The Breathing Mode of the Hoyle State in ^{12}C

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Excited states of the Hoyle state in ^{12}C are discussed by taking into account the correct boundary condition for three-α resonant states with the complex scaling method. The Coulomb interaction is considered to play an important role for stability of a breathing 3α motion. It is expected that such a breathing mode exists in other nα systems as well. The development of a new method for solving nα systems in the complex scaling method is reported.

1 Introduction

As shown in the Ikeda diagram[1], light 4N nuclei are considered to have various α cluster structures in the vicinity of the corresponding threshold energies. As a typical example, it has been studied that the second 0^+ state, so-called the Hoyle state[2], of ^{12}C located at 0.38 MeV above the 3α-decay threshold ($E_x = 7.65$ MeV) has a well-developed 3α-cluster structure. Detailed theoretical analyses have been carried out in various frameworks, such as the resonating group method (RGM)[3, 4], the generator coordinate method (GCM) [5], the orthogonality condition model (OCM)[6, 7], the antisymmetrized molecular dynamics (AMD)[8], and so on, which have succeeded in describing the intriguing properties of ^{12}C.

All of the theoretical results for the Hoyle state showed a gas-like state in which three α-clusters were well developed and weakly bonded. On the other hand, significant progress was made in understanding this Hoyle state in the early 2000s. This new understanding was based on the research by Schuck and Röpke on the α-clustering in the low-density nuclear matter [9], and considered the Hoyle state to be an α-cluster condensed state in finite nuclei [10]. This idea is extremely important as it provides a unified understanding not only for the 3α system but also for $n\alpha$ states in general. In particular, it is interesting to investigate what kind of structure the neighboring excited states other than the 0_2^+ Hoyle state in the ^{12}C nucleus have, and what kind of excitation mode the α condensed state has.

Experimentally, Ito and his group of RCNP has reported that the 0_3^+ and 2_2^+ states coexist at the excitation energies of $E_x = 9 - 11$ MeV [11]. They have also measured decay widths, which are 1.0 ± 0.3 MeV and 2.7 ± 0.3 MeV for the 2_2^+ and 0_3^+ states, respectively. From the theoretical side, since the observed 2_2^+ state has a large α-decay width, it can be considered to have a similar structure to the 0_2^+ state, which has recently been interpreted as the α-condensed state with a new type of wave functions [10, 12, 13]. The 2_2^+ state has been predicted in many previous 3α calculations, but the 0_3^+ state with such a large decay width has never been obtained near the 2_2^+ state.

Since these states of the $n\alpha$ system are considered to exist near the $n\alpha$ threshold or in a higher energy region, the boundary conditions for the unbound states must be treated appropriately in the theory. In our previous papers [14, 15], we performed a complex-scaled 3α OCM calculations for ^{12}C. The 3α system describing ^{12}C has only two bound states, the ground 0^+ and excited 2^+ states, below the 3α threshold, but all other states are resonances. In Ref. [14], using the method of the Analytic Continuation in the Coupling Constant combined with the Complex Scaling Method (ACCC+CSM) for the three-body resonant states, we predicted the existence of the 0_3^+ state near the observed energy [11]. This result of 0_3^+ state was also obtained at $E_{res} = 0.79 - i0.84$ MeV by Ohtsubo *et al.* [16] applying the complex-range Gaussian basis function to the CSM calculation.

In this article, we discuss the properties of the 0_3^+ state as an excited state of the 0_2^+ Hoyle state. Coulomb interactions are thought to create a barrier that confines the three α clusters outside the nuclear potential. It is expected that condensed states of α clusters will exist even in the systems where the number of α clusters increases, and in such systems, the Coulomb force will become stronger and more excited states of condensed states of α clusters will appear due to the strong Coulomb barrier. Therefore, it becomes very interesting to study the many-body resonant states of $n\alpha$ systems ($n > 3$).

Recently, we have proposed a new method using generalized coherent states as a basis for OCM calculations of $n\alpha$ systems [18]. An outline of the method is shown below. This method based on a dilation generator is expected to describe breathing-like motion of clusters effectively and to solve the problem of a large number of the basis states in the $n\alpha$ model calculations.

