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Antiparallel spin polarization induced by current in a bilayer

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The antiparallel current-induced spin polarization (CISP) in an inversion-symmetric double-quantum-well structure with the antiparallel Rashba effective magnetic field is calculated by solving the Boltzmann equation which is derived for the distribution operator in layer-pseudospin space in the relaxation-time approximation. The antiparallel CISP in an analytical form, obtained for constant momentum relaxation time in the limit of large Fermi energy, exhibits factor-two increase with increasing the interlayer-tunneling strength. This increase originates from the correction (to the semiclassical CISP) created by the subband mixing due to the momentum dependence of the Rashba effective magnetic field. The present interlayer-coupling dependence of the antiparallel CISP is successfully explained as the layer-pseudospin Hanle effect in which the spin in the original Hanle effect is replaced by the layer pseudospin. The present finding suggests that the correction by band mixing can be significant in transport properties of various systems with the momentum-dependent effective magnetic field.

Keywords: current-induced spin polarization, bilayer, quantum well, Rashba effective magnetic field, Boltzmann equation, pseudospin

I. INTRODUCTION

The broken inversion symmetry in crystals is known to have significant impacts on their physical properties. As a typical example the absence of inversion symmetry can create, through the spin splitting due to the spin-orbit interaction (SOI), the current-induced spin polarization (CISP) [1–7], which has been used for the magnetization switching [8], one of basic actions in spintronics [9].

Recent studies have shown that even *locally* broken inversion symmetry in a system with the *global* inversion symmetry affects superconducting [10–12] and spintronic [13] properties in metals and semiconductors. The presence of both the global inversion symmetry and the time reversal symmetry ensures that two states with opposite spins at a given wave vector in crystals are degenerate. However, these two degenerate states, when occupied, can give a staggered spin density if an inversion-symmetric crystal consists of two sectors with a broken inversion symmetry between which is the inversion center of the crystal [13]. Such local spin density, which is called hidden spin polarization [13], has recently been investigated by many theoretical [14, 15] and experimental [16–21] studies. Among others, a theoretical study [22] has shown that the local spin density of doubly degenerate eigenstates at each wave vector leads to a local spin polarization when contributions from all occupied states are summed in the presence of current. Such a local CISP, generated by a staggered current-induced effective magnetic field, can be used for the sublattice-magnetization switching in antiferromagnet, which has been proposed in the above-mentioned study [22] and has actually been demonstrated in a subsequent experiment [23]. Extensive studies on the local CISP are desirable in both physics and application. From an application point

of view, a quantitative and systematic calculation of the local CISP, including the robustness against the thermal disturbance, is indispensable. From a physical point of view, many fundamental questions will arise about the present local staggered CISP since the global and local CISPs correspond to ferromagnet and antiferromagnet, respectively. One of such questions is the magnitude of the local CISP compared to that of the global CISP.

In this paper we consider, as a simple system with locally broken inversion symmetry, a symmetric double-quantum-well (DQW) structure [12, 24–32] as illustrated in Fig. 1. This system is characterized by a double-well potential along the z direction and an additional orbital degree of freedom (layer pseudospin) consisting of $|L\rangle$ and $|R\rangle$, localized states in the left and right wells. These two wells have opposite potential gradients along z , giving rise to antiparallel Rashba effective magnetic field acting on spin. Such a bilayer system has the advantage in studying the antiparallel local CISP because we can compare the local CISP in a coupled bilayer with the global CISP in a separate layer with no inversion symmetry by changing only the width or the height of the barrier separating the two wells. In order to clarify the difference in magnitude between the local CISP and the global CISP, we calculate the antiparallel CISP in the limit of large Fermi energy as a function of the interlayer coupling strength represented by Δ_{SAS} , which is the energy separation between the symmetric ground state and the antisymmetric first-excited state in the absence of the Rashba SOI. We also investigate the temperature dependence and the influence of the layer-pseudospin decoherence in order to clarify the robustness of the antiparallel CISP.

We find that the magnitude of the antiparallel CISP in the strongly-coupled bilayer (with locally broken in-

version symmetry) is twice as large as that of a decoupled bilayer (with no inversion symmetry in each layer) and that such increase with the interlayer coupling occurs around $\omega\tau_p = 1$ in the case of $\omega_{so}\tau_p \ll 1$ where $\omega \equiv \Delta_{SAS}/\hbar$, τ_p is the momentum relaxation time and ω_{so} represents the strength of the Rashba SOI. We also show that the temperature dependence is absent at temperatures much lower than the Fermi temperature, and that a strong pseudospin decoherence increases the antiparallel CISP to the value of strongly-coupled bilayer at any ω as long as $\omega \gg \omega_{so}$.

We explain the calculated variation of the antiparallel CISP as a function of $\omega\tau_p$ by dividing the antiparallel CISP into the semiclassical value and the correction. The semiclassical value, which is the same as the value at the strong pseudospin decoherence, has no dependence on ω as long as $\omega \gg \omega_{so}$. The correction is opposite in sign to the semiclassical value and significant in the region of $\omega\tau_p < 1$, leading to the reduction of the antiparallel CISP with decreasing $\omega\tau_p$. In the region of $\omega\tau_p < 1$, the correction is comparable to the semiclassical value and the semiclassical calculation is not accurate. We describe the suppression of the correction with increasing $\omega\tau_p$ by the pseudospin Hanle effect in which the spin in the original Hanle effect [33–35] is replaced by the pseudospin. The original spin Hanle effect has been playing an important role in spin physics and spintronics (see Ref. [9] for a review and [36–43] for examples of recent achievements).

