



HOKKAIDO UNIVERSITY

Title	On limit strength of frozen clay undergoing triaxial tension
Author(s)	Nishimura, Satoshi; Kawasaki, Hidetoshi; Sato, Issei
Citation	Géotechnique, 1-13 https://doi.org/10.1680/jgeot.22.00192
Issue Date	2023-01-19
Doc URL	https://hdl.handle.net/2115/91068
Type	journal article
File Information	manuscript_HUSCAP.pdf



1 ON LIMIT STRENGTH OF FROZEN CLAY UNDERGOING TRIAXIAL TENSION

2
3 Satoshi Nishimura^{1,*}, Hidetoshi Kawasaki² and Issei Sato³

4
5 ¹Faculty of Engineering, Hokkaido University, Japan (*Corresponding author)

6 ²Formerly School of Engineering, Hokkaido University, Japan

7 ³Kajima Technical Research Institute, Kajima Corporation, Japan

8
9
10 **ABSTRACT**

11 A new interpretation of failure of frozen clay in tension is presented, through a unique test program
12 applying ‘triaxial tension’, in which the axial stress was reduced to negative values while zero or
13 positive radial stress was maintained. The limit strength is determined by a meeting point of two
14 competing mechanisms – strain-hardening in shear, and progressively degrading tensile strength
15 as the shear deformation disturbs the soil structure. In some conditions, the limit strength observed
16 in tension is actually given by the shear resistance limit, not by a true tensile strength. This
17 interpretation is successful in explaining apparently unclear confining stress-dependency of the
18 observed failure envelopes at low stresses. The influence of the pre-freezing effective stress, post-
19 freezing total stress and pre-freezing pore water pressure on the strength was also investigated.
20 The tensile strength degradation was formulated by using the specific work input to estimate the
21 operational tensile strength and the strain at which a tensile rupture occurs. Cooling after shear
22 deformation apparently healed the damage and recovered the initial tensile strength, allowing the
23 stress-strain curve to significantly overshoot the proposed tensile strength line. Explaining the
24 combined influence of shear and temperature history on the tensile strength – work relationships
25 requires further study.

26
27 **INTRODUCTION**

28 Recent underground projects involve complex excavation processes at greater depths reaching
29 40-50 m more commonly than ever, often adjacent to existing substructures and/or under soft and
30 unstable ground conditions. Some processes, such as tunnel connection and enlargement, require
31 prior stabilisation of surrounding ground. Artificial Ground Freezing (AGF) is an effective technique
32 to stabilise selected localities with high accuracy and reliability. As the recent innovation allows
33 freezing at lower temperatures (for example, circulation of CO₂, in lieu of conventional brine, can
34 cool the chiller pipes down to -45 °C; Tsuji and Yoshida, 2019), frozen domains are expected to
35 mobilise higher strength. Even colder liquid nitrogen (LN) has long been available for smaller-scale,
36 short-term work, but it is usually produced in remote factory plants, and the freezing was typically
37 one-off and expensive. The above development opens up more possibilities in design, for example
38 by using thinner frozen soil as load-bearing roofs or earth-retaining walls. While keeping the frozen

39 soil thinner is highly cost-effective due to significant reduction in the required heat-conduction time,
40 it risks subjecting the soil to bending in some parts. In this scenario, the design based only on the
41 compressive or shear strength of frozen soil is inappropriate, as the tensile strength limits the
42 bending resistance of the frozen soil. These facts point to importance of an appropriate testing
43 method to characterise the tensile failure in frozen soils, and mechanical theory and insights to
44 interpret the results.

45

46 In the context of deep underground application, it is important to note that considerable total and
47 effective stresses, as well as large pore pressure, exist at the time of freezing, and that they may
48 partially remain even during the excavation processes. As will be reviewed, conventional tension
49 tests on frozen soils have been almost all performed in unconfined states, mostly with soil samples
50 frozen unconfined too. This conventional approach severely limits the possibility of interpreting the
51 frozen soil's failure behaviour in tension by drawing a parallel to the effective-stress-based unfrozen
52 soil mechanics, as will be discussed in the next section. A particularly relevant example of deep
53 AGF application is Trans-Tokyo Bay (Aqua-Line) Expressway in Japan (e.g. Uchida et al., 1993).
54 This project involved construction of submarine tunnels passing through soft alluvial clay at some
55 40-55 m below the sea level (20-35 m below the sea bottom) and vertical shafts connected to them,
56 assisted partially by AGF (e.g. Akagawa, 2021). The pore pressure at the invert level was about
57 600 kPa. A high pore (back) pressure usually prevents tensile failure in unfrozen soft clays by
58 preventing cavitation. It is not clear, however, how this condition affects the tensile failure of frozen
59 clay, nor is it possible to address this issue by the conventional testing methods with no control of
60 confining stress and pore pressure. AGF case histories under similarly deep settings with high
61 water pressure or soft soil conditions have also been reported elsewhere (e.g. Ou et al., 2009;
62 Viggiani et al., 2015).

63

64 This study adopted a new laboratory test approach by developing 'triaxial tension' testing, in which
65 tensile loading was undertaken with lateral confinement. In contrast to conventional triaxial
66 extension, it allows the minor (total) principal stress σ_3 to go negative (i.e. active pulling). Freezing
67 was performed directly after consolidation while the specimen remained under confinement, by
68 improving the method by Wang et al. (2017). To the authors' knowledge, no similar attempt has
69 been reported so far. This study, after considerable trial and error, eventually found a set-up
70 allowing stable temperature control and providing repeatable results.

71

72 This paper will firstly review the basic mechanics of unfrozen soils under tensile loading, which may
73 involve either shear failure or tensile failure. A review of previous studies on tensile strength of
74 frozen soils will be then given to underline the need for this study's approach. In the authors' view,
75 investigation on the tensile failure of frozen soils must answer the following questions:

76 (i) Is the limit strength of frozen soils in tension determined by a 'tensile strength' (i.e. σ_3 limit),

- 77 independently from shear strength (deviator stress q limit)?
78 (ii) Is the tensile strength affected by the initial, pre-freezing pore pressure and the effective stress?
79 (iii) Is the peak tensile limit, whatever (i) reveals, to be relied upon in design, given the eventually
80 inevitable strain-softening and fracturing?
81 (iv) How are all the above affected by conditions such as water content w (intertwined with the
82 above issue of the consolidation effective stress), temperature and strain rate?

83 The present study provides new, if not comprehensive, insights to these questions through a
84 systematic series of high-quality, repeatable triaxial tension tests in which the pre- and post-freezing
85 stress and the initial pore water pressure were systematically varied. Additional tests adopted
86 complex loading histories involving temperature changes, lateral stress changes and a loading
87 pause.

88

89 This study only considers saturated clay, as initial phase of more thorough investigation. This paper
90 uses 'extension' to mean geometric elongation, and 'tension' to mean imposition of an absolute
91 negative normal stress.

92

93 TENSILE STRENGTH OF UNFROZEN AND FROZEN SATURATED SOILS: A REVIEW

94 *Basic mechanics of (unfrozen) saturated soils under tensile loading*

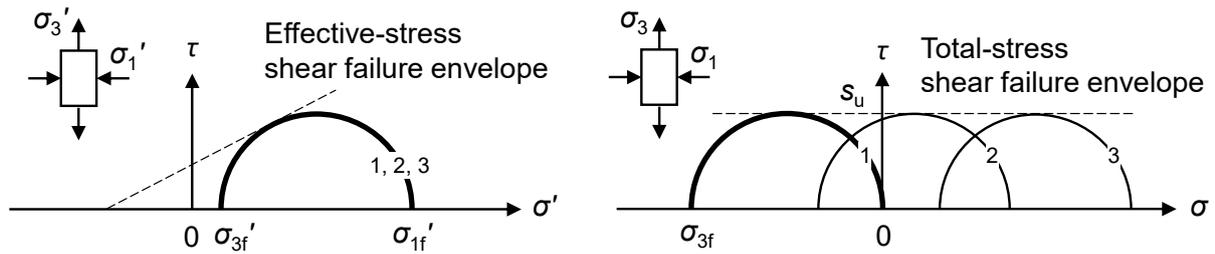
95 It may be helpful to briefly review elementary unfrozen soil mechanics concerning tensile failure.
96 The discussion shall be limited to saturated soil in undrained triaxial conditions. In the simplest case
97 (**Figure 1(a)**), when the initial pore water pressure (or the back pressure) u_0 has ample room for
98 reduction, any total stress change, including the average confining total stress decrease due to
99 tensile loading, is compensated by a negative excess pore water pressure under undrained
100 conditions. As a result, the eventual effective stress is unaffected by the total stress changes, and
101 thus the failure occurs when the deviator stress q reaches $2s_u$, where s_u is the undrained shear
102 strength. The pattern is basically a shear failure in triaxial extension, even when the soil is
103 undergoing tension in total stress term (i.e. negative σ_3).

104

105 The second pattern, illustrated in **Figure 1(b)**, occurs when the pore pressure u decreases to a
106 limit value u_{\min} , and ceases to compensate the total stress changes for any more σ_3 reduction. In
107 coarse-grained soils, this may correspond to cavitation pressure of the pore fluid. For finer-grained
108 soils, u_{\min} may be smaller than the vacuum and sustain high suction. If the pore pressure remains
109 at u_{\min} for further tensile loading, the total stress reduction is directly reflected on the effective stress,
110 and when the latter reaches the failure envelope, failure will occur. It is more likely in reality that,
111 when the pore pressure reaches u_{\min} , the system cannot maintain undrained conditions, and u_{\min}
112 rebounds to the atmospheric pressure, causing abrupt tensile failure. For this scenario, the total
113 stress application under undrained conditions before tensile loading influences the eventual tensile
114 strength by raising u , and allowing σ_3 to decrease more before u decreases to u_{\min} .

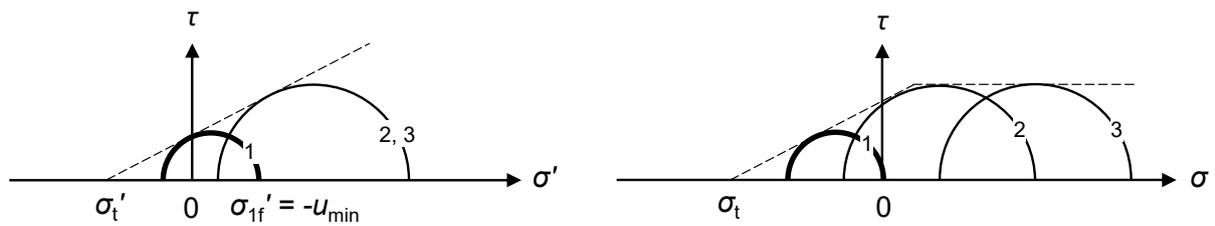
115
 116
 117
 118
 119
 120
 121
 122
 123

In fact, there is no a priori reason to assume that the effective-stress failure envelope is linear. The true tensile strength σ_t' on the effective stress σ' -axis may not be where extrapolation of the shear failure line would suggest. The third pattern, illustrated in **Figure 1(c)**, is a likely response in this case. When the failure is caused by u meeting u_{\min} , and σ' meeting σ_t' , this is considered to be a pure tensile failure, with σ_3 at failure unaffected by initial confining stress (Mohr's stress circles 1-3 in **Figure 1(c)** right).



