, rank  $C$  may be less than  $p$ , and  $p$  may be larger than  $n$ .

The control system includes a sensor network and an actuator network. Each network communicates information of inputs and measurements of (1), respectively. We represent the sensor network as a directed graph  $G(V, E)$ , where  $V := \{1, 2, \dots, N\}$  denotes a set of nodes which are information sources of the networks and  $E \subseteq V \times V$  denotes a set of edges which constitute communication paths between the nodes.

We categorize the nodes into sensor, actuator and relay nodes. A sensor node has own sensors and provides its measurements to the sensor network. Let us define the scalable communication as follows.

**Definition 1** *When the dimension of communication data on all edges in  $E$  is independent of  $N = |V|$ , it is said that the communication is scalable.*

This paper considers aggregations of the sensor measurements by scalable communications. The redundant raw output  $y$  is aggregated into a  $q$ -dimensional vector

$$y_{\text{agr}} = Fy = C_{\text{agr}}x, \quad (2)$$

where  $F \in \mathbb{R}^{q \times p}$  represents the relationship between  $y$  and  $y_{\text{agr}}$ , and  $C_{\text{agr}} = FC$ . By renumbering the elements of  $y$ , we can decompose the matrix  $F$  as

$$F = [F_1 | \dots | F_N],$$

where  $F_i$  corresponds to the output of  $i$ -th node. Therefore, the outputs of the nodes, which are mapped to the aggregated output space, are defined as

$$\begin{aligned} y_1 &= [F_1 | 0 | \dots | 0]y = C_1x \\ &\vdots \\ y_N &= [0 | \dots | 0 | F_N]y = C_Nx, \end{aligned} \quad (3)$$

where  $C_i = [0 | \dots | 0 | F_i | 0 | \dots | 0]C$ . If one can obtain the current outputs  $y_i$  with no transmission delay, the aggregated output of  $y_i$  coincides with  $y_{\text{agr}}$ , i.e.

$$y_{\text{agr}} = \sum_{i=1}^N y_i. \quad (4)$$

It is assumed that there exist  $\gamma_{i,j,k} \geq 0$  such that

$$\text{row}(C_i)_k = \gamma_{i,j,k} \text{row}(C_j)_k \quad (\text{row}(C_j)_k \neq 0),$$

and therefore a weighted aggregated output  $\bar{w}_1 y_1 + \dots + \bar{w}_N y_N$  ( $\bar{w}_i > 0$ ) has the same physical meaning as  $y_{\text{agr}}$ . We suppose that the physical meaning of measurements should be preserved in the aggregation. For example, the aggregations of positions and velocities are prohibited by this assumption. In addition, we also prohibit the aggregations which cancel the measurements each other.

The actuator and relay nodes have no sensor, which means that these nodes provide no information to the sensor network, and the corresponding  $F_i$  has no column. Therefore, we model them as the nodes whose output matrices are zero matrices, i.e.  $C_i = 0$ .

In the sensor network, each node communicates the information of the measurements to the other nodes via the paths in  $E$ . If node  $i$  can directly send data to the node  $j$ , the pair  $(i, j)$  is included in  $E$ . We also define a neighborhood set as  $J_i := \{j; (j, i) \in E\}$ , which means that node  $i$  can receive the information from the nodes in  $J_i$ . We assume that the communication on the edge  $(i, j)$  has a sufficient bandwidth but there exists a constant communication delay  $d_{(i,j)}$ .

The actuator network has the same node set as the sensor network, and broadcasts the information about the input values of all actuators. We make an assumption that the actuator network is a real-time high-speed network with a limited capacity, and the dimension of the input  $u$  is smaller than the one of the aggregated output  $y_{\text{agr}}$ . Some legacy field-bus networks are the examples of limited-bandwidth networks. Therefore, we ignore the communication delay in the broadcast of the actuator network and all the nodes can obtain the value of  $u$  instantly.

**Remark 1** *In general, there is a trade-off between the control performances and constraints of networks (e.g. bit rates and communication capacities), which affect the selection of the aggregation matrix  $F$  and its dimension  $q$ . We should select the aggregation matrix under considerations on these effects. Some works, for example [13], propose the frameworks which determine the aggregation matrix under the trade-off. In this paper, we assume that  $F$  and  $q$  are given.*

### 2.3 Problem Formulation

The objective of this subsection is to formulate problems to be solved in this paper. The first problem is about the scalable communication which realizes the real time synchronization. This problem is formulated as follows.