2 3α OCM and Complex Scaling Method

In this section, we briefly explain the 3α OCM and Complex Scaling Method to solve the three-body resonant states. The details are given in Refs. [14, 15]. The Hamiltonian of the 3α OCM is given as

$$H = \sum_{i=1}^{3} t_i - T_G + \sum_{i=1}^{3} V_{\alpha\alpha}(\xi_i) + V_{3\alpha}(\xi_1, \xi_2, \xi_3) + V_{Pauli}. \tag{1}$$

The operators t_i and T_G stand for the kinetic energy of each α-cluster and the center of mass motion, respectively. The α-α interaction $V_{\alpha\alpha}$ is given by a folding type potential for nuclear and Coulomb potentials. The relative coordinates between α-clusters are expressed as $\xi_i = |\vec{r}_j - \vec{r}_k|$ for $(i, j, k) = (1, 2, 3)$. We introduce a 3α potential $V_{3\alpha}$ to take into account effects from non-central interactions such as $\ell \cdot s$ or tensor forces, and overall antisymmetrization effect among 3α clusters. V_{Pauli} is a pseudo potential to remove the Pauli forbidden states between 2α.

The Schrödinger equation for the total spin J state,

$$H\Psi_J = E\Psi_J, \tag{2}$$

is solved by using the basis function method,

$$\Psi_J = \sum_{N=1}^{N_{max}} \sum_{n=1}^{n_{max}} \sum_{L}^{L_{max}} \sum_{l}^{l_{max}} \alpha_{NLnl}^{J} \sum_{c=1}^{3} u_{NL}(\xi_c) u_{nl}(\eta_c) \left[Y_L(\hat{\xi}_c) \otimes Y_l(\hat{\eta}_c) \right]^J, \tag{3}$$

where $c = 1,\ 2$ and 3 represent three Jacobian coordinate systems with ξ_c and η_c, whose superposition is done due to the bosonic symmetry of the α cluster. The radial wave functions are described by the Gaussian basis functions of $u_{NL}(\xi_c)$ and $u_{nl}(\eta_c)$.

This basis function method is available only for bound states, but cannot solve resonant states correctly, which have the divergent behavior at the asymptotic distance region. Then we apply the complex scaling method (CSM) [17] to the Hamiltonian, in which radial coordinates are transformed by

$$\xi_c \longrightarrow \xi_c e^{i\theta}, \qquad \eta_c \longrightarrow \eta_c e^{i\theta}. \tag{4}$$

Due to this transformation, the resonance with the complex energy eigenvalue of $\tan^{-1}(\Gamma/2E_r) < 2\theta$ changes to have a damping behavior in the asymptotic region [17]. Therefore, we can obtain the resonant states by solving the following complex-scaled eigenvalue equation:

$$\sum_{\gamma'}^{\gamma_{max}} \left[H_{\gamma\gamma'}(\theta) - E(\theta)N_{\gamma\gamma'}\right]\alpha_{\gamma'}(\theta) = 0, \tag{5}$$

where for $\gamma = \{NLnl\}$ and $\psi_\gamma^J = \sum_{c=1}^{3} u_{NL}(\xi_c)u_{nl}(\eta_c)\left[Y_L(\hat{\xi}_c)\otimes Y_l(\hat{\eta}_c)\right]^J$,

$$H_{\gamma\gamma'}(\theta) = \langle\psi_\gamma^J|H(\theta)|\psi_{\gamma'}^J\rangle, \qquad N_{\gamma\gamma'} = \langle\psi_\gamma^J|\psi_{\gamma'}^J\rangle \tag{6}$$

The complex energy of resonance is obtained independently of θ [17] as

$$E_{res} = E_r - i\frac{\Gamma}{2}. \tag{7}$$

3 The Breathing Mode of the Hoyle State

Here we will concentrate on the discussion of the 0^+ states, especially the 0_2^+ and 0_3^+ states in ^{12}C. In Table 1, we show the results of resonance energies and widths, amplitudes of $[^8\mathrm{Be}(L)\otimes l]_J$ channels and r.m.s. radii of the 0^+ solutions which were obtained with the 3α complex scaled OCM [15]. The 0_3^+ state could not be solved within the real-range Gaussian basis functions, and we discuss this state separately.