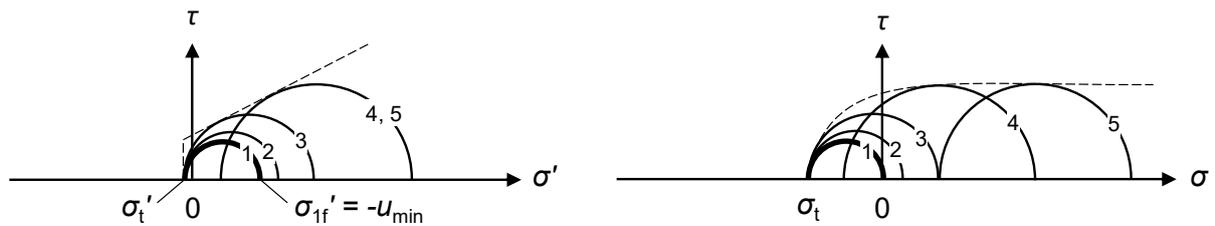
124
 125

(a) Case of sufficient back pressure u_0 relative to s_u : Shear failure



126
 127

(b) Case of small back pressure u_0 relative to s_u : Shear failure with pore cavitation



128
 129
 130

(c) Case of small back pressure u_0 relative to s_u and tension cutoff: Tensile failure with pore cavitation

131
 132
 133
 134
 135

Figure 1: Failure scenarios in triaxial tension in unfrozen saturated soils
 (subscript f denotes failure state)

136
 137

These scenarios are relatively simple to discern when the pore water pressure is measured in unfrozen soil. In frozen soil, however, measuring liquid and ice water pressures accurately is not

138 an established technique, and no direct manner of observation is possible. The idea in this study
139 is therefore to observe how the σ_3 limit in tensile loading varies with the major principal stress σ_1 .
140 If σ_3 at failure, σ_{3f} , shifts in parallel with σ_1 , what is perceived as ‘tensile strength’ in frozen soil may
141 in reality be governed by the shear strength. If σ_{3f} is unaffected by σ_1 , on the contrary, the pore
142 materials (ice and liquid) and the soil skeleton may be truly being pulled apart on the perpendicular
143 plane. This logic must also be checked with visual observation of the failed specimen.

144

145 *Testing methods for tensile strength*

146 The tensile strength of frozen soils has been conventionally investigated by three testing methods;
147 direct tension, Brazilian splitting and beam-bending, as illustrated in **Figure 2**. Haynes and Karalius
148 (1977), Eckardt (1982) and Zhu and Carbee (1985; 1987), among others, adopted cylindrical
149 specimens with enlarged ends in direct tension. Eckardt (1982) penetrated nails to the specimen
150 ends to ensure the soil-machine coupling. Zhang et al. (2019) tested regular cylindrical specimens,
151 attached to loading plates with epoxy adhesive. In the other studies, the coupling mechanisms are
152 not clearly explained. Akagawa and Nishisato (2009) and Li et al. (2018) adopted ‘8-shaped’
153 specimens (the leftmost in **Figure 2**), following Tamrakar et al. (2007). All these studies performed
154 tension tests in unconfined states. One possible exception is a study by ‘Jessberger/Ebel’
155 mentioned in Jessberger’s (1981) state-of-the-art paper, of which drawing suggests application of
156 radial pressure. However, the source is not cited, and more detail could not be found. Apart from
157 this, there has been no systematic study conducting ‘triaxial tension’ testing, to the authors’
158 knowledge.

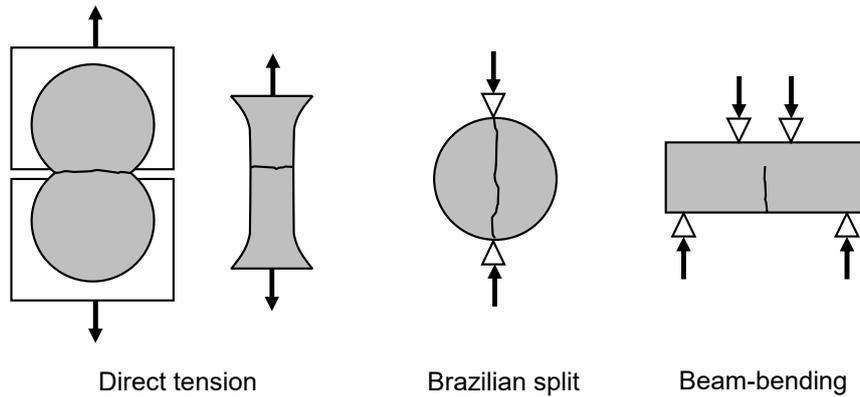
159

160 Brazilian split (Bragg and Andersland, 1982; Zhou et al., 2015; You et al., 2021) and beam-bending
161 (Azmatch et al., 2011; Yamamoto and Springman, 2017) tests have also been performed to obtain
162 the tensile strength or observe fracture processes of frozen soils. While these tests are relatively
163 straightforward in principle, the stress field is non-uniform within the specimen. Namikawa and
164 Koseki (2007) compared the above two testing methods with direct tension for cement-treated sand
165 both experimentally and numerically, and concluded that the direct tension test gives the most
166 accurate tensile strength, while the other two are affected by local shear and progressive failure. It
167 would be also difficult to apply confining stress in these two testing methods.

168

169 The literature review suggests that there has been no experimental data or attempt of applying
170 tension from confined states, which, as discussed earlier, would clarify at what conditions the frozen
171 soil fails in shear or reaches a true tensile limit. This background led to development of triaxial
172 tension testing in this study.

173



Direct tension

Brazilian split

Beam-bending

Figure 2: Conventional tensile strength tests for soils

174

175

176

177

178

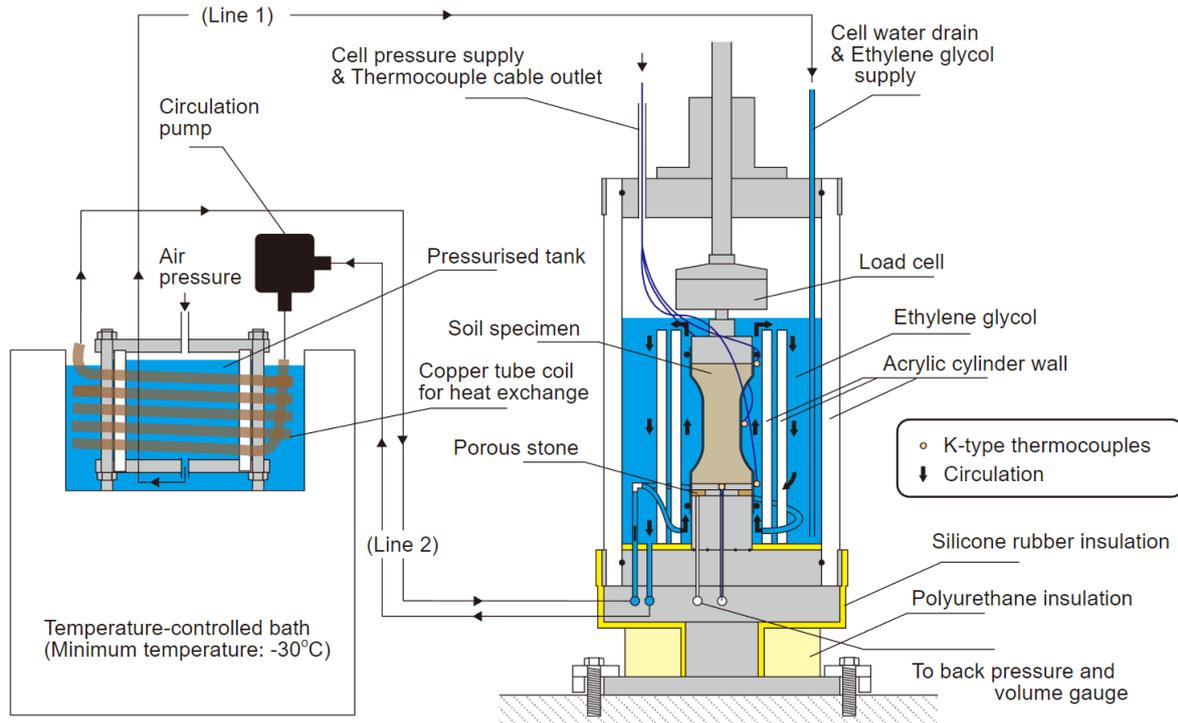
179 MATERIAL AND TESTING METHOD

180 *Apparatus*

181 Triaxial apparatus was modified to allow direct tension testing of frozen soils under confining
 182 pressure. The system is illustrated in **Figure 3**. The whole system was installed and operated in a
 183 room at a temperature of 25 °C. A unique feature of this apparatus is that it allows freezing a soil
 184 specimen within the cell, unlike in many other studies in which samples pre-frozen in a separate
 185 freezer were transferred to testing machines. This feature permits freezing without releasing the
 186 stress after consolidation in unfrozen states, as it happens in-situ. This freezing method prevents
 187 unnatural cracking and inhomogenisation due to unconfined freezing (Wang et al., 2017). The
 188 freezing is performed by firstly draining out the confining water slowly via an adjustable valve lest
 189 the cell pressure drops, and then by pumping in pre-cooled refrigerant into the cell, again slowly
 190 not to change the cell pressure. The cooling mechanism was modified from the earlier study by
 191 Wang et al. (2017). The refrigerant is controlled via two lines. Line 1 is connected to a pressurised
 192 tank in the cold bath, from which the pre-cooled refrigerant (ethylene glycol) is sent to the
 193 pressurised triaxial cell. This line was used to adjust the refrigerant level in the triaxial cell. Line 2
 194 circulates the refrigerant through the heat exchange coil. This line was used to maintain and adjust
 195 the temperature in the triaxial cell. The temperature control of the refrigerant directly in contact with
 196 the specimen by Line 2 is much more responsive than more indirect manners, such as controlling
 197 the room ambient temperature or placing a heat exchanger within the cell. All through these
 198 operations, the cell pressure could be controlled independently (kept constant in this study).

199

200



201

202

203

204

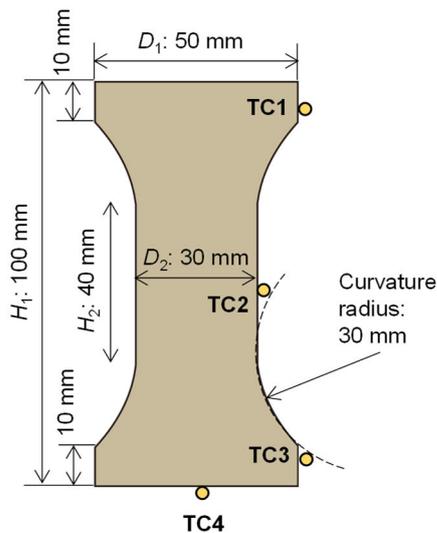
205

Figure 3: Triaxial tension test apparatus for frozen soil

206 Higher cooling efficiency was achieved by setting a double acrylic wall around the specimen and
 207 restricting the circulation path. This cell-in-a-cell proved very effective in insulating the specimen
 208 from the room environment. The refrigerant comes to contact with the specimen first as it enters
 209 the triaxial cell, and thus effectively deprives the heat from the specimen before receiving heat from
 210 outside. For $-30\text{ }^{\circ}\text{C}$ set at the cold bath, $-23\text{ }^{\circ}\text{C}$ was achieved at the specimen surface. This is much
 211 higher cooling efficiency than in the previous design (Wang et al., 2017). The refrigerant outside
 212 the inner double-wall works as a buffer, and was only mildly cold. This helps limiting the outer cell
 213 surface frosting that hinders visual inspection. Another improvement was to use a taller pedestal.
 214 The specimen bottom was slightly warmer in this apparatus due to the heat leak through the
 215 pedestal. A taller pedestal alleviated this problem. The temperature was measured by K-type
 216 thermocouples at four locations, three at the specimen's outer surface and one at the bottom end
 217 centre (**Figure 4**). TC1 was at $-14\sim-13\text{ }^{\circ}\text{C}$ when TC2-TC4 were at $-15\text{ }^{\circ}\text{C}$. This difference was
 218 smaller for higher temperatures.