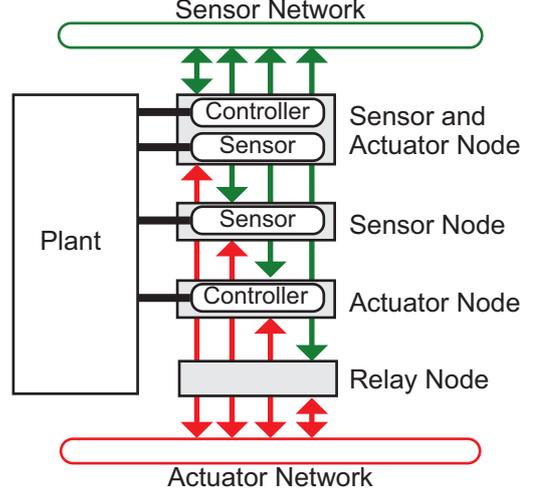


Fig. 1. Plants and Networks

**Problem 1** *Given the system (1) and the network graph  $G(V, E)$ , find scalable communication laws over the sensor network*

$$\hat{y}_i(t) = g_i(y_i(t), \{\hat{y}_j(t - d_{(j,i)}); j \in J_i\}, \{u(\tau); t - d_i^{\max} \leq \tau \leq t\})$$

and nonzero matrices  $\tilde{C}_i$  ( $i = 1, \dots, N$ ) such that

$$\lim_{t \rightarrow \infty} \hat{y}_i(t) = \tilde{C}_i x(t),$$

where  $d_i^{\max} = \max_{j \in J_i} d_{(j,i)}$ . The communication law is described by a functional  $g_i(\cdot)$ .

The second problem is about the distributed estimation at each node. It is formulated as follows.

**Problem 2** *Suppose that a solution of Problem 1 is given. Then, construct an observer*

$$\dot{\hat{x}}_i(t) = f_i(\hat{x}_i(t), \hat{y}_i(t), u(t), t)$$

such that

$$\lim_{t \rightarrow \infty} (\hat{x}_i - x) = 0,$$

where  $\hat{x}_i(t) \in \mathbb{R}^n$  is an estimate of  $x(t)$ .

## 3 Real-Time synchronization over sensor networks

### 3.1 Preliminaries

Delay compensation [14] is utilized through this paper. For understanding the proposed method, that result is

described in this subsection. Let us consider a linear time-invariant system with multi delays in sensors

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (5a)$$

$$y(t) = \sum_{i=1}^N C_i x(t - d_i). \quad (5b)$$

For the system (5), the following lemma holds.

**Lemma 1** *For the system (5),*

$$y(t) + \sum_{i=1}^N C_i e^{-Ad_i} \int_{t-d_i}^t e^{A(t-\tau)} Bu(\tau) d\tau = \hat{C}x(t),$$

holds, where  $\hat{C} = \sum_{i=1}^N C_i e^{-Ad_i}$ .

**Proof 1** *From the solution of (5) whose initial state is  $x(t - d_i)$ , we obtain*

$$x(t) = e^{Ad_i} x(t - d_i) + \int_{t-d_i}^t e^{A(t-\tau)} Bu(\tau) d\tau. \quad (6)$$

We solve (6) with respect to  $x(t - d_i)$  and substitute it into (5b), which directly derives Lemma 1.

### 3.2 Synchronization Over Directed Graph

The scalable synchronization of the measurements over the tree network are proposed in [11], [2]. In this paper, we extend these results to the case with any graph topology, where a weighted-data-aggregation method with delay compensation is proposed. In the proposed method, each node compensates communication delays of received values from neighbor nodes. Each node aggregates these data with some weights and sends the other nodes. We will show that the compensated data can be represented as a function of the current state  $x(t)$ . We will also show the stability condition of the communication law, which gives a selection guideline of the weight coefficients.

