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Lyapunov-Based Approach to Event-Triggered Control with Self-Triggered Sampling

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ABSTRACT
In this paper, a Lyapunov-based design method for event-triggered control with self-triggered sampling is proposed. In the proposed method, update of the control input and sampling of the state are individually determined based on two kinds of upper bounds of the Lyapunov function. By the proposed method, a non-monotonic Lyapunov function can be constructed. By non-monotonic decrease of a Lyapunov function, the number of communications can be reduced. In the neighborhood of the origin, we consider a control method such that the state stays in a certain set. As a result, it is guaranteed that the closed-loop system is uniformly ultimately bounded. Through a numerical example, we demonstrate the effectiveness of the proposed method.

KEYWORDS
cyber-physical systems; event-triggered control; self-triggered sampling; Lyapunov function; uniformly ultimately boundedness

1. Introduction

With developments in IoT (Internet of Things) and digital twins technologies, there has been much attention paid to researches on a cyber-physical system (CPS) that physical subsystems and information subsystems such as cloud servers are connected through communication networks [1,2]. Several systems such as smart grid, healthcare, distributed robotic systems, and automobile systems can be regarded as a CPS (see, e.g., [3,4]).

Event-triggered and self-triggered control methods are well known as typical control methods for CPSs [5,6]. In event-triggered control, communications occur (i.e., the control input is updated) only when a measured signal is widely changed (i.e., an event occurs) (see, e.g., [7–10]). In event-triggering conditions, we evaluate the error between the newly measured signal and the signal that was recently sent to the controller. If this error is bigger than a certain threshold, then a sensor sends the measured signal to a controller. In self-triggered control, the next sampling time (i.e., the next updated time of the control input) is determined based on the current measured signal (see, e.g., [11,12]).

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In event-triggered control, continuous sensing is required (i.e., a certain signal must be measured at each time). In self-triggered control, continuous sensing may not be necessarily required. However, the interval until the next update time of the control input may be unnecessarily shorter for dynamical systems with disturbances.

In [13], a method of event-triggered control with self-triggered sampling has been proposed. In this method, the next sampling time is calculated based on the policy of self-triggered control. When the state is measured at the sampling time, the controller decides if the control input is updated based on the event-triggering condition. If the control input is updated, then its value is sent to the actuator, and the next sampling time is calculated. If the control input is not updated, then communications from the controller to the actuator do not occur, and the next sampling time is calculated. Hence, this method has merits of both event-triggered and self-triggered control methods. However, in [13], in design of the event-triggering condition, the cost for one control update is evaluated. This cost is not related to a control specification such as uniformly ultimately boundedness [14]. It is important to develop a simpler approach where a control specification is directly considered.

In this paper, we propose a Lyapunov-based method for event-triggered control with self-triggered sampling. The proposed method is based on the basic result in the Lyapunov stability method such as the discrete-time Lyapunov equation. A control specification on uniformly ultimately boundedness can be directly evaluated by using a Lyapunov function. In the proposed method, update of the control input and sampling of the state are individually determined based on two kinds of upper bounds of the Lyapunov function. By the proposed method, a non-monotonic Lyapunov function can be constructed. By non-monotonic decrease of a Lyapunov function, the number of communications from the sensor (controller) to the controller (actuator) can be reduced. In the neighborhood of the origin, we consider a control method such that the state stays in a certain ellipsoid characterized by the Lyapunov function. As a result, it is guaranteed that the closed-loop system is uniformly ultimately bounded.

This paper is organized as follows. In Section 2, we formulate the problem studied in this paper. The outline of event-triggered control with self-triggered sampling is also summarized. In Section 3, we derive two kinds of upper bounds of the Lyapunov function. Based on these upper bounds, we propose a procedure of event-triggered control with self-triggered sampling. In Section 4, we demonstrate the effectiveness of the proposed method by a numerical example. In Section 5, we conclude this paper.

Notation: The set of real numbers is denoted by $\mathcal{R}$. For the matrix M, the transpose matrix of M is denoted by $M^\top$. For the matrix M, the minimum (maximum) eigenvalue of M is denoted by $\lambda_{\min(\max)}(M)$. For the vector x, let $\|x\|$ denote the Euclidean norm (2-norm) of x.