219

220 The axial ram was driven by a very precise and powerful direct-drive motor capable of applying
 221 compression and extension loads equally. Pore water drainage during consolidation was allowed
 222 towards the porous stone ring (**Figure 3**) embedded in the pedestal via filter paper strips attached
 223 to the specimen's lateral surface (**Figure 4** right).



(a) Dimensions and thermocouples (TC)

(b) Specimen, as trimmed, and as enclosed by rubber membrane with filter paper strips

Figure 4: Specimen geometry and dimensions

The soil specimens were trimmed into a dumbbell shape with enlarged ends, as illustrated in **Figure 4**. The diameters of the middle part and the enlarged ends were 30 mm and 50 mm, respectively. The total height was 100 mm. The ends were trimmed smoothly, and simply put into contact with smooth metal platen surfaces. The soil-platen coupling to transmit a tensile load was established solely by freezing adhesion. It was therefore unnecessary to perform any special treatment during specimen set-up in this study. The bottom platen was polished stainless steel, and the top platen was anodised aluminum. The cross-sectional area ratio between the specimen ends and middle is 2.8. The tensile failure is therefore forced to occur in the middle part as long as the platen-specimen adhesion is greater than $1/2.8$ of the tensile strength. The transition from the middle to the enlarged parts had a gentle curvature with a radius of 30 mm to avoid stress concentration. This geometry was successful in causing the failure in the middle part. By adopting the quick freezing under confining pressure, the water content ratio of the middle part to the ends, as measured after freezing, was 0.996-0.1011, meaning almost no internal axial water migration. Wang et al. (2017) observed slightly greater radial water migration, with a rim-to-core water content ratio of 1.03 on average for 30 mm-diameter Kasaoka clay specimens. This is still small and accepted in this study.

Material and specimens

Reconstituted saturated samples of Kasaoka clay were tested in this study. The clay has been extensively tested in frozen states by the first author in the past. The physical properties are shown

251 in **Table 1**. The gravimetric unfrozen water contents w_u measured at several temperatures by
 252 Nuclear Magnetic Resonance (NMR) are shown in **Figure 5**, along with the calculated degree of
 253 liquid saturation S_l . The clay was made into slurry at the water content $w=100\%$, then mixed and
 254 poured into a 220mm-diameter consolidometer under vacuum for de-airing, and then one-
 255 dimensionally pre-consolidated. Thus prepared samples were saturated, showing B value > 0.95 .
 256 Nine specimens were taken from each consolidated cake. The sample pre-consolidated at $\sigma_{vp}'=100$
 257 and 200 kPa were used for tests with pre-freezing isotropic effective stress $p_c'=100$ and 600 kPa,
 258 respectively. Ideally, σ_{vp}' should have been smaller than 50 kPa for $p_c'=100$ kPa so that the effect
 259 of one-dimensional consolidation would be eventually erased at p_c' . However, such a soft specimen
 260 was difficult to be prepared into the desired shape. The specimens were formed by carefully
 261 trimming along curved guides in a rotary soil lathe.

262
 263 Wang et al. (2017) and Nishimura and Wang (2019) presented the triaxial compression strength of
 264 Kasaoka clay prepared in a similar manner. At -10, -5 and -2 °C, the clay did not exhibit any change
 265 of strength against post-freezing total stress p_f in the range of 200-600 kPa, as shown in **Figure 6**
 266 (solid markers; note that the ultimate strength, not the peak strength, was published by Nishimura
 267 and Wang, 2019), unlike against the pre-freezing stress (open markers). This insensitivity to p_f is a
 268 characteristic of fine-grained soils, seen in other earlier studies too (e.g. Chamberlain et al., 1972;
 269 Wang et al., 2004). Inspired by Ladanyi and Morel (1990), Nishimura and Wang (2019) explained
 270 this by less dilative nature of fine-grained soils leading to shear failure in the pattern of **Figure 1(a)**
 271 right. Although there has been clear evidence that polycrystalline ice's strength is pressure-
 272 dependent (Rist and Murrell, 1994; Singh and Jordaan, 1996; Shulson, 2001), the ice in smaller
 273 pores in fine-grained soils may have less micro-defects that respond to confining pressure.

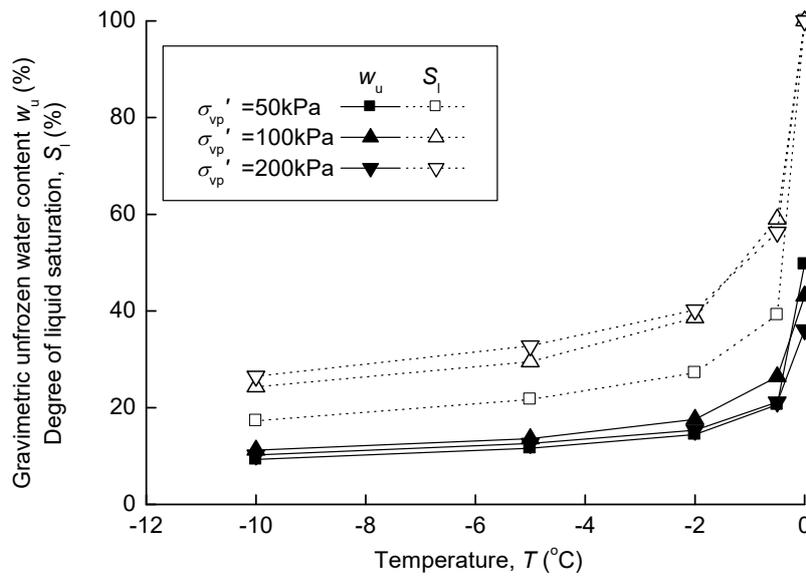
274
 275

276 **Table 1:** Physical properties of Kasaoka clay

277

Plastic limit (%)	Liquid limit (%)	Particle density (Mg/m ³)	Percentage finer than 2/5/63/75 μm (%)
28	62	2.65	40/60/98/100

278



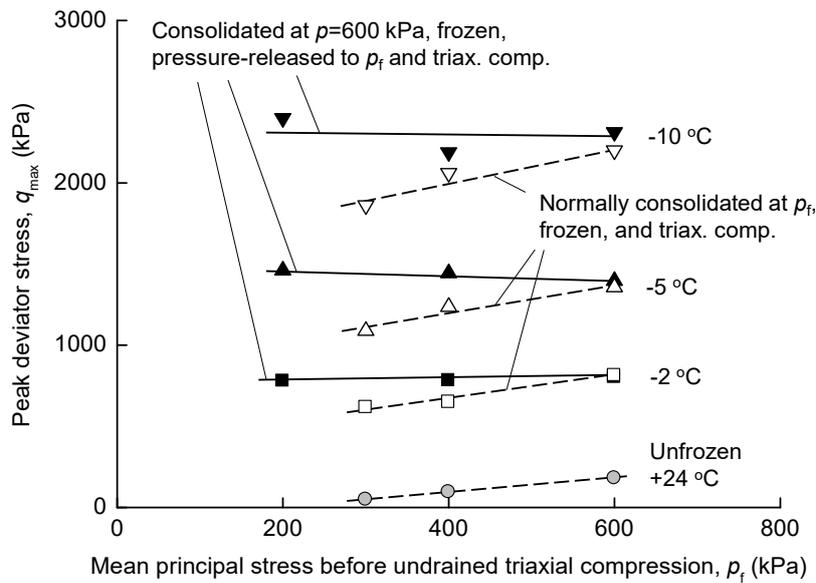
279

280

Figure 5: Gravimetric unfrozen water content w_u and degree of liquid saturation S_l in frozen Kasaoka clay one-dimensionally consolidated at σ_{vp}'

281

282



283

284

Figure 6: Triaxial compression peak strength of frozen Kasaoka clay: Influence of pre- and post-freezing stress changes (pre-freezing back pressure=200 kPa in all cases)

285

286

287 *Test program*

288

289

290

291

30 triaxial tension tests were performed in total, as summarised in **Table 2**. The main variables were the temperature T (-1, -4, -9 or -15 °C), pre-freezing effective consolidation stress p_c' (100 or 600 kPa) and post-freezing total stress p_f (0, 200 or 650 kPa). A series consisting of Tests 1-23 covers all the combinations of these, except $p_f=0$ kPa at -1 °C. The adhesion at the bottom end

292 turned out to be too weak in this condition to apply tension. Note that the T values cited are nominal,
 293 and the measured values are shown in **Table 2**. They are taken from thermocouple TC2, since the
 294 failure occurred in the specimen middle.

295
 296

Table 2: List of performed triaxial tension tests

ID	T (°C)		p_c (kPa)	p_c' (kPa)	u_c (kPa)	p_f (kPa)	$e_c^{(2)}$	Note
	Nominal	Measured ¹⁾						
1		-0.71	200	100	100	200	1.133	
2		-0.63	200	100	100	650	1.129	
3	-1	-0.78	650	600	50	200	0.819	
4		-0.75	650	600	50	650	0.802	
5		-0.63	650	600	50	200	0.802	(Duplicate of Test 3)
6		-4.2	200	100	100	0	1.123	
7		-4.0	200	100	100	200	1.131	
8	-4	-4.4	200	100	100	650	1.126	
9		-4.3	650	600	50	0	0.796	
10		-4.4	650	600	50	200	0.797	
11		-4.3	650	600	50	650	⁻³⁾	
12		-8.3	200	100	100	0	1.109	(Slightly warm)
13		-9.0	200	100	100	200	1.129	
14	-9	-9.2	200	100	100	650	1.114	
15		-8.9	650	600	50	0	0.798	
16		-9.2	650	600	50	200	0.801	
17		-9.0	650	600	50	650	⁻³⁾	
18		-14.4	200	100	100	0	1.104	
19		-14.7	200	100	100	200	1.099	
20	-15	-14.5	200	100	100	650	1.105	
21		-14.6	650	600	50	0	0.794	
22		-14.6	650	600	50	200	0.798	
23		-14.6	650	600	50	650	0.795	
24		-14.7	650	100	550	0	1.116	
25	-15	-14.7	650	100	550	200	1.102	
26		-14.7	650	100	550	650	1.112	
27	-4 to -15	-4.2 to -14.7	650	600	50	650	0.792	Pause and T change
28	-4	-4.3	650	600	50	650	⁻³⁾	Pause
29	-15	-14.7	650	600	50	650	0.799	Pause
30	-4	-4.4	650	600	50	σ_1 : 650 to 0	0.791	σ_1 change

297
 298
 299
 300
 301
 302

¹⁾ Mid-height thermocouple (TC2), average over 2,000 sec around the tensile peak

²⁾ e_c : Void ratio after consolidation

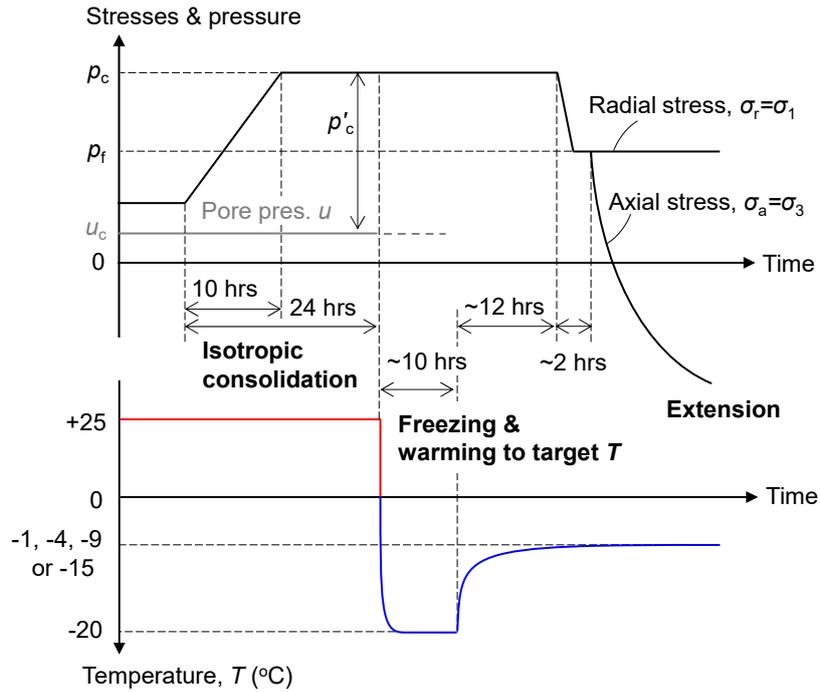
³⁾ Final water content could not be measured accurately due to membrane rupture when removing the specimen