The organization of the paper is as follows. Section II describes the Hamiltonian of the DQW structure and introduces layer-pseudospin operators and basis vectors. Section III derives the Boltzmann equation in the relaxation-time approximation for the distribution function which is expressed by the operator in the pseudospin space. The solution for the distribution operator in the linear response to the applied inplane electric field is obtained in Sec. IV and is employed to derive the analytical formula for the antiparallel CISP in Sec. V. In Sec. V we also explain the obtained dependence on the interlayer coupling ω of the antiparallel CISP by dividing it into the semiclassical CISP and the correction due to subband mixing, and describe the ω dependence of the correction by introducing the pseudospin analog of the Hanle effect. In Sec. VI, conclusions are given.

II. HAMILTONIAN

The system we consider is the DQW structure having the mirror symmetry with respect to the plane between two wells, which is chosen as the xy plane ($z = 0$) (see Fig. 1). We describe electron states in the conduction band within the effective-mass formalism including the SOI (the effective-mass formalism has been developed to include the SOI originating from electrostatic potential [44] and from band offsets in a heterostructure [27, 45]). We assume that the state vector with respect to the motion along the z direction is given by a linear combination

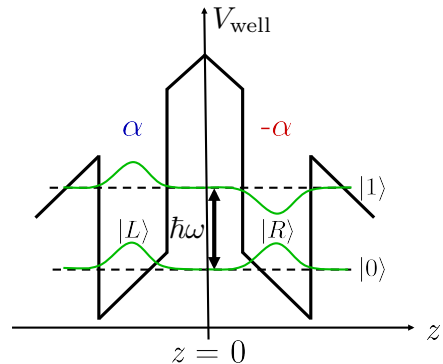


FIG. 1. Potential $V_{\text{well}}(z)$ and electronic states in a symmetric DQW structure (schematic). State vectors $|0\rangle$ and $|1\rangle$ represent the symmetric ground state and the antisymmetric first-excited state, respectively, for the motion perpendicular to the DQW (along z) in the absence of the SOI, each of which is composed of $|L\rangle$ and $|R\rangle$, bound states in the left and right wells. The energy separation $\hbar\omega = \Delta_{\text{SAS}}$ between $|0\rangle$ and $|1\rangle$ is a measure of the strength of the coupling between $|L\rangle$ and $|R\rangle$. The symmetric DQW has antiparallel Rashba effective magnetic field with the Rashba coefficient α (> 0) and $-\alpha$ in the left and right wells, respectively.

of $|L\rangle$ and $|R\rangle$, the lowest-energy state tightly bound to the left and right wells, respectively. Thus the motion along z can be represented by the pseudospin degree of freedom for which the corresponding Pauli operators, $\hat{\tau}_\gamma$ ($\gamma = 1, 2$, and 3), and the identity operator, $\hat{\tau}_0$, are defined by

$$\begin{aligned}\hat{\tau}_0 &= |L\rangle\langle L| + |R\rangle\langle R|, \\ \hat{\tau}_1 &= |R\rangle\langle L| + |L\rangle\langle R|, \\ \hat{\tau}_2 &= i|R\rangle\langle L| - i|L\rangle\langle R|, \\ \hat{\tau}_3 &= |L\rangle\langle L| - |R\rangle\langle R|.\end{aligned}\tag{1}$$

The Hamiltonian describing the electron motion in the DQW structure is

$$H_W = \frac{\hat{\mathbf{p}}^2}{2m} + V_{\text{well}}(z) + H_{\text{well}}^{\text{so}},\tag{2}$$

$$H_{\text{well}}^{\text{so}} = -\hbar^{-1}\eta \hat{\boldsymbol{\sigma}} \cdot (\nabla \tilde{V}_{\text{well}} \times \hat{\mathbf{p}}),\tag{3}$$

where $\hat{\mathbf{p}}$ is the momentum operator, $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y, \hat{p}_z) = -i\hbar\nabla = -i\hbar(\nabla_x, \nabla_y, \nabla_z)$, $\hat{\boldsymbol{\sigma}}$ is the Pauli spin operator, $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$, m is the effective mass of the conduction band, and η is the effective coupling constant of the SOI for an electron in the conduction band. The potential $V_{\text{well}}(z) [= V_{\text{bo}}^c(z) + V_{\text{es}}(z)]$ consists of $V_{\text{bo}}^c(z)$ given by the conduction-band offset between the well and barrier materials and $V_{\text{es}}(z)$ induced by the dopant charge distribution and the electron-electron interaction in the Hartree approximation. We take into account the Rashba SOI [46–48] due to $V_{\text{well}}(z)$, which is denoted by $H_{\text{well}}^{\text{so}}$. In Eq. (3) for $H_{\text{well}}^{\text{so}}$ appears $\tilde{V}_{\text{well}}(z) [= b_{\text{off}}V_{\text{bo}}^c + V_{\text{es}}]$ which is different from $V_{\text{well}}(z)$ in that the band-offset component of $\tilde{V}_{\text{well}}(z)$ includes a factor b_{off} depending on the

well and barrier materials [49, 50]:

$$b_{\text{off}} = \frac{\Delta E_v / [E_g(E_g - \Delta E_v)] - \Delta E_s / [E_g^s(E_g^s - \Delta E_s)]}{\Delta E_c \{1/(E_g)^2 - 1/(E_g^s)^2\}},$$

where $E_g = E_c - E_v$, $E_g^s = E_g + \Delta_{\text{so}}$ ($\Delta_{\text{so}} = E_v - E_s$), $\Delta E_c = E_c^b - E_c$, $\Delta E_v = E_v^b - E_v$, and $\Delta E_s = E_s^b - E_s$ with E_c (E_c^b) the energy of the conduction-band bottom, E_v (E_v^b) that of the valance-band top, E_s (E_s^b) that of the split-off-band top in the well layer (the barrier layer).