2. Problem Formulation

2.1. Plant

As a plant, consider the following discrete-time linear system:

$$x(t+1) = Ax(t) + Bu(t) + Dw(t), \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state of the plant, $u(t) \in \mathcal{R}^m$ is the control input, $w(t) \in \mathcal{R}^l$ is the disturbance, $t \in \{0, 1, 2, \dots\}$ is the discrete time, and A, B, and D are given matrices with the appropriate size. The i-th element of $w(t)$ and the i-th column of the matrix D are denoted by $w_i(t)$ and D_i, respectively. Assume that $|w_i(t)| \leq W_i$ is satisfied, where W_i is a given scalar. Assume also that the system (1) is stabilizable in the case of $W_i = 0$.

2.2. Event-Triggered Control with Self-Triggered Sampling

In this paper, we consider event-triggered control with self-triggered sampling [13] as a control method. We summarize here this method. See [13] for further details.

In this method, the controller calculates the next sampling time using the measured (sampled) state. Let t_k, $k = 0, 1, 2, \ldots$ denote the sampling time (assume $t_0 = 0$). The update law of the sampling time is given by

$$t_{k+1} = \begin{cases} t_k + S_1(x(t_k)) & \text{if a triggering condition is satisfied,} \\ t_k + S_2(x(t_k)) & \text{otherwise,} \end{cases} \tag{2}$$

where a triggering condition and functions $S_1(x(t_k))$, $S_2(x(t_k))$ are determined by a certain method. When the state is measured, the controller determines whether the control input is updated or not. The update law of the control input is given by

$$u(t_k) = \begin{cases} u_{\text{new}} & \text{if a triggering condition is satisfied,} \\ u(t_k - 1) & \text{otherwise,} \end{cases} \tag{3}$$

where u_{new} is determined by a certain method (assume that $u(t_0)$ is given). Regardless of updating the control input, the controller calculates the next sampling time by using (2).

In conventional event-triggered control methods, the control input may not be necessarily updated at each time, but continuous sampling is required (i.e., $t_k = k$ holds). In conventional self-triggered control methods, the control input is updated at the sampling time $(t_0, t_1, \ldots)$. In this method, owing to the effect of the disturbance, the number of times updating the control input may be redundant. We can take advantages of these two methods and can overcome disadvantages, by using a method of event-triggered control with self-triggered sampling.

2.3. Uniformly Ultimate Boundedness Control Problem

We define uniformly ultimate boundedness (UUB) proposed in [14] as follows.

Definition 2.1. The closed-loop system composed of the plant (1) and a certain controller is said to be uniformly ultimately bounded (UUB) in a convex and compact set $\mathcal{X}$ containing the origin in its interior, if for every initial condition $x(0) = x_0$, there exists $T(x_0)$ such that for $k \geq T(x_0)$ and $T(x_0) \in \{0, 1, 2, \ldots\}$, the condition $x(k) \in \mathcal{X}$ holds.

UUB is a weaker notion than asymptotic stability. In [13], $S_1(x(t_k))$ and $S_2(x(t_k))$ are designed considering both the cost for one control update and the cost for one sampling. In this paper, we do not consider these costs, and we consider achieving UUB of the closed-loop system. From this purpose, we assume $S_1(x(t_k)) = S_2(x(t_k)) =: S(x(t_k))$. Under the above preparation, we consider the following problem.

Problem 2.2. For the system (1), consider the event-triggered control mechanism (3) with self-triggered sampling (2). Assume that $S_1(x(t_k)) = S_2(x(t_k)) =: S(x(t_k))$ in (2). Find a function $S(x(t_k))$, u_{new} in (3), and an event-triggering condition such that the closed-loop system is UUB in a certain convex and compact set $\mathcal{X}$.

In also [13], a similar problem has been studied for discrete-time linear systems with bounded uncertainties and bounded disturbances. In this paper, we consider only the bounded disturbance (see (1)). We will propose a simpler approach using Lyapunov functions.