303 The back pressure after consolidation u_c was basically 100 kPa for $p_c'=100$ kPa, and 50 kPa for
 304 $p_c'=600$ kPa. Ideally, the latter would have been set at 100 kPa too – however, the slightly smaller
 305 value had to be accepted due to the cell capacity of 700 kPa. In Tests 24-26, $u_c=550$ kPa was set

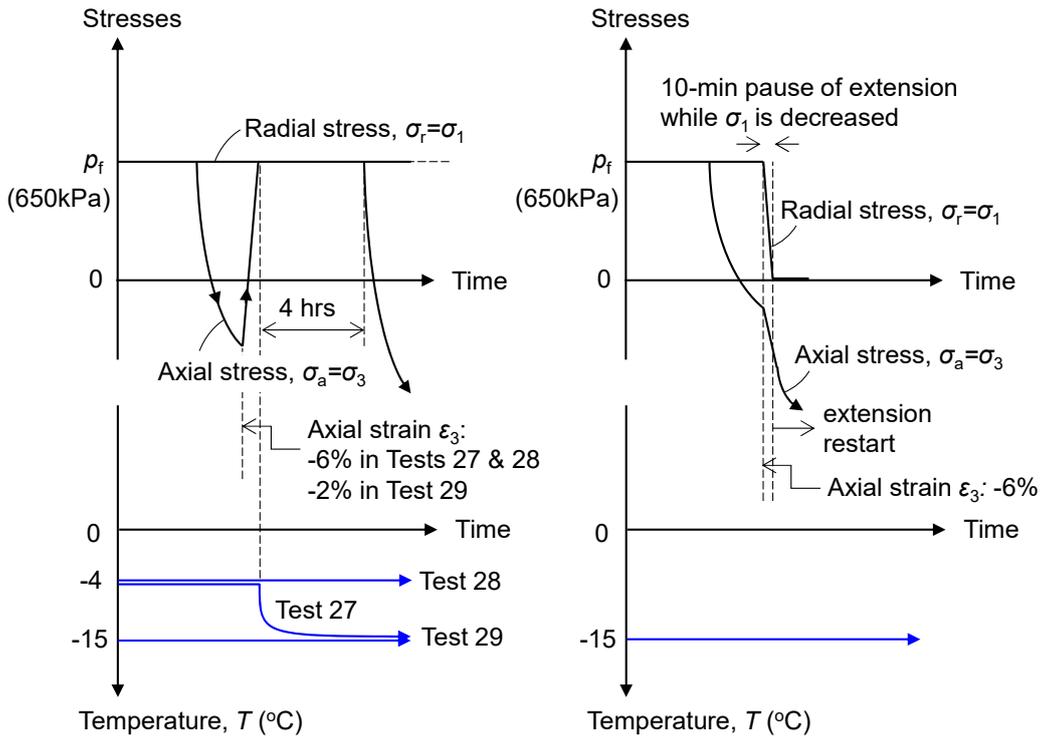
306 for $p_c'=100$ kPa. Test 5 is a duplicate test of Test 3 performed early in the program, out of a concern
307 about test repeatability, as a slight temperature fluctuation has relatively large influence on the
308 result at such high temperatures as -1 °C. Tests 18-26 constitute an interesting comparative set,
309 allowing observation of influence of p_c' , p_r and u_c independently while keeping the other two
310 constant ($u_c=50$ kPa and 100 kPa are assumed nearly equal here).

311
312 The above basic series (Tests 1-26) were conducted in the following steps. Firstly, the pre-
313 consolidated, unfrozen specimen was set up in the apparatus after being covered with a rubber
314 membrane with a diameter of 30 mm. The enlarged part therefore underwent greater membrane
315 force, as the membrane size was chosen for the middle part. This influence was not deemed
316 important, as the failure always occurred at the specimen middle. After being isotropically
317 consolidated to p_c' , the specimen was frozen under constant cell pressure, as described earlier.
318 This process was undrained. The convergence of axial displacement can be considered to mark
319 temperature equilibrium, and the freezing was complete in 20 minutes. At this moment, the
320 specimen was frozen at -20 °C, and this state was kept at least for 10 hours. The temperature was
321 then increased to a target value. The change was fully transferred to the specimen core within an
322 hour (as inferred from TC4 and numerical analysis), but about 12 hours were allowed for ensuring
323 equilibration. The total stress was then changed to p_r in an hour, and after the axial strain rate
324 became smaller than 10^{-4} %/min, the extension stage was initiated. These stages are illustrated in
325 **Figure 7(a)**.

326
327 The extension was applied at a constant axial strain rate of -0.05 %/min. This rate was sufficiently
328 slow for the frozen clay to remain well in the static, ductile regime (e.g. Parameswaran 1980) but
329 allows a failure to be reached within the day under the operator's constant watch. In this paper, the
330 nominal axial strain ϵ_a (=minor principal strain ϵ_3) is defined by the axial displacement d divided by
331 the total specimen height H_1 (see **Figure 4**). Given the enlarged ends, this definition may
332 underestimate the true strain in the middle, and is only nominal. The axial stress σ_a ($=\sigma_3$) is defined
333 by the axial load divided by the mid-height cross-sectional area A_2 ($=\pi D_2^2/4$) after consolidation,
334 without update for horizontal thinning due to extension. The deviator stress, q , is defined as the
335 axial stress minus the radial stress (i.e. negative in axial tension).



(a) Tests 1-26: Constant T and σ_1 in extension stage



(b) Tests 27-29: 4-hour pause with or without T change

(c) Test 30: Decrease of σ_1 in extension stage

Figure 7: Variable control schemes in triaxial tension tests
(Pre-extension stages in (b) and (c) are same as (a))

345 Tests 27-30 were designed to investigate the influence of more complex state history on the failure.
346 They followed the same steps except in the extension stage. Tests 27-29 involved unloading and
347 reloading of deviator stress, with a 4-hour pause in between with or without temperature decrease,
348 as illustrated in **Figure 6(b)**. The purpose of the pause was to allow temperature equilibration in
349 Test 27, and to have the same time to failure as this in Tests 28 and 29. The intension will be
350 discussed more later. The unloading in these tests started at $\epsilon_a=-6\%$ in Tests 27 and 28 ($T=-4\text{ }^\circ\text{C}$)
351 and $\epsilon_a=-2\%$ in Test 29 ($T=-15\text{ }^\circ\text{C}$), approximately 75% of the tensile rupture strain observed in Tests
352 11 and 23, respectively. In Test 30, extension was paused at $\epsilon_a=-6\%$, the radial stress σ_1 was
353 reduced from 650 to 0 kPa in 10 minutes, then the extension was resumed. This is illustrated in
354 **Figure 6(c)**.

355

356 PEAK STRENGTH OBSERVED IN EXTENSION

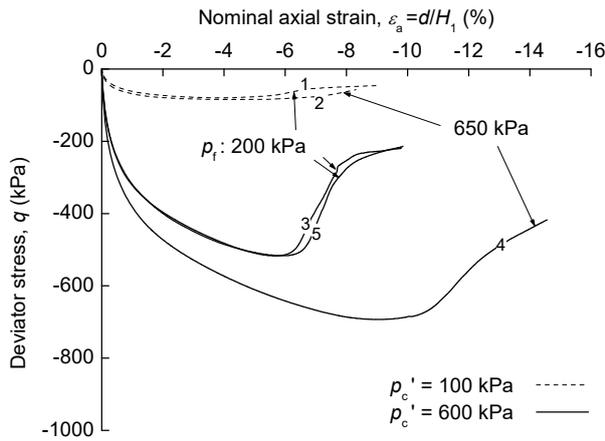
357 *Stress-strain relationships and failure patterns*

358 The observed relationships between the deviator stress q and the nominal axial strain ϵ_a from Tests
359 1-23 are shown in **Figure 8** for different temperatures T . Consistency of Tests 3 and 5 assures the
360 repeatability, even in sensitive temperature ranges. For $T=-4, -9$ and $-15\text{ }^\circ\text{C}$, the pre-peak parts of
361 the curves are unique to each p_c' , unaffected by the post-freezing stress p_f . This is consistent with
362 the earlier results in **Figure 6** showing no influence of the post-freeze stress change on the triaxial
363 compression strength at moderate stress levels. Beyond the initial part, the influence of p_f becomes
364 clearer, as the initially same stress-strain relationships start branching out one after another,
365 starting from lower p_f , due to failure. In some cases ($T=-4$ and $-15\text{ }^\circ\text{C}$, $p_c'=100\text{ kPa}$), this occurred
366 after common q peaks (-950 kPa and -2950 kPa , respectively) were reached, meaning that the
367 minimum q value is unaffected by p_f , and only the post-failure strength reduction behaviour is
368 different. For the rest of conditions, lower p_f led to slightly lower strength (i.e. higher q) as the stress-
369 strain curves fell short of travelling up to the common q peak.

370

371 The peak strength (minimum q) is shown as Mohr's stress circles in **Figure 9**, along with earlier
372 triaxial compression data obtained by the authors' team where the conditions are same (the
373 difference in the strain rate, $0.01\text{ } \%/ \text{min}$ vs $0.05\text{ } \%/ \text{min}$, is corrected by the observed rate-
374 dependency; Wang et al., 2017). The failure envelopes are ambiguous when viewed with **Figure**
375 **1(a)-(c)** right as possible templates. As discussed above, some cases showed no influence of p_f on
376 the strength, leading to a flat envelope in line with **Figure 1(a)** right. The others show only gentle
377 slopes – far gentler than expected from the effective stress-based shear resistance angle ϕ' .
378 Testing with negative σ_1 would lead to a clearer picture, but this all-round negative stress is an
379 unlikely regime in field, and hard to achieve in tests too. Whilst it is probably reasonable to
380 approximate ϕ as 0, except at $T=-1\text{ }^\circ\text{C}$, from **Figure 8** in practice, this 'as-plotted' Mohr's stress
381 circle leaves ambiguity.