Owing to the translational symmetry in the xy plane, the unperturbed Hamiltonian H_W is diagonal in the in-plane wave vector, $\mathbf{k} = (k_x, k_y)$, which is defined by $\hat{p}_x|\mathbf{k}\rangle = \hbar k_x|\mathbf{k}\rangle$ and $\hat{p}_y|\mathbf{k}\rangle = \hbar k_y|\mathbf{k}\rangle$ and corresponding eigenvectors $|\mathbf{k}\rangle$ are chosen as basis vectors for the motion in the xy plane. As basis vectors for the spin degree of freedom, we choose $|\sigma\rangle$ with $\sigma = \pm 1$ ($\sigma = \uparrow, \downarrow$) defined by

$$\hat{\sigma} \cdot \mathbf{e}_b|\sigma\rangle = \sigma|\sigma\rangle, \quad (4)$$

where $\mathbf{e}_b$ is the unit vector in the direction of the $\mathbf{k}$ -dependent effective magnetic field, given by

$$\mathbf{e}_b = k^{-1}(k_y, -k_x), \quad (5)$$

with $k = \sqrt{k_x^2 + k_y^2}$.

Then H_W is also diagonal in σ and diagonal elements are

$$\langle \mathbf{k}\sigma | H_W | \mathbf{k}\sigma \rangle = \varepsilon_k + H_{\perp}, \quad (6)$$

where

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \quad (7)$$

and $H_{\perp}$ describes the motion along z at given $\mathbf{k}$ and σ and is expressed by

$$\begin{aligned} H_{\perp} &= \frac{\hat{p}_z^2}{2m} + V_{\text{well}}(z) + \langle \mathbf{k}\sigma | H_{\text{well}}^{\text{so}} | \mathbf{k}\sigma \rangle \\ &= -\frac{1}{2}\hbar\omega\hat{\tau}_1 + \frac{1}{2}\hbar\omega_{\text{so}}\sigma\hat{\tau}_3. \end{aligned} \quad (8)$$

Here $\hbar\omega = \Delta_{\text{SAS}}$ represents the interlayer coupling which is chosen to be positive ($\omega > 0$) so that $(|L\rangle + |R\rangle)/\sqrt{2}$ is the ground state at $\omega_{\text{so}} = 0$. On the other hand $\hbar\omega_{\text{so}}$ represents the spin splitting due to the SOI in the decoupled bilayer ($\omega = 0$) and is given by

$$\hbar\omega_{\text{so}} = 2\alpha k, \quad (9)$$

with α the coefficient of the Rashba SOI which is defined by

$$\alpha = \eta \left\langle L \left| \nabla_z \tilde{V}_{\text{well}} \right| L \right\rangle > 0. \quad (10)$$

Even when we take into account the linear-in- $\mathbf{k}$ Dresselhaus SOI [51, 52], the Dresselhaus term in a (111)-oriented DQW with negligible $\langle L | \hat{p}_z^2 | R \rangle$ has the form of

$\beta\sigma k\hat{\tau}_0$ and can be canceled out with the symmetric part of the Rashba term, $\alpha_s\sigma k\hat{\tau}_0$, by introducing and adjusting the antisymmetric part of $\tilde{V}_{\text{well}}$ in Eq. (3) so that the total effective magnetic field is antiparallel.

We denote the ground-state and first-excited-state eigenvectors of $H_{\perp}$ by $|G\rangle$ and $|E\rangle$, respectively:

$$H_{\perp}|n\rangle = \varepsilon_n(k)|n\rangle \quad (n = G, E), \quad (11)$$

where $\varepsilon_n(k)$ is the corresponding eigenenergy,

$$\varepsilon_G(k) = -\frac{1}{2}\hbar\omega_1, \quad \varepsilon_E(k) = \frac{1}{2}\hbar\omega_1, \quad (12)$$

with

$$\omega_1 = \sqrt{(\omega_{\text{so}})^2 + \omega^2}. \quad (13)$$

Then the eigenvector of H_W is $|n\mathbf{k}\sigma\rangle \equiv |n\rangle|\mathbf{k}\rangle|\sigma\rangle$:

$$H_W|n\mathbf{k}\sigma\rangle = \varepsilon_{nk}|n\mathbf{k}\sigma\rangle, \quad (14)$$

with the corresponding eigenenergy

$$\varepsilon_{nk} = \varepsilon_k + \varepsilon_n(k). \quad (15)$$

III. BOLTZMANN EQUATION

In this paper we derive the formula for the local CISP by solving the Boltzmann equation for the density operator $\hat{\rho}$ [53]. Since off-diagonal matrix elements $\langle n'\mathbf{k}\sigma | \hat{\rho} | n\mathbf{k}\sigma \rangle$ ($n' \neq n$) is nonzero in the presence of the phase coherence in the pseudospin degree of freedom, it is convenient to introduce an operator in the pseudospin space,

$$\hat{\rho}_{\mathbf{k}\sigma} = \langle \mathbf{k}\sigma | \hat{\rho} | \mathbf{k}\sigma \rangle. \quad (16)$$

We call $\hat{\rho}_{\mathbf{k}\sigma}$ the distribution operator because it is reminiscent of the distribution function for a state with wave vector $\mathbf{k}$ and spin σ . In this paper we neglect the spin coherence (that is, assume $\langle \mathbf{k}, -\sigma | \hat{\rho} | \mathbf{k}\sigma \rangle = 0$) in order to focus on the effect of the pseudospin coherence in the antiparallel CISP. In uniform steady states, the distribution operator $\hat{\rho}_{\mathbf{k}\sigma}$ satisfies the following Boltzmann equation

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_{\text{force}} = \left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_{\text{p}} + \left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_{\text{c}} + \left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_{\text{d}}. \quad (17)$$

On the left-hand side of this equation is the force term representing the change due to the force, $\mathbf{F} = (-e)\mathbf{E}$, acting on electrons, where $\mathbf{E} = (E_x, E_y, 0)$ is the in-plane electric field and $e > 0$ is the absolute value of the electronic charge. On the right-hand side, the first, second, and third terms describe the precession of pseudospin, the collision with impurities and phonons, and the decoherence of pseudospin, respectively.