Four 0^+ states shown in Table 1 are obtained even in bound-state calculations because the decay widths of $0_{2,4,5}^+$ are not so large. The ground 0_1^+ state of a small r.m.s. radius indicates the $0p$-shell dominant configuration in ^{12}C. The values of channel amplitudes are close to those of the SU(3) $(\lambda,\mu) = (0,4)$ configuration. The compact structure with small r.m.s. radii is also seen in the 0_5^+ state at a high excitation energy of $E_x > 15$ MeV where the excitation of α clusters can occur. On the other hand, the 0_2^+ state corresponding to the Hoyle

Table 1: Resonance energies (E_r), decay widths (Γ), channel amplitudes ($C_{L,l}^2$) and r.m.s. radii ($R_{r.m.s}$) of the 0^+ states calculated with the 3α complex scaled OCM [15].

	Energy	Width	$C_{0,0}^2$		$C_{2,2}^2$		$C_{4,4}^2$		$R_{r.m.s.}$ [fm]	
	E_r [MeV]	Γ [MeV]	Re	Im	Re	Im	Re	Im	Re	Im
0_1^+	-7.29	0	0.36	0	0.38	0	0.25	0	2.36	0
0_2^+	0.76	0.0024	0.78	0.03	0.15	-0.019	0.08	-0.014	4.23	0.49
0_4^+	4.58	1.1	0.50	0.17	0.31	-0.017	0.19	-0.15	3.49	0.75
0_5^+	14.3	1.5	0.34	0.079	0.51	0.013	0.15	-0.092	2.89	0.20

state has a large r.m.s. radius and a dominant amplitude of $[{}^8\mathrm{Be}(0^+) \otimes l = 0]_{J\pi=0^+}$. These results are consistent with Ref.[13] where the 0_2^+ state is interpreted as the α-condensate state.

In order to find out how many 0^+ states can exist in the energy region of relatively small excitation of the 3α system, we performed the calculation (ACCC+CSM)[14] by introducing the auxiliary 3α potential as

$$H(\delta) = H + \delta V_{aux}, \qquad V_{aux} = \exp\{-\mu(\xi_1^2 + \xi_2^2 + \xi_3^2)\}. \tag{8}$$

The auxiliary 3α potential is the same potential form as $V_{3\alpha}$ in Eq.(1) but the strength δ is changed as a parameter. The obtained results show five 0^+ states in the energy region of $E_r < 15$ MeV, adding the 0_3^+ state to the results shown in Table 1. Our results of the 0_3^+ resonant state obtained using the Pade approximation are $E_r = 1.66$ MeV and $\Gamma = 1.48$ MeV, and after that, as mentioned in the Introduction, more precise predictions are reported by Ohtsubo *et al.*[16].

Applying the analytical continuation of the coupling constant (ACCC) to the calculation of the channel amplitudes and the r.m.s. radius for the 0_3^+ state, we find that the 0_3^+ state has a large amplitude in the $[{}^8\mathrm{Be}(0^+) \otimes l = 0]_{J\pi=0^+}$ channel and a large r.m.s radius. This result shows that although the radius of the 0_3^+ state is much larger than that of 0_2^+, the 0_2^+ state and the 0_3^+ state have very similar structures [15]. There is a speculation that the 0_3^+ state has a $[{}^8\mathrm{Be}(0^+) \otimes l = 0]_{J\pi=0^+}$ structure[16]. However, we think that this state is a breathing mode excited from the 0_2^+ state because of the result that this state is strongly dependent on the 3α potential. Since the 0_2^+ state with a large radius exists in a potential pocket created by the nuclear and Coulomb potentials just above the threshold, there is good possibility that a breathing mode from such a state will occur within the Coulomb barrier. The important role of the Coulomb potential barrier for spatially extended cluster states has also been discussed for mirror three-cluster nuclei [19], ^{8}He-^{8}C[18, 20, 21] and ^{9}Be-^{9}B[22].