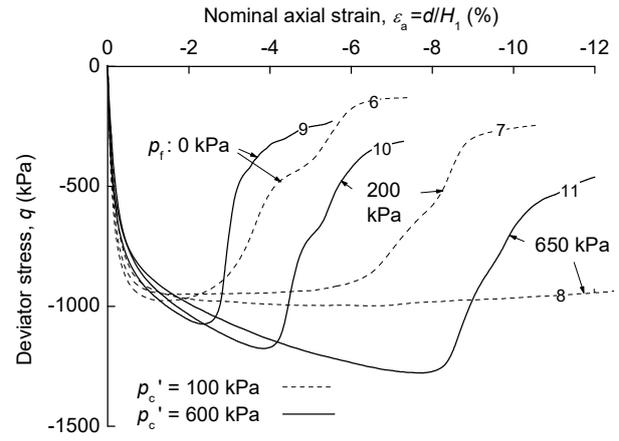
382



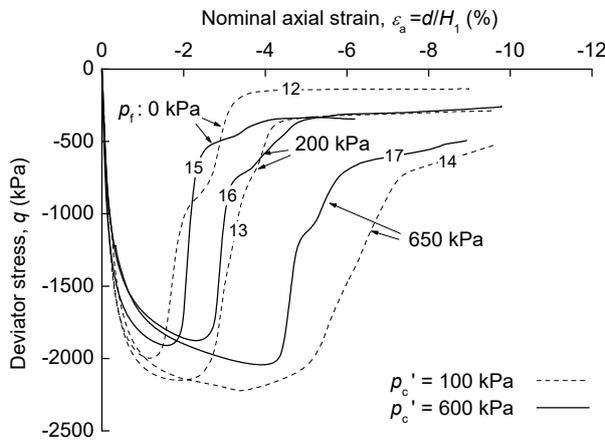
384

385

(a) $T = -1\text{ }^\circ\text{C}$



(b) $T = -4\text{ }^\circ\text{C}$

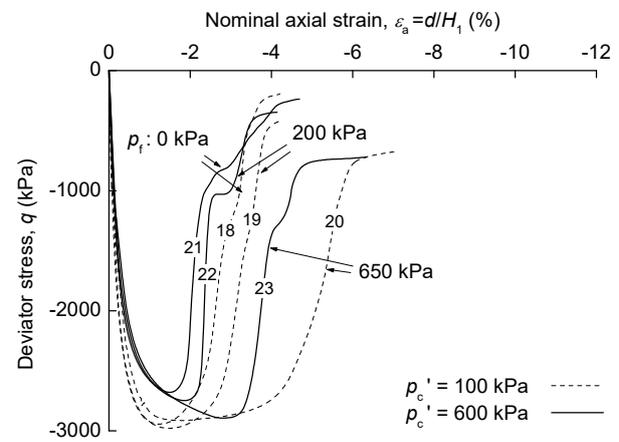


386

387

388

(c) $T = -9\text{ }^\circ\text{C}$



(d) $T = -15\text{ }^\circ\text{C}$

Figure 8: Deviator stress – axial strain relationships from Tests 1-23

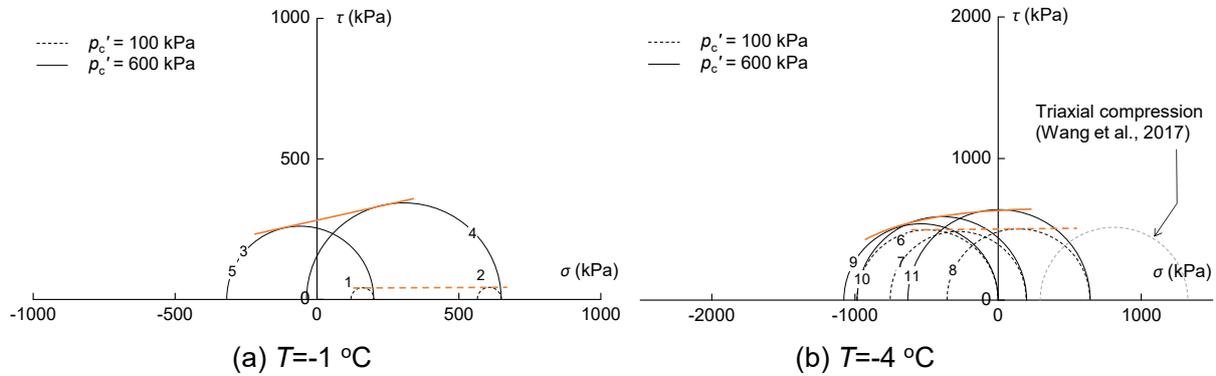
(The numbers indicate test ID)

389

390

391

392



393

394

395

396

(a) $T = -1\text{ }^\circ\text{C}$

(b) $T = -4\text{ }^\circ\text{C}$

(Figure 9: Continues to next page)

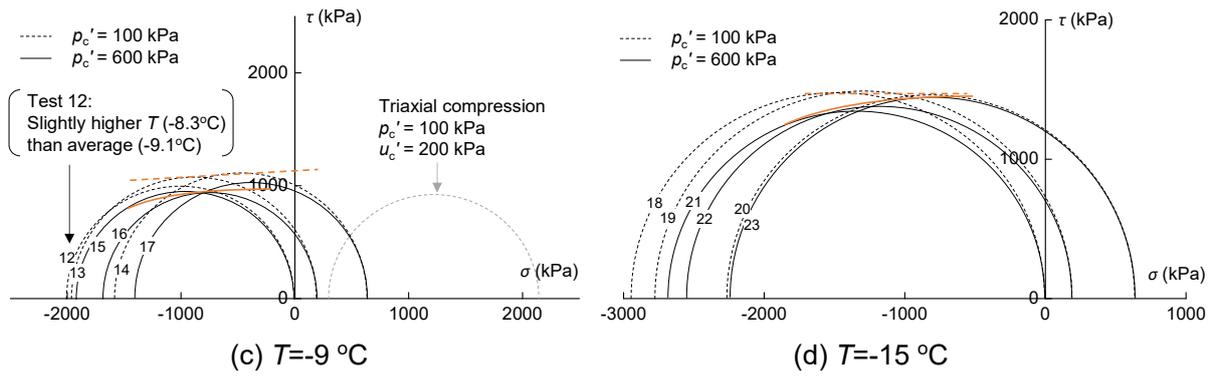


Figure 9: Mohr's stress circles for peak deviator stress from Tests 1-23
(The numbers indicate test ID)

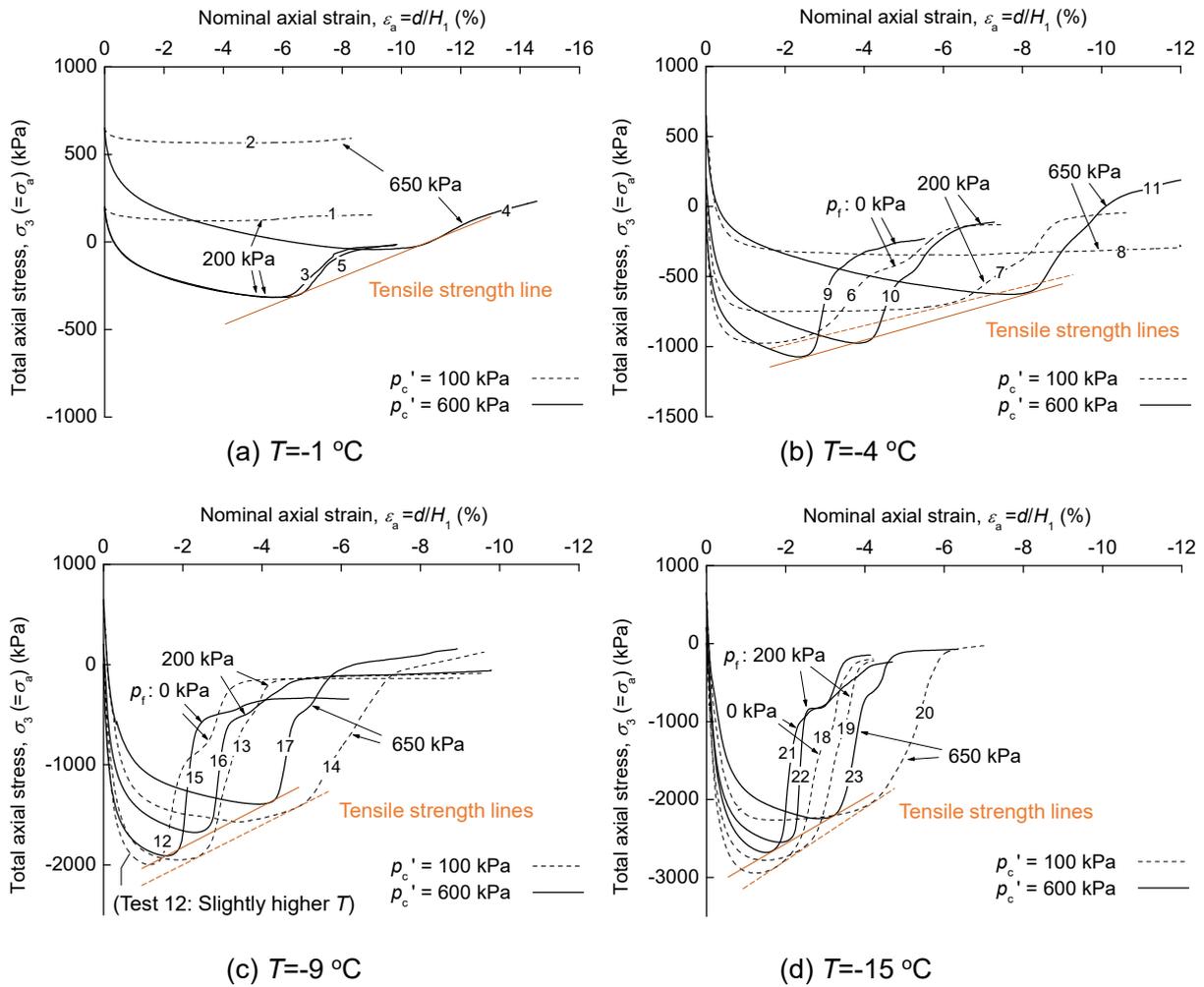
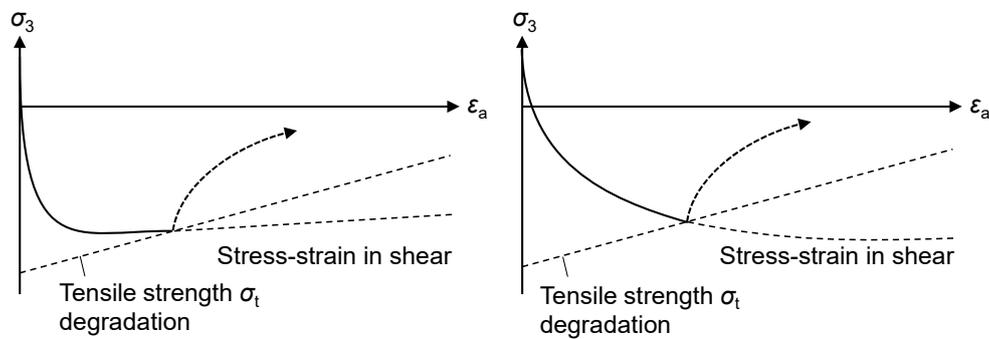


Figure 10: Axial stress – axial strain relationships from Tests 1-23
(The numbers indicate test ID)

412 Another perspective offers a clearer view of the possible process of failure. **Figure 10** replots
 413 **Figure 8** in terms of axial stress $\sigma_a = \sigma_3$. For each p_f and T condition, the failure points are neatly
 414 aligned along a single line. A possible mechanism hypothesised from this observation is that the
 415 tensile strength is not constant but decreases as the shear strain accumulates. If the stress-strain
 416 relationship shows comparatively higher initial stiffness and smaller strain to peak shear resistance,
 417 the peak q value is dictated by the shear resistance, and only the post-peak behaviour is affected
 418 by tensile failure, as illustrated in **Figure 11(a)**. This was the case for $T = -4$ and -15 °C, $p_c' = 100$ kPa.
 419 In the other cases, the intersection of the decreasing tensile strength line with the stress-strain
 420 curve occurred before the latter reached the peak, as shown in **Figure 11(b)**. In such cases, lower
 421 p_f (and hence lower σ_3 for a given q since $\sigma_3 = p_f + q$) leads to higher q (lower $|q|$) as it meets the
 422 tensile strength line earlier.
 423



