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Modelling of opposed flame spread over thin electric wires: Estimation of flame spread rates and limiting oxygen concentrations

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Modelling of opposed flame spread over thin electric wires: Estimation of flame spread rates and limiting oxygen concentrations

A model of opposed flame spread along thin electric wires is developed to estimate the variation of flame spread rate (V_f) with oxygen concentrations and oxidizer flow velocity. The length of pyrolysis zone (L_{py}) and V_f are treated as eigenvalues of the problem, and one-dimensional energy balance equations for the wire core and insulation are formulated to derive their solutions. To obtain a set of steady solutions for L_{py} and V_f , the macroscopic energy balance equation for the entire system is calculated from the one-dimensional energy balance equations with appropriate boundary conditions. Since the equations are nonlinear function of L_{py} and V_f , multiple solutions appear. The physical meaning of each solution is evaluated based on the deviation of energy from the equilibrium. Comparison of the temperature distribution along the wire with experimental data confirms that the model reproduces well the actual flame spread process. The variation of V_f with oxygen concentration shows a C-shaped curve. The upper branch is the stable solution, and the lower branch is the unstable solution and the merging point of two branches correspond to the limit condition for flame spread. Therefore, the model can be used to estimate the limiting oxygen concentration based on the existence of a solution for V_f . Furthermore, the variation of V_f with oxidizer flow velocity shows O-shaped curve, and it is confirmed that radiative extinction under low flow velocity and blow-off under high flow velocity can also be estimated with the model. Mapping of variations of radiative extinction and blow-off extinction with oxygen concentration reveals the U-shaped flammability diagram. The predicted flame spread limits are in good agreement with the limiting oxygen concentrations measured in both microgravity and normal gravity, demonstrating the validity of using this model to predict flammability characteristic of electric wires under various oxidizer flow velocity.

Keywords: spacecraft fire safety; electric wires; flame spread; extinction; material flammability

Nomenclature

A cross-sectional area, (m^2)

1	c_p	specific heat, (J/(kg·K))
2	D	mass diffusivity, (m ² /s)
3	Da	Damköhler number, (-)
4	E	activation energy, (J/mol)
5	h	heat loss intensity, (W/(m ³ ·K))/ heat transfer coefficient, (W/(m ² ·K))
6	Δh_v	heat of vaporization, (J/kg)
7	H	heat loss parameter, (-)
8	L	length, (m)
9	m	mass, (kg)
10	Nu	Nusselt number, (-)
11	P	perimeter, (m)
12	q	heat, (J)
13	r	radius, (m)
14	R	gas constant, (J/(mol·K))
15	Re	Reynolds number, (-)
16	S	molar basis stoichiometric ratio, (-)
17	T	temperature, (K)
18	U	deviation of energy from equilibrium, (W)
19	V	velocity, (m/s)
20	x_d	development length of velocity boundary layer, (m)
21	X	mole fraction, (-)
22	Y	mass fraction, (-)
23	Z	pre-factor, (m ³ /(kg·s))
24		
25	<i>Greeks</i>	
26	α	thermal diffusivity, (m ² /s)
27	δ_v	thickness of velocity boundary layer, (m)
28	ε	emissivity, (-)
29	λ	thermal conductivity, (W/(m·K))
30	ν	kinematic viscosity, (m ² /s)
31	ρ	density, (kg/m ³)
32	σ	Stefan-Boltzmann constant, (W/(m ² ·K ⁴))
33	Ω	correction factor of flame temperature, (-)

1

2 *Subscripts*

3 b1 burned zone 1

4 b2 burned zone 2

5 c core

6 cond conduction

7 eff effective

8 enth enthalpy

9 f flame

10 F fuel

11 g gas

12 pr preheat zone

13 py pyrolysis zone

14 rad radiation

15 s insulation

16 u unburned zone

17 ∞ ambient condition

18

19 *Superscripts*

20 + upstream

21 – downstream

22 " per unit area, (m^{-2})

23 * fixed point

24 $\cdot$ per unit time, (s^{-1})

25 **1. Introduction**

26 Fire safety is one of the most important issues in manned space missions [1–3]. Because
27 firefighting and evacuation are not easy in spacecraft, it is extremely important to
28 prevent fires from occurring or to minimize damage after a fire breaks out. The first line
29 of defence against fire is to keep the spacecraft free of either heat sources, oxidants, or
30 combustible materials. From this perspective, the materials to be installed on manned

1 spacecraft are carefully selected through material flammability tests specified by
2 NASA-STD-6001B Test 1 and Test 4 [4]. The test method is a pass/ fail test and
3 observes the upward flame spread along the material under the cabin conditions to
4 which the material is exposed in the spacecraft. The materials that pass a certain test
5 criterion may be brought into the cabin of a spacecraft.

6 NASA-STD-6001B [4] has a long history of ensuring fire safety for manned
7 space missions and is reliable and track record. However, in recent years, some issues
8 have been raised by scientists and engineers against the test protocol [5,6]. One of these
9 is that results of pass/fail test are applicable only to the same test conditions as those in
10 the test, and it does not provide further information on the actual flammability of the
11 material under the cabin conditions [5]. An additional potential issue is that since
12 flammability tests are conducted on the ground, the effect of gravity on the flammability
13 of material cannot be evaluated.

14 With the recent progress of manned space exploration, such as the Artemis and
15 Gateway programs, the cabin environment in which astronauts operate is expected to
16 change significantly from the current ISS onboard environment. In fact, it is planned to
17 set the cabin conditions to low pressure and high oxygen concentration (called
18 exploration atmosphere) to reduce the risk of decompression sickness associated with
19 extravehicular activity (EVA) and to shorten transition time to EVA [7]. Furthermore,
20 manned space missions will be conducted not only in the microgravity field of Earth
21 orbit, but also the Lunar and Martian surfaces, and space agencies are reviewing hazard
22 and risk management and life support systems for previously unforeseen situations.

23 Current fire safety regulations under NASA STD 6001B [4] require that the
24 flammability of all materials must be re-tested under expected cabin conditions if the
25 environmental conditions in the cabin changes [5]. To address these shortcomings of

existing fire safety standards, the Japan Aerospace Exploration Agency (JAXA) is developing new options for fire safety standards for manned space missions in the Flammability Limits at Reduced-g experiment (FLARE) project [8]. The FLARE project proposes the use of the Oxygen Index (OI) [9], which is already widely used in industry to evaluate the flame resistance of plastic materials and is developing a conversion formula to predict how OI will change under various atmospheric conditions [10,11]. The OI is the minimum oxygen concentration that can sustain self-sustained combustion of the material, as determined by downward flame spread test, and corresponds to the flammability limit of the material on the ground. Therefore, one of the challenges of the FLARE project is to predict how OI will change in a low gravity field, where the flow field near the flame is slower than natural convection induced at ground level.

Takahashi et al. [12–14] developed a method to predict the limiting oxygen concentration (LOC) as a function of opposed flow velocity for rectangular sheet samples, which are also used as standard specimens for the OI test. An overview of the method for predicting the variation of LOC with opposed flow velocity is shown in Figure 1, along with a typical flammability map of solid materials [15]. As shown in Figure 1, the flammability limit of diffusion flame, not limited to diffusion flame spreading over solid materials, is known to have a U-shaped curve with respect to the inverse of the flow residence time (e.g., gas flow velocity, gravity, stretch rate, etc.) [16], as has been clarified theoretically [17,18], experimentally [12,19–23], and numerically [24–28]. The left branch is the radiative extinction branch, and the right branch is the blow-off branch. We can understand that the flame resistance over the material at the merging point of the two branches causes worst situation from the view of fire safety, since the flame spread is sustained at the lowest oxygen concentration

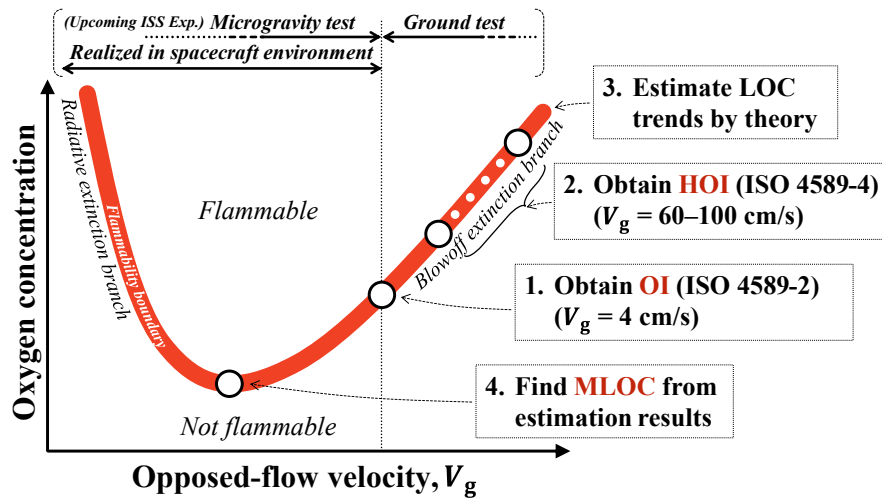


Figure 1. Conceptual diagram of flammability map for solid materials and overview of MLOC prediction method [15].

(minimum limiting oxygen concentration: MLOC). Therefore, the benefit of the fire safety standard that the FLARE project is developing is that it can clarify under what flow velocity conditions the MLOCs of various materials exist. To quantitatively predict the MLOC of the solid materials, Takahashi et al. [14] proposed to acquiring LOC under high gas flow velocity conditions (60–100 cm/s) in addition to the OI described above. The method for determining LOC under high flow velocity conditions has already been newly standardized as ISO 4589-4 [29] (called the high gas flow oxygen index (HOI) test) and its role is to adjust parameters (such as the constants governing heat losses and chemical reaction rates) that appear in theoretical equations predicting LOC trends.

The authors have been working on the development of a method to predict MLOC for electric wires, which are also the subject to flammability tests specified in NASA-STD-6001B Test 4 [4]. This paper reports the full details of a flame spread model for estimating the flame spread rate and LOC of electric wires. Note that since TEST 4 of NASA-STD-6001B [30] recommends the use of wires with a diameter of 0.9

mm, this study targets thin wires. Furthermore, TEST 4 specifies that electric wires should be set at a 15-degree angle for upward flame spread tests and that the internal wire temperature should be set to 125°C or the maximum operating temperature. However, in this study, the OI [9] and HOI [29] methods are applied to electric wires from the viewpoint of unifying the evaluation method for flammability of materials used in spacecraft. This facilitates comparison of the flammability of the plastic material alone with that of the wire insulation and allows discussion of how the flammability of various plastic materials changes when combined with the conductor.

Because electric wires are a source of fire not only in spacecraft but also in our daily life [31], there are tremendous previous studies that have investigated their combustion characteristics under various environmental conditions. To summarize the previous studies, the presence of a metal conductor in a combustible material and its cylindrical shape are important characteristics governing the flammability of electric wires. Therefore, it is essential to make it possible to evaluate the effects of these factors on the flammability of the wire insulation materials. For example, previous studies have reported that the presence of a metal conductor makes the transition from ignition to flame spread more difficult due to predominant heat conduction in the solid phase [32,33], and that the shape of the flame enveloping the metallic core governs the flammability of the wire insulation materials [34–39]. As for the effect of solid surface curvature, it has been clarified that the flame spread rate increases in microgravity [40] and low-pressure fields [41], and that radiative extinction is less likely to occur compared to a flat shaped solid materials [42]. In addition, since highly flexible synthetic polymers are used for the insulation of electrical wires, melting, deformation, and dripping of the insulation during combustion is often observed. It has been reported that the melting phenomenon can dominate the combustion of electrical wires,

especially those with thick insulation, complicating the understanding of the flame spread and its extinction [43,44]. Until now, wire combustion phenomena have been experimentally driven, and there is no physical model that can comprehensively discuss the aforementioned effects on flame spread rate and LOC of wire insulation materials.

Again, this research is being conducted as part of the FLARE project [8], and the ultimate goal of this study is to develop a methodology that can predict the velocity dependence of LOC for various wire insulation materials. To achieve this goal, the present study developed a model of opposed flame spread over electric wire which can estimate the flame spread rate and LOC. This paper presents the details of the flame spread model and discusses the validity of the model by comparing calculated and experimental results.

2. Analysis

This section describes a model of opposed flame spread over electric wires. The aim of

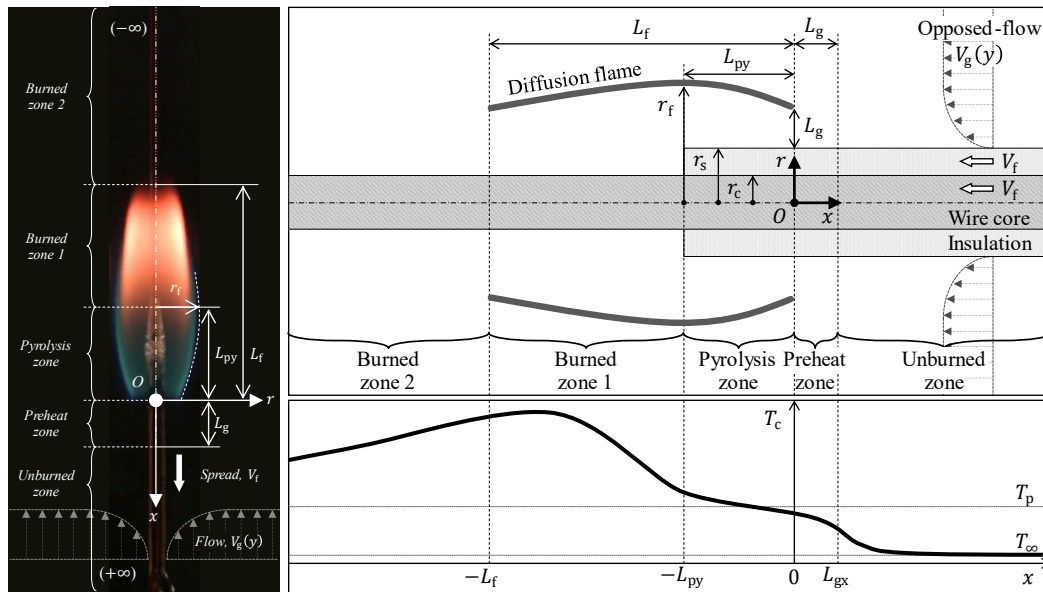


Figure 2. Schematic description of opposed-flow flame spread over an electric wire and the temperature profile along a wire core during steady flame spread.

the analysis is to estimate the flame spread rate (V_f) and limit conditions of flame spread (LOC) over wire insulation materials. Figure 2 shows a conceptual diagram of opposed flame spread over an electric wire. An electric wire with a core diameter of $2r_c$ and an insulation thickness of $r_s - r_c$ is placed in a uniform flow field with velocity V_g and a diffusion flame travels along the insulation surface with a constant velocity V_f to the opposite direction of the surrounding flow. In the present model, the development of the velocity boundary layer on the wire surface is considered. Since the gas flow field of interest is low velocity (less than 1 m/s), a thick velocity boundary layer may form on the surface of a thin wire and the flame may penetrate into the velocity boundary layer. Thus, the effective opposed-flow velocity seen by the flame leading edge is adopted in the analysis. Moreover, the effect of flow residence time on the flame temperature (T_f) is also considered in the energy balance equation for solid phase using the same method as in the previous study [42]. In the analysis, the length of the pyrolysis zone (L_{py}) and the flame spread rate (V_f) are treated as unknown quantities (eigenvalues) to be solved. Thus, a system of two equations is derived to compute a set of solutions for L_{py} and V_f corresponding to the steady-state flame spread. As described below, the model is based on the one dimensional steady-state energy balance equation for the solid phase. The gas phase problems, such as flow field, flame geometries, and flame temperature, are not analysed but are modelled using simple mathematical equations developed by previous studies.

2.1. Assumptions

To simplify the problem, the model is developed based on the following assumptions.

(a) The flame spread is steady-state and insulation is burned out completely within the spreading flame. Thus, the correlation between V_f and the mass loss rate ($\dot{m}_F$) of the insulation material is expressed as follows

$$\dot{m}_F = \rho_s A_s V_f. \quad (1)$$

In Eq. (1), subscripts F, s, and f denote fuel, insulation, and flame, respectively. ρ is the density and A is the cross-sectional area, respectively. It is noted that this assumption restricts the length of the pyrolysis zone as $0 < L_{py} < L_f$. L_f denotes the flame length.

(b) A coordinate system moves at V_f and its origin is attached to the center of the wire at the location of the flame leading edge ($x = 0$ and $r = 0$ in Figure 2).

(c) The electric wire is sufficiently long that end effects are negligible.

(d) The diameter of the electric wire is sufficiently small that the wire can be treated as a one-dimensional axisymmetric system.

(e) The heat conduction in the axial direction for the insulation is negligible since its thermal conductivity is much smaller than that of the wire core.

(f) There is no thermal contact resistance between the wire core and insulation. It is noted that the wire in this study is single-core and the contact condition between the insulation and metallic core is good.

(g) Ignore the deformation of the insulation due to the melting and thermal expansion.

(h) A flame enveloping the wire has a uniform flame temperature (T_f), and its shape is an axisymmetric hollow cylinder. The flame radius (r_f) is calculated based on

the mass balance between $\dot{m}_F$ and the mass flow rate of pyrolysis gas which passes through the cross-sectional area of the flame

$$\dot{m}_F = \pi(r_f^2 - r_s^2)\rho_F(V_g + V_f): \Rightarrow r_f = \{r_s^2 + \dot{m}_F/[\pi\rho_F(V_g + V_f)]\}^{1/2}. \quad (2)$$

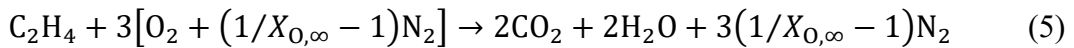
The flame length (L_f) is calculated by the Roper [45] and Annamalai [46] models for the analytical prediction of the axisymmetric laminar diffusion flame

$$L_f = \frac{\dot{m}_F(T_\infty/T_{\text{eff}})^{0.67}}{4\pi\rho_{F,\infty}D_\infty \ln(1 + 1/S)}. \quad (3)$$

In Eq. (3), subscripts ∞ and eff denote the ambient condition and the effective quantity, respectively. T is the temperature, D is the mass diffusivity, and S is the molar basis stoichiometric ratio between oxidizer and fuel, respectively. In the present study, T_{eff} is evaluated by considering the average temperature of the pyrolysis temperature of the insulation (T_p) and flame temperature (T_f)

$$T_{\text{eff}} = T_p + 0.5(T_f - T_p). \quad (4)$$

Regarding D , the mass diffusivity of O_2 diffusing into diluent gas (N_2) is used in the calculation. In addition, because the solid fuel considered in the present study is low-density polyethylene (LDPE), the stoichiometric combustion of ethylene (C_2H_4) in the mixture gas of O_2 and N_2 gas is considered



where $X_{O,\infty}$ is the mole fraction of oxygen. Thus, S is calculated as

$$S = 3/X_{O,\infty}. \quad (6)$$

- (i) The preheat zone (L_{gx}) and the flame standoff distance (L_{gr}) are defined upstream side of the flame leading edge. Each length scale is defined as follows [47]

$$L_g = L_{gx} = L_{gr} = \alpha_g / (V_g + V_f) \quad (7)$$

where α is the thermal diffusivity.

- (j) The physical properties of the wire core, insulation, and gas are assumed to be independent of temperature.
- (k) The temperature of unburned insulation must equal T_p before the flame spreads over it ($T_s = T_p$ at $x = 0$).
- (l) The insulation pyrolyzed at the constant temperature with T_p ($T_s = T_p$ for $-L_{py} \leq x < 0$).

Assumptions (k) and (l) are identical with the classical flame spread theory [48] and those are essential conditional expressions to derive L_{py} and V_f . It is noted that the present model is only applicable when the insulation is burned out completely as mentioned in assumption (a). When the insulation heated by the flame is left over during the flame spread process, $\dot{m}_F$ must be treated as unknown quantity instead of L_{py} and the present model cannot be used. This situation may be seen, for instance, in charring materials. This is one of the limitations of the present model. It should also be noted that if the insulation melts and drips significantly, it will be difficult to apply this

model. Numerous studies [49–53] have repeatedly pointed out that the melting of the thermoplastic materials changes the dynamics of flame spread by forming a liquid ball in the burning region or by flowing down to the unburned side. However, modelling the deformation and dripping phenomena associated with the phase change process is challenging, and assumption (g) is adopted in this study in favour of simplicity.

2.2. Energy balance equations for solid phase

With the assumptions described in Section 2.1, one-dimensional axisymmetric energy balance equations for the wire core and insulation are formulated as follows

$$\rho_c c_{p,c} A_c (-V_f) \frac{dT_c}{dx} = A_c \lambda_c \frac{d^2 T_c}{dx^2} + P_c \Omega \dot{q}_{gc}'' - P_c \dot{q}_{cs}'' - P_c \dot{q}_{loss,c}'', \quad (8)$$

$$\rho_s c_{p,s} A_s (-V_f) \frac{dT_s}{dx} + P_s \dot{m}_F'' \Delta h_v = P_s \Omega \dot{q}_{gs}'' + P_c \dot{q}_{cs}'' - P_s \dot{q}_{loss,s}''. \quad (9)$$

where subscripts c denotes the wire core. Ω is the dimensionless number that corrects the heat flux from the flame to the solid surface [42], $\dot{q}_{gc}''$ is the heat flux from the flame to the surface of the wire core, $\dot{q}_{cs}''$ is the heat flux at the interface between the wire core and insulation, $\dot{q}_{loss,c}''$ is the heat loss from the surface of the wire core, $\dot{m}_F''$ is the mass flux of the pyrolysis gas from the insulation surface, Δh_v is the heat of vaporization of the insulation, $\dot{q}_{gs}''$ is the heat flux from the flame to the surface of the insulation, and $\dot{q}_{loss,s}''$ is the heat loss from the surface of the insulation.

2.2.1. Correction of adiabatic flame temperature

The role of Ω in Eqs. (8) and (9) is to evaluate the flame temperature reduction due to the volumetric radiation heat loss in the gas phase and the finite rate chemistry. In

1 accordance with the previous study [42], Ω is defined as follows

$$\Omega = 1 - 1/Da - H, \quad 0 \leq \Omega \leq 1. \quad (10)$$

2 where Da is the Damköhler number and H is heat loss parameter. The relations of Da
 3 and H to the dimensional counterparts are as follows

$$Da = \frac{\alpha_g \rho_g Y_{O,\infty} Z e^{-E/(RT_f)}}{(V_{g,\text{eff}} + V_f)^2}, \quad (11)$$

$$H = \frac{h \lambda_g}{\rho_g^2 c_{p,g}^2 (V_{g,\text{eff}} + V_f)^2}, \quad (12)$$

4 where $Y_{O,\infty}$ is the mass fraction of oxygen in oxidizer, Z is the pre-factor, E is the
 5 activation energy, R is the gas constant, $V_{g,\text{eff}}$ is the effective gas flow velocity affecting
 6 the spreading flame, and h is the volumetric heat loss intensity.

7 $V_{g,\text{eff}}$ in Eqs. (11) and (12) are calculated by applying the Pohlhausen method
 8 [54] to account for the development of velocity boundary layer at the wire surface as
 9 follows [42]

$$V_{g,\text{eff}} = \begin{cases} V_g \left[\frac{3}{2} \left(\frac{L_g}{\delta_v} \right) - \frac{1}{2} \left(\frac{L_g}{\delta_v} \right)^3 \right] & \text{for } L_g < \delta_v, \\ V_g & \text{for } L_g \geq \delta_v, \end{cases} \quad (13)$$

10 where $\delta_v = 4.64 x_d Re_{x_d}^{-0.5}$ is the thickness of the velocity boundary layer, x_d is the
 11 development length, $Re_{x_d} = x_d V_g / \nu_g$ is the Reynolds number, and ν is the kinematic
 12 viscosity.

2.2.2. Heat and mass fluxes at the solid surfaces and interfaces

As depicted in Figure 2, a single electric wire is exposed to the various surrounding condition in the different regions during the flame spread process. To model this situation, the wire is divided into five zones, i.e., an unburned zone ($x \geq L_g$), a preheat zone ($0 \leq x \leq L_g$), a pyrolysis zone ($-L_{py} \leq x \leq 0$), a burned zone 1 ($-L_f \leq x \leq -L_{py}$), and a burned zone 2 ($x \leq -L_f$). Then, by considering the appropriate surrounding conditions in each zone, heat and mass fluxes in Eqs. (8) and (9) are defined as follows

$$\dot{q}_{gc}'' = \begin{cases} h_{gc}(T_f - T_c) & \text{for } -L_f \leq x \leq -L_{py}, \\ 0 & \text{otherwise,} \end{cases} \quad (14)$$

$$\dot{q}_{cs}'' = \begin{cases} h_{cs}(T_c - T_p) & \text{for } -L_{py} \leq x \leq L_g, \\ h_{cs}(T_c - T_s) & \text{for } x \geq L_g, \\ 0 & \text{otherwise,} \end{cases} \quad (15)$$

$$\dot{q}_{loss,c}'' = \begin{cases} h_{T,b2}(T_c - T_\infty) & \text{for } x \leq -L_f, \\ h_{rad,c}(T_c - T_\infty) & \text{for } -L_f \leq x \leq -L_{py}, \\ 0 & \text{otherwise,} \end{cases} \quad (16)$$

$$\dot{q}_{gs}'' = \begin{cases} h_{gs,py}(T_f - T_p) & \text{for } -L_{py} \leq x \leq 0, \\ h_{gs,pr}(T_f - T_p) & \text{for } 0 \leq x \leq L_g, \\ 0 & \text{otherwise,} \end{cases} \quad (17)$$

$$\dot{q}_{loss,s}'' = \begin{cases} \varepsilon_s \sigma (T_p^4 - T_\infty^4) & \text{for } -L_{py} \leq x \leq L_g, \\ h_{T,u}(T_s - T_\infty) & \text{for } x \geq L_g, \\ 0 & \text{otherwise,} \end{cases} \quad (18)$$

$$\dot{m}_F'' = \begin{cases} d(\dot{m}_F/P_s)/dx & \text{for } -L_{py} \leq x \leq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

The h in Eqs. (14)–(18) denotes the reduced heat transfer coefficients. The form of h in Eqs. (14)–(18) is summarized in Appendix A.

For simplicity, Eqs. (14)–(19) are approximated as a linear function of T_c or a constant. Further, in the unburned zone ($x \geq L_g$), Eqs. (8) and (9) are analysed in the same way as our previous work [55]. Namely, assuming that T_c is equal to T_s to avoid a multi-temperature model that simultaneously tracks T_c and T_s , and the coupled energy-balance equation of Eqs. (8) and (9) is used for the analysis. For a similar reason, $\dot{q}_{gs}''$ in Eq. (17) and $\dot{q}_{loss,s}''$ in Eq. (18) for the insulation in the preheat zone ($0 \leq x \leq L_g$) are approximated as constant, which allows a single temperature model of T_c in the preheat zone.

Four unknown quantities are defined here: L_{py} , V_f , T_c , and T_s . These unknowns are solved in the latter section by Eqs. (8) and (9) and two conditional expressions provided by assumptions (k) and (l), as well as some essential boundary conditions.

2.3. *Analytical solutions of core temperature*

We first analyse Eq. (8) to obtain analytical solutions of T_c and macroscopic energy-balance equations for the entire system. As mentioned previously, the electric wire is divided into five zones and the form of Eq. (8) is transformed using Eqs. (14)–(18). Therefore, to satisfy the continuity of T_c , the boundary and matching conditions are given as follows

$$\begin{cases} x \rightarrow -\infty: & T_c = T_\infty, \\ x = -L_f: & T_c^+ = T_c^-, \quad dT_c^+/dx = dT_c^-/dx, \\ x = -L_{py}: & T_c^+ = T_c^-, \quad dT_c^+/dx = dT_c^-/dx, \\ x = L_g: & T_c^+ = T_c^-, \quad dT_c^+/dx = dT_c^-/dx, \\ x \rightarrow +\infty: & T_c = T_\infty \end{cases} \quad (20)$$

1 where the superscripts + and - denote the upstream and downstream side at the
 2 interface of each zone, respectively. Using Eqs. (14)–(18) and (20), solutions to Eq. (8)
 3 are obtained analytically as follows

$$T_c = \begin{cases} T_\infty + T_1 e^{x/L_1} & \text{for } x \leq -L_f, \\ T_k + T_2 e^{-x/L_2} + T_3 e^{x/L_3} & \text{for } -L_f \leq x \leq -L_{py}, \\ T_p + T_4 e^{-x/L_4} + T_5 e^{x/L_5} & \text{for } -L_{py} \leq x \leq L_g, \\ T_\infty + T_6 e^{-x/L_6} & \text{for } x \geq L_g. \end{cases} \quad (21)$$

4 In Eq. (21), L_1, L_2, L_3, L_4, L_5 , and L_6 are characteristic roots of Eq. (8). T_1, T_2, T_3, T_4 ,
 5 T_5 , and T_6 are integral constants determined by Eq. (20), and T_k is the reduced
 6 parameter. The form of the coefficients in Eq. (21) is summarized in Appendix B.

7 **2.4. Macroscopic energy balance for the entire system**

8 Because analytical solutions of T_c were obtained, the macroscopic energy-balance
 9 equation for the system can be calculated by integrating Eqs. (8) and (9) with Eq. (21).
 10 The results for the wire core in the unburned zone ($x \geq L_g$), preheat zone ($0 \leq x \leq L_g$),
 11 pyrolysis zone ($-L_{py} \leq x \leq 0$), burned zone 1 ($-L_f \leq x \leq -L_{py}$), and burned zone 2
 12 ($x \leq -L_f$) are obtained as follows

$$\dot{q}_{\text{enth,w,u}} = \dot{q}_{\text{cond,u}} - \dot{q}_{\text{loss,u}}, \quad (22)$$

$$\dot{q}_{\text{enth,c,pr}} = \dot{q}_{\text{cond,pr}} - \dot{q}_{\text{cs,pr}}, \quad (23)$$

$$\dot{q}_{\text{enth,c,py}} = \dot{q}_{\text{cond,py}} - \dot{q}_{\text{cs,py}}, \quad (24)$$

$$\dot{q}_{\text{enth,c,b1}} = \dot{q}_{\text{cond,b1}} + \dot{q}_{\text{gc,b1}} - \dot{q}_{\text{loss,b1}}, \quad (25)$$

$$\dot{q}_{\text{enth,c,b2}} = \dot{q}_{\text{cond,b2}} - \dot{q}_{\text{loss,b2}}, \quad (26)$$

1 In the above equations, subscripts u, pr, py, b1, and b2 denote the unburned zone,
 2 preheat zone, pyrolysis zone, burned zone 1, and burned zone 2, respectively. In
 3 addition, $\dot{q}_{\text{enth}}$ is the enthalpy change, $\dot{q}_{\text{cond}}$ is the heat conduction, $\dot{q}_{\text{loss}}$ is the heat
 4 loss, $\dot{q}_{\text{cs}}$ is the heat exchange between the wire core and insulation, $\dot{q}_{\text{gc}}$ is the heat input
 5 from the flame to the wire surface, respectively. The derivations of each term are
 6 summarized in Appendix C.

7 The results for the insulation in the preheat zone ($0 \leq x \leq L_g$) and pyrolysis
 8 zone ($-L_{\text{py}} \leq x \leq 0$) are obtained as follows

$$\dot{q}_{\text{enth,s,pr}} = \dot{q}_{\text{gs,pr}} + \dot{q}_{\text{cs,pr}} - \dot{q}_{\text{loss,pr}}, \quad (27)$$

$$\dot{q}_{\text{enth,s,py}} = \dot{q}_{\text{gs,py}} + \dot{q}_{\text{cs,py}} - \dot{q}_{\text{loss,py}}. \quad (28)$$

9 Regarding the unburned zone ($x \geq L_g$), the coupled energy-balance equation of Eqs. (8)
 10 and (9) are used in the analysis [55]. Thus, Eq. (22) includes the energy balance for the
 11 insulation as well as the wire core. In the above equations, $\dot{q}_{\text{gs}}$ is heat input from the
 12 flame to the insulation surface. As in the case of the wire core, the derivations of each
 13 term are summarized in Appendix C.

Once again, L_{py} and V_f are treated as unknowns in this analysis. To obtain a steady solution of L_{py} , we use Eq. (28) and redefine it for future discussion as follows

$$U_{py}(L_{py}, V_f) = \dot{q}_{gs,py} + \dot{q}_{cs,py} - \dot{q}_{loss,py} - \dot{m}_F \Delta h_v. \quad (29)$$

where U_{py} is the deviation of energy from the equilibrium state in the pyrolysis zone, which is a nonlinear function of L_{py} and V_f . Namely, L_{py} satisfying $U_{py} = 0$ corresponds to the steady solution of L_{py} .

Furthermore, to find a steady solution of V_f , Eqs. (22) through (28) are added together to derive the energy balance equation for the entire system as follows

$$U_T(L_{py}, V_f) = \dot{q}_{input} - \dot{q}_{loss} - \dot{m}_F [\Delta h_v + c_{p,s}(T_p - T_\infty)], \quad (30)$$

where U_T is the deviation of energy from the equilibrium state in the entire system, which is also a nonlinear function of L_{py} and V_f . Namely, V_f satisfying $U_T = 0$ corresponds to the steady solution of V_f . $\dot{q}_{input}$ and $\dot{q}_{loss}$ in Eq. (30) are calculated as follows

$$\dot{q}_{input} = \dot{q}_{gc,b1} + \dot{q}_{gs,py} + \dot{q}_{gs,pr}, \quad (31)$$

$$\dot{q}_{loss} = \dot{q}_{loss,b2} + \dot{q}_{loss,b1} + \dot{q}_{loss,py} + \dot{q}_{loss,pr} + \dot{q}_{loss,u}. \quad (32)$$

Equation (30) can be transformed in the form of a flame spread rate, which can also be expressed as follows

$$V_f = f(L_{py}, V_f), \quad f(L_{py}, V_f) = \frac{\dot{q}_{input} - \dot{q}_{loss}}{\rho_s A_s [\Delta h_v + c_{p,s}(T_p - T_\infty)]}. \quad (33)$$

As a result, the set of solutions for L_{py} and V_f corresponding to the steady-state flame spread can be determined by solving Eqs. (29) and (30) (or (33)) simultaneously.

3. Calculation methods and conditions

To find a set of solutions of L_{py} and V_f , iterations are performed using Eqs. (29) and (30). An algorithm for computing solutions is illustrated in Figure 3. As shown in Figure 3, a two-step iterative calculation is developed to find L_{py}^* and V_f^* , respectively. Note that superscript $*$ denotes the solution of the system. Eqs. (29) and (30) are nonlinear equations for L_{py} and V_f , thus multiple solutions can exist. The physical meaning of each solution given by Eqs. (29) and (30) is discussed in the latter section.

In the calculations, LDPE insulated Cu and NiCr wires are considered. The material properties used in the calculations are summarized in Table 1. It is noted that

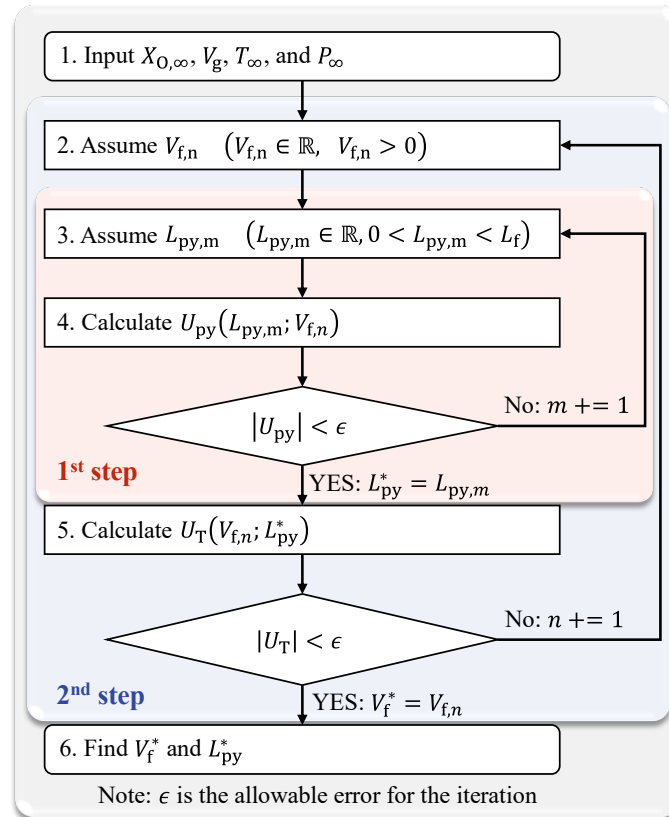


Figure 3. Flowchart of an algorithm for calculating a set of solutions for V_f and L_{py} .

the kinetic parameters (Z and E in Da in Eq. (11)) and the heat loss intensity (h in Eq. (12)) were estimated empirically based on the LOCs obtained in the previous study [15]. The wire dimension applied in the calculation are a core wire diameter of 0.5 mm and insulation thickness of 0.15 mm. A mixture gas of O_2 and N_2 is considered as the oxidizer. The adiabatic flame temperature is calculated using NASA CEA2 [56].

Table 1. The value of input parameter used in calculation.

Item	LDPE	Cu	NiCr
Density, ρ (kg/m ³)	920	8890	7850
Specific heat, c_p (J/(kg·K))	2300	385.1	470.0
Thermal conductivity, λ (W/(m·K))	0.34	376.6	15.0
Surface emissivity, ε (-)	1	1	1
Pyrolysis temperature, T_p (K)	688	—	—
Heat of combustion, Δh_c (MJ/kg)	46.024	—	—
Heat of vaporization, Δh_v (MJ/kg)	0.962	—	—
Activation energy, E (kJ/mol)	—	154	230
Pre-exponential factor, Z (m ³ /(kg·s))	—	3.46×10^6	7.59×10^7
Heat loss intensity, h (W/(m ³ ·K))	—	231	231

4. Results and Discussion

4.1. Characteristic of steady solution

There are multiple solutions that satisfy $U_{py}(L_{py}, V_f) = 0$ and $U_T(L_{py}, V_f) = 0$ because Eqs. (29) and (30) are nonlinear equations for L_{py} and V_f . To identify which solution is physically meaningful in discussing actual flame spread phenomena, we discuss the stability of the steady solutions for L_{py} and V_f in a physical way using a graphical analysis. The discussion is based on typical calculation results for LDPE insulated Cu wire at $V_g = 4$ cm/s and $X_{O,\infty} = 0.19$, but note that essentially the same argument holds

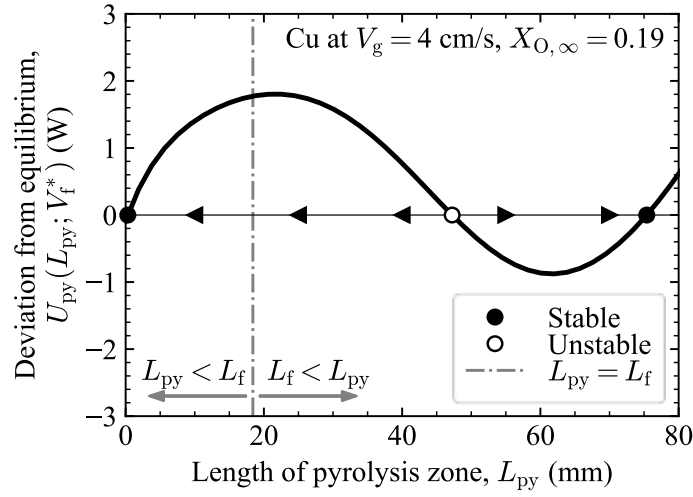


Figure 4. Mapping of U_{py} . V_f used in the calculation is the stable solution.

1 for other conditions.

2 4.1.1. Length of pyrolysis zone (L_{py}^*)

3 Figure 4 plots $U_{py}(L_{py})$ in Eq. (29). As can be seen from Figure 4, U_{py} follows a cubic-
4 like trend, and there are at most three solutions that satisfy $U_{py} = 0$. Equation (29)

5 shows that $U_{py} > 0$ represents an energy surplus state in the pyrolysis zone, while

6 $U_{py} < 0$ represents an energy deficit state in the pyrolysis zone. When $U_{py} > 0$, L_{py}

7 should decrease because the gasification rate of the insulation material must be

8 increased, and vice versa. This dynamic behaviour of L_{py} is governed by the change in

9 core temperature and heat input from the flame to the insulation surface due to the

10 change in L_{py} relative to L_f . Because the wire core in the burned zone 1 (see Figure 2) is

11 exposed to the flame, the core temperature increases as the ratio of the burned zone 1 to

12 the flame length ($1 - L_{py}/L_f$) increases. However, heat input from the flame to the

13 insulation surface ($\dot{q}_{gs,py}$ in Eq. (29)) is proportional to L_{py} , thus $\dot{q}_{gs,py}$ can decrease

14 with decreasing L_{py} . This competition between heat exchange in the gas and solid

15 phases causes U_{py} to exhibit a non-monotonic trend.

We can then use a graphical approach to identify the stability of the solution by observing the sign of U_{py} in the vicinity of the solution. As shown by the arrows in Figure 4, the flow of L_{py} is to the left for $U_{py} > 0$ and to the right for $U_{py} < 0$. At the points where $U_{py} = 0$, there is no flow, and they represent the steady solutions of L_{py} . As seen in Figure 4, there are two kinds of solutions: filled circles represent stable solutions and open circles represent unstable solutions. However, the present model does not allow the insulation to remain unburned, so the solutions which exist for $L_f < L_{py}$ do not satisfy the model. Therefore, the smallest of the three solutions can be judged as the physically meaningful solution that can be observed in the experiments.

4.1.2. Flame spread rate (V_f^*)

Similarly, $U_T(V_f)$ in Eq. (30) is plotted in Figure 5 to discuss the qualitative nature of solutions. As can be seen from Figure 5, U_T shows an upward convex parabolic-like trend, indicating the existence of two flame spread rates under certain calculation conditions. The same argument holds for U_T as for U_{py} , and we can determine the energy surplus or deficiency of the entire system based on the sign of U_T . When $U_T >$

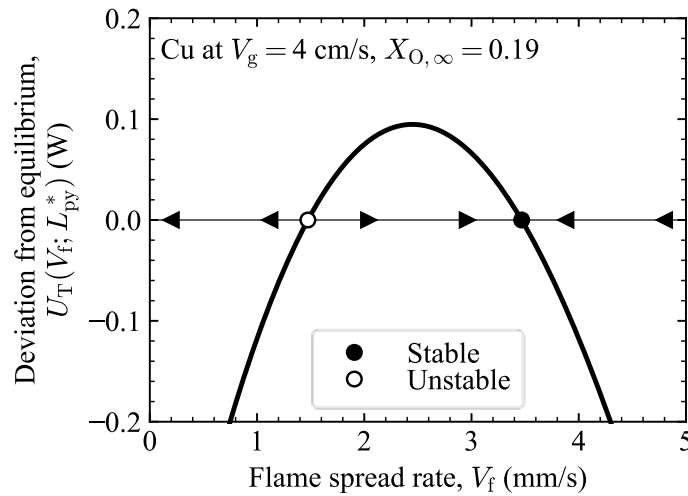


Figure 5. Mapping of U_T . L_{py} used in the calculation is the stable solution.

0, i.e., the energy balance in the entire system is in a surplus state, V_f should increase and vice versa. Therefore, as represented by the arrows in Figure 5, the flow of V_f is to the right where $U_T > 0$ and to the left where $U_T < 0$. At the points where $U_T = 0$, there is no flow, and they represent the steady solutions of V_f . In the case of the spread rate, the larger solution indicated by a filled circle represents the stable solution and the smaller one indicated by an open circle represents the unstable solution.

The controlling mechanism of the dynamic behaviour of V_f is somewhat complex, but the appearance of the unstable solution is dominated by heat losses from the system. In essence, the behaviour of the solution is due to the existence of a negative feedback mechanism in the system, which is inherent characteristic of the propagating flame under the influence of heat losses, where a decrease in the traveling velocity of the flame causes an increase in heat loss from the system [57,58]. For example, as the flame traveling velocity decreases, the flame thickness increases or the residence time of the flame at the heat loss field increases, thus increasing heat losses and changing the system in the direction of decay. In the previous study, we have experimentally and theoretically confirmed that an equivalent phenomenon is caused by the axial heat conduction through the wire core [55]. This is also an important point in the discussion of the limit conditions of the flame spread.

4.2. Temperature distribution along the wire core

Now that the characteristics of the steady solutions are understood, the next step is to examine the validity of the model by comparing experimental and calculation results. Axial temperature distributions along the wire core during the downward flame spread were measured by Konno et al. [55] using K type thermocouple of 0.1 mm diameter. Here, a comparison is made using experimental data obtained on the ground under

1 forced flow velocity of 4 cm/s and oxygen volume fraction of 0.20. However, in the
 2 calculation, $V_g = 35$ cm/s [12] was set to account for the effect of natural convection
 3 since the experiment were performed in normal gravity.

4 The measured and predicted axial distributions of temperature along Cu and
 5 NiCr wires are presented in Figure 6(a) and (b), respectively. The dotted and solid lines

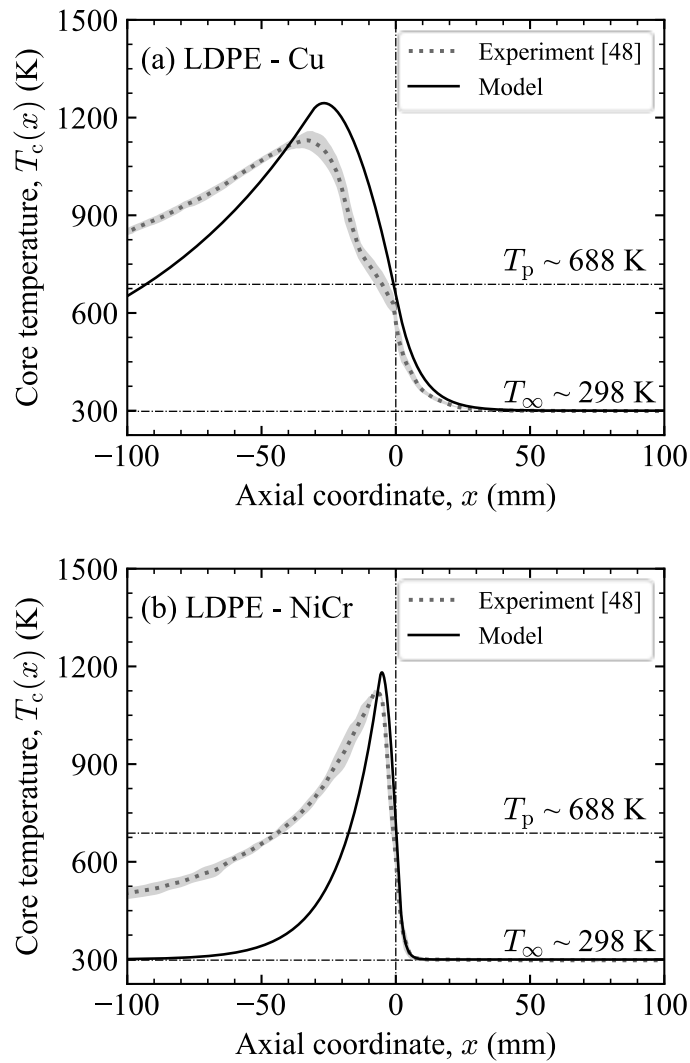


Figure 6. Comparison of axial temperature distributions along the wire core obtained by experiments [55] and theoretical analysis. Experimental conditions: $V_g = 4$ cm/s and $X_{O,\infty} = 0.20$. Calculation conditions: $V_g = 35$ cm/s and $X_{O,\infty} = 0.20$. Note that the calculations use a flow velocity of 35 cm/s to account for gravity-induced natural convection.

are measured and predicted data, respectively. The errors of the experimental data are shown by the shaded areas. In Figure 6, $x = 0$ corresponds to the location of the flame leading edge and $x > 0$ to the upstream unburned side. Although there is a discrepancy between measured and predicted data at the maximum temperature and the downstream of the spreading flame, the overall characteristics of the temperature distribution and the change in temperature distribution when the core material is changed are well reproduced. The surface of the wire core downstream of the spreading flame is modelled to be exposed to the ambient temperature (T_∞) as defined in Eq. (16), which may explain the lower tendency than the measured temperature distribution. The trend should be improved if the heat flux is defined taking into account the temperature distribution of the gas phase downstream of the flame. Although this problem needs to be solved, especially when the model is extended to concurrent flame spread, the upstream temperature distribution, which controls the opposed flame spread, is well reproduced, and the model is therefore considered to be appropriate for predicting flame spread rates and LOCs of wire insulation materials. How the temperature distribution of the core wire, flame spread rate, and extinction limit change with the presence of heat loss downstream of the flame is discussed in Appendix D for reference.

4.3. Effect of oxygen concentration

Here, we discuss the behaviour of the steady solutions for V_f obtained from Eqs. (29) and (30) under different flame temperatures with oxygen concentration as a variable. First, V_f obtained under various oxygen concentrations are plotted in Figure 7 as a function of Da . In the calculation, opposed flow velocity was fixed at 35 cm/s. As described in section 4.1.2, two types of solutions are obtained from Eqs. (29) and (30): stable and unstable solutions. Therefore, two solutions obtained under each calculation

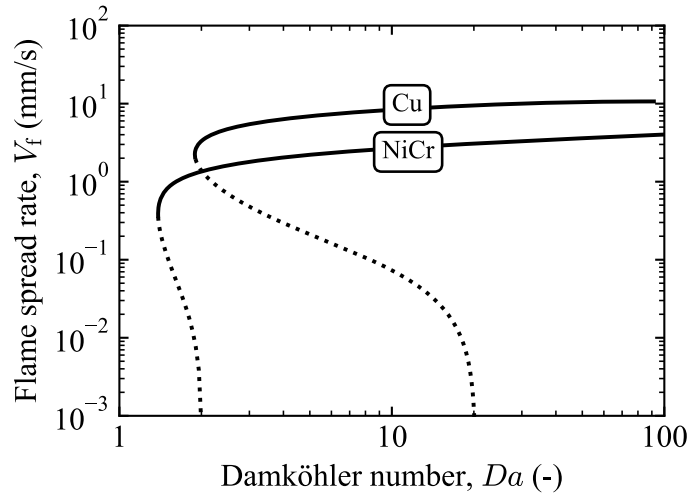


Figure 7. Flame spread rate as a function of Damköhler number for Cu and NiCr at $V_g = 35$ cm/s. Oxygen concentration is varied in the calculations. Solid line, stable solution; dotted line, unstable solution.

condition are plotted in Figure 7. The upper solid line is the stable solution and lower dotted line is the unstable solution.

As can be seen from Figure 7, the locus of the two solutions showed a similar trend to the well-known S-shaped extinction curve [59]. Namely, as Da decreases, the stable solution at the upper branch decreases while the unstable solution at the lower branch increases. At a certain Da , the stable and unstable solutions coincide, and for conditions where Da is lower than this critical point, there is no solution for V_f . The condition under which the stable and unstable solutions coincide corresponds to the limiting condition for the flame spread, i.e. LOC. Therefore, the model can be used to predict both the flame spread rate and LOC by determining whether a real solution to V_f exists or not.

Comparing the results for Cu and NiCr, V_f is always greater for Cu. This is because, as can be seen from the temperature distribution in Figure 6, the unburned insulation is preheated by the highly conductive core, since axial heat conduction in the wire core is predominant for Cu. Also, looking at the value of Da under the critical

condition, Cu exhibits a larger value than NiCr. In other words, when low conductive material is used as the wire core, the flame spread can be maintained even under conditions with small Da compared to the high conductive material. Differences in flame extinction nature are also due to differences in heat conduction through the wire core. In the case of Cu, the predominant axial heat conduction results in greater heat loss from the unburned zone [55], which causes the flame spread to be terminated before Da is as small as for NiCr wire. Thus, the extinction phenomenon under the same flow velocity conditions is dominated by the gas phase extinction factor (radiation heat loss in the gas phase and finite rate chemistry) for low conductive wire, whereas the solid phase extinction factor (heat loss from the solid surface) is more dominant for high conductive wire.

For comparison with the experimental data obtained in previous studies [15][55], the variations of V_f with oxygen concentration with different opposed flow velocity are presented in Figure 8. The relationship between V_f and oxygen concentration obtained by the model draws a C-shaped curve regardless of core materials and flow velocity. As in Figure 7, the upper branch corresponds to the stable

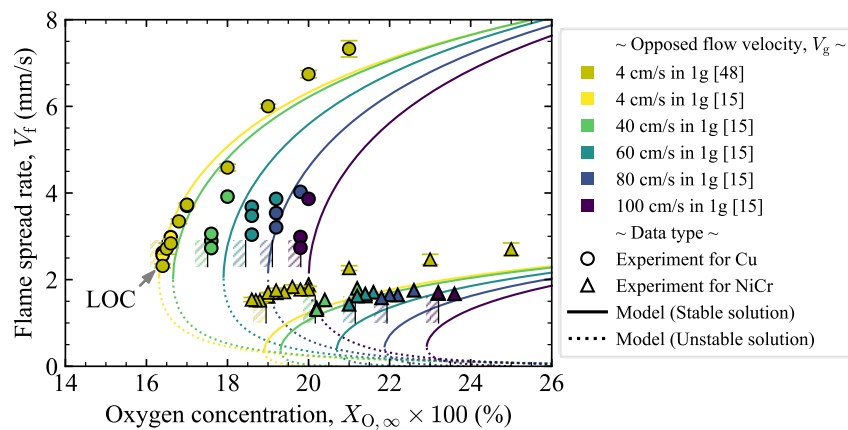


Figure 8. Comparison of flame spread rates as a function of oxygen concentration obtained by experiments [15][55] and theoretical analysis.

solution, the lower branch to the unstable solution, and bifurcation point to the limit condition for the flame spread. The stable solution reproduces well the changes in measured V_f when the core material, oxygen concentration, and opposed flow velocity are changed and the bifurcation point reproduces well the LOC.

As in the previous section, it was confirmed that V_f was greater for Cu wire than for NiCr wire under the same conditions. Furthermore, the dependence of V_f on oxygen concentration is greater for Cu wire than for NiCr wire. The reason for the greater dependence of V_f for Cu wire on oxygen concentration is that high conductive wire has a greater heat transport from the downstream side of the burned zone 1 to the upstream side of the unburned zone. For example, as the oxygen concentration increases, the flame temperature increases and V_f increases. Then, as V_f increases, L_f increases and the heat input from the flame to the wire core increases further. This feedback mechanism becomes apparent when the thermal conductivity of the core material is large [39], which increases the sensitivity of V_f to the oxygen concentration.

4.4. Effect of gas flow velocity

We next discuss the behaviour of the steady solutions for V_f when the oxygen concentration is fixed, and the opposed flow velocity is the variable. Figure 9 show variations of V_f with opposed flow velocity. The calculation was performed using Cu as the core material and $X_{O,\infty} = 0.19$. As can be seen in Figure 9, the locus of two solutions of V_f shows O-shaped curve. As in previous discussion, the upper and lower branches are stable and unstable solutions, respectively. Under a certain oxygen concentration condition, bifurcation points appear for low and high flow velocity conditions, respectively. These bifurcation points reproduce radiative extinction and blow-off, respectively.

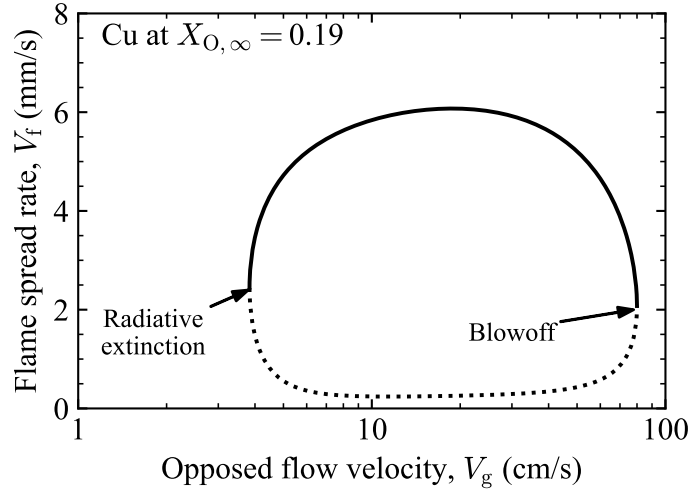


Figure 9. Flame spread rate as a function of opposed flow velocity for Cu at $X_{O,\infty} = 0.19$. Solid line, stable solution; dotted line, unstable solution.

To understand the behaviour of steady solutions for V_f as a function of opposed flow velocity, extinction factors in the gas phase (Ω , $1/Da$, and H), rate of heat flow ($\dot{q}_{input}$ and $\dot{q}_{loss}$) and fraction of heat loss (f_{loss}) as a function of opposed flow velocity are plotted in Figure 10(a) and (b). f_{loss} is calculated as follows

$$f_{loss} = \dot{q}_{loss}/\dot{q}_{input}. \quad (34)$$

As shown in Figure 10(a), Ω shows an upward convex trend with respect to the opposed flow velocity. This is because H , which evaluates the decrease in flame temperature due to radiation heat loss in the gas phase, increases under low flow velocity conditions while $1/Da$, which evaluates the decrease in flame temperature due to the finite rate chemistry, increases under high flow velocity conditions. As a result of the heat flux from the flame to the solid surface being modified by Ω , $\dot{q}_{input}$, which gives the stable solution, shows an upward convex trend as well (see Figure 10(b)). $\dot{q}_{loss}$ also shows a similar trend because the core temperature, which is controlled by $\dot{q}_{input}$, dominates the absolute amount of heat loss. On the other hand, f_{loss} , increased for low and high flow

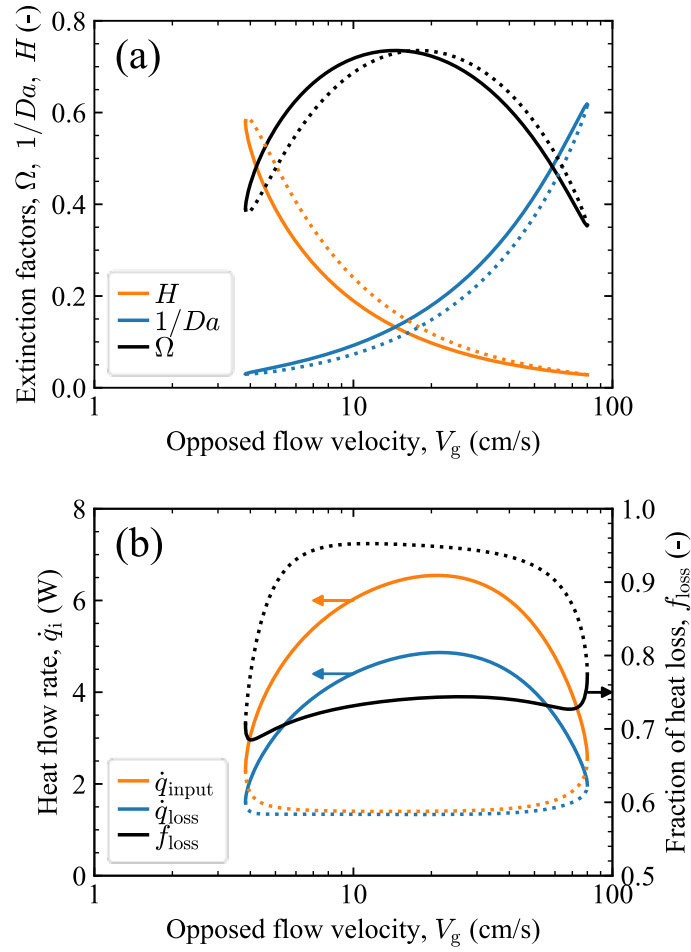


Figure 10. Extinction factors in the gas phase (a) and rate of heat flow and fraction of heat loss (b) as a function of opposed flow velocity for Cu at $X_{O,\infty} = 0.19$. Solid line, stable solution; dotted line, unstable solution.

1 velocity conditions (see Figure 10(b)). This means that $\dot{q}_{loss}$ does not decrease as much
 2 as $\dot{q}_{input}$ before the limit condition of flame spread is reached. This is due to the
 3 expansion of a high temperature region in the unburned zone when V_f decreases.

4 A comparison of the temperature distribution along the wire core just before
 5 radiative extinction and blow-off and under away from extinction is shown in Figure 11.
 6 As can be seen in Figure 11, the temperature distribution in the unburned zone for $V_g = 4$
 7 cm/s and 80 cm/s shows that a higher temperature region develops upstream compared
 8 to $V_g = 20$ cm/s. As the spreading flame approaches extinction and V_f decreases, the

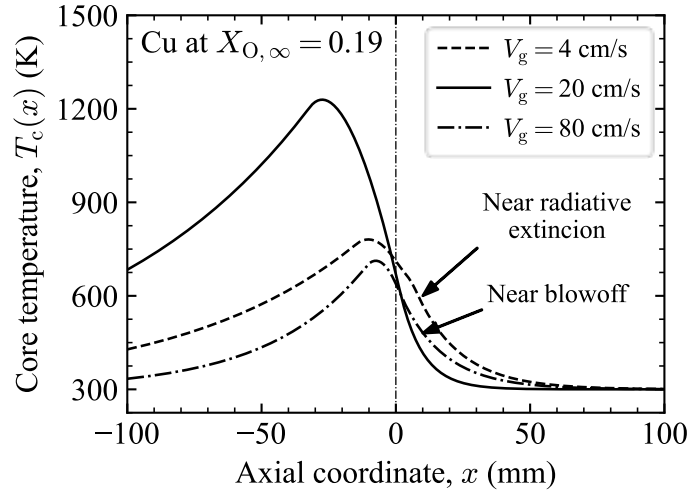


Figure 11. Temperature distributions along Cu wire at various opposed flow velocity. $X_{O,\infty} = 0.19$.

residence time of the flame over the wire surface increases and thermal diffusion to the upstream side occurs. Thus, in the unburned region, as V_f decreases, there is a negative feedback of increasing heat loss, which increases the fraction of heat loss in the entire system. As discussed in section 4.3, this is one of the main reasons why the steady solution of V_f disappears before it reaches zero, and well replicating the self-extinguishing process of a flame spreading over solid material.

It is interesting to note that f_{loss} increases even under high flow velocity conditions. This is a unique phenomenon observed in systems which axial heat conduction in the solid phase is not negligible. In other words, the blow-off of the flame spreading over high conductive material cannot be explained by a decrease in Da alone; a decrease in V_f due to a decrease in Da causing an increase in convective heat loss from the unburned zone also dominates the flame extinction. It has been shown by previous studies [60,61] that radiative extinction under low flow velocity conditions is caused by radiative heat loss from the solid surface, but it is noteworthy that convective heat loss from the solid surface is also important for extinction under high flow velocity

conditions. It should be noted, however, that some of the convective heat loss upstream of the flame in an opposed-flow field is used to preheat the gas flowing into the flame, resulting in heat recycling. Therefore, the current model assumes that all upstream convective heat losses are losses from the system, but this may overestimate the heat losses. It is recommended to investigate the relationship between the thermal boundary layer formed upstream of the flame and the flame height by numerical analysis, and to develop a more sophisticated model of heat loss.

Figure 12(a) and (b) show V_f as a function of opposed flow velocity for Cu and NiCr, respectively for varying oxygen concentration. The steady solution of V_f is confirmed to exhibit an O-shaped curve regardless of oxygen concentration and the core material. The bifurcation points for low and high flow velocity conditions represent radiative extinction and blow-off, respectively. Here, focusing on the trend of upper solutions of V_f with respect to the opposed flow velocity, we can see that a peak appears under high flow velocity conditions for Cu and under low flow velocity conditions for NiCr. The dominant mechanism for the increase in V_f for Cu under high flow velocity conditions is the increase in heat flux from the flame to the wire core as the flame location approaches the wire surface under high flow velocity conditions. As mentioned earlier, highly conductive wire can transport heat from the downstream of the flame to the upstream unburned insulation, so the increased heat flux from the flame is advantageous to the flame spread process. On the other hand, the cause of the higher V_f for NiCr under low flow velocity conditions has been clarified in previous study and is due to the curvature of the solid surface [40][42].

4.5. Flammability limits

The variations of the radiative extinction and blow-off limits with oxygen concentration

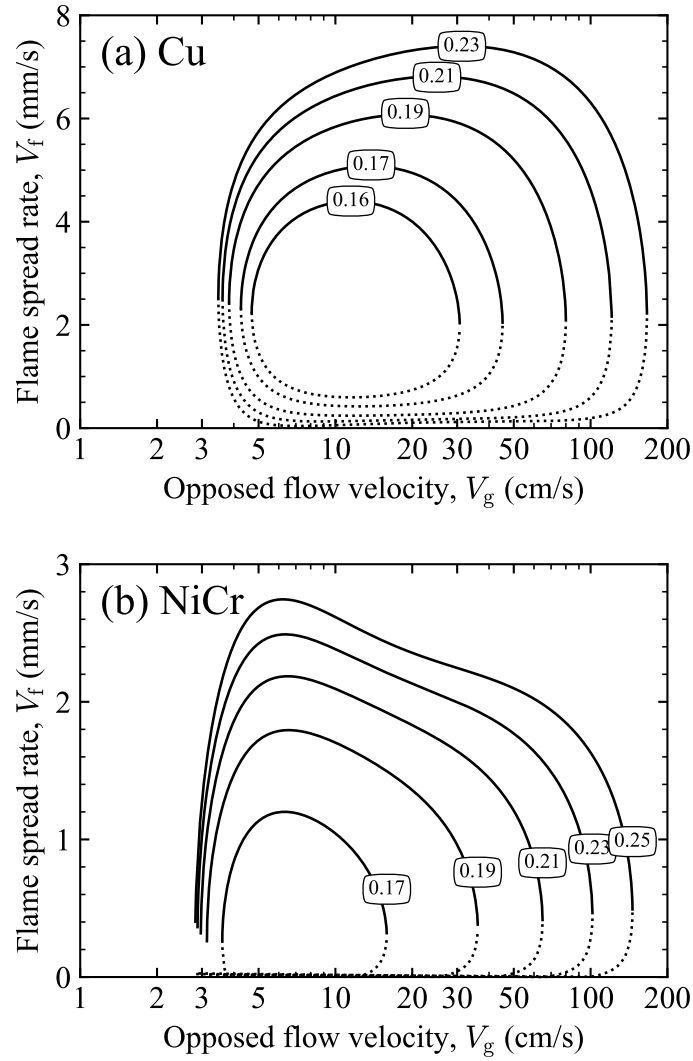


Figure 12. Flame spread rate as a function of opposed flow velocity for Cu (a) and NiCr (b). Solid line, stable solution; dotted line, unstable solution.

1 shown in Figure 12 are plotted in Figure 13 together with experimental data obtained in
2 microgravity [62] and normal gravity [15]. As can be seen in Figure 13, there are two
3 branches of flame spread limits. The left branch is the radiative extinction branch while
4 the right branch is the blow-off branch. The merging point of the two branches
5 corresponds to the minimum limiting oxygen concentration (MLOC). Although the
6 radiative extinction branch was not confirmed in the experiment, the trends of the
7 experimental and predicted LOC are in good agreement, indicating the validity of the
8 model developed in this study. The trend of LOC under low velocity conditions will be

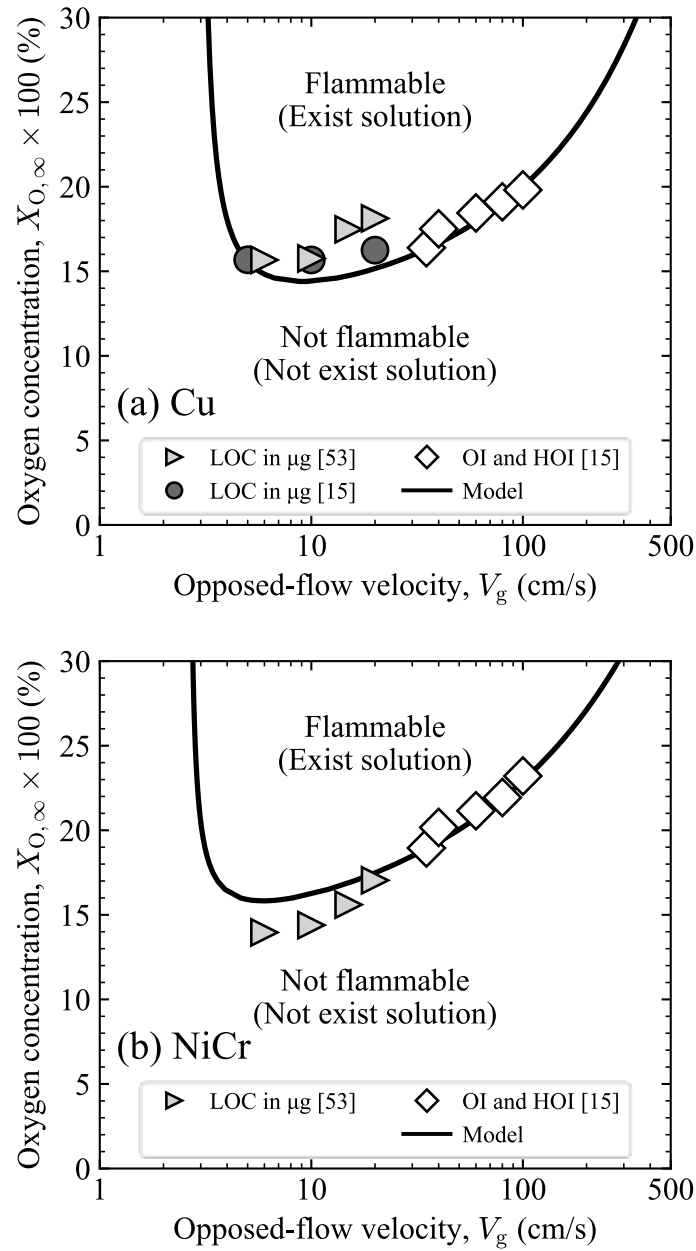


Figure 13. Flame spread limits as a function of opposed flow velocity for Cu (a) and NiCr (b).

- 1 further verified by microgravity experiments to be conducted in international space
- 2 station in the near future.
- 3 **5. Conclusions**
- 4 A model describing the opposed flame spread over electric wire is developed based on

the one-dimensional energy balance equation for the solid phase. To reproduce the effect of flow residence time of chemical species in the flame on the flame spread process, a correction factor of flame temperature (Ω) which consists of Damköhler number (Da) and heat loss parameter (H) is introduced into the energy balance equation. In the model, the length of the pyrolysis zone (L_{py}) and flame spread rate (V_f) are treated as eigenvalues, and the physical meaning of the steady solution for each eigenvalue obtained from the energy balance equation is clarified. It is found that the flame spread model can not only predict the flame spread rate and limiting oxygen concentration, but also simulate the temperature distribution along the wire core during flame spread. The temperature distribution obtained from the model reproduces the experimental results well, indicating that the one-dimensional model can be effectively used to investigate the structure of the burning region of the electric wires in the flame spread process.

The variation of flame spread rate with oxygen concentration shows a C-shaped curve, where the upper branch is the stable solution and the lower branch is the unstable solution, and the point where the stable and unstable solutions coincide corresponds to the limiting oxygen concentration. Furthermore, the variation of flame spread rate with opposed flow velocity shows a O-shaped curve and typical two types of extinction phenomena are predicted; the radiative extinction at low flow velocity condition and the blow-off at high flow velocity condition. Under low flow velocity conditions, radiation quenching appears as H increase, while under high flow velocity conditions, blow-off appears as Da decreases, both of which reproduce the actual flame extinction characteristic well. However, analysis on the energy balance of the solid surface reveals that heat loss from the solid surface also plays an important role in both radiation extinction and blow-off. In particular, in the unburned zone upstream of the flame, the

high-temperature region expands as the flame spread rate decreases, resulting in negative feedback. Moreover, since this heat loss is governed by the flame spread rate, it is found that fraction of heat loss (f_{loss}) from the system increases near the flame spread limits regardless of the flow velocity conditions. Finally, it is confirmed that the predicted flammability maps are in good agreement with the experimental data, indicating the validity of the flame spread model developed in the present study for predicting LOC of wire insulation material under various flow velocities. Both experimental and theoretical results show that the minimum value of LOC appears under low gas flow velocity conditions of less than 10 cm/s, indicating that the flammable range of the target wire in this study is extended in a microgravity environment rather than on the ground.

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Declaration of interest statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References:

- [1] J.M. Citerne, H. Dutilleul, K. Kizawa, M. Nagachi, O. Fujita, M. Kikuchi, G. Jomaas, S. Rouvreau, J.L. Torero, and G. Legros, *Fire safety in space – Investigating flame spread interaction over wires*, Acta Astronaut. 126 (2016), pp. 500–509.
- [2] G. Jomaas, J.L. Torero, C. Eigenbrod, J. Niehaus, S.L. Olson, P. V. Ferkul, G. Legros, A.C. Fernandez-Pello, A.J. Cowland, S. Rouvreau, N. Smirnov, O. Fujita,

- 1 J.S. T'ien, G.A. Ruff, and D.L. Urban, *Fire safety in space – beyond flammability*
2 *testing of small samples*, Acta Astronaut. 109 (2015), pp. 208–216.
- 3 [3] A. Guibaud, G. Legros, J.-L. Consalvi, and J. Torero, *Fire safety in spacecraft:*
4 *Past incidents and Deep Space challenges*, Acta Astronaut. 195 (2022), pp. 344–
5 354.
- 6 [4] NASA-STD-6001B, *Flammability, Offgassing, and Compatibility Requirements*
7 *and Test Procedures*, Washington, DC, 2011.
- 8 [5] O. Fujita, *Solid combustion research in microgravity as a basis of fire safety in*
9 *space*, Proc. Combust. Inst. 35 (2015), pp. 2487–2502.
- 10 [6] A. Hosogai and Y. Nakamura, *Overview of Flammability Test for the*
11 *International Space Station Program; Inherent Problems and Potential*
12 *Improvements*, Int. J. Microgravity Sci. Appl. 32 (2015), pp. 320406.
- 13 [7] K.E. Lange, A.T. Perka, B.E. Duffield, and F.F. Jeng, *Bounding the Spacecraft*
14 *Atmosphere Design Space for Future Exploration Missions*, Nasa/Cr (2005), pp.
15 213689.
- 16 [8] M. Kikuchi, *Overview of the “Solid Combustion” Experiment and the Solid*
17 *Combustion Experiment Module*, Int. J. Microgravity Sci. Appl. 32 (2015), pp.
18 320407.
- 19 [9] ISO 4589-2, *Plastics — Determination of burning behaviour by oxygen index —*
20 *Part 2: Ambient-temperature test*, 2017.
- 21 [10] S. Takahashi, *Prediction of Flammability Limit of Flat Materials in Microgravity*
22 *Environments*, Int. J. Microgravity Sci. Appl. 32 (2015), pp. 320403.
- 23 [11] Y. Nakamura and A. Hosogai, *A Classification of Flammability of Thermoplastic*
24 *Materials in Space - Challenges to Utilize ISO4589-2 to Estimate the*
25 *Flammability Limit under Reduced Gravity Environments-*, Int. J. Microgravity
26 Sci. Appl. 32 (2015), pp. 320405.
- 27 [12] S. Takahashi, T. Ebisawa, S. Bhattacharjee, and T. Ihara, *Simplified model for*
28 *predicting difference between flammability limits of a thin material in normal*
29 *gravity and microgravity environments*, Proc. Combust. Inst. 35 (2015), pp.
30 2535–2543.

- 1 [13] S. Takahashi, M.A.F. Bin Borhan, K. Terashima, A. Hosogai, and Y. Kobayashi,
2 *Flammability limit of thin flame retardant materials in microgravity*
3 *environments*, Proc. Combust. Inst. 37 (2019), pp. 4257–4265.
- 4 [14] S. Takahashi, K. Terashima, M.A.F. bin Borhan, and Y. Kobayashi, *Relationship*
5 *Between Blow-Off Behavior and Limiting Oxygen Concentration in Microgravity*
6 *Environments of Flame Retardant Materials*, Fire Technol. 56 (2020), pp. 169–
7 183.
- 8 [15] Y. Konno, M.Z. Bin Zainal, N. Hashimoto, and O. Fujita, *Application of ISO*
9 *4589-4 to determine limiting oxygen concentrations for opposed-flow flame*
10 *spread over thin electric wires and their comparison with microgravity data*, Fire
11 Saf. J. 141 (2023), pp. 103989.
- 12 [16] Y. Ju, *Understanding cool flames and warm flames*, Proc. Combust. Inst. 38
13 (2021), pp. 83–119.
- 14 [17] B.H. Chao, C.K. Law, and J.S. T'ien, *Structure and extinction of diffusion flames*
15 *with flame radiation*, Symp. Combust. 23 (1991), pp. 523–531.
- 16 [18] T.-K. Oh, J.S. Lee, and S.H. Chung, *Effect of gas-phase and surface radiation on*
17 *the structure and extinction of diffusion flames stabilized on a condensed fuel*,
18 Int. J. Heat Mass Transf. 37 (1994), pp. 2893–2900.
- 19 [19] S.L. Olson, P. V. Ferkul, and J.S. T'ien, *Near-limit flame spread over a thin solid*
20 *fuel in microgravity*, Symp. Combust. 22 (1989), pp. 1213–1222.
- 21 [20] S.L. Olson, *Mechanisms of Microgravity Flame Spread Over a Thin Solid Fuel:*
22 *Oxygen and Opposed Flow Effects*, Combust. Sci. Technol. 76 (1991), pp. 233–
23 249.
- 24 [21] K. Maruta, M. Yoshida, H. Guo, Y. Ju, and T. Niioka, *Extinction of low-stretched*
25 *diffusion flame in microgravity*, Combust. Flame 112 (1998), pp. 181–187.
- 26 [22] K. Mizutani, K. Miyamoto, N. Hashimoto, Y. Konno, and O. Fujita, *Limiting*
27 *Oxygen Concentration Trend of ETFE-Insulated Wires under Microgravity*, Int.
28 J. Microgravity Sci. Appl. 35 (2018), pp. 350104.
- 29 [23] S.L. Olson and P. V. Ferkul, *Microgravity flammability boundary for PMMA*
30 *rods in axial stagnation flow: Experimental results and energy balance analyses*,
31 Combust. Flame 180 (2017), pp. 217–229.

- 1 [24] J.S. T'ien, *Diffusion flame extinction at small stretch rates: The mechanism of*
2 *radiative loss*, Combust. Flame 65 (1986), pp. 31–34.
- 3 [25] C.T. Yang and J.S. T'ien, *Numerical Simulation of Combustion and Extinction of*
4 *a Solid Cylinder in Low-Speed Cross Flow*, J. Heat Transfer 120 (1998), pp.
5 1055–1063.
- 6 [26] J.S. T'ien, *The possibility of a reversal of material flammability ranking from*
7 *normal gravity to microgravity*, Combust. Flame 80 (1990), pp. 355–357.
- 8 [27] A. Kumar, H.Y. Shih, and J.S. T'ien, *A comparison of extinction limits and*
9 *spreading rates in opposed and concurrent spreading flames over thin solids*,
10 Combust. Flame 132 (2003), pp. 667–677.
- 11 [28] J.S. T'ien and M. Endo, *Material flammability: A combustion science*
12 *perspective*, Procedia Eng. 62 (2013), pp. 120–129.
- 13 [29] ISO 4589-4, *Plastics — Determination of burning behaviour by oxygen index —*
14 *Part 4: High gas velocity test*, 2021.
- 15 [30] ISO14624-2:2003(E), *Space systems — Safety and compatibility of materials —*
16 *Part 2: Determination of flammability of electrical-wire insulation and accessory*
17 *materials*, (2003), .
- 18 [31] X. Huang and Y. Nakamura, *A Review of Fundamental Combustion Phenomena*
19 *in Wire Fires*, Fire Technol. 56 (2020), pp. 315–360.
- 20 [32] X. Huang, Y. Nakamura, and F.A. Williams, *Ignition-to-spread transition of*
21 *externally heated electrical wire*, Proc. Combust. Inst. 34 (2013), pp. 2505–2512.
- 22 [33] M. Nagachi, F. Mitsui, J.-M. Citerne, H. Dutilleul, A. Guibaud, G. Jomaas, G.
23 Legros, N. Hashimoto, and O. Fujita, *Effect of Ignition Condition on the*
24 *Extinction Limit for Opposed Flame Spread Over Electrical Wires in*
25 *Microgravity*, Fire Technol. 56 (2020), pp. 149–168.
- 26 [34] N.N. Bakhman, L.I. Aldabaev, B.N. Kondrikov, and V.A. Filippov, *Burning of*
27 *polymeric coatings on copper wires and glass threads: I. Flame propagation*
28 *velocity*, Combust. Flame 41 (1981), pp. 17–34.

- 1 [35] N.N. Bakhman, L.I. Aldabaev, B.N. Kondrikov, and V.A. Filippov, *Burning of*
2 *polymeric coatings on copper wires and glass threads: II. Critical conditions of*
3 *burning*, Combust. Flame 41 (1981), pp. 35–43.
- 4 [36] Y. Nakamura, N. Yoshimura, T. Matsumura, H. Ito, and O. Fujita, *Opposed-wind*
5 *Effect on Flame Spread of Electric Wire in Sub-atmospheric Pressure*, J. Therm.
6 Sci. Technol. 3 (2008), pp. 430–441.
- 7 [37] Y. Nakamura, K. Azumaya, J. Iwakami, and K. Wakatsuki, *Scale Modeling of*
8 *Flame Spread Over PE-Coated Electric Wires*, in *Progress in Scale Modeling,*
9 *Volume II*, K. Saito, A. Ito, Y. Nakamura and K. Kuwana, eds., Springer
10 International Publishing, Cham, 2015, pp. 275–292.
- 11 [38] L. Hu, Y. Zhang, K. Yoshioka, H. Izumo, and O. Fujita, *Flame spread over*
12 *electric wire with high thermal conductivity metal core at different inclinations*,
13 Proc. Combust. Inst. 35 (2015), pp. 2607–2614.
- 14 [39] Y. Konno, N. Hashimoto, and O. Fujita, *Downward flame spreading over*
15 *electric wire under various oxygen concentrations*, Proc. Combust. Inst. 37
16 (2019), pp. 3817–3824.
- 17 [40] O. Fujita, K. Nishizawa, and K. Ito, *Effect of low external flow on flame spread*
18 *over polyethylene-insulated wire in microgravity*, Proc. Combust. Inst. 29 (2002),
19 pp. 2545–2552.
- 20 [41] Y. Nakamura, N. Yoshimura, H. Ito, K. Azumaya, and O. Fujita, *Flame spread*
21 *over electric wire in sub-atmospheric pressure*, Proc. Combust. Inst. 32 (2009),
22 pp. 2559–2566.
- 23 [42] Y. Konno, N. Hashimoto, and O. Fujita, *On the radiation extinction of opposed*
24 *flame spread over curved solid surface in low flow velocity conditions*, Combust.
25 Flame 254 (2023), pp. 112836.
- 26 [43] Y. Kobayashi, X. Huang, S. Nakaya, M. Tsue, and C. Fernandez-Pello, *Flame*
27 *spread over horizontal and vertical wires: The role of dripping and core*, Fire
28 Saf. J. 91 (2017), pp. 112–122.
- 29 [44] Y. Kobayashi, Y. Konno, X. Huang, S. Nakaya, M. Tsue, N. Hashimoto, O.
30 Fujita, and C. Fernandez-Pello, *Effect of insulation melting and dripping on*

- opposed flame spread over laboratory simulated electrical wires, *Fire Saf. J.* 95 (2018), pp. 1–10.
- [45] F.G. Roper, *The prediction of laminar jet diffusion flame sizes: Part I. Theoretical model*, *Combust. Flame* 29 (1977), pp. 219–226.
- [46] K. Annamalai and P. Durbetaki, *Characteristics of an open diffusion flame*, *Combust. Flame* 25 (1975), pp. 137–139.
- [47] M.A. Delichatsios, R.A. Altenkirch, M.F. Bundy, S. Bhattacharjee, L. Tang, and K. Sacksteder, *Creeping flame spread along fuel cylinders in forced and natural flows and microgravity*, *Proc. Combust. Inst.* 28 (2000), pp. 2835–2842.
- [48] J.N. De Ris, *Spread of a laminar diffusion flame*, *Symp. Combust.* 12 (1969), pp. 241–252.
- [49] R.K. Wharton, *Factors that influence the critical oxygen index of various solids*, *Fire Mater.* 3 (1979), pp. 39–48.
- [50] Y. Nakamura, K. Kizawa, S. Mizuguchi, A. Hosogai, and K. Wakatsuki, *Experimental Study on Near-Limiting Burning Behavior of Thermoplastic Materials with Various Thicknesses Under Candle-Like Burning Configuration*, *Fire Technol.* 52 (2016), pp. 1107–1131.
- [51] Y. Kim, A. Hossain, and Y. Nakamura, *Numerical modeling of melting and dripping process of polymeric material subjected to moving heat flux: Prediction of drop time*, *Proc. Combust. Inst.* 35 (2015), pp. 2555–2562.
- [52] S. Singh and Y. Nakamura, *A Numerical Study of Dripping on the Ignitability of a Vertically Oriented Thermoplastic Material Locally Heated by an Irradiation Source*, *Fire Technol.* 58 (2022), pp. 75–105.
- [53] P. Sun, Y. Liu, and X. Huang, *Exploring the brachistochrone (shortest-time) path in fire spread*, *Sci. Rep.* 12 (2022), pp. 13600.
- [54] K. Pohlhausen, *Zur näherungsweisen Integration der Differentialgleichung der laminaren Grenzschicht*, *ZAMM - J. Appl. Math. Mech. / Zeitschrift für Angew. Math. und Mech.* 1 (1921), pp. 252–290.

- [55] Y. Konno, N. Hashimoto, and O. Fujita, *Role of wire core in extinction of opposed flame spread over thin electric wires*, Combust. Flame 220 (2020), pp. 7–15.
- [56] S. Gordon and B.J. McBride, *Computer Program for Calculation of Complex Chemical Equilibrium*, NASA Ref. Publ. 1311 (1994), pp. 1–2.
- [57] D.B. Spalding, *A theory of inflammability limits and flame-quenching*, Proc. R. Soc. London. Ser. A. Math. Phys. Sci. 240 (1957), pp. 83–100.
- [58] J. Buckmaster, *The effects of radiation on stretched flames*, Combust. Theory Model. 1 (1997), pp. 1–11.
- [59] F.E. Fendell, *Ignition and extinction in combustion of initially unmixed reactants*, J. Fluid Mech. 21 (1965), pp. 281–303.
- [60] S. Bhattacharjee and R.A. Altenkirch, *The effect of surface radiation on flame spread in a quiescent, microgravity environment*, Combust. Flame 84 (1991), pp. 160–169.
- [61] S. Takahashi, M. Kondou, K. Wakai, and S. Bhattacharjee, *Effect of radiation loss on flame spread over a thin PMMA sheet in microgravity*, Proc. Combust. Inst. 29 (2002), pp. 2579–2586.
- [62] S. Takahashi, H. Ito, Y. Nakamura, and O. Fujita, *Extinction limits of spreading flames over wires in microgravity*, Combust. Flame 160 (2013), pp. 1900–1902.
- [63] Y. Sano and S. Nishikawa, *The heat transfer coefficient of fine wires in air flow*, Chem. Eng. 28 (1964), pp. 275–284.

Appendix A. Heat transfer coefficients

Heat transfer coefficients in Eqs. (14)–(18) are calculated as follows

$$h_{gc} = \lambda_g / [r_c \ln(r_f/r_c)], \quad (\text{A-1})$$

$$h_{cs} = \lambda_s / [r_c \ln(r_s/r_c)], \quad (\text{A-2})$$

$$h_{T,b2} = Nu_c \lambda_g / (2r_c) + 4\varepsilon_c \sigma T_\infty^3, \quad (A-3)$$

$$h_{rad,c} = 4\varepsilon_c \sigma T_\infty^3, \quad (A-4)$$

$$h_{gs,py} = \lambda_g / [r_s \ln(r_f/r_s)], \quad (A-5)$$

$$h_{gs,pr} = \lambda_g / [r_s \ln(1 + L_g/r_s)], \quad (A-6)$$

$$h_{T,u} = Nu_s \lambda_g / (2r_s) + 4\varepsilon_s \sigma T_\infty^3, \quad (A-7)$$

- 1 where Nu is the Nusselt number, ε is surface emissivity, and σ is the Stefan-Boltzmann
2 coefficient, respectively. Nu_c and Nu_s in Eqs. (A-3) and (A-7) are calculated by
3 considering the thin cylinder in a co-axial flow field [63]

$$Nu_c = 0.32 + 0.155 Re_c^{0.50}, \quad Re_c = 2r_c V_g / \nu_g, \quad (A-8)$$

$$Nu_s = 0.32 + 0.155 Re_s^{0.50}, \quad Re_s = 2r_s V_g / \nu_g. \quad (A-9)$$

4 **Appendix B. Characteristic roots and integral constants**

- 5 Characteristic roots in Eq. (21) are calculated by Eq. (8) as follows

$$L_1 = 2 \left[-\frac{V_f}{\alpha_c} + \sqrt{\left(\frac{V_f}{\alpha_c}\right)^2 + \frac{4P_c h_{T,b2}}{A_c \lambda_c}} \right]^{-1}, \quad (B-1)$$

$$L_{2,3} = 2 \left[\pm \frac{V_f}{\alpha_c} + \sqrt{\left(\frac{V_f}{\alpha_c}\right)^2 + \frac{4P_c h_{T,b1}}{A_c \lambda_c}} \right]^{-1}, \quad (\text{B-2})$$

$$L_{4,5} = 2 \left[\pm \frac{V_f}{\alpha_c} + \sqrt{\left(\frac{V_f}{\alpha_c}\right)^2 + \frac{4P_c h_{cs}}{A_c \lambda_c}} \right]^{-1}, \quad (\text{B-3})$$

$$L_6 = 2 \left[\frac{V_f}{\alpha_w} + \sqrt{\left(\frac{V_f}{\alpha_w}\right)^2 + \frac{4P_c h_{T,u}}{A_c \lambda_c}} \right]^{-1}, \quad (\text{B-4})$$

1 where

$$h_{T,b1} = \Omega h_{gc} + h_{rad,c}, \quad (\text{B-5})$$

$$\alpha_w = A_c \lambda_c / (\rho_c c_{p,c} A_c + \rho_s c_{p,s} A_s). \quad (\text{B-6})$$

2 In addition, integral constants in Eq. (21) are calculated using Eq. (20). These constants
 3 were computed by organizing the multiple simultaneous equations for the six constants
 4 as the following matrix equation

$$\mathbf{Ax} = \mathbf{b}; \Rightarrow \mathbf{x} = \mathbf{A}^{-1} \mathbf{b}. \quad (\text{B-7})$$

5 Each matrix is defined as follows

$$\mathbf{A} = \begin{pmatrix} e^{-L_f/L_1} & -e^{L_f/L_2} & -e^{-L_f/L_3} & 0 & 0 & 0 \\ \frac{e^{-L_f/L_1}}{L_1} & \frac{e^{L_f/L_2}}{L_2} & -\frac{e^{-L_f/L_3}}{L_3} & 0 & 0 & 0 \\ 0 & e^{L_{py}/L_2} & e^{-L_{py}/L_3} & -e^{L_{py}/L_4} & -e^{-L_{py}/L_5} & 0 \\ 0 & \frac{e^{L_{py}/L_2}}{L_2} & -\frac{e^{-L_{py}/L_3}}{L_3} & -\frac{e^{L_{py}/L_4}}{L_4} & \frac{e^{-L_{py}/L_5}}{L_5} & 0 \\ 0 & 0 & 0 & e^{-L_g/L_4} & e^{L_g/L_5} & -e^{-L_g/L_6} \\ 0 & 0 & 0 & \frac{e^{-L_g/L_4}}{L_4} & -\frac{e^{L_g/L_5}}{L_5} & -\frac{e^{-L_g/L_6}}{L_6} \end{pmatrix}, \quad (\text{B-8})$$

$$\mathbf{x} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} T_k - T_\infty \\ 0 \\ T_p - T_k \\ 0 \\ T_\infty - T_p \\ 0 \end{pmatrix} \quad (\text{B-9})$$

1 where T_k is calculated as follows

$$T_k = (\Omega h_{gc} T_f + h_{rad,c} T_\infty) / h_{T,b1}. \quad (\text{B-10})$$

2 Appendix C. Integration of the energy balance equations

3 The derivation of each term comprising the macroscopic energy balance equation given

4 in section 2.4 are summarized here.

5 The wire core and insulation in the unburned zone ($x \geq L_g$):

$$\begin{aligned} \dot{q}_{\text{enth,w,u}} &= \int_{L_g}^{\infty} (\rho_c c_{p,c} A_c + \rho_s c_{p,s} A_s) (-V_f) \frac{dT_c}{dx} dx \\ &= (\rho_c c_{p,c} A_c + \rho_s c_{p,s} A_s) V_f [T_c(L_g) - T_\infty], \end{aligned} \quad (\text{C-1})$$

$$\dot{q}_{\text{cond,u}} = \int_{L_g}^{\infty} A_c \lambda_c \frac{d^2 T_c}{dx^2} dx = -A_c \lambda_c \frac{dT_c(L_g)}{dx}, \quad (\text{C-2})$$

$$\dot{q}_{\text{loss,u}} = \int_{L_g}^{\infty} P_s h_{T,u} (T_c - T_{\infty}) dx = P_s h_{T,u} L_6 T_6 e^{-L_g/L_6}. \quad (\text{C-3})$$

1 The wire core in the preheat zone ($0 \leq x \leq L_g$):

$$\dot{q}_{\text{enth,c,pr}} = \int_0^{L_g} \rho_c c_{p,c} A_c (-V_f) \frac{dT_c}{dx} dx = \rho_c c_{p,c} A_c (-V_f) [T_c(0) - T_c(L_g)], \quad (\text{C-4})$$

$$\dot{q}_{\text{cond,pr}} = \int_0^{L_g} A_c \lambda_c \frac{d^2 T_c}{dx^2} dx = A_c \lambda_c \left[\frac{dT_c(L_g)}{dx} - \frac{dT_c(0)}{dx} \right], \quad (\text{C-5})$$

$$\begin{aligned} \dot{q}_{\text{cs,pr}} &= \int_0^{L_g} P_c h_{cs} (T_c - T_p) dx \\ &= P_c h_{cs} [T_4 L_4 (1 - e^{-L_g/L_4}) - T_5 L_5 (1 - e^{L_g/L_5})]. \end{aligned} \quad (\text{C-6})$$

2 The wire core in the pyrolysis zone ($-L_{py} \leq x \leq 0$):

$$\dot{q}_{\text{enth,c,py}} = \int_{-L_{py}}^0 \rho_c c_{p,c} A_c (-V_f) \frac{dT_c}{dx} dx = \rho_c c_{p,c} A_c V_f [T_c(-L_{py}) - T_c(0)], \quad (\text{C-7})$$

$$\dot{q}_{\text{cond,py}} = \int_{-L_{py}}^0 A_c \lambda_c \frac{d^2 T_c}{dx^2} dx = A_c \lambda_c \left[\frac{dT_c(0)}{dx} - \frac{dT_c(-L_{py})}{dx} \right], \quad (\text{C-8})$$

$$\begin{aligned}
\dot{q}_{\text{cs,py}} &= \int_{-L_{\text{py}}}^0 P_c h_{\text{cs}} (T_c - T_p) dx \\
&= P_c h_{\text{cs}} [L_4 T_4 (e^{L_{\text{py}}/L_4} - 1) - L_5 T_5 (e^{-L_{\text{py}}/L_5} - 1)].
\end{aligned} \tag{C-9}$$

1 The wire core in the burned zone 1 ($-L_f \leq x \leq -L_{\text{py}}$):

$$\begin{aligned}
\dot{q}_{\text{enth,c,b1}} &= \int_{-L_f}^{-L_{\text{py}}} \rho_c c_{p,c} A_c (-V_f) \frac{dT_c}{dx} dx \\
&= \rho_c c_{p,c} A_c V_f [T_c(-L_f) - T_c(-L_{\text{py}})],
\end{aligned} \tag{C-10}$$

$$\dot{q}_{\text{cond,b1}} = \int_{-L_f}^{-L_{\text{py}}} A_c \lambda_c \frac{d^2 T_c}{dx^2} dx = A_c \lambda_c \left[\frac{dT_c(-L_{\text{py}})}{dx} - \frac{dT_c(-L_f)}{dx} \right], \tag{C-11}$$

$$\begin{aligned}
\dot{q}_{\text{gc,b1}} &= \Omega \int_{-L_f}^{-L_{\text{py}}} P_c h_{\text{gc}} (T_f - T_c) dx \\
&= \Omega P_c h_{\text{gc}} [(T_f - T_k)(L_f - L_{\text{py}}) + L_2 T_2 (e^{L_{\text{py}}/L_2} - e^{L_f/L_2}) \\
&\quad - L_3 T_3 (e^{-L_{\text{py}}/L_3} - e^{-L_f/L_3})],
\end{aligned} \tag{C-12}$$

$$\begin{aligned}
\dot{q}_{\text{loss,b1}} &= \int_{-L_f}^{-L_{\text{py}}} P_c h_{\text{T,b2}} (T_c - T_\infty) dx \\
&= P_c h_{\text{T,b2}} [(T_k - T_\infty)(L_f - L_{\text{py}}) - L_2 T_2 (e^{L_{\text{py}}/L_2} - e^{L_f/L_2}) \\
&\quad + L_3 T_3 (e^{-L_{\text{py}}/L_3} - e^{-L_f/L_3})].
\end{aligned} \tag{C-13}$$

2 The wire core in the burned zone 2 ($x \leq -L_f$):

$$\dot{q}_{\text{enth,c,b2}} = \int_{-\infty}^{-L_f} \rho_c c_{p,c} A_c (-V_f) \frac{dT_c}{dx} dx = -\rho_c c_{p,c} A_c V_f [T_c(-L_f) - T_\infty], \tag{C-14}$$

$$\dot{q}_{\text{cond,b2}} = \int_{-\infty}^{-L_f} A_c \lambda_c \frac{d^2 T_c}{dx^2} dx = A_c \lambda_c \frac{dT_c(-L_f)}{dx}, \quad (\text{C-15})$$

$$\dot{q}_{\text{loss,b2}} = \int_{-\infty}^{-L_f} P_c \dot{q}_{\text{loss,c}}'' dx = P_c L_1 h_{\text{T,b2}} [T_c(-L_f) - T_\infty]. \quad (\text{C-16})$$

1 The insulation in the preheat zone ($0 \leq x \leq L_g$):

$$\dot{q}_{\text{enth,s,pr}} = \int_0^{L_g} \rho_s c_{p,s} A_s (-V_f) \frac{dT_s}{dx} dx = \rho_s c_{p,s} A_s V_f [T_p - T_w(L_g)], \quad (\text{C-17})$$

$$\dot{q}_{\text{gs,pr}} = \Omega \int_0^{L_g} P_s h_{\text{gs,pr}} (T_f - T_p) dx = \Omega P_s L_g h_{\text{gs,pr}} (T_f - T_p), \quad (\text{C-18})$$

$$\dot{q}_{\text{loss,pr}} = \int_0^{L_g} P_s \varepsilon_s \sigma (T_p^4 - T_\infty^4) dx = P_s L_g \varepsilon_s \sigma (T_p^4 - T_\infty^4). \quad (\text{C-19})$$

2 The insulation in the pyrolysis zone ($-L_{\text{py}} \leq x \leq 0$):

$$\dot{q}_{\text{enth,s,py}} = \int_{-L_{\text{py}}}^0 P_s \frac{d}{dx} \left(\frac{\dot{m}_F}{P_s} \right) \Delta h_v dx = \rho_s A_s V_f \Delta h_v, \quad (\text{C-20})$$

$$\dot{q}_{\text{gs,py}} = \Omega \int_{-L_{\text{py}}}^0 P_s h_{\text{gs,py}} (T_f - T_p) dx = \Omega P_s L_{\text{py}} h_{\text{gs,py}} (T_f - T_p), \quad (\text{C-21})$$

$$\dot{q}_{\text{loss,py}} = \int_{-L_{\text{py}}}^0 P_s \varepsilon_s \sigma (T_p^4 - T_\infty^4) dx = P_s L_{\text{py}} \varepsilon_s \sigma (T_p^4 - T_\infty^4). \quad (\text{C-22})$$

Appendix D. Effect of heat loss in the downstream side of flame

In the discussion of the temperature distribution along the wire core in section 4.2, a discrepancy between calculated and experimental results was observed in the downstream of the spreading flame. Therefore, here we discuss the influence of the treatment of the heat balance downstream of the flame (burned zone 2) on the flame spread model. Two extreme cases are taken here. One is the case where the wire core in the burned zone 2 is adiabatic, and the other is the case where the wire core is non-adiabatic. When calculating the adiabatic condition, $\dot{q}'_{\text{loss},c}$ in Eq. (16) was set to 0 in the region $x \leq -L_f$.

Figure D1 compares the temperature distribution along Cu and NiCr wires with and without heat loss in the burned zone 2. The calculation conditions are the same as in Figure 6, with $V_g = 35$ cm/s and $X_{O,\infty} = 0.20$. With heat loss, the downstream core temperature steadily decreases until it finally reaches the ambient temperature due to the continuous action of heat loss. Without heat loss, on the other hand, the downstream core temperature shows a constant value. Comparing the two results, it can be seen that the downstream heat loss lowers the maximum temperature by 124 K for Cu and 113 K

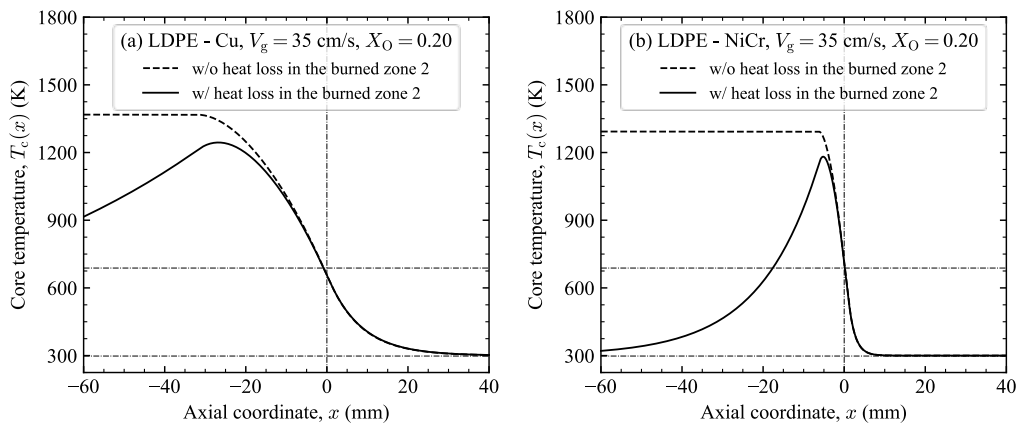


Figure D1. Effect of heat loss from the wire surface in the burned zone 2 on the temperature distribution along the wire core.

for NiCr, but the effect on the temperature distribution upstream of the flame is very small.

Figure D2 shows the flame spread rate as a function of gas flow velocity with and without heat loss in the burned zone 2 for Cu and NiCr wires. The difference in V_f with and without heat loss is small for conditions away from the extinction limit, but as V_f decreases closer to the limit condition, the rate of decrease in V_f becomes larger when downstream heat loss is taken into account, further narrowing the range of flammable gas flow velocity. Therefore, the treatment of downstream heat loss has little influence on the steady-state spread model, but it becomes important when trying to accurately predict the extinction limit. In the present model, the downstream wire core is exposed to gas of ambient temperature, but a more accurate description of the phenomenon would be possible if the axial temperature distribution of the gas phase were modelled in some functional form. In modelling opposed-flow flame spread over solid materials, the widely accepted approach is to discuss the energy balance of the preheating zone ahead of the propagating flame, but when inert materials that are good conductors, such as electric wires, are left unburned, the core temperature downstream of the flame affects the flame extinction limit, so even in an opposed-flow field, the energy

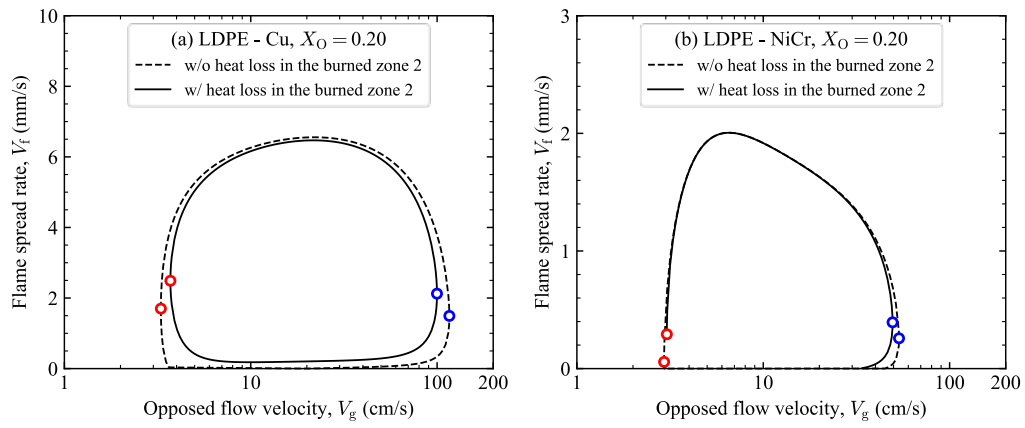


Figure D2. Effect of heat loss from the wire surface in the burned zone 2 on flame spread rate, radiative extinction, and blow-off.

- 1 conservation of the entire system must be taken into account, making it very
- 2 challenging to simplify the model.
- 3

Figure captions

Figure 1. Conceptual diagram of flammability map for solid materials and overview of MLOC prediction method [15].

Figure 2. Schematic description of opposed-flow flame spread over an electric wire and the temperature profile along a wire core during steady flame spread.

Figure 3. Flowchart of an algorithm for calculating a set of solutions for V_f and L_{py} .

Figure 4. Mapping of U_{py} . V_f used in the calculation is the stable solution.

Figure 5. Mapping of U_T . L_{py} used in the calculation is the stable solution.

Figure 6. Comparison of axial temperature distributions along the wire core obtained by experiments [55] and theoretical analysis. Experimental conditions: $V_g = 4$ cm/s and $X_{O,\infty} = 0.20$. Calculation conditions: $V_g = 35$ cm/s and $X_{O,\infty} = 0.20$. Note that the calculations use a flow velocity of 35 cm/s to account for gravity-induced natural convection.

Figure 7. Flame spread rate as a function of Damköhler number for Cu and NiCr at $V_g = 35$ cm/s. Oxygen concentration is varied in the calculations. Solid line, stable solution; dotted line, unstable solution.

Figure 8. Comparison of flame spread rates as a function of oxygen concentration obtained by experiments [15][55] and theoretical analysis.

Figure 9. Flame spread rate as a function of opposed flow velocity for Cu at $X_{O,\infty} = 0.19$. Solid line, stable solution; dotted line, unstable solution.

Figure 10. Extinction factors in the gas phase (a) and rate of heat flow and fraction of heat loss (b) as a function of opposed flow velocity for Cu at $X_{O,\infty} = 0.19$. Solid line, stable solution; dotted line, unstable solution.

Figure 11. Temperature distributions along Cu wire at various opposed flow velocity. $X_{O,\infty} = 0.19$.

- 1 Figure 12. Flame spread rate as a function of opposed flow velocity for Cu (a) and NiCr
2 (b). Solid line, stable solution; dotted line, unstable solution.
- 3 Figure 13. Flame spread limits as a function of opposed flow velocity for Cu (a) and
4 NiCr (b).
- 5 Figure D1. Effect of heat loss from the wire surface in the burned zone 2 on the
6 temperature distribution along the wire core.
- 7 Figure D2. Effect of heat loss from the wire surface in the burned zone 2 on flame
8 spread rate, radiative extinction, and blow-off.