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Abstract of Doctoral Dissertation

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Non-very generic intersections and their combinatorics
(ノンベリージェネリック交叉とその組合せ論)

The discriminantal arrangement $\mathcal{B}(n,k,\mathcal{A})$ defined by Manin and Schechtman is an arrangement of hyperplanes constructed from a generic arrangement $\mathcal{A}$ of n hyperplanes in a k-dimensional space generalizing the classical braid arrangement. In particular, $\mathcal{B}(n,1,\mathcal{A})$ is the Braid arrangement.

The combinatorics of $\mathcal{B}(n,k,\mathcal{A})$ is known to be constant when $\mathcal{A}$ varies in an open Zariski set $\mathcal{Z}$ in the space of all generic arrangements. In particular $\mathcal{A}$ is called very generic if $\mathcal{A} \in \mathcal{Z}$ and non-very generic otherwise. In the later case, when $\mathcal{A}$ is a non-very generic arrangement, the combinatorics of the discriminantal arrangement $\mathcal{B}(n,k,\mathcal{A})$ can be studied by exploiting its relation with the special configurations of points in the space.

In this dissertation, we begin by reviewing the definition of a discriminantal arrangement and some basic properties of its combinatorics. We rewrite the orchard and the Sylvester's problems in terms of the combinatorics of $\mathcal{B}(n,2,\mathcal{A})$ and then we build two arrangements of 12 lines in the projective real plane having, respectively, the minimum number of ordinary points and the maximum number of multiplicity 3 intersections. These arrangements of 12 lines in the projective real plane can help finding solutions for the orchard and the Sylvester's problems in a purely combinatorial way using the combinatorics of the intersection lattice of the discriminantal arrangements.

An intersection in $\mathcal{B}(n,k,\mathcal{A})$ when $\mathcal{A} \in \mathcal{Z}$ is called a very generic intersection and an intersection which doesn't exist in $\mathcal{B}(n,k,\mathcal{A})$ when $\mathcal{A} \in \mathcal{Z}$ is called a non-very generic intersection. We distinguish between the two types of non-very generic intersections as *arithmetic non-very generic* or *geometric non-very generic* considering whether the non-very generic intersection satisfies a condition (which we term as the BBA-condition) provided by Athanasiadis, Bayer and Brandth and then study the *arithmetic non-very generic intersections* which can be classified using the combinatorics of the discriminantal arrangement $\mathcal{B}(n,k,\mathcal{A})$.