424

(a) Smaller p_c' and larger ice content:
 Smaller shear strength and larger initial
 stiffness

(b) Larger p_c' and smaller ice content:
 Larger shear strength and smaller initial
 stiffness

425

426 **Figure 11:** Schematic illustration of shear hardening and tensile strength degradation
 427 simultaneously at work

428

429

430

431

432

433

434

435



Test 1

Test 2

Test 3

Test 4

Test 5

436

437

438

(a) $T = -1\text{ }^{\circ}\text{C}$



Test 8

Test 21

(Similar in all the rest)

439

440

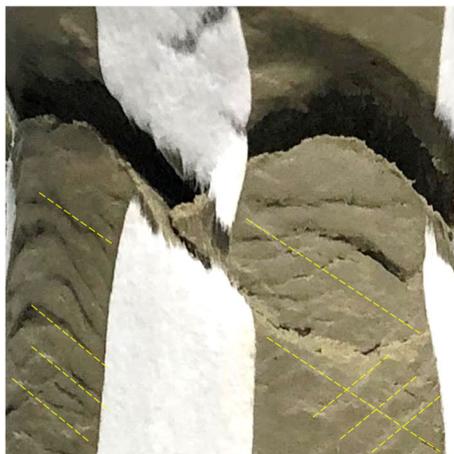
441

(b) $T = -4, -9\text{ and }-15\text{ }^{\circ}\text{C}$

Figure 12: Failure modes of specimens in extension

442

443



444

(a) Test 4: Conjugate shear surfaces

(b) Test 22: Typical rupture surface

445

446

Figure 13: Close-up of failure

447 Almost all the tests exhibited a clear tensile failure surface at the end, as shown in **Figure 12(b)**,
448 right) and **Figure 13(b)**. The surface was perpendicular to the axis, and the surface texture was
449 rough with no trace of shear. The exceptions include Test 8, which did not reach clear failure despite
450 significant extension beyond -12%. The other exceptions are Test 1-5, performed at $T=-1$ °C (more
451 accurately, around -0.7 °C). These tests exhibited only marginally higher strength than unfrozen
452 states, due to the significant unfrozen water content. It is probably also relevant that the sample
453 had been frozen initially to -20 °C and thawed back to -0.7 °C. This freezing and undrained 'half-
454 thawing' might have led to significant softening by generating excess pore water pressure
455 (Nishimura et al., 2020). As a result, the strength was very low, and σ_3 remained positive at failure
456 in Tests 1 and 2. The failure in these tests was therefore triaxial extension shear without involving
457 eventual tensile failure, as shown in **Figure 12(a)**. In the tests with $p_c'=600$ kPa (Tests 3-5), σ_3 did
458 decrease to negative values (i.e. tensile stress). While these tests eventually showed tensile failure
459 in form of rupture, diagonal traces of triaxial extension shear had developed as seen in **Figure**
460 **12(a)** due to the large shear strain before reaching the tensile failure line; $\epsilon_a=-6.2\%$, -10.8% and $-$
461 6.5% in Tests 3, 4 and 5, respectively. A close-up view of the shear traces is shown in **Figure 13(a)**.
462 Conjugate shear surfaces with approximately 30° dip were formed, along which the tensile rupture
463 occurred, unlike at lower temperatures where the tensile rupture was almost horizontal. These tests
464 almost failed in triaxial extension shear, only engaging the decreasing tensile strength after shear
465 localisation occurred.

466

467 *Initial tensile strength and degradation*

468 The above observations suggest that the rather ambiguous nature of the failure envelopes in
469 **Figure 9** – neither completely flat nor obviously stress-dependent – can be explained by a fine
470 balance of two competing factors; the rate of strain-hardening in shear deformation, and the rate
471 of tensile strength degradation, as illustrated in **Figure 11**. In general, smaller p_c' led to faster initial
472 strain-hardening. For same Kasaoka clay, Wang et al. (2017; 2019) found that the initial stiffness
473 was higher for lower p_c' due to larger w and hence larger ice content, while the peak shear strength
474 was lower for lower p_c' due to less inter-soil particle friction. The pre-peak strain in shear therefore
475 tended to be smaller for lower p_c' , as seen in **Figure 8**. Mobilising the shear strength, which was
476 not post-freezing stress-dependent, before engaging the tensile strength led to generally flatter
477 peak strength envelope for $p_c'=100$ kPa.

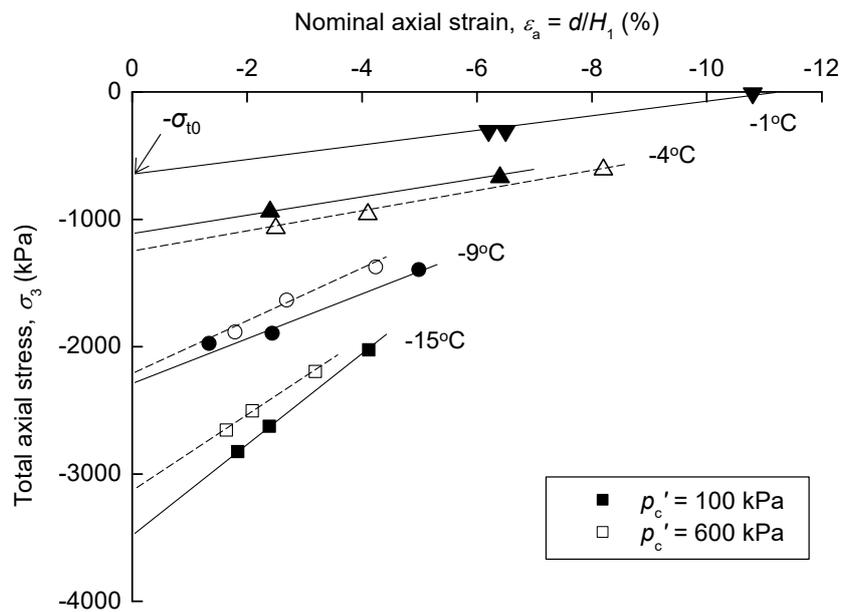
478

479 One of the key issues in understanding the limit strength is the tensile strength degradation during
480 shear. **Figure 14(a)** summarises the tensile strength lines taken from **Figure 10** for all the
481 conditions. Although there is a consistent trend, different slopes make the description difficult. When
482 the same envelopes are replotted against the specific shear work input (i.e. the area between the
483 $q - \epsilon_a$ curve and $q=0$ line in **Figure 10**), the relationships have an almost constant slope, as shown
484 in **Figure 14(b)**. This slope can be used for tensile strength degradation modelling. The intercept

485 at $\varepsilon_a=0$ is the initial tensile strength σ_{t0} , or a hypothetical tensile strength that would be observed if
 486 the shear stress-strain relationship were perfectly rigid plastic. The σ_{t0} values are plotted against
 487 temperature T in **Figure 15**, along with the actually observed maximum $|q|$ ($=|q|_{\max}$) in the triaxial
 488 tensile Tests 1-2, 6-8, 14, 18-20. These tests are likely to have mobilised the shear strength before
 489 reaching the tensile failure. Also shown are $|q|_{\max}$ values from triaxial compression tests (Wang et
 490 al., 2017), corrected for the strain rate difference. Although the anisotropy and the intermediate
 491 principal stress (σ_2)-effect lead to different trends, these $|q|_{\max}$ values represent the shear strength.
 492 They are seen to be only slightly lower than σ_{t0} . Therefore, whether the limit strength in extension
 493 is determined by reaching the potential maximum shear strength or being cut short by degrading
 494 tensile strength (i.e. shear-induced reduction from σ_{t0}) depends on the fine balance of these
 495 mechanisms.

496
 497 Interestingly, σ_{t0} is higher than the ice tensile strength compiled by Cuda and Ash (1984), despite
 498 the strain rate slower by one order (0.05%/min against 0.9%/min) and significant unfrozen water
 499 content in the clay. The microscopic failure of ice in the clay's small pores is probably harder to
 500 develop, just as the pore liquid water is harder to cavitate.

501



502

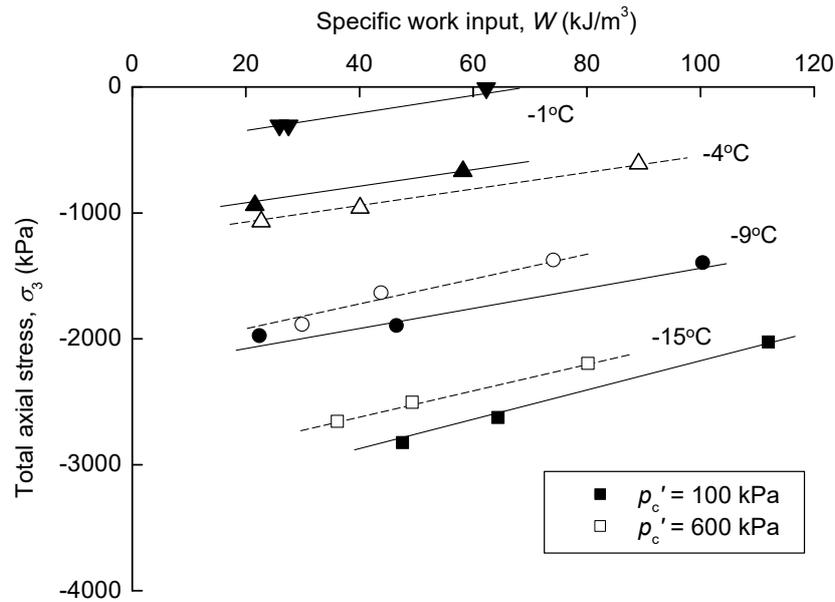
503

504

505

(a) Degradation against (nominal) axial strain

(Figure 14: Continues to next page)



(b) Degradation against specific work input

Figure 14: Summary of all the tensile strength lines and the strength degradation due to shear