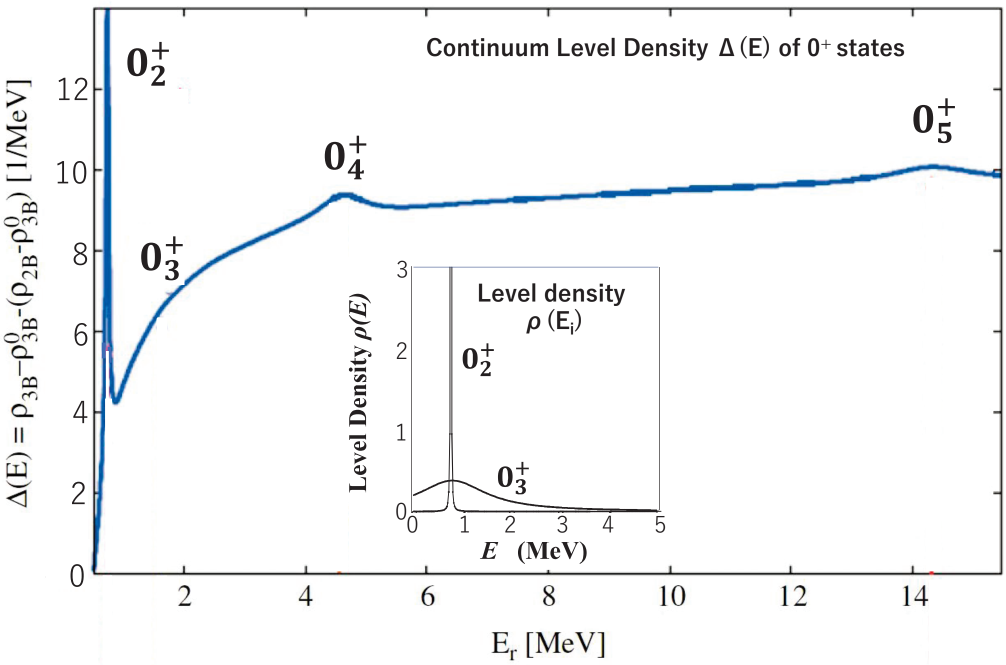

Figure 1: The 3α CLD of 0^+ states of ^{12}C [24]. The inner diagram shows the level density ρ of 0_2^+ ($E_r = 0.76$ MeV, $\Gamma = 0.0023$ MeV[14]) and 0_3^+ ($E_r = 0.79$ MeV, $\Gamma = 1.68$ MeV[16]) resonances.

For the purpose to see how isolated the 3α 0^+ resonant states exist, we calculated the three-body continuum level density (CLD)[23] defined as

$$\Delta(E) = -\frac{1}{\pi} Im\left\{ Tr\left[\frac{1}{E-H_{3B}} - \frac{1}{E-H^0_{3B}} - \left(\frac{1}{E-H_{2B}} - \frac{1}{E-H^0_{3B}} \right) \right] \right\}. \qquad (9)$$

The Hamiltonian H_{3B} is given in Eq.(1), and the asymptotic Hamiltonian H^0_{3B} consists of the kinetic energy and the point-charge Coulomb potential between two α's. The two-body CLD term in Eq.(9) is calculated for the two-α sub-system in the 3α system by using

$$H_{2B} = T_{3\alpha} + V_{\alpha\alpha}(\xi_3) + \sum_{i=1}^{2} \frac{2\cdot 2e^2}{\xi_i} + V_{Paul}, \qquad H^0_{3B} = T_{3\alpha} + \sum_{i=1}^{3} \frac{2\cdot 2e^2}{\xi_i} + V_{Pauli}, \quad (10)$$

where $T_{3\alpha} = \sum_{i=1}^{3} t_i - T_G$.