3. Main Result

3.1. Preliminaries

Consider the case where the system (1) has no disturbance (i.e., $w(t) = 0$). Since the system (1) is stabilizable from the assumption, there exist a symmetric positive-definite matrix P and a stabilizing state-feedback gain K for the following discrete-time Lyapunov equation:

$$(A+BK)^\top P(A+BK) - P = -Q, \tag{4}$$

where Q is an arbitrary symmetric positive definite matrix, and $A+BK$ is stable. In this paper, we consider a controller based on the state-feedback $u(t) = Kx(t)$. The quadratic Lyapunov function is defined by

$$V(t) := x^\top(t)Px(t) = \|P^{\frac{1}{2}}x(t)\|^2. \tag{5}$$

For any time t, the following inequality holds:

$$x^\top(0)((A+BK)^t)^\top P(A+BK)^t x(0) \leq (1-\lambda)^t V(0), \quad \lambda := \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}. \tag{6}$$

See, e.g., [15] for further details.

3.2. Upper Bound of the Lyapunov function Based on the Current State

In this subsection and the next subsection, we derive two kinds of upper bounds of the Lyapunov function. These upper bounds are used in design of functions $S_1(x(t_k))$, $S_2(x(t_k))$ and an event-triggering condition.

First, let $V(t+N|t)$ denote the future Lyapunov function for the system (1) when the current state $x(t)$ is given (t is the current time). Let $\hat{V}(t+N|t)$ denote the upper bound of $V(t+N|t)$ when the control input is fixed as $u(t) = u(t+1) = \cdots = u(t+N-1)$. The upper bound $\hat{V}(t+N|t)$ is used for evaluating the performance when the control input is not updated in self-triggered sampling and event-triggered control. Consider deriving $\hat{V}(t+N|t)$. The upper bound $\hat{V}(t+N|t)$ must satisfy

$$\max_{w(t+i),\ i=0,1,\ldots,N-1} V(t+N|t) \leq \hat{V}(t+N|t).$$

From (1) and (5), the Lyapunov function at time $t+N$ can be derived as

$$V(t+N|t) = \left\| P^{\frac{1}{2}} \left(A^N x(t) + \sum_{i=0}^{N-1} A^{N-1-i} \left(Bu(t+i) + Dw(t+i) \right) \right) \right\|^2 .$$

Using $|w_j(t)| \leq W_j$, $j = 1, 2, \ldots, l$ and $u(t) = u(t+1) = \cdots = u(t+N-1)$, we can obtain $\hat{V}(t+N|t)$ as follows:

$$\begin{aligned}
\max_{\substack{w(t+i),\\ i=0,1,\ldots,N-1}} V(t+N|t) \leq & \max_{\substack{w(t+i),\\ i=0,1,\ldots N-1}} \Bigg(\left\| P^{\frac{1}{2}} (A^N x(t) + \sum_{i=0}^{N-1} (A^{N-1-i} Bu(t)) \right\| \\
& + \left\| P^{\frac{1}{2}} \sum_{i=0}^{N-1} A^{N-1-i} Dw(t+i) \right\| \Bigg)^2 \\
= & \max_{\substack{w(t+i),\\ i=0,1,\ldots,N-1}} \Bigg(\left\| P^{\frac{1}{2}} (A^N x(t) + \sum_{i=0}^{N-1} (A^{N-1-i} Bu(t)) \right\|^2 \\
& + 2 \left\| P^{\frac{1}{2}} (A^N x(t) + \sum_{i=0}^{N-1} (A^{N-1-i} Bu(t)) \right\| \\
& \cdot \left\| P^{\frac{1}{2}} \sum_{i=0}^{N-1} A^{N-1-i} Dw(t+i) \right\| \\
& + \left\| P^{\frac{1}{2}} \sum_{i=0}^{N-1} A^{N-1-i} Dw(t+i) \right\|^2 \Bigg) \\
\leq & \left\| P^{\frac{1}{2}} (A^N x(t) + \sum_{i=0}^{N-1} (A^i Bu(t))) \right\|^2 \\
& + 2 \left\| P^{\frac{1}{2}} (A^N x(t) + \sum_{i=0}^{N-1} (A^i Bu(t))) \right\| \cdot \sum_{j=1}^{l} \sum_{i=0}^{N-1} \left\| P^{\frac{1}{2}} A^i D_j \right\| W_j \\
& + \left(\sum_{j=1}^{l} \sum_{i=0}^{N-1} \left\| P^{\frac{1}{2}} A^i D_j \right\| W_j \right)^2 \\
=: & \hat{V}(t+N|t).
\end{aligned}$$

