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Mode Converter Using Bent and Twisted Coupled-Multicore Fibers for Group Delay Spread Reduction

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Abstract

A mode converter using bent and twisted coupled-multicore fiber for the reduction of the group delay spread is proposed. The bent and twisted fiber, which is one of the periodic structures, can work as a long-period grating mode converter by properly setting the twist rate. To investigate the mode conversion characteristics, a simple and effective eigenmode analysis for three-dimensional periodic structures is proposed. We found that to convert the specified modes, the core layout is also important as well as the twist rate. For example, the mode conversion behaviors of 4CFs with square lattice and triangular lattice layouts are greatly different although each 4CF enables the specified mode conversion. It is revealed that, by switching the higher and lower order modes, the group delay can be equalized, resulting in the suppression of the group delay spread. It leads to reducing the complexity of the multiple-input multiple-output receiver in the mode division multiplexing system.

Keywords: Coupled multicore fiber, group delay spread, space division multiplexing, coupled-mode theory

1. Introduction

Space division multiplexing (SDM) technique can provide a way to overcome the physical capacity limits of the single-mode fiber [1]. Multicore fiber (MCF) is one of the transmission mediums used in the SDM system. Uncoupled-type MCFs, in which the cores are arranged with enough core-to-core distance, can independently transport the lightwave in each core. On the other hand, coupled MCFs (CMCFs) become attractive because the spatial density is higher than uncoupled MCFs although the multiple-input multiple-output digital signal processor (MIMO-DSP) is required. Compared with few-mode fibers (FMFs), CMCFs tend to have the lower attenuation characteristics and the lower group-delay spread (GDS) [2].

To reduce the MIMO complexity, much lower GDS is preferable. There are two plans to drastically reduce the GDS of CMCF. One is utilizing the strong coupling state as in [3]. In general, the GDS of the straight fiber increases linearly along with propagation distance L , while the GDS of the strong coupling state is proportional to $\sqrt{L}$. Such a strong coupling state appears by the mode coupling with the random perturbation [3-8]. However, the previous report [5] clarified that, if the core-to-core distance is too close, the random mode coupling is weakened and the GDS increases. Depending on the radius, the minimum core pitch to make the strong coupling state seems to be approximately from 17 to 20 μm [5,8]. To realize both the dense SDM transmission and the GDS suppression, the use of the mode converter as an optical relay component is another

promising way. Assuming the two-mode transmission system by using CMCFs, if their modes are switched to each other at the halfway point, the GDS is suppressed because of the compensation of differential modal delay (DMD). If the structural perturbation is enough small, the GDS in such a type could be lower than that in the strong random coupling fiber [9]. In addition, a smaller bending radius tends to be required to make the strong random coupling. Such a requirement could also be eased by using the mode converter.

Various types of fiber-based mode converters have been studied. By bending the FMF, the modes coupling occurs due to the distortion of the electric field distribution of the guided modes [10] although the mode selective conversion is difficult. The long-period grating (LPG) based on periodical modulation by the structural stress or material deforming is also one of the candidates. In [11], the LPG using the FMF is reported, where the one-period length must be around 1 mm. To appropriately select the one-period length, the converted mode can be controlled. If the LPG is formed in a CMCF, it is expected that the one-period length becomes larger than FMF due to the small difference between the propagation constants, which makes the fabrication easier. In [12], it is reported that only twisting the fiber also makes the mode conversion because the twisted fiber can be considered as one of the LPG, where the structural asymmetry is necessary instead of the bending. In this paper, we focus on the LPG-based mode converter using the bent and twisted CMCF. This type of mode converter can also be called supermode converter more accurately. The twisted CMCF can be fabricated by spinning the fiber preform [13], otherwise, it can be formed by simply rotating the standard CMCF [14]. Compared to the LPG which requires a mechan-

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ical grating plate with a precise periodic groove, our proposed bent and twisted type mode converter has an advantage that the one-period length can be fine-tuned by controlling the amount of torsion, although the fabrication or equipment cost may increase. If the mode converter of the CMCF is deployed as an optical relay in the MDM transmission system, the GDS will be successfully suppressed. We would like to repeat that the proposed mode converter is urged to be required to reduce the GDS for the transmission system based on CMCF, which has a small core pitch of approximately less than 17 μm .

This paper is organized as follows. In Section 2, we explain an eigenmode analysis to investigate the LPG-based mode conversion. In this method, the transfer matrix method (TMM) is mainly used. In Section 3, the mode conversion characteristics in the CMCFs are investigated. We show that the mode conversion occurs even if the bending radius is larger than 1 m. In Section 4, the example of the GDS suppression by using the CMCF is shown. Section 5 is dedicated to conclusions.

2. Numerical Method and Theoretical model

2.1. Eigenmode analysis of periodic waveguide

A transfer matrix for a one-period waveguide is defined as $\mathbf{T}_{1P}$, where we assume that the straight (non-periodic) waveguides are connected before and after the periodic waveguide. If the matrix is diagonalizable, it can be decomposed as

$$\mathbf{T}_{1P} = \mathbf{P}_{S \leftarrow P} \mathbf{D}_{1P} \mathbf{P}_{P \leftarrow S}, \quad (1)$$

and the transfer matrix for the N -period waveguide is given as

$$(\mathbf{T}_{1P})^N = \mathbf{P}_{S \leftarrow P} (\mathbf{D}_{1P})^N \mathbf{P}_{P \leftarrow S}, \quad (2)$$

where the orthogonal matrix $\mathbf{P}_{S \leftarrow P}$ has the series of eigenvectors and the diagonal matrix $\mathbf{D}_{1P}$ has the series of eigenvalues. $\mathbf{P}_{P \leftarrow S}$ is given as the inverse matrix of $\mathbf{P}_{S \leftarrow P}$. Suppose that the i th eigenvalue is expressed as $A_i \exp(j\theta_i)$, θ_i and A_i indicate the amount of phase shift and attenuation (or gain) for the i th eigenmodes, respectively. While $\mathbf{P}_{P \leftarrow S}$ indicates the basis conversion matrix from the input straight waveguide to the periodic waveguide. In this paper, such a 3D eigenmode of the periodic waveguide is called a periodic mode. The i th electric field vector $\mathbf{E}_i^{(P)}$ of the periodic mode is virtually visualized by the basis conversion matrix $\mathbf{P}_{S \leftarrow P}$ and the electric field vector of the input straight waveguide, $\mathbf{E}_i^{(S)}$, as

$$\mathbf{E}_i^{(P)} = \sum_{j=1}^n (\mathbf{P}_{P \leftarrow S})_{ij} \mathbf{E}_j^{(S)}, \quad (3)$$

where $(\cdot)_{ij}$ denotes the (i, j) element of the matrix, and n is the number of modes. Please note that the electric field given by Eq. (3) is valid only for the cross section at $z = N\Lambda$, where Λ is a one-period length. Also, if θ_i is defined as $-\gamma_i \cdot \Lambda$, γ_i is equivalent to the propagation constant of the periodic mode. It is not easy to obtain γ_i , since $|\theta_i|$ generally becomes greater than 2π . In other words, we cannot distinguish whether γ_i or $\gamma_i + 2\pi/\Lambda$ is a genuine solution. By carrying out some procedures, γ_i can be properly estimated in most cases (see Appendix A).

Although there is another method to directly obtain the periodic mode [15], the analysis proposed above is simple and adequate for the low- Δ optical waveguide.

It is known that two guided modes can convert to each other by the modulation with the periodicity of

$$\Lambda_{\text{beat}} = \frac{2\pi}{\Delta\beta}, \quad (4)$$

where $\Delta\beta$ is the difference between two propagation constants of guided modes in the straight waveguides. When setting $\Lambda = \Lambda_{\text{beat}}$, the following two conditions should be satisfied. First, for the i th and j th mode conversion, two guided modes should excite the same ratio of periodic modes, namely,

$$|(\mathbf{P}_{P \leftarrow S})_{ki}| = |(\mathbf{P}_{P \leftarrow S})_{kj}|, \quad (5)$$

where the order of the periodic mode, k , is an arbitrary integer. Second, after the propagation of a cycle of p_1 , the phase difference of periodic modes excited by two guided modes should be

$$p_1 \arg \left[\frac{(\mathbf{D}_{1P})_{kk}}{(\mathbf{D}_{1P})_{11}} \right] = \arg \left[\frac{(\mathbf{P}_{P \leftarrow S})_{kj}}{(\mathbf{P}_{P \leftarrow S})_{1j}} \right] - \arg \left[\frac{(\mathbf{P}_{P \leftarrow S})_{ki}}{(\mathbf{P}_{P \leftarrow S})_{1i}} \right] \quad (6)$$

for any k , where $\arg[\cdot]$ denotes the argument. If $\mathbf{P}_{P \leftarrow S}$ is a 2×2 unitary matrix, the conditions of Eqs. (5) and (6) can be rewritten as

$$|\mathbf{P}_{P \leftarrow S}| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (7)$$

$$p_1 (\theta_2 - \theta_1) = (2p_2 + 1)\pi, \quad (8)$$

where p_2 is an integer. Eq. (8) leads to the coupling length L_c as

$$L_c = \frac{\pi}{\Delta\gamma}, \quad (9)$$

where $\Delta\gamma$ is the difference between two propagation constants of periodic modes. From Eq. (9), the coupling coefficient for periodic modes can be expressed as

$$\kappa_c = \frac{\pi}{2L_c}. \quad (10)$$

If L_c calculated by Eq. (9) is much lower than Λ , it may not be a reasonable value. From Eq. (8), to make the phase difference of π , the condition of

$$p_1 \Lambda \frac{\Delta\gamma}{\pi} \approx 2p_2 + 1 \quad (11)$$

is required. Thus we can calculate the modified coupling length, L'_c , as

$$L'_c = p_1 \Lambda. \quad (12)$$

In Eq. (12), a candidate solution of L'_c becomes the discrete value. To obtain the solution of the continuous value, we found the other form of Eqs. (11) and (12) as

$$L''_c = \frac{\Lambda}{2\chi}, \quad (13)$$

$$\chi = \left\lfloor \frac{2L_c}{\Lambda} + p_3 \right\rfloor, \quad (14)$$

where p_3 is an integer, which makes χ minimum value.

If $|\Lambda - \Lambda_{\text{beat}}| \gg 0$, the condition of mode conversion is broken. By evaluating the similarity of the eigenmode vector in Eq. (1), the degradation of conversion efficiency can be estimated. The maximum conversion efficiency, T_{cmax} , can be calculated as

$$T_{\text{cmax}} = \left[\sum_k^n |(\mathbf{P}_{\text{P} \leftarrow \text{S}})_{ki}| |(\mathbf{P}_{\text{P} \leftarrow \text{S}})_{kj}| \right]^2. \quad (15)$$

According to the coupled-mode theory, Eq. (15) forms the Lorentzian curve in the ideal case [16]. If the full width at half maximum (FWHM) is defined as Δk , we can find that Δk is also related to the coupling length as

$$L_c''' = \frac{2\pi}{\Delta k}. \quad (16)$$

In this way, the coupling length cannot be only obtained by eigenvalues but also by eigenvectors. Eq. (16) is useful except for a situation, in which the various modes are simultaneously coupled and the FWHM cannot be obtained. To confirm the validity of the eigenmode analysis, the LPG-based mode converter with a silica planar lightwave circuit (PLC) is investigated as an example in Appendix B.

2.2. Periodic mode in bent and twisted CMCFs

To explore the characteristics of the CMCF, the transfer matrix corresponding to one-period CMCF is required. As in [7], by formularizing the mode coupling equations and solving the simultaneous linear differential equations, we can easily obtain the transfer matrix of the bent and twisted CMCF. Strictly speaking, in [7], the bending direction is rotated along to the z -direction as shown in Fig. 1(a), where α and R_b are the twist rate and the bending radius. In the numerical simulation, the effect of twisting and bending is treated as a refractive index change ratio δn as follows:

$$\delta n(x, y, z) = 1 + \frac{\cos(\alpha z)x + \sin(\alpha z)y}{R_b}. \quad (17)$$

For example, for the homogeneous single-mode two-core bent and twisted CMCF, the simultaneous equations can be represented as

$$\frac{d}{dz} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = -j\mathbf{M} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (18)$$

$$\mathbf{M} = \begin{pmatrix} \beta \cdot \delta n_1 & \kappa \\ \kappa & \beta \cdot \delta n_2 \end{pmatrix}, \quad (19)$$

$$\delta n_i = \delta n(x_i, y_i, z), \quad (20)$$

where a_i is a linear-polarized modal amplitude in the i th core, β is the propagation constant, κ is the coupling coefficient, and (x_i, y_i) is the i th core position. If $R_b^{-1} = 0$ (without bending), δn

becomes 1, and hence, Eqs. (18), (19), and (20) can be rewritten as

$$\frac{d}{dz} \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = -j\tilde{\mathbf{M}} \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix}, \quad (21)$$

$$\tilde{\mathbf{M}} = \begin{pmatrix} \beta + \kappa & 0 \\ 0 & \beta - \kappa \end{pmatrix}, \quad (22)$$

$$\begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = \mathbf{P}_{\text{C} \leftarrow \text{D}} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (23)$$

$$\mathbf{P}_{\text{C} \leftarrow \text{D}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad (24)$$

where $\tilde{a}_i$ is the modal amplitudes of eigenmodes for no-bent fiber, which are also called supermodes (even and odd modes). In contrast, the mode for a_i is called the discrete core mode. $\mathbf{P}_{\text{C} \leftarrow \text{D}}$ ($\mathbf{P}_{\text{D} \leftarrow \text{C}}$) is basis conversion matrix between the supermodes and the discrete core modes. Eqs. (21) and (22) indicate that no coupling between supermodes occurs for any α . One may consider that the propagation characteristics depend on the twist rate α because the modal coupling occurs even in the no-bent twisted fiber as shown in Fig. 1(b). It is known that the propagation constant split to $\beta \pm q\alpha$ according to the twist rate and the order of the hybrid modes, q , due to the polarization coupling [12]. These modes are circularly polarized modes. In this study, each core supports one spatial mode (HE11 mode), and hence, $q = 1$. Note that, such a split is observed in the orthogonal coordinate system. From the view of the helicoidal coordinate system, the propagation constant is not splitted for HE11 mode, and the linearly polarized mode propagates as a linearly polarized mode as shown in Figs. 2(a) and (b), where both modes have the same propagation constants. Therefore, we considered one of the polarized modes, and the scalar finite-element method [17] developed in our laboratory was used to obtain the propagation constant and the coupling coefficients in this paper.