The force term is in general given [54] by

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}}}{\partial t} \right)_{\text{force}} = \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \hat{\rho}_{\mathbf{k}}}{\partial \mathbf{k}}, \quad (18)$$

where $\hat{\rho}_{\mathbf{k}}$ is the operator in the space of the additional degree of freedom, in the present paper, consisting of spin and pseudospin. The incoherence of spin, $\langle -\sigma | \hat{\rho}_{\mathbf{k}} | \sigma \rangle = 0$, leads to

$$\left\langle \sigma \left| \frac{\partial \hat{\rho}_{\mathbf{k}}}{\partial \mathbf{k}} \right| \sigma \right\rangle = \frac{\partial}{\partial \mathbf{k}} \langle \sigma | \hat{\rho}_{\mathbf{k}} | \sigma \rangle. \quad (19)$$

Then the force term becomes

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_{\text{force}} = \frac{\mathbf{F}}{\hbar} \cdot \left\langle \sigma \left| \frac{\partial \hat{\rho}_{\mathbf{k}}}{\partial \mathbf{k}} \right| \sigma \right\rangle = \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial \mathbf{k}}. \quad (20)$$

The term describing the pseudospin precession in Eq. (17) is derived by taking the element diagonal in $\mathbf{k}\sigma$ of

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar} [H_W, \hat{\rho}], \quad (21)$$

to be

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_p = \frac{1}{i\hbar} [H_{\perp}, \hat{\rho}_{\mathbf{k}\sigma}]. \quad (22)$$

Now we determine the collision term in Eq. (17). There are a variety of electron scattering models which are described by different collision terms. Because the purpose of this paper is not to study dependences of CISP on scattering models, but to study the effect of pseudospin coherence in CISP. Therefore we have chosen the simplest collision term, that is the collision term in the relaxation-time approximation, which is expressed by

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_c = -\frac{1}{\tau_p} \hat{\rho}_{\mathbf{k}\sigma}^{(1)}, \quad (23)$$

where $\hat{\rho}_{\mathbf{k}\sigma}^{(1)}$ is the deviation of $\hat{\rho}_{\mathbf{k}\sigma}$ in the first order of $E = |\mathbf{E}|$ from that in $\mathbf{E} = 0$, $\hat{\rho}_{\mathbf{k}\sigma}^{(0)}$:

$$\hat{\rho}_{\mathbf{k}\sigma} = \hat{\rho}_{\mathbf{k}\sigma}^{(0)} + \hat{\rho}_{\mathbf{k}\sigma}^{(1)}, \quad (24)$$

and τ_p is assumed to have no dependence on $k = |\mathbf{k}|$ and σ . In this approximation the distribution function obtained from the balance between the collision term and the force term is $\hat{\rho}_{\mathbf{k}\sigma}^{(0)} + \hat{\rho}_{\mathbf{k}\sigma}^{(1)} = \hat{\rho}_{\mathbf{k}\sigma}^{(0)} - \tau_p \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \hat{\rho}_{\mathbf{k}\sigma}^{(0)}}{\partial \mathbf{k}} = \hat{\rho}_{\mathbf{k}-\mathbf{k}_1\sigma}^{(0)}$ with $\mathbf{k}_1 = \tau_p \frac{\mathbf{F}}{\hbar}$, which represents the distribution obtained by shifting the equilibrium distribution $\hat{\rho}_{\mathbf{k}\sigma}^{(0)}$ by $\mathbf{k}_1$. This is the simplest distribution in the presence of the current flow. The term describing the decoherence of pseudospin in Eq. (17) is given, according to the widely-used decoherence model for a two-level system [55], by

$$\left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_d = -\frac{1}{\tau_{\text{ps}}} (|G\rangle\langle G| \hat{\rho}_{\mathbf{k}\sigma} |E\rangle\langle E| + |E\rangle\langle E| \hat{\rho}_{\mathbf{k}\sigma} |G\rangle\langle G|), \quad (25)$$

with τ_{ps} a constant coherence time. Here the projection operators $|G\rangle\langle G|$ and $|E\rangle\langle E|$ can be expressed by $H_{\perp}$:

$$|G\rangle\langle G| = \frac{1}{2} - \frac{H_{\perp}}{\hbar\omega_1}, \quad |E\rangle\langle E| = \frac{1}{2} + \frac{H_{\perp}}{\hbar\omega_1}. \quad (26)$$

Then the Boltzmann equation Eq. (17) reduces to

$$\frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial \mathbf{k}} = \frac{1}{i\hbar} [H_{\perp}, \hat{\rho}_{\mathbf{k}\sigma}] - \frac{1}{\tau_p} \hat{\rho}_{\mathbf{k}\sigma}^{(1)} + \left(\frac{\partial \hat{\rho}_{\mathbf{k}\sigma}}{\partial t} \right)_d. \quad (27)$$