3.3. Upper Bound of the Lyapunov function Based on the Initial State

Next, let $\bar{V}(t)$ denote the upper bound of the Lyapunov function $V(t)$ for the system (1) when the initial state $x(0)$ is given. In this paper, we consider evaluating the best performance under the worst disturbance, that is, the control input is updated at each time, and is given by $u(t) = Kx(t)$. The upper bound $\bar{V}(t)$ must satisfy

$$\max_{w(i),\ i=0,1,\ldots,t-1} V(t) \leq \bar{V}(t).$$

From (1) and (5), the Lyapunov function at time t can be derived as

$$V(t) = \left\| P^{\frac{1}{2}}(A+BK)^t x(0) + P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\|^2 .$$

Using $|w_j(t)| \leq W_j$, $j = 1, 2, \ldots, l$ and (6), we can obtain $\bar{V}(t)$ as follows:

$$\begin{aligned}
\max_{\substack{w(i),\\ i=0,1,\ldots,t-1}} V(t) \leq & \max_{\substack{w(i),\\ i=0,1,\ldots t-1}} \left(\left\| P^{\frac{1}{2}}(A+BK)^t x(0) \right\| + \left\| P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\| \right)^2 \\
= & \max_{\substack{w(i),\\ i=0,1,\ldots,t-1}} \Bigg(\left\| P^{\frac{1}{2}}(A+BK)^t x(0) \right\|^2 \\
& + 2 \left\| P^{\frac{1}{2}}(A+BK)^t x(0) \right\| \cdot \left\| P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\| \\
& + \left\| P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\|^2 \Bigg) \\
\leq & \max_{\substack{w(i),\\ i=0,1,\ldots,t-1}} \Bigg((1-\lambda)^t V(0) \\
& + 2 \left\| P^{\frac{1}{2}}(A+BK)^t x(0) \right\| \cdot \left\| P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\| \\
& + \left\| P^{\frac{1}{2}} \sum_{i=0}^{t-1} (A+BK)^{t-1-i} Dw(i) \right\|^2 \Bigg) \\
\leq & (1-\lambda)^t V(0) \\
& + 2 \left\| P^{\frac{1}{2}}(A+BK)^t x(0) \right\| \cdot \left(\sum_{j=1}^{l} \sum_{i=0}^{t-1} \left\| P^{\frac{1}{2}}(A+BK)^i D_j \right\| W_j \right) \\
& + \left(\sum_{j=1}^{l} \sum_{i=0}^{t-1} \left\| P^{\frac{1}{2}}(A+BK)^i D_j \right\| W_j \right)^2 \\
=: & \bar{V}(t)
\end{aligned}$$

3.4. Derivation of Sampling Interval Function and Event-Triggering Condition

Using $\hat{V}(t+N|t)$ and $\bar{V}(t)$, we derive the functions $S_1(x(t_k))$, $S_2(x(t_k))$ in (2) and an event-triggering condition.

First, the set $\mathcal{X}$ in Problem 2.2 is given by

$$\mathcal{X} = \{x \mid x^\top Px \leq \beta\},$$

where β is a given scalar satisfying

$$\beta > \lim_{t \to \infty} \bar{V}(t). \tag{7}$$

Then, under an appropriate control law, there exists $T(x(0))$ such that $x(t) \in \mathcal{X}$ holds for any $t \geq T(x(0))$.