3. Mode Conversion in CMCF

3.1. Coupled two-core fiber

In this section, the coupled single-mode two-core CMCF (2CF) is investigated by using the eigenmode analysis derived in Section 2. Figs. 3 (a), (b), and (c) show the cross section of the bent and twisted fiber of CMCF with various numbers of cores, where a is the core radius, g is the distance between cores (core pitch). The wavelength is fixed to 1550 nm throughout this paper. To satisfy the single-mode operation, the relative refractive index difference (Δ) and a are set to 0.35% and 4.5 μm , respectively. When the number of cores is n , we define the i th ($i = 1, \dots, n$) core position (x_i, y_i) as

$$x_i = r_n \cos[(i-1)\theta_n], \quad (25)$$

$$y_i = r_n \sin[(i-1)\theta_n], \quad (26)$$

$$r_n = \frac{g}{2 \sin\left(\frac{\pi}{n}\right)}, \quad (27)$$

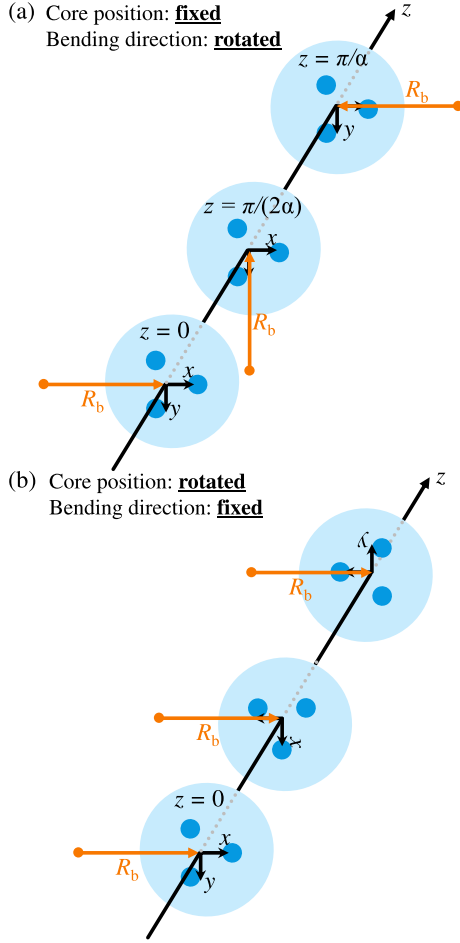


Figure 1: Comparison of two models for the bent and twisted CMCF.

$$\theta_n = \frac{2\pi}{n}, \quad (28)$$

which give the ring layout. By solving the modal evolution in the i th core by Eqs. (18) and (19), the transfer matrix for discrete core mode, $\mathbf{T}_D$, can be obtained. After that, the transfer matrix for supermodes can be obtained by

$$\mathbf{T}_C = \mathbf{P}_{C \leftarrow D} \mathbf{T}_D \mathbf{P}_{D \leftarrow C}. \quad (29)$$

Note also that, the supermodes of CMCF are equivalent to the eigenmodes of the straight waveguides (subscription of “S”) in Section 2.1. Figs. 4(a) and (b) show the field distributions of two supermodes of the straight 2CF, where $g = 14 \mu\text{m}$. In order to convert from the 1st (even) mode to the 2nd (odd) mode, we apply the structural modulation by bending and twisting with the period of the beat length. Fig. 4 (c) shows the propagation constants of two supermodes and the beat length for various g . The larger the core pitch is, the larger the beat length becomes.

Fig. 5 shows the T_{cmax} as a function of α for various R_b , where g is fixed to $14 \mu\text{m}$ corresponding to $\Lambda_{\text{beat}} = 10.6 \text{ mm}$ ($2\pi/\Lambda_{\text{beat}} = 189\pi \text{ rad/m}$). We can see the peak for each R_b located at $\alpha = 2\pi/\Lambda_{\text{beat}}$. The smaller R_b gives the strong refractive index modulation, resulting in the broadening of the peak width (Δk) and the shortening of the coupling length (L_c). If $R_b < 500 \text{ mm}$, the spectrum is deformed because the modulation is too strong, in other words, the perturbation theory seems not to be held. Though, the perfect mode conversion occurs as long as $T_{\text{cmax}} = 1$. Fig. 6 shows the coupling length as a function of R_b . The coupling length gets close to Λ_{beat} for $R_b < 500 \text{ mm}$, which indicates that the modulation is too strong. In such a condition, the conversion efficiency greatly depends on the initial bending angle. Figs. 7(a) and (b) show the difference of T_{cmax} for $R_b = 1000$ and 200 mm when the initial bending angle changes from αz to $\alpha z + \pi/2$. For $R_b = 1000 \text{ mm}$, the spectra almost agree. For $R_b = 200 \text{ mm}$, the peak around $\alpha = \Lambda_{\text{beat}}$ is shifted and the spectral shapes for $\alpha < \Lambda_{\text{beat}}$ are greatly changed. To retain the stable conversion condition, we should select the parameter satisfying $L_c \gg \Lambda_{\text{beat}}$. Figs. 8(a) and (b) show the T_{cmax} as a function of α for various g , where $R_b = 1000 \text{ mm}$. For $g = 12$ and $14 \mu\text{m}$, ideal Lorentzian curves appear at $\alpha = 2\pi/\Lambda_{\text{beat}}$. Increasing g , the peaks are shifted to the left as expected from Λ_{beat} , and the spectra are deformed due to the increase of Λ_{beat} . As shown in Fig. 9, for $g > 18 \mu\text{m}$, L_c is expected to be smaller than Λ_{beat} . This is the reason for the deformation of spectra for $g > 18 \mu\text{m}$. Fig. 10 shows the T_{cmax} as a function of R_b for various g , where α is fixed to $2\pi/\Lambda_{\text{beat}}$. We can see that the increase of g limits the range of R_b for making the complete mode conversion.

Figs. 11(a) and (b) show the propagation constants of the periodic modes and the squared absolute value of the basis conversion matrix when changing α , where $g = 14 \mu\text{m}$ and $R_b = 1000 \text{ mm}$. Note that, 1st and 2nd modes in the legend do not indicate the supermodes. Figs. 11(a) and (b) are similar to Figs. B.2(a) and (b) in Appendix B, which indicates that the bent and twisted CMCF works as same as the standard LPG. In the inset of Fig. 11(b), the field distributions calculated by Eq. (3) are displayed. When $\alpha = 180\pi \text{ rad/m}$, the 1st (2nd) periodic mode

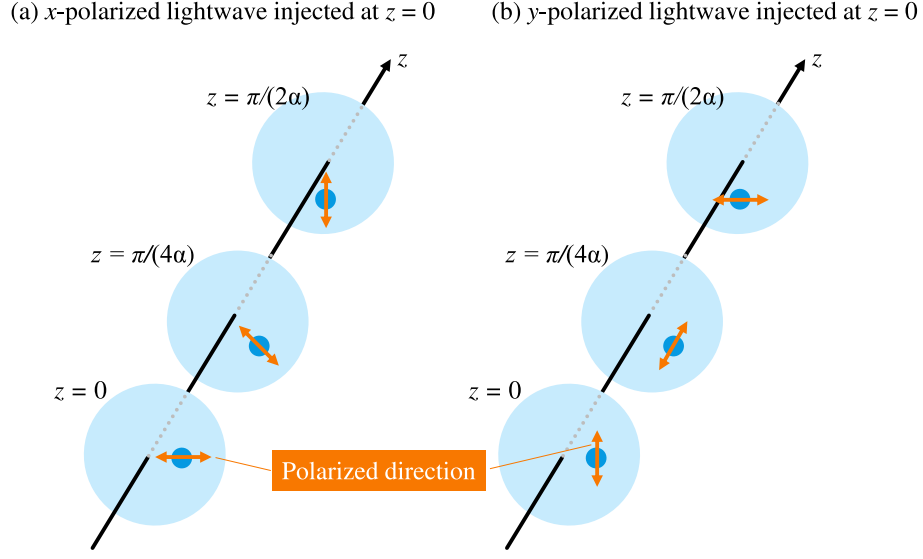


Figure 2: Polarization states when the (a) x- and (b) y-polarized mode propagate in the bent and twisted CMCF as shown in Fig. 1(b).

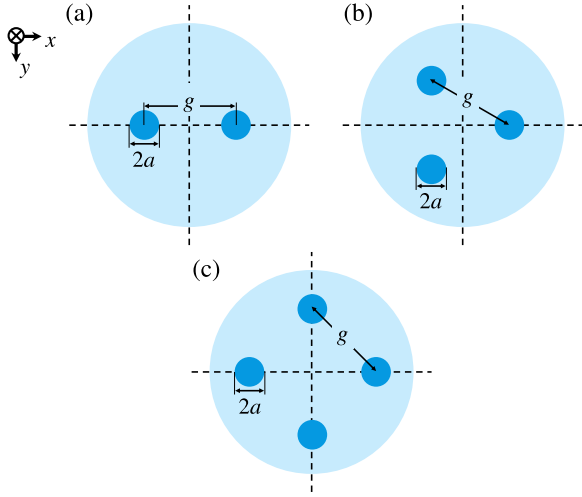


Figure 3: Cross section of bent and twisted fiber of CMCF with the ring layout for (a) 2CF, (b) 3CF, and (c) 4CF.

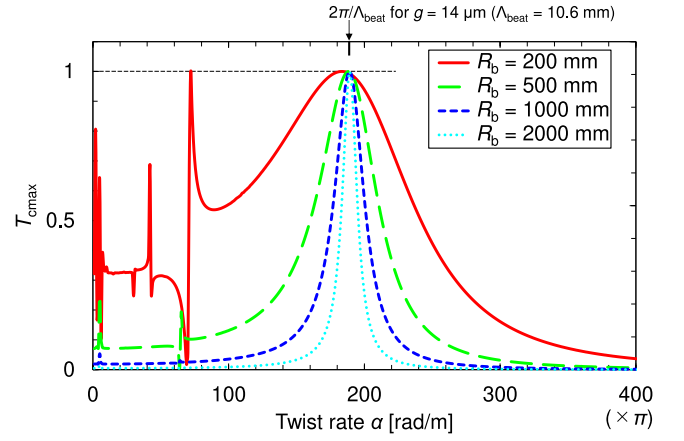


Figure 5: T_{cmax} as a function of α for the 2CF, where $g = 14 \mu\text{m}$.

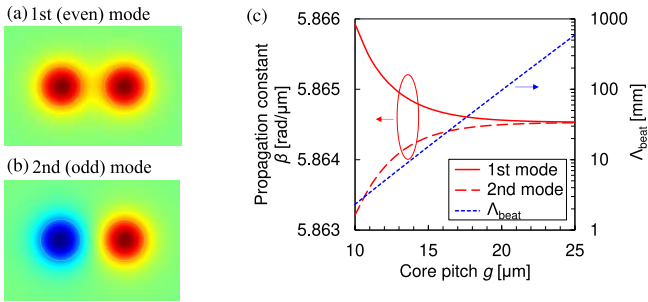


Figure 4: Characteristics of 2CF. (a), (b) Field distributions of even and odd modes of straight 2CF when $g = 14 \mu\text{m}$. (c) Propagation constants of even and odd modes and the beat length.

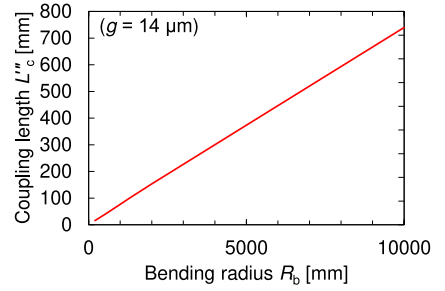


Figure 6: Coupling length for the 2CF as a function of R_b , where $g = 14 \mu\text{m}$.

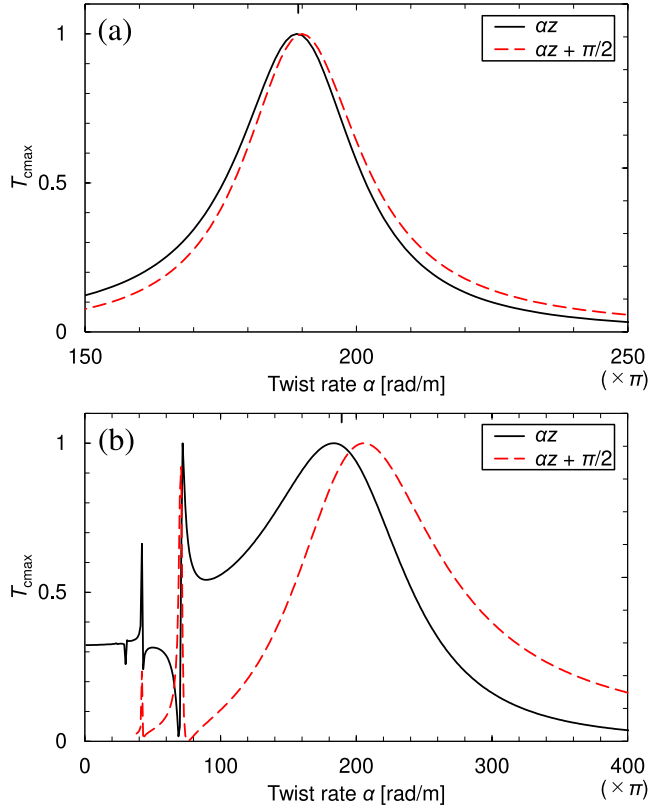


Figure 7: Difference of $T_{c\max}$ for $R_b =$ (a) 1000 mm and (b) 200 mm when the initial bending angle changes from αz to $\alpha z + \pi/2$, where $n = 2$ and $g = 14 \mu\text{m}$.