In Fig.1, we show the complex scaled CLD of the 0^+ states obtained with the 3α OCM. The large decay width of the 0^+_3 resonance makes it difficult to identify the state as isolated one. However, the fact that the decay width is large and the CLD extends close to the threshold energy suggests that the 0^+_3 state contributes to an increase in the capture cross section of a low energy α.

4 Generalized Coherent States for the Pauli-Allowed States

Although we have discussed the breathing mode of the 0^+_3 state in the 3α system above, it needs to be investigated in more detail and quantitatively. And, it is very interesting to study such states in heavier $n\alpha$ systems. Recently, the α condensed states in the 4α [25, 26, 27] and 5α [28, 29] systems have been studied to realize near the $n\alpha$ threshold energy[30]. Since the Coulomb barrier becomes larger in heavier α systems, the excited states of a breathing mode in the $n\alpha$ condensed states are expected. However, numerical calculations become very heavy with increasing of the number n of α clusters because of the treatment of the Pauli-allowed states for every α cluster pair. To solve this problem, new methods have continued to be developed [31], and we also proposed a new basis states using the generalized coherent states (GCS) in the Pauli-allowed state space [18].

The GCS are defined as

$$\begin{aligned} \phi^\beta_{nlm}(\boldsymbol{r},\nu) &= \exp\left(\frac{1}{2}\beta\hat{D}^\dagger\right)\phi_{nlm}(\boldsymbol{r},\nu) \\ &= \frac{1}{\sqrt{(1+\beta)^{N+3/2}}}\cdot\exp\left(\frac{\beta}{2(1+\beta)}\nu r^2\right)\phi_{nlm}(\boldsymbol{r},\frac{\nu}{1+\beta}), \end{aligned} \qquad (11)$$

where $\phi_{nlm}(\boldsymbol{r},\nu)$ is the harmonic oscillator (HO) wave function with the oscillator constant ν and quantum numbers $\{nlm\}$ ($N = 2n + l$). Using the HO creation operator ($\hat{\boldsymbol{a}}^\dagger$), the raising operator is given by

$$\hat{D}^\dagger = \hat{\boldsymbol{a}}^\dagger \cdot \hat{\boldsymbol{a}}^\dagger, \qquad (12)$$

and β is a parameter of $|\beta| < 1$ and plays a role of the radial dilation of the HO basis function.

It is a very important property of the GCS that

$$\langle \phi_{n'lm}(\boldsymbol{r},\nu)|\phi^\beta_{nlm}(\boldsymbol{r},\nu)\rangle = 0 \qquad \text{for} \qquad n' < n. \qquad (13)$$

Table 2: Resonance energies (E_r) and decay widths (Γ) of the 2α system (^{8}Be). GCS and PPM are results calculated in the coherent basis method and the pseudo potential method, respectively. The experimental values (Exp) are taken from Refs.[32, 33].

J^π	0^+		2^+		4^+	
	E_r(MeV)	Γ(MeV)	E_r(MeV)	Γ(MeV)	E_r(MeV)	Γ(MeV)
GCS	0.294	0.014	3.01	1.65	12.13	5.19
PPM	0.296	0.015	3.00	1.67		
Exp.	0.0918	5.57(25) $\times 10^{-6}$	3.12(1)	1.513(15)	22.44(15)	≈ 3.5

This property means that the GCS are always the Pauli-allowed states when $\phi_{n'lm}(\boldsymbol{r},\nu)$ are the Pauli-forbidden ones. Therefore, we can use the GCS as basis states in the OCM calculation without the pseudo potential V_{Pauli} used in Eq. (1).