Next, consider the update law (2) of the sampling time. Note that in Section 2.3, we assume $S_1(x(t_k)) = S_2(x(t_k)) =: S(x(t_k))$. Using $\hat{V}(t+N)$ and $\bar{V}(t)$, the function $S(x(t_k))$ is given by

$$S(x(t_k)) = \min \left\{ \tau \geq 1 \mid \hat{V}(t_k + \tau + 1 | t_k) > \max(\bar{V}(t_k + \tau + 1), \beta) \right\}. \tag{8}$$

Consider the update law (3) of the control input. Using $\hat{V}(t)$ and $\bar{V}(t)$, (3) is rewritten as

$$u(t_k) = \begin{cases} Kx(t_k) & \text{if } \hat{V}(t_k + 1 | t_k) > \max(\bar{V}(t_k + 1), \beta) \text{ is satisfied,} \\ u(t_k - 1) & \text{otherwise,} \end{cases} \tag{9}$$

Finally, we present the proposed procedure as follows.

Procedure of event-triggered control with self-triggered sampling:
Step 1: Set $t_k = 0$, $k = 0$, and $x(0) = x_0$.
Step 2: Apply $u(t_k) = Kx(t_k)$ to the plant.
Step 3: Calculate $S(x(t_k))$ of (8).
Step 4: Update $t_{k+1} = t_k + S(x(t_k))$.
Step 5: Wait until t_{k+1}, measure $x(t_{k+1})$, and set $t_k := t_{k+1}$.
Step 6: If the event-triggering condition in (9) is satisfied, then go to Step 2, otherwise go to Step 3.

See also Fig. 1. This figure is made based on the procedure in [13], and the explanation about computation of the upper bound of the Lyapunov function is added.

Consider replacing Step 6 with the following Step 6':
Step 6': Go to Step 2.
This implies that (9) is replaced with $u(t_k) = K(x(t_k))$, i.e., the event-triggering condition is ignored. In this case, the above procedure becomes the self-triggered control method. We can obtain the following theorem.

Theorem 3.1. *The closed-loop system driven by the above procedure, which is replaced Step 6 with Step 6', is UUB in the set $\mathcal{X}$.*

Proof. If the control input is not updated at time $t_k + S(x(t_k))$, then the Lyapunov function $V(t_k + S(x(t_k)) + 1)$ may be higher than $\max(\bar{V}(t_k + S(x(t_k)) + 1), \beta)$. The inequality $V(t_k + S(x(t_k)) + 1) < \max(\bar{V}(t_k + S(x(t_k)) + 1), \beta)$ holds by updating the control input at time $t_k + S(x(t_k))$. Hence, from (7), the closed-loop system is UUB in the set $\mathcal{X}$. □

From this theorem, we see that even if the event-triggering condition is ignored, it is achieved that the closed-loop system is UUB. However, since we evaluate the worst

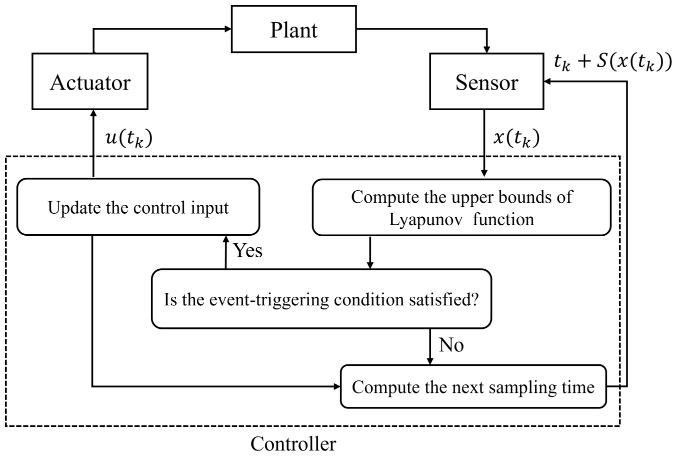

Figure 1. Outline of the procedure in the controller (this figure is made based on the procedure in [13]).

case in computation of $\bar{V}(t)$, the control input may not need to be updated at time $t_k + S(x(t_k))$. Then, we can also obtain the following theorem immediately.