It is convenient [56, 57] to decompose the distribution operator $\hat{\rho}_{\mathbf{k}\sigma}$ into a linear combination of the pseudospin operator $\hat{\tau}_{\gamma}$,

$$\hat{\rho}_{\mathbf{k}\sigma} = \frac{1}{2} \sum_{\gamma=0}^3 \tau_{\gamma\mathbf{k}\sigma} \hat{\tau}_{\gamma}, \quad (28)$$

according to the similar decomposition for the spin degree of freedom [58–60]. Then $\tau_{\gamma\mathbf{k}\sigma}$ is given by the statistical average of $\hat{\tau}_{\gamma}$ at given momentum $\mathbf{k}$ and spin σ ,

$$\tau_{\gamma\mathbf{k}\sigma} = \sum_{\ell=L,R} \langle \ell | \hat{\rho}_{\mathbf{k}\sigma} \hat{\tau}_{\gamma} | \ell \rangle. \quad (29)$$

Using $\tau_{\gamma\mathbf{k}\sigma}$ the Boltzmann equation Eq.(27) is expressed by

$$\begin{aligned} \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \tau_{0\mathbf{k}\sigma}}{\partial \mathbf{k}} &= -\tau_p^{-1} \tau_{0\mathbf{k}\sigma}^{(1)}, \\ \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \tau_{1\mathbf{k}\sigma}}{\partial \mathbf{k}} &= -\sigma \omega_{\text{so}} \tau_{2\mathbf{k}\sigma} - \tau_p^{-1} \tau_{1\mathbf{k}\sigma}^{(1)} + \left(\frac{\partial \tau_{1\mathbf{k}\sigma}}{\partial t} \right)_d, \\ \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \tau_{2\mathbf{k}\sigma}}{\partial \mathbf{k}} &= \omega \tau_{3\mathbf{k}\sigma} + \sigma \omega_{\text{so}} \tau_{1\mathbf{k}\sigma} - \tau_p^{-1} \tau_{2\mathbf{k}\sigma}^{(1)} + \left(\frac{\partial \tau_{2\mathbf{k}\sigma}}{\partial t} \right)_d, \\ \frac{\mathbf{F}}{\hbar} \cdot \frac{\partial \tau_{3\mathbf{k}\sigma}}{\partial \mathbf{k}} &= -\omega \tau_{2\mathbf{k}\sigma} - \tau_p^{-1} \tau_{3\mathbf{k}\sigma}^{(1)} + \left(\frac{\partial \tau_{3\mathbf{k}\sigma}}{\partial t} \right)_d, \end{aligned} \quad (30)$$

where

$$\begin{aligned} \left(\frac{\partial \tau_{1\mathbf{k}\sigma}}{\partial t} \right)_d &= -\tau_{\text{ps}}^{-1} (s_{\omega}^2 \tau_{1\mathbf{k}\sigma} + \sigma s_{\omega} c_{\omega} \tau_{3\mathbf{k}\sigma}), \\ \left(\frac{\partial \tau_{2\mathbf{k}\sigma}}{\partial t} \right)_d &= -\tau_{\text{ps}}^{-1} \tau_{2\mathbf{k}\sigma}, \\ \left(\frac{\partial \tau_{3\mathbf{k}\sigma}}{\partial t} \right)_d &= -\tau_{\text{ps}}^{-1} (\sigma s_{\omega} c_{\omega} \tau_{1\mathbf{k}\sigma} + c_{\omega}^2 \tau_{3\mathbf{k}\sigma}), \end{aligned} \quad (31)$$

with

$$s_{\omega} = \frac{\omega_{\text{so}}}{\omega_1}, \quad c_{\omega} = \frac{\omega}{\omega_1}, \quad s_{\omega}^2 + c_{\omega}^2 = 1. \quad (32)$$

IV. DISTRIBUTION OPERATOR

A. Distribution operator in the absence of the inplane electric field

We can choose, as the density operator in the zeroth order of $E = |\mathbf{E}|$,

$$\hat{\rho}^{(0)} = f_0(H_W), \quad f_0(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) + 1 \right]^{-1}, \quad (33)$$

where $f_0(\varepsilon)$ is the Fermi-Dirac distribution function with the chemical potential μ and the temperature T . Since

$$\hat{\rho}_{\mathbf{k}\sigma}^{(0)} = \sum_{n=G,E} |n\rangle f_0(\varepsilon_{nk}) \langle n|, \quad (34)$$

we obtain $\tau_{\gamma\mathbf{k}\sigma}^{(0)}$, the γ component of $2\hat{\rho}_{\mathbf{k}\sigma}^{(0)}$, as follows:

$$\begin{aligned} \tau_{0\mathbf{k}\sigma}^{(0)} &= f_0(\varepsilon_{Gk}) + f_0(\varepsilon_{Ek}), \\ \tau_{1\mathbf{k}\sigma}^{(0)} &= c_\omega [f_0(\varepsilon_{Gk}) - f_0(\varepsilon_{Ek})], \\ \tau_{2\mathbf{k}\sigma}^{(0)} &= 0, \\ \tau_{3\mathbf{k}\sigma}^{(0)} &= \sigma s_\omega [-f_0(\varepsilon_{Gk}) + f_0(\varepsilon_{Ek})]. \end{aligned} \quad (35)$$

B. Distribution operator in the linear response to the inplane electric field

To obtain the γ component, $\tau_{\gamma\mathbf{k}\sigma}^{(1)}$, corresponding to the distribution operator in the first order of E , we retain first-order terms in the Boltzmann equation and seek the solution for $\tau_{\gamma\mathbf{k}\sigma}^{(1)}$ of the form

$$\tau_{\gamma\mathbf{k}\sigma}^{(1)} = b_{\gamma\mathbf{k}\sigma} \mathbf{F} \cdot \hbar \mathbf{k}, \quad (36)$$

where $b_{\gamma\mathbf{k}\sigma}$ has no dependence on the direction of $\mathbf{k}$.