We can also apply the complex scaling method to GCS easily. The complex scaled wave function is expressed by using the GCS as

$$\Phi^\theta = \sum_{i=1}^{N_{base}} C_i^\theta \phi_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu), \qquad \phi_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu) = \phi_{nlm}^{\beta_i}(\boldsymbol{r},\nu e^{-i\theta}). \tag{14}$$

The expansion coefficient C_i^θ and energy E^θ are determined by solving the equation

$$\sum_{i=1}^{N_{base}} (H_{ij}^\theta - E^\theta N_{ij}^\theta) C_j^\theta = 0, \tag{15}$$

where

$$\begin{aligned}
H_{ij}^\theta &= \langle \tilde{\phi}_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu)|H^\theta|\phi_{nlm}^{\beta_j,\theta}(\boldsymbol{r},\nu)\rangle \\
&= \sum_{p,q}^{N_p} \langle \tilde{\phi}_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu)|u_p\rangle\langle u_p|H^\theta|u_q\rangle\langle u_q|\phi_{nlm}^{\beta_j,\theta}(\boldsymbol{r},\nu)\rangle = \sum_{p,q}^{N_p} D_{ip}^\beta H_{pq}^\theta D_{qj}^\theta, \qquad (16)\\
N_{ij}^\theta &= \langle \tilde{\phi}_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu)|\phi_{nlm}^{\beta_j,\theta}(\boldsymbol{r},\nu)\rangle = \sum_{p,q}^{N_p} D_{ip}^\beta D_{qj}^\theta, \\
& D_{ip}^\theta = \langle \tilde{\phi}_{nlm}^{\beta_i,\theta}(\boldsymbol{r},\nu)|u_p(\boldsymbol{r})\rangle, \qquad D_{qj}^\theta = \langle u_q(\boldsymbol{r})|\phi_{nlm}^{\beta_j,\theta}(\boldsymbol{r},\nu)\rangle. \qquad (17)
\end{aligned}$$

The wave function $\tilde{\phi}$ is the biorthogonal state of ϕ, and $\{u_p\}$ ($p = 1, 2, \cdots$) are a set of appropriate orthonormal functions.

To examine the reliability of this new basis states, we apply this method to relative motion of the 2α system of ^{8}Be and show the results by comparing with the previous method (PPM) of the pseudo potential V_{Pauli}. The results of resonance parameters are shown in Table 2 together with the observed data[32, 33]. Good agreement is observed, confirming the effectiveness of the method using the GCS.

Applications to the $n\alpha$ systems are highly anticipated. For example, using the lowest Pauli-allowed state $\Phi_{Q,(\lambda,\mu),JKM}$ of the 3α system with the SU(3) basis state[7], the GCS can be given as

$$\Phi_{Q,(\lambda,\mu),JKM}^\beta = \exp\left(\frac{1}{2}\beta \hat{D}^\dagger\right) \Phi_{Q,(\lambda,\mu),JKM}, \qquad \hat{D}^\dagger = \sum_{i=1}^{2} \hat{D}_i^\dagger, \tag{18}$$

where $\hat{D}_i^\dagger$ is a raising operator for each of two Jacobi coordinates of the 3α system.

5 Summary

We discussed the state in ^{12}C as a breathing excitation of the 0_2^+ Hoyle state. The 0_3^+ state, which has the similar structure as the 0_2^+ Hoyle state, is a broad resonant state of the 3α system, and it seems difficult to observe it as an isolated resonance. However, this state, which has a width extending to the 3α threshold, may contribute to the low-energy ^{12}C nucleosynthesis more than the Hoyle state.

The 0_3^+ state with a spatially extended configuration is thought to be stabilized by the Coulomb barrier. Such states may also exist in heavier α-particle systems where the effect of the Coulomb force increases, so it is very interesting to investigate heavier α systems. We proposed the generalized coherent basis states (GCS) for the OCM calculations of $n\alpha$ systems applying the complex scaling method. We demonstrated its applicability to the α-α system by comparing it with the result of the previous pseudo potential method.

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References

[1] K. Ikeda, N. Takigawa, H. Horiuchi, Prog. Theor. Phys. Suppl. Extra Number, (1968) 464.

[2] F. Hoyle, Astrophys. J. Suppl. **1**, (1954) 121.

[3] Y. Fukushima and N. Kamimura, J. Phys. Soc. Jpn **44**, (1977) 225; M. Kamimura, Nucl. Phys. **A 351**, (1981) 456.