Theorem 3.2. *The closed-loop system driven by the above procedure is UUB in the set $\mathcal{X}$.*

Proof. Even if $\hat{V}(t_k + S(x(t_k)) + 1|t_k) > \max(\bar{V}(t_k + S(x(t_k)) + 1), \beta)$ holds, then

$$\hat{V}(t_k + S(x(t_k)) + 1|t_k + S(x(t_k))) > \max(\bar{V}(t_k + S(x(t_k)) + 1), \beta) \qquad (10)$$

may not hold. When (10) does not hold, it is not necessary to update the control input at time $t_k+S(x(t_k))$. Hence, even if the event-triggering condition in (9) is introduced, the closed-loop system is still UUB in the set $\mathcal{X}$. □

Introducing the event-triggering condition in (9) enables us to reduce unnecessary updates of the control input in the sense that UUB is achieved. We remark that if we set $S(x(t_k))$ as $S(x(t_k)) = 1$, then the state is measured at each time, and the above procedure becomes the event-triggered control. Also in this case, it is achieved that the closed-loop system is UUB.

4. Numerical Example

Through a numerical example, we present the effectiveness of the proposed method. As a plant, consider the following discrete-time linear systems with two states and a single input/disturbance:

$$x(t+1) = \begin{bmatrix} 1.1 & 0.9 \\ 0 & 1.2 \end{bmatrix} x(t) + \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} w(t),$$

where $|w(t)| \leq 1$ is satisfied (i.e., $W = 1$). The matrix P in (5) and the gain K in (9) are derived by solving the linear quadratic regulator problem minimizing the following

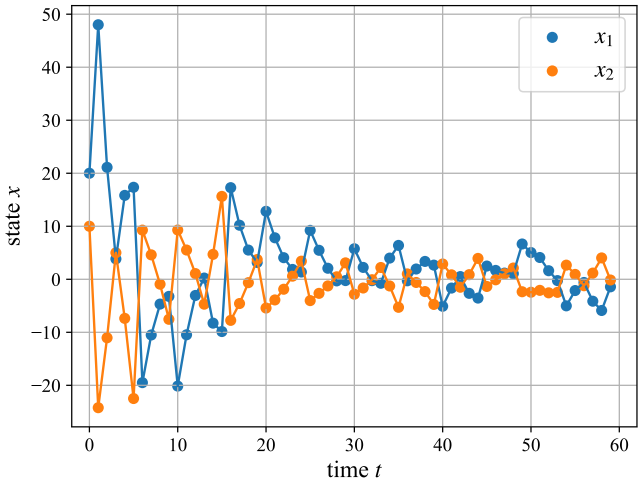

Figure 2. Time response of the state.

cost function:

$$J = \sum_{t=0}^{\infty} x^\top(t) \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} x(t) + u^2(t).$$

The matrix P in (5) is given by a symmetric positive definite solution of the discrete-time algebraic Riccati equation, and can be derived as

$$P = \begin{bmatrix} 444.3629 & 470.4353 \\ 470.4353 & 744.0974 \end{bmatrix}.$$

The gain K in (9) can be derived as

$$K = \begin{bmatrix} -0.7079 & -2.1622 \end{bmatrix}.$$

Using the obtained P and K, the matrix Q in (4) can be derived as

$$Q = \begin{bmatrix} 100.5011 & 1.5307 \\ 1.5307 & 104.6752 \end{bmatrix}.$$

In addition, we set β as $\beta = 20000$.

We present the simulation result. The initial state is given by $x(0) = [20\ 10]^\top$. From $\beta = 20000$, $T(x(0))$ in UUB is derived as $T(x(0)) = 39$. The disturbance is generated by uniformly distributed random numbers in the interval $[-1, 1]$. Fig. 2 shows the time response of the state. Fig. 3 shows the time response of V and $\bar{V}$. Fig. 4 shows the control input. Fig. 5 shows the event occurrence.

From Fig. 2, we see that the state reaches the neighborhood of the origin. From Fig. 3, we see that the closed-loop system is UUB. We also see that the time change of the Lyapunov function $V(t)$ is non-monotonic decrease. From Fig. 4, we see that the control input is sometimes not changed. From Fig. 5, we see that even if the state is sampled, then the control input may not be updated.