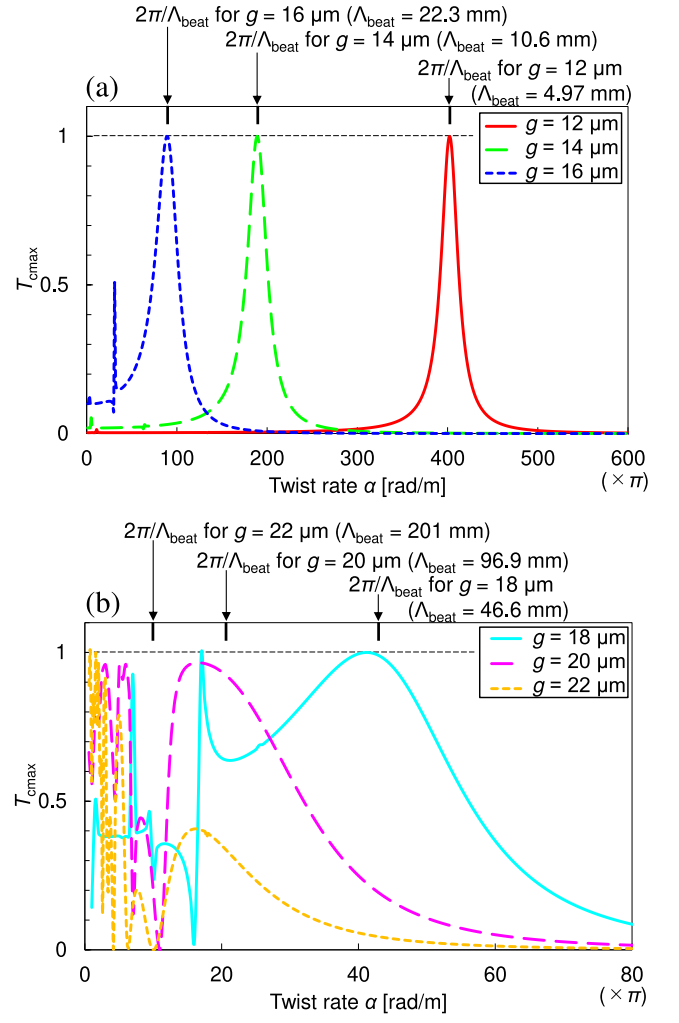


Figure 8: $T_{c\max}$ as a function of α for 2CF, where $R_b = 1000$ mm. The core-to-core distance g is ranged (a) from $12 \mu\text{m}$ to $16 \mu\text{m}$ and (b) from $18 \mu\text{m}$ to $22 \mu\text{m}$.

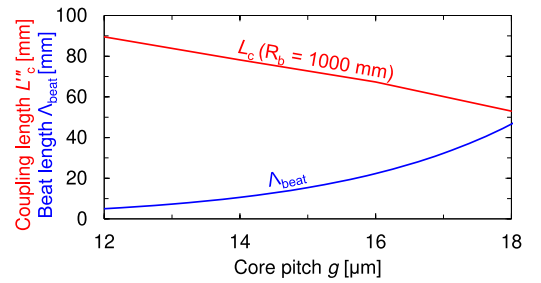


Figure 9: Coupling length and the beat length for 2CF as a function of g where $R_b = 1000$ mm.

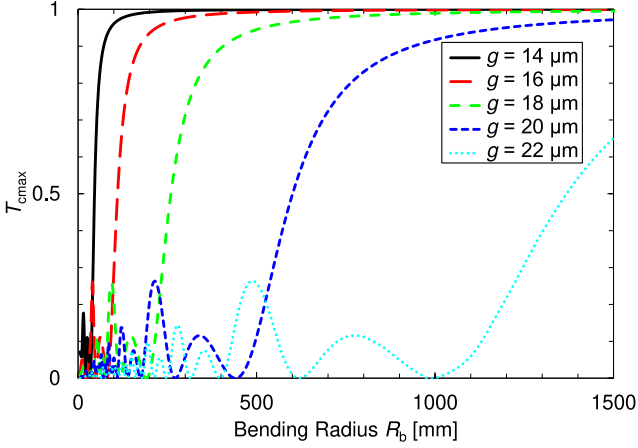


Figure 10: $T_{c\max}$ of 2CF as a function of R_b for various g , where α is fixed to $2\pi/\Lambda_{\text{beat}}$.

is expressed by even mode – odd mode (even mode + odd mode). Conversely, we can understand that the launched even mode excites 1st and 2nd periodic modes almost equally. Since these periodic modes have different propagation constants, when the phase shift becomes π , only the odd mode can be exploited. Fig. 12 shows the transmission as a function of the propagation distance when the even mode is launched, where $g = 14 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 189\pi \text{ rad/m}$. The launched even mode is fully converted to the odd mode around z from 75 to 78 mm. As discussed in Section 2, this coupling length can be estimated by some equations. Fig. 13 shows the comparison of the coupling lengths estimated by Eqs. (9), (13), and (16). We can see that the original L_c obtained by Eq. (9) must be modified by Eq. (13) or (16) for periodic modes. L'_c obtained by Eq. (16) is 76.2 mm, which agrees well with Fig. 12. The discontinuity of L'_c by Eq. (13) arises from the uncertainty of γ . In practice, we can estimate the plausible coupling length from Eq. (13) by averaging the high and low L'_c in the vicinity of the discontinuous point. In this case, the high L'_c and the low L'_c are 81.8 and 71.2 mm, respectively. As the result, we can obtain a reasonable value of 76.5 mm without investigating FWHM of $T_{c\max}$.

3.2. Coupled three-core fiber

In this section, similarly to the 2CF, the coupled single-mode three-core CMCF (3CF) is investigated. The core positions are given by Eqs. (25) and (26), where $n = 3$. It gives the core layout as shown in Fig. 3 (b). Other structural parameters are the same with 2CF. According to the coupled-mode theory, we defined the matrix $\mathbf{M}$ used in Eq. (18) for $n = 3$ as

$$\mathbf{M}_{R3} = \begin{pmatrix} \beta \cdot \delta n_1 & \kappa & \kappa \\ \kappa & \beta \cdot \delta n_2 & \kappa \\ \kappa & \kappa & \beta \cdot \delta n_3 \end{pmatrix}, \quad (30)$$

where κ is the coupling coefficient for adjacent cores. The basis conversion matrix, which has countless forms because $\mathbf{M}_{R3}$ has

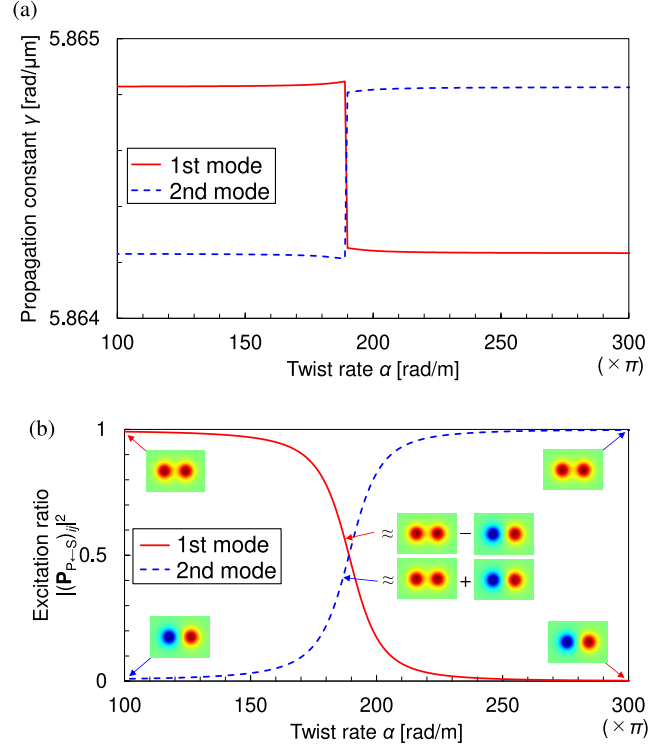


Figure 11: Eigenmodes of the 2CF, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 14 \mu\text{m}$, and $R_b = 1000 \text{ mm}$. (a) Propagation constants and (b) squared absolute value of the basis conversion matrix as a function of the twist rate α . The 1st and 2nd (periodic) modes correspond to $(i, j) = (1, 1)$ and $(2, 1)$, respectively. The field distributions of the inset are the virtually visualized fields of the periodic modes obtained by Eq. (3).

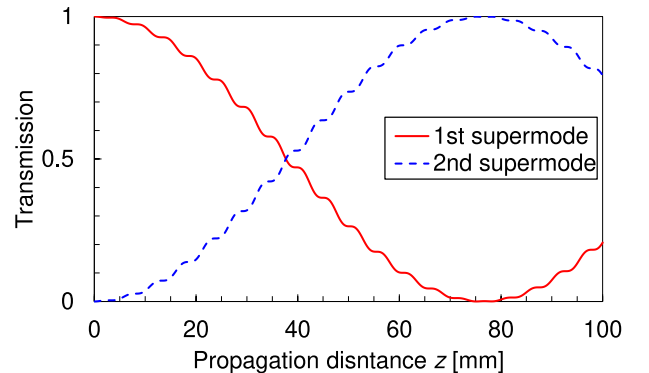


Figure 12: Mode evolution in the 2CF when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 14 \mu\text{m}$, $R_b = 1000 \mu\text{m}$, and $\alpha = 189\pi \text{ rad/m}$.

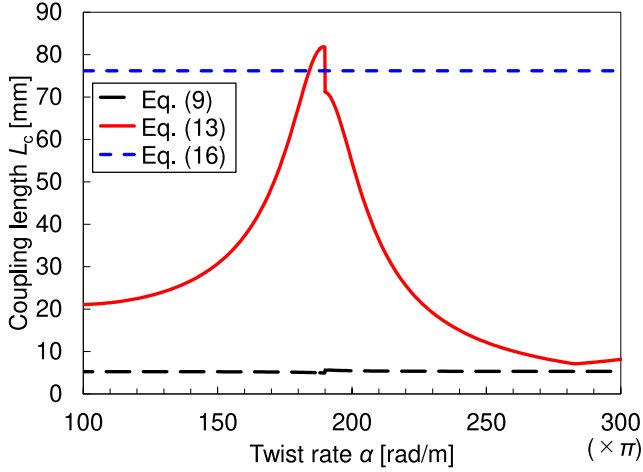


Figure 13: Comparison of the coupling lengths estimated by Eqs. (9), (13), and (16), where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 14 \mu\text{m}$, and $R_b = 1000 \text{ mm}$.

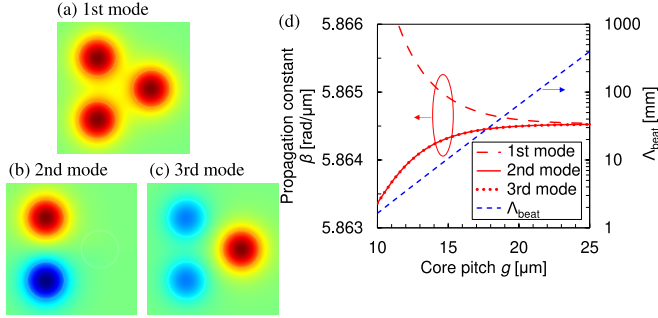


Figure 14: Characteristics of 3CF. (a), (b), and (c) Field distributions of three supermodes of straight 3CF when $g = 15 \mu\text{m}$. (d) Propagation constants of three supermodes and the beat length.

multiple roots for $R_b^{-1} = 0$, is fixed to

$$\mathbf{P}_{\text{R3,C} \leftarrow \text{D}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix}, \quad (31)$$

which is normalized and orthogonalized.

Figs. 14(a), (b), and (c) show the field distributions of three supermodes of the straight 3CF, where $g = 15 \mu\text{m}$. By bending and twisting with the period of the beat length, the 1st mode can be converted into 2nd or 3rd modes. Fig. 14(d) shows the propagation constants of three supermodes and the beat length for various g . When $R_b^{-1} = 0$, the 2nd supermode and the 3rd supermode degenerate.

Fig. 15 shows the excitation ratio of eigenmodes in the 3CF (squared absolute value of the basis conversion matrix) when changing α , where $g = 15 \mu\text{m}$ and $R_b = 1000 \text{ mm}$, which corresponds to $\Lambda_{\text{beat}} = 10.5 \text{ mm}$ ($2\pi/\Lambda_{\text{beat}} = 191\pi \text{ rad/m}$). In the bent and twisted 3CF, the degeneration of 2nd and 3rd supermodes is broken as displayed in the left (right) bottom corner in Fig. 15. In a similar manner with the 2CF, when $\alpha = 191\pi \text{ rad/m}$, the

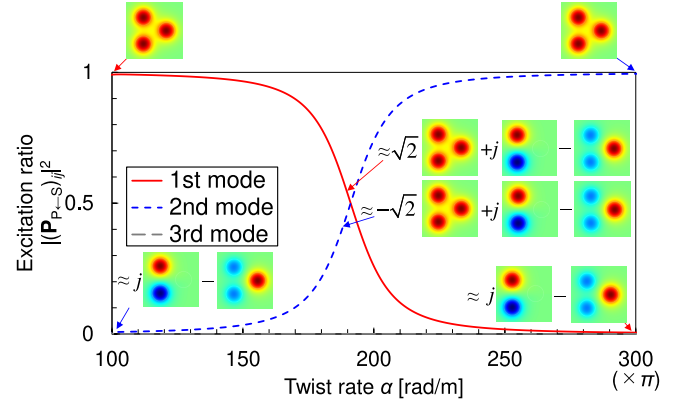


Figure 15: Excitation ratio of eigenmodes in the 3CF (squared absolute value of the basis conversion matrix) as a function of the twist rate α , where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 15 \mu\text{m}$, and $R_b = 1000 \text{ mm}$. The 1st, 2nd, and 3rd (periodic) modes correspond to $(i, j) = (1, 1)$, $(2, 1)$, and $(3, 1)$, respectively. The field distributions of the inset are the virtually visualized field of the periodic modes obtained by Eq. (3).

launched 1st supermode can excite 1st and 2nd periodic modes almost equally. After the propagation of the coupling length, 2nd/3rd supermodes can be exploited. Fig. 16 shows the transmission as a function of the propagation distance when one of supermodes is launched, where $g = 15 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 191\pi \text{ rad/m}$. The launched 1st mode is fully converted to 2nd and 3rd supermodes around $z = 62 \text{ mm}$. When the 2nd supermode is launched, because of the optical linear reciprocity, it is impossible to exploit only 1st supermode. To convert from 2nd and 3rd supermodes into the 1st supermode, 2nd and 3rd supermodes must be almost equally launched with the 90-degree phase difference.

3.3. Coupled Four-core fiber

In this section, the coupled single-mode four-core CMCF (4CF) is investigated. The basic principle is the same as the 2CF and 3CF. However, as discussed below, the 1st mode cannot be directly converted into 4th mode even if the core arrangement is equally spaced as in Eqs. (25) and (26). Instead, we examine the conversion from the 1st supermode to the 4th supermode via 2nd/3rd supermodes.