Then the Boltzmann equation expressed with use of $\tau_{\gamma\mathbf{k}\sigma}$, Eq. (30), gives the following coupled equations for $b_{\gamma\mathbf{k}\sigma}$:

$$C_{0k} = -\tau_p^{-1} b_{0k\sigma}, \quad (37)$$

$$C_{1k} = -\sigma \omega_{s0} b_{2k\sigma} - \tau_p^{-1} b_{1k\sigma} - \tau_{ps}^{-1} (s_\omega^2 b_{1k\sigma} + \sigma s_\omega c_\omega b_{3k\sigma}), \quad (38)$$

$$0 = \omega b_{3k\sigma} + \sigma \omega_{s0} b_{1k\sigma} - (\tau_p^{-1} + \tau_{ps}^{-1}) b_{2k\sigma}, \quad (39)$$

$$\sigma C_{3k} = -\omega b_{2k\sigma} - \tau_p^{-1} b_{3k\sigma} - \tau_{ps}^{-1} (\sigma s_\omega c_\omega b_{1k\sigma} + c_\omega^2 b_{3k\sigma}), \quad (40)$$

with

$$\begin{aligned} C_{0k} &= \frac{1}{\hbar^2 k} \frac{\partial}{\partial k} [f_0(\varepsilon_{Gk}) + f_0(\varepsilon_{Ek})], \\ C_{1k} &= \frac{1}{\hbar^2 k} \frac{\partial}{\partial k} \{c_\omega [f_0(\varepsilon_{Gk}) - f_0(\varepsilon_{Ek})]\}, \\ C_{3k} &= \frac{1}{\hbar^2 k} \frac{\partial}{\partial k} \{s_\omega [-f_0(\varepsilon_{Gk}) + f_0(\varepsilon_{Ek})]\}. \end{aligned} \quad (41)$$

Here we perform the following rotation in pseudospin space:

$$\begin{pmatrix} \tilde{b}_{1k\sigma} \\ \tilde{b}_{2k\sigma} \\ \tilde{b}_{3k\sigma} \end{pmatrix} = \begin{pmatrix} c_\omega & 0 & -\sigma s_\omega \\ 0 & 1 & 0 \\ \sigma s_\omega & 0 & c_\omega \end{pmatrix} \begin{pmatrix} b_{1k\sigma} \\ b_{2k\sigma} \\ b_{3k\sigma} \end{pmatrix}. \quad (42)$$

Then we obtain, from Eqs. (38), (39) and (40), a set of equations for $\tilde{b}_{\gamma\mathbf{k}\sigma}$

$$\tilde{C}_{1k} = -\tau_p^{-1} \tilde{b}_{1k\sigma}, \quad (43)$$

$$0 = \omega_1 \tilde{b}_{3k\sigma} - \tau_1^{-1} \tilde{b}_{2k\sigma}, \quad (44)$$

$$\sigma \tilde{C}_{3k} = -\omega_1 \tilde{b}_{2k\sigma} - \tau_1^{-1} \tilde{b}_{3k\sigma}, \quad (45)$$

with

$$\tau_1^{-1} = \tau_p^{-1} + \tau_{ps}^{-1}, \quad (46)$$

and

$$\begin{aligned} \tilde{C}_{1k} &\equiv c_\omega C_{1k} - s_\omega C_{3k} = \frac{1}{\hbar^2 k} \frac{\partial}{\partial k} [f_0(\varepsilon_{Gk}) - f_0(\varepsilon_{Ek})], \\ \tilde{C}_{3k} &\equiv s_\omega C_{1k} + c_\omega C_{3k} = -\frac{c_\omega s_\omega}{\hbar^2 k^2} [f_0(\varepsilon_{Gk}) - f_0(\varepsilon_{Ek})]. \end{aligned} \quad (47)$$

Equations (43), (44) and (45) show that the pseudospin precesses around the new axis $\hat{\mathbf{l}}$ with frequency ω_1 and that the transverse time τ_1 is shorter than the longitudinal time τ_p in the presence of the pseudospin decoherence ($\tau_{ps}^{-1} \neq 0$). The solution is immediately obtained to be

$$\begin{aligned} \tilde{b}_{1k\sigma} &= -\tau_p \tilde{C}_{1k}, \\ \begin{pmatrix} \tilde{b}_{2k\sigma} \\ \tilde{b}_{3k\sigma} \end{pmatrix} &= \frac{(-\sigma) \tau_1 \tilde{C}_{3k}}{1 + \omega_1^2 \tau_1^2} \begin{pmatrix} \omega_1 \tau_1 \\ 1 \end{pmatrix}, \end{aligned} \quad (48)$$

and the original coefficients become

$$b_{1k\sigma} = -c_\omega \tau_p \tilde{C}_{1k} - s_\omega \frac{\tau_1 \tilde{C}_{3k}}{1 + \omega_1^2 \tau_1^2}, \quad (49)$$

$$b_{2k\sigma} = \frac{(-\sigma) \tau_1 \tilde{C}_{3k}}{1 + \omega_1^2 \tau_1^2} \omega_1 \tau_1, \quad (50)$$

$$b_{3k\sigma} = \sigma s_\omega \tau_p \tilde{C}_{1k} - c_\omega \frac{\sigma \tau_1 \tilde{C}_{3k}}{1 + \omega_1^2 \tau_1^2}. \quad (51)$$