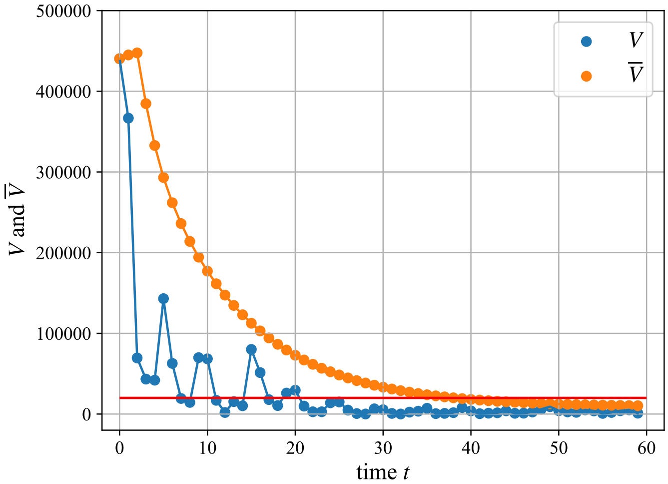

Figure 3. Time response of V and $\bar{V}$.

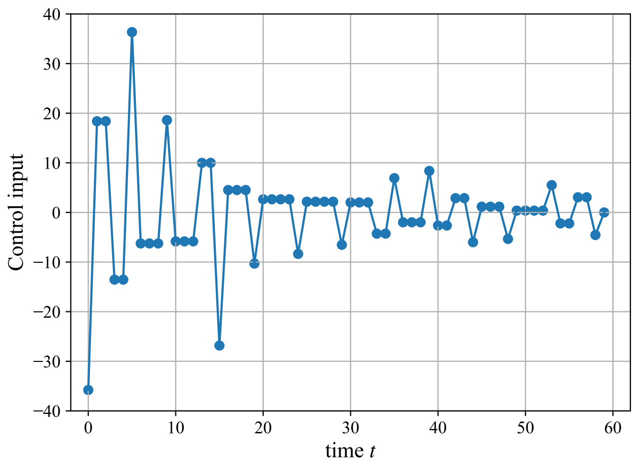

Figure 4. Control input.

5. Conclusion

In this paper, we proposed a Lyapunov-based approach to event-triggered control with self-triggered sampling. Based on two kinds of upper bounds of the Lyapunov function, the event-triggering condition and the function for finding the sampling interval were derived. By the proposed procedure, we can guarantee that the closed-loop system is UUB in the ellipsoid characterized by the Lyapunov function.

One of the future efforts is to consider a large-scale system with many sensors and actuators. In such cases, it is important to consider a self-triggered multi-rate sampling mechanism. An extension of the proposed method to switched systems and nonlinear systems is also one of the future efforts.

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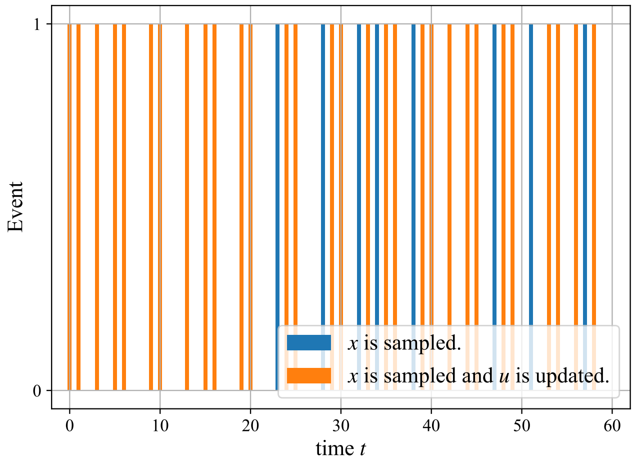

Figure 5. Event occurrence, where the orange line implies that the state is sampled and the control input is updated, and the blue line implies that the state is sampled (the control input is not updated).

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