At first, the core positions are assumed to be set by Eqs. (25) and (26), where $n = 4$. It gives the core layout as shown in Fig. 3 (c). Other structural parameters are the same with the 2CF and 3CF. According to the coupled-mode theory, we defined the matrix $\mathbf{M}$ used in Eq. (18) for $n = 4$ as

$$\mathbf{M}_{\text{R4}} = \begin{pmatrix} \beta \cdot \delta n_1 & \kappa & \kappa' & \kappa \\ \kappa & \beta \cdot \delta n_2 & \kappa & \kappa' \\ \kappa' & \kappa & \beta \cdot \delta n_3 & \kappa \\ \kappa & \kappa' & \kappa & \beta \cdot \delta n_4 \end{pmatrix}, \quad (32)$$

where κ and κ' are the coupling coefficient for adjacent cores and opposite cores, respectively. It is noted that κ' should be carefully calculated by following the procedure in Appendix

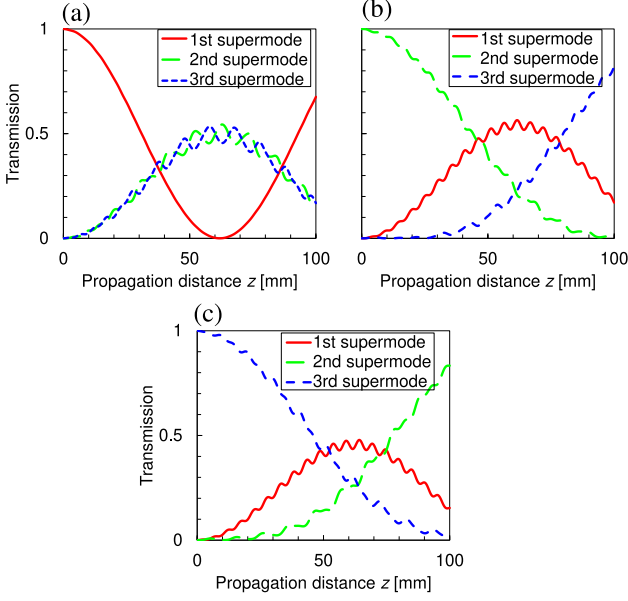


Figure 16: Mode evolution in the 3CF when launching (a) 1st, (b) 2nd, and (c) 3rd supermodes, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 15 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 191\pi \text{ rad/m}$.

C. Since $\mathbf{M}_{R4}$ has also multiple roots for $R_b^{-1} = 0$, the basis conversion matrix is fixed to

$$\mathbf{P}_{R4,C \leftarrow D} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}, \quad (33)$$

which is normalized and orthogonalized.

Figs. 17 (a), (b), (c), and (d) show the field distributions of four supermodes of the straight 4CF, where $g = 15 \mu\text{m}$. Here, we emphasize that asymmetric refractive index modulation is required for the interband transition [16]. That is,

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta n(x, y) (\mathbf{E}_i^{(s)}(x, y) \cdot \mathbf{E}_j^{(s)}(x, y)) dx dy \neq 0 \quad (34)$$

for the conversion between the i th and j th mode. For the 4CF, 1st and 4th supermodes are 90- and 180-degree symmetry. If we define $f_{ij}(r, \phi) = \mathbf{E}_i^{(s)}(r, \phi) \cdot \mathbf{E}_j^{(s)}(r, \phi)$ with the cylindrical coordinate system, $f_{14}(r, \phi)$ is 180-degree symmetry, namely $f_{14}(r, \phi) = f_{14}(r, \phi + \pi)$. Whereas, the refractive index change can be expressed as $\delta n(r, \phi) = -\delta n(r, \phi + \pi)$. As a result, the left-hand term in (34) for 1st and 4th supermodes becomes 0. For 1st and 2nd/3rd supermodes, we can find $f_{12}(r, \phi) = -f_{12}(r, \phi + \pi)$, which satisfies (34). For 4th and 2nd/3rd supermodes, we can find $f_{24}(r, \phi) = -f_{24}(r, \phi + \pi)$, which also satisfies Eq. (34). Therefore, the 1st supermode can be converted into the 4th supermode via 2nd/3rd supermodes. Fig. 17(e) shows the propagation constants of four supermodes and the beat length for various g , where the beat length is calculated by averaging the propagation constant difference of 1st-2nd and 2nd-4th supermodes. When $R_b^{-1} = 0$, the 2nd supermode and the 3rd supermode degenerate.

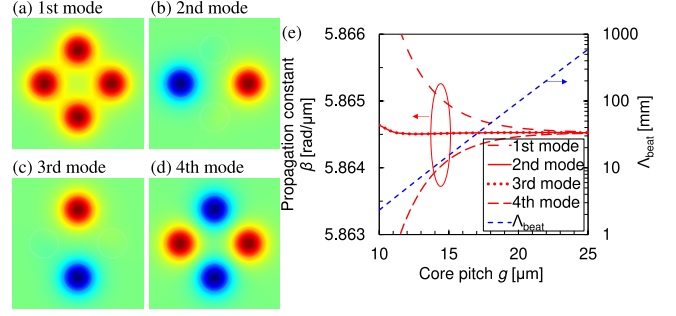


Figure 17: Characteristics of 4CF. (a), (b), (c), and (d) Field distributions of four supermodes of straight 4CF when $g = 16 \mu\text{m}$. (e) Propagation constants of four supermodes and the beat length calculated by averaging the propagation constant difference of 1st-2nd and 2nd-4th supermodes.

In the beginning, we apply the approximation of $\kappa' = 0$ for simplicity. Figs. 18(a) and (b) show the propagation constants of the periodic modes and the squared absolute value of the basis conversion matrix when changing α , where $g = 16 \mu\text{m}$ and $R_b = 1000 \text{ mm}$, which corresponds to $\Lambda_{\text{beat}} = 22.3 \text{ mm}$ ($2\pi/\Lambda_{\text{beat}} = 89.5\pi \text{ rad/m}$). In Fig. 18(a), 1st and 2nd periodic modes seem to be coupled. But, as seen in Fig. 18(b), the 3rd periodic mode is also related to the mode conversion. Such an operation resembles the directional coupler with the in-line arrangement (See Appendix D). In this situation, instead of using the coupling length defined by Eq. (9), the following definition should be used:

$$\tilde{L}_c = \frac{2\pi}{\gamma_1 - \gamma_2}. \quad (35)$$

It should also be modified by Eq. (13). In addition, L_c''' in Eq. (16) cannot be used because it is derived when only two modes are coupled. Fig. 19 shows T'_{cmax} calculated by Eq. (15) and L_c'' calculated by Eqs. (13), (14), and (35) as a function of α , where $g = 16 \mu\text{m}$. We can see the ideal Lorentzian curve for $\alpha = 2\pi/\Lambda_{\text{beat}}$. The coupling length of 67 mm is estimated around the discontinuous point. Fig. 20 shows the transmission as a function of the propagation distance when the 1st supermode is launched, where $g = 16 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 90\pi \text{ rad/m}$. The launched 1st mode is successfully converted to the 4th supermode via 2nd/3rd supermodes around $z = 67 \text{ mm}$.

From now, κ' is taken into account for the simulation, which cannot be neglected. Fig. 21 shows the comparison of propagation constants of the 3rd and 4th periodic modes, where the parameters are the same as Fig. 18(a) except for κ' . The degeneration of 3rd and 4th periodic modes is slightly broken. It means that there will be an imbalance between 1st-3rd and 2nd-3rd periodic modes. For example, in the standard directional coupler as shown in Fig. D.1, if the left waveguide can directly couple to the right waveguide, the regularity of power transition is broken. It may deteriorate the conversion efficiency from the left waveguide to the right waveguide. The same situation occurs in the 4CF. As shown in Fig. 22, the transmission as a function of the propagation distance obviously deteriorates from Fig. 20. Around $z = 67 \text{ mm}$, we can see roughly 97%

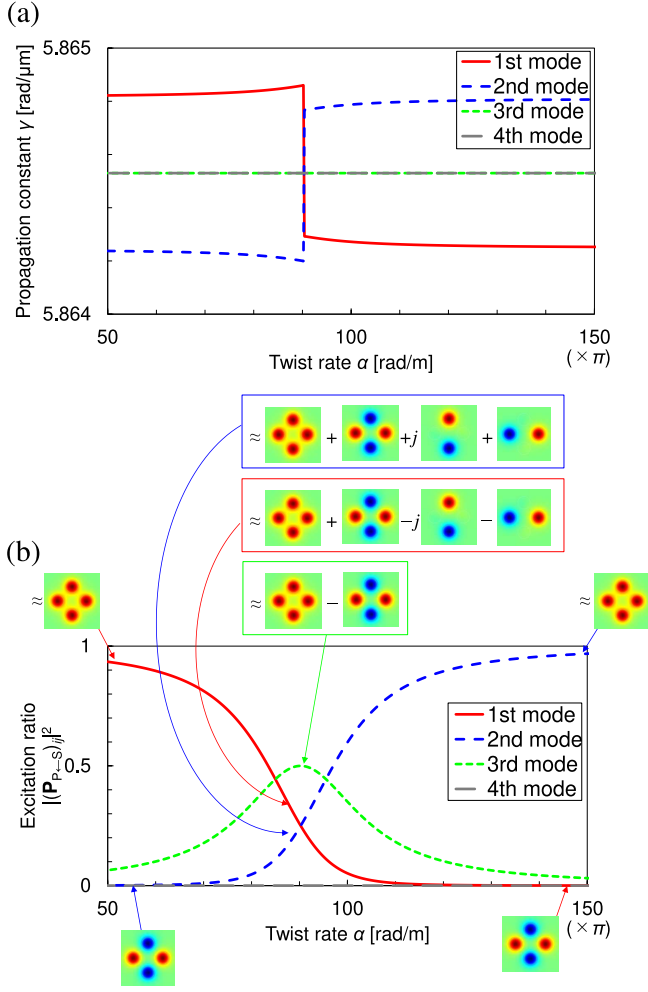


Figure 18: Eigenmodes of the 4CF (ring layout) on the condition of $\kappa' = 0$, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, and $R_b = 1000 \text{ mm}$. (a) Propagation constants and (b) squared absolute value of the basis conversion matrix as a function of the twist rate α . The 1st, 2nd, 3rd, and 4th (periodic) modes correspond to $(i, j) = (1, 1), (2, 1), (3, 1)$, and $(4, 1)$, respectively. The field distributions of the inset are the virtually visualized field of the periodic modes obtained by Eq. (3).

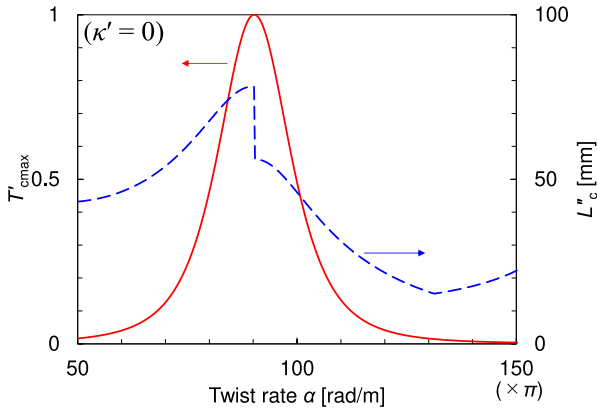


Figure 19: T'_{cmax} and L'_c of 4CF (ring layout) on the condition of $\kappa' = 0$, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, and $R_b = 1000 \text{ mm}$.

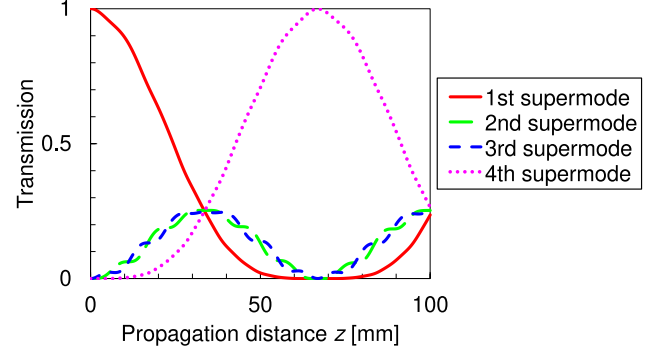


Figure 20: Mode evolution in the 4CF (ring layout) on the condition of $\kappa' = 0$ when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 90\pi \text{ rad}/\text{m}$.

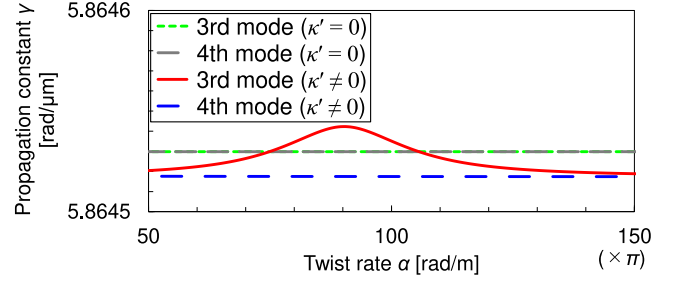


Figure 21: Comparison of propagation constants of eigenmodes of the 4CF (ring layout) for $\kappa' = 0$ and $\kappa' \neq 0$, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, and $R_b = 1000 \text{ mm}$.

of the 4th supermode and the remaining 3% of the 1st supermode. For $z < 1000 \text{ mm}$, the maximum conversion efficiency does not exceed 97%. If the coupling between opposite cores is suppressed by making a small air hole at the center, or the bending radius is appropriately adjusted, such deterioration may be avoided.

Whereas, the 1st supermode seems to couple to 4th supermode if Eq. (34) becomes non-zero by adding the structural asymmetry. For the sake of checking the effect of structural asymmetry, we examine that the spacing angle of the core position given by Eq. (28) is only changed from θ_4 to $0.95\theta_4$ (namely, 85.5°) without changing all other parameters including κ and κ' . Fig. 23 shows the T'_{cmax} for different θ_4 as a function of α , where the parameters are the same as Fig. 19 except for κ' and θ_4 . If $\theta_4 = 85.5^\circ$, T'_{cmax} reaches 100% around $\alpha = 178\pi \text{ rad}/\text{m}$. It corresponds to $2\pi/\Lambda_{beat}$, in which Λ_{beat} is the propagation constant difference of 1st-4th supermodes instead of 1st-2nd supermode. Fig. 24 shows the transmission as a function of the propagation distance when the 1st supermode is launched, where α is fixed to $177.5\pi \text{ rad}/\text{m}$ and other parameters are the same as Fig. 23. As clearly seen from Fig. 24, the 1st supermode is directly converted to the 4th supermode. Although it is not a precise simulation (only θ_4 is changed without modifying κ and κ'), the effectiveness of introducing the structural asymmetry for the mode conversion in the 4CF is demon-

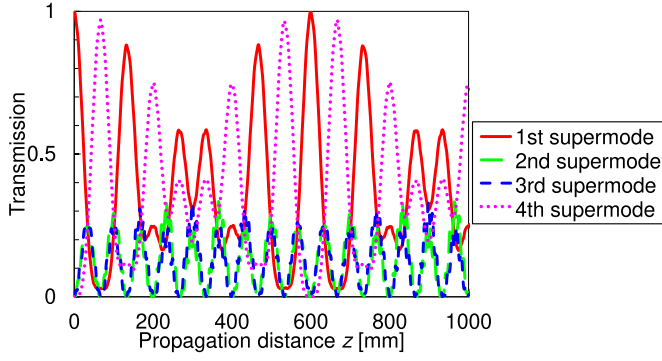


Figure 22: Mode evolution in the 4CF (ring layout) for $\kappa' \neq 0$ when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 90\pi \text{ rad/m}$.