[4] Y. Fujiwara, H. Horiuchi, K. Ikeda, M. Kamimura, K. Kato, Y. Suzuki, E. Uegaki, Prog. Theor. Phys. Suppl. **68**, (1980) 60.

[5] E. Uegaki, S. Okabe, Y. Abe, H. Tanaka, Prog. Theor. Phys. **57**, (1977) 1262; **59**, (1978) 1031; **62**, (1979) 1621.

[6] H. Horiuchi, Prog. Theor. Phys. **51**, (1974) 1266; **53**, (1975) 447.

[7] K. Fukatsu, K. Katō and H. Tanaka, Prog. Theor. Phys. **80**, (1988) 663.

[8] Y. Kanada-En'yo, Phys. Rev. Lett. **81**, (1998) 5291.

[9] G. Röpke, A. Schnell, P. Schuck and P. Noziéres, Phys. Rev. Lett. **80**, (1998) 3177; M Beyer,et al., Phys. Lett. B **488**, (2000) 247.

[10] A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke, Phys. Rev. Lett. **87**, (2001) 192501.

[11] M. Itoh, et al., Nucl. Phys. **A 738**, (2004) 268.

[12] Y. Funaki, H. Horiuchi, A. Tohsaki, P. Shuck, G. Röpke, Prog. Theor. Phys. **108**, (2002) 297.

[13] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, Phys. Rev. **C 67**, (2003) 051306(R).

[14] C. Kurokawa and K. Katō, Phys. Rev. **C 71**, (2005) 021301(R).

[15] C. Kurokawa and K. Katō, Nucl. Phys. **A 792**, (2007) 87.

[16] S. Ohtsubo, Y. Fukushima, M. Kamimura, and E. Hiyama, Prog. Theor. Exp. Phys. **2013**, (2013) 073D02.

[17] J. Aguilar and J.M. Combes, Commun. Math. Phys. **22**, (1971) 269; E. Balslev and J. M. Combes, Commun. Math. Phys. **22**, (1971) 280.

[18] T. Myo and K. Katō, submitted to Phys. Rev. C.

[19] A. D. Duisenbay, N. Kalzhigitov, K. Katō, V. O. Kurmangaliyeva, N. Takibayev, V. S. Vasilevsky, Nucl. Phys. **A 996**, (2020) 121692.

[20] T. Myo and K. Katō, Prog. Theor. Exp. Phys. **2020**, (2020) 12A101.

[21] T. Myo, M. Odsuren, and K. Katō, Phys. Rev. **C 104**, (2021) 044306; T. Myo and K. Katō, Phys. Rev. **C 107**, (2023) 034305.

[22] M. Odsuren, Y. Kikuchi, T. Myo and K. Katō, private communication.

[23] T. Myo, Y. Kikuchi, H. Masui, K. Katō, Prog. Part. Nucl. Phys. **79**, (2014) 1.

[24] H. Horiuchi, K. Ikeda and K. Katō, Prog. Theor. Phys. Suppl. **192**, (2012), 1.

[25] Y. Funaki *et al.*, Phys. Rev. Lett. **101**, (2008) 082502.

[26] Y. Funaki *et al.*, Phys. Rev. C **80**, (2009) 064326.

[27] Y. Funaki,H. Horiuchi and A. Tohsaki, Prog. Part. Nucl. Phys. **82**, (2015) 78.

[28] B. Zhou, Y. Funaki, H. Horiuchi, Y. G. Ma, G. Röpke, P. Schuck, A. Tohsaki and T. Yamada, Nature Commun. **14**, (2023) 8206.

[29] Y. Funaki, Presentation in Workshop on Nuclear Clsuter Physics (WNCP2023), (2023).

[30] T. Yamada and P. Schuck, Eur. Phys. J. A **26**, (2005) 185.

[31] H. Moriya, W. Horiuchi and B. Zhou, Eur. Phys. J. A **59**, (2023) 197.

[32] D. R. Tilley, *et al.*, Nucl. Phys. A **745**, (2004) 155.

[33] https://www.nndc.bnl.gov/nudat3/.