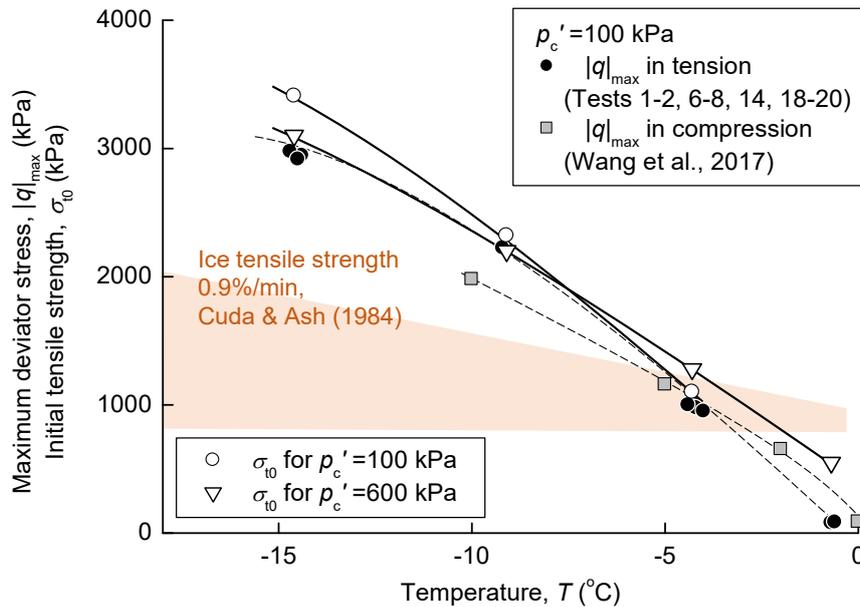


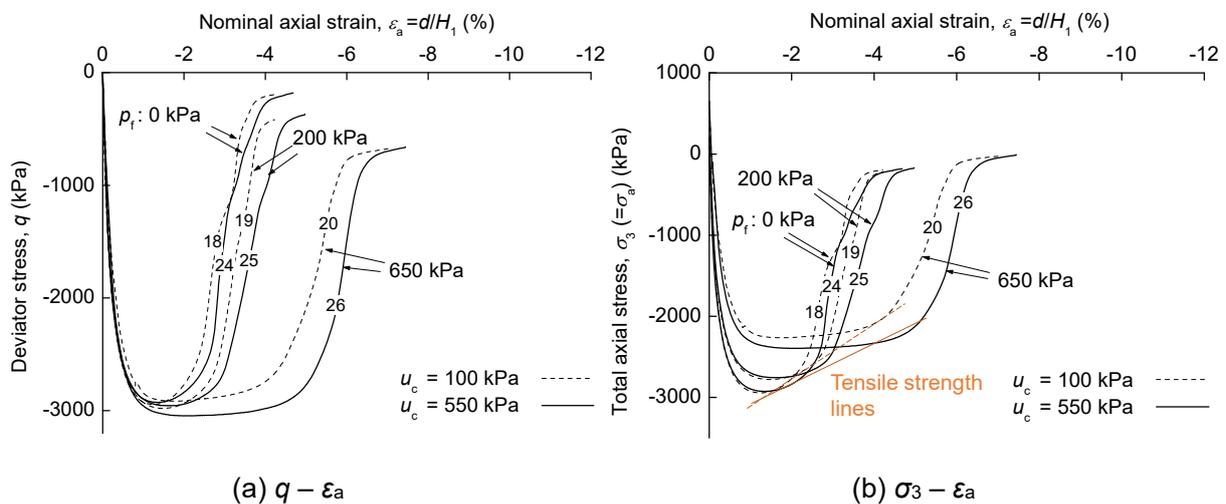
Figure 15: Initial tensile strength against temperature, in comparison with shear strength

TENSILE STRENGTH FOR MORE GENERAL STATES

Pore water pressure effects: Tests 24-26

Test 24-26 applied a higher pre-freezing back pressure u_c of 550 kPa, while the samples were consolidated at $p'_c=100$ kPa. Figure 16 compares these tests with Tests 18-20, with same $p'_c=100$

519 kPa and $T=-15\text{ }^{\circ}\text{C}$ but with $u_c=100\text{ kPa}$. The higher back pressure only marginally affected the
 520 failure, providing a small extra strain before the tensile rupture at each p_r . The multi-phase coupled
 521 system theory by Nishimura and Wang (2019) does not differentiate these cases for shear
 522 behaviour, as a frozen state is essentially an undrained state, and the pore water (ice + liquid)
 523 pressure at frozen state will be only dependent on the initial effective stress, just as in the unfrozen
 524 soil mechanics. In Test 24, for example, the pore water pressure would decrease by 450 kPa in this
 525 theory when eventually p_r was reduced from 650 kPa to 200 kPa, bringing it to the same level as
 526 in Test 19. The almost same results seen for Tests 18-20 and 24-26 confirm the theory. A possible
 527 explanation for the slight difference is that less air escapes from the pore water when it turns ice
 528 under higher pressure, giving the pore ice with less defect. **Figure 16** suggests, however, that this
 529 effect is negligible in practice.
 530

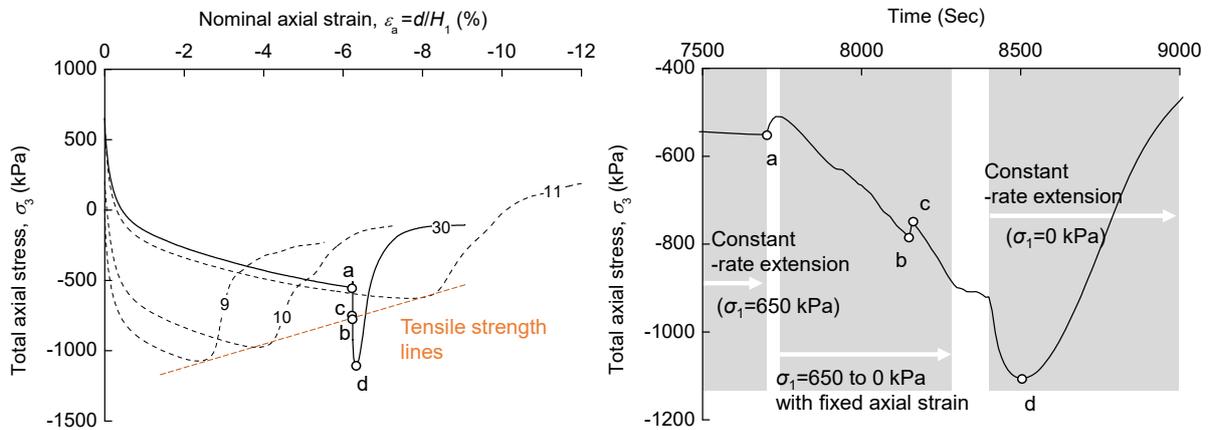


534 **Figure 16:** Influence of initial (pre-freezing) back pressure on stress-strain curves at $-15\text{ }^{\circ}\text{C}$

537 *Major principal stress σ_1 changes: Test 30*

538 The framework of shear-induced degradation of the tensile strength was tested for more general
 539 testing conditions. Test 30 involved reduction of the radial stress σ_r (= the major principal stress σ_1)
 540 from 650 kPa to 0 kPa after reaching $\epsilon_a=-6\%$, or 75% of ϵ_a at tensile rupture. When the extension
 541 was resumed with $\sigma_1=0\text{ kPa}$, the $q - \epsilon_a$ curve obviously overshoot the tensile strength line established
 542 from Tests 9-11 with same $T=-4\text{ }^{\circ}\text{C}$ and $p_c'=600\text{ kPa}$, as shown in **Figure 17(a)**. However, a closer
 543 look at the σ_1 -reduction stage in **Figure 17(b)** reveals that the failure actually had initiated at Point
 544 b. As the axial ram was fixed, the σ_3 change is a spontaneous reaction to σ_1 reduction. It usually
 545 changes parallelly with σ_1 to keep q constant (because there is no shear strain change), but at
 546 Point b, there was a temporary rebound of σ_3 to Point c. These points fall exactly on the expected
 547 tensile strength line. The fact that the test reached tensile failure immediately after the overshooting

548 to point d suggests the strength is still bound by this line, despite the temporary excessive strength.
 549 This overshoot is similar to TESRA behaviour proposed by Tatsuoka et al. (2002), reflecting the
 550 temporary effect of strain acceleration due to sudden loading restart.
 551
 552



553
 554 **Figure 17:** Effect of σ_1 reduction: Stress-strain relationship in Test 30
 555
 556

557 *Time to failure and temperature changes: Tests 27-29*

558 A temperature change from $T=-4$ °C to -15 °C was conducted during extension in Test 27. As
 559 described earlier, a pause of 4 hours was set between the unloading and reloading to allow
 560 temperature equilibrium at the specimen core. Reloading after cooling to -15 °C resulted in
 561 considerably high tensile stress, as shown in **Figure 18**, unbound by the established tensile
 562 strength line. The peak q , observed at around -3000 kPa, is the value expected if Test 23 could
 563 continue shear deformation without being bound by the tensile strength line. Obviously, 4-hour
 564 cooling 'healed' the tensile strength degradation induced by the previous shear up to $\epsilon_a=-6\%$. It was
 565 not clear, however, whether the cooling or the pause was responsible for this. Tests 28 and 29 were
 566 therefore conducted, in which similar unloading, 4-hour pause and reloading was conducted but
 567 without T change. **Figure 18** shows that the pause alone has limited healing effect, only moderately
 568 affecting the tensile strength line. The observed full recovery of the tensile strength by cooling
 569 cannot be explained just by the additional freezing of unfrozen pore water, because **Figure 5**
 570 suggests S_i reduction of only 10% from -4 °C to -15 °C. Regelation of the damaged ice must have
 571 also occurred. A longer pause, or more generally, a longer time to failure including slower loading
 572 rates, may have greater influence on the tensile strength line. The different loading rate conditions
 573 affect the shear stress-strain curves too. Obviously more study is necessary in incorporating the
 574 combined effects of temperature and deformation rate into the tensile strength framework.
 575

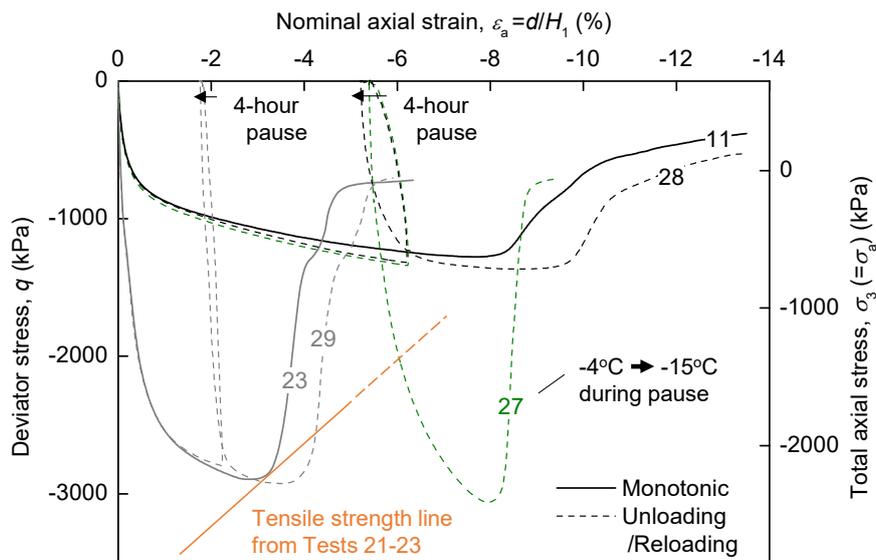


Figure 18: Influence of pause and/or temperature decrease (Tests 27-29) on stress-strain relationship: Comparison at $p_c=600$ kPa and $p_f=650$ kPa

576

577

578

579

580

581 CONCLUSIONS AND IMPLICATIONS

582

583

584

585

586

587

588

589

590

591

592

593

594

595

596

597

598

599

600

A series of 'triaxial tension' tests was conducted on frozen Kasaoka clay, in which the axial stress was reduced to negative values while zero or positive radial stress was maintained. The samples were consolidated to different stresses in the triaxial cell and frozen without releasing the pressure before the tension tests, as expected in artificial ground freezing. This testing method enabled investigating the influence of the pre-freezing consolidation effective stress, post-freezing total radial stresses as well as the initial pore water pressure – a feature not attained in the conventional uniaxial tensile, split or bending tests.

Apparently flat to sub-flat failure envelopes, showing undecisive influence of stress, to Mohr's stress circle were obtained. It was shown, however, that they are explicable by considering two competing mechanisms; shear strain-hardening and tensile strength degradation. The former is independent of the confining (or radial) stress in Kasaoka clay for the studied stress range (0-650 kPa), while the latter is uniquely correlated to the specific work input as shear deformation progresses. A peak shear strength may be mobilised before tension rupture eventually occurs, or the tensile loading may be cut short by engaging the degrading tensile strength before attaining the peak shear strength. Question (i) raised in the introduction has been thus answered – both tensile and shear strengths play a role.

The tensile strength is described by two factors; the initial tensile strength, or a hypothetical strength

601 mobilised at zero shear strain, and its degradation rate as the soil structure is disturbed by shearing.
602 They are strongly dependent on the temperature, although the degradation rate can be normalised
603 by using the specific work input. More modest influence comes from the initial pore water pressure
604 and time to failure. The higher the pore water pressure, and the longer the time to failure, the frozen
605 clay endured to slight larger strains before eventually failing in tensile rupture. An interesting
606 phenomenon was observed that cooling from -4 °C to -15 °C just before the tensile failure
607 apparently healed the damage and fully recovered the tensile strength even at large strain.
608 Elucidating more generally the combined effect of temperature changes and loading rate (or time
609 to failure) requires much further investigation. These observations answer questions (ii) and (iv),
610 while leaving a room for further research.