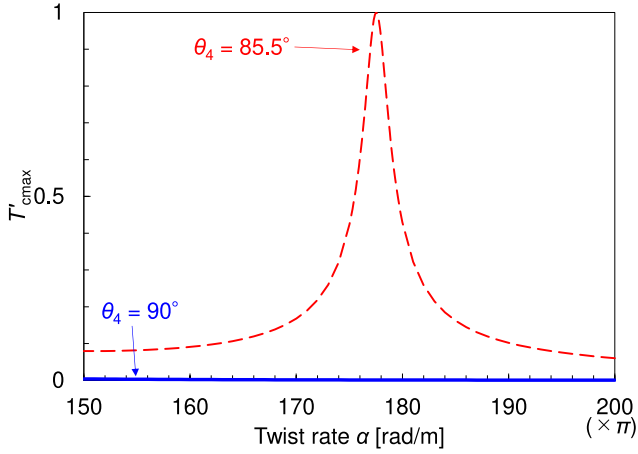


Figure 23: T'_{cmax} of 4CF (ring layout) for $\kappa' \neq 0$ for different θ_4 , where the parameters are the same as Fig. 19 except for κ' and θ_4 .

strated.

Here, we reconsider the core layout. The core layout of 4CF as in Fig. 3 (c) is categorized as a square lattice arrangement. A triangular lattice arrangement for 4CF, as shown in Fig. 25, can be defined as:

$$(x_1, y_1) = \left(\frac{g}{2}, 0\right), \quad (36)$$

$$(x_2, y_2) = \left(0, \frac{\sqrt{3}}{2}g\right), \quad (37)$$

$$(x_3, y_3) = \left(-\frac{g}{2}, 0\right), \quad (38)$$

$$(x_4, y_4) = \left(0, -\frac{\sqrt{3}}{2}g\right), \quad (39)$$

and hence, the matrix $\mathbf{M}$ used in Eq. (18) and the basis conver-

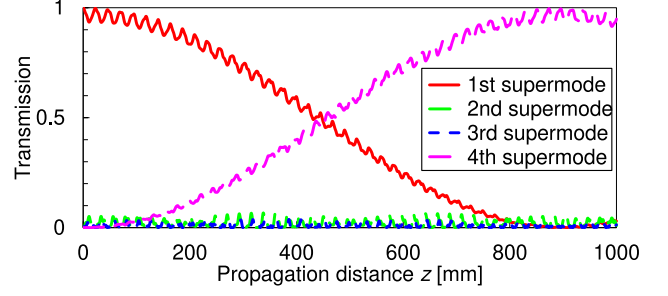


Figure 24: Mode evolution in the asymmetric ($\theta_4 = 85.5^\circ$) 4CF (ring layout) for $\kappa' \neq 0$ when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha = 90\pi \text{ rad/m}$.

sion matrix are given by

$$\mathbf{M}_{T4} = \begin{pmatrix} \beta \cdot \delta n_1 & \kappa & \kappa & \kappa \\ \kappa & \beta \cdot \delta n_2 & \kappa & \kappa' \\ \kappa & \kappa & \beta \cdot \delta n_3 & \kappa \\ \kappa & \kappa' & \kappa & \beta \cdot \delta n_4 \end{pmatrix}, \quad (40)$$

and

$$\mathbf{P}_{T4, C \leftarrow D} = \begin{pmatrix} \frac{1+\sqrt{17}}{2} & 2 & \frac{1+\sqrt{17}}{2} & 2 \\ \frac{1}{\sqrt{17+\sqrt{17}}} & \frac{1}{\sqrt{17+\sqrt{17}}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1-\sqrt{17}}{2} & 2 & \frac{1-\sqrt{17}}{2} & 2 \\ \frac{1}{\sqrt{17-\sqrt{17}}} & \frac{1}{\sqrt{17-\sqrt{17}}} & \frac{1}{\sqrt{17-\sqrt{17}}} & \frac{1}{\sqrt{17-\sqrt{17}}} \end{pmatrix}, \quad (41)$$

where κ' should be calculated as in Appendix C, and Eq. (41) is normalized and orthogonalized. In this case, the propagation constants of four supermodes are different from each other as follows:

$$(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = \left(\beta + \frac{1+\sqrt{17}}{2}\kappa, \beta, \beta - \kappa, \beta + \frac{1-\sqrt{17}}{2}\kappa \right), \quad (42)$$

which is obtained for $\alpha = R_b^{-1} = \kappa' = 0$. From Eqs. (34), (41), and (42), 1st and 4th supermodes can couple to either of 2nd or 3rd mode, and these conversions do not occur simultaneously. In Figs. 26 (a), (b), (c), and (d), the transmission as a function of the propagation distance in the triangular-lattice 4CF for various α are shown, where $g = 16 \mu\text{m}$. Figs. 26 (a) and (b) show the mode evolution when $\alpha = 113\pi$ and $159\pi \text{ rad/m}$ corresponding to the beat length of 1st-2nd and 1st-3rd supermodes, respectively. Almost ideal conversion between two modes are observed. Figs. 26 (c) and (d) show the mode evolution when $\alpha = 73\pi$ and $27\pi \text{ rad/m}$ corresponding to the beat length of 4th-2nd and 4th-3rd supermodes, respectively. Similarly, the conversion between two modes can be seen. By combining these mode converters, mode conversions for any mode are possible.

3.4. Effect of structural errors on mode conversion characteristics

The fabrication accuracy's effect on the performance is described here. We consider the situation that one of the cores in

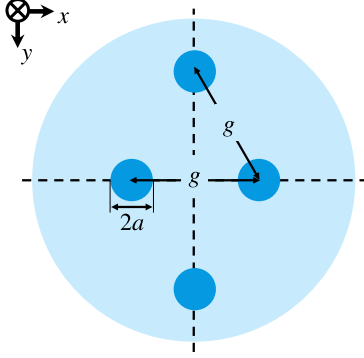


Figure 25: Cross section of bent and twisted fiber of 4CF with the triangular lattice layout.

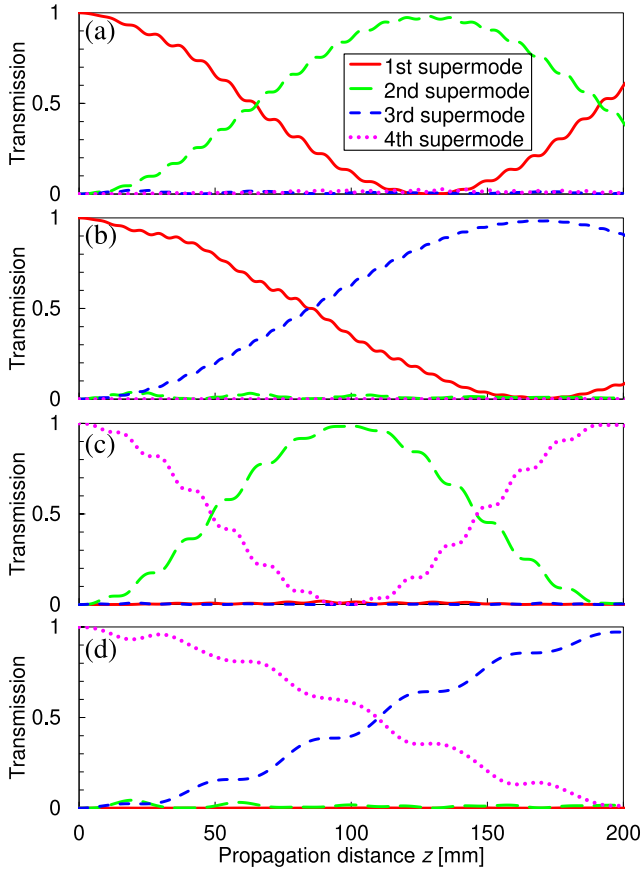


Figure 26: Mode evolution in the 4CF (triangular lattice layout) when launching 1st or 4th supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, $R_b = 1000 \text{ mm}$, and $\alpha =$ (a) 113π , (b) 159π , (c) 73π , and (d) $27\pi \text{ rad/m}$.

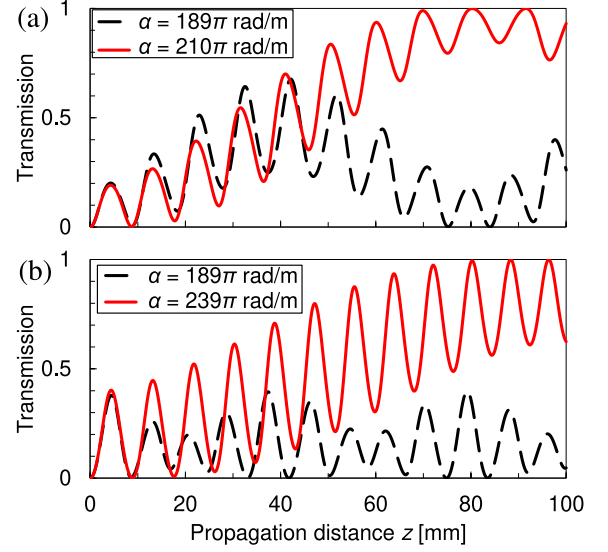


Figure 27: 2nd supermode evolution in the 2CF with fabrication errors when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 14 \mu\text{m}$, and $R_b = 1000 \text{ mm}$. (a) The radius of one of the cores was increased by $0.1 \mu\text{m}$. (b) Δ of one of the cores was increased by 0.01% .

the 2CF (as in Fig. 12) has some fabrication error. Fig. 27(a) shows the 2nd supermode evolution in the 2CF with the structural error of the radius of $0.1 \mu\text{m}$. As seen from the black dashed line in Fig. 27(a), the maximum conversion is degraded to 70%. However, if the twist rate α is adjusted from $189\pi \text{ rad/m}$ to $210\pi \text{ rad/m}$, the maximum conversion is improved to 100%, where $210\pi \text{ rad/m}$ is almost the same as the beat length calculated by Eq. (4) for 2CF with core radius variance. In such asymmetric 2CF, there is an imbalance of the power confinement in each core of supermodes, leading to the large fluctuation of the mode conversion ratio along the z direction. Fig. 27(b) shows the mode evolution for the structural error of Δ of 0.01% . Similarly to Fig. 27(b), if the twist rate α is adjusted from $189\pi \text{ rad/m}$ to $239\pi \text{ rad/m}$, the maximum conversion is improved from 40% to 100%.

4. GDS Characteristics

The GDS suppression using the mode converter based on the bent and twisted CMCF is investigated. As shown in Fig. 28, we assume that the mode converter is used as an optical relay for GDS suppression. The GDS is defined as the standard deviation of the group delays [18], which is given by the eigenmode of a group delay operator (GDO) [19]:

$$\mathbf{F} \equiv j\mathbf{T}^{-1} \frac{d\mathbf{T}}{d\omega}, \quad (43)$$

where $\mathbf{T}$ is the transfer matrix of the system, and ω is the angular frequency. Eigenvalues of Eq. (43), τ_i , indicate the group delay of the i th principal mode. Thus, the GDS is calculated by

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (\tau_i - \bar{\tau})^2}, \quad (44)$$

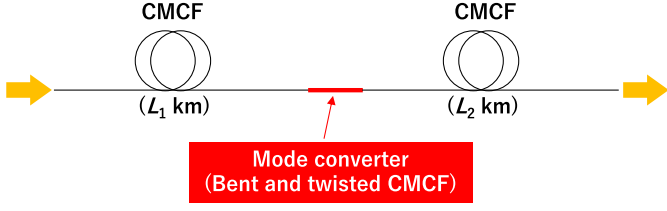


Figure 28: Schematics of the transmission system using two straight CMCFs and the mode converter (bent and twisted CMCF) to suppress the GDS.

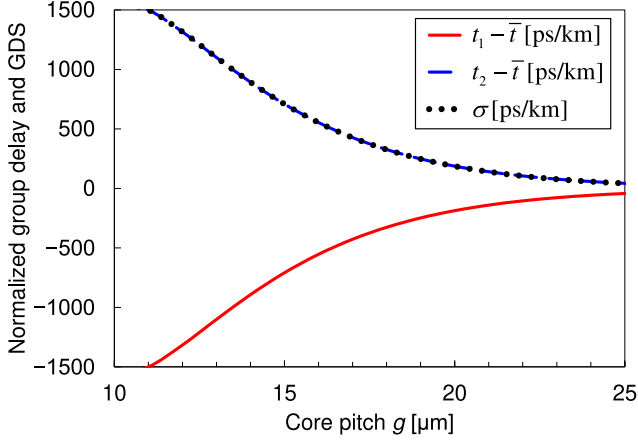


Figure 29: Normalized group delay and GDS for the straight 2CF.

where $\bar{\tau}$ is the mean value of τ_i . In the configuration of Fig. 28, $\mathbf{T}$ can be expressed as $\mathbf{T}_2 \mathbf{T}_C \mathbf{T}_1$, where $\mathbf{T}_1$ corresponds to L_1 -km fiber and $\mathbf{T}_C$ is given by Eq. (29). $\mathbf{T}_1$ is the diagonal matrix. The mode converter is much shorter than the L_1 -km transmission line, namely $d\mathbf{T}_C/d\omega \ll d\mathbf{T}_1/d\omega$. Assuming that $\mathbf{T}_1$ and $\mathbf{T}_C$ are unitary matrices, Eq. (43) can be simply rewritten as

$$\tilde{\mathbf{F}} = \frac{L_2}{L_1} \mathbf{T}_C^{-1} \text{diag}[t_i] \mathbf{T}_C + \text{diag}[t_i], \quad (45)$$

$$\text{diag}[t_i] = j\mathbf{T}_1^{-1} \frac{d\mathbf{T}_1}{d\omega} = \frac{L_1}{L_2} j\mathbf{T}_2^{-1} \frac{d\mathbf{T}_2}{d\omega}, \quad (46)$$

where the eigenvalues agree with Eq. (43), but $\mathbf{F} \neq \tilde{\mathbf{F}}$ (the eigenvectors of Eq. (43) are not kept). t_i denotes the group delay of the i th supermode in the straight CMCF for the L_1 -km propagation. From Eqs. (45) and (46), if L_2/L_1 is fixed, we can understand that the GDS of the system in Fig. 28 linearly increases with total length $L = L_1 + L_2$. In practice, the number of segments of the transmission line, as well as the lengths of L_1 and L_2 , are unknown. According to the given transmission lines, by investigating the relation of Eq. (43) and mode conversion characteristics, and adapting an optimization scheme, we can obtain the best GDS characteristics. Hereafter, we demonstrate that even one mode converter is effective to reduce the GDS.

Fig. 29 shows the normalized group delay ($t_i - \bar{t}$) and GDS (σ) for the straight 2CF. When $n = 2$, the GDS coincides with the

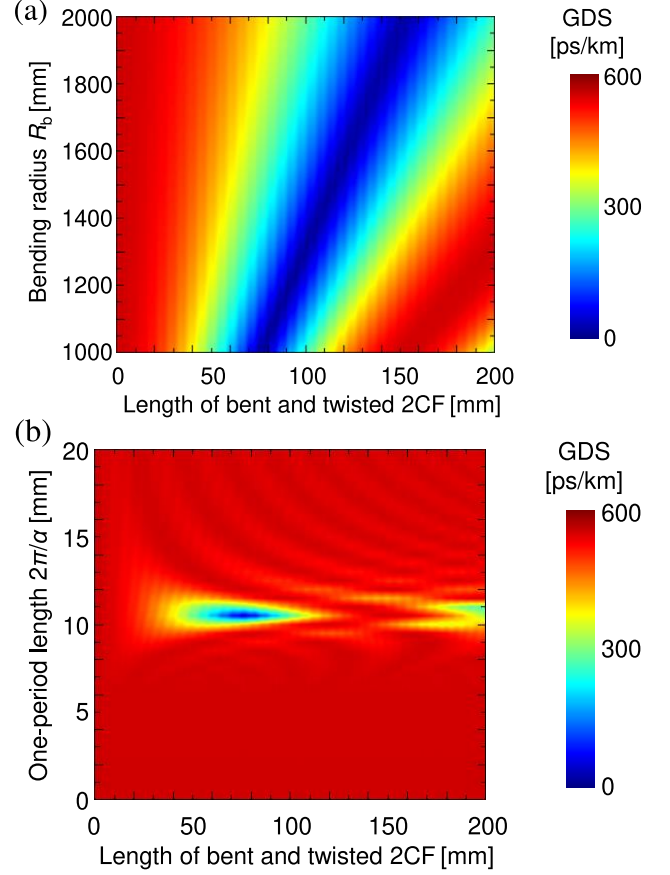


Figure 30: Structural dependence of the GDS characteristics of the 2CF for various structural parameters, where $g = 14 \mu\text{m}$ and $L_1 = L_2$. (a) Dependency on the bending radius R_b and the length of the mode converter for $\alpha = 189\pi$ rad/m. (b) Dependency on the one-period length Δ and the length of the mode converter for $R_b = 1000$ mm.

normalized group delay, in other words, the GDS agrees with half of the DMD. Ideally, the DMD is completely compensated if two modes are switched in the relay point. Fig. 30(a) shows the GDS as a function of the length of the bent and twisted 2CF and the bending radius R_b , where $g = 14 \mu\text{m}$ and $\alpha = 189\pi$ rad/m. The GDS becomes almost 0 for any R_b by setting the length to the coupling length ($L_c'' = 76.5$ mm when $R_b = 1000$ mm). Fig. 30(b) shows the GDS as a function of the length of 2CF and the one-period length $\Lambda (= 2\pi/\alpha)$, where $g = 14 \mu\text{m}$ and $R_b = 1000$ mm. The GDS is well suppressed by setting Λ to the beat length ($\Lambda_{\text{beat}} = 10.6$ mm), where the GDS is less than 100 ps/km for the coupling length.

The 4CF with the ring layout (equivalent to the square lattice layout) is also investigated. Fig. 31 shows the normalized group delay and GDS for the straight 4CF. We can expect that, if 1st and 4th supermodes are switched, the group delays are averaged, and the GDS is greatly reduced. Fig. 32(a) shows the GDS as a function of the length of the bent and twisted 4CF and the bending radius R_b , where $g = 16 \mu\text{m}$, $\alpha = 90\pi$ rad/m, and κ' is nonzero. Although the fluctuations are observed as shown in Fig. 22, even if R_b is larger than 1000 mm, the GDS

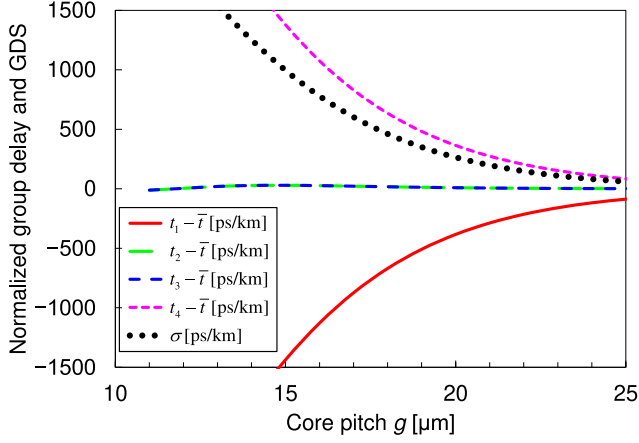


Figure 31: Normalized group delay and GDS for the straight 4CF (ring layout), where κ' is nonzero.

can be reduced by properly selecting R_b and the length of the twisted fiber. Fig. 32(b) shows the GDS as a function of the length of 4CF and one-period length Λ , where $g = 16 \mu\text{m}$ and $R_b = 2300 \text{ mm}$. The GDS can be suppressed by setting Λ to the beat length ($\Lambda_{\text{beat}} = 22.3 \text{ mm}$), where the GDS is less than 100 ps/km for the length of 600 mm . Although only certain parameters are demonstrated here, similar characteristics can be obtained with different values of g , R_b , and α as long as the appropriately designed parameters are used. Note that, the larger value of R_b leads to larger T_{cmax} , whereas the dependence on α becomes more sensitive.

As described in the introduction, the GDS reduction by the mode converter is effective for small core pitch because the CMCF with a larger core pitch can successfully suppress the GDS by the strong random coupling state. Let us compare the GDS characteristics in 2CFs with and without strong random coupling. Fig. 33 shows the GDS characteristics in 2CFs, where $g = 14$ or $20 \mu\text{m}$ and $R_b = 80$ or 1000 mm . The simulation method with random perturbation is the same as [8], where the parameters are as follows: twist rate is $\pi \text{ rad/m}$, the twist rate of standard deviation is $0.1\pi \text{ rad/m}$, the step size is 1 mm , and 30 realizations are averaged. In the green long-dashed line (i) and blue dashed line (ii), the mode converter is inserted at 5 km ($L_1 = L_2 = 5 \text{ km}$) and $10/3 \text{ km}$ ($L_1 = 10/3 \text{ km}$, $L_2 = 20/3 \text{ km}$), respectively, where the parameter of the mode converter is the same as in Fig. 12 and its length is 78 mm . As seen in Fig. 33, for $g = 20 \mu\text{m}$ and $R_b = 80 \text{ mm}$, the strong random coupling appears and the GDS is less than 100 ps at 10 km . If the bending radius becomes larger to $R_b = 1000 \text{ mm}$, the GDS linearly increases along with the propagation distance, which means that the strong coupling state is no longer seen due to the weak bending condition. For the small core pitch $g = 14 \mu\text{m}$, the GDS linearly increases even in the strong bending condition of $R_b = 80 \text{ mm}$. On the other hand, for the 2CF with the mode converter where $g = 14 \mu\text{m}$ and $R_b = 1000 \text{ mm}$, the GDS is drastically reduced due to the GDS compensation by the mode conversion. When the mode converter is inserted at $10/3 \text{ km}$,

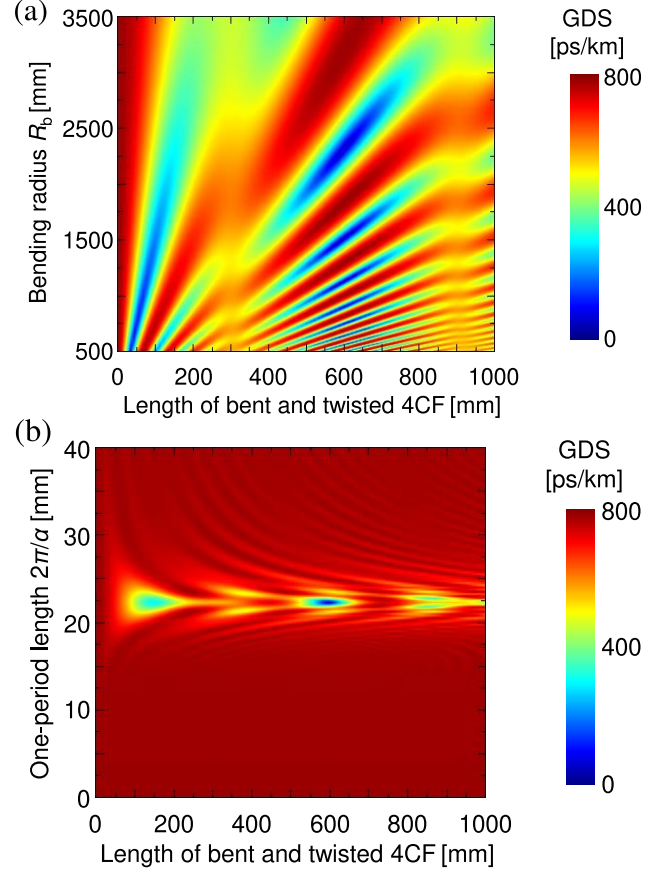


Figure 32: Structural dependence of the GDS characteristics of the 4CF (ring layout) for various structural parameters, where $g = 16 \mu\text{m}$, $L_1 = L_2$, and κ is nonzero. (a) Dependency on the bending radius R_b and the length of the mode converter for $\alpha = 90\pi \text{ rad/m}$. (b) Dependency on the one-period length Λ and the length of the mode converter for $R_b = 2300 \text{ mm}$.

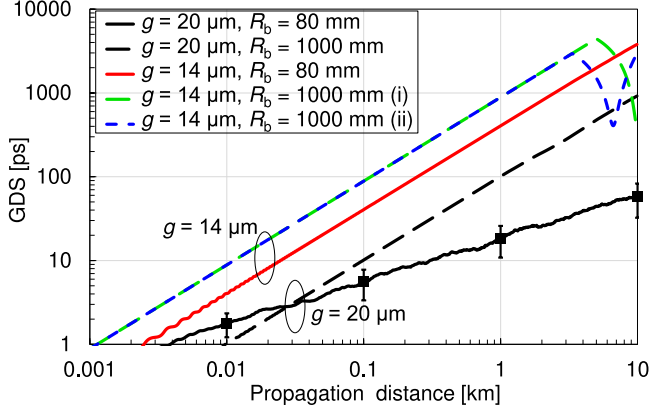


Figure 33: GDS characteristics in 2CFs, where $g = 14$ or $20 \mu\text{m}$ and $R_b = 80$ or 1000 mm . In the green long-dashed line (i) and blue dashed line (ii), the mode converter is inserted at 5 km and $10/3 \text{ km}$, respectively. The random perturbation is given as a twist rate is $\pi \text{ rad/m}$, the twist rate of standard deviation is $0.1\pi \text{ rad/m}$, the step size is 1 mm , and 30 realizations are averaged. For $g = 20 \mu\text{m}$ and $R_b = 80 \text{ mm}$, the standard deviation is displayed.

the GDS is compensated at $20/3 \text{ km}$. After this, the GDS increases again. The appearance of the floor around GDS of 300 ps arises from the imperfection of mode conversion and the difference between former and latter transmission lines due to the random perturbation. It can be concluded that the GDS reduction by the mode converter is effective especially for dense core layouts although it does not improve up to the characteristics of the strong coupling state.

Finally, we briefly describe about the loss in the proposed system. The bending losses of CMCFs as shown in Figs. 30 and 32 are less than 0.001 dB/km (it does not include the material loss) because the bending radius is enough large. Although the loss due to twisting depends on the fabrication method and experimental setup, the propagation distance is very short, and hence the connection loss is probably the main factor of loss.

5. Conclusion

The CMCF-based mode converter for the reduction of the group delay spread is proposed. The bent and twisted fibers are worked as the LPG-type mode converter. The characteristics of the mode conversion are investigated by the eigenmode analysis based on TMM. The specified mode conversion can be done by applying the asymmetric refractive index modulation. We would like to emphasize that the complete mode conversion arises even in the small core pitch condition of $g < 17 \mu\text{m}$ and the weak bending condition of $R_b \geq 1000 \text{ mm}$. To make sure to reduce the GDS, it is effective that the higher mode is switched to the lower mode. It is revealed that the GDSs of 2CF and 4CF are well suppressed by deploying the mode converter in the relay point. If the core layout is properly designed, the mode conversion chain is controlled to some extent, which enables the GDS suppression of the many-core CMCF.

Appendix

A. Calculation of propagation constants for periodic modes

The transfer matrix of the one-period waveguide is given by Eq. (1). As a function of $z (\approx \Lambda)$, Eq. (1) is rewritten as

$$\mathbf{T}(z) = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}, \quad (\text{A.1})$$

$$\mathbf{D} = \text{diag}[\exp(-j\gamma_i z)], \quad (\text{A.2})$$

where $\text{diag}[f(i)]$ denotes the diagonal matrix, in which the (i, i) element has $f(i)$, and off-diagonal components of $\text{diag}[f(i)]$ are zero. If the propagation length is added by Δz , Eq. (A.1) can be rewritten as

$$\mathbf{T}(z + \Delta z) \approx \mathbf{P}\text{diag}[\exp(-j\gamma_i \Delta z)]\mathbf{D}\mathbf{P}^{-1}, \quad (\text{A.3})$$

where we assume that $(\partial/\partial z)\mathbf{P}$ is sufficiently smaller than $(\partial/\partial z)\mathbf{D}$. By calculating the eigenvalue of Eq. (A.3) divided by the eigenvalue of Eq. (A.1), we can obtain $\text{diag}[\exp(-j\gamma_i \Delta z)]$. And thus, the estimated propagation constant is obtained by

$$\tilde{\gamma}_i = \frac{1}{-j\Delta z} \arg[(\mathbf{P}^{-1}\mathbf{T}(\Lambda + \Delta z)\mathbf{T}(\Lambda)\mathbf{P})_{ii}]. \quad (\text{A.4})$$

But Eq. (A.4) includes a non-negligible error by ignoring $(\partial/\partial z)\mathbf{P}$ in Eq. (A.3). If the error e_i is expressed as

$$\gamma_i = \tilde{\gamma}_i + e_i, \quad (\text{A.5})$$

the error e_i can be calculated by

$$e_i = \frac{1}{-j\Lambda} \arg[(\mathbf{D}\text{diag}[\exp(+j\tilde{\gamma}_i \Lambda)])_{ii}]. \quad (\text{A.6})$$

In this way, the propagation constant is almost precisely obtained.

B. Numerical example by eigenmode analysis for periodic waveguide

To confirm the validity of the proposed method described in Section 2.1, the standard LPG-based mode converter with silica PLC [20] is analyzed.

Fig. B.1 shows a schematic of the PLC-based LPG. The core and cladding materials are Ge-doped and pure silica, where $\Delta = 1.1\%$. A rectangular waveguide with the width of $w = 11 \mu\text{m}$ and the height of $h = 10 \mu\text{m}$ has periodic grooves, where one groove length is $\Lambda/2$, and the depth of the groove is $d = 0.5 \mu\text{m}$. Here, we call two straight waveguides with the width of w and $w - d$ as the WA and WB, respectively. According to the TMM, the transfer matrix for one period, $\mathbf{T}_{1P}$, can be expressed as:

$$\mathbf{T}_{1P} = \mathbf{C}_{A \leftarrow B}\mathbf{T}_B\mathbf{C}_{B \leftarrow A}\mathbf{T}_A, \quad (\text{B.1})$$

$$\mathbf{T}_{A,B} = \text{diag}\left[\exp\left(-j\beta_i^{(A,B)}\frac{\Lambda}{2}\right)\right], \quad (\text{B.2})$$

$$(\mathbf{C}_{B \leftarrow A})_{ij} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[\begin{array}{c} \mathbf{E}_i^{(A)}(\mathbf{r}) \times \mathbf{H}_j^{\dagger(B)}(\mathbf{r}) \\ + \mathbf{E}_j^{\dagger(B)}(\mathbf{r}) \times \mathbf{H}_i^{(A)}(\mathbf{r}) \end{array} \right] \cdot \mathbf{i}_z dx dy, \quad (\text{B.3})$$

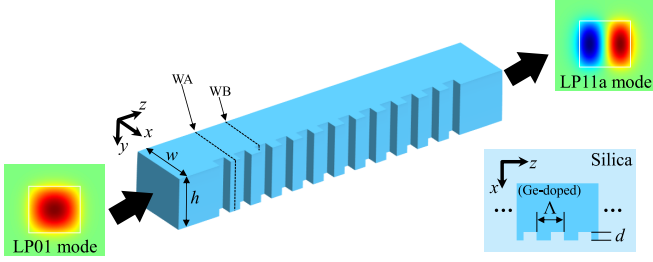


Figure B.1: Schematic of the PLC-based LPG where the launched LP01 mode from the left side converts to LP11a mode by the appropriate structural parameters.

$$\mathbf{C}_{A \leftarrow B} = \mathbf{C}_{B \leftarrow A}^\dagger, \quad (\text{B.4})$$

where $\mathbf{T}_A$ is the transfer matrix for the WA. $\mathbf{C}_{B \leftarrow A}$ is the mode coupling matrix for the butt-joint coupling from the WA to the WB. $\mathbf{E}_i^{(A)}$, $\mathbf{H}_i^{(A)}$, and $\beta_i^{(A)}$ are the electric field vector, the magnetic field vector, and the propagation constant for the i th 2D eigenmode in the WA. $\mathbf{r}$, $\mathbf{i}_z$, and $\dagger$ denote the position vector, the unit vector for z -direction, and the complex conjugate transpose. Vice versa for the WB.

Figs. B.2(a) and (b) show the propagation constant and the squared absolute value of the basis conversion matrix when changing Λ , where a waveguide width modulation is defined as $k = 2\pi/\Lambda$. For simplifying the calculation, we show the mode conversion only between LP01 and LP11 modes. We can see the cross point around $\Lambda = 438 \mu\text{m}$ in Fig. B.2(b). At this point, we can find the basis conversion of

$$\mathbf{C}_{P \leftarrow S} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}, \quad (\text{B.5})$$

which means that the electric field distributions of periodic modes form even- and odd-like modes as shown in the inset in Fig. B.2(b). If the LPG length, namely the number of periods, N , is chosen as two modes have the phase difference of π , the transfer matrix can be expressed as

$$\begin{aligned} (\mathbf{T}_{1P})^N &= \frac{1}{2} \begin{pmatrix} 1 & +j \\ +j & 1 \end{pmatrix} \begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix} \begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned} \quad (\text{B.6})$$

It makes the mode conversion between LP01 and LP11a modes. When calculating $\Delta\beta$ for the WA and the WB, we can obtain $\Lambda_{\text{beat}} = 454$ and $422 \mu\text{m}$, respectively. The average value of beat lengths becomes $438 \mu\text{m}$, which agrees well with Λ at the cross point in Fig. B.2(b). Note that, although we can also see the discontinuous point around $\Lambda = 438 \mu\text{m}$ in Fig. B.2(a), this discontinuity is not important. It is shown as one example, and we can shift the discontinuous point according to the calculation of γ as above explained (γ may be $\gamma + 2n\pi/\Lambda$ where n is an integer).

Fig. B.3 shows the mode evolution for launching LP01 mode when $\Lambda = 440 \mu\text{m}$. In this simulation, in addition to TMM analysis by Eqs. (2) and (B.1), the beam propagation method

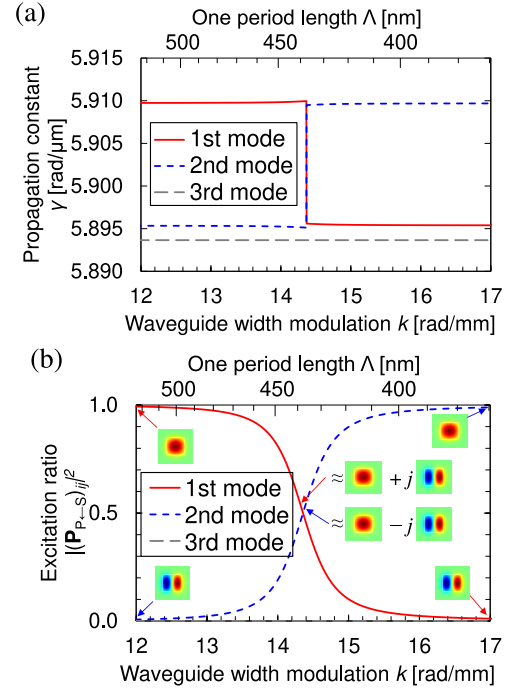


Figure B.2: Eigenmodes of the PLC-based LPG where $\Delta = 1.1\%$, $w = 11 \mu\text{m}$, $h = 10 \mu\text{m}$, and $d = 0.5 \mu\text{m}$. (a) Propagation constants and (b) squared absolute value of the basis conversion matrix as a function of waveguide width modulation k . The 1st, 2nd, and 3rd (periodic) modes correspond to $(i, j) = (1, 1), (2, 1),$ and $(3, 1)$, respectively. The field distributions of the inset are the virtually visualized fields of the periodic modes obtained by Eq. (3).

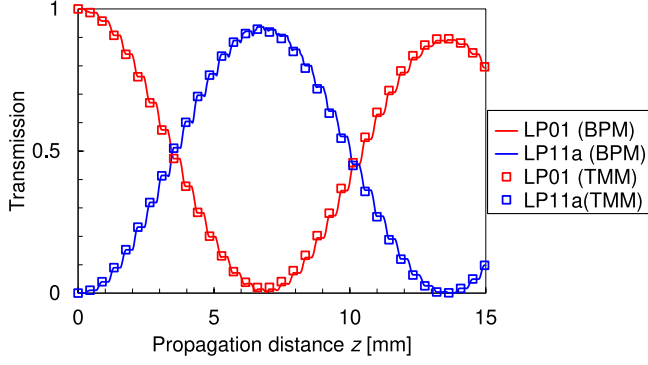


Figure B.3: Mode evolution when launching LP01 mode, where $\Delta = 1.1\%$, $w = 11 \mu\text{m}$, $h = 10 \mu\text{m}$, $d = 0.5 \mu\text{m}$, and $\Lambda = 440 \mu\text{m}$.

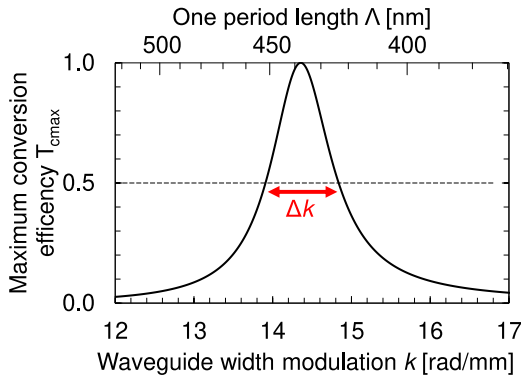


Figure B.4: Maximum conversion efficiency corresponding to Fig. B.2(b).

(BPM [21]) is also performed to confirm the validity. These plots agree well, where we can see the coupling losses at the discontinuous points ($z = n\Lambda/2$) in both simulations. We can see that the LP01 mode is completely converted to the LP11 mode when z is from 6.5 to 7.0 mm. However, L_c defined by Eq. (9) becomes $213 \mu\text{m}$ ($\Delta\gamma = 0.015 \text{ rad}/\mu\text{m}$) at $\Lambda = 440 \mu\text{m}$. This value seems not to correct because L_c is lower than Λ . By using Eqs. (11) and (12), we can find $p_1 = 16$, $p_2 = 15$, and $L'_c = 7040 \mu\text{m}$ for $\Lambda = 440 \mu\text{m}$. Also, by using Eqs. (13) and (14), we can find $p_3 = -1$ and $L''_c = 6951 \mu\text{m}$ for $\Lambda = 440 \mu\text{m}$. These values are reasonable.

When $|\Lambda - \Lambda_{\text{beat}}| \gg 0$, even and odd modes are deformed. Fig. B.4 shows T_{cmax} calculated by Fig. B.2(b) and Eq.(15), in which the FWHM is $\Delta k = 9.29 \times 10^{-4} \text{ rad}/\mu\text{m}$. By using Eq. (16), we can obtain the coupling length as $L'''_c = 6763 \mu\text{m}$. It also gives a reasonable value.

C. Calculation of coupling coefficients

According to the standard coupled-mode theory, the coupling coefficient between i th and j th modes ($i \neq j$) is defined as [22]

$$\kappa_{ij} = \frac{k_0}{4Z_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\varepsilon - \varepsilon_j) \mathbf{E}_i^\dagger \cdot \mathbf{E}_j dx dy, \quad (\text{C.1})$$

where k_0 is the free-space wavenumber, Z_0 is the free-space impedance, ε is the relative permittivity of the whole structure,

and ε_j is the relative permittivity in which there is only core for j th mode. Whereas, we can also obtain the coupling coefficient as follows. If $R_b^{-1} = 0$, the propagation constants of periodic modes are obtained by the standard guided-mode analysis. According to the relations from Eqs. (18) to (24), we found the following relation:

$$\kappa_{ij} = (\mathbf{C}_{D \leftarrow C} \text{diag} [\gamma_i] \mathbf{C}_{C \leftarrow D})_{ij}, \quad (\text{C.2})$$

where $(\mathbf{C}_{D \leftarrow C})$ is the mode coupling matrix from discrete core modes to supermodes, which is defined as similar to Eq. (B.3). This is the same procedure in [23] except for the use of a pseudo unitary conversion. For 2CF and 3CF, the difference between Eqs. (C.1) and (C.2) is negligible. However, especially in κ' and κ'' (κ'' is defined in Appendix E) of triangular and square lattice layout, it becomes a non-negligible value.

Here, one example of the 4CF (ring layout; equivalent to square lattice layout) is shown, where g is fixed to $16 \mu\text{m}$. By using Eq. (C.1), κ and κ' are calculated as $1.41 \times 10^{-4} \text{ rad}/\mu\text{m}$ and $1.25 \times 10^{-5} \text{ rad}/\mu\text{m}$, respectively. When using Eq. (C.2), κ can be calculated by averaging $\kappa_{12}, \kappa_{14}, \kappa_{21}, \kappa_{23}, \kappa_{32}, \kappa_{34}, \kappa_{14}$, and κ_{34} , resulting in $1.41 \times 10^{-4} \text{ rad}/\mu\text{m}$, which agrees well with the result of Eq. (C.1). Since the standard deviation of these eight κ_{ij} is less than $10^{-11} \text{ rad}/\mu\text{m}$, the procedure of averaging is reasonable. However, by using Eq. (C.2) and by averaging $\kappa_{13}, \kappa_{24}, \kappa_{31}$, and κ_{42} , $\kappa' = 5.25 \times 10^{-6} \text{ rad}/\mu\text{m}$ is obtained, which is about half the result of Eq. (C.1). The standard deviation of these four κ_{ij} is less than $10^{-13} \text{ rad}/\mu\text{m}$. In the condition without twisting and bending, the expression of supermodes is more plausible than discrete core modes. According to the consistency between the discrete core modes and the supermodes, we conclude that the use of κ' and κ'' from Eq. (C.2) is more reasonable. Therefore, in this paper, Eq. (C.2) is used for κ' and κ'' instead of Eq. (C.1).

D. Behavior of mode conversion in 4CF

Eigenmodes of the bent and twisted 4CF (ring layout) is similar with the standard directional coupler having three waveguides with the in-line arrangement as shown in Fig. D.1. In the directional coupler, the lightwave launched from the left waveguide can couple to the right waveguide via the center waveguide. Similarly, in the bent and twisted 4CF, the 1st supermode can couple to the 4th supermode via the 2nd/3rd supermode. But more accurately, as shown in the field distribution in Fig. 18(b), the 2nd/3rd supermode indicates (2nd supermode) + j (3rd supermode). To make it easier to understand, it is better to take a basis conversion from the degenerated 2nd and 3rd supermodes to the $((2\text{nd supermode}) \pm j(3\text{rd supermode}))/\sqrt{2}$. In these basis, the amplitudes in the cores are changed with clockwise and anti-clockwise in a complex plane, and hence, here we call them clockwise and anti-clockwise modes. We can understand that the 4th periodic mode in Fig. 18 corresponds to the anti-clockwise mode, and that the clockwise refractive index modulation does not affect the propagation constant of the anti-clockwise mode.

In the directional coupler as shown in Fig. D.1, the matrix $\mathbf{M}$ used in Eq. (18) can be defined as

$$\mathbf{M}_{\text{DC}} = \begin{pmatrix} \beta & \kappa & 0 \\ \kappa & \beta & \kappa \\ 0 & \kappa & \beta \end{pmatrix} = \mathbf{P}_{\text{DC}} \mathbf{D}_{\text{DC}} \mathbf{P}_{\text{DC}}^{-1}, \quad (\text{D.1})$$

$$\mathbf{D}_{\text{DC}} = \begin{pmatrix} \beta + \sqrt{2}\kappa & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta - \sqrt{2}\kappa \end{pmatrix}, \quad (\text{D.2})$$

$$\mathbf{P}_{\text{DC}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix} = \mathbf{P}_{\text{DC}}^{-1}, \quad (\text{D.3})$$

where β and κ indicate the propagation constant of three waveguides and the coupling coefficient between neighboring waveguides, respectively. Eqs. (D.2) and (D.3) correspond to the illustration of the field distributions in Fig. D.1. When the light-wave is launched from the left waveguide, 1st, 2nd, and 3rd supermodes are excited with the amplitude ratio of 1: $\sqrt{2}$:1 (power ratio of 1:2:1). When the phase difference between 1st and 2nd supermodes becomes π , the phase difference between 2nd and 3rd supermodes also becomes π . At this time, the amplitude of 1st, 2nd, and 3rd supermodes becomes 1: $-\sqrt{2}$:1, which leads to the complete transfer from the left waveguide to the right waveguide. Whereas, 1st, 2nd, and 3rd supermodes in Fig. D.1 correspond to 1st, 3rd, and 2nd periodic modes in Fig. 18, respectively. In the situation of the 4CF, 1st and 4th supermodes excites 1st, 2nd, and 3rd periodic modes with the amplitude ratio of 1:1: $\pm\sqrt{2}$ (power ratio of 1:1:2). As explained above, the 3rd periodic mode is equivalent to ((2nd supermode) + j (3rd supermodes))/ $\sqrt{2}$, resulting in the field distribution of (1st supermode) + (4th supermode) $\pm j$ (3rd supermodes) $\pm$ (2nd supermode) for 1st and 2nd periodic modes in the vicinity of $\alpha = 90\pi$. Similar to the case of directional coupler, when the phase difference between 1st and 3rd periodic modes becomes π , the 1st supermode will completely convert to the 4th supermode.

From Eq. (D.2), the propagation constant of 2nd supermode in Fig. D.1 does not change in Fig. D.1 regardless of the coupling strength. This is the reason why the propagation constant of the 3rd periodic mode in Fig. 18(a) is constant for any α and it agrees with that of 4th periodic mode.

E. Larger number of cores

Here, the conversion characteristics for CMCF having larger number of cores are summarized.

E.1. Ring layout

The matrix $\mathbf{M}$ used in Eq. (18) for the ring layout is approximately given as

$$(\mathbf{M}_{\text{Rn}})_{ij} = \begin{cases} \beta \cdot \delta n_i & (i = j) \\ \kappa & (|i - j| = 1) \\ \kappa & (|i - j| = n - 1) \\ 0 & \text{otherwise} \end{cases}. \quad (\text{E.1})$$

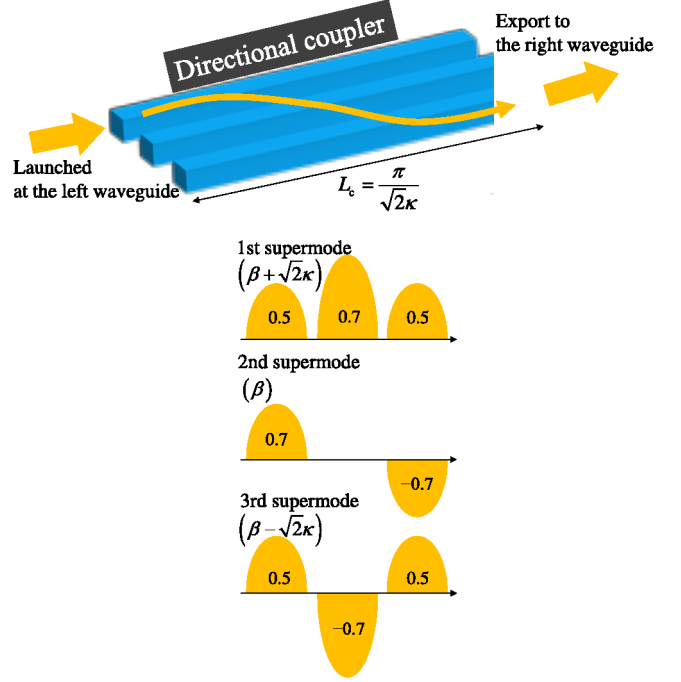


Figure D.1: Schematics of the standard directional coupler having three waveguides with the in-line arrangement and three supermodes in the coupler.

By solving the eigenvector of Eq. (E.1) for $R_b^{-1} = 0$, we can find the circumferential order of the field distribution for the i th supermode is given as

$$m_i = \left\lfloor \frac{i}{2} \right\rfloor, \quad (\text{E.2})$$

where $\lfloor \cdot \rfloor$ denotes the floor function. From Eqs. (34) and (E.2), the condition to enable the mode conversion between the i th and j th supermodes is given as

$$|m_i - m_j| = 1. \quad (\text{E.3})$$

For example, the circumferential order for 1st and 4th supermodes are $m_1 = 0$ and $m_4 = 2$, respectively, which means that 1st and 4th supermodes cannot be directly converted for any n as long as the ring core layout with equally core-to-core distance is used. The propagation constants of the supermodes for various n are summarized in Table E.1. The chained conversion as for 4CF seems not to occur when $n \neq 4$.

E.2. Triangular lattice layout

Here, we consider 3CF, 4CF, and 7CF. The matrix $\mathbf{M}$ used in Eq. (18) for the triangular lattice layout when $n = 7$, as shown in Fig. E.1(a), can be defined as

$$\mathbf{M}_{\text{T7}} = \begin{pmatrix} \cdot & \kappa & \kappa & \kappa & \kappa & \kappa & \kappa \\ \kappa & \cdot & \kappa & \kappa' & \kappa'' & \kappa' & \kappa \\ \kappa & \kappa & \cdot & \kappa & \kappa' & \kappa'' & \kappa' \\ \kappa & \kappa' & \kappa & \cdot & \kappa & \kappa' & \kappa'' \\ \kappa & \kappa'' & \kappa' & \kappa & \cdot & \kappa & \kappa' \\ \kappa & \kappa' & \kappa'' & \kappa' & \kappa & \cdot & \kappa \\ \kappa & \kappa & \kappa' & \kappa'' & \kappa' & \kappa & \cdot \end{pmatrix}, \quad (\text{E.4})$$

Table E.1: The propagation constants of the supermodes in the CMCF with ring layout for various n .

n	Order of supermode							
	1	2	3	4	5	6	7	8
1	β							
2	$\beta + \kappa$	$\beta - \kappa$						
3	$\beta + 2\kappa$	$\beta - \kappa$	$\beta - \kappa$					
4	$\beta + 2\kappa$	β	β	$\beta - 2\kappa$				
5	$\beta + 2\kappa$	$\beta + \frac{\sqrt{5}-1}{2}\kappa$	$\beta + \frac{\sqrt{5}-1}{2}\kappa$	$\beta - \frac{\sqrt{5}+1}{2}\kappa$	$\beta - \frac{\sqrt{5}+1}{2}\kappa$			
6	$\beta + 2\kappa$	$\beta + \kappa$	$\beta + \kappa$	$\beta - \kappa$	$\beta - \kappa$	$\beta - 2\kappa$		
8	$\beta + 2\kappa$	$\beta + \sqrt{2}\kappa$	$\beta + \sqrt{2}\kappa$	β	β	$\beta - \sqrt{2}\kappa$	$\beta - \sqrt{2}\kappa$	$\beta - 2\kappa$

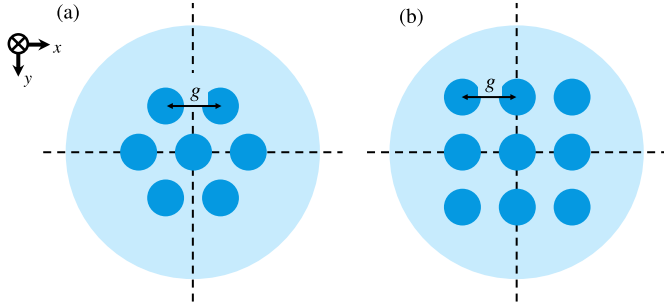


Figure E.1: Cross sections of bent and twisted fiber of (a) 7CF with the triangular lattice layout and (b) 9CF with the square lattice layout.

where κ' and κ'' should be calculated as in Appendix C, and these values cannot be neglected. In Eq. (E.4), diagonal elements are omitted because of the space limitations. When $\alpha = R_b^{-1} = \kappa' = \kappa'' = 0$, Eq. (E.4) leads to the supermodes similar to the linear polarized mode of LP01, LP11a/b, LP21a/b, LP31, LP02 modes. From Eq. (34), the LP01-like mode can couple to LP11-like modes. Whereas, the LP11-like modes can couple to LP21- and LP02-like modes as well as LP01-like mode. For 3CF and 4CF, the matrix $\mathbf{M}$ is already defined in Eqs. (30) and (40). The propagation constants of the supermodes for various n are summarized in Table E.2. Except for 4CF, the degeneration can be seen, which makes difficult the mode conversion between the selected modes.

E.3. Square lattice layout

Here, we consider 4CF and 9CF. The matrix $\mathbf{M}$ used in Eq. (18) for the triangular lattice layout when $n = 9$, as shown in

Fig. E.1 (b), can be defined as

$$\mathbf{M}_{S9} = \begin{pmatrix} \cdot & \kappa & \kappa'' & \kappa & \kappa' & 0 & \kappa'' & 0 & 0 \\ \kappa & \cdot & \kappa & \kappa' & \kappa & \kappa' & 0 & \kappa'' & 0 \\ \kappa'' & \kappa & \cdot & 0 & \kappa' & \kappa & 0 & 0 & \kappa'' \\ \kappa & \kappa' & 0 & \cdot & \kappa & \kappa'' & \kappa & \kappa' & 0 \\ \kappa' & \kappa & \kappa' & \kappa & \cdot & \kappa & \kappa' & \kappa & \kappa' \\ 0 & \kappa' & \kappa & \kappa'' & \kappa & \cdot & 0 & \kappa' & \kappa \\ \kappa'' & 0 & 0 & \kappa & \kappa' & 0 & \cdot & \kappa & \kappa'' \\ 0 & \kappa'' & 0 & \kappa' & \kappa & \kappa' & \kappa & \cdot & \kappa \\ 0 & 0 & \kappa'' & 0 & \kappa' & \kappa & \kappa'' & \kappa & \cdot \end{pmatrix}, \quad (\text{E.5})$$

where κ' and κ'' cannot be neglected similarly to Eq. (E.4). For 4CF, the matrix $\mathbf{M}$ is already defined in Eq. (32). Assuming $\alpha = R_b^{-1} = \kappa' = \kappa'' = 0$, Eq. (E.5) gives the supermodes similar to LP01, LP11a/b, LP21a/b, LP02, LP31a/b, and LP12 modes. As summarized in Table E.3, the propagation constants of these modes are equally spaced. Figs. E.2(a) and (b) show the transmission as a function of the propagation distance when the 1st supermode is launched, where $g = 16 \mu\text{m}$, $R_b = 2000 \text{ mm}$, and $\alpha = 64\pi \text{ rad/m}$. The power of each mode group is summed up. In Fig. E.2(a), we apply the approximation of $\kappa' = \kappa'' = 0$. The launched LP01-like mode can be coupled to the LP12-like mode. On the other hand, κ' and κ'' are nonzero in Fig. E.2(b), which makes the deterioration of the balance of the mode coupling similar to the 4CF with the square lattice layout. It can be used as a mode scrambler [20].

Table E.2: The propagation constants of the supermodes in the CMCF with triangular layout for various n , where the approximation of $\kappa' = \kappa'' = 0$ is applied.

n	Order of supermode						
	1	2	3	4	5	6	7
3	$\beta + 2\kappa$	$\beta - \kappa$	$\beta - \kappa$				
4	$\beta + \frac{\sqrt{17}+1}{2}\kappa$	β	$\beta - \kappa$	$\beta - \frac{\sqrt{17}-1}{2}\kappa$			
7	$\beta + (\sqrt{7}+1)\kappa$	$\beta + \kappa$	$\beta + \kappa$	$\beta - \kappa$	$\beta - \kappa$	$\beta - 2\kappa$	$\beta - (\sqrt{7}-1)\kappa$

Table E.3: The propagation constants of the supermodes in the CMCF with square lattice layout for various n , where the approximation of $\kappa' = \kappa'' = 0$ is applied.

n	Order of supermode								
	1	2	3	4	5	6	7	8	9
4	$\beta + 2\kappa$	β	β	$\beta - 2\kappa$					
9	$\beta + 2\sqrt{2}\kappa$	$\beta + \sqrt{2}\kappa$	$\beta + \sqrt{2}\kappa$	β	β	β	$\beta - \sqrt{2}\kappa$	$\beta - \sqrt{2}\kappa$	$\beta - 2\sqrt{2}\kappa$

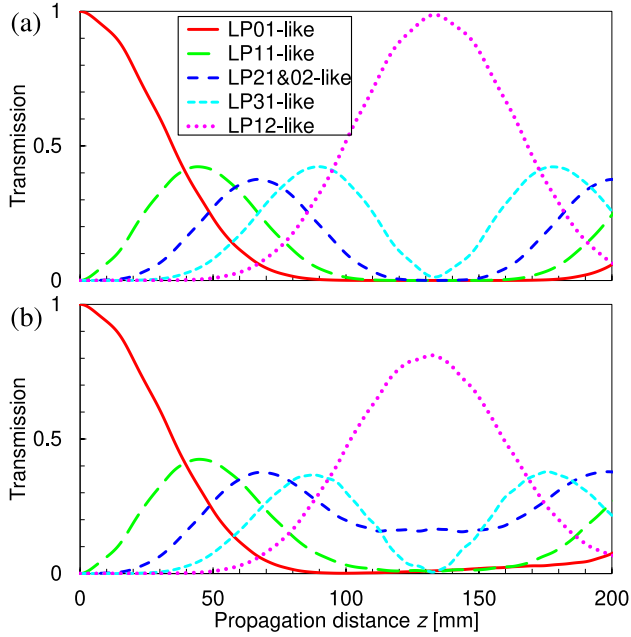


Figure E.2: Mode evolution in the 9CF when launching 1st supermode, where $\Delta = 0.35\%$, $a = 4.5 \mu\text{m}$, $g = 16 \mu\text{m}$, $R_b = 2000 \mu\text{m}$, and $\alpha = 64\pi \text{ rad/m}$. In (a), the assumption of $\kappa' = 0$ is applied. In (b), κ' and κ'' are correctly simulated.

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