V. CURRENT-INDUCED SPIN POLARIZATION

We define the spin polarization along the direction μ ($= x, y$) in the pseudospin component γ ($= 1, 2, 3$) by

$$\sigma_{\mu\gamma} = S^{-1} \sum_{\ell=L,R} \sum_{\mathbf{k}\sigma} \langle \ell \mathbf{k}\sigma | \hat{\rho} \hat{\tau}_\gamma \hat{\sigma}_\mu | \ell \mathbf{k}\sigma \rangle. \quad (52)$$

Since $\langle \sigma | \hat{\rho}_{\mathbf{k}} | -\sigma \rangle = 0$ (spin incoherence) is assumed and the spin polarization is absent in the zeroth order of E , we have

$$\sigma_{\mu\gamma} = S^{-1} \sum_{\mathbf{k}\sigma} \tau_{\gamma\mathbf{k}\sigma}^{(1)} \langle \mathbf{k}\sigma | \hat{\sigma}_\mu | \mathbf{k}\sigma \rangle, \quad (53)$$

with

$$\langle \mathbf{k}\sigma | \hat{\sigma}_x | \mathbf{k}\sigma \rangle = \sigma k^{-1} k_y, \quad \langle \mathbf{k}\sigma | \hat{\sigma}_y | \mathbf{k}\sigma \rangle = -\sigma k^{-1} k_x. \quad (54)$$

Now we choose the x axis in the direction of the inplane electric field. Then $\tau_{\gamma\mathbf{k}\sigma}^{(1)} = b_{\gamma\mathbf{k}\sigma} \mathbf{F} \cdot \hbar \mathbf{k}$ is proportional to k_x and

$$\begin{aligned} \sigma_{x\gamma} &= 0, \\ \sigma_{y\gamma} &= -F_x \hbar (2S)^{-1} \sum_{\mathbf{k}\sigma} k_\sigma b_{\gamma\mathbf{k}\sigma}. \end{aligned} \quad (55)$$

Since the spin polarization originates from the spin dependence of $b_{\gamma k\sigma}$, the CISP only appears in $\gamma = 2$ and 3 components.

In this paper we focus on the CISP in $\gamma = 3$ (antiparallel) component σ_{y3} . To obtain σ_{y3} in an analytical formula, we restrict our calculation to the case where $\hbar\omega_1 \ll \varepsilon_F$ and $k_B T \ll \varepsilon_F$ with $\varepsilon_F = \hbar^2 k_F^2 / 2m$ the Fermi energy. Then we can replace k , including that in ω_{so} , with the Fermi wavenumber k_F and obtain

$$\sigma_{y3} = 2\sigma_{yL} \left(2 - s_\omega^2 - c_\omega^2 \frac{\tau_1}{\tau_p} \frac{1}{1 + \omega_1^2 \tau_1^2} \right), \quad (56)$$

where σ_{yL} represents the CISP of layer L in the decoupled bilayer ($\omega = 0$), given by

$$\sigma_{yL} = -\tau_p e E_x \frac{m\alpha}{2\pi\hbar^3}. \quad (57)$$

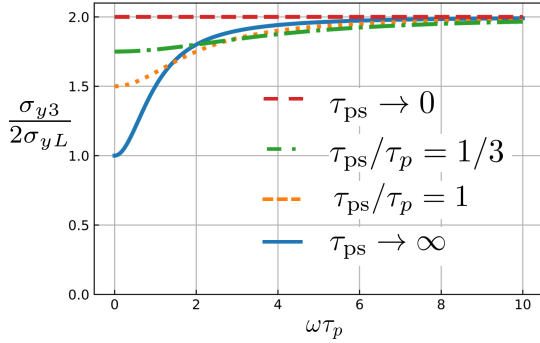


FIG. 2. Normalized antiparallel CISP $\sigma_{y3}/(2\sigma_{yL})$ [Eq. (56)] as a function of $\omega\tau_p$ in the case where $\omega_{so}\tau_p \ll 1$ and $\hbar\omega, \hbar\omega_{so}, k_B T \ll \varepsilon_F$. Here σ_{yL} is the CISP of decoupled layer L [Eq. (57)], ω and ω_{so} represent the strength of the interlayer coupling and that of the Rashba SOI, respectively, τ_p is the momentum relaxation time, and τ_{ps} is the pseudospin coherence time. Note that $\sigma_{y3}/(2\sigma_{yL}) \approx 1$ in the region where ω is so small that $\omega \ll \omega_{so}$ (not shown in the figure).

We have $\sigma_{y3} = 2\sigma_{yL}$ in the absence of the interlayer coupling ($\omega = 0$). On the other hand, when the interlayer coupling is so strong that $\omega\tau_1 \gg 1$ and $\omega \gg \omega_{so}$, we have

$$\sigma_{y3} = 4\sigma_{yL} \quad (\omega \gg \tau_1^{-1}, \omega_{so}). \quad (58)$$

This might be considered as an unexpected result from the fact that the increase of ω at a fixed ω_{so} gives the decrease of the local spin density produced by two degenerate states, $|n\mathbf{k}\uparrow\rangle$ and $|n\mathbf{k}\downarrow\rangle$. The reason why σ_{y3} does not vanish at $\omega \rightarrow \infty$ is that the effect of such decrease in the local spin density is cancelled out by that of the increase in the energy separation between the ground and first-excited states, $|G\rangle$ and $|E\rangle$. The value of $\sigma_{y3}/(2\sigma_{yL})$ in the case of $\omega_{so}\tau_p \ll 1$ is plotted in Fig. 2 as a function of $\omega\tau_p$ for different values of the pseudospin coherence time τ_{ps} .

A. Semiclassical model and the correction due to subband mixing

In this section we first derive, from Eq. (56), the CISP σ_{y3} in the semiclassical model, which neglects subband mixing, and then show that the correction due to subband mixing leads to the increase of σ_{y3} with the interlayer coupling $\omega\tau_p$, which has been found in Fig. 2. We also show that the $\omega\tau_p$ dependence of the correction can be described as the Hanle effect [33–35] for the pseudospin precession.