611
612 In a more practical perspective, it may be acceptable to approximate the clay's failure envelope,
613 such as shown in **Figure 9**, as flat and stress-independent, except at very warm, half-thawing
614 temperatures (above -1 °C, for example). It should be remembered, however, that such envelopes
615 do not reflect the deformability after reaching the limit tensile stress. Given the abrupt loss of
616 resistance in tensile failure, a safety margin in design should take account of the deformability. The
617 ductile behaviour such as observed in Test 8 (-4 °C) in **Figure 8** is assuring when exploiting the
618 tensile strength in design, while colder temperatures lead to much more brittle behaviour. The
619 proposed framework involving the work-induced tensile strength degradation model will be useful
620 in estimating not just the peak strength, but also the strain to reach it. This is a tentative answer to
621 question (iii).

622

623 ACKNOWLEDGMENTS

624 This study was conducted with JSPS Kakenhi Grant-in-Aid 22H01582. The authors are grateful for
625 the funding.

626

627 REFERENCES

- 628 Akagawa, S. (2021) Artificially frozen ground and related engineering technology in Japan.
629 *Sciences in Cold and Arid Regions* 13 (2) 77-86. DOI: 10.3724/SP.J.1226.2021.20046.
- 630 Akagawa, S. and Nishisato, K. (2009) Tensile strength of frozen soil in the temperature range of
631 the frozen fringe. *Cold Regions Science and Technology* 57 13-22. DOI:
632 10.1016/j.coldregions.2009.01.002
- 633 Azmatch, T. F., Sego, D. C., Arenson, L. U. and Biggar, K. W. (2011) Tensile strength and stress-
634 strain behaviour of Devon silt under frozen fringe conditions. *Cold Regions Science and*
635 *Technology* 68 85-90. DOI: 10.1016/j.coldregions.2011.05.002
- 636 Bragg, R. A. and Andersland, O. B. (1981) Strain rate, temperature, and sample size effects on
637 compression and tensile properties of frozen sand. *Engineering Geology* 18 (1) 35-46. DOI:
638 10.1016/0013-7952(81)90044-2

639 Chamberlain, E., Groves, C. and Perham, R. (1972) The mechanical behaviour of frozen earth
640 materials under high pressure triaxial test conditions. *Géotechnique* 22 (3) 469–483. DOI:
641 10.1680/geot.1972.22.3.469

642 Cuda, V. Jr. and Ash, R. L. (1984) Development of a uniaxial ice tensile specimen for low
643 temperature testing. *Cold Regions Science and Technology* 9 47-52. DOI: 10.1016/0165-
644 232X(84)90047-8

645 Eckardt, H. (1982) Creep tests with frozen soils under uniaxial tension and uniaxial compression.
646 The Proceedings of the 4th Canadian Permafrost Conference, Calgary, Alberta, Eds. French, H.
647 M., National Research Council of Canada, Ottawa, Canada 394-405.

648 Haynes, D. and Karalius, J. A. (1977) Effect of temperature on the strength of frozen silt. CRREL
649 Report 77-3 27p.

650 Jessberger, H. L. (1981) A state-of-the-art report. Ground freezing: Mechanical properties,
651 processes and design. *Engineering Geology* 18 (1) 5-30. DOI: 10.1016/B978-0-444-42010-
652 7.50006-2

653 Ladanyi, B. and Morel, J.-F. (1990) Effect of internal confinement on compression strength of frozen
654 sand. *Canadian Geotechnical Journal* 27 (1) 8–18. DOI: 10.1139/t90-002

655 Li, Y., Ling, X., Su, L., An, L., Li, P. and Zhao, Y. (2018) Tensile strength of fiber reinforced soil under
656 freeze-thaw condition. *Cold Regions Science and Technology* 146 53-59. DOI:
657 10.1016/j.coldregions.2017.11.010

658 Namikawa, T. and Koseki, J. (2007) Evaluation of tensile strength of cement-treated sand based
659 on several types of laboratory tests. *Soils and Foundations* 47 (4) 657-674. DOI:
660 10.3208/sandf.47.657

661 Nishimura, S. and Wang, J. (2019) A simple framework for describing strength of saturated frozen
662 soils as multi-phase coupled system. *Géotechnique* 69 (8) 659-671 DOI: 10.1680/jgeot.17.P.104

663 Nishimura, S., Okajima, S., Joshi, B. R., Higo, Y. and Tokoro, T. (2020) Volumetric behaviour of
664 clays under freeze-thaw cycles. *Géotechnique* 71 (12) 1150-1164. DOI: 10.1680/jgeot.20.P.047

665 Ou, C.-Y., Kao, C.-C. and Chen, C.-I. (2009) Performance and analysis of artificial ground freezing
666 in the shield tunneling. *Journal of GeoEngineering* 4 (1) 29-40. DOI: DOI:
667 10.6310/jog.2009.4(1).4

668 Parameswaran, V. R. (1980) Deformation behaviour and strength of frozen sand. *Canadian*
669 *Geotechnical Journal* 17 (1) 74-88. DOI: 10.1139/t80-007

670 Rist, M. A. and Murrell, S. A. F. (1994) Ice triaxial deformation and fracture. *Journal of Glaciology*
671 40 (135) 305-318. DOI: 10.3189/S0022143000007395

672 Shulson, E. M. (2001) Brittle failure of ice. *Engineering Fracture Mechanics* 68 1839-1887. DOI:
673 10.1016/S0013-7944(01)00037-6

674 Singh, S. K. and Jordaan, I. J. (1996) Triaxial tests on crushed ice. *Cold Regions Science and*
675 *Technology* 24 153-165. DOI: 10.1016/0165-232X(95)00017-6

676 Tamrakar, S. B., Mitachi, T. and Toyosawa, Y. (2007) Measurement of soil tensile strength and

677 factors affecting its measurements. *Soils and Foundations* 47 (5) 911-918. DOI:
678 10.3208/sandf.47.911

679 Tatsuoka, F., Ishihara, M., di Benedetto, H. and Kuwano, R. (2002) Time-dependent shear
680 deformation characteristics of geomaterials and their simulation. *Soils and Foundations* 42 (2)
681 103-129. DOI: 10.3208/sandf.42.2_103

682 Tsuji, R. and Yoshida, T. (2019) Rational design of the artificial ground freezing with state-of-the-art
683 techniques. *Proceedings of the 16th Asian Regional Conference on Soil Mechanics and*
684 *Geotechnical Engineering, Taipei, Paper No. SA10-01-007.*

685 Uchida, K., Shioi, Y., Hirukawa, T. and Tatsuoka, F. (1993) The Trans-Tokyo Bay Highway Project
686 – A huge project currently under construction. *The Proceedings of Transportation Facilities*
687 *through Difficult Terrain, Snowmass, Colorado, USA, Eds. Wu, J. T. H. & Barrett, R. K., Balkema,*
688 *Rotterdam, The Netherlands* 57-87.

689 Viggiani, G. M. B., Casini, F., Winter, M. G., Smith, D. M., Eldred, P. J. L. and Toll, D. G. (2015)
690 Artificial ground freezing: from applications and case studies to fundamental research.
691 *Proceedings of the 16th ECSMGE Geotechnical Engineering for Infrastructure and Development,*
692 *ICE Publishing* 65–92. DOI:10.1680/ecsmge.60678

693 Wang, D., Ma, W. and Chang, X. (2004) Analyses of behavior of stress-strain of frozen Lanzhou
694 loess subjected to K_0 consolidation. *Cold Regions Science and Technology* 40 (1) 19-29. DOI:
695 10.1016/j.coldregions.2004.03.002

696 Wang, J., Nishimura, S. and Tokoro, T. (2017) Laboratory study and interpretation of mechanical
697 behavior of frozen clay through state concept. *Soils and Foundations* 57 (2) 194-210. DOI:
698 10.1016/j.sandf.2017.03.003

699 Wang, J., Nishimura, S., Joshi, B. R. and Okajima, S. (2019) Small-strain deformation
700 characteristics of frozen clay from static testing. *Géotechnique* 69 (9) 816-827. DOI:
701 10.1680/jgeot.18.P.115

702 Yamamoto, Y. and Springman, S. M. (2017) Three- and four-point bending tests on artificial frozen
703 soil samples at temperature close to 0 °C. *Cold Regions Science and Technology* 134 20-32.
704 DOI: 10.1016/j.coldregions.2016.11.003

705 You, Z., Ma, Y., Wang, Z. and Ma, J. (2021) Tensile strength variation of a silty clay under different
706 temperature and moisture conditions. *Cold Regions Science and Technology* 189 103314. DOI:
707 10.1016/j.coldregions.2021.103314

708 Zhang, J., Fu, H., Huang, Z., Wu, Y., Chen, W. and Shi, Y. (2019) Experimental study on the tensile
709 strength and failure characteristics of transversely isotropic rocks after freeze-thaw cycles. *Cold*
710 *Regions Science and Technology* 163 68-77. DOI: 10.1016/j.coldregions.2019.04.006

711 Zhou, G., Hu, K., Zhao, X., Wang, J., Liang, H. and Lu, G. (2015) Laboratory investigation on tensile
712 strength characteristics of warm frozen soils. *Cold Regions Science and Technology* 113 81-90.
713 DOI: 10.1016/j.coldregions.2015.02.003

714 Zhu, Y. and Carbee, D. L. (1985) Strain rate effect on the tensile strength of frozen silt. *The*

715 Proceedings of the 4th International Symposium on Ground Freezing, Sapporo, Japan, Eds.
716 Kinoshita, S. and Fukuda, M., Balkema, Rotterdam, The Netherlands 153-157.
717 Zhu, Y. and Carbee, D. L. (1987) Tensile strength of frozen silt. CRREL Report 87-15. 23p.

718
719

720 LIST OF NOTATIONS

721 A_2 : Area of the middle part of the specimen
722 D_1, D_2 : Diameters of the end and middle parts of the specimen
723 e_c : Void ratio after consolidation
724 H_1, H_2 : Total and middle part heights of the specimen
725 p_c' : The effective mean effective stress at consolidation
726 p_r : The total mean effective stress after freezing and before extension
727 $q, |q|_{\max}$: Deviator stress (=axial stress – radial stress) and its absolute maximum value
728 S_i : Degree of liquid saturation (=volume of ice / volume of pore)
729 s_u : Undrained shear strength
730 T : Temperature
731 u : Pore water pressure
732 u_0 : Back pressure
733 u_c : Pore water pressure after consolidation (= back pressure in the experiment)
734 u_{\min} : The minimum sustainable pore water pressure
735 w : Water content
736 w_u : Gravimetric unfrozen water content
737 ϵ_1, ϵ_3 : Major and minor principal strains
738 ϵ_a, ϵ_r : Axial and radial strains (= ϵ_3 and ϵ_1 , respectively)
739 σ, σ' : Generic expressions for total and effective normal stresses
740 $\sigma_1, \sigma_2, \sigma_3$: Major, intermediate, and minor principal stresses
741 $\sigma_{1f}, \sigma_{3f}, \sigma_{1f}', \sigma_{3f}'$: Major and minor principal stresses at failure, and their effective values
742 σ_a, σ_r : Axial and radial stresses (= σ_3 and σ_1 , respectively)
743 σ_t, σ_t' : Total and effective tensile strength
744 σ_{t0} : (Total) tensile strength at zero shear strain
745 σ_{vp}' : One-dimensional pre-consolidation effective stress
746 τ : Shear stress