Let  $w_{(j,i)} \in (0, 1]$  denote the weight constant with respect to each edge  $(j, i) \in E$ , which is a design parameter. The weighted adjacency matrix  $W$  for  $G(V, E)$  is defined as

$$W_{j,i} = \begin{cases} w_{(j,i)} & \text{if } (j, i) \in E \\ 0 & \text{otherwise.} \end{cases}$$

We also define  $\widehat{W}_{(j,i)} \in \mathbb{R}^{|V| \times |V|}$  as a matrix such that its  $(j, i)$  element is  $w_{(j,i)}$ , and the other elements are 0.

We also define a communication law based on the weights for sharing the measurements in sensor network. In the

sensor network, transmission data between nodes are the aggregated measurements  $\hat{y}_i(t)$  and these output matrices  $\hat{C}_i(t) \in \mathbb{R}^{q \times n}$ . After the compensation of communication delays based on [14], node  $i$  aggregates these data into  $\hat{y}_i(t)$  with the weights.

We assume that the sensor network starts at  $t = 0$ , and set  $\hat{y}_i(\tau) = 0_q$  ( $\tau < 0$ ). Input signals for  $t < 0$  is also assumed to be zero. We define  $\hat{y}_i(t)$  for  $t \geq 0$  as

$$\hat{y}_i(t) = y_i(t) + \sum_{j \in J_i} w_{(j,i)} \left( \hat{y}_j(t - d_{(j,i)}) + \hat{C}_j(t - d_{(j,i)}) e^{-Ad_{(j,i)}} \int_{t-d_{(j,i)}}^t e^{A(t-\tau)} Bu(\tau) d\tau \right), \quad (7)$$

where  $\hat{C}(\cdot)$  is defined later.

If  $G(V, E)$  has some cyclic structure, list of the nodes that affect  $\hat{y}_i(t)$  is persistently updated. Therefore, each node also needs to update the output matrix of the aggregated outputs. The matrix  $\hat{C}_i(t)$  is determined by own output matrix  $C_i$  and the received aggregated output matrices as

$$\hat{C}_i(t) = C_i + \sum_{j \in J_i} w_{(j,i)} \hat{C}_j(t - d_{(j,i)}) e^{-Ad_{(j,i)}} \quad (8)$$

for  $t \geq 0$ , while  $\hat{C}_i(t) = 0$  for  $t < 0$ . Since the dimension of  $\hat{y}_i(t)$  and  $\hat{C}_i(t)$  are  $q$  and  $q \times n$ , respectively, which are independent from  $|V| = N$ , it is obvious that the proposed control law is scalable.

We show that  $\hat{y}_i(t)$  is expressed by the function of  $x(t)$ , and that the construction method of  $\hat{y}(\cdot)$ , which are defined by (7) and (8), provides a solution of Problem 1. The delay compensation of (7) is based on the result of the delay compensated observer [14]. Therefore, the following lemma holds.

**Lemma 2** *Under the communication law (7) with (8),*

$$\hat{y}_i(t) = \hat{C}_i(t)x(t) \quad (9)$$

holds for  $i = 1, \dots, N$ .

**Proof 2** *From the definition of (7) and (8), it is obvious that  $\hat{y}_i(t)$  satisfies (9) for  $t < 0$ .*

*Next, we consider the case for  $t \geq 0$ . Assume that  $\{\hat{y}_j(t - d_{(j,i)}); j \in J_i\}$ , which are sets of information received by the node  $i$ , satisfy (9), i.e.  $\hat{y}_j(t - d_{(j,i)}) = \hat{C}_j(t -$*

$d(j, i)x(t - d(j, i))$  for  $j \in J_i$ . Then,  $\hat{y}_i(t)$  satisfies

$$\hat{y}_i(t) = C_i x(t) + \sum_{j \in J_i} w_{(j,i)} \left( \hat{C}_j(t - d_{(j,i)}) x(t - d_{(j,i)}) + \hat{C}_j(t - d_{(j,i)}) e^{-Ad_{(j,i)}} \int_{t-d_{(j,i)}}^t e^{A(t-\tau)} B u(\tau) d\tau \right).$$

Since  $x(t) = e^{Ad_{(j,i)}} x(t - d_{(j,i)}) + \int_{t-d_{(j,i)}}^t e^{A(t-\tau)} B u(\tau) d\tau$  is satisfied, we obtain

$$\begin{aligned} \hat{y}_i(t) &= \left( C_i + \sum_{j \in J_i} w_{(j,i)} \hat{C}_j(t - d_{(j,i)}) e^{-Ad_{(j,i)}} \right) x(t) \\ &= \hat{C}_i(t) x(t), \end{aligned}$$

and (9) holds for the node  $i$  at  $t$ .

From the above discussion, we conclude that  $\hat{y}_i(t)$  recursively satisfies Lemma 2.

This lemma shows that the results of [14], which is summarized in Lemma 1 in Appendix, can be performed over the sensor network in the distributed manner. From Lemma 2, it has been shown that each node can obtain a part of state  $\hat{C}_i(t)x$ . Hence, the problem remaining is to find the weights to stabilize (8). The problem remaining is formulated as follows.

**Problem 3** Let  $\hat{C}(t) = [\hat{C}_1(t), \dots, \hat{C}_N(t)]$  and consider the system

$$\dot{\hat{C}}(t) = \sum_{e \in E} \hat{C}(t - d_e) \widehat{W}_e \otimes e^{-Ad_e} + \bar{C}, \quad (10)$$

where  $\bar{C} = [C_1, \dots, C_N]$ . Then, find  $w_{(j,i)}$  such that  $\hat{C}(\infty) = \lim_{t \rightarrow \infty} \hat{C}(t) < \infty$  exists.

To simplify the problem, we assume that all  $d_{(j,i)}$  are equal to  $d$ . By adding virtual relay nodes to  $G(V, E)$ , we can reconstruct a sensor network with uniform communication delay. Therefore,  $d_{(i,j)} = d$  is a natural assumption. Then, we obtain the solution of the Problem 3 as the following theorem.

**Theorem 1** Suppose that  $d_{(j,i)} = d$  and  $\hat{C}_i(t)$  is given by (8). Then,  $\lim_{t \rightarrow \infty} \hat{C}(t) = \bar{C}(I_{nN} - W \otimes e^{-Ad})^{-1}$  if the spectral radius of  $W \otimes e^{-Ad}$  is smaller than 1.

**Proof 3** Since (10) updates  $\hat{C}(t)$  at every time period  $d$ , (10) behaves as a discrete-time system. Therefore, we can express  $\hat{C}(t)$  on discrete time  $k \in \mathbb{N}$ . Let  $k$  be  $k = \lceil t/d \rceil$ , and then  $\hat{C}(k)$  is expressed by

$$\hat{C}(k) = \hat{C}(k-1)W \otimes e^{-Ad} + \bar{C}. \quad (11)$$

The solution of (11) is given by

$$\hat{C}(k) = \bar{C} \sum_{l=0}^{k-1} (W \otimes e^{-Ad})^l. \quad (12)$$

Therefore,  $\tilde{C}_i = \lim_{k \rightarrow \infty} \hat{C}(k) = \bar{C}(I_{nN} - W \otimes e^{-Ad})^{-1}$  if the spectral radius of  $W \otimes e^{-Ad}$  is smaller than 1.

We also provide a design method of the weights. The weights are determined from an eigenvalue of  $A$ , delay  $d$ , and output degree of the network. Let  $\lambda_{\min}$  be an eigenvalue of  $A$  whose real part is minimum. An output degree of  $i$ -th node is denoted by  $d_{\text{out}}(i)$ . Let  $\hat{\lambda} = \text{Re}(\lambda_{\min})$  and  $0 < \alpha < 1$ . If we select the weight as

$$w_{(j,i)} = e^{\hat{\lambda}d} \frac{1}{d_{\text{out}}(j)} \alpha, \quad (13)$$

(11) becomes stable. The above summarised the following lemma.

**Lemma 3** If the weights are given by (13), the spectral radius of  $W \otimes e^{-Ad}$  is smaller than 1.

**Proof 4** Let  $\lambda_W^i$  for all  $i = 1, \dots, N$  be eigenvalues of  $W$  given by (13). Due to Gershgorin circle theorem, all  $\lambda_w^i$  satisfy

$$|\lambda_W^i| < e^{\hat{\lambda}d} \alpha. \quad (14)$$

Let  $\lambda_A^i$  for all  $i = 1, \dots, n$  be eigenvalues of  $A$ . Using a property of Kronecker product, the eigenvalue of  $W \otimes e^{-Ad}$  is obtained as

$$\lambda_W^i e^{-\lambda_A^j d} \quad (15)$$

for all  $i = 1, \dots, N$  and  $j = 1, \dots, n$ . From (14) and (15), the spectral radius of  $W \otimes e^{-Ad}$  is smaller than  $\alpha < 1$ .

### 3.3 Synchronization on Undirected Graph

In previous subsection, we define communication laws for the real time synchronization over the directed sensor network. It is obvious that the communication laws of the previous subsection can be applied to the undirected sensor network. However, re-transmissions on the undirected edges are harmful for the estimation. The data that are transmitted via an undirected edge are reflected at the other node immediately, and they contain past measurements of own sensors. In addition, the reflection generates a short cyclic path, and makes the design of the weight coefficients restrictive. Therefore, we modify (7) and (8) to prevent the reflection on the undirected edges. Concretely speaking, in the calculation of the transmission data to node  $j$  at node  $i$ , the received data from node  $j$  is omitted from the sum in (7). We

show the stability condition of the weights based on the modified communication laws.

Let  $\hat{y}_{(i,j)}(t)$  and  $\hat{C}_{(i,j)}(t)$  be the information which are sent from node  $i$  to  $j$ . Then, the communication law of each node is defined as

$$\begin{aligned} \hat{y}_{(i,j)}(t) = & y_i(t) + \sum_{k \in J_i \setminus \{j\}} w_{(k,i),(i,j)} \left( \hat{y}_{(k,i)}(t - d_{(k,i)}) \right. \\ & \left. + \hat{C}_{(k,i)}(t - d_{(k,i)}) e^{-Ad_{(k,i)}} \int_{t-d_{(k,i)}}^t e^{A(t-\tau)} Bu(\tau) d\tau \right), \end{aligned} \quad (16)$$

$$\hat{C}_{(i,j)}(t) = C_i + \sum_{k \in J_i \setminus \{j\}} w_{(k,i),(i,j)} \hat{C}_{(k,i)}(t - d_{(k,i)}) e^{-Ad_{(k,i)}} \quad (17)$$

for  $t \geq 0$ , while  $\hat{y}_{(i,j)}(t) = 0$  and  $\hat{C}_{(i,j)}(t) = 0$  for  $t < 0$ .

We can regard the modified rule (16) with (17) as the weighted data-aggregation on a modified sensor network  $G(\tilde{V}, \tilde{E})$ . In  $G(\tilde{V}, \tilde{E})$ , the node means the transmission information on each edge in  $G(V, E)$ . Therefore, the modified node set  $\tilde{V}$  is expressed by  $\tilde{V} = E$ . Since node  $(i, j) \in \tilde{V}$  is the transmission information from node  $i$ , the nodes  $(i, *)$  play as the sensor nodes which share the measurement  $y_i = C_i x(t)$ .

While  $\tilde{V}$  means the set of the transmission information on  $G(V, E)$ ,  $\tilde{E} \subset \tilde{V} \times \tilde{V}$  means the relationship between each  $(i, j) \in \tilde{V}$ . Since  $\hat{y}_{(i,j)}$  does not include  $\hat{y}_{(j,i)}$  in (16), we can represent  $\tilde{E}$  as  $\tilde{E} = \{((k, i), (i, j)); k \neq j, (i, j) \in \tilde{V}, (j, k) \in \tilde{V}\}$ . The edge  $((k, i), (i, j)) \in \tilde{E}$  means that node  $i$  relays the information from node  $k$  to node  $j$ , where  $i, j, k \in V$ . Because the real entities of  $(k, *)$  and  $(i, *)$  are the  $k$ -th node and  $i$ -th node, respectively, the real connection of  $((k, i), (i, j))$  is  $(k, i)$ . Therefore, the delay on  $((k, i), (i, j))$  is  $d_{(k,i)}$ .

Since (16) and (17) can be explained by the the weighted data-aggregation on  $G(\tilde{V}, \tilde{E})$ , the stability of (17) follows Theorem 1. In this case, The weighted adjacency matrix on  $G(\tilde{V}, \tilde{E})$  is given by

$$\tilde{W} = \begin{cases} w_{(k,i),(i,j)} & \text{if } ((k, i), (i, j)) \in \tilde{E} \\ 0 & \text{otherwise.} \end{cases}$$

On the communication defined by (16) and (17), each node does not need to aggregates the information with the weights to calculate  $\hat{y}_i(t)$  and  $\hat{C}_i(t)$ . Therefore, the

definition of  $\hat{y}_i(t)$  and  $\hat{C}_i(t)$  are

$$\begin{aligned} \hat{y}_i = & y_i(t) + \sum_{k \in J_i} \left( \hat{y}_{(k,i)}(t - d_{(k,i)}) \right. \\ & \left. + \hat{C}_{(k,i)}(t - d_{(k,i)}) e^{-Ad_{(k,i)}} \int_{t-d_{(k,i)}}^t e^{A(t-\tau)} Bu(\tau) d\tau \right), \\ \hat{C}_i(t) = & C_i + \sum_{k \in J_i} \hat{C}_{(k,i)}(t - d_{(k,i)}) e^{-Ad_{(k,i)}} \end{aligned}$$

for  $t \geq 0$ , while  $\hat{y}_i(t) = 0$  and  $\hat{C}_i(t) = 0$  for  $t < 0$ . If  $(i, j) \in E$  and  $(j, i) \notin E$ , then  $\hat{y}_i$  and  $\hat{C}_i$  are equivalent to  $\hat{y}_{(i,j)}$  and  $\hat{C}_{(i,j)}$ , respectively. However, in general,  $\hat{y}_i$  and  $\hat{C}_i$  are used for the state estimation at the  $i$ -th node and these are not transmitted to other nodes.

#### 4 Distributed Estimation over Sensor Network

In the previous section, we have proposed the communication laws which synchronize the measurements in real-time over the network with any topology. In this section, we propose distributed observers to estimate the state from the information on the sensor network. Lemma 2 shows that  $\hat{y}_i(t)$  is an output of the system (1), which is linear with respect to the state with a time-varying coefficient matrix. Therefore, the observer which is proposed in this section makes an adaptation of the observer gain according to the updates of the output matrix  $\hat{C}_i(t)$ . For the adaptation algorithm, we utilize the calculation of Kalman gain. The proposed observer is formulated as follows.

Let  $Q_i$ ,  $R_i$  and  $P_i(0)$  be symmetric positive-definite matrices with appropriate dimensions. Then, we define an observer for the system (1) as

$$\begin{aligned} \dot{\hat{x}}_i(t) = & A\hat{x}_i(t) + Bu(t) + K_i(t)(\hat{C}_i(t)\hat{x}_i(t) - \hat{y}_i(t)) \\ K_i(t) = & -P_i(t)\hat{C}_i^T(t)R_i^{-1} \\ \dot{P}_i(t) = & AP_i(t) + P_i(t)A^T + B_iQ_iB_i^T \\ & - P_i(t)\hat{C}_i^T(t)R_i^{-1}\hat{C}_i(t)P_i(t). \end{aligned} \quad (18)$$

When the distributed observer in each node is given by (18), the stability of these estimation error is given by the following theorem.

**Theorem 2** *Suppose that the pairs  $(\tilde{C}_i, A)$  for all  $i = 1, \dots, N$  are the observable. Then, the estimation error  $\xi_i(t) = x(t) - \hat{x}_i(t)$  of the observer (18) tends to 0 as  $t \rightarrow \infty$ .*

**Proof 5** *Let us consider the dynamics of  $\xi_i(t)$  as*

$$\dot{\xi}_i(t) = (A + K_i(t)\hat{C}_i(t))\xi_i(t).$$

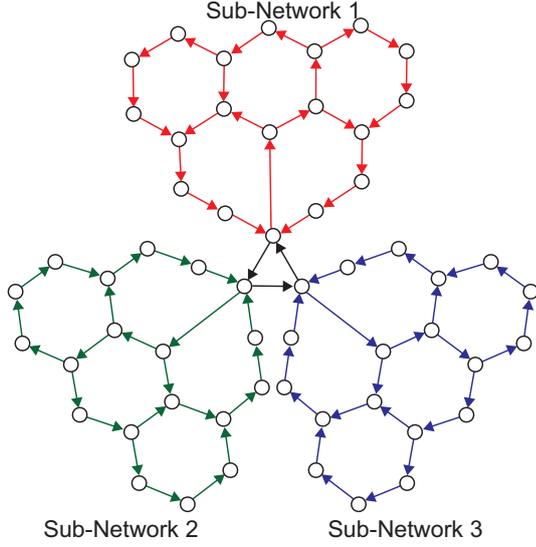


Fig. 2. Topology of Sensor Network in the Simulation

Since  $\lim_{t \rightarrow \infty} \hat{C}_i(t) = \tilde{C}_i$ , which is shown in Theorem 1,  $(A + K_i(t)\hat{C}_i(t))$  for all  $i \in V$  converge to stable matrices. Therefore,  $\lim_{t \rightarrow \infty} \xi_i(t) = 0$  for all  $i = 1, \dots, N$ .

As discussed in Section 2, weighted output matrices have the same physical meaning of  $C_{agr}$  when the delays are ignorable. Therefore, when the network is strongly connected and the delay is sufficiently small, the observability condition is equivalent to that of  $(C_{agr}, A)$ .

According to (7), (8) and (18), the distributed estimation over the sensor network is executed. This process is summarized as follows:

- Step 1** Node  $i$  receives  $\hat{y}_j(t - d_{(j,i)})$  and  $\hat{C}_j(t)$  from  $j$ -th node, where  $j \in J_i$ .
- Step 2** Node  $i$  updates  $\hat{y}(t)$  and  $\hat{C}_i(t)$  based on (7) and (8), respectively.
- Step 3** The estimate  $\hat{x}_i(t)$  is updated based on (18).
- Step 4** Node  $i$  sends  $\hat{y}(t)$  and  $\hat{C}_i(t)$  as messages to the neighborhood.

## 5 Numerical Simulation

In this section, we show a result of a numerical simulation to evaluate the effectiveness of the proposed method. Let

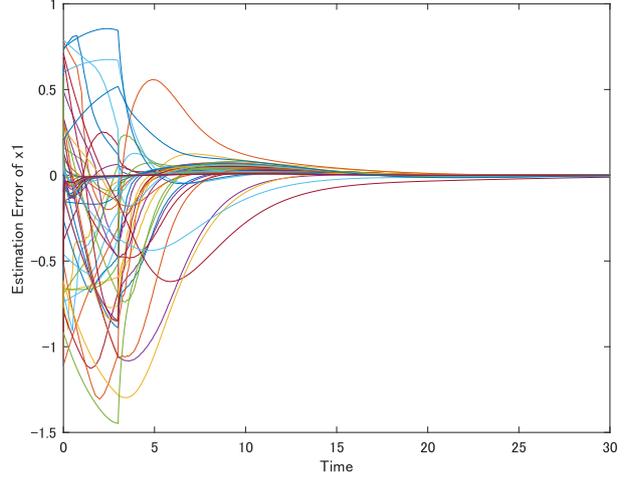


Fig. 3. Estimation Errors on  $x_1$

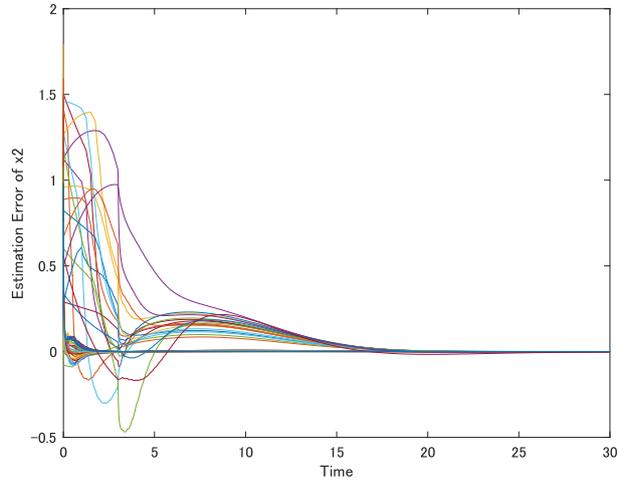


Fig. 4. Estimation Errors on  $x_2$

us consider the quadruple tank system [5] as follows:

$$A = \begin{pmatrix} -0.15 & 0 & 0.5 & 0 \\ 0 & -0.25 & 0 & 0.5 \\ 0 & 0 & -0.15 & 0 \\ 0 & 0 & 0 & -0.25 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.1 & 0 & 0 & 0.1 \\ 0 & 0.1 & 0.1 & 0 \end{pmatrix}^T,$$

$$u = \begin{pmatrix} \cos(\pi t) + 1 & \sin(\pi t) + 1 \end{pmatrix}^T.$$

A sensor network in this simulation has the topology illustrated by Fig. 2. All nodes in sub-network 1 and 3 measure  $x_1$ . The other nodes measure  $x_2$ . At least two types of sensors are needed for this system to satisfy the observability. Therefore, each node needs to communicate the other nodes to estimate all state variables.

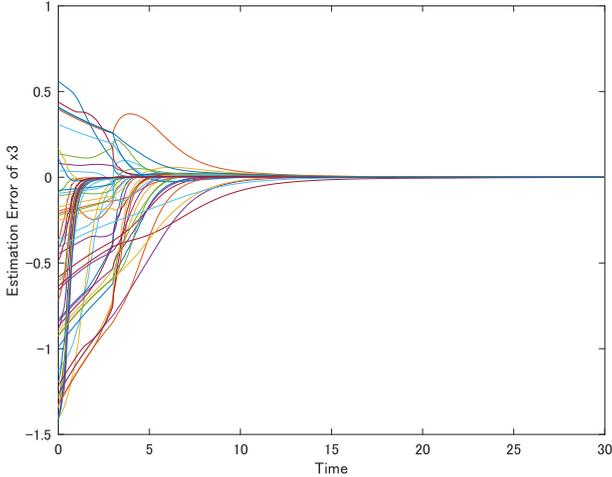


Fig. 5. Estimation Errors on  $x_3$

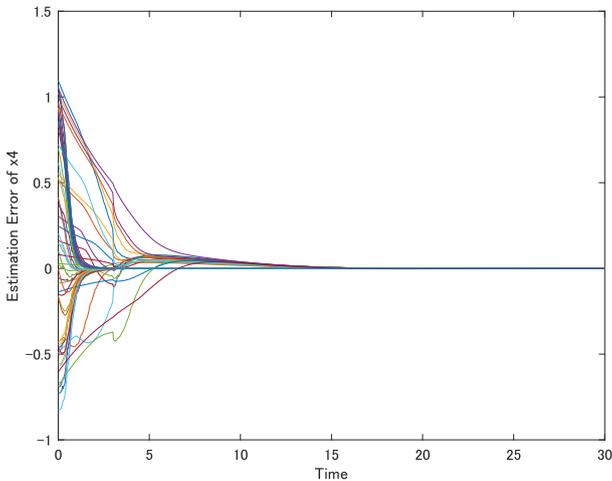


Fig. 6. Estimation Errors on  $x_4$

Parameters in this simulation are summarized as follows: Initial states of the system are given by random uniform random numbers between 0 and 1. We set the delay of each edge as  $d = 0.25$ . The weights are design based on Lemma 2, where  $\alpha$  is selected by 0.9. Initial estimates of all nodes are given by uniform random numbers between 0 and 1. The parameters of the distributed observers are given by  $P_i(0) = I_4$ ,  $Q_i = 0_4$  and  $R_i = \text{diag}(0.005, 0.005)$ .

The results of the simulation under above condition are shown by Fig. 3 - 6, where each curve indicates the estimation at a node. From these figures, we can confirm that the estimation errors at all nodes converge to zero. Therefore, it is confirmed that our proposed method can estimate all state in each node.

## 6 Conclusions

In this paper, we proposed scalable communication laws to synchronize the measurements over a delayed sen-

sor network with any topology. Each node compensates communication delays of the received data, and aggregates these data with the multiplication of weight coefficients. The stability condition of the proposed law was given by using the weighted adjacency matrix. The distributed observers which provide the estimates from the aggregated measurements were also proposed. The effectiveness of the proposed method was shown by a numerical simulation.

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