In the semiclassical model of electron dynamics in general (see, for example, Ref. [61]) the band index n is a constant of motion, namely the band mixing in wave function is ignored. Such absence of the band mixing in wave function is equivalent to the vanishing of off-diagonal density-matrix elements: $\langle n' | \hat{\rho}_{\mathbf{k}\sigma} | n \rangle = 0$ ($n' \neq n$). In our DQW we can obtain the semiclassical CISP by taking the limit of $\tau_{ps} \rightarrow 0$ because no pseudospin coherence, $\tau_{ps} \rightarrow 0$, and no subband mixing, $\langle G | \hat{\rho}_{\mathbf{k}\sigma} | E \rangle = \langle E | \hat{\rho}_{\mathbf{k}\sigma} | G \rangle = 0$, are equivalent on the basis of the Boltzmann equation Eq. (27) with Eq. (25).

By taking the limit of $\tau_{ps} \rightarrow 0$ in Eq. (56), we obtain the semiclassical CISP:

$$\sigma_{y3}^{\text{cl}} = 2\sigma_{yL} (2 - s_\omega^2). \quad (59)$$

Then the correction to the semiclassical CISP is given by

$$\sigma_{y3}^{\text{corr}} \equiv \sigma_{y3} - \sigma_{y3}^{\text{cl}} = -2\sigma_{yL} c_\omega^2 \frac{\tau_1}{\tau_p} \frac{1}{1 + \omega_1^2 \tau_1^2}. \quad (60)$$

The semiclassical model has been widely used in calculating the CISP in various systems (see, for example, Refs. [62, 63]). However we must pay attention to the correction when off-diagonal elements of the density matrix, $\langle n' | \hat{\rho}_{\mathbf{k}} | n \rangle$ ($n' \neq n$), can not be ignored. In our DQW, the correction in σ_{y3} is as large as the semiclassical CISP except the case where $\omega_1\tau_1 \gg 1$, $\tau_{ps} \ll \tau_p$, or $\omega \ll \omega_{so}$. The subband mixing $\langle E | \hat{\rho}_{\mathbf{k}\sigma} | G \rangle$ stems from the k dependence of $|G\rangle$ and $|E\rangle$ which originates from the k dependence of ω_{so} [Eq. (9)] in the Hamiltonian describing the pseudospin precession $H_\perp$ [Eq. (8)].

The suppression of the correction $\sigma_{y3}^{\text{corr}}$ with increasing $\omega_1\tau_1$ in Eq. (60) can be explained in terms of the Hanle effect [33–35] by replacing spin with pseudospin. In fact equations for $\tilde{b}_{2k\sigma}$ and $\tilde{b}_{3k\sigma}$ [Eqs. (44) and (45)] have the same form as equations describing the Hanle effect [35] with a precession frequency ω_1 and a coherence time τ_1 . Then $\tilde{b}_{2k\sigma}$ and $\tilde{b}_{3k\sigma}$ decrease with increasing $\omega_1\tau_1$ [Eq. (48)]. Therefore the correction, which is produced by $\tilde{b}_{2k\sigma}$ and $\tilde{b}_{3k\sigma}$, shows the suppression with $\omega_1\tau_1$.

Such suppression of the correction due to the pseudospin Hanle effect manifests itself as the increase of σ_{y3} with increasing ω in Fig. 2. To see this, we consider the strong scattering case where $\omega_{so}\tau_p \ll 1$ (as in Fig. 2) and focus on σ_{y3} as a function of $\omega\tau_p$ in the region of $\omega\tau_p \gg \omega_{so}\tau_p$. In such region Eq. (59) reduces to $\sigma_{y3}^{\text{cl}} = 4\sigma_{yL}$, which does not depend on ω , and Eq. (60)

becomes $\sigma_{y3}^{\text{corr}} = -2\sigma_{yL}(\tau_1/\tau_p)(1 + \omega^2\tau_1^2)^{-1}$. Therefore we obtain the increase of σ_{y3} with ω as shown in Fig. 2.

VI. CONCLUSIONS

We have theoretically investigated the CISP in an inversion-symmetric DQW structure with the antiparallel Rashba effective magnetic field by deriving and solving the Boltzmann equation in the relaxation-time approximation for the distribution function which is described as an operator in layer-pseudospin space. Although the CISP summed over two layers vanishes due to the inversion symmetry, the locally broken inversion symmetry in each layer gives rise to the local CISP such that the bilayer CISP is antiparallel. We have obtained the antiparallel CISP in an analytical form in the limit of large Fermi energy in the case of constant momentum relaxation time, and found that the antiparallel CISP increases by factor two with increasing the interlayer-tunneling strength. The present increase originates from the correction to the semiclassical CISP when $\omega_{\text{so}}\tau_p \ll 1$. This correction is due to the subband mixing, which is

induced by the momentum dependence of the state vector in the layer-pseudospin space in the presence of the momentum-dependent Rashba effective magnetic field. We have successfully explained the interlayer-coupling dependence of the antiparallel CISP by the pseudospin analog of the Hanle effect.

The semiclassical model, which neglects band mixing, has been widely used in calculating the CISP in various systems (see, for example, Refs. [62, 63]). However, we have demonstrated in a DQW structure that, the subband mixing, which is created by momentum-dependent effective magnetic field through the force term of the Boltzmann equation, produces a significant correction to the semiclassical CISP. This suggests that the correction by the band mixing as well as the subband mixing can be significant in transport properties of various systems with the momentum-dependent effective magnetic field.

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