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# **A Computer Program for the Calculation of Piezomagnetic Field due to a Spherical Pressure Source (Mogi Model) in the Inhomogeneously Magnetized Crust**

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## **Abstract**

We developed an analytical method for evaluating the piezomagnetic field considering the inhomogeneously magnetized crust. In this paper, we present a FORTRAN program to calculate the piezomagnetic effect due to a spherical pressure source (the Mogi model) in the inhomogeneously magnetized crust.

## **1. Introduction**

In the previous studies (Sasai, 1991; Utsugi et al., 1999), the analytical solution of piezomagnetic effect has been obtained in the case of uniformly magnetized crustal model. However, it is self-evident that the earth's crust is inhomogeneously magnetized. Then, to represent the inhomogeneity of the crustal magnetization, we divide the crust into a number of compartments. Each compartment is assumed to have its own uniform magnetic properties such as the magnetization and the stress sensitivity. The geomagnetic field change at a certain point on the earth's surface may be approximated by the sum of the piezomagnetic field derived from each compartment. The piezomagnetic field is generally expressed by the surface integral of displacement and its derivation over the boundary surface of magnetized region. In the case of the compartment model, the surface integral becomes finite one. This integral cannot be solved analytically. However the line integral with respect to either coordinate can be represented analytically using elliptic integrals. There are several simple algorithms to make rapid and exact evaluation of the elliptic integrals. Using these algorithms, the piezomagnetic field can be expressed by line inte-

gral. Through numerical evaluations of this line integral by using the double exponential method (e.g. Takahashi and Mori, 1974), we can obtain the piezomagnetic field considering the inhomogeneity of the crustal magnetization.

In this paper, we present a FORTRAN program which calculates the piezomagnetic effect due to a spherical pressure source (the Mogi model: Mogi, 1958) in the inhomogeneously magnetized crust.

## 2. Geomagnetic field changes due to Mogi model

We consider the coordinate system as shown in Fig. 1. A semi-infinite elastic medium occupies  $z > 0$ . We assume that the region  $V_1$  is uniformly magnetized cube and outside  $V_2$  is demagnetized. We also assume that the elastic properties such as Lamé constants  $\lambda, \mu$  are common in the regions  $V_1$  and  $V_2$ . The analytical solution of the displacement  $\mathbf{u}$  due to the Mogi model is obtained by Mindlin and Cheng (1950) and Yamakawa (1955) as follows:

$$u_x = \frac{C}{2\mu} \left[ \frac{x}{R_1^3} + 2\frac{x}{R_2^3} - \frac{6xz(z+D)}{R_2^5} \right] \quad (1)$$

$$u_y = \frac{C}{2\mu} \left[ \frac{y}{R_1^3} + 2\frac{y}{R_2^3} - \frac{6yz(z+D)}{R_2^5} \right] \quad (2)$$

$$u_z = \frac{C}{2\mu} \left[ \frac{z-D}{R_1^3} - 2\frac{D}{R_2^3} - \frac{6z(z+D)^2}{R_2^5} \right] \quad (3)$$

where  $R_1 = \sqrt{x^2 + y^2 + (z-D)^2}$ ,  $R_2 = \sqrt{x^2 + y^2 + (z+D)^2}$  and  $D$  is the source depth. The moment  $C$  is given by

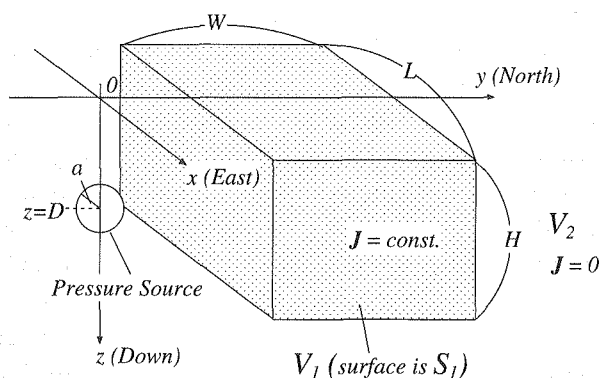


Fig. 1. Coordinate system, source and crustal model are shown.

A semi-infinite elastic medium occupies  $z > 0$ . The cubic region  $V_1$  ( $L \times W \times H$   $\text{km}^3$ ) is uniformly magnetized and outside  $V_2$  is demagnetized. In this medium, a spherical pressure source (the Mogi model) is assumed.

$$C = -\frac{1}{2}a^3\Delta P,$$

where  $a$  is the radius of the sphere and  $\Delta P$  is the hydrostatic pressure acting on the surface of the sphere.

According to the representation theorem for the piezomagnetic field (Sasai, 1991), the geomagnetic change  $\Delta \mathbf{M}$  is expressed by the integral over the boundary surface  $S_1$  (Fig. 1) of the cubic region :

$$\begin{aligned} \Delta \mathbf{M}^k(\mathbf{r}_0) &= -C_k \iint_{S_1} \left[ \left\{ -\frac{\partial u_k(\mathbf{r})}{\partial n} + \frac{2(\lambda + \mu)}{3\lambda + 2\mu} \Delta \mathbf{m}^k \cdot \mathbf{n} \right\} \frac{\partial}{\partial \mathbf{r}_0} \frac{1}{\rho} \right. \\ &\quad \left. + u_k(\mathbf{r}) \frac{\partial^2}{\partial \mathbf{r}_0 \partial n} \left( \frac{1}{\rho} \right) \right] dS_r, \quad (4) \\ C_k &= \frac{1}{2} \beta \mu \frac{3\lambda + 2\mu}{\lambda + \mu} J_k, \\ \Delta m_i^k &= \frac{3}{2} \left( \frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k} \right) - \delta_{ij} \text{div} \mathbf{u}, \\ \rho &= |\mathbf{r}_0 - \mathbf{r}|, \end{aligned}$$

where  $\delta_{ij}$  is the Kronecker's delta,  $J_k$  is the  $k$ -th component of initial magnetization within  $V_1$  and  $\mathbf{u}$  is the displacement vector.  $\mathbf{r}_0$  and  $\mathbf{r}$  indicate the observation point and the arbitrary point within the medium, respectively.

Substituting eqs. (1) to (3) into eq. (4), the geomagnetic field changes are written in the form of surface integral of function  $f(1/R, 1/\rho)$ . In the present case, we have to make the finite surface integral because  $S_1$  is finite. As mentioned in the previous chapter, we cannot solve this finite surface integral analytically. However, using elliptic integrals, the integral with respect to either coordinate can be represented by the analytical form. Then we can transform the surface integral of eq. (4) to the line integral as shown in the following chapter.

### 3. Line integrals of $f(1/R, 1/\rho)$

In eq. (4), we see the following integrals of function  $f(1/R, 1/\rho)$  :

$$\Phi_{ij}(y, z, x_0, y_0, z_0; D) = \int \frac{1}{R^i \rho^j} dx, \quad (5)$$

$$\Psi_{ij}(y, z, x_0, y_0, z_0; D) = \int \frac{x}{R^i \rho^j} dx, \quad (6)$$

These integrals can be solved analytically using the following elliptic integrals :

$$F(\varphi, k) = \int_0^\varphi \frac{1}{\sqrt{1 - k^2 \sin^2 \varphi'}} d\varphi' \quad (\text{First Kind}), \quad (7)$$

$$E(\varphi, k) = \int_0^\varphi \sqrt{1 - k^2 \sin^2 \varphi'} d\varphi' \quad (\text{Second Kind}). \tag{8}$$

For example, we consider the following integral :

$$\Phi_{11} = \int \frac{1}{R\rho} dx. \tag{9}$$

For the convenience, we denote  $R$  and  $\rho$  as follows :

$$\begin{cases} R^2 = x^2 + \zeta^2, \\ \rho^2 = x^2 - 2x_0x + c_0^2, \\ \zeta^2 = y^2 + (z - D)^2, \\ c_0^2 = x_0^2 + c^2, \\ c^2 = (y_0 - y)^2 + (z_0 - z)^2, \end{cases}$$

where  $(x, y, z)$  is the arbitrary point within the magnetized region of the medium, and  $(x_0, y_0, z_0)$  is the observation point.

To express eq. (9) using the elliptic integrals, we introduce the following variable transform :

$$x \rightarrow \frac{\alpha t + \beta}{t + 1}, \tag{10}$$

where

$$\begin{cases} \alpha = \frac{c_0^2 - \zeta^2 - D}{2x_0}, \\ \beta = \frac{c_0^2 - \zeta^2 + D}{2x_0} \quad (\alpha \leq \beta), \\ D^2 = (c_0^2 - \zeta^2)^2 + 4x_0^2 \zeta^2. \end{cases}$$

Using eq. (10),  $R$  and  $\rho$  become more simply :

$$R^2 = \frac{1}{(t + 1)^2} \{ (\alpha^2 + \zeta^2)t^2 + (\beta^2 + \zeta^2) \},$$

$$\rho^2 = \frac{1}{(t + 1)^2} \{ (\alpha^2 - 2x_0\alpha + c_0^2)t^2 + (\beta^2 - 2x_0\beta + c_0^2) \}.$$

Therefore the terms  $1/R$  and  $1/\rho$  are written as follows :

$$\frac{1}{R} \rightarrow \frac{1}{\sqrt{p(\alpha)}} \frac{|t + 1|}{\sqrt{t^2 + \xi^2}}, \quad \frac{1}{\rho} \rightarrow \frac{1}{\sqrt{q(\alpha)}} \frac{|t + 1|}{\sqrt{t^2 + \eta^2}},$$

where

$$\begin{cases} p(s) = s^2 + \zeta^2, \\ q(s) = (x_0 - s)^2 + c^2, \\ \xi^2 = p(\beta)/p(\alpha), \\ \eta^2 = q(\beta)/q(\alpha) \quad (\xi^2 > \eta^2), \end{cases}$$

and eq. (9) is rewritten as follows :

$$\Phi_{11} = \frac{\alpha - \beta}{\sqrt{p(\alpha)q(\alpha)}} \int \frac{1}{\sqrt{t^2 + \xi^2} \sqrt{t^2 + \eta^2}} dt. \tag{11}$$

Transforming  $t \rightarrow \varphi = \tan^{-1}\left(\frac{t}{\eta}\right)$ , we obtain the following result :

$$\Phi_{11} = \frac{\alpha - \beta}{\sqrt{p(\alpha)q(\alpha)}} \frac{1}{\xi} F(\varphi, k),$$

where  $k^2 = (\xi^2 - \eta^2) / \xi^2$ . To evaluate the elliptic integrals, there are some algorithms (Cayley, 1961 ; Byrd and Friedman, 1954) without solving eqs. (7) and (8) directly. Using these algorithms, we can evaluate  $\Phi_{11}$  easily. With the same manner,  $\Phi_{ij}$  and  $\Psi_{ij}$  are solved analytically. The exact forms of  $\Phi_{ij}$  and  $\Psi_{ij}$  are given by Utsugi (1999).

The geomagnetic change is written by the following line integral of  $\mathbf{g}$  which is an arbitrary function of  $\Phi_{ij}$  and  $\Psi_{ij}$  :

$$\Delta \mathbf{M}^k(\mathbf{r}_0) = C_k \int \mathbf{g}(\Phi_{ij}(\mathbf{r}_0, \mathbf{r}), \Psi_{ij}(\mathbf{r}_0, \mathbf{r})) dl r. \tag{12}$$

From numerical calculation of this integral, we can evaluate the piezomagnetic change considering the inhomogeneously magnetized crust. To evaluate eq. (12), we use the double exponential integral method (DEM).

#### 4. Programs

The source list of the programs for calculating the piezomagnetic field is given in Appendix. The subroutine 'MGINHOMO' calculates the geomagnetic

Table 1. Input parameters.

X0, Y0, Z0	$x_0, y_0, z_0$	(km)	Observation point
CX0, CY0, CZ0		(km)	Location of the center of the cube
CL, CW, CH	L, W, H	(km)	Length, width and height of the cube
C0	$a^3 \Delta P / 2$	( $\text{km}^3 \cdot \text{bar}$ )	Moment of Mogi model
D0	D	(km)	Source depth
AMU	$\mu$	(cgs)	Rigidity
POI	$\nu$		Poisson ratio
CMZX, CMZY, CMZZ	$\mathbf{J} = (J_x, J_y, J_z)$	(A/m)	Magnetization vector within the region $V_1$
BETA	$\beta$	( $\text{bar}^{-1}$ )	Stress sensitivity

change due to a magnetized cubic block. This subroutine requires the parameters as shown in Table 1 and returns eastward (DMX), northward (DMY), downward (DMZ) and total force (DMF) components of geomagnetic change.

The double precision functions 'MGXYX<sub>i</sub>', 'MGXZX<sub>i</sub>' and 'MGYZX<sub>i</sub>' ( $X_i = X, Y$  or  $Z$ ) calculate the contributions from the  $x-y$ ,  $x-z$  and  $y-z$  plane of  $V_1$ , respectively. The subroutine 'PHSIIJ' calculates  $\Phi_{ij}$  and  $\Psi_{ij}$ . The elliptic integrals which appear in  $\Phi_{ij}$  and  $\Psi_{ij}$  are calculated by the subroutine 'ELLIPFE'. The subroutine 'DEMINT' calculates the numerical line integral of the functions of  $\Phi_{ij}$  and  $\Psi_{ij}$  numerically using DEM.

### 5. Numerical example

In Fig. 2, we show a numerical example. This figure shows the profiles of the total force of geomagnetic change along the  $y$  axis ( $x_0=0, z_0=1$ )(m). The case A in Fig. 2 is based on the uniformly magnetized crust: a layer  $0 < z < H_c = 15$  km ( $H_c$  indicates the Curie depth) is uniformly magnetized by 1 A/m. The case B is based on the inhomogeneously magnetized crust as shown in Fig. 1. The intensity of magnetization within the cube is assumed as 1 A/m. In both

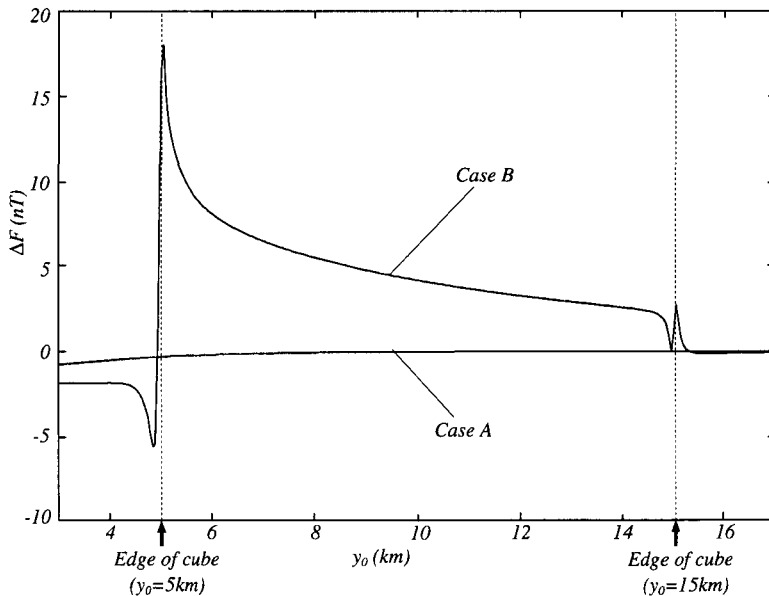


Fig. 2. Profiles of the total force of geomagnetic change along the  $y$  axis ( $x_0=0, z_0=1$  m). Cases A and B indicate the geomagnetic changes based on an uniformly magnetized crustal model and a cubic model, respectively.

Table 2. Cubic model parameters.

(X0, Z0)	(0, 1)	(m)
(CX0, CY0, CZ0)	(10, 0, 7.5)	(km)
(CL, CW, CH)	(10, 10, 15)	(km)
C0	$10^9$	( $\text{km}^3 \cdot \text{bar}$ )
D0	5	(km)
AMU	$3.5 \times 10^{11}$	(cgs)
POI	0.25	
(CMZX, CMZY, CMZZ)	(0, $1/\sqrt{2}$ , $1/\sqrt{2}$ )	(A/m)
BETA	$10^{-4}$	( $\text{bar}^{-1}$ )

cases, the magnetic inclination and declination are assumed as  $45^\circ$  and  $0^\circ$ , respectively. The model parameters are given in Table 2. Through this calculations, it becomes clear that the piezomagnetic effect is enhanced around the edges of the cubic block (at  $y_0=5$  and  $y_0=15$  km). This is caused by the fact that, unlike a uniform medium, the magnetic fields arising from stress-induced magnetic dipoles do not cancel with one another around the edges. The mechanism of this enhancement is well discussed in Oshiman (1990) and Utsugi (1999).

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1 SUBROUTINE MGINHOM(CX0, Y0, Z0, CX0, CY0, CZ0,
2 & CL, CW, CH, C0, D0, AMU, POI, CMZX, CMZY, CMZZ,
3 & DMX, DMY, DMZ, DMF)
4 C*****
5 C
6 C GEOMAGNETIC FIELD CHANGE AT OUTSIDE THE MEDIUM
7 C DUE TO THE MOGI MODEL
8 C BASED ON THE CUBIC BLOCK MODEL
9 C CODED BY M. UTSUGI....DEC 1999
10 C
11 C*****
12 C**** INPUT PARAMETERS
13 C X0, Y0, Z0 : LOCATION OF THE OBSERVATION POINT
14 C CX0, CY0, CZ0 : LOCATION OF CENTER OF CUBIC BLOCK
15 C CL, CW, CH : LENGTH, WIDTH AND HEIGHT OF CUBE
16 C C0 : MOMENT OF MOGI MODEL
17 C D0 : SOURCE DEPTH
18 C AMU : RIGIDITY
19 C POI : POISSON RATIO
20 C CMZX, CMZY, CMZZ : MAGNETIZATION GAP THROUGH THE BOUNDARY
21 C SURFACE OF CUBE
22 C BETA : STRESS SENSITIVITY
23 C
24 C**** OUTPUT VALUES
25 C DMX,DMY,DMZ,DMF : EAST, NORTH, DOWNWARD COMPONENTS
26 C AND TOTAL FORCE OF GEOMAGNETIC
27 C FIELD CHANGE (NT)
28 C
29 C IMPLICIT REAL*8 (A-H, O-Z)
30 C INTEGER NEND, NPOW
31 C COMMON /COM1/ NEND, NPOW, AO(2),
32 & APC(608, 2), AM(608, 2), B0, BB(608)
33 C COMMON /COM2/ C00, C0X, C0Y, C0Z
34 C
35 C EXTERNAL MGXYX, MGXY, MGXYZ
36 C EXTERNAL MGZX, MGZY, MGZZ
37 C EXTERNAL MGYX, MGYZ, MGYZZ
38 C
39 C EPS = 1.0-15
40 C L = 0
41 C CMZ0 = SQRT(CMZX**2+CMZY**2+CMZZ**2)
42 C SHM=CMZ0*BETA*AMU*(1.00+POI)
43 C C00=0.500*SHM*1.0-7
44 C C0X=CMZX*C00
45 C C0Y=-CMZY*C00
46 C C0Z=CMZZ*C00
47 C
48 C CALL FIAB (NEND, NPOW, AO, AM, AP, B0, BB)
49 C
50 C*** DMX : CONTRIBUTIONS FROM X-Y PLANE ***
51 C
52 C DMXYX : EASTWARD COMPONENTS
53 C
54 C CALL DEMINT (MGXYX, X0, Y0, Z0, D0, CZ-0.500*CH,
55 & CX-0.500*CL, CX+0.500*CL,
56 & CY-0.500*CW, CY+0.500*CW,
57 & EPS, L, DMXYX0)
58 C CALL DEMINT (MGXYX, X0, Y0, Z0, D0, CZ+0.500*CH,
59 & CX-0.500*CL, CX+0.500*CL,
60 & CY-0.500*CW, CY+0.500*CW,
61 & EPS, L, DMXYXH)
62 C
63 C DMXYX=DMXYX0-DMXYXH
64 C
65 C DMXYZ : NORTHWARD COMPONENTS
66 C
67 C CALL DEMINT (MGXY, X0, Y0, Z0, D0, CZ-0.500*CH,
68 & CX-0.500*CL, CX+0.500*CL,
69 & CY-0.500*CW, CY+0.500*CW,

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70 & EPS, L, DMXYH0)
71 C CALL DEMINT (MGYZ, X0, Y0, Z0, D0, CZ+0.500*CH,
72 & CX-0.500*CL, CX+0.500*CL,
73 & CY-0.500*CW, CY+0.500*CW,
74 & EPS, L, DMXYH)
75 C
76 C DMXYX-DMXYH0-DMXYH
77 C
78 C DMXYZ : DOWNWARD COMPONENTS
79 C
80 C CALL DEMINT (MGYZ, X0, Y0, Z0, D0, CZ-0.500*CH,
81 & CX-0.500*CL, CX+0.500*CL,
82 & CY-0.500*CW, CY+0.500*CW,
83 & EPS, L, DMXYH0)
84 C CALL DEMINT (MGYZ, X0, Y0, Z0, D0, CZ+0.500*CH,
85 & CX-0.500*CL, CX+0.500*CL,
86 & CY-0.500*CW, CY+0.500*CW,
87 & EPS, L, DMXYH)
88 C
89 C DMXYX-DMXYH0-DMXYH
90 C
91 C*** DMZ : CONTRIBUTIONS FROM X-Z PLANE ***
92 C
93 C DMZX : EASTWARD COMPONENTS
94 C
95 C CALL DEMINT (MGZX, X0, Y0, Z0, D0, CY-0.500*CW,
96 & CX-0.500*CL, CX+0.500*CL,
97 & CZ-0.500*CH, CZ+0.500*CH,
98 & EPS, L, DMZX0)
99 C CALL DEMINT (MGZX, X0, Y0, Z0, D0, CY+0.500*CW,
100 & CX-0.500*CL, CX+0.500*CL,
101 & CZ-0.500*CH, CZ+0.500*CH,
102 & EPS, L, DMZXH)
103 C
104 C DMZX-DMZX0-DMZXH
105 C
106 C DMZY : NORTHWARD COMPONENTS
107 C
108 C CALL DEMINT (MGZY, X0, Y0, Z0, D0, CY-0.500*CW,
109 & CX-0.500*CL, CX+0.500*CL,
110 & CZ-0.500*CH, CZ+0.500*CH,
111 & EPS, L, DMZY0)
112 C CALL DEMINT (MGZY, X0, Y0, Z0, D0, CY+0.500*CW,
113 & CX-0.500*CL, CX+0.500*CL,
114 & CZ-0.500*CH, CZ+0.500*CH,
115 & EPS, L, DMZYH)
116 C
117 C DMZY-DMZY0-DMZYH
118 C
119 C DMZZ : DOWNWARD COMPONENTS
120 C
121 C CALL DEMINT (MGZZ, X0, Y0, Z0, D0, CY-0.500*CW,
122 & CX-0.500*CL, CX+0.500*CL,
123 & CZ-0.500*CH, CY+0.500*CW,
124 & EPS, L, DMZZ0)
125 C CALL DEMINT (MGZZ, X0, Y0, Z0, D0, CY+0.500*CW,
126 & CX-0.500*CL, CX+0.500*CL,
127 & CZ-0.500*CH, CY+0.500*CW,
128 & EPS, L, DMZZH)
129 C
130 C DMZZ-DMZZ0-DMZZH
131 C
132 C*** DMYZ : CONTRIBUTIONS FROM Y-Z PLANE ***
133 C
134 C DMYZ : EASTWARD COMPONENTS
135 C
136 C CALL DEMINT (MGYZ, X0, Y0, Z0, D0, CX-0.500*CL,
137 & CY-0.500*CW, CY+0.500*CW,
138 & CZ-0.500*CH, CZ+0.500*CH,

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139 & EPS, L, DMYZX0)
140 CALL DEMINT (MGYZX, X0, Y0, Z0, D0, CX+0.5D0*CL,
141 & CY-0.5D0*CW, CY+0.5D0*CW,
142 & CZ-0.5D0*CH, CZ+0.5D0*CH,
143 & EPS, L, DMYZXH)
144 C
145 DMYZX=DMYZX0-DMYZXH
146 C
147 C DMYZY: NORTHWARD COMPONENTS
148 C
149 CALL DEMINT (MGYZY, X0, Y0, Z0, D0, CX-0.5D0*CL,
150 & CY-0.5D0*CW, CY+0.5D0*CW,
151 & CZ-0.5D0*CH, CZ+0.5D0*CH,
152 & EPS, L, DMYZY0)
153 CALL DEMINT (MGYZY, X0, Y0, Z0, D0, CX+0.5D0*CL,
154 & CY-0.5D0*CW, CY+0.5D0*CW,
155 & CZ-0.5D0*CH, CZ+0.5D0*CH,
156 & EPS, L, DMYZYH)
157 C
158 DMYZY=DMYZY0-DMYZYH
159 C
160 C DMYZZ: DOWNWARD COMPONENTS
161 C
162 CALL DEMINT (MGYZZ, X0, Y0, Z0, D0, CX-0.5D0*CL,
163 & CY-0.5D0*CW, CY+0.5D0*CW,
164 & CZ-0.5D0*CH, CZ+0.5D0*CH,
165 & EPS, L, DMYZZ0)
166 CALL DEMINT (MGYZZ, X0, Y0, Z0, D0, CX+0.5D0*CL,
167 & CY-0.5D0*CW, CY+0.5D0*CW,
168 & CZ-0.5D0*CH, CZ+0.5D0*CH,
169 & EPS, L, DMYZZH)
170 C
171 DMYZZ=DMYZZ0-DMYZZH
172 C
173 DMX=DMXYX+DMXZX+DMYZX
174 DMY=DMXYX+DMYZY+DMYZY
175 DMZ=DMXYZ+DMXZZ+DMYZZ
176 C
177 DMF=(DMX*CMZX+DMY*CMZY+DMZ*CMZZ)/CMZ0
178 C
179 RETURN
180 END
181 C
182 C
183 C
184 SUBROUTINE DEMINT (FUNC, X0, Y0, Z0, D0,
185 & Z, X1, XZ, A, B, EPS, L, V)
186 C
187 IMPLICIT REAL*8 (A-H, O-Z)
188 INTEGER NEND, NPOW
189 COMMON /COM1/ NEND, NPOW, AO(2),
190 & AP(608, 2), AM(608, 2), BO, BB(608)
191 C
192 DATA HALF, EPS0 / 0.5D0, 1.0D-32 /
193 DATA EPSM, EPSP / 0.D0, 0.D0 /
194 C
195 FAC = (B - A) * HALF
196 IF (L .EQ. 0) THEN
197 SHFM = (B + A) * HALF
198 SHFP = SHFM
199 ELSE
200 SHFM = 0.D0
201 SHFP = 0.D0
202 ENDIF
203 C
204 L1 = L + 1
205 C
206 IF (ABS(EPS) .GE. EPS0) THEN
207 EPSV = ABS(EPS)

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```

208 ELSE
209 EPSV = EPS0
210 ENDIF
211 C
212 EPSQ = 0.2D0 * SQRT(EPSV)
213 C
214 H = HALF
215 C
216 IS = 2**NPOW
217 IM = IS
218 C
219 KM = 0
220 KP = 0
221 NM = 0
222 NP = 0
223 C
224 VNEW = FUNC(X0, Y0, Z0, D0,
225 & AO(L1)*FAC+SHFP, Z, X1, XZ) * BO
226 *
227 * ---- INITIAL STEP ----
228 * INTEGRATE WITH MESH SIZE = 0.5
229 * AND CHECK DECAY OF INTEGRAND
230 *
231 DO 10 I = IS, NEND, IM
232 C
233 IF (KM .LE. 1) THEN
234 NM = FUNC(X0, Y0, Z0, D0,
235 & AM(I, L1)*FAC + SHFM, Z, X1, XZ) * BB(I)
236 VNEW = VNEW + NM
237 IF (ABS(NM) .LE. EPSV) THEN
238 KM = KM + 1
239 IF (KM .GE. 2) NM = I - IM
240 ELSE
241 KM = 0
242 ENDIF
243 ENDIF
244 C
245 IF (KP .LE. 1) THEN
246 NP = FUNC(X0, Y0, Z0, D0,
247 & AP(I, L1)*FAC + SHFP, Z, X1, XZ) * BB(I)
248 VNEW = VNEW + NP
249 IF (ABS(NP) .LE. EPSV) THEN
250 KP = KP + 1
251 IF (KP .GE. 2) NP = I - IM
252 ELSE
253 KP = 0
254 ENDIF
255 ENDIF
256 C
257 IF (KM .EQ. 2 .AND. KP .EQ. 2) GOTO 11
258 C
259 10 CONTINUE
260 C
261 11 CONTINUE
262 C
263 IF (NM .EQ. 0) THEN
264 NM = NEND
265 EPSM = SQRT (ABS(NM))
266 ENDIF
267 C
268 IF (NP .EQ. 0) THEN
269 NP = NEND
270 EPSP = SQRT (ABS(NP))
271 ENDIF
272 C
273 EPSQ = MAX (EPSQ, EPSM, EPSP)
274 *
275 * ---- GENERAL STEP ----
276 *

```

```

277 VOLD = H * FAC * VNEW
278 C
279 DO 20 MSTEP = 1, NPOW
280 C
281 VNEW = 0.0
282 C
283 IH = IS
284 IS = IS / 2
285 C
286 DO 540 I = IS, NM, IH
287 VNEW = VNEW
288 & + FUNC(X0, Y0, Z0, D0,
289 & AM(I, L1)*FAC + SHFP, Z, X1, X2) * BB(I)
290 540 CONTINUE
291 C
292 DO 550 I = IS, NP, IH
293 VNEW = VNEW
294 & + FUNC(X0, Y0, Z0, D0,
295 & AP(I, L1)*FAC + SHFP, Z, X1, X2) * BB(I)
296 550 CONTINUE
297 C
298 VNEW = (VOLD + H * FAC * VNEW) * HALF
299 C
300 IF (ABS(VNEW - VOLD) .LT. EPSQ) THEN
301 *
302 * ----- CONVERGED AND RETURN -----
303 *
304 V = VNEW
305 RETURN
306 ENDF
307 C
308 H = H * HALF
309 VOLD = VNEW
310 C
311 20 CONTINUE
312 C
313 V = VNEW
314 RETURN
315 C
316 END
317 C
318 C
319 C
320 SUBROUTINE FIAB (NEND, NPOW, AO, AM, AP, BO, BB)
321 C
322 IMPLICIT REAL*8 (A-H, O-Z)
323 DIMENSION AM(608,2), AO(2), AP(608,2), BB(608)
324 C
325 PARAMETER (ONE = 1)
326 PARAMETER (HALF = ONE / 2)
327 PARAMETER (NG = 6)
328 PARAMETER (H = ONE / 2**(NG + 1))
329 C
330 A9 = 0.9999 9999 9999 9998 D0
331 *
332 * ----- START COMPUTATION OF POINTS AND WEIGHTS -----
333 *
334 PH = 2 * ATAN (ONE)
335 C
336 NPOW = NG
337 NEND = 608
338 C
339 AO(1) = 0.D0
340 AO(2) = 1.D0
341 BO = PH
342 C
343 EH = EXP (H)
344 EN = 1.D0
345 C

```

```

346 DO 10 I = 1, NEND
347 EN = EH * EN
348 ENI = 1.D0 / EN
349 SH = (EN - ENI) * HALF
350 CH = (EN + ENI) * HALF
351 EXS = EXP (PH * SH)
352 EXSI = 1.D0 / EXS
353 CHSI = 2.D0 / (EXS + EXSI)
354 AP(I,1) = ((EXS - EXSI) * HALF) * CHSI
355 IF (AP(I,1) .GE. A9) AP(I,1) = A9
356 AP(I,2) = EXSI * CHSI
357 AM(I,1) = - AP(I,1)
358 AM(I,2) = - AP(I,2)
359 BB(I) = PH * CH * CHSI**2
360 10 CONTINUE
361 C
362 AP(608,2) = AP(607,2)
363 AM(608,2) = AM(607,2)
364 C
365 RETURN
366 END
367 C
368 C
369 C
370 DOUBLE PRECISION FUNCTION MGXYX(X0, Y0, Z0, D0, Y, Z, X1, X2)
371 IMPLICIT REAL*8 (A-H, O-Z)
372 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
373 COMMON /COM2/ C00, C0X, C0Y, C0Z
374 C
375 ZTZ = Y*Y + (Z-D0)*(Z-D0)
376 CCZ = (Y0-Y)*(Y0-Y) + (Z0-Z)*(Z0-Z)
377 ZT = SQRT(ZTZ)
378 CC = SQRT(CCZ)
379 C
380 CALL PHSII(X0, ZT, CC, X1, X2, PHI, PSI)
381 C
382 MGXYX = C0X*(3.D0*(Z0-Z)*(X0*PHI(3,5)-PSI(3,5)))
383 & + C0Y*(3.D0*(Z0-Z)**2*PHI(3,5))
384 & + C0Z*(3.D0*(Z0-Z)**2*PHI(3,5))
385 RETURN
386 END
387 C
388 C
389 C
390 DOUBLE PRECISION FUNCTION MGXYX(X0, Y0, Z0, D0, Y, Z, X1, X2)
391 IMPLICIT REAL*8 (A-H, O-Z)
392 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
393 COMMON /COM2/ C00, C0X, C0Y, C0Z
394 C
395 ZTZ = Y*Y + (Z-D0)*(Z-D0)
396 CCZ = (Y0-Y)*(Y0-Y) + (Z0-Z)*(Z0-Z)
397 ZT = SQRT(ZTZ)
398 CC = SQRT(CCZ)
399 C
400 CALL PHSII(X0, ZT, CC, X1, X2, PHI, PSI)
401 C
402 MGXYX = C0X*(3.D0*(Z0-Z)*(Y0-Y)*(PHI(3,5)-PSI(3,5)))
403 & + C0Y*(3.D0*(Z0-Z)**2*(Y0-Y)*PHI(3,5))
404 & + C0Z*(3.D0*(Z0-Z)**2*(Y0-Y)*PHI(3,5))
405 RETURN
406 END
407 C
408 C
409 C
410 DOUBLE PRECISION FUNCTION MGXYZ(X0, Y0, Z0, D0, Y, Z, X1, X2)
411 IMPLICIT REAL*8 (A-H, O-Z)
412 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
413 COMMON /COM2/ C00, C0X, C0Y, C0Z
414 C

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```

415 ZT2=Y*Y+(Z-D0)*(Z-D0)
416 CC2=(Y0-Y)*(Y0-Y)+(Z0-Z)*(Z0-Z)
417 ZT=SQRT(ZT2)
418 CC=SQRT(CC2)
419 C
420 CALL PHSI1J(X0, ZT, CC, X1, X2, PHI, PSI)
421 C
422 MGXYZ=C0X*(3.D0*(Z0-Z)*(Y0-Y)*(PHI(3,3)-PSI(3,5)))
423 & +C0Y*(3.D0*(Z0-Z)*Y*(Y0-Y)*PHI(3,5))
424 & +C0Z*(3.D0*(PHI(1,1)-(Z0-Z)*(Z0-Z)*PHI(3,5)))
425 RETURN
426 END
427 C
428 C
429 C
430 DOUBLE PRECISION FUNCTION MGXZX(X0, Y0, Z0, D0, Z, Y, X1, X2)
431 IMPLICIT REAL*8 (A-H, O-Z)
432 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
433 COMMON /COM2/ C00, C0X, C0Y, C0Z
434 C
435 ZT2=Y*Y+(Z-D0)*(Z-D0)
436 CC2=(Y0-Y)*(Y0-Y)+(Z0-Z)*(Z0-Z)
437 ZT=SQRT(ZT2)
438 CC=SQRT(CC2)
439 C
440 CALL PHSI1J(X0, ZT, CC, X1, X2, PHI, PSI)
441 C
442 MGXZ=C0X*(3.D0*(Y0-Y)*(X0*PHI(3,3)-PSI(3,5)))
443 & +C0Y*(3.D0*(Y0-Y)*Y*PHI(3,5))
444 & +C0Z*(3.D0*(Y0-Y)*Z*PHI(3,5))
445 RETURN
446 END
447 C
448 C
449 C
450 DOUBLE PRECISION FUNCTION MGXZY(X0, Y0, Z0, D0, Z, Y, X1, X2)
451 IMPLICIT REAL*8 (A-H, O-Z)
452 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
453 COMMON /COM2/ C00, C0X, C0Y, C0Z
454 C
455 ZT2=Y*Y+(Z-D0)*(Z-D0)
456 CC2=(Y0-Y)*(Y0-Y)+(Z0-Z)*(Z0-Z)
457 ZT=SQRT(ZT2)
458 CC=SQRT(CC2)
459 C
460 CALL PHSI1J(X0, ZT, CC, X1, X2, PHI, PSI)
461 C
462 MGXYZ=C0X*(3.D0*(Y0-Y)*(Y0-Y)*(PHI(3,3)-PSI(3,5)))
463 & +C0Y*(3.D0*(Y0-Y)*Y*(Y0-Y)*PHI(3,5))
464 & +C0Z*(3.D0*(Y0-Y)*Z*(Y0-Y)*PHI(3,5))
465 RETURN
466 END
467 C
468 C
469 C
470 DOUBLE PRECISION FUNCTION MGXZZ(X0, Y0, Z0, D0, Z, Y, X1, X2)
471 IMPLICIT REAL*8 (A-H, O-Z)
472 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
473 COMMON /COM2/ C00, C0X, C0Y, C0Z
474 C
475 ZT2=Y*Y+(Z-D0)*(Z-D0)
476 CC2=(Y0-Y)*(Y0-Y)+(Z0-Z)*(Z0-Z)
477 ZT=SQRT(ZT2)
478 CC=SQRT(CC2)
479 C
480 CALL PHSI1J(X0, ZT, CC, X1, X2, PHI, PSI)
481 C
482 MGXZZ=C0X*(3.D0*(Y0-Y)*(Y0-Y)*(PHI(3,3)-PSI(3,5)))
483 & +C0Y*(3.D0*Y*(Y0-Y)*(Y0-Y)*PHI(3,5))

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484 & +C0Z*(3.D0*(Y0-Y)*PHI(3,3))
485 RETURN
486 END
487 C
488 C
489 C
490 DOUBLE PRECISION FUNCTION MGYZX(X0, Y0, Z0, D0, X, Z, Y1, Y2)
491 IMPLICIT REAL*8 (A-H, O-Z)
492 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
493 COMMON /COM2/ C00, C0X, C0Y, C0Z
494 C
495 ZT2=X*X+(Z-D0)*(Z-D0)
496 CC2=(X0-X)*(X0-X)+(Z0-Z)*(Z0-Z)
497 ZT=SQRT(ZT2)
498 CC=SQRT(CC2)
499 C
500 CALL PHSI1J(Y0, ZT, CC, Y1, Y2, PHI, PSI)
501 C
502 MGYZ=C0X*(3.D0*(X0-X)*X*PHI(3,5))
503 & +C0Y*(3.D0*(X0-X)*(X0*PHI(3,3)-PSI(3,5)))
504 & +C0Z*(3.D0*(X0-X)*Z*PHI(3,5))
505 RETURN
506 END
507 C
508 C
509 C
510 DOUBLE PRECISION FUNCTION MGYZY(X0, Y0, Z0, D0, Z, Y, X1, X2)
511 IMPLICIT REAL*8 (A-H, O-Z)
512 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
513 COMMON /COM2/ C00, C0X, C0Y, C0Z
514 C
515 ZT2=X*X+(Z-D0)*(Z-D0)
516 CC2=(X0-X)*(X0-X)+(Z0-Z)*(Z0-Z)
517 ZT=SQRT(ZT2)
518 CC=SQRT(CC2)
519 C
520 CALL PHSI1J(Y0, ZT, CC, Y1, Y2, PHI, PSI)
521 C
522 MGYZY=C0X*(3.D0*(X0-X)*(X0-X)*PHI(3,5))
523 & +C0Y*(3.D0*(X0-X)*(Y0*PHI(3,3)-PSI(3,5)))
524 & +C0Z*(3.D0*(X0-X)*Z*(X0-X)*PHI(3,5))
525 RETURN
526 END
527 C
528 C
529 C
530 DOUBLE PRECISION FUNCTION MGYZZ(X0, Y0, Z0, D0, Z, Y, X1, X2)
531 IMPLICIT REAL*8 (A-H, O-Z)
532 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)
533 COMMON /COM2/ C00, C0X, C0Y, C0Z
534 C
535 ZT2=X*X+(Z-D0)*(Z-D0)
536 CC2=(X0-X)*(X0-X)+(Z0-Z)*(Z0-Z)
537 ZT=SQRT(ZT2)
538 CC=SQRT(CC2)
539 C
540 CALL PHSI1J(Y0, ZT, CC, Y1, Y2, PHI, PSI)
541 C
542 MGYZZ=C0X*(3.D0*(X0-X)*(X0-X)*PHI(3,5))
543 & +C0Y*(3.D0*(X0-X)*(Y0*PHI(3,3)-PSI(3,5)))
544 & +C0Z*(3.D0*(X0-X)*Z*(X0-X)*PHI(3,5))
545 RETURN
546 END
547 C
548 C
549 C
550 SUBROUTINE PHSI1J(X0, ZT, CC, X1, X2, PHI, PSI)
551 IMPLICIT REAL*8 (A-H, O-Z)
552 DIMENSION DI(10,10), PHI(10,10), PSI(10,10)

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```

553 C ZT2-ZT*ZT
554 CC2-CC*CC
555
556 C
557 CALL PQAB(X0,ZT2,CC2,X1,X2,ALP,BET,PALP,PBET,
558 & QALP,QBET,DK,XI,ET,T1,T2,PH1,PH2)
559 CALL ELLIPFE(PH1,DK,ELPF1,ELPE1)
560 CALL ELLIPFE(PH2,DK,ELPF2,ELPE2)
561 ELPF=ELPF2-ELPF1
562 ELPE=ELPE2-ELPE1
563 C
564 XI2=XI*XI
565 ET2=ET*ET
566 TX1=T1*T1+XI2
567 TX2=T2*T2+XI2
568 TE1=T1*T1+ET2
569 TE2=T2*T2+ET2
570 C
571 I11
572 DI(1,1)=ELPF/XI
573 I13
574 DI(1,3)=(XI2*ELPE-ET2*ELPF)/(XI*ET2*(XI2-ET2))
575 I31
576 DI(3,1)=(ELPF-ELPE)/(XI*(XI2-ET2))
577 & +T2/(XI2*SQRT(TX2*TE2))-T1/(XI2*SQRT(TX1*TE1))
578 I33
579 DI(3,3)=((XI2+ET2)*ELPE-2.D0*ET2*ELPF)/(XI*ET2*
580 & (XI2-ET2)*(XI2-ET2))
581 & +(T2/(XI2*SQRT(TX2*TE2)))
582 & -T1/(XI2*SQRT(TX1*TE1)))/(XI2*(XI2-ET2))
583 C
584 I51
585 DI(5,1)=(3.D0*XI2-ET2)*ELPF
586 & -2.D0*(2.D0*XI2-ET2)*ELPE)/(3.D0*XI2*XI*(XI2-ET2)*(XI2-ET2))
587 & +(3.D0*XI2-2.D0*ET2)*(T2/(XI2*SQRT(TX2*TE2)))
588 & -T1/(XI2*SQRT(TX1*TE1)))/(3.D0*XI2*(XI2-ET2))
589 & +(T2/(XI2*SQRT(TX2*TE2)*TE1))/(3.D0*XI2)
590 I35
591 DI(3,5)=(DI(3,3)-DI(5,1))/(XI2-ET2)
592 C
593 AB=ALP-BET
594 C
595 C
596 PHI11,PSI11
597 PHI(1,1)=AB*DI(1,1)/SQRT(PALP*QALP)
598 PSI(1,1)=AB*(QBET*DI(1,1)
599 & +0.500*ALP*(LOG(XI2*ET2-2.D0*T2*T2+2.D0*SQRT(TE2*TX2))
600 & -LOG(XI2*ET2-2.D0*T1*T1-2.D0*SQRT(TE1*TX1))))/SQRT(PALP*QALP)
601 C
602 PHI33
603 PHI(3,3)=AB*(DI(1,1)-(XI2*ET2-6.D0)*DI(1,3)
604 & +(XI2*XI2-6.D0*XI2+1.D0)*DI(3,3)
605 & +2.D0*(SQRT(TX2*TE2)
606 & +(XI2-1.D0)/((ET2-XI2)*(ET2-XI2)*TX2)
607 & +(ET2-1.D0)/(ET2-XI2)*(ET2-XI2)*TE2))
608 & -2.D0*(SQRT(TX1*TE1)
609 & +(XI2-1.D0)/((ET2-XI2)*(ET2-XI2)*TX1)
610 & +(ET2-1.D0)/((ET2-XI2)*(ET2-XI2)*TE1)))/(SQRT(PALP*QALP)**3)
611 C
612 PHI35,PSI35
613 PHI(3,5)=AB*(DI(1,1)-(XI2+2.D0*ET2-15.D0)*DI(3,1)
614 & +3.D0*(ET2*ET2-10.D0*ET2+5.D0)*DI(3,3)
615 & -(ET2**3-15.D0*ET2*ET2+15.D0*ET2-1.D0)*DI(3,5))
616 PSI(3,5)=AB*(ALP*DI(1,1)-(ALP*(XI2+2.D0*ET2)
617 & -5.D0*(2.D0*ALP*BET))*DI(3,1)
618 & +(3.D0*ALP*ET2*ET2-10.D0*ET2*(2.D0*ALP*BET)
619 & +5.D0*(ALP+2.D0*BET))*DI(3,3)
620 & -(ALP*ET2**3-5.D0*ET2*ET2*(2.D0*ALP*BET)
621 & +5.D0*ET2*(ALP+2.D0*BET)+BET)*DI(3,5))
622 C
623 C
624 SUBROUTINE PQAB(X0,ZT2,CC2,X1,X2,ALP,BET,PALP,PBET,
625 & QALP,QBET,DK,XI,ET,T1,T2,PH1,PH2)
626 IMPLICIT REAL*8 (A-H,O-Z)
627 DATA F0,F1,F0,D0,1.D0/
628 C
629 X02=X0*X0
630 IF(X0.NE.F0) THEN
631 DET2=(CC2+X02-ZT2)*(CC2+X02-ZT2)+4.D0*X02*ZT2
632 DET=SQRT(DET2)
633 ALP = -2.D0 * X0 * ZT2 / (CC2+X02-ZT2+DET)
634 BET=(CC2+X02-ZT2-DET)/(2.D0*X0)
635 ALP2=ALP*ALP
636 BET2=BET*BET
637 C
638 PALP=ALP2+ZT2
639 PBET=BET2+ZT2
640 QALP=(X0-ALP)*(X0-ALP)+CC2
641 QBET=(X0-BET)*(X0-BET)+CC2
642 C
643 T1=(X1-BET)/(ALP-X1)
644 T2=(X2-BET)/(ALP-X2)
645 XI2=PBET/PALP
646 ET2=QBET/QALP
647 XI=SQRT(XI2)
648 ET=SQRT(ET2)
649 ELSE
650 T1=X1
651 T2=X2
652 IF(ZT2.GT.CC2) THEN
653 XI2=ZT2
654 ET2=CC2
655 ELSE
656 XI2=CC2
657 ET2=ZT2
658 ENDF
659 XI=SQRT(XI2)
660 ET=SQRT(ET2)
661 ALP=F1
662 BET=F0
663 PALP=F1
664 QALP=F1
665 ENDF
666 DK2=(XI2-ET2)/XI2
667 DK=SQRT(DK2)
668 PH1=DATAN(T1/ET)
669 PH2=DATAN(T2/ET)
670 C
671 RETURN
672 END
673 C
674 C
675 C
676 SUBROUTINE ELLIPFE(PH0,G0,ELPF,ELPE)
677 IMPLICIT REAL*8 (A-H,O-Z)
678 DATA F0,F1,PI,EPS,INF/0.D0,1.D0,3.1415926535897932D0,
679 & 2.22044604925031D-16,2.D16/
680 C
681 G=ABS(G0)
682 GZ=G*G
683 PH=PH0
684 C
685 IF(C.EQ.F1) THEN
686 ELPE = DSIN(PH)
687 IF(ABS(PH0).LT.PI) ELPF=DLOG((F1+DSIN(PH))/DCOS(PH))
688 IF(ABS(PH0).GE.PI) ELPF=INF
689 GOTO 300
690 ENDF

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```

622 C
623 C
624 SUBROUTINE PQAB(X0,ZT2,CC2,X1,X2,ALP,BET,PALP,PBET,
625 & QALP,QBET,DK,XI,ET,T1,T2,PH1,PH2)
626 IMPLICIT REAL*8 (A-H,O-Z)
627 DATA F0,F1,F0,D0,1.D0/
628 C
629 X02=X0*X0
630 IF(X0.NE.F0) THEN
631 DET2=(CC2+X02-ZT2)*(CC2+X02-ZT2)+4.D0*X02*ZT2
632 DET=SQRT(DET2)
633 ALP = -2.D0 * X0 * ZT2 / (CC2+X02-ZT2+DET)
634 BET=(CC2+X02-ZT2-DET)/(2.D0*X0)
635 ALP2=ALP*ALP
636 BET2=BET*BET
637 C
638 PALP=ALP2+ZT2
639 PBET=BET2+ZT2
640 QALP=(X0-ALP)*(X0-ALP)+CC2
641 QBET=(X0-BET)*(X0-BET)+CC2
642 C
643 T1=(X1-BET)/(ALP-X1)
644 T2=(X2-BET)/(ALP-X2)
645 XI2=PBET/PALP
646 ET2=QBET/QALP
647 XI=SQRT(XI2)
648 ET=SQRT(ET2)
649 ELSE
650 T1=X1
651 T2=X2
652 IF(ZT2.GT.CC2) THEN
653 XI2=ZT2
654 ET2=CC2
655 ELSE
656 XI2=CC2
657 ET2=ZT2
658 ENDF
659 XI=SQRT(XI2)
660 ET=SQRT(ET2)
661 ALP=F1
662 BET=F0
663 PALP=F1
664 QALP=F1
665 ENDF
666 DK2=(XI2-ET2)/XI2
667 DK=SQRT(DK2)
668 PH1=DATAN(T1/ET)
669 PH2=DATAN(T2/ET)
670 C
671 RETURN
672 END
673 C
674 C
675 C
676 SUBROUTINE ELLIPFE(PH0,G0,ELPF,ELPE)
677 IMPLICIT REAL*8 (A-H,O-Z)
678 DATA F0,F1,PI,EPS,INF/0.D0,1.D0,3.1415926535897932D0,
679 & 2.22044604925031D-16,2.D16/
680 C
681 G=ABS(G0)
682 GZ=G*G
683 PH=PH0
684 C
685 IF(C.EQ.F1) THEN
686 ELPE = DSIN(PH)
687 IF(ABS(PH0).LT.PI) ELPF=DLOG((F1+DSIN(PH))/DCOS(PH))
688 IF(ABS(PH0).GE.PI) ELPF=INF
689 GOTO 300
690 ENDF

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```

691 C
692 A0=F1
693 B0=DSQRT(F1-G2)
694 P=F1
695 ARG=F0
696 CE1=F1
697 CE2=F0
698 100 A = 0.5D0*(A0+B0)
699 B = DSQRT(A0*B0)
700 C = 0.5D0*(A0-B0)
701 CE1 = CE1-(2.D0**P)*(A*C)
702 IF(PH0.NE.90.D0) THEN
703 ARG = REAL(NINT(PH/PI))
704 IF(PH.GT.F0.AND.MOD(PH/PI+0.5D0,F1).EQ.F0) ARG=ARG-F1
705 PH = PH-DATAN(B0*DTAN(PH)/A0)+PI*ARG
706 CE2 = CE2+C*DSIN(PH)
707 ENDIF
708 C
709 IF(ABS(A-A0).LE.EPS.AND.ABS(B-B0).LE.EPS.
& AND.ABS(A-B).LE.EPS) THEN
710 GOTO 200
711 ELSE
712 A0 = A
713 B0 = B
714 P = P-F1
715 P = P-F1
716 GOTO 100
717 ENDIF
718 C
719 200 IF(PH0.NE.PI) AGMPH = PH/(2.D0**P)
720 IF(PH0.EQ.PI) AGMPH = PI/2.D0
721 ELPE = AGMPH/A
722 ELPE = CE1*AGMPH/A+CE2
723 300 RETURN